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Quantized Feedback for Slow Fading Channels

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Abstract

Two topics in fading channels with a strict delay constraint and a resolution-constrained feedback link are treated in this thesis.

First, a multi-layer variable-rate single-antenna communication system with quantized feedback, where the expected rate is chosen as the performance measure, is studied under both short-term and long-term power constraints. Iterative algorithms exploiting results in the literature of parallel broadcast channels are developed to design the system parameters. A necessary and sufficient condition for single-layer coding to be optimal is derived. In contrast to the ergodic case, it is shown that a few bits of feedback information can improve the expected rate dramatically. The role of multi-layer coding, however, reduces quickly as the resolution of the feedback link increases.

The other part of the thesis deals with partial power control systems utilizing quantized feedback to minimize outage probability, with an emphasis on the diversity-multiplexing tradeoff. An index mapping with circular structure is shown to be optimal and the design is facilitated with a justified Gaussian approximation. The diversity gain as a function of the feedback resolution is analyzed. The results are then extended to characterize the entire diversity-multiplexing tradeoff curve of multiple-antenna channels with resolution-constrained feedback. Adaptive-rate communication is also studied, where the concept of minimum multiplexing gain is introduced. It is shown that the diversity gain of a system increases significantly even with coarsely quantized feedback, especially at low multiplexing gains.

Keywords: Fading channels, information rates, feedback communications, diversity methods, MIMO systems, power control, adaptive systems, quantization.

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Contents

Part I

Introduction

Introduction

Driven by an increasing demand for high-performance information systems that allow reliable and convenient access to various forms of applications practically anywhere, anytime, wireless communication has become one of the most rapidly developing research areas over the last couples of decades. Recent advances in the field not only opened new opportunities to achieve these ambitious goals but also gave rise to many new interesting and challenging problems in both practice and theory. This thesis aims at a better understanding of the theoretical limitations of a certain adaptive communication systems over wireless environments. Before presenting the main contributions of the thesis, we begin with a brief introduction to some pivotal information-theoretic concepts and properties of communications over wireless channels along with some key references. The relations to the topics treated in this work will be discussed when appropriate.

1 Communication Systems

To introduce some important information-theoretic concepts, we consider the simple but quite general discrete-time model of a communication system depicted in Figure 1. The mission of the system is to send a message from a transmitter to a receiver over a *channel*. The channel is characterized by a conditional probability density of the output given the input, $p(y_1^T|x_1^T)$. This essentially represents the randomness added to the transmitted signals, which may come from e.g., thermal noise, interference, and the physical medium. To protect the message from the possibly detrimental effect of the channel, some redundancy is added to the actually transmitted signals in the form of *channel coding*.

More precisely, at the transmitter, an integer *message* m, assumed to take equally likely values on the set $\{1, \ldots, 2^{RT}\}$, is mapped (encoded) into a sequence of symbols of length T to be transmitted over the channel, x_1^T. We say that the transmission consumes T *channel uses*. Such a sequence is referred to as a *codeword*, and the integer T is the *codeword length*. The set of all possible 2^{RT} codewords is called a *codebook*, which is known to both sides

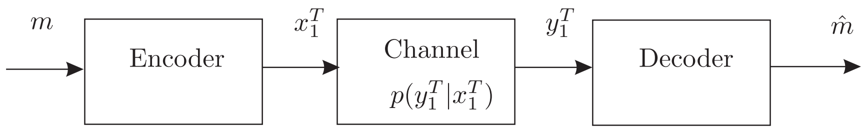

Figure 1: A communication system.

of the communication link. Normally some cost functions are associated with the codewords to represent the physical limitations of the transmission. An example of a cost function is the typical power constraint that keeps the average, or the peak transmit power below a certain threshold. At the receiver, a decoder attempts to detect which message has been sent, based on its received sequence y_1^T, and the result is the decoded message $\hat{m} \in \{1, \ldots, 2^{RT}\}$. The communication system attempts to convey RT bits of information through the channel after T channel uses, thus the rate of the code is said to be R bits per channel use. This definition of code rate should clearly be distinguished from other definitions used elsewhere, e.g. in [Pro95] where the code rate, a quantity less than unity, is used to indicate the level of redundancy of a code. The simple model Figure 1 indeed includes the basic building blocks of a quite general communication system using channel coding (but perhaps too general to be actually implemented).

The above model is perhaps the most classical way to represent a point-to-point communication system, but is not the only one. In Paper A, we will deal with a system employing *multi-layer coding* where multiple messages $m_1, \ldots, m_L$ are mapped into a single sequence to be transmitted and then successively decoded at the receiver. At a first look, it seems that such an approach is a special case of the above model because multiple messages can be combined into a single message with a larger codebook, and therefore the chance of a transmission failure (error) can only increase. This is not necessarily true, however, because "failure" can be measured in different ways depending on the applications and characteristics of the channel considered. Such information-theoretic performance measures will be discussed in more details in Section 5.

2 Fading Channels

We will now review the typical characteristics of the physical medium in a wireless environment and relate these to a more specific model of the "channel" in Figure 1. In particular, the additive Gaussian noise model where the transmitted signals, after travelling through a medium, is corrupted by the addition of white Gaussian noise at the receiver, will be considered. However, one of the most distinguishing features of a wireless channel does not come from the properties of the noise, but from the time-varying nature of the underlying physical media.

The time-varying nature of wireless channels is generally governed by two dominating terms. The so-called large-scale fading term is caused by path loss and shadowing as the transmit signals travel over distance and get obstructed by large obstacles [TV05b]. This however happens in a much larger time scale (i.e., changing much slower) than the duration of a symbol or a codeword. In this work we are more interested in smaller-scale effects, described as follows.

In a wireless environment, the transmitted signals normally propagate to the receiver via many different paths. For example, the transmitted signals from a mobile station can be reflected from buildings, cars and other obstacles before reaching the receiver. At the receiver, these signal components may add destructively, as they undergo different attenuations and arrive at different delays. The fluctuation of received signal strength due to multipath is known as (small-scale) *fading*. If the bandwidth of the transmitted signal is much smaller than the coherence bandwidth B_{m} [Pro95] of the channel, all the frequency components of the transmitted signal will suffer almost the same attenuation and phase shift. Therefore, in this case the channel is called frequency-nonselective or *flat fading*. A flat fading channel is well modelled as an equivalent time-varying one-tap filter with complex-valued coefficient, illustrated in Figure 2. We usually encounter the case when this coefficient, or *channel gain*, is modeled as a zero-mean complex Gaussian random variable. This represents a rich scattering environment with a lot of reflection paths and no direct line-of-sight component. Such a channel is called a Rayleigh fading one because the amplitude of the channel gain is Rayleigh distributed.

On the other hand, when the bandwidth of the transmitted signal is larger than the coherence bandwidth, the components that separate more than B_{m} in frequency will suffer almost uncorrelated gain and phase offset. The channel in this case is called *frequency-selective*, and is usually modeled as a time-varying tapped delay line with complex-valued coefficients. Transmission over a frequency-selective channel results in inter-symbol interference (ISI) and may require complicated equalization in the time domain. Nowadays, it is generally agreed that a common technique known as orthogonal frequency division multiplexing (OFDM) can often be applied to convert a frequency-selective channels into a set of parallel narrow-band, flat-fading channels (as usual, under some optimistic assumptions). The thesis therefore focuses only on the flat fading case.

Fading is traditionally seen as problematic for communication systems, as they may cause significant degradation in the performance, especially in deep fades (when the received signal power drops too low to be useful). A conventional method to combat fading is through *diversity* techniques. The basic idea is to provide several independent copies of the transmitted signal

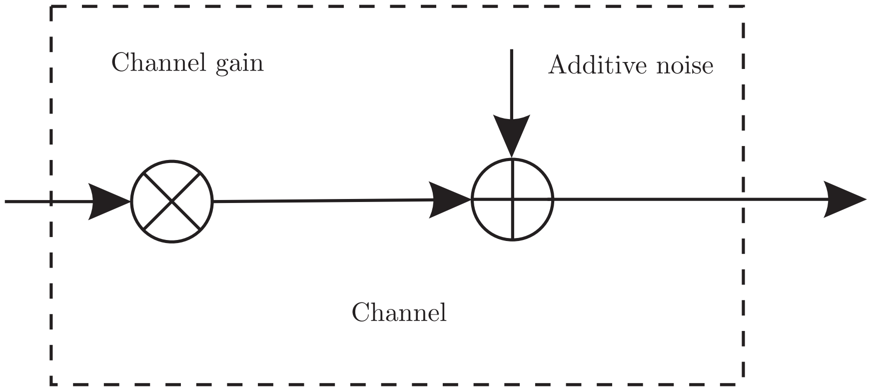

Figure 2: An additive-noise flat fading channel.

to the receiver so that the probability that all these copies simultaneously suffer deep fades is very small. Common diversity techniques include time, frequency and space diversity, as well as the combinations of these methods. With frequency diversity, the signals carrying the same information are transmitted on several carrier frequencies. If the separation between any two carrier frequencies exceeds the coherence bandwidth B_{m}, then each received version can be considered to undergo independent fades. With time diversity, the signals carrying the same information can be transmitted at different time instants such that the time separation between any two copies exceeds the coherence time T_{d} of the channel. One way to achieve this in wireless communications is by *interleaving* a codeword before its transmission.

3 Slow Fading

The classification of fading channels into fast and slow ones is critical in order to determine a suitable information-theoretic performance measure for the system of interest, as will be elaborated in Section 5. Throughout this work, the term "slow fading" does not necessarily reflect the speed of change of the underlying physical medium like in e.g., [Pro95], but relates to the *delay constraints* of the transmission. For applications completely insensitive to delay constraint, the receiver can, in principle, wait for an unlimited amount of time before attempting to decode. A codeword therefore can be assumed to span an infinite number of independent fading blocks. In practice that models a system with a relaxed delay constraint so that it enjoys near perfect interleaving, and thus a codeword can span a large number of independent fading states to exploit a significant amount of time diversity.

One can think about the download of a large file for several hours, even days. We will refer to such a channel as an ergodic one, or less technically, a *fast* fading channel. On the other hand, for applications that require a stricter delay constraint such as real-time voice and video transmission, a codeword can only span a finite, typically *small*, number of fading blocks. The length of each fading block, where the channel gain remains constant, is typically large enough to average out the effect of noise, thus studying the system behavior in the limit of infinite block length still makes sense, even though this is seemingly contradicting to the delay-limited assumption [BCT01]. (This is a good example showing that some information-theoretic concepts are *rather* abstract, to say the least!) We refer to this kind of channel as a *slow* (or slowly) fading one to distinguish this from the fast fading case. Furthermore, in this thesis, we exclusively focus on the extreme case where a codeword spans a *single* fading block, i.e., the so-called quasi-static fading channel.

4 Channel State Information

The performance of a communication system is greatly influenced by the assumptions on the available channel state information (CSI) at both sides of the links. The term CSI in this thesis refers to the possibly imperfect information about the *realization* of the channel gain (or a channel matrix in a multi-antenna channel, as presented later in Section 6). We then distinguish between CSI at the transmitter (CSIT) and CSI at the receiver (CSIR).

In both theory and practice, CSIR is considered “easier” to acquire. A typical way to obtain CSIR is by sending a training sequence known a priori to the receiver so that the it can estimate the channel gain with a certain level of accuracy (assuming that the channel gain does not change significantly until the next training sequence is sent). A thorough analysis of training schemes is presented in [HH03]. On the other hand, CSIT is more difficult to obtain. In Time Division Duplex (TDD) systems where uplink and downlink transmission takes place in the same frequency, the reciprocity of the channel can be exploited to estimate the reverse channel gain (provided that the time separation between uplink and downlink slots is considerably smaller than the coherence time). The reciprocal properties generally cannot be exploited in Frequency Division Duplex (FDD) systems, where obtaining CSIT requires some form of *feedback*.

To simplify the analysis and highlight the effect of partial CSIT, throughout the thesis perfect CSIR is always assumed. Of course, in practice, the imperfectness of CSIR must also be taken into careful consideration because this may lead to remarkable changes in the behavior of some information-theoretic measures [LM03]. This thesis exclusively focuses on an explicit quantized feedback model where CSIT is obtained via a noiseless, zero-delay dedicated feedback link, depicted in Figure 3. In particular, given a channel gain, the receiver employs an index mapping to obtain an integer feedback index belonging to a finite set $\{1, \ldots, K\}$ and sends it back to the transmitter prior to the transmission of a codeword. The constant K is referred to as the *feedback resolution*. Clearly for that approach to work the transmitter and receiver must agree on a common strategy with the parameters designed *off-line*. More realistic assumptions regarding the feedback link should be taken into account for any practical system, e.g., the case of noisy feedback link is treated in [JS04]. Nevertheless, the feedback resolution that we considered in this work is generally low (corresponding to 1-2 bits of feedback per channel use) so that the feedback delay can be considered insignificant and low-complexity forms of channel coding in the feedback link are also possible, making the zero-delay noiseless feedback link a relatively reasonable assumption.

The explicit quantized feedback model in Figure 3 is not the only model for limited feedback. Other models may impose different assumptions, for example, that a noisy estimate of the channel is available at the transmitter [JSO02]. Another line of thought assumes that only the long-term statistics of the channel gain is available at the transmitter, for example, the case of correlated channels with covariance matrix known at the transmitter is studied thoroughly in e.g., [VM01, JB04, JG04, VP06]. A hybrid model combining both long-term statistics and short-term information regarding the channel gain realization is presented in [KBLS06]. There are also the interesting and challenging cases when CSI is *not* known by any party of the communication link. Such noncoherent communication systems, see e.g., [MH99, ZT02], are outside the scope of this thesis.

5 Some Information-theoretic Performance Measures on Fading Channels

Perhaps the most important information-theoretic limitation of a communication channel is the *channel capacity*, introduced in Shannon's seminal work [Sha48] (see also [CT91]). Roughly speaking, the capacity C of a channel lets us know the upper limit on the rate of reliable communication over that channel. That is, with an error is defined as the event that the transmitter and receiver disagree on what has been sent, i.e., $\hat{m} \neq m$ in the model in Figure 1, then for any positive number $R < C$, it is possible to find codes of rate R that yield arbitrarily small probability of error, $\Pr(\hat{m} \neq m)$, provided that the codeword length T is sufficiently large. For a memoryless channel, i.e., if $p(y_1^T|x_1^T) = \prod_{i=1}^{T}(y_i|x_i)$, the capacity is given by Shannon's famous maximum-mutual-information formula.

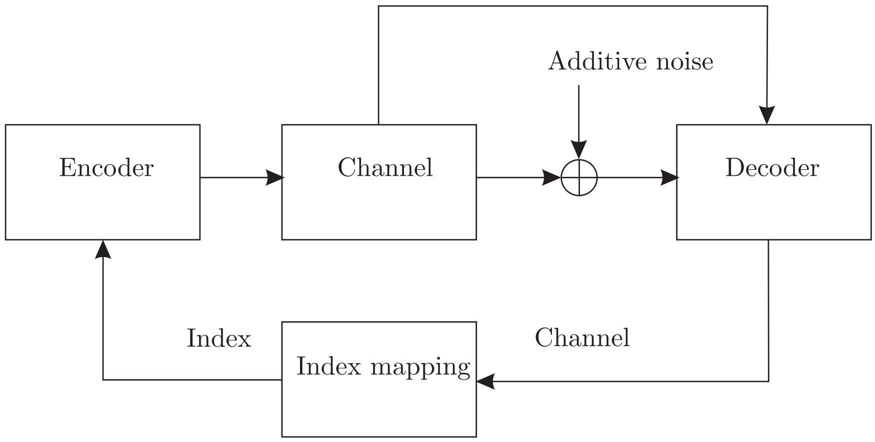

Figure 3: A quantized feedback model.

Fast fading channels belong to a general class of information-stable channels, that is, roughly speaking, channels for which a capacity-achieving input and the resulting output behave ergodically [VH94]. Naturally, in the literature, the capacity of a fast fading channel may also be explicitly termed the ergodic capacity.

On the other hands, for slow fading channels the ergodicity assumption does not hold. To compute the capacity of a slow fading channel, one therefore should use the general formula introduced in [VH94], which holds for an arbitrary channel. Slow fading channels are often discussed in the framework of *compound channels*, see e.g., [BBT59] [RV68], where the transition distribution is parameterized by some θ, i.e., $p(y_1^T|x_1^T;\theta)$. A compound channel where the parameter θ is associated with an a priori distribution is sometimes called a composite channel [EG98, BPS98]. For example, in the flat-fading model in Figure 2, the amplitude of the channel gain may take the role of θ and be associated with e.g., a Rayleigh distribution.

However, in slow fading channels, the channel capacity generally does not give a useful and complete picture. For example, a slow fading Rayleigh channel with perfect CSIR and no CSIT has a pessimistic *zero capacity.* This therefore motivates the framework of capacity versus outage, first mentioned in [OSW94]. The instantaneous mutual information between the input and the output of a slow fading channel is a *random variable,* depending on the actual realization of the channel gain. Given a code rate, there is a probability that the current realization of the mutual information is strictly smaller than the code rate, thus reliable communication is not possible no matter how large the codeword length T is and how good the codes are. In such a situation, the system is said to be in *outage.* There is obviously a tradeoff between the code rate and the probability of an outage event: High code rates lead to a high outage probability (more unreliable communication), while a lower code rate increases reliability but also reduces overall throughput.

The capacity versus outage framework, however, is not the only performance measure for a slow fading channel. For certain applications, it may be better to split a message into several ones so that any of them, if successfully decoded at the receiver, can improve the performance. For example, a coarse version of an image can be obtained if some messages are correctly decoded, and a finer, higher-quality image can be reproduced if more information is available. It should be clearly emphasized that such a *multi-layer* approach is *not* suitable for many applications. For instance, in data communications an error is declared whenever *any* layer is incorrectly decoded, thus adding extra layers is generally not an appealing choice.

A frequent performance measure of multi-layer coding is the *expected rate*. Herein we avoid the term "expected capacity" as used in e.g., [BPS98] and adopt the more moderate term "expected rate" from [VH94, Cov72] instead. This is both to avoid confusion with capacity in the traditional sense of the word and to emphasize that, to our knowledge, the multi-layer coding approach has not been shown to be optimal in any sense. Expected rate can be seen as the rate that can be correctly *received*, averaged over the randomness of the channel and the noise. This is therefore also called reliably received rate in [EG98]. Interestingly, one of the main motivations of Cover's seminal work on broadcast channels [Cov72] was to improve the expected rate over a compound channel. This interesting concept has reemerged recently in [Sha97, SS03], where the asymptotic case with a continuum of layers using differential rate and power is studied. Later, Liu et al. showed that most of the gain of infinitely many layers of codes can be realized by a simple two-layer coding scheme for many common channel distributions [LLTF02]. Multi-layer coding is closely related to the study of the capacity region for a general broadcast channel, a long standing problem in information theory [VH94].

All the aforementioned work assumes perfect CSIR and no CSIT. The presence of CSIT changes the picture dramatically. For fast fading channels, Goldsmith and Varaiya studied a single-antenna Gaussian channel with perfect CSI at both sides of the link and showed that allocating power in a water-filling manner is optimal in a capacity sense [GV97]. However, the benefit of CSIT in terms of capacity is not significant, especially at high signal-to-noise ratio (SNR). The achievability part in [GV97] relies on the multiplexing of *multiple* codebooks. It was later clarified that a simpler combination of a *single* codebook and a CSIT-dependent power amplifier is sufficient to achieve capacity [CS99]. Furthermore, such a separated structure is optimal even when CSIT is causal and imperfect, under certain assumptions. This holds also for multiple-antenna scenarios, where CSIT-dependent "transmit weighting" and coding are separated [SJ03].

The presence of CSIT in slow fading channels gives rise to the interesting concept of power control. If the transmit power can be varied according to the current channel gain, outage can be completely avoided even for a strictly positive code rate. Under certain assumptions, this is possible with a *finite average* transmit power over infinitely many codewords. In other words, the capacity of such a channel is strictly positive, even though it is a slow fading one. To distinguish this notion from the (ergodic) capacity in the fast fading case, this is referred to as *delay-limited capacity* in [HT98, CTB99]. Of course, in connection to the discussion in this section, delay-limited capacity is precisely the capacity in a traditional (Shannon's) sense, applied to a special channel model.

6 Multiple-antenna Systems

Using multiple antennas is identified as a promising approach to improve the performance of wireless communications in fading environments. Although space diversity has long been utilized by means of multiple receive antennas, only recently has knowledge about communications with multiple antennas placed at both the transmitter and the receiver reached a new level of maturity. Such so-called multiple-input multiple-output (MIMO) systems have been an extremely hot research topic over the last decade.

Seminal work by Telatar [Tel99] (see also Foschini [Fos96]) showed that in a system with only CSIR, using N_t transmit antennas and N_r receive antennas, where the components of the channel matrix are independent identically distributed (i.i.d.) zero-mean complex Gaussian, the ergodic capacity at high SNR is approximately

$$C \approx \min(N_\mathrm{r}, N_\mathrm{t}) \log \mathrm{SNR}.$$

That is, in terms of capacity a significant gain of $\min(N_\mathrm{r}, N_\mathrm{t})$ can be expected at high SNR compared to a single-antenna system. Unsurprisingly, their promising results have sparked great interests in MIMO communications.

A codeword in a MIMO communication system is a matrix, with both spatial and temporal dimensions to be exploited, giving rise to the term *space-time coding*. In [TSC98], a sufficient condition for a space-time code to achieve "full diversity" is presented. The developed criterion is quite mild, requiring all codeword difference matrices to be full rank. A surprisingly simple but extremely powerful space-time block code for two transmit antennas is introduced by Alamouti in [Ala98]. Among the attractive properties of Alamouti's code are its simplicity in combining and decoding and its ability to extract the full diversity of the channel. Later, it is shown in [TJC99] that Alamouti's codes belong to a general class of orthogonal space-time block codes (OSTBC). Unfortunately, in [TJC99], it is also shown that "full rate" OSTBC's using symbols drawn from a complex constellation (such as QAM) do not exist for more than two transmit antennas. Some extensions of OSTBC's are also proposed in [Jaf01], compromising receiver complexity and performance. However, except for the setting of two transmit and one receive antennas, OSTBC's display a performance loss compared to the more general linear dispersion codes designed to maximize mutual information in [HH02], over certain ranges of SNR. Decoding linear dispersion codes generally requires a complicated maximum likelihood receiver (assuming equally likely codewords), or some near-maximum likelihood such as the sphere decoder.

The aforementioned space-time codes are of relatively short length. Combining multiple-antenna and more sophisticated error-correcting codes such as trellis codes [TSC98], turbo codes [BGT93], low-density parity check codes [Gal62] and variations such as repeat-accumulate codes also provides significant extra gains. For fast fading MIMO channels, very close to capacity performance can be achieved with long random-like codes and joint iterative detection-decoding, see e.g., [SD01, HtB03, tKA04, tK03].

Let us briefly review some work in MIMO channels with some forms of CSIT. In the frontier of fundamental limits, for a constant channel matrix with full CSI at both sides, a singular value decomposition converts the channel matrix into a set of parallel spatial channels, and therefore power allocation in a water-filling manner is optimal in a capacity sense [Tel99]. This is readily extendable to fast fading channels with full CSI, where water-filling over both time and space is optimal. With limited feedback, capacity results for fast fading channels are reported in [SJ03, LLC04b, LLC04a]. For slow fading channels, the probabilistic power control framework in [CTB99] is extended to the MIMO case in [BCT01]. Their scheme is not suitable for exploiting time diversity because of a noncausality assumption. The causal case is solved under a dynamic programming framework in [NC02]. The concept of minimum rates is independently proposed in [LLYS03] for a single-user channel and in [JG03] for a broadcast channel, which leads to an interesting solution combining both water-filling and channel inversion.

More practical use of partial CSIT in multi-antenna systems has attracted a great deal of attention recently. Given that a large number of complex channel coefficients needed to be quantized, it becomes more difficult to "imagine" what to send back to the transmitter with e.g., 1 bit. Early studies include the design of a precoding matrix influenced by possibly impaired CSIT to improve the performance of OSTBC's [JSO02, JS04]. Vector quantization techniques are applied in [NLTW98] to design feedback schemes under different optimization criteria. Limited feedback design using an elegant geometrical framework is pursued in [LHS03, LH05, MSEA03].

7 Diversity–Multiplexing Tradeoff

Most early work on space-time coding either tried to extract a "full" diversity gain [Ala98, TJC99, TSC98], or to achieve "high rates," e.g., the vertical Bell Labs Layered Space-Time (BLAST) structure [TV05b] and the linear dispersion codes [HH02]. A new line of thought is pursued in [ZT03], where it is shown that both types of gains can be *simultaneously* achieved over a slow fading channel, with a fundamental tradeoff between them. Such an elegantly characterized tradeoff is referred to as the diversity-multiplexing tradeoff, and has sparked a great deal of attentions, even if it is rather *coarse* (defined in the limit of SNR $\to \infty$). Roughly speaking, the diversity gain d lets us know about the asymptotic slope of the error probability while the multiplexing gain r reflects how large the code rate is compared to the capacity of a single-antenna channel at high SNR. At high SNR, given a code rate $R = r \log \mathrm{SNR}$, an error probability in the order of SNR^{-d} can be achieved with "good" codes. In other words, this is a high-SNR tradeoff between reliability and throughput of a multi-antenna system. Notice that the notion of diversity gain in [ZT03] should not be interpreted (in a traditional way) as the number of independently faded copies of the transmit signals as seen at the receiver.

The diversity-multiplexing tradeoff is closely related to the theory of error exponent [Gal65, Gal68]. Error exponent techniques however involve an optimization over all probability distributions that is very difficult to solve in general. By restricting to a Gaussian distribution and letting the SNR grow unbounded, Zheng and Tse have been able to characterize exactly the asymptotic SNR exponent of an error event. The key idea is to analyze the asymptotic behavior of the joint probability density function of the singular values of an i.i.d. complex Gaussian matrix, under a powerful large-deviations framework.

In the original work [ZT03], it is shown that there exist codes with *finite length* that can achieve the optimal diversity-multiplexing tradeoff. In particular, for a channel matrix of size $N_\mathrm{r} \times N_\mathrm{t}$, the codeword length $T \geq N_\mathrm{t} + N_\mathrm{r} - 1$ is sufficient to achieve an *error* probability that decays as fast as the *outage* probability does. This is rather surprising, as one may have expected that it is only asymptotically achievable with infinitely long codewords. However, that conclusion is based on a random coding argument, with the only practical coding scheme known to be diversity-multiplexing optimal at that time was (again, somewhat surprisingly) the simple Alamouti's scheme with a QAM constellation for the 2×1 channel. As an example (from [ZT03]), the diversity-multiplexing tradeoffs achieved by some space-time codes together with the optimal one over a 2×2 channel are plotted in Figure 4. As can be seen Alamouti's codes are strictly better in a tradeoff sense than are the simple repetition codes, even though both can achieve "full diversity," i.e., the diversity at very small rates compared

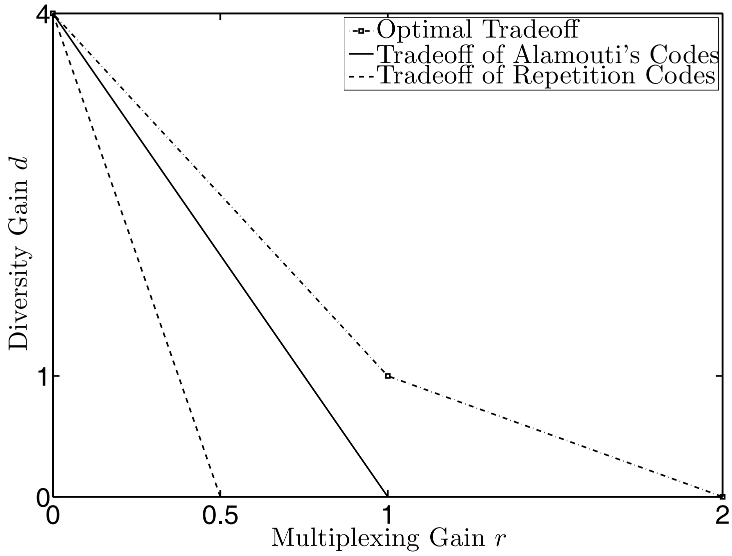

Figure 4: Diversity-multiplexing tradeoff over a 2×2 channel.

to log SNR. However, none of these schemes are tradeoff optimal, especially at high multiplexing gains (they cannot be used to achieve "high-rate").

Subsequently, the design of other short-length space-time codes that achieve the *entire* diversity-multiplexing tradeoff has then quickly become a very active research area. Among the first codes designed towards that goal are the lattice space-time (LAST) codes [ECD04a] and their variances [ECD04b]. To be precise, LAST is still a random ensemble, albeit is more structured than a Gaussian ensemble and thus allows for generally more efficient algorithms than a maximum likelihood search such as the sphere decoding, see e.g., [AEVZ02, JO05]. Even randomly generated LAST codes are shown to perform very well. In [YW03], Yao and Wornell explicitly constructed a family of codes for the 2-transmit-antenna case ($N_{\mathrm{t}} = 2$), using a carefully chosen rotation matrix and symbols taken from QAM constellations. Interestingly, they showed that there exist codes of length $T = 2$ that can achieve the entire tradeoff, for any number of receive antennas $N_{\mathrm{r}} \geq 2$, while the Gaussian coding argument in [ZT03] can only show the existence of codes with length $T \geq N_{\mathrm{t}} + N_{\mathrm{r}} - 1 = N_{\mathrm{r}} + 1$ in similar settings. One of the key ideas in [YW03] is to find a sequence of codes so that all codeword difference matrices have a nonvanishing or sufficiently slow decaying determinant as the code rate grows. Explicit code design based on that nonvanishing determinant criterion is studied extensively, see e.g., [BRV05, RBV04, EKP+04]. The tradeoff optimality of these codes can be explained in the framework of approximately universal codes [TV05a], which characterizes codes having all pairwise error probabilities decay exponentially as SNR $\to \infty$, as long as the channel is not in outage. An alternative, geometric interpretation of such a class of codes together with their applications in MIMO channels with feedback are presented in [KS06].

8 Contributions and Outline

The theme of the thesis is the design and analysis of communication systems with quantized feedback over slow fading channels. The contributions of the thesis can be grouped into two major areas

- the study of single-antenna quantized-feedback systems where expected rate is a relevant performance measure (Paper A).
- the study of outage-minimizing power control schemes using quantized feedback, with an emphasis on the asymptotic behavior of the outage probability under the diversity-multiplexing tradeoff framework (Papers B, C, D) [1].

Short summaries of the papers are given below.

Paper A: On the Expected Rate of Slowly Fading Channels with Quantized Side Information

T. T. Kim and M. Skoglund. Under review for possible publication in *IEEE Transactions on Communications.*

We consider a multiple-layer variable-rate system employing quantized feedback to maximize the expected rate over a single-input single-output slow fading Gaussian channel. The transmitter utilizes partial channel-state information, which is obtained via an optimized resolution-constrained feedback link, to adapt the power and to assign code layer rates, subject to different power constraints. To systematically design the system parameters, we develop a simple iterative algorithm that successfully exploits results in parallel broadcast channels. We present the necessary and sufficient condition for single-layer coding to be optimal, irrespective of the number of code layers that the system can afford. Unlike in the ergodic case, even coarsely quantized feedback is shown to improve the expected rate considerably. Our results also indicate that with as few as one bit of feedback information, the role of multi-layer coding reduces significantly.

[1] The achievability part with finite-length codes is presented in [KS06].

Paper B: Partial Power Control for Slowly Fading MIMO Channels

T. T. Kim and M. Skoglund. To appear in *Proc. 2006 IEEE International Conference on Communications*, Istanbul, Turkey, scheduled in June 2006.

We study transmit power control to minimize the outage probability over a slow fading multiple-antenna channel utilizing partial channel state information at the transmitter. The optimal index mapping is shown to have a "circular" structure. The design problem is then explicitly formulated and numerically solved based on a justified Gaussian approximation. Asymptotic behavior of the outage probability achieved with a finite-size power codebook is investigated. It is shown that the diversity gain of such a system is a K-order polynomial of the product of the number of transmit and receive antennas, where K is the feedback resolution.

Paper C: Diversity–Multiplexing Tradeoff of MIMO Systems with Partial Power Control

T. T. Kim and M. Skoglund. In *Proc. 2006 Zurich Seminar on Communications*, Zurich, Switzerland, pages 106–109, February 2006.

We extend the diversity analysis in Paper B to characterize the entire tradeoff between the diversity and multiplexing gains that can be simultaneously achieved over a slow fading multiple-antenna channel with partial transmit side information. It is assumed that the code rate grows as the long-term average transmit power increases, but does not adapt to the channel state, i.e., a single-rate system is considered. Partial power control is shown to be instrumental in achieving the optimal tradeoff over such a system. Our results indicate that the diversity gain can be increased considerably even with coarsely quantized channel state information, especially at low multiplexing gains.

Paper D: Outage Behavior of MIMO Channels with Partial Feedback and Minimum Multiplexing Gains

T. T. Kim and M. Skoglund. Under review for possible publication in *Proc. 2006 IEEE Symposium on Information Theory,* scheduled in July 2006.

We carry out the diversity-multiplexing tradeoff analysis for a variable-rate MIMO system with quantized feedback. To make "reliability" more meaningful, the concept of minimum guaranteed multiplexing gain in the forward link is introduced and shown to influence the tradeoff remarkably. The results suggest that the optimal diversity-multiplexing tradeoff can be achieved by a single high-rate codebook and multiple low-rate codebooks. It is shown that the multiplexing gain of the system is determined by that of the high-rate codebook, and that partial power control over the low-rate codebooks is instrumental in achieving a quickly decaying outage probability.

In addition to papers A-D, some contributions on feedback design for fast fading channels that are not formally included in the thesis are summarized below.

Paper 1: Weighted space-time bit-interleaved coded modulation.

T. T. Kim, G. Jöngren, and M. Skoglund. In *Proc. IEEE Information Theory Workshop*, San Antonio, TX, pages 375–380, October 2004.

A simple structure to exploit CSIT is proposed. When combined with turbo-coded modulation, the proposed scheme performs very close to the capacity limits. With only a few bits per channel use to feedback CSIT, we can achieve a substantial portion of the possible gain with perfect CSIT.

Paper 2: On the convergence behavior of weighted space-time bit-interleaved coded modulation.

T. T. Kim, G. Jöngren, and M. Skoglund. In *Proc. Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, pages 1257–1261, November 2004.

The converge behavior of the scheme proposed in Paper 2 is analyzed using extrinsic information transfer (EXIT) charts. Our results show that with the proposed technique, a fixed outer code can interact efficiently with the inner detector under different assumptions about the quality of CSIT.

Paper 3: Combining long-term and short-term channel state information over correlated MIMO channels.

T. T. Kim, M. Bengtsson, E. G. Larsson, and M. Skoglund. To appear in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing*, Toulouse, France, scheduled in May 2006.

A transmission scheme combining both short-term and long-term channel state information at the transmitter of a MIMO communication system is proposed. Partial short-term CSIT in the form of a weighting matrix is obtained via a resolution-constrained feedback link, combined with a unitary transformation based on the long-term channel statistics. The feedback link is optimized under different power constraints, using vector quantization techniques. Simulations indicate the benefits of the proposed scheme in all scenarios considered.

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