Bandwidth–reduced Linear Models of Non–continuous Power System Components

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To Eva and Elias
Abstract

This thesis is focused in modelling of power system components and their representation in simulations of power systems.

The thesis compares different linearization techniques. These techniques are both numerical as well as analytical and are utilized when linearization of a dynamic system is desired. After a linearization it is possible to calculate the eigenvalues of the linearized system as well as to perform other applicable activities on a linear system. In the thesis it is shown how the linearization techniques influence the calculation of eigenvalues of the linear system.

In the thesis bandwidth-reduced linear models of a power system component are developed using two techniques. The simulations with the linear models are done with an introduced interface system. The studied power system component is a Thyristor-Controlled Series Capacitor (TCSC).

One advantage with using a linear representation of a power system component is that it simplifies the simulations. The size of the complexity of a simulation when solving the equations decreases and the consumed physical time to simulate becomes shorter. A disadvantage of a linear model is that its validity might be limited.

The need of building linear models of power systems will continue to attract interest in the future. With the horizon of today (year 2006) there is a need of among other models to build linear models of detailed models of High Voltage Direct Current-links (HVDC-links), Static Var Compensators (SVCs), as well as Thyristor-Controlled Series Capacitors (TCSCs). How should these be represented when we want to study the dynamics of a whole power system and it is necessary to reduce their complexity? This question rises when we want to perform time-domain simulations with a not too detailed level of complexity of each individual power system component or if we want to linearize the power system and study it within small-signal stability analysis.
Sammanfattning

Denna avhandling är fokuserad på modellering av elkraftsystemkomponenter och deras representation vid simuleringar av elkraftsystem.

Avhandlingen jämför olika linjäriseringstekniker. Dessa tekniker är såväl numeriska som analytiska och används vid linjärisering av ett dynamiskt system. Efter en linjärisering är det möjligt att beräkna egenvärdena av det linjäriserade systemet samt använda andra verktyg ämnade för studier av linjära system. I avhandlingen visas hur olika linjäriseringstekniker influerar egenvärdesberäkningen av det linjära systemet.


En fördel med att använda en linjär representation av en kraftsystemkomponent är att det förenklar simuleringsarna. Storleken på komplexiteten av en simulering vid lösandet av ekvationerna minskar och den konsumerade fysiska tiden att simulera minskar. En nackdel med en linjär modell är att dess giltighet kan vara begränsad.

Behovet av att bygga linjära modeller av kraftsystemkomponenter torde även finnas i framtiden. Med dagens horisont (år 2006) finns behov av att bygga linjära modeller utgående från detaljerade modeller av bl a högspända likströmslänkar (HVDC-länkar), reaktiva effektkompensatorer (SVC) samt tyristorstyrd seriekondensatorer (TCSC). Hur skall dessa representeras när vi vill studera dynamiken av ett helt kraftsystem och det då är nödvändigt att reducera deras komplexitet? Denna frågeställning kommer upp när vi vill genomföra tidsdomänsimuleringar på en inte alltför detaljerad nivå av de individuella kraftsystemkomponenterna eller när vi vill linjärisera kraftsystemet för att studera dess stabilitet med hjälp av småsignalanalys.
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Now I hope I will have more time for my family and friends and hopefully focus on another stage in my life.

Jonas Persson, Solna, a wonderful sunny day, May, 2006.
Chapter 1

Introduction

1.1 Background

Simulation of power systems has been an area of interest since decades. Simulation programs are used to study future alternative solutions in power systems or to analyze scenarios that have already occurred in order to understand how it is possible to avoid them in the future.

When performing power system studies it is necessary to have appropriate models. With this means that the models should be on an enough detailed level to describe necessary behaviour of a power system component to match the aim of the study or the simulation.

Today power systems are often driven closer to their limits and during such circumstances knowledge of the systems are even more necessary. During recent years some blackouts of power systems have happened partly as consequences of lacking knowledge of the dynamics of the power system. After events like blackouts, system operators are analyzing the reason for the blackout and during such analysis even simulation models of components in the power system have been upgraded since it has been shown that they have behaved different during power failures compared to what simulations of the power system have shown, see for instance Recommendation 24 in U. S.-Canada Power System Outage Task Force [84].

This thesis is focused on representation of complex power system components in simulations of power systems. Complex power system components are technically advanced equipment that make them computationally heavy to simulate. The aim of the project is to neglect the complexity of detailed models of components and to build simplified linear models which are still accurate but easier to simulate. The linear models are in the thesis investi-
In the thesis we are studying a power system component within the group of Flexible AC Transmission Systems (FACTS) components. We have selected the Thyristor-Controlled Series Capacitor (TCSC) as our study object.

As the title *Bandwidth-reduced Linear Models of Non-continuous Power System Components* of the thesis is formulated we want to emphasize on that we here develop *linear models* that have a *reduced bandwidth*. In the thesis we have focused on fundamental low-frequency behaviour of a power system component and therefore excluded the representation of higher frequencies. The bandwidth can be expanded to also include higher frequencies, however, the here developed linear models exclude the possibility to study higher frequencies like e.g. subsynchronous resonance (SSR) of a power system including a Thyristor-Controlled Series Capacitor (TCSC).

In the title of the thesis we have used the word *non-continuous* which means that we analyze models that are in a class of components that suddenly change setup of differential equations at some certain events. These events are non-continuous. Another word to describe *non-continuous* (or *not continuous*) is *discontinuous*. In the components we study we have variables that contain "breaks" at specific time points as described in Gill [18], pp. 46-47.

The differential equations that the studied components contain are also non-linear which makes the components non-linear. Normally there is a number of non-linear models in a power system where a synchronous machine is one example. However, a synchronous machine is modelled with the same set of differential equations during a simulation. This is opposite with the models we mean with *non-continuous* which are suddenly changing setup of differential equations.

The studied component in the project is a Thyristor-Controlled Series Capacitor (TCSC) which contains thyristor switchings which are computationally *heavy* to simulate since they create several events in each period of the power system frequency. In the simplified linear models we do not model each event (as e.g. a thyristor switch) during a time-domain simulation, like high-frequent changes of signals. Instead, the aim is to find linear models that describe the behavior of the non-linear power system component within a certain bandwidth (in our case the TCSC). We are interested in investigating the behaviour of the TCSC based on a detailed level of modelling and from that build a linear model containing the behavior within a certain bandwidth of the TCSC.

When the linear models once have been developed they do simplify and
speed up time-domain simulations.

To make investigations through small-signal stability analysis of a power system containing non-linear components which during normal conditions (such as a steady state) is periodically changing working point, can be done more adequately if such components are replaced by linear models.

An interface system has been developed in the thesis project. It connects the linear models to the rest of the power system by transforming currents and voltages between the coordinate system of the network and the local coordinate system of the linearized component. The interface system can be actual for use also in other applications of modelling of power system components.

Since the thesis is focused on linear models, an extensive part of the thesis is discussing linearization of dynamic systems applied to power systems. Three linearization methods are investigated and compared and it is shown how these methods influence the result obtained from linearizations. A practical consequence is that three engineers working in parallel, studying the same power system but using three different linearization routines, will get different results.

1.2 Problem formulation

When simulating dynamic behavior of power systems it is necessary to have models that represent the real components as close as possible, and that the results correspond to what would also appear in the real power system. To get simulated results as close as possible to the real power system demands the following,

1. Models of the power system components that have enough accuracy depending on the aim of the simulation.

2. A simulator which,

   a) accurately solves the power system simulation including solution of differential and algebraic equations in time-domain simulations, i.e., accurate numerical methods, as well as a time step in the simulation which is set short enough to enable the demanded level of accuracy of the study. With accuracy is here meant both in time resolution as well as the tolerances with which the values of the variables are calculated; and
b) accurately linearize the power system when performing small-
signal stability analysis (linear analysis). Here different tools
should be included to enable small-signal stability analysis.

3. Accurate information of all components’ parameter values.

Point 2 in the list above can be expanded with more subpoints. However, within the work of this thesis subpoints $a$ and $b$ are the most important. This thesis deals with points 1 and 2b in the list above.

In point 1 the thesis elaborates on the possibility to; from a detailed model create simpler linear models and compare the performance with the original model and investigate their accuracy. Here it should also be mentioned that often the following relation appear; the higher level of detail a simulation is performed with, the longer time it takes to simulate. In point 2b the thesis compares three linearization methods.

The power system simulations in the thesis are made with STRI’s power system simulation software Simpow version 10.2.078, see Fankhauser [16] and [75]. When building linear models, functions of the System Identification Toolbox provided by Matlab version 6.5.1 is used, see Ljung [41]. Also Siemens’ power system simulation software PSS/E version 26, see [67], is used when comparing linearization methods between different software. This thesis is written in \LaTeX.

1.3 Main contributions of the thesis

The main contributions of the thesis are:

• Analysis of how three different linearization methods influence the linearization of a power system when performing small-signal stability analysis (linear analysis). The linearized power system is later used when studying the locations of the eigenvalues of the power system.

• Use of two methods (Transient Analysis and Frequency Analysis) to develop linear models of a non-linear power system component. The two methods have been applied in the thesis to create linear models of a Thyristor-Controlled Series Capacitor (TCSC).

• Development of an interface system connecting the linear models to the rest of the power system.
Application and analysis of the created linear models of a TCSC in
time-domain simulations in instantaneous value mode of the power
system.

Application and analysis of the created linear models of a TCSC in
small-signal stability analysis (linear analysis).

Evaluation of the developed linear models.

1.4 Previous research in the area

The contributions in what other have done in related works to this thesis
can be found in sections 2.2 and 4.2.

1.5 Outline of the thesis

The outline of the thesis is as follows. Chapter 2 contains a background
of linearization and describes three techniques in how to linearize a power
system.

Chapter 3 describes the modelling of the power system in the thesis and
details about the modelling of the TCSC. Chapter 4 describes the develop-
ment of the linear models and the linearity of the original TCSC model.
Chapter 5 introduces the interface system between the identified linear mod-
els and the surrounding power system.

Chapter 6 contains comparisons in time-domain simulations and chapter
7 compare linearizations of the power system including the linear models of
the TCSC.

Chapter 8 contains a summary and makes conclusions of the thesis.

1.6 List of publications

The work during the project has been described in the following publications.
The material presented in chapter 5 is not yet published in any publication.

Conference papers

[57] Jonas Persson, Kjell Aneros, Jean-Philippe Hasler, *Switching a Large
Power System Between Fundamental Frequency and Instantaneous Va-
alue Mode*, in *Proceedings of the 3rd International Conference on Digital
28th, 1999.


Technical reports


**Licentiate thesis**


Conference papers [28, 78, 60] and the technical report [54] are included in chapter 2. Conference paper [57] and the technical report [51] are included in chapter 3 and appendix C. Chapter 5 and 6 contain new material that has not been published yet. Conference papers [62, 59] are included in chapter 6. Conference papers [61, 58] are included in chapter 7.
Chapter 2

Linearization

In this chapter different methods to linearize a dynamic system are studied, both generally as well as specifically for power systems.

2.1 Introduction

Linearization of a power system is necessary to perform if its small-signal stability should be examined. Small-signal stability of a power system is the ability of the power system to maintain in synchronism when subjected to small disturbances. In this context, a disturbance is considered to be small if the equations that describe the resulting response of the system may be linearized for the purpose of the analysis, Kundur [35]. See Kundur [36] for definitions of different types of power system stability.

To be able to linearize the system, a linearization method has to be utilized. In this chapter three such linearization methods are documented and compared. Also two software which use two of the linearization methods are compared and evaluated. It is shown and also explained in which way the linearization methods influence the results, i.e., the location (the real and imaginary parts) of the obtained eigenvalues in the complex plane. The conclusions drawn from eigenvalue analysis are thus not only dependent on the properties of the investigated system, but also on which linearization method that is used since software packages use different linearization methods. Therefore, conclusions can differ when studying the same power system with different software packages.
2.2 Background

Eigenvalues of a power system give a picture of the stability in the current operating point. The eigenvalues are calculated from the system matrix of a dynamic system which is here referred to as the $A$-matrix. To create the $A$-matrix, the dynamic system has to be linearized and to do that, linearization methods are utilized when performing small-signal stability analysis of dynamic systems. A power system is one example of a dynamic system.

In this chapter differences between three linearization methods are analyzed as well as consequences thereof.

In the chapter all linearizations and simulations are made when simulating in fundamental frequency mode. The models of the power system components are identical when evaluating the linearization methods. It then becomes easier to see the pure influence of the linearization methods.

Calculations are done by hand for the three linearization methods as well as with two power system simulation software which use two of the linearization methods. These are Siemens’ PSS/E [67] and STRI’s Simpow [75]. Software that can be used for this purpose can be found in Paserba [50].

In different software, models of the same power system component differ. This is one reason why there are different results when studying eigenvalues in small-signal stability with different software. However, such comparison of different models is not done here. In this chapter the models of the power system components are identical in all linearizations and software.

Related works where linearization routines are discussed is Martins [43] where the first linearization method below, namely the analytical linearization method, is utilized. The second method, the forward-difference approximation method is a numerical method which can be found in for instance Gill [18]. In Taylor [81] the numerical method center-difference approximation is utilized and it is shown how it is used for a special software. Also the truncation error is discussed in Taylor [81]. In works of Kaberere [30, 31] comparisons are made with different software for small-signal stability analysis.

In the literature there has not yet been any systematic comparison of these three methods applied to power systems.
2.3 Problem formulation

There are several software packages on the market that can extract eigenvalues, see Paserba [50]. However, the various software are seldom evaluated and compared. Since the software packages both use different linearization methods as well as different models of the power system components it is important to understand that the consequence thereof is that there are differences in the extraction of the eigenvalues between the software packages.

Differences between models which represent a power system component but in different software is a source which produces differences when small-signal stability analysis is performed in software. There are numerous of different synchronous machine models reported in the literature such as Kundur [35], Bühler [5], Laible [38], and Canay [6]. Investigations in finding differences between machine models can be found in Kundur [35], Johansson [28], Persson [60], and Slootweg [78] where differences in representation of synchronous machines have been documented.

In this chapter a linearization of the classical synchronous machine model in a power system is done in order to show how the linearization methods influence the results. The models of the classical synchronous machine and the power system are identical in all linearizations.

A more complete model of a synchronous machine to linearize would be a high-order synchronous machine model equipped with an exciter and a turbine model, but since differences exist between the representation of the high-order machine models in the software on the market, such comparison is hard to draw conclusions from since both the models and the linearization routines are different. However, comparisons can be found in Persson [60] and Slootweg [78].

2.4 Linearization methods

In this section three linearization methods are described. These are,

- Analytical Linearization, abbreviated as the AL method in the following text, see Martins [43];

- Forward-Difference Approximation, the FDA method, see Gill [18]; and

- Center-Difference Approximation, the CDA method, see Gill [18].
PSS/E (version 26) currently uses the forward-difference approximation method, see [68] and Simpow (version 10.2) currently uses the analytical linearization method, see [75].

Since we later will study the linearization of a power system containing a classical model of a synchronous machine, we here mention one of the differential equations (actually one of the acceleration equations in per unit) of that component,

\[
\Delta \dot{\omega} = \frac{1}{2H} (T_m - T_e - D \cdot \Delta \omega) \tag{2.1}
\]

where \( \Delta \omega \) is the per unit speed deviation, \( T_m \) is the mechanical torque in per unit, \( T_e \) is the electrical torque in per unit, \( H \) is the rotor inertia in seconds, and \( D \) is the damping torque coefficient of the synchronous machine in p.u. torque/p.u. speed deviation. The speed deviation \( \Delta \omega \) is equal to \( \Delta \omega = \omega - \omega_0 \) where \( \omega \) is the rotor speed of the synchronous machine in radians per second and \( \omega_0 \) is nominal rotor electrical speed in radians per second (\( \omega_0 = 2\pi \cdot f_0 \)).

In equation (2.1) a value can be set to the damping torque coefficient \( D \) to include damping in the synchronous machine model make it behave like a more detailed representation of a synchronous machine containing damping provided from the field winding and the damper windings, i.e., a high-order model of a synchronous machine, see Kundur [35] and Johansson [28].

Equation (2.1) contains one state variable, \( \Delta \omega \), and one algebraic variable, \( T_e \). State variables are variables that are time-derived in the system of equations, for instance speed deviation \( \Delta \omega \) and rotor angle \( \delta \) of a synchronous machine. Algebraic variables are variables that are not time-derived in the system of equations, for instance, network variables as voltages and currents when simulating a power system in fundamental frequency mode. The mechanical torque \( T_m \) (provided from a turbine) is constant for the classical machine model and is therefore not defined as an algebraic variable.

We will later come back to equation (2.1). First we will generalize the form on which we express the system of equations.

The equations of a dynamic system consists of both differential equations, see for instance equation (2.1), as well as algebraic equations. Algebraic equations of the dynamic system are for instance expressions of node voltages and line currents when simulating in fundamental frequency mode.

All differential equations can be re-organized and written on the following form,
\[
\dot{x}_i = f_i(x_1, \ldots, x_n; u_1, \ldots, u_p; v_1, \ldots, v_r; t) \land i = 1, \ldots, n. \tag{2.2}
\]

Equation (2.2) is an expression for the time derivative of state variable \( i \); \( \dot{x}_i \). Expression \( f_i \) may contain the \( n \) state variables \( \{x_1, \ldots, x_n\} \), the \( p \) input variables \( \{u_1, \ldots, u_p\} \), and the \( r \) algebraic variables \( \{v_1, \ldots, v_r\} \). Expression \( f_i \) can be non-linear and of any form.

All \( r \) algebraic equations can be combined to the following form,

\[
0 = g_i(x_1, \ldots, x_n; u_1, \ldots, u_p; v_1, \ldots, v_r; t) \land i = 1, \ldots, r \tag{2.3}
\]

Equation (2.3) consists of an expression \( g_i \) which may contain the \( n \) state variables \( \{x_1, \ldots, x_n\} \), the \( p \) input variables \( \{u_1, \ldots, u_p\} \), and the \( r \) algebraic variables \( \{v_1, \ldots, v_r\} \). Expression \( g_i \) can be non-linear and of any form.

Time is denoted by \( t \) in equations (2.2) and (2.3). If \( f \) and \( g \) are not explicit functions of time, i.e., \( \dot{x}_i \) is only a function of the state variables, the input signals, and the algebraic variables, the system is called time invariant and we can then ignore the notation of \( t \) in equations (2.2) and (2.3). Most dynamic systems in power systems are time invariant and we will in the following assume that our setup is time invariant.

Here we put the algebraic variables in an algebraic variable vector \( v \) and the state variables in a state vector \( x \) to make the following steps more structured and easier to understand in the chapter as,

\[
v = \begin{bmatrix} v_1 \\ \vdots \\ v_r \end{bmatrix} \tag{2.4}
\]

\[
x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}. \tag{2.5}
\]

The time-derivatives of the state variables are put in a vector \( \dot{x} \) as,

\[
\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_n \end{bmatrix}. \tag{2.6}
\]
The algebraic variable $T_e$ and the state variable $\Delta \omega$ of the earlier mentioned synchronous machine are therefore elements of the $v$-vector and $x$-vector respectively. The time-derivative of the state variable $\Delta \omega$ is an element of the $\dot{x}$-vector.

The aim when performing linearization of a dynamic system is to create a dynamic model on a linear form from an operating point, an equilibrium point $x_0$ (a steady-state solution of the power system). The linear form shows how the system reacts linearly on small disturbances $\Delta$, i.e., a form that shows how the time-derivatives of the state variables $\dot{x}$ are varying depending on the state variables $x$ for small disturbances $\Delta$. The relation between $\dot{x}$ and $x$ is then described with a linear system matrix $A$ and since it might be valid only for small disturbances in $\dot{x}$ and $x$ we add $\Delta$ for both $\dot{x}$ and $x$.

We get the following equation,

$$\Delta \dot{x} = A \Delta x$$  \hspace{1cm} (2.7)

where $\Delta \dot{x}$ describes the contribution in the time-derivatives for the state variables for small disturbances of the state variables in $\Delta x$. To create such a form, we need to consider the feedback of the algebraic variables $v$ when the state variables are perturbed.

The process when generating the linear form in equation (2.7) is done in different ways for the three linearization methods and that is in the following described in detail.

### 2.4.1 Analytical linearization, AL method

When performing linearization with the AL method, all differential equations and algebraic equations are linearized by their analytical expressions. For instance, the differential equation describing the time-derivative of the speed deviation, $\Delta \dot{\omega}$, for a synchronous machine (in this case a classical machine model) is done as,

$$\Delta \dot{\omega} = \frac{1}{2H} \Delta T_e - \frac{D}{2H} \cdot \Delta \omega.$$  \hspace{1cm} (2.8)

The linearization procedure as described with equations (2.1) and (2.8) is also called symbolic differentiation.

For notational reasons we write the differentiation of speed deviation $\Delta \omega$ as $\Delta \omega$. A more correct notation would be to write $\Delta(\Delta \omega)$ but it is more convenient to exclude the extra $\Delta$. The same have been done for the time
derivative of the differentiation of speed deviation, i.e., we write \( \Delta \dot{\omega} \) instead of \( \Delta(\Delta \dot{\omega}) \)

In a software package, analytical linearization is possible because mathematical rules prescribing the differentiation of the various operators are implemented. Expressions including several operators are solved using the chain rule, Aneros [3].

In a steady-state situation we can assume that the \( p \) input signals \( \{u_1, \ldots, u_p\} \) in equations (2.2) and (2.3) are constant and therefore they can be excluded in the linearization. Then, the linearized form of equations (2.2) – (2.3) can be written as,

\[
\Delta \dot{x}_i = \frac{\partial f_i}{\partial x_1} \Delta x_1 + \ldots + \frac{\partial f_i}{\partial x_n} \Delta x_n + \frac{\partial f_i}{\partial v_1} \Delta v_1 + \ldots + \frac{\partial f_i}{\partial v_r} \Delta v_r \quad \forall i = 1, \ldots, n
\]

\[
0 = \frac{\partial g_i}{\partial x_1} \Delta x_1 + \ldots + \frac{\partial g_i}{\partial x_n} \Delta x_n + \frac{\partial g_i}{\partial v_1} \Delta v_1 + \ldots + \frac{\partial g_i}{\partial v_r} \Delta v_r \quad \forall i = 1, \ldots, r.
\]

An often made normalization of the system of equations (2.3) makes the term \( \frac{\partial g_i}{\partial v_i} \Delta v_i \) in equation (2.10) equal to \( \frac{\partial g_i}{\partial v_i} \Delta v_i = -1 \cdot \Delta v_i \), see equations (2.51) – (2.59) in section 2.5.1.

The differentiation of the \( p \) input signals \( \{u_1, \ldots, u_p\} \) cannot be found in equations (2.9) and (2.10) since they are all equal to zero.

In equations (2.9) – (2.10) we have differentiated equations (2.2) – (2.3) with respect to the \( n \) state variables and the \( r \) algebraic variables.

The linearized system of equations (2.9) – (2.10) can be identified with the following equation,

\[
\begin{bmatrix}
0 \\
\Delta \dot{x}
\end{bmatrix} = 
\begin{bmatrix}
J_{aa} & J_{as} \\
J_{sa} & J_{ss}
\end{bmatrix}
\begin{bmatrix}
\Delta \v \\
\Delta x
\end{bmatrix}
\]

(2.11)

where

\[
\Delta \v = 
\begin{bmatrix}
\Delta v_1 \\
\vdots \\
\Delta v_r
\end{bmatrix}
\]

(2.12)
\[
\Delta x = \begin{bmatrix}
\Delta x_1 \\
\vdots \\
\Delta x_n
\end{bmatrix}
\] (2.13)

\[
\Delta \dot{x} = \begin{bmatrix}
\Delta \dot{x}_1 \\
\vdots \\
\Delta \dot{x}_n
\end{bmatrix}
\] (2.14)

The 0 in equation (2.11) is a vector containing 1 column and \( r \) rows with all elements equal to 0.

For the earlier mentioned synchronous machine included in a power system, one of the elements in vector \( \Delta v \) is the differentiated electrical torque, \( \Delta T_e \), one of the elements in vector \( \Delta x \) is the differentiated speed deviation, \( \Delta \omega \), and one of the elements in vector \( \Delta \dot{x} \) is the differentiated time-derivative of the speed deviation, \( \Delta \dot{\omega} \).

The linearized system in equation (2.11) is described with a Jacobian-matrix \( J \), consisting of four sub-matrices \( J_{aa}, J_{as}, J_{sa}, \) and \( J_{ss} \) as in equation (2.15), see Martins [43, 44],

\[
J = \begin{bmatrix}
J_{aa} & J_{as} \\
J_{sa} & J_{ss}
\end{bmatrix}
\] (2.15)

The sub-matrix \( J_{aa} \) contains partial derivatives of the algebraic variables in the algebraic equations, i.e., row \( i \) contains factors \( \left( \frac{\partial g_i}{\partial v_1}, \ldots, \frac{\partial g_i}{\partial v_{r-1}}, -1, \frac{\partial g_i}{\partial v_r} \right) \) from equation (2.10). Sub-matrix \( J_{as} \) contains partial derivatives of the state variables in the algebraic equations, i.e., row \( i \) contains factors \( \left( \frac{\partial g_i}{\partial x_1}, \ldots, \frac{\partial g_i}{\partial x_n} \right) \) from equation (2.10). Examples of such algebraic equations are expressions expressing node voltages, which are algebraic variables when simulating a power system in fundamental frequency mode.

The sub-matrix \( J_{sa} \) contains partial derivatives of the algebraic variables in the differential equations, i.e., row \( i \) contains factors \( \left( \frac{\partial f_i}{\partial v_1}, \ldots, \frac{\partial f_i}{\partial v_r} \right) \) from equation (2.9), among other elements factor \( -\frac{1}{2H} \) from equation (2.8). The sub-matrix \( J_{ss} \) contains partial derivatives of the state variables in the differential equations, i.e., row \( i \) contains factors \( \left( \frac{\partial f_i}{\partial x_1}, \ldots, \frac{\partial f_i}{\partial x_n} \right) \) from equation (2.9), among other elements factor \( -\frac{D}{2H} \) from equation (2.8).

Since the number of algebraic variables are equal to the number of expressions \( g \), the sub-matrix \( J_{aa} \) is quadratic \([r \times r]\). Since the number of state variables are equal to the number of expressions \( f \), the sub-matrix \( J_{ss} \) is quadratic \([n \times n]\). The sub-matrix \( J_{as} \) has the size \([r \times n]\) and the sub-matrix \( J_{sa} \) has the size \([n \times r]\).
Equation (2.11) gives the relation,

$$0 = J_{aa} \Delta v + J_{as} \Delta x$$

(2.16)

which can be re-written as,

$$\Delta v = -J_{aa}^{-1} J_{as} \Delta x.$$  

(2.17)

When the matrix $J_{aa}$ is singular, which can be the case in the presence of network elements containing time delays such as HVDC models, more complex routines are necessary to arrive at equation (2.17), Aneros [3].

Equation (2.11) also gives the relation,

$$\Delta \dot{x} = J_{sa} \Delta v + J_{ss} \Delta x.$$  

(2.18)

Using equation (2.17) in equation (2.18) gives,

$$\Delta \dot{x} = -J_{sa} J_{aa}^{-1} J_{as} \Delta x + J_{ss} \Delta x = (-J_{sa} J_{aa}^{-1} J_{as} + J_{ss}) \Delta x$$

(2.19)

and in the most right-hand side of equation (2.19) the $A$-matrix can be identified as,

$$A = J_{ss} - J_{sa} J_{aa}^{-1} J_{as}.$$  

(2.20)

Equation (2.20) can also be found in Martins [43, 44].

Later in section 2.5.1 a numerical example when linearizing the earlier mentioned classical model of a synchronous machine in a small power system using the AL method is shown.

2.4.2 Forward-difference approximation, FDA method

The forward-difference approximation method (mentioned as the FDA method from now on) is a numerical method, see Gill [18], p. 54. Starting from a valid equilibrium condition $x_0$, a second state vector is created, $x_{i+}$, in which the $ith$ component of the state vector $x$ is perturbed from the equilibrium point $x_0$ by adding a small perturbation $h$ for state variable $i$. The difference between $x_{i+}$ and $x_0$ is denoted as,

$$\Delta x_i = x_{i+} - x_0.$$  

(2.21)

All elements except for element $i$ are equal to zero in the vector $\Delta x_i$. Element $i$ is equal to the perturbation size $h$, 

\[ \Delta x_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ h \\ 0 \\ \vdots \\ 0 \end{bmatrix} . \quad (2.22) \]

With the new state vector \( x_{i+} \), new values of the \( r \) algebraic variables in vector \( v \) are solved using the non-linear system of equations (2.3). All state variables except state variable \( i \) remain constant at the values from the equilibrium point \( x_0 \). In an equilibrium point the \( p \) input signals \( \{u_1, \ldots, u_p\} \) in equations (2.2) and (2.3) are constant.

With the new values of the \( r \) algebraic variables and state variable \( i \), time-derivatives are calculated for all \( n \) state variables with equation (2.2). These \( n \) time-derivatives when perturbing state variable \( i \) are denoted \( \dot{x}_{i+} \) and the difference between \( \dot{x}_{i+} \) and \( \dot{x}_0 \) is denoted as,

\[ \Delta \dot{x}_{i+} = \dot{x}_{i+} - \dot{x}_0 = \dot{x}_{i+} . \quad (2.23) \]

Since all elements of \( \dot{x}_0 \) are equal to zero we have deleted \( \dot{x}_0 \) in the most right-hand side of equation (2.23) this since we linearize the system in an equilibrium point where all time-derivatives are equal to zero.

Now, when \( \Delta \dot{x}_{i+} \) is known we can calculate each element of column \( i \) of the \( A \)-matrix, denoted as \( A_i \), by using,

\[ \Delta \dot{x}_{i+} = A_i h \quad (2.24) \]

and equation (2.24) can be re-written so that \( A_i \) can be identified as,

\[ A_i = \frac{1}{h} \Delta \dot{x}_{i+} \quad (2.25) \]

or if we use equation (2.23) in equation (2.25), then \( A_i \) is,

\[ A_i = \frac{1}{h} \dot{x}_{i+} \quad (2.26) \]

where \( A_i \) is column \( i \) of system matrix \( A \).

Equation (2.25) is used to calculate values of the \( i \)th column of the system matrix \( A \) and \( h \) is the perturbation that we add to state variable \( i \).
By sequentially perturbing all entries of state vector $x$ and get all $\Delta \dot{x}_i$ for all $i$:s we can identify all columns of the $A$-matrix and create the full $A$-matrix as formulated in equation (2.7).

The size of the perturbation $h$ affects the obtained result, i.e., the elements of the $A$-matrix and as a consequence, its eigenvalues. This is shown later in section 2.5.2.

### 2.4.3 Center-difference approximation, CDA method

The center-difference approximation (mentioned as the CDA method from now on) is as the FDA method (which was shown in section 2.4.2) a numerical method, see Gill [18], p. 55. Starting from a valid equilibrium condition $x_0$, two other state vectors are created, $x_{i+}$ and $x_{i-}$ respectively. In $x_{i+}$ the $i$th component of the state vector $x$ is perturbed from the equilibrium point $x_0$ by adding a small perturbation $h$ for state variable $i$, and in $x_{i-}$, the $i$th component of the state vector $x$ is perturbed from the equilibrium point $x_0$ by subtracting the same small perturbation $h$ for state variable $i$. The difference between $x_{i+}$ and $x_{i-}$ is denoted as,

$$\Delta x_i = \frac{1}{2} \cdot (x_{i+} - x_{i-}).$$

(2.27)

All elements except for element $i$ in $\Delta x_i$ are equal to zero. Element $i$ is equal to the perturbation size $h$.

With the new state vectors $x_{i+}$ and $x_{i-}$ respectively, new values of the $r$ algebraic variables in vector $v$ are calculated using the non-linear system of equations (2.3). All state variables except state variable $i$ remain constant at the values from the equilibrium point $x_0$.

Here we can see that the non-linear system of equations (2.3) has to be solved twice as many times when using the CDA method compared to when using the FDA method.

With the new values of the $r$ algebraic variables and state variable $i$, time-derivatives are calculated for all $n$ state variables with equation (2.2). These $n$ new time-derivatives when perturbing state variable $i$ are denoted as $\dot{x}_{i+}$ and $\dot{x}_{i-}$ respectively.

The difference between $\dot{x}_{i+}$ and $\dot{x}_0$ is denoted as earlier in section 2.4.2 as,

$$\Delta \dot{x}_{i+} = \dot{x}_{i+} - \dot{x}_0 = \dot{x}_{i+}.$$  

(2.28)

Since all elements of $\dot{x}_0$ are equal to zero we have deleted $\dot{x}_0$ in the most right-hand side of equation (2.28).
The difference between $\dot{x}_0$ and $\dot{x}_{i-}$ is denoted as,

$$\Delta \dot{x}_{i-} = \dot{x}_0 - \dot{x}_{i-} = -\dot{x}_{i-}. \quad (2.29)$$

The average of the two vectors $\Delta \dot{x}_{i+}$ and $\Delta \dot{x}_{i-}$ is,

$$\Delta \dot{x}_i = \frac{1}{2} \cdot (\Delta \dot{x}_{i+} + \Delta \dot{x}_{i-}) = \frac{1}{2} \cdot (\dot{x}_{i+} - \dot{x}_{i-}). \quad (2.30)$$

Now when $\Delta \dot{x}_i$ is known we can calculate each element of column $i$ of the $A$-matrix by using,

$$\Delta \dot{x}_i = A_i h \quad (2.31)$$

where $A_i$ is column $i$ of system matrix $A$. Equation (2.31) can be re-written so that $A_i$ can be identified as,

$$A_i = \frac{1}{h} \Delta \dot{x}_i \quad (2.32)$$

or if we use equation (2.30) in equation (2.32), then $A_i$ is,

$$A_i = \frac{1}{2h} [\dot{x}_{i+} - \dot{x}_{i-}] \quad (2.33)$$

Equation (2.33) is used to calculate values of the $i$th column of system matrix $A$ and $h$ is the perturbation.

By sequentially perturbing all entries of state vector $x$ with perturbations $h$ and $-h$ and get all $\Delta \dot{x}_{i+}$ and $\Delta \dot{x}_{i-}$ for all $i$'s we can identify all columns of the $A$-matrix and create the full $A$-matrix as formulated in equation (2.7).

The size of the perturbation $h$ affects the obtained result, i.e., the elements of the $A$-matrix and as a consequence, its eigenvalues. This is shown later in section 2.5.3.

## 2.5 Linearization of a classical machine

With the three linearization methods a small test system containing an infinite bus, a transmission impedance $x_{line}$, and a synchronous machine represented by the classical machine model are modelled and linearized, see figure 2.1 and Kundur [35], p. 732, where all system data can be found. In appendices A.1 – A.2 the system data as well as the power-flow solution are given.

The reason for applying the three different linearization methods to a power system is to come as close as possible to a real situation when engineers...
are working with linearization but utilizing different linearization methods. A practical consequence is that we in this section show results obtained from three engineers working in parallel, studying the same power system but using three different linearization methods.

Two of the linearization methods can be found in two software that exist on the market. The AL method can be found in Simpow and the FDA method can be found in PSS/E. The results in the following are both done by hand and compared with these software.

The models of the power system components are identical in all linearizations and therefore it is possible to observe what impact the different linearization methods have on the obtained eigenvalues. In figure 2.2 the classical machine model and its connection to the rest of the power system is shown.

The machine contains two state variables, the speed deviation $\Delta \omega$ and the machine angle $\delta$ (DELTA in figure 2.2). There are no other state variables in the power system. The initial values of all variables are calculated in the power-flow solution and in the initialisation of the dynamic simulation, see appendix A.2. It can also be found in Kundur [35], pp. 732-733. The machine parameters are given in table 2.1 where $x'_d$ is the transient reactance of the $d$-axis of the machine.

\[
\begin{array}{cccc}
  x'_d \text{ (p.u.)} & D \text{ (1/s)} & H \text{ (s)} \\
  0.30 & 0.00 & 3.50 \\
\end{array}
\]

Table 2.1: Settings of the classical machine model in section 2.5.

For a classical machine model the following two differential equations exist,
The speed deviation $\Delta \omega$ is equal to $\Delta \omega = \omega - \omega_0$ where $\omega$ is the rotor speed of the synchronous machine in radians per second and $\omega_0$ is nominal rotor electrical speed in radians per second ($\omega_0 = 2\pi \cdot f_0$).

The following two state variables are included in the state vector $x$ as,

$$
\begin{bmatrix}
\Delta \omega \\
\delta
\end{bmatrix}
$$

(2.36)

The nine algebraic equations for the machine and the rest of the power system are,

$$0 = -T_e + u_d i_d + u_q i_q$$

(2.37)

$$0 = -u_d + u_re \sin \delta - u_im \cos \delta$$

(2.38)

$$0 = -u_q + u_re \cos \delta + u_im \sin \delta$$

(2.39)
\[ 0 = -i_d - \frac{u_q - E_{fd0}}{x_d'} \]  
\[ 0 = -i_q + \frac{u_d}{x_d'} \]  
\[ 0 = -u_{re} + u_{re infinite bus} - x_{line} i_{im} \]  
\[ 0 = -u_{im} + u_{im infinite bus} + x_{line} i_{re} \]  
\[ 0 = -i_{re} + i_d \sin \delta + i_q \cos \delta \]  
\[ 0 = -i_{im} + i_q \sin \delta - i_d \cos \delta \]

where \( i_d \) and \( i_q \) are the stator current of the machine in \( d \)- and \( q \)-axis respectively, \( u_d \) and \( u_q \) are the stator voltage of the machine in \( d \)- and \( q \)-axis respectively, \( u_{re} \) and \( u_{im} \) are the real and imaginary parts of the positive-sequence voltage of the machine bus, \( E_{fd0} \) is a constant electric field voltage, \( i_{re} \) and \( i_{im} \) are the real and imaginary parts of the current through the line reactance \( x_{line} \). \( u_{re infinite bus} \) and \( u_{im infinite bus} \) are the real and imaginary parts of the positive-sequence voltage of the infinite bus.

\( E_{fd0} \) is an internal voltage and can in the literature often be found as \( E' \) as in Kundur [35] or as \( E'_q \) as in Andersson [2].

It would have been possible to decrease the number of algebraic equations and variables above but here we have chosen to follow Johansson [28].

All left-hand sides of equations (2.37) – (2.45) are equal to zero since we want to have the left-hand side of these equations equal to zero later during the linearization using the AL method, see equation (2.10).

We assume that the corresponding nine algebraic variables are included in the algebraic variable vector \( \mathbf{v} \) as,

\[ \mathbf{v} = \begin{bmatrix} T_e \\ u_d \\ u_q \\ i_d \\ i_q \\ u_{re} \\ u_{im} \\ i_{re} \\ i_{im} \end{bmatrix} \]
In the following sections 2.5.1, 2.5.2, and 2.5.3 the classical machine model in figure 2.2 is linearized with the three methods. The results concerning the AL method is obtained by hand and from Simpow, the results concerning the FDA method is obtained by hand and from PSS/E, and the results concerning the CDA method is obtained only by hand.

In the following subsections it is the \( A \)-matrix in equation (2.47) that should be derived when using the three methods. It contains linear relations of the left-hand sides \( \Delta \dot{\omega} \) and \( \Delta \dot{\delta} \) and the right-hand sides \( \Delta \omega \) and \( \Delta \delta \),

\[
\begin{bmatrix}
\Delta \dot{\omega} \\
\Delta \dot{\delta}
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta \omega \\
\Delta \delta
\end{bmatrix}
\tag{2.47}
\]

which we recognize as the earlier defined equation (2.7) which we here give a second time.

\[
\Delta \dot{x} = A \Delta x. \tag{2.48}
\]

## 2.5.1 Linearization of a classical machine using the AL method

Here we apply the AL method that was described in section 2.4.1. In the AL method we linearize equations (2.34) – (2.35), and (2.37) – (2.45) with respect to the algebraic variables and the state variables as shown in section 2.4.1. We get,

\[
\Delta \dot{\omega} = -\frac{1}{2H} \Delta T_e - \frac{D}{2H} \cdot \Delta \omega
\]

\[
\Delta \dot{\delta} = \omega_0 \cdot \Delta \omega
\]

\[
0 = -\Delta T_e + i_{d0} \Delta u_d + i_{q0} \Delta u_q + u_{d0} \Delta i_d + u_{q0} \Delta i_q
\]

\[
0 = -\Delta u_d + (u_{re0} \cos \delta_0 + u_{im0} \sin \delta_0) \Delta \delta + \sin \delta_0 \Delta u_{re} - \cos \delta_0 \Delta u_{im}
\]

\[
0 = -\Delta u_q + (u_{im0} \cos \delta_0 - u_{re0} \sin \delta_0) \Delta \delta + \cos \delta_0 \Delta u_{re} + \sin \delta_0 \Delta u_{im}
\]

\[
0 = -\Delta i_d - \frac{1}{x_d} \Delta u_q
\]
\[ 0 = -\Delta i_q + \frac{1}{x_d} \Delta u_d \]  \hspace{1cm} (2.55)

\[ 0 = -\Delta u_{re} - x_{line} \Delta i_{im} \]  \hspace{1cm} (2.56)

\[ 0 = -\Delta u_{im} + x_{line} \Delta i_{re} \]  \hspace{1cm} (2.57)

\[ 0 = -\Delta i_{re} + (i_d 0 \cos \delta_0 - i_q 0 \sin \delta_0) \Delta \delta + \sin \delta_0 \Delta i_d - \cos \delta_0 \Delta i_q \]  \hspace{1cm} (2.58)

\[ 0 = -\Delta i_{im} + (i_q 0 \cos \delta_0 + i_d 0 \sin \delta_0) \Delta \delta - \cos \delta_0 \Delta i_d + \sin \delta_0 \Delta i_q. \]  \hspace{1cm} (2.59)

In equations (2.49) – (2.59) we have inserted values from the operating point. These values are indicated with a ”0” in the sub-index of the variable. They can be found in appendix A.2.

The expressions of the eleven partial derivatives in equations (2.49) – (2.59) are identified with the four sub-matrices of the Jacobian-matrix \(J\) in equation (2.15). The rows of the \(J_{aa}\)-matrix is constructed in the same order as of equations (2.51) – (2.59). The order of the columns follows the order of the algebraic variables in the \(v\)-vector.

\[
\begin{bmatrix}
-1 & i_d 0 & i_q 0 & u_d 0 & u_q 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & \sin \delta_0 & -\cos \delta_0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & \cos \delta_0 & \sin \delta_0 & 0 & 0 \\
0 & 0 & -\frac{1}{x_d} & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{x_d} & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -x_{line} \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & x_{line} & 0 \\
0 & 0 & 0 & \sin \delta_0 & \cos \delta_0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & -\cos \delta_0 & \sin \delta_0 & 0 & 0 & 0 & -1 \\
\end{bmatrix}
\]  \hspace{1cm} (2.60)

The rows of the \(J_{as}\)-matrix is constructed in the same order as of equations (2.51) – (2.59). The order of the columns follows the order of the state variables in the \(x\)-vector.
\[
J_{\text{sa}} = \begin{bmatrix}
0 & 0 \\
0 & u_{re0} \cos \delta_0 + u_{im0} \sin \delta_0 \\
0 & u_{im0} \cos \delta_0 - u_{re0} \sin \delta_0 \\
0 & 0 \\
0 & 0 \\
0 & \cos \delta_0 \\
0 & -\sin \delta_0 \\
0 & 0 \\
0 & i_{d0} \cos \delta_0 - i_{q0} \sin \delta_0 \\
0 & i_{q0} \cos \delta_0 + i_{d0} \sin \delta_0
\end{bmatrix}
\] (2.61)

The rows of the \( J_{\text{sa}} \)-matrix is constructed in the same order as of equations (2.49) – (2.50). The order of the columns follows the order of the algebraic variables in the \( v \)-vector.

\[
J_{\text{sa}} = \begin{bmatrix}
-\frac{1}{2H} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\] (2.62)

The rows of the \( J_{\text{ss}} \)-matrix is constructed in the same order as of equations (2.49) – (2.50). The order of the columns follows the order of the state variables in the \( x \)-vector.

\[
J_{\text{ss}} = \begin{bmatrix}
-\frac{D}{2H} & 0 \\
\omega_0 & 0
\end{bmatrix}
\] (2.63)

The final \( A \)-matrix and its eigenvalues with the AL method

In the four sub-matrices in equations (2.60) – (2.63), values from the current operating point as well as machine and network parameters are entered to get values of the elements of the sub-matrices. The values can be found in appendices A.1 – A.2. The \( A \)-matrix is then constructed as with equation (2.20). The \( A \)-matrix becomes in the current operating point,

\[
A = J_{\text{ss}} - J_{\text{sa}}J_{\text{aa}}^{-1}J_{\text{as}} = \begin{bmatrix}
0 & -0.108131 \\
376.991118 & 0
\end{bmatrix}
\] (2.64)

The eigenvalues of matrix \( A \) are then calculated by hand,

\[
\lambda_{1,2} = 0 \pm j6.38471(1/s, \text{rad/s}).
\] (2.65)

The eigenvalues of the same power system are when calculated in Simpow,

\[
\lambda_{1,2} = 0 \pm j6.38471(1/s, \text{rad/s}).
\] (2.66)
The frequency of the imaginary parts of the eigenvalues in equations (2.65) and (2.66) is 1.01616 (Hz).

The eigenvalues are in Simpow calculated using the Quick Response-method (QR-method), see Watkins [85] and the inverse iteration method, see Peters [63] and Wilkinson [86] is used to calculate the eigenvectors as well as to improve the real and imaginary parts of the eigenvalues. In Simpow the imaginary part of an eigenvalue is calculated in (Hz), therefore a multiplication of $2\pi$ is necessary to arrive to the result in equation (2.66).

As can be seen, the results calculated by hand using Matlab and when using Simpow are consistent and can also be found in Kundur [35], p. 735. In Kundur [35], the results have been rounded off several times in the calculation process and since then, it is calculated as $\lambda_{1,2} = 0 \pm j6.39$ (1/s, rad/s). However, if six digits accuracy are used in example 12.2 in Kundur [35] during the whole calculation process, then the bus voltage at the machine bus, indicated as $E_t$ in Kundur [35] is $E_t = 0.999818 \cdot 36.0185^\circ$. By re-calculating example 12.2, the state matrix in Kundur [35] is equal to equation (2.64) and since then also the eigenvalues. In appendix A.3 the calculations of example 12.2 in Kundur [35] can be found using six digits accuracy.

### 2.5.2 Linearization of a classical machine using the FDA method

Here we apply the FDA method as described in section 2.4.2. For the FDA method the $A$-matrix is column by column identified by applying a perturbation in each state variable from their steady-state values. In the example the state variables are $\Delta \omega$ and $\delta$. In this section the size of the applied perturbation $h$ is selected to 0.01 (p.u. and rad) since it is the default perturbation size in PSS/E. By adding it to each state variable, it is calculated from the change of the left-hand sides of equations (2.67) – (2.68) including feedback of equations (2.37) – (2.45).

$$
\Delta \dot{\omega} = \frac{1}{2H}(T_m - T_e - D \cdot \Delta \omega) \quad (2.67)
$$

$$
\dot{\delta} = \omega_0 \cdot \Delta \omega \quad (2.68)
$$

Equations (2.67) – (2.68) are here repeated and have earlier been introduced as equations (2.34) and (2.35).

Equations (2.67) – (2.68) together with the algebraic equations (2.37) – (2.45) are used to derive a state matrix describing linear relations of the left-hand sides $\Delta \dot{\omega}$ and $\Delta \dot{\delta}$ and the right-hand sides $\Delta \omega$ and $\Delta \delta$ as described
with equation (2.69) below. In the following it is shown how it is done for each state variable.

The linear system that should be identified is,

$$
\begin{bmatrix}
\Delta \dot{\omega} \\
\Delta \delta
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta \omega \\
\Delta \delta
\end{bmatrix}
$$

(2.69)

where the $A$-matrix is,

$$
A = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}.
$$

(2.70)

**Perturbing the speed deviation $\Delta \omega$**

The left-hand sides of equations (2.67) and (2.68) when perturbing $\Delta \omega$ are used to identify matrix-elements $A_{11}$ and $A_{21}$ in equation (2.69). When perturbing $\Delta \omega$, all other state variables are assumed to be unperturbed, see section 2.4.2. The feedback of $\delta$ to the variable $T_e$ in equation (2.67) is therefore neglected. Since the damping constant $D$ is zero in this example, the left-hand side of equation (2.67) is equal to zero when perturbing $\Delta \omega$. Therefore, matrix-element $A_{11}$ is,

$$
A_{11} = \frac{\Delta \dot{\omega}}{\Delta \omega} = \frac{0}{0.01} = 0
$$

(2.71)

When perturbing $\Delta \omega$, the left-hand side of equation (2.68) changes proportional to the perturbation multiplied with $\omega_0$, see figure 2.2. Therefore matrix-element $A_{21}$ is,

$$
A_{21} = \frac{\Delta \dot{\delta}}{\Delta \omega} = \frac{0.01 \cdot \omega_0}{0.01} = \omega_0 = 376.991118
$$

(2.72)

**Perturbing the machine angle $\delta$**

The left-hand sides of equations (2.67) and (2.68) when perturbing $\delta$, noted as $\Delta \delta$, are used to identify matrix-elements $A_{12}$ and $A_{22}$ in equation (2.69). $\delta$ is not direct included in the differential equation (2.67) and therefore $\delta$’s feedback to the variable $T_e$ must be calculated using the algebraic equations (2.37) – (2.45). The matrix-element $A_{12}$ is,

$$
A_{12} = \frac{\Delta \dot{\omega}}{\Delta \delta} = \frac{-1.07480 \cdot 10^{-3}}{0.01} = -0.107480
$$

(2.73)
The feedback of the state variable $\Delta \omega$ in equation (2.68) is neglected when perturbing the state variable $\delta$, see section 2.4.2, and therefore the matrix-element $A_{22}$ is,

$$A_{22} = \frac{\Delta \delta}{\Delta \delta} = \frac{0}{0.01} = 0$$

(2.74)

The final $A$-matrix and its eigenvalues with the FDA method

The $A$-matrix is composed of the four matrix-elements $A_{11}$, $A_{12}$, $A_{21}$, and $A_{22}$ as in equation (2.75),

$$A = \begin{bmatrix} 0 & -0.107480 \\ 376.991118 & 0 \end{bmatrix}.$$  

(2.75)

The eigenvalues of $A$ are then calculated by hand,

$$\lambda_{1,2} = 0 \pm j6.36545(1/s, \text{rad}/s).$$

(2.76)

In PSS/E the $A$-matrix is,

$$A = \begin{bmatrix} 0 & -0.107370 \\ 376.991118 & 0 \end{bmatrix}.$$  

(2.77)

The eigenvalues are in PSS/E calculated using the Quick Response method (the QR-method), see Watkins [85]. The eigenvalues of $A$ in equation (2.77) are,

$$\lambda_{1,2} = 0 \pm j6.3622(1/s, \text{rad}/s).$$

(2.78)

The eigenvalues in equations (2.76) and (2.78) were calculated when the state matrix $A$ was identified with a perturbation size $h$ of 0.01.

The frequency of the imaginary parts of the eigenvalues in equations (2.76) and (2.78) are 1.01309 and 1.0126 (Hz) respectively.

The obtained differences between the eigenvalues shown in equations (2.76) and (2.78) has not been possible to investigate further. The differences can be a result from propagation of roundoff errors in PSS/E, both when calculating the elements of the $A$-matrix as well as when applying the QR-method to calculate the eigenvalues. However, the differences are very small and the linearization procedure as described in section 22.2 in the Application Guide [68] is indeed the FDA method.

In section 2.6, eigenvalues are shown derived from state matrices constructed with different perturbation sizes, both manually as in this section and by PSS/E.
2.5.3 Linearization of a classical machine using the CDA method

Here we apply the CDA method that was described in section 2.4.3. For the CDA method the A-matrix is column by column identified by applying two perturbations in each state variable. As in section 2.5.2, the perturbation is $h$ but for the CDA method also perturbation $-h$ is added to the state variables, see below. In the example the state variables are $\Delta \omega$ and $\delta$. In this section the size of the applied perturbations $h$ are 0.01 and -0.01 respectively. By adding it to each state variable from an equilibrium point, it is calculated how the left-hand sides of differential equations (2.67) and (2.68) change including feedback of the algebraic equations (2.37) – (2.45).

Equations (2.67) – (2.68) together with the algebraic equations (2.37) – (2.45) are used to derive a state matrix describing linear relations of the left-hand sides $\Delta \dot{\omega}$ and $\Delta \dot{\delta}$ and the right-hand sides $\Delta \omega$ and $\Delta \delta$ as described with equation (2.69). In the following it is shown how it is done for each state variable.

**Perturbing the speed deviation $\Delta \omega$**

The left-hand sides of equations (2.67) and (2.68) when perturbing $\Delta \omega$ are used to identify matrix-elements $A_{11}$ and $A_{21}$ in equation (2.69). When perturbing $\Delta \omega$, all other state variables are assumed to be unperturbed, see section 2.4.3. The feedback of $\delta$ to the variable $T_e$ in equation (2.67) is therefore neglected. Since the damping constant $D$ is zero, the left-hand side of equation (2.67) is equal to zero when perturbing $\Delta \omega$. Therefore, matrix-element $A_{11}$ is by using the earlier derived equation (2.33) equal to,

$$A_{11} = \frac{\Delta \dot{\omega}_i}{\Delta \omega} = \frac{\Delta \dot{\omega}_{i+} - \Delta \dot{\omega}_{i-}}{2 \cdot \Delta \omega} = \frac{0 - 0}{2 \cdot 0.01} = 0 \quad (2.79)$$

When perturbing $\Delta \omega$, the left-hand side of equation (2.68) changes proportional to the perturbation multiplied with $\omega_0$, see figure 2.2. Therefore, matrix-element $A_{21}$ is,

$$A_{21} = \frac{\Delta \dot{\delta}_i}{\Delta \omega} = \frac{\dot{\delta}_{i+} - \dot{\delta}_{i-}}{2 \cdot \Delta \omega} = \frac{0.01 \cdot \omega_0 - (-0.01 \cdot \omega_0)}{2 \cdot \omega_0} = \frac{2 \cdot 0.01 \cdot \omega_0}{2 \cdot 0.01} = 376.991118 \quad (2.80)$$
Perturbing the machine angle $\delta$

The left-hand sides of equations (2.67) and (2.68) when perturbing $\delta$, noted as $\Delta \delta$, are used to identify matrix-elements $A_{12}$ and $A_{22}$ in equation (2.69). $\delta$ is not direct included in differential equation (2.67) and therefore $\delta$'s feedback to the variable $T_e$ must be calculated using the algebraic equations (2.37) – (2.45). The matrix-element $A_{12}$ is,

$$A_{12} = \frac{\Delta \dot{\omega}_i}{\Delta \delta} = \frac{\dot{\Delta} \omega_i - \Delta \dot{\omega}_i}{2 \cdot \Delta \delta} = \frac{-1.07480 \cdot 10^{-3} - 1.08766 \cdot 10^{-3}}{2 \cdot 0.01} = -0.108123$$ (2.81)

The feedback of the state variable $\Delta \omega$ in equation (2.68) is neglected when perturbing the state variable $\delta$, see section 2.4.3, and therefore matrix-element $A_{22}$ is,

$$A_{22} = \frac{\Delta \delta_i}{\Delta \delta} = \frac{\delta_i^+ - \delta_i^-}{2 \cdot \Delta \delta} = \frac{0 - 0}{2 \cdot 0.01} = 0$$ (2.82)

The final A-matrix and its eigenvalues with the CDA method

The A-matrix is composed of the four matrix-elements $A_{11}$, $A_{12}$, $A_{21}$, and $A_{22}$ as in equation (2.83).

$$A = \begin{bmatrix} 0 & -0.108123 \\ 376.991118 & 0 \end{bmatrix}$$ (2.83)

The eigenvalues of matrix $A$ are then calculated by hand,

$$\lambda_{1,2} = 0 \pm j6.38446 (1/s, \text{rad/s}).$$ (2.84)

The eigenvalues in equation (2.84) were calculated when the state matrix $A$ was identified with a perturbation size $h$ of 0.01.

The frequency of the imaginary parts of the eigenvalues in equation (2.84) is 1.01612 (Hz).

In section 2.6, eigenvalues are shown derived from state matrices constructed with different perturbation sizes.
2.6 Comparison of the three linearization methods

In this section the three linearization methods are compared for the example in section 2.5.

Table 2.2 contains results from sections 2.5.1 – 2.5.3. The table contains eigenvalues also for three other disturbances. The following 4 different perturbation sizes $h = 0.0001, 0.001, 0.01,$ and $0.1$ are summarized in table 2.2.

<table>
<thead>
<tr>
<th>Perturbation $h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
</tr>
<tr>
<td>FDA</td>
</tr>
<tr>
<td>CDA</td>
</tr>
<tr>
<td>AL</td>
</tr>
</tbody>
</table>

Table 2.2: Eigenvalues of the classical machine in subsections 2.5.1 – 2.5.3. The eigenvalue’s real and imaginary parts are given in $[1/s]$ and $[\text{rad}/s]$ respectively. All results are calculated by hand.

When comparing the columns, differences are seen only in the imaginary parts of the eigenvalues and therefore we proceed in this subsection with varying the perturbation size $h$ and observing only the imaginary parts of the obtained eigenvalues.

Figure 2.3 shows the absolute value of the imaginary parts of the eigenvalues obtained from the three linearization methods.

The black curve indicated as 1) in figure 2.3 shows the absolute value of the imaginary part of the eigenvalues obtained from the AL method as described in section 2.5.1. Linearization with the AL method is done without any perturbation $h$ and therefore the result is independent of the perturbation size $h$.

The blue curve indicated as 2) in figure 2.3 shows the absolute value of the imaginary part of the eigenvalues obtained from the FDA method as described in section 2.5.2 for perturbation sizes $h$ varied in the range between 0.0001 and 0.1000 in 1000 steps.

The red curve indicated as 3) shows results obtained from PSS/E wherein also the FDA method is used. $h$ in curve 3) is varied from 0.0001 to 0.1000 in 32 steps. Since curves 2) and 3) are very close it indicates that PSS/E’s linearization method is as shown in section 2.5.2. The differences can be a result from propagation of roundoff errors in PSS/E, both when calculating the elements of the $A$-matrix as well as when applying the QR-method to
Figure 2.3: Imaginary part of the eigenvalue when using the AL method (in black) by hand and with Simpow, FDA method (in blue by hand and in red with PSS/E), and CDA method (in green).

calculate the eigenvalues.

Figure 2.3 shows that the smaller $h$, the closer the result from the FDA method (curves 2) and 3)) is to the AL method (curve 1)).

The green curve indicated as 4) in figure 2.3 shows the absolute value of the imaginary part of the eigenvalues from the CDA method described in section 2.5.3 for perturbation sizes $h$ varied in the range between 0.0001 and 0.1000 in 1000 steps. The resulting imaginary part is very close to the AL method in figure 2.3. The results from the CDA method is almost equal to the result from the AL method. This is due to the symmetry created by the CDA method when centering the perturbation in the equilibrium point, see below concerning truncation error.

In Smed [79] it has been shown that the perturbation size $h$ has to be set according to whether the variables are defined with single or double precision. As a rule of thumb the author advice that $h$ should not be smaller than $\sqrt{tol}$ where $tol$ is the used precision with which the variables are calculated when utilizing the FDA method.

It is not shown in this chapter and has to be further studied if a small
perturbation size $h$ can be utilized with the FDA and CDA methods when studying large power systems. For small perturbations the accuracy of which the equations are calculated is predicted to play a major role and therefore the size of the perturbations are supposed impossible to set to a too small value.

### 2.6.1 Truncation errors

In figure 2.3 it is clearly shown that there exist differences in the results obtained with the three methods and that the size of the perturbation $h$ influences the calculated eigenvalues. If all equations of the system had been linear, curves 2), 3), and 4) in figure 2.3 had perfectly followed curve 1) also for large values of $h$, but since non-linear block diagrams are included when the four matrix-elements in equation (2.69) are identified, the result is depending on the perturbation size $h$. As can be seen by comparing the $A$-matrices in equations (2.64) and (2.75) it is in fact only element $A_{12}$ that is depending on the perturbation size $h$.

Figure 2.4 shows how matrix-element $A_{12}$ from equations (2.73) and (2.81), varies with the perturbation size $h$ for the three linearization methods.

As can be seen in figure 2.4, matrix element $A_{12}$'s variation with $h$ is also a consequence of whether the FDA method or CDA method is utilized: The numerical expression (2.73) includes a truncation error since higher orders of the Taylor expansion is neglected in the FDA and CDA methods, see Gill [18], pp. 54-55. The truncation errors $\Delta A_{12_{FDA}}$ and $\Delta A_{12_{CDA}}$ can be seen in figure 2.4. In appendix B expressions of the truncation errors for the FDA and CDA methods are derived.

#### Truncation error obtained from the FDA method

The truncation error obtained with the FDA method $\Delta A_{12_{FDA}}$ is,

$$\Delta A_{12_{FDA}} = \frac{E_{fd0} \cdot u_{\text{reinfinitebus}}}{4H \cdot (x_d' + x_{\text{line}})} (h \cdot \sin \delta_0 + \frac{h^2}{3} \cos \delta_0) + O(h^3). \quad (2.85)$$

Equation (2.85) is derived in appendix B.1. In equation (2.85) we have neglected third and higher orders of $h$.

In general when using the FDA method, truncation errors $\Delta$ are of first order for small values of $h$ as,
Figure 2.4: Matrix element $A_{12}$ derived when using the FDA method (in blue) and the CDA method (in green). The truncation error when using the FDA method $\Delta A_{12FDA}$ is marked as the distance between the black and the blue curve. The truncation error when using the CDA method $\Delta A_{12CDA}$ is marked as the distance between the black and the green curve. The red curve shows $A_{12}$ when having a linear relation with $h$.

\[ \Delta = O(h). \]  

(2.86)

In figure 2.4 we can see that the truncation error $\Delta A_{12FDA}$ indeed has almost a linear relation with $h$ as shown in equation (2.85). If the truncation error would consist only of the first term in equation (2.85), as,

\[ \Delta A_{12FDA-mod} = \frac{E_{fd0} \cdot u_{reinfinitebus}}{4H \cdot (x_d + x_{line})} h \cdot \sin \delta_0 \]  

(2.87)

we would obtain the red curve in figure 2.4 describing how $A_{12}$ would develop with $h$. In figure 2.4 it is possible to see that the distance between the blue
and the red curve is equal to \( \Delta A_{12CDA} \). This can also be observed by comparing equations (2.85), (2.87), and equation (2.88) below.

The matrix-elements other than \( A_{12} \) are independent of the size of \( h \) since the equations are linear.

**Truncation error obtained from the CDA method**

The truncation error obtained with the CDA method \( \Delta A_{12CDA} \) is,

\[
\Delta A_{12CDA} = \frac{E_{fd} \cdot u_{reinfinitebus}}{4H \cdot (x_d' + x_{line})} \cos \delta_0 \cdot \frac{h^2}{3} - \frac{h^4}{60} + O(h^6).
\]  

Equation (2.88) is derived in appendix B.2. In equation (2.88) we have neglected sixth and higher orders of \( h \).

In general when using the CDA method, truncation errors \( \Delta \) are of second order for small values of \( h \) as,

\[
\Delta = O(h^2).
\]  

In figure 2.4 we can see that the truncation error \( \Delta A_{12CDA} \) indeed is very small as shown in equation (2.88).

In Taylor [81, 82] a method is shown to decrease the truncation error when using the CDA method. The method sets the size of the perturbation to minimize the truncation error as much as possible.

### 2.7 Large-disturbance stability analysis

In small-signal stability analysis, conclusions are drawn from small disturbances \( \Delta \). In this section we will apply large disturbances for the same power system and see if we obtain similar results as in sections 2.5 and 2.6.

Curves 2), 3), and 4) in figure 2.3 showed the result when perturbations were large. The FDA and CDA methods include non-linear feedback from the algebraic equations and we have seen in figure 2.3 that the imaginary part of the eigenvalue-pair is as low as 6.2 (rad/s) for large perturbations \( h \) when using the FDA method.

The results from the linearization has in this section been compared with time-domain simulations of the power system where a solid three-phase fault has been applied on the machine bus in figure 2.2 with different fault durations \( t_c \). In figure 2.5 the rotor angle \( \delta \) and speed deviation \( \Delta \omega \) for
Figure 2.5: Rotor angle $\delta$ and speed deviation $\Delta \omega$ for different fault durations $t_c$.

different values of $t_c$ are shown. The simulations have been made with Simpow.

In figure 2.5 we can see that the longest fault duration $t_c$ possible is 0.10 (s). For longer fault durations the generator looses synchronism and falls out of step.

By measuring the periods of the remaining oscillations in $\Delta \omega$ in figure 2.5 it is possible to see that the remaining oscillations are slower the longer the fault duration $t_c$ is. For the largest fault durations we can also observe that the response is not purely sinus-formed. This is a consequence of the non-linear feedback from system of equations.

Often the system Single Machine Infinite Bus (SMIB) is compared with the motion for a mass connected to a wall with a spring as in figure 2.6, see Andersson [2], p. 125. The spring constant $K$ in such mechanical system corresponds to matrix element $A_{12}$ in equation (2.7) (after a multiplication
of a factor $-2H$, see equation (B.21) in appendix B). The spring constant $K$ in the analogously with the mechanical system varies and for the positive swing (when $\delta > 50^\circ$ in the uppermost diagram of figure 2.5) the spring constant is smaller the farther we are from the equilibrium point $\delta_0 = 50^\circ$ and since then, the curve of $\delta$ is smooth.

For the negative swing (when $\delta < 50^\circ$ in the uppermost diagram of figure 2.5) the spring constant is larger the farther we are from $\delta_0 = 50^\circ$ and since then, the curve of $\delta$ is sharp, Andersson [1].

![Figure 2.6: Mass and spring in analogy with a single machine infinite bus.](image)

Figure 2.6: Mass and spring in analogy with a single machine infinite bus.

![Figure 2.7: The remaining oscillation frequency after a solid three-phase fault with different time duration has been applied to the power system.](image)

Figure 2.7: The remaining oscillation frequency after a solid three-phase fault with different time duration has been applied to the power system.

For large perturbations $h$ in the system, the oscillation frequency is in-
deed decreasing as shown earlier in figures 2.3 and 2.5 and is as slow as 4.6 (rad/s) for very large perturbations, see Fault duration = 0.10 (s) in figure 2.7. In figure 2.7 the remaining oscillation frequency in $\Delta \omega$ has been plotted versus the fault duration. The fault duration has been varied between 0.0002 (s) and 0.10 (s) in steps of $\Delta t_c = 0.005$ between 0.005 < $t_c$ < 0.10 (s) as shown on the horizontal axis. The oscillation frequency of the recovering system has been calculated with Fast Fourier Transform (FFT) calculations. A time window of 500 (s) has been used in the calculations with a desired peak frequency of 50 (Hz) and a rectangular window. A time window of 500 (s) gives the frequency solution $1/500 = 0.002$ (Hz).

From figure 2.7 we can see that the result is similar as with the FDA method; the larger perturbations, the smaller imaginary parts of the eigenvalue-pair and the slower the remaining oscillation in the power system is when it is recovering after the fault has been removed. When the FDA and CDA methods are utilized for linearizations, non-linear relations in the algebraic equations are included when deriving the $A$-matrix. The non-linear equations are also included when performing time-domain simulations.

Earlier in figure 2.3 we could see that the perturbation $h = 0.1$ provided an imaginary part of the eigenvalue-pair of $\omega \approx 6.18$ (rad/s). In figure 2.7 a fault duration of $\approx 0.05$ (s) gives the same imaginary part, see the marked upper left corner.

We could further investigate how to interpret the perturbations we have added to one state variable when linearizing the system of equations and how that can be compared with a time-domain simulation. Therefore, in figure 2.8 the upper left corner of figure 2.7 has been enlarged. The figure shows the oscillation frequency in $\Delta \omega$ as a function of the magnitude of the disturbance in the rotor angle $\delta$ (on the x-axis). This would be reasonable to do since it was when we disturbed $\delta$ that we had a non-linear response of the system of equations when identifying matrix element $A_{12}$ in section 2.5.2.

In figure 2.8 we can see that the magnitude of the disturbance in the oscillating rotor angle $\delta$ is equal to $\approx 0.375$ (rad) for values of $\omega \approx 6.18$ (rad/s). Therefore we can see that a perturbation size of $h = 0.1$ in figure 2.3 for the FDA method gives the same oscillation frequency as a magnitude of the disturbance in the oscillating rotor angle $\delta \approx 0.375$ (rad).

One conclusion that would be attractive to draw here is that the FDA method gives an interval of possible values of the imaginary part of the eigenvalues that we will experience in the power system for small and large disturbances, depending on the perturbation size $h$. However, based on the reservations above we have not enough material to draw such a conclusion.
Figure 2.8: The remaining oscillation frequency in $\Delta \omega$ as a function of the magnitude of the disturbance in the oscillating rotor angle $\delta$.

For instance, if we had used the perturbation size $h = -0.01$ in section 2.5.2, the matrix-element $A_{12}$ had become $A_{12} = -0.108766$ (see section 2.5.3 where $A_{12}$ was calculated for the CDA method) and the eigenvalue-pair had then become,

$$\lambda_{1,2} = 0 \pm j6.40342(1/s, \text{rad/s})$$

which is an imaginary part with a faster oscillation.

Whether the same relation exists between the FDA method with large perturbation sizes $h$ and time-domain simulations with large disturbances in studies of other, (larger) power systems is difficult to investigate since the larger a power system is, the more eigenvalues exist and as a consequence; in the plots from a time-domain simulation it is more or less impossible to observe the influence from one single eigenvalue.

However, conclusions from this chapter is that when small-signal behaviors of systems are studied, the perturbation is assumed to be as small so that the system has a linear response in the current working point. Figure 2.3 shows that the FDA and CDA methods are sensitive to the degree of
non-linearity of the equations of the studied system as well as to the size of the perturbation $h$. The sensitivity is largest for the FDA method.

2.8 Summary

Three linearization methods have been evaluated in this chapter. The analytical linearization is achieved by writing the models in terms of the most elementary components for which a linearization formula is known. The forward-difference and center-difference approximation methods are numerical methods.

The chapter analyzes results obtained from three engineers working in parallel, studying the same power system but using three different linearization methods. The aim in the chapter has been to show that the used linearization method in a power system simulation software has an impact on the obtained eigenvalues. Attention should be paid to this when analyzing a power system with different software since differences can be generated from the used linearization methods.

It has been shown that the perturbation size $h$ in the forward-difference approximation method influences the calculated eigenvalues. However, for small values of the perturbation, results from all the methods are very similar. When analyzing the electromechanical mode of a small power system, the impact of the difference in linearization methods were investigated.

The results from the numerical center-difference approximation method are very close to the analytical linearization method.

It is not shown in this chapter and has to be further studied if a small perturbation size $h$ can be utilized when studying large power systems. For small perturbations the accuracy of which the equations are calculated is predicted to play an important role.

The larger perturbation size when using the forward-difference approximation method, the smaller imaginary parts of the eigenvalue-pair is obtained. This is in conformity with time-domain simulations of the power system in the chapter; the larger perturbations the power system is going through, the slower the responding oscillation is when the system is recovering after the fault has been removed. Whether the same conformity are obtained also in studies of other power systems could be further investigated for the forward-difference approximation method.

The forward-difference approximation and center-difference approximation methods are sensitive to the degree of non-linearity of the equations of the studied system as well as to the size of the perturbation $h$. The
sensitivity is largest for the forward-difference approximation method.
Chapter 3
Modelling the power system in the thesis

Some formulations and fundamentals of the modelling of the power system in the thesis has to be mentioned. In this chapter we explain some modelling of the power system as well as comment specific models that are simulated.

3.1 Simulations in instantaneous value mode

The electrical state of the three-phase power system is in the thesis modelled in instantaneous value mode. When using this mode, phase currents and phase voltages do not necessary have to be sinusoidal, they can have any arbitrary form. The detailed possibility of modelling is of certain importance for us since we want to model the thyristor switchings in the power grid in detail. Other names that can be found in the literature for characterizing this mode of simulation are momentary values, electromagnetic transients, and alternative transients program simulations (ATP-simulations), see PSCAD [66], and Persson [57].

3.1.1 The $dq0$-representation

To speed up used computer time, the used software in the thesis transforms three-phase voltages and currents, below indicated as $abc$-quantities, and represent them into a coordinate system called the $dq0$-system, [75]. Figure 3.1 shows phase voltage $u_a$ which we here refer to as one of the $abc$-quantities of a three-phase voltage. During normal conditions the $abc$-quantities are sinus formed with the power frequency, see the phase voltage for $1.033 <$
Figure 3.1: Phase voltage $u_a$ of a three-phase node.

$t \leq 1.50 \text{ (s)}$ in figure 3.1.

The $dq0$-system refers in the used software to a synchronously rotating machine which is called the reference machine when solving the electrical quantities during the simulation.

Equation (3.1) shows the transformation from $abc$- to $dq0$-quantities for a three-phase voltage.

$$
\begin{bmatrix}
  u_d \\
  u_q \\
  u_0
\end{bmatrix}
= \frac{2}{3}
\begin{bmatrix}
  \cos(\theta_{ref}) & \cos(\theta_{ref} - \frac{2\pi}{3}) & \cos(\theta_{ref} + \frac{2\pi}{3}) \\
  -\sin(\theta_{ref}) & -\sin(\theta_{ref} - \frac{2\pi}{3}) & -\sin(\theta_{ref} + \frac{2\pi}{3}) \\
  \frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\begin{bmatrix}
  u_a \\
  u_b \\
  u_c
\end{bmatrix}
$$

Equation (3.1) is referred to as Park’s transformation, see Park [49]. In equation (3.1) $u_a$, $u_b$, and $u_c$ are the momentary values of the three-phase voltages and $u_d$, $u_q$, and $u_0$ are the voltages in $d$-axis, $q$-axis, and zero sequence respectively.

The coordinate system in the $dq0$-representation rotates with the rotor of the reference machine $\theta_{ref}$ anywhere in the synchronously rotating power system. Such reference machine must exist in each AC part of the simulated power system. It is recommended to select a large generator or an infinite bus in the synchronously rotating power system to represent such a reference
machine, see Aneros [4], p. 18. The abbreviation AC stands for Alternating Current.

The angle $\theta_{\text{ref}}$ is defined as,

$$\theta_{\text{ref}} = 2\pi f_0 \cdot t + \theta_r$$  \hspace{1cm} (3.2)

where $f_0$ is the power frequency (Hz), $\theta_r$ is the load angle of the reference machine (radians), and $t$ is actual simulated time (s). $\theta_{\text{ref}}$ is given in radians.

The load angle $\theta_r$ has the following relation with the per unit speed deviation $\Delta \omega$ of the reference machine,

$$\dot{\theta}_r = \omega_0 \Delta \omega$$  \hspace{1cm} (3.3)

where $\omega_0$ is nominal rotor speed ($\omega_0 = 2\pi f_0$) in radians/s. The per unit speed deviation $\Delta \omega$ is in (p.u.).

From equation (3.3) we can see that the time-derivative of $\theta_r$ is related to the speed deviation $\Delta \omega$ of the reference machine and therefore also the actual frequency of the power system; If the frequency decreases in the power system, $\Delta \omega$ will become negative which gives a negative derivative to $\theta_r$, according to equation (3.3), so that $\theta_r$ decreases.

From equation (3.2) we can then see that a decreasing $\theta_r$ ”slows down” the development of $\theta_{\text{ref}}$ and as a consequence, the $dq\theta$-system rotates slower since it rotates with $\theta_{\text{ref}}$ as shown in equation (3.1). Therefore, the $dq\theta$-system follows the actual frequency of the power system.

In the upper part of figure 3.2 it is shown how $\Delta \omega$ decreases for the reference machine and becomes negative when the frequency in a power system decreases at $t = 5$ (s) according to a sudden increase in the real power consumption in the power system. This gives a negative derivative to $\theta_r$ according to equation (3.3) so that $\theta_r$ decreases as well. This is shown in the lower part of figure 3.2. The power system contains two areas containing one synchronous machine each, feeding a load placed between the two areas.

Since the reference machine in this case is equipped with a secondary frequency control, $\Delta \omega$ returns soon to $\approx 0$ which gives almost zero time-derivative to $\theta_r$ according to equation (3.3) and therefore $\theta_r$ remains at the end ($t \approx 50$ (s)) almost to a constant value $\approx -8$ (radians).

As a consequence of that $\theta_r$ has decreased as shown in figure 3.2, $\theta_{\text{ref}}$ in equation (3.2) has ”slowed down” and the $dq\theta$-system is at $t = 50$ (s) again rotating with power frequency but nowadays $\theta_r$ (radians) behind, compared to the position it would have had if the power consumption of the power system would had remained undisturbed.
Figure 3.2: Speed deviation $\Delta \omega$ and $\theta_r$ of a reference machine.

The used software, [75] has changeable settings of the maximum allowed speed deviation $\Delta \omega$ for the reference machine and the default value is $|\Delta \omega|_{\text{max}} = 0.05$ (p.u.). If $|\Delta \omega| > |\Delta \omega|_{\text{max}}$, the simulation stops. It is practical to have such a limitation in a simulation tool.

The inverse transformation from $dq0$- to $abc$-quantities is done for a three-phase voltage with,

$$
\begin{pmatrix}
    u_a \\
    u_b \\
    u_c
\end{pmatrix}
= \begin{bmatrix}
    \cos \theta_{\text{ref}} & -\sin \theta_{\text{ref}} & 1 \\
    \cos(\theta_{\text{ref}} - \frac{2\pi}{3}) & -\sin(\theta_{\text{ref}} - \frac{2\pi}{3}) & 1 \\
    \cos(\theta_{\text{ref}} + \frac{2\pi}{3}) & -\sin(\theta_{\text{ref}} + \frac{2\pi}{3}) & 1
\end{bmatrix}
\begin{pmatrix}
    u_d \\
    u_q \\
    u_0
\end{pmatrix}
$$

(3.4)

Figure 3.1 shows phase voltage $u_a$ of a three-phase node in a small power system of radial character and figure 3.3 shows the $dq0$-components of the same three-phase node. Between $1.00 \leq t \leq 1.033$ (s), a solid three-phase fault is applied to the node.

In figure 3.1 we can see that before and after the fault, phase voltage $u_a$ is more or less sinusoidal with the power frequency. During such circumstances we observe the advantage with representing the power system with the $dq0$-representation, see figure 3.3, compared to the $abc$-representation. In figure 3.3 we can see that the $dq0$-components are varying very slow compared...
to the phase voltage $u_a$ in figure 3.1 during the time when the fault is not applied, i.e., during the time intervals $0.95 \leq t < 1.00 \text{ (s)}$ and $1.033 < t \leq 1.50 \text{ (s)}$.

### 3.2 Per unit system

Some words has to be mentioned about the per unit system which is used in the simulations of the thesis.

Generally in simulations of power systems, it is convenient to use a per unit system to normalize system variables. Compared to the use of physical units such as amperes, volts, ohms, etc., the per unit system offers computational simplicity by eliminating units and expressing system quantities as dimensionless ratios, see Kundur [35], p. 75. Thus,

$$x_{\text{p.u.}} = \frac{x_{\text{phys}}}{x_{\text{base-value}}}.$$  \hspace{1cm} (3.5)

In equation (3.5) the quantity $x$ is given in physical units in $x_{\text{phys}}$ and dimensionless in per unit in $x_{\text{p.u.}}$ after it has been scaled with a base value $x_{\text{base-value}}$.

When simulating electric power systems a base power $S_{\text{base}}$ is selected for the whole power system to be used at all voltage levels. $S_{\text{base}}$ is a three-phase
apparent power.

A normal value of $S_{\text{base}}$ when including voltage levels of transmission systems is $S_{\text{base}} = 100$ (MVA). All calculations of power ($P$, $Q$, and $S$) in per unit (p.u.) at all voltage levels are then referred to (scaled with) the base power $S_{\text{base}}$.

For all three-phase nodes a phase-to-phase rms base voltage $U_{\text{base}}$ are set. This value is depending on which voltage level the node is located at. On transmission level typical values of $U_{\text{base}}$ are 500 and 100 (kV). $U_{\text{base}}$ for a node should be set to a value close to the node’s nominal voltage level. The abbreviation rms stands for root mean square.

In the following we will introduce the per unit system for the instantaneous value mode.

The per unit system for instantaneous value mode is different compared to the one used in fundamental frequency mode which can be found in textbooks such as Elgerd [14], p. 47.

### 3.2.1 Per unit system used in instantaneous value mode

When simulating in instantaneous value mode, instantaneous values of voltages and currents are calculated. It is therefore convenient to select their peak values as base values and since then the following bases are used for phase voltages, (phase) currents, and impedances when they are calculated in p.u.,

$$U_{\text{base-phase-inst}} = U_{\text{base-phase-rms}} \cdot \sqrt{2} = \frac{U_{\text{base}}}{\sqrt{3}} \cdot \sqrt{2} \quad (3.6)$$

$$I_{\text{base-inst}} = I_{\text{base-phase-rms}} \cdot \sqrt{2} = \frac{S_{\text{base}}}{U_{\text{base}} \cdot \sqrt{3}} \cdot \sqrt{2} \quad (3.7)$$

$$Z_{\text{base-inst}} = \frac{U_{\text{base-phase-inst}}}{I_{\text{base-inst}}} = \frac{U_{\text{base}}^2}{S_{\text{base}}} = Z_{\text{base-rms}} \quad (3.8)$$

We can see that the base values of voltages and currents are $\sqrt{2}$ larger in instantaneous value mode compared to base values in fundamental frequency mode ($U_{\text{base-phase-rms}} = \frac{U_{\text{base}}}{\sqrt{3}}$ and $I_{\text{base-rms}} = \frac{S_{\text{base}}}{U_{\text{base}} \cdot \sqrt{3}}$). However, the base value for impedances $Z_{\text{base-inst}}$ is the same in instantaneous value mode as in fundamental frequency mode.

The base values $U_{\text{base-phase-inst}}$ and $I_{\text{base-inst}}$ are used also when calculating $dq0$-components of voltages and currents in p.u. in instantaneous value mode.
3.3 The original model of the Thyristor-Controlled Series Capacitor

The basic scheme of the Thyristor-Controlled Series Capacitor (TCSC) is shown in figure 3.4. It consists of a series compensating capacitor $C$ shunted by a Thyristor-Controlled Reactor (TCR) $L$.

Assume that the thyristor valve is initially open and that the line current $i_{\text{line}}$ produces a voltage $u_{\text{cap}}$ across the fixed series compensating capacitor $C$ as illustrated in figure 3.5 between $5.01 < t < 5 (\text{s})$. Suppose that the thyristor-controlled reactor $L$ is to be turned on at point of time $E$. As seen, at the instant of turn-on $E$, the capacitor voltage $u_{\text{cap}}$ is negative, the line current $i_{\text{line}}$ is positive and thus charging the capacitor in the positive direction, see Hingorani [20]. Because of the negative voltage $u_{\text{cap}}$ over the capacitor $C$ the current $i_{\text{rea}}$ becomes negative when the thyristor-controlled reactor is turned on. Here the reverse thyristor is conducting, see the upper thyristor in figure 3.4.

The reverse thyristor stops conducting when the current $i_{\text{rea}}$ crosses zero as shown in point of time $F$ in figure 3.5.

When none of the thyristors are conducting, the currents $i_{\text{line}}$ (in red) and $i_{\text{cap}}$ (in blue) are equal, see for instance time interval $F \leq t \leq G$ in figure 3.5.

One half period later the procedure described above repeats but now...
Figure 3.5: Line current $i_{\text{line}}$ (in red), capacitor current $i_{\text{cap}}$ (in blue), and reactor current $i_{\text{rea}}$ (in green) in the upper part. Capacitor voltage $u_{\text{cap}}$ in the lower part. The power frequency is 60 (Hz).

The voltage drop $u_{\text{cap}}$ over the capacitor is positive at the instant of turn-on $G$, the line current $i_{\text{line}}$ is negative and thus charging the capacitor in the negative direction, Hingorani [20]. Because of the positive voltage $u_{\text{cap}}$ over the capacitor $C$, the current $i_{\text{rea}}$ becomes positive when the thyristor-controlled reactor is turned on at point of time $G$. Here the forward thyristor is conducting, see the lower thyristor in figure 3.4.

The forward thyristor stops conducting when the current $i_{\text{rea}}$ crosses zero as shown in point of time $H$ in figure 3.5.

By regulating the conducting time for the thyristors, the total fundamental capacitive reactance between node A and node B in figure 3.4 can be varied. From a fundamental frequency point of view the inserted capacitive reactance of the TCSC is therefore controllable as $C_{\text{TCSC}}$ in figure 3.6.

When successfully varying the inserted capacitance $C_{\text{TCSC}}$ in figure 3.6, power oscillation damping can be carried out in a transmission system. Power oscillations have oscillation frequencies around $0.2 < f < 2$ (Hz). However, to damp these, the TCSC model in the thesis must be completed with a reactance unit that provides a varying reactance reference value, see parameter $Ref$ in table C.1 in appendix C and Persson [51].

In this thesis the reactance reference $Ref$ (the desired reactance) is set
to a constant value and the control algorithm controls the TCSC so that it provides this reactance.

With $Ref = 2$, the fundamental reactance of the TCSC is set to twice the fundamental reactance of the series capacitor which is equal to 1 (p.u.).

With a TCSC it is also possible to create a fundamental inductive reactance, see Hingorani [20] and Ångquist [88], but this has not been implemented in the TCSC model of this work. The normal operating mode of a TCSC is known as "capacitive boosting mode" which is modelled in this thesis, see Ångquist [88].

Figure 3.7 shows a characteristic picture on how the TCSC works. As can be seen, the series capacitor voltage $u_{cap}$ is "deformed" and is not completely sinusoidal with power frequency as a result of that the thyristors are conducting around the points of time $A$ and $B$. The series capacitor voltage would have been sinusoidal if the thyristors would have been blocked.

In figure 3.7 it can also be seen that if the series capacitor voltage $u_{cap}$ is positive at the turn-on instant, then the current through the reactor $i_{rea}$ is positive (as it is defined in figure 3.4) when the forward thyristor starts to conduct. This is the situation in point of time $G$ in figure 3.7.

During the time interval $E < t < F$ in figure 3.7, the reverse thyristor is conducting. The reverse thyristor is the upper thyristor in figure 3.4. During the time interval $G < t < H$, the forward thyristor is conducting. The forward thyristor is the lower thyristor in figure 3.4.

The implementation of the TCSC control follows the Synchronous Voltage Reversal-control algorithm developed by ABB, see Ångquist [88, 89].

The instantaneous value mode in Simpow [75] enables modelling of a TCSC in detail, i.e., that it includes the switches of the thyristors as in figure 3.7. The instantaneous value mode is similar to the simulation modes that exist in power system simulation software such as PSCAD/EMTDC, DlgsILENT, and NETOMAC.
Figure 3.7: Current through the reactor $i_{rea}$ and series capacitor voltage $u_{cap}$ in steady state. The figure describes the situation for one phase of the TCSC. The power frequency is 60 (Hz).

The TCSC can also be modelled as a continuously variable capacitance as in figure 3.6 in the fundamental frequency mode of a software. In the scope of this thesis, we are interested in investigating the behaviour of the TCSC based on a detailed level of modelling and from that, to build simplified models.

### 3.3.1 The structure of the TCSC control

The series capacitor $C$ and the thyristor-controlled reactor $L$ in figure 3.4 are modelled using standard components in the used power system simulation software [75]. The rest of the three-phase TCSC (the control algorithm and the thyristors) are split up in three equal parts, one for each phase. Such a part has been implemented as a user-defined system in the software and each part has four underlying functions which represent the control algorithm and the thyristors. These are,

- Phase Locked Loop (PLL);
- Booster (BOO);
- Thyristor Pulse Generator (TPG); and,
- Thyristors (THY)

which are all shown in figure 3.8 except for the function THY. The four functions are documented in appendix C and in Persson [51].

![Control system diagram](image)

Figure 3.8: Control system of one phase of the TCSC. The control system uses a phase current of $i_{line}$ and a phase voltage of $u_{cap}$ to calculate the point of time when the thyristors should start to conduct. The control systems for every phase are exactly alike.

The control algorithm is formulated by using the in-built simulation language Dynamic Simulation Language (DSL) in Simpow. Basically, the aim of the control algorithm is to calculate in what point of time the thyristors should start to conduct. The thyristors are automatically blocked at the following zero-crossing of the phase current through the series reactor. The thyristors are included in the user-defined system in the function Thyristors (THY) which is not shown in figure 3.8.

### 3.3.2 Assumptions of the TCSC

In figure 3.10 we can see how the $dq0$-components of the line current $i_{line}$ are initialized and how they reach steady-state values during the initialization of the TCSC which is activated at $t = 1$ (s). This steady state will later be used as a base case in the simulations in chapters 6 and 7. A one-line
diagram of the actual power system can be found in figure 3.9. A description of the power system can be found in section 6.3.

In figure 3.11 we can see how the \( dq0 \)-components of the voltage \( u_{cap} \) vary during the initialization of the TCSC. From figures 3.10 and 3.11 we can see that the TCSC is initialized between \( 1 < t < 3 \) (s). Basically it is the TCSC control that is reaching its initialization values. How that is done is described in appendix C.7.

When modelling the original model of the TCSC we can observe that the \( 0 \)-component of the incoming current to the TCSC, \( i_{line} \) in figure 3.4 is almost zero, see \( i_0 \) in figure 3.10. It contains however a small magnitude of the third harmonic 180 (Hz) which in steady state is \( 10.7 \cdot 10^{-6} \) (p.u.) (22.6 (mA)). Later in this section we will mention that the third harmonic of the line current is created from the thyristor switchings and the reactor current \( i_{rea} \) in figure 3.7.

The third harmonic in the line current \( i_{line} \) comes from the current \( i_{rea} \). Since the zero-sequence impedance of the series capacitor, see \( C \) in figure 3.4, is much smaller than the zero-sequence impedance of the surrounding system in figure 3.9, a majority of the third harmonic rotates internally in the TCSC circuit.

During transients in the power system, the magnitude of the third harmonic in the \( 0 \)-component of \( i_{line} \) do vary but is still very small compared to
Figure 3.10: $i_d$, $i_q$, and $i_0$ of $i_{line}$ when the power system is reaching steady state during the initialization. The TCSC is turned on at $t = 1$ (s).

Figure 3.11: $u_d$, $u_q$, and $u_0$ of $u_{cap}$ when the power system is reaching steady state. $u_0$ is enlarged in figure 3.12.
the \( d \)- and \( q \)-components of \( i_{\text{line}} \). The third harmonic in the \( \theta \)-component of \( i_{\text{line}} \) comes from the third harmonic in the phase currents, see table 3.1 where it is shown how harmonics in the \( abc \)-components are transformed to \( dq\theta \)-components.

Since we want to develop bandwidth-reduced linear models, frequencies as high as 180 (Hz) is not important to include in our modelling. Also, since we are not interested in studying un-symmetries in the examples, we can of this reason omit the \( \theta \)-component of \( i_{\text{line}} \) since the \( \theta \)-component is only necessary to include if unbalanced conditions should be studied, see Kundur [35].

Therefore, in the scope of this thesis, it is not necessary to represent the \( \theta \)-component of the incoming current to the TCSC and we will exclude its representation in the linear models.

During the initialization, the \( d \)- and \( q \)-components of \( i_{\text{line}} \) reach an almost constant level of \( i_d = 0.094 \) (p.u.) (200 (A)) and \( i_q = 0.169 \) (p.u.) (358 (A)) respectively, see figure 3.10.

Locally in the TCSC, a rotating third harmonic 180 (Hz) in the \( \theta \)-component of the currents \( i_{\text{cap}} \) and \( i_{\text{rea}} \) exists. In figure 3.7 \( i_{\text{rea}} \) has been plotted and from there a trained eye can see that it contains the fundamental frequency (60 (Hz)) as well as the third, fifth, seventh, and eleventh... harmonics. An FFT calculation results in these frequencies.
In steady state the frequency component for the third harmonic creates the same magnitude and phase within all three phases and since the $\theta$-component is defined as with equation (3.1); the $\theta$-component is equal to the sum of the three phases multiplied with a factor of $\frac{1}{3}$.

The third harmonic in the local TCSC current causes a third harmonic in the $\theta$-component of the voltage drop $u_{cap}$ over $C$, see figure 3.12. It can be shown that if the magnitude of a third harmonic is $M_3$ with the same phase in all three phase voltages (as in our steady-state case), then we will get the same magnitude $M_3$ of the third harmonic in the $\theta$-component of $u_{cap}$. This is shown in table 3.1 and in figure 3.13. In the upper part of figure 3.13 we can see that we have 0.0036 (p.u.) of the third harmonic 180 (Hz) in all three phases and in the lower part of the figure we have the same magnitude of the third harmonic 180 (Hz) in the $\theta$-component. The $\theta$-component is the red graph in the back in the lower part of figure 3.13. Observe that the scales are not equal in the upper and lower parts of the figure.

However, such a high frequency is not interesting when describing the low-frequency behavior of the TCSC and therefore also the $\theta$-component of the voltage drop $u_{cap}$ is omitted in the linear model of the TCSC.

Table 3.1 shows how existing harmonics in the three-phase voltages $abc$ appears in the corresponding $dq\theta$-components. Observe that both 300 (Hz) and 420 (Hz) in the phase voltages $abc$ appear as 360 (Hz) in the $d$- and $q$-components.

In the table it has been assumed that an harmonic appear with the same magnitude in all three phases.

In figure 3.13, $|M_3|$ is 13.5% of $|M_1|$, $|M_5|$ is 5.1% of $|M_1|$, $|M_7|$ is 1.5% of $|M_1|$, $|M_{11}|$ is 0.4% of $|M_1|$, and $|M_{13}|$ is 0.3% of $|M_1|$.

In table 3.1 the fundamental frequency (60 Hz) is symmetric in the three phases, i.e., with forward phase order and 120° phase difference and with the same magnitude in all phases.

The third harmonic (180 Hz) has no phase difference, i.e., 0° phase difference. The fifth harmonic (300 Hz) and eleventh harmonic (660 Hz) appear with reverse phase order and 120° phase difference. The seventh (420 Hz) and thirteenth harmonic (780 Hz) appear with forward phase order and 120° phase difference.

The constant $k$ in table 3.1 is a scalar with a value between zero and one. The results in table 3.1 have been derived by studying equation (3.1). The results are also summarized in figure 3.13.

Since both the fifth and seventh harmonics in the $abc$-components are extracted to the same frequency (300 (Hz)) in the $dq$-components, they can
Figure 3.13: Frequency spectra of the phase voltages $abc$ and the $dq0$-components of $u_{cap}$ in steady state.
Table 3.1: Harmonics in the three-phase voltages of \( u_{cap} \) (\( abc \)-components) of the TCSC and how they appear in the \( dq\theta \)-components.

<table>
<thead>
<tr>
<th>Frequency and magnitude</th>
<th>Phase order</th>
<th>Frequency and magnitude in ( d )-component</th>
<th>Frequency and magnitude in ( q )-component</th>
<th>Frequency and magnitude in ( \theta )-component</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 Hz (</td>
<td>M_1</td>
<td>)</td>
<td>+120°</td>
<td>0 Hz ( k \cdot</td>
</tr>
<tr>
<td>180 Hz (</td>
<td>M_3</td>
<td>)</td>
<td>0°</td>
<td>–</td>
</tr>
<tr>
<td>300 Hz (</td>
<td>M_5</td>
<td>)</td>
<td>−120°</td>
<td>360 Hz (</td>
</tr>
<tr>
<td>420 Hz (</td>
<td>M_7</td>
<td>)</td>
<td>+120°</td>
<td>360 Hz (</td>
</tr>
<tr>
<td>660 Hz (</td>
<td>M_{11}</td>
<td>)</td>
<td>−120°</td>
<td>720 Hz (</td>
</tr>
<tr>
<td>780 Hz (</td>
<td>M_{13}</td>
<td>)</td>
<td>+120°</td>
<td>720 Hz (</td>
</tr>
</tbody>
</table>

strike out the other. In figure 3.13 we can see that amount of 300 (Hz) in the \( d \)-component is equal to \(|M_5| + |M_7|\) at the same time as the amount of 300 (Hz) in the \( q \)-component is equal to \(|M_5| − |M_7|\). In the upper part of figure 3.13 we can see that the magnitudes \(1.36 \cdot 10^{-3}\) (p.u.) and \(0.40 \cdot 10^{-3}\) (p.u.) respectively of the fifth harmonic 300 (Hz) and the seventh harmonic 420 (Hz) in the phase voltages have been extracted to 360 (Hz) in the \( d \)- and \( q \)-components with the magnitudes \(1.76 \cdot 10^{-3}\) (p.u.) and \(0.96 \cdot 10^{-3}\) (p.u.) respectively. Here we can see that the magnitudes of the harmonics of the \( abc \)-components are added to the \( d \)-component while they are subtracted for the \( q \)-component.

The situation is the same for the eleventh and thirteenth harmonics in the \( abc \)-components when they are transformed to \( dq \)-components.

We can also see a twelfth harmonic 720 (Hz) in the \( d \)-component in the lower part of the figure. It has been extracted from the eleventh harmonic 660 (Hz) and thirteenth harmonic 780 (Hz) in the phase components. From the upper part of figure 3.13 it is not possible to detect these frequencies since the scale of the upper figure is too large.

The lower part of figure 3.13 is scaled so that the frequency component of 0 (Hz) of the \( q \)-component does not appear correct. The correct value is
0.0267 (p.u.) which is higher than the maximum value of the y-axis in the figure.

3.4 Use of filter to remove harmonics

In section 3.3.2 it has been shown from where the harmonics in the $dq0$-components are derived. We could see that in the $d$- and $q$-components we have 360 and 720 (Hz) during normal conditions. The amount of 360 (Hz) in the $d$- and $q$-components appeared from the fifth 300 (Hz) and seventh harmonic 420 (Hz) in the three phase voltages $abc$.

In this work the harmonics are not of interest since we want to create models of the TCSC which represent its slow behavior. Later we will therefore low-pass filter the $d$- and $q$-components to enable focusing on the slow behavior of the component. The used low-pass filter has the characteristic as shown in figure 3.14.

![Bode Diagram](image)

Figure 3.14: Frequency characteristic for the used low-pass filter.

The used low-pass filter is a first-order filter,

$$H(s) = \frac{1}{1 + sT}.$$  \hspace{1cm} (3.9)

For our study object we have filtered the voltages $u_d$ and $u_q$ with the
filter constant $T = 0.01$ (s). The time constant $T = 0.01$ (s) gives a 3 (dB) bandwidth at 15.9 (Hz).

3.5 The accuracy of the model of the Thyristor-Controlled Series Capacitor

The TCSC model in the thesis has been developed with the support from ABB, Ångquist [87] and it follows the Synchronous Voltage Reversal-control algorithm developed by ABB, see Ångquist [88, 89]. Through the development of the model the aim has been to derive a TCSC model that works and is representative for a TCSC. The aim has not been to model an exact existing physical installation of a TCSC although the here developed model is influenced from existing TCSC installations. The aim of the developed TCSC model is to represent a study object in the thesis to which we apply different ideas of simplifying the modelling when developing linear models.

Whether the model is representing an existing real TCSC model is not investigated in the thesis. However, the functionality of the TCSC model covers the fundamentals of a TCSC.

3.6 Summary

In this chapter some formulations and fundamentals of the modelling of the power system in the thesis has been mentioned. In the chapter it has been explained both the modelling of the power system as well as comments about specific models that are simulated in the thesis. Also the modelling of the original TCSC model have been outlined.
Chapter 4

Developing linear models

In this chapter it is generally described how to develop simplified linear models of a system. It contains discussions of different methods when linearizing a power system containing the non-linear power system component TCSC. It also includes the processes of how the linear models for our study object, the TCSC, are created.

4.1 Introduction

When developing a model of an existing system it can be done with knowledge about the physical system and to investigate the physical system to build a model.

One problem that might occur is how to set numerical values of the parameters in the model? Hopefully some of the parameters are documented in data sheets of the physical system. This process can be complicated if it is necessary to derive the dynamics for the real system that might have a complex structure.

To overcome problems as mentioned above it can be easier to use measurements of the system. By sending a well-defined input signal into the system and record the output signal from it, it is possible to analyze the system. In that analysis it is possible to identify the dynamical response of the system, for instance its transfer function(s). That process is called system identification, Schmidtbauer [74].

In other words, instead of trying to get a known output signal by trying to build a model from the physical relations of the system we in system identification try to do it in the opposite way; to use the input and output signals and with that knowledge create a system, without including the
physical relations of the system (to treat the system as a \textit{black box}).

In this work we have made the system identification both in the time domain as well as in the frequency domain. We have used the following two techniques,

- Transient Analysis, i.e., in a time-domain simulation disturb input by input from a steady-state situation; and

- Frequency Analysis, i.e., in a time-domain simulation from a steady-state situation disturb input by input with sinus-formed signals of varying frequencies.

One difference between the two techniques is that the measurements with the frequency analysis demands longer simulation time since the system should be excited with a number of input signals of different frequencies.

A general list of how to structure an identification work is listed below. Both general comments are given as well as specific comments for our study object.

\textbf{i) A component and its state}

There are different reasons for identifying a simplified model for a component. One reason can be its high complexity or that it has to be simplified to fit in the actual modelling possibilities for a certain software. The component might be operated in different states, with different behavior that can be hard to represent in the software. When identifying the model it is necessary to decide which state the model should be valid for.

In our case the study object is a TCSC with a certain control algorithm. The state of the TCSC in the thesis is restricted only to its capacitive region.

This point has a connection to point \textit{ii} in the list.

\textbf{ii) Do pre-studies}

Is it meaningful to build a linear model? It is recommended to do some pre-studies to control if the component has the possibility to be represented by a linear model. These investigations should represent normal conditions that we later want to simulate with the linear model.

In sections 4.4 – 4.6 such pre-studies of our study object are found.
iii) Preparation of signals – Select bandwidth

Before starting the identification process of a model it might be necessary to prepare the signals. It can be for instance to analyze the noise that can be included in the input signals. The aim of the identified model influence this point. It has to be clear what the aim of the studies are. Is it a model that should represent a certain bandwidth?

For our study object the focus have been to represent frequencies up to some Hertz.

The obtained models consist of two parts, one part which represent the steady-state part of the component and another part which represent the dynamic behavior of the component.

iv) Choose input and output signals

To select input and output signals for a component demands knowledge about the studied component.

After studies documented in section 3.3.2 we have decided to represent the linear models of our study object with two input signals (currents $i_d$ and $i_q$) and two output signals (voltages $u_d$ and $u_q$).

v) Choose environment

The environment should be easy to analyze. A simple understandable environment makes the response in the studied component easy and clear to understand. To select environment for a component demands knowledge about it and its response for certain scenarios.

In this work we have used two different environments. One for identifying the linear models and another to investigate their validity.

vi) Identification methods

In sections 4.7 and 4.8 the two identification processes used in this thesis can be found.

vii) Validation

The obtained linear models have to be validated. For instance, to examine their properties towards the original models.

In Ljung [41] basic steps of system identification can be found including similar points as above.
4.2 Different methodologies

In this section we discuss earlier published methods when linearizing a power system containing a TCSC.

4.2.1 Poincaré map

In Jalali [23, 26], state space analysis of circuits containing Thyristor Controlled Reactors (TCRs) have been applied. The Poincaré map has been used to study the non-linear dynamics of different TCR circuits. It has been shown that a TCR circuit can damp small perturbations even when the resistances of the TCR circuit are ignored and the thyristors are assumed to be ideal.

In Jalali [23, 26] the installed Kayenta Substation controller of a TCSC in northeastern Arizona, USA (see Christl [7, 8]) has been studied. It is in Jalali [23, 26] stated that the dynamic response of a TCSC changes as a function of the thyristor conduction time $\sigma$. This dependence is also discussed in other papers describing the design of the Kayenta system, see for instance Christl [7, 8].

In Jalali [26] it is shown that the dynamics of the TCSC depends on the TCSC’s conduction time $\sigma$ during steady state in the operating point. By using the Jacobian of the Poincaré map it is in Jalali [26] possible to analyze the small-signal stability of the system. The eigenvalues of the system in Jalali [26] are shown in the unit circle.

The TCSC model in Jalali [26] does not represent the impact of the line current on the TCSC voltage and does not have the capability to represent the synchronization system or the higher-level control loops, see Othman [48].

The Poincaré map have also been used in Dobson [11] and Othman [48].

4.2.2 Identification from Bode plots

In Christl [8] the design of the Kayenta controller of a TCSC was achieved by first finding the open loop response of line current to changes in the firing point using a detailed EMTP model. The resulting envelope of the line current response enabled control engineers to find a transfer function which approximates the dynamics of the system around an operating point. The resulting fourth order transfer function has two poles on the real axis and a complex pair of poles. The dependence of the complex poles on the
thyristor conduction time $\sigma$ is shown in table 4.1. The operating points are specified by $\sigma$ (thyristor conduction time in electrical degrees), Jalali [26].

<table>
<thead>
<tr>
<th>$\sigma$ (thyristor conduction time)</th>
<th>$\sigma \pm j\omega \ [1/\text{s}, \ \text{rad/s}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$20^\circ$</td>
<td>$-14.5 \pm j56.2$</td>
</tr>
<tr>
<td>$40^\circ$</td>
<td>$-13.5 \pm j48.1$</td>
</tr>
<tr>
<td>$50^\circ$</td>
<td>$-10.6 \pm j30.2$</td>
</tr>
<tr>
<td>$60^\circ$</td>
<td>$-13.5 \pm j11.9$</td>
</tr>
</tbody>
</table>

Table 4.1: Complex poles for Kayenta System, see Jalali [26].

Also the real-valued double pole has a dependence on $\sigma$.

Unfortunately $\sigma$ is used both for the thyristor conduction time as well as for the real part of the eigenvalues in table 4.1.

### 4.2.3 Linearised discrete-time model

In Ghosh [17] a strictly line current synchronization of the TCSC has been published. The discretised model is synchronised to the zero-crossings of the line current once in each half-cycle. The method is presented as "Linearised discrete-time model of a thyristor-controlled series-compensated transmission line". The authors build up two linear models by analyzing the two different circuits that one phase circuit of the TCSC-compensated line can model; either that none of the thyristors are conducting or that one of them does. For each circuit a state equation is formulated.

The aim of the method is to predict the change in the line current when a thyristor is conducting.

In Ghosh [17] it is not described how the firing angles to the thyristors are regulated. Change in the line current or change in the real power through the TCSC can be modelled as the output signal from the model. The model is a fundamental frequency model and it predicts either a round mean square value of the line current or the real power through the TCSC one half-period later. Ghosh [17] provides a linear model of the system when it is in steady-state. The linear model varies as a function of operating point of the system.

### 4.2.4 Analytical Modelling of TCSC for SSR Studies

In Othman [48] a proposed method is described which remains of the method described in Ghosh [17] in section 4.2.3 but differ in the sense that the authors use the $dq0$-representation. The two possible circuits that the TCSC
can represent in one phase are described. The model is valid in the frequency range from 0 (Hz) to twice the operating frequency.

The inputs to the TCSC model are the capacitor voltage at the present instant $t_0$, the $dq0$-components of the line current, and the upcoming thyristor triggering instant $\phi_{1/2}$, see figure 4.1. The output of the TCSC model is the capacitor voltage one half cycle later from time $t_0$ at time $t_{1/2}$, see figure 4.1.

![Figure 4.1: One phase line current $i_{\text{line}}$ and an associated thyristor current $i_{\text{thyr}}$ (earlier introduced as $i_{\text{rea}}$).](image)

The obtained linearized model is dependent on the thyristor-triggering instant $\phi_{1/2}$ and the operating point.

The analytical linearized model of the single-phase TCSC seeks to predict the changes in the capacitor voltage at time $t_{1/2}$ from its equilibrium value. The value of the capacitor voltage at $t_{1/2}$ is a function of the change in the capacitor voltage at time $t_0$, the change in the thyristor triggering instant $\phi_{1/2}$, and the change in the line current during the half cycle between $t_0$ and $t_{1/2}$.

Later, the single-phase model of the TCSC is expanded straightforward to a three-phase model assuming that the line current is symmetric and that the thyristor triggering instants are equally spaced for the three phases with $\frac{\pi}{3}$ (rad), i.e., $\frac{1}{6}$ of the fundamental period.
4.2.5 Phasor Dynamics

Models have been derived using a general averaging theory for converter switching circuits in Mattavelli [46]. These models are based on forming dynamical equations, which describe the development of the essential Fourier components that characterize the operation of the TCSC. One assumption of Mattavelli [46] is that the line current is essentially sinusoidal.

The method Phasor Dynamics develops a continuous-time model for the TCSC. The model is based on a representation of voltages and currents as time-varying Fourier coefficients. The Fourier coefficients (Fourier series) are truncated to keep only the fundamental component of it.

Mattavelli [46] derives a TCSC model based on time-varying Fourier coefficients that capture the phasor dynamics of the TCSC. The approach can be viewed as intermediate between instantaneous value mode and the quasi-static sinusoidal steady-state approximation. The TCSC’s phasor dynamics represent the dynamics of the fundamental currents and voltages. The developed model of the TCSC is a phasor model of fourth-order. The model can be linearized and in different operating points it provides different linear models.

4.2.6 Prony Analysis

In Dolan [13] an application of Prony analysis is shown when identifying a TCSC system of Slatt 500 kV substation, Arlington Oregon, USA.

Dolan [13] shows good result for closed loop identification but these are only accurate for small variations near the operating point used for the identification of the system.

4.2.7 Discrete-time model

In Jalali [25] a method is presented to create a stability model of a TCSC to be used in time-domain simulations. The purpose of the stability model is to correctly represent the fundamental component of the series capacitor voltage for line currents which contain variation in both magnitude and phase. The stability model is essentially the solution to the differential equations of the TCSC circuit. It updates the capacitor voltage every half-cycle based on the line current, the thyristor firing time and the initial capacitor voltage and is therefore a discrete-time model. The TCSC circuit is obtained by representing the thyristor valve by an equivalent current source. By introducing an internal state variable to keep track of the capacitor voltage, a discrete-time equation for the current in this equivalent source is
formulated. The range of frequencies is limited to power swing values of 0.1 to 3.0 (Hz). The model is tested in a small power system consisting of a transmission line, a TCSC, an infinite bus, and a classical representation of a synchronous machine.

The method can only be implemented into stability programs which utilize a fixed time step integration routine.

4.2.8 State-space model depending on the firing angle

In Jovcic [29] a second order transfer function is identified using frequency response (frequency analysis) to represent the TCSC. The characteristic frequency of the denominator has a linear relationship with the firing angle.

4.2.9 Summary of different methodologies

In sections 4.2.1 – 4.2.8 different methods in the literature with the aim to build simplified models of a TCSC, have been listed.

In Christl [8] a fourth-order linear model identified from Bode Plots (frequency analysis) was derived. The poles of the model were dependent on the thyristor conduction angle $\sigma$. Time-domain comparisons showed good agreement between the identified model and the original detailed model. Also in Jovcic [29] a linear model (of second order) was identified by using frequency analysis.

In Jalali [26] a fourth-order linear model have been developed but here it has been done by analyzing the circuit of the TCSC and applying the Poincaré map.

Also in Ghosh [17], Othman [48], Mattavelli [46], and Jalali [25] the circuit of the TCSC is analyzed in order to build a linear model.

In Dolan [13] identification of a TCSC have been done with transient analysis by using Prony analysis.

Contribution from this work

Compared to previous studies, this thesis contains development of two linear models with transient and frequency analysis from a detailed TCSC model. As mentioned above, similar works have been reported in Christl [8], Jovcic [29] and Dolan [13]. In Christl [8] and Jovcic [29] the identification process is not documented and in Dolan [13] the transient analysis is done using Prony Analysis. In this work the identification processes are well-documented. In the identification processes we have treated the study ob-
ject as a black box, i.e., we have not analyzed the circuit of the TCSC to build linear models.

The developed models have been investigated in time-domain simulations and small-signal stability analysis.

4.3 Introduction to linearization of the TCSC

In the thesis we have modelled a detailed original model of a TCSC of which we build simplified linear models. The original TCSC model including a small surrounding power system contains in total 69 state variables of which the original TCSC model contains 22. The state variables of the original TCSC model are for instance variables of the control algorithm described in appendix C.

The created simplified models are abstract linear models that should represent the detailed original model of the TCSC although they are limited in bandwidth. The state variables in the created linear models do not represent real physical quantities such as measurable angles and voltages, instead they are representing abstract quantities.

To create the linear models we study the response of the original TCSC model by disturbing it with well-defined input signals. Then we record the output signals which we use for developing linear models. We will develop the linear models with two techniques namely,

- transient analysis, i.e., in a time-domain simulation disturb the inputs with a step in a steady-state situation; and

- frequency analysis, i.e., in a time-domain simulation of a steady-state situation disturb the inputs with sinus-formed signals with constant amplitude and varying frequencies.

In both items above we record the output signals from the model and identify the TCSC with the two techniques.

The task in this chapter is to describe the behavior of a physical component and investigate whether it is possible to simplify its representation with a linear model and in that case how that model should look like. A physical component consists of several interactive parts and is therefore represented as a "system" below. In the following we will step by step motivate our progress in this process.

Let us say that we have a system containing \( p \) input signals \( \{u_1, \ldots, u_p\} \) and \( m \) output signals \( \{y_1, \ldots, y_m\} \) as shown in figure 4.2.
Systems with several input and output signals occur in many practical situations such as; driving a car where the driver receives different input signals such as light, actual speed, and weather conditions and has to decide about how to continue the driving (the output signals) by accelerate or stop the vehicle, turn it, or do something else.

In a power system a process with a number of input signals and output signals is, e.g., a synchronous machine which has input signals such as mechanical torque, electric field voltage, and actual terminal voltage and where the output signals are delivered real and reactive power (or real and imaginary part of the current) to the network.

A system with more than one input signal and more than one output signal is called a Multi-Input Multi-Output system, a MIMO system, see figure 4.2.

A MIMO system as described with figure 4.2 can be split into $m$ Multi-
Input Single-Output subsystems, MISO subsystems, as described in figure 4.3. The difference between figure 4.2 and figure 4.3 is that the $m$ subsystems in figure 4.3 are not influenced from each other, i.e., the output signal from one subsystem cannot influence another subsystem. If our real system has such cross couplings, for instance that the output signal $y_1(t)$ from subsystem 1 influence subsystem $m$, then we will with the structure shown in figure 4.3 be forced to include a local copy of subsystem 1 inside subsystem $m$ to calculate a copy of the output signal from subsystem 1 inside subsystem $m$.

If we go one step further we could divide each of the $m$ subsystems in figure 4.3 into $p$ underlying Single-Input Single-Output subsystems, SISO subsystems, for each of the $m$ output signals as shown in figure 4.4. Earlier we said that if our real system has cross couplings such that the output signal from one subsystem, for instance $z_{11}(t)$ in figure 4.4 when calculating output signal $y_1(t)$ should influence another subsystem, for instance the subsystem that calculates the output signal $z_{1p}(t)$ in figure 4.4, then for this structure we cannot include a local copy of the calculation of output signal $z_{11}(t)$ inside subsystem $1p$ since $u_1(t)$ is not an input signal to subsystem $1p$. So, before we can split the model as in figure 4.4 we have to investigate whether it is possible to split the representation of the model into $m \times p$ subsystems.
or not. We have to make sure that the \( m \times p \) subsystems are independent of each other. This investigation will take place in section 4.4.

### 4.4 Investigations of our component

Before we can start to create simplified models of a component we first have to do investigations based on section 4.3.

Earlier in section 3.3.2 we discussed which signals that are necessary to model for our component. We came to the conclusion that \( i_d, i_q, u_d, \) and \( u_q \) have to be included.

Since the voltage drop over the series capacitor \( u_{cap} \) is a most volatile signal, see the lower part of figure 3.5 and figure 3.11, containing harmonics, we have decided to use the \( d- \) and \( q- \)components of the line current \( i_{line} \) into the TCSC as input signals while the \( d- \) and \( q- \)components of the voltage drop over the series capacitor \( u_{cap} \) are output signals. Whether it is possible to select the voltage drop \( u_{cap} \) as input signal and the line current \( i_{line} \) as output signal has not been investigated in the thesis.

Our model of the component looks as drawn in figure 4.5 containing the
two input signals \(i_d(t)\) and \(i_q(t)\) and the two output signals \(u_d(t)\) and \(u_q(t)\). As stated in section 4.3 we though have to check if this is possible and this is done in the following.

Earlier in this chapter and also in chapter 2, \(u\) represented input signals but from now on we will have voltages as output signals and therefore it must be emphasized that \(u_d\) and \(u_q\) represent output signals in the following text.

Later in chapters 6 and 7 we will use a base case of a power system including our component. In that base case the TCSC is in steady state when it is disturbed with different changes in the power system. In the steady state of the base case the \(d\)- and \(q\)-components of the line current \(i_{\text{line}}\) have reached the following values in (p.u.),

\[
i_{d0} = 0.094 \quad (4.1)
\]

\[
i_{q0} = 0.169 \quad (4.2)
\]

As discussed in section 3.3.2, the \(\theta\)-component of \(i_{\text{line}}\) in steady state is equal to 0 except for small amounts of the third harmonic (180 (Hz)) which we neglect.

In a \(dq\)-frame the steady-state current can be sketched as in figure 4.6. The absolute value of the steady-state current \(i(0)\) in figure 4.6 is in (p.u.),

\[
|i(0)| = \sqrt{i_d^2 + i_q^2} = \sqrt{0.094^2 + 0.169^2} = 0.193 \quad (4.3)
\]

\[
i_q \begin{cases} i(0) \\ i(0) = i_d + ji_q \end{cases}
\]

\[
=i_d
\]

Figure 4.6: Line current \(i_{\text{line}}\) into the TCSC in steady state.

The component we are interested in is placed in a small stiff power system to simplify the analyze of the dynamics of the studied component, see figure 4.7. The TCSC is isolated in a power system containing an infinite bus on the left side of the TCSC-compensated line and a load of constant current character on the right side of the TCSC in node B as shown in figure 4.7.
Figure 4.7: System used for identifying linear models of the TCSC.

The load in node B is set to produce the same working point as described above, i.e., so that the magnitude of the line current $i_{\text{line}}$ is equal to the value calculated in equation (4.3) above. The current $i_{\text{line}}$ is equal to the current $i_{\text{load}}$.

The steady-state current as described with equations (4.1) and (4.2) will later represent the steady-state situation in the studied cases.

Earlier in section 3.3.2 we have mentioned why we can omit the $\theta$-component of the current and the voltage in the modelling of the TCSC.

### 4.4.1 Isolated perturbations

The steady-state current in equation (4.3) is in the following assumed to be of pure $d$-component character, given as $i_{d0}$ in figure 4.8. By turning the axis so that the current has the same magnitude but being of pure $d$-component character makes the analysis of the disturbed TCSC easier. Physically it is the same steady state as in section 3.3.2 since the magnitude of the current is the same. In other words, we have introduced an angle reference (other than $\theta_{\text{ref}}$ that was introduced in section 3.1.1) that gives us a pure $d$-component current through the TCSC.

Figure 4.8: Line current $i_{\text{line}}$ into the TCSC is disturbed in both its $d$- and $q$-components.
It would be possible to let the current be of pure \( q \)-component character or a mix of both \( d \)- and \( q \)-components but here we have selected a pure \( d \)-component character. Since then, a disturbance in the \( d \)-direction is equivalent with a change in the magnitude of the current and a disturbance in the \( q \)-direction is equivalent with a change in the angle of the current.

In the stiff power system, the TCSC is perturbed both in the current’s \( d \)- and \( q \)-component, one by one as in figure 4.8. The load in Node B in figure 4.7 models the load wherein the changes of current takes place.

![Graphs showing response of \( u_q \) and \( u_d \) when \( i_q \) and \( i_d \) are perturbed.](image)

Figure 4.9: Response from the original model of the TCSC when the current has been disturbed with a 10%-steps in its \( d \)- and \( q \)-component.

In figure 4.9 the resulting voltage drop over the series capacitor is plotted when the current through the TCSC has been disturbed both in its \( d \)- and \( q \)-component. The amount of the 6th and 12th harmonics (360 and 720 (Hz) respectively) have been decreased by applying the filter shown in section 3.4.

In the upper part of figure 4.9 the current has been increased with a step of 10% in its \( d \)-component. In the upper left corner we can see how the \( d \)-component of the voltage drop \( u_{cap} \) responds and in the upper right


Figure 4.10: Line current $i_{line}$ into the TCSC is disturbed in both its $d$- and $q$-components at the same time.

corner we can see how the $q$-component of the voltage drop $u_{cap}$ responds. As can be seen, the response in the $q$-component of $u_{cap}$ is larger than the response in the $d$-component. In the upper left corner we can see how the earlier introduced subsystem 11 (figure 4.5) reacts for our component and in the upper right corner we can see how subsystem 21 (figure 4.5) reacts. The $q$-component of the current has been constantly equal to zero throughout the studied time interval.

In the lower part of figure 4.9 the current has been increased with a step of 10% in its $q$-component. In the lower left corner we can see how the $d$-component of the voltage drop $u_{cap}$ responds and in the lower right corner we can see how the $q$-component of the voltage drop $u_{cap}$ responds. Also here the response in the $q$-component of $u_{cap}$ is larger than the response in the $d$-component. In the lower left corner we can see how subsystem 12 reacts for our component and in the lower right corner we can see how subsystem 22 reacts. The $d$-component of the current has been constantly equal to 0.193 (p.u.) throughout the studied time interval.

### 4.4.2 Simultaneous perturbations

In section 4.4.1 we have disturbed one input signal while the other was undisturbed. By this routine we cannot be sure of if the component behaves in the same way if both input signals are disturbed at the same time which is the situation in normal operation. Does the component has the same response for a combination of simultaneous perturbations in the two input signals $i_d$ and $i_q$ as in figure 4.10, i.e., are the summation block diagrams in figure 4.5 valid?

Therefore we here make a verification to prove that our results also are valid when we have disturbances simultaneously in both input signals. Then we have the possibility to observe whether the four subsystems 11, 12, 21, and 22 are consistent, no matter if one or both input signals are perturbed at the same time. This investigation was mentioned earlier at the end of
Figure 4.11: Responses of the TCSC in $u_d$ and $u_q$ when $i_d$ and $i_q$ have been disturbed with 10%. The sum (indicated as "The sum") of two separate disturbances (in blue and red) in the upper half of the figure are equivalent with a simultaneous disturbance in $i_d$ and $i_q$ which is shown in the lower half of the figure.

In the upper left corner of figure 4.11 the perturbation of $u_d$ is shown when we have disturbed it as in section 4.4.1 separately with $i_d$ and $i_q$ in blue. The sum of the responses in $u_d$ has also been plotted and is indicated with "The sum" in black in the upper left corner. Since the response in $u_d$ is very small when $i_d$ is disturbed the total sum is very close to the response in $u_d$ when $i_q$ was disturbed, see section 4.4.1. Therefore it is impossible to see the second graph in blue since it is hidden under the graph indicated as "The sum" in the upper left corner of the figure.

In the upper right corner of figure 4.11 the perturbation of $u_q$ is shown when we have disturbed it as in section 4.4.1 separately with $i_d$ and $i_q$ in red. The sum of the responses in $u_q$ has also been plotted and is indicated
with "The sum" in black.

In the lower left corner of figure 4.11 we can see the response in $u_d$ when $i_d$ and $i_q$ have been disturbed *simultaneously* with the same disturbances as in section 4.4.1, i.e., a 10% step in the $d$- and $q$-component at the same time as in figure 4.10. Since the response in $u_d$ in the lower left corner is following the curve indicated with "The sum" in the upper left corner we come to the conclusion that subsystems 11 and 12 are consistent, no matter if there are perturbations in one of the input signals or simultaneously in both of them.

In the lower right corner of figure 4.11 we can see the response in $u_q$ when $i_d$ and $i_q$ are disturbed *simultaneously* with the same disturbances as in section 4.4.1, i.e., a 10% step in the $d$- and $q$-component at the same time as in figure 4.10. Since the response in $u_q$ in the lower right corner is following the curve indicated with "The sum" in the upper right corner we come to the conclusion that also subsystems 21 and 22 are consistent, no matter if there are perturbations in one of the input signals or simultaneously in both of them.

Above it has been shown that the component reacts in the same way for both simultaneous disturbances in the two input signals as well as the sum of isolated disturbances in the two input signals. This is called the principle for superposition, see Glad [19] and IEEE [21], which indicates that our system (the TCSC) behaves linearly in this steady state and for these perturbations, see section 4.5.

Since then we have also seen that our model can be divided into four subsystems in this steady state and for these perturbations as shown in figure 4.5.

Since we for our study object have the possibility to disturb one input signal while the other remains constant, we do not need to consider identification of a MIMO system. Instead we can identify transfer functions for two output signals while we are disturbing one input signal, in other words, we start with identifying two Single-Input Single-Output systems, SISO systems and then we repeat this for the other input signal.

If it would have been impossible to disturb the input signals one by one we could had identified our model as a MIMO system, see Skogestad [77].

### 4.5 Definition of a linear system

A system is linear if,
\[
\begin{align*}
\{ u_1(t) & \rightarrow y_1(t) \\
u_2(t) & \rightarrow y_2(t) \} \implies c_1 \cdot u_1(t) + c_2 \cdot u_2(t) \rightarrow c_1 \cdot y_1(t) + c_2 \cdot y_2(t) \quad (4.4)
\end{align*}
\]

for any values of the real constants \( c_1 \) and \( c_2 \), see Petersson [64]. The notation \( \rightarrow \) means that the expression on the left-hand side of the arrow passes a system. The output from the system is written on the right-hand side of the arrow. Equation (4.4) is called superposition, see Glad [19] and IEEE [21].

Most real circuit elements are non-linear to some extent but they can often be accurately represented by a linear approximation, IEEE [21]. The smaller disturbances that are applied to the input signals of the element, the better the linear approximation represents the element, Skogestad [76].

In our case, we have disturbed the TCSC with two different disturbances and recorded the responses of which we then have built the sum, see figure 4.11. We also disturbed the TCSC with a new disturbance which was equal to the sum of the two earlier disturbances. The response from the new disturbance was the same as the sum of the responses from the two earlier individual disturbances. This shows us that a linear approximation is valid for the TCSC at least for these disturbances and this steady state.

In figure 4.12 the disturbances in the current \( i_{\text{line}} \) is one fifth of the disturbances that were applied in figure 4.11. The responses in figure 4.12 are one fifth of the responses in figure 4.11. This shows that the TCSC behaves with the same response also for smaller disturbances in the current \( i_{\text{line}} \).

So, then our goal, which is to build linear models of the TCSC, seems to be possible to accomplish. However, we have through more analysis of the component found that the response of the TCSC changes for one of the subsystems as we will discuss in the following section 4.6.

### 4.6 Subsystem 12

By continuing to disturb our component for disturbances as recommended in Skogestad [77], p. 70, we can see that the component has the same response in all subsystems for both combined disturbances in the two input signals as well as for increases and decreases in the input signals except for subsystem 12 in figure 4.5.

The TCSC has been disturbed in a spectra of possible directions with the following 16 disturbances,
Figure 4.12: Responses of the TCSC in $u_d$ and $u_q$ when $i_d$ and $i_q$ have been disturbed with 2%. The sum (indicated as "The sum") of two separate disturbances (in blue and red) in the upper half of the figure are equivalent with a simultaneous disturbance in $i_d$ and $i_q$ which is shown in the lower half of the figure. The responses are one fifth of the result in figure 4.11.

\[
\begin{bmatrix}
\Delta i_d \\
\Delta i_q
\end{bmatrix} =
\begin{bmatrix}
0.1 \cdot |i(0)| \cdot \cos (n \cdot \pi / 8) \\
0.1 \cdot |i(0)| \cdot \sin (n \cdot \pi / 8)
\end{bmatrix} \quad \forall \ n = 0, \ldots, 15. \quad (4.5)
\]

The disturbances are drawn in figure 4.13 and the responses in $u_d$ and $u_q$ are drawn in figure 4.14.

Disturbances with number $n = k$ and $n = k + 8$ for $k = 0, \ldots, 7$ are opposite each other, i.e., symmetric in both the horizontal and the vertical axis, see figure 4.13. Since the disturbance in current is opposite each other for these values of $k$, also the response in the voltage, see figure 4.14, should be opposite each other for the same values of $k$.

In figure 4.14 we can see that this is the situation only for disturbance $n = 0$ and $n = 8$ and not for the other disturbances. For instance, disturbance
Figure 4.13: Line current $i_{\text{line}}$ into the TCSC is with 16 disturbances disturbed in both its $d$- and $q$-components to cover a selection of possible directions.

Number $n = 4$ (in blue) has a $d$-component $\Delta u_d \approx 0.0033$ (p.u.) on its path to the new steady-state point at $(\Delta u_d \approx 0.0027$ and $\Delta u_q \approx 0.0$) while disturbance number $n = 12$ (in red) has a $d$-component $\Delta u_d \approx -0.0039$ (p.u.) on its path to the new steady-state point at $(\Delta u_d \approx -0.0027$ and $\Delta u_q \approx 0.0$) which shows that the graphs are not symmetric in the vertical axis (in the response of $u_d$). The maximal magnitude in the response for $n = 4$ is 18% larger than the maximal magnitude in the response for $n = 12$.

Here we have omitted to plot the $16 \times 2$ graphs as functions of time in order to simplify the comparison of the simulated cases.

The new steady-state points are correct placed for all the curves, see points $n = 0, \ldots, 15$ in figure 4.14, but the paths from the center of the figure to the new steady-state points are as mentioned above slightly different for
Figure 4.14: Responses of the capacitor voltage $u_{cap}$ over the TCSC for 16 disturbances. The steady state before the disturbances are applied is positioned in the center of the figure.

the disturbances. The differences are seen only in the $u_d$-direction, i.e., the differences are between the right half plane and the left half plane of figure 4.14.

By further analyzing the disturbances in figures 4.13 and 4.14 we can see that the difference in response from our system can be derived from the relation between $u_d$ and $i_q$. Since it is the relation between $u_d$ and $i_q$ that gives different response as shown in figure 4.15 we can see that it is subsystem 12 in figure 4.5 that is producing this "phenomenon".

If we plot the absolute value of the response in $u_d$ when we disturb with an increasing or decreasing $i_q$ then we can draw the two responses as in figure 4.15.

As can be seen in figure 4.15, the response is different and this is the
cause for that the response of the TCSC is not symmetric in the vertical axis of figure 4.14.

In order to check whether this difference in response can be derived from different values of the capacitor reference \( Ref \) of the TCSC the response in \( u_d \) for increases and decreases in \( i_q \) is plotted for four different values of the boost factor \( Ref \) in figure 4.16. From top to bottom \( Ref \) is 2.0 (in yellow), 1.8 (in green), 1.6 (in blue), and 1.4 (in black).

We can see that the response in \( u_d \) depending on if \( i_q \) is increased or decreased is the same for the different values of \( Ref \), so that the value of the boost factor \( Ref \) does not influence the response. If we divide the responses with the value of \( Ref \) for the four responses we will have the same curves as earlier plotted in figure 4.15.

It would be possible for us to later include the different responses as shown in figure 4.15 for our model and identify one linear model for positive changes in \( i_q \) and another linear model for negative changes in \( i_q \). The selection between which model that should be used to represent subsystem
12 could then be handled in the interface that is described in chapter 5; in figure 5.1 we can detect the sign of $\Delta \tilde{I}_q$ and from that information choose which linear model of subsystem 12 that should be utilized. However, this has not been done in the thesis.

### 4.7 Transient Analysis

Transient analysis of power system components have been reported in works such as Sanchez-Gasca [73] wherein linear models have been created from time-domain simulations. In Sanchez-Gasca [73] it is shown that it is possible to get a good matching between the linear model and the original power system already for low orders of the generated linear models. However, in Sanchez-Gasca [73] no details about the used methods are provided.

By sending a well-defined input signal into a system in a time-domain simulation and record the output signal(s) from it, it is possible to analyze the system. The input signal can for instance have the character of an im-

Figure 4.16: Absolute value of the responses of the $d$-component of the capacitor voltage $u_{cap}$ when the $q$-component of $i_{line}$ is increased and decreased respectively for different values of the boost factor $Ref$. 
pulse, step, or a ramp. In the analysis it is possible to identify the dynamical response of the system, its transfer function(s), Schmidtbauer [74].

Figure 4.17 describes the flow chart when performing identification of a model with transient analysis.

![Flow chart for system identification in transient analysis.](image)

To obtain the dynamic response of the TCSC the current into it is disturbed, see \( i_{\text{line}} \) in figure 4.7. The \( d \)- and \( q \)-components of the incoming current \( i_{\text{line}} \) to the TCSC are disturbed with isolated steps as was done earlier in section 4.4.1.

By subtracting the initial values of the voltages shown in figure 4.9 from the present values of the voltages, the dynamic response of the changed voltage drop over the TCSC according to steps in both the currents \( d \)- and \( q \)-component can be generated, i.e., the relations,

\[
\Delta u_d(\Delta i_d), \Delta u_q(\Delta i_q), \Delta u_d(\Delta i_q), \Delta u_q(\Delta i_q)
\]

(4.6)
can be constructed.

### 4.7.1 Preparation of the signals

The signals shown in figure 4.9 are non-uniformly spaced, i.e., they are not given with a fixed time-step from the power system simulation software, see Fankhauser [16] and [75]. Since we are using an identification method which
demands equidistant sampled signals, see Ljung [41], we have to make the signals equidistant sampled. So, first the four voltage signals in figure 4.9 are sampled with uniformly spaced sampling. By studying the signals it can be observed that they, after have been filtered with the filter mentioned in section 3.4, still contain harmonics, but the harmonics have been significantly reduced. Frequencies higher than the sixth harmonic (360 (Hz)) have significantly been removed from the signals. The sampling frequency is set to $f_s = 10$ (kHz) to be sure of that no frequencies are folded into the frequency spectrum of interest. With sampling frequency $f_s = 10$ (kHz), the Nyquist frequency is 5 (kHz) according to the sampling theorem, i.e., frequencies above 5 (kHz) could be folded - aliased into the desired frequency spectrum, see Proakis [65]. This is however not the situation since we have significantly reduced frequencies higher than 360 (Hz) in the signals with the earlier introduced filter.

Figure 4.18 shows a detailed flow chart of the identification process in transient analysis.

Before continuing, the signals are again filtered with a first-order filter, this time with the time constant $T = 0.005$ (s), to decrease the magnitude of the sixth harmonic. Afterwards the signals are re-sampled with the sampling frequency $f_s = 500$ (Hz) so that the signals are given with the time step 0.002 (s). Re-sampling has here been made with the Nyquist frequency 250 (Hz) and the small amount of the sixth harmonic 360 (Hz) that still exists in the signals may have been folded into the frequency spectrum and appears as 140 (Hz). However, this has been checked in the re-sampled signals and is not causing any problem in the identification process, see section 4.7.2.

The last re-sampling is made since the number of samples is unnecessary large to describe the slow dynamics of the signals and it has been found that it is easier to identify difference equations for the transfer functions as in equation (4.7) when the signals are given with a larger time step.

### 4.7.2 Identification of the four subsystems

By using Matlab’s System Identification Toolbox, Ljung [40, 41] to build ARX-models for the four relations, transfer functions have been found. The method works in the time-discrete domain. The input to the method is uniformly spaced sampled values of input and output signals of an unknown system. The following difference equation is then identified,

$$y(t) + e_1 y(t-1) + \ldots + e_n y(t-n_e) = f_1 u(t-1) + \ldots + f_n f u(t-n_f) + v(t).$$

(4.7)
Simulation Program

First-order filter, $T = 0.01$ (s)

Sampling with $f_s = 10$ (kHz)

First-order filter, $T = 0.005$ (s)

Re-sampling with $f_s = 500$ (Hz)

Identification Program, choose order of $n_e$ and $n_f$

Enough, small error?

Yes

$\Delta x = A \Delta x + B \Delta u$

$\Delta y = C \Delta x + D \Delta u$

Change parameters in $A$, $B$, $C$, and $D$ and compare with *.

Enough, small error?

No

Yes

Finish

No

Figure 4.18: Detailed flow chart of system identification in transient analysis of the TCSC.

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When using the method, the required orders of the polynomials in (4.7) are given, i.e., $n_e$ and $n_f$. By varying the settings of $n_e$ and $n_f$ manually, values of parameters $e_1, \ldots, e_{n_e}$ and $f_1, \ldots, f_{n_f}$ are automatically found using the least squares method for each of the four subsystems 11, 21, 12, and 22. The term $v(t)$ is white noise.

The four generated time-discrete models in equation (4.7) are converted to continuous-time models and after that, the continuous-time models are converted to state-space models, see Ljung [40, 41].

Then the derived state-space models $G_{11}(s)$, $G_{21}(s)$, $G_{12}(s)$, and $G_{22}(s)$ have been multiplied with the Laplace transform of the disturbance which was a step, $(0.0193 \cdot \theta(t) \supset \frac{0.0193}{s})$. The product $G(s) \cdot \frac{0.0193}{s}$ have been transformed to a time-domain expression.

The values of the parameters $a_1, \ldots, a_6$, $b_1, \ldots, b_5$, $c_1, \ldots, c_5$, and $d_1, \ldots, d_4$ in the transfer functions, see $G_{11}(s)$, $G_{21}(s)$, $G_{12}(s)$, and $G_{22}(s)$ in equations (4.9) – (4.12) have then been adjusted so that they have the closest possible matching with the time-domain responses earlier recorded and shown in figure 4.9. The signals shown in figure 4.9 are the signals indicated as the output signals from the box with the sampling frequency $f_s = 10$ (kHz) marked with * and a dash-dot line in figure 4.18. The values of the parameters in equations (4.9) – (4.12) have been adjusted so that the least square error $y_{ls}$ is minimized as shown in equation (4.8) in the time interval $(3.0 \leq t \leq 5.0 \text{ (s)})$,

$$y_{ls} = \sum_{k=1}^{N} \left( y_{\text{real}}(k) - y_{\text{identified}}(k) \right)^2$$  \hspace{1cm} (4.8)

where $N = 20 \, 001$ since the sampling frequency is 10 (kHz). Similar least square criterion can be found in the literature, Ljung [41], p. 204.

With this matching, the values of the parameters which were delivered from the Identification Program in figure 4.18 are changed.

Here it should be clarified that; to modify the values of the parameters in equations (4.9) – (4.12) is the same as modifying the matrix-elements of matrices $A$, $B$, $C$, and $D$ in figure 4.18.

The errors $(y_{\text{real}}(k) - y_{\text{identified}}(k))$ are denoted as $\text{Difference} \ G_{11}$, $\text{Difference} \ G_{21}$, $\text{Difference} \ G_{12}$, and $\text{Difference} \ G_{22}$ shown in figures 4.19 and 4.20. The least square error $y_{ls}$ is largest for $G_{22}$ for which it is $2.00 \cdot 10^{-4}$.

After adjustment of the parameter values the following four transfer functions (4.9) – (4.12) have been identified,
Figure 4.19: Subsystems $G_{11}$ and $G_{21}$ when identified with transient analysis. The recorded responses are in green. The identified responses and the differences in the final models are in blue.

$$G_{11}(s) = \frac{\Delta u_d(s)}{\Delta i_d(s)} = d_1 \frac{s + a_1}{(s - (b_1 + j c_1))(s - (b_1 - j c_1))}$$  \hspace{1cm} (4.9)

where

$$a_1 = -0.1035 \quad b_1 = -25.1746 \quad c_1 = 32.4144 \quad d_1 = 1.4472$$

$$G_{21}(s) = \frac{\Delta u_q(s)}{\Delta i_d(s)} = d_2 \frac{s + a_2}{(s - (b_2 + j c_2))(s - (b_2 - j c_2))}$$  \hspace{1cm} (4.10)

where

$$a_2 = 43.3154 \quad b_2 = -14.6100 \quad c_2 = 23.6539 \quad d_2 = -2.4711$$

$$G_{12}(s) = \frac{\Delta u_d(s)}{\Delta i_q(s)} = d_3 \frac{s + a_3}{(s - (b_3 + j c_3))(s - (b_3 - j c_3))}$$  \hspace{1cm} (4.11)
Figure 4.20: Subsystems $G_{12}$ and $G_{22}$ when identified with transient analysis. The recorded responses are in green. The identified responses and the differences in the final models are in blue.

where

\[
G_{22}(s) = \frac{\Delta u_q(s)}{\Delta i_q(s)} = d_4 \frac{(s + a_4)}{(s - (b_4 + jc_4))(s - (b_4 - jc_4))} \cdot \frac{(s + a_5)(s + a_6)}{(s - (b_5 + jc_5))(s - (b_5 - jc_5))}
\]

(4.12)

where

\[
a_3 = 14.9983 \quad b_3 = -10.8263 \quad c_3 = 13.6453 \quad d_3 = 2.7826
\]

In equations (4.9) – (4.12) all poles (roots of the denominator) are in the left half plane.

In equations (4.9) and (4.12) there are two right half plane zeros (roots of the numerator) at 0.1035 and 0.7740 (1/s) respectively. If the aim had
been to control the studied component in a certain way we could take these zeros into consideration when designing a controller, Skogestad [77]. But since the aim is to identify our component and to model it with a linear model and see if it behaves in the same way as our original model, we will include these right half plane zeros in our linear model.

Since there are in total ten poles in the four transfer functions above, a tenth-order linear model describes the behaviour of the four relations. Constants $c_1, \ldots, c_5$ are given in (rad/s).

In figures 4.19 and 4.20 the responses from the real subsystems as well as the identified subsystems are shown for $3.0 \leq t \leq 3.5$ (s). The graphs of the real subsystems in green have been low-pass filtered with the filter introduced in section 3.4.

In figure 4.21 the responses from the real subsystems in green as well as the identified subsystems $G_{11}$ and $G_{21}$ are shown for $3.0 \leq t \leq 3.5$ (s). The graphs of the real subsystems are here unfiltered.

![Graph 1](image1)

![Graph 2](image2)

Figure 4.21: Responses of the identified subsystems $G_{11}$ and $G_{21}$ when identified with transient analysis (in bold blue) compared with the unfiltered voltages $u_d$ and $u_q$ (in thin green).

It should be mentioned that it might be possible to find (identify) another setup of linear subsystems that would create a matching similar to the one
shown in figures 4.19 and 4.20. However, we have here seen that the least square errors are acceptably small for the identified linear model. Therefore we continue in chapters 6 and 7 to investigate whether it represent the dynamics of the TCSC.

4.7.3 A check of the linear model developed from transient analysis

Before continuing we plot the frequency response of the developed model with transient analysis here in section 4.7. The results are shown in figure 4.22.

When comparing figure 4.22 with the figures 4.27 and 4.28 (that will be created in the next section) we can see that the frequency responses are very similar for the two linear models.

4.8 Frequency Analysis

By sending a sinus-formed input signal into a system in a time-domain simulation and record the output signal(s), and repeat this for a number of frequencies and create a frequency spectra, it is possible to analyze the system. In the analysis it is possible to identify the dynamical response of the system, its transfer function(s), see Schmidtbauer [74].

Figure 4.23 describes the flow chart when performing identification of a model with frequency analysis.

If the input signal of a linear system is excited (perturbed) by a periodic signal composed of specific harmonic(s) then the output signal will also be periodic and contain the same harmonic(s). Unlike linear systems, output signals from non-linear system may contain harmonics that do not appear in the input sequence, see Docter [12]. Since it is not possible for a linear model to produce such harmonics, these harmonics are impossible to model with linear techniques and should be removed from the data. This is easily accomplished by decomposing the output data with the Fast Fourier Transform (FFT) into two parts; the harmonic(s) that are in the input sequence, and those that are not.

The linearity of the TCSC has been investigated in sections 4.4 – 4.6 so therefore it should be conceivable to apply frequency analysis to our study object.

To obtain the dynamic response of the TCSC, the current into it, see \( i_{line} \) in figure 4.7, is disturbed. The \( d \)- and \( q \)-components of the incoming
Figure 4.22: Frequency response of the linear model identified with transient analysis.
current $i_{line}$ to the TCSC are disturbed with a frequency-variant sinusoidal signal $g_{dist}(t)$ as given in equation (4.13) and shown in figure 4.24.

$$g_{dist}(t) = K_{dist} \cdot \sin(2\pi f_{dist} t)$$ (4.13)

The frequency $f_{dist}$ in equation (4.13) have been varied with the frequencies given in table 4.2.

<table>
<thead>
<tr>
<th>frequency step $\Delta f$ (Hz)</th>
<th>frequency interval (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>$0.1 \leq f_{dist} \leq 1.0$</td>
</tr>
<tr>
<td>0.5</td>
<td>$1.5 \leq f_{dist} \leq 5.0$</td>
</tr>
<tr>
<td>1.0</td>
<td>$6.0 \leq f_{dist} \leq 70.0$</td>
</tr>
</tbody>
</table>

Table 4.2: Frequencies used to disturb the TCSC.

The signal $g_{dist}(t)$ is added to the steady-state current $i_{d0} = 0.193$ (p.u.) and $i_{q0} = 0$ (p.u.) respectively. This steady-state current is selected since it is the initial steady-state current of the TCSC in the later chapters 6 and 7. $K_{dist}$ in equation (4.13) is 10% of the magnitude of the steady-state current, i.e., 0.0193 (p.u.). Figure 4.7 describes the actual situation where the current $i_{load}$ through the load in node B is disturbed with $g(t)$ in both
its $d$- and $q$-component respectively. The current through the load $i_{load}$ in node B is the same as the incoming current $i_{line}$ to the TCSC into node A.

The signal $g_{dist}(t)$ is first added to the $d$-component of the current while the $q$-component remains constantly equal to zero. Later $g_{dist}(t)$ is added to the $q$-component of the current while the $d$-component remains constant. $g(t)$ is varied in the frequency range $0.1 \leq f_{dist} \leq 70.0$ (Hz) [$0.628 < w < 440$ (rad/s)] with the frequency steps shown in table 4.2.

Figure 4.25 shows a detailed flow chart of the identification process in frequency analysis.

For every frequency, $g_{dist}(t)$ is applied until the TCSC reaches a stationary status, see Skogestad [77], then the TCSC’s frequency response in $u_d$ and $u_q$ is calculated using Fast Fourier Transform (FFT). The desired frequency spectra is chosen from 0 (Hz) up to $f_c = 127$ (Hz). An Anti-aliasing filter is applied before the FFT-computation.

### 4.8.1 Anti-aliasing filter

The Anti-aliasing filter filters the signal from higher frequencies than the desired peak frequency $f_c$. An FFT-computation without using an Anti-aliasing filter could result in that higher frequencies are aliased (folded) into the desired frequency spectrum, see Proakis [65]. An FFT-computation with the Anti-aliasing filter results in that the highest complex Fourier coefficients
Figure 4.25: Detailed flow chart of system identification in frequency analysis of the TCSC.
might be calculated with an error in phase with some degrees caused by the filter. However, the magnitude of the Fourier coefficients are not affected, see below. The used Anti-aliasing filter is in our work a seventh-order low-pass Butterworth filter.

The equation of the Butterworth filter is given in equation (4.14).

\[ N(s) = \frac{1}{1 + \left( \frac{\omega}{\omega_c} \right)^{2n}} \]  

(4.14)

The frequency response of the filter is pictured in figure 4.26 with a cut-off frequency \( f_c \) set to \( f_c = 100 \) (Hz).

![Bode Diagram](image)

Figure 4.26: The frequency response from a seventh-order low-pass Butterworth filter with \( f_c = 100 \) (Hz).

The filter in figure 4.26 is designed so that the amplification of the filter is \(-0.19\) (dB) at the cut-off frequency \( f_c = 100 \) (Hz). At the stop band 200 (Hz) the filter amplification is \(-34.4\) (dB).

In frequency analysis of the TCSC the cut-off frequency \( f_c \) has been set to \( f_c = 127 \) (Hz).
The FFT-calculation in our frequency analysis is performed with the sampling frequency $f_s$ set equal to,

$$f_s = 4(f_c + \Delta f) = 4(127 + 1) = 512(Hz)$$  \hspace{1em} (4.15)

for disturbances in the frequency interval $(6.0 \leq f_{\text{dist}} \leq 70.0)$ according to table 4.2. In equation (4.15), $\Delta f$ is

$$\Delta f = \frac{1}{t_{\text{stop}} - t_{\text{start}}} = \frac{1}{\Delta t} = \frac{1}{1} = 1(Hz).$$  \hspace{1em} (4.16)

In equation (4.16) $t_{\text{start}}$ is the start point of the studied time interval and $t_{\text{stop}}$ is the end point of the studied time interval $\Delta t$. For disturbances in the frequency interval $(6.0 \leq f_{\text{dist}} \leq 70.0)$, $\Delta f$ is $\Delta f = 1$ (Hz).

In our case, the sampling $f_s$ is set equal to 512 (Hz) for disturbances in the frequency interval $(6.0 \leq f_{\text{dist}} \leq 70.0)$ according to table 4.2. The sampling frequency $f_s = 512$ (Hz) enables the possibility to detect frequencies up to,

$$f_{\text{detect}} = \frac{f_s}{2} - \Delta f = \frac{512}{2} - 1 = 255(Hz).$$  \hspace{1em} (4.17)

However, the magnitudes of the frequencies in the interval $127 < f < 255$ (Hz) have been reduced with the characteristic of the Butterworth filter. Since then we can assume that no frequencies higher than 255 (Hz) have been aliased (folded) into the desired frequency spectra which was selected to $0 < f < 127$ (Hz).

As mentioned earlier, an FFT-computation with an Anti-aliasing filter results in that the complex Fourier coefficients for the frequencies just below $f_c$ are calculated with an error in phase with some degrees caused by the filter characteristic. (The frequencies $f_c < f \leq f_{\text{detect}}$ are also affected by the Anti-aliasing filter but these frequencies are not in our interest.) In this work, the calculated magnitudes of the Fourier coefficients are not affected since they are divided with the magnitude of the filter characteristic of the Butterworth filter, see equation (4.14). However, the phase of the highest Fourier coefficients might have some error in phase, see Persson [53].

### 4.8.2 Window function

A rectangular window function is used when performing the FFT-computations. Window functions are used to avoid disturbances generated when the studied time interval does not contain a whole number of periods of the studied signal. However, in our computations the studied time interval is
selected so that a whole number of periods of the studied signal is inside the studied time interval and therefore the rectangular window can be used, see Proakis [65].

4.8.3 Identification of the four subsystems

The resulting frequency responses are shown in figures 4.27 – 4.28.

The first-order filter with time constant \( T = 0.01 \) (s) that was used earlier in section 4.7 is not used when doing frequency analysis in this section.

![Frequency response figures](image)

Figure 4.27: The frequency response of the original TCSC (un-smooth and in blue) and the response from the identified model (in green) when \( i_d \) is disturbed. \( u_d(i_d) \) and \( u_q(i_d) \) are shown.

The frequency response of the \( d- \) and \( q- \)components of the voltage drop \( u_{cap} \) over the TCSC when disturbing the \( d- \) and \( q- \)components of the current respectively are shown in figures 4.27 and 4.28. The un-smooth curve (in blue) is the recorded response from the original detailed TCSC model. Also the response from the identified models are shown in the figures in green.
The curves from the identified responses are smoother. In the right-hand diagrams of figure 4.27 ($u_q/i_d$), the identified response is so close to the recorded one so that it is hard to see any difference between the two curves. The situation is the same for the left-hand diagrams of figure 4.28.

![Graphs showing frequency response](image)

Figure 4.28: The frequency response of the original TCSC (in blue) and the response from the identified model (in green) when $i_q$ is disturbed. $u_d(i_q)$ and $u_q(i_q)$ are shown.

From the recorded responses in figures 4.27 and 4.28, four transfer functions are built by using a frequency response function which fits a frequency spectra with a suggested order of the transfer function by the user. The function identifies the transfer function with a least-squares fitting, no weighting of the frequency spectra have been selected. The Matlab function 'fitsys' can be found in the System Identification Toolbox, see Ljung [40, 41].

The difference in matching in the upper right corner of figure 4.28 between the identified and recorded response for low frequencies is not important since the magnitude is very small ($\approx -50$ (dB)).

By selecting different order of an identified transfer function, a polyno-
mial with a response as close as possible to the recorded response can be found. This is done for each of the four transfer functions.

The identified responses demand at least a linear model of order 17. For orders lower than 17, the matching between the identified and the recorded responses in figures 4.27 and 4.28 are less.

**Perturbing the d-component of the current**

Figure 4.27 shows the frequency response of the TCSC when the d-component of the current has been disturbed with 10% of the steady-state value of $i_{line}$. The q-component of the current has remained equal to zero during the perturbation of the d-component.

The following two transfer functions have been identified for the TCSC,

\[
G_{11}(s) = \frac{\Delta u_d(s)}{\Delta i_d(s)} = 0.0631 \frac{(s + 0.4985)(s^2 - 89.96s + 6.013 \cdot 10^4)}{(s + 751.8)(s^2 + 126.6s + 5355)} \tag{4.18}
\]

and

\[
G_{21}(s) = \frac{\Delta u_q(s)}{\Delta i_d(s)} = -0.021901 \frac{(s + 159)(s + 12.14)}{(s^2 + 37.66s + 499.3)} \cdot \frac{(s^2 - 36.87s + 3.067 \cdot 10^4)}{(s^2 + 167.7s + 1.884 \cdot 10^4)}. \tag{4.19}
\]

In equations (4.18) and (4.19) all poles (roots of the denominator) are in the left half plane.

In equation (4.18) there are right half plane zeros (roots of the numerator) at 45.0 $\pm j241.1$ (1/s, rad/s). These zeros are probably of little significance since the frequency of them (38.4 Hz) is higher than what we want to control/send into the linear model. The bandwidth we are interested in is below this frequency. In general, zeros in the right half plane that are ”far away” from the origin are often of little significance for control, Skogestad [76].

In equation (4.19) there are right half plane zeros at 18.4 $\pm j174.2$ (1/s, rad/s). Also these zeros are probably of little significance since the frequency of them (27.7 Hz) is high compared to what we want to control/send into the linear model.
Perturbing the $q$-component of the current

Figure 4.28 shows the frequency response of the TCSC when the $q$-component of the current has been disturbed with 10% of the steady-state value of $i_{line}$. The $d$-component of the current has remained equal to the steady-state value during the perturbation of the $q$-component.

The following two transfer functions have been identified for the TCSC,

$$G_{12}(s) = \frac{\Delta u_d(s)}{\Delta i_q(s)} = 0.014482 \frac{(s + 439.5)(s + 11.39)}{(s^2 + 50.94s + 754.6)} \frac{(s^2 - 109.5s + 5.188 \cdot 10^4)}{(s^2 + 238.9s + 3.703 \cdot 10^4)}$$ (4.20)

and

$$G_{22}(s) = \frac{\Delta u_q(s)}{\Delta i_q(s)} = 0.02088 \frac{(s + 658.4)(s + 2.885)}{(s^2 + 11.6s + 48.08)} \frac{(s^2 - 1.342s + 1.156)(s^2 + 382s + 4.997 \cdot 10^4)}{(s^2 + 44.93s + 567.8)(s^2 + 210.2s + 2.377 \cdot 10^4)}$$ (4.21)

In equations (4.20) and (4.21) all poles are in the left half plane.

In equation (4.20) there are right half plane zeros at $54.8 \pm j221.1$ (1/s, rad/s). These zeros are probably of little significance since the frequency of them (35.2 (Hz)) is higher than what we want to control/send into the linear model.

In equation (4.21) there are right half plane zeros at $0.67 \pm j0.84$ (1/s, rad/s). This is how our component has been identified and if the aim had been to control the component in a certain way we could include this in a design of a controller, but here the aim is to model the TCSC with a linear model and see if it behaves in the same way as our original TCSC model. Therefore we will include these zeros in the linear model.

4.8.4 A check of the linear model developed from frequency analysis

Before continuing we disturb the developed model in section 4.8 with the same disturbance as we disturbed the original TCSC model in section 4.7 when performing transient analysis. The results from the linear model are shown in figure 4.29 in red and the curves from the original TCSC model (earlier plotted in figure 4.9) are plotted in blue.
When comparing the blue and red curves in figure 4.29 we can see that the time-domain responses are very similar.

![Figure 4.29: Time-domain response of the linear model that has been identified with frequency analysis (in red). The response from the original TCSC model (in blue).](image)

### 4.9 Summary

By disturbing the current into the TCSC in its $d$- and $q$-component we have in sections 4.7 and 4.8 identified the four elements of the $[2 \times 2]$-matrix in equation (4.22).

$$
\begin{bmatrix}
\Delta u_d \\
\Delta u_q
\end{bmatrix} = 
\begin{bmatrix}
G_{11}(s) & G_{12}(s) \\
G_{21}(s) & G_{22}(s)
\end{bmatrix}
\begin{bmatrix}
\Delta i_d \\
\Delta i_q
\end{bmatrix}
$$

(4.22)

By disturbing the current into the TCSC in its $d$- and $q$-components ($\Delta i_d$ and $\Delta i_q$) separately it has been possible to identify the four elements of the $[2 \times 2]$-matrix in equation (4.22) containing the four transfer functions.
Our approach has been to disturb input by input. First we disturbed the $d$-component of the current $\Delta i_d$ and studied the $d$- and $q$-components of the voltage $\Delta u_d$ and $\Delta u_q$ and with that, identified $G_{11}(s)$ and $G_{21}(s)$. Then we disturbed the $q$-component of the current $\Delta i_q$ and studied the $d$- and $q$-components of the voltage $\Delta u_d$ and $\Delta u_q$ and with that identified $G_{12}(s)$ and $G_{22}(s)$. In the chapter we have seen the following differences when using two methods to develop linear models,

- subsystem 12 for our model of the TCSC had different response whether the $q$-component of the current $i_q$ was increased or decreased,

- when developing the linear models we have used a first-order filter to reduce the amounts of harmonics for the methodology transient analysis in section 4.7. When using the methodology frequency analysis in section 4.8 we used an Anti-aliasing filter,

- when using the methodology frequency analysis in section 4.8 we did not have the possibility to observe the different response of $u_d$ of the TCSC for increases/decreases in $i_q$, i.e., to observe the different response in subsystem 12.
Chapter 5

Interface to a linear model

In this chapter it is described how the created linear models are simulated with the rest of the power system. To use them, an interface system which connects them to the surrounding power system is utilized.

5.1 Introduction

The interface system described in this chapter is necessary to have since the linear models have different response depending on if there is a change in the $d$- or $q$-component of the incoming current. Depending on if the incoming current has an angle change or a magnitude change, the changes will be transformed to a local reference system where the current in a steady-state situation is of pure $d$-component character.

The interface has to consider under which circumstances the developed linear models are valid. The interface can be summarized as,

A) The developed linear models describe impact of changes from a steady-state operating point created by the interface system; and,

B) The steady state is defined as of pure $d$-component character with different behavior of changes in $d$- and $q$-direction.

The interface system transforms a change in the magnitude of the incoming current as a change in the $d$-component of the input to the linear model. A change in the angle of the incoming current is transformed to a change in the $q$-component of the input to the linear model. In a steady-state situation, the current through the linear model is zero.
It would be possible to let the current be of pure $q$-component character or a mix of both $d$- and $q$-component but here as well as when we identified the linear models in chapter 4 we have selected a pure $d$-component character.

The interface system is also slowly sliding from one steady state to another. This behavior means that the linear model, for the instant of a difference in magnitude of the incoming current or an angle difference in the incoming current, has a response which is timely for the dynamics of the real TCSC. This feature of the interface makes the linear model having a timely response for subsequent disturbances.

The linear models that are created are developed in the $dq0$-coordinate system, i.e., the so-called Park’s transformation, see Park [49]. All simulations are done in the instantaneous value mode, i.e., instantaneous values of phase voltages and phase currents can be shown although they are calculated in a $dq0$-coordinate system. As mentioned in chapter 3 there is no $0$-component in the studied system. Therefore we exclude the $0$-component of the currents and voltages. So from now on we simply write $dq$-coordinate system.

The linear models are representing a small-signal model of the power system component and represent the dynamic behaviour of the component from a steady state. The steady state of the power system component is represented by the steady-state model in figure 5.1. The sum of the response from the steady-state model and the linear model is the total response from the studied power system component.

## 5.2 Coordinate system

The coordinate system for the linear models is representing a local $dq$-coordinate system which is standing still in a steady-state situation. Between the local coordinate system and the surrounding power system exists an interface which is described in this chapter.

One advantage of representing a power system simulation with instantaneous values using the $dq$-coordinated frame is that the $dq$-frame is standing still in steady state and is slowly varying close to a steady-state situation, i.e., the currents and voltages are in the $dq$-frame varying very slow compared to phase quantities which are in a steady state in instantaneous value mode varying sinusoidal with the power frequency of the network.
5.3 Interface system

In figure 5.1, the interface system is shown. Observe that the interface system is only used when simulations are done with the linear models. Simulations with the original TCSC model are done without the interface system.

The input signals to the interface system are the three-phase currents shown in the upper left part of figure 5.1. The output signal is the complex-valued signal $U$ that is shown in the right part of figure 5.1.

The interface system is connected both to the linear model of the TCSC as well as to the steady-state model of the TCSC as in figure 5.1. The linear model and the steady-state model may represent any power system component but is in the thesis representing a TCSC.

Bold lines in figure 5.1 represent complex-valued quantities.

The input signal $\tilde{i}(t)$ consists of a real and an imaginary part, $i_{re}(t)$ and $i_{im}(t)$ and is defined as in equation (5.1) below,

$$\tilde{i}(t) = i_{re}(t) + ji_{im}(t) = \frac{2}{3}(i_a(t) + e^{j\frac{2\pi}{3}} \cdot i_b(t) + e^{-j\frac{2\pi}{3}} \cdot i_c(t))$$  

(5.1)

where $i_a(t)$, $i_b(t)$, and $i_c(t)$ are instantaneous phase currents, scalars.

This transformation keeps the magnitude of the phase quantities during symmetrical conditions, i.e., the length of the space vector (the magnitudes of $i_{re}(t)$ and $i_{im}(t)$) is identical with the magnitudes of the phase quantities $i_a(t)$, $i_b(t)$, and $i_c(t)$, Ångquist [88].

Equation (5.1) is often referred to as the space-vector approach, Kovács [34] and $\tilde{i}(t)$ is a current vector for the three instantaneous phase-currents, see also Nee [47] and Ångquist [88]. $i_a(t)$, $i_b(t)$, and $i_c(t)$ in figure 5.1 are the incoming instantaneous phase-currents from the surrounding power system.

As mentioned in chapter 3, the $\theta$-component is not necessary to include in the simulations and as a function of that, transformation (5.1) is possible. The three-phase currents are represented in the $dq$-frame of the surrounding power system, see $i_d + ji_q$ in figure 5.1.

The angle $\theta$ in figure 5.1 is the angle of the steady-state current $I_{SS}$. $\theta$ is calculated in the block diagram $\mathcal{L}I_{SS}$ and is used as an input signal to two of the block diagrams in figure 5.1. The signal $\omega_{ref}$ is the rotor speed of the reference machine in the power system.

The real and imaginary parts of $\tilde{i}(t)$; $i_{re}(t)$ and $i_{im}(t)$, are in steady state sinus formed with the power frequency of the network and contain information about all three phases as shown in the following three equations.
\[ i_a(t) = Re\{\tilde{i}(t)\} \] \hspace{1cm} (5.2)

\[ i_b(t) = Re\{e^{-j\frac{2\pi}{3}} \cdot \tilde{i}(t)\} \] \hspace{1cm} (5.3)

\[ i_c(t) = Re\{e^{j\frac{2\pi}{3}} \cdot \tilde{i}(t)\} \] \hspace{1cm} (5.4)

The three equations above are valid under the assumption that we have a zero-sequence free power system, i.e., no \( 0 \)-components.

### 5.3.1 Phasor estimation

It is important to select what should be sent into the linear model, i.e., what signals should be used as input signals to represent the small-signal behaviour from a steady state of the power system component and what should be handled as the steady-state level. The total contribution from the steady-state model and the linear model represent the total behaviour of the power system component.

In the block diagrams marked as \( PE_{\omega_{max}} \) and \( PE_{\omega_{SS}} \) in figure 5.1, phasor estimation takes place. The output signal \( I_{SS} \) from block diagram
\( PE \omega_{SS} \) represents a steady-state current which is allowed to vary slowly. \( I_{SS} \) is as slow so that its dynamic is uninteresting to represent with any other model than a steady-state model. The steady-state model with the output signal \( U_{SS} \) is a constant capacitive reactance which is modelled within fundamental frequency mode as in equation (5.12). See section 5.3.3 for settings of the block diagram \( PE \omega_{SS} \).

The output signal \( I_{\text{var}} \) from block diagram \( PE \omega_{max} \) represents a faster varying current. This signal is allowed to vary faster than \( I_{SS} \). The dynamics of \( PE \omega_{max} \) (the value of the time constant \( T_{F1} \), see section 5.3.3) sets the bandwidth of the input signals we send into the linear model of the TCSC. In section 5.3.3 more is said about the extraction of \( I_{SS} \) and \( I_{\text{var}} \). In section 5.3.4 we will continue to discuss how we split the representation of the study object; the linear model and the steady-state model respectively.

### 5.3.2 Combined coordinate transformation and filtering

Let us assume that the complex-valued signal \( \tilde{i}(t) \) is built up with the following three terms:

\[
\tilde{i}(t) = I_{av} + I_p \cdot e^{j\omega_{ref} \cdot t} + I_n^* \cdot e^{-j\omega_{ref} \cdot t}
\]  

(5.5)

where \( I_{av} \) is a complex-valued average value of \( \tilde{i}(t) \), \( I_p \) is the positive-sequence component multiplied with \( e^{j\omega_{ref} \cdot t} \), \( I_n \) is the negative-sequence component and \( I_n^* \) is the complex conjugate of the negative-sequence component multiplied with \( e^{-j\omega_{ref} \cdot t} \), see Ångquist [88].

The factor \( e^{j\omega_{ref} \cdot t} \) is rotating counter-clockwise with the system frequency in the complex plane and the factor \( e^{-j\omega_{ref} \cdot t} \) is rotating clockwise with the system frequency in the complex plane. \( \omega_{ref} \) is the angular speed of the reference machine in the power system (\( \omega_{ref} \approx 2\pi f_0 \text{[rad/s]} \)).

In the following we want to create estimates of \( I_{av}, I_p, \) and \( I_n \) from knowledge of \( \tilde{i}(t) \). These estimates we note as \( \hat{I}_{av}, \hat{I}_p, \) and \( \hat{I}_n \). Later we will write formulas for how these estimates can be calculated.

By re-writing equation (5.5) we can build rough estimates \( \hat{x}_{av}, \hat{x}_p, \) and \( \hat{x}_n \) of \( I_{av}, I_p, \) and \( I_n \) respectively as below

\[
\hat{x}_{av} = \tilde{i}(t) - \hat{I}_n^* \cdot e^{-j\omega_{ref} \cdot t} - \hat{I}_p \cdot e^{j\omega_{ref} \cdot t}
\]  

(5.6)

\[
\hat{x}_p = (i(t) - \hat{I}_{av} - \hat{I}_n^* \cdot e^{-j\omega_{ref} \cdot t}) \cdot e^{-j\omega_{ref} \cdot t}
\]  

(5.7)

\[
\hat{x}_n = ((i(t) - \hat{I}_{av} - \hat{I}_p \cdot e^{j\omega_{ref} \cdot t}) \cdot e^{j\omega_{ref} \cdot t})^*.
\]  

(5.8)
Figure 5.2: Phasor estimator used in the block diagrams $PE\omega_{SS}$ and $PE\omega_{max}$ in figure 5.1. All signals are complex-valued.

The rough estimates $\hat{x}_{av}$, $\hat{x}_p$, and $\hat{x}_n^*$ vary rapidly for any change in the input signal $\tilde{i}(t)$, see figure 5.2. In order to 'slow down' the variation of the estimates in equations (5.6) – (5.8) we low-pass filter $\hat{x}_{av}$, $\hat{x}_p$, and $\hat{x}_n^*$ to create our estimates $\hat{I}_{av}$, $\hat{I}_p$, and $\hat{I}_n^*$ as,

$$\hat{I}_{av} = \frac{\hat{x}_{av}}{1+sT} \quad (5.9)$$

$$\hat{I}_p = \frac{\hat{x}_p}{1+sT} \quad (5.10)$$

$$\hat{I}_n^* = \frac{\hat{x}_n^*}{1+sT}. \quad (5.11)$$

Figure 5.2 shows the block diagrams of the phasor estimator containing equations (5.6) – (5.11). Equations (5.6) – (5.11) are solved both for the real and imaginary part of the variables $\hat{x}_{av}$, $\hat{x}_p$, $\hat{x}_n^*$, $\hat{I}_{av}$, $\hat{I}_p$, and $\hat{I}_n^*$.

The estimate $\hat{I}_p$ represents the positive-sequence component (a fundamental frequency component) of $\tilde{i}(t)$ which is allowed to vary depending on the filter time $T$. In section 5.3.3 an example is shown in how the estimates respond to a sudden step in the magnitude of the input signals $i_a(t)$, $i_b(t)$, and $i_c(t)$.

After a period of time in a symmetric steady-state situation, $\hat{I}_{av} = \hat{I}_n^* = 0$ after some time depending on the time constant $T$. 

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5.3.3 Time constants to filter $I_{SS}$ and $I_{var}$

To calculate $I_{SS}$ and $I_{var}$ we use the phasor estimator introduced in sections 5.3.1, 5.3.2, and figure 5.2. When we estimate $I_{SS}$ we set a time constant $T_{F2}$ (indicated as $T$ in figure 5.2) to a value between $0.5 < T_{F2} < 2.0$ (s), Ångquist [87]. The output signal $\hat{I}_p$ from the phasor estimator in figure 5.2 then represents $I_{SS}$, a slow phasor of $\tilde{i}(t)$.

$I_{SS}$ is also indicated in figure 5.1 as the output signal from the block diagram $PE\omega_{SS}$.

By using the same phasor estimator but with a faster time constant $T$ we can estimate the quantity $\hat{I}_p$ but now allow it to vary faster. The time constant $T_{F1}$ is here set to a value in the interval $0.004 < T_{F1} < 0.4$ (s), Ångquist [87]. The output signal $\hat{I}_p$ from the phasor estimator in figure 5.2 then represents $I_{var}$.

$I_{var}$ is representing a faster phasor of $\tilde{i}(t)$ than $I_{SS}$. $I_{var}$ can also be found in figure 5.1 as the output signal from block diagram $PE\omega_{max}$.

The difference between $I_{var}$ and $I_{SS}$ is later sent into the linear model to represent the input signals to the small-signal model of the TCSC, see section 5.3.4.

Figure 5.3: Time-domain response to a sudden step in the magnitudes of the three input signals $i_a(t)$, $i_b(t)$, and $i_c(t)$ from 0 to 1, see figure 5.1. $i_a(t)$, $i_b(t)$, and $i_c(t)$ are symmetric.
Figure 5.4: In the upper left corner the signals $I_{var}$ and $I_{SS}$ are shown in steady state, see 1). In the upper right corner and in the lower half a small initial angle difference in the current is added, see 2) and 3). See also figure 5.1.

In figure 5.3 the responses in $\hat{I}_{av}$, $\hat{I}_p$, and $\hat{I}_n$ are shown for a sudden and simultaneous step in the magnitudes of the three currents $i_a(t)$, $i_b(t)$, and $i_c(t)$ from 0 to 1, see figure 5.1. $i_a(t)$, $i_b(t)$, and $i_c(t)$ are symmetric and the time constant $T = 0.01(s)$ in the figure. That time constant gives the three filters in the phasor estimator a 3 (dB) bandwidth at 15.92 (Hz), see Lennartson [39] and Schmidtbauer [74]. Therefore, the setting of time constants $T_{F1}$ and $T_{F2}$ sets a bandwidth of frequencies that we send into the linear model of the TCSC.

Later in chapter 6 the values of $T_{F1}$ and $T_{F2}$ are modified to get a good matching between the original TCSC model and the linear models. The values of $T_{F1}$ and $T_{F2}$ are in the suggested intervals earlier mentioned in this section.

### 5.3.4 Input signals to the linear model

By creating the difference between $I_{var}$ and $I_{SS}$, see figure 5.1, we get a current $\Delta I$ that is $\neq 0$ during non steady-state situations and = 0 after some time in a steady state.

As mentioned earlier, $I_{var}$ and $I_{SS}$ are in steady-state pointing in the
same direction with the same length, see 1) (in the upper left corner) in figure 5.4.

**Angle difference in the current**

When an angle difference occurs in the system which means that the magnitude of the incoming current $i_{\text{line}}$ to the TCSC is constant but its angle changes, then the lengths of $I_{\text{var}}$ and $I_{SS}$ remain the same but the angles of them change and $I_{\text{var}}$ changes faster than $I_{SS}$, see the upper right corner marked as 2) in figure 5.4. The difference between $I_{\text{var}}$ and $I_{SS}$ is marked as $\Delta I$ in the lower part marked as 3) in figure 5.4. When a small angle difference occurs in the current, this should be represented as a pure $\Delta I_q$ deviation into the linear model in figure 5.1. This is achieved by turning $\Delta I$ clockwise (formally the coordinate system is turned) with the angle of $I_{SS}$, i.e., $\theta$. That signal is represented with $\Delta \tilde{I}$, see 3) in figure 5.4 and figure 5.1. In the lower part of figure 5.4 we can see that $\Delta \tilde{I}$ is pointing in a pure $q$-axis direction.

Finally, the real and imaginary parts of $\Delta \tilde{I}$ are sent into the linear model, $\Delta \tilde{I}_d$ and $\Delta \tilde{I}_q$ respectively.

From the above we can see that when it occurs a small angle difference in the current, only $\Delta \tilde{I}_q$ is influenced.

In a steady-state, the current through the TCSC is in local TCSC coordinates of pure $d$-component character. Since then, the local $q$-component represents an angle difference and a change in the $d$-component represents a change in the magnitude of the current through the TCSC.

**Magnitude difference in the current**

When a magnitude difference occurs in the current, the angles of $I_{\text{var}}$ and $I_{SS}$ remain the same but the lengths of them change and $I_{\text{var}}$ changes faster than $I_{SS}$, see the upper right corner marked as 2) in figure 5.5. The difference between $I_{\text{var}}$ and $I_{SS}$ is marked as $\Delta I$, see 3) in figure 5.5. When a small magnitude difference occur in the current, this should be represented as a pure $\Delta I_d$ deviation into the linear model in figure 5.1. This is also here done by turning $\Delta I$ clockwise with the angle of $I_{SS}$, i.e., $\theta$. That signal is represented with $\Delta \tilde{I}$, see the lower part marked as 3) in figure 5.5. In the figure we can see that $\Delta \tilde{I}$ is pointing in a pure $d$-axis direction.

From the above we can see that when it occurs a small magnitude difference in the current, only $\Delta \tilde{I}_d$ is influenced.
Figure 5.5: In the upper left corner the signals $I_{\text{var}}$ and $I_{SS}$ is shown in steady state, see 1). In the upper right corner and in the lower half a small initial magnitude difference in the current is added, see 2) and 3). See also figure 5.1.

### 5.3.5 Output signals from the linear model

The output signals from the linear model are $\Delta \tilde{U}_d$ and $\Delta \tilde{U}_q$ respectively, see figure 5.1. They create the real and imaginary parts of the complex signal $\Delta \tilde{U}$. That signal has to be turned counter-clockwise with the angle $\theta$ so that the result from the linear model is transformed to the same reference frame as the steady-state model. The output from that transformation is $\Delta \bar{U}$ where $\theta$ is the argument of the steady-state current $I_{SS}$ in figure 5.1.

### 5.3.6 Steady-state model

In the steady-state model of the power system component, a rather simple fundamental frequency calculation is done. Since the studied power system component is a Thyristor-Controlled Series Capacitor, TCSC, the equation for $U_{SS}$ is:

$$U_{SS} = \frac{-jX_c \cdot \text{Ref}}{Z_{\text{base}}} I_{SS}$$

(5.12)

where $U_{SS}$ is the complex steady-state voltage in p.u., $I_{SS}$ is the complex...
steady-state current in p.u., $X_c$ is the capacitive reactance of the capacitor in $\Omega$, $Re f$ is a reference value specified for the TCSC which fundamental capacitive reactance the TCSC should produce, and $Z_{\text{base}}$ is the base impedance of the component. ($Z_{\text{base}} = \frac{U_{\text{base}}^2}{S_{\text{base}}}$, where $U_{\text{base}}$ is the phase-to-phase rms base voltage of the TCSC-nodes and $S_{\text{base}}$ is the three-phase base power of the network).

The sum of $U_{SS}$ and $\Delta U$ in figure 5.1 represents the total response from the power system component, see $U$ in figure 5.1.

The $d$- and $q$-components of $U$ express the voltage drop of the TCSC and represents the dynamic behaviour of the TCSC.

5.4 Summary

In this chapter an interface has been introduced which aim is to connect the linear models to the surrounding power system.

The interface system is slowly sliding from one steady state to another. This feature of the interface makes the linear model having a timely response for subsequent disturbances.

Earlier in publications of the subject of the thesis, see Persson [55, 58, 59, 61, 62] another interface system has been used. However, in chapter 6 it can be seen that the interface system developed here in the thesis provides a better agreement between the developed linear models and the original TCSC model compared to the earlier publications.
Chapter 6

Time-domain simulations

This chapter compares time-domain simulations with the original TCSC model and the created linear models. First, we tune the interface system between the linear models and the rest of the power system and then comparisons are made for three different cases. The event in the first case is in the same range as was used when the linear models were developed in chapter 4. The second case contains a larger disturbance and the third case simulates a disturbance wherein the power system is in another operating point.

6.1 Background

To control the accuracy of the linear models of the TCSC, comparisons are made with the linear models and the original TCSC model. Since the linear models have been developed under the assumptions for being valid for small-signal perturbations, the perturbations cannot be too large. The assumption on which a linearization is built is that in a narrow interval of the input signals, the component behaves linearly. Therefore, the created linear models are valid for such small disturbances and the accuracy may decrease when larger disturbances are applied.

6.2 Settings of the interface system

Before we can start using our linear models and validate them we have to set the filter constants of the interface system that was introduced in chapter 5. To do this we return to our stiff identification system that was used earlier in chapter 4.
In the interface system we have to separate the 'steady-state behavior' of the studied component that should be sent into the Steady-state model (see the lower part of figure 5.1) from the 'faster dynamic response' of the studied component that should be sent into the linear model (shown in the upper part of figure 5.1).

For our study object the steady-state part of the TCSC models a constant capacitive impedance.

Therefore we start investigating how slow variations in the line current $i_{\text{line}}$ has to be to get a response from the original TCSC model equal to a constant capacitive impedance, i.e., a fixed capacitance since the control of the TCSC is set to produce a constant capacitance.

### 6.2.1 Setting the time constant $T_{F2}$ in the interface

In figure 6.1 the response in the impedance of the original TCSC is shown for four different variations in $i_{\text{line}}$. $i_{\text{line}}$ has been varied as the earlier introduced disturbance number $n = 2$ in figure 4.13 but with four different speeds of the disturbance.

![Figure 6.1: Investigations in how time constant $T_{F2}$ should be set in PE$\omega_{SS}$.](image)

The graph marked with $T = 0.00001$ (s) in figure 6.1 shows the impedance that the TCSC creates when the current through it is varied almost as a
step, it is a step filtered through a first-order filter with the time constant 
\( T = 0.00001 \) (s); \( \frac{1}{1+sT} \) so that the disturbance is \( \frac{0.193 \cdot 0.1}{\sqrt{2}}(\theta(t-3) - e^{-\frac{1}{T}(t-3)}) \) 
for \( t \geq 3 \) (s) in both the \( d \)- and \( q \)-direction of \( i_{\text{line}} \), as shown in figure 6.2.

The impedance of the TCSC, \( Z_{\text{TCSC}} \), which is shown in figure 6.1 is 
calculated as,

\[
Z_{\text{TCSC}} = \frac{u_d + ju_q}{i_d + ji_q} \cdot \frac{U_{\text{base}}}{S_{\text{base}}}
\]

where \( u_d, u_q, i_d, \) and \( i_q \) have been filtered through the filter introduced in section 3.4 to decrease the amount of harmonics.

<table>
<thead>
<tr>
<th>( T ) (s)</th>
<th>0.00001</th>
<th>0.05</th>
<th>0.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \max \left</td>
<td>\frac{\Delta X}{X_{t=0}} \right</td>
<td>) (p.u.)</td>
<td>0.179</td>
<td>0.106</td>
</tr>
</tbody>
</table>

Table 6.1: Maximum difference in reactance in figure 6.1.

Figure 6.2: Slow variations in the current that produce an almost constant impedance of the TCSC, here \( T = 2.0 \) (s).

As can be seen, the TCSC changes its impedance during the response
and is creating both a negative and a positive resistance during the response.

The loop of the impedance of the TCSC indicated as $T = 0.00001$ (s) takes
$\approx 0.5$ (s) to complete and it is done in a clockwise direction starting and
ending in the center of the figure at $(R_{t=0} = 0$ (\Omega) and $X_{t=0} = -26.54$ (\Omega)).

The currents for the slow variation when $T = 2.0$ (s) are shown in figure
6.2.

By studying the response of the linear models of the TCSC in section 6.4,
an adequate value of the time constant $T_{F2}$ has been found in an empirical
way and has been set to $T_{F2} = 0.5$ (s). This value is in resemblance with
the results shown in figure 6.1. The time constant $T_{F2} = 0.5$ (s) gives a 3
(dB) bandwidth at 0.32 (Hz).
6.2.2 Setting the time constant $T_{F1}$ in the interface

We will continue to use the stiff identification system to set the time constant $T_{F1}$ which together with the linear models sets the bandwidth of the linear models.

The time constant $T_{F1}$ in the interface system limits/decreases the amount of high frequencies that will be sent into the linear models of the TCSC, i.e., $T_{F1}$ in combination with the frequency spectra (see figures 4.22, 4.27, and 4.28) of each linear model set the bandwidths of the linear models, see figure 6.3.

![Graphs](image)

Figure 6.4: The response in $u_d$ and $u_q$ for disturbances in the stiff power system, $T_{F1} = 0.020$ (s) and $T_{F2} = 2$ (s).

In figures 6.4 and 6.5 we can see the response of the system including the interface, Steady-state model, and the linear model that was created from frequency analysis. In figure 6.4 the time constant $T_{F1} = 0.020$ (s) and in figure 6.5 the time constant $T_{F1} = 0.015$ (s). In both figures $T_{F2} = 2$ (s). From figures 6.4 and 6.5 we can see that the larger time constant $T_{F1}$, the slower response in $u_d$ and $u_q$. This is easiest to see when comparing the lower right corner of figures 6.4 and 6.5. From Ångquist [87] the time constant $T_{F1}$ has been recommended to be something in the interval $0.004 < T_{F1} < 0.4$ (s).
Figure 6.5: The response in $u_d$ and $u_q$ for disturbances in the stiff power system, $T_{F1} = 0.015$ (s) and $T_{F2} = 2$ (s).

By studying the response of the linear models of the TCSC in section 6.4, an adequate value of the time constant $T_{F1}$ has been found in an empirical way so that the error is as small as possible between the simulated curves of the linear models and the original TCSC model. During this process it has been found that $T_{F1} = 0.005$ (s) gives a good matching. The value of $T_{F1}$ gives a 3 (dB) bandwidth at 31.8 (Hz).

### 6.3 Description of the test network

The used power system for the time-domain comparisons is shown in figure 6.6.

The two synchronous machines S1 and S2 are modelled with realistic ninth-order machine models including saturation, see Johansson [27]. Also exciters, governors, and turbines are modelled with their dynamic models, see appendix D.

The machines are connected directly to the 500 kV level, i.e., no transformers are included in the system. The impedance of the line between Bus D and Bus E is $R = 17.7$ (Ω) and $X_L = 266.0$ (Ω) representing a 1 000 km
long transmission line. The impedance of the line between Bus A and Bus B is $X_L = 0.266 \, \Omega$. The control algorithm of the TCSC is set to produce the fundamental capacitive reactance $X_C = 26.54 \, \Omega$ in order to compensate 10% of the line reactance $X_L$. The real power consumed at Bus A is $P = 500 \, \text{MW}$ at nominal voltage and of constant impedance character and the consumed reactive power is $Q = 0 \, \text{Mvar}$.

One difference between the system used during the identification procedure in chapter 4 and the system used here in the three time-domain simulation cases is that the latter one contains dynamic models in the surrounding power system, for instance the two synchronous machines. In the identification procedure in chapter 4 the power system was modelled as stiff, i.e., no dynamics existed outside the TCSC. That is not the situation in the system shown in figure 6.6. So from now on the input signals to the linear models will vary rather freely.

6.3.1 Description of the three cases of time-domain simulation

The following three cases have been constructed to investigate the validity of the linear models with the original TCSC model. They are,

a) the event in the power system reminds of the event that took place
when the linear models were identified, i.e., the load in Bus A is increased with some 10% at \( t = 16 \) (s).

\( b) \) a larger disturbance takes place in the power system than in case \( a \). The load in Bus A is increased with some 22% at \( t = 16 \) (s) (\( \Delta P = 100 \) (MW) and \( \Delta Q = 100 \) (Mvar)). This is a larger event than the one that took place when the linear models were identified.

\( c) \) the event is the same as in case \( a \) but the power system is before the event in another operating point. Case \( c \) has been constructed to check the validity of the linear models in another operating point of the power system.

For all cases three simulations are done, one with the original TCSC model as in figure 6.6, and two with each of the linear models, see figure 6.7. In case \( a \) and case \( b \), the initial conditions are the same as the ones used when the linear models were developed in sections 4.7 – 4.8. In case \( c \) the initial conditions are not the same as in case \( a \) and \( b \).

In the simulations with the linear models a fixed series capacitance \( X_C \) models the TCSC until \( t = 4 \) (s). The fixed series capacitance has the same capacitive reactance as the TCSC earlier modeled, i.e., \( X_C = 26.54 \) (Ω). At
\( t = 4 \) (s), the fixed series capacitance \( X_C \) is replaced by \( M(s) \) as the two switches indicate in figure 6.7. After \( t = 4 \) (s) the block diagram marked with \( M(s) \), including a) the selected linear model, b) the Steady-state model as well as the c) interface system (described in chapter 5) is switched on. \( M(s) \) replaces the series impedance \( X_C \).

In all the following diagrams, results are compared from time-domain simulations with the original TCSC model and the linear models, i.e., figures 6.6 and 6.7.

6.4 Case a

In case a the real power of the load in Bus A is increased at \( t = 16 \) (s) with 10%, \( \Delta P = 50 \) (MW). However, the load is modelled with constant impedance character and therefore the load increment is less than 50 (MW) since the bus voltage in the load node decreases when the load is increased. Before \( t = 16 \) (s), the load is initially \( P_0 = 500 \) (MW) and \( Q_0 = 0 \) (Mvar).

Three simulations are done, one with the original TCSC model, one with the linear model developed from Transient Analysis, and one with the linear model developed from Frequency Analysis.

Since the load is increased with 10% and the initial steady-state is the same as when the linear models of the TCSC were identified, case a is in the same range as when the linear models of the TCSC were identified.

In figure 6.8, the current into the original TCSC model and the voltage drop over the TCSC are shown. The signals from the original TCSC model have been filtered before plotting. The used filter is the one shown in section 3.4.

In figure 6.9, \( i_q \) is plotted as a function of \( i_d \). In steady state \( (t \leq 16 \text{ s}) \), the current is in the center of the circle and during the disturbance the change in current gets just a little larger than 10% of the steady-state value of the current, see the dashed circle that has the diameter equal to 10% of the current’s steady-state value. The change in current is therefore in the same range as when the linear models of the TCSC were identified in sections 4.7 and 4.8.

In figure 6.10 the real and reactive power through the TCSC into node B is shown.

In figure 6.11 we have added the graphs when the TCSC is represented only with the Steady-state model, see the lower part of figure 5.1. We can see that it is during the first seconds that the TCSC behaves different compared to the Steady-state model. The functionality of the linear model
Figure 6.8: $i_d$, $i_q$, $u_d$, and $u_q$ for the original TCSC model in case a.

Figure 6.9: The currents $i_d$ and $i_q$ through the TCSC from figure 6.8 depicted in one diagram, case a.
Figure 6.10: Real and reactive power through the TCSC into node B, case a.

Figure 6.11: $i_d$, $i_q$, $u_d$, and $u_q$ for the original TCSC model (in blue) and the Steady-state model (in black) in case a.
Figure 6.12: $i_d$, $i_q$, $u_d$, and $u_q$ for the original TCSC model (in blue), the linear model identified from Transient Analysis (in black), and the linear model identified from Frequency Analysis (in red) in case $a$.

Figure 6.13: Focused picture of $u_d$ and $u_q$ for the original TCSC model (in blue), the linear model identified from Transient Analysis (in black), and the linear model identified from Frequency Analysis (in red) in case $a$. 

is here neglected and the time constant \( T_{F2} = 0.5 \) (s) obtaining that the Steady-state model solely represents the TCSC.

In figures 6.12 and 6.13 the results from the original TCSC model (in blue), the linear model identified from Transient Analysis (in black), and the linear model identified from Frequency Analysis (in red) are shown. In the graphs in figure 6.13 we can see that there are small differences between the results from the two linear models. The two linear models follow the original TCSC model except for the two time points \( \approx 16.3 \) and \( \approx 17.3 \) (s).

### 6.5 Case b

In case b, the real power at Bus A is \( P = 500 \) (MW) until \( t = 16 \) (s) when it is increased with 100 (MW). Also the reactive power at Bus A is increased at \( t = 16 \) (s) from 0 to 100 (Mvar). This implies the same initial state as in case a, but the disturbance is larger (the load change is 22\% of the initial load). However, the load is modelled with constant impedance character and therefore the load increment is less than 100 (MW) and 100 (Mvar) respectively since the bus voltage in the load node decrease when the load is increased.

In figures 6.14 and 6.15 the results from the original TCSC model, the linear model identified from Transient Analysis (in black), and the linear model identified from Frequency Analysis (in red) are shown.

Figure 6.14 shows that the identified linear models have less matching for disturbances that are larger than was previously used in the identification process. The linear models have larger responses in \( u_d \) and \( u_q \) compared to what the original TCSC model has. The resemblance are less between the original TCSC model and the linear models compared to in case a.

### 6.6 Case c

It is important to check if the linear model is valid in another initial steady state than the one used during the identification in sections 4.7 and 4.8. Therefore in case c, the real power at Bus A is \( P = 700 \) (MW) until \( t = 16 \) (s) when it is increased with 50 (MW) (7.1\%). The load is modelled with constant impedance character and therefore the load increment is less than 50 (MW) since the bus voltage in the load node decrease when the load is increased.

In figures 6.16 and 6.17 the results from the original TCSC model, the linear model identified from Transient Analysis (in black), and the linear
Figure 6.14: $i_d$, $i_q$, $u_d$, and $u_q$ for the original TCSC model (in blue), the linear model identified from Transient Analysis (in black), and the linear model identified from Frequency Analysis (in red) in case $b$.

Figure 6.15: Focused picture of $u_d$ and $u_q$ for the original TCSC model (in blue), the linear model identified from Transient Analysis (in black), and the linear model identified from Frequency Analysis (in red) in case $b$. 

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Figure 6.16: $i_d$, $i_q$, $u_d$, and $u_q$ for the original TCSC model (in blue), the linear model identified from Transient Analysis (in black), and the linear model identified from Frequency Analysis (in red) in case $c$.

Figure 6.17: Focused picture of $u_d$ and $u_q$ for the original TCSC model (in blue), the linear model identified from Transient Analysis (in black), and the linear model identified from Frequency Analysis (in red) in case $c$. 
model identified from Frequency Analysis (in red) are shown.

Figure 6.14 shows that the identified linear models have less matching in another working point that was previously used in the identification process. The resemblance are less between the original TCSC model and the linear models compared to in case a.

6.7 Summary

In this chapter we have set the filter constants of the interface system that was earlier introduced in chapter 5.

In the interface system we have separated the 'steady-state behavior' of the TCSC from the 'faster dynamic response'. The 'steady-state behavior' of the TCSC was sent into the Steady-state model and the 'faster dynamic response' was sent into the linear models. The steady-state part of the TCSC models a constant capacitive impedance.

The created linear models have been compared in time-domain simulations. Both of the linear models provide results which are similar to the original TCSC model although the linear models showed more resemblance in case a than in cases b and c.

The responses from the linear models are close in the three cases. This similarity between the linear models have earlier been shown in the models' frequency responses, see figures 4.22, 4.27, and 4.28 as well as in the time-domain responses shown in figures 4.19, 4.20, and 4.29.

Time-domain simulations with the linear models are much faster to perform compared to time-domain simulations with the original TCSC model. The simulations with the linear models demand a few per cent of physical time compared to the simulations with the original TCSC model.

In time-domain simulations with the linear models, the harmonics that were included in simulations with the original TCSC model are not present since the bandwidth of the linear models is reduced. The slow dynamic of the original TCSC model is however represented by the linear models.

The interface system developed in the thesis provides good agreement between the developed linear models and the original TCSC model. In earlier publications carried out within this thesis work another interface system has been used. The one used herein is better than the previous one used in Persson [55].

The interface developed in this thesis contains a 'sliding' effect, i.e., the interface is slowly moving to a new steady state at the same time as it is separating two bandwidths; allowing frequencies up to one bandwidth to be
sent into the Steady-state model of the component and frequencies up to a second bandwidth to be sent into the linear model representing the 'faster dynamic response' of the component. The interface in Persson [55] did not contain any of these characteristics. It was a static interface, valid only for one steady state and did not separate any bandwidths.
Chapter 7

Small-signal stability analysis

This chapter contains comparisons of small-signal stability analysis (linear analysis) of the two linear models earlier developed in chapter 4.

7.1 Background

Linear models allow for the application of small-signal stability analysis techniques (linear analysis) to complement the information obtained from non-linear time-domain simulations and often allow for a better understanding of the dynamic characteristic of the system than that obtained from the inspection of time-domain simulations alone. Although the non-linear nature of a power system must be recognized, in many cases a linearized system representation allows for a more efficient means of analysis, Sanchez-Gasca [73].

When modelling the TCSC in detail it contains power electronics (thyristors) that switches in and out several times for each cycle. As a result of the switchings and the TCSC’s non-linear control algorithm, the TCSC behaves non-linear. A time-domain model of a TCSC can be made very accurate and is essential for studying large disturbances in the vicinity of the TCSC. However, these models are not suitable for small-signal stability analysis (linear analysis) of a power system including a TCSC, Othman [48].

If we would perform a linearization of the power system including the detailed original TCSC model we would have problems with the thyristor switchings and the change in setup of differential equations of the TCSC and the result from the linearization would be hard to interpret. The linearization would depend on the actual thyristor status as well as the actual status of the control algorithm. Eigenvalues would not occur consistent between different time points of linearization within the same steady state.
The control algorithm of the TCSC contains non-rational functions like delays and that makes the linearization troublesome. In Persson [58] this has been shown and there we can see why it is necessary to represent the TCSC with a linear model to make it possible to linearize the power system.

In section 7.2, small-signal stability analysis (linear analysis) is performed for the following representations of the TCSC:

1. Linear model of the TCSC identified with transient analysis.
2. Linear model of the TCSC identified with frequency analysis.

The used power system is the same as in chapter 6. The power system is simulated in instantaneous value mode (momentary values mode).

7.2 Studying the two linear models

When performing linear analysis of a power system, it is linearized in the current operating point. To do so, the state matrix (the $A$-matrix) has to be constructed. In a power system simulation software a process, as presented in chapter 2, is implemented to obtain such an $A$-matrix.

In the following, the power system in figure 7.1 is linearized. $M(s)$ represents a steady-state model, a linear model as well as the surrounding interface of the TCSC. The linear model is in section 7.2.1 represented by the identified model from transient analysis and in section 7.2.2 represented by the identified model from frequency analysis. The power system is linearized in the pre-disturbance steady state as earlier described in cases $a$ and $b$ in section 6.3.1.

The interface that earlier was shown in figure 5.1 (repeated here in figure 7.2) includes two block diagrams indicated as $PE\omega_{SS}$ and $PE\omega_{max}$. In figure 5.2 a detailed picture of these block diagrams can be found.

It can be shown that the block diagram $PE\omega_{SS}$ (which creates a slow phasor of $\hat{i}(t)$ indicated as $I_{SS}$) introduces two eigenvalues in $\lambda \approx -3/T_{F2} = -3/0.5 = -6$. It can also be shown that the block diagram $PE\omega_{max}$ (which creates a faster varying phasor of $\hat{i}(t)$ indicated as $I_{var}$, varying faster than $I_{SS}$) introduces two eigenvalues in $\lambda \approx -3/T_{F1} = -3/0.005 = -600$.

7.2.1 Studying the linear model derived from transient analysis

The calculated complex eigenvalues are shown in table 7.1 when the TCSC is represented by the linear model developed from transient analysis. Real-
Figure 7.1: The power system wherein the TCSC is modelled with $M(s)$ containing a steady-state model, a linear model as well as an interface system.

Figure 7.2: Interface to the Linear model and the Steady-state model. Bold lines represent complex quantities.
Table 7.1: Complex poles with real part greater than $-200$ with the linear model identified from transient analysis. Eigenvalues $\lambda_1$ – $\lambda_{14}$ are found in phasor mode when the TCSC is modelled as a constant series capacitor.

The eigenvalue-pair $\lambda_{1,2}$ in table 7.1 is an inter-area oscillation. After a disturbance, power oscillates between the two areas of the power system with the frequency $1.13$ (Hz) during the recovery of the power system. The TCSC interconnects the two areas. That oscillation can be observed in time-domain simulations of the power system, see figures for case $a$ – case $c$ in sections 6.4 - 6.6.

Eigenvalues $\lambda_3$ – $\lambda_{12}$ in table 7.1 occur when the synchronous machines include exciters, i.e., when the synchronous machines are modelled without exciters these eigenvalues do not exist. Eigenvalue-pair $\lambda_{13,14}$ occurs when the synchronous machines include turbines and governors.

The less damped eigenvalue-pair in table 7.1 is $\lambda_{11,12}$. However, the imaginary part of that eigenvalue-pair is very small and therefore impossible to detect in a time-domain simulation.

Eigenvalues $\lambda_{15}$ – $\lambda_{24}$ are derived from the poles of the transfer functions of the linear model that were identified in section 4.7.

Eigenvalue-pairs $\lambda_{25,26}$ and $\lambda_{27,28}$ come from the system’s power frequency and are generated from the actual implementation of the interface.
together with the linear models.

The total number of eigenvalues is 57 where 10 comes from the linear model of the TCSC.

7.2.2 Studying the linear model derived from frequency analysis

The calculated complex eigenvalues are shown in table 7.2 when the TCSC is represented by the linear model developed from frequency analysis. Real-valued eigenvalues are not listed.

Eigenvalues $\lambda_1 - \lambda_{14}$ occur also in this subsection as in subsection 7.2.1. See subsection 7.2.1 for comments about them.

Eigenvalues $\lambda_{15} - \lambda_{30}$ are derived from the poles of the transfer functions of the linear model that were identified in section 4.8.

The total number of eigenvalues is 64 where 17 comes from the linear model of the TCSC.

Also the real-valued pole derived from $G_{11}(s)$, see equation (4.18), is included but not shown in table 7.2.

Eigenvalue-pairs $\lambda_{31,32}$ is a result of a fusion of two real-valued eigenvalues. Earlier in section 7.2.1 this eigenvalue-pair constituted of two real-valued eigenvalues $\lambda = -14.63$ and $\lambda = -18.70$.

7.3 Summary

In this chapter, two linear models of a TCSC have been utilized. The small-signal stability analysis (linear analysis) of the studied power system have been made in instantaneous value mode.

In Persson [58] is has been shown how troublesome it is to perform linear analysis using the original TCSC model. By replacing the original TCSC model with a linear model, linear analysis is possible to perform.

By comparing table 7.1 and 7.2 we can see that the result from linear analysis of the system is the same, independent of which linear model that is used; the eigenvalues $\lambda_1 - \lambda_{14}$ are in both tables consistent.

Among the listed eigenvalues we can find the poles from the linear models which have been identified from the original TCSC model.

When representing the TCSC with the linear models, all eigenvalues exist, no matter when linear analysis is done in a steady-state. This is in contradiction with linear analysis performed with the original TCSC model.
Table 7.2: Complex poles with real part greater than -200 with the linear model identified from frequency analysis. Eigenvalues $\lambda_1 - \lambda_{14}$ are found in phasor mode when the TCSC is modelled as a constant series capacitor.
Chapter 8

Summary and conclusions of the thesis

Summary and conclusions are given in this chapter as well as a list of possible future work.

8.1 Conclusions

Since the thesis is focused on two main subjects the conclusions are divided in the following two subsections.

8.1.1 Linearization methods

Three linearization methods have been evaluated in the thesis. These are,

- Analytical Linearization,
- Forward-Difference Approximation,
- Center-Difference Approximation.

The analytical linearization is achieved by writing the models of the power system components in terms of the most elementary components for which a linearization formula is known. The forward-difference and center-difference approximation methods are numerical methods.

When analyzing the electromechanical mode of a small power system, the impact of the difference in linearization methods have been investigated.

In the thesis it has been shown that the perturbation size $h$ influences the calculated eigenvalues when the forward-difference approximation and
center-difference approximation methods are used. The perturbation size $h$ influences more the forward-difference approximation method than the center-difference approximation method. However, for small values of the perturbation size $h$, results from all the three methods are very similar.

The results from the numerical center-difference approximation method are very close to the analytical linearization method also for large values of $h$.

In the studied system it has been shown that the larger perturbation size $h$ when using the forward-difference approximation method, the smaller imaginary parts of the eigenvalue-pair is obtained. This is in conformity with time-domain simulations of the power system studied in the thesis; the larger perturbations the power system is going through, the slower the responding oscillation is when the system is recovering after the fault has been removed. Whether the same conformity are obtained also in studies of other power systems could be further investigated for the forward-difference approximation method.

The forward-difference approximation and center-difference approximation methods are sensitive to the degree of non-linearity of the equations of the studied system as well as to the size of the perturbation $h$. This sensitivity is largest for the forward-difference approximation method.

### 8.1.2 Bandwidth-reduced linear models of non-continuous power system components

In the thesis a detailed original model of a TCSC has been modelled and two simplified linear models have been developed. To do this the responses of the TCSC have been studied by applying disturbances. Input and output signals to the linear models have been selected. The input signals have been disturbed with well-defined input signals. The output signals have been recorded and used for identifying linear models.

The linear models have been identified with the following two techniques,

- Transient analysis; in a time-domain simulation disturb input by input with a step from a steady-state situation,
- Frequency analysis; in a time-domain simulation from a steady state disturb input by input with sinus-formed signals with constant amplitude and varying frequency.

In the thesis the created linear models have been compared both in time-domain simulations and in small-signal stability analysis (linear analysis).
Both linear models provide results which are similar to the original TCSC model in time-domain simulations although the linear models showed more resemblance in case a than in cases b and c.

The responses from the linear models are close in the three cases. This similarity between the linear models have earlier been shown in the frequency responses as well as in time-domain responses of the linear models in chapter 4.

Time-domain simulations with the linear models are much faster to perform compared to time-domain simulations with the original TCSC model.

In time-domain simulations with the linear models, the harmonics that were included in simulations with the original TCSC model are not present since the bandwidth of the linear models is reduced. The slow dynamic of the original TCSC model is however represented by the linear models.

In earlier publications in the project, another interface system has been used. The interface system developed in the thesis provides a better agreement between the developed linear models and the original TCSC model than the earlier used interface system. The interface system is slowly sliding from one steady state to another. This feature of the interface makes the linear model having a timely response for subsequent disturbances.

When performing small-signal stability analysis (linear analysis) of the system with the linear models of the TCSC, all eigenvalues exist, no matter when it is done in a steady-state. This is in contradiction with linear analysis performed with the original TCSC model. In Persson [58] is has been shown how troublesome it is to perform linear analysis using the original TCSC model.

In chapter 7 we could see that the result from linear analysis of the system was the same, independent of which linear model that was used. Among the listed eigenvalues we could find the poles from the linear models.

With the linear models the fundamental behavior of the TCSC is included in the linearization.

8.2 Possible future work

Here follows a list of possible future work that can be done.

- It can be further investigated whether the forward-difference approximation method provides results that are related to real (non-linear) time-domain simulations with different sizes of perturbations. In this thesis it has been investigated whether there is a coupling between a) linearization using the forward-difference approximation method by
varying the size of the disturbance and b) time-domain simulations of small and large disturbances. This has been investigated for a small power system. It can be further investigated also for other and larger power systems.

- Translate the linear models to fundamental frequency models. To convert the models from $dq0$-representation to symmetrical components.

- Include the reference signal to the input vector of the linear models of the TCSC and with that build expanded linear models. Later on, add a regulator that changes the reference value of the produced reactance of the TCSC. In this thesis the reference signal has been constant in the created linear models of the TCSC. By doing this expansion of the models it will be possible to see how the linear models can include the modelling of damping of power oscillations.

- Apply the identification methods to other power system components than a TCSC.

- Continue the validation of the linear models of the TCSC developed in the thesis and explore how valid they are for larger perturbations as well as for other power systems.
Appendix A

System data for the classical machine

This appendix contains system data and power-flow solution for the linearization of the classical machine model in section 2.5. The example is given as example 12.2 in Kundur [35], p. 732. It is used in section 2.5 as an example of a linearization of a power system. The calculation in Kundur [35] is done with round-offs. Therefore, the example here is re-calculated using six digits accuracy.

A.1 System data

The system data of the classical machine shown in figure A.1 are shown in table A.1.

<table>
<thead>
<tr>
<th>$x_d$ (p.u.)</th>
<th>$D$ (1/s)</th>
<th>$H$ (s)</th>
<th>$\omega_0$ (rad/s)</th>
<th>$x_{line}$ (p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.30</td>
<td>0.0</td>
<td>3.50</td>
<td>376.991118</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Table A.1: Settings of the classical machine model in section 2.5.

A.2 Power-flow solution

The power-flow solution of the classical machine and the small power system shown in figure A.1 is shown in table A.2. The power-flow solution is the same as in Kundur [35], here given with six digits.
Figure A.1: The classical machine model in a small test system.

<table>
<thead>
<tr>
<th>$i_d0$ (p.u.)</th>
<th>0.507750</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_q0$ (p.u.)</td>
<td>0.801572</td>
</tr>
<tr>
<td>$i_{re0}$ (p.u.)</td>
<td>0.904523</td>
</tr>
<tr>
<td>$i_{im0}$ (p.u.)</td>
<td>0.286645</td>
</tr>
<tr>
<td>$\delta_0$ (rad)</td>
<td>0.871538 (49.9354°)</td>
</tr>
<tr>
<td>$u_d0$ (p.u.)</td>
<td>0.240471</td>
</tr>
<tr>
<td>$u_q0$ (p.u.)</td>
<td>0.970469</td>
</tr>
<tr>
<td>$u_{re0}$ (p.u.)</td>
<td>0.808680</td>
</tr>
<tr>
<td>$u_{im0}$ (p.u.)</td>
<td>0.587940</td>
</tr>
<tr>
<td>$u_{re\infty\text{finitebus}}$ (p.u.)</td>
<td>0.9950</td>
</tr>
<tr>
<td>$u_{im\infty\text{finitebus}}$ (p.u.)</td>
<td>0.0</td>
</tr>
<tr>
<td>$E_{f0}$ (p.u.)</td>
<td>1.122794</td>
</tr>
<tr>
<td>$T_{e0}$ (p.u.)</td>
<td>0.90</td>
</tr>
<tr>
<td>$T_{m0}$ (p.u.)</td>
<td>0.90</td>
</tr>
<tr>
<td>$P_{m0}$ (p.u.)</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Table A.2: Power-flow results of the classical machine model in section 2.5.
A.3 Linearization of example 12.2 in Kundur [35]

The power system shown in figure A.1 is used in section 2.5. Since the calculation in Kundur [35] is done with round-offs, the result gets different compared to section 2.5 in this thesis even though the method is the same. Therefore, the example is here re-calculated using six digits accuracy. Below, the variable names are following Kundur [35].

The bus voltage at the machine bus, indicated as $E_t$ in Kundur [35] is

$E_t = 0.999818 \angle 36.0185^\circ$  \hspace{1cm} (A.1)

which is derived from the power-flow calculation.

With $E_t$ as reference phasor, the generator stator current $\tilde{I}_t$ in Kundur [35] is,

$$
\tilde{I}_t = \frac{(P + jQ)^*}{E_t^*} = \frac{0.9 - j0.3}{0.999818} = 0.900164 - j0.300055
$$

(A.2)

The stator current $\tilde{I}_t$ in Kundur [35] corresponds to $i_{re} + j i_{im}$ in figure A.1 which is calculated with the infinite bus as reference phasor as,

$$
i_{re} + j i_{im} = \tilde{I}_t \cdot e^{36.0185^\circ} = (0.900164 - j0.300055) \cdot e^{36.0185^\circ} = 0.904523 + j0.286645
$$

(A.3)

With the stator current $\tilde{I}_t$, the voltage behind the transient reactance, indicated as $\tilde{E}'$ in Kundur [35] can be calculated as,

$$
\tilde{E}' = \tilde{E}_t + j X' \cdot \tilde{I}_t = 0.999818 + j0.3 \cdot (0.900164 - j0.300055) = 1.089834 + j0.270049 = 1.122794 \angle 13.916961^\circ
$$

(A.4)

The angle by which $\tilde{E}'$ leads $E_B$ (the rotor angle $\delta_0$) is,

$$
\delta_0 = 13.916961^\circ + 36.0185^\circ = 49.935461^\circ
$$

(A.5)

$E_B$ is the voltage for the infinite bus in Kundur [35]. The synchronizing torque $K_S$ in Kundur [35] gets,
\[ K_S = \frac{E' \cdot E_B}{X_T} \cos \delta_0 = \frac{1.122794 \cdot 0.995}{0.95} \cos 49.935461^\circ = 0.756919 \] (A.6)

The linear system that should be identified is,

\[
\begin{bmatrix}
\Delta \dot{\omega} \\
\Delta \dot{\delta}
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta \omega \\
\Delta \delta
\end{bmatrix}
\] (A.7)

where the A-matrix is,

\[ A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}. \] (A.8)

Matrix element \( A_{12} \) is equal to,

\[ A_{12} = -\frac{K_S}{2H} = -\frac{0.756919}{2 \cdot 3.5} = -0.108131 \] (A.9)

The other matrix elements are the same in Kundur [35] as in section 2.5.1.

The A-matrix gets,

\[ A = \begin{bmatrix} 0 & -0.108131 \\ 376.991118 & 0 \end{bmatrix} \] (A.10)

which is the same as in section 2.5.1. The A-matrix gives the following eigenvalues,

\[ \lambda_{1,2} = 0 \pm j6.38471(1/s, rad/s) \] (A.11)

which are the same as in section 2.5.1.
Appendix B

Truncation errors for the FDA and CDA methods

This appendix contains a detailed description of the truncation errors obtained when utilizing the FDA and CDA methods as described in chapter 2.

B.1 Truncation error for the FDA method

When $A_{12}$ was calculated in section 2.5.2 with equation (2.73) we included non-linear feedback from the algebraic equations (2.37) – (2.45) to calculate the value of the time-derivative of the speed deviation $\Delta \dot{\omega}$ when the machine angle $\delta$ had been disturbed with $h$ from the equilibrium point.

When $\delta$ was disturbed, we calculated the feedback of the perturbation to the electrical torque $T_e$ in equation (2.67), below repeated as equation (B.1).

$$\Delta \dot{\omega} = \frac{1}{2H} (T_m - T_e - D \cdot \Delta \omega) \quad (B.1)$$

Since the damping constant $D$ is equal to zero in our example the last term $D \cdot \Delta \omega$ is deleted in equation (B.2).

$$\Delta \dot{\omega} = \frac{1}{2H} (T_m - T_e) \quad (B.2)$$

In the following we will rewrite equation (B.2) to derive $\Delta \dot{\omega}$'s relation with $\delta$. If we re-write equation (2.37) we get,

$$T_e = u_d i_d + u_q i_q. \quad (B.3)$$
Equation (B.3) often can be seen containing the electrical power $P_e$ (in p.u.) in the left-hand side. Since we here do not include any stator resistance and that the rotor speed is $\omega = 1.0$ we can write the electrical torque $T_e$ (in p.u.) in the left-hand side of equation (B.3), see Kundur [35], p. 175.

Instead of $u_d$ and $u_q$ we can apply equations (2.38) and (2.39) in equation (B.3). Then we get,

$$T_e = u_{re}(i_d \sin \delta + i_q \cos \delta) + u_{im}(i_q \sin \delta - i_d \cos \delta). \quad (B.4)$$

Instead of $u_{re}$ and $u_{im}$ we can apply equations (2.42) and (2.43) in equation (B.4). Since $u_{iminfinitebus} = 0$ we have excluded it in equation (B.5) below,

$$T_e = (u_{reinfinitebus} - x_{line} i_{im})(i_d \sin \delta + i_q \cos \delta) + x_{line} i_{re}(i_q \sin \delta - i_d \cos \delta). \quad (B.5)$$

In equation (B.5) we can replace $i_{re}$ and $i_{im}$ with equations (2.44) and (2.45). With that, the equation for $T_e$ gets rather simplified and becomes,

$$T_e = u_{reinfinitebus}(i_q \cos \delta + i_d \sin \delta). \quad (B.6)$$

In equation (B.6) we then replace $i_d$ and $i_q$ with the following two equations,

$$i_d = \frac{E_{f}d_0 - u_{reinfinitebus} \cdot \cos \delta}{x_d' + x_{line}} \quad (B.7)$$

$$i_q = \frac{u_{reinfinitebus} \cdot \sin \delta}{x_d' + x_{line}} \quad (B.8)$$

Equation (B.7) has been derived by replacing $u_q$ in equation (2.40) with equation (2.39). Later $u_{re}$ and $u_{im}$ have been replaced with equations (2.42) and (2.43) respectively. Finally, the relation

$$i_d = i_{re} \sin \delta - i_{im} \cos \delta \quad (B.9)$$

has been used. It can be identified from the block diagram with the exponential function $e^{-j(\frac{\pi}{2} - \delta)}$ in the lower part of figure 2.2.

Equation (B.8) has been derived by replacing $u_d$ in equation (2.41) with equation (2.38). Later $u_{re}$ and $u_{im}$ have been replaced with equations (2.42) and (2.43) respectively. Finally, the relation
\[ i_q = i_{re} \cos \delta + i_{im} \sin \delta \]  \hspace{1cm} (B.10)

has been used. It can be identified from the block diagram with the exponential function \( e^{-j(\frac{\pi}{2} - \delta)} \) in the lower part of figure 2.2.

When we replace \( i_d \) and \( i_q \) in equation (B.6) with equations (B.7) and (B.8) respectively we get,

\[ T_e = \frac{E_{fd0} \cdot u_{reinfinitebus}}{x_d' + x_{line}} \sin \delta. \]  \hspace{1cm} (B.11)

Equation (B.11) shows \( T_e \)'s relation with \( \delta \).

As mentioned earlier, the electrical torque \( T_e \) (in p.u.) in the left-hand side of equation (B.11) can be replaced with electrical power \( P_e \) (in p.u.), see Kundur [35], p. 175.

The mechanical torque \( T_m \) is equal to the electrical torque \( T_e \) since the time-derivative of the speed deviation \( \Delta \dot{\omega} \) of the machine is zero in the equilibrium point, see equation (B.2). Since \( T_m \) is constant for the classical machine it will therefore remain constantly equal to the value of equation (B.11) in the equilibrium point, i.e.,

\[ T_m = \frac{E_{fd0} \cdot u_{reinfinitebus}}{x_d' + x_{line}} \sin \delta_0 \]  \hspace{1cm} (B.12)

during the perturbation of the machine angle \( \delta \).

Now finally we can write an equation of how the speed deviation will develop when we perturb the machine angle \( \delta \) with \( h \). We put equation (B.12) of \( T_m \) into equation (B.2). \( T_e \) in equation (B.2) we replace with equation (B.11) with the perturbation \( h \) added to the machine angle \( \delta \). We get,

\[ \Delta \dot{\omega} = \frac{E_{fd0} \cdot u_{reinfinitebus}}{2H \cdot (x_d' + x_{line})} (\sin \delta_0 - \sin(\delta_0 + h)) \]  \hspace{1cm} (B.13)

where \( \delta_0 \) is the machine angle in the equilibrium point.

The matrix-element \( A_{12} \) is identified as with equation (2.73),

\[ A_{12} = \frac{\Delta \dot{\omega}}{\Delta \delta} \]  \hspace{1cm} (B.14)

where \( \Delta \dot{\omega} \) is equal equation (B.13) and \( \Delta \delta \) is equal to the perturbation \( h \). We get,

\[ A_{12} = \frac{E_{fd0} \cdot u_{reinfinitebus}}{2H \cdot (x_d' + x_{line}) \cdot h} (\sin \delta_0 - \sin(\delta_0 + h)). \]  \hspace{1cm} (B.15)
The expression \( \sin(\delta + h) \) we replace with,

\[
\sin(\delta_0 + h) = \sin \delta_0 \cos h + \cos \delta_0 \sin h \tag{B.16}
\]

and \( \sin h \) and \( \cos h \) can be Maclaurin developed as,

\[
\sin h = h - \frac{h^3}{6} + \frac{h^5}{120} + O(h^7) \tag{B.17}
\]

\[
\cos h = 1 - \frac{h^2}{2} + \frac{h^4}{24} + O(h^6). \tag{B.18}
\]

Therefore equation (B.16) can be re-written as,

\[
\sin(\delta_0 + h) = \sin \delta_0 \left(1 - \frac{h^2}{2} + \frac{h^4}{24}\right) + \cos \delta_0 \left(h - \frac{h^3}{6} + \frac{h^5}{120}\right) + O(h^6) \tag{B.19}
\]

If we put equation (B.19) in equation (B.15) and neglect orders higher than \( O(h^3) \) of the perturbation \( h \) we get the equation for \( A_{12FDA} \) which was earlier drawn as a function of \( h \) in figure 2.4,

\[
A_{12FDA} = \frac{E_{fd0} \cdot u_{reinfinitebus}}{2H \cdot (x_d' + x_{line})} (-\cos \delta_0 + \frac{h^2}{2} \sin \delta_0 + \frac{h^2}{6} \cos \delta_0) + O(h^3) \tag{B.20}
\]

To obtain the truncation error \( \Delta A_{12FDA} \), we have to subtract equation (B.20) with the analytical linearization of the same matrix element, i.e., \( A_{12AL} \). The analytical linearization of equation (B.2) with respect to the machine angle \( \delta \) is,

\[
A_{12AL} = \frac{\partial}{\partial \delta}(\Delta \omega) = \frac{\partial}{\partial \delta}(\frac{1}{2H}(T_m - T_e)) = \frac{\partial}{\partial \delta}(\frac{1}{2H}(T_m - \frac{E_{fd0} \cdot u_{reinfinitebus} \cdot \sin \delta}{x_d' + x_{line}})) = \frac{1}{2H}(\frac{E_{fd0} \cdot u_{reinfinitebus} \cdot \cos \delta_0}{x_d' + x_{line}}) \tag{B.21}
\]

In equation (B.21) we have replaced \( T_e \) with the earlier derived equation (B.11).

The truncation error obtained with the FDA method, \( \Delta A_{12FDA} \), is,

\[
\Delta A_{12FDA} = A_{12FDA} - A_{12AL} = \frac{E_{fd0} \cdot u_{reinfinitebus}}{4H \cdot (x_d' + x_{line})} (h \cdot \sin \delta_0 + \frac{h^2}{3} \cos \delta_0) + O(h^3) \tag{B.22}
\]


B.2 Truncation error for the CDA method

In equation (B.13) we could see how the speed deviation develops when we perturb the machine angle $\delta$ with $h$. When using the CDA method we want a similar expression when we perturb the load angle but with a negative perturbation $-h$. Therefore we modify equation (B.13) to,

$$\Delta \dot{\omega}_i = \frac{E_{fd0} \cdot u_{reinfinitebus}}{2H \cdot (x'_d + x_{line})} \left( \sin \delta_0 - \sin(\delta_0 - h) \right)$$

(B.23)

where $\delta_0$ is the machine angle in the equilibrium point.

The matrix-element $A_{12}$ is when using the CDA method identified as with equation (2.81),

$$A_{12} = \frac{\Delta \dot{\omega}_i}{\Delta \delta} = \frac{\Delta \dot{\omega}_{i+} - \Delta \dot{\omega}_{i-}}{2 \cdot \Delta \delta}$$

(B.24)

where $\Delta \dot{\omega}_{i+}$ is equal to $\Delta \dot{\omega}$ in equation (B.13), $\Delta \dot{\omega}_{i-}$ is derived with equation (B.23), and $\Delta \delta$ is equal to the perturbation $h$.

$A_{12}$ can then be calculated as,

$$A_{12} = \frac{E_{fd0} \cdot u_{reinfinitebus} \left( \sin(\delta_0 - h) - \sin(\delta_0 + h) \right)}{2H \cdot (x'_d + x_{line}) \cdot 2 \cdot h}$$

(B.25)

sin($\delta_0 + h$) in equation (B.25) is replaced with equation (B.19) and sin($\delta_0 - h$) is replaced with,

$$\sin(\delta_0 - h) = \sin \delta_0 \cos h - \cos \delta_0 \sin h.$$ 

(B.26)

Then equation (B.25) can be re-written as,

$$A_{12} = -\frac{E_{fd0} \cdot u_{reinfinitebus} \cdot \cos \delta_0 \sin h}{2H \cdot (x'_d + x_{line}) \cdot h}$$

(B.27)

and sin $h$ can be Maclaurin developed as in equation (B.17). If we neglect orders higher than 6 of the perturbation $h$ we get the equation for $A_{12CDA}$ which is drawn in figure 2.4,

$$A_{12CDA} = -\frac{E_{fd0} \cdot u_{reinfinitebus} \cdot \cos \delta_0 (1 - \frac{h^2}{6} + \frac{h^4}{120} + O(h^6))}{2H \cdot (x'_d + x_{line})}$$

(B.28)

To obtain the truncation error $\Delta A_{12CDA}$, we have to subtract equation (B.28) with the analytical linearization of the same matrix element, i.e., $A_{12AL}$ which also can be found in figure 2.4. $A_{12AL}$ was earlier derived.
in equation (B.21). The truncation error obtained with the CDA method, $\Delta A_{12CD\!\!A}$, is finally,

\[
\Delta A_{12CD\!\!A} = A_{12CD\!\!A} - A_{12AL} = \frac{E_{f0} \cdot u_{reinfinitebus}}{4H \cdot (x_d' + x_{line})} \cos \delta_0 \left( \frac{h^2}{3} - \frac{h^4}{60} \right) + O(h^6). \tag{B.29}
\]

By comparing equation (B.22) with equation (B.29) we can see that the term $h \cdot \sin \delta_0$ in equation (B.22) does not exist in equation (B.29). However, the term $\cos \delta_0 \cdot \frac{h^2}{3}$ exist in both equations.
Appendix C

Implementation of the TCSC

This appendix contains a detailed description of the implemented original TCSC model which has been the study object in the thesis when building linear models.

C.1 Structure of the TCSC control

The series capacitor $C$ and the series reactor $L$, sketched in figure C.1, are modelled using the corresponding standard components in the used power system simulation software Simpow, see Fankhauser [16] and [75]. The rest of the three-phase TCSC (the control algorithm and the thyristors) are split up in three parts, one for each phase. Such a part has been implemented as a user-defined system in the software and each part has four underlying functions which represent the control algorithm and the thyristors. These are,

- Phase Locked Loop (PLL)
- Booster (BOO)
- Thyristor Pulse Generator (TPG)
- Thyristors (THY)

which are shown in figure C.2 except for the function THY.

The implementation of the TCSC follows the Synchronous Voltage Reversal control algorithm developed by ABB, see Angquist [88, 89]. The control system and its functionality are formulated by using the in-built simulation language Dynamic Simulation Language (DSL) in Simpow. One
Figure C.1: Basic scheme of the Thyristor-Controlled Series Capacitor (TCSC).

Figure C.2: Control system of one phase of the TCSC. The control system uses a phase current $i_{\text{line}}$ and a phase voltage over the series capacitor $u_{\text{cap}}$ to calculate the point of time when the thyristors should start to conduct. The control systems for all phases are exactly alike.
process have been programmed for each of the four functions. The processes are connected inside a system which models one phase.

The aim of the total control algorithm is to calculate in what point of time the thyristors should start conducting and automatically block them at the following zero-crossing of the phase current through the series reactor $L$. The thyristors are included in the here sketched algorithm.

Currents and voltages in the following sections are phase currents and phase voltages. Input parameters to the TCSC system are given in table C.1.

### C.2 Phase Locked Loop

The aim of the Phase Locked Loop function (PLL) is to provide synchronization of the control system to the power system. To do this the line current $i_{\text{line}}$ is used. The capacitor voltage $u_{\text{cap}}$ is a most volatile signal which cannot be used to provide such a stable and robust reference signal for the control system, see Ångquist [88] and Jalali [26]. However, the capacitor voltage $u_{\text{cap}}$ is later used as an input signal to the Booster (BOO) and the Thyristor Pulse Generator (TPG), see sections C.3 and C.4.

The PLL extracts the argument of the line current $i_{\text{line}}$, see the signal $TetaPLL$ in the following text. In figure C.3 the phase current $i_{\text{line}}$ is shown in the upper part and $TetaPLL$ in the lower part.

Into the PLL there is basically one single input signal,

- one phase of the line current, see $i_{\text{line}}$ in figure C.2 which corresponds to $\text{Current}$ in figure C.4.

The output signals from the PLL are,

- the angle $TetaPLL$
- the complex-valued estimate of the current $\text{CurrEstimate}$
- $\text{CountRev}$, sets the Thyristor Pulse Generator (TPG) to start to calculate the next point of time when the reverse thyristor should close
- $\text{CountFor}$, sets the TPG to start to calculate the next point of time when the forward thyristor should close
- $\text{TimeRev}$, a prediction of the time point of the next minimum in the reactor current
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>2.5 (p.u.)</td>
<td>$\lambda$ explains how the TCSC is tuned. $\lambda = 2.5$ means that the oscillation frequency of the TCSC is 2.5 times the power frequency (60 (Hz)). In this case $2.5 \cdot 60 = 150$ (Hz).</td>
</tr>
<tr>
<td>$K_{PLL}$</td>
<td>18.8496 (rad/s/rad)</td>
<td>Proportional gain in the PLL.</td>
</tr>
<tr>
<td>$T_{PLL}$</td>
<td>0.3 (s)</td>
<td>Time constant in the PLL.</td>
</tr>
<tr>
<td>Phase</td>
<td>1, 2, or 3</td>
<td>Phase that the TCSC system represents.</td>
</tr>
<tr>
<td>Capac</td>
<td>$1.999 \cdot 10^{-4}$ (F)</td>
<td>Capacitance of the series capacitor, input parameter to the BOO and TPG.</td>
</tr>
<tr>
<td>$Ref$</td>
<td>2 (p.u.)</td>
<td>The reference, the value of the fundamental reactance that the TCSC creates. With $Ref = 2$, the fundamental reactance between node A and node B will be two times the fundamental reactance of the series capacitor. $Ref$ is an input parameter to BOO.</td>
</tr>
<tr>
<td>$Tau1$</td>
<td>$6.366 \cdot 10^{-3}$ (s)</td>
<td>Time constant in the PLL and BOO for phasors.</td>
</tr>
<tr>
<td>$Tau2$</td>
<td>$10.61 \cdot 10^{-3}$ (s)</td>
<td>Time constant in the PLL and BOO for averages.</td>
</tr>
<tr>
<td>$T_{BOO}$</td>
<td>0.15 (s)</td>
<td>Time constant in the BOO.</td>
</tr>
<tr>
<td>$K_{BOO}$</td>
<td>0.1 (rad/p.u.)</td>
<td>Proportional gain in the BOO.</td>
</tr>
<tr>
<td>Freeze</td>
<td>0 (s)</td>
<td>Freezes the functionality of the TCSC at a specific point of time $Freeze$. If $Freeze = 0$ the functionality of the TCSC is never frozen.</td>
</tr>
<tr>
<td>TimeOn</td>
<td>1 (s)</td>
<td>Point of time when the TCSC is activated.</td>
</tr>
</tbody>
</table>

Table C.1: Input parameters to the TCSC system.
Figure C.3: The upper part shows one phase current of $i_{\text{line}}$ and the lower part shows $TetaPLL$. $TetaPLL$ is synchronized so that it is 0 when $i_{\text{line}}$ reaches its maximum, see $t = 5.03$ (s). $TetaPLL$ is varying in the interval $-\frac{\pi}{2} \leq TetaPLL < 3\frac{\pi}{2}$.

- $TimeFor$, a prediction of the time point of the next maximum in the reactor current.

See figures C.4 and C.5 for a better understanding of the output signals in the list above.

In steady state, the real part of the complex-valued $CurrEstimate$ is equal to the magnitude of the phase current $i_{\text{line}}$ in p.u. and the imaginary part of $CurrEstimate$ is equal to 0.

As mentioned in the introduction of this subsection, the PLL extracts the argument of one phase of the line current $i_{\text{line}}$. In figure C.3 the phase current $i_{\text{line}}$ is shown in the upper part and $TetaPLL$ in the lower part. $TetaPLL$ is synchronized so that it is 0 when $i_{\text{line}}$ reaches its maximum and $-\frac{\pi}{2}$ when $i_{\text{line}}$ has its positive zero-crossing (when $i_{\text{line}} = 0$ and $\frac{di_{\text{line}}}{dt} > 0$). $TetaPLL$ is varying in the interval $-\frac{\pi}{2} \leq TetaPLL < 3\frac{\pi}{2}$.

Both output signals $CountRev$ and $CountFor$ from the PLL are integers. They are only calculated if simulated time is greater than $TimeOn$, i.e., $Time > TimeOn$. The output signal $TimeRev$ is calculated when $CountRev = 1$ and the output signal $TimeFor$ is calculated when $CountFor = 1$.

In figure C.4 it looks like $TimeRev$ and $TimeFor$ are calculated contin-
Figure C.4: Block diagram of the PLL.
Figure C.5: The upper part shows that the value of $Teta_{PLL}$ sets $Count_{Rev}$ and $Count_{For}$. When $Count_{Rev} = 1$ the value of $Time_{Rev}$ is calculated and when $Count_{For} = 1$ the value of $Time_{For}$ is calculated, see the middle part of the figure. The lower part of the figure shows that $Time_{Rev}$ and $Time_{For}$ are predicted time points for next occurring minimum, see $A_1$ and $A_2$, and maximum, see $B_1$ and $B_2$ for the reactor current $i_{rea}$. 
Figure C.6: The series capacitor voltage $u_{\text{cap}}$ and the current through the reactor $i_{\text{rea}}$ in steady state. The figure describes the situation for one phase of the TCSC.

.. figure:: figure6.png
   :alt: Figure C.6: The series capacitor voltage $u_{\text{cap}}$ and the current through the reactor $i_{\text{rea}}$ in steady state. The figure describes the situation for one phase of the TCSC.

In figure C.4 the signals marked in bold; $\text{CurrEstimate}$, $X_3$, $X_4$, $X_6$, and $X_7$ are complex-valued signals.

The block diagram with the output signals $\text{CountRev}$ and $\text{CountFor}$ in figure C.4 has the following functionality,

- if $TetaPLL$ is within the interval $-\frac{\pi}{\lambda} < TetaPLL < 0$, the output signal $\text{CountRev}$ is equal to 1, otherwise 0. With $\lambda = 2.5$ the interval becomes $-1.26 < TetaPLL < 0$, see the upper part of figure C.5 where $\text{CountRev} = 1$ for the mentioned interval of $TetaPLL$.

- if $TetaPLL$ is within the interval $\frac{\pi(\lambda-1)}{\lambda} < TetaPLL < \pi$, the output signal $\text{CountFor}$ is equal to 1, otherwise 0. With $\lambda = 2.5$ the interval becomes $1.88 < TetaPLL < \pi$, see the upper part of figure C.5 where $\text{CountFor} = 1$ for the mentioned interval of $TetaPLL$.

In the upper part of figure C.5, dashed lines mark from bottom to top the interval limits $-\frac{\pi}{2}$, $-1.26$, $1.88$, $\pi$, and $3\frac{\pi}{2}$. 

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If $CountRev = 1$, then the prediction of next point of time A, see $A_1$ and $A_2$ in the lower part of figure C.5, is calculated with equation,

$$TimeRev = Time - \frac{TetaPLL}{2\pi f_0}$$  \hspace{1cm} (C.1)

where $Time$ is actual simulated time and $f_0$ is the power frequency.

If $CountFor = 1$, then the prediction of next point of time B, see $B_1$ and $B_2$ in the lower part of figure C.5, is calculated with equation,

$$TimeFor = Time + \frac{\pi - TetaPLL}{2\pi f_0}.$$  \hspace{1cm} (C.2)

At points of time A and B, the reactor current $i_{rea}$ is at its minimum and maximum respectively. At the same time the voltage drop $u_{cap}$ over the capacitor has its zero crossings, see figure C.6.

The predictions of the time points $TimeRev$ and $TimeFor$ are later improved in the function BOO, see section C.3.

### C.3 Booster

The Booster (BOO) has the following input signals,

- one phase of the series capacitor voltage, see $u_{cap}$ in figure C.2 which corresponds to $Voltage$ in figure C.7
- the complex-valued current $CurrEstimate$
- the angle $TetaPLL$
- $TimeRev$, predicted time point from the PLL for the next positive zero-crossing in the series capacitor voltage $u_{cap}$
- $TimeFor$, predicted time point from the PLL for the next negative zero-crossing in the series capacitor voltage $u_{cap}$.

The last four input signals in the list above are calculated in the PLL function.

The output signals from the BOO are,

- $TzRevNew$, boosted points of time for the next positive zero-crossing in the series capacitor voltage $u_{cap}$
Figure C.7: Block diagram of the BOO.
• \( TzForNew \), boosted points of time for the next negative zero-crossing in the series capacitor voltage \( u_{cap} \).

As mentioned above the results from the BOO are boosted points of time for the next two zero-crossings of the series capacitor voltage, \( TzRevNew \) and \( TzForNew \), see figure C.7. \( TzRevNew \) is the adjusted point of time A and \( TzForNew \) is the adjusted point of time B in figure C.6. BOO is therefore boosting (improving) the predicted points of time \( TimeRev \) to \( TzRevNew \) and \( TimeFor \) to \( TzForNew \) from the function PLL, see section C.2. The BOO function is set to stand-by mode until \( Time \geq TimeOn \). When it is in stand-by mode, both \( TzRevNew \) and \( TzForNew \) are set equal to 0.

The functionality of the BOO can be frozen in a specific moment by specifying a positive value (> 0) to the input parameter \( Freeze \). If \( Freeze \) is specified to a positive value, the BOO function will at the same moment stop calculating new values of adjustments from the predictions of \( TimeRev \) and \( TimeFor \) which were calculated inside the PLL. In other words, when \( Time > Freeze \), the signal X27 in figure C.7 will be frozen to its current value. If the input parameter \( Freeze \) is not specified then it is set to the default value 0 which means that the BOO function is active throughout the whole simulation.

In figure C.7 the signals marked in bold; \textit{CurrEstimate}, \textit{VoltEstimate}, \textit{X3}, \textit{X4}, \textit{X6}, \textit{X7}, \textit{Z_{pu}}, and \textit{Z_{Ohm}} are complex-valued signals.

Since the phase voltage of the series capacitor voltage \( u_{cap} \) is not perfectly sinusoidal, see the lower part of figure C.6, the real and imaginary part of the complex-valued \textit{VoltEstimate} does not find constant values during a steady state wherein the TCSC is activated. However, the real part of \textit{VoltEstimate} is pending around 0 and the imaginary part of \textit{VoltEstimate} is pending around a negative value of the magnitude of the series capacitor voltage \( u_{cap} \). This since the voltage drop over a capacitor is 90\(^\circ\) behind its current. The steady-state current is estimated in the PLL and is in steady state of pure real character as was mentioned earlier in section C.2.

### C.4 Thyristor Pulse Generator

The aim of the Thyristor Pulse Generator (TPG) is to calculate the points of time when the thyristors should start conducting. Pulse signals are then generated to fire each thyristor (\textit{FireRev} and \textit{FireFor}). The pulse signals are set when the forward and the reverse thyristor should start to conduct.
respectively. As long as \((\text{Time} < \text{TimeOn})\), the TPG is in stand-by mode and \(\text{FireRev}\) and \(\text{FireFor}\) are both set equal to 0.

The equations of the TPG are shown in figure C.8 but can also be found in Ångquist [88], pp. 51-54.

The TPG has the following input signals,

- \(TzRev\text{New}\), boosted points of time from the BOO for the next positive zero-crossing in the series capacitor voltage \(u_{cap}\)
- \(TzFor\text{New}\), boosted points of time from the BOO for the next negative zero-crossing in the series capacitor voltage \(u_{cap}\)
- one phase of the series capacitor voltage \(u_{cap}\) referred to as \(\text{Voltage}\) in figure C.8
- one phase of the line current \(i_{line}\) referred to as \(\text{Current}\) in figure C.8
- \(\text{BlowRev}\), order from the function Thyristors (THY) to open the reverse thyristor
- \(\text{BlowFor}\), order from the function THY to open the reverse thyristor.

The last two input signals in the list above are not viewed in figure C.8 of the TPG.

The output signals from the TPG are,

- \(\text{FireRev}\), command to the reverse thyristor to start to conduct
- \(\text{FireFor}\), command to the forward thyristor to start to conduct.

When the calculated conducting time \(TfRev\) or \(TfFor\) in figure C.8 are equal to actual simulated time, the output signals \(\text{FireRev}\) and \(\text{FireFor}\) are set equal to 1 respectively. These output signals are input signals to the function THY and they are commands to the thyristors to start conducting. Once a thyristor has start to conduct it conducts until the following zero-crossing of the current.

\(\text{FireRev}\) and \(\text{FireFor}\) are equal to 1 as long as the corresponding thyristor is conducting and remain equal to 1 until they are reset by the signals \(\text{BlowRev}\) and \(\text{BlowFor}\) which are generated in the function THY. When \(\text{BlowRev}\) or \(\text{BlowFor}\) are equal to 1, \(\text{FireRev}\) or \(\text{FireFor}\) are set equal to 0 and the corresponding thyristors are then blocked in the function THY.
Figure C.8: Block diagram of the TPG.

Figure C.9: A block diagram inside the TPG containing an implicit equation when solving $BF$. 

BF: $U_{zFor} = \text{iline} \times X_0 \times (\Lambda \times BF - \tan(\Lambda \times BF))$

BR: $U_{zRev} = \text{iline} \times X_0 \times (\Lambda \times BR - \tan(\Lambda \times BR))$

UzFor
iline
X0
BF
UzFor = iline*X0*(Lambda*BF-tan(Lambda*BF))
UzRev
X0
BF
UzRev = iline*X0*(Lambda*BR-tan(Lambda*BR))

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Table C.2: The order of the thyristor switches.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Thyristor</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Forward</td>
</tr>
<tr>
<td>C</td>
<td>Reverse</td>
</tr>
<tr>
<td>B</td>
<td>Forward</td>
</tr>
<tr>
<td>A</td>
<td>Reverse</td>
</tr>
<tr>
<td>C</td>
<td>Forward</td>
</tr>
<tr>
<td>B</td>
<td>Reverse</td>
</tr>
</tbody>
</table>

The block diagram showed in figure C.9 is an enlargement of the block diagram in figure C.8 which solves the forward variable $BF$ with the implicit equation,

$$U_{zFor} = i_{line} \cdot X_0 \cdot (\lambda \cdot BF - \tan(\lambda \cdot BF)).$$  \hspace{1cm} (C.3)

The output signal from the block diagram in figure C.9 is $BF$.

A similar block diagram is used for the reverse variable $BR$ in figure C.8.

C.5 Thyristors

Into the function Thyristors (THY) the signals $FireRev$ and $FireFor$ are input signals which are generated in the TPG. These signals command the thyristors to start to conduct. The thyristors are conducting until the following zero-crossing of the current through the thyristors. When the current through the thyristor reaches zero the signals $BlowRev$ and $BlowFor$ respectively are set equal to 1. $BlowRev$ and $BlowFor$ are sent to the TPG and when they are equal to 1, $FireRev$ and $FireFor$ respectively are reset in the TPG and as a consequence, the thyristors are opened.

C.6 The rhythm of the thyristors

The thyristors turn on within one period in the order as given in table C.2. Every thyristor is automatically turned off when the current through it has its zero crossing, see the upper part of figure C.6.
C.7 Initialization of the TCSC control

The control of the TCSC is initialized as described below. The current $i_{line}$ and the voltage $u_{cap}$ are the main input signals to the TCSC control. Since $i_{line}$ and $u_{cap}$ are influenced when the thyristors start to conduct, the control system is influenced thereof at $t = TimeOn$. This can be seen during $1 < t < 3$ (s) after the TCSC has been turned on at $t = 1$ (s) in figures 3.10 and 3.11.

C.7.1 Initialization of the Phase Locked Loop, PLL

The angle $TetaPLL$ and the complex-valued estimate of the current $CurrEstimate$ are calculated even before the point of time $TimeOn$.

The integers $CountRev$ and $CountFor$ are not set before $TimeOn$. The predictions of time points $TimeRev$ and $TimeFor$ are not calculated before $TimeOn$.

The block diagram for the PLL can be found in figure C.4.

C.7.2 Initialization of the Booster, BOO

During $t < TimeOn$ all calculations are made in the BOO except for the two output signals $TzRevNew$ and $TzForNew$ which are set constantly equal to 0. When $t \geq TimeOn$, they are calculated.

The block diagram for the BOO can be found in figure C.7.

C.7.3 Initialization of the Thyristor Pulse Generator, TPG

During $t < TimeOn$ almost all calculations are by-passed in the TPG. The reason for this is that the result of the TPG depends on the signals $TzRevNew$ and $TzForNew$ provided from the BOO. Since $TzRevNew$ and $TzForNew$ are equal to 0 when $t < TimeOn$, the TPG cannot make any calculations. Since then, the output signals $FireRev$ and $FireFor$ both are equal to zero.

The block diagram for the TPG can be found in figure C.8.

C.7.4 Initialization of the Thyristors, THY

No thyristor switchings are executed during $t < TimeOn$. 

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Appendix D

Exciter, governor, and turbine models

This appendix contains details concerning the exciters, governors, and turbine models used in chapters 6 and 7.

D.1 Exciter model

The used exciter model in chapters 6 and 7 is shown in figure D.1 and its settings in table D.1.

The signal $V_C$ in figure D.1 is the terminal voltage, $V_S$ is an input signal from a power system stabilizer (not used in the simulations), and $REF$ is a reference adjusted initially to match the initial value of the field voltage $E_{FD}$ to get equal to $E_{FD0}$ (calculated in the associated power-flow calculation).

<table>
<thead>
<tr>
<th>$K_A$</th>
<th>20</th>
<th>$T_F$</th>
<th>1.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_A$</td>
<td>0.055</td>
<td>$T_R$</td>
<td>0.05</td>
</tr>
<tr>
<td>$T_E$</td>
<td>0.36</td>
<td>$VR_{MAX}$</td>
<td>4.0</td>
</tr>
<tr>
<td>$K_F$</td>
<td>0.125</td>
<td>$VR_{MIN}$</td>
<td>-4.0</td>
</tr>
<tr>
<td>$K_{E-S_1}$</td>
<td>0.034</td>
<td>$K_{E-S_2}$</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Table D.1: Data for exciter model shown in figure D.1.
D.2 Governor model

Both synchronous machines in the test system use the same governor setting, see figure D.2 and table D.2. The governors include a primary frequency control. The signal $P_0$ in figure D.2 is the initial mechanical power at $t = 0$ (s), $V_{REF}$ is a reference adjusted initially to match the initial value of the gate opening $Y$ to get equal to $Y_0$ (calculated in the associated power-flow calculation), and $W$ is the speed of the machine.

The gate opening $Y$ is the output signal from the governor. The gate opening $Y$ is later the input signal to the turbine model, see figure D.3.
Figure D.3: Block diagram of the steam turbines for S1 and S2.

**D.3 Turbine model**

The output signal $TM$ in figure D.3 is the mechanical torque which is an input signal to the synchronous machine. Both synchronous machines are using the same steam turbine model except for different setup of parameter $K$, see $K_{S1}$ and $K_{S2}$ in table D.3

<table>
<thead>
<tr>
<th>TC</th>
<th>TR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>7.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$K_{S1}$</th>
<th>$K_{S2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table D.3: Data for turbine model shown in figure D.3.
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