Transverse Anisotropy in Softwoods

Modelling and Experiments

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Abstract

Transverse anisotropy is an important phenomenon of practical and scientific interest. Although the presence of ray tissue explains the high radial modulus in many hardwoods, experimental data in the literature shows that this is not the case for pine. It is possible that anisotropy in softwoods may be explained by the cellular structure and associated deformation mechanisms.

An experimental approach was developed by which local radial modulus in spruce was determined at sub-annual ring scale. Digital speckle photography (DSP) was used, and the density distribution was carefully characterized using x-ray densitometry and the SilviScan apparatus. A unique set of data was generated for radial modulus versus a wide range of densities. This was possible since earlywood density shows large density variations in spruce. Qualitative comparison was made between data and predictions from stretching and bending honeycomb models. The hypothesis for presence of cell wall stretching was supported by data.

A model for wood was therefore developed where both cell wall bending and stretching are included. The purpose was a model for predictions of softwood moduli over a wide range of densities. The relative importance of the deformation mechanisms was investigated in a parametric study. A two-phase model was developed and radial and tangential moduli were predicted. Comparison with experimental data showed good agreement considering the nature of the model (density is the only input parameter). Agreement is much better than for a regular honeycomb model. According to the model, cell wall bending dominates at both low and high densities during tangential loading. In radial loading, cell wall stretching dominates at higher densities.
Preface

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Chapter 1

Introduction

Wood is a heterogeneous material. This apparent from on the cross-section of coniferous wood. The annual rings form light and dark patterns. If the same surface is viewed in larger magnification, the previously homogeneous rings become heterogeneous where the tracheids form a cellular material. The cellular structure and annual ring structure are two important microstructural scales controlling wood properties.

Transverse anisotropy is an important phenomenon in wood science of both scientific and industrial interest. Simple hexagonal honeycomb models for transverse moduli of softwoods may offer improved understanding through identification of critical material parameters and their influence on transverse moduli.

1.1 Mechanics of honeycombs

Gibson and Ashby [4] calculated the mechanical properties of structures having the same hexagonal geometry as the honeycombs in bee hives. This kind of structure is found in many places in nature and in man-made products. The following discussion essentially follows Gibson and Ashby [4]

The idealised geometry of a honeycomb cell is presented in Figure 1.1. A general honeycomb has four geometrical parameters. The four parameters are two cell wall lengths, \( h \) and \( l \), controlling the dimensions of the cell, the cell shape angle \( \theta \) controlling the shape of the honeycomb and finally cell wall thickness \( t \). A honeycomb where \( h = l \) and \( \theta = 30^\circ \) is termed a regular honeycomb. Such a honeycomb has an interesting symmetry, since it will look exactly the same if rotated \( 60^\circ \). Figure 1.1 also contains a coordinate system. The axes \( R \) and \( T \) correspond to the radial and tangential directions found in wood. The \( R \) and \( T \)-directions correspond to the \( x_1 \) and \( x_2 \)-directions in the notation used by Gibson and Ashby. There is also a thickness dimension of the honeycomb out of the plane. Throughout this text, this thickness is assumed to be unity.
Figure 1.1: Geometry of a honeycomb cell.

Figure 1.1 shows one cell of the honeycomb. This geometry is useful in order to understand the structure of the honeycomb. There is however an even simpler geometrical representation of the honeycomb, that is useful when deriving properties of the honeycomb. Figure 1.2 shows this representation of the smallest repeat unit of a honeycomb structure. This unit consists of only three cell walls. The dimensions of the repeat unit are \( h + l \sin \theta \) and \( 2l \cos \theta \).

Figure 1.2: Geometry of the smallest repeat unit in a hexagonal honeycomb.

A very important property of a honeycomb is the relative density. This is calculated as the density of the cellular structure \( \rho \) divided by the density of the solid cell wall \( \rho_s \). The relative density is thus given as the volume fraction of material in the repeat unit. From Figure 1.2 it clear that the relative density can be expressed as

\[
\frac{\rho}{\rho_s} = \frac{2tl + th}{2l \cos \theta (h + l \sin \theta)} = t \frac{h/l + 2}{2 \cos \theta (h/l + \sin \theta)}
\]  

(1.1)

In the second step, the aspect ratio of the cell wall \( t/l \) is separated from the rest of the geometry. As expected, the relative density is directly proportional to the
aspect ratio of the cell wall. In the case of regular honeycombs, the density simplifies to

$$\rho = \frac{t}{l} \frac{2}{\sqrt{3}}$$  \hspace{1cm} (1.2)

Classical beam theory can be used to derive the mechanical properties of honeycombs in the plane. This means that the members will not stretch and cross-sections of the walls will be plane during beam deformation. The corners of the honeycomb are assumed to be rigid, i.e. there is no hinging in the corners. The corners can translate, but rotation of the corners are limited, because of the repeated structure, so that all corners must rotate equally. Due to symmetry, no rotation of the corners is possible at all if the honeycomb is loaded parallel to any of the coordinate axes. Symmetry also prevents deformation of the cell walls parallel to the $T$-axis for the cases when load is parallel to any of the two coordinate axes.

Figure 1.3 a) shows a honeycomb loaded in the $R$-direction. The relationship between the global stress $\sigma_R$ and the force $P_R$ in one cell wall is

$$\sigma_R = \frac{P_R}{h + l \sin \theta}$$  \hspace{1cm} (1.3)

The denominator, $h + l \sin \theta$, is the size of the smallest repeat unit in the $T$-direction. The component of the load perpendicular to the cell wall, $P\sin \theta$ will cause the wall to bend. According to beam theory, the deflection of the wall will be

$$\delta = \frac{P_R \sin \theta l^3}{12 E_s I}$$  \hspace{1cm} (1.4)

Where $E_s$ is the elastic modulus of the cell wall and $I$ is the second moment of inertia of the cell wall. For a rectangular cross section with thickness $t$ and unity width this is $I = t^3/12$.

Strain is displacement divided by the length over which the displacement is distributed. In the case of deformation of a honeycomb in the $R$-direction, the displacement is the component of beam deflection in the $R$-direction, $\delta \cos \theta$. This is the total deformation of a unit cell and hence strain in the $R$-direction is related to deflection $\delta$ as

$$\varepsilon_R = \frac{\delta \sin \theta}{l \cos \theta} = \frac{P_R \sin^2 \theta l^2}{E_s \cos \theta l^3}$$  \hspace{1cm} (1.5)

Hooke's law, $\sigma = E\varepsilon$, tells us the relationship between stress and strain for a linear elastic material. Since we have derived relations for $\sigma_R$ and $\varepsilon_R$, it is easy to derive the Young's modulus in the $R$-direction.

$$E_R = \frac{\sigma_R}{\varepsilon_R} = \frac{E_s t^3 \cos \theta}{(h + l \sin \theta) \sin^2 \theta l^2}$$  \hspace{1cm} (1.6)

which can be rewritten to express the relative modulus

$$\frac{E_R}{E_s} = \left( \frac{t}{l} \right)^3 \frac{\cos \theta}{(h/l + \sin \theta) \sin^2 \theta}$$  \hspace{1cm} (1.7)
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The same arguments can be used for deriving expressions for stresses and strains when the honeycomb is loaded in the $T$-direction, see Figure 1.4. The stress in the $T$-direction is

$$\sigma_T = \frac{P_T}{l \cos \theta} \quad (1.8)$$

Relation between beam deflections and force $P_T$ on the beam is

$$\delta = \frac{P_T \cos \theta l^3}{12E_s I} \quad (1.9)$$
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The strains will be
\[ \varepsilon_T = \frac{\delta \cos \theta}{h + l \sin \theta} = \frac{P_T \cos^2 \theta l^3}{E_s (h + l \sin \theta) t^3} \]  
(1.10)

Modulus in the T-direction is
\[ E_T = \frac{\sigma_T}{\varepsilon_T} = \frac{E_s (h + l \sin \theta) t^3}{l^2 \cos^3 \theta} \]  
(1.11)

Again this expression can be rewritten to express the relative modulus of the honeycomb in the T-direction.
\[ \frac{E_T}{E_s} = \left( \frac{t}{l} \right)^3 \frac{h/l + \sin \theta}{\cos^3 \theta} \]  
(1.12)

Two interesting special cases are a honeycomb with regular hexagons and a honeycomb with \( \theta = 0^\circ \). In the case of a regular honeycomb
\[ \frac{E_R}{E_s} = \frac{E_T}{E_s} = \left( \frac{t}{l} \right)^3 \frac{4}{\sqrt{3}} \]  
(1.13)

Since \( E_R = E_T \) and because of symmetry for rotations of 60°, the regular honeycomb is isotropic. In order to study the other interesting special case, we study the limit when \( \theta \) approaches 0.
\[ \lim_{\theta \to 0} \frac{E_R}{E_s} = \lim_{\theta \to 0} \left( \frac{t}{l} \right)^3 \frac{\cos \theta}{(h/l + \sin \theta) \sin^2 \theta} = \infty \]  
(1.14)
\[ \lim_{\theta \to 0} \frac{E_R}{E_s} = \lim_{\theta \to 0} \left( \frac{t}{l} \right)^3 \frac{h/l + \sin \theta}{\cos^3 \theta} = \left( \frac{t}{l} \right)^3 \frac{h}{l} \]  
(1.15)

The relative modulus \( E_R/E_s \) is infinite when cell walls are parallel to R-direction and this consequence is not discussed by Gibson and Ashby. The infinite modulus does clearly not exist in a real material, but is a limitation caused by the assumption that cell walls only deform through bending. We will return to this problem when we discuss the applications of honeycomb theories to wood.

1.2 Application of honeycombs to wood

Honeycomb models have been applied to wood. Easterling et al.[3] studied the mechanics of balsa using regular honeycombs. They proposed that the ray cells would act as stiffeners in the radial direction, thus explaining the anisotropy in balsa.

Gibson and Ashby [4] generalised the model and applied it to all species of wood. They also assume a honeycomb with regular hexagons and use data from the literature and fit a line following the equation
\[ \frac{E_T}{E_s} = C_1 \left( \frac{\rho}{\rho_s} \right)^3 \]  
(1.16)
They use axial data from Cave [2] for the cell wall modulus, $E_s = 35$ GPa. The constant $C_1$, fitted to data will include the cell wall anisotropy and effects from cellular arrangements, and is reported as to 0.54. The figure from Gibson and Ashby is reproduced in Figure 1.5.

![Figure 1.5: Wood modulus as a function of density. After Gibson and Ashby [4]. The points are experimental data in three directions from various species. The fitted lines are proportional to density for the axial data, and proportional to the cube of radial and tangential data. Note the discrepancy between radial data (filled circles) and the theory.]

Since Gibson and Ashby assume regular hexagons, the model behaves isotropic. The higher modulus in the radial direction is explained by the existence of rays. Gibson and Ashby argue that the radial modulus is

$$E_R = V_R R^3 E_T + (1 - V_R) E_T$$

(1.17)
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where $V_R$ is the volume fraction and $R$ is the density ratio of rays and tracheids. A typical value for the radial modulus is $E_R = 1.5E_T$.

A problem with the argument used by Gibson and Ashby is that the rays might not act as stiffeners, at least not in softwoods. Boutelje [1] measured the anisotropy in swelling during water sorption of small pine specimen of varying density. Some of the specimens had the rays removed whereas the rays were still present in other samples. Boutelje showed that anisotropy is clearly dependent on density, but not dependent on whether the rays were removed or not.

Boutelje’s [1] observations lead to the conclusion that Gibson and Ashby’s [4] assumptions of rays, that increase the radial modulus, is wrong for pine, and possibly most softwoods. It is necessary to find another analytical method to model the anisotropy of transverse modulus. One way of doing this is to remove the restriction of a regular honeycomb by changing the cell shape angle $\theta$.

Kahle and Woodhouse [5] implemented the model by Gibson and Ashby in a numerical approach, and adjusted $\theta$ according to experimental data. Watanabe et al. [7, 6] determined cell geometries in earlywood for seven species of softwood. They used a honeycomb where bending and stretching of the cell walls was included. The agreement between predictions and data was good.

1.3 Thesis objective

The objective of the present study is to improve the understanding of transverse anisotropy in softwoods through the combination of experimental characterisation and theoretical modelling. In particular, the hypothesis of cell wall stretching as a contributing deformation mechanism is investigated. This mechanism has not received much attention previously.
Bibliography


