Power Dissipation in Car Tyres

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Abstract

Traffic is a major source of green house gases. The transport field stands for 32% of the energy consumption and 28% of the total CO2 emissions, where road transports alone causes 84% of these figures. The energy consumed by a car traveling at constant speed, is due to engine inefficiency, internal friction, and the energy needed to overcome resisting forces such as aerodynamic drag and rolling resistance. Rolling resistance plays a rather large role when it comes to fuel economy. An improvement in rolling resistance of 10% can yield fuel consumption improvements ranging from 0.5 to 1.5% for passenger cars and light trucks and 1.5 to 3% for heavy trucks.

The objective of this thesis is to estimate the power consumption in the tyres. To do this a car tyre is modeled with waveguide finite elements. A non-linear contact model is used to calculate the contact forces as the tyre is rolling on a rough road. The contact forces combined with the response of the tyre is used to estimate the input power to the tyre structure, which determines a significant part of the rolling resistance.

The tyre model accounts for: the curvature, the geometry of the cross-section, the pre-stress due to inflation pressure, the anisotropic material properties and the rigid body properties of the rim. The model is based on design data. The motion of the tyre belt and side wall is described with quadratic anisotropic, deep shell elements that includes pre-stress and the motion of the tread on top of the tyre by quadratic, Lagrange type, homogenous, isotropic two dimensional elements.

To validate the tyre model, mobility measurements and an experimental modal analysis has been made. The model agrees very well with point mobility measurements up to roughly 250 Hz. The eigen-frequency prediction is within five percent for most of the identified modes. The estimated damping is a bit too low especially for the anti-symmetric modes. Above 500 Hz there is an error ranging from 1.5 dB up to 3.5 dB for the squared amplitude of the point mobility.

The non proportional damping used in the model is based on an ad hoc curve fitting procedure against measured mobilities.

The contact force predictions, made by the division of applied acoustics, Chalmers University of Technology, are based on a non-linear contact model in which the tyre structure is described by its flexibility matrix. Topographies of the surface are scanned, the tread pattern is accounted for, and then the tyre is ’rolled’ over it. The contact forces are inserted into the tyre model and the response is calculated. The dissipated power is then calculated through the injected power and the power dissipated within each element. Results are promising compared to literature and measurements.
Licentiate Thesis

The thesis consists of an introduction and the following two papers:

**Paper A**

**Paper B**

**Contribution from the author of this thesis**

**Paper A**
Experimental modal analysis and the mobility measurements. Performed simulations. Fine tuning of the model developed by the supervisor. Writing the paper.

**Paper B**
Performed the power calculations. Literature study on rolling resistance. Writing the paper.

The material from this thesis has been presented at five workshops in the ITARI project plus at three conferences:


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1 Introduction

1.1 Background

For over fifty years traffic has been an irritating noise polluter. For higher speeds tyres have been found to be the major contributor for traffic noise. Also the interior noise in the vehicle due to the tyres are becoming more important as other noise sources such as engines, exhaust systems and gear boxes are better managed.

The negative effect on the environment has been highlighted for a number of years, given that traffic is a major source of green house gases. The transport field is representing 32% of the energy consumption and 28% of the total CO2 emissions, where road transports alone stands for 84 % of these figures [1].

When it comes to the dynamics of the car the tyres are crucial, as they provide the grip required for cornering, braking and acceleration. In addition, tyres are also highly involved in the cars handling abilities. As a final point it is the tyres and the suspension system that assures a comfortable ride.

The energy consumed by a car traveling at constant speed, is due to engine ineffiency, internal friction, and the energy needed to overcome resisting forces such as aerodynamic drag and rolling resistance, which is the topic of this thesis.

The rolling resistance \( F_r \) is defined as the energy consumed per unit of distance traveled [2]. The unit is \( Nm/m = N \) which is equivalent to a drag force in Newtons. Tyres are made of reinforced rubber, which is a viscoelastic material. As it deforms a part of the energy is stored elastically but the remainder is dissipated as heat. These hysteretic losses, as well as aerodynamic drag and friction in the contact patch and with the rim are losses that contribute to the total drag force on a moving vehicle. Rolling resistance has a rather large impact when it comes to fuel economy. A 10 % improvement in rolling resistance can give fuel consumption reductions ranging from 0.5 to 1.5 % for passenger cars and light trucks and 1.5 to 3 % for heavy trucks [3].

Normally the rolling resistance is given as a dimensionless constant times the gravity force,

\[
F_r = C_r m g,
\]
where \( m \) is the mass, \( g \) is the constant of gravity and \( C_r \) is the rolling resistance coefficient. \( C_r \) is normally in the range 0.01-0.02 with a typical value of 0.012 for a passenger car tyre on dry asphalt [4]. The power consumed by this force is

\[
P = V F_r = V C_r m g
\]

where \( V \) is the speed of the vehicle. In equation (1) the only explicit parameter is the load. The variation with other parameters are concealed in \( C_r \). Studies has shown that the rolling resistance coefficient is influenced by a number of parameters such as speed, driving torque, acceleration, rubber compound, internal and ambient temperature, road texture, road roughness, and wear. The model is however usually sufficient for some applications.

The aim of this thesis is to model a radial car tyre with waveguide finite elements and to use this model to estimate the power dissipation as the tyre is rolling on a rough road. These losses determine a significant part of the rolling resistance. The model was originally designed for tyre road noise predictions.


The model used in this study has the correct geometry and stiffness parameters as it is based on design data provided by the tyre manufacturer Goodyear. None of the models above are treating a rough road even though the road texture and roughness have a significant effect on the rolling resistance [9].

### 1.2 Car tyres

Car tyres are made of several different materials including steel, fabric and of course numerous rubber compounds, see Figure 1. To get different dynamic properties in the tyre sub regions the materials are used in many ways. The three major sub regions of the tyre are the upper side wall, the lower side wall and the central area. The ply is a layer of embedded fabric in the rubber. At the lower side walls the ply encloses a volume filled with both steel wires and hard rubber materials, this makes the lower side wall areas relatively stiff. The upper side wall areas are on the other hand quite flexible, since the ply layer there is simple and there is less steel in there. The central area consists of the belt and the tread. The belt consists of a rubber embedded steel lining (breakers) in the circumferential direction to give support and rigidity. The tread is an about 13 mm thick rubber layer which is there to provide the grip. This makes the central area rigid with
respect to bending waves in the circumferential direction but fairly flexible when it comes to motion within the cross-section. The high loss factor of the tread rubber makes the latter motion highly damped.

The tyre studied here is a Goodyear, radial, passenger car tyre, with the dimensions 205/55ZR16, mounted on an Argos rim. The tyre is 'slick', i.e. it does not have a tread pattern or groves, but in all other aspects has properties typical of a production tyre.

To make use of the rotational symmetry of the tyre a waveguide finite element approach is employed, where only the cross-section is discretised, and hence the calculation time is reduced. The model accounts for: the curvature, the geometry of the cross-section, the pre-stress due to inflation pressure, the anisotropic material properties and the rigid body properties of the rim.

1.3 Waveguide finite elements

A waveguide is a wide-ranging term for a device, which, constrains or guides the propagation of mechanical waves along the waveguide. Here it is also assumed that a waveguide has constant geometrical and material properties along one direction.

Waveguide FE yield equations of motion for systems with wave-propagation along a single direction in which the structure is uniform. It is then possible
to separate the solution to the wave equation into one part depending on
the cross-section, one part depending on the coordinate along the waveguide
and one part depending on time.

As an example of a waveguide a generalised beam, in which longitudinal,
torsional, shearing and flexural waves can travel, can be considered. The
main idea with a waveguide approach is to study waves propagating in the
structure.

The most important benefit with waveguide FE is that it decreases the
calculation time compared to ordinary finite elements since only the cross-
section has to be discretised and the number of degrees of freedom is re-
duced. Another advantage compared to conventional FE methods is that it
is straightforward to identify and analyse different wave types, which allows
a physical understanding of the structure under investigation. The ability
to handle infinite waveguides is an additional good feature of this method.

Forced response solutions for waveguide FE models can be handled in
several different ways. Four of these methods for forced responses will be
briefly explained.

For infinite waveguides an approach based on Fourier transforms may
be used. The equations of motion are transformed to the wave number do-
main through a spatial Fourier transform. The solution in the wave number
domain then has to be transformed back to the spatial domain through an
inverse Fourier transform which generally involves residue calculus [10].

‘Super Spectral Elements’, (SSE), are derived by using wave solutions,
given from a generalised eigenvalue problem, as test and shape functions
in the variational form of the wave equation [11]. At the ends, the spec-
tral elements can be coupled to other spectral elements or to regular finite
elements.

A modal solution is suited for a structure with rotational symmetry, such
as a car tyre. The response is assumed to be a sum of the waves (eigenvectors) with real integer wave numbers, resulting from a twin parameter
eigenvalue problem. The amplitude of these waves are then treated as un-
knowns in the strong form of the wave equation. The wave equation is then
multiplied with one specific eigenvector and the result is integrated over the
length of the waveguide. The orthogonality between the eigenvectors, over
the length of the waveguide, filters out the coefficients corresponding to the
eigenvector. The non-proportional damping used in the present analysis,
however, leads to non-orthogonal eigenvectors and therefore this method is
not used.

In an assumed modes procedure the response is assumed to be an expo-
nential Fourier series in the spatial domain. This approach is suitable, since
the tyre is a circular structure and the solutions to the wave equation will be
periodic with respect to the circumferential angle. The sum is inserted into
the variational statement, and upon variation follows the equations of mo-
tion. The advantage with this direct methodology in the frequency domain
is that it is uncomplicated to handle fluid-structure interactions. The car tyre including the air cavity has been modeled successfully by Nilsson [10] with a waveguide FE approach similar to the one presented here. Also, frequency dependant materials are easily included. This is an especially good quality when considering a structure such as a car tyre, which is built from rubber, whose material properties show a strong frequency dependency. This is the procedure used in the present analysis.


2 Summary of the papers

2.1 A waveguide finite element model of a pneumatic tyre

A waveguide finite elements model based on design data is used to describe the dynamic properties of a passenger car tyre. The response of the tyre belt and side wall is described with quadratic anisotropic, deep shell elements that include pre-stress and the motion of the tread on top of the tyre by quadratic, Lagrange type, isotropic two dimensional elements.

To validate the tyre model, mobility measurements and an experimental modal analysis has been made. The calculations agrees very well with measurements up to roughly 250 Hz for the radial point mobilities, see Figures 2 and 3 for excitation in the middle of the tread. The eigenfrequency prediction are within five percent for the identified modes, except for the axial semi rigid body mode (error 12 %), the anti-symmetric mode of order two (error 10 %) and the anti-symmetric mode of order seven (error 7 %). The estimated damping, especially for the anti-symmetric modes, is a bit too low.

The 'cut-on' frequency, of the belt bending modes, is the lowest frequency at which the corresponding waves are propagated. It comes earlier in the prediction than in the measurement. This is perhaps due to ageing of the tyre since comparable measurements performed in the spring of 2001 [16] is in agreement with the calculation. In the range 500 -1000 Hz there is an error ranging from 1.5 dB up to 3.5 dB for the squared amplitude of the point mobility. For the transfer mobilities, the error is larger since they are more sensitive to the exact position of the accelerometer, particularly so for the anti-resonances, see Figure 4.

The non proportional damping is found with an ad hoc curve fitting procedure based on the measured mobilities.
2.2 Power dissipation in car tyres

The tyre model described in Paper A is used to estimate the power consumed by visco elastic losses. External forces resulting from a non-linear contact model, for three different roads are inserted and the responses are calculated. The dissipated power is then equated to the injected power as well as to the sum of the power dissipated within the elements.

The contact force predictions are made by Frédéric Wullens of the division of applied acoustics, Chalmers University of Technology (CTH) as described in reference [17]. It is based on a non-linear contact model in which the response of the tyre is described with its flexibility matrix. Topographies of the surface are scanned, the tread pattern is accounted for, and then the tyre is 'rolled' over it in the time domain. The nonlinear conditions used are: i) the tyre cannot indent into the road, ii) if a point is not in contact the force is zero and iii) the force cannot be negative (road pulling tyre down). Only forces acting normal to the road is considered.

The contact forces are used to calculate the response of the tyre. When the forces and the motion is known the injected power can be calculated. The predicted power dissipation compares favorably with those from literature [4] and with measurements. The power dissipation is larger on the rough road than on the smooth road, this showing the great influence of the road on the rolling resistance. To the best of the author’s knowledge, this influence is neglected in all previous works.

The dissipated power for a test road managed and scanned by Renault, as a function of frequency and wave order can be seen in Figure 5 and 6 respectively. The reason that the frequency spectrum looks so rough is that only two revolutions have been used for the calculation. If the contact forces were truly periodic every other frequency component would cancel out. By using more non identical revolutions the result would probably look much smoother. A significant part of the dissipation occurs below 100 Hz and at a wave order around 3.

By studying the power dissipated within the elements it can be concluded that there are nearly no losses occurring in the side wall, see Figures 7 and 8, which is in conflict with [3] who says that roughly 30% of the total dissipated power appears in the upper and lower side wall. The overall damping level in the model is estimated quite accurately (see Paper A), but the distribution of the damping, in the different parts of the tyre, is probably wrong. Since the visco elastic data is very important for a rolling resistance prediction, the damping should be established in a more scientific way, and this development will be reported at a later stage.
3 Future Work

Future work consists in fine tuning the tyre model with regards to damping and to use longer contact forces in the time domain. Based on measurements of the dynamic shear modulus a frequency dependant tread will be introduced. The damping of the belt and side wall will also be estimated in a more scientific way based on an optimisation routine where the modal damping ratios will be used as an error criterion.

Longer contact forces will lead to a finer frequency resolution, which is needed for the accurate evaluation of the power consumed at the tyre resonances in the 100 Hz region. Also, more revolution would lead to a better and perhaps smoother power spectrum.

An investigation of the influence of certain tyre parameters would also be interesting. It would be possible to change the speed, the load on the tyre and perhaps also to model wear of the tyre.

Preliminary tests with a frequency dependant tread have been made and will briefly be explained. The tyre model presented in paper A is updated to include a frequency dependent tread resulting from a dynamic shear modulus measurement. The shear modulus data is fitted to a fractional Kelvin-Voigt Model, described in for example [18], which has the following appearance,

\[ \hat{G} = G_0 (1 + \left( \frac{i \omega}{\omega_0} \right) \alpha). \]  

(3)

In equation (3) the parameters that are fitted to the measured data is \( G_0 \), \( \omega_0 \) and \( \alpha \). \( G_0 \) is equivalent to the static shear modulus parameter, \( \omega_0 \) has dimension \([\text{rad/s}]\) while \( \alpha \) is dimensionless. Note that the Fourier transform of the fractional derivative of order \( \alpha \) of \( x(t) \) is \( (i \omega)^\alpha \) times the Fourier transform of \( x(t) \) [19]. The values of the fitted parameters are in Table 1

\[
\begin{array}{ccc}
G_0 & \omega_0 & \alpha \\
5.25 \times 10^6 & 3.84 \times 10^3 & 0.40
\end{array}
\]

Table 1: Fitted parameters in fractional Kelvin-Voigt model.

The loss factor is defined as:

\[ \eta = \frac{\text{Im}(\hat{G})}{\text{Re}(\hat{G})}. \]

(4)

The frequency dependence of the tread is such that the loss factor is zero at zero frequency and then increases. The real part and the loss factor of the dynamic shear modulus is seen in Figures 9 and 10.

The damping of the belt is tampered a bit to get a similar agreement with the point mobility measurement as the original model, see Figures 11 and 12. The rolling resistance calculation is then re-done with the new model but
with the old contact forces. The contact forces calculation depends on the flexibility matrix of the tyre, so the result should be interpreted with care. For the Renault road the original model gave a total power loss of 805.7 Watts whereas the model with frequency dependent tread gives a value of 645.0 Watts.

The main part of the power loss occur around 50 Hz where the loss factor, from the measurements of the tread is much smaller ($\eta = 0.16$) than the one used in the original model ($\eta = 0.3$) the losses are consequently reduced. See Figure 13 for the power loss versus frequency for the original and the new model.

4 Conclusion

A car tyre is modeled with wave guide finite elements. The model is employed to calculate the power dissipation as the tyre is rolling on a rough road showing promising agreement with measurements. The road roughness is seen to have a significant effect on the dissipated power, which, to the best of the authors’ knowledge, is neglected in all previous works.

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References


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**Figure 2:** Magnitude of point mobility for excitation in the middle position. Measured (solid) and calculated (dashed).
Figure 3: Phase of point mobility for excitation in the middle position. Measured (solid) and calculated (dashed).

Figure 4: Magnitude of transfer mobility for excitation in the middle position. The response is measured 23.5 cm away in the circumferential direction and 4.3 cm above the geometric centre. Measured (solid) and calculated (dashed).
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Figure 11: Magnitude of point mobility for excitation in the middle position. Measured (solid), original model (dashed) and new model with frequency dependent tread (dotted).
Figure 12: Phase of point mobility for excitation in the middle position. Measured (solid), original model (dashed) and new model with frequency dependent tread (dotted).
Figure 13: Dissipated power as a function of frequency. The bandwidth is 5.6 Hz. Original model (solid) and model with frequency dependent tread (dashed).