ROYAL INSTITUTE OF TECHNOLOGY

The Black-Litterman Model

- mathematical and behavioral finance approaches towards its use in practice

Licentiate thesis by

CHARLOTTA MANKERT

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School of Industrial Engineering and Management
Department of Industrial Economics and Management
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SWEDEN

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Abstract

The financial portfolio model often referred to as the Black-Litterman model is analyzed using two approaches; a mathematical and a behavioral finance approach. After a detailed description of its framework, the Black-Litterman model is derived mathematically using a sampling theoretical approach. This approach generates a new interpretation of the model and gives an interpretable formula for the mystical parameter τ, the weight-on-views. Secondly, implications are drawn from research results within behavioral finance. One of the most interesting features of the Black-Litterman model is that the benchmark portfolio, against which the performance of the portfolio manager is evaluated, functions as the point of reference. According to behavioral finance, the actual utility function of the investor is reference-based and investors estimate losses and gains in relation to this benchmark. Implications drawn from research results within behavioral finance indicate and explain why the portfolio output given by the Black-Litterman model appears more intuitive to fund managers than portfolios generated by the Markowitz model. Another feature of the Black-Litterman model is that the user assigns levels of confidence to each asset view in the form of confidence intervals. Research results within behavioral finance have, however, shown that people tend to be badly calibrated when estimating their levels of confidence. Research has shown that people are overconfident in financial decision-making, particularly when stating confidence intervals. This is problematic. For a deeper understanding of the use of the Black-Litterman model it seems that we should turn to those financial fields in which social and organizational context and issues are taken into consideration, to generate better knowledge of the use of the Black-Litterman model.

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Charlotta Mankert

Department of Industrial Economics and Management

Royal Institute of Technology

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1 Introduction

In 1952 Markowitz published the article *Portfolio Selection,* which can be seen as the genesis of modern portfolio theory. Portfolio models are tools intended to help portfolio managers of investors decide the weights of the assets within a fund or a portfolio. The ideas of Markowitz have had a great impact on portfolio theory and have, theoretically, withstood the test of time. However, in practical portfolio management the use of Markowitz' model has not had the same impact as it has had in academia. Many fund and portfolio managers consider the composition of the portfolio given by the Markowitz model as unintuitive (Michaud, 1989; Black & Litterman, 1992). The practical problems in using the Markowitz model motivated Fisher Black and Robert Litterman (1992) to develop a new model in the early 1990s. The model, often referred to as the Black-Litterman model (hereafter the B-L model), builds on Markowitz' model and aims at handling some of its practical problems. While optimization in the Markowitz model begins from the null portfolio, the optimization in the B-L model begins from, what Black and Litterman refer to as, the equilibrium portfolio (often assessed as the benchmark weights of the assets in the portfolio). "Bets" or deviations from the equilibrium portfolio are then taken on assets to which the investor has assigned views. To each view, the manager assigns a level of confidence, indicating how sure he/she is of that particular view. The level of confidence affects how much the weight of that particular asset in the B-L portfolio differs from the weights of the equilibrium portfolio.

The studies presented in this thesis, are intended to investigate, develop and test the B-L model in an applied perspective. This is done by (1) carefully and methodologically describing and mathematically deriving the model, (2) searching for and locating relevant research results within the field of behavioral finance and discussing their implications in relation to the use of the B-L model and, in conclusion, (3) reflecting on and discussing the research results and also presenting and discussing some theoretical starting points for future research.

1.1 Aim and Purpose

The overall aim of this research project is to make contributions to the practical use (or, if indicated, the rejection) of the B-L model.

The research and the thesis consist of two main parts or steps. Each step has its own, more specific aim and purpose, but they point in the same direction, toward the rewarding use of the B-L model.[1]

The aim of the first part is to develop the B-L model and thereby complement the literature with a careful description and mathematical derivation of the model using a sampling theoretical approach, in the hope that it will facilitate a more profound understanding of the model.

The aim of the second part is to draw conclusions from research results within the field of behavioral finance and discuss their implications in relation to the use of the B-L model.

1.2 Methodological Considerations

Working with the research presented in this thesis has meant dealing with many methodological considerations. The following items summarize the most important methodological choices.

The B-L Model

The study of the B-L model has both academical and practical motivations. Portfolio theory is well established within the academic field of finance. Integrating quantitative portfolio models with judgments of portfolio managers, as is done in the B-L model *"…has been motivated by various discussions on increasing the usefulness of quantitative models for global portfolio management"* (Giacometti et.al., 2005, p. 3). There is also interest in the B-L model in the financial industry. For example, the model is claimed to be "*key tool in the Investment Management Division's asset allocation process*" at Goldman Sachs (Litterman,

[1] Note that I may find that it could be most rewarding not to use the B-L model at all.

2006). The amount of literature, academic and non-academic, relating to the model is, however, limited. There is a lack of research regarding both the mathematical characteristics behind the model and its practical use. The model appears appealing, but my impression is that the scarcity of literature discussing the B-L model hinders its development and testing and thereby its fruitful use. For this reason, one aim of this thesis is to provide a thorough and methodological mathematical derivation of the model.

A Tool

The B-L model has its origins in traditional quantitative financial theory. Traditional finance is based on neoclassical theory, the assumptions of the efficient market hypothesis and homo economicus. I am however influenced by research traditions within the social sciences. A consequence has been that I view the B-L model as a tool to be used in a social and organizational context. If the model is good, then it has the possibility of becoming a good tool. The value of a tool used in an investment context is not only dependent on its theoretical characteristics. It might be impossible to use, in practice, the most theoretically advanced and elegant model. By not fulfilling the requirements of a good tool, it is not, in my view, a good model.

Social Constructivism

During the research process I have tried to maintain a quite independent and distanced attitude to both the B-L model and to financial theory. I believe that neither financial research results, nor the B-L model, are "discovered". Instead, I view them as being constructed over time. I am influenced by social constructivism (Berger & Luckmann, 1966 and Czarniawska, 2005). Czarniawska claims that social constructivism means accepting that reality is under constant reconstruction and that it is therefore meaningless to search for its essence. This influence has resulted in me perceiving the B-L model not as a finished model that needs to be demystified, but as an idea that has been and is being constructed. The article *Global Portfolio Optimization* by Black and Litterman (1992) is a central contribution to the construction of the B-L model but its development continues. When writing about the B-L model I therefore refer not only to the model as explained by Black and Litterman. I consider everyone working with the B-L model, including myself, as contributors to and participants in its development. Hence, when I move from the mathematical step to the behavioral finance step in the research process, the model is different from what it was before. The model is in a way reconstructed: some variables have new formulas and the model is interpreted and derived in another way.

Practical Use and Application

Although the model is theoretically and mathematically derived in the first part of the thesis, the *overall* aim of the research project is to contribute to the practical use and application of the B-L model, not to make theoretical contributions. Practical use and application are the driving forces behind the derivation. Since the research aspires to relate to the practical use and application of the model, the thesis will contain normative elements. Descriptive theories such as behavioral finance will however be used as input. The reasons for making practical use and application of the model the overall aim are related to empirical experiences achieved during a project in 2002 (see appendix 1). The main experience from this project was that although the theoretical model may seem appealing, its practical use can be, and probably often is, problematic.

Four Steps

This thesis presents my contribution to the construction of the B-L model. It concludes two steps in a research project consisting of four steps:

1. Develop the B-L model and thereby complement the literature with a careful description and mathematical derivation of the model using a sampling theoretical approach
2. Draw conclusions from research results within the field of behavioral finance and discuss their implications in relation to the use of the B-L model

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3. Practical decision-making in portfolio management.
4. Empirical tests of the B-L model in portfolio management situations as real as possible.

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The two first steps are presented in this thesis. In the last chapter, the third and fourth steps are touched upon. The third and fourth steps will constitute the research for my doctoral thesis.

The steps are quite different in character and draw upon different research traditions and fields. However there exists both a clear progression in the different studies and a clear connection between them. The four steps all concern the use of the B-L model. The steps are also based on each other. It would not have been possible to draw implications from research within behavioral finance if the theoretical characteristics of the model were not thoroughly explained. Hence, the first step (mathematical in character) lays the foundations for the second step (behavioral in character). Performing empirical research in practical portfolio management (step three and four) also builds on the knowledge and experience from the two first steps. Steps one and two have made me realize the importance of performing practical empirical tests and evaluations of the use of the B-L model. The theories, both portfolio theory and behavioral finance, are individualistic in character and with little reference to social and organizational considerations.

These arguments also concern the fact that the research is presented in a monograph instead of as separate articles. The progression between the steps would have made it difficult to divide the research into separate articles. The ambition is, however, that the different steps in the thesis will serve as bases for articles to be published in academic journals.

Step 1: Develop the B-L model and thereby complement the literature with a careful description and mathematical

The decision to derive the B-L model theoretically and mathematically may seem surprising when the overall aim of the research project is to make contributions concerning the *use* of the B-L model. However, the theoretical contributions serve as a prerequisite for the performance of applied research concerning the use of the model. Arguments for deriving the model are:

a. Fruitful research and application of quantitative financial models requires the researcher and the user to be familiar with the theoretical foundations of the model.

b. The insufficiency of literature concerning the theoretical characteristics of the B-L model.

c. The absence of mathematical explanations of some of the variables within the model, which therefore can be interpreted by neither a researcher nor a user.

The B-L model will be derived using a sampling theoretical approach. Existing literature concerning the B-L model takes a Bayesian approach. I have found it very difficult to understand the B-L model on the basis of the existing literature.[2] Although suggested by Black and Litterman (1992) the sampling theory approach does not appear in the literature. Hopefully, a derivation using this approach will provide a way for people unfamiliar with Bayesian theory to understand the theoretical characteristics of the model.

[2] I have also met other academics and practitioners claiming that they find the literature difficult to understand.

Step 2: Develop the B-L model and thereby complement the literature with a careful description and mathematical

Using the B-L model demands actions: judgments and estimations. Since there is a large and active field, behavioral finance, involving research into the behavior of individuals in investment situations, it seems motivated to search here for research results within the field relevant to the use of the B-L model.

Since using the B-L model requires action of its users, most of the research results within behavioral finance may have some relevance to the use of the B-L model. The aim of this research project has, however, not been to search for all research results, which might have implications for the use of the B-L model. Instead the focus lies on:

- Results pointing directly to features specific to the B-L model.
- Results that are robust and well established.

The process of evaluating the relevance and importance of research results within behavioral finance has not been formalized. Instead, while reading literature within behavioral finance parallel with working with the B-L model, I have tried to make judgments as careful as possible about whether different research results are relevant to this study or not.

To find research results within behavioral finance that might have implications for the use of the B-L model I have prepared a literature review presented in Appendix 4. This does not aspire to be exhaustive. My main literature sources concerning behavioral finance are Gilovich, Griffin & Kahneman (2002) and Kahneman & Tversky (2002). During the search for research results that may have implications for the use of the B-L model I have had the main characteristics of the B-L model in mind. There exist research results within behavioral finance concerning the main characteristics in the B-L model, those that differentiate the B-L model from other portfolio models.

Step 3: Practical decision-making in portfolio management

Having mathematically derived the B-L model and obtained implications for the use of the model from research results within behavioral finance I have become aware of the individualistic perspective characterizing both behavioral finance and traditional financial theory. My ambition in the third step is to investigate how allocation decisions are actually made in practical portfolio management. There are several small but active groups of researchers focusing on the organizational and social context of financial actors and my proposal is, both in step 3 and step 4, to take theoretical starting points from their fields and perform empirical research on practical portfolio management.

My hope is that the step will contribute to a deeper understanding of the social and organizational context of portfolio managers and how decisions are made in portfolio allocation situations.

Step 4: Empirical tests of the B-L model in as real portfolio management situations as possible

The fourth step concerns the use of the Black-Litterman model in practical portfolio management. It appears that such a project will integrate the other three steps. The research will make possible the consideration of theoretical, technical, social and organizational issues.

A Broad Perspective

The research project has a broad perspective. It aims at going “all the way” from the theoretical characteristics of the model to its practical use in organizations. Doing this demands interaction with different research traditions and cultures. The project may hence be seen as a cross disciplinary project or perhaps rather an interdisciplinary project (Vetenskapsrådet, 2005). Cross or interdisciplinary research can be difficult, demanding both methodological and theoretical knowledge from different research fields. One of my strategies for the solution of problems that may arise is to have supervision from two different academic cultures. I have had two supervisors, one, with a more mathematical focus, working in the Mathematics department and the other, my main supervisor, with a more organizational perspective, working, as I do, in the Industrial management and organization department. I have attended seminars and other activities with a group of researchers with a qualitative research approach, which has been very instructive. My impression is that this arrangement has helped me and will help me in navigating between and working within the mathematical characteristics of the B-L model as well as the more qualitative characteristics that constitute its use.

Performing research under these circumstances has influenced my work in many ways. Interaction with the mathematical department facilitated the necessary mathematical derivations. Working with researchers in the Department of industrial management and organization with a more qualitative and organizational approach helped me keep a distanced and critical attitude towards research results within the different financial fields. This distance has been a prerequisite to taking the steps I am taking in parts two and three.

1.3 Outline

In this *first chapter* I have described the aim of the study presented in this thesis and the main methodological considerations.

Chapters two and *three* constitute part one of this research project. In *chapter two* I describe the basis of portfolio theory, the Markowitz model. Markowitz formed the foundations of portfolio theory and the B-L model builds on the Markowitz model. To be able to truly understand the B-L model it is hence important to understand the Markowitz model. The *third chapter* explains the B-L model. The chapter begins with a broad description of the framework and the idea of the B-L model. The second part is more detailed. The B-L model is presented and derived from a sampling theoretical approach instead of the more common Bayesian approach. A rigorous mathematical derivation of the B-L model is presented

The second step of the research project is presented in *Chapter four*. The field of behavioral finance is described briefly and then implications from research results within this field are drawn, analyzed and discussed in relation to the use of the B-L model.

The *fifth chapter* serves two purposes. Firstly it provides a presentation of financial research fields in which the social and organizational context perspectives are given importance. The second purpose of this chapter is to present an introduction to the forthcoming third and fourth steps of the research project.

2 The Markowitz Model

Portfolio theory took form as an academic field when Harry Markowitz published the article *Portfolio Selection* in 1952. Markowitz focuses on a portfolio as a whole; instead of security selection he discusses portfolio selection. Previously, little research concerning the mathematical relations within portfolios of assets had been carried out. Markowitz began from John Burr Williams' *Theory of Investment Value*. Williams (1938) claimed that the value of a security should be the same as the net present value of future dividends. Since the future dividends of most securities are unknown, Markowitz claimed that the value of a security should be the net present value of *expected* future returns. Markowitz claims that it is not enough to consider the characteristics of individual assets when forming a portfolio of financial securities. Investors should take into account the co-movements represented by covariances of assets. If investors take covariances into consideration when forming portfolios, Markowitz argues that they can construct portfolios that generate higher expected return at the same level of risk or lower level of risk with the same level of expected return than portfolios ignoring the co-movements of asset returns. Risk, in Markowitz' model (as well as in many other quantitative financial models) is assessed as the variance of the portfolio. The variance of a portfolio in turn depends on the variance of the assets in the portfolio and on the covariances between its assets.

Markowitz' mean-variance portfolio model is the base on which much research within portfolio theory is performed. It is also from this model that the B-L model was developed. The B-L model builds on the Markowitz model and it is hence important to understands Markowitz' model. A detailed review of Markowitz' model for portfolio choice is therefore provided in Appendix 2. A summary of the model is provided in this chapter, with focus on the practical problems encountered in the use of the model. The practical problems in using Markowitz' model prompted Black and Litterman to continue the development of portfolio modeling.

Markowitz shows that investors under certain assumptions, *theoretically*, can build portfolios that maximize expected return given a specified level of risk, or minimize the risk given a level of expected return. The model is primarily a normative model. The objective for Markowitz has been not to explain how people select portfolios, but how they should select portfolios (Sharpe, 1967). Even before 1952 diversification was a well-accepted strategy to lower the risk of a portfolio, without lowering the expected return, but until then, no thorough foundation existed to validate diversification. Markowitz' mean-variance portfolio model has remained to date the cornerstone of modern portfolio theory (Elton & Gruber, 1997).

2.1 The Model

According to Markowitz (1952), inputs needed to create optimal portfolios are: expected returns[3] for every asset, variances for all assets and covariances between all of the assets handled by the model.

In Markowitz' model investors are assumed to want as high expected future returns as possible but at as low risk as possible. This seems quite reasonable. There may be many other factors that investors would like to consider, but this model focuses on risk and return.

To derive the set of attainable portfolios (derived from the expected return and the covariance matrix estimated by the investor) that an investor can reach, we need to solve the following problem:

$$\begin{cases} \min\limits_{\mathbf{w}} \mathbf{w}^T \Sigma \mathbf{w} \\ \mathbf{w}^T \bar{\mathbf{r}} = \bar{r}_p \end{cases} \tag{2.1}$$

or

$$\begin{cases} \max\limits_{\mathbf{w}} \mathbf{w}^T \bar{\mathbf{r}} \\ \mathbf{w}^T \Sigma \mathbf{w} = \sigma_p^2 \end{cases} \tag{2.2}$$

$\mathbf{w}$ - the column vector of portfolio weights

$\mathbf{w}^*$- the Markowitz' optimal portfolio

[3] For simplicity expected return will refer to the expected excess return over the one-period risk-free rate.

σ_p^2 - the variance of the portfolio

$\bar{r}_p$ - the expected return of the portfolio

$\bar{\mathbf{r}}$ - the column vector of expected returns

$\boldsymbol{\mu}$ - the column vector of expected (excess) returns

$\boldsymbol{\Sigma}$ - the covariance matrix.

δ - the risk aversion parameter stated by the investors. States the trade-off between risk and return. Equals $\frac{\mu_P}{\sigma_P^2}$[4]. This is consistent with Satchell and Scowcroft (2000, p. 139). Economists would call this parameter the standard price of variance.

Often the following problem is solved instead of the above ones:

$$\max_{\mathbf{w}} \mathbf{w}^T \boldsymbol{\mu} - \frac{\delta}{2} \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w} \qquad (2.3)$$

This is actually the same as solving problem (2.1) or (2.2) (proof in Appendix 2).
Solving these equations generates:

$$\mathbf{w}^* = (\delta \boldsymbol{\Sigma})^{-1} \boldsymbol{\mu} \qquad (2.4)$$

This is the formula for the Markowitz' optimal portfolio.

2.2 Problems in the Use of Markowitz' Model

Although Markowitz' mean-variance model might seem appealing and reasonable from a theoretical point of view, several problems arise when using the model in practice. In the article *The Markowitz optimization Enigma: Is "Optimized" Optimal?* (1989), Michaud thoroughly discusses the practical problems of using the model. He claims that the model often leads to irrelevant optimal portfolios and that some studies have shown that even equal weighting can be superior to Markowitz optimal portfolios. Michaud argues that the most important reason for many financial actors not to use Markowitz' model is "political". The fact that the quantitatively oriented specialists would have a central role in the investment process would intimidate more qualitatively oriented managers and top level managers, according to Michaud. The article was however written 15 years ago and this may no longer be the most important reason for not using Markowitz' model. In the article Michaud also reviews other disadvantages of using the model.

[4] For a derivation please see appendix 1.

The most important problems in using theMarkowtiz' model are:

- According to Michaud (1989) and Black and Litterman (1992), Markowitz' optimizers maximize errors. Since there are no correct and exact estimates of either expected returns or variances and covariances, these estimates are subject to estimation errors. Markowitz' optimizers overweight securities with high expected return and negative correlation and underweight those with low expected returns and positive correlation. These securities are, according to Michaud, those that are most prone to be subject to large estimation errors. The argument appears however somewhat contradictory. The reason for investors to estimate high expected return on assets should be that they believe that this asset is prone to return well. It then seems reasonable that the manager would appreciate that the model overweighs this asset in the portfolio (taking covariances into consideration).

- Michaud claims that the habit of using historical data to produce a sample mean and replace the expected return with the sample mean is not a good one. He claims that this line of action contributes greatly to the error-maximization of the Markowitz mean-variance model.

- Markowitz' model doesn't account for assets' market capitalization weights. This means that if assets with a low level of capitalization have high-expected returns and are negatively correlated with other assets in the portfolio, the model can suggest a high portfolio weight. This is actually a problem, especially when adding a shorting constraint. The model then often suggests very high weights in assets with low level of capitalization.

- The Markowitz mean-variance model does not differentiate between different levels of uncertainty associated with the estimates input to the model.

- Mean-variance models are often unstable, meaning that small changes in input might dramatically change the portfolio. The model is especially unstable in relation to the expected return input. One small change in expected return on one asset might generate a radically different portfolio. According to Michaud this mainly depends on an ill-conditioned covariance matrix. He exemplifies ill-conditioned covariance matrixes by those estimated with "*insufficient historical data*".

Michaud also discusses further problems with Markowitz mean-variance model. These are: non-uniqueness, exact vs. approximate mean-variance optimizers, inadequate approximation power and default settings of parameters.

One of the most striking empirical problems, in using the Markowitz model, is that when running the optimizer without constraints, the model almost always recommends portfolios with large negative weights in several assets (Black & Litterman, 1992). Fund or portfolio managers using the model are often not permitted to take short positions. Because of this, a shorting constraint is often added to the optimization process. What happens then is that when optimizing a portfolio with constraints, the model gives a solution with zero weights in many of the assets and therefore takes large positions in only a few of the assets and unreasonable large weights in some assets. Many investors find portfolios of this kind unreasonable and although it seems, as though many investors are appealed to the idea of mean-variance optimization, these problems appear to be among the main reasons for not using it. In a world in which investors are quite sure about the input to an optimization model, he output of the model would not seem so unreasonable. In reality however, every approximation about future return and risk is quite uncertain and the chance that it is "absolutely correct" is low. Since the estimation of future risk and return is uncertain, it seems reasonable that investors wish to invest in portfolios which are not prospective disasters if the estimations prove incorrect. Markowitz' model has been shown, however, to generate portfolios that are very unstable i.e. sensitive to changes in input (Fisher & Statman, 1997), meaning that a small change in input radically changes the structure of the portfolio. Michaud (1989) claims that better input estimates could help bridge problems of the unintuitiveness of Markowitz' portfolios. Fisher and Statman, however, maintain that although good estimates are better then bad, better estimates will not bridge the gap between mean-variance optimized portfolios and "intuitive" portfolios, in which investors are willing to invest, since estimation errors can never be eliminated. It is not possible to predict future expected returns, variances and covariances with 100 % confidence.

Estimating covariances between assets is also problematic. In a portfolio containing 50 assets the number of variances that need to be estimated is 50, but the number of covariances that need to be estimated is 1225. This seems to be much for a single portfolio manager to handle. It also seems much for an investment team, consisting of several persons. According to Markowitz (1990, p. 102) "*in portfolios involving large numbers of correlated securities, variances shrink in importance compared to covariances*".

Although there exist several quite severe disadvantages in the use of the Markowitz mean-variance model, the idea of maximizing expected return; minimizing risk or optimizing the trade-off between risk and expected return is so appealing that the search for better-behaved models has continued. The B-L model is one of these and the model has gained much interest in recent years.

2.3 Historical Data

There seems to exist a common misconception saying that Markowitz' theories and model build solely on historical data. This, however, is not the case. Markowitz asserts that various types of information can be used as input to a portfolio analysis:

> "*One source of information is the past performance of individual securities. A second source of information is the beliefs of one or more security analysts concerning future performances*" (Markowitz, 1991, p.3).

> "*Portfolio selection should be based on reasonable beliefs about future returns rather than past performances per se. Choices based on past performances alone assume, in effect, that average returns of the past are good estimates of the 'likely' return in the future; and variability of return in the past is a good measure of the uncertainty of return in the future.*" (Markowitz 1991, p.14).

Markowitz (1991) is quite clear that he focuses on portfolio analysis and not security analysis. He claims that he does not discuss how to arrive at a reasonable belief about securities since this is the job of a security analyst. Markowitz' contribution begins where the contribution of the security analysis leaves off. While Markowitz time and time again repeats that historical data alone is inadequate as a basis for estimating future returns and covariances, we can often read about the importance of historical data in modern financial theory. It is hard to question the fact that historical time series have had great impact on financial decision-makings.

> "*…covariance matrices determined from empirical financial time series appear to contain such a high amount of noise that their structure can essentially be regarded as random. This seems, however, to be in contradiction with the fundamental role played by covariance matrices in finance, which constitute the pillars of modern investment theory and have also gained industry-wide applications in risk management*" (Pafka & Kondor, 2002, Abstract).

There seems to be a general confusion between the covariances of future returns and covariances estimated from historical data. This is problematic and may affect the discussion and the development of portfolio theory. The discussion whether historical data is a good approximation for future covariance matrices is, to me, interesting and also important. Also, I believe that it is of importance to discuss whether it is possible at all to make reasonable estimates of future covariances and how this affects the use of portfolio modeling. Separating the two discussions would however probably be productive.

3 The Black-Litterman Model

The problems encountered when using Markowitz' model in practical portfolio management and the fact that mean-variance optimization hasn't had such a high impact in practice motivated Fisher Black and Robert Litterman to work on the development of models for portfolio choice. Black and Litterman (1992) proposed a means of estimating expected returns to achieve better-behaved portfolio models. However they require the portfolio to be at the efficient frontier. If this is not the case, it may be possible to obtain a "better" portfolio from a mean-variance perspective. The B-L model is often referred to as a completely new portfolio model. Actually the B-L model differs only from the Markowitz model with respect to the expected returns. The B-L model is otherwise theoretically quite similar to Markowitz' mean-variance model. How the B-L expected returns are to be estimated has been found to be quite complicated. The model generates portfolios differing considerably from portfolios generated by using Markowitz' model.

This chapter begins with a description of the concept and the framework behind the B-L. A brief presentation of the Bayesian approach – the more commonly used approach to the B-L model follows before a sampling theory approach to the B-L model is presented and derived. Although suggested by Black and Litterman (1992) this approach does not seem to appear in the literature. The chapter is concluded with a summary and discussion of the results.

3.1 The Framework and the Idea

The B-L model was developed to make portfolio modeling more useful in practical investment situations (Litterman 2003c, p. 76). To do this, Black and Litterman (1992) apply, what they call, an equilibrium approach. They set the idealized market equilibrium as a point of reference. The investor then specifies a chosen number of market views in the form of expected returns and a level of confidence for each view. The views are combined with the equilibrium returns and the combination of these constitutes the B-L expected returns. The B-L expected returns are then optimized in a mean-variance way, creating a portfolio where bets are taken on assets where investors have opinions about future expected returns but not elsewhere. The size of the bets, in relation to the equilibrium portfolio weights, depends on the confidence levels specified by the user and also on a parameter specifying the weight of the collected investor views in relation to the market equilibrium, the weight-on-views.

The following notation is used:

$\mathbf{w}^*$ - the weight vector of the B-L unconstrained optimal portfolio.

$\mathbf{w}^M$ - the weight vector of the market capitalized portfolio, referred to as the equilibrium portfolio or the market portfolio.

δ - the risk aversion factor. It is according to Black and Litterman (1991, p. 37) proportionality constant based on the formulas in Black (1989). $\delta = \frac{\mu_P}{\sigma_P^2}$ (Satchell and Scowcroft 2000, p. 139). See Appendix 2 for a derivation. In He and Litterman (1999) the authors use *"$\delta = 2.5$ as the risk aversion parameter representing the world average risk tolerance"*.

$\mathbf{\Sigma}$ - the covariance matrix containing variances of and covariances between all the assets handled by the model.

$\mathbf{P}$ - a matrix representing a part of the views. Each row in the matrix contains the weights of assets of one view. The maximum number of rows, i.e. the maximum number of views is the number of assets in the portfolio.

$\overline{\mathbf{q}}$ - a column vector that represents the estimated expected returns in each view.

ω_i - the level of confidence assigned to view i. It is the standard deviation around the expected return of the view so that the investor is 2/3 sure that the return will lie within the interval.

$\mathbf{\Omega}$ - a diagonal matrix consisting of $\omega_1^2, \ldots, \omega_k^2$.

τ - A parameter often referred to as the *weight-on-views*. τ is a constant, which together with $\mathbf{\Omega}$ determines the weighting between the view portfolio and the equilibrium portfolio.

$\boldsymbol{\mu}^*$ - This is the B-L modified vector of estimated expected returns.

$\mathbf{\Pi}$ - The column vector of equilibrium expected excess returns.

To derive the B-L expected returns estimated by the market, the following problem is solved:

$$\max_{\Pi}(\mathbf{w}^M)^T\mathbf{\Pi} - \frac{\delta}{2}(\mathbf{w}^M)^T\mathbf{\Sigma}\mathbf{w}^M$$

equilibrium excess returns, $\mathbf{\Pi}$ is

$$\mathbf{\Pi} = \delta\mathbf{\Sigma}\mathbf{w}^M \tag{3.1}$$

This formula represents the expected returns estimated by the market. Many managers, however, do not wish to invest in the market portfolio. They have views that differ from the market returns. The market returns are then combined with investor views and a modified vector of expected returns constituting the B-L vector of expected returns is created. This new vector of B-L expected returns is then optimized in a mean-variance manner, yielding the formula for the weights of the optimal portfolio. The formula for the Black-Litterman optimal portfolio, without constraints, is presented below. Readers need not understand this formula at this point - a detailed derivation and explanation will be given further on in this chapter. However let us no just have a look at the formula to know where we are heading:

$$\mathbf{w}^* = \mathbf{w}^M + \frac{\tau}{\delta}\mathbf{P}^T(\mathbf{\Omega} + \tau\mathbf{P}\mathbf{\Sigma}\mathbf{P}^T)^{-1}(\mathbf{q} - \delta\mathbf{P}\mathbf{w}^M) \tag{3.2}$$

For the full derivation of this formula, please see the section 3.3.2. The intuition here can however be that by just looking at the formula we can see that the model takes the market weights and then ads a component, hence the model starts of from the market weights.

3.1.1 Equilibrium

What do Black and Litterman mean by equilibrium? In the book "*Modern Investment Management – An Equilibrium Approach*", (Litterman et. al. 2003), Litterman discusses the concept of the equilibrium approach. Equilibrium, according to Litterman, is an idealized state in which supply equals demand. He stresses that this state never actually occurs in financial markets, but argues that there are a number of attractive characteristics about the idea. According to Litterman there are "natural forces", in the form of arbitrageurs, in the economic system that function to eliminate deviations from equilibrium. Even if there are disturbances in markets – such as noise traders, uncertain information and lack of liquidity that result in situations in which deviations are large and in which adjustment takes time, there is a tendency that mispricing will, over time, be "corrected". Hence, the markets are not assumed to be in equilibrium (Litterman 2003a). Equilibrium is instead viewed as a "centre of gravity". Markets deviate from this state, but will forces in the system will push markets towards equilibrium. The idea of an equilibrium as a point of reference for the B-L model is hence a kind of ideal condition for the model. In order to apply the model to real life investment situations we need to make a reasonable approximation of this state.

Litterman (Litterman 2003a) claims that the reason for recommending the equilibrium approach is the belief that it is a favorable and appropriate point of reference from which identification of deviations can be made and taken advantage of. He admits that no financial theory can ever capture the complexity of financial markets. Still, "*Financial theory has the most to say about markets that are behaving in a somewhat rational manner. If we start by assuming that markets are simply irrational, then we have little more to say*" (Litterman 2003a). He refers to the extensive amount of literature we can access if we are willing to accept the assumption of arbitrage-free markets. According to Litterman, we also need to add the assumption that markets, over time, move toward a rational equilibrium in order to take advantage of portfolio theory. He states that portfolio theory makes predictions about how markets will behave, tells investors how to structure their portfolios, how to minimize risk and also how to take maximum advantage of deviations from equilibrium.

Much literature concerning the B-L model assumes a global asset allocation model, and because of this Litterman (2003c) argue that the global Capital Asset Pricing Model (CAPM) is a good starting point for a global equilibrium model. Black (1989) discusses an equilibrium model providing a framework from which the B-L global asset allocation model has emerged. However, the B-L model is not used only in global asset management, but also in domestic equity portfolio management and fixed income portfolio management. In such cases the equilibrium weights are easier to find by using the domestic CAPM.

There is an obvious problem in using equilibrium weights as a point of reference since these weights are not observable and hence must be estimated. Bevan and Winklemann (1998), present a way of dealing with this. If the market is in equilibrium, a representative investor will hold a part of the capitalization-weighted portfolio. Many investors are evaluated according to a benchmark portfolio. Often the benchmark is a capitalization-weighted index (Litterman 2003b). The equilibrium portfolio is then approximated as the benchmark portfolio. These estimated expected returns could be seen as the expected returns estimated by the market if all actors on the market act in a mean-variance manner. Expected equilibrium returns are calculated from the benchmark weights using formula 4 in chapter 2.1. As Schachter et al. (1986 p. 254) write: *"[T]he price of a stock is more than an objective, rationally determined number; it is an opinion, an aggregate opinion, the moment-to-moment resultant of the evaluation of the community of investors.*" For each asset, to which the investor has no view, this is what will be handed over to the optimizer. For the assets to which the investor has views, modified expected returns are calculated as a combination of the benchmark weights and the investor views. This way of estimating the equilibrium portfolio is what will be used in this chapter. From now on the equilibrium portfolio often will be referred to as the market portfolio.

3.1.2 Investor Views and Levels of Confidence

The B-L idea is to combine the equilibrium with investor-specific views. To each view a level of confidence is to be set by the manager. The model allows the investor to express both absolute and relative views. An example of an absolute view is "*I expect that equities in country A will return X%"* an example of a relative view is "*I believe domestic bonds will outperform domestic equities by Y%*". In traditional mean-variance portfolio optimization, relative views cannot be expressed. To each view, whether stated in the relative or absolute form the investor also shall assign a level of confidence. The level of confidence is expressed as the standard deviation around the expected return of the view. If managers feel confident in one view the standard deviation should be small and if they are not confident in a view, the standard deviation should be large. The confidence level affects the influence of a particular view. The weaker confidence that is set to a view the less the view affects the portfolio weights. This is considered as an attractive feature since views most often are incorrect. Views however indicate on which assets investors want to take bets and in which direction the bets ought to be taken.

3.1.3 Combining Views with the Equilibrium Expected Returns

The B-L optimal portfolio is a weighted combination of the market portfolio and the views of the investor. The views are combined with the equilibrium, and positions are taken in relation to the benchmark portfolio on assets to which investors have expressed views. The size of the bet taken depends on three different variables: the views, the level of confidence assigned to each view and the weight-on-views. It depends on the views specified by the investor. Views that differs much from the market expected returns contributes to larger bets. If the level of confidence assigned to a view is strong, this also contributes to larger bets. The more confidence the investor assigns to a view, the larger the bets are on that particular asset. The matrix $\mathbf{\Omega}$ represents the levels of confidence of the views. There is however one more variable that affects the size of the bets taken in relation to the equilibrium portfolio. The variable τ, the *weight-on views* (Bevan & Winkelmann, 1998), determines, with $\mathbf{\Omega}$, how much weight is to be set on the set of view portfolios specified by the investor in relation to the equilibrium portfolio. I have found no clear description of this variable in existing literature. There seem to be quite different ideas on how to set this variable. Black and Litterman (1992, p.17) propose that the constant should be set close to zero "*because the uncertainty in the mean is much smaller than the uncertainty in the return itself*". Satchell and Scowcroft (2000) however claim that τ often is set to 1, but they also claim that this is not always successful in reality. Bevan and Winkelmann (1998, p.4), on the other hand, suggest that τ can be set so that the *information ratio*[5] does not exceed 2.0. They have found that τ most often lies between o.5 – 0.7. He and Litterman (1999, p. 6), on the other hand, claim that τ need not be set at all, since only $\tau^{-1}\mathbf{\Omega}$ enters the model. Mathematically, this is correct, but then there would be no point in specifying these two different variables from the beginning. The reasoning concerning τ is hence quite weak in existing literature. The articles don't express any associations to normative and descriptive argumentation. There are totally different suggestions on what τ ought to be set to and explanations of why these are reasonable values of τ is not given properly.

By the end of this chapter an interpretable formula to the weight-on-views will however be derived and explained. One of the great advantages of taking a sampling theoretical approach to the B-L model is that it provides an interpretable formula to the weight-on-views. The chapter won't however result in a recommended value of τ, the formula however will give the user of the B-L model guidance in setting this variable.

When no investor views are specified, the B-L model recommends holding the market portfolio. If investors have no opinion about the market they should not place bets in relation to the equilibrium weights. However, if they have opinions about assets, it seems reasonable that the bets are placed in those assets and the rest of the assets have weights close to the market-capitalized portfolio. The stronger confidence assigned, to both the individual view and the weight-on-views, the more the output portfolio deviates from the market portfolio.

Below a brief description of the Bayesian approach to the B-L model is given before the sampling theoretical approach is presented. The sampling theoretical approach will then provide a detailed derivation of the B-L expected returns and the B-L portfolio.

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[5] A risk measure, measuring how well a fund is paid for the active risk taken, hence how much extra the fund returns by deviating from the index portfolio.

3.2 The Bayesian Approach to the B-L Model

Most of the literature concerning the B-L model makes use of a Bayesian[6] approach to construe the B-L model. The approach combines prior information (information considered as relevant although not necessarily in the form of sample data) with sample data. Through repeated use of Bayes' theorem[7], the prior information is updated. Although the Bayesian approach to inference, conceptually, is quite different from the sampling theory approach to inference, the results of the two methods are generally nearly identical. An example of an important difference between the approaches is that in the sampling theory approach we consider θ, the estimate of the unknown parameter μ, to be an unknown constant, while the Bayesian approach views θ as a random variable.

As mentioned, the most frequent way of interpreting the B-L model is from a Bayesian point of view. Since the idea is to update information from the market with information from the investor, the Bayesian approach lays easy at hand. Two articles that clearly use the Bayesian approach are: *A Demystification of the B-L model: Managing quantitative and traditional portfolio construction* by Stephen Satchell and Alan Scowcroft (2000) and *Bayesian Optimal Portfolio Selection: the B-L Approach* by George A Christodoulakis and John Cass (2002).

Satchell and Scowcroft claim that the B-L model is, in fact, based on a Bayesian methodology and also that this "*methodology effectively updates currently held opinions with data to form new opinions*" (p.139). The authors point out that despite the importance of the model, it appears, as if there is no comprehensible description of the mathematics underlying the model.

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6 The theory of Bayesian inference rests primarily on Bayes' theorem. Thomas Bayes' contribution to the literature on probability theory was only two papers published in the Philosophical Transactions in 1763-1764. Still, his work has had a major impact on probability theory and the theory of statistics. Both papers where published after his death and there is still some disagreement on exactly what Bayes' was suggesting in the second article, called "*Essay*". There are however aspects within the articles that are widely agreed upon and three important features of his theory are: the use of continuous frameworks rather than discrete, the idea of inference (essentially estimation) through assessing the chances that an informed guess about a practical situation will be correct, and in proposing a formal description of what is meant by prior ignorance.

7 $P(A|B) = \frac{P(B|A)}{P(B)} P(A)$

The prior information that is to be entered into a Bayesian model is represented by a probability P(A), the prior probability. This information is then updated by the information of B, that is supposed to be sample data and represented in the form of likelihood. The resulting probability is referred to as the posterior probability. However, there are two well-known difficulties within the Bayesian theory of inference. First, there is a problem in the interpretation of the probability idea in a particular Bayesian analysis. Second, it is often difficult to specify a numerical representation of the prior probabilities used in the analysis. How do we proceed when the quantities $P(A|B)$ and $P(B|A)$ are unknown? In a Bayesian framework we would answer that the best we can do is to compute the quantities with all the information we have at our disposal. The central problem in Bayesian theory is how to use a sample drawn independently according to the fixed but unknown probability distribution $P(B)$ to determine $P(A|B)$.

In the Bayesian approach we need to decide what is to be considered as prior information and what is to be considered as sample information. Satchell and Scowcroft use the investor views as prior information and information from the market is seen as sample data with which they update the investor views to receive the posterior distribution. Satchell and Scowcroft admit that their interpretation of what is prior information and what is the sample data *may* differ from that of others. It might be questioned whether this is a good way to *demystify* the B-L model. The authors also claim that the aim of Black and Litterman was to form a model that made the idea of combining investor views with market equilibrium sensible to investors. I argue that neither Black and Litterman nor Satchel and Scowcroft have succeeded with this task. If Black and Litterman had produced a text that made the idea of combining investor views with the market equilibrium comprehensible to investors, there would be no need for Satchell and Scowcroft to write an article intended to demystify the model. Satchell and Scowcroft however assert that the Bayesian approach has been undermined by the problems in specifying a numerical distribution representing the view of an individual. It is claimed in the article that the parameter $\boldsymbol{\tau}$ is a "*known scaling factor that often is set to one*" (p.140). The parameter is not explained in any further way.

Christodoulakis and Cass also interpret the B-L model in a Bayesian manner. They claim that the articles by Black and Litterman provide more of a framework for combining investor views with the market equilibrium, than a sensible and clear description of the model. Christodoulakis and Cass argue as Satchell and Scowcroft for using the investor views as the prior information and the market equilibrium returns for updating these to receive the posterior expected returns. The fact that the model assumes that the investor views are formed independently of each other is discussed. The assumption that the returns are normally distributed together with the fact that $\boldsymbol{\Omega}$ is a diagonal matrix implies this. The B-L model assumes a diagonal $\boldsymbol{\Omega}$-matrix. This is however an inconsistency in the model, which is, discussed in section 3.2.3 Christodoulakis and Cass refer to $\boldsymbol{\tau}$ as a scalar known to the investor that scales the "*historical covariance matrix* $\boldsymbol{\Sigma}$" (p. 5). That they refer to $\boldsymbol{\Sigma}$ as the historical covariance matrix is questionable. My interpretation of the B-L model is that $\boldsymbol{\Sigma}$ is the same covariance matrix as that in the Markowitz model and neither Markowitz nor Black and Litterman claim that this should be anything else than the estimated future covariances between the assets that the model handles.

3.3 Sampling Theory Approach and the Black-Litterman Model

So far, the explanation of the B-L model has focused on the idea and the framework behind the B-L model. Some parts of the B-L model are difficult to understand on the basis of existing literature. Articles concerning the model have titles such as: *The Intuition behind the B-L model* (He & Litterman 1999), *A step By Step Guide to the B-L model* (Idzorek, 2004), *A Demystification of the B-L model* (Satchell & Scowcroft, 2000). These titles suggest that others have encountered such problems with the model as I have experienced. It seems relatively easy to grasp the framework, but understanding how the formula for the B-L vector of expected returns is derived is quite a challenge. As discussed, the articles discussing the B-L model begin from a Bayesian perspective. The idea of trying to derive the model from a sampling theory point of view was actually presented by Black and Litterman (1992):

> "*One way we think about representing that information is to act as if we had a summary statistic from a sample of data drawn from the distribution of future returns – data in which all that we're able to observe is the difference between the returns of A and B. Alternatively, we can express this view directly as a probability distribution for the difference between the means of the excess returns of A and B. It doesn't matter which of these approaches we want to use to think about our views; in the end we get the same result.*" (pp. 34-35)

As mentioned most people seem to have chosen the Bayesian approach, but the quotation implies that the authors also had the sampling theoretical approach in mind.

The following section will present a detailed exposition of the B-L model. A full and detailed derivation from a sampling theoretical approach will be provided. Until now the mathematics in this thesis has been on a relatively low level to enable as many readers as possible to follow its parts. In the following section the level of mathematical complexity will rise, this being necessary to derive the model mathematically. However, to enable as many readers as possible to follow the steps in this derivation I have tried to be very explicit and perform the derivation in many small steps. My impression is that often, in mathematical literature, important steps may be considered so obvious that they need not be shown. It is then easy to lose some readers and to avoid this happening, the mathematical steps taken in this section are small but many.

3.3.1 Sampling Theory[8]

In sampling theory, the study of sample data is supposed to shed light on an unknown parameter. The unknown parameter can for example be the variance or the expected value of a stochastic variable or stochastic vector. Point estimation is a well-known concept of sampling theory. Point estimates of the expected excess returns are what we want to estimate in the B-L model.

Different realizations of the stochastic variable or the vector of stochastic variables may generate different values yielding different estimates. The resulting probability distribution is called the sampling distribution of the statistic. Sampling theory cannot yield statements of final precision. Let us clarify this with an example. Consider a dice of uncertain symmetry i.e. with an unknown probability function. The true probability function of the dice is $p_Y(y)$. The sampling theoretical way of getting information of the probability function of the dice is by throwing it a number of times and studying the results. Here the sample data, y, is represented by the outcome of one toss of the dice. Different tosses will generate different outcomes $y_1,y_2,\ldots,y_n$. If the unknown parameter, μ, is the expected value, we can estimate this by calculating its sample mean. The sample mean could then act as an estimate θ of the unknown parameter μ. The sample mean is:

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

All estimates of unknown parameters are not accurate. Characteristics that an estimator should possess according to sampling theory are freedom from bias, consistency, sufficiency, efficiency, low variance etc. [9] One of the most recognized methods for point estimation within sampling theory is the maximum likelihood method. The estimates generated by this method possess many of the characteristics of a good estimator.

"Likelihood" or rather the likelihood function is a central element within classical statistics. It is simply a re-interpretation of the density function and expresses how the probability (density), $p_\theta(x)$, of the data x fluctuates with different values of the parameter

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[8] Sampling theory – the classical approach to statistical inference. The approach stems from the work of Fisher, Neyman, E. S. Pearson and others. It relies solely on sample data, which is being represented by their likelihood (Garthwaite et. al. 2002). Sampling theory is considered as the classical approach to inference since historically it was the first of three approaches to take form. The other approaches to inference often discussed are the Bayesian approach, which is the common approach to apply to the B-L model, and was discussed in 3.2. The third approach to inference is the Decision theory approach to inference. The Decision theory approach to inference will not be discussed in this text. Development of sampling theory can be traced to the early 1800s whilst the rivalling approaches have taken form during the last 50 years.

[9] To read more about characteristics of estimators please see Barnett (1999).

θ. However, the likelihood is not the probability (density) of θ for a given sample x. The likelihood function for θ based on the sample data X=x is given by:

$$L_\theta(x) = p_\theta(x_1) \cdot p_\theta(x_2) \cdot \ldots \cdot p_\theta(x_n)$$

The maximum likelihood method is a statistical method for estimating parameters from sample data. The parameter values that maximize the probability of obtaining the observed data are selected as estimates. The method is one of the most widely used for constructing estimators. The estimates, resulting from this approach, often possess the desirable properties originating from the classical approach. Maximum likelihood methods possess many attractive features (Barnett 1999, p. 153).

In general, the idea is that the value of the parameter under which the obtained data would have had highest probability (density) of occurring must be the best estimator of θ. Intuitively we can think of the estimates as the value of θ that best supports the observed sample.

Almost always when working with the maximum likelihood method we choose to work with the logarithm of the likelihood function instead of the likelihood function itself. We do this because the log-likelihood function is easier to work with and both the likelihood function and the log-likelihood function have their maximum values for the same θ. To obtain the maximum likelihood estimator, we differentiate the log-likelihood function, set it equal to zero and solve for θ. The maximum likelihood method generates good estimators when we have a good model for the underlying distributions and their dependence of the parameter θ. A poor model for underlying distributions may, not surprisingly, generate bad estimates.

Although the classical approach to inference seems to make sense and is widely applied, it does not lack critics. Criticism is focused on two fundamental factors within sampling theory. The first is the preoccupation with a frequency-based probability concept providing justification for assessing the behavior of statistical procedures in terms of their long-term behavior. The criticism questions the validity of assigning aggregate properties to specific inferences. The second type of criticism of sampling theory relates to the restrictions applied by the approach on what is regarded as relevant information, namely sample data (Barnett 1999, p. 197).

3.3.2 The Sampling Theory Approach to the Black-Litterman Model

One reason for trying a sampling theoretical approach to the B-L model has to do with the problems I have experienced when trying to get a deeper understanding of the model from the existing literature. Since sampling theory is just another way of considering inference and point estimation, the idea of using the approach appeared interesting. At first sight, readers might find this a bit odd. Sampling theory builds on sample data as information for inference, but in this case we have no sample data. The two approaches, Bayesian and sampling theory, will however be seen to generate the same result. I will begin by giving a conceptual explanation of the B-L model from a sampling theoretical point of view. After this a more thorough mathematical derivation will be presented.

To handle the fact that we have no sample data while sampling theory depends on this as the sole source of information, we will suppose that both the market and the individual investor have observed samples of future returns. The sample returns observed by the market will then represent the equilibrium portfolio, while the sample returns observed by the investor will represent the views of the investor. The samples observed by the market are different from those observed by the investor.

Suppose that the market has observed a number of samples of future asset returns. With the method of maximum likelihood we derive the markets' estimated expected returns, referred to as the equilibrium or market returns. We also suppose that the investor has observed a number of samples of returns. The investor has observed returns on a number of portfolios of assets instead of the assets themselves. These portfolios can relate to all the assets in the investor universe or just one or a few of them. We use the maximum likelihood method to estimate the expected returns of the investor views. We assume that the observations of future asset returns are normally distributed. This is a common assumption within quantitative finance and also an assumption fundamental to the following derivation. This assumption is sometimes criticized and this will be shortly discussed in chapter 4.2. For the present, we just accept that this is one of the assumptions within the B-L model. We then derive the maximum likelihood estimates of the asset returns observed by the market together with the portfolio returns observed by the individual investor. The estimator we get is hence the B-L estimator of the expected excess returns.

The following pages of this chapter will provide the mathematical derivation and description of the sampling theoretical approach to the B-L model.

The Equilibrium Portfolio

Let us suppose that the market has observed m samples of asset returns and that the investment universe contains d assets. We then suppose that the market has observations in the following form:

$$\mathbf{r}_1 = \begin{bmatrix} r_1^1 \\ \vdots \\ r_1^d \end{bmatrix}, \quad \mathbf{r}_2 = \begin{bmatrix} r_2^1 \\ \vdots \\ r_2^d \end{bmatrix}, \ldots, \quad \mathbf{r_m} = \begin{bmatrix} r_m^1 \\ \vdots \\ r_m^d \end{bmatrix}$$

From these we will derive the market estimated expected returns, equilibrium returns

$$\mathbf{\Pi} = \bar{\mathbf{r}}^M = \begin{bmatrix} \bar{r}^1 \\ \vdots \\ \bar{r}^d \end{bmatrix} = \frac{1}{m}\sum_{i=1}^{m} \mathbf{r}_i$$

by using the method of maximum likelihood. Assume that the observed samples of the market are "drawn" from a normal distribution with the true vector of expected value equal to $\boldsymbol{\mu}$ and the covariance matrix equal to $\boldsymbol{\Sigma}$. Then the vector of sample means is normally distributed with the vector of expected returns, $\boldsymbol{\mu}$ and the covariance matrix, $\boldsymbol{\Sigma}/m$, i.e.:

$$\mathbf{r}_i \in N(\boldsymbol{\mu}, \boldsymbol{\Sigma}),\ i = 1 \ldots m$$

$$\bar{\mathbf{r}}^M \in N(\boldsymbol{\mu}, \frac{\boldsymbol{\Sigma}}{m})$$

The probability function of the return is then:

$$p(\mathbf{r}_i) = \frac{1}{(2\pi)^{d/2}\sqrt{\det \boldsymbol{\Sigma}}} \exp(-\frac{1}{2}(\mathbf{r}_i - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\mathbf{r}_i - \boldsymbol{\mu}))$$

Since we are only interested in for which value of $\boldsymbol{\mu}$ the likelihood function, i.e. the product of the probability functions, takes its maximum value, we do not need to consider the constants. Instead we will work with:

$$\varphi(\mathbf{r}_i) = \exp(-\frac{1}{2}(\mathbf{r}_i - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\mathbf{r}_i - \boldsymbol{\mu}))$$

The likelihood function is then:

$$\mathbf{L} = \varphi(\mathbf{r}_1) \cdot \varphi(\mathbf{r}_2) \cdot \ldots \cdot \varphi(\mathbf{r}_m)$$

As mentioned the logarithm of the likelihood function is easier to work with and the log-likelihood function is then:

$$\ell = \ln \mathbf{L} = \ln\left[\varphi(\mathbf{r}_1) \cdot \varphi(\mathbf{r}_2) \cdot \ldots \cdot \varphi(\mathbf{r}_m)\right] = \ln \varphi(\mathbf{r}_1) + \ln \varphi(\mathbf{r}_2) + \ldots + \ln \varphi(\mathbf{r}_m)$$

$$\left\{ \ln \varphi(\mathbf{r}_i) = \ln\left[\exp\left(-\frac{1}{2}(\mathbf{r}_i - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\mathbf{r}_i - \boldsymbol{\mu})\right)\right] = -\frac{1}{2}(\mathbf{r}_i - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\mathbf{r}_i - \boldsymbol{\mu}) \right\}$$

$$\ell = \frac{1}{2}\left(-\sum_{i=1}^{m}(\mathbf{r}_i - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\mathbf{r}_i - \boldsymbol{\mu})\right)$$

We want to maximize the log-likelihood function:

$$\max_{\boldsymbol{\mu}} \ell = \max_{\boldsymbol{\mu}} \frac{1}{2}\left(-\sum_{i=1}^{m} (\mathbf{r}_i - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{r}_i - \boldsymbol{\mu}) \right)$$

Let us differentiate the function with respect to $\boldsymbol{\mu}_{\mathrm{j}}$ and set the derivative equal to zero. We use the notation

$$\mathbf{e}_j = \left[0\ldots 0\underset{\uparrow\ \text{entry j}}{1}0\ldots 0\right],\ m \text{ elements}$$

$$\frac{\partial}{\partial \mu^{\mathbf{j}}} \ell = \frac{1}{2}\sum_{i=1}^{m}\left(-\mathbf{e}_j^T \boldsymbol{\Sigma}^{-1} (\mathbf{r}_i - \boldsymbol{\mu}^{*M}) - (\mathbf{r}_i - \boldsymbol{\mu}^{*M})^T \boldsymbol{\Sigma}^{-1} \mathbf{e}_j \right) = 0$$

$$\left\{ (\mathbf{r}_i - \boldsymbol{\mu}^{*M})^T \boldsymbol{\Sigma}^{-1} \mathbf{e}_j = \left[(\mathbf{r}_i - \boldsymbol{\mu}^{*M})^T \boldsymbol{\Sigma}^{-1} \mathbf{e}_j \right]^T = \mathbf{e}_j^{\;T} \boldsymbol{\Sigma}^{-1} (\mathbf{r}_i - \boldsymbol{\mu}^{*M}) \right\}$$

$$-\mathbf{e}_j^T \boldsymbol{\Sigma}^{-1} \sum_{i=1}^{m} (\mathbf{r}_i - \boldsymbol{\mu}^{*M}) = 0$$

$$-\mathbf{e}_j^T \boldsymbol{\Sigma}^{-1} \left(\sum_{i=1}^{m} \mathbf{r}_i - \sum_{i=1}^{m} \boldsymbol{\mu}^{*M} \right) = 0$$

$$m\mathbf{e}_j^T \boldsymbol{\Sigma}^{-1} (\bar{\mathbf{r}}^M - \boldsymbol{\mu}^{*M}) = 0$$

Since this holds for all j=1,…,d it follows that

$$\boldsymbol{\mu}^{*M} = \bar{\mathbf{r}}^M = \frac{1}{m}\sum_{i=1}^{m} \mathbf{r}_i$$

$$\boldsymbol{\Pi} = \boldsymbol{\mu}^{*M}$$

$\boldsymbol{\mu}^{*M}$ is hence the expected future excess return estimated by the market.

The Views of the Manager

Let us assume that an investor has observed *n* other samples of returns. These observations are however, as mentioned, not observations of returns on individual assets. Instead they are observations of returns on portfolios of assets. As described above, the investor need not state views about every asset in his or hers investment universe. Instead a number of portfolios are chosen and the investor postulates that he/she observes a number of samples of the future returns of these portfolios. The weights of the portfolios are expressed in a matrix, $\mathbf{P}$, in which each position represents the weight of a certain asset in a certain view portfolio. Each row in the matrix represents one view portfolio and for each view portfolio the investor expresses an expected return $\bar{q}_i$ and a level of confidence ω_i. Suppose that the investor has opinions about *k* portfolios, $k \leq d$, where *d* is the number of assets handled by the model.

In the B-L model, $\mathbf{P}$ is the matrix

$$\mathbf{P} = \begin{bmatrix} w_1^1 & \cdots & w_1^d \\ \vdots & \ddots & \vdots \\ w_k^1 & \cdots & w_k^d \end{bmatrix}$$

where w_j^i is the weight of asset *i* in view portfolio *j*.

The expected returns to each portfolio are referred to as

$$\overline{\mathbf{q}} = \begin{bmatrix} \overline{q}_1 \\ \vdots \\ \overline{q}_k \end{bmatrix}$$

Where

$$\overline{\mathbf{q}} = \mathbf{P}\overline{\mathbf{r}}^I$$

From this formula we can hence derive the expected returns to each asset estimated by the investor:

$$\overline{\mathbf{r}}^I = \mathbf{P}^{-1}\overline{\mathbf{q}}$$

To clarify how to set $\mathbf{P}$ and $\overline{\mathbf{q}}$, let us consider an example of the two easiest and perhaps most used views.

Consider a portfolio holding just three assets, assets A, B and C. The investor can hence express three or fewer views. In this example only two views are expressed:

View 1: "I believe that asset A will return 3%"

View 2: "I believe that asset B will outperform asset C with 2%"

$\mathbf{P}$ and $\overline{\mathbf{q}}$ will then appear as follows:

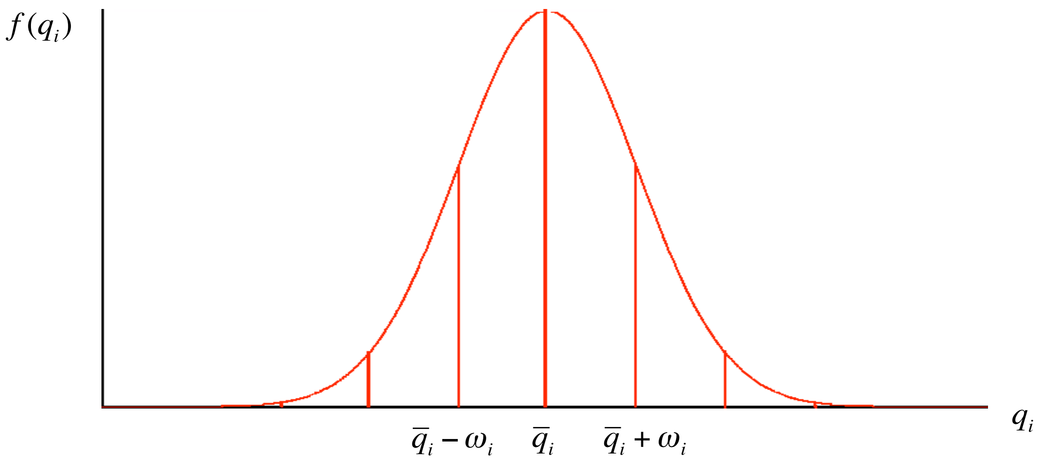
$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \qquad \overline{\mathbf{q}} = \begin{bmatrix} 3\% \\ 2\% \\ 0 \end{bmatrix}$$

Each row in $\mathbf{P}$ represents one view portfolio. Each column represents the weights of a specific asset.

The diagonal matrix represents the investor's levels of confidence $\mathbf{\Omega}$. $\omega_1^2, \ldots, \omega_k^2$ constitute the diagonal of $\mathbf{\Omega}$. The number of rows and columns equals of course the number of views stated by the investor.

$$\mathbf{\Omega} = \begin{bmatrix} \omega_1^2 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \omega_\kappa^2 \end{bmatrix}$$

The possibility to express a level of confidence to each view is, to many, considered to be the most attractive feature of the B-L model. But what is a level of confidence? How is this supposed to be estimated? Let us remind ourselves of the samples of portfolio returns observed by the investor. We assumed that the investor had observed *n* samples of the returns of the view portfolios and that the samples were normally distributed. The level of confidence, $\omega_i^{\,2}$, is the variance of $\overline{q}_i$. ω_i can be interpreted as an interval around $\overline{q}_i$, so that 2/3, of the postulated samples lie within the interval $\overline{q}_i \pm \omega_i$, where $i = 1,\ldots,k$, see figure 3.1.



Figur 3.1 The level of confidence, $\omega_i^{\,2}$, is the variance of $\overline{q}_i$. ω_i can be interpreted as an interval around $\overline{q}_i$, so that 2/3, of the postulated samples lie within the interval $\overline{q}_i \pm \omega_i$, where $i = 1,\ldots,k$

The samples observed by the investor are also supposed to be drawn from a normally distributed set. The vector of expected values is the same as for the market i.e. $\boldsymbol{\mu}$. The covariance matrix however is not the same.

$$\underbrace{\mathbf{r}_1,\ldots,\mathbf{r}_m}_{\substack{m \text{ observation} \\ \text{by the market}}},\underbrace{\mathbf{r}_{m+1},\ldots,\mathbf{r}_{m+n}}_{\substack{n \text{ observations} \\ \text{by the investor}}}$$

Since $\mathbf{r}_j \in N(\boldsymbol{\mu},\boldsymbol{\Sigma})$ and $\mathbf{q}_j = \mathbf{P}\mathbf{r}_j$ then $\mathbf{q}_j$ should be $N(\mathbf{P}\boldsymbol{\mu},\mathbf{P}^T\boldsymbol{\Sigma}\mathbf{P})$[10]. However, in the B-L model, the distribution of $\mathbf{q}_j$ is $\mathbf{q}_j \in N(\mathbf{P}\boldsymbol{\mu},\boldsymbol{\Omega})$. Hence, this is an inconsistency since $\boldsymbol{\Omega} \neq \mathbf{P}^T\boldsymbol{\Sigma}\mathbf{P}$. $\boldsymbol{\Omega}$ is a diagonal matrix implying that returns of the portfolios observed by the investor are uncorrelated. This is an inconsistent assumption because the returns of the assets from which the portfolios are formed are has the covariance matrix $\boldsymbol{\Sigma}$ and $\boldsymbol{\Sigma}$ is not diagonal.

I will not derive the maximum likelihood estimator of the investor observations. The procedure is the same as for the market, the only difference being the number of observations. The market has observed m samples and the investor has observed n samples. The maximum likelihood estimator of the expected excess return of the investor is hence:

$$\boldsymbol{\mu}^{*I} = \overline{\mathbf{q}} = \frac{1}{n}\sum_{j=1}^{n}\mathbf{q}_j = \frac{1}{n}\sum_{j=1}^{n}\mathbf{P}\overline{\mathbf{r}}_j = \mathbf{P}\frac{1}{n}\sum_{j=1}^{n}\mathbf{r}_j = \mathbf{P}\overline{\mathbf{r}}^I$$

Combining Investor Views with Market Equilibrium

Let us now derive the maximum likelihood estimator of the expected returns from the returns observed by the market together with the returns observed by the investor.

$$\max_{\boldsymbol{\mu}} \sum_{i=1}^{m} -\frac{1}{2}(\mathbf{r}_i - \boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\mathbf{r}_i - \boldsymbol{\mu}) + \sum_{j=m+1}^{m+n} -\frac{1}{2}(\mathbf{q}_j - \mathbf{P}\boldsymbol{\mu})^T\boldsymbol{\Omega}^{-1}(\mathbf{q}_j - \mathbf{P}\boldsymbol{\mu})$$

We will use:

$$\mathbf{e}_k = \big[\underset{\uparrow\ \text{entry } k}{0\ldots010\ldots0}\big],\ n{+}m \text{ elements}$$

Let us differentiate the function with respect to $\boldsymbol{\mu}_{\mathrm{j}}$ and set the derivative equal to zero.

$$\frac{\partial}{\partial \mu^k}\left(\sum_{i=1}^{m} -\frac{1}{2}(\mathbf{r}_i - \boldsymbol{\mu}^*)^T\boldsymbol{\Sigma}^{-1}(\mathbf{r}_i - \boldsymbol{\mu}^*) + \sum_{j=m+1}^{m+n} -\frac{1}{2}(\mathbf{q}_j - \mathbf{P}\boldsymbol{\mu}^*)^T\boldsymbol{\Omega}^{-1}(\mathbf{q}_j - \mathbf{P}\boldsymbol{\mu}^*))\right)$$

$$\frac{1}{2}\sum_{i=1}^{m}\left(-\mathbf{e}_k{}^T\boldsymbol{\Sigma}^{-1}(\mathbf{r}_i - \boldsymbol{\mu}^*)^T - (\mathbf{r}_i - \boldsymbol{\mu}^*)\boldsymbol{\Sigma}^{-1}\mathbf{e}_k\right) + \frac{1}{2}\sum_{j=m+1}^{m+n}\left(-\mathbf{e}_k{}^T\mathbf{P}\boldsymbol{\Omega}^{-1}(\mathbf{q}_j - \mathbf{P}\boldsymbol{\mu}^*)^T - (\mathbf{q}_j - \mathbf{P}\boldsymbol{\mu}^*)\boldsymbol{\Omega}^{-1}\mathbf{P}\mathbf{e}_k\right) = 0$$

$$e_k{}^T\boldsymbol{\Sigma}^{-1}\sum_{i=1}^{m}(\mathbf{r}_i - \boldsymbol{\mu}^*) + e_k{}^T\mathbf{P}\boldsymbol{\Omega}^{-1}\sum_{j=m+1}^{m+n}(\mathbf{q}_j - \mathbf{P}\boldsymbol{\mu}^*) = 0$$

[10] Some articles actually suggest $\mathbf{q}_j \in N(\mathbf{P}\boldsymbol{\mu},\mathbf{P}^T\boldsymbol{\Sigma}\mathbf{P})$. This is mathematically correct but my impression is however that this impairs one of the main ideas of the B-L model, namely that the investor can specify the confidence in each view portfolio.

$$e_k^T\left(m\mathbf{\Sigma}^{-1}(\mathbf{\Pi}-\boldsymbol{\mu}^*)+n\mathbf{P\Omega}^{-1}(\overline{\mathbf{q}}-\mathbf{P}\boldsymbol{\mu}^*)\right)=0$$

Since this is true for all *k=1,…,,n+m* we get

$$\frac{m}{n}\mathbf{\Sigma}^{-1}(\mathbf{\Pi}-\boldsymbol{\mu}^*)+\mathbf{P\Omega}^{-1}(\overline{\mathbf{q}}-\mathbf{P}\boldsymbol{\mu}^*)=0$$

We then set

$$\tau=\frac{n}{m}$$

$$\boldsymbol{\mu}^*\left(\mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P}+\tau^{-1}\mathbf{\Sigma}^{-1}\right)=\mathbf{P}^T\mathbf{\Omega}^{-1}\overline{\mathbf{q}}+\tau^{-1}\mathbf{\Sigma}^{-1}\mathbf{\Pi}$$

$$\boldsymbol{\mu}^*=\left[(\tau\mathbf{\Sigma})^{-1}+\mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P}\right]^{-1}\cdot\left[(\tau\mathbf{\Sigma})^{-1}\mathbf{\Pi}+\mathbf{P}^T\mathbf{\Omega}^{-1}\overline{\mathbf{q}}\right]$$

This gives us the B-L formula for the modified vector of expected returns

$$\boldsymbol{\mu}^*=\left[(\tau\mathbf{\Sigma})^{-1}+\mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P}\right]^{-1}\cdot\left[(\tau\mathbf{\Sigma})^{-1}\mathbf{\Pi}+\mathbf{P}^T\mathbf{\Omega}^{-1}\overline{\mathbf{q}}\right] \tag{3.3}$$

This is the form most often used in the literature. Another way of expressing the B-L vector of modified expected returns is:[11]

$$\boldsymbol{\mu}^*=\mathbf{\Pi}+\tau\mathbf{\Sigma}\mathbf{P}^T(\mathbf{\Omega}+\tau\mathbf{P\Sigma P}^T)^{-1}(\overline{\mathbf{q}}-\mathbf{P\Pi}) \tag{3.4}$$

This way of presenting the B-L modified vector of expected returns may appear as more intuitive than the original formula. We see here that the modified vector of expected returns consists of first the vector of expected returns estimated by the market, $\mathbf{\Pi}$, and then another expression $\tau\mathbf{\Sigma}\mathbf{P}^T(\mathbf{\Omega}+\tau\mathbf{P\Sigma P}^T)^{-1}(\overline{\mathbf{q}}-\mathbf{P\Pi})$. Hence the expected returns estimated by the market are updated with another expression. If the last part of (3.4) $(\overline{\mathbf{q}}-\mathbf{P\Pi})$ equals zero, i.e. if the view of the investor is the same as the market view, then the modified vector of the expected return is only $\mathbf{\Pi}$. It is not obvious, however, that equation (3.3) and equation (3.4) are equal and it is not at all easy to deduce expression (3.4) of the modified vector of expected returns from expression (3.3). I therefore will show how this is done.

$$\begin{aligned}
\boldsymbol{\mu}^*&=\left[(\tau\mathbf{\Sigma})^{-1}+\mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P}\right]^{-1}\cdot\left[(\tau\mathbf{\Sigma})^{-1}\mathbf{\Pi}+\mathbf{P}^T\mathbf{\Omega}^{-1}\overline{\mathbf{q}}\right]\\
&=\left[(\tau\mathbf{\Sigma})^{-1}+\mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P}\right]^{-1}(\tau\mathbf{\Sigma})^{-1}(\tau\mathbf{\Sigma})\left[(\tau\mathbf{\Sigma})^{-1}\mathbf{\Pi}+\mathbf{P}^T\mathbf{\Omega}^{-1}\overline{\mathbf{q}}\right]\\
&=\left[\mathbf{I}+\tau\mathbf{\Sigma}\mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P}\right]^{-1}\cdot\left[\mathbf{\Pi}+\tau\mathbf{\Sigma}\mathbf{P}^T\mathbf{\Omega}^{-1}\overline{\mathbf{q}}\right]\\
&=\left[\mathbf{I}+\tau\mathbf{\Sigma}\mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P}\right]^{-1}\cdot\left[(\mathbf{I}+\tau\mathbf{\Sigma}\mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P})\mathbf{\Pi}+\tau\mathbf{\Sigma}\mathbf{P}^T\mathbf{\Omega}^{-1}(\overline{\mathbf{q}}-\mathbf{P\Pi})\right]\\
&=\mathbf{\Pi}+(\mathbf{I}+\tau\mathbf{\Sigma}\mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P})^{-1}\cdot(\tau\mathbf{\Sigma}\mathbf{P}^T\mathbf{\Omega}^{-1}(\overline{\mathbf{q}}-\mathbf{P\Pi}))\\
&=\mathbf{\Pi}+(\mathbf{I}+\tau\mathbf{\Sigma}\mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P})^{-1}\cdot\tau\mathbf{\Sigma}\mathbf{P}^T\mathbf{\Omega}^{-1}\left\{(\mathbf{\Omega}+\mathbf{P}^T\tau\mathbf{\Sigma}\mathbf{P})(\mathbf{\Omega}+\mathbf{P}^T\tau\mathbf{\Sigma}\mathbf{P})^{-1}\right\}(\overline{\mathbf{q}}-\mathbf{P\Pi})
\end{aligned}$$

[11] This was brought to my attention by Dr.F Armerin

$$= \mathbf{\Pi} + (\mathbf{I} + \tau\mathbf{\Sigma}\mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P})^{-1}(\tau\mathbf{\Sigma}\mathbf{P}^T + \tau\mathbf{\Sigma}\mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P}^T\tau\mathbf{\Sigma}\mathbf{P})(\mathbf{\Omega} + \mathbf{P}^T\tau\mathbf{\Sigma}\mathbf{P})^{-1}(\overline{\mathbf{q}} - \mathbf{P\Pi})$$
$$= \mathbf{\Pi} + (\mathbf{I} + \tau\mathbf{\Sigma}\mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P})^{-1}(\mathbf{I} + \tau\mathbf{\Sigma}\mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P})\tau\mathbf{\Sigma}\mathbf{P}^T(\mathbf{\Omega} + \mathbf{P}^T\tau\mathbf{\Sigma}\mathbf{P})^{-1}(\overline{\mathbf{q}} - \mathbf{P\Pi})$$
$$= \mathbf{\Pi} + (\mathbf{I} + \tau\mathbf{\Sigma}\mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P})^{-1}(\mathbf{I} + \tau\mathbf{\Sigma}\mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P})\tau\mathbf{\Sigma}\mathbf{P}^T(\mathbf{\Omega} + \mathbf{P}^T\tau\mathbf{\Sigma}\mathbf{P})^{-1}(\overline{\mathbf{q}} - \mathbf{P\Pi})$$

Here one parenthesis is multiplied by its own inverse. Hence we get

$$\boldsymbol{\mu}^* = \mathbf{\Pi} + \tau\mathbf{\Sigma}\mathbf{P}^T(\mathbf{\Omega} + \tau\mathbf{P}\mathbf{\Sigma}\mathbf{P}^T)^{-1}(\overline{\mathbf{q}} - \mathbf{P\Pi})$$

or

$$\boldsymbol{\mu}^* = \mathbf{\Pi} + \mathbf{\Sigma}\mathbf{P}^T(\frac{\mathbf{\Omega}}{\tau} + \mathbf{P}\mathbf{\Sigma}\mathbf{P}^T)^{-1}(\overline{\mathbf{q}} - \mathbf{P\Pi})$$

Using the formula

$$\mathbf{W}^* = (\delta\mathbf{\Sigma})^{-1}\boldsymbol{\mu}^*$$

we get

$$\mathbf{W}^* = \mathbf{W}^M + \mathbf{P}^T(\frac{\mathbf{\Omega}}{\tau} + \mathbf{P}\mathbf{\Sigma}\mathbf{P}^T)^{-1}(\frac{\overline{\mathbf{q}}}{\delta} - \mathbf{P}\mathbf{\Sigma}\mathbf{W}^M)$$

representing the unconstrained optimal portfolio.

The derivation of the B-L model from the sampling theoretical approach is hereby completed. We have arrived at the same formula for the B-L modified expected returns as reached in articles taking a Bayesian approach. The formula for the B-L modified expected returns are also reformulated and the formula for the weights of the optimal unconstrained portfolio is shown as well.

Readers may wonder whether this approach is really new. Have these calculations not been published previously? Black and Litterman already suggested this method in 1992! However, after extensive web search it appears, as the sampling theoretical derivations of the B-l model haven't been published before.

3.4 Results

The main results of the first part of the study are summarized below.

A detailed derivation of the B-L model from a sampling theoretical approach

It has been shown that the sampling theory approach offers an alternative way to derive the B-L model. The derivation leads to the same formula for the B-L modified vector of expected return as obtained by using a Bayesian approach.

A new way to interpret the model

The sampling theory approach provides a new way to interpret the B-L model. Sampling theory depends solely on sample data, but since we have no sample data, users are required to postulate a number of sample returns. Investors postulate that the market has observed a number of samples of asset returns and that they themselves have observed a number of samples of returns of portfolios of assets. The number of observations need not be specified, but the number of samples observed by the investor in relation to the number of samples observed by the market must be estimated.

A formula for the parameter τ, the weight-on-views

The derivation has generated a formula for τ:

$$\tau = \frac{n}{m}$$

It seems possible to interpret the formula. n represents the number of samples observed by the investor and m represents the number of samples observed by the market. Hence, τ is the ratio between these numbers and it is only this ratio that need be estimated. If investors postulate the number of samples they have observed to be the same as the number of samples observed by the market, then τ, should equal 1. If investors postulate the numbers of samples observed by the investor to be more numerous than the number of samples observed by the market τ, should be larger than one and vice versa. So, the more confident investors are in all the views, the higher τ ought to be set.

As suggested in 3.1.3 it appears that there is no clear description of the variable τ in the existing literature. Hopefully the sampling theory approach presented here will help investors to set τ and help academics as well as practitioners to continue the process of testing and further developing the B-L model.

A new interpretation of the matrix $\mathbf{\Omega}$

The sampling theory approach to the B-L model generates an interpretation of the matrix $\mathbf{\Omega}$ that differs somewhat from the Bayesian approach. The level of confidence in an expected return on view i is seen as the value of ω_i^2 so that one standard deviation, about 2/3, of the postulated observed samples of a certain view portfolio lies within the interval $q_i \pm \omega_i$. Note that also here investors need not postulate how many samples they have observed, they need only postulate a confidence interval around the expected return of the portfolio so that 2/3 of the postulated samples lie within this interval. It is however possible to implement the model so that investors estimate both an interval and another percentage. The investor could then, for instance, claim that he/she believes that in 90% of the n trials, the true return of the view will lie within the interval $q_i \pm \gamma_i$. ω_i^2 is then calculated from these data.

An inconsistency in the distribution of $\mathbf{q}_j$

The distribution of $\mathbf{q}_j$ is $\mathbf{q}_j \in N(\mathbf{P\mu},\mathbf{\Omega})$, but for the model to be consistent the distribution should be $N(\mathbf{P\mu},\mathbf{P}^T\mathbf{\Sigma P})$. Those trying to understand the B-L model should benefit from knowing of this inconsistency. If unaware it is probable that people will be confused, believing that there is something they have misunderstood. It will probably be easier to handle $\mathbf{\Omega}$ knowing of this inconsistency.

The reason for deriving the B-L model from a sampling theoretical approach was to facilitate a thorough understanding of the model, both for myself and for others interested in the model. So, is this derivation a contribution in this direction?

It would seem that the results presented above might contribute to a more thorough understanding of the B-L model. New ways of deriving models should constitute a contribution both to academics and practitioners. A derivation of the B-L model from a sampling theoretical approach hopefully facilitates understanding of the B-L model by individuals not familiar or comfortable with Bayesian theory. The fact that the approach generates an interpretable formula for τ, the weight-on-views, should also contribute to the development of the model. How would it be possible to understand, use, test and/or evaluate a model consisting of one parameter of which no clear and interpretable description exists? However, the practical contribution of this derivation will not be known until it is tested "in use". Studying the use of the B-L model can generate knowledge about how users relate to this way of interpreting the model.

The construction of the B-L model continues and I will continue to take part in this process. The derivation of the sampling theory approach to the B-L model is one contribution to its construction. Since I believe that the contributions are useful I choose to take this interpretation of the model as a starting point in the next step of this study.

4 Behavioral Finance and the Black-Litterman Model – Implications

In its purest form, the B-L model is nothing but a mathematical model, more particularly, a mathematical optimization model. Such a model is supplied with some kind of input data, which it mathematically combines and optimizes in accordance with certain rules. There are no obvious implications between behavioral finance and a mathematical model with any predetermined application. The aim of this second step the research project is therefore to analyze and discuss implications that can be drawn from research results within behavioral finance with respect to the *use* of the B-L model. Fortunately, the B-L model is not only a mathematical model; the literature concerning the B-L is quite explicit when it comes to the application and the use of the model. The B-L model is a mathematical portfolio model intended for use as a tool in investment situations. The use of the model demands action from its users. Investors are required to make estimates and judgments. There is, however, in the existing literature concerning the B-L model, little discussion of the behavior of the people using the model and the context in which the model is to be used. It appears that research concerning the *use* of quantitative financial models in general and the B-L model in particular is quite limited.

This chapter will begin with a short presentation of behavioral finance. A more detailed review of the field is provided in Appendix 4. A discussion of behavioral finance in relation to quantitative models in general is given before implications from behavioral finance with respect to the use of the B-L model is examined and discussed.

4.1 Behavioral Finance

Behavioral finance can be seen as a response to the severe criticism leveled at traditional finance theory and the efficient market hypothesis (EMH) during recent decades. Many people find the common assumptions regarding homo economicus and efficient markets problematic. Behavioral finance has now become one of the most active fields in today's economic research (The royal Swedish academy of sciences, 2002).

Behavioral finance is commonly divided into two main parts, as in Barberis and Thaler (2003). One part of behavioral finance is referred to as Limits to arbitrage or Inefficient markets. The other part focuses on the individual investor and the impacts of psychological factors on investment decisions and is commonly divided into two sub parts: The heuristics and biases approach to judgments under uncertainty, and Frame dependence.

4.1.1 The History of Behavioral Finance

Camerer and Loewenstein (2004) neatly put behavioral finance, or behavioral economics as the field is also called, in its historic context. The following historic description of behavioral finance is based mainly on their article.

The ideas within behavioral finance are not new. Instead they originate from the roots of neoclassical economic theory: "*When economics first became identified as a distinct field of study, psychology did not exist as a discipline.*" (Camerer & Loewenstein, 2004, p. 4). Many of the well-known early economists, however, had in fact psychological insights. For example, in his book, *The Theory of Moral Sentiments*, Adam Smith points at the psychological principles of individual behavior. According to Camerer and Loewenstein, many ideas in the book foreshadow the current developments in behavioral economics. These include Smith's comment (1759) "*we suffer more… when we fall from a better to a worse situation, than we ever enjoy when we rise from a worse to a better*". This is consistent with the concept loss aversion (see Appendix 4), one of the major theories within behavioral finance. Jeremy Bentham (1789) developed the utility theory at the end of the eighteenth-century. Utility theory is the foundation of the neoclassical theory concept, but Bentham also wrote about the psychological support of utility. Some of these insights are now gaining wider appreciation.

According to Camerer and Loewenstein, the neoclassical revolution was the beginning of the rejection of academic psychology by economists. At the beginning of the 20th century, economists such as Irving Fisher and Vilfred Pareto, incorporated discussions about how people feel and think about economic choices in economic theory. In the middle of the century, however, the discussion of psychology had largely disappeared from the economic agenda. At the beginning of the 1960's the metaphor of the brain as an information-processing device became dominant in cognitive psychology. This metaphor allowed studies of subjects such as memory, problem solving and decision-making. With this, "*Psychologists such as Ward Edwards, Duncan Luce, Amos Tversky and Daniel Kahneman, began to use economic models as a benchmark against which to contrast their psychological models*"(Camerer & Loewenstein, 2004, p. 6). Interest in the field of behavioral finance has expanded tremendously during recent years. This might have to do with the fact that Daniel Kahneman, one of the front figures of the field, was awarded the Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel in 2002.

4.1.2 The Parts of Behavioral Finance

As mentioned behavioral finance is often divided into two parts; one part concerns markets and the other part concerns individual investors. The part concerning the individual investor is then divided into two different parts. This gives us three areas or parts (Shleifer 2002):

1) **Limits to arbitrage** – The efficient market hypothesis states that real-world financial markets are efficient in a sense that prices always reflect fundamental values. In the last 20 years this view of markets has been challenged. The main finding in this part of behavioral finance is that in an economy in which rational and irrational traders[12] interact, irrational prices – i.e., prices that differ from their fundamental value – can be significant and long lasting. It is argued that the forces that are supposed to maintain market efficiency, such as arbitrage trading, are likely to be much weaker than the defenders of the hypothesis stress (Shleifer, 2000). Behavioral finance, both theoretically and empirically, offer an alternative approach. See Appendix 4 for more information regarding limits to arbitrage.

2a) **Heuristics and biases** – While limits to arbitrage concerns markets, both "Heuristics and biases" and "Frame dependence" concern the behavior of the individual investor. Considerable empirical research, within this field, has shown, not surprisingly, that people do not always act according to the rational model as suggested by the neoclassical theory. It is worth noting is that traditional economists have assumed that the behavior of people differs from the rational model in a non-systematic way and therefore it is considered impossible to incorporate this behavior in models. Behavioral finance claims to have found clear systematic patterns in the ways in which people deviate from "rational" behavior. In 1974 Tversky and Kahneman's article "*Judgment under Uncertainty: Heuristics and*

[12] In traditional finance and behavioral finance a rational trader is a trader acting in accordance with the efficient market hypothesis. An irrational trader is hence a trader not acting according to this hypothesis.

Biases" was published in the journal *Science*. It made a significant impression in the area of social sciences. This was the starting point of the field, within behavioral finance, often referred to as the "*Heuristics and biases approach to judgment under uncertainty*". The core idea of the field is that complex probability judgments are often based on simplified heuristics instead of formal and extensive algorithms, as suggested by the rationality paradigm. This can give rise to series of systematic "errors"[13], often referred to as biases. According to the heuristics and biases approach to judgment under uncertainty, people do not estimate likelihood and risk according to the laws of probability. Tversky and Kahneman (1974) present three heuristics: Representativeness, availability, and anchoring and adjustment. Heuristics give rise to a number of biases. Some of the most well established biases are: overconfidence, conservatism, sample size neglect and home bias. See Appendix 4 for explanations of the heuristics and biases mentioned here.

2b) **Frame Dependence** – According to modern finance the framing of a problem should not affect the behavior of investors. The framing should always be transparent and investors are always assumed not to be affected by how different financial problems are described. However, research within behavioral finance has generated profound results implying that people are sensitive to the framing of problems. Examples of well established research results concerning frame dependence are: The disposition effect, mental accounting, prospect theory and loss aversion. See Appendix 4 for explanations of these frame dependences.

4.2 Behavioral Finance and Quantitative Financial Models

As discussed, behavioral finance, as a field, is a reaction to traditional financial theory. While traditional quantitative financial models assume rational investors, arbitrage-free markets, normally distributed returns etc., research within behavioral finance claims that these assumptions do not apply in the real financial world. People are "irrational" (in relation to the assumptions of the efficient market hypothesis) in many different ways and this affects how financial models are used and how they should be used.

As mentioned, *Limits to arbitrage*, claims that the theory of arbitrage-free markets is often inapplicable in the real world. In real life, arbitrage traders can far from always

[13] The term "Systematic errors" is used within behavioral finance referring to the systematic divergence of investors from "rational" behaviour according to homo economicus

eliminate what seem to be arbitrage possibilities within the market and hence these "risk arbitrage"[14] possibilities can exist in the market for long periods. .

Traditional finance assumes normally distributed returns, but this is a disputed assumption. Discussions about fat tailed[15] and skewed[16] distributions are common but the normality assumption of returns lies behind many quantitative financial models. Certain research within behavioral finance indicates that returns are not necessarily normally distributed. DeBondt and Thaler (1985) compare two types of stocks: extreme losers and extreme winners. Each year from 1933 until 1980 they form one portfolio containing stocks with the worst performance during the most recent three years and another portfolio containing stocks that have performed the best during the same period. For each year they have then computed the returns of the two portfolios over the five following years. On average, the loser portfolio has had higher returns for every period of five years than the winner portfolio. The reason for this, according to DeBondt and Thaler, is that prices overreact. Since investors are likely to extrapolate past returns into the future, firms becomes undervalued or overvalued. This continues up to a point at which investors begin questioning the market value of the stocks and their price development changes direction. This implies that stock prices are not really normally distributed. They are said to be skewed. Underreaction however suggests that stock prices underreact to information in the short run. According to the efficient-market hypothesis, prices should immediately react "correctly" to new information. However, Abarbanell and Bernard (1992) show that stocks, in general, have higher returns after surprisingly good information than after surprisingly bad information. This also contradicts the normality assumption.

Research results within behavioral finance can be seen as both opposing and supporting the use of quantitative models. Opponents may claim that quantitative financial models are built upon unrealistic assumptions regarding both the investors using them and the markets. Proponents on the other hand may claim that since humans are prone to act "irrationally", the use of quantitative models may help them to overcome this failing.

4.2.1 Research Concerning Portfolio Models

Research has been performed within behavioral finance with respect to portfolio modeling. This research however focuses on how private investors invest their own capital. The research presented here focuses more on how a fund or portfolio manager,

[14] I choose to refer to these real life arbitrage possibilities as "risk arbitrage", an expression used by Shleifer (2002). This relates to the fact that real life arbitrage is not risk free.
[15] The tails of distributions or returns are often thicker than theory predicts. There are more extreme events, a larger number of very high and very low values.
[16] The statistical distribution of returns is not always symmetrical. Frequently the curve shows an asymmetry. This means that it is distorted towards one side, an anomaly compared to the theory.

managing other people's money, acts. Shefrin and Statman (1997) present a theory they refer to as behavioral portfolio theory. The theory is not normative as traditional modern portfolio theory but descriptive. Shefrin and Statman discuss how private investors act and how these actions diverge from Markowitz' portfolio theory. Massa and Simonov (2003) show that behavioral biases affect portfolio choices in different ways. Risk taking, for instance, is argued as being affected by prior gains and losses.

According to Shefrin and Statman, investors divide their wealth into different mental accounts and "*build portfolios as pyramids of assets, layer by layer*" (Shefrin & Statman, 1997, p. 3). To each layer they apply different goals with different attitudes towards risk and return. According to the authors, the layer-by-layer style used by investors leads to covariances being disregarded. In behavioral portfolio theory the relation between the upside potential and the downside protection is what matters.

4.3 Behavioral Finance and the Black-Litterman Model

As explained in chapter 3, the B-L model is a development of the Markowitz model. The two most recognized and important features of the B-L model are that:

1. The model begins from what is called the equilibrium portfolio, in the literature, most often approximated by the weights of the benchmark portfolio against which the fund manager is evaluated. This portfolio acts as a point of reference.

2. The investor inputs "views" and to each view he/she assigns a level of confidence. The resulting portfolio is then a combination of the benchmark portfolio and the view portfolio input by the investor. The weighting depends on the levels of confidence assigned to each view and the weight-on-views.

As argued in chapter 1.2 most of the research results within behavioral finance may have some implication for the use of the B-L model. But, since the above two features of the B-L model are the most, important the search for research results within behavioral finance has been focused on research results that might have implications for these two in particular.

4.4 The B-L model and the Utility Function

The traditional theory of finance assumes a quadratic utility function. This is also the case for Markowitz' model. The traditional utility function is defined in absolute terms, with decreasing marginal utility of wealth and the function is concave for all wealth. The shape of the function assumes that investors should evaluate investments in terms of absolute wealth. According to traditional financial theory investors have no references with which they compare returns. The utility function according to behavioral finance differs both in shape and in the domain in which it is defined (Tversky and Kahaneman 1984; Kahneman, Knetsch and Thaler 1991). According to Tversky and Kahaneman (1984) the utility functions of investors are not defined in absolute terms, instead they are defined for losses and for gains in relation to a certain point of reference. The function is concave in the domain of gains and convex in the domain of losses. It is also considerably steeper for losses than for gains. The function has a kink at the reference point (origin). The shape of the utility function of behavioral finance implies loss aversion[17], meaning that the investor is risk-averse in the domain of gains but risk-seeking in the domain of losses.

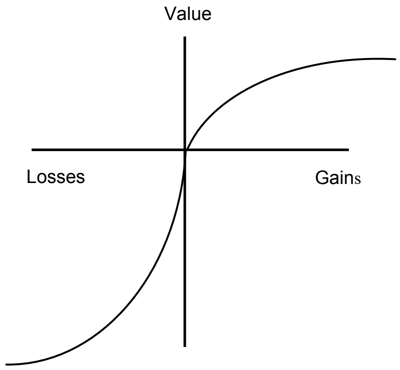


Figure 5.1a Utility function suggested in behavioral finance

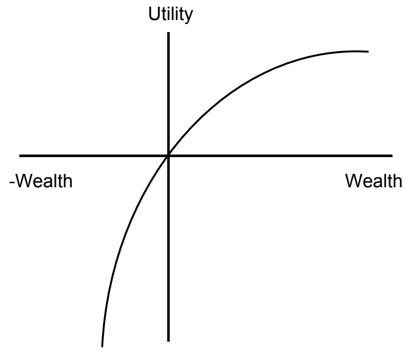


Figure 5.1b Traditional utility function

In the B-L model, the market portfolio acts as a point of reference. This is the portfolio often approximated to the benchmark portfolio against which the portfolio manager is evaluated. If the market portfolio would act as the point of reference it would mean, for example, that if the value of the fund has decreased by 3% in a month while the value of the benchmark portfolio has decreased by 4%, the fund has outperformed the benchmark portfolio and the manager could be satisfied. In traditional finance, a fund manager should be unhappy with a loss and happy with a gain, but according to behavioral finance, the manager rates his or her success relative to a point of reference.

The B-L model builds, as we know, on the Markowitz model. It is thus easily assumed that the utility function of the B-L model should be exactly the same as the utility

[17] Loss-aversion expresses the reluctance of people to bet on a fair coin and is implied by the kink and the difference in the rake of the value function of prospect theory. Research has shown that the attractiveness of winning X € is not nearly sufficient to compensate for the fear of losing the same amount. Loss aversion however implies that the value function is convex in the domains of losses.

function in Markowitz' model. In the B-L model, we optimize a quadratic function, similar to that of the Markowitz model. The shape of the function is hence the same in the B-L model as in Markowitz' model. There is, however, one important difference between the utility functions of these models. The difference lies in the domains in which the utility functions are defined. The utility function of the B-L model is not defined on total wealth, in absolute terms. Instead the utility function of the B-L model is defined in terms of deviations from a certain point of reference, as losses and gains relative to the benchmark portfolio (market portfolio) in relation to which the investor is evaluated: see figure 5.2 below.

The utility function assumed within the B-L model can thereby be seen as a step from the traditional, modern finance toward behavioral finance. It is defined in domains similar to the domain of the value function of behavioral finance.

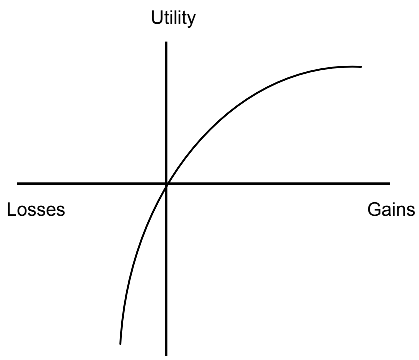


Figure 5.2 The B-L utility function

On the other hand, the utility function of the B-L model still has the same shape as that of traditional financial models. It is concave for the whole domain and there is no kink at the reference point. Hence, the utility function does not represent loss-aversion. If the portfolio manager is loss-averse, the bets taken in the portfolio output given by the B-L model in relation to the benchmark portfolio may not necessarily correspond with that given by the intuitive feelings of the portfolio manager.

The S-shaped utility function of behavioral finance implies that individuals are loss-averse. According to behavioral finance, loss-aversion leads to biases in relation to traditional, modern finance. Biases that seem to be of interest in relation to the utility function of the B-L model are: regret, the status-quo bias, the endowment effect and herd behavior. These four biases will be presented below and their implications for the B-L model will be discussed.

4.4.1 Regret

Much research within the area of behavioral finance has shown that the fear to regret a decision is psychologically strong and influences the decision-making of individuals.

Assume that you have gone with your friends to Nice in France on holiday, planning to spend most of the time at the beach and strolling around in the city shopping and drinking espresso. One day, however, you plan to take the bus up to Grasse to experience the beautiful village. The plan has been to leave for Grasse on Thursday, but on Tuesday afternoon the group decides to go on Wednesday instead. So, on Wednesday you all take the buss up to Grasse, but the buss crashes and two of your friends are seriously injured. This would be a very tragic outcome, but is it more tragic because the group originally planned to go the day after? Would you feel any "if only"-thoughts if this had happened to you? Many people find this kind of experience more psychologically painful than if the same accident had happened when keeping to the original plan.

> "*Regret theory (Loomes and Sugden, 1982) and disappointment aversion (Gul, 1991) are both based on the idea that agents value (either in a backward-looking or in a forward-looking manner) the emotional cost of being disappointed and of having made a mistake, which they might have avoided.*" (Stracca, 2002, p. 11).

In prospect theory (Kahneman & Tversky 1979), the degree of pain of having made a mistake leading to a certain amount of loss is psychologically greater than the degree of happiness gained by doing the right thing, which yields a return of the same amount of money. The pain of regret caused by making mistakes is represented by the kink at the reference point in the utility function of behavioral finance (Shiller, 1998, pp 7-8). When the recall of past experiences is biased, the ability to assess the likelihood that a course of action will lead to a certain outcome is affected. But, decision-making can also be affected even if experience is not biased or when likelihood is not affected. The likelihood might be assessed correctly, but the experience of counterfactual regret can be so psychologically uncomfortable that the action is still avoided (Miller and Taylor 1995). Odean (1998a) and Shefrin and Statman (1985) have found that to avoid the feeling of regret traders tend to sell winners and hold on to losers. It seems that investors evaluate their original purchase decisions not on the basis of the accrued returns but on the realized return. By selling winners and holding on to losers investors will consider themselves as having made fewer poor decisions. This way of acting facilitates positive self-evaluation since the feedback from losers is delayed.

Numerous studies from a broad range of theoretic fields have shown that regret can affect people's decision making[18]. Regret can affect decisions (Shefrin, 2002, p. 31) both when planning a vacation and planning an investment. Even Harry Markowitz admits that he acts according to the unwillingness to feel regret. Markowitz was asked) if his choice of equity-fixed income allocations in a retirement plan was an example of seeking optimum trade-off between risk and return. He answered that this was not the case, instead his intention was to minimize future regret *"so I split my contributions fifty-fifty between bonds and equities"* (Shefrin, 2002, p. 31).

4.4.2 The Status Quo Bias and the Endowment Effect

Knetsch and Sinden (1984), Samuelson and Zeckhauser (1988), and Knetsch (1989) introduce the status quo bias.

> *"One implication of loss- aversion is that individuals have a strong tendency to remain at the status quo, because the disadvantages of leaving it appear larger than the advantages. Samuelson and Zeckhauser (1998) have demonstrated this effect, which they term the status quo bias"* (Kahneman et. al., 1991)

Knetsch and Sinden (1984), and Knetsch (1989) demonstrate the status quo bias by randomly handing out mugs and candy bars to students. The students were subsequently given an opportunity to trade mugs for candy. 90% of both the mug owners and the candy bar owners chose not to trade. The authors claim that because the commodities where handed out randomly and transaction costs were low, the preference of the students must depend on the allocation of the commodities. So the commodities that students were allocated were considered as the status quo and very few were willing to leave this position regardless of which commodity they where holding, the mug or the candy bar. In another study by Samuelsons and Zeckhausers (1998), the subjects where asked a hypothetical multiple-choice question. Some of the subjects had the possibility to choose a status quo answer while others did not. Those not offered the status quo alternative were asked the following question:

> *"You are a serious reader of the financial pages but until recently have had few funds to invest. That is when you inherited a large sum of money from your great-uncle. You are considering different portfolios. Your choices are to invest in: a moderate-risk company, a high risk company, treasury bills, municipal bonds."*

The others were asked a similar question but one alternative was designed with a status quo bias. It could be that the opening sentence was followed by:

[18] "(Bell, 1982, 1983, 1985a,; Fishburn, 1983; Janis & Mann, 1977; Kahneman & Tversky, 1982a; Loomes, 1988; Loomes & Sugden, 1982, 1987a, 1987b; Simonson, 1992; Sugden, 1985; Walster, Walster, Piliavin, & Schmidt, 1973; see also Gleicher, Boninger, Strathman, Amor, Hetts, and Ahn, 1995)" (Miller and Taylor 1995 p. 379).

> *"…That is when you inherited a portfolio of cash and securities from your great-uncle. A significant portion of this portfolio is invested in a moderate risk company…(The tax and broker commission consequences of any change are insignificant.)"*

The subjects who were presented with a question in which one of the investment alternative was presented as a status quo choice chose this alternative considerably more often than those who were presented with a question with no status quo choice (Kahneman et. al., 1991). In a test carried out by Hartman, Doane and Woo (1990) California electric power consumers were asked about their preferences in service reliability and rates. The customers were told that their answers would help to determine the future service policy of the company. The consumers fell into two groups. One group of consumers had a much more reliable service contract with the company than the other. Each consumer was asked to state a preferred combination of service and rates among six different combinations. One of the six alternatives was always the status quo choice. The test showed that the status quo choice had a much higher rate of response than the others for both groups of respondents; hence most of the respondents preferred the status quo choice implying that the consumers were status quo biased.

The status-quo bias can, according to Kahneman et. al. (1991) be explained by loss-aversion. Loss-aversion and the status quo bias are closely related to the endowment effect identified by Thaler (1980, 1985). The endowment effect tells us that once a person comes to possess a commodity, he/she instantly values it more than previously (Rabin 1996, p 5). In an experiment by Kahneman and Tversky (1991), some of the students in a class were given a commodity. The commodity here was also a mug. One third of the students randomly received a mug worth $5. These students where then handed a questionnaire.

> "*You now own the object in your possession. You have the option of selling it at a price, which will be determined later. For each of the possible prices below indicate whether you wish to (x) Sell your object and receive this price; (y) Keep your object and take it home with you....*" (Kahneman & Tversky, 1991, p. 145)

The students were also asked to specify their decision at prices ranging from $0.50 to $9.50 in steps of 50 cents. The students not receiving a mug were then informed that they would subsequently receive either a mug or an amount of money to be decided later on. They were asked to specify their preference between a mug and an amount of money. These subjects were also to indicate their decision at prices ranging from $0.50 to $9.50 in steps of 50 cents. Here both groups of students face the same decision problem. However their state of reference differs. The students receiving a mug at the beginning of the test must chose between keeping the mug and giving up the mug and receiving money instead, hence they must chose between remaining in the status quo or leaving it. The exchange rate between the mug and money was quite different between the two groups. The group receiving the mug at the beginning required $7.00 to give up the mug while the other group felt that they where indifferent between the mug and the money at a rate of $3.50. The difference in the prices depends, according to the authors of the endowment effect, which appears almost directly when individuals are given property rights over consumption goods.

4.4.3 Herd Behavior

What happens when each decision maker considers the decision taken by previous decision makers before making their own decision? This is what is often referred to as herd behavior or herd effects. Consider the following often used explanation for the October 1987 bull market (Scharfstein & Stein 1990, pp. 465):

> *"The consensus among professional money managers was that price levels were too high – the market was, in their opinion, more likely to go down rather than up. However, few money managers were eager to sell their equity holding. If the market did continue to go up, they were afraid of being perceived as lone fools for missing out on the ride. On the other hand, in the more likely event of a market decline, there would be comfort in numbers – how bad could they look if everybody else had suffered the same fate?"*

Denow and Welch (1996, p. 604) claim that there needs to be a coordination mechanism for herding to occur. It might be so that behavioral patterns between individuals are correlated but it might also be so that correlated information arrives independently to investors.

Professional money managers may choose portfolios that are excessively close to the benchmark against which they are evaluated to minimize the risk of underperforming this benchmark. Investors may also herd and select stocks that other managers select, to avoid falling behind and losing their reputation (Scharfstein & Stein, 1990). They may also artificially add to their portfolios stocks that have recently done well, and sell stocks that have recently done poorly to look good to investors in connection with fund reports circulated to customers. Pension and mutual fund managers on the average, consistently underperform passive investment strategies (Shleifer, 2000).

4.4.4 Implications for the B-L Model

The fact that the utility function in the B-L model is defined in the same domain as the behavioral utility function should make portfolios output by the B-L model seem more intuitive to investors. Fund and portfolio managers are often evaluated in relation to a benchmark portfolio and hence often evaluate their own performance in relation to this benchmark portfolio. Since many financial managers are evaluated to a reference point it seems reasonable that the fund manager would appreciate working with a portfolio model taking this reference point into consideration. The taking of bets in relation to benchmark could be one of the reasons why managers find the portfolios generated by the B-L model more realistic than portfolios generated by the Markowitz' model. If status quo biased, investors should be more comfortable working with a model using the same point of reference, as they are themselves to avoid feeling regret.

However, the shape of the utility function in the B-L model still has the traditional shape of a quadratic function. The bets taken in relation to the benchmark portfolio in the output portfolios will hence probably often differ from the gut feeling of the investor. Regret, the status quo bias, the endowment effect and herding are consequences of loss-aversion. Since loss-aversion is not taken into account in the utility function of the B-L model, expected returns in relation to the risk will not always be high enough for investor to risk leaving the status quo, falling behind the benchmark, feeling regrets and leaving the herd. This implies that portfolios generated by the B-l model also may appear unintuitive to managers, although probably more reasonable than portfolios generated by the Markowitz model.

Since the benchmark portfolio is the point of reference for the fund manager, deviations from this portfolio should generate anxiety. If a bet taken during an investment period proves wrong, it would not be surprising if the manager is subjected to the "if only"-feeling discussed earlier. He/she could easily ask why he took this bet at this particular time and why he/she chose not to keep to the benchmark weights. There has been a recent debate in Sweden about why people pay fund managers the price they charge while holding portfolios very close to the benchmark portfolio. The price a fund charges should be related to the active management and hence the expected excess return provided by the specific fund. Why pay extra for almost no management? The fact that many funds have weights close to the benchmark portfolio might have many explanations. It would be reasonable to believe that the status quo bias might push managers in this direction. Loss aversion may help us to explain this behavior. Since losses in relation to the reference point have negative psychological effects, which are greater than the positive psychological effects of corresponding gains, the status quo bias implies that an investor would frequently prefer to keep to the benchmark weights. Herding and the fear of falling behind in performance should have the same effect.

Now we may ponder upon whether we believe that it would also be better if the shape of the B-L model were similar to the shape of the behavioral utility function. One could argue for this by saying that the portfolio manager would probably not use a model if it contradicts his/her intuition. On the other hand, the investor may wish to use a model that helps him/her avoid acting in accordance with his/her biases. I will not discuss this further, leaving the reader with these thoughts, and move on to discussing another important feature of the B-L model in relation to behavioral finance – the level of confidence.

4.5 The B-L model and Overconfidence

Let us now move on to the other much recognized and interesting idea of the B-L model; the level of confidence and the weight-on-views. Together, these two factors decide how much weight to allocate to the market portfolio in relation to the view portfolio. There are however considerable research results within behavioral finance indicating that the levels of confidence expressed by people are often misleading. People are most often overconfident.

The following section will begin with a general review of the research on overconfidence and then implications from this research results to the use of the B-L model are discussed and analyzed.

4.5.1 Overconfidence

What is Overconfidence?

A definition of overconfidence is that when estimated probabilities have a tendency to exceed the "accurate portion," then the judgments on which they are based are said to be overconfident. In behavioral finance, overconfidence relates to the exaggerated belief of people and investors in their ability to correctly forecast returns and future asset prices.

Within behavioral finance and in other fields studying overconfidence it is common to discuss two ways in which people tend to be overconfident. Consider two questions of the form:

Which country has the greater population?

1. Argentina

2. Egypt

How sure are you that your answer is correct?

In answering such questions, people tend to be overconfident in judging how sure they are of being correct. A typical study, with questions similar to this, has shown that when respondents believe 73% of their answers to be correct, they have actually only answered correctly 65% of the questions asked (Yates et. al., 2002). Another way in which people have been shown to be overconfident is when deeming their confidence interval. Questions testing this can be of the following form:

> *State an interval of the age of Kofi Annan, so that you are 90% sure that his correct age lies in the interval.*

In a study by Klayman et. al. (1999) the correct answers were within the stated interval only 43% of the time when subjects were asked to state an interval in which they were 90% confident their answer lied within the interval. This implies that people are more overconfident when stating confidence intervals than in answering two-choice questions. Many studies have shown that the confidence people assign their judgments exceeds their accuracy. An often-used example of overconfidence is that typically, when asked how good drivers they are relative to other drivers, 65% to 80% of the people answering the question consider themselves as above average. People tend to be overconfident when estimating their own capabilities in many situations. According to Shefrin (2002) investors are as overconfident in their investment decisions as they are in their driving abilities. Overconfidence has been found in several studies to be just as prevalent in the area of finance as in others. [19]

People overweight salient information i.e. information that captures attention and stands out (Kahneman & Tversky, 1973). Odean (1998b) claims, that in general we expect people to rely too heavily on less relevant and more attention drawing information and we expect people to underweight important abstract information. Odean also discusses the fact that traders try to invest in assets generating higher returns than others and that this is a quite difficult task. He reminds us that it is in these difficult tasks that people display most overconfidence. Odean (1998b) also shows that overconfident investors trade more than "rational investors" and that doing so reduces their expected utility. He models overconfidence, as traders' belief that their information is more precise than it actually is.

Although an appreciation of overconfidence as an important consideration in behavioral finance is well established, research on overconfidence otherwise remains as a subject of debate. Klayman et. al. (1999) suggest that the overconfidence apparently demonstrated by researchers is due to the nature of the questions asked in the experiments. The questions are claimed to be harder-than-normal questions. Klayman

[19] "Examples include psychologists (Oskamp 1965), physicians and nurses (Christensen-Szalanski and Bushyhead 1981, Baumannm Deber, and Thompson 1991), engineers (Kidd 1970) attorneys (Wagenaar and Keren 1986), negotiators (Neale and Bazerman 1990), entrepreneurs (Cooper, Woo, and Dunkelber 1988), managers (Russo and Schoemaker 1992), investment bankers (Stael von Holstein 1972), and market professionals such as security analysts and economicc forecasters (Ahlers and Lakonishok 1983, Elton, Gruber and Gultekin 1984, Froot and Frankel 1989, DeBondt and Thaler 1990, DeBondt 1991)." (Daniel et. al. 1997 p. 8)

et. al. (1999) present a study in which they find little general overconfidence in two-choice problems but explicit overconfidence in problems requiring the subject to state confidence intervals. They find that within easy tasks, overconfidence is not common. Within very simple tasks even underconfidence may appear. The authors supply two commonly used explanations for overconfidence: biases in information processing and effects of unbiased judgmental error. Early researchers within behavioral finance explained overconfidence with biases in information processing. When a person makes a judgment he/she first searches his/her memory to select a preliminary answer. After this, memory is searched again to find evidence supporting the preliminary answer. The retrieval of information supporting the initial idea is facilitated by mechanisms of associative memory (conservatism) and therefore a person making a judgment subconsciously observes more consistent support for the tentative answer than is justified. (Klayman et. al., 1999). The other explanation for overconfidence is the effects of unbiased judgmental error. Shortcomings in learning the predictive power of different sources of information are one source of judgmental error considered by Klayman et. al. (1999). According to Klayman et. al. the debate about biased confidence in judgment seemed settled in the 1980s: People appeared to be systematically overconfident in the easiest of questions. In the 1990s overconfidence was given another explanation. It was then claimed that people judged questions of confidence imperfectly, but without bias. The questions asked were instead considered to be biased. But in many practical situations many people who are required to make judgments receive biased samples of questions. Doctors, financial managers, lawyers and others are asked questions that are more difficult to answer than questions asked in the world at large. Klayman et. al. (1999) point to the openness of the question of overconfidence, but their study shows, as a large majority of previous studies, that people are overconfident. The more confident they are the more overconfident they are. They also find support for systematism in the way people are overconfident, hence supporting the concept of overconfidence as a heuristic driven bias.

When are Investors Overconfident?

As mentioned, studies have shown more general overconfidence in estimating confidence intervals than in two-choice questions. Odean (1998b) refers to the extensive research within cognitive psychology, which establishes that people are especially overconfident in judging the precision of their knowledge. As Klayman et. al (1999) Odean also finds that exceptions to overconfidence can be found when people are answering very easy questions. He writes that individuals tend to be well-calibrated when asked repetitive questions with fast and clear feedback. When people are asked very easy questions they even can show signs of being underconfident. According to

Odean, these exceptions do not, however, apply in financial markets. Traders and investors in the financial markets try to buy assets with higher returns than others and they try to sell assets with lower returns. Odean argues that this is a difficult task and in performing difficult tasks people are prone to be overconfident. Odean also asserts that security markets are not good places in which to calibrate one's confidence. Good places in which to calibrate confidence are environments in which feedback is quick and correct. In financial markets, however, feedback is neither quick nor correct. There may also be a trade-off between quick and correct feedback in financial markets. According to Odean, short-term traders may get quicker but noisier feedback while long-term traders get less noisy feedback but must wait for it instead. Research has shown that people overestimate their capability to perform tasks well and that this overestimation increases with the personal importance of the task. People overestimate their own contribution to past positive outcomes and underestimate their contribution to past negative outcomes (Odean, 1998b).

Research has shown differences in the overconfidence between groups of people. Gender and cultural differences have been found. Barber and Odean (1998, p. 1) assert:

> *"Psychological research has established that men are more prone to overconfidence than women. Thus, models of investor overconfidence predict that men will trade more and perform worse than women. Using account data for over 35,000 households from a large discount brokerage firm, we analyze the common stock investments of men and women from February 1991 through January 1997. Consistent with the predictions of the overconfidence models, we document that men trade 45 percent more than women and earn annual risk-adjusted net returns that are 1.4 percent less than those earned by women."*

Yates et.al. (2002) discuss probability judgment across cultures. Wright et. al. (1978) find that Asian students tend to be more overconfident than British students. In the article Yates et. al. summarize what they and others have learned about probability judgments across cultures. They present several studies and almost all of these show that people in western countries (in this case most often USA) are less prone to be overconfident than those in Asian counties.

Wang (2001) takes up the discussion on whether overconfident investors, over time, learn and therefore eventually acquire "rational" beliefs. He refers to Kahneman et. al. (1982) showing that people actually do not update beliefs and hence do not achieve rationality. Research has shown that experience is an important factor in investors' expectations about the market. The results showed that novice investors are more confident that they will beat the market than the more experienced investors. Since most investors have difficulties beating the market, we have reason to believe that novice investors are often overconfident (Shefrin, 2002). Not only novices exhibit overconfidence. Griffin and Tversky (1992) find that when predictability is very low, as in the stock market, experts have theories and models, which they tend to overweight.[20]

What Does Overconfidence Lead To?

Barber and Odean (1999) find that investors who began trading online during the period 1991-1996 generally earned less after switching to online trading. When trading over the Internet they increased their trading activity, traded more speculatively and performed less successfully. Overconfident investors trade more excessively than rational traders. Barber and Odean argue that several biases lead to the overconfidence of online investors. Investors who performed well before going online are likely to attribute this to their own ability instead of luck. Also, online investors get access to data and information that can give an impression of knowledge, which in turn increases overconfidence. The authors also point at the illusion of control investors get when managing their own stock portfolios and can execute a trade with just "a click of a mouse". This illusion of control also encourages overconfidence. Statman and Thorely (2001) agree with Odean (1998b) in that high returns make investors overconfident and that overconfident investors increase their trading volume. They find strong relations between trading volume and past returns. Shefrin (2002) sees two main implications of investor overconfidence. Firstly, investors fail to realize that they are at an informational disadvantage and therefore take on bad bets. Secondly, investors trade too much and therefore reduce their expected utility. Barber and Odean (1999) agree with Shefrin, saying that overconfidence is a simple and powerful explanation for the high levels of trading on financial markets. They claim that humans are overconfident about their abilities, their knowledge and their future prospects.

Odean (1998b) finds that overconfident traders have lower expected utility than well-calibrated traders. It is not so that overconfident traders necessarily have lower expected returns than others. Overconfident investors take on a more risky portfolio than would others. It may therefore be so that overconfident investors are rewarded, with higher expected returns, for the extra risk taken. The expected utility, however, is lower. Wang (2000) points at the different views of non-rational traders within financial theory. Black (1986) claims that financial markets are dependent on noise traders. If all investors were to perceive information in the "correct" way, there would be very little trading in progress since well-informed traders have no interest in trading with each other. Black claims that the financial markets depend on noise traders to provide

[20] This does not apply to experts who adhere to computer-based quantitative models, see (Dawes, Faust, & Meehl 1989)

liquidity in the markets. Friedman (1953), on the other hand, argues that traders who trade on noise are irrelevant to financial markets since they are driven out of the markets by informed investors (in a process of natural selection). Wang (2000) studies whether or not noise traders can survive and especially if overconfident traders survive. He finds that the group of overconfident investors survives at the expense of the rational investors. This is because the overconfident investor has a higher expected return than the rational investor and also because he/she has a higher variance i.e. higher risk than the rational investor. This also implies that the overconfident investor, as an individual, is more likely to become a bankrupt, but as a group the overconfident investors survive.

Odean (1998b) takes up what he calls the selection bias and the survivorship bias. The selection bias may cause the financial markets to attract people with a higher degree of overconfidence then the overall population. People differ in their ability to make judgments in situations characterized by uncertainty. Odean claims that those who believe that they have a high ability to make these kinds of judgments will probably seek jobs as traders to a higher degree than others. And, if people are bad at judging their own ability, the financial markets should be populated with those with the most ability and those who are most prone to overestimate their ability. The Survivorship bias, also discussed by Odean (1998b), causes the financial markets to continue to be populated by individuals who are more overconfident than the remainder of the population. Unsuccessful traders lose their jobs or choose to leave the financial market place. Unsuccessful investors will therefore, on an average, manage less money than successful investors. If investors, to a high degree, as is common, attribute their success as investors to their personal characteristics, they may become increasingly overconfident the more they trade and overconfident investors will control more and more wealth. Gervais and Odean (1997) claim that self-enhancing bias makes wealthy traders, not afraid of being driven out of the marketplace, overconfident. Overconfidence does not make them rich - it is rather the process of becoming wealthy that makes investors overconfident.

4.5.2 Implications of Overconfidence and the Levels of Confidence

Although there is some criticism of the methods used to prove that humans are often prone to overconfidence, overconfidence is still one of the most recognized ideas within behavioral finance. So, if we now accept that people are often overconfident in their judgments, does this affect the use of the B-L model?

In the B-L model investors allocate levels of confidence to each view as explained in chapter 3.3. A level of confidence is expressed as an interval around the expected return of the view. With the sampling theoretical interpretation of the B-L model, investors should estimate the interval so that about 2/3 of the postulated observed samples lie within the interval. In the above we have learned that people tend to be overconfident. If people are as poorly calibrated when estimating their own level of confidence as is claimed, it seems reasonable to question whether the feature in the B-L model that requires investors to input a level of confidence is such a good idea.

Klayman (1999) claim that people are more prone to overconfidence when estimating confidence intervals than in answering two-choice questions. Remember that Klayman et. al. (1999) find that when stating their 90% confidence intervals, the correct answer was only within the interval on 43% occasions. Remember also that this is exactly what the B-L model demand of the investor. The investor must state the 2/3 confidence interval in which the expected return lies. So, when stating these confidence intervals, managers can be expected to assign too narrow confidence intervals.

Research has shown that the overconfidence of a person differs depending on the different characteristics of the task. Odean (1998b) observed more overconfidence in the performance of difficult than in easy tasks. Estimating future returns on assets is claimed to be a quite difficult task (Odean 1998b) and hence people are prone to overconfidence in judging their ability to estimate returns. This therefore suggests that people should be overconfident when estimating levels of confidence in the B-L model. Odean (1998b) also claims that confidence levels can be calibrated in situations where feedback is correct and quick, but that feedback in the financial markets is neither correct nor quick, implying that investors act in an environment in which it is difficult to calibrate confidence. The B-L model is intended for use in investment situations in financial markets in which people are unable to calibrate their levels of confidence and hence the users of the B-L model tend to remain overconfident.

The levels of confidence that should be assigned to views are not the only parameters related to confidence in the B-L model. τ, the weight-on-views, must also be considered. The higher weight-on-views is set; the more weight is allocated to the views in relation to the market portfolio or the benchmark portfolio. τ scales the matrix $\mathbf{\Omega}$. With the sampling theory approach presented in 3.3.2, τ represents the number of samples observed by the investor divided by the number of samples observed by the market. Setting the weight-on-views means neither answering a two-choice problem nor estimating a confidence interval. However, if a person is overconfident when allocating confidence levels to each view, it appears probable that investors are also overconfident when allocating the weight-on-views. How well we can estimate future returns, a difficult task according to Odean (1998b), is still dependent on the weight on views. . Hence it seems realistic to believe that investors using the B-L model are prone to express overconfidence both when setting the confidence levels to each view and when setting the weight-on-views.

The B-L model is characterized by the way the views of the investors are combined with the market equilibrium or the benchmark portfolio. In this sense the views of the investor are scaled by the weight-on-views. If the investor is equally overconfident in each view, then it is possible to adjust the influence of the confidence levels when setting the weight-on-views. The levels of confidence that the investor must assign have very similar characteristics. They are a measure of the certainty the investor feels with respect to a view. The views can be of different forms as mentioned. They can be absolute or relative but all concern the future expected returns of different assets or portfolios of assets. Since the confidence levels that should be stated are of similar types, the difficulty of the tasks should be almost the same and hence the extent to which investors are overconfident should also be almost the same. If the investor is as much overconfident in each view, this may be handled when setting the weight/on/views. The levels of confidence estimated by the investor are tools for ranking the bets taken, in relation to the other bets and to the equilibrium portfolio. Thus if one level of confidence is biased toward overconfidence and the other levels of confidences are biased in similar ways, we have at least the possibility of dampening this overconfidence by lowering the weight-on-views, since that which is actually input to the model is $\frac{\mathbf{\Omega}}{\tau}$.

4.5.3 Confidence Levels and the Home Bias

Expressing views and levels of confidences in each view is of course a tool with which users can give expression to many heuristic-driven biases. Since the model provides the portfolio manager with the opportunity to express views and since the model takes these views into account when forming portfolios, the portfolio manager will quite obviously feel that the portfolio output given by the model is more intuitive than the output given by a model not taking these views into consideration.

One example of a heuristic-driven bias that can be expressed when using the B-L model is the home bias. Even though the U.S. stock market only capitalizes 45% of the total global stock market, American investors still hold most U.S. stocks. In the same way European investors hold mostly European stocks and Japanese investors hold mostly Japanese stocks (Shefrin 2002, p. 136). Investors might overweight domestic assets because the domestic stocks and markets *feel* more familiar than the foreign – they are home-biased. Of course investors have more information about domestic assets, but it seems as though they tend to exaggerate this information. Massa and Simonov (2003) claim that familiarity may depend on either some behavioral bias or better information about the specific stock. When dependent on a behavioral bias, it is availability or saliency that drives it. Saliency and availability mean that investors focus on information that is salient or often mentioned even though this information may not generate any informational advantage in relation to other investors. When underweighting foreign assets depends on an informational disadvantage, the underweighting is of course not a bias (if we do not believe in the strong form of market efficiency). But within the field of behavioral finance the "home bias" is a well-accepted behavioral bias when it comes to investing.

I believe that there are ways in which to act according to the home bias when using the Markowitz model. It seems that investors assume that investment in foreign assets is a risk. Thus when estimating covariances and variances for the Markowitz' model, the feeling that investing in foreign assets is more risky than domestic investment should be reflected there. Good or bad; this should be the case. Often variances and covariances are estimated from historical data. When estimating covariances and variances in this way the home bias cannot affect the portfolio weights.

In the case of the B-L model, it is possible to increase the estimated risk characteristics of a foreign asset and hence incorporate the home bias as well. But when using the B-L model there is yet another way of incorporating the home bias in the portfolio weights – via the levels of confidence. It appears reasonable to believe that an investor who is prone to be home biased has less confidence in the views concerning foreign assets than in those concerning domestic assets. Hence, he/she might feel less confident in the views concerning foreign assets, this leading to the weights in these assets being closer to the benchmark weights than the weights of the domestic assets.

Note that I am not discussing whether this is an advantage or a disadvantage in using the Black-Litterman model. I am just arguing that it in fact is the case. The B-L model enables home-biased portfolio managers to give expression to this when using the model.

4.6 Behavioral Finance and the B-L model – What it gave and didn't give?

The implications drawn from Behavioral finance concern both the attributes that distinguish the B-L model from the Markowitz' mean-variance model: (1) the equilibrium portfolio as a neutral point of reference and (2) the levels of confidence together with the weight-on-views. Research within behavioral finance gives support for a reference based portfolio model such as the B-L model. The equilibrium portfolio approximated as the benchmark portfolio seems also to be a reasonable point of reference since this is the portfolio against which the fund or portfolio manager is evaluated. However, with respect to the use of levels of confidence and the weight-on-views; implications seem more critical. Nothing in behavioral finance implies that we should not use parameters to weigh the portfolio weights between the market portfolio and the investor views. But, according to behavioral finance, people have difficulty in estimating their levels of confidence accurately. They are prone to overconfidence and hence implications from research regarding overconfidence do not favor the use of confidence levels when weighting between the benchmark portfolio and the view portfolio.

These implications appear important. They should be useful to an organization considering the use of, or already using the B-L model. However, it should be noticed that these implications are quite individualistic. They focus on the individual investor and do not take into consideration the social context in which the investor acts. This is actually quite typical of research within behavioral finance. Organizational and social questions are ignored. My impression is that researchers within behavioral finance focus on the individual investor as actually being only one single person. In this thesis I have considered the typical investor as a fund- or portfolio manager. Fund- and portfolio managers work most often in an organization and hence they affect and are affected by social and organizational activities in this context. It should also be noted that researchers within the field do not specify limits to their research, which exclude these questions. They express, in fact, no awareness of these issues at all. It is as if they are forgotten, as if the social context in which investors exist has no effect on their professional activities. I consider, as others have previously, this to be a serious omission from research in the field of behavioral finance. *Actors on the Financial Markets – an organizational finance perspective* (Finansmarknadens aktörer – ett organizational finance perspektiv, Blomberg 2005) was published toward the conclusion of this research project. Blomberg criticizes the individualistic perspective of behavioral finance, but he also criticizes behavioral finance for its structural functionalism. He argues that the individualism within behavioral finance leads to a reduced possibility to describe and explain complex social processes. The structural functionalism within behavioral finance leads, according to Blomberg, to individuals being not only detached from other individuals but also from other structures within the community. I agree with Blomberg. Different social situations should lead to different actions by investors. Hence, the social context and its influences on the actions of the investor seem interesting and relatively unexplored. Another weakness in behavioral finance is the lack of real-world studies. Much of the research is performed on aggregated data of different stock prices or empirical material from experiments performed with subjects, often students, making quite unrealistic financial decisions. The subjects of these experiments are often students. This is also emphasized by Blomberg (2005).

Does the criticism of behavioral finance suggest that the search for implications from this field for the use of the B-L model has been disappointing? Yes and No! Reading, studying and searching for implications to draw from behavioral finance have been rewarding. The implications drawn are both interesting and should be useful when using the model. The limitations of individual actions are still interesting and important when it comes to taking financial decision. Yet, extending the analysis with an organizational perspective seems essential and adds important dimensions

Frankfurter and McGoun (2002) seriously criticize behavioral finance. In the article *Resistance is futile: The Assimilation of Behavioral Finance* they claim that behavioral finance, as a field, is being assimilated by modern finance. Frankfurter and McGoun make a very appealing analogy with the television series Star Trek in which a creature called the Borg appears. The Borg is a creature consisting of other organisms but acting as one. The Borg aims at development by assimilating other species of the universe into the Borg. Frankfurter and McGoun liken the interaction between behavioral finance and modern finance to the meeting between species and the Borg. When meeting new species the Borg says: "*Resistance is Futile. You will be assimilated*". The authors claim that modern finance is now attempting to assimilate behavioral finance in the same way. Behavioral finance has often been referred to as the "anomalies literature". Now, as behavioral finance gains more and more appreciation, modern finance is no longer trying to exile the field "to a remote planet". Instead modern finance is assimilating behavioral finance. According to Frankfurter and McGoun this process is retrograde since behavioral finance is now becoming a prisoner of the forms and methods of modern finance. Adhering too closely to the EMH, they have been unable to establish a new paradigm of finance. What seem to surprise the authors most is that the supposed proponents of the field are in no way resisting the process of assimilation. However, Frankfurter and McGoun provide one explanation of the unresisted assimilation of behavioral finance into modern finance:

> *"Behavioral finance is allowing itself to be assimilated into the modern finance paradigm, because that is the only possible way research can be done today and still be called finance"* (Frankfurter & McGoun, 2002, p 20)

The individualistic perspective within behavioral finance might have been inherited from modern finance and its future existence might depend on, as Frankfurter and McGoun assert, behavioral finance clinging to the "*underlying tenets, forms, and methods of modern finance*" (Frankfurter & McGoun ,2002, p. 4).

Remember the background of this research project; the project I worked on in 2002, described in Appendix 1. Behavioral finance cannot provide tools to analyze the commissioner's way of acting. To be able to do this we need to move away from the individualistic perspective of behavioral finance and complement the analysis with a social and organizational perspective.

5 Further Research

This chapter serves two purposes. It provides a brief presentation of financial fields considering the social and organizational context as important perspectives when performing financial research and building financial theories. The other purpose of the chapter is to serve as a preview of and starting point for further work in the subject of the thesis. The two steps presented in previous chapters have led to the insights that traditional financial theory and behavioral finance are limited by the individualistic perspective and that it seems to be of great importance to take the social and organizational context into account when studying the B-L model and its use. These insights prompt an interest in possible studies of real life portfolio management situations.

5.1 Other Types of Finance

Given the individualistic perspective of behavioral finance and the importance of social and organizational perspectives I have searched for and located fields that I believe may provide different and complementing approaches to both behavioral finance and traditional financial theory. There already exist small but interesting fields in financial research with social and organizational starting points. Blomberg (2005) aims at introducing a new field of financial research, which he refers to as "Organizational finance". Examples of other fields closely related to organizational finance include: Social studies of finance, post-modern finance and post-autistic finance. I will briefly present these fields and organizational finance in the following. The fields have many similarities. They all aim at changing or complementing the way financial research is performed today. The researchers adopt a critical standpoint in relation to the assumptions and methodology of traditional finance. These "other types of finance" will act as important sources of inspiration in the third and fourth steps of this research project.

5.1.1 Social Studies of Finance

When searching for financial studies drawing on organizational theory or other social sciences one is surprised by how little research has been performed (excluding research within behavioral finance). Behavioral finance is the financial field, taking another social science into account that has earned most acceptances by traditional financial theory. This might be related to the tendency within behavioral finance to allow itself to be assimilated by traditional finance (Frankfurter and McGoun, 2002) as discussed above. The researchers within the field referred to as Social studies of finance consist mainly of sociologists and anthropologists interested in the financial markets.

> "*Social Studies of Finance, offers new and powerful research methodologies that can potentially address questions that lie beyond the reach of traditional economic treatment.*" (Stark, 2002)

On the homepage[21] of the social studies of finance network (SSFN) it is claimed that the complex world of modern finance is dominating economic research and that there is a need for new research approaches to study the financial markets and its industry. Donald MacKenzie holds a professorial fellowship to carry out "social studies of finance". On MacKenzie's homepage he writes:

> "*To understand the creation, development and effects of financial markets we need more than the perspectives of economics or of a 'behavioral' finance that is rooted in individual psychology. Markets are cultures.*" (MacKenzie 2005a)

de Goede (2005) concludes that social studies of finance is and ought to be a flexible research program. Social studies of finance should be:

> "*…an interdisciplinary forum for discussion and debate, enabling dialogue and disagreement between researchers in a diversity of disciplines who share a fascination for money, and who may otherwise not have easily engaged.*"

According to de Goede (2005), one of the most important aspects of the social studies of finance is the opening of the *"late-modern 'black box' of financial statistics, models and technology"*. According to MacKenzie (2005b) the only way of opening a black box is to

[21] www.ssfn.org/intro.htm; 2005-09-20

interact with those involved in the construction of the box. de Goede (2005) articulate three "concerns" central to the field of social studies of finance. These are:

1. Resocialisation of financial practices – Populate abstract financial models with social human creatures.
2. Performativity – Meaning that economic theory itself contributes to the construction of the phenomena it describes. Although the precise meaning of performativity is under debate, financial reporting has made financial data a part of news broadcasts.
3. Repoliticisation of financial practices – Writing cultural histories and opening the black boxes shows that markets and money are socially constructed.

Social studies of finance is a constellation of different research areas, using different approaches and methodologies for studying the financial markets, organizations and people. Many methods are however borrowed from sociology and field studies, anthropological and ethnographical methods are often used. Benunza and Stark (2004) conduct ethnographic field research in the Wall Street trading room of a major international investment bank. They make a careful and detailed description of the social-technology of arbitrage. Willman et al. (2002) make a field study in an investment bank.

5.1.2 Organizational Finance

Blomberg (2005) declares that organizational finance is closely related to the social studies of finance. However, while normative questions are absent in social studies of finance, organizational finance takes both normative business administrative and purely descriptive sociological problems into consideration. When it comes to theoretical starting points and tools the differences between organizational finance and social studies of finance are almost non-existent.

Blomberg (2005) defends the idea that competence studies and critical analyses of power are compatible in the same research area. He means that there are no pure paradigms and hence it cannot be claimed that it is more rewarding to be pragmatic orthodox than to move between different paradigms. As long as analyses are based on conscious choices and a reasonably high level of theory, Blomberg claims that it is unimportant whether the analysis is normative or descriptive. He makes a thorough exposition of the models within organizational theory that he finds suitable for producing a new financial field of research. He concludes the exposition by stating what is required of an organizational analysis of the stock market (pp. 84-85)[22]:

1. "*For organizational analysis of stock markets to result in a new and better knowledge than modern finance can generate; research must include analysis of the relations between actors' thoughts and actions; between actors themselves; and between actors and artifacts*."

2. "*In order to generate knowledge about the organization of the stock market, the analyses ought to contain analysis of power and processes of influences*."

3. "*An organizational analysis of the actors in the stock market should contain the (re)constructional process of the identities of the actors including their (variable) motives and interests under certain circumstances.*"

4. "*An organizational analysis of the actors of the stock market should try to answer questions about what kind of people are constructed because of the organization of the stock market, how they are constructed and why*."

5. Even if empirical analyses of the stock markets would miss processes of power and influence it doesn't mean that we shouldn't ask for them. "*Blindness for power is a worse starting point than a critical organizational perspective as the first prevents an empirical investigation of whether power and influence are significant aspects of the organization of stock markets.*"

6. "*From an academic perspective the constructionist perspective is an essential and not an arbitrary choice*."

Blomberg motivates the practical relevance of organizational finance by claiming that organizational finance, in contrast to traditional finance, can show that development is not spontaneous but depends on active actors. Results from organizational finance may be used instrumentally, but Blomberg suggests rather an emancipated use of the results.

5.1.3 Post-Modern and Post-Autistic Finance

Post-modern finance attempts to take financial research "one-step further", leaving modern finance behind and developing a post-modern type of finance. According to Frankfurter (Frankfurter et. al., 1997, p. 134):

> "*Modernity begins with things (objects) and the properties of the things, and the purpose of science is to discover the fact about them; that is, laws that govern how properties change and how things relate. In post-modernity, what is important is how things and so-called facts are used within culture, which, of course, changes as culture changes. In modernity,*

[22] Translated from swedish to englinsh by the author.

there is an inherent meaning to objects; in post-modernity, the meaning lies in their appearances."

McGoun claims in the same article that post-modern finance differs from modern finance in the sense that while modern finance seeks to discover the reality, post-modern finance believes that the reality is not there to be discovered but is instead something that is constructed: "*We don't discover finance; we invent finance*" (Frankfurter et. al., 1997, p.148). McGoun (1996) makes an interesting analysis, taking a fashion theory approach to finance, both the industry and the academy. He points to the fact that finance is a part of the popular culture, following trends and fashion, both in activities in the markets and in the way research is performed.

Bondio (2003) suggests four new assumptions for a new economic theory:

1. *"People search for meaning and value in their own existence and everything around them.*"
2. "*We are all subject to social norms.*"
3. "*People will adopt the norms of the groups most important to them at any point in time*"
4. "*The importance of groups (and their norms, which are followed) will depend on two, sometimes opposing, forces – personal development and context.*"

According to Bondio (2003) these assumptions imply at least the following foundations for a new economic theory:

1. "*Bring both consumer and producer analysis together through the analysis of people*
2. *Require less emphasis on mathematical logic and more on observing reality*
3. *Shift methodology towards group analysis away from individualism (this is important in explaining group hostility and cooperation, gender, class, societal and cultural characteristics)*
4. *Require explicit historical perspectives when analyzing the development and emergence of groups and their norms."*

A movement referred to as Post-Autistic finance has taken form during recent years. It began in 2000 when a group of students "*associated with France's 'Grandees Coles', whose enormous academic prestige and selectivity surpasses that of other higher education institutions in France*" (Fullbrook, 2002) distributed a manifest on the Internet protesting against the lack of realism in economics. They protested against the way mathematics is used within economics with the result that economics has become an "autistic science". In the manifest they demanded a change in the teaching of the subject that would leave space for critical and reflective thoughts. The students chose to call the movement post-autistic economics because "*allegiance to a single narrative necessarily means that in the main it refuses to look at economic reality*" (Fullbrook 2005). Beginning in Paris in 2000 among students, the post-autistic economics movement now involves thousands of economists all over the world. The post-autistic movement now wishes to free economics from the neoclassical approach as the only approach to economics. They aim at opening up economics with pluralism and critical thinking.

The discussed fields of finance share many common characteristics. Most importantly, they share the social and organizational approach to the field of finance, an approach that I hope to use when performing the empirical research that will constitute the two last steps of this research project.

5.2 Organizational Structure and the Use of the B-L model

Ideas concerning the organization of the use of a model such as the B-L have arisen during the two first steps. These ideas will be presented and discussed generally in the following. They will serve as initial thoughts and concepts for the following studies.

5.2.1 Who should Set the Weight-On-Views?

The dilemma with overconfidence when stating the levels of confidence in the B-L model might be solved to some extent with a well-designed and well-functioning organization. When reading existing literature concerning the B-L, it can be considered that it is assumed that the same person should state both the confidence levels and the weight-on-views. The organizational discussion within the literature regarding the B-L model is, as mentioned, limited. It is stated that parameters should be set, but not how nor by whom. That some parameters might be difficult to set in any way is not mentioned. It should be possible however, and might be interesting to consider whether one person could state the views and the levels of confidence allocated to each view while another person sets the weight-on-views. The person setting the weight-on-views could for example be the fund manager's boss. The boss can then focus more on studying the fund managers, who they are, how they have performed and how well they may be able to estimate future returns and confidence levels. If an inexperienced investor has been fortunate and performed very well for some months, he/she may become overconfident, attributing his/her success to his/her own skills rather than to chance (see 4.5). If the boss notes this, he/she has a tool with which to cope with the overconfidence of the fund manager. The boss can lower the weight-on-views and thereby reduce the impact of the overconfidence of the fund manager on the portfolio. Evaluating fund managers in this way may appear quite difficult, but it may be an interesting way to use the model. It has been said above that behavioral finance has shown that humans are bad at estimating their own level of confidence. I have found no research results indicating whether people are good or bad at estimating the confidence levels of others.

5.2.2 Decision Groups

Both behavioral finance and portfolio theory often refer to "the investor". I find it interesting to consider the appearance of a team managing a fund using the B-L model. My, so far, limited experience in practical fund management makes it difficult for me to envisage such a team. The complexity of managing a fund with the help of an advanced quantitative tool such as the B-L model, would, I believe, demand a group of individuals with different positions and special knowledge. The team can be seen as a "dream team" based on impressions I have gained during my research. The team could consist of the following participants:

- Asset analysts who analyze in detail the assets the portfolio or fund contains.
- A macro specialist focused in macro economic prognosis and the effects of macro economic events.
- A risk specialist focused on future risk and hence not only on the ARCHing, GARCHing and EGARCHing (expression from Frankfurter 2003) of historical time series, but on risk *forecasts*, forecasts of covariances and variances.
- A B-L specialist focused in the model itself and in the particular implementation of the model used in the organization.
- A boss or group leader who is specialized in economic philosophy and organizational issues - The group leader could be specialized in organizational questions and also have knowledge about economic philosophical questions. The group leader might be able to widen the group's viewpoint and perhaps reduce the risk of the group following un-fruitful perspectives.

These are examples of roles in a fund management team that seems appealing. For allocation of the portfolio I imagine a meeting between these specialists presenting their knowledge, prognoses and ideas of asset returns, risks and macro economic events during the following investment period. A dynamic discussion involving all these persons would increase the probability that data input to the B-L model would be as well thought through as possible. The team could then, together, evaluate the reasonability of the portfolio output by the model. The team members could test different inputs and investigate how these would affect the output portfolio. It is an interesting question to what degree this way of working would be fruitful and rewarding.

5.3 Further Research

The discussions above are examples of ideas that have emerged during the two first steps in this research project and they are also examples of the kind of research project I aim at working with in the last two steps of this research project; projects in which the social and organizational context of the use of portfolio models are taken into account. These steps will build on "real-world studies" and be carried-out in interaction with active financial managers. This kind of empirical research seems to be infrequent within the financial area and hence I believe that its contributions would be of interest to practitioners and to different kinds of academic financial fields. I am interested in the use of the B-L model *within* the financial industry. How do/would they: work with the B-L model, estimate the covariance matrix, set investor views, set the levels of confidence, set the weight-on-view? Do/Would they: consider overconfidence as a dilemma, find the model easy or difficult to work with, find the output portfolio reasonable, invest directly in the output portfolio or use as input in the decision on how to weigh the portfolio? Do/Would "real-world" financial managers find the output portfolio generated by the B-L model more realistic than the output portfolio generated by the Markowitz' mean-variance model? Do/Would they be working in a team or would a single investor take all the fund management decisions alone?

The items below are examples of possible studies concerning decision-making within fund management.

1. A study focusing on a real world application of the B-L model taking the social and organizational context into account as well as the technical aspects. To further analyze the Black-Litterman model I believe it would be of great interest to study a real world project within a bank or other financial institution using or beginning to use the B-L model.

2. A study to determine the skill of investors in estimating their level of confidence. This may be done by comparing the level of confidence set with the actual outcome.

3. A study of how portfolio allocation decisions are taken in practical portfolio management, to learn what tools are used in the analyses and what internal and external factors affect the decision-making appears interesting. Are portfolio optimizers used? If not, why and would fund managers like to use portfolio

optimizers? What are deemed to be as portfolio optimizers' main advantages and disadvantages?

4. A study researching a development of the B-L model with which the users have the possibility to express views to variances and covariances similar to the views that users give on expected returns in the existing model.

5. I have discussed the relations of the utility functions in the traditional Markowitz' model, the B-L model and the utility function within behavioral finance. It would be interesting to study what happens if the utility function in the B-L model was changed to one expressing loss aversion.

These are examples of possible projects concerning the B-L model that I believe would produce interesting and fruitful research results. The following two steps in this research project will, as explained in chapter 1, probably concern to items 1,2 and 3.

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Appendix 1

Background

This research project was largely motivated by my experiences during the implementation of the B-L model in a major Swedish bank in 2002. I was commissioned to attempt this in the absence of the expertise within the bank. The documentation of the project became a part of my master's thesis. After preparatory reading of the literature concerning the model it became clear that no methodological and detailed description of the B-L model was available. Several parameters were puzzling and difficult to understand on the basis of the existing literature.

During the execution of the project several difficulties of different character were encountered. One problem concerned input data. As in most quantitative financial models it was necessary to estimate the covariances between all the assets handled by the model. In university courses dealing with the subject, covariances had always been given and taken for granted, but in practice they must be estimated. The first, and most obvious, alternative was to calculate the covariances from historical data, this being an easy process according to university courses. During the project questions arose concerning the implications of calculating covariances from historical data alone. It is not intended that the covariances to be input into portfolio models, both the Markowitz model and the B-L model, should be estimated from historical data alone (Markowitz 1991). Instead, it is the estimated *future* covariances, which should be estimated. Markowitz claims however that historical data could constitute as one input to the estimation of variances and covariances. To me, this has been an important insight. Estimating the future covariances is not easy. The commissioning instance was not particularly concerned how the input data to the B-L model was estimated, all that was of importance was that the computer program could be run without much effort by the user and gave acceptable results.

Toward the end of the project, I doubted whether the bank would gain much from the use of the program. The B-L model seemed to have both advantages and disadvantages and the estimations and implementation obviously also had both advantages and disadvantages. To be able to use the model successfully, I deem it necessary that the user should understand both the model itself and the way it is implemented. At the bank, however, there appeared to be little interest in any questions regarding the theoretical characteristics or implementation of the model. One of the reasons for the bank implementing it, according to bank sources, was to obtain a better understand the B-L model. The low level of active participation of the bank personnel in the implementation of the model meant that they knew little more about the B-L model than they did before the project began.

I began to reflect on how the estimations could affect the output of the model. That the bank was not interested in the kind of approximations used was a most disturbing fact. Was the bank unaware of the problems? What was the purpose of implementing the model if the future users cared neither about the theoretical foundation of the model nor how it was implemented?

Experiences from the project have had much influence on the studies presented in this thesis. I was prompted to study not only a theoretical model, but also its *use*. People act in a social and organizational context that influences the use of models. For example, the organizational structure in which the model is used is of great importance for the use of a model such as the B-L model. My experiences from the implementation of the B-L model resulted in the following conclusions:

1. Portfolio models in general and the B-L model in particular are tools. These tools have both advantages and disadvantages. Hence, to gain from the use of a tool (or to be able to choose to use it or not to use it) in an investment context, it is of great importance that the user understands the theoretical characteristics of the tool as well as its practical use. Sellstedt (2002) comments positively on models and their use. He stresses that the shortcomings of a model need not invalidate its use. Sellstedt means that all models are tools and it all comes down to using the model and the results from its use in a sensible way.

2. Estimating input data to the B-L model is difficult. Since the input data to the B-L model are difficult to estimate, it is of great importance that the investor using the model understands the implemented estimations and their inherent problems.

3. Portfolio models are used in a social and organizational context. This context is of great importance for the use of the model.

Since my interest lies in the *use* of the B-L model I searched for literature discussing the use of portfolio models and/or other quantitative financial models. I was impressed by the volume of purely theoretical and mathematical research results available and, at the same time, the scarcity of literature discussing the use of the models in real life investment situations. In searching for articles about the application of portfolio models I became aware of the field most often referred to as behavioral finance. Behavioral finance, as an academic field, has expanded in recent of years, especially since 2002 when one of the leaders in the field, Daniel Kahneman, was awarded the Bank of

Sweden Prize in Economic Sciences in Memory of Alfred Nobel. Researchers within behavioral finance question many of the assumptions of traditional finance and are interested in what motivates and influences people when making financial decisions. I chose to study behavioral finance and to search for areas within the field, which appear to be of interest for the use of the B-L model and then to try to draw implications from the empirical research performed within the field for the use of the B-L model. My expectation is that the implications drawn from behavioral finance for the use of the B-L model will shed light on the use of the model and help financial institutions to use the model more successfully.

In searching for interesting research results within behavioral finance and implications for the use of the B-L model, I carefully read important articles and books in the field. My main references have been *Choices, Values and Frames* (Kahneman and Tversky 2002) and *Heuristics and Biases*: *The Psychology of Judgment* (Gilovich, Griffin and Kahneman 2002).

Appendix 2

Markowitz' Mean-Variance Model

According to Markowitz (1952), the inputs needed to create optimal portfolios are: expected returns[23] for every asset, variances for all assets and covariances between all of the assets handled by the model.

Markowitz does not state exactly how these parameters should be estimated although his discussion of some alternatives is quite detailed. He sees past performances as one source of information, but he emphasizes that portfolio selection solely based on historical data assumes that past data are reasonable approximation of the future ditto. Instead, Markowitz prefers the "probability beliefs" of experts as inputs to the portfolio analysis (Markowitz 1991, p. 27). He compares the way a security analyst arrives at probability beliefs with the way a meteorologist arrives at a weather forecast and calls the security analyst the meteorologist of stocks and bonds (Markowitz 1991, p. 28). But, Markowitz also emphasizes that portfolio analysis begins where security analyses ends. In Markowitz' model, expected future returns are to be estimated as the expected return of every asset during the investment period. Investors specify the length of the investment period.

Risk, in the Markowitz model, as well as in many other financial models, is approximated by the variances and covariances of future returns. When considering only one asset, it is sufficient to estimate and evaluate only its expected future return and the future variance. When evaluating a portfolio of assets, however, we should consider how the assets within the portfolio covariate to be able to estimate the variance of the portfolio as a whole. The covariance is a measure of how the values of two random variables move up and down together. In this case the random variables are any pair of assets in a portfolio. The covariance is crucial to portfolio theory and increases the possibilities of getting a well-diversified portfolio.

Choosing a Portfolio

In portfolio theory, investors are assumed to want as high expected future return as possible but at a risk as low as possible. There are many other factors, which investors might consider, but risk and return are what this model focuses on.

We use the following notation:

$\mathbf{w}$ - the column vector of portfolio weights
$\mathbf{w}^*$ - the Markowitz' optimal portfolio

[23] For simplicity expected return will refer to the expected excess return over the one-period risk-free rate

σ^2 - the variance of the portfolio

$\bar{r}_i$ - the expected return of asset number i

r_{rf} - the return of the risk free asset

$\bar{r}$ - the expected return of the portfolio

w_{rf} - the weight of the risk free asset in percent of the portfolio as a whole

$\boldsymbol{\mu}$ - the column vector of expected (excess) returns

$\boldsymbol{\Sigma}$ - the covariance matrix.

δ - the risk aversion parameter stated by the investors. States the trade-off between risk and return

We set:

$$\bar{\mathbf{r}} = \begin{bmatrix} \bar{r}_1 \\ \vdots \\ \bar{r}_d \end{bmatrix}, \qquad \mathbf{e} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

hence we get:

$$\mathbf{e}r_{r_f} = \begin{bmatrix} r_{r_f} \\ \vdots \\ r_{r_f} \end{bmatrix}$$

To derive the set of attainable portfolios (derived from the expected return and the covariance matrix estimated by the investor) that an investor can reach, we need to solve the following problem:

$$\begin{cases} \min\limits_{\mathbf{w}} \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w} \\ \mathbf{w}^T \bar{\mathbf{r}} = \bar{r} \end{cases} \tag{A.1}$$

or

$$\begin{cases} \max\limits_{\mathbf{w}} \mathbf{w}^T \bar{\mathbf{r}} \\ \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w} = \sigma^2 \end{cases} \tag{A.2}$$

We minimize the variance of the portfolio given a certain level of expected return or we maximize the expected return of the portfolio for a certain level of risk (variance).

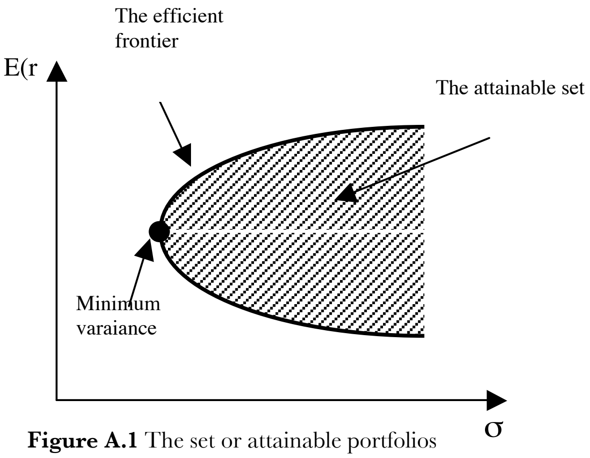

Figure A.1 The set or attainable portfolios

Figure 1 shows the function of all attainable portfolios i.e. combinations of expected return and, in this case, standard deviation. All combinations to the right of the curve are attainable whereas those at the left of the curve are not attainable. The combinations on the curve are called the minimum variance set, since for every level of expected return, the point on the curve represents the minimum variance attainable. The upper part of the curve is called the efficient set or the efficient frontier. Portfolios on this part of the curve are referred to as efficient since they represent portfolios generating maximum expected return according to certain levels of standard deviations or minimum risk according to a certain level of expected return. For every portfolio lying on the lower part of the curve it is possible to choose another portfolios with better characteristics, either higher expected return or lower standard deviation. Because of this, all portfolios not lying on the efficient frontier are called inefficient.

Let us now include a risk-free asset[24]. Assume we have *d* risky assets. The weight of the risk-less asset in the portfolio is hence:

$$w_{r_f} = 1 - \mathbf{e}^T \mathbf{w}$$

The expected return of the portfolio, r_P is then

$$\bar{r}_P = \mathbf{w}^T \bar{\mathbf{r}} + w_{r_f} r_{r_f}$$

and we can write the expected return as

$$\bar{r}_P = \mathbf{w}^{\mathbf{T}} \bar{\mathbf{r}} + (1 - \mathbf{w}^{\mathbf{T}} \mathbf{e}) r_{r_f} = \mathbf{w}^{\mathbf{T}} (\bar{\mathbf{r}} - \mathbf{e} r_{r_f}) + r_{r_f}$$

We define the vector of expected (excess) returns as:

24 There are no real risk-free asset often the 5-year government bond is used as an estimation. But let us however use the expression risk-free asset since it is most often used in the theory.

$$\boldsymbol{\mu} \equiv \bar{\mathbf{r}} - \mathbf{e}r_{r_f} = \begin{bmatrix} \bar{r}_1 - r_{r_f} \\ \vdots \\ \bar{r}_d - r_{r_f} \end{bmatrix}$$

Hence now the universe of available portfolios has been expanded and the efficient frontier is moved.

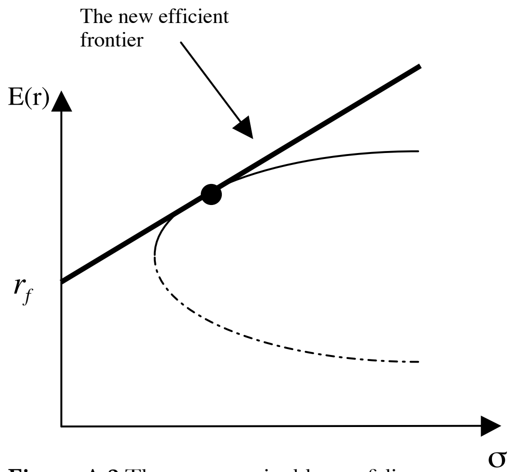

Figure A.2 The set or attainable portfolios

The new efficient frontier is a weighted combination of the risk-free asset and the portfolio in which a straight line drawn from the risk-free rate or return is a tangent to the efficient frontier when no risk-free asset is available. This is also quite reasonable because, in this model, we always want an expected return as high as possible when taking a certain level of risk or as low level of risk as possible for a certain level of expected return.

Let us introduce the parameter δ, often referred to as the risk-aversion parameter. This parameter is a measure of the risk the trade-off between risk and expected return of the portfolio. We are to solve the following problem:

$$\max_{\mathbf{w}} r_{r_f} + \mathbf{w}^T\boldsymbol{\mu} - \frac{\delta}{2}\mathbf{w}^T\Sigma\mathbf{w}$$

Since r_{r_f} is constant, we can exclude it and still get the same result. The problem to be solved is hence:

$$\max_{\mathbf{w}} \mathbf{w}^T\boldsymbol{\mu} - \frac{\delta}{2}\mathbf{w}^T\Sigma\mathbf{w} \qquad \text{(A.3)}$$

This problem is solved by setting:

$\mathbf{e}_k^{\,T} = [00\ldots \underbrace{1}_{\text{entry k}} 0\ldots 0]$, number of elements equals number of assets

Differentiate the function and set it equal to zero:

$$\mathbf{e}_k^T\boldsymbol{\mu} - \frac{\delta}{2}\mathbf{e}_k^T\boldsymbol{\Sigma}\mathbf{w} - \frac{\delta}{2}\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{e}_k = \mathbf{0}$$

$$\mathbf{e}_k^T(\boldsymbol{\mu} - \delta\boldsymbol{\Sigma}\mathbf{w}) = \mathbf{0}$$

This is true for all $k = 1,\ldots,d \Rightarrow$

$$\mathbf{w}^* = (\delta\boldsymbol{\Sigma})^{-1}\boldsymbol{\mu} \qquad \text{(A.4)}$$

Where $\mathbf{w}^*$ represents the Markowitz optimal portfolio given the risk aversion coefficient, covariance matrix and vector of expected returns estimated by the investor.

Problem (A.4) is actually the same as solving problem (A.1). Hence:

$$\begin{cases} \max\limits_{\mathbf{w}} \mathbf{w}^T\boldsymbol{\mu} \\ \mathbf{w}^T\Sigma\mathbf{w} = \sigma^2 \end{cases}$$

The Lagrange function is then:

$$L = \mathbf{w}^T\boldsymbol{\mu} - \lambda(\mathbf{w}^T\Sigma\mathbf{w} - \sigma^2)$$

Differentiate and we get:

$$\mathbf{e}_k^T\boldsymbol{\mu} - 2\lambda\mathbf{e}_k^T\Sigma\mathbf{w} = 0$$

This is the same as differentiating (A.3), which is

Let

$$\lambda = \frac{\delta}{2}$$

then

$$\mathbf{e}_k^T\boldsymbol{\mu} - \delta\mathbf{e}_k^T\Sigma\mathbf{w} = 0$$

$$\boldsymbol{\mu} = \delta\Sigma\mathbf{w}$$

$$\mathbf{w} = (\delta\Sigma)^{-1}\boldsymbol{\mu}$$

$$\sigma^2 = \mathbf{w}^T\Sigma\mathbf{w} = \delta^{-2}\boldsymbol{\mu}^T\Sigma^{-1}\Sigma\Sigma^{-1}\boldsymbol{\mu} = \delta^{-2}\boldsymbol{\mu}^T\Sigma^{-1}\boldsymbol{\mu}$$

This shows that when we select the value of the parameter σ the value of δ is given. We can also choose a value of δ and we then get the value of σ.

$$\lambda = \frac{\delta}{2}$$

$\frac{\delta}{2}$ is thus just the Lagrange multiplier.

When:

$$\boldsymbol{\mu} = \delta\Sigma\mathbf{w}$$

$$\mathbf{w}^* = (\delta\Sigma)^{-1}\boldsymbol{\mu}$$

$$\mu_P = \mathbf{w}^{*T}\boldsymbol{\mu} = \boldsymbol{\mu}^T(\delta\Sigma)^{-1}\boldsymbol{\mu} = \delta^{-1}\boldsymbol{\mu}^T\Sigma^{-1}\boldsymbol{\mu}$$

$$\sigma_P^2 = \mathbf{w}^{*T}\Sigma\mathbf{w}^* = \boldsymbol{\mu}^T(\delta\Sigma)^{-1}\Sigma(\delta\Sigma)^{-1}\boldsymbol{\mu} = \delta^{-2}\boldsymbol{\mu}^T\Sigma^{-1}\boldsymbol{\mu} = \delta^{-1}\delta^{-1}\boldsymbol{\mu}^T\Sigma^{-1}\boldsymbol{\mu} = \delta^{-1}\mu_P$$

then:

$$\delta = \frac{\mu_P}{\sigma_P^2}$$

This is also consistent with Satchell and Scowcroft (2000, p. 139). Economists would call this parameter the standard price of variance.

Hence the Markowitz optimized portfolio is:

$$\mathbf{w}^* = (\delta\Sigma)^{-1}\boldsymbol{\mu}$$

Appendix 3

Assumptions

It seems relevant to list some of the assumptions of the B-L model. This is not easy since many of the assumptions are the same for portfolio modeling in general or hence for quantitative financial models in general. It is also difficult since many of the assumptions are implicit. The list presented below is not aiming at being exhaustive. It presents some assumptions that might be interesting to have in mind while reading the rest of the thesis.

Assumptions common for many *quantitative financial models*:

- Returns are normally distributed
- Investors are rational
- Absence of arbitrage
- Decreased marginal utility of wealth
- Increased risk is concerned as negative
- Increased expected return is concerned as positive
- There is a trade-off between expected return and risk
- Capital markets are efficient in that the prices of securities reflect all available information and that prices of individual securities adjust very rapidly to new information;

Assumptions common for *quantitative portfolio models*:

- Each possible investment has a probability distribution of expected returns over some holding period.
- Only risk and expected return are used in investment decisions.
- Investors will choose the combination of asset weights that generates the highest expected return for a given risk level. Or, investors will choose the combination of asset weights that generates the lowest risk for a given level of expected return.
- The investor is risk averse
- A portfolio's risk can be measured by the future variance of and the covariance between the assets' rate of return.
- Taxes and other transaction costs like cuortage and similar aren't taken into account.

Assumptions specific to the *B-L model*:

- Investors have views about assets that they believe can lead to a better portfolio
- The market isn't totally efficient (Litterman, 2003).
- Risk ought to be taken in the assets to which investors have views
- Funds or portfolios are evaluated according to a benchmark portfolio.
- To every opinion a level of confidence can be estimated
- Investors aren't absolutely sure on any view

Appendix 4

Behavioral Finance

A more detailed description of the three parts within behavioral finance is given in the following. The description is not exhaustive but, hopefully, it will provide readers not familiar with the field of behavioral finance with an overview of the field and a feeling for its main ideas and research results. The overview will describe some central and well-established research results from the field.

The presentation begins with a description of "Limits to arbitrage", one of the main parts of behavioral finance. Following this, "Heuristic-driven biases" and "Frame dependence" will be presented. These two parts of behavioral finance concern how psychological factors affect individual investors whereas the part "Limits to arbitrage" is concerned with how psychology and "irrationality" affect markets.

Limits to Arbitrage

Whether markets behave "*rationally*" or not is the subject of a continuous debate. . The efficient market hypothesis (EMH) has dominated economic theory since Fama (1970) presented the efficient financial theory as one in which securities are always priced in consideration of all available information. The efficient market hypothesis then states that real-world financial markets are efficient according to this definition. In the last 20 years this view of markets has been challenged. It is argued that the forces supposed to attain this efficiency, such as arbitrage trading, are likely to be much weaker than the defenders of the hypothesis claim (Shleifer, 2000). Behavioral finance, both theoretically and empirically, offers an alternative approach. The efficient market hypothesis rests, according to Shleifer (2000), on three arguments relying on progressively weaker assumptions:

1. Investors are assumed to be rational and hence to value securities rationally.
2. If some investors are not rational, their irrational trades are random and therefore cancel each other out.
3. If investors should be acting irrationally in similar ways, rational arbitrageurs act on the market and eliminate the influence of irrational investors on prices.

A rational investor is, according to the EMH, defined as an investor who values securities on the basis of their fundamental value, the expected net present value of their future cash flows, discounted using their risk characteristics. According to EMH, rational investors only consider expected return and risk when evaluating investment strategies.

During the last 20 years, this view of markets has been challenged. It is argued that the forces that are supposed to attain the efficiency, such as arbitrage trading, are likely to be much weaker than the defenders of the hypothesis stress (Shleifer, 2000). Behavioral finance claims that errors, as they are discussed in EMH, are both systematic and significant and also that they can persist for long periods of time.

Let us begin by considering the first argument of EMH. It is difficult to sustain the belief that investors act fully rationally. Black (1986) shows that investors often trade on noise rather than on information, fail to diversify, sell winning securities and hold on to losers etc. People deviate from the standard decision-making model in many essential ways (Kahneman & Reipe, 1998). One of the most widely known examples of this is what Kahneman and Tversky (1979) call *loss aversion*, saying, among other things, that the value function is steeper for losses than for gains and that the value function is concave for losses and convex for gains. Kahneman and Tversky (1973) show that individuals violate Bayes' rule and other rules of probability theory. Kahneman and Tversky (1979) also show that people assume that the empirical mean value of small and large samples has the same probability distribution. This bias they refer to as *the law of small numbers*.

Kahneman and Tversky also question the second argument in the efficient market hypothesis, saying that irrational investors' trades are random and therefore cancel each other. Kahneman and Tversky (1979) dispose this entirely by claiming that most often people deviate from rationality in the same way. For example investors are often evaluated according to a benchmark and therefore often act to minimize the risk of falling behind. They also often act as a herd and select the same stock as other investment managers, again to avoid falling behind.

The last of the three arguments of the efficient market hypothesis says that even if the trades of noisy investors are correlated, arbitrageurs act to bring prices back to their fundamental values. However, researchers within behavioral finance claim that arbitrage trades are risky and because of this, limited. Arbitrage relies heavily on the existence of close substitutes. Yet, in many cases securities do not have good substitutes and therefore arbitrage trading cannot work to push prices back to fundamental values. For example an investor believing that stocks are overpriced cannot go short in stocks and buy a substitute portfolio. But even if there are almost perfect substitutes and the prices of the two securities ultimately converge, the trade may lead to temporary losses. Most arbitrageurs do not manage their own money; acting instead as agents for other people. These investors evaluate their portfolios regularly and quite frequently. If the evaluation horizon is shorter than the trade, the investor may not be satisfied with the performance of the arbitrageur and therefore withdraw money. If many people withdraw money from the fund, the arbitrageur may have to liquidate the position, leading to further performance problems. These losses may result in the arbitrageur being unable to maintain the position.

Empirical evidence supporting the efficient market hypothesis in the 1960s and 1970s was overwhelming. Shleifer (2000) divides the empirical evidences for the hypothesis into two categories. First, when news affecting the value of a security hits the market, it should *quickly* and *correctly* affect the price of the security. *Quickly* means that an investor who receives the information late should not be able to profit from this information. *Correctly*, means that the price movement in response to the new information should be accurate on average. Second, since rational investors, according to the efficient market hypothesis, value securities on the basis of their fundamental value, prices should not be affected by changes in supply and demand of the security.

According to the first category money cannot be made on the basis of stale information. This argument is somewhat difficult to challenge. To do this, we need to define the meaning of "*stale information*" and "making *money*". "*Making money*" is hard to define. In finance "*making money*" means earning surplus returns after adjustment for risk. Showing that a strategy, based on stale information, earns on average a positive return is not enough to show market inefficiency. The profit may only be a fair market compensation for risk taking, but to evaluate this, we need a model for a fair relationship between risk and return etc. Still, when researchers suggest that they have found ways of "making money" on the basis of stale information, critics suggest that these profits are only fair compensation for risk-taking. One empirical result suggesting that information is not always quickly and correctly reflected in security prices is the so-called "*January effect*". Returns are seen to be superior in January, especially for small stocks but there is no evidence that stock or small stocks are riskier in January than the rest of the year.

According to the second category, rational investors only evaluate securities according to their fundamental values, meaning that changes in demand or supply should not affect prices. Research has however shown that prices react to inclusion of stocks in the Standard and Poor's 500 Index (Shleifer, 2000). According to the efficient market hypothesis, inclusion of an asset in the Index is not supposed to convey any information to the market, but the asset price increases substantially and the increase is shown to be sustainable. According to Schole's theory, inclusion of a security in an index should not affect its price because of increased demand. When the price of an asset begins to rise because of index inclusion the initial holders should want to sell and thereby stabilize the prices.

Heuristic-driven Biases

The other part of behavioral finance focuses on investor behavior and psychology. Extensive empirical research within this field has shown that people do not always act according to the rational model as suggested by neoclassical theory. This, however, is probably not surprising. What is worth noting is that traditional economists have assumed that people differ from the rational model in a non-systematic way and therefore consider it impossible to incorporate this in models. Behavioral finance claims to have found clear systematic patterns in some of the ways in which people deviate from rational behavior.

1974, the article *Judgment under Uncertainty: Heuristics and Biases* was published in the journal Science. It made a significant impression in the area of social sciences. The two authors, Amos Tversky and Daniel Kahneman, had written a number of articles on human judgment in the late 1960s and the early 1970s. This was the starting point in the field, within behavioral finance, often referred to as the *Heuristics and biases approach to judgment under uncertainty*. The core idea of the field is that complex probability judgments are often based on simplified heuristics instead of formal and extensive algorithms, as suggested by the rationality paradigm. This can give rise to series of systematic errors[25], often referred to as biases. (Gilovich, Griffin & Kahneman, 2002). According to the heuristics and biases approach to judgment under uncertainty, people do not estimate likelihood and risk according to the laws of probability. Already in 1954, Paul Meehl compiled evidence saying that actuarial methods almost always outperformed expert predictions.

Kahneman and Tversky (1974) present three heuristics that give rise to a number of biases. These heuristics: representativeness, availability, and anchoring and adjustment will be described below. It should however be mentioned that when reading literature regarding heuristic-driven biases, heuristics and biases are frequently not distinguished (see for example Shefrin, 2004). Instead both heuristics and biases are referred to as heuristic-driven biases and hence representativeness, availability, and anchoring and adjustment are also referred to as biases.

Heuristics

In this context, heuristics are the trial-and-error processes that lead people to develop rules of thumb. "*It's like back-of-the-envelope calculations that sometimes come close to providing the right answer*" (Shefrin, 2002, p. 13). Heuristics help people reduce complex probability judgments into more simple judgment processes (Kahneman & Tversky, 1974). The use

[25] Systematic errors is used within behavioral finance and refers to the systematic divergence of people from "rational" behaviour according to homo economicus.

of heuristics is often advantageous, but it can give rise to some systematic errors, or biases.

Representativeness – Representativeness refers to judgments based on stereotypes. Kahneman and Tversky (1974) show that when people try to determine the probability that a model B generated a data set A or that an object D belongs to a class C, they often use the representativeness heuristic. To illustrate, I will give an example of a bias derived from the representative heuristic referred to as base rate neglect. Kahneman and Tversky (1983) present this description of a person named Linda:

> *Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice and also participated in anti-nuclear demonstrations.*

When asked which of the statements "*Linda is a bank teller*" (statement A) and "*Linda is a bank teller and is active in the feminist movement*" (statement B) is more likely to be the correct statement, subjects typically assign greater probability to B. This is of course impossible since B is a subset from A. Here people fail to apply Bayes' law, saying that:

$$p(statementB|description) = \frac{p(description|statementB)\,p(statementB)}{p(description)}$$

People put too much weight on $p(description|statementB)$ which captures representativeness and too little weight on the base rate $p(statementB)$. Representativeness provides a simple explanation. The description of Linda sounds like the description of a feminist – it is representative of a feminist – leading subjects to pick B. Representativeness also leads to another bias, sample size neglect. People often fail to take the size of the sample into account. In situations where people do know the data-generating process in advance, the law of small numbers generates a gambler's fallacy effect (see the section "*Sample size neglect and the law of small numbers*").

Availability – When judging the probability of an event – say the likelihood of getting mugged in Chicago – people often search in their memories for relevant information. While this is a perfectly sensible procedure, it can produce biased estimates because not all memories are equally retrievable or available. More recent events and more salient events – the mugging of a close friend in Chicago – will weight more heavily and distort the estimate. Whenever we use this kind of information and not only the frequency of the event, our assessment of the probability of the event will systematically be biased (Barberis & Thaler, 2003).

Anchoring and adjustment – Tversky and Kahneman (1974) argue that when forming estimates, people often start with some initial value and then adjust away from it. Experimental evidence shows that this is not beneficial. Tversky and Kahneman performed a test, asking two groups of subjects to estimate various percentages. Before determining their answers, a wheel of fortune was spun that settled at an arbitrary value. The student groups were then first asked to estimate whether their answer was lower or higher than the value on the wheel of fortune. After this they were asked to determine the final guess of the percentage. The median estimates of the percentage were 25 and 45 for the groups obtaining spin results of 10 and 65 respectively on the wheel of fortune. This indicates that the groups were affected by the value given by the wheel of fortune even though they knew it to be an arbitrary value.

<u>Biases</u>

The use of heuristics to solve complex problems can lead to systematic errors. These errors are referred to as biases. The dictionary explanations of a bias are[26]: (1*) "Bias: a personal and sometimes unreasoned judgment."* (2) *"Bias: deviation of the expected value of a statistical estimate from the quantity it estimates."* (3) *"Bias: systematic error introduced into sampling or testing by selecting or encouraging one outcome or answer over others."*

In the following, I will present three biases that are quite well established within behavioral finance.

Overconfidence - People have been shown to be overconfident in their judgments. The confidence intervals people assign to their estimates of quantities are frequently far too narrow. Their 98% confidence intervals, for example, include the true quantity only about 60% of the time. People have also been shown to be poorly calibrated with respect to estimating probabilities: events they believe are certain to occur actually occur only approximately 80% of the time and events they deem impossible occur approximately 20% of the time. According to Odean (1998b) overconfidence leads investors to trade too often and thereby reduce their returns.

Another is that, typically, over 90% of those surveyed think they are above average in such domains as driving skill, ability to get along with people and sense of humor.

DeBondt and Thaler (1995) state, "*perhaps the most robust finding in the psychology of judgment is that people are overconfident.*"

Most people are not as well-calibrated as they should be according to the efficient market hypothesis. They are overconfident and when they are overconfident, people set

[26] www.merriam-webster.com

their confidence bands overly narrow, setting their high guess too low and their low guess to high (Shefrin, 2002). In a study by Werner DeBondt (1993) he finds that people tend to formulate their predictions by naively projecting trends that they perceive in the charts. He also found that people tend to be overconfident of their ability to predict accurately and that their confidence intervals are skewed, meaning that their best guesses do not lie midway between their low and high guesses (Shefrin, 2002, p. 51).

Conservatism - Once people have formed an opinion, they cling to it too tightly and for too long (Lord, Ross & Lepper, 1979). People are reluctant to search for evidence that contradicts their beliefs. Even if they find such evidence, they treat it with excessive skepticism. In the context of academic finance, belief perseverance predicts that if people begin believing in the Efficient Markets Hypothesis they may continue to believe in it long after compelling evidence to the contrary has emerged.

While representativeness leads to an underweighting of base rates, there are situations in which base rates are over-emphasized relative to sample evidence. In an experiment performed by Edwards (1968) there are two urns, one containing 3 blue balls and 7 red ones, and the other containing 7 blue balls and 3 red ones. A random draw of 12 balls with replacement from one of the urns yields 8 red and 4 blue. What is the probability that the draw was made from the first urn? While the correct answer is 0.97, most people estimate a number around 0.7, thus overweighting the base rate of 0.5. It appears that if a data sample is representative of an underlying model, people react too little to the data and rely too much on their prior information.

Sample size neglect or the law of small numbers **-** Sample size neglect originates from the representative heuristic. Research has shown that people assess the same probability distribution to the empirical mean value of small and large samples. The phenomenon is related to the under-use of base rates. By this people expect close to the same probability distribution of types in small groups as they do in large groups. People also exaggerate the likelihood that a short sequence of flips of a fair coin will yield roughly the same number of heads as tails (Rabin, 1998). 1969 Kahneman and Frederick performed a study on 84 participants at meetings of the Mathematical Psychology Society and the American Psychological Association (Kahneman & Tversky, 1971). The respondents were asked realistic questions about the robustness of statistical estimates and the reliability of research results. The survey showed a belief that the law of large numbers applies to small numbers as well. The respondents showed little sensitivity to sample size and therefore placed too much confidence in the results of small samples. Most of the respondents had the capability to easily compute the correct answers, hence they had access to two distinct approaches for answering statistical questions, one spontaneous and fast, and one rule-governed, laborious and slow. These results raised questions about the educability of statistical intuition.

A concept known as "*the gambler's fallacy*" is regarded as a manifestation of the law of small numbers. If a fair coin has not come up tails after 2-3 tosses, people think it is "due" for a tails, because a sequences of flips with a fair coin ought to result in nearly as many tails as heads. The fallacy leads people to over-infer the probability distribution from short sequences (Rabin, 1998).

One more implication of the law of small number is that people expect too few lengthy strikes (series of associated events) in a random sequence. This has been shown in several tests. Most series imagined by subject contains too many short sequences of the same events and hence too few long sequences of the same event. (Falk & Konold, 1997). In basketball there is a widespread belief in the "hot hand" phenomenon. This implies that a particular basketball player has "on" nights, when he or she plays very well, and "off" nights, when he or she plays poorly. It is not believed that these "on" and "off" nights can be explained by randomness. Gilovich, Vallone and Tversky (1985) and Tversy and Gilovich (1989) have argued that this phenomenon does not exist. The "hot hand" idea can be explained by the problems we have in believing in lengthy strikes (Rabin, 1998).

Home Bias – Investors might tend to overweight domestic assets because the domestic stocks and markets feels more familiar and are maybe often are more familiar than the foreign ones Availability or saliency that drives home bias (Massa & Simonov, 2003). People focus heavily on information that is salient or is often mentioned.

Frame Dependence

According to traditional the framing of a problem should not affect the behavior of investors. The framing of financial problems should always be transparent investors. However, researchers within behavioral finance have obtained convincing research results implying that people are, in fact, sensitive to the framing of problems.

The disposition effect – The disposition effect is one of the results of extending prospect theory to investments. It builds on the S-shaped value function of prospect theory. The disposition effect refers to the tendency of investors to hold losers too long and to sell winners too soon. Consider an investor who holds two stocks, one is up and the other is down. If the investor has a liquidity problem, she/he is more likely to sell the stock that is up (Odean, 1998a). Investors are thus disposed toward realizing their gains but not selling their losers. The disposition effect is similar to the overconfidence hypothesis but where overconfidence is market-wide and implies an increase in trading volume; the disposition effect is stock-specific (Statman & Thorley, 2001).

Mental Accounting – It has been shown that individuals and households divide their wealth into mental accounts to organize their financial activities. One example that I believe many people might recognize is the winning of money on a gamble. When money is won on a gamble, it is quite common for people to mentally put this money in a specific account to be spent on further gambling.

Prospect Theory – In the mid-seventies Tversky and Kahneman presented a new theory called *Prospect Theory*. Prospect theory builds on the results from research performed on judgment under uncertainty and on frame dependence. Prospect theory asks questions on how consumer choices are formed by probabilities and related outcomes (Laibson & Zeckhauser, 1998). According to Tversky and Kahneman (1979), prospect theory is to be considered as an alternative model to the expected utility theory. According to Shiller (1998), prospect theory is probably the behavioral theory that has had the most influence on economic research. Rabin (1998) also gives prospect theory the second place, after expected utility theory, as the most frequent subject for research in economics.

It is well known that human behavior systematically deviates from that predicted by expected utility theory (Shiller, 1998). Kahneman and Tversky (1979) demonstrate how people systematically violate the theory:

First, subjects were asked to choose between buying tickets in two lotteries. One lottery offered a 25% chance of winning 3,000 and the other lottery offered a 20% chance of winning 4,000. When choosing between these two lotteries, 65% of the subjects chose the latter. Second, subjects were asked to choose between two other lotteries, offering a 100% chance of winning 3,000 and an 80% chance of winning 4,000. 80% chose the former lottery (loss aversion). According to expected utility theory people should be indifferent to these two pairs of lotteries because the choices are the same except that the probabilities (25% and 20%) are multiplied by the same constant (4) in the second pair of lotteries. This example illustrates what is called "certainty effect", the fact that people have a preference for outcomes, which are certain.

Prospect theory is a mathematical theory that is said to capture the results from experimental outcomes and is to be considered as an alternative to the expected utility maximization. Prospect theory is similar in some ways to expected utility theory. In prospect theory "*individuals are represented as maximizing a weighted sum of 'utilities', although the weights are not the same as probabilities and the 'utilities' are determined by what they call a 'value* *function' rather than a utility function*" (Shiller 1998, p. 4). Shiller also suggests that by substituting the Kahneman and Tversky weights for the probabilities in expected utility theory, a number of puzzling phenomena in observed human behavior in relation to risk might be explained. . Shiller claims that the Kahneman-Tversky value function can explain overpricing of out-of-the-money and in-the-money options. The options smile might be explained in terms of the distortion in probabilities represented by the Kahneman-Tversky value function since prospect theory suggests that people act as if they overestimate the small probability that the price of the underlying crosses the strike price and underestimate the high probability that the price remains on the same side of the strike price (Shiller, 1998).

The shape of the value function differs between prospect theory and expected utility theory. In prospect theory the value function is:

- Defined on losses and gains instead on total wealth
- Concave in the domain of gains and convex in the domain of losses
- Considerably steeper for losses than for gains
- The kink at the reference point (origin)

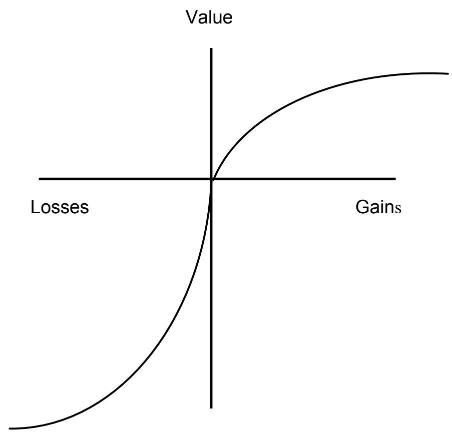

Figure A.3 Utility function suggested in behavioral finance

Loss Aversion – An important concept both within behavioral finance as a field and in prospect theory is *Loss aversion*. Loss aversion is an expression of the unwillingness of many people to bet on a fair coin and is implied by the kink and the difference in the slope of the value function of prospect theory. Research has shown that the attractiveness of winning € X is not nearly sufficient to compensate for the risk of loosing the same amount. Risk aversion has played a central role in economic theory. Loss aversion however implies that the value function is convex in the domains of losses, see figure 4.1, and therefore represents a risk-seeking behavior in the case of loss. Consider a situation in which a person must choose from a sure loss off €800 and an 85 % risk of loosing €1000 . Most people would accept the 85 % risk of loosing €1000 instead of the sure loss. This is a risk-seeking behavior. Risk-seeking behavior has been confirmed by several investigations. "*A person who has not made peace with her/his losses is likely to accept gambles that would be unacceptable to her /him otherwise*" (Kahneman and

Tversky, 1979). Loss aversion accounts for the endowment effect and the status quo bias. The Status quo bias means that individuals tend to remain at the status quo because of the asymmetry in the utility function. The endowment effect means that people are prone to demand more to give up an object than they are to acquire the same object. For a more elaborative explanation of the status quo bias and the endowment effect please see 4.4.2. Regret can also be associated with loss aversion. On making a mistake that could have been avoided, individuals tend to feel regret. Research has shown that the fear of regretting a decision affects the behavior of individuals. Regret is also, in some sense, embodied in the utility function of behavioral finance (see section 4.4.1).