Internal Erosion Phenomena in Embankment Dams

Throughflow and internal erosion mechanisms

FARZAD FERDOS
INTERNAL EROSION PHENOMENA IN EMBANKMENT DAMS

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Dedicated to my beloved family -
Hashem, Zarrin, Fariba and Parisa
ABSTRACT

Internal erosion phenomena occur in granular material when particles within the porous matrix are transported by seeping fluid due to a hydraulic load that exceeds the erosion resistance of the material. Such phenomena pose a particular threat to embankment dams since in most cases they are concealed processes taking place within the dam’s body or its foundation. However, these processes can evolve, cause sink holes and even lead to dam failure. This PhD thesis aims to accomplish a better quantitative understanding of two major internal erosion initiation processes, suffusion and concentrated leak mechanisms, which lead to both defect formation in a dam’s body and its foundation and also high throughflow in dams subjected to internal erosion. This understanding has the potential to facilitate numerical modelling and expedite dam safety assessment studies.

One of the most likely breach mechanisms for large, zoned embankment dams is the unravelling and instability of the downstream slope of the dam initiated by internal erosion and leakage through the core of the dam. Leakage through an erosion tunnel results in extreme flows in the rockfill downstream of the tunnel and it is therefore of crucial importance to determine the flow resistance in such rockfill under these extremely high throughflows. For this purpose, the throughflow properties of coarse rockfill material were studied under flow conditions similar to those prevailing under assumed leakage scenarios in zoned embankment dams by 1) analysing pump test data from Trängslet rockfill dam in Sweden, 2) performing extensive laboratory experiments with a large-scale apparatus and 3) numerically simulating the three-dimensional flow through coarse, cobble-sized to boulder-sized rock materials, replicating the material used in the laboratory experiments by using Flow-3D software.

Results from the field and laboratory tests demonstrate that the parameters of the nonlinear momentum equation of the flow depend on the Reynolds number for pore Reynolds numbers lower than 60,000. The laboratory experiments in this study covered pore Reynolds numbers as large as 220,000 for diameter distributions in the range 100-160 mm and as large as 320,000 for the range 160-240 mm.

Numerical Computational Fluid Dynamics (CFD) studies were also carried out to model the complex flow through the cobbles by overcoming the limits of the Reynolds number in the laboratory experiments (i.e. pore Reynolds number range of 10 to $10^6$). The novelty of the numerical approach lies in the use of results from the large-scale experiments to constrain the three-dimensional numerical simulations with calibration and validation. The validated numerical three-dimensional model was used to conduct numerical experiments which extended the investigation of the parameters of the flow law to Reynolds numbers as large as $10^5$. By applying a Lagrangian particle tracking method, a model for estimating the lengths of the flow channels in the porous media was also developed. Additionally, the shear forces exerted on the coarse particles in the porous media were found to be significantly dependent on the inertial forces of the flow. These shear forces could be estimated using the proposed equation for developed turbulent flows in porous media.

Suffusion and concentrated leak mechanisms were studied for this PhD thesis by means of laboratory experiments to develop a theoretical framework for continuum-based numerical modelling of the evolution of these mechanisms
over time. An erosion apparatus was designed and constructed with the capability of applying simultaneous hydraulic and mechanical loading on the test specimens and providing continuous monitoring with mounted instrumentation. Results from the experiments were then used to develop constitutive laws of the soil erosion rate as a function of the applied hydromechanical load for both suffusion and concentrated leak mechanisms. These constitutive laws can be used to define both the initiation of erosion as well as the mass-removal rate. Both the initiation and mass removal rate of suffusion and concentrated leak mechanisms were found to be dependent on the soil in-situ stresses. To reflect this dependency, a new, modified hydromechanical envelope model was developed based on the Mohr-Coulomb criterion for defining the initiation of internal erosion.

A three-dimensional electrical-resistivity-based tomography method was also adopted for the internal erosion apparatus and was found to be successful in visualising the porosity evolution occurring in the soil due to suffusion in the test soil. These three-dimensional topographical data could also be used to validate numerical models of the suffusion phenomenon.

**Keywords:** Internal erosion modelling in porous material; Constitutive law of erosion for closure of continuum-mechanics-based balance equations for numerical modelling; High Reynolds-number turbulent flow; Flow laws; Coarse rockfill material throughflow; Computational Fluid Dynamics.
Internal erosion phenomena in embankment dams

Preface

This thesis, an original and independent work by the author, Farzad Ferdos, is written in fulfilment of the thesis requirements for the degree of Doctor of Philosophy at the Department of Civil and Architectural Engineering at KTH Royal Institute of Technology, Stockholm, Sweden.

The research presented in this thesis was carried out as a part of a PhD programme financed by the Swedish Hydropower Centre (Svenskt Vattenkraft Centrum, SVC, http://www.svc.nu). SVC has been established by the Swedish Energy Agency, Elforsk and Svenska Kraftnät together with Luleå University of Technology, KTH Royal Institute of Technology, Chalmers University of Technology and Uppsala University. Participating companies and industry associations are: Alstom Hydro Sweden, Andritz Hydro, E.ON Vattenkraft Sverige, Falu Energi & Vatten, Fortum Generation, Holmen Energi, Jämtkraft, Jönköping Energi, Karlstads Energi, Mälarenergi, Norconsult, Skellefteå Kraft, Sollefteåforsens, Statkraft Sverige, Sweco Energuide, Sweco Infrastructure, SveMin, Umeå Energi, Vattenfall Research and Development, Vattenfall Vattenkraft, Voith Hydro, WSP Sverige and AF Industry.
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Farzad Ferdos

September 2016, Stockholm, Sweden
LIST OF APPENDED PAPERS

The thesis is based on the following papers:


VII. Masi, M., Ferdos, F., Solari, L., Losito, G. Detecting internal erosion processes in porous media by an Arduino-based electrical impedance tomography system developed on laboratory scale (Draft manuscript).

Notes on the contribution of the author for each paper:

I. The author was responsible for the scientific planning, experimental design, conducting the experiments, theoretical development, and evaluation. The author was responsible for most of the writing.

II. The author was responsible for the experimental design and conducting the experiments (except for the field work), scientific planning, theoretical development, and evaluation. The author was responsible for most of the writing.

III. The author was responsible for the scientific planning, numerical model development, modelling, and evaluation of the results. The author was responsible for most of the writing.

IV. The author was responsible for the scientific planning, numerical model development, modelling, evaluation of the results and the theoretical development. The author was responsible for most of the writing.
V. The author was responsible for the experimental design and conducting the experiments, scientific planning, evaluation of the results and theoretical development. The author was responsible for most of the writing.

VI. The author was responsible for the experimental design and conducting the experiments, scientific planning, evaluation of the results and theoretical development. The author was responsible for most of the writing.

VII. The author was responsible for some of the experimental design, conducting the experiments, some of the scientific planning and theoretical development, some of the evaluation and some of the writing.
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CHAPTER 1

INTRODUCTION: AIMS AND SCOPE OF THE THESIS

1.1 Hydropower and dams

Hydropower is a renewable energy source whereby electricity is produced from running water. Hydropower accounts for about 50% of electricity production in Sweden (Svensk energi, 2014) and 16% of electricity production in the world (IEA, 2015). The most common and efficient way to facilitate hydropower is to build dams and create elevated water reservoirs that allow for a localisation of the energy abstraction and a reservoir volume for the regulation of the available water.

Dams can be classified as “single-purpose” or “multipurpose”. Reservoirs created by dams may not only be used for hydropower, but also provide water for other major activities such as flood suppression, water storage, irrigation, human consumption, industrial use, aquaculture, and navigability. However, regardless of their purpose(s), the design and construction of these dams must ensure that they are safe and stable for years to come.

Dams are hydraulic structures built across a river to create a reservoir on its upstream side for impounding water. They are often divided into two major groups based on construction material: embankment dams and concrete dams. Both groups can then be classified more elaborately according to how the dams achieve their strength and stability. The type and the shape of a dam for a new project are carefully decided, taking into account the dam’s safety and suitability for the site and the aims of the project.

1.2 Embankment dams

Embankment dams have been used for at least 5,000 years, as evidenced by archeological finds in Babylonia, Persia, Egypt, India, and the Far East. Some of these structures were of considerable size, such as the triple gravity dam built by the Achaemenians on the river Kur south of Persepolis in 600 B.C (Jansen, 1980). Today, as in the past, these dams are the most common type of dams, principally because their construction involves using natural materials which are obtained from near the dam site.

Embankment dams are made by placing and compacting mounds of compositions of natural soil materials such as clay, silt, sand gravel, boulders and rocks. The two main types of embankment dams are rockfill and earthfill dams. Earthfill dams consist mainly of compacted earth material, while rockfill dams consist mainly of crushed and compacted rockfill. The materials used in their construction are usually excavated or quarried from nearby sites, preferably within the reservoir basin.
1.3 **Internal erosion phenomena in embankment dams**

Internal erosion is a major cause of failure in embankment dams. Indeed, statistics on the failure of large dams constructed between 1800 and 1986 show that 94% of failures were related to erosion, and that internal erosion was responsible for approximately 48% of embankment dam failures (Foster et al. (1998, 2000a); ICOLD (1974, 1983, 1995)). Compared to embankment slides and failures due to earthquakes, which account for only 4% and 1.7% respectively, internal erosion is a much greater threat to existing embankment dams. This susceptibility to failure by internal erosion originates from the nature of internal erosion processes, which can evolve under case-specific conditions and occur concealed inside the dam’s body or its foundation. This makes the erosion process hidden until it has either progressed enough to be visible as sink holes on the surface of the structure or detected by dam monitoring instruments.

Internal erosion, being the most common cause of failure in embankment dams, can occur due to the high reservoir water level. The hydraulic load, directly related to the water head difference between the upstream and downstream of the dam, works against the resistance of the materials of the dam’s structure. The reservoir water level increases during higher river inflows in comparison to the outflows, and this can arise due to normal management, a structural malfunctioning, a flooding event, storm surges or high tides in the sea on the downstream side. Internal erosion may also occur as a result of structural defects such as cracks and other unstable zones formed during construction, an earthquake or even during a dam’s normal operating condition. Although these processes can occur in all natural soils, dams that are built of widely-graded glacial materials, such as moraines, have been found to be more susceptible to internal erosion than dams of other soil types. It is noteworthy that the cores of embankment dams in Sweden are typically built of moraines (Vattenfall, 1998).

The need for the accurate and timely prediction of the initial stages of internal erosion is great, but it cannot be achieved only by adopting empirical rules, since each dam is built according to a unique design suiting its site-specific conditions. The evolution of internal erosion from its initiation to complete dam failure is also of significant importance. To be able to design mitigation and intervention action plans and decide on warning and evacuation procedures to minimize the consequences, it is crucial to know how much time it takes for a scar to develop on the dam structure that might result in a breach.

The need is evident for a greater understanding of these structural scars caused by internal erosion processes and their subsequent failure mechanisms. This understanding should be based not only on empirical inquiry and quantitative assessments but also on an evaluation of the relevant physics and physical processes.
1.3.1 General categories of internal erosion and their mechanisms

The evolution process of internal erosion in granular material has been broadly broken down into four phases in the literature: initiation, continuation, progression, and breach (ICOLD, 2013).

When the initiation of the erosion process is considered, this process itself is grouped into four distinctive mechanisms: concentrated leak, backward erosion, contact erosion, and suffusion (Bonelli 2013).

The Concentrated leak

The concentrated leak mechanism may happen when certain local defects are present within the porous material. These defects can be cracks caused by frost, differential settlement during the construction or the operation of the dam, hydraulic fractures caused by low stresses around conduits, or through desiccation at high levels in the fill. Concentrated leaks may also occur because of the settlement of poorly compacted fill material adjacent to other structural elements such as conduits, walls and spillways. Uncontrolled leaks can even be initiated by rotten tree roots and animal burrows dug into the embankments. However, for this mechanism to progress to form a breach, the material needs to hold open the erosion scar throughout the process (i.e. be able to “hold a roof”). For this purpose, certain granulometric and compaction characteristics for the soil as well as a certain proportionality of fine grains are essential.

Backward erosion

A backward erosion mechanism can be initiated in two different ways: backward erosion piping and global backward erosion. Backward erosion piping occurs where the erosion process begins on a free surface on the downstream zone of an embankment dam due to critically high hydraulic gradients. At this region of the dam, the erosion scar develops backwards downstream of the dam through small erosion channels, and water transports the eroded particles and the process progresses into the dam’s body or its foundation.

Global backward erosion occurs in dams with a sloping core, where the core is not efficiently protected by filters or a transition zone, thereby enabling the detachment and transport of soil grains from the downstream surface of the core. This type of internal erosion can arise in even less cohesive soils since gravity is in favour of the process and the formation of the pipe. Global backward erosion is known to be one of the major causes of sinkhole formation in dams constructed of glacial tills. Such material is typical in Swedish embankment dam cores, where 9 out of 40 dams (40%) with moraine core material are documented with internal erosion (Nilsson, 1999).
Contact erosion

Contact erosion occurs when a coarse soil such as gravel comes into contact with a finer soil, and the fluid flow transports the finer soil grains through the coarser matrix of the coarser soil. This type of erosion is triggered by any of these three conditions: by an unsatisfactory drain and filter design for zoned embankment dams, by embankment dams being built directly on gravel alluvium foundations or by inhomogeneous zones being formed by the segregation of material during the construction (ICOLD, 2013).

Suffusion

The suffusion mechanism occurs within a mixture of soil when fine aggregates are transported by the flow through the pores that are supported by the coarse aggregates. The coarser particles are not transported. This mechanism commences more commonly in internally unstable soil mixes where the material gradation is widely graded or gap graded (see ASTM D6913 (2009)) for the soil classification). Cohesion of the soil prevents this mechanism to a high degree; therefore, suffusion occurs mostly in cohesionless soils such as poorly-graded materials for filter and drain layers and cores constructed from glacial tills (Bonelli, 2013). This poor gradation may also occur due to the segregation of non-plastic materials during the placement of various layers in the dam construction. This segregation could create areas that are prone to developing suffusion even though the design grading of the soil mixes has been internally stable.

In suffusion, soil stresses carried by the coarse grains of the soil matrix are higher than the finer fraction of the soil (i.e. the stress reduction concept (Skempton and Brogan, 1994)), enabling the finer grains to be transported more easily through the porous structure with less noticeable soil deformation. The suffusion process causes a porosity growth in the soil, which leads to an increase in permeability, seepage velocities, and higher hydraulic gradients.

1.3.2 Embankment dam breach

After internal erosion is initiated in a dam body or its foundation, weakening of the material starts and, if the erosion progresses and the material transport is not hindered locally by filtering or clogging processes, scars caused by internal erosion can evolve and cause sink holes or even lead to dam breach.

Embarkment dam breaches can be classified into five mechanisms: gross enlargement of the erosion scar; overtopping due to either settlement of the crest or minor slope instabilities in the upstream slope; instability of the downstream slope; unravelling of the downstream slope; and static liquefaction, an example of sudden destabilisation (Fell and Fry 2007).

A conceptual model of an embankment dam failure from initiation to breach is shown in Figure 1. Similar failure path diagrams for failure through the embankment dam’s body and foundation can
be found in Fell et al. (2005) or Foster (1999). A more detailed description of internal erosion phenomena and a thorough literature survey is given in Mattsson and Hellström (2008).

Figure 1. A failure path diagram through an embankment dam.

Despite the idiosyncratic nature of internal erosion processes, methodological and statistical investigations of dam failure confirm that unravelling and instability of the downstream slope due to a sudden release of water is the most common breach mechanism for large zoned rockfill dams (Fell and Fry 2007; Fell et al. 2008). Release of sudden flow and severe leakage into the downstream zone of the embankment dam can cause turbulent throughflows which can lead to downstream slope instabilities of the shoulder and result in a dam breach.

1.4 Thesis aim and scope

Internal erosion phenomena occur in porous material when soil particles within the porous matrix are transported by seeping fluid due to a hydraulic load that exceeds the erosion resistance of the material. These phenomena imply a particular threat to embankment dams.

In order to design dams with measures against erosion processes, to assess dam safety, and to develop effective emergency action plans preventing casualties and damages, it is necessary to develop relevant engineering models to analyse internal erosion processes and to see how they develop in porous material. Furthermore, numerical modelling of both throughflow and the destabilisation of downstream slopes when faced with a sudden leakage is of crucial importance.

This PhD thesis aims to accomplish a better quantitative understanding of the processes that lead to internal erosion formation with a specific focus on suffusion and concentrated leak phenomena that can lead to high throughflows in embankment
dams. This research relies on laboratory experiments, numerical modelling and theoretical studies to present new constitutive laws to facilitate continuum-based numerical modelling and expedite dam safety assessment studies.

1.5 Structure of the thesis

The study presented in this thesis is the result of a research project financed by the Swedish Hydropower Centre (SVC) and it has been conducted in two phases. Phase I, the major part of the project, includes a thorough study of the hydraulic behaviour of coarse rockfill material subjected to heavy and turbulent throughflow conditions. For this purpose, the throughflow properties of coarse rockfill material were studied under extreme flow conditions similar to those prevailing under assumed leakage scenarios in zoned embankment dams by:

- analysing in-situ pump test data from Trängslet rockfill dam in Sweden
- performing extensive laboratory experiments in a large-scale permeameter apparatus, and
- numerically simulating the three-dimensional flow through coarse materials and replicating the ones used for the laboratory experiments by using Flow-3D software.

To this end, the interactions between the rockfill material and the flowing fluid were studied (Paper I), momentum equations of the flow were defined (Paper II), results were used to calibrate 3D numerical models and to conduct validation studies (Paper III), and the validated numerical model was used to overcome the flow and monitoring limitations (e.g. high and low discharge limitations and 3D data of velocity and exerted forces in the medium) present in the laboratory experiments (Paper IV). This work was done with the aim of providing a solid basis for numerical modelling whereby turbulent throughflows and the destabilisation of the downstream slopes can be modelled when a sudden and heavy leakage occurs due to internal erosion.

Phase II of this PhD project addresses two major mechanisms of internal erosion, “suffusion” and “the concentrated leak” by means of conducting laboratory experiments and developing a theoretical framework for continuum-based numerical modelling. An erosion apparatus was designed and constructed with the capability of applying simultaneous hydraulic and mechanical loading on the test specimens. The results from these experiments were then used to develop constitutive laws for the erosion rate of soil material as a function of the applied hydromechanical load for suffusion (Paper V) and the concentrated leak (Paper VI). A miniature resistivity probe system was designed to produce 3D electrical resistivity profiles of soil material, so that changes in the porous matrix could be visualised and quantified (Paper VII).
The rest of the thesis is structured as follows: Chapter 2 provides a theoretical background to flow in porous media and a summary of previous studies on this particular subject. Chapter 3 provides a theoretical background to internal erosion in porous media and a similar summary of previous research. Chapter 4 presents the aim, methodology and the findings of the studies conducted in phase I of this project, which is based on the theoretical framework of flow in porous media presented in Chapter 2. Chapter 5 presents the aim, methodology, and the findings of the studies conducted in phase II of this project, which is based on the theoretical framework of internal erosion in porous media presented in Chapter 3. Chapter 6 provides a summary of the main findings from both phases of the project.
CHAPTER 2

THEORETICAL BACKGROUND: FLOW IN POROUS MEDIA

2.1 General theory

Flow through porous media is the main mechanism of many natural processes in engineering applications, such as ground water flow, flow through earth structures (i.e. embankment dams, dykes and levees) and any process that involves filtering.

The dynamics of fluids in porous media is dominated by the porous matrix structure pervades the entire volume and that leads to an efficient dissipation of the fluid’s kinetic energy. In continuum physics, the motion of fluids can be described by the Navier-Stokes (N-S) equations, which for an incompressible fluid read as:

$$\rho \left[ \frac{\partial (\bar{V})}{\partial t} + (\bar{V} \cdot \nabla) \bar{V} \right] = -\nabla p + \rho \bar{g} + \mu \nabla^2 \bar{V}$$

(1)

where $\bar{V}$ is the velocity vector of the fluid, $p$ is the pressure, $\rho$ is the fluid’s density, $\bar{g}$ the acceleration vector due to gravity and $\mu$ is fluid dynamic viscosity.

The Navier-Stokes equations can be adopted for a porous medium by applying the volume-averaging method to the conservation equations for a representative elementary volume (REV) of the porous medium (Figure 2), similar to Whitaker’s approach (Whitaker 1996). This results in:

$$\rho \left[ \frac{\partial \langle \bar{V} \rangle}{\partial t} + \langle \bar{V} \rangle \nabla \langle \bar{V} \rangle + \varphi^{-1} \nabla \langle \bar{V} \bar{V} \rangle \right] =$$

$$-\nabla \langle p \rangle + \rho \bar{g} + \mu \nabla^2 \langle \bar{V} \rangle + \frac{1}{V} \int \left[ -\bar{p} + \mu \nabla \bar{V} \right] dA$$

(2)

where $\langle \bar{V} \rangle$ is spatially averaged intrinsic (real) velocity, $\bar{V}$ is the spatial deviation of flow velocity (intrinsic velocity $\bar{V}$ minus superficial velocity, $\bar{U}$), $\varphi$ is the active porosity of the media, $\bar{n}$ is a unit vector directed from the fluid phase to the solid phase, $\bar{p}$ is the spatial deviation of pressure, $\bar{g}$ is gravitational acceleration, $\mathcal{V}$ is the volume of REV, $A$ is the contact area between the fluid phase and solid phase within the REV, and $I$ is the unit tensor.

A number of similar adaptations to the Navier-Stokes equations have been proposed (e.g. de Lemos and Pedras, 2001) which apply a range of averaging procedures.

These averaging adaptations, however, are complicated both by the intricacy of the porous matrix and the difficulty of identifying the precise boundary conditions required for solving the governing...
Consequently, the momentum equation is often simplified by disregarding some terms and/or replacing them with empirical relations to account for the energy losses of the fluid within porous media.

2.2 Flow laws for porous media

Darcy’s law (Eq. 3) is the most commonly used momentum equation for a fluid flow (flow law) through a porous medium. This law disregards all the inertia terms on the left side of Eq. 2 and adopts a linear form of induced force in the porous medium per unit medium volume. In doing so, the flow through a porous medium is defined as a linear relation between the water head loss and the mean fluid velocity, proportioned by hydraulic conductivity as the medium’s property:

\[ \bar{U} = \rho g \frac{K}{\mu} \nabla (z + p/\rho g) \quad (3) \]

where \( K \) is the intrinsic permeability tensor, \( z \) is the (vertical position) elevation and \( \nabla (z + p/\rho g) \) is the gradient of hydraulic head where \( (z + p/\rho g) \) is the hydraulic head \( H \).

Darcy’s flow law is useful for modelling sub-surface, saturated flow cases for which the inertia terms can be neglected and a linear friction force can be assigned (i.e. laminar flows), such as moderate throughflow in embankment dams.

The upper limit of validity of Darcy’s flow law, in terms of the limit of the throughflow in porous materials, has been established by a number of experimental studies. These limits are defined based on the pore Reynolds number of the throughflow \( (Re = \frac{Vd}{\nu}, \text{where } V \text{ is the intrinsic velocity magnitude, } \nu \text{ is fluid kinematic viscosity and } d \text{ is the characteristic/representative diameter of the porous material}) \). These limits, which are slightly different for each study, suggest that the friction force in coarse porous materials is seldom governed by laminar flow. In fact, wide
gaps between grains in coarse media can allow the flow to reach velocities that cause turbulent bursts, whose representation as pore Reynolds numbers exceeds the aforementioned limits (Hansen et al. 2012). Subsequently, most seepage cases in rockfill dams cannot be precisely described by Darcy’s flow law because the structure includes coarse material.

Phillip Forchheimer (1901) suggested the following equation, which adds an inertial term representing the kinetic energy of the fluid to the flow law:

\[ \nabla (z + p / \rho g) = aU + \beta U |U| \]

(4)

In Eq. 4, \( |U| \) is the velocity magnitude and parameters \( a \) and \( \beta \) are the Forchheimer coefficients with units \([\text{sm}^{-1}]\) and \([\text{s}^2\text{m}^{-2}]\) respectively.

A second-order term is added to represent turbulent forces. This offsets the disregarded divergence of momentum on the left side of Eq. 2 by introducing a representative inertia force. This empirical adaptation of the Navier-Stokes equations done by Forchheimer was later confirmed by Ahmed and Sunada (1969) based on Ward’s dimensional analysis method (1964).

In a similar vein, Wilkins (1955) employed a power law to express non-Darcy flow, given in the form of:

\[ \nabla (z + p / \rho g) = \dot{i} = b \bar{V} |\bar{V}|^{(a-1)} \]

(5)

where \( \bar{V} \) is the pore velocity and is defined as \( \bar{V} = \dot{U} / \phi \), \( b \) is a coefficient depending on the fluid and porous medium properties and \( a \) is an energy dissipation index.

Similar forms to this power law were also suggested by Missbach (1967) and Scheidegger (1963).

Equations 4 and 5 denote the effect of convective acceleration, viscous stress and the terms that are the result of the phase averaging, known as the Brinkman effect (Whitaker 1996). However, as not all the actual terms are embodied in this method, the development of specific empirical relations is necessary to solve the governing system of equations. Consequently, both the coefficients for a quadratic flow law (e.g. Forchheimer’s equation) and the manner to obtain the parameters for power-function flow laws (e.g. Wilkins’ equation) must include experiments and curve-fitting by regression analysis. The experiments must involve the measurements of head losses and the flow rates to define the flow laws (Teng and Zhao 2000).

### 2.3 Forces on soil particles in porous media

The force exerted on soil grains, inside this densely packed column of particles in the porous media, is of crucial importance in order to evaluate the soil stresses in earth structures and examine the stability of embankments.
Saturated soils are porous media that can be considered as a collection of densely packed submerged objects. Within this framework, the point of departure for the examination of exerted forces is the Stokes equation, which provides an equation for the viscous force on a spherical particle subjected to a creeping (viscous) motion:

\[ F = 6\pi\mu RU_0 \]  \hspace{1cm} (6)

where \( F \) is the induced force on the particle, \( R \) is the particle radius, and \( U_0 \) is the undisturbed fluid velocity in a uniform flow field at a distance far from the particle.

Einstein (1905), Burgers (Van Wijngaarden, 1996), and Brinkman (1947) adapted Eq. 6 to account for particle interactions. Einstein and Burgers accomplished this interaction by solving a boundary value problem for an incompressible fluid containing a number of identical, rigid particles, demonstrating the reciprocated influence of each particle’s flow field on the other particles. However, Brinkman’s model considers a single embedded particle in a continuous porous medium. Equation 7 gives Burgers’ solution for one particle by disregarding the wall effects and supposing that the mean diameter of the particles is smaller than the diameter of the column of porous medium:

\[ F = 6\pi\mu_{\text{eff}} RU \left[ 1 + \left( \lambda_\gamma + \lambda_\eta \right) S / \forall \right] \]  \hspace{1cm} (7)

where \( \mu_{\text{eff}} \) is the effective dynamic viscosity, \( S \) is the particle volume, \( \forall \) is the volume of the porous media, and \( \lambda_\gamma \) and \( \lambda_\eta \) are material parameters that are dependent on the arrangement of particles.

Depending on various statistical positioning probabilities (Bishop, 1996), these parameters can take different values. For a statistically uniform packed column with equal particle position probability in the \( x \)-, \( y \)-, and \( z \)-dimensions, \( \lambda_\gamma = 15/8 \) and \( \lambda_\eta = 5 \) can be assigned (Van Wijngaarden, 1996). \( \mu_{\text{eff}} \) can be calculated based on Eq. 8 from Einstein (1905):

\[ \mu_{\text{eff}} = \mu \left[ 1 + \xi \left( 1 - \varphi \right) \right] \]  \hspace{1cm} (8)

where \( \xi \) is a coefficient reflecting the momentum transport contribution of the particles in the fluid flow. For a dilute suspension, \( \xi \) is equal to 2.5 according to Einstein (1905).

Brinkman (1947) calculated the viscous forces on a single particle by assuming it was isolated from the surrounding particles. The influence of the other particles was represented by a continuous porous medium. Moreover, he addressed the interactions between the particle and the porous media by applying appropriate boundary conditions, yielding Eq. 9:

\[ F = 6\pi\mu_{\text{eff}} RU \left[ 1 + \left[ \gamma R + \left( \gamma^2 R^2 / 3 \right) \right] \right] \]  \hspace{1cm} (9)
where $K$, denoted earlier, is the intrinsic permeability, which is a geometrical property of the porous media and can be calculated using the Kozney-Carman equation (Carman, 1997) and $\gamma$ is calculated using Eq. 10.

$$\gamma = \sqrt{\frac{\mu}{\mu_{\text{eff}}} K}$$  \hspace{1cm} (10)
CHAPTER 3

THEORETICAL BACKGROUND: INTERNAL EROSION IN POROUS MEDIA

3.1 Background

Internal erosion processes can begin in numerous and different ways depending on material type, structural properties and even operational activities. However, when internal erosion processes are analysed with a focus on the physics, they can be expressed collectively if a fundamental understanding exists of what a porous medium is and how seepage occurs.

All types of soil and aggregate mixtures, which come under the common name of porous media, are materials that are made up of relatively large numbers of discrete but adjoining grain particles. These particles have interactions and, due to their edges, arrangement, size and shape variation, pore space with interconnected pathways is left between grains which makes it possible for fluids to pass through the medium. Internal erosion is initiated when some of the particles of the porous medium located within the flow path start to sway, become detached and move with the flow. These particles, passing through the pore channel system and leaving their position, trigger a flow of particles, which are removed and not replaced and, therefore create a bigger opening for the fluid and more grains to flow through, resulting in the continuation and propagation of the erosion inside the medium. This internal erosion process occurs in a contact-dominated multiphase flow scheme.

Analysing the physics of such phenomena and their relation to many other similar engineering and mechanical processes in porous media requires either a continuum framework or a discrete framework.

3.1.1 Discrete mechanics and its adaptability to model a porous medium

Discrete mechanics and discrete element methods (DEM) are a family of numerical methods to compute and analyse the motion, interactions and effect of a large number of particles.

Within this framework and with the help of both recently advanced computational capacity and numerical algorithms for behaviour capturing and nearest-neighbour sorting, it has become possible to numerically simulate the mechanics of the motion of thousands of individual particles and their interactions simultaneously. This ability represents a great stride forward since the first discrete element simulations were conducted more than twenty years ago (e.g. Cundall and Strack, 1979, 1982; Moreau, 1988).

DEM is becoming widely accepted as an effective method for analysing engineering problems in granular and discontinuous
materials, especially in granular flows, powder mechanics, and its original subject, soil and rock mechanics (Munjiza, 2004).

This method was implemented by Helström (2009) to model internal erosion phenomena with CFD studies combined with the discrete modelling of the mechanics of the motion of soil particles with a grain-size scale, i.e. “micro-scale” approach. The limitation of this approach, however, is that the predictions of behaviour are substantially dependent on how the particle interacting forces are implemented. More importantly, the major drawback is the limited assembly population that can be built. This framework, exhibiting as it does the potential to be used successfully both as an investigation tool of the microscopic mechanics and a support for the development of continuum models (Helström 2009), raises the question of whether the behaviour of diverse and extremely populated assemblies involved in embankment dams and many other civil engineering applications can ever realistically be simulated by the DEM (see Figure 3).

3.1.2 Theory of porous media - a continuum perspective

The theory of porous media (TPM) is used in branches of engineering dealing with fluid flow in soil, fractured rock or other types of porous media where problems cannot be neatly classified as solid mechanics or fluid mechanics. This theory, which began as the classical mixture theory, is a special characterisation with a unified treatment of volumetrically coupled and averaged solid-fluid aggregates. TPM was first introduced by Truesdell (1957) and extended with volume fractions as described by Bowen (1980 and 1982), de Boer (2000), and Ehlers and Bluhm (2002). It represents an extension of the classical single-phase continuum mechanics for a multiphase modelling approach, in which a wide selection of models for granular materials has been developed, mostly in the context of Noll’s simple material, defined in Noll (1958).

The continuum-based theory of porous media creates the opportunity to achieve a thermodynamically consistent modelling framework, where all the conventional conservation equations can
be written and mathematically solved. In this framework, a mean characteristic of attributes of material within the REV is taken, whereby individual characteristics of members are replaced by that representative average value instead of considering the individual behaviour of members within this volume. An example of this is to use average velocity, average porosity, and average strength instead of each and every particle’s particular status (Wolfgang and Bluhm 2002)).

3.2 Continuum-based internal erosion process

In a continuum-mechanics-based modelling framework, the erosion phenomena can be accounted for by modelling the mass exchange between the continuous phases of pore liquid and solid skeleton. For each continuum, balance equations can be written and the mass exchange interaction between the phases, such as the driving force of the erosion, can be included. Therefore, as internal erosion continues, the continuous portion of the solid phase is reduced in mass and volume over time and the fluid-phase continuum increases in volume by filling the created space (i.e. the material becomes increasingly porous). This continuous approach can be described as “macro-scale” modelling. For models with a macro-perspective of a system, the major focus is on the overall output and behaviour of that system instead of the behaviour of individuals within it. An analogy for this concept is the image on a television screen or a monitor, which is achieved not by focusing on each and every pixel and analysing its colour status and variation at each moment, but rather by the combined assembled outcome as illustrated in Figure 4.

![Figure 4. A framework for continuum-based modeling. The dam structure is divided into elements and within each element the physics are continuous, the media are considered homogeneous and the properties are averaged.](image)

Vardoulakis et al. (1996) were the first to suggest a continuum-based theoretical framework for the hydromechanical aspect of the erosion phenomena for sand production, which is a borehole wall erosion phenomenon in hydrocarbon reservoir sandstones during fluid extraction. They introduced a three-component mixture theory for a REV consisting of a solid skeleton, fluidized particles and the fluid. In their model, the porous medium was defined as consisting of three continuous compartments for the REV of solid
grains with volume $V_s$, fluid with volume $V_f$, and eroded fluidized grains with volume $V_e$. The fluid and eroded particles fill the void space $V_p$ so that $V_p = V_f + V_e$. Based on this conceptual model, the equation below is a definition of the active porosity of the medium, $\phi$, the concentration of eroded and fluidized particles in the fluid phase, $c$, the partial densities of the solid, fluid and eroded, and suspended compartments, $\rho[i], i=1,2,3$ respectively and the mean density, $\rho$: \\
\[
\phi = \frac{V_p}{V} = \frac{V_f + V_e}{V} \\
c = \frac{V_p}{V_f + V_e} = \frac{V_f}{V_f + V_e} \\
\rho[i] = \frac{m_i}{V} = \rho_s \frac{V_f - V_e}{V} = \rho_s (1 - \phi) \\
\rho[2] = \frac{m_f}{V} = \rho_f \frac{\phi V_f - V_e}{V} = \rho_f \phi (1 - c) \\
\rho[3] = \frac{m_e}{V} = \rho_e \frac{c \phi V_e}{V} = \rho_e \phi c \\
\rho = \frac{m}{V} = \frac{m_s + m_f + m_e}{V} = \rho[1] + \rho[2] + \rho[3] = \rho_s (1 - \phi + c \phi) + \rho_f \phi (1 - c)
\]

In Eqs. 11-16, $m_s$ and $\rho_s$ are mass and density for the solid constituent, $m_f$, $\rho_f$ is the fluid constituent and $m_e$ is the mass of the eroded solids.

Mass balance for the solid compartment can then be written within the REV on the basis of the partial densities and the velocities of the constituents as:

\[
\frac{\partial \rho[i]}{\partial t} + \nabla \cdot (\rho[i] \vec{v}) = \dot{m}[i] \\
\frac{\partial \rho[2]}{\partial t} + \nabla \cdot (\rho[2] \vec{v}) = 0 \\
\frac{\partial \rho[3]}{\partial t} + \nabla \cdot (\rho[3] \vec{v}) = -\dot{m}
\]

In Eqs. 17-19, $\vec{v}$ is the velocity vector of each phase and the term $\dot{m}$ on the right-hand side of Eqs. 17 and 19 is the mass exchange rate per volume per time (mass removal rate).

For momentum and force balances, Vardoulakis et al. (1996) used a simplified equation by neglecting the momentum contribution from the solid phase and considering a linear equation for the fluid phase (Darcy’s law) as:
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\[ \varphi (p^{[2]} - p^{[l]}) = -\frac{K}{\mu} p \]  

(20)

where \( p \) is the pore water pressure, \( K \) is the intrinsic permeability of the medium and \( \mu \) is the viscosity of the fluid phase.

Incorporating Eq. 20 with the mass balance equation, the constitutive laws for the mass removal rate and the permeability need to be defined in order to close the equation system (Eq. 17-20). Vardoulakis et al. (1996) have adopted the Kozney-Carman equation for the medium’s permeability (Carman, 1997) which is defined as:

\[ K = k_0 \frac{\varphi^3}{(1 - \varphi)^2} \]  

(21)

In Eq. 21, \( k_0 \) is the permeability parameter independent of the porosity of the medium.

For the mass removal rate, the Einstein-Sakthivadivel erosion model (Einstein, 1937 and Sakthivadivel, 1966) was adopted by Vardoulakis et al. (1996), which is defined as:

\[ \dot{m} = \omega \varphi \left( 1 - \varphi \right) c \varphi \left| v^{[3]} - v^{[l]} \right| \]  

(22)

In Eq. 22, \( \omega \) is an experimental and empirical coefficient.

This continuum-based internal erosion model framework was extended for other types of internal erosion in porous media in more recent work by Vardoulakis et al. (2001), Papumichos et al. (2001), Papumichos and Vardoulakis (2005), Steeb and Diebels (2003), Xu et al. (2006), Wan and Wang (2002a,b), Brivois et al. (2007), Bonelli and Marrot (2008), Steeb et al. (2005b and 2012) and Golay and Bonelli (2011).

### 3.2.1 Suffusion mechanism

The three-compartment model was originally adopted by Steeb et al. (2003) to describe suffusion processes and has since been extended by Steeb et al. (2004, 2005a,c and 2007) and Steeb and Scheuermann (2012) to account for a fully coupled, thermodynamically consistent approach based on the TPM.

This model accounts for all the mass and momentum exchanges between the compartments and considers both the mechanical behaviour of the solid phase and an extra volumetric compartment in the REV for solids. This is important, because it takes into account the immobile soil particles and their granulometric properties. Adopting these adaptations, the mass conservation equations are slightly modified compared to Vardoulakis et al. (1996) and this accounts for the immobile portion of the solid compartment as:

\[ \frac{\partial \varphi}{\partial t} + \nabla \cdot \left( \bar{v}^{[l]} \right) - \nabla \cdot \left( \varphi v^{[l]} \right) = \dot{m} \]  

(23)
In Eqs. 23-26, all the variables are similar to the previous model and the new added variable for volume, $n$, accounts for the portion of the volume of the erodible fine material in the solid constituent. The momentum conservation, however, has been modified substantially to account for both the momentum exchange that occurs between constituents and the mechanical behaviour of the solid phase. If a linear strain model for the deformation is adopted and the balance of momentum of the mixture is considered valid for the REV (Steeb and Diebels 2003; Steeb et al. 2005b and 2007), the resulting equation will be:

$$-\nabla(\sigma) = \frac{\rho}{\varphi} \dot{m} \left[ \bar{v}^{[2]} - \bar{v}^{[1]} \right]$$

$$\sigma = \sum_i \sigma^{[i]}$$

In Eqs. 27 and 28, $\sigma^{[i]}$ is the stress tensor of each compartment and $\sigma$ accounts for the summation of the stress tensor components in the REV (i.e. the concept of effective stress by summing up the soil stresses and the pore water pressure and accounting for shear stresses).

Including the deformation of the solid compartments and accounting for soil stress requires an extra constitutive law for the closure. This constitutive equation, which accounts for the stress-strain condition of the solid skeleton, needs to be considered for the equation system (Eqs. 23-28).

Steeb and Diebels (2013) defined their constitutive law of erosion rate as:

$$\dot{m} = \bar{\omega} (1 - \varphi) c \left( \bar{v}^{[3]} - \bar{v}^{[1]} \right) \cdot \left( \bar{v}^{[3]} - \bar{v}^{[1]} \right)$$

where $\bar{\omega}$ is similar to $\omega$ is an experimental and empirical coefficient.

However, Eq. 29 has a nonlinear term of seepage velocity (i.e. $[ \bar{v}^{[3]} - \bar{v}^{[1]} ]$) compared to the Einstein-Sakthivadivel erosion model given in Eq. 22.

### 3.2.2 Concentrated leak mechanism

For concentrated leak erosion, Bonelli et al. (2006) presented a model in a straight cylindrical hole with length $l$ and initial radius
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\( r_0 \) within a homogeneous and saturated porous medium, following the continuum-based fluid-soil interface erosion framework introduced by Vardoulakis et al (1996). In this conceptualisation, the solid compartment is eroded by the flow and is carried away. The only difference between this model and the suffusion model is the adaptation of the hole’s diameter growth instead of the porosity change for the model when forming the balance equations. The enlargement of the erosion hole due to this phenomenon can be described by the following mass and momentum balance equations:

\[
\rho' \frac{\partial r}{\partial t} = \dot{m}_{HET} \tag{30}
\]

\[
\rho'[2] \frac{r}{2} \frac{\partial \rho'[2]}{\partial t} + \nu'[2] \dot{m}_{HET} = \frac{r}{r_0} p - \tau_b \tag{31}
\]

In Eqs. 30 and 31, \( r \) is the radius of the hole, \( p \) is the driving pressure which is equal to \( r_0 (p_{in} - p_{out} / 2L) \) in which \( p_{in} \) and \( p_{out} \) are the respective fluid pressure at the inlet and outlet of the erosion hole at a given time.

Similar to the balance equations for the suffusion phenomenon, two constitutive laws are needed in order to close the equation system: the flow-induced shear force on the interface, \( \tau_b \), and the mass removal rate, \( \dot{m}_{HET} \). The shear stresses at the interface can be calculated by \( \tau_b = \rho'[2] f \nu'[2] \), in which \( f \) is the surface friction factor that is equal to 1/64 for the laminar flow condition. The mass removal rate, \( \dot{m}_{HET} \), can be defined in an analogy to the surface erosion equation (Shields 1936) as:

\[
\dot{m}_{HET} = C_{HET} (\tau_b - \tau_c) \quad \text{if } \tau_b > \tau_c \tag{32}
\]

\[
\dot{m}_{HET} = 0 \quad \text{if } \tau_b \leq \tau_c \tag{33}
\]

In Eqs. 32 and 33, \( \tau_c \) is the critical shear strength of the solid compartment at the interface and \( C_{HET} \) is the coefficient of the mass erosion rate. These are both soil-specific, empirical properties that need to be evaluated experimentally.

3.3 Constitutive laws for closing the mathematical models

Constitutive laws are necessary to relate various physical properties of soil material to the erosion rate in continuum-based descriptions of internal erosion phenomena. These are specific to each material and provide estimations of the response or the rate of response (i.e. erosion rate) of that material to the external load, usually defined as forces. These can close the problem formulation to solve erosion problems. For internal erosion phenomena, a constitutive law for a material needs to be defined considering the granulometric and hydromechanical criteria for both the initiation of erosion as well as its continuation (i.e. mass
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removal rate) in order to get an appropriate solution. Similar to flow laws, the empirical coefficient of these constitutive laws needs to be evaluated experimentally.

3.3.1 Initiation of internal erosion

Granulometric criteria for the constitutive law for suffusion

For internal instability and the suffusion phenomenon to happen in a porous material, the medium needs to fail, providing self-filtering characteristics where the fine fraction of the soil mix can pass through the constrictions between its coarser fractions. If the constrictions formed by the coarse fraction of the soil are not large enough for the finer fractions to pass, mechanical filtration occurs. The pore structure, therefore, has to be taken into account when defining the constitutive law for numerical calculations of internal erosion. Some of the major granulometric criteria that define thresholds for suffusion are those of the following researchers:

Kezdi (1979)

Kezdi’s method assesses the potential for the internal stability of soils. In this method, at an arbitrary grain diameter, the grain size distribution (GSD) curve is split into its coarse and fine elements where the coarser fraction is defined based on its capacity to retain the finer fraction. Both parts are then used to assess the internal stability by applying the stability threshold of \( D_{15}/d_{85} \leq 4 \) according to Terzaghi (1939), where \( D_{15} \) is the diameter of the 15% mass passing in the coarser fraction and \( d_{85} \) is the diameter of the 85% mass passing in the finer fraction. Only if the soil satisfies the limiting criterion is the soil considered internally stable. One element of Kezdi’s method that is unclear is the point at which the grain size distribution curve should be divided.

Kovacs (1981)

Kovacs (1981) adapted the capillary tube model to assess internal stability. In this model, the porosity is considered as a bundle of parallel cylinders with an average diameter calculated from Kozeny’s effective diameter, which is based on the coarse fraction of the soil mix. The coarse fraction is decided from the GSD shape. The average diameter of tubes, \( \bar{d} \), is calculated by

\[
\bar{d} = 4\alpha_s D_{15} (\varphi_c/(1-\varphi_c))
\]

where \( \alpha_s \) is a shape coefficient, \( \varphi_c \) is the porosity of the coarse fraction, and \( D_{15} \) is the effective diameter for the coarse fraction, calculated with a weighted average of the coarse fraction of the soil mix. The internal instability threshold is evaluated by \( d_{85} \) of the finer fraction passing through the \( \bar{d} \) (i.e. \( d_{85} \leq \bar{d} \)).

Kenny and Lau (1986)

In Kenny and Lau’s method, a passing increment, \( H \), occurring over the designated \( D \) to \( 4D \) interval of grain size is compared to a passing increment of \( F \) at grain size \( D \). A stability index is defined
as $H/F$. The boundary for instability was initially defined as $H/F = 1.3$ (Kenney and Lau, 1985), but later revised by the authors to be $H/F = 1.0$ (Kenney and Lau, 1986).

**Burenkova (1993)**

Burenkova (1993) proposed a method to assess internal stability based on the negligible extent of the contribution of the soil’s fine fraction to the formation of its skeleton. In this method, the content limit of fine fraction is defined as the proportion not to cause a volume increase when mixed with the coarser fraction. The method is based on defining the heterogeneity of the soil mix with the help of three representative fractions of the soil, $d_{15}$, $d_{60}$, and $d_{90}$, by two ratios defined as: $H' = d_{90}/d_{15}$; $H' = d_{60}/d_{15}$ named as conditional factors of uniformity. Based on these two ratios, boundaries of separating the stable soils from instable ones for suffusion are defined by Eq. 34 (see Burenkova (1993) for more details):

$$0.76 \log(d_{90}/d_{15}) + 1 < d_{90}/d_{60} < 1.86 \log(d_{90}/d_{15}) + 1 \quad (34)$$

**Steeb and Scheuermann (2012)**

Steeb and Scheuermann presented a mathematical model that can be used to evaluate quantitatively the volume fraction of the available erodible fine fraction from the pore constriction size distribution (CSD) of the soil calculated from its GSD. This mathematical model facilitates the numerical modelling approach further by setting not only the threshold for internal instability, but defining the amount of the soil that can potentially be eroded. In this method, CSD is calculated based on the bedding of the particles with an analogy introduced to predict the pore constrictions from the porosity of the soil within a probabilistic approach. A comprehensive mathematical description of the model can be found in Scheuermann et al. (2010) and Steeb and Scheuermann (2012).

**Granulometric criteria for the constitutive laws for concentrated leaks**

For a concentrated leak mechanism, the granulometric parameters are different to the parameters evaluated for the suffusion phenomenon, and they are expressed qualitatively rather than quantitatively. The five main granulometric criteria for a concentrated leak mechanism to occur are: the soil mixture’s ability to “hold a roof” for the erosion scar, the cohesive content of the soil mix, the dispersivity of this cohesive content, the degree of compaction, and the moisture content when compacted (Bonelli, 2013).

The major geometrical parameter for a soil to “hold a roof” is the fine content of the soil mix (i.e. finer than 0.075 mm in diameter). Soil mixes with 15% or higher fine content are likely to hold a roof, as are soil mixes with a plastic content (15% or higher). The dispersivity of the cohesive content is highly significant and this is dependent on a soil’s chemical properties such as pH value and
sodium ion percentage. Lastly, compacted soils have a greater tendency to support a roof for the formation of an erosion scar than loose soil mixes. Compaction of the soil on the “wet side” of the optimum moisture content lessens the risk of micro-crack formation and increases the likelihood of roof support.

**Hydromechanical criteria for the constitutive law**

Despite the importance of granulometric criteria for internal erosion and the assessment of the erosion vulnerability of various soil mixtures (e.g. extensive study of Swedish morane by Rönnqvist (2015)), it is important to remember these criteria are geometrical soil properties and their influence on the constitutive law is therefore limited to setting thresholds that can be assigned from the findings of the previous studies that have been carried out. In this thesis, however, the main focus is on defining the constitutive law for internal erosion processes (i.e. suffusion and the concentrated leak), identifying the main affecting parameters governing physical laws, and facilitating the numerical modelling of the processes which are presented in Chapter 5 of this thesis.

Hydromechanical criteria consist of two sub-criteria of the hydraulic load and mechanical load applied on the soil mix and their effect on the initiation of internal erosion as well as the mass removal rate from the soil mix due to the persistence of the internal erosion. A hydraulic load applied to the soil material is defined as the hydraulic pressure head, which is interpreted as the pore pressure within the REV and the flow-induced shear force on the solid compartment due to the fluid movement. Mechanical load is defined as the force applied on the soil surface, which is then translated to principal stresses and the stress field within the solid compartment and merged with the resultant stresses from the weight of the soil.

Considering an REV, the stresses acting on its surfaces can be expressed by the stress tensor. This tensor has nine stress components: three for normal and six for shear stress. When the REV is at rest, the number of the shear components is reduced to three due to the internal equilibrium and the equality of respective shear stress components (i.e. \( \tau_{ij} = \tau_{ji} \)). Thus the state of stress at the REV can be specified by six independent components of the stress tensor \( \tau \) as given in Eq. 35:

\[
\tau = \begin{bmatrix}
\sigma_{xx} & \tau_{xy} & \tau_{xz} \\
\tau_{yx} & \sigma_{yy} & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & \sigma_{zz}
\end{bmatrix}
\]

(35)

If the axes are turned to make the shear stresses equal to zero, these three axes are called principal axes and the associated planes for this no-shear condition are called the principal planes. If we reduce the analysis to a two-dimensional REV and consider the equilibrium only in two directions (i.e. \( x \) and \( y \) directions), the normal stresses associated with these directions are called principal...
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stresses (Brady and Brown, 1999). These stress components can be calculated by Eqs. 36 and 37 as illustrated in Figure 5.

\[ \sigma_1 = \frac{1}{2}(\sigma_x + \sigma_y) + \sqrt{\tau_{xy}^2 + \frac{1}{4}(\sigma_x - \sigma_y)^2} \]  
\[ \sigma_3 = \frac{1}{2}(\sigma_x + \sigma_y) - \sqrt{\tau_{xy}^2 + \frac{1}{4}(\sigma_x - \sigma_y)^2} \]

where \( \sigma_1 \) and \( \sigma_3 \) are the maximum and minimum principal stresses respectively.

If the co-ordinate system is oriented in such a way that the \( x \)-axis is parallel to the maximum principal stress and the \( y \)-axis to the minimum principle stress, a Mohr stress circle can be drawn for the system by plotting the corresponding values of \( \sigma \) and \( \tau \) in a diagram that has a radius of \( (\sigma_1 - \sigma_3)/2 \) cantered at \( (\sigma_1 + \sigma_3)/2 \) on the \( \sigma \)-axis. Stresses on any arbitrary plane can be extracted from this circle if the orientation from the principal axis is known. \( \sigma \) and \( \tau \) in a general direction \( \theta \) relative to the \( x \)-axis can be expressed by Eqs. 38 and 39:

\[ \sigma_\theta = \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3)\cos 2\theta \]  
\[ \tau_\theta = -\frac{1}{2}(\sigma_1 - \sigma_3)\sin 2\theta \]

For concentrated leak erosion, the principal stresses around a vertical hole in porous media need to be known. Stresses on the wall of a vertical borehole are more easily presented in a cylindrical coordinate system in Figure 6.
Applying the force and momentum balances for an REV on the wall of a vertical hole results in the following principal stresses:

\[ \sigma_1 = \sigma_v - 2\nu(\sigma_H - \sigma_h)\cos 2\theta \]  \hspace{1cm} (40)
\[ \sigma_2 = p_f \]  \hspace{1cm} (41)
\[ \sigma_3 = \sigma_H + \sigma_h - 2(\sigma_H - \sigma_h)\cos 2\theta - p_f \]  \hspace{1cm} (42)

In Eqs. 40-42, \( \sigma_1, \sigma_2, \) and \( \sigma_3 \) are the principal stresses in the cylindrical coordinate system, \( \sigma_v \) is the applied vertical stress, \( \sigma_H \) and \( \sigma_h \) are the minor lateral (horizontal) stresses, and \( p_f \) is the fluid pressure in the hole and for this system of equations \( \tau_{r\theta} = \tau_{r\phi} = \tau_{r\phi} = 0 \).

### 3.3.2 Mass erosion in porous media

In order to predict the time-dependent physical changes of the porous material, an understanding of the mass removal rate (i.e. \( \dot{m} \) for suffusion and \( \dot{m}_{HET} \) for the concentrated leak) from the soil matrix over time is necessary. This rate refers to the porosity change over time for the suffusion mechanism and the increase of the size of the erosion scar for the concentrated leak mechanism.

The rate of erosion can be expressed either as volumetric content of eroded material (kg m\(^{-3}\)s\(^{-1}\)) or as a rate of the volumetric content leaving the interacting surface (kg m\(^{-2}\)s\(^{-1}\)). The preferred method depends on the way the system of the differential equations used for modelling the media is defined. The rate of erosion leaving the surface is a more conventional way of presenting the erosion rates.
and is more appropriate for interface-erosion phenomena such as contact erosion, concentrated leaks, or piping.

The same rate can also be successfully employed for suffusion types of erosion; however, a volumetric content of eroded material is more appropriate for the volumetrically occurring suffusion mechanism. The priority when expressing these rates must be the careful definition of the coefficients for the constitutive law of erosion so that they are consistent with the expected dimension for the constitutive law used to enclose the balance equation system.

3.3.3 Monitoring mass erosion of suffusion and concentrated leak phenomena

The continuous removal of mass in the suffusion phenomenon has not been sufficiently monitored in the available literature. The handful of available examples did so in three ways. Firstly, by retrieving the cumulative eroded mass only when the experiments were completed (e.g. Moffat (2005) and Li (2008)); secondly, by comparing the initial sample weight that was not subjected to erosion to the total dry mass of the post-test specimen (e.g. Wan (2006) and Rönnqvist (2015)); finally, by stopping the experiments at certain intervals and continuing after the cumulative eroded mass was collected (e.g. Marrot and Bendahmane (2014)). No continuous monitoring seems to have been done in previous studies.

Erosion monitoring for the concentrated leak mechanism is normally done by adopting the standard method used for HET tests, which is developed based on the force equilibrium on the body of eroding fluid along a differential length of an axial circular hole in an HET set-up (Wan and Fell 2002, 2004a, 2004b). In this method, the pressure drop along the differential length of the hole can be expressed by the energy head loss due to wall friction, or friction head loss (i.e. Eq. 46). The hole is assumed to have a uniform circular cross section and the flow is considered to be fully developed. This force balance can be simply integrated over the total length of the hole by adopting four simplifying assumptions: i) assuming that a negligible throughflow happens in the soil matrix and in most of the flow path through the opening, ii) the shear resistance is solely present along the soil and water interface, iii) energy losses at the entrance and exit of the hole are negligible, and iv) the hole remains uniformly circular along the length during the tests. This integration results in a set of ordinary differential equations (i.e. Eq. 47) whereby the rate of growth for the erosion hole can be calculated following these steps:

1) Defining the initial and final flow regimes using the continuity, and calculating the Reynolds number of the flow by using Eq. 43.

\[
Re = \frac{2\rho Q}{\pi \mu r}
\]  

(43)

2) Estimating the initial and final friction factors \((f_0\) and \(f_t\)) based on the flow regime (i.e. laminar (subscript \(L\))
versus turbulent (subscript $T$)), the pressure loss over the
length of the hole, $s$, and the initial and final radii of the
hole (i.e. $r_0$ and $r_f$) using Eq. 44 and linearly
interpolating it over time.

4) $f_L = \frac{\pi \rho g s r_0^3}{2Q}$ and $f_T = \frac{\pi^2 \rho gs r_f^5}{2Q^2}$

(44)

for laminar and turbulent flow respectively.

5) Calculating the radius of the erosion hole, $r(t)$, at any
time during the test using Eq. 45, and plotting the curve
of calculated $r(t)$ against time, $t$.

6) $r(t) = \left( \frac{2Qf_L}{\pi \rho gs} \right)^{\frac{1}{3}}$ and $r(t) = \left( \frac{2Q^2 f_T}{\pi^2 \rho gs} \right)^{\frac{1}{5}}$

(45)

for laminar flow and turbulent flow respectively.

7) Evaluating the slope of the $r(t)$ curve versus time, $t$

8) Estimating the interface shear stress, $\tau_{HET}$, using Eq. 46.

9) $\tau_{HET} = \rho gs r(t)/8$

(46)

10) Extracting the mass erosion rate, $m_{HET}$, using Eq. 47.

11) $m_{HET} = \frac{\rho_s}{C_{HET}} \frac{dA}{dt} = \rho_s \frac{dr(t)}{dt} \approx \rho_s \frac{\Delta r(t)}{\Delta t}$

(47)

12) Plotting $m_{HET}$ against $\tau_{HET}$, adopting a linear regression
to fit the curve, and determining the coefficient of soil
erosion, $C_{HET}$, using Eq. 32.

Alternatively, the method introduced by Benahmed and Bonelli
(2012) can be used for HET tests where, similarly to the
aforementioned method, the erosion rate can be extracted with an
indirect method from the pressure and flow measurements. In this
method, the rate of growth for the erosion hole is derived by
considering two additional constraints: a no-slip condition at the
interface between solid and fluid phases and the Nikuradse
equation for the radial profile of the velocity field (Nikuradse,
1950). Adopting the constraints, this equation system can be
solved analytically by adopting three further assumptions:

1) the concentration of the fluidised particles is low and
its influence on the density, the inertia, the velocity, or
the stress can be neglected.

2) the flow is quasi-steady.

3) the velocity of the fluid- and solid-phase interface
movement is low and does not contribute to inertia.

This model has only two unknown parameters which correlate the
radius of the hole with critical shear stress, hydraulic gradient and
time, and provide an easy way to determine erosion characteristics
without measurements or interpolation of the hole diameter. It can
be solved with a numerical non-linear solver and accounts for the
energy losses at the entrance and exit to the erosion hole. However, its applicability is further restricted compared to the
previous method (Wan and Fell 2002, 2004a, 2004b) in that it is only applicable to a HET set-up with a constant pressure drop and prevailing turbulent flow conditions which, according to Wan and Fell (2002, 2004a, 2004b), occurs at $Re > 5000$.

The rate of growth for the erosion hole can be calculated using Eq. 48 (Bonelli, 2013):

$$r(t) = r_0 \left[ \frac{Q(t)}{Q_0} \right]^2 \left[ \frac{f_b(\alpha^* \Delta P_{r0})}{f_{b0} \Delta P_r(t)} \right]^{\frac{1}{3}}$$  \hspace{1cm} (48)

where $r(t)$ is the radius of the hole at time $t$, $Q(t)$ is the flow rate at time $t$, $f_b$ is the friction factor of the interface, $\alpha^*$ is the singular head loss factor accounting for the pressure loss at the entrance and exit of the erosion hole and $\Delta P_{r0}$ is the pressure loss inside the erosion hole at time $t$. The subscript, 0, for the parameters denotes the initial values for the parameters at the moment of initiation of the erosion. The rate of growth of the radius can be evaluated by fitting the pressure and flow data collected from the experiments to Eq. 48. The erosion rate is then calculated in a similar way by fitting the data to Eq. 47.
CHAPTER 4

PHASE I OF THE PROJECT: FLOW IN POREOUS MEDIA

Turbulent throughflows under heavy leakage in the downstream of embankment dams can be a consequence of internal erosion processes occurring inside the dam, whereby an erosion-related scar has developed within the body or foundation of the dam. Therefore, a systematic and quantitative understanding of the throughflow is crucial for the design, safety assessment and erosion protection of embankment dams, especially to reduce the risk of a breach. One way of achieving this understanding is by numerical modelling. However, to model throughflow numerically, friction losses in coarse rockfill material and flow behaviour under these critical conditions need to be understood and evaluated empirically.

To facilitate the numerical modelling, all the interactions between the rockfill material and the flowing fluid need to be studied and the drag forces and friction factors need to be assessed. Perhaps more importantly, the flow law within the rockfill material and its parameters must be defined in accordance with the Navier-Stokes equations and also the forces exerted by fluid on the media need to be understood and quantified. The flow law, combined with the continuity equation, can be used to build numerical engineering models by applying relevant initial and boundary conditions.

The flow law parameters, similar to other common constitutive laws in engineering practices, are empirically extracted by conducting experiments and applying data fitting techniques. Therefore, throughflow properties of coarse rockfill material were studied as part of this thesis project under flow conditions similar to those prevailing under heavy leakage scenarios in embankment dams by means of three approaches:

a) analysing pump test data from Trängslet rockfill dam,
b) constructing a large-scale apparatus and performing extensive laboratory tests, and
c) simulating three-dimensional CFD studies through coarse materials, resembling those used in the laboratory experiments, by using Flow-3D software.

Results from the field, laboratory and numerical studies were used to address the research questions, which would facilitate the numerical modelling of these heavy leakage throughflows in embankment dams. The drag forces and friction factors from the laboratory experiments were evaluated in Paper I. The flow law for the course porous media and its coefficients were defined and their values were extracted from the field and laboratory experiments in paper II in accordance with the Navier-Stokes equations. The interactions between the rockfill material and the flowing fluid were studied in depth in Papers III and IV. In Paper
III, three-dimensional models of the laboratory experimental set-up were replicated numerically, the numerical model parameters were studied extensively, and a three-dimensional validated model was created. This validated model was used in Paper IV to conduct extensive numerical experimenting, where the discharges below and above both the field and laboratory experiments limits were studied, the flow law and the coefficients for this extended discharge interval were defined, the 3D flow field inside the course porous media was evaluated, and the forces exerted by fluid on the media were quantified.

At the beginning of this chapter, a summary of the field and laboratory experiments is presented with their aim and set-up, followed by a brief presentation of the main results taken from Papers I and II. Afterwards, the CFD studies are briefly presented with their aim and set-up, followed by a summary of the main results from the numerical studies taken from Papers III and IV. A more comprehensive account of the conducted studies is provided in Papers I-IV.

4.1 Pumping field tests

For the purposes of this work, field pumping tests were used to extract the hydraulic properties of rockfill material used in earthfill hydraulic structures. Although the objectives of the tests were similar for both the laboratory and the field tests, the procedures for data collection, extraction and the interpretation of results were case-specific, based on the type of the tests conducted (ASTM 2004).

In order to evaluate the hydraulic conductivity of the coarse rockfill material of the downstream zone of Trängslet dam, eight injection tests were conducted (Figure 7). These in-situ tests were done by using in-situ pumping with a constant water head and free fall (balanced infiltration with a constant water head of 0 meter).

In this chapter, a short description of the conducted tests and analysis basis is presented. A detailed description of the pump test and the data evaluation is presented in Paper II.
The pumping tests were carried out at the four boreholes shown in Figure 7, slightly downstream of the dam toe in a supporting, horizontal berm in which the filling material was the same as in the dam shoulder. The pumping wells were located 16 m, 32 m, 65 m and 72 m upstream of the measuring weir of the dam to achieve a representative average of the conductivity of the entire downstream rockfill material. Water was pumped into the rockfill vertically through a plastic pipe into the shoulder material of the dam. The berm used for the injection tests was assumed to be representative of the main bulk of the rockfill in the downstream shoulder of the dam. This assumption is based on a comparison of photographs taken during the dam’s construction in the 1960s.

According to the available data, the rockfill material used in the shoulders of the dam is well-graded. The maximum grain size in the shoulder is about 1 meter and the fine material content contains fine sand and silt. The grain size distribution was confirmed by studying samples from large test pits and conducting digital image analysis to determine the particle size distribution. From the conducted analysis, the representative diameter for seepage (i.e. $d_{10}$) was found to be 128 mm.

Two types of tests were conducted: 1) tests with a constant water head of 200-240 mm, and 2) free infiltration tests, in which the head was kept balanced at the ground surface. Both the water head in boreholes and pump discharges were constant for each test. Similarly, discharges and the water head were monitored and data was recorded at the measuring weir located further downstream from the dam toe.

To analyse data from the field tests for the purposes of this study, an analytical method similar to Ahmed and Sunada (1969) was
developed. This theoretical model was developed to consider two flow regimes simultaneously, i.e. a nonlinear specific discharge-hydraulic gradient relationship for parts of the medium close to the well and a linear relationship for the remainder of the medium. This method allowed for the coupling of the turbulent flow domain (near the well) to the laminar flow domain (further from the well) at a critical radius at which the flow regimes change from linear to nonlinear. The model development and its description are described in greater detail in Paper II. The final developed analytical equation reads:

\[
{h}_{t}(r) = \left[\frac{Q}{\pi K} \ln \left(\frac{r}{r_w}\right) + \left[\frac{h_0^{(1-a_w)}}{(1-a_w)} + b_w' \left(\frac{Q}{2\pi}\right)^{1-a_w}\left(\frac{r}{r_w}\right)^{(1-a_w)} - 1\right]^{1/2}\right]^{1/2}
\]  (49)

where \(h_{t}(r)\) is the piezometric water head at the region with laminar flow, \(Q\) is the pump discharge, \(r\) is the radial distance from the well, \(K\) is the hydraulic conductivity of Darcy's law, \(h_0\) the piezometric water head at the well’s radius, and \(a_w\) and \(b_w'\) are parameters for the flow law.

Regression analysis was then used to fit Eq. 49 to the recorded data from the well and measuring weir (the head and discharge values) during the pumping tests to extract both turbulent permeability parameters \(a_w\) and \(b_w'\) and Darcy’s flow hydraulic conductivity \(K\). This is presented in greater detail in Paper II.

### 4.2 Large-scale permeameter laboratory studies

The advantage of conducting laboratory tests rather than field tests is the ability to test within a controlled environment with selected granular material. Conducting such controlled experiments is very difficult in the field. Due to such limitations, a customised large-scale permeameter was designed and built for improved testing. This apparatus enabled throughflow tests to evaluate the turbulent permeability of coarse rockfill material.

The section below presents a short description of the design, instrumentation and operation of the apparatus as well as a summary of the results. Paper I provides this information in greater detail.

#### 4.2.1 Experimental apparatus

High-flow discharge experiments in coarse rock materials were carried out in a large-scale experimental apparatus made of heavy stainless steel. The apparatus was custom designed to resist the high hydraulic heads applied and the process of loading and unloading the rockfill materials. The apparatus has a total length of 5.7 m and consists of three discrete parts: an inlet (a conical
decelerator 1.5 m in length with a 17° divergence angle diffuser), a main pipe (2.0 m in length with a diameter of 1.0 m), and an outlet (a 1.5 m-long conical contraction pipe). A photograph of the experimental apparatus with dimensions is given in Figure 8.

![Experimental apparatus](image)

**Figure 8.** The experimental set-up with a total length of 5.7 metres. Flow enters the apparatus from the left-hand side and leaves from the right-hand side.

**Procedure and measurements**

The rockfill materials were carefully loaded in the main pipe, then packed and fastened by two metallic gratings placed on either side of the main pipe. An automatic traverse unit and hydraulic jacks were used to operate the system. The apparatus was connected to the recirculation reservoir system in the laboratory. The principal experimental variables were the size and grading of the material and the flow discharge. The apparatus, supported by two large pumps and assembled from steel pipes and plates, could perform tests with discharge rates ranging from 20 litres per second up to a maximum of 650 litres per second. Adequate water pressure was applied to reach these high flow rates for packed rockfill columns, making sure that hydraulic gradients higher than 1 and highly turbulent regimes in the porous media were reached.

The flow discharge was measured using a calibrated magnetic flow meter with an accuracy of ±0.01 litres per second. Pressure measurements were taken at 6 sections (1 to 6 in Figure 9) along the main pipe. Sections 1 through 6 were located at 0.1 m, 0.4 m, 0.7 m, 1.3 m, 1.6 m and 1.9 m from the main pipe entrance. Sixteen pressure measurement units were installed diagonally in these sections. Two pressure measurement units were installed in sections 1, 3, 4 and 6 diagonally, together with 4 pressure measurement units in sections 2 and 5.
The mean porosity of the packed column inside the apparatus was measured by checking the water volume filled in the experimental compartment for each test. Three sets of pumping tests were conducted on four different materials. These materials were of two types (i.e. crushed rockfill and river-rounded cobbles) and in two size ranges (100-160 mm and 160-240 mm). The bulk material was

Figure 9. The experimental set-up with the pump station and apparatus. The apparatus is assembled in three parts. The pressure cells were installed in 6 sections in the main part.
sieved carefully to achieve a uniform material size distribution, reflecting the representative grain dimension of the coarse rockfill material used in Trängslet dam’s downstream shoulder.

For the first and second sets, the initial pumping intensity was set to 50 litres per second and increased in 12 steps to reach a flow rate of 600 litres per second. Based on preliminary studies of the results taken from the two sets performed, the third set was conducted with an expanded testing range of discharge from 20 litres per second to 650 litres per second.

At every step, data pertaining to flow rate and water pressure was recorded. These readings were collected through analogue ports and processed with LabVIEW data acquisition software at intervals of 0.6 seconds.

4.2.2 Results and analysis

Studies from the field and laboratory tests, presented in Papers I and II, demonstrate that, if the flow law (i.e. the momentum conservation equation relating the hydraulic gradient and the flow velocity) is written as a power function (e.g. Eq. 5), the exponent of this power flow law is dependent on the Reynolds number for pore Reynolds numbers lower than 60,000. The power remains constant (independent of the Reynolds number) above this Reynolds number asymptote for the fully developed turbulent regime.

Figure 10 shows the variation of the coefficient for the power law in a dimensionless form (i.e. $c_p$) for the entire range of pore Reynolds numbers in both the laboratory and field tests. As can be seen in the figure, the medium constant, $c_p$, becomes a constant, Reynolds-number-independent value for each test after the flow reaches a certain degree of turbulence, despite constant statistical variation in the values of these medium constants from the different tests. The statistical variation envelope begins to widen as the pore Reynolds number falls below 60,000.

In Figure 10, the in-situ pumping field tests are plotted in red with the interval of the $c_p$ as well as the interval of the Reynolds number covered by the tests. These tests did not achieve a fully developed turbulence, and this was due not only to the moderate discharge capacity of the pumps used for these filed tests compared to the ones used for the laboratory tests, but also the significant and rapid decrease of the radial flow velocity in field pumping tests (see Paper II for more detail). When compared to the laboratory test results and considering both the range of Reynolds numbers (maximum value close to the well and decreasing with increased radius) and the porosity threshold for the material (90% confidence interval is estimated to be $0.3 \leq \phi \leq 0.5$), the field test results fall within the variation envelope of the material parameter for crushed rock (coated in grey in Figure 10) in the transient flow regime zone and are in very good agreement with the laboratory test results.
This constant behaviour and the validity threshold also applies if the flow law is written as a quadratic form (Forchheimer (1901), Ergun (1949) and Engelund (1953)).

A similar Reynolds-number dependency and asymptote can be observed for the friction factors. This behaviour stems from the flow patterns and the boundary layer instabilities around aggregates. After reaching this asymptote, the boundary layers around the aggregates stabilize and the Reynolds-number dependency becomes negligible. As a result, the friction factors can be described as a function only of the surface roughness without introducing significant errors for flows with a pore Reynolds number exceeding 60,000 (see paper I).

The aforementioned asymptote lies beyond the ranges investigated experimentally by previous researchers. The experiments in this study examined pore Reynolds numbers as large as 220,000 for grain diameter distributions in the range 100-160 mm and as large as 320,000 in the range 160-240 mm.

4.3 Numerical CFD studies

Numerical solutions of the Navier-Stokes equations in 3D can be used for numerical experimentation if the governing equations, modelling parameters, discretisation scheme as well as the numerical solver routines are correctly assigned and the output results are found to be realistic. For this purpose, numerical simulations replicating the experiments were also conducted and the results were used for validation studies.

Various numerical parameter effects such as discretisation, the turbulence model, the advection term’s order and numerical solver type have been studied (see paper III). The outcome of this study is a fully validated numerical model, which was then used to conduct numerical experimentation. The numerical experimentation was conducted to better understand the physics.
of the throughflow in coarse porous media and explore flow magnitudes beyond the limits explored experimentally.

4.3.1 Model set-up

For the numerical model set-up, three-dimensional AutoCAD models of the cobbles, which kept both the packing and the actual size range used in the experiments, were created and positioned randomly by manually placing individual stones in the AutoCAD-3D environment using similar packing and size variations as in the laboratory experiment (as illustrated in Figure 11). Here, the aim was to model the fluid flow as a continuum that circumvented 3D, rigid, discrete, and solid particles.

![Figure 11. The corresponding three-dimensional CAD model parts replicating the apparatus segments, created for the numerical simulations.](image)

Flow-3D, a specialised CFD computer programme, was used to model the flow through the rockfill materials. It used the Finite-Volume Method to provide results for the full three-dimensional Reynolds-averaged, Navier-Stokes equations in a xyz-Cartesian coordinate system. In this section, a short description of the set-up, the boundary conditions and the numerical scheme for the developed validated numerical model of porous media flow is presented. For a more elaborate description of the numerical model and the calibration and validation procedures, see Paper III.

4.3.2 Grid and boundary conditions

Equal-sized cubic cells were used for the discretisation of the model. Eight different discretisation schemes with the corresponding total numbers of cells \((250 \times 10^3, 300 \times 10^3, 500 \times 10^3, 1 \times 10^6, 2 \times 10^6, 4 \times 10^6, 5 \times 10^6, \text{ and } 8 \times 10^6)\) were adopted for
validation studies. The significant variations in cell size made it possible to address the model discretisation dependency in detail.

For the boundary conditions of the numerical models, the upstream boundary condition was set to a “fixed flow rate” of a predetermined magnitude and direction. An outlet-type boundary condition was set at the end of the outlet, where the pressure flow condition prevailed. An initial condition was also set for the model by considering a fluid region that filled the entire model with water to facilitate the convergence of the numerical solutions. This region was initiated with zero fluid velocity under an atmospheric pressure condition.

4.3.3 Numerical solver

For the model calibration and validation studies, the Generalized Minimum Residual (GMRES) implicit solver was used. Transient incompressible flow simulations with no sharp interfaces were carried out until the solution converged and steady-state conditions were reached. A total simulation time of 300 seconds was maintained in all simulations to ensure prevailing steady-state conditions. For the numerical validation studies, the influences of the following numerical variables on the solution were studied:

1) momentum advection terms, including first order and second order,
2) grid dependency,
3) turbulence model choice, including k-ε, Renormalized Group (RNG), k-ω, and Large Eddy Simulation (LES) models, and
4) solver type, i.e. explicit and implicit.

The calibration simulations were performed at a constant discharge of 600 litres per second for various discretisation schemes (i.e. the various numbers of cells).

4.3.4 Model calibration and validation

The Flow-3D simulation outputs were used to analyse the various flow characteristics. The focus was to compare the hydraulic heads calculated from the numerical simulations with the measurements taken during the laboratory experiments (presented in Paper I and Paper II). From these validation studies, the 5M RNG-explicit model, yielding a minimum error of 0.84%, was the most representative model in terms of capturing the actual nature of the flow field. Figure 12 is a sample from the validation studies.
The 5M RNG-explicit model was used as the working model in the numerical experimental studies presented in Paper IV.

### 4.3.5 Numerical experimentation

CFD simulations were conducted using the validated, 5M-cell, RNG-explicit numerical model. All the flow features and the solid-fluid interactions were extracted from the simulation results to provide a set of new relationships and findings. The results were used to validate the momentum and force exchanges and to analyse the channelisation process in the porous media. The simulations covered viscous, transitional and also turbulent flows with high pore Reynolds numbers.

The validated numerical model developed for porous media flow studies was therefore run using 47 different flow discharges ranging from low to high, including values below and above the pore Reynolds number ranges explored both in literature and also in Papers II and III. The low discharges were in the range of 0.1-50 liters per second, and the high discharges were in the range of 50-3000 litres per second. The model set-up, the choice of numerical variables, the turbulence model and the operation of the numerical model were identical to the validated numerical model developed for porous media flow. To simulate low discharges, a viscous (laminar flow) formulation was used. The results of this study are presented in Paper IV.

The intrinsic permeability parameter is an important porous media property to determine. This determination can, however, be difficult, specifically if based on laboratory experiments. One
possible approach is the use of the travelled length in a flow tube, as suggested by Kozney-Carman (Carman 1997). In numerical models, this task is more viable and can be performed using the Lagrangian particle-tracking model, which can be integrated with flow equations. For the purposes of numerical experimentation, the integrated particle-tracking method was also applied. This study was done at flow discharges of 10, 40, 200 and 600 litres per second by releasing 100 weightless particles (drogues) in the inlet section of the test tube during each simulation.

A broad flow range was chosen to study potential variations in characteristics based on the flow regime. For each simulation, the path and travelled lengths were extracted. The resultant data was then used to analyse the stochastic characteristics of the flow tube lengths in the porous media, the pathways’ dependence on the pore Reynolds number, and the intrinsic permeability parameter of the porous media based on the Kozney-Carman concept.

Variation of the flow resistance coefficient (the conventional drag coefficient) with the pore Reynolds number, specifically the sharp drop of this coefficient at high pore Reynolds numbers (e.g. $5 \times 10^5$ for a smooth spherical particle (Perry, 1950)) was also studied numerically. Similar behaviour for packed porous media was studied, whereby numerical simulations were performed at different pore Reynolds numbers in the range of $10^5$-$10^6$.

4.3.6 Analysis and the results from the numerical experimentation

The novelty of these numerical throughflow studies lie in their use of a combined large-scale experimental and three-dimensional numerical approach to develop a fully calibrated and validated CFD model that was applicable to flows through cobble-sized materials at pore Reynolds numbers in the range of $10^1$-$10^6$. Conducting numerical experiments with the validated CFD model led to a new set of findings regarding flow channelisation within porous media in terms of passive particles pathways.

Flow simulations in a broad range of Reynolds numbers made it possible to observe flow transitions from viscous to turbulent, and to compare them to the laboratory throughflow studies as presented in Figure 13.
Internal erosion phenomena in embankment dams

The simulations provided modified flow laws to estimate the flow behaviour in coarse porous media, which are presented in greater detail in Paper IV. By studying the flow field in the porous media from the numerical simulations (of which Figure 14 is an example), it was understood that the interactions between flow fields, which are induced by coarse particles, limit the formation of coherent structures within the porous media.

**Figure 13.** Hydraulic gradients versus pore Reynolds number using the viscous and turbulent models. The x-axis is the Reynolds number and the y-axis is the hydraulic gradient; both are plotted using a logarithmic scale. This figure is taken from Paper IV.

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**Figure 14.** Streamline and z-vorticity contours plotted for positive vortices at $R = 2 \times 10^5$ taken from Paper IV.

From the particle-tracking studies it was found that the length of flow paths in porous media can be demonstrated using a Gamma distribution where the scale and shape parameters of the distribution are dependent on the pore Reynolds number. A normalised tube length parameter, $l_n$, was proposed that could be used to evaluate various processes, such as energy dissipation, induced forces, diffusion, and permeability from the particle-tracking studies.

From the studies of the flow-induced forces on the particles embedded in the porous media, it was understood that the shear forces exerted on particles are significantly dependent on the inertial forces of the flow, as illustrated in Figure 15. These shear...
forces can be estimated using the new proposed equation presented in Paper IV, which reads as:

\[
F = 6\pi \mu u^0 RU \left[ 1 + \left( \lambda_1 + \lambda_2 \right) S / \forall \right] + T_D \rho U^2 \left[ \left( \lambda_1 + \lambda_2 \right) S^2 \right]
\]  

(50)

In Eq. 50, all the variables are similar to Burgers’ equation (i.e. Eq. 7). \( T_D \) is defined as the volumetric turbulent drag coefficient in porous media and can be evaluated by fitting the data to the experimental results.

**Figure 15. A comparison of the simulated shear forces and theoretical equations as a function of Reynolds number. The x-axis is the Reynolds number and the y-axis is the shear force; both are plotted using a logarithmic scale. This figure is taken from Paper IV.**
CHAPTER 5

PHASE II OF THE PROJECT: INTERNAL EROSION IN POROUS MEDIA

The definition of a relevant constitutive law for internal erosion processes in porous media is a fundamental step in the continuum-based numerical modelling of these phenomena in soil structures. In order to predict the physical changes of the soil material over time (i.e. the porosity change over time for suffusion processes and the increase of the size of the erosion scar for concentrated leak processes), these constitutive laws need to be carefully defined and used to close the balance equations.

Despite the extensive work done to understand internal erosion phenomena, most studies conducted previously on suffusion were qualitative rather than quantitative in nature and were focused on geometrical parameters and their role in initiating suffusion. The effect of soil in-situ stresses is an important parameter that was apparently never considered. More specifically, these studies were more focused on the effect of compaction and the optimal moisture content to minimize the erodibility of core material rather than quantitatively considering the effect of in-situ soil effective stresses around an existing hole on the critical shear stresses to start the instability and its erosion rate.

Therefore, these two major mechanisms of internal erosion, “suffusion” and “the concentrated leak”, were studied by conducting laboratory experiments and developing a theoretical framework to facilitate continuum-based numerical modelling for this thesis.

This study benefits from the findings of the previous studies, specifically the granulometric criteria, by adopting what are now widely accepted criteria to prepare granulometrically susceptible soil mixes. These soil mixes were used to study the effect of the hydromechanical loading aspect, including the effect of flow velocities and also the concept of stress reduction within the soil matrix for both the initiation and continuation of suffusion and the concentrated leak internal erosion phenomena in porous media. In particular, the experimental data was elaborated with the aims of:

1) detecting the material failure point (i.e. initiation of internal erosion) and all affecting parameters (including soil stresses and flow-induced shear stress due to fluid velocity), and
2) detecting the mass removal rate and its affecting parameters after the internal erosion processes were initiated.

Obtained results were further used to propose new constitutive laws for these phenomena. Such constitutive laws provide a
scientific explanation for seepage-induced internal erosion, and represent a development in engineering practice that can be used for continuum-based numerical modelling.

The adopted methods consisted of:

i) reviewing and analysing the current knowledge on constitutive laws for internal erosion,

ii) conducting laboratory experiments to investigate the initiation of material instability, “erosion initiation”, as well as the continuation of the phenomenon “mass removal rate”,

iii) devising a theoretical formulation of a ‘hydromechanical envelope’ in order to account for the soil in-situ stress dependency of internal erosion initiation and its continuation in porous media, and

iv) conducting auxiliary numerical simulations for soil stress-strain behaviour from the experimental apparatus, with the packed soil specimen inside, to be used in formulating the theoretical framework.

For these laboratory experiments, an erosion apparatus was designed and constructed with the capability of applying simultaneous hydraulic and mechanical loading on the test specimens. The apparatus was also equipped with extensive mounted instrumentation to carefully monitor the pressure, discharge, continuous eroded material, turbidity and temperature of the tests. Additionally, an electrode probing for three-dimensional electrical resistivity tomography studies (3D-ERT) was designed and installed.

In this Chapter, a short description of the set-up of the experimental apparatus, the design for the 3D-ERT and the conducted experiments is presented, as well as a summary of the findings. A more detailed description of the erosion tests and the constitutive laws are given in Paper V and Paper VI for suffusion and the concentrated leak mechanisms respectively. The ERT-3D set-up, data acquisition and visualisation are presented in Paper VII.

5.1 Laboratory studies on internal erosion

The erosion apparatus was specifically designed to assess internal erosion mechanisms under a range of hydromechanical conditions. As shown in Figure 16, the apparatus comprises of an inlet, an erosion chamber, an outlet and a sedimentation tank, which are all made of Plexiglas and framed by stainless steel for extra reinforcement. The erosion chamber has inner dimensions of 350×200×150 mm and a wall thickness of 50 mm.
5.1.1 Experimental apparatus

In the apparatus, the test specimen is confined laterally by the rigid walls of the erosion rig and vertically by a perforated top plate pressed down by a piston rod. An inlet port in the inlet section allows water to flow into the erosion rig, passing through a chamber of plastic balls that diffuse the water flow before entering the erosion chamber. This ensures a uniform flow through the perforated plate in the erosion chamber. A plastic filter mesh between the specimen and the perforated rigid bottom plate facilitates the throughflow of water without soil grains falling down into the inlet chamber. Material flowing out of the outlet section is captured in the sedimentation tank as illustrated in Figure 16.

Hydraulic loading of the test specimen is applied by means of water head control, using inlet and outlet tanks. A municipality water system was used to supply the inlet tank with clean water. An overflow on the inlet tank bypassed the excess flow to the reservoir tank in the lab, and hence maintained a constant water head at the bottom of the test specimen.

An overflow piano key weir in the sedimentation tank was used to maintain a constant water head at the top of the specimen and the whole outlet section. The height of the inlet tank above the outlet section establishes the applied differential water head.

For the concentrated leak tests, a specific top plate with a 50-mm central opening and an add-on plate for the bottom plate with similar features (see Figure 17) were used to eliminate the effect of the perforation features on the development and growth of the erosion hole.
5.1.2 Instrumentation

The water head distribution along the length of the specimen was monitored using three pressure transducers (PTs) located at 20 mm, 100 mm and 180 mm above the bottom plate on the wall of the erosion chamber. An additional PT at the inlet section on the inlet pipe monitored the water head at the inlet, and the inlet and outlet tanks’ water level were kept constant for the experiments by means of spills. Mechanical loading was applied to the specimen through the piston rod to impose a target value of vertical stress. It was measured using a compression load cell of a maximum 1000 kg-force capacity.

The water discharge rate was measured using a magnetic flow meter appropriate for low flows. Flow rates for discharge values smaller than the minimum measurable discharge for the flow meter (0.01 litres per second) were measured by a magnetostrictive level transmitter by intercepting the collected outflow level in a cubic tank for a specific time interval. The discharge rate was then used to deduce the initiation of erosion, the granulometric changes of the reconstituted soil and its evolution time and also the estimation of flow velocities through the specimens during the tests.

Eroded grains from the suffusion test specimens were captured in the sedimentation tank located parallel to the erosion rig and mounted on an online digital weighing scale with a resolution of ± 0.01 g. The output signals of the weighing scale were directly read by the data logger. This set-up allowed for the accumulation of finer particles during the tests and for online eroded mass weight monitoring.
To facilitate the visualisation of the erosion processes, the erosion chamber was equipped with 96 stainless steel electrodes to perform electrical resistivity analyses and provide 3D tomography.

Figure 18. The 96 stainless steel electrodes installed in the erosion chamber for 3D-ERT acquisitions. The electrodes were installed in 3 rows on two broader sides of the erosion chamber.
images of the specimen (Figure 18). A detailed description of the design, application and performance of the tomography system is presented in Paper VII.

5.1.3 Data acquisition system

An electronic data logger was used to record all outputs from the pressure transducers, the magnetic flow meter, the thermometer, the load cell and the weighing scale. This system comprises a power supply, a signal-conditioning unit and a Metrabyte DAS-16 board connected to a laptop computer. The operation of the 3D-ERT system was controlled by a separate set-up and all the outputs from the resistivity sequences were recorded in a separate laptop computer using eight single-ended analogue input channels. The software used for both the data acquisition systems was LabVIEW software.

5.1.4 Erosion rig calibration

Prior to the erosion tests, the erosion apparatus had been used to run for several rounds of different flow rates and various magnitudes of hydraulic loading in order to obtain apparatus-specific parameters. The data acquired from these calibration tests was used to calibrate the sedimentation tank’s weight measurements with regard to variation in the flow rate. This calibration was necessary to be able to monitor the weight of the accumulated eroded mass in the sedimentation tank.

The elevation readings of the flow tank (by the magnetostrictive level transmitter, as shown in Figure 19) were also calibrated with the discharge. This calibration was done in order to prepare the flow tank to be used for flow rates below the range of the magnetic flow meter. Results from these calibration studies were also used for evaluating the efficiency of the apparatus for various flow rates and quantifying the energy losses through hoses and connections in the set-up.
5.1.5 Description

Gravel and sand with a range of grain sizes were obtained in order to produce the soil mixes for both the suffusion and concentrated leak erosion tests. A sample of each obtained soil type was sieved to ascertain its GSD as illustrated in Figure 20. The silt and clay materials were taken from a local site close to the laboratory and, once oven dried and crushed to powder, were then sieved and separated based on the GSD to be used for producing soil mixes. Test specimens were prepared by mixing different size ranges to attain the target gradation curve for each test type. The specific gravity of all material types and the mixes were also measured separately.

For the suffusion tests, materials were mixed proportionally to reproduce a similar GSD of “material-A” tested by Skempton and Brogan (1994) and subsequently also used by Li (2008). This soil mix grading and its properties are presented in Figure 21.
The soil mixes for both tests were selected because of their notorious susceptibility to internal erosion. For the concentrated leak mechanism test, for example, the core material of Teton dam in Idaho, U.S.A., which failed due to internal erosion in 1976, was replicated for the tests by reproducing the GSD of the dam’s core material as presented in Table 1. Other considerations for the soil mixes were the granulometric criteria for internal erosion and the comparability with results from other studies that used the same soil mixes.
Internal erosion phenomena in embankment dams

Table 1. The core material property for Teton dam as replicated for the concentrated leak tests. This table is taken from Paper VI.

<table>
<thead>
<tr>
<th>Soil Name</th>
<th>USCS</th>
<th>Coarse Fraction (%)</th>
<th>Fine Fraction (%)</th>
<th>Liquid Limit</th>
<th>Plastic Limit</th>
<th>Optimum moisture content (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teton Dam Core Material</td>
<td>CL-ML</td>
<td>0</td>
<td>16</td>
<td>70</td>
<td>14</td>
<td>84</td>
</tr>
</tbody>
</table>

5.1.6 Testing procedure

The test procedure for the suffusion tests consisted of two main phases: 1) consolidation, and 2) multi-stage seepage flow. The purpose of the first phase was to attain the target vertical stress on the specimen. The purpose of the second phase was to gradually increase the hydraulic loading and monitor the initiation of internal instability and, once the erosion was initiated, to monitor the mass removal rate due to the erosion.

Three types of tests were performed under four distinct mechanical loadings equivalent to 0 kPa, 10 kPa, 25 kPa and 50 kPa which compressed the specimen in the vertical direction. These test types had the following aims:

1) to determine the initiation of erosion due to internal instability. In these tests, under a constant mechanical load, the hydraulic head on the upstream tank was increased stepwise until the initiation of internal erosion was detected. These tests are referred to as Test 1.1-Test 4.2 in the text and in Paper V.

2) to examine suffusion under continuous hydraulic loading. For these tests, the hydraulic loading of the specimen was stepwise increased until it reached the initiation of erosion and, thereafter, the mechanical and the upstream water head were kept constant and the mass removal was monitored over time. These tests are referred to as suffusion tests type (A) in the text and in Paper V.

3) to examine the suffusion rate under a range of prescribed hydraulic loadings. In contrast to suffusion test type (A), the hydraulic head was increased stepwise after the initiation of erosion, and was only kept constant for a prescribed period of time for each step before being raised to a higher level. The mass removal was monitored for each step. These tests are referred to as suffusion tests type (B) in the text and in Paper V.

The concentrated leak test procedure, by conducting hole erosion tests (HET), also consisted of the same two phases: 1) consolidation, and 2) multi-stage seepage flow. The aim of the first phase was to attain the target vertical stress on the specimen where a metal rod was used to drill the erosion hole, 6 mm in diameter,
through the longitudinal axis of the compacted specimen. The aim of the second phase was to determine the critical hydraulic head at which internal instability occurred. HET tests were performed under 5 different mechanical loadings of 0 kPa, 25 kPa, 50 kPa, 75 kPa and 100 kPa where, under constant mechanical loading, the hydraulic loading was increased stepwise by lifting up the upstream tank until the initiation of the erosion was detected and kept constant while the erosion hole grew and the discharge increased.

5.1.7 Monitoring of erosion

The removal of mass in the suffusion phenomenon was recorded by an online monitoring of the sedimentation tank weight. Adopting the calibrations, the submerged, accumulative and eroded mass over time was extracted from the recorded data from the tank’s weight. The removal of mass by suffusion was also monitored by conducting 3D-ERT acquisitions. These acquisitions were taken for each suffusion test at the start, at every hour interval after the initiation of erosion, and after the test was completed.

Turbidity measurements were also taken after each hydraulic loading step until the initiation of erosion and after the initiation at fixed-time intervals until the visual turbidity diminished. At the end of every test, the eroded mass from the sedimentation tank was gathered, dewatered, oven dried and weighed to be used both as calibration data and to extract the weight of the “dry” cumulative eroded mass from the “submerged” eroded mass recordings available from the sedimentation tank weight monitoring.

Erosion monitoring for the concentrated leak mechanism was carried out by adopting the standard method used for HET tests (Wan and Fell 2002, 2004a, 2004b). The Bonelli (2013) method was not adopted because, for this method to be adopted correctly, a turbulent flow regime (Reynolds number >5000) needs to prevail in the erosion hole to fulfil the method’s assumptions. However, the conducted experiments for this study mostly had smaller relative Reynolds numbers and therefore the standard method was found to be more appropriate for data analysis.

5.2 Supplementary numerical studies

5.2.1 Effective stress distribution in a soil volume

Effective stress has been shown to have a significant role in the initiation of internal erosion (Moffat, 2005; Li, 2008; Crawford-Flett, 2014), specifically when suffusion phenomena are considered. In order to establish a relationship between the erosion initiation and effective stress, the effective principal stress magnitude at any location within the porous media must be accurately described.

In the laboratory tests, vertical stress ($\sigma_v$) was applied to the top surface of the specimen and kept constant during testing. This
applied vertical stress varied along the depth of the specimen because of the friction between the sidewall and the soil, yielding a depth-dependent vertical stress pattern decreasing by depth. Horizontal stresses ($\sigma_H$ and $\sigma_A$) are dependent on the vertical stress and sidewall displacement and any change in vertical stress also results in a change in the horizontal stress at the same point.

The relationships between vertical and horizontal (lateral) stresses are complex. The most common theories of lateral earth pressure are based on the classical methods of Coulomb (1776) and Rankine (1857), which are both based on the limit equilibrium theory. They are widely used because of their simplicity. However, these methods provide insufficient information regarding the distribution and magnitude of lateral earth stress produced by different magnitudes of wall displacement and soil wall friction. Therefore, they are only valid for the limiting condition of substantial ground and wall movements. In order to evaluate the actual principal soil stresses from the soil and sidewall interactions, numerical modelling is necessary and therefore included in the current study in order to estimate the principal stresses in the packed soil samples.

### 5.2.2 Numerical modelling of the soil stresses

For estimating the principal stresses in the packed soil samples, a 3D numerical model of the erosion rig was developed in COMSOL Multiphysics software where a plane-strain condition was considered for the behaviour. The Plexiglas sidewalls were considered to be elastic and a linear isotropic elastic model was used to model them. The extended Drucker-Prager plasticity model was used with a non-associated flow rule to analyse the behaviour of the packed soil. Analyses were carried out using COMSOL Multiphysics software.

The soil parameters of this model were taken from the triaxial test results conducted on similar soil mixtures from Carter and Bentley (2016). A complete interaction of the soil and wall interfaces was taken into account and any arbitrary motion of the surfaces was allowed. Tangential interaction between the wall and the soil was defined using the static-kinetic exponential decay function. Several amounts of sidewall movement (lateral displacement) were considered and the horizontal and vertical effective stress distributions by the wall were evaluated (see Figure 22).
For the HET tests, the effect of stress reduction due to the lateral movement of the walls and the friction between the walls and the soil was similarly extracted from the numerical simulations on COMSOL and the vertical and lateral stress coefficients were assigned as 0.57 and 0.40 respectively. For this mechanism of erosion, however, the effect of the special top perforated capping (see Figure 23) on the vertical stress distribution on the soil underneath was also found to be very significant. It was found that the big opening of the top cap (diameter =50 mm) results in a conical zone directly underneath the opening with reduced vertical stress. This zone, which extends for approximately 22 mm into the soil, affected the stress field around the erosion hole significantly. The applied stresses were much lower in this field compared to the stresses at a greater depth.

Figure 23. The effect of the top capping on vertical stress distribution in the HET tests. a) and b) The stress redistribution residual contours simulated in COMSOL in x and y planes from the centre of the hole. c) Photograph of mould from the erosion scar taken from HET test no.7 that clearly demonstrates the effect of the stress reduction on top of the sample.
This effect of the top capping in directing the applied stresses was taken into account while developing the constitutive law for concentrated leak erosion and its empirical coefficients.

5.3 Results and analysis

5.3.1 Initiation of internal erosion

From both the tests for suffusion and concentrated leak erosion mechanisms, data for the initiation of erosion as well as the erosion rate were extracted. Three types of tests with a total of 16 sets of testing were conducted for the suffusion mechanism: 8 tests for the initiation of erosion, 4 tests with constant hydraulic head and 4 with increments of hydraulic head. For the concentrated leak mechanism, 8 tests of HET were performed: 3 commission tests and 5 with a constant hydraulic head applied.

For tests that examine the initiation of erosion on suffusion, a new criterion was considered for detecting the initiation. In this method, in addition to incorporating initiation to the sudden changes in local hydraulic conductivity (i.e. the method adopted by Moffat, 2005; Li, 2008; Crawford-Flett, 2014), the changes of flow were monitored at constant upstream water-head conditions in order to detect the initiation of internal erosion by means of a continuous change in the flow magnitude under that constant hydraulic loading condition. This was monitored for each level of the upstream water head for each test that was conducted to detect the initiation of suffusion by monitoring the continuous slope change in the measurements of the discharge curve versus time under constant hydromechanical loading.

Figures 24 and 25 present the detection of initiation for the results from Test 1.1 of the initiation detection tests with both methods. All the results from the suffusion initiation detection tests are presented in Table 2.

![Figure 24. The local and global hydraulic conductivity monitoring of a specimen during Test 1.1. The y-axis to the right in red is for the K_BC which is plotted in a secondary axis. This figure is taken from Paper V.](image-url)
Table 2. A summary of the set-up and the results from suffusion onset determination tests. This table is taken from Paper V.

<table>
<thead>
<tr>
<th>Test ID</th>
<th>Reconstituted sample</th>
<th>Onset of internal erosion - Local</th>
<th>Onset of internal erosion- continuous</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Load</td>
<td>Length</td>
<td>Dry Unit weight</td>
</tr>
<tr>
<td></td>
<td>$F$ (kN)</td>
<td>$L$ (mm)</td>
<td>$\gamma_d$ (g/cm$^3$)</td>
</tr>
<tr>
<td>Test-1.1</td>
<td>0</td>
<td>187</td>
<td>1.66</td>
</tr>
<tr>
<td>Test-1.2</td>
<td>0</td>
<td>199</td>
<td>1.76</td>
</tr>
<tr>
<td>Test-2.1</td>
<td>10</td>
<td>196</td>
<td>1.86</td>
</tr>
<tr>
<td>Test-2.2</td>
<td>10</td>
<td>197</td>
<td>1.91</td>
</tr>
<tr>
<td>Test-3.1</td>
<td>25</td>
<td>196</td>
<td>1.83</td>
</tr>
<tr>
<td>Test-3.2</td>
<td>25</td>
<td>195</td>
<td>1.85</td>
</tr>
<tr>
<td>Test-4.1</td>
<td>50</td>
<td>193</td>
<td>1.88</td>
</tr>
<tr>
<td>Test-4.2</td>
<td>50</td>
<td>190</td>
<td>1.87</td>
</tr>
</tbody>
</table>

The initiation of erosion for the concentrated leak erosion mechanism (i.e. HETs) was simply set to the moment before the occurrence of progressive erosion, when the flow rate grew under constant upstream water head until the test stopped.

5.3.2 Rate of internal erosion

For suffusion tests, the rate of erosion was evaluated by monitoring the continued weight of the sedimentation tank’s data, and by extracting the cumulative sediment mass weight curve in this tank. Time differentiating this cumulative curve gave the eroded mass over time (mass removal rate). For this differentiation, two methods were adopted. In the first method, the rate was extracted directly from the measured data. For the second method, a curve fitting was done to the collected data,
where the rate of erosion was evaluated by time differentiating these fitted curves. These two methods and their implementation are described in detail in Paper V. The maximum mass removal rates for all the four suffusion tests type (A) are presented in Table 3.

**Table 3. A summary of the results from suffusion tests type (A) where the mass removal rates were evaluated with two methods. “Span” refers to the data smoothing interval applied to the test data. This table is taken from Paper V.**

<table>
<thead>
<tr>
<th>Test name</th>
<th>Cumulative dried mass</th>
<th>Experiment time</th>
<th>Logistic function ( a/[1+e^{(t-t_0)}] )</th>
<th>Natural logarithmic function (-Ae^{(-Bt)}+C)</th>
<th>Maximum erosion rate from the curve ( \Delta m/\Delta t ) ((kgs^{-1}))</th>
<th>Maximum erosion rate from the smoothed data ( \Delta m/\Delta t ) ((kgs^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( m ) g</td>
<td>( t ) s</td>
<td>( a )</td>
<td>( B )</td>
<td>( c )</td>
<td>( t &lt; )</td>
</tr>
<tr>
<td>Test-A-1</td>
<td>432.68</td>
<td>92000</td>
<td>0.371</td>
<td>1.597</td>
<td>3.03</td>
<td>31060</td>
</tr>
<tr>
<td>Test-A-2</td>
<td>417.63</td>
<td>160000</td>
<td>0.235</td>
<td>3.358</td>
<td>1.837</td>
<td>20730</td>
</tr>
<tr>
<td>Test-A-3</td>
<td>214.87</td>
<td>200000</td>
<td>0.099</td>
<td>8.588</td>
<td>0.433</td>
<td>9076</td>
</tr>
<tr>
<td>Test-A-4</td>
<td>379.61</td>
<td>346000</td>
<td>0.291</td>
<td>4.229</td>
<td>0.195</td>
<td>4750</td>
</tr>
</tbody>
</table>

The rate of erosion for a concentrated leak mechanism calculated from the hole’s radius growth rate was described as the mass removal per unit surface in order to include the effect of the porosity and density of the soil material. All the results from the HET tests are summarised in Table 4 where both the maximum and the average erosion rates are calculated from all the HET test results. The final hole radius for the HET tests is taken from the measurements after each test. The experimental results and the evaluation are presented in detail in Paper VI.

**Table 4. A summary of the set-up and results from all the HET tests. This table is taken from Paper VI.**

<table>
<thead>
<tr>
<th>Test ID</th>
<th>Mechanical load ( F ) ((kPa))</th>
<th>Length ( L ) ((mm))</th>
<th>Saturated unit weight ( \gamma_{sw} ) ((g/cm^3))</th>
<th>Initial hole radius ( r_0 ) ((mm))</th>
<th>Final hole radius ( r ) ((mm))</th>
<th>Flow regime ( Re ) range</th>
<th>Maximum erosion rate ((kgm^{-3}s^{-1}))</th>
<th>Average erosion rate ((kgm^{-3}s^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>HET-1</td>
<td>0</td>
<td>197</td>
<td>2.04</td>
<td>3</td>
<td>11</td>
<td>200-5013</td>
<td>4.14E+04</td>
<td>2.72E+03</td>
</tr>
<tr>
<td>HET-2</td>
<td>0</td>
<td>201</td>
<td>2.04</td>
<td>3</td>
<td>11</td>
<td>200-6400</td>
<td>4.01E+04</td>
<td>2.37E+03</td>
</tr>
<tr>
<td>HET-3</td>
<td>0</td>
<td>181</td>
<td>2.04</td>
<td>3</td>
<td>25</td>
<td>200-3500</td>
<td>3.77E+04</td>
<td>2.32E+03</td>
</tr>
<tr>
<td>HET-4</td>
<td>0</td>
<td>181</td>
<td>2.04</td>
<td>3</td>
<td>20</td>
<td>220-1800</td>
<td>3.12E+04</td>
<td>2.48E+03</td>
</tr>
<tr>
<td>HET-5</td>
<td>75</td>
<td>225</td>
<td>2.04</td>
<td>3</td>
<td>29</td>
<td>300-3150</td>
<td>3.28E+04</td>
<td>1.73E+03</td>
</tr>
<tr>
<td>HET-7</td>
<td>50</td>
<td>215</td>
<td>2.04</td>
<td>3</td>
<td>15</td>
<td>400-2400</td>
<td>2.06E+04</td>
<td>2.44E+03</td>
</tr>
<tr>
<td>HET-8</td>
<td>100</td>
<td>217</td>
<td>2.04</td>
<td>3</td>
<td>15</td>
<td>350-7300</td>
<td>2.34E+04</td>
<td>2.77E+03</td>
</tr>
<tr>
<td>HET-10</td>
<td>25</td>
<td>217</td>
<td>2.04</td>
<td>3</td>
<td>19</td>
<td>400-2800</td>
<td>2.28E+04</td>
<td>2.68E+03</td>
</tr>
</tbody>
</table>
5.3.3 Constitutive law for erosion for both mechanisms

The constitutive law for the suffusion mechanism \( \dot{m} \) was defined to be in a similar form to the concentrated leak erosion mechanism (i.e. \( \dot{m}_{\text{HET}} \) as presented in Eq. 32) and to be driven by excess shear stress application. This equation reads:

\[
\dot{m} = C \left( \tau_w - \tau_c \right) , \quad \dot{m}_{\text{HET}} = C_{\text{HET}} \left( \tau_w - \tau_c \right)
\]

(51)

where \( C \) and \( C_{\text{HET}} \) are the coefficients of soil erosion for suffusion and concentrated leak mechanisms respectively, \( \tau_w \) is the in-situ total shear stress in the media and \( \tau_c \) is the critical shear stress to trigger the erosion process.

From the conducted studies, it was found that \( \tau_c \) is not a constant value for either of the mechanisms and is, rather, dependent on both the soil stresses and the hydraulic loading applied on the specimen. For this, a new, modified hydromechanical envelope was suggested based on the Mohr-Coloumb shear strength envelope to define and quantify this critical shear stress level. The definition of this new theoretical model (i.e. a modified hydromechanical envelope, MHE) and a description of the method of extracting the critical shear stress level are described in detail in Paper V.

Applying the theoretical model of the modified hydromechanical envelope to the results from the suffusion and concentrated leak tests resulted in the behaviour presented in Figures 26 and 27 for two of the conducted tests from each erosion phenomenon. A complete analysis of the data to produce the hydromechanical path from a single test’s results and the hydromechanical envelope for a set of tests undergoing internal erosion is presented for the suffusion phenomenon in Paper V and for the concentrated leak phenomenon in Paper VI.

![Figure 26. The hydromechanical path of Soil-A from suffusion Test 1.1. The half-circles show the in-situ stress status at the 1\(^{\text{st}}\), 3\(^{\text{rd}}\), 8\(^{\text{th}}\) and 13\(^{\text{th}}\) (last) hydraulic loading step. The principal stresses are marked with dotted-line circles and the total induced shear stresses, including the flow-induced shear, with solid-line circles. This figure is taken from Paper V.](image)
This definition of $\tau_c$ and its adaptation in Eq. 50 implies a stress dependency of the constitutive law of erosion for both of the initiation and the mass removal rates of both the studied internal erosion phenomena.

The initiation is controlled by the implementation of the modified hydromechanical envelope and the evaluation of the in-situ shear stresses compared to the shear resistance for the material. In this framework, when the total shear stresses in the media from both the mechanical and hydraulic loadings reach the erosion initiation envelope, the constitutive law is activated and mass removal is applied.

The erosion rate is controlled by the excess shear stress applied above this critical shear stress for the material, evaluated from the modified hydromechanical envelope. The coefficient of this law is assigned by fitting Eq. 50 to the data from the experiments for various erosion mechanisms as presented in Figures 28 and 29 for suffusion and concentrated leak mechanisms respectively.
Interesting findings were attained from the 3D resistivity tomography studies, conducted using the 3D-EIT system and designed and constructed to study internal erosion phenomena in porous media. A detailed description is presented in Paper VII.

In this study, images of the electrical properties of conductive materials on a laboratory scale were successful in capturing the suffusion process in granular cohesionless porous media. These 3D visualisations were successfully achieved by discretising the media to elements and extracting material resistivity maps at certain times for these elements by applying inverse modelling. The changes of the resistivity values over time were then related to

\[ \dot{m} = 1.9 \times 10^{-3} \times (\tau_b - \tau_{\text{cm}}) \]

\[ \dot{m} = 6.012 \times 10^{-6} \times (\tau_b - \tau_{\text{cm}}) \]

\[ \dot{m} = 6.012 \times 10^{-6} \times (\tau_b - \tau_{\text{cm}}) \]

Figure 28. Extracting the coefficient of the mass erosion rate for suffusion (C) by applying linear regression to mass removal versus induced excessive shear stresses. For Soil-A from Skempton and Brogan (1994) and suffusion test type (B), C = 1.9 \times 10^{-3} \text{ (m}^2 \text{s)} was extracted. This figure is taken from Paper V.

Figure 29. Extracting the coefficient of the mass erosion rate for a concentrated leak mechanism (\( C_{\text{HET}} \)) by applying linear regression to mass removal versus induced excessive shear stresses. For core material of Teton dam from HET tests, \( C_{\text{HET}} = 6.012 \times 10^{-6} \text{ (m}^2 \text{s)} \) was extracted. This figure is taken from Paper VI.
the soil structural changes which occurred due to internal erosion processes. This coupling was achieved by adopting Archie’s law (Archie, 1942) and calibrating its parameters with the experimental results from the laboratory tests as presented for a case of the suffusion type (A) tests in Figure 30. Further details of the design of the 3D-ERT system, data acquisition, as well as the analysis of the results are given in Paper VII.

Figure 30. The resistivity changes compared to the initial resistivity after 20, 22, 27, 32, 37 and 43 hours after erosion started, and a comparison of the last image with photographic evidence of the eroded material. This figure is taken from Paper VII.
CHAPTER 6

CONCLUSION: SUMMARY AND RECOMMENDATIONS

6.1 Summary of research findings

Results from the pumping field tests and large-scale laboratory experiments demonstrate that the coefficients of the nonlinear flow law (i.e. the momentum equation) depend on the Reynolds number for pore Reynolds numbers less than 60,000. Above this Reynolds number asymptote, these coefficients were found to be constant values. The magnitude of the throughflows studied lies far beyond the ranges investigated experimentally by previous researchers on the subject. The laboratory experiments with the large permeameter filled with coarse rockfill material covered pore Reynolds numbers as large as 220,000 for grains with diameter size distributions in the range 100-160 mm and as large as 320,000 for the range 160-240 mm.

Numerical Computational Fluid Dynamics (CFD) studies were done, which successfully modelled the complex flow around virtual cobbles, thereby overcoming the limits and constraints of the laboratory experiments. The novelty of the numerical approach lies in the use of results from the large-scale permeameter experiments to constrain the three-dimensional numerical simulations, leading to a fully calibrated and validated model that is applicable to flows through the cobble-sized material. This fully calibrated and validated numerical three-dimensional model was then used to conduct numerical experiments. One major aim was to extend the investigation of the parameters of the flow law to pore Reynolds numbers as large as $10^6$. Further, a Lagrangian particle tracking method was applied in order to estimate the length distribution of the flow channels in the porous media.

A Gamma distribution was fitted to the histogram of the normalised particle trajectory lengths between the inflow and outflow sections for particles released at various discharges. The shape of the fitted Gamma distribution and consequently its scale and shape parameters were found to be dependent on pore Reynolds number. The proposed normalised length model can be used to evaluate permeability, energy dissipation, induced forces and diffusion in porous media. Additionally, the shear forces exerted on the coarse particles in the porous media were found to be significantly dependent on the inertial forces of the flow which could be estimated using the proposed equation for developed turbulent flows in porous media.

For the conducted laboratory experiments on the two internal erosion mechanisms of suffusion and concentrated leaks, it was observed that both the initiation and the mass removal rate of suffusion and concentrated leaks were dependent on the soil
stresses. To reflect this dependency, a new, modified hydromechanical envelope model was developed based on the Mohr-Coulomb failure criterion for detecting the initiation of internal erosion. This model involves both the granulometric and shear strength properties of the soil. The coefficients of the hydromechanical envelope model were successfully evaluated from the experimental data. The hydromechanical envelope $\tau_{en} = 0.0385 + \sigma \tan(25.37)$ was extracted for Soil-A from suffusion Tests 1.1 to 4.2. From the data taken from the suffusion test type (B), the coefficient for the constitutive law of erosion $C = 1.9 \times 10^{-3}$ was extracted by data fitting. The constitutive law of erosion for the suffusion mechanism $\dot{m} = 1.9 \times 10^{-3} \left[ \tau_s - \tau_{en} \right]$ was suggested where $\tau_s$ and $\tau_{en}$ are in (kN m$^{-2}$) and $\dot{m}$ is in (kg m$^{-3}$ s$^{-1}$).

The hydromechanical envelope $\tau_{en} = 2.471 + \sigma \tan(21.77)$ was extracted for the concentrated leak mechanism from the HET-1 to HET-10 tests. From the HETs’ data, the coefficient for the constitutive law of erosion $C_{HET} = 6.012 \times 10^{-6}$ ($I_{HET} = 5.221$) was extracted by data fitting. The constitutive law of erosion for the concentrated leak mechanism $\dot{m}_{HET} = 6.012 \times 10^{-4} \left[ \tau_s - \tau_{en} \right]$ was suggested where $\tau_s$ and $\tau_{en}$ are in (Nm$^{-2}$) and $\dot{m}_{HET}$ is in (kgm$^{-2}$ s$^{-1}$).

A three-dimensional electrical-resistivity-based tomography method adopted for the erosion rig successfully visualized the porosity evolution occurring in the soil due to suffusion in the tested soil (i.e. Soil-A). Data fitting with unbounded adaptation for Archie’s law parameters resulted in $\alpha = 6.65$ and $m = 0.11$ for the tortuosity factor and the cementation exponent respectively. With bounded calibration, where the cementation exponent was bound to be higher than 1.0, $\alpha = 3.35$ and $m = 1.09$ were extracted. The bound data fitting resulted in more reliable parameters for Archie’s law, and a better estimation of the porosity value of both the media and the cumulative eroded mass due to the suffusion phenomenon. This three-dimensional tomographic data could also be used to validate numerical models for modelling the suffusion phenomenon.

### 6.2 Recommendations for future studies

- Adopt the developed flow laws for numerical models to study scenarios of dams with high flow at their tails.

- Implement the suggested experimentation method and conduct more experimental studies on various types of rockfill material to extract parameters for the flow laws.

- Conduct 3D numerical CFD studies of crushed rockfill material to study the numerical model parameters and to create a three-dimensional validated model. This validated model could then be used to conduct extensive numerical experiments on coarse crushed rockfill material.
• Implement the suggested experimentation method and conduct more experimental studies on various types of soil to extract parameters for the constitutive law of erosion.

• Conduct numerical simulations by adopting both the modified hydromechanical envelope model and the constitutive laws of erosion and validate the models with the 3D-ERT images.

• Conduct extensive experimentation using the 3D-ERT set-up to achieve a better calibration/validation database.
Internal erosion phenomena in embankment dams

REFERENCES


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APPENDED PAPERS