Investment Opportunities for Swedish Life Insurance Companies

PONTUS RUFELT
Investment Opportunities for Swedish Life Insurance Companies

PONTUS RUFELT

Master’s Thesis in Financial Mathematics (30 ECTS credits)
Master Programme in Applied and Computational Mathematics (120 credits)
Royal Institute of Technology year 2016
Supervisor at Towers Watson: Marcus Granstedt
Supervisor at KTH: Fredrik Armerin
Examiner: Boualem Djehiche

TRITA-MAT-E 2016:68
ISRN-KTH/MAT/E--16/68--SE
Investment Opportunities for Swedish Life Insurance Companies

Abstract
Since the new risk sensitive regulation Solvency II was enabled the 1st of January 2016 the European insurance companies have to review their investment strategies. Insurance companies are among the largest institutional investors in Europe holding EUR 6.7 trillion assets, thus major changes in their asset management can impact the capital markets. To investigate how the investing opportunities have changed for life insurance companies, a representative Swedish life insurance company with an occupational pension portfolio was simulated for thirty years. This was made by first simulating the money market, bonds, equities and real estate for the simulated time by a stochastic multivariate process. Using Modern Portfolio Theory the portfolio weights was constructed for the financial asset portfolios for the model of the company. To determine future liabilities a representative ITP 2 pension portfolio was modelled where the pension policies was priced using traditional life insurance pricing theory in continuous time. For the company to be representative actuarial assumptions and as well as a consolidation policy was constructed in line with the major traditional life insurance companies in Sweden. The simulations of the company resulted in monthly cash flows, development of life insurance mathematical functions and the solvency capital requirements. The solvency capital requirement by Solvency II was calculated by applying the standard formula handed by EIOPA, where for life insurance companies the market risk module dominates in contribution to the capital requirement. By comparing the new risk sensitive capital requirement with the solvency capital requirement by the old regulations a change of structure dependent on time and asset allocation was observed. The Solvency II capital requirement for life insurance companies is clearly more dependent on the financial asset strategy for the company whereas the old capital requirement is not. The structure of the new capital requirement follows the same structure as the solvency market risk module where it is clear that low risk portfolios does not necessarily correspond to a lower capital requirement. The conclusion of this thesis is that life insurance companies in Sweden have tightened financial investing opportunities. This is due to Solvency II since this regulation is more risk sensitive than the old regulation.
Investeringsmöjligheter för svenska livförsäkringsbolag

Sammanfattning
Acknowledgements

I would personally like to thank my supervisor Marcus Granstedt for the opportunity and insight in the insurance business from an actuarial perspective as well as support throughout the project. I am very grateful to Ph.D Fredrik Armerin at KTH Royal Institute of Technology for his supervision and valuable support during this thesis project.

I would also like to thank my friends and study partners from my studies in Vehicle Engineering, Engineering Physics and Financial Mathematics at KTH Royal Institute of Technology for the support and all the great laughs during my five years of studies. Last but far from least, I want to direct my dearest gratitude and love to my family for all the love and support throughout my studies.

Stockholm, September 2016
Pontus Rufelt
# Contents

1 Background .......................................................... 1

2 Data Analysis ......................................................... 4
   2.1 Geometric Brownian Motion .................................. 4
   2.2 Multivariate Geometric Brownian Motion .................. 5
   2.3 Ornstein-Uhlenbeck Process ................................. 7
   2.4 Choosing Historical Data ...................................... 8
   2.5 Checking Financial Modelling Assumptions ................. 11
   2.6 Parameter Estimation and Simulation of Future Returns 17

3 Solvency Capital Requirement ................................. 25
   3.1 Best Estimate of Liabilities Calculation ................. 26
   3.2 Calculation of the Solvency Capital Requirement .......... 31
      3.2.1 Basic Solvency Capital Requirement .................. 31
      3.2.1.1 Market Risk Module ............................ 33
      3.2.1.2 Life Underwriting Risk Module .................. 36
   3.3 Risk Margin .................................................. 38

4 Modelling ............................................................. 39
   4.1 Modern Portfolio Theory ..................................... 39
   4.2 Actuarial Assumptions and Model Points ................. 40
   4.3 Asset Liability Management ............................... 50

5 Results and Discussion ............................................ 55
   5.1 Portfolio Optimization ...................................... 55
   5.2 Solvency Analysis .......................................... 59
   5.3 Conclusions and Further Studies ......................... 69

A Appendix ............................................................ 71
   A.1 Chapter 2 .................................................. 71
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.2 Chapter 3</td>
<td>79</td>
</tr>
<tr>
<td>A.3 Chapter 4</td>
<td>82</td>
</tr>
<tr>
<td>A.4 Chapter 5</td>
<td>85</td>
</tr>
</tbody>
</table>
Chapter 1

Background

The 1st of January 2016 a new directive from the European Union was enabled, namely the Solvency II Directive (2009/138/EC) \[8\], with an aim of modernizing the regulation regarding risk exposure for insurance companies in the member states of the European Union. Solvency can be described as a company’s ability to meet its long term liabilities and to accomplish long term expansion and growth. It is important for insurance companies to be solvent to ensure that the company has resources to pay its claims. The EU-Directive specifies that the insurance company needs to fulfill a solvency capital requirement which can be determined based on a standard formula or a (partial-)internal model. The new capital requirement is risk sensitive meaning that the size of solvency capital requirement is linked to the risk exposures of the insurance company.

The solvency capital requirement is based on the familiar risk measure Value-at-Risk (VaR). Value-at-Risk at level \( p \in (0, 1) \) at time 0 of a random variable \( X \) at time 1 is defined as

\[
\text{VaR}_p(X) = \min\{m \in \mathbb{R} : P(mR_0 + X < 0) \leq p\}
\]

where \( R_0 \) is the percentage return of a risk-free asset. In other words \( \text{VaR}_p(X) \) can be interpreted as the smallest amount of money \( m \) such that the probability of the loss being at most \( m \) is at least \( 1 - p \). Suppose that an insurance company has an insurance portfolio with a current market value of insurance liabilities \( L \) and a current market value of assets denoted by \( A \). By EIOPA \[5\] the solvency capital requirement should correspond to the 99.5%-percentile of Value-at-Risk of the difference between \( A \) and \( L \) of an insurance undertaking over a one year period, i.e. \( p = 0.005 \) in the definition of Value-at-Risk above. Another common interpretation, but not strictly true, is that the risk of defaulting should not be larger than one in two hundred years.
For a typical life insurance company the insurer, i.e. the insurance company, promises to pay a sum of money (benefit), as set out in the contract, in exchange for a regular premium or as a single premium, by the policy holder, i.e. the insured. In life insurance these policies are often long term. One example is a pension product such as occupational pension. In this case the company for which an individual works for pays premiums regularly to an insurance company during the time the person is employed. The individual will then receive a cash flow, often monthly, from the time of retirement till maturity age or when the person dies. As one can suspect from this scenario, the time between employment and retirement age (and maturity time) is long and hence long term investments need to be made by the life insurance company to fulfill its obligations. Since they are long term, the investment strategy for the life insurance company has to involve risk free financial instruments due to the long investment horizon. Investing in only risky assets can be considered as a too "risky" strategy for long term investments. One can therefore consider a life insurance company as a somewhat risk averse investor which means that the insurance company is not interested in large portions of risk and therefore aims to minimize the variance for a given expected return for its financial portfolio.

Examining investment opportunities for insurance companies in light of the new regulations is an important matter due to the fact that insurance companies are among the largest institutional investors in Europe. According to Fitch Ratings [17], the European insurance companies combined hold EUR 6.7 trillion assets, thus major changes in their asset management due to new regulations can impact pricing of assets in capital markets. From the same report we also see that the investments in the financial markets dominates in contribution to the solvency capital requirement for a life insurance company followed by risks due to life risk exposure.

In addition to the Solvency II regulatory the IFRS 4 Phase II (International Financial Reporting Standards) issued by the International Accounting Standards Board (IASB) [13] is assumed to take affect in the beginning of 2018. This financial reporting standard is an accounting guidance for insurance contracts and will be constructed in light of IFRS 9 which will be effective for annual periods on January 2018, which replaces IAS 39. IFRS 9 [12] addresses the accounting for financial instruments and contains of three main topics; classification and measurement of financial instruments, impairment of financial assets and hedge accounting.

One may suspect that these accounting changes yields adjusting the asset management for insurance companies. The insurance companies will not only be required to fulfill the capital requirement from the Solvency II regulation, they also need to follow the new

---

1. In this example we neglect the fact that the insured may have a death cover or survivor pension, which means that e.g. the family of a dead individual can receive cash flow if the person dies before the maturity of the contract.
financial reporting standards from IASB. It follows that the insurance companies faces new major challenges besides competition within the line of business. In this thesis we choose to examine how the investment opportunities for a fictive Swedish life insurance company will change in light of the new solvency regulations and in comparison to the old solvency regulations. Note that the upcoming IFRS 4 accounting standard will not be considered since information regarding this standard is yet not sufficient to be implemented in the thesis.
Chapter 2

Data Analysis

Since a fictive life insurance company will be considered the analysis of the financial assets will be crucial. A common problem for life insurance companies is that each individual policy will be alive for a long time which implies long investment horizons in order to match future liabilities. Long investment periods implies that the company needs to make a forecast or construct a model of how each financial asset will evolve over a long time period. In this report the financial assets will be simulated using the geometric Brownian motion (GBM) approach. This approach is the most common model for stock prices and also used in the Black-Scholes model \([3]\). Using the GBM method to model stock prices is powerful since the expected returns are independent of the stock price which is expected in real life \([15]\) and the calculations are straightforward since it assumes normally distributed logarithmic return within non overlapping time steps. The STIBOR rate will also be considered and modelled using the Vasiček model which is an Ornstein-Uhlenbeck process (OU). This chapter first introduces the reader to stochastic calculus, the GBM-model and the OU-model. Then the theory will be applied to the nine time series considered in this thesis.

2.1 Geometric Brownian Motion

Consider a stochastic process \(X\). This stochastic process is said to follow a geometric Brownian motion if it satisfies the stochastic differential equation (SDE) \([3]\)

\[
dX_t = \mu X_t dt + \sigma X_t dW_t,
\]
where $\mu \in \mathbb{R}$ and $\sigma > 0$ are the parameters corresponding to drift and volatility respectively and $W_t$ is a one-dimensional Wiener process (Brownian motion). The first term of the right hand side controls the upwards or downwards trend of the process while the latter one controls the randomness in the process. In order to solve this equation Itô’s formula is applied to the dynamics of $\ln X$. Applying Itô’s formula yields

$$d\ln X_t = \frac{1}{X_t} dX_t - \frac{1}{2X_t^2} (dX_t)^2 = \left( \mu - \frac{1}{2} \sigma^2 \right) dt + \sigma dW_t.$$ 

Integration from time 0 to time $t$ yields

$$\ln X_t = \ln X_0 + \left( \mu - \frac{1}{2} \sigma^2 \right) t + \sigma W_t,$$

We finally arrive at the solution of this SDE:

$$X_t = X_0 e^{\left( \mu - \frac{1}{2} \sigma^2 \right) t + \sigma W_t}.$$ 

Recall that $W_t \sim N(0, \sqrt{t})$ using the $N(\mu, \sigma)$ notation. The expected value and the variance of the stochastic process $X$ is then given by

$$E [X_t] = X_0 e^{\mu t} \quad \text{and} \quad \text{Var} [X_t] = X_0 e^{2\mu t} \left( e^{\sigma^2 t} - 1 \right).$$

respectively. The stochastic process is log-normally distributed with expected value $X_0 e^{\mu t}$ and variance $X_0 e^{2\mu t} \left( e^{\sigma^2 t} - 1 \right)$.

Since financial assets will be considered it is of interest to examine how the price changes per time step. In the GBM-model the logarithmic returns $\ln \frac{X_t}{X_{t-\Delta t}}$ are normally distributed which can be seen directly from the SDE for $\ln X_t$. We conclude that

$$R_t := \ln \frac{X_t}{X_{t-\Delta t}} \sim N \left( (\mu - \frac{1}{2} \sigma^2) \Delta t, \sigma \sqrt{\Delta t} \right).$$

2.2 Multivariate Geometric Brownian Motion

In this thesis we consider multiple financial assets. Assuming that these assets, or the stochastic processes in this model, are uncorrelated is a very naive assumption. Thus, the
multivariate GBM-model with correlated Brownian motions will be considered. Consider the system

\[ dX_t^k = \mu_k X_t^k dt + \sigma_k X_t^k dW_t^k, \quad k = 1, \ldots, n, \]

where \( W_t^1, \ldots, W_t^n \) are correlated Brownian motions with \( dW_t^i dW_t^j = \rho_{i,j} dt \), where \( \rho \) is the constant (Pearson's) correlation matrix given by

\[
\rho = \begin{pmatrix}
1 & \rho_{1,2} & \cdots & \rho_{1,n} \\
\rho_{2,1} & 1 & \cdots & \rho_{2,n} \\
\vdots & \vdots & \ddots & \vdots \\
\rho_{n,1} & \rho_{n,2} & \cdots & 1
\end{pmatrix}
\]

The correlation matrix \( \rho \) is a symmetric positive semidefinite matrix. We rewrite \( \rho \) using the Cholesky decomposition, i.e. write \( \rho = L L^T \), where \( L \) is on the form

\[
L = \begin{pmatrix}
L_{1,1} & 0 & \cdots & 0 \\
L_{2,1} & L_{2,2} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
L_{n,1} & L_{n,2} & \cdots & L_{n,n}
\end{pmatrix}
\]

Note that \( L_{1,1} \) will always be equal to one since we are considering a correlation matrix. If we introduce independent Brownian motions \( B_t^1, \ldots, B_t^n \) then the correlated Brownian motions can be written as

\[
\mathbb{W}_t = L \mathbb{B}_t,
\]

where \( \mathbb{W}_t = (W_t^1, \ldots, W_t^n)^T \) and \( \mathbb{B}_t = (B_t^1, \ldots, B_t^n)^T \) corresponds to the \( n \)-dimensional dependent and independent Brownian motions respectively. Thus, the multivariate system can be written as

\[
dX_t^k = \mu_k X_t^k dt + \sigma_k X_t^k \sum_{j=1}^n L_{k,j} dB_t^j.
\]

Note that the distribution of the individual processes does not change and the log-returns are still normally distributed. This can be shown by once again looking at the dynamics of \( d\ln X_t^k \) and applying Itô's formula

\[
d\ln X_t^k = \frac{1}{X_t^k} dX_t^k - \frac{1}{2}(X_t^k)^2 \left( dX_t^k \right)^2
\]

\[
= \left( \mu_k - \frac{1}{2} \sigma_k^2 \sum_{j=1}^n L_{k,j}^2 \right) dt + \sigma_k \sum_{j=1}^n L_{k,j} dB_t^j
\]

\[
= \left( \mu_k - \frac{1}{2} \sigma_k^2 \right) dt + \sigma_k \sum_{j=1}^n L_{k,j} dB_t^j.
\]
In the latter expression we used the fact that the quadratic row-sum of the Cholesky decomposition matrix is equal to one. We can directly from this expression see that the drift term of this process is independent of the correlation. Recall that $B^j_t - B^j_{t-\Delta t} \sim N(0, \sqrt{\Delta t})$. Thus, the distribution of the logarithmic returns will be

$$R^k_t := \ln \frac{X^k_t}{X^k_{t-\Delta t}} \sim N \left( \left( \mu_k - \frac{1}{2}\sigma_k^2 \right) \Delta t, \sigma_k \sqrt{\Delta t} \right) \sum_{j=1}^{n} L^2_{k,j}$$

$$\sim N \left( \left( \mu_k - \frac{1}{2}\sigma_k^2 \right) \Delta t, \sigma_k \sqrt{\Delta t} \right),$$

where $R^k_t$ denotes the return of asset number $k$.

### 2.3 Ornstein-Uhlenbeck Process

Consider a stochastic process $X$. This is said to follow an Ornstein-Uhlenbeck Process or a Vasiček model if it satisfies the stochastic differential equation [3]

$$dX_t = \lambda (\mu - X_t) dt + \sigma dW_t,$$

where $\lambda > 0$, $\mu \in \mathbb{R}$ and $\sigma > 0$ are the parameters for mean reversion rate, long term mean and volatility respectively. To solve this SDE we consider the dynamics of $f(X_t, t) = X_t e^{\mu t}$.

Itô’s formula yields

$$df(X_t, t) = \lambda X_t e^{\mu t} dt + e^{\mu t} dX_t$$

$$= \mu \lambda e^{\mu t} dt + \sigma e^{\mu t} dW_t.$$ 

Integration from 0 to $t$ yields

$$X_t = X_0 e^{-\lambda t} + \mu \left( 1 - e^{-\lambda t} \right) + \sigma e^{-\lambda t} \int_0^t e^{\lambda s} dW_s.$$ 

Thus the stochastic process is normally distributed with mean and variance equal to

$$E [X_t] = X_0 e^{-\lambda t} + \mu \left( 1 - e^{-\lambda t} \right) \quad \text{and} \quad \text{Var} [X_t] = \frac{\sigma^2}{2\lambda} \left( 1 - e^{-2\lambda t} \right)$$

respectively. One can also see that

$$\lim_{t \to \infty} E [X_t] = \mu \quad \text{and} \quad \lim_{t \to \infty} \text{Var} [X_t] = \frac{\sigma^2}{2\lambda}.$$
2.4 Choosing Historical Data

In order to determine what types of assets Swedish life insurance companies invest in investigations were made. We studied annual reports and financial information published by the following major life insurance companies in Sweden: Skandia Liv, AMF Pension, Alecta and Folksam. In this thesis we only consider traditional life insurance companies and portfolios since the liability model used in the asset liability management model is a model based on traditional occupational pension, which is the pension one receives from employment. We observed that the majority of the assets was invested in bonds, stocks, real estate, private equity/hedge funds and on the money market, where the two first mentioned financial assets were in clear majority. Thus, we want to find historical data for each asset class respectively.

Regarding investing in bonds the reports showed that the life insurance companies invested in both government and corporate bonds, where the majority of the bond allocations was placed in government bonds that were both inflation and non-inflation linked bonds. In the case of the allocation of the stocks the traditional life insurance companies invest in both Swedish and world wide stocks. To investigate how the investing opportunities have changed we choose asset indexes rather than individual assets to replicate a benchmark of the market. Multiple indexes for stocks and bonds was used as benchmark of the bond and stock market respectively of the different types. For the bond market we chose bond indexes from Standard & Poor’s (S&P) to replicate the Swedish non-inflation- and inflation-linked government bonds and an international corporate bond index. For the stock market we choose OMX Stockholm 30, Deutsche Boerse AG German Stock Index and Dow Jones Industrial Average to replicate the Swedish, European and American stock market respectively. Further we choose OMX Stockholm Real Estate Price Index from NASDAQ OMX Nordic for historical data of real estate price and STIBOR Fixed 1M from the Swedish National Bank (Riksbanken) for the money market. Finally Credit Suisse’s hedge fund index was chosen to replicate the private equity market. To summarize, the financial assets in which our fictive life insurance company will invest in can be found in Table 2.1.
<table>
<thead>
<tr>
<th>Asset number</th>
<th>Abbreviation</th>
<th>Asset name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>STIBOR</td>
<td>STIBOR Fixed 1M</td>
</tr>
<tr>
<td>2</td>
<td>SPGOV</td>
<td>S&amp;P Sweden Sovereign Bond Index</td>
</tr>
<tr>
<td>3</td>
<td>SPRGOV</td>
<td>S&amp;P Sweden Sovereign Inflation-linked Bond Index</td>
</tr>
<tr>
<td>4</td>
<td>SPCORP</td>
<td>S&amp;P International Corporate Bond Index</td>
</tr>
<tr>
<td>5</td>
<td>OMXS30</td>
<td>OMX Stockholm 30</td>
</tr>
<tr>
<td>6</td>
<td>DAX</td>
<td>Deutsche Boerse AG German Stock Index</td>
</tr>
<tr>
<td>7</td>
<td>DJIA</td>
<td>Dow Jones Industrial Average</td>
</tr>
<tr>
<td>8</td>
<td>OMXREPI</td>
<td>OMX Stockholm Real Estate Price Index</td>
</tr>
<tr>
<td>9</td>
<td>CSHFI</td>
<td>Credit Suisse Hedge Fund Index</td>
</tr>
</tbody>
</table>

Table 2.1: The STIBOR rate and the financial assets our fictive Swedish life insurance company will invest in. Historical rates and prices have been gathered from 2006-02-28 to 2016-01-29. The asset number in the table will correspond to asset number \( k \) in the multivariate geometric Brownian motion notation. The abbreviations will be used later in the report and does not necessarily correspond to the official ones.

The asset numbers for each financial asset in Table 2.1 corresponds to \( k \) in the multivariate geometric Brownian motion notation. Since we have a long investment horizon we want to gather as much historical data for each index as possible. We were only able to gather ten years of historical data for the bond indexes from S&P and therefore had to limit our observation window to ten years since we want to match the historical windows due to correlation between assets. In Figure 2.1 the STIBOR rate and the financial assets for the ten year time span are shown. Note that the vertical axis of the STIBOR sub figure corresponds to percentage since STIBOR is an interest rate and has no actual price and is therefore not tradeable. To trade with STIBOR we constructed a zero coupon bond with maturity in one month and face value 1. Since we use the Vasiček model under the risk neutral measure \( \mathbb{Q} \) for the STIBOR rate, the price of a zero coupon bond with maturity in one month is determined by

\[
P(t, T) = e^{A(t, T) - B(t, T)X_{t}^1},
\]

where \( X_{t}^1 \) is the monthly STIBOR rate at time \( t \) with maturity \( T = t + \frac{1}{12} \) since it has maturity in one month. The functions \( A(t, T) \) and \( B(t, T) \) are given by

\[
A(t, T) = \left( \mu - \frac{1}{2\lambda^2}\sigma^2 \right) (B(t, T) - (T - t)) + \frac{1}{4\lambda}\sigma^2 B^2(t, T)
\]

and

\[
B(t, T) = \frac{1}{\lambda} \left(1 - e^{-\lambda(T-t)}\right)
\]
respectively. For simplicity we assume that the equivalent martingale measure $Q$ (risk neutral measure) is equal to the objective probability measure $\mathcal{P}$ (real world measure).

Figure 2.1: Figure showing the development of the STIBOR rate and the financial assets. The price of the financial assets are denoted in SEK where foreign exchange risks are neglected. The time series contains historical financial data gathered from 2006-02-28 to 2016-01-29. The index on the horizontal axis corresponds to observed daily prices for the first eight sub plots, for the CSHFI index corresponds to monthly prices since that index only had public monthly data. Note that the vertical axis of the STIBOR rate is denoted in percentage.
2.5 Checking Financial Modelling Assumptions

Since the financial assets was modelled using geometric Brownian motions the log-returns are assumed to be normally distributed and, as well as the price process, independent within the increments. Note that for the STIBOR rate the logarithmic return was not used. Instead, the returns was calculated as the differences between the interest rate at time $t$ and $t - \Delta t$. When return is mentioned further in this report we refer to the change in interest rate for STIBOR and log-return for the rest of the assets.

Figure 2.2 and Figure 2.3 shows the monthly returns and the histograms of the monthly returns of the assets respectively. In Figure 2.2 we observe that the returns reminds us of a random noise. Figure 2.3 shows that the histograms for the government bonds and STIBOR rate have far less deviation around its mean return than the risky assets, which is expected since government bonds and interest rate for stable countries is often considered as less risky assets in comparison for instance a risky stock. In the histogram for the corporate bonds the deviations lies somewhat in between the risky and ”non-risky” assets, which is also expected since investing in a corporate is in most cases more risky than investing in a stable government bond.
Figure 2.2: Monthly returns for the ten year period of historical data of the STIBOR rate and the financial assets. Due to the limits of the vertical axis of the returns of the STIBOR rate does not seem to be normally distributed, however if we look more closely we see that this is the case.
Figure 2.3: Histograms for the monthly returns for the ten year period of historical data of the STIBOR rate and the financial assets.

In order to check for normality for the returns we study the Quantile-Quantile plots (QQ-plots) for monthly returns for each asset. Figure 2.4 shows the QQ-plots for the financial assets where we observe that the normal approximation approach is reasonable around its mean and deviates from the theoretical quantile in the extreme tails. This is a known fact and complication with modelling asset returns as normally distributed random variables. For the government bonds the assumption of normally distributed returns is a good approach, even in the tails. We conclude that the normality approach is acceptable despite the deviations since the goal of this thesis is to investigate the investing possibilities rather than constructing the best model for financial assets.
A Jarque-Bera test was also performed in order to test if the normality assumption is reasonable. The test checks whether the sample data have skewness and kurtosis matching the normal distribution. The test statistic is defined as 

\[\chi^2_{JB} = \frac{n}{6} \left( \gamma^2 + \frac{1}{4}(\kappa - 3)^2 \right)\]

where \(n\) is the number of observations and \(\gamma\) and \(\kappa\) is the sample skewness and kurtosis defined as

\[\gamma = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^3 \quad \text{and} \quad \kappa = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^4\]
respectively. The test statistic is assumed to be approximately \( \chi^2 \)-distributed with \( n \) degrees of freedom. The null hypothesis of the test is that the sample is normally distributed. The result from this analysis is found in Table 2.2. The table shows that for the bonds and CSHFI the hypothesis cannot be rejected at a significance level of 5%. The other assets does not clearly fulfill the test at a level of 5%. However, note that for some samples of the standard normally distributed random variable the Jarque-Bera test implied that the variable was not normally distributed, which is not expected. This behaviour was also seen for higher number of samples.

<table>
<thead>
<tr>
<th>Asset</th>
<th>( \chi^2_{JB} )</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>STIBOR</td>
<td>9.0507</td>
<td>0.0108</td>
</tr>
<tr>
<td>SPGOV</td>
<td>0.3616</td>
<td>0.8346</td>
</tr>
<tr>
<td>SPRGOV</td>
<td>0.6605</td>
<td>0.7188</td>
</tr>
<tr>
<td>SPCORP</td>
<td>3.1640</td>
<td>0.2056</td>
</tr>
<tr>
<td>OMXS30</td>
<td>7.0446</td>
<td>0.0295</td>
</tr>
<tr>
<td>DAX</td>
<td>8.4071</td>
<td>0.0149</td>
</tr>
<tr>
<td>DJIA</td>
<td>6.8588</td>
<td>0.0324</td>
</tr>
<tr>
<td>OMXREPI</td>
<td>12.0320</td>
<td>0.0024</td>
</tr>
<tr>
<td>CSHFI</td>
<td>5.9391</td>
<td>0.0513</td>
</tr>
<tr>
<td>( N(0, 1) )</td>
<td>0.0938</td>
<td>0.9542</td>
</tr>
</tbody>
</table>

Table 2.2: The result from the Jarque-Bera test on the monthly returns. We also added how 1000 simulations of a standard normal random variable performed in the test as an reference point.

To check whether the returns are time independent within the time steps we study the auto-correlation function (ACF) and partial auto-correlation function (PACF) for the assets. Figure A.1- Figure A.3 in Appendix shows the ACFs and PACFs plotted for the assets. By these plots we suspect that CSHFI can be modelled as an AR(1)-process and it is not clear that the ACF for STIBOR does not have any significant lags. For the other assets we observe that they do not have any significant lags in the historical returns. A Ljung-Box test was also performed to test whether the sample of historical data is independently distributed or not. The null hypothesis for the test is that the data are independent and identically distributed, i.e. the auto-correlation parameter \( \rho \) for the sample is zero for each lag \( k \) in the test statistic \( Q \) defined as \[ Q = n(n + 2) \sum_{k=1}^{m} \frac{\hat{\rho}^2(k)}{n - k}. \]

Here \( n \) is the sample size, \( \hat{\rho}(k) \) is the sample auto-correlation at lag \( k \) and \( m \) is the number...
of lags tested. Here the sample auto-correlation of lag $k$ is defined as

$$\hat{\rho}(k) = \frac{\sum_{j=k+1}^{n} (r_j - \bar{r})(r_{j-k} - \bar{r})}{\sum_{k=1}^{n} (r_k - \bar{r})^2},$$

where $\bar{r}$ is the mean of the time series and $r_k$ is the value of the sample at time $k$. Under the null hypothesis $H_0$ the $Q$-statistic follows a $\chi^2$ distribution with $m$ degrees of freedom. For a given significance level $p$, the region for rejection of the hypothesis is $Q > \chi^2_{1-p,m}$ where $\chi^2_{1-p,m}$ is the $p$-quantile of the $\chi^2(m)$-distribution.

The result from this test applied to the monthly data can be found in Table 2.3. The test results implies that we cannot reject the null hypothesis of an independent and identically distributed for all of the assets except for STIBOR and CSHFI at a level of 5%. The test implies that the returns for the STIBOR rate is not independent and identically distributed, therefore the Ornstein-Uhlenbeck approach is an suitable method since the returns are not independent and identically distributed in that model. By this analysis we conclude that the assumptions that the returns are independent and normally distributed are fulfilled and that our financial modelling approach using geometric Brownian motions is plausible.

We choose to model the returns of CSHFI using the geometric Brownian motion approach even though the Ljung-Box test implies otherwise.

<table>
<thead>
<tr>
<th>Asset</th>
<th>$\chi^2$</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>STIBOR</td>
<td>9.3716</td>
<td>0.0220</td>
</tr>
<tr>
<td>SPGOV</td>
<td>0.4107</td>
<td>0.5216</td>
</tr>
<tr>
<td>SPRGOV</td>
<td>0.0976</td>
<td>0.7547</td>
</tr>
<tr>
<td>SPCORP</td>
<td>0.0180</td>
<td>0.8933</td>
</tr>
<tr>
<td>OMXS30</td>
<td>1.2193</td>
<td>0.2695</td>
</tr>
<tr>
<td>DAX</td>
<td>2.6935</td>
<td>0.1008</td>
</tr>
<tr>
<td>DJIA</td>
<td>2.3093</td>
<td>0.1286</td>
</tr>
<tr>
<td>OMXREPI</td>
<td>0.0395</td>
<td>0.8425</td>
</tr>
<tr>
<td>CSHFI</td>
<td>17.414</td>
<td>3.006e-04</td>
</tr>
</tbody>
</table>

Table 2.3: The result from Ljung-Box test applied to the monthly returns.
2.6 Parameter Estimation and Simulation of Future Returns

The log-returns are normally distributed with mean \( m = (\mu - \frac{1}{2} \sigma^2) \Delta t \) and standard deviation \( v = \sigma \sqrt{\Delta t} \) under the geometric Brownian motion model. The drift and the volatility of the stochastic processes are estimated by using the unbiased estimators

\[
\hat{m} = \frac{1}{n} \sum_{i=1}^{n} x_i \quad \text{and} \quad \hat{v}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \hat{m})^2
\]

respectively, where \( x_i \) denotes observed historical (monthly) returns. Thus, the drift and volatility parameter can be estimated as

\[
\hat{\mu} = \frac{\hat{m}}{\Delta t} + \frac{1}{2} \hat{v}^2 \quad \text{and} \quad \hat{\sigma} = \frac{\hat{v}}{\sqrt{\Delta t}}
\]

respectively. This is the same result one would have got using the maximum likelihood method.

To estimate the parameters for the Ornstein-Uhlenbeck process, which are used for the STIBOR rate, we use the maximum likelihood approach. For an Ornstein-Uhlenbeck process the log-likelihood function of observations \( X_0, \ldots, X_n \) can be derived from the conditional density function:

\[
L(\mu, \lambda, \hat{\sigma}) = \sum_{i=1}^{n} \ln f (X_i | X_{i-1}; \mu, \lambda, \sigma)
\]

\[
= -\frac{n}{2} \ln 2\pi - n \ln \hat{\sigma} - \frac{1}{2\hat{\sigma}^2} \sum_{i=1}^{n} \left[ X_i - X_{i-1} e^{-\lambda \Delta t} - \mu \left( 1 - e^{-\lambda \Delta t} \right) \right]^2,
\]

where \( \Delta t \) denotes the time step and

\[
\hat{\sigma}^2 = \frac{\sigma^2}{2\lambda} \left( 1 - e^{-2\lambda \Delta t} \right).
\]

Taking the first order conditions to \( L \) we get that the parameters \( \mu, \lambda \) and \( \sigma \) can be estimated as

\[
\mu = \frac{X_y X_{xx} - X_x X_{yy}}{n \left( X_{xx} - X_{xy} \right) - \left( X_x^2 - X_x X_y \right)},
\]

\[
\lambda = \frac{-1}{\Delta t} \ln \frac{X_{xy} - \mu X_x - \mu X_y + n\mu^2}{X_{xx} - 2\mu X_x - n\mu^2}
\]

and

\[
\sigma^2 = \frac{2\lambda}{n \left( 1 - \alpha^2 \right)} \left[ X_{yy} - 2\alpha X_{xy} + \alpha^2 X_{xx} - 2\mu (1 - \alpha) (X_y - \alpha X_x) + n\mu^2 (1 - \alpha)^2 \right]
\]
respectively, where \( \alpha = e^{-\lambda \Delta t} \) and \( X_{jl}, j, l \in \{x, y\} \), denotes the following sums

\[
X_x = \sum_{i=1}^{n} X_{i-1} \quad X_y = \sum_{i=1}^{n} X_i \quad X_{xx} = \sum_{i=1}^{n} X_i^2 \quad X_{xy} = \sum_{i=1}^{n} X_{i-1}X_i \quad X_{yy} = \sum_{i=1}^{n} X_i^2.
\]

The \( k \):th asset’s future return is simulated from

\[
\ln \frac{X^k_t}{X^k_{t-\Delta t}} = \left( \mu_k - \frac{1}{2} \sigma_k^2 \right) \Delta t + \sigma_k \sqrt{\Delta t} \sum_{j=1}^{n} L_{k,j} Z_j
\]

where \( Z_1, \ldots, Z_n \) are independent standard normally distributed random variables. In order to calculate the price of the assets at the next time step we take the exponential of the return multiplied with the price of the asset at the previous time step. For the STIBOR rate we simulate the rate from

\[
X^1_t = \alpha X^1_{t-\Delta t} + \mu (1 - \alpha) + \sigma \sqrt{\frac{1 - \alpha^2}{2\lambda}} \sum_{j=1}^{n} L_{k,j} Z_j.
\]

Currently the STIBOR rate is negative and has a downwards slope as seen in upper left subfigure in Figure 2.1. This is very rare from an economic point of view. Due to this fact we applied a constraint to the simulations that the rate at each time step would be resimulated if it was below zero. If this was not applied the rate could go below two percent for instance, which from an economic point of view is not very realistic especially for a stable economy such as Sweden.

Regarding the data points for the parameter estimation a weighting system of the data points was implemented. Using only latter data points, e.g. last 120 monthly returns, implies that the price of the asset may tend to accelerate too fast exponentially. It is expected that the price will grow exponentially since the geometric (exponential) Brownian motion approach is used. This is shown in Figure A.4 in Appendix where 100 simulations where made when only the latter 120 returns were used to estimate the parameters. The lines of different colors in this figure represents the outcome of one simulation while the thicker black line corresponds to an arithmetic mean of the different paths at given time steps. As expected, the risky assets are far more volatile in comparison to the less risky assets. From this figure the price of risky assets tend to accelerate fast and go towards infinity very fast. The exponential growth seem to accelerate for the less risky assets for later time steps. This can be seen in detail in Figure 2.5 where more detailed plots for SPGOV and DAX are shown.
Figure 2.5: The result of 100 simulated paths for SPGOV and DAX when considering the latter 120 data points for parameter estimation. The other time series can be found in Figure A.4. The thicker black and red line corresponds to the arithmetic mean and median respectively.

From a mathematical point of view the mean path of the simulations is reasonable since the constructed scenarios have the same probability occurring. However, from an economic point of view the fast accelerating asset prices are not reasonable. By studying the median path, represented by the thicker red line, we observe that this path is more realistic than the mean path. Using this as an "average" path of our assets can be plausible since the returns was modelled from a normal distribution. Due to the symmetry of the normal distribution the mean value and the median value is expected to be equal. It is well known
that the median is a more robust, especially when considering noisy data. The median is an unbiased estimator of the mean for the normal distribution. Suppose that 100 simulations has been made. Then the median value of a sorted sample $X_1 \leq X_2 \leq \cdots \leq X_{100}$ is

$$M = \frac{X_{50} + X_{51}}{2},$$

by taking expected value and using that $X_{50}$ and $X_{51}$ are equally distributed with mean $\mu$ we get that

$$E[M] = \frac{E[X_{50}] + E[X_{51}]}{2} = \frac{\mu + \mu}{2} = \mu.$$

In Figure A.4 we observe that the mean paths and median paths respectively are very similar for the STIBOR rate and the government bonds, which is expected by the result from the Jarque-Bera test. The larger deviations from the mean and the median for the financial assets coincides with the test results.

By considering the first (observed) 120 monthly returns the exponential growth is not as extreme as in the former case, which can be observed in Figure A.5 in Appendix. Basing a 30 year forecast on the first 120 returns can be more questionable than the former case since it yields that the latter estimations will be based on that the development of the market will stay the same as it did 30 years ago. Figure A.6 shows the result of 100 simulations where the parameter estimation was based on all known data points. This yields a result that grows exponentially rather quick, but not as the case when considering only the latter returns. It is more reasonable to use all the known data points in the parameter estimation since it is contradictory to statistical philosophy not to use all the data points.

Instead of ignoring earlier data points their significance was weighted down in comparison to the latter points. The observation window was divided into two separate windows of equal length and weighted the parameter estimations based on the different windows as

$$\hat{\mu}_k = (1 - \lambda) \mu^1_k + \lambda \mu^2_k,$$

where $\lambda \in (0, 1)$ is a user-input weight and $\mu^1_k$ and $\mu^2_k$ denotes the latter and earlier window respectively. We let $\lambda = 0.6$ and simulate 1000 sample paths, then the simulations reminds of the simulations based on using all historical data equally weighted, which can be found in Figure A.7 in Appendix. Figure 2.6 is a more detailed plot for SPGOV and DAX in the case of 1000 sample paths with this weighting system. Once again the price seem to grow exponentially.
Figure 2.6: The result of 1000 simulated paths for SPGOV and DAX when considering all the observed and simulated data points, weighted according to the weighting system, for parameter estimation. The other time series can be found in Figure A.7. The thicker black and red line corresponds to the arithmetic mean and median respectively.

Figure A.8 shows the returns of each month in a 30 year forecasting horizon combined with the observed time span. The returns does not seem to explode which one could suspect by only studying the price plots. As introduced in the price plots, the mean path for the returns are close to zero which is expected since the return "noise" is normally distributed with mean close to zero. A reason behind that the mean path in the price plots tend to grow so fast could be the fact that there existed some extreme cases where the price grew very fast, i.e. stacking high positive returns, unlike the other simulation paths and
thus the mean path were driven upwards. The geometric Brownian motion path cannot be negative and thus there does not exist paths that weigh the extreme high value cases down. The empirical kernel density function of the latest prices (and rate for STIBOR) for each individual asset from the simulations, e.g. the price at the latest time step, shows that there exist scenarios where there exists asset prices far away represented by small peaks in Figure 2.7 from the majority of the asset. The extreme value peaks have a small density relative far out in the right tail and may be hard to spot in Figure 2.7. Figure 2.8 shows a detailed version of the density plot for OMXREPI where we clearly see extreme value peaks. These peaks (prices) will push the arithmetic mean upwards and some prices are very questionable if they are reasonable from a economic perspective, while from a mathematical perspective they are since every path have the same probability to occur by the model.
Figure 2.7: Density plot of the latest price of the 1000 simulations. In this figure it may be hard to locate the peaks in the far out on the horizontal axis. By studying the density plot for OMXREPI in Figure 2.8, these peaks become more clear. In the figure the black and red points corresponds to the last mean and median price respectively.
Figure 2.8: Density plot of the latest price of the 1000 simulations for OMXREPI. In this figure the peaks are more clear. In the figure the black and red points corresponds to the last mean and median price respectively.

To summarize, we choose the median path for the returns and prices respectively. They will be used in the portfolio optimization algorithm and hence we are prioritizing the realism of the economic perspective rather than the mathematical perspective.
Chapter 3

Solvency Capital Requirement

In this chapter we introduce how the solvency capital requirement (SCR) is calculated using the Solvency II standard formula. In order to do so we first look at how a balance sheet is constructed. A balance sheet contains of two columns, assets and liabilities, both of equal total value. A simplified Solvency II balance sheet can be seen in Figure 3.1 [5]. In this figure MVA denotes the market value of assets, which will be the value of the assets in the portfolio we will construct later and excess capital (EC) is equal to the difference between the market value of assets and the sum of the best estimate of liabilities (BEL), the risk margin (RM) and the solvency capital requirement (SCR). The EC can be calculated as

\[ EC = MVA - BEL - RM - SCR \]

The basic own funds (BOF), which will be used in order to calculate the solvency capital requirement, is equal to the difference between MVA and the sum of BEL and RM, i.e.

\[ BOF = MVA - BEL - RM. \]

The sum BEL + RM is often referred to as the technical provisions (TP). We also introduce the terms degree of solvency (DoS) and solvency ratio (SR) as

\[ \text{DoS} = \frac{MVA}{TP} \quad \text{and} \quad \text{SR} = \frac{BOF}{SCR} \]

respectively as measures of financial stability of the life insurance company. The insurance company will be considered as solvent if the solvency ratio is larger than or equal to one.
We will compare the investing opportunities in comparison to the old solvency regulation, Solvency I, where the capital requirement can be simplified as the sum of 4% of the technical provisions$^1$ and 0.1% of the positive risk sums$^1$, i.e.

$$SCR_{SI} = 0.04 \cdot \text{Technical Provisions}_{SI} + 0.001 \cdot \text{Positive risk sums}.$$ 

Figure 3.1: Simplified Solvency II balance sheet$^5$. In this figure BEL is denoted MVL.

### 3.1 Best Estimate of Liabilities Calculation

As mentioned earlier the best estimate of liabilities needs to be calculated in order to calculate the solvency capital requirement. The BEL contains cash flow projections calculated in gross, within the contract boundaries, and should reflect realistic future developments over the lifetime of the insurance obligations. The gross cash in-flows used to determine the best estimate are future premiums and receivables for salvage and subrogation, where the latter will not be considered in the model used in this thesis. The cash out-flows are based on future benefits, such as claims and annuity payments and maturity, death and surrender benefits, and future expenses, such as administrative, investment management and claims management expenses$^9$.

In this thesis monthly cash flows are considered and each future cash flow is discounted according to a discount rate curve constructed according to the technical documentation.

$^1$Note that the technical provisions for the old regulations are calculated differently and we will not present in detail how it is calculated.
of the construction of a discount rate curve described by the Swedish financial supervisory
authority Finansinspektionen [19]. Since we simulate our representative life insurance
company for thirty years we need to construct discount rate curves for each simulated
month. We gathered daily historical data from NASDAQ OMX for Swap Fixing rates
with maturities of one to ten years and performed principal component analysis (PCA) to
study the swap rate changes. PCA uses orthogonal transformation to transform a set of
correlated variables to a set of linearly uncorrelated variables, which are called principal
components. This analysis saves computational time for constructing future values of the
observed variables since the number of principal components used are generally less than
the number of variables.

With this analysis we construct estimations of future swap rates which will be used to
determine a new discount rate curve for every month. From the daily data we construct
vectors of monthly changes in swap rates, i.e. a vector of swap rate changes as \( \Delta r = \langle \Delta r_1, \ldots, \Delta r_n \rangle \) for each monthly change where \( n = 10 \). Now assume that this vector is
stochastic with a symmetric and positive definite covariance matrix. Since the covariance
matrix is assumed to be symmetric and positive definite the covariance matrix can be
written as
\[
\text{Cov} (\Delta r) = ODO^T,
\]
where \( O \) is an orthogonal matrix and \( D \) is a diagonal matrix with strictly positive di-
agonal entries. Since the covariance matrix can be rewritten on this form, the columns
\( o_1, o_2, \ldots, o_n \) of \( O \) are eigenvectors of the covariance matrix of the swap rate changes and
the diagonal elements in \( D \) is the corresponding eigenvalues. Furthermore, we assume that
the diagonal elements in \( D \) are ordered in decreasing order.

Now define the vector \( \Delta r^\ast \) as
\[
\Delta r^\ast = O^T (\Delta r - E[\Delta r]).
\]
Then the expected value and the covariance matrix of \( \Delta r^\ast \) is equal to
\[
E[\Delta r^\ast] = O^T (E[\Delta r] - E[\Delta r]) = 0 \quad \text{and} \quad \text{Cov}(\Delta r^\ast) = \text{Cov}(O^T (\Delta r - E[\Delta r])) = D
\]
respectively. The covariance matrix of \( \Delta r^\ast \) is a diagonal matrix, hence the elements in this
stochastic vector are uncorrelated. If we let \( e_k \) be the unit vector in direction \( k \), then the
swap fixing rate changes can be rewritten as
\[
\Delta r = OO^T \Delta r = O (\Delta r^\ast + O^T E[\Delta r]) = E[\Delta r] + O \sum_{k=1}^{n} \Delta r_k^\ast e_k = E[\Delta r] + \sum_{k=1}^{n} \Delta r_k^\ast o_k,
\]
where the vectors \( o_1, \ldots, o_n \) are the principal components which creates an orthonormal
basis in \( \mathbb{R}^n \). As mentioned earlier, often the number of principal components used are less
than the number of original variables. In order to choose the number of variables that are significant in this analysis we study the ratio \[ \frac{\sum_{k=1}^{j} \lambda_k}{\sum_{k=1}^{n} \lambda_k} \]

where the \( \lambda_k \)'s, \( k = 1, \ldots, n \), are the eigenvalues and the diagonal entries in \( D \), and \( j \) is number the of principal components. This ratio shows how much of the variability that can be explained by the first \( j \) components [16]. For our swap rates the three first principal components explains 99.5% of the randomness which corresponds to 93.2%, 5.5% and 0.8% respectively for the three components. Thus we choose these three components to simulate future vectors \( \Delta r^* \). The three principal components are plotted in Figure 3.2 and corresponds approximately to parallel shifts, changes in slope and changes in curvature respectively. The principal components \( o_1, o_2 \) and \( o_3 \) can also be found in Table A.1 in Appendix.

![Principal Components](image)

Figure 3.2: The first three principal components \( o_1 \) (black line), \( o_2 \) (red line) and \( o_3 \) (green line) which explain 99.5% of the randomness for maturity in one to ten years.
A future change in swap fixing rates, i.e. future $\Delta r$ vectors, can be estimated by

$$
\Delta r \approx E[\Delta r] + \sum_{k=1}^{j} \Delta r_k^* o_k.
$$

Using this with $j = 3$ and drawing samples of $r_k^*$ with replacement for every month for thirty years we obtain future swap fixing rates which can be used to construct discount rate curves when calculating the best estimate of liabilities.

We construct the discount rate curve using the procedure presented by the Finansinspektionen [19]. Using this method, 35 basis points are deducted from the swap rates and if the swap rates are negative after deduction these rates are set to zero [19]. This is a fixed deduction which is determined by the current market. We use the adjusted swap rates, denoted $r(t)$, to first determine the implied zero coupon rate $\tilde{z}(t)$ for maturities from one to ten years by solving

$$
r(t) \sum_{k=1}^{t} \frac{1}{(1 + \tilde{z}(k))^k} = 1 - \frac{1}{(1 + \tilde{z}(t))^t},
$$

where $t$ is the maturity time in years. Then we calculate the forward rate $\tilde{f}(s,t)$, $s \leq t$, consistent with current market quotations for interest rate swaps, for each maturity by

$$
\tilde{f}(t-1,t) = \frac{(1 + \tilde{z}(t))^t}{(1 + \tilde{z}(t-1))^{t-1}} - 1.
$$

The forward rates on the discount rate curve is then calculated by

$$
f(t-1,t) = (1 - \omega(t)) \tilde{f}(t-1,t) + \omega(t)UFR,
$$

where UFR is the ultimate forward rate which is equal to 4.2% for SEK [19] and the parameter $\omega(t)$ is a weight parameter determined by the last liquid point (LLP) and the convergence period (CP) by

$$
\omega(t) = \begin{cases} 
0, & t \leq LLP, \\
\frac{t-LLP}{CP-LLP+1}, & LLP < t \leq CP \\
1, & t > CP.
\end{cases}
$$

The parameters LLP and CP depends on the currency. For SEK the last liquid point is equal to 10 and the convergence period is equal to 20 [19]. The liquid point is the last fully liquid maturity with full weight and the speed of convergence is the time from the last liquid point to the ultimate forward rate. The discount rate $z(t)$ at time $t$ is then calculated by

$$
z(t) = \left[ (1 + f(t-1,t)) \cdot (1 + z(t-1))^{t-1} \right]^\frac{1}{t} - 1.
$$
Using this method the discount rate curve was calculated for each month and the different discount rate curves are plotted in Figure 3.3. This figure shows that the discount rate curve may differ between the years. Thus it would not be realistic to use the same discount rate curve for the simulated years. Also, since we consider monthly cash flows the discount rate for cash flows with maturity in between the incremental years are obtained by linear interpolation.

Figure 3.3: Discount rate curves for each month for the upcoming thirty years constructed for simulated and observed data using the procedure presented by Finansinspektionen [19]. The discount rate curves are created for maturities from 0 to 100 years.
3.2 Calculation of the Solvency Capital Requirement

The solvency capital requirement is based on assumptions such as the insurance company will continue its services by the going concern basis and the SCR should correspond to the 99.5\%–percentile of the own funds within the company. In this thesis the solvency capital requirement will be calculated using the standard formula from EIOPA [9]

\[
SCR = BSCR + \text{Adjustment} + SCR_{\text{operational}}
\]

where BSCR is the basic solvency capital requirement, which is the primary capital requirement, Adjustment corresponds to future discretionary bonuses, adjustments for technical provisions and adjustment for taxes and SCR_{\text{operational}} is the operational risk charge. In this thesis we will review the BSCR since it is the main part of the SCR and we will assume that the terms Adjustment and SCR_{\text{operational}} are both equal to zero.

3.2.1 Basic Solvency Capital Requirement

The basic solvency capital requirement is calculated by [9]

\[
BSCR = \sqrt{\sum_{i,j} \text{Corr}_{i,j} \text{SCR}_i \text{SCR}_j + \text{SCR}_{\text{intangible}}}.
\]

Here SCR_{\text{intangible}} is the solvency capital requirement based on intangible assets, which will be neglected in this thesis. SCR_i and SCR_j, i, j \in \{\text{market, default, life, health, non-life}\}, is the capital requirement based on the market, default, life, health and non-life risk exposures and Corr_{i,j} is the correlation between the sub modules for risk. The correlations Corr_{i,j} are provided by EIOPA [9]. The correlation matrix Corr_{i,j} can be found in Table 3.1. Figure 3.4 [5] shows that these modules for risk also consists of multiple sub modules respectively. In this thesis the focus is on the market risk since it is has the largest impact on the solvency capital requirement, especially for life insurance companies due to long term policies. The capital requirement for the life underwriting risk module is also calculated. For the simulated life insurance company the health and non-life risk exposure are insignificant since it has no exposure to these modules. The risk module for default risk is chosen to be neglected in this thesis. Below we study how the capital requirement modules due to market and life underwriting risk exposure are calculated, for the other risk modules we refer to the regulations by EIOPA [9].
Table 3.1: Table showing correlations between sub modules for risk for a insurance company.

<table>
<thead>
<tr>
<th>Corr_{i,j}</th>
<th>SCR_{market}</th>
<th>SCR_{default}</th>
<th>SCR_{life}</th>
<th>SCR_{health}</th>
<th>SCR_{non-life}</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCR_{market}</td>
<td>1</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>SCR_{default}</td>
<td>0.25</td>
<td>1</td>
<td>0.25</td>
<td>0.25</td>
<td>0.5</td>
</tr>
<tr>
<td>SCR_{life}</td>
<td>0.25</td>
<td>0.25</td>
<td>1</td>
<td>0.25</td>
<td>0</td>
</tr>
<tr>
<td>SCR_{health}</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>SCR_{non-life}</td>
<td>0.25</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 3.4: Graphical description of how the solvency capital requirement is derived. The BSCR is divided further into risk modules regarding market, default, life, health and non-life risk exposures.
3.2.1.1 Market Risk Module

As seen in Figure 3.4 the basic solvency capital requirement is constructed by capital requirements for different risk modules. The procedure for calculating SCR\textsubscript{market} will be shown below. The equations below are simplified and corresponds to how the regulations affects the model we consider in this thesis and they may differ if one considers other models. We refer to regulations by the European Union \cite{9}, if one wants to study the risk module in more detail.

In the life insurance business the largest financial risk is often the risk due to the financial market. As seen in Figure 3.4 the market risk module can be divided into sub modules of interest rate, equity, property, spread, currency, concentration and illiquidity risk. For a life insurance company the sub modules of interest rate, equity, property and spread is the majority of their market risk exposure. Thus, our solvency capital requirement for market risk will be based on these factors.

In order to calculate the capital requirement for each individual risk exposure we perform stress tests constructed by EIOPA. For the interest rate risk module we perform a stress test with two scenarios, an upwards and a downwards shock. The shock will result in a change in basic own funds for the company, which contributes to the SCR for this risk sub module. The maximum of the change to the BOF due to the interest rate shock will be the capital the insurance company need to put into the calculation of the total capital requirement due to market risk. In other words, let $r_{t,T}$ be the spot interest rate at time $t$ with maturity $T$, then the stress factor should be

$$r_{t,T|s_T} = r_{t,T}(1 + s_T),$$

where $s_T$ denotes the applied shock. For the interest rate stress test the stress factor $s_T$ is dependent on the maturity $T$ of cash flows for both the upwards and downwards case. These factors are given by EIOPA \cite{9} yearly from maturities for 1 to 20 years and for 90 years. Table A.2 in Appendix shows the stress factors for the upwards and the downwards case respectively. Maturities below one year will be stressed by the same factor as if it matured in one year and maturities over 90 years will be stressed as cash flows with maturity in 90 years. For cash flows between the incremental years in the table will be stressed by a factor determined by linear interpolation between the years. The capital requirement implied by the interest rate stress test will be equal to the change in basic own funds for the two scenarios, denoted $\text{SCR}_{\text{up \ interest rate}}$ and $\text{SCR}_{\text{down \ interest rate}}$.

For the equity risk module the assets are divided into two categories, Type 1 and Type 2. Type 1-equities contains stocks that are listed in an EEA or OECD country \cite{9}. Since we in this report use OMXS30, DJIA and DAX as stock indices these will be considered
as equities of Type 1. The hedge fund index, CSHFI, will be considered as Type 2. To calculate the capital requirement implied by the equities we must perform a two-step calculation since the equities is divided into two categories. We apply the same procedure to stress the individual types as for the interest rate. Let \( X_t \) denote the price of an equity at time \( t \), then \( X_t|_s = X_t(1 + s) \). For \( i = 1, 2 \) the capital requirement will be calculated for each category of equity as

\[
SCR_{\text{equity}_i} = \max \{ \Delta BOF|_s, 0 \},
\]

where \( s_1 = -0.39 \) and \( s_2 = -0.49 \), where \( s_1 \) and \( s_2 \) denotes the shock for Type-1 and Type-2 equity respectively. The total capital requirement for the equity module is

\[
SCR_{\text{equity}} = \sqrt{SCR_{\text{equity}_1}^2 + 2 \cdot 0.75 \cdot SCR_{\text{equity}_1} \cdot SCR_{\text{equity}_2} + SCR_{\text{equity}_2}^2}.
\]

To calculate the property risk sub module we let \( X_t \) denote the price of the property index at time \( t \) and apply the stress as \( X_t|_s = X_t(1 + s) \). The SCR for the property sub module is

\[
SCR_{\text{property}} = \max \{ \Delta BOF|_s, 0 \},
\]

where \( s = -0.25 \) for the property stress. For the spread shock stress test, in this case bonds, we have to perform a slightly different two step approach [9]. The spread shock on bonds is calculated as

\[
\text{Spread shock} = \sum_{i=1}^{n} MV_i \cdot \text{dur}_i \cdot F(\text{rating}),
\]

where \( MV_i \) denotes the market value of asset \( i \), \( \text{dur}_i \) denotes the modified duration of asset \( i \) and \( F \) is a function of the security’s external rating. As can be seen in Table A.6 in Appendix the modified duration for all of our bond lies within (5, 10] years which implies that \( F_i = a_i + b_i(\text{dur}_i - 5) \), where \( a_i \) and \( b_i \) are constants based on each bond’s credit quality step (CQS) [9] which in turn is related to its credit rating. In our case SPGOV and SPRGOV received CQS equal to 0 and SPCORP had 1, where 0 is the least risky credit step, by following the regulations [9]. The capital requirement for the spread risk is calculated as

\[
SCR_{\text{spread}} = \max \{ \Delta BOF|_{\text{spread shock}}, 0 \},
\]

The total capital requirement for market risk \( SCR_{\text{market}} \) is calculated as [9]

\[
SCR_{\text{market}} = \max \left\{ \sqrt{\sum_{i,j} \text{Corr}SCR_{\text{up}_i}^{\text{up}_j}SCR_{\text{up}_i}^{\text{up}_j}SCR_{\text{down}_j}^{\text{down}_j}}, \sqrt{\sum_{i,j} \text{Corr}SCR_{\text{down}_i}^{\text{down}_j}SCR_{\text{down}_i}^{\text{down}_j}SCR_{\text{down}_j}^{\text{down}_j}} \right\},
\]

\(^{2}\text{In this thesis the symmetric equity adjustment mentioned in [9] is excluded.}\)
where $i, j \in \{\text{interest rate, equity, property, spread}\}$ and $\text{CorrSCR}^{\text{up}}$ and $\text{CorrSCR}^{\text{down}}$ corresponds the correlation matrices which can be found in Table A.3 and Table A.4 in Appendix respectively [9]. The shock factors for the equity and property can be found in Table A.5 in Appendix and the input parameters for the spread risk stress test can be found in Table A.6 in Appendix. In this thesis the duration is assumed to remain constant throughout our simulations of the representative Swedish life insurance company. Figure 3.5 shows that the observed duration is more or less constant over the time span. This is the case for the other corporate bond index as well. The duration changes when coupons are paid to the holder. Once a coupon is paid out there is an immediately increase of the duration the day it is paid. The duration will then decrease as the time to maturity decreases. However, since we will consider bond indices in this thesis they do not have a specific maturity and when maturity is met for bonds in the bond index portfolio new bonds will added with new maturities. As seen in Figure 3.5 the time span of that figure is only around two years and we see that the duration is more or less constant. Applying a prediction line by linear regression on the duration plots results in a slope that has the first significant digit in the fourth decimal point. Thus, we assume that the duration will be equal to the observed modified durations that can be seen in Table A.6 for the simulations.
Figure 3.5: Plotted durations for the government bonds. We see that the duration is more or less constant over the observed time span.

3.2.1.2 Life Underwriting Risk Module

The second essential risk module for a traditional life insurance company is the life underwriting risk module. In this thesis the standard formula for the life underwriting risk module is applied, as is done for the market risk module. The life underwriting risk module, as shown in Figure 3.4 contains of mortality, longevity, disability-morbidity, life expense, revision, lapse and life catastrophe risk sub modules. In the model used for the liabilities for the life insurance company they are not affected by the disability-morbidity risk sub module and therefore refer to the regulations by the European Union [9] if one is interested in the calculations for this sub module.

The capital requirement for the sub module for mortality risk shall be equal to the loss in BOF that would be resulted by an instantaneous permanent increase of 15 percent in the
mortality rates used for the calculations of technical provisions, i.e. the capital requirement for the mortality risk can be calculated as

$$\text{SCR}_{\text{mortality}} = \max \left\{ \Delta \text{BOF} | \text{mortality increase}, \ 0 \right\}.$$  

The capital requirement for the longevity risk is calculated similarly. Instead of an increase of 15 percent in mortality rates we now decrease the mortality rates with 20 percent, i.e. the risk capital required for the longevity module can be calculated as

$$\text{SCR}_{\text{longevity}} = \max \left\{ \Delta \text{BOF} | \text{mortality decrease}, \ 0 \right\}.$$  

For the life expense sub module the required capital is calculated by stressing the amount of expenses in the model upwards of 10 percent and also increase the expense inflation rate by 1 percentage point. The capital requirement for life expense risk can thus be calculated as

$$\text{SCR}_{\text{life expense}} = \max \left\{ \Delta \text{BOF} | \text{expense amount decrease and expense inflation increase}, \ 0 \right\}.$$  

The risk capital for the revision risk sub module is equal to the loss in BOF by applying a permanent and instantaneous increase of the amount of annuity benefits of 3 percent, i.e.

$$\text{SCR}_{\text{revision}} = \max \left\{ \Delta \text{BOF} | \text{annuity for benefits increase}, \ 0 \right\}.$$  

For the lapse risk sub module, this is calculated as the maximum loss in BOF of the three following cases:

1. A permanent increase in lapse rates of 50 percent, but the rates shall not exceed 100 percent in total;
2. A permanent decrease in lapse rates of 50 percent, but the rates shall not exceed 20 percent in total;
3. A mass lapse event, which is described more in detail in [9].

Thus, the capital requirement for the lapse risk sub module can be calculated as

$$\text{SCR}_{\text{lapse}} = \max \left\{ \Delta \text{BOF} | \text{lapse increase}, \ \Delta \text{BOF} | \text{lapse decrease}, \ \Delta \text{BOF} | \text{mass lapse}, \ 0 \right\}.$$  

For the life catastrophe sub risk module the capital requirement is equal to the loss in BOF that is the result from an instantaneous increase of 0.15 percentage points to reflect the mortality experience in the following twelve months, i.e.

$$\text{SCR}_{\text{life catastrophe}} = \max \left\{ \Delta \text{BOF} | \text{mortality increase in twelve months}, \ 0 \right\}.$$
The solvency capital requirement for the life underwriting risk module, \( \text{SCR}_{\text{life}} \), is then calculated by:

\[
\text{SCR}_{\text{life}} = \sqrt{\sum_{i,j} \text{CorrSCR}_{ij} \text{SCR}_i \text{SCR}_j},
\]

where \( i, j \in \{ \text{mortality, longevity, life expense, revision, lapse, life catastrophe} \} \) and \( \text{CorrSCR}_{ij} \) corresponds to the correlation matrix between the different sub risk modules in the life underwriting risk module and is found in Table A.7 in Appendix.

### 3.3 Risk Margin

The risk margin (RM) is defined as:

\[
\text{RM} = \text{CoC} \sum_{t \geq 0} \frac{\text{SCR}'(t)}{(1 + r(t + 1))^{t+1}},
\]

where \( \text{CoC} \) corresponds to the cost of capital rate, set to 6 percent \( [9] \), and \( r(t + 1) \) is the discount rate for maturity \( t + 1 \) years. Note that the sum covers integer incremental of one (year) including zero. \( \text{SCR}' \) denotes the solvency capital requirement for the reference undertaking. This capital requirement consists of the underwriting risk with respect to existing business. It should also include counter party default risk, operational risk and unavoidable market risk, excluding interest rate risk \( [9] \).

Note that only the business existing at the valuation date, \( t = 0 \), is taken into account in \( \text{SCR}' \) and that the underwriting risk with respect to non-obligated future business is not taken into account. This solvency capital requirement is a subset of the actual SCR, ignoring new business and some portion of market risk. Since the default risk and operational risk are neglected in this thesis \( \text{SCR}' \) will be equal to the solvency capital requirement for the life underwriting risk \( \text{SCR}_{\text{life}} \) for our problem due to simplicity and since we can see the investment strategy for the company as an avoidable market risk exposure.
Chapter 4

Modelling

Now that the analysis of the financial assets have been made to create their future developments and we know how to calculate the solvency capital requirement using the standard formula, we are one step closer to be able to simulate our representative life insurance company. In this chapter we first introduce Modern Portfolio Theory which is used to construct minimum variance portfolios for our life insurance company. To be able to produce reasonable and realistic results accordingly to the Swedish life insurance market we establish actuarial assumptions for the simulated life insurance company based on major Swedish life insurance companies with traditional occupational pension products. In the simulations we need representative policies to study the monthly cash flows for our company. The policies were constructed based on age, sex and yearly salary distribution of the population in Sweden. In order to calculate policy annual benefits and premiums we introduce the traditional theory regarding Swedish life insurance pricing. Finally we introduce the term asset liability management and how it is applied to this specific problem.

4.1 Modern Portfolio Theory

For this thesis quadratic investment principles was applied in order to optimize our investment portfolio for our fictive Swedish life insurance company. Due to regulations of life insurance companies they can be seen as a somewhat risk averse investor, i.e. the goal is to minimize the risk for a given return. The given return can for example correspond to
the goal of the strategic asset allocation for an insurance company. Thus the quadratic optimization problem will be

$$\begin{align*}
\min_w & \quad \frac{1}{2} w^T \Sigma w, \\
\text{s.t} \quad & w^T \mu = \mu_0 V_0, \\
& w^T 1 = V_0, \\
& w_i \geq 0, \quad i \in \{1, \ldots, 9\},
\end{align*}$$

(P)

where \( w, \Sigma, \mu \) and \( V_0 \) denotes the portfolio weight vector, covariance matrix, expected return vector and the initial wealth respectively [16]. The optimal solution to this optimization problem yields the minimum variance portfolio subject to a given return \( \mu_0 \), that all the money \( V_0 \) should be invested in assets and that no short selling is allowed. The expected return and the variance of the portfolio will be given by \( \mu_p := w^T \mu \) and \( \sigma_p^2 := w^T \Sigma w \) respectively. Note that this quadratic optimization problem has no analytic solution and thus must be solved numerically. Furthermore we have no restrictions regarding how the money should be distributed among the financial assets for the Solvency II regulation. For the further simulations we will use an initial wealth \( V_0 \) that corresponds to a degree of solvency of 125 percent, which is a reasonable starting point based on annual reports from numerous top contenders in the Swedish tradition life insurance line of business.

4.2 Actuarial Assumptions and Model Points

In this thesis we proceed from a given liability model for Swedish traditional life insurance that have been in use as a Swedish pension system. The liability model is built on traditional occupational pension and we will in this thesis consider a pension portfolio without any new business, i.e. no new policies are expected. Note that Swedish traditional occupational pension companies does not necessarily need to follow the new Solvency II Directive and by Swedish law [18] they may follow transitional rules instead. Thus they may apply the old solvency regulations if the company was in business before January 1 2016 and follow these regulations till December 31 2019 when new adjusted regulations for occupational pensions will take affect. If this was not the case the companies would have to first adapt to the new solvency regulations 2016 and then once again 2019 for the adjusted regulations. Note that in Sweden there exists life insurance companies with occupational pensions that follows the new solvency regulations. Since the adjusted regulations has yet to be settled we proceed under the assumption that our life insurance company follows the new Solvency II Directive during the thirty years of projection.

The pension policies have input parameters such as issue year, age, retirement age, premiums and status. The status parameter explains whether the policy holder is active (paying
premiums), paid up (the policy holder stops paying premiums but will receive pension corresponding to amount paid in earlier), payout (insurance company is paying out insurance claims, i.e. pension payout) and family (the (dead) policy holder have survivor pension, i.e. the dead policy holders family will receive the pension payments). The liability model will affect the premiums, expenses, commission and claims yielded by the policies in the insurance company.

In order to construct realistic results from our simulations of the life insurance company we need to go over the actuarial assumptions in the model and make the liability model consistent with our constructed economic scenario. Since the majority of the life insurance companies in Sweden are mutual we also need to construct a consolidation policy for our life insurance company, otherwise the end results may be unrealistic. For instance, without a consolidation policy the solvency ratio and degree of solvency will increase to unrealistic levels. Mutual Swedish life insurance companies follow the contribution principle by law which means that the bonus gained for an insurance policy should be proportional to how the specific policy have contributed to the surplus. Hence the consolidation policy must be constructed according to this.

Regarding the actuarial assumptions we study data from major Swedish life insurance companies to construct realistic assumptions such as premium fees and guaranteed interest rate for our pension portfolio. We collected data from Collectum which is an administrative link between the occupational pension ITP, the employer that pays premiums to the insurance company that handles the pensions and the insurance company. Collectum’s website shows brief information regarding fees, expenses and guaranteed rate on the pension. We created actuarial assumptions partly based on this information. The actuarial assumptions in our model can be found in Table A.8 in Appendix. The assumptions are divided in first order, experience and second order basis. The first order basis is used when doing prospective calculations while second order basis is used when doing retrospective calculations. The experience basis can be seen as the company’s assumption how e.g. the actual mortality will differ from the theoretical or technical mortality.

We choose the consumer price index inflation rate to be fixed at 2 percent since it is the target rate of the Swedish National Bank. As shown in Figure A.9 in Appendix, the inflation rate the last 26 years have not been constant. Deviations from the assumed inflation rate will for instance have affect expenses and the solvency capital requirement for life underwriting risk in the model. The average inflation rate during this time period is 2 percent therefore we proceed with this assumption. We also assume a constant transfer and paid-up rate at 3 and 5 percent respectively, i.e. the rate that the individual changes status from active to transfer or paid-up respectively.

Since we consider a Swedish life insurance company with no new issued policies we consider a variety of the pension product ITP 2 which is the employer pension for individuals born
year 1978 or earlier. The modelled pension plan in this thesis is a simplified version of this pension plan where the major benefits are considered. Individuals born year 1979 or later will in most cases follow the pension plan ITP 1 which is a defined contribution pension, i.e. the premium payments are determined in forehand. ITP 2 is a defined benefit pension, which means that the benefits of the pension are determined in forehand. If the individual works at least 360 months the annual annuity of the pension will be determined by Table 4.1. However, if a person works less than 360 months the annuity will be decreased by $\frac{1}{360}$ per month less than 360. We also include survivor pension or family pension which leads to that if the individual dies, the individuals partner or family will receive annual annuity determined by Table 4.2 if they are alive. In these tables the yearly salary is the salary the individual have at year of retirement, with possible decrements depending on the salary development for the individual the last 60 months of employment. These decrements can be found in Table A.9 in Appendix and prevent large salary rises during the last months of employment to affect the pension benefit. Otherwise, theoretically, one could get a large raise the last month of employment and get a significantly higher pension benefit. The income base amount varies each year and the pension will be based on the income base amount at the year of retirement, where we let the retirement age to be be equal to 65.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Yearly salary & Pension % of part of salary \\
\hline
0 – 7.5 income base amount & 10 \\
7.5 – 20 income base amount & 65 \\
20 – 30 income base amount & 32.5 \\
\hline
\end{tabular}
\caption{The benefit plan for a ITP 2 pension product [14].}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Yearly salary & Pension % of part of salary \\
\hline
7.5 – 20 income base amount & 32.5 \\
20 – 30 income base amount & 16.25 \\
\hline
\end{tabular}
\caption{The benefit plan for survivor pension for a ITP 2 pension product [14].}
\end{table}

In order to determine the premium cost for the pension policy we first introduce the basics of Swedish traditional life insurance pricing. Let $T_x$ be the non-negative random variable corresponding to the remaining life span of an arbitrary individual at age $x$ with the distribution function \cite{2}

$$F_x(t) = P(T_x \leq t), \quad t \geq 0,$$

and let $f_x$ denote the density function of this variable. The survivor function $l_x$ is defined as \cite{2}

$$l_x(t) = 1 - F_x(t) = P(T_x > t), \quad t \geq 0,$$

42
which corresponds to the probability of an individual of age \( x \) be alive at least \( t \) more years. One can also write the survivor function \( l_x(t) \) as

\[
l_x(t) = \frac{l_0(x + t)}{l_0(x)}, \quad t \geq 0.
\]

To simplify the notation for the survivor function we denote \( l_0(t) \) as \( l(t) \) instead. The mortality rate for an individual of age \( x \) is defined and can be rewritten as

\[
\mu_x = \frac{f_x(x)}{1 - F_x(x)} = -\frac{l'(x)}{l(x)} = -\frac{d\ln(l(x))}{dx}
\]

since \( l'(x) = -f_x(t) \). Thus we can write the survivor function as

\[
l(x) = \exp \left\{ -\int_0^x \mu_s ds \right\}.
\]

Determining the mortality rate by using the density function and the distribution function may be complicated. A more convenient method is to use Makehams model \[2\] to determine the mortality rate. By this approach we model the mortality rate at age \( x \) as

\[
\mu_x = \alpha + \beta 10^{\gamma(x-f)},
\]

where \( \alpha, \beta > 0, (\alpha + \beta > 0) \) and \( \gamma \geq 0 \) are parameters to be estimated and \( f \) denotes the adjustment for females, which is equal to 6 if it is a female and 0 otherwise. The parameters are estimated using the modified minimum \( \chi^2 \) method based on historical statistics regarding life span from Statistics Sweden (SCB) \[6\]. In the modified minimum \( \chi^2 \) method we minimize

\[
Q = \sum_{i=1}^{n} \omega_{x_i} (\hat{\mu}_{x_i} - \alpha - \beta 10^{\gamma x_i})^2
\]

where \( \omega_{x_i} \) denotes the weights

\[
\omega_{x_i} = \frac{1}{\text{Var}[\hat{\mu}_{x_i}]} = \frac{D_{x_i}(t)}{\hat{\mu}_{x_i}^2} = \frac{N_{x_i}(t)}{\hat{\mu}_{x_i}},
\]

where \( D_x(t) \) denotes the number of individuals that decease under year \( t \) at age \((x, x+1)\) and \( N_x(t) \) denotes the average number of individuals aged \( x \) at year \( t \). We minimize \( Q \) by fixing \( \gamma \) and solving the following system of equations

\[
\frac{\partial Q}{\partial \alpha} = 0, \quad \frac{\partial Q}{\partial \beta} = 0,
\]

which has the solution

\[
\hat{\alpha} = \frac{m_{01} - \hat{\beta} m_{10}}{\omega}, \quad \hat{\beta} = \frac{\omega m_{11} - m_{10} m_{01}}{\omega m_{20} - m_{10}^2},
\]

43
where
\[
\omega = \sum_{i=1}^{n} \omega_{x_i}, \quad m_{10} = \sum_{i=1}^{n} \omega_{x_i} 10^\gamma x_i, \quad m_{01} = \sum_{i=1}^{n} \omega_{x_i} \hat{\mu}_{x_i},
\]
\[
m_{20} = \sum_{i=1}^{n} \omega_{x_i} 10^{2\gamma} x_i \quad \text{and} \quad m_{11} = \sum_{i=1}^{n} \omega_{x_i} 10^\gamma x_i \hat{\mu}_{x_i},
\]
respectively. Using this combined with historical data for different \(\hat{\gamma}\) we obtain the result presented in Table 4.3 that minimizes \(Q\).

<table>
<thead>
<tr>
<th>Makeham parameter</th>
<th>(\hat{\alpha})</th>
<th>(\hat{\beta})</th>
<th>(\hat{\gamma})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter estimation</td>
<td>0</td>
<td>(3 \cdot 10^{-5})</td>
<td>(4.4 \cdot 10^{-2})</td>
</tr>
</tbody>
</table>

Table 4.3: Parameter estimations for the Makeham model for mortality we will use in our model.

To determine the premium of the policy we use traditional Swedish life insurance pricing technique in continuous time. We introduce the commutation functions \(D(x)\) and \(N(x)\) defined as [2]
\[
D(x) = l(x)e^{-\delta x}, \quad \text{and} \quad N(x) = \int_x^\infty D(t)dt
\]
respectively for \(x \geq 0\). Note that these functions converge to zero as \(x \to \infty\). In the above definitions \(\delta\) corresponds to the interest rate intensity and is defined as [2]
\[
\delta = \ln (1 + i),
\]
where \(i\) denotes the interest rate. The commutation functions \(D\) and \(N\) are often referred to as the livings discounted value and the sum of the livings discounted value and are essential in the traditional method of life insurance pricing in Sweden. To calculate \(D(x)\) is not complicated since we know how to compute the mortality rate due to the parameter estimations and thus also the survival function. The interest rate intensity is also known since we previously modelled the STIBOR rate which we use as our interest rate \(i\) to consistent price the pension product. Calculating \(N(x)\) can however be complicated. Using Euler-Maclaurin summation this function can be approximated as [2]
\[
N(x) \approx \sum_{i=0}^{\omega} D(x + 1) - \frac{D(x)}{2} - \frac{\mu_x + \delta}{12} D(x),
\]
where \(\omega\) here denotes an age high enough for the function to converge to zero, typically \(\omega\) is around 100. Up till now we only considered life insurance for one life. However, since we consider a pension product with survivor pension we also need to introduce the
commutation functions for two lives. The commutation functions for two lives of age \( x \geq 0 \) and \( y \geq 0 \) respectively are defined as \[2\]

\[
D(x, y) = l(x)l(y)e^{-\frac{\delta}{2}(x+y)} \quad \text{and} \quad N(x, y) = \int_0^\infty D(x + t, y + t)dt
\]

respectively. For life insurance considering two lives we assume that the random variables for remaining life span \( T_x \) and \( T_y \) are independent random variables and the age \( x \) and \( y \) denotes the age of a male and female respectively. Working with two dimensional commutation functions may not be convenient. Instead, the theory of joint life have been introduced. Joint life means that we transform the individual ages \( x \) and \( y \) to one age \( \omega \) and consider the two individuals as one individual of age \( \omega \) \[2\]. Using Makehams formula the mortality rate of the two individuals of age \( x \) and \( y \), \( x \geq y \) is given by

\[
\mu'_x = \alpha' + \beta'e^{\gamma x} \quad \text{and} \quad \mu''_y = \alpha'' + \beta''e^{\gamma y}
\]

respectively for \( x, y \geq 0 \). If we assume that the parameter \( \gamma \) is equal for the two individuals, then the joint life \( \omega \) is defined as \[2\]

\[
\beta' e^{\gamma x} + \beta'' e^{\gamma y} = \beta'' e^{\gamma \omega}.
\]

The age \( \omega \) can then be calculated by

\[
\omega - y = \frac{1}{\gamma}\ln \left(1 + \frac{\beta'}{\beta''}e^{\gamma(x-y)}\right).
\]

For simplicity we will further assume that \( x = y \) for the issued policies in our pension portfolio. The joint age \( \omega \) will correspond to the age such that the relations

\[
\frac{D(x + t, y + t)}{D(x, y)} = \frac{D(\omega + t)}{D(\omega)}, \quad \frac{N(x, y)}{D(x, y)} = \frac{N(\omega)}{D(\omega)}
\]

and

\[
\frac{N(x, y) - N(x + t, y + t)}{D(x, y)} = \frac{N(\omega) - N(\omega + t)}{D(\omega)}
\]

holds. Note however that \( D(x, y) \neq D(\omega) \) and \( N(x, y) \neq N(\omega) \).

Now let \( X(t) \) and \( Y(t) \) denote the random variables representing the discounted value of the insured individual’s and the insurer’s future obligations at time \( t \) respectively. We introduce the value function of the insurance policy \( V(t) \) issued at time 0 as \[2\]

\[
V(t) = E[X(t)] - E[Y(t)], \quad V(0) = 0.
\]

This function is often referred to as the reserve of the insurance policy. Future obligations for the insurer and insured can for instance be annual annuity payments to the retired
individual and premium payments respectively. Note that we in this pricing model exclude costs such as fees and other expenses. By this pricing theory the goal is to determine the benefits and premiums such that \( E[X(t)] - E[Y(t)] = 0 \), this is often referred to as the premium equation \([2]\).

Now consider the pension product we use in this thesis. Assume that we have an individual at age \( x \) which after \( k \) years will get a yearly benefit of \( L' \geq 0 \) SEK given that the policy holder is alive and a yearly survivor benefit \( L'' \geq 0 \) SEK. Let \( L \geq 0 \) SEK denote the yearly salary for the individual at retirement age. Then by using \( L \) with Table 4.1 and Table 4.2 we can write the benefits \( L' \) and \( L'' \) as

\[
L' = \min \{ L, 7.5I \} \pi'_1 \\
+ \min \{ (20 - 7.5) I, \max [L - 7.5I, 0] \} \pi'_2 \\
+ \min \{ (30 - 20) I, \max [L - 20I, 0] \} \pi'_3
\]

and

\[
L'' = \min \{ (20 - 7.5) I, \max [L - 7.5I, 0] \} \pi''_2 \\
+ \min \{ (30 - 20) I, \max [L - 20I, 0] \} \pi''_3
\]

respectively, where \( I \) denotes the income base amount and

\[
\pi'_1 = 10\%, \quad \pi'_2 = 65\%, \quad \pi'_3 = 32.5\%, \quad \pi''_2 = \frac{\pi'}{2} \quad \text{and} \quad \pi''_3 = \frac{\pi'}{2}
\]

respectively. The expected value of the insurer’s future obligations at time 0, i.e. the expected future discounted cash flows to the insured individual or the survivor of the individual, can by using the commutation functions written as

\[
E[Y] = L' \frac{N(x + k) - N(x + k + s)}{D(x)} + L'' \frac{N(x + k, y + k) - N(x + k + s, y + k + s)}{D(x, y)},
\]

where \( x + k + s \) and \( y + k + s \) denote a age high enough for the commutation function to converge to zero. Using the theory of joint life the expected value of \( Y \) can be rewritten using only one dimensional commutation functions as

\[
E[Y] = L' \frac{N(x + k) - N(x + k + s)}{D(x)} + L'' \frac{N(\omega + k) - N(\omega + k + s)}{D(\omega)},
\]

where \( \omega \) denotes the age of the joint life. Assume that the insured individual will pay monthly premiums corresponding to the annual premium \( P \). Then the expected value of the insured future obligations at time 0, i.e. the expected future discounted cash flows from the policy holder to the insurance company, can be written as

\[
E[X] = P \frac{N(x) - N(x + n)}{D(x)},
\]

46
where \( x + n \) denotes the age till the individual will pay premiums. This annual premium is determined using the premium equation which yields

\[
P = D(x) \frac{L' \left[ N(x+k) - N(x+k+s) \right]}{D(x)} + D(x) \frac{\mu N(\omega+k) - N(\omega+k+s)}{D(\omega)} \frac{N(x) - N(x+n)}{N(x) - N(x+n)}.
\]

Using this knowledge we construct model points or in other words representative insured individuals for our simulations. We want to construct a portfolio of policy holders that is representable for Swedish life insurance companies. The size of the benefits for the insurance policies will be based on the average salary of individuals in Sweden. Note that the benefits for our simplified ITP 2 product are dependent on the salary of the individual right before retirement age. We base our benefit plan, and hence also the size of the annual premiums, on statistics from Statistics Sweden regarding yearly salaries for men and women in Sweden.

To create a representable portfolio of policy holders we also study the age and sex distribution of individuals in Sweden based on data from Statistics Sweden. We choose to create policy groups for computational time benefits by age and sex. We constructed six age groups: 25-34, 35-44, . . . , 75-84 for males and females respectively. We also let one model point, or in other words one policy holder, represent the whole group. This policy was scaled based on age, sex and expected yearly salary. For instance we let a thirty year old represent the age group 25-34 with a yearly salary that is scaled up. Assume that we have a thirty year old scaled up by a factor ten. If we compare this policy with ten identical thirty year olds we will get the exact same result in the simulations. If we compare the policy with ten policies of varying age from 25 to 34 we see that the input reserve will deviate with around 0.1 percent, which we will see as an acceptable deviation. If we look at the best estimate of liabilities we see a deviation of around 0.8 percent, which we also accept. If we plot the value of the BEL for both cases, Figure 4.1, we observe that the scaled policy follows the individual policies well. In this figure the red dashed and solid black line corresponds to the scaled up policy and the individual policies respectively. Hence, we will consider upwards scaled policies rather than individual policies due to computational time and since it does not seem to result in unrealistic results.

---

1. Today the most common pricing method of life insurance in practice is by using a cash flow model where you study the future cash flows (often monthly) in discrete time. The continuous technique applied in this thesis is the traditional way of pricing life insurances in Sweden and is powerful since it provides a good theoretical basis.
Figure 4.1: Comparison of the value of BEL for the case of one scaled policy (red dashed) versus ten individual policies (solid black). The scaled policy are of age 30 and the individuals are aged from 25 to 34, one policy for each age.

The scaling parameters were constructed using data from Statistics Sweden [6] based on data gathered from 2015 and can be found in Table A.10 and Table A.11 in Appendix. Based on these statistics, men make more money than women which corresponds to the fact that they expect to have higher benefits from the pension product. We also observe that there are fewer men alive for higher ages, which is expected since it is known that women tend to live longer than men. We use our 12 model points as input data for our policy portfolio found in Table 4.4. In this model we choose to have 100 females in the policy portfolio and distribute them in the age groups accordingly to the age distribution in Table A.10. For the male group we scale the number of males by using the male/female ratio. We assume that the retirement age is 65 and that the policy is matured at age 99. We also assume that all policies are issued in January 2015 and that all the policy
holders have the same birth day and month. The status ACTIVE corresponds to premium paying policy while the statuses PAYOUT and FAMILY corresponds to benefit payout for the alive individual and for the survivor/family of the insured individual respectively. The ingoing reserve, annual benefit and annual premium have been calculated according by the definition of the value function and the benefit plan for this pension.

Table 4.4: Input policies for our representative Swedish life insurance company. We assume that all the policies have the same birth day and month. The column ”Num. of policies” corresponds to the number of policies issued. In the column ”Sex” M and F corresponds to male and female respectively. The benefits, premiums and reserves are scaled up based on the number of policies and are denoted in SEK. Note that when the insured age is above retirement age, no premiums are payed and the reserve is decreasing due to benefit payouts.

Last, but not least, we also need to introduce a consolidation policy. Since the majority of all the life insurance companies in Sweden are mutual the surplus gained will be distributed over the policies that contributed to the profit. We introduce the keywords collective degree of consolidation (CDoC) and individual degree of consolidation (IDoC) and define the CDoC and IDoC at time \( t \) as

\[
\text{CDoC}(t) = \frac{\text{AS}_t}{V_w(t)} - 1 \quad \text{and} \quad \text{IDoC}(t) = \frac{\text{AS}_i}{V_w(t)} - 1
\]

respectively. In these definitions \( V_w \) and \( V_w^i \) denotes the retrospective reserve for the whole pension portfolio and individual \( i \) respectively. \( \text{AS}_t \) and \( \text{AS}_i \) corresponds to the asset shares for the whole pension portfolio and individual \( i \) respectively. We also introduce the bonus rate \( r_w \) defined as

\[
r_w(t) = r_g(t) + \frac{\text{DoC}(t) - m}{d}
\]

where \( r_g \) denotes the guaranteed interest rate by the pension agreement, \( m \) denotes the target degree of consolidation and \( d \) corresponds to the attenuation factor. DoC in this definition can either be the collective or individual degree of consolidation. Due to competition between life insurance companies we want the DoC to be constant at a level \( m \)
which is close to one since we want to pay out the profit to the policy holders which in most cases generates more new business\footnote{Even though we do not consider new business in the model we still act according to this.}. The attenuation factor determines how fast the bonus rate should react to the investment income.

Based on the major traditional life insurance companies we see that the companies have had average bonus rates between 4.8 to 7.6 percent the last five years. In order for the simulated life insurance company to be representative a consolidation policy that corresponds to bonus rates within this scope was constructed. We set the target level and the attenuation factor to \( m = 1.05 \) and \( d = 2 \) years and designed the consolidation policy as follows:

- If \( DoC \in (0.90, 1.15) \), no need for change in \( r_w \)
- If \( DoC \geq 1.15 \), increase \( r_w \) to match target level
- If \( DoC \leq 0.90 \), decrease \( r_w \) to match target level but not below \( r_g \).

### 4.3 Asset Liability Management

Asset liability management (ALM) is a tool to manage risks due to mismatches between asset and liabilities in a long term perspective. One may consider the ALM framework as a method to model the asset and the liability part of a balance sheet. One of the goals for a life insurance company is to have enough capital, which in most cases contains of financial assets, for the best estimate of future liabilities and fulfill the solvency capital requirement by the new regulations. Thus considering an ALM model is appropriate for this problem.

The simulations begin with a degree of solvency of 125 percent, which is equal to the market value of the assets divided by the technical provisions. This is based on DoS-levels for the major Swedish traditional life insurance companies, which in turn is based on data from Collectum \footnote{Even though we do not consider new business in the model we still act according to this.} lies between 162 and 212 percent (2015-12-31). The data from Collectum are based on DoS-levels based on the Solvency I regulation and we expect the levels to be similar in light of the new solvency regulation. There exists traditional life insurance companies with a lower degree than this interval and we choose a lower degree as a starting point for our model. It is also preferable to aim for higher degree of solvency (and a higher solvency ratio for that matter) in the sense that it enables for e.g. business expansion and riskier strategies. If the solvency ratio is below 100 percent, the solvency capital requirement is not fulfilled for the company and therefore cannot be considered as
solvent, which in turn is very important for insurance companies. For each year a Solvency II-balance sheet for the company will be constructed to determine if the company fulfills the solvency capital requirement for a given asset allocation. For each time step and asset allocation for the simulations of the representative life insurance company we will do the following:

- Determine best estimate of liabilities and the risk margin (from the liabilities model)
- Determine the market value of assets
- Construct a Balance Sheet for the base case (no Solvency II-stresses)
- Perform Solvency II-stress tests for market and life risk exposures in order to calculate the SCR
- Construct a Solvency II-Balance Sheet
- Determine if the company is solvent for the time step.

The best estimate of liabilities, the risk margin and the solvency capital requirement is determined according to the methods described in Chapter 3. To receive a more accurate result we construct actuarial assumptions such as guaranteed interest rate on the pension and fees to match the levels the major traditional life insurance companies in Sweden have. In order to determine whether the company is solvent or not is determined by the solvency ratio.

The value of the assets at time $t$ will be equal to

$$AS_t = AS_{t-1} + (1 + R_t) + \text{Premiums}_t - \text{Expenses}_t - \text{Commission}_t - \text{Claims}_t + \text{Charges}_t$$

where $AS_t$ denotes the asset shares at time $t$ and $R_t$ denotes the return of the investments at time $t$. The investment return will be the return of the investments from $t - 1$ to $t$ from the minimum variance portfolio. Premiums, expenses, commission, claims and charges at every monthly time step is determined within the model and depends on number of active policies, which state the policy is in (e.g. active, survivor’s pension) and gender to name a few.

We simulate 30 years in to the future for our traditional occupational pension portfolio using RiskAgility FM (RAFM) which is a financial modelling software used for life insurance modelling [21]. In RAFM the representative Swedish life insurance company was simulated in modules found in Figure 4.2. In Asset the market value of assets are determined based on the minimum variance portfolios for different target rates $\mu_0$. In Liability the development
of the insurance policies are simulated. For each monthly time step the output from Asset and Liability is the cash flows from that simulated month. The cash flows from these modules is the input in the module Control for the next time step. First, this module calculates the result from the previous time step based on the cash flows from Asset and Liability. Then the result is then used to calculate the ingoing market value of assets for the next time step, the bonus rate based on the ingoing degree of consolidation for the next time step. By ingoing market value of assets we refer to the market value before the monthly returns of the financial portfolio for that time step. Suppose that we simulate from time $t = m$ to $t = m + 1$, then the model will do the following:

- $t = m - 1$ in Control
  - Calculate the result from Asset and Liability from $t = m - 1$
  - Calculate the degree of consolidation for $t = m$
  - Calculate the bonus rate based for $t = m$

- $t = m$ in Asset
  - Calculate the market value of the assets

- $t = m$ in Liability
  - Calculate the cash flows from the insurance policies

- $t = m$ in Control
  - Calculate the result from Asset and Liability from $t = m$
  - Calculate the degree of consolidation for $t = m + 1$
  - Calculate the bonus rate based for $t = m + 1$

- $t = m + 1$ in Asset
  - Calculate the market value of the assets

- $t = m + 1$ in Liability
  - Calculate the cash flows from the insurance policies
The procedure above is for the base case, i.e. when no Solvency II-stresses are applied. Depending on which stress that is applied, for instance in the case of an equity stress the equities in the model will be stressed when calculating the market value of assets. For convenience we choose to calculate the solvency capital requirements separately in Solvency. This modules collects the result from all monthly time steps $t = 0, \ldots, 360$ for all asset allocations and all stressed scenarios for the company. The most important output from this module are the Solvency I and Solvency II capital requirements.

![Diagram](image)

Figure 4.2: Simplified version of the modelling structure in RAFM. Control, Asset and Liability are connected for every simulated time step. The module Solvency is used for solvency calculations such as the capital requirements based on Solvency I and Solvency II.

In the ALM model we study the monthly cash flows for the fictive life insurance company from December 2015 to December 2045. These cash flows were used to calculate the best estimate of liabilities by applying a long term interest rate term structure. For instance, if we consider year 2016, in order to calculate the BEL for that year we consider all the cash flows from January 2016 and to December 2045 (to maturity). The upcoming cash flows
was discounted using the interest rate term structure in order to get the present value of the future liabilities to get the BEL. This was made from every start of the year up to maturity. Since we consider a pension portfolio with no new policies we expect the number of policies decrease over time and hence the BEL to decrease when the simulated year is close to maturity year. We also expect that the BEL will increase when the simulated year is close to claim payouts, since the discount rate are often smaller for closer maturities. Note that this term structure, used to construct the discount rate curve, was recalculated every month since we are moving in time in the simulations and hence have new "observed" values.

A huge part of the thesis work was to implement the data analysis and the solvency capital requirement calculations to RiskAgility FM and construct the ALM model from scratch in the software. Even though we used the established liability model as a starting point, it had to be adjusted in order to fit in to our newly constructed ALM framework and Solvency II framework. We have implemented the solvency capital requirement calculations by the standard formula for future annual reports for every projected year, which is a major achievement itself since these annual calculations is something insurance companies rarely or cannot do in practice. By performing such analysis an insurance company for instance can see a detailed expected development of the insurance portfolio(s) parameters and its effect to the solvency capital requirement.
Chapter 5

Results and Discussion

In this chapter we first study the results of the minimum variance portfolio optimization for the financial assets the representative Swedish life insurance company invests in. This study is done by looking at the efficient frontier and how the portfolio weights changes based on the target return. From the simulations of the representative life insurance company we study the development of the basic own funds, the Solvency I capital requirement and the Solvency II capital requirement and its risk modules. Finally further studies and room for improvements in the model are discussed.

5.1 Portfolio Optimization

The optimization problem $\mathcal{P}$ was solved for different target returns $\mu_0$ for the strategic asset allocation in the range of 0 to 0.71 percent per month\footnote{Higher returns corresponded to infeasible solutions due to the short selling constraint.}. The target return corresponds to the average return of the portfolio for each time step, which in this case is a month. Recall the financial assets found in Table 5.1 for the representative Swedish life insurance company. Transforming the monthly returns to annual returns corresponds to a range from 0 to 8.86 percent. By using different annual targets we get an efficient frontier for the financial portfolio which is presented in Figure 5.1. The figure shows that higher target returns correspond to higher risk, which is expected since assets with higher return tend to be more risky. The portfolios on the frontier in this figure are efficient in that sense that the asset allocation corresponds to the minimum risk (standard deviation) for the given investment return. The colored dots in the plots corresponds to the expected return and
standard deviation of the financial portfolio if the portfolio manager choose to invest all the money in that specific asset. This figure shows that the stock indices lies far more to the right, i.e. more volatile, and that there exists efficient financial portfolios with higher expected return than these assets at a lower risk. We also observe that investing in only real estate, based on this model for our real estate index, is a more risky investment in comparison to the other assets. As expected, by Figure 5.1 the expected return for the money market is close to zero for a significant small risk and that investing in bonds is less risky than investing in risky assets.

Figure 5.2 shows how the asset allocation changes for the different efficient portfolios combined with the standard deviation for the financial portfolios. Table 5.2 shows a sample of portfolio weights for the different financial portfolios. As expected, for lower target returns the less risky assets dominates since they have less standard deviation. For the higher target portfolios the wealth tend to be invested in more risky assets. Note that SPCORP and OMXS30 does not have any non-zero portfolio weights and that DJIA only have small portfolio weights. This is due to the fact that there exists portfolio combinations that meets the target return for a smaller amount of risk, which from a mean variance perspective is preferable since it minimizes the risk (standard deviation) for a given return. By this figure we also observe a problem with applying the minimum variance approach in practice. If we only consider the stock indices in the graph we see that DAX is the only stock index with significant portfolio weights. This means that we do not distribute our invested wealth in stocks on different geographical markets which in practice would be preferable to construct a well diversified portfolio, this may be a hint that an investor should not trust the mathematical model blindly. By studying both Figure 5.1 and Figure 5.2 we observe the appearance of diversification since there exists efficient portfolios where e.g. the majority of the money is invested in risky real estate assets and still have a significant smaller risk exposure than investing all the money in real estate assets.
<table>
<thead>
<tr>
<th>Asset number</th>
<th>Abbreviation</th>
<th>Asset name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>STIBOR</td>
<td>STIBOR Fixed 1M</td>
</tr>
<tr>
<td>2</td>
<td>SPGOV</td>
<td>S&amp;P Sweden Sovereign Bond Index</td>
</tr>
<tr>
<td>3</td>
<td>SPRGOV</td>
<td>S&amp;P Sweden Sovereign Inflation-linked Bond Index</td>
</tr>
<tr>
<td>4</td>
<td>SPCORP</td>
<td>S&amp;P International Corporate Bond Index</td>
</tr>
<tr>
<td>5</td>
<td>OMXS30</td>
<td>OMX Stockholm 30</td>
</tr>
<tr>
<td>6</td>
<td>DAX</td>
<td>Deutsche Boerse AG German Stock Index</td>
</tr>
<tr>
<td>7</td>
<td>DJIA</td>
<td>Dow Jones Industrial Average</td>
</tr>
<tr>
<td>8</td>
<td>OMXREPI</td>
<td>OMX Stockholm Real Estate Price Index</td>
</tr>
<tr>
<td>9</td>
<td>CSHFI</td>
<td>Credit Suisse Hedge Fund Index</td>
</tr>
</tbody>
</table>

Table 5.1: The nine financial assets our fictive Swedish life insurance company will invest in. The asset number in the table corresponds to asset number $k$ in the multivariate geometric Brownian motion notation.

![Efficient Frontier](image)

Figure 5.1: The Efficient frontiers for the financial portfolios presented for yearly returns and standard deviations. The numbered and colored dots corresponds to the expected return and standard deviation by investing all the capital in one asset, where the numbers corresponds to asset number $k$ in Table 5.1.
Table 5.2: Table showing a sample of financial portfolio weights. Here $\mu_0$ denotes the annual target return, $\sigma$ denotes the corresponding annual standard deviation of the financial portfolio. The portfolio weights are denoted $w_k$, $k = 1, \ldots, 9$. In this table all values are denoted in percent.

<table>
<thead>
<tr>
<th>$\mu_0$</th>
<th>$\sigma$</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$\omega_3$</th>
<th>$\omega_4$</th>
<th>$\omega_5$</th>
<th>$\omega_6$</th>
<th>$\omega_7$</th>
<th>$\omega_8$</th>
<th>$\omega_9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.09</td>
<td>0.34</td>
<td>73.10</td>
<td>16.82</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.34</td>
<td>0</td>
<td>0</td>
<td>9.74</td>
</tr>
<tr>
<td>2.06</td>
<td>0.63</td>
<td>49.19</td>
<td>31.84</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.57</td>
<td>0</td>
<td>18.40</td>
<td></td>
</tr>
<tr>
<td>3.04</td>
<td>0.93</td>
<td>25.29</td>
<td>46.86</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.80</td>
<td>0</td>
<td>27.05</td>
<td></td>
</tr>
<tr>
<td>4.03</td>
<td>1.22</td>
<td>1.38</td>
<td>61.89</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.03</td>
<td>0</td>
<td>35.70</td>
<td></td>
</tr>
<tr>
<td>5.03</td>
<td>2.31</td>
<td>0</td>
<td>49.14</td>
<td>29.31</td>
<td>0</td>
<td>0</td>
<td>9.73</td>
<td>0</td>
<td>11.82</td>
<td></td>
</tr>
<tr>
<td>6.04</td>
<td>4.43</td>
<td>0</td>
<td>1.98</td>
<td>56.85</td>
<td>0</td>
<td>0</td>
<td>8.98</td>
<td>0</td>
<td>32.19</td>
<td></td>
</tr>
<tr>
<td>7.06</td>
<td>7.54</td>
<td>0</td>
<td>0</td>
<td>30.92</td>
<td>0</td>
<td>0</td>
<td>9.20</td>
<td>0</td>
<td>59.88</td>
<td></td>
</tr>
<tr>
<td>8.08</td>
<td>9.49</td>
<td>0</td>
<td>0</td>
<td>14.11</td>
<td>0</td>
<td>0</td>
<td>9.36</td>
<td>0</td>
<td>76.53</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.2: Figure showing how the portfolio weights change for the different financial portfolios. The black line represents the standard deviation in percent for the efficient financial portfolios (right vertical axis). Only the annual returns of every fifth portfolio are displayed on the horizontal axis, a complete list can be found in Table A.12 in Appendix.
5.2 Solvency Analysis

From the simulations of the representative Swedish life insurance company we obtain monthly observations, for each asset allocation, of elements such that how the cash flows in and out of the company and how life insurance mathematical functions such as survival functions develop over time, and also on a yearly basis, how the solvency capital requirement and its sub modules develops for the projected years. We choose to limit the presentation of all of the results and will not present the results for the risk sub modules for life underwriting risk and market risk since it is in this report more interesting to look at the risk modules and the basic solvency capital requirement rather than the specific sub modules.

Before we review the capital requirement, we start this section by studying the development of the basic own funds over time and for the financial portfolios determined in the previous section. In Figure 5.3 this development is shown. For the asset allocations with low target return the basic own funds becomes negative in the latter years. This is expected since these asset allocations generates a low return on the investments and therefore if the company offers a guaranteed interest rate above this return there will not be enough capital to pay out the pension benefits. In portfolios with high annual return the basic own funds will increase over time. This is due to the fact that the parameters in our ingoing consolidation policy was not aggressive enough to keep up with the aggressive strategic asset allocation. This can for instance be solved by lowering the attenuation factor, however using these strategic asset allocations are unrealistic especially for a life insurance company and therefore we will not resolve this issue since insurance companies in general do not seek unnecessary risks. It can also be solved by implementing allocation and real allocation in the consolidation policy where the company adjusts the value the insurance policy by increasing or decreasing (but not below the guaranteed amount) by a lump sum if the degree of consolidation is above or below certain levels. Also, for the latter years more policies becomes matured in the sense that the individuals have died or the pension plan is fully paid out which corresponds to smaller liabilities.
As mentioned in Chapter 3, the solvency capital requirement based on the previous regulations is more or less four percent of the technical provisions under the Solvency I regulation. The capital requirement based on the old regulations will therefore over the simulated years for our company follow the development of the Solvency I technical provisions. Hence if the liability portfolio decreases, i.e. pensions are paid out, the capital requirement will decrease. In other words, the old capital requirement will not be as risk sensitive as the new regulations since the risk sensitivity in the old regulations is only one percent of the positive risk sums which is relatively small part of the capital requirement. Figure 5.4 shows the structure and development of the SCR based on the old regulations. In this figure we observe that the capital requirement decreases over time which is expected since the technical provisions decreases over time due to pension payments. Also, note that this figure shows that capital requirement is somewhat independent of asset allocation strategy and can therefore be seen as a poor measure of the risk exposure of the insurance company.

Figure 5.3: Figure showing the development of the basic own funds for the fictive Swedish life insurance company over time for the different financial portfolios. The values in the color bar on the right side is denoted in SEK. Only every fifth year and annual return of every fifth portfolio are displayed on the axes, a complete list of the annual returns can be found in Table A.12 in Appendix.
Figure 5.4: Figure showing the development of the solvency capital requirement based on the old regulations for the fictive Swedish life insurance company over time for the different financial portfolios. The values in the color bar on the right side is denoted in SEK. Only every fifth year and annual return of every fifth portfolio are displayed on the axes, a complete list of the annual returns can be found in Table A.12 in Appendix.

Interestingly, if we study the development of the solvency capital requirement due to life underwriting risk, see Figure 5.5, we see a structure that reminds us of the structure for the capital requirement based on the old regulations. The life underwriting risk looks independent of asset strategy except for the area where we expect bankruptcy due to low return investments. The higher capital requirement in this area is due to the interest rate risk which comes into play for the life underwriting risk module as well is much larger for the lower target portfolios since more is invested in interest rate assets.
Figure 5.5: Figure showing the development of the solvency capital requirement for life underwriting risk for the fictive Swedish life insurance company over time for the different financial portfolios. The values in the color bar on the right side is denoted in SEK. Only every fifth year and annual return of every fifth portfolio are displayed on the axes, a complete list of the annual returns can be found in Table A.12 in Appendix.

In Figure 5.6 the development of the solvency capital requirement for market risk is shown. In this figure we observe that a well diversified financial portfolio is beneficial for this capital requirement module. We also conclude that the portfolios that are not well diversified will suffer since the risk exposure is not distributed over several markets. For instance, the lower target portfolios will correspond to the company being highly exposed to interest rate risks. Note the capital requirement for these kinds of financial portfolios decreases over time, however this is due to the fact that the portfolio value decreases.
Figure 5.6: Figure showing the development of the solvency capital requirement for market risk for the fictive Swedish life insurance company over time for the different financial portfolios. The values in the color bar on the right side is denoted in SEK. Only every fifth year and annual return of every fifth portfolio are displayed on the axes, a complete list of the annual returns can be found in Table A.12 in Appendix.

By studying the solvency capital requirement development, which in this report is equal to the basic solvency capital requirement, we observe in Figure 5.7 that this clearly reminds us of the market risk module both in structure and absolute size. This result is expected and is in line with previous studies [17] where it is shown that the market risk dominates the contribution to the solvency capital requirement in the new regulations. We conclude that solvency capital requirement have increased in comparison to the old regulations and also its structure is more dependent on the market risk exposure. In line with the market risk module we see that the well diversified financial portfolios are beneficial. In Figure A.10 and Figure A.11 in Appendix the indicators regarding the company is solvent or insolvent for Solvency II and Solvency I respectively are shown. By comparing these figures we conclude that the area of insolvency is larger due to the new regulation.
The different asset allocations correspond to different solvency capital requirements. For instance, a well diversified financial portfolio will have a smaller capital requirement than a more risky strategy with riskier assets. So, why do the insurance companies invest in risky assets? Of course, the solvency capital requirement will be significantly lower for the more diversified asset allocation. But, such a strategy does not necessarily correspond to a higher degree of solvency or a higher solvency ratio. One can also argue that this is due to competition between insurance companies. Even though the life insurance companies are mostly mutual companies the companies wants more policy holders. From more policy holders follows, in most cases, more premium payments and a more diversified insurance portfolio.

From the figures in this section we conclude that using a well diversified portfolio is beneficial in the sense that it corresponds to a lower capital requirement due to the match of assets and liabilities. We now study more in depth the development of the one of the
more diversified financial portfolios we consider in this thesis since they are more realistic financial portfolio for a life insurance company. For instance, investing all the money in real estate or in interest rate may not be a fair view of the typical developments of the capital requirement and hence we will not investigate further into these portfolios. Figure 5.8 shows how the standard deviation for the financial portfolio, the basic solvency capital requirement, the solvency capital requirement for market risk and the solvency capital requirement based on the old regulations change over time for the financial portfolio corresponding to an annual return of 4.91 percent. The portfolio weights for this financial portfolio can be found in Table 5.3. We observe that the capital requirement for market risk follows the change in basic solvency capital requirement which is expected since it dominates in contribution to the solvency capital requirement. In the same figure we observe deviations between the capital requirement for market risk and the standard deviation of the financial portfolio, while the change in solvency capital requirement based on the old regulations stays the same for the different asset allocations, which is in line with Figure 5.4. One could study the corresponding plots for all of the beneficial portfolios, however this corresponds to a similar result and thus we choose not to present these plots in this report.

<table>
<thead>
<tr>
<th>$\mu_0$</th>
<th>$\sigma$</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$\omega_3$</th>
<th>$\omega_4$</th>
<th>$\omega_5$</th>
<th>$\omega_6$</th>
<th>$\omega_7$</th>
<th>$\omega_8$</th>
<th>$\omega_9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.91</td>
<td>2.11</td>
<td>0</td>
<td>55.02</td>
<td>25.86</td>
<td>0</td>
<td>9.83</td>
<td>0</td>
<td>9.29</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5.3: Table showing the portfolio weights of the financial portfolio corresponding to an annual return of 4.91 percent. Here $\mu_0$ denotes the annual target return, $\sigma$ denotes the corresponding annual standard deviation of the financial portfolio. The portfolio weights are denoted $\omega_k$, $k = 1, \ldots, 9$. In this table all values are denoted in percent.
Figure 5.8: Figure showing the development of the standard deviation for the financial portfolio, the basic solvency capital requirement, the solvency capital requirement for market risk and the solvency capital requirement based on the old regulations change over time for the financial portfolio with an annual return of 4.91 percent.

Figure 5.9 shows the ratio between the market risk solvency capital requirement and the asset value of the company and the ratio between the annual standard deviation of the financial portfolio and its asset value. This is shown for calendar years 2016, 2021, 2026 and 2031. Note the dip around 5 percentage in annual return for the market risk module. This dip is in line with the diversification effect observed in previous figures when studying the structure and development of the capital requirement and its modules. We choose to exclude the solvency capital requirement by the old regulations in this plot since, as seen in Figure 5.4, the capital requirement for the old regulations are not that dependent
of the market risk exposure. This was also one of the major issues and reasons behind implementing the new solvency regulation [10], being more risk sensitive and potentially rewarding insurance companies with better risk management. Note that the ratio for the annual standard deviation is equal to the ratio found for the efficient frontier found in Figure 5.1. By studying Figure 5.9 and Figure 5.8 combined we see that there exists deviations in development between the standard deviation of the financial portfolio and the capital requirement for market risk. Previously we concluded that the market risk module dominates in contribution to the solvency capital requirement, thus one could have expected that the solvency capital requirement for the market risk should coincide with the standard deviation of the financial portfolio. We are not stating that it should be of the same size, however if the market risk dominates in the capital requirement one could have expected the same characteristics as the standard deviation. This may be because of the fact that the regulatory have constructed the correlations between the asset types in general, rather than for the specific assets we consider in this thesis. Thus, minimizing a financial portfolio does not necessarily correspond to a "good" portfolio in light of the solvency regulations. This may force the insurance companies to act in a non-efficient way, in the minimum variance perspective, in order to fulfill the capital requirements from the regulator.

It may be interesting to construct a portfolio optimization algorithm that focuses on minimizing the solvency capital requirement rather than the portfolio variance, or a combination of those two. One should however keep in mind that minimizing the solvency capital requirement in a portfolio optimization algorithm in light of the standard formula does not necessarily correspond to a good portfolio in light of minimizing the actual market risks for the insurance company. Due to these mentioned problems it is obvious that larger companies tend to construct internal or partial internal models for computing the solvency capital requirement. However, in this thesis we do not focus on constructing such a internal model since it may be very complex and is more than a master’s thesis. Instead we state that there may exist flaws in the standard formula for solvency capital requirement. It would be interesting to construct internal models for the capital requirement and compare it to the standard formula.
Figure 5.9: Figure showing how the solvency capital requirement for market risk and the standard deviation of the financial portfolio relates to the asset value of the company for different calendar years. The dots in this figure is the ratio between the capital requirement due to market risk and the asset value and the ratio between the standard deviation of the financial portfolio and its asset value. Note that the ratio for the annual standard deviation is equal to the ratio found for the efficient frontier found in Figure 5.1.
5.3 Conclusions and Further Studies

In the previous section we conclude that the Solvency II capital requirement is more risk sensitive than the Solvency I capital requirement. As concluded earlier, the Solvency II capital requirement has the same structure as the risk module for market risk and thus depend on the asset allocation of the insurance company. Meanwhile, the structure of the old solvency capital requirement is not as dependent on the asset management. As expected, the life underwriting risk is not as dependent on the asset allocation, however when interest bearing securities are dominating the asset allocation there are significant effects on the life underwriting risk module. By comparing the development of the standard deviation of the minimum variance portfolio with the new capital requirement for the one of the more diversified portfolios we conclude that there exists some deviations, but the Solvency II capital requirement follows the risk measure better in comparison to the Solvency I capital requirement. We once again observed that the development of the market risk module follows the solvency capital requirement which was expected. Therefore, life insurance companies in Sweden have tightened financial investing opportunities due the new regulations. This is due to the fact that the capital requirement now is far more risk sensitive and the market risk module dominates the contribution to the capital requirement.

It is worth mentioning that there exists room for improvement regarding the simulation of assets as well as the asset allocation. Using minimum variance to determine portfolio weights is a straightforward and powerful tool. Even though insurance companies are risk seeking, they do not seek unnecessary risks. Therefore minimizing the risk, or the standard deviation from a MPT-perspective, for a given return is reasonable. However, constructing a whole investment strategy based on only a minimum variance portfolio is in practice not recommended. As observed in the asset allocations for the different target returns we saw that trusting the model blindly does not necessarily correspond to a well diversified portfolio in the real world. It is also worth mentioning that the model considers risk from a variability of return over one period whereas the insurance contracts have much longer duration. A interesting strategy would be to duration match future guaranteed liabilities with e.g. government bonds, inflation and non-inflation linked, and use a minimum variance strategy for the excess since the insurers main risk is inability to match future liabilities and objectives rather than variability in assets. This would probably result in a even better match to future liabilities and higher return than using one minimum variance portfolio to match the liabilities and it would be interesting to perform this asset allocation instead and compare the results. It would also be interesting to investigate further regarding adding a solvency capital requirement constraint to the minimum variance optimization. Regarding the asset simulations, using the GBM approach is a commonly used and powerful tool. The approach may be questionable since it assumes that the logarithmic returns are normally
distributed which is not always the case, which we also saw in the Jarque-Bera test. One improvement could be to implement a stochastic volatility model combined with a jump diffusion model to model unexpected jumps in the asset prices.

There exists some improvements regarding the model used for the mortality rate. The Makeham model used in this thesis can for instance be replaced by a more modern model such as the Lee-Carter model [2] or by using the mortality rates directly from DUS14 [11]. However, we concluded earlier that the life module does not dominate in the contribution to the solvency capital requirement, therefore this change will not affect the result of this study as much as the model for financial assets and portfolio optimization. Regarding pricing the policies one should preferably use the deterministic model, but then again the continuous technique is effective and powerful for these kinds of analyses.
Appendix A

Appendix

A.1 Chapter 2

Figure A.1: Auto-Correlation Function (left side) and Partial Auto-Correlation function (right side) plotted for STIBOR, SPGOV and SPRGOV up to lag 50 based on the historical data.
Figure A.2: Auto-Correlation Function (left side) and Partial Auto-Correlation function (right side) plotted for SPCORP, OMXS30 and DAX up to lag 50 based on the historical data.
Figure A.3: Auto-Correlation Function (left side) and Partial Auto-Correlation function (right side) plotted for DJIA, OMXREPI and CSHFI up to lag 50 based on the historical data.
Figure A.4: The result of 100 simulated paths for the STIBOR rate and the financial assets when considering the latter 120 data points for parameter estimation. The thicker black and red line corresponds to the arithmetic mean and median respectively.
Figure A.5: The result of 100 simulated paths for the STIBOR rate and the financial assets when considering the first 120 data points for parameter estimation. The thicker black and red line corresponds to the arithmetic mean and median respectively.
Figure A.6: The result of 100 simulated paths for the STIBOR rate and the financial assets when considering all the observed and simulated data points, equally weighted, for parameter estimation. The thicker black and red line corresponds to the arithmetic mean and median respectively.
Figure A.7: The result of 1000 simulated paths for the STIBOR rate and the financial assets when considering all the observed and simulated data points, weighted according to the weighting system, for parameter estimation. The thicker black and red line corresponds to the arithmetic mean and median respectively.
Figure A.8: The result of 1000 simulated paths for the returns for STIBOR rate and the financial assets when considering all the observed and simulated points, weighted according to the weight system, for parameter estimation. The thicker black points corresponds to the arithmetic mean. In this figure we do not show the points corresponding to the median returns since they more or less lie on top of each other.
A.2 Chapter 3

<table>
<thead>
<tr>
<th>o_i</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>o_1</td>
<td>-0.102</td>
<td>-0.183</td>
<td>-0.237</td>
<td>-0.296</td>
<td>-0.336</td>
<td>-0.355</td>
<td>-0.366</td>
<td>-0.376</td>
<td>-0.383</td>
<td>-0.388</td>
</tr>
<tr>
<td>o_2</td>
<td>-0.438</td>
<td>-0.471</td>
<td>-0.418</td>
<td>-0.296</td>
<td>-0.166</td>
<td>-0.033</td>
<td>0.106</td>
<td>0.223</td>
<td>0.31</td>
<td>0.371</td>
</tr>
<tr>
<td>o_3</td>
<td>0.556</td>
<td>0.295</td>
<td>0.028</td>
<td>-0.306</td>
<td>-0.416</td>
<td>-0.335</td>
<td>-0.123</td>
<td>0.097</td>
<td>0.264</td>
<td>0.358</td>
</tr>
</tbody>
</table>

Table A.1: Table of the first principal components, o_1, o_2 and o_3, which explain 99.5% of the randomness from maturity in one to ten years. The components explain the parallel shifts, changes in slope and changes in curvature for the swap fixing rate respectively.

<table>
<thead>
<tr>
<th>Maturity (in years)</th>
<th>Stress Shock Down (in percent)</th>
<th>Stress Shock Up (in percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-75</td>
<td>70</td>
</tr>
<tr>
<td>2</td>
<td>-65</td>
<td>70</td>
</tr>
<tr>
<td>3</td>
<td>-56</td>
<td>64</td>
</tr>
<tr>
<td>4</td>
<td>-50</td>
<td>59</td>
</tr>
<tr>
<td>5</td>
<td>-46</td>
<td>55</td>
</tr>
<tr>
<td>6</td>
<td>-42</td>
<td>52</td>
</tr>
<tr>
<td>7</td>
<td>-39</td>
<td>49</td>
</tr>
<tr>
<td>8</td>
<td>-36</td>
<td>47</td>
</tr>
<tr>
<td>9</td>
<td>-33</td>
<td>44</td>
</tr>
<tr>
<td>10</td>
<td>-31</td>
<td>42</td>
</tr>
<tr>
<td>11</td>
<td>-30</td>
<td>39</td>
</tr>
<tr>
<td>12</td>
<td>-29</td>
<td>37</td>
</tr>
<tr>
<td>13</td>
<td>-28</td>
<td>35</td>
</tr>
<tr>
<td>14</td>
<td>-28</td>
<td>34</td>
</tr>
<tr>
<td>15</td>
<td>-27</td>
<td>33</td>
</tr>
<tr>
<td>16</td>
<td>-28</td>
<td>31</td>
</tr>
<tr>
<td>17</td>
<td>-28</td>
<td>30</td>
</tr>
<tr>
<td>18</td>
<td>-28</td>
<td>29</td>
</tr>
<tr>
<td>19</td>
<td>-29</td>
<td>27</td>
</tr>
<tr>
<td>20</td>
<td>-29</td>
<td>26</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>90</td>
<td>-20</td>
<td>20</td>
</tr>
</tbody>
</table>

Table A.2: Table of interest rate shocks that will be used in the Solvency II stress tests in order to calculate the solvency capital requirement.
<table>
<thead>
<tr>
<th>CorrSCR&lt;sup&gt;up&lt;/sup&gt;</th>
<th>Equity</th>
<th>Interest rate</th>
<th>Property</th>
<th>Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>1</td>
<td>0</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Property</td>
<td>0.75</td>
<td>0</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>Spread</td>
<td>0.75</td>
<td>0</td>
<td>0.5</td>
<td>1</td>
</tr>
</tbody>
</table>

Table A.3: Solvency II standard formula correlation matrix for the upward stress scenario.

<table>
<thead>
<tr>
<th>CorrSCR&lt;sup&gt;down&lt;/sup&gt;</th>
<th>Equity</th>
<th>Interest rate</th>
<th>Property</th>
<th>Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>1</td>
<td>0.5</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Property</td>
<td>0.75</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>Spread</td>
<td>0.75</td>
<td>0.5</td>
<td>0.5</td>
<td>1</td>
</tr>
</tbody>
</table>

Table A.4: Solvency II standard formula correlation matrix for the downward stress scenario.

<table>
<thead>
<tr>
<th>Risk sub module</th>
<th>Stress shock (in percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity - Type 1</td>
<td>-39</td>
</tr>
<tr>
<td>Equity - Type 2</td>
<td>-49</td>
</tr>
<tr>
<td>Property</td>
<td>-25</td>
</tr>
</tbody>
</table>

Table A.5: Table of shocks that will be used in the Solvency II stress tests in order to calculate the solvency capital requirement due to equity and property risk.

<table>
<thead>
<tr>
<th>Bond index</th>
<th>Duration</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPGOV</td>
<td>5.98</td>
<td>0.045</td>
<td>0.005</td>
</tr>
<tr>
<td>SPRGOV</td>
<td>7.38</td>
<td>0.045</td>
<td>0.005</td>
</tr>
<tr>
<td>SPCORP</td>
<td>6.39</td>
<td>0.055</td>
<td>0.006</td>
</tr>
</tbody>
</table>

Table A.6: Table of parameters for the bond indices that will be used in the Solvency II stress tests in order to calculate the solvency capital requirement due to spread risk. The duration for the bond indices was found in the fact sheet from Standard and Poor’s website and is measured in years.
<table>
<thead>
<tr>
<th>CorrSCR$^{\text{life}}$</th>
<th>Mortality</th>
<th>Longevity</th>
<th>Life expense</th>
<th>Revision</th>
<th>Lapse</th>
<th>Life catastrophe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mortality</td>
<td>1</td>
<td>-0.25</td>
<td>0.25</td>
<td>0</td>
<td>0</td>
<td>0.25</td>
</tr>
<tr>
<td>Longevity</td>
<td>-0.25</td>
<td>1</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0</td>
</tr>
<tr>
<td>Life expense</td>
<td>0.25</td>
<td>0.25</td>
<td>1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.25</td>
</tr>
<tr>
<td>Revision</td>
<td>0</td>
<td>0.25</td>
<td>0.5</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Lapse</td>
<td>0</td>
<td>0.25</td>
<td>0.5</td>
<td>0</td>
<td>1</td>
<td>0.25</td>
</tr>
<tr>
<td>Life catastrophe</td>
<td>0.25</td>
<td>0</td>
<td>0.25</td>
<td>0</td>
<td>0.25</td>
<td>1</td>
</tr>
</tbody>
</table>

Table A.7: Solvency II standard formula correlation matrix for the life underwriting risk module.
### Chapter 4

<table>
<thead>
<tr>
<th>First order basis</th>
<th>Value</th>
<th>Second order basis</th>
<th>Value</th>
<th>Experience basis</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premium frequency</td>
<td>12</td>
<td>Mortality factor</td>
<td>1</td>
<td>Mortality factor</td>
<td>1</td>
</tr>
<tr>
<td>Fee on capital, benefit (percentage)</td>
<td>0.4</td>
<td>Acquisition fee (fixed)</td>
<td>0</td>
<td>Premium index rate (percentage)</td>
<td>0.5</td>
</tr>
<tr>
<td>Fee on capital, benefit (fixed)</td>
<td>0</td>
<td>Admin fee (fixed)</td>
<td>100</td>
<td>Admin expense (fixed)</td>
<td>139</td>
</tr>
<tr>
<td>Fee on capital, prognosis (percentage)</td>
<td>0.4</td>
<td>Admin fee (percentage)</td>
<td>0.7</td>
<td>Admin expense (percentage)</td>
<td>0</td>
</tr>
<tr>
<td>Fee on capital, prognosis (fixed)</td>
<td>0</td>
<td>Commission on premium (percentage)</td>
<td>0</td>
<td>Acquisition expense (fixed)</td>
<td>0</td>
</tr>
<tr>
<td>Fee on capital, reserve (percentage)</td>
<td>0.4</td>
<td>Premium fee (fixed)</td>
<td>0</td>
<td>Transfer expense (fixed)</td>
<td>200</td>
</tr>
<tr>
<td>Fee on capital, reserve (fixed)</td>
<td>0</td>
<td>Premium fee (percentage)</td>
<td>0.4</td>
<td>Transfer expense (percentage)</td>
<td>0</td>
</tr>
<tr>
<td>Guaranteed interest rate, benefit (percentage)</td>
<td>3.222</td>
<td>Premium fee, external (fixed)</td>
<td>0</td>
<td>Premium fee, external (percentage)</td>
<td>0</td>
</tr>
<tr>
<td>Guaranteed interest rate, prognosis (percentage)</td>
<td>1.57</td>
<td>Premium fee, external (fixed)</td>
<td>0</td>
<td>Transfer fee (fixed)</td>
<td>400</td>
</tr>
<tr>
<td>Guaranteed interest rate, reserve (percentage)</td>
<td>1.57</td>
<td>Transfer fee (fixed)</td>
<td>400</td>
<td>Transfer fee (percentage)</td>
<td>0</td>
</tr>
</tbody>
</table>

Table A.8: Table showing the actuarial assumptions used in the model. The values of the fixed costs are in SEK and each assumption are per policy. Premium frequency 12 corresponds to monthly premium payments.

![Development of the Inflation Rate](image)

Figure A.9: The development of the inflation rate in Sweden. The data is gathered from SCB [6]. Even though the inflation rate is not constant at 2 percent the mean inflation rate of the observed time period is 2 percent.
Table A.9: Table showing salary raise that can contribute to the pension benefit for the individual. For example, if the individual receives a pay raise when it is 40 months left till retirement, then only the part of the raise that is 1.15 multiplied with the increase of the income base amount can contribute to the pension benefit. Thus, large salary raises during the last years of employment will not affect the pension benefit as much [14].

### Table A.9

<table>
<thead>
<tr>
<th>Months to retirement</th>
<th>Maximum salary raise for pension benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td>59 – 49</td>
<td>1.20 income base amount increase</td>
</tr>
<tr>
<td>48 – 37</td>
<td>1.15 income base amount increase</td>
</tr>
<tr>
<td>36 – 25</td>
<td>1.10 income base amount increase</td>
</tr>
<tr>
<td>24 – 13</td>
<td>1.05 income base amount increase</td>
</tr>
<tr>
<td>12 – 1</td>
<td>1.00 income base amount increase</td>
</tr>
</tbody>
</table>

Table A.10: The ratio between males and females and the distribution of ages for the different group of ages based on data from Statistics Sweden [6] from 2015.

### Table A.10

<table>
<thead>
<tr>
<th>Age group</th>
<th>Male/Female</th>
<th>Age distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>25-34 y/o</td>
<td>1.051212382</td>
<td>0.194353364</td>
</tr>
<tr>
<td>35-44 y/o</td>
<td>1.036427495</td>
<td>0.188235227</td>
</tr>
<tr>
<td>45-54 y/o</td>
<td>1.031483701</td>
<td>0.195160355</td>
</tr>
<tr>
<td>55-64 y/o</td>
<td>1.005142968</td>
<td>0.170065540</td>
</tr>
<tr>
<td>65-74 y/o</td>
<td>0.968936585</td>
<td>0.164280265</td>
</tr>
<tr>
<td>75-84 y/o</td>
<td>0.821381717</td>
<td>0.087905248</td>
</tr>
</tbody>
</table>

### Table A.11

<table>
<thead>
<tr>
<th>Age group</th>
<th>Sex</th>
<th>Average salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>25-34 y/o</td>
<td>Male</td>
<td>418231</td>
</tr>
<tr>
<td></td>
<td>Female</td>
<td>368369</td>
</tr>
<tr>
<td>35-44 y/o</td>
<td>Male</td>
<td>542366</td>
</tr>
<tr>
<td></td>
<td>Female</td>
<td>449329</td>
</tr>
<tr>
<td>45-54 y/o</td>
<td>Male</td>
<td>582947</td>
</tr>
<tr>
<td></td>
<td>Female</td>
<td>468270</td>
</tr>
<tr>
<td>55-64 y/o</td>
<td>Male</td>
<td>563077</td>
</tr>
<tr>
<td></td>
<td>Female</td>
<td>439697</td>
</tr>
</tbody>
</table>

## A.4 Chapter 5

<table>
<thead>
<tr>
<th>Annual Returns (in percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
</tr>
<tr>
<td>0.12</td>
</tr>
<tr>
<td>0.24</td>
</tr>
<tr>
<td>0.36</td>
</tr>
<tr>
<td>0.48</td>
</tr>
<tr>
<td>0.60</td>
</tr>
<tr>
<td>0.72</td>
</tr>
<tr>
<td>0.84</td>
</tr>
<tr>
<td>0.96</td>
</tr>
<tr>
<td>1.09</td>
</tr>
<tr>
<td>1.21</td>
</tr>
<tr>
<td>1.33</td>
</tr>
<tr>
<td>1.45</td>
</tr>
<tr>
<td>1.57</td>
</tr>
<tr>
<td>1.69</td>
</tr>
<tr>
<td>1.81</td>
</tr>
<tr>
<td>1.94</td>
</tr>
<tr>
<td>2.06</td>
</tr>
<tr>
<td>2.18</td>
</tr>
<tr>
<td>2.30</td>
</tr>
<tr>
<td>2.43</td>
</tr>
<tr>
<td>2.55</td>
</tr>
<tr>
<td>2.67</td>
</tr>
<tr>
<td>2.80</td>
</tr>
<tr>
<td>2.92</td>
</tr>
<tr>
<td>3.04</td>
</tr>
<tr>
<td>3.17</td>
</tr>
<tr>
<td>3.29</td>
</tr>
<tr>
<td>3.41</td>
</tr>
<tr>
<td>3.54</td>
</tr>
<tr>
<td>3.66</td>
</tr>
<tr>
<td>3.78</td>
</tr>
<tr>
<td>3.91</td>
</tr>
<tr>
<td>4.03</td>
</tr>
<tr>
<td>4.16</td>
</tr>
<tr>
<td>4.28</td>
</tr>
</tbody>
</table>

Table A.12: Table showing the complete list of annual returns for the financial portfolios.
Figure A.10: Figure showing if the development regarding if the representative life insurance company is solvent or insolvent over time for the different financial portfolios. A white and black box corresponds to solvent and insolvent respectively. Only every fifth year and annual return of every fifth portfolio are displayed on the axes, a complete list of the annual returns can be found in Table A.12.

Figure A.11: Figure showing if the development regarding if the representative life insurance company is solvent or insolvent by the old regulations over time for the different financial portfolios. A white and black box corresponds to solvent and insolvent respectively. Only every fifth year and annual return of every fifth portfolio are displayed on the axes, a complete list of the annual returns can be found in Table A.12.
Bibliography


87


