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Contact-free Measurement of Currents in Two-dimensional Parallel Conductors Using the Green Identity Approach

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Abstract—The Green identity approach is used to reconstruct currents in very long parallel conductors above the ground from magnetic field data measured at some points under the conductors. By assuming that the conductor positions are known, this method can be used for contact-free measurement of currents in transmission lines in power system. This method gives the possibility of measuring high-frequency currents which cannot be measured with current transformers.

I. INTRODUCTION

Monitoring voltage and current at different locations of power grid is critical to have a reliable system. Measurement transformers that are used as an established technique to measure voltage and current are installed as integral parts of the grid and only at few locations in the grid. They are rather expensive and have limited bandwidths. Thus it is not possible to detect high-frequency voltage and current due to the fast transients, which may be needed for assessing the state of the grid.

To facilitate measuring voltage and current at different locations of the grid and in a wider range of frequencies, contact-free measurement techniques which are proposed in [1]–[3] could be a complement to measurement transformers. These techniques are based on determining voltage and current from measuring the electric and magnetic fields surrounding the conductors by solving an inverse source problem.

Handling the external fields due to the external sources in the vicinity of the conductors is a critical issue in this kind of inverse source problem. In [1] where the reconstruction of both positions and currents of conductors are discussed, the impact of two-dimensional disturbance fields has been canceled completely by using a Green identity approach and measuring the magnetic field on a closed contour that circumscribes the conductors. This method needs magnetic field data on a closed contour that circumscribes the conductors. Nevertheless, from practical point of view, the measurement points must be limited both in the angular coverage and in the number of measurement points. Moreover, in electric power engineering, in many situations the conductor positions are known or determinable by other means than magnetic field measurements. Thus, in this work by assuming that the positions of conductors are known, the method proposed in [1] is modified in a way that knowing the magnetic field data

Fig. 1. Cross section of the problem.

at some points under the conductors are enough to reconstruct the currents.

II. PROBLEM FORMULATION AND METHOD

The configuration is shown in Fig. 1. There are a finite number of long, parallel and straight conductors above the ground. There is no z -dependence and we want to reconstruct the currents in the conductors inside the counter $\mathcal{C}$. Other two-dimensional line sources exist outside of the counter $\mathcal{C}$ and the images of all currents in the ground are considered as the external sources in this problem. A harmonic time-dependence $e^{j\omega t}$ is assumed and suppressed. We restrict to quasi-static regime in which the propagation effects can be neglected. In other words, the wavelength along the conductors is considerably larger than the distances between the conductors and their distances to the ground.

A. Current Reconstruction Method Using Green Identity

Expressing current density as $\mathbf{J} = J\hat{\mathbf{z}}$ and using the Green identity for a scalar and a vector function over the surface S bounded by counter $\mathcal{C}$ for this problem (See Fig. 1) yield [1]

$$\int_S J \nabla G ds = \frac{-1}{\mu_0} \hat{\mathbf{z}} \times \oint_{\mathcal{C}} [(\hat{\mathbf{n}} \cdot \mathbf{B}) \nabla G + (\hat{\mathbf{n}} \times \mathbf{B}) \times \nabla G] dl, \quad (1)$$

Fig. 2. Cross section of the problem with a semicircle as contour $\mathcal{C}$. The black dots under the x -axis are the poles of analytical function $f_m(w(\mathbf{r}))$.

where $\hat{\mathbf{n}}$ is a normal unit vector on $\mathcal{C}$ (see Fig. 1), G is a smooth test function which is harmonic both over $\mathcal{S}$ and in the neighbourhood of $\mathcal{C}$, and $\mathbf{B}$ is the magnetic field distribution. To have a harmonic and reasonably simple function as G , [1] proposes

$$G(\mathbf{r}) = G(w(\mathbf{r})), \quad (2)$$

where $\mathbf{r} \in \mathcal{S}$ and $w(\mathbf{r}) = x + jy$ or $w(\mathbf{r}) = x - jy$. Defining $dG/dw = f(w)$, (1) can be simplified to [1]

$$\int_{\mathcal{S}} J f(w(\mathbf{r})) ds = \frac{1}{\mu_0} \oint_{\mathcal{C}} [\mathbf{j}\hat{\mathbf{n}} + \hat{\mathbf{t}}] \cdot \mathbf{B} f dl, \quad (3)$$

where $\hat{\mathbf{t}}$ is a tangent unit vector on $\mathcal{C}$ (see Fig. 1), and $s = 1$ if $w(\mathbf{r}) = x + jy$ and $s = -1$ if $w(\mathbf{r}) = x - jy$. In this study, we choose $s = 1$ and $w(\mathbf{r}) = x + jy$.

Substituting

$$J(\mathbf{r}) = \sum_{n=1}^N I_n \delta(\mathbf{r} - \mathbf{r}_n) \quad (4)$$

in (3), where N is the number of conductors inside $\mathcal{C}$, and $\{I_n\}_{n=1}^N$ and $\{\mathbf{r}_n\}_{n=1}^N$ are currents and positions of these conductors respectively, for M different harmonic function $f(w(\mathbf{r}))$ yields

$$\sum_{n=1}^N I_n f_m(w(\mathbf{r}_n)) = \frac{1}{\mu_0} \oint_{\mathcal{C}} (\mathbf{j}\hat{\mathbf{n}} + \hat{\mathbf{t}}) \cdot \mathbf{B} f_m(w(\mathbf{r})) dl = b_m, \quad (5)$$

where $m = 1, 2, \dots, M > N$. Solving these equations, currents $\{I_n\}_{n=1}^N$ can be obtained. In general, knowing the magnetic field data over the whole contour $\mathcal{C}$ is needed to reconstruct the currents. However, in practice, to reconstruct currents in transmission lines in power system, the measurement points must be limited in the angular coverage under the conductors. In next section, we will explain how to choose harmonic functions $f_m(w(\mathbf{r}))$ to overcome this difficulty.

B. Choosing Harmonic Function $f_m(w(\mathbf{r}))$

From now on, we assume that the contour $\mathcal{C}$ is a semicircle with radius R as shown in Fig. 2. In a case that external sources are not too close to the arc part of the semicircle,

Fig. 3. Reconstructed currents for different values of R .

by choosing the harmonic function $f_m(w(\mathbf{r}))$ in a way that putting more weight on magnetic field data obtained from the diameter of the semicircle, one can approximate the integral in (5) as

$$b_m \cong \frac{1}{\mu_0} \int_{\mathcal{C}_1} (\mathbf{j}\hat{\mathbf{n}} + \hat{\mathbf{t}}) \cdot \mathbf{B} f_m(w(\mathbf{r})) dl, \quad (6)$$

where $\mathcal{C}_1$ is the diameter of the semicircle (see Fig. 2).

Defining the harmonic functions $f_m(w(\mathbf{r}))$ as

$$f_m(w(\mathbf{r})) = \frac{1}{(w - w_m)^2} \quad (7)$$

and setting the poles w_m close to the center and the diameter of the semicircle, as shown in Fig. 2 with black dots, the approximation in (6) holds and currents can be reconstructed by solving

$$\sum_{n=1}^N I_n f_m(w(\mathbf{r}_n)) = b_m, \quad (8)$$

where b_m obtained from (6). The integral in (6) can be calculated numerically by measuring $\mathbf{B}$ at enough points along $\mathcal{C}_1$.

III. RESULT

As a test case, we consider a three-conductor power transmission line above the ground whose currents are to be reconstructed and two other line sources as external sources. In addition, the images of the currents in the ground are considered as external sources. See Fig. 2 and assume $N = 3$ and the origin of the coordinate system is 0.5 m above the ground. The positions of conductors inside the contour $\mathcal{C}$ are $(-1, 1.5)$ m, $(0, 1.5)$ m, and $(1, 1.5)$ m. The positions of conductors outside the contour $\mathcal{C}$ are $(-7, 1.5)$ m and $(7, 1.5)$ m. The phasor currents in these conductors are 1 A, $e^{j2\pi/3}$ A, $e^{-j2\pi/3}$ A, $e^{j2\pi/3}$ A, and $e^{-j2\pi/3}$ A respectively.

To put more weight on magnetic field data measured on $\mathcal{C}_1$ (see Fig. 2; $\mathcal{C}_1 : y = 0, -R < x < R$, where R is the radius of the semicircle), we choose the poles of w_m in (7) as $w_1 = -0.5 - 0.2j$, $w_2 = -0.2j$, and $w_3 = 0.5 - 0.2j$. In the simulation to find the currents, we calculate $\mathbf{B}$ on $\mathcal{C}_1$ and the

integral in (6) analytically. The result of current reconstruction for different values of radius R is depicted in Fig. 3. The result shows that there is an optimal value for R . The reason is that the approximation in (6) is valid if the arc part of the contour $\mathcal{C}$ in Fig. 2 is far enough from both currents inside and outside the contour $\mathcal{C}$. In this test case, a value between 3 m and 4 m is optimal for R .

IV. CONCLUSIONS AND FUTURE WORK

In this study, using the Green identity, we have proposed a contact-free measurement method to reconstruct currents in power transmission lines from the magnetic field measured below the conductors. The proposed method reduce the effect of two-dimensional disturbance fields and facilitate the mea-

surement of high-frequency currents in power transmission lines. The result of the test case studied shows the method can provide reliable result. However, more work is needed regarding the condition for validity of approximation in (6), choosing the optimal positions for the poles of $f_m(w(\mathbf{r}))$ and choosing the optimal value for the radius R .

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