The Effect of GPS Signal Quality on the Ambiguity Resolution Using the KTH Method

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Abstract

Since the last decade the integer phase ambiguity resolution has become one of the most important elements in the GPS field because of its impact on the accuracy. Several scientific researches have been carried out for resolving the integer ambiguity. One example of such kind of researches is the Quick GPS ambiguity resolution for a short baseline (KTH method). The study focus on investigation of the impact of the selected elements: the GPS signal, the observations elevation angles and the GPS satellites azimuth on the KTH method performance for resolving the integer phase ambiguity.

The investigation has covered carrier phase measurements for three baselines with length less than one km for each and “GeoGenius” software package as the quality control for the fixed ambiguities of the KTH method results. The KTH method achieved more than 84 percent of success in the level of signal strength 8; about 90 percent for elevation angle between (55 to 65)° and approximately 94 percent for GPS satellite azimuth in range of (130 to 150)°.

The overall results showed a clear correlation of about 0.9 for most of the cases between KTH method and the selected components. In such a case it leads to a high performance of the method under healthy reliable observations conditions, and the method has the capability to yield outcomes expected to be within a specific accuracy by knowing the level of group elements (signal strength, elevation angle and satellite azimuth) that have been used.
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<tr>
<td>A-S</td>
<td>Anti-Spoofing</td>
</tr>
<tr>
<td>C/A code</td>
<td>Coarse / Acquisition code</td>
</tr>
<tr>
<td>CSTG</td>
<td>Coordination of Space Techniques for Geodesy and Geodynamics</td>
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<tr>
<td>DD</td>
<td>Double Differences</td>
</tr>
<tr>
<td>DoD</td>
<td>Department of Defense</td>
</tr>
<tr>
<td>DOP</td>
<td>Dilution of Precision</td>
</tr>
<tr>
<td>FARA</td>
<td>Fast Ambiguity Resolution Approach</td>
</tr>
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<td>FASF</td>
<td>Fast Ambiguity Search Filter</td>
</tr>
<tr>
<td>GDOP</td>
<td>Geometric Dilution of Precision</td>
</tr>
<tr>
<td>GeG</td>
<td>GeoGenius Software</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>HDOP</td>
<td>Horizontal Dilution of Precision</td>
</tr>
<tr>
<td>IAG</td>
<td>International Association of Geodesy</td>
</tr>
<tr>
<td>IGS</td>
<td>International GPS Service</td>
</tr>
<tr>
<td>KTH</td>
<td>The Royal Institute of Technology (Kungl Tekniska Högskolan)</td>
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<tr>
<td>LSAST</td>
<td>Least-Squares Ambiguity Search Technique</td>
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<td>LAMBDA</td>
<td>Least-squares AMBiguity Decorrelation Adjustment</td>
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<tr>
<td>OTF</td>
<td>On The Fly</td>
</tr>
<tr>
<td>P-code</td>
<td>Precision code</td>
</tr>
<tr>
<td>PDOP</td>
<td>Position Dilution of Precision</td>
</tr>
<tr>
<td>PPS</td>
<td>Precise Positioning Service</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Full Form</td>
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<td>--------------</td>
<td>-----------</td>
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<tr>
<td>PRN</td>
<td>Pseudo Random Noise</td>
</tr>
<tr>
<td>RINEX</td>
<td>Receiver INdependent EXchange format</td>
</tr>
<tr>
<td>SA</td>
<td>Selective Availability</td>
</tr>
<tr>
<td>SD</td>
<td>Single Differences</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>SPS</td>
<td>Standard Positioning Service</td>
</tr>
<tr>
<td>SV</td>
<td>Space Vehicle</td>
</tr>
<tr>
<td>TDOP</td>
<td>Time Dilution Of Precision</td>
</tr>
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<td>TD</td>
<td>Triple Differences</td>
</tr>
<tr>
<td>VDOP</td>
<td>Vertical Dilution Of Precision</td>
</tr>
<tr>
<td>UTC</td>
<td>Universal Time Coordinate system</td>
</tr>
<tr>
<td>WGS</td>
<td>World Geodetic System</td>
</tr>
<tr>
<td>Y-code</td>
<td>Encrypted P-code</td>
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Chapter One

Introduction

Resolving the Global Positioning System (GPS) carrier-phase ambiguities has been one of the most important research fields in GPS over the last decade which has been carried out by many scientific groups from all over the world.

Application of the carrier phase observations enables the differential GPS to achieve centimetre level accuracy. The main issue is that the phase lock loop cannot measure the full cycle part of the carrier phase. This unmeasured part is known as an integer ambiguity that requires to be resolved using integer least-squares estimation algorithms. Since the ambiguity inherent with the phase measurements rely upon both the receiver and the satellite, sometimes the integer ambiguity resolution may not be possible. There are some factors that have an impact on the determination of the ambiguity resolution such as: baseline length, ionospheric and tropospheric effects, satellite geometry, time, satellite orbit, multipath, antenna phase centre, the elevation angle and the strength of the signal. In the case of short baseline, most of these errors can be neglected or eliminated (i.e. ionospheric effect) and can be disregarded.

Nowadays, there are different methods that have been developed to resolve the ambiguity. The basic idea of these techniques is to focus on bringing the most favourable combination of fixed sets of phase ambiguities that minimize the residuals between fixed phase and float phase ambiguities. Examples of ambiguity resolution techniques are: the Fast Ambiguity Resolution Approach (FARA) by Frei and Beutler (1990); the Least-Squares Ambiguity Search Technique (LSAST) by Hatch (1990); The Fast Ambiguity Search Filter (FASF) by Chen and Lachapelle (1994); the Least-squares AMBiguity Decorrelation Adjustment (LAMBDA) by Teunissen (1994) and the KTH method presented by Sjöberg (1996), (1997), (1998a), (1998b), Almgren (1998), Horemuž and Sjöberg (1999).
The thesis contains six chapters starting with the introduction chapter, which states the objectives of the work and the general background of the thesis work. Chapter two shows the basic concept of the GPS observable including the differencing technique as well as the possible errors sources that can be carried out by GPS observations. Chapter three presents examples of the phase ambiguity techniques including the main method what is used in the thesis to determine the integer ambiguity resolution. Then comes chapter four, which in its turn represents what is done in this study from the practical perspective. After that chapter five containing how all the analysis has been performed and its results. Finally, chapter six presents the conclusions and the recommendations for the thesis. The references are given at the end of the work.

1.1 Thesis Objectives

For high precision of GPS positioning precise tracking of the carrier phase is required. The carrier phase includes the directly measured fractional part and an unknown integer part, which is known as the integer ambiguity. The key to precise carrier phase based positioning is to resolve the integer ambiguity.

The objectives of this thesis rely upon the impact of the ambiguity resolutions on precise GPS positioning; and the success of the KTH methods to resolve the integer ambiguity from different aspects.

The KTH method for near real-time GPS ambiguity resolution has been tested in several projects and the method has been given a positive result. However, the results were negatively affected by un-modelled systematic errors, mainly multipath. The presence of the systematic effects can be signalled by the low signal strength. The main objective of this thesis is to determine the correlation between signal strength and the success rate of the KTH method of ambiguity resolution.
The thesis objective is also to determine the correlation between the elevation angle and the success rate of the KTH method of ambiguity resolution, as well as the correlation with satellite azimuth.

1.2 The Principle of GPS

The Global Positioning System (GPS) is a space-based 24 hours a day system, originally intended for global military navigation system authorised by the U.S. military. Lately the system has been further developed to be used for civilian applications; the first application formed in this direction for GPS was for geodetic surveys.

The principle method of GPS is the measurement of distance or range between the satellites and the receiver, by the receiving satellites broadcasting data the receiver uses to compute their positions.

i. GPS Elements

The GPS consists of three elements: the space segment, the user segment and the control segment. The space segment consists of a constellation of 24 satellites in 6 orbital planes divided into 4 satellites in each orbit with orbital inclination angle 55° with elevation of 20200 km above the earth. This allows the space segment to provide global coverage with at least 4 observable satellites above elevation mask 15°.

The user segment consists of receivers. The receivers classified based on the type of observables that can receive (code pseudoranges or carrier phases) and the received code; C/A code, P code or Y code.

The control segment consists of three main stations such as: ground stations (six of them located around the world), the master control station and the monitor stations.
ii. GPS Satellite Signal

The GPS satellites transmit two microwave carrier signals depending on the basic frequency $10.23 \, MHz$. The L1 frequency ($1575.42 \, MHz$) with the wavelength $19 \, cm$ carries the navigation message and the standard positioning service (SPS) code signals. The L2 frequency ($1227.60 \, MHz$) with the wavelength $24.4 \, cm$ is used to measure the ionospheric delay by precise positioning service (PPS) equipped receivers (Leick 2004, p. 76).

Three binary codes modulate the L1 and/or L2 carrier phase:

- The C/A Code (Coarse/Acquisition) modulates only the L1 carrier phase. The C/A code is a repeating $1 \, MHz$ Pseudo Random Noise (PRN) Code. This noise-like code modulates the L1 carrier signal, "spreading" the spectrum over a $1 \, MHz$ bandwidth. The C/A code repeats every 1023 bits (one millisecond). There is a different C/A code PRN for each space vehicle (SV). GPS satellites are often identified by their PRN number, the unique identifier for each pseudo-random-noise code. The C/A code that is modulated on L1 carrier is the basis for the civil SPS.

![Biphase modulation of carrier wave](image.png)

**Fig.(1):** Biphase modulation of carrier wave, (Hofmann et al. 2001, p.73)

- The P-Code (Precise) modulated on both the L1 and L2 carrier phases. The P-Code is a very long (37 weeks, repeated weekly) $10 \, MHz$ PRN
code. In the Anti-Spoofing (AS) mode of operation, the P-Code is encrypted into the Y-Code. The encrypted Y-Code requires a classified AS Module for each receiver channel and is for use only by authorized users with cryptographic keys. The P (Y)-Code is the basis for the precise positioning service (PPS). The Navigation Message also modulates the L1-C/A code signal.

- The Navigation Message is a 50 Hz signal consisting of data bits that describe the GPS satellite orbits, clock corrections, and other system parameters.

### iii. GPS Data

Refer to Dana (1994) the GPS satellite provides data required to support the positioning process which includes information needed to determine the following elements.

- Satellite time of transmission.
- Satellite position.
- Satellite health.
- Satellite clock correction.
- Propagation delay effects.
- Constellation status.

### iv. RINEX

The Receiver INdependent EXchange format (RINEX), is a set of standard definition and formats for ASCII data files to promote the free exchange of GPS data and facilitate the use of data from any GPS receiver with any software package. The RINEX format is recommended to be used internationally as the standard exchange format geodetic GPS data after a GPS user meeting organized by the GPS sub-commission of the
International Coordination of Space Techniques for Geodesy and Geodynamics (CSTG) during the IAG Symposium in Edinburgh, August 1989 (Gurtner et al. 1989).

v. GPS Positioning Types

GPS uses a system of coordinates called WGS 84, which stands for World Geodetic System 1984. The positioning with GPS can be divided into two types which are absolute and relative positioning.

a. Absolute Positioning

The absolute positioning determines the position from a single receiver station to collect data from multiple satellites in order to resolve the user's geo-referenced position. The position can be determined by measuring the vector between the satellite and the receiver and minus it from the geocentric position vector for the satellite.

b. Relative Positioning

The relative positioning determines the position of unknown point by using two receivers or more, with one being placed at a known reference point in the baseline, with requirements of simultaneous observations at both the reference and the unknown point. Relative positioning can be performed with code ranges or phase ranges.

The use of a control point as a differencing method gives the opportunity of reducing some errors such as satellite orbit errors and signal propagation biases, and allows for a correction factor to be calculated and applied to other roving GPS units used in the same area and in the same time series. This helps to improve the accuracy of the new point position (Sjöberg 2006).
Chapter Two

Pseudorange and Carrier Phase Observables

Pseudorange and carrier phase are the most important GPS observations (observables) used for positioning. Solutions are available that use pseudorange only, carrier phase only, or both types of observations. The early solutions for navigation relied on pseudorange. More recently, even point positioning often includes the carrier phase observable. Carrier phases are always required for accurate surveying at the centimetre level (Leick 2004, p.170).

2.1 Pseudorange and Carrier Phase Observables Basics

2.1.1. Pseudorange

The pseudorange can be defined as the distance measurement between the receiver’s antenna and the satellites at the epochs of transmit ($t^s$) and receive time ($t_R$) of the code. Both codes are generated by the receiver and satellite transmissions are based on their own clock. Pseudorange observable is suitable to P(Y) code and C/A codes. The basic equation for pseudorange observable can be obtained by the time difference ($\Delta t$) with considering the correction of clock biases for the satellite $\delta^s$ and the receiver $\delta_R$.

$$\Delta t = t_R - \delta_R - (t^s - \delta^s) \quad (2.1)$$

$$\Delta t = \Delta \tilde{t} - \Delta \delta \quad (2.2)$$

where

$\Delta t$ the corrected time difference, $\Delta \tilde{t}$ the observed time difference and ($\Delta \delta = \delta^s - \delta_R$) the clock bias difference.

The observed pseudorange calculated from the light time equation can be:
\[
\tilde{R} = c\Delta t = \rho + c\Delta \delta
\]  

(2.3)

where; \( c \) the speed of the signal in the vacuum, \( \rho = c\Delta t \approx \rho(t^s, t_R) = \rho(t^s) + \dot{\rho}(t^s)\Delta t \) is the difference of the position of the receiver at the true receiver time minus the position of the satellite at the true transmitted time.

Of course one cannot forget to include into the pseudorange observable equation the affects of the ionosphere and troposphere, as well as to be aware of the delays of hardware at the satellite and the receiver.

### 2.1.2. Carrier Phase

The carrier is a radio wave having at least one characteristic (e.g., frequency, amplitude, phase) that can be varied from a known reference value by modulation. In the case of GPS there are two transmitted carrier waves; L1 and L2, amplitude modulated by the Navigation Message (both L1 and L2), the P-Code (both L1 and L2) and the C/A-Code (L1).

The phase observable based on the carrier phase of the signal as fractional part of the \( L_1 \) or \( L_2 \) carrier wavelength, is represented in units of meters, cycles, or fractions of a wavelength. Phase observables accumulated or integrated measurements which are also included in the fractional part add to the number of cycles.

The mathematical expression for phase observable can be shown as the difference between the received phases \( \varphi^i(t) \) from satellite \( i \) and the internal receiver \( A \) generated phase \( \varphi_A(t) \) which leads to the mathematical expression for the observed phase difference \( \tilde{\varphi}_A^i(t) \) (Sjöberg 2006).

\[
\tilde{\varphi}_A^i(t) - \varepsilon_A^i(t) = \varphi^i(t) - \varphi_A(t) + N_A^i(1) - (\varphi_0^i - \varphi_{0A})
\]  

(2.4)

where

\[ N_A^i(1) : \text{phase ambiguity.} \]
\( \varepsilon_A^i(t) \): random observations error.

\( \phi_0^i, \phi_{0A} \): phase lags at epoch \( t = 0 \), for \( \phi^i(t) \) and \( \phi_A(t) \).

\( N \) represents the integer number of cycles and it refers to the first epoch of observation. As long as the signal tracked continuously between the receiver and the satellite, \( N \) remains constant along the observation period with changes on the fractional phase. Cycle slips causes integer jumps on the observations, and this leads to a new integer constant and must be solved separately from the previous ambiguity.

### 2.2. Differencing

Differencing techniques need simultaneous observations between two receivers and a set of satellites, by observing the same satellite at nominal times to eliminate some bias correction by forming linear combinations.

There are different types of differencing such as: single-differences (SD); double-differences (DD) and triple-differences (TD). The differencing techniques generally performed between two receivers and one or more satellites, in the same or in different epoch of observations.

#### 2.2.1. Single difference

By observing satellite \( i \) from receiver \( A \) and \( B \) at the nominal times \( t_A \) and \( t_B \) one can obtain two pseudorange equations and two carrier phase equations. In this section, only the phase equations are represented. The differencing equation of the carrier phase observation can be shown as:

\[
\Delta \phi_{AB}^i(t) = \phi_B^i(t) - \phi_A^i(t)
\]

from Eq.(2.4) one can rewrite (2.5) as;

\[
\Delta \phi_{AB}^i(t) = \phi_B(t) - \phi_A(t) - \frac{f}{c} \left[ \rho_B^i(t) - \rho_A^i(t) \right] + N_{AB}^i(1) + f(\delta_B - \delta_A)
\]
where

\[ N^i_{AB}(1) = N^i_B - N^i_A \]

The single difference helps to eliminate the satellite clock correction as it is shown in Eq.(2.6).

### 2.2.2. Double difference

Double difference can be formed by observing satellite \( i \) and \( j \) from receiver A and B at the same time. In other words, double difference is differencing of two single differences. The double difference equation can be formed as:

\[ \Delta \varphi_{AB}^{ij}(t) = \Delta \varphi_{AB}^j(t) - \Delta \varphi_{AB}^i(t) \quad (2.7) \]

from Eq.(2.6) one can form (2.7) as;

\[ \Delta \varphi_{AB}^{ij}(t) = \frac{f}{c} \left[ \{ \rho^j_B(t) - \rho^j_A(t) \} - \{ \rho^i_B(t) - \rho^i_A(t) \} \right] + N_{AB}^{ij}(1) \quad (2.8) \]

where

\[ N_{AB}^{ij}(1) = N^j_{AB} - N^i_{AB} \]

In double difference one can also eliminate the receivers clock biases \( \delta_A \) and \( \delta_B \) as it is presented in the equation above.

### 2.2.3. Triple difference

The triple difference is the difference between two double differences Eq.(2.8) at different epochs \( t_1 \) and \( t_2 \) of observations. The triple difference equation can be written as:

\[ \nabla \varphi_{AB}^{ij}(t_1, t_2) = \nabla \varphi_{AB}^{ij}(t_2) - \nabla \varphi_{AB}^{ij}(t_1) \quad (2.9) \]

\[ \nabla \varphi_{AB}^{ij}(t_1, t_2) = \frac{f}{c} \left[ \{ \rho^j_B(t_2) - \rho^j_A(t_2) \} - \{ \rho^i_B(t_2) - \rho^i_A(t_2) \} ight] \\
+ \{ \rho^j_A(t_1) - \rho^j_B(t_1) \} - \{ \rho^i_A(t_1) - \rho^i_B(t_1) \} \quad (2.10) \]
In triple difference, the integer phase ambiguity $N^i_{AB}(t)$ is eliminated unless there are cycle slips between the epochs $t_1$ and $t_2$. The triple differences have an advantage in detecting cycle slips.

### 2.3. GPS Observation Error Sources

The precise GPS observations are obtained from the processing of dual frequency signals. The observable is processed for precise GPS is carrier phase differences between a satellite and a receiver. This observable is subject to a number of effects of different nature, some of which can be modelled, the others being considered as error sources.

The phase delay measurement cannot distinguish the number of entire wavelengths (ambiguity) between the transmitter and the receiver, but takes into account only the instantaneous fractional part of the phase delay. The ambiguity of a phase measurement is an integer number and remains constant as long as the phase measurement is not interrupted (i.e. cycle slip). One from Eq.(2.6) can show the ambiguity as component of the phase difference model between receiver $A$ and satellite $i$ such as:

$$\lambda \Delta \varphi^i_A(t) = \rho^i_A(t) + \lambda N^i_A + c \Delta \delta^i_A(t)$$  \hspace{1cm} (2.11)

where

$\lambda$ is the wavelength, and the range difference is $\rho^i_A(t) = c(t_A - t^i)$ and $c \Delta \delta^i_A(t) = c(\delta t_A - \delta t^i)$ the bias difference of the satellite and receiver clocks, and $N^i_A$ the ambiguity, then one can rewrite (2.11) refer to Walpersdorf (2007) as:

$$\lambda \Delta \varphi^i_A(t) = \rho^i_A + \Delta l_{\text{mono}} + \Delta l_{\text{Trop}} + c(\delta t_A - \delta t^i) + \lambda N^i_A$$  \hspace{1cm} (2.12)

where $\delta t_A \& \delta t^i$ are clocks errors on receiver $A$ and satellite $i$, $\rho^i_A = |X_A - X^i|$ the geometrical distance between the receiver $X_A$ and the satellite $X^i$, and
Pseudorange and Carrier Phase Observables

\( \Delta l_{\text{iono}} \) and \( \Delta l_{\text{Trop}} \) are the refraction of the electromagnetic signal in the Earth’s ionosphere and troposphere.

The complete phase delay is due to the differences of the time of reception and the time of emission of the signal, the signal travel time. However, clock errors on both the receiver and the transmitter sides are also included in the complete phase delay. The signal travel time is due to the geometrical distance, between the receiver and the satellite, with additional delays created by the refraction of the electromagnetic signal in the Earth’s ionosphere and troposphere.

Eq. (2.12) contains a number of effects that are either corrected a priori or adjusted during the data analysis with the help of specific models. There are other effects, not explicit in this equation, which act mainly as error sources, such as multipath near the GPS receiver antenna. Among the modelled effects are antenna phase centre variations, variations in station height due to geophysical phenomena (Earth tides, ocean loading, atmospheric loading, varying hydrological conditions) and variations in tropospheric delay with satellite-viewing angle. The ionospheric delay depends on the baseline length; it can be neglected for short baselines, which is the case in this study. Clock errors can be reduced significantly using double-differenced phase observations. The analysis of GPS data requires also the knowledge of precise satellite orbits (term \( X_i \) in the geometrical distance). The precise satellite orbits provide by the International GPS Service (IGS); they computed the precise satellite orbits based on the GPS satellite accurate orbit ephemerides, which are collected by the IGS permanent GPS tracking network.

The sources of these errors can be related to the satellites, receivers or propagations, these errors affecting the GPS observations processes can be divided into three types: gross, systematic and random errors.
The random errors can never be eliminated because of the effect of the nature of the measurements, but it can be reduced by least-squares adjustment, and these errors are usually small.

Systematic errors are errors that vary systematically in sign and/or magnitude. Systematic errors are particularly dangerous because they tend to accumulate. If the errors are known, the observation can be corrected before making the adjustment; otherwise, one might attempt to model and estimate these errors. Success is not at all guaranteed (Leick 2004, p.95).

Gross errors can be mistakes caused by the operator or it could be technical collapse and these errors are usually not small.

### 2.3.1. Satellite and Receiver Clock Error

The codes generated by the receiver are based on the receiver's own clock, and the codes of the satellite transmissions are generated by the satellite clock. Unavoidable timing errors at the satellite and the receiver will cause the measured pseudorange to differ from the geometric distance corresponding to the instants of emission and reception.

Therefore, the synchronization of the receiver clock with the satellite time during the observation detects clock bias. This clock bias represents the combined clock offset of the receiver and the satellite clock with respect to GPS time.
2.3.2. Atmospheric Errors

The transmission signal from the satellite is affected by many elements. One of them is atmospheric delay, which contain the upper layer ionosphere and lower troposphere.

Ionosphere has an altitude range from 50 to 1500 km above the earth. It consists largely of ionized particles, which cause a disturbing effect on the GPS signals. Since the density of the ionosphere is affected by the sun, there is less ionospheric influence during night time. In addition, low elevation satellite signals (anywhere between the horizon and up to 15 degrees above) will be affected by a longer ionospheric delay as the distance the signal has to travel is larger and generally noisier. In the more sophisticated GPS receivers an elevation mask can be set, so that satellites below the mask are not used in computing position.

Most of the atmosphere mass is located in the troposphere. The troposphere is the lower part of the atmosphere, with effective height of about 40 km above mean sea level. The tropospheric delay of pseudoranges and carrier phase are caused by the tropospheric refraction. The refraction includes the effects of the neutral, gaseous atmosphere. The tropospheric refraction can be divided into two parts; the dry component that follows the laws of ideal gases, and the wet component, which is responsible of delay in the zenith direction. Computing the wet delay is a difficult task because of the spatial and temporal variation of water vapour. About 90% of the tropospheric refraction arises from the dry and about 10% from the wet component.

The troposphere influences all GPS-frequencies in the same way and for that reason the size of its influence on the passing signal is directly related to the travelling distance through the tropospheric layer (Andersson 2006).
2.3.3. Multipath Error

Multipath error is one of the major error sources affecting the positional accuracy of GPS. Multipath error can happen when a part of the transmitted signal from satellite is reflected by the earth surface or any surface that have high power of reflection near the receiver. Multipath can be also happen when the receiver received more secondary path signal from various directions, which is neither coded translated nor understandable by GPS receiver and resulted from reflection from earth surface or any high objects for instance buildings around the observation station.

The signals from low altitude satellite have more tendencies to cause multipath error than signals from higher altitude satellite. The resulting error for code pseudoranges lies in range of a few meters, while for carrier phase the error can be around few centimetres.

The multipath effect can be reduced by choosing sites without reflecting surfaces around the receivers or using proper antennas to mitigate the reflected signal. It is difficult to eliminate all multipath effects from GPS observations, but they can be reduced through a variety of different techniques which are available for this purpose.

2.3.4. Cycle Slips

Cycle slip is a sudden jump in the carrier phase observable by an integer number of cycles. Cycle slips are caused by the loss-of-lock of the phase lock loops. This loss of lock can last for two epochs or more and when the signal is locked-on the phase ambiguity will be changed. If the receiver software would not attempt to correct for cycle slips, it would be a characteristic of a cycle slip that all observations after the cycle slip would be shifted by the same integer unless there is another cycle slip occurred (Leick 2004, p.179).
The causes of cycle slips can be different, such as obstructions of the satellite signal due to object disturbance, or too low signal to noise ratio (SNR), or it can also be due to a failure in the receiver software or of the satellite oscillator.

The cycle slip disturbs the carrier phase measurement, causing the unknown Ambiguity \( N \) value to be different after the cycle slip compared with its value before the slip. It must be "repaired" (the unknown number of "missing" cycles determined and the carrier observation subsequent to the cycle slip all corrected by this amount) before the phase data is processed in double-differenced observables for GPS Surveying techniques. The cycle slip can be detected by triple differences (TD) because they are differences over time.

### 2.3.5. Satellite Geometry

One can follow the definition of Ludwig (1999) where he states that the satellite geometry is the relative position of the satellites at a specific moment from the view of the receiver. When the satellites are located at wide angles relative to each other, the possible error margin is small. In the case of satellites being grouped together or located in a line, the geometry will be poor which in the worst case; no position determination is possible at all. The effect of the geometry of the satellites on the position error is measured by DOP factors, which are simple functions of the diagonal elements of the covariance matrix of the adjusted parameters. The effect of the geometry is called Geometric Dilution of Precision (GDOP). GDOP a composite (3-D) measure of the vertical, horizontal and time dimensions, and comprises also the components shown below:
- PDOP: Position Dilution of Precision (3-D).
- HDOP: Horizontal Dilution of Precision (Latitude, Longitude).
- VDOP: Vertical Dilution of Precision (Height).
- TDOP: Time Dilution of Precision (Time).

### 2.3.6. Selective Availability (SA)

Since the GPS system was created for military purpose in the first place, the Department of Defense (DoD) created SA to degrade the accuracy for non-U.S. military and government users. Dana and Foote (1999) define Selective Availability as the intentional degradation of the SPS signals by a time varying bias. The potential accuracy of the C/A code of around 30 meters is reduced to 100 meters (two standard deviations).

The SA bias on each satellite signal is different, and so the resulting position solution is a function of the combined SA bias from each SV used in the navigation solution. Because SA is a changing bias with low frequency terms in excess of a few hours, position solutions or individual SV pseudo-ranges cannot be effectively averaged over periods shorter than a few hours. Differential corrections must be updated at a rate less than the correlation time of SA (and other bias errors).

### 2.3.7. Summary

Table (1) summarizes the different errors sources affecting the GPS observations described above. Most of these errors at short baselines are almost eliminated and can be disregarded in the algorithm as the case in this study. Systematic errors, which affect all observations from a satellite or a receiver, such as clock errors, are eliminated by using double difference technique. The atmospheric errors are baseline dependent, where the ionospheric bias is neglected for short baseline, as well as the troposphere biases because the troposphere affects all observables by the
same amount and in the same time was not disturbing the ambiguity estimation it disturbs only the estimation of the geometrical distance ($\rho$). The multipath is station dependent, and its effects are not reduced in the double differences, but can be reduced by good satellite geometry and reasonably long observation interval or by physical or mathematical methods.

**Table (1):** Common error sources by GPS surveying from Sjöberg (2006)

<table>
<thead>
<tr>
<th>Source</th>
<th>Type</th>
<th>Order</th>
<th>Method of reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Satellite:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orbit</td>
<td>systematic</td>
<td>5 m</td>
<td>Relative positioning; precise orbits</td>
</tr>
<tr>
<td>Clock</td>
<td></td>
<td>1 m</td>
<td></td>
</tr>
<tr>
<td><strong>Signal propagation:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ionosphere</td>
<td></td>
<td>30 m</td>
<td>Modelling; -“-”; 2-freq. Data.</td>
</tr>
<tr>
<td>Troposphere</td>
<td></td>
<td>10 m</td>
<td>-“-</td>
</tr>
<tr>
<td><strong>Receiver:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ant. Ph. Centre</td>
<td></td>
<td>mm-cm</td>
<td>Calibration</td>
</tr>
<tr>
<td>Hardware delay</td>
<td></td>
<td>mm-dm</td>
<td>-“-</td>
</tr>
<tr>
<td>Multipath</td>
<td></td>
<td>mm-dm</td>
<td>Avoid bad stations! Groundplane.</td>
</tr>
<tr>
<td><strong>Computations</strong></td>
<td>(Only phase observables)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-detected slips</td>
<td>gross</td>
<td>2 dm/cycle</td>
<td>Efficient detection algorithm</td>
</tr>
<tr>
<td>Wrong ambiguity</td>
<td>gross</td>
<td>2 dm/cycle</td>
<td>-“-</td>
</tr>
<tr>
<td>Observation noise</td>
<td>random</td>
<td>mm(phase), m(code)</td>
<td>Least squares adjustment, Long observation period.</td>
</tr>
</tbody>
</table>
In phase measurement the emission time is unknown, and this leads to an unknown number of whole wavelength cycles the carrier signals contain in a set of measurements from a satellite to a receiver. These integer values are called ambiguities and these values hold the same as long as no loss of the signal lock occurs from satellite to receiver (e.g. by cycle slip).

The process of resolving the unknown cycle ambiguities of the carrier phase data as integer values is known as fixing the ambiguity. Resolving the initial phase ambiguities of GPS carrier phase observations was always considered as the key to fast and high precision relative GPS positioning.

The basic idea for the ambiguity resolution can be considered in three steps. Start by generation of potential integer ambiguity combination by search space technique. The search space can be realized from float ambiguity solution in case of static positioning and from a code range solution for kinematic positioning case. The size of the search is very important, because it will affect the efficiency. The second step involves the identification of the correct integer ambiguity combinations that minimizes the sum of squared residuals in the sense of least squares adjustment. These combinations are defined as the best fit of the data. The last step is the validation of the ambiguities (Hofmann-Wellenhof et al. 2001, p.214).

Nowadays, there are different methods available to fix the integer ambiguity. These methods apply different techniques see Table (2) such as: using single to dual frequency phase data; combining dual frequency carrier phase data and code data; combinations between triple frequency carrier phase and code data. Also, some of the methods implement search techniques, which are advantageous.
in reducing the number of integer candidates. This section contains a review of ambiguity resolution techniques including the main method that is applied in this thesis.

Table (2): Characteristics of ambiguity resolution techniques from Kim & Langley (2000)

<table>
<thead>
<tr>
<th>Technique</th>
<th>Principal Author(s)</th>
<th>Ambiguity Search Method</th>
<th>Data Processing Method</th>
<th>Search Space Handling Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSAST</td>
<td>Hatch</td>
<td>Independent</td>
<td>Single-epoch</td>
<td>None</td>
</tr>
<tr>
<td>FARA</td>
<td>Frei and Beutler</td>
<td>All</td>
<td>Multi-epoch</td>
<td>Conditional</td>
</tr>
<tr>
<td>LAMBDA</td>
<td>Teunissen</td>
<td>All</td>
<td>Multi-epoch</td>
<td>Transformation /Conditional</td>
</tr>
<tr>
<td>FASF</td>
<td>Chen and Lachapelle</td>
<td>All</td>
<td>Multi-epoch</td>
<td>Conditional</td>
</tr>
</tbody>
</table>

The ambiguity success rate depends on three factors, the observation equations; the precision of the observables and the method of integer ambiguity estimation. If we assume that two receivers $A$ and $B$ are observing simultaneously satellite $i$ and $j$, one can simplify the double-differenced carrier phase observations equation to:

$$
\varphi_{AB}^{ij}(t) = \frac{f}{c} \rho_{AB}^{ij}(t) + N_{AB}^{ij} + noise
$$

all variables in the model (3.1) hold the same as in Eq.(2.1) to Eq.(2.8).

3.1 Fast Ambiguity Resolution Approach

The Fast Ambiguity Resolution Approach (FARA) was developed in the early 1990’s by Frei and Beutler and was refined by some development later. The approach deals with double-difference phases and computation of the float carrier phase solution $\hat{x}$. In addition to that, the cofactor matrix of unknown parameters, their variance covariance and the computation of standard deviation of the ambiguities can be done.
By assuming that the number of observations is \( n \) and unknowns \( k \), then the ambiguity float solution \( \mathbf{x} \) (in vector form of all unknowns) can be calculated by an adjustment procedure, which also computes the cofactor matrix of the unknown parameters and the standard error of unit weight. From that one can form the inequality for any vector \( x_A \), which is the solution to the adjustment system when some or all ambiguities have been resolved (Sjöberg 2006):

\[
(x_A - \mathbf{x})^T Q_{xx}^{-1} (x_A - \mathbf{x}) \leq k S_0^2 F_{f,1-\alpha}
\]

where

- \( Q_{xx} \) the covariance matrix for the unknowns.
- \( S_0 \) the standard error of unit weight of the adjustment.
- \( F \): the value of the F-distribution with \( f = n - k \) number of degree of freedom of the adjustment system and significance level \( \alpha \).

If the vector \( x_A \) satisfies Eq.(3.2), then this will be a candidate for final solution. However it will not be the only candidate yield from this search volume test, therefore all candidates within this search must be checked by applying this test. The vector \( x_A \) satisfying the test and that provides the smallest quadratic term is the best candidate for \( x \).

The FARA method can be, generally, formed in four steps such as:

i. Computing the float carrier phase solution.
ii. Selecting ambiguity sets to form a test.
iii. Computing a fix solution for each ambiguity set.
iv. Statistically testing the fixed solutions with the smallest variance.
3.2 On The Fly

The ambiguity resolution On The Fly (OTF) technique deals with kinematic cases. This technique is used to resolve the integer ambiguities in the real-time kinematic environment. This technique is used in many methods.

OTF technique is based on a search space using differential phase and pseudorange positioning, and the size of the search space is defined by the standard deviations of the relative code range position. To minimize the search, the least-squares search can be used as one technique to define the correct solution within the search space (Hofmann-Wellenhof et al. 2001, p.226).

If dual frequency data is available, the wide-lane technique can reduce the observation time span required to a few seconds (Lachapelle et al. 1992b). The use of wide-lane has become more common to resolve integer ambiguities, which has only disadvantage: using the wide-lane technique, the measurement is significantly noisier than $L_1$.

3.3 Least-Squares Ambiguity Search Technique

Following Hofmann-Wellenhof et al. (2001, p.232), the Least-Squares Ambiguity Search Technique (LSAST) requires an approximate solution for the position (due to the linearization of the observation equation) which may be obtained from a code range solution. The search area is defined by $3\sigma$ region around the approximate position.

The main idea of this approach is to divide the satellites into two groups: the first group with good PDOP which consists of four satellites shows where the possible ambiguity sets are determined. The second group contain the remaining satellites, which are used to eliminate candidates of the possible ambiguity sets.

The set of potential solution follows the simplified double difference model (3.1). By treating the ambiguities ($N$) as if they were known variables in this model,
and are moved to the left side then (3.1) can be re-written in the simple way such as \((\lambda \Phi - N = \rho)\). From four satellites one can form three equations for the unknown as the station coordinates on the right side of the equation which is the solution can be optioned by linearization of the unknowns. Note that Hatch (1990) does not use double differences but un-differenced phases to avoid any biasing.

### 3.4 Fast Ambiguity Search Filter

According to Chen and Lachapelle (1994), the Fast Ambiguity Search Filter (FASF) is applied Kalman filter to a unique search range for the ambiguities. The search of the ambiguities is performed at every epoch until they are fixed. The search ranges for the ambiguities are computed recursively and are related to each other. To avoid very large search ranges a computational threshold is used. Ambiguities which cross this threshold are not fixed but computed as real numbers. Thus, an attempt to fix the ambiguities is only made if the number of potential ambiguity sets is below this threshold (Hofmann-Wellenhof et al. 2001, p.235-237).

In the Kalman filter, the ambiguity parameters are included in the state vector to be estimated as the float solution if they cannot be fixed. In FASF approach, the ambiguity ranges are determined recursively and are related to each other. Also there is an important element in this technique that all observations from the initial to the current epoch are taken into account by Kalman filtering. Once the ambiguities have been fixed, they are removed from the state vector, and the normal equations are modified accordingly.

FASF is useful for real time applications using: either single or dual frequency receivers in a high data collection rate.
3.5 LAMBDA Method

The Least-squares AMBiguity Decorrelation Adjustment (LAMBDA) method is a strict implementation of the integer least-squares principle. This method one of the most used technique for ambiguity determination.

The LAMBDA technique, which has been referred to as the integer least-square estimator, is the estimator that has the highest probability of correct integer estimation among all possible admissible integer estimators (Teunissen 1999).

The main idea of the method is to apply a transformation $Z$ that decorrelates the ambiguities, which means that the transformed covariance matrix of the ambiguities becomes a diagonal matrix. $Z$ is a regular and square matrix and it is necessary that the elements of both matrix $Z$ and the inverse $Z^{-1}$ are integers. The transformation can be shown as in Eq.(3.3); the mathematical model here refer to Teunissen et al. (1994):

$$ z = Z^T N, \quad \hat{z} = Z^T \hat{N} \quad \text{and} \quad Q_z = Z^T Q_N Z $$ \hspace{1cm} (3.3)

The LAMBDA method minimizes the residuals of the ambiguities by using the least squares principle of adjustment by parameters such as:

$$ (\hat{N} - N)^T Q_N^{-1} (\hat{N} - N) = \text{minimum} $$ \hspace{1cm} (3.4)

where $\hat{N}$ is the vector of adjusted float ambiguities, $N$ is the vector of the corresponding integer ambiguities and the difference between the two vectors $(\hat{N} - N)$ may be regarded as residuals of ambiguities and $Q_N$ is the cofactor matrix, which is in this case denoted as covariance matrix of the adjusted float ambiguities. Then by substituting Eq. (3.3) in Eq.(3.4) gives

$$ (\hat{z} - z)^T (Z^{-1})^T Q_N^{-1} Z^{-1} (\hat{z} - z) = \text{minimum} $$

$$ (\hat{z} - z)^T Q_z^{-1} (\hat{z} - z) = \text{minimum} $$ \hspace{1cm} (3.5)
Now we would like the new covariance matrix to be diagonal, which is help the new ambiguities to become completely decorrelated, and solving the integer least squares problem can be done by rounding to the nearest integer. Considering two ambiguities and their variance-covariance matrix are given as:

$$\tilde{N} = \begin{bmatrix} \tilde{N}_1 \\ \tilde{N}_2 \end{bmatrix} \text{ and } Q_N = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{bmatrix}$$  \hspace{1cm} (3.6)

The transformation $Z = Z^T N$ utilizes a transformation matrix of the special form diagonalizes $Q_N$:

$$Z^T = \begin{bmatrix} 1 & -\sigma_{12}\sigma_2^{-2} \\ 0 & 1 \end{bmatrix}$$  \hspace{1cm} (3.7)

To fulfill the condition of $Z$ contains only integers, this can be done by rounding $-\sigma_{12}\sigma_2^{-2}$ to the nearest integer $\text{int}[-\sigma_{12}\sigma_2^{-2}]$ and applying this value in Eq.(3.7) this yield:

$$Z^T = \begin{bmatrix} 1 & -\text{int}[\sigma_{12}\sigma_2^{-2}] \\ 0 & 1 \end{bmatrix}$$  \hspace{1cm} (3.8)

where the operator (int) refer to the rounding to the nearest integer.

This transformation reduces the correlation and improves the precision of the first ambiguity. Then, we can apply the same transformation Eq.(3.8) to the second ambiguity. And, their covariance matrix is given by:

$$Q_z = Z^T Q_N Z$$  \hspace{1cm} (3.9)

The variances of the transformed ambiguities decrease compared to the original one. The property of decreasing the variance while preserving the integer makes the transformation Eq.(3.8) a favourite to resolve the ambiguities because it minimizes the search (Leick 2004 p.284).

Since $Z$ is an integer transformation, and from the previous points one can obtain the requested ambiguities by the inverse of the transformation, i.e. $N = Z^{-1} z$.  \hspace{1cm} (3.10)
3.6 KTH Method

Quick GPS ambiguity resolution for short and long baselines which is known as KTH method is used in this thesis as a basic method to estimate the ambiguities.

The KTH method by Horemuz and Sjöberg (1999) has its roots in the work of Sjöberg (1996), (1997), (1998 a, b) and Almgren (1998). First of all, in order to apply the equations for this method, there is an assumption that dual frequency code and phase observables are available. The method can be divided into two parts, for the short baseline which are less than 10 km and for long baseline. All theory and mathematical models in this part refer to Horemuz and Sjöberg (1999).

Since in this study all observation done within one km range, then the ionosphere effects are neglected. The tropospheric effects also neglected since they affect all observables by the same amount and hence it disturbs only the estimation of \( \rho \) and not the ambiguity estimation. Multipath effects are also omitted. Then double difference observation equations can be written for code and phase such as:

\[
\begin{align*}
\Phi_1 &= u + \lambda_1 \cdot N_1 + \varepsilon_{11} \\
\Phi_2 &= u + \lambda_2 \cdot N_2 + \varepsilon_{12} \\
R_1 &= u + \varepsilon_{21} \\
R_2 &= u + \varepsilon_{22}
\end{align*}
\] (3.10)

Where \( \Phi_1 \) and \( R_1 \) are phase and code observables on the frequency \( L_1 \) with wavelength \( \lambda_1 = 0.1903 \, \text{m} \) and frequency \( f_1 = 1575.42 \, \text{MHz} \). Likewise \( \Phi_2 \) and \( R_2 \) are the phase and code observables on the frequency \( L_2 \) with wavelength \( \lambda_2 = 0.2442 \, \text{m} \) and frequency \( f_2 = 1227.6 \, \text{MHz} \). \( N_1 \) and \( N_2 \) are integer ambiguities, \( \varepsilon_{ij} \) are random observation errors and \( u = \rho + c\Delta \delta (t) \).
Note that the study is built on the determination of the ambiguity and not the position; therefore the least-squares solution of (3.10) will estimate the ambiguity.

The equation above can be written in matrix expression as:

\[ L - \varepsilon = Ax \] (3.11)

Where; \( L \) is the vector of observations, \( x \) represents the vector of unknowns, and \( A \) is the design matrix.

\[
A = \begin{bmatrix}
1 & \lambda_1 & 0 \\
1 & 0 & \lambda_2 \\
1 & 0 & 0 \\
1 & 0 & 0
\end{bmatrix},
\quad x = \begin{bmatrix} u \\ N_1 \\ N_2 \end{bmatrix},
\quad L = \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ R_1 \\ R_2 \end{bmatrix}
\] (3.12)

One can assume that the observables \( \Phi_1, \Phi_2, R_1 \) and \( R_2 \) are not correlated and there the ratio \( K \) of the standard deviations is constant for each frequency:

\[ K = \frac{\sigma_{R_1}}{\sigma_{\Phi_1}} = \frac{\sigma_{R_2}}{\sigma_{\Phi_2}} \] (3.13)

Moreover, it is assumed that both carriers have the same phase resolution and with regard to this assumption the covariance matrix of observations can be written as:

\[
\Sigma_L = 4 \sigma_{\Phi_1}^2 \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \frac{f_1^2}{f_2^2} & 0 & 0 \\
0 & 0 & K^2 & 0 \\
0 & 0 & 0 & \frac{f_1^2}{f_2^2} \cdot K^2
\end{bmatrix}
\] (3.14)

The standard deviations of unknown parameters \( \sigma_u, \sigma_{N_1} \) and \( \sigma_{N_2} \) can be computed from the cofactor matrix below:
$$Q_x = (A^T \Sigma^{-1} A)^{-1} \quad (3.15)$$

In the case of applying $K = 154$ and $\sigma_{\phi_1} = 2$ mm (Leick 1995), the matrix $Q_x$ gives the standard deviations of unknowns as:

$$\sigma_u = 0.49 \ m, \sigma_{N_1} = 2.55 \ and \ \sigma_{N_2} = 1.99 \quad (3.16)$$

and the correlation matrix is:

$$C_x = \begin{bmatrix}
1 & -0.9999988 & -0.9999677 \\
-0.9999988 & 1 & 0.999954 \\
-0.9999677 & 0.999954 & 1
\end{bmatrix} \quad (3.17)$$

It is impossible to have correct integer values of ambiguity because this solution is unstable with their large standard deviation and high correlations, so one can form linear combinations of the original phase equations and instead of $N_1$ and $N_2$, estimate their linear combination $N_{ij} = i \ N_1 + j \ N_2$. To obtain this expression, the phase observables linear combination can be written as:

$$\Phi_{ij} = u + \lambda_{ij} N_{ij} \quad (3.18)$$

where

$$\Phi_{ij} = \left( \frac{i \ \phi_{i1} + j \ \phi_{i2}}{\lambda_1 + \lambda_2} \right) \ \lambda_{ij} \ \text{and} \ \lambda_{ij} = \left( \frac{i \ \lambda_1 + j \ \lambda_2}{\lambda_1 + \lambda_2} \right)^{-1} \quad (3.19)$$

Using matrix notation, we can rewrite Eq.(3.11) generally for any linear combinations $\Phi_{ij}$ and $\Phi_{mn}$ of the phase observables as follows:

$$L_c - \varepsilon_c = A_c x_c \quad (3.20)$$

where
\[ L_c = TL = \begin{bmatrix} \Phi_{ij} \\ \phi_{mn} \\ R_1 \\ R_2 \end{bmatrix}, \quad x_c = \begin{bmatrix} u \\ N_{ij} \\ N_{mn} \end{bmatrix} \]  

(3.21)

\[ T = \begin{bmatrix} i \lambda_{ij} & j \lambda_{ij} & 0 & 0 \\ \lambda_1 & \lambda_2 & 0 & 0 \\ m \lambda_{mn} & n \lambda_{mn} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{, and } A_c = \begin{bmatrix} 1 & \lambda_{ij} & 0 \\ 1 & 0 & \lambda_{mn} \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \]  

(3.22)

and the covariance matrix of \( L_c \) is:

\[ \Sigma_{Lc} = T \Sigma L T^T \]  

(3.23)

The least squares solution of the system (3.20) can be formed:

\[ \hat{X}_c = (A^T P_c A)^{-1} A^T P_c L_c \]  

(3.24)

where

\[ P_c = \Sigma_{Lc}^{-1} \]

If we apply \( i = 1, j = 0 \) and \( m = 0, n = 0 \) then Eq.(3.20) becomes identical with Eq.(3.11). By changing \( i \) and \( j \) and keeping \( m = 1 \) and \( n = 0 \), one can form different linear combinations. Applying \( i = 4, j = -5 \) yield standard deviation \( \sigma_{N4,-5} = 0.30 \), and for \( i = 4, j = -5 \) the standard deviation will be \( \sigma_{N1,-1} = 0.56 \) from direct estimation.

To round the estimated ambiguity \( \hat{N} \) to nearest integer, the standard deviation has to be sufficiently small. This is valid only if random errors are present in the observations.
Hence, it is easy to fix the integer value of $N_{4,-5}$, as it has the smallest standard deviation. Then, one can consider $N_{4,-5}$ as known and can be moved to the observation vector $L_c$. By using the fixed $N_{4,-5}$ one can estimate the wide-lane ambiguity $N_{1,-1}$. Likewise, using the fixed wide-lane ambiguity to estimate $N_1$, which yield small standard deviation allows to fix $N_1$ to integer, see Fig.(2).

$$\begin{align*}
\text{First: } & [i=1, j=0] \\
& [m=4, n=-5] \\
& \text{Fix } N_{m,n}
\end{align*}$$

$$\begin{align*}
\text{Second: } & [i=1, j=-1] \\
& [m=4, n=-5] \text{ known} \\
& \text{Fix } N_{i,j} (N_w)
\end{align*}$$

$$\begin{align*}
\text{Third: } & [i=1, j=0] \\
& [m=1, n=-1] \text{ known} \\
& \text{Fix } N_{i,j} (N_1)
\end{align*}$$

$$\begin{align*}
\text{Finally: } & N_w \text{ known} \\
& N_1 \text{ known} \\
& \text{Fix } N_2
\end{align*}$$

**Fig. (2):** The least squares iteration to fix the ambiguity

$N_2$ can be also fixed by using the fixed wide-lane ambiguity $N_w$. All steps above have to be repeated for each satellite epoch by epoch.

Note that, weak signal on lower elevation mask lead to higher probability of large errors in estimating the ambiguity. Furthermore, multipath and ionosphere errors can shift the estimated $\hat{N}$ by fraction or even several cycles. Rounding the ambiguity to nearest integer yields incorrect results.

To perform best estimation for ambiguity one can follow the suggested algorithm for quick ambiguity resolution by Horemuz and Sjöberg (1999).
Chapter Four

Data Processing

In order to fully exploit the high accuracy of the carrier phase observables the ambiguity must be resolved to their correct integer values. Fixing the integer phase ambiguity can be done by using one of the available methods. The main goal of the used techniques to fix the integer phase ambiguity is to identify the combination of a fixed set of phase ambiguities, which are the best fits, by using least squares adjustment to minimize the residual between the fix and the float phase ambiguities.

4.1. Data Collection

Ambiguity resolution is more difficult for shorter time intervals. Therefore the longer time the satellite is observed, the easier it will be to fix the ambiguity to correct integer.

The thesis is based on dual frequency phase and code observations for a minimum period of two hours continuously of matched data between the reference and rover receiver. The GPS receivers must observe the same satellites simultaneously.

4.1.1 GPS Observations

The measurements in this project are done by using two GPS receivers for each of the three baselines. The observations are done simultaneously between the reference receiver KTH2 control point located in the rooftop of the L-building in The Royal Institute of Technology (KTH) the main
campus, and the rover receivers are observing in three different positions around the KTH campus.

![Diagram](image)

**Fig.(3): The study area (KTH main camps)**

The three observation points for the rover receivers are; KBR1, KBR2 and KBR4. They are located at different positions around the reference point KTH2, and with different surrounding area conditions, see Fig.(3).

KBR1 is located in the forest North-West L-building under tree cover; KBR2 North L-building, in open area surrounded by forest and KBR4 which is close to V-building. These points have created the baselines for the measurements such as; [KTH2 – KBR1], [KTH2 – KBR2] and [KTH2 – KBR4] so that all of them are within the range of one km length, as defined in previous chapter as short baseline.
Table (3): The specifications of the GPS instruments in use

<table>
<thead>
<tr>
<th>Station</th>
<th>Antenna</th>
<th>Receiver</th>
</tr>
</thead>
<tbody>
<tr>
<td>KHT2 - Reference</td>
<td>Ashtech choke ring</td>
<td>Trimble 4000 ssi</td>
</tr>
<tr>
<td>KBR1 - Rover</td>
<td>Trimble L1/L2 compact</td>
<td>Trimble 4000 ssi</td>
</tr>
<tr>
<td>KBR2 - Rover</td>
<td>Trimble L1/L2 compact</td>
<td>Trimble 4000 ssi</td>
</tr>
<tr>
<td>KBR4 - Rover</td>
<td>Trimble L1/L2 compact</td>
<td>Trimble 4000 ssi</td>
</tr>
</tbody>
</table>

Long observation periods are needed to increase the probability of correct ambiguity resolution. The observations in this study have been done statically for about two hours for each baseline. Both receivers the reference and the rover were set to observe with elevation mask $+10^\circ$ and synchronize interval time $15\text{ sec}$ with a minimum of 4 satellites to be observed.

The need of simultaneous and continuous observations at both the reference and the rover receiver in this study, make it very important to secure power sources (batteries) that provide the receivers with power for at least two hours.

4.1.2 Data Transfer

After the measurements have been done in the field, all collected data had to be transferred from the GPS receivers to the computer to be available for processing.

The GPload software v2.70 was used to transfer the data. The values of the elements in GPload interface see Table (4), have to be adjusted to match the same values in the GPS receivers instruments are in use before one can transfer the data.
Table (4): Port settings in GPLoad interface

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Baud Rate</td>
<td>38400</td>
</tr>
<tr>
<td>Data Bits</td>
<td>8</td>
</tr>
<tr>
<td>Stop Bits</td>
<td>1</td>
</tr>
<tr>
<td>Parity</td>
<td>odd</td>
</tr>
</tbody>
</table>

The data transferred from the receiver is in binary format, which must be converted into RINEX format to be used in the processing steps. This operation has been performed by using Trimble Office Total Control software.

4.2. Data Post-processing

After converting the data into RINEX format, phase and code observables on the frequencies $L_1$ and $L_2$ are available for processing. Then the observations in both receivers need to be synchronized and double differencing can be done to eliminate the clock terms. The determination of the integer ambiguity has been done based on the KTH algorithm as well as by using Geogenius software as the control component to compare the fixed ambiguities results that carried out by KTH method.
4.2.1. Synchronization of the data

The requirement of simultaneous relative observations between the reference and rover receiver makes it very important to correct the bias in the receiver's clocks. The clock bias can be defined by the general equations:

\[ t_A = \ddot{t}_A - \delta t_A \]

\[ t^s = \ddot{t}^s - \delta t^s \]  

(4.1)

where:

\( \ddot{t}_A \) nominal time of the signal reception measured by the clock of receiver A.

\( \ddot{t}^s \) nominal time of signal transmission measured by the clock of satellite S.

The synchronization for the observation time has been done in this study by using prepared programme script written in Matlab code by Andersson (2006).

4.2.2. GeoGenius 2000

The GeoGenius 2000 (GeG) is GPS post processing software; it is designed to work with GPS and GLONASS data in RINEX format. The GeG is multitasking software and has a number of user defined features. This software supports processing in all survey methods: Static, Short static (Rapid static), Kinematic and continuous surveys.

The GeG has modules for planning, processing, advanced Graphical editing, network adjustment, OTF ambiguity resolution as well as supports total station and digital level data in the network adjustment by least squares method (Spectra precision 2000).
Static project was created in GeG software to process the baselines. By using the tools available on the software, processing of the baselines have been done see Fig.(4) and baseline solutions are available. GeG creates a report containing the details of the baseline processing. This report includes the integer values of the ambiguities for each satellite in certain time and if there are any detected cycles slips. The report also shows the reference satellite that was used to compute double differences.

![Fig. (4): Processing the baselines in GeoGenius software](image)

4.2.3. Differencing

The use of double differences for carrier phase processing is very important, because of its elimination of the clock term. After synchronization, filtering for the data has been done using database tools in Excel, combined with visual basic programming. The filtering procedure was based on finding the match pairs of satellites that observed simultaneously between the reference and rover receiver in each epoch.
By applying Eq.(2.5) and Eq.(2.7) single and double differences have been computed. The reference satellite for DD computations in each epoch, it has to be the same satellite that used as reference in the processing of GeG software. The differences have been obtained by applying script written in Matlab. The output results are represented in text format contains the double differences for the phase and the code ($\Delta \varphi_{AB}^{ij}(t)$ and $\Delta R_{AB}^{ij}(t)$) on the frequency $L_1$ and $L_2$, which is represent the variables of the observations vector matrix (L). These double differences are needed for estimation the ambiguity by using KTH method algorithm.

4.2.4. KTH Method

Now all variables needed in Eq.(3.10) for the least squares solution to estimate the ambiguities are available. By using the KTH method algorithm described in Sect. 3.6, a Matlab code has been created to estimate the ambiguities. The code base on the least squares solutions can be simplified in matrix notation in three steps as:

$$\hat{X}_c = (A^T P_c A)^{-1} A^T P_c L_c$$

First: by applying the combinations $i = 1$, $j = 0$ and $m = 4$, $n = -5$;

$$X_c = \begin{bmatrix} u \\ N_1 \\ N_{4-5} \end{bmatrix} \xrightarrow{\text{Fix } N_{4-5}}$$

Second: $N_{4-5}$ known moved to other side ($L_c$ matrix); $i = 1$, $j = -1$, $m$ and $n$ hold the same;

$$X_c = \begin{bmatrix} u \\ N_{1-1} \end{bmatrix} \xrightarrow{\text{Fix } N_{1-1}}$$

Third: $N_{1-1}$ known moved to other side ($L_c$ matrix); $i = 1$, $j = 0$, $m = 1$, $n = -1$
\[ X_c = \left| \frac{u}{N_1} \right| \rightarrow \text{Fix } N_1 \]

Iteration for the three steps above has been done for each satellite, epoch by epoch to fix the integer ambiguity. \( N_2 \) can be fixed also by using \( N_w \).

To ensure that the Matlab code processed correctly, we used simulated data first to run the Matlab code. The test for the programme gave positive results in the end. The integer values of \( N_w, N_1 \) and \( N_2 \) for all baselines printed in text format to be easy for sorting and use in the analysis.
Chapter Five

Analysis and Results

Investigations of the effect of the three components that were selected in this study: signal strength, elevation angle and satellite azimuth on the performance of the KTH method for fixing the integer ambiguity have been done. The GeoGenius software (GeG) is in use as a reference for the ambiguities results.

The analysis is based on finding out the correct integer ambiguities by comparing the results of the integer ambiguities from the KTH method with the ones from GeoGenius software, see Table (5). The differences of ambiguities vary from correct results to few cycles. But this is not always the case, some of these ambiguities differences were huge compared to the GeG results, these ambiguities behave as it was affected by cycle slips. Therefore their differences were compared with the cycle slips detected by GeG software. Most of these ambiguity differences matched with the cycle slips values. Note that the comparison for the cycle slip was done related to their specific satellite and their epoch of appearance.

These incorrect results by KTH method that were influenced by cycle slips, have been corrected by adding or subtracting these values of the cycle slips depending on their signs as shown in GeG report. These corrections minimized the differences between the compared results, and most of them gave reasonable results by the end.

There are another group of incorrect ambiguity results behaves as if they were influenced by cycle slips, but they do not have any references in the static report related to their epochs of appearance. Therefore these ambiguities they cannot be corrected and were neglected from the analysis part.
Table (5): The results of fixing ambiguities by KTH method

<table>
<thead>
<tr>
<th>Baseline: KTH2 -</th>
<th>KBR1 (N)</th>
<th>KBR2 (N)</th>
<th>KBR4 (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time span of Observations (h:m:sec)</td>
<td>08:43:45</td>
<td>11:06:45</td>
<td>13:30:45</td>
</tr>
<tr>
<td>Total number of ambiguities results are tested in the analysis</td>
<td>1021</td>
<td>2247</td>
<td>2337</td>
</tr>
<tr>
<td>Correct results of N1 compared to GeG $dN_1 = N_{1_{KTH}} - N_{1_{GeG}} = 0$</td>
<td>151</td>
<td>933</td>
<td>1020</td>
</tr>
<tr>
<td>Correct results of N1 compared to GeG $dN_2 = N_{2_{KTH}} - N_{2_{GeG}} = 0$</td>
<td>165</td>
<td>933</td>
<td>1020</td>
</tr>
<tr>
<td>Correct results of Nw compared to GeG $dN_w = N_{w_{KTH}} - N_{w_{GeG}} = 0$</td>
<td>210</td>
<td>941</td>
<td>1028</td>
</tr>
<tr>
<td>Correct ambiguities results under signal strength ($\geq 6$) for rover receiver on $L_1$</td>
<td>93</td>
<td>887</td>
<td>976</td>
</tr>
<tr>
<td>Correct ambiguities results under signal strength ($&lt; 6$) for rover receiver on $L_1$</td>
<td>58</td>
<td>46</td>
<td>44</td>
</tr>
<tr>
<td>Correct ambiguities results under signal strength ($\geq 4$) for reference receiver on $L_1$</td>
<td>119</td>
<td>582</td>
<td>466</td>
</tr>
<tr>
<td>Correct ambiguities results under signal strength ($&lt; 4$) for reference receiver on $L_1$</td>
<td>32</td>
<td>351</td>
<td>554</td>
</tr>
<tr>
<td>Correct ambiguities results under signal strength ($\geq 6$) for rover receiver on $L_2$</td>
<td>123</td>
<td>910</td>
<td>986</td>
</tr>
<tr>
<td>Correct ambiguities results under signal strength ($&lt; 6$) for rover receiver on $L_2$</td>
<td>42</td>
<td>23</td>
<td>34</td>
</tr>
<tr>
<td>Correct ambiguities results under signal strength ($\geq 4$) for reference receiver on $L_2$</td>
<td>141</td>
<td>640</td>
<td>625</td>
</tr>
<tr>
<td>Correct ambiguities results under signal strength ($&lt; 4$) for reference receiver on $L_2$</td>
<td>24</td>
<td>293</td>
<td>395</td>
</tr>
</tbody>
</table>

The results of the filtering process of ambiguities differences have been divided into groups depending on which level of signal strength they belong to.

The levels of signal strength used for grouping followed the standard of RINEX format. Furthermore, groups of ambiguities differences were created depending on their elevation angles and GPS satellites azimuth.

40
5.1 The Signal Strength

To detect the influence of the signal strength on the KTH method performance for fixing the ambiguity, groups of fixed ambiguities differences are created following the RINEX format related to their received signal strength. The RINEX format projects the signal strength into interval from 1 to 9 (Gurtner 2007). Fig. (5) to Fig. (12) show the success rate of KTH method among the different signal strengths in the two signal frequencies ($L_1$ and $L_2$) based on GeG software under the rover and reference receivers for the three observed baselines. All graphs, which are presented here, are based on tables in Appendix A.

- **The Rover Receiver**

![Graph](image)

**Fig. (5):** The percentages of correct $N_w$ in each signal strength on the signal frequency ($L_1$) for the observed baselines
**Fig. (6):** The percentages of correct ambiguities \( N_1 \) results in each signal strength for the observed baselines.

**Fig. (7):** The percentages of correct \( N_w \) in each signal strength on the signal frequency \( L_2 \) for the observed baselines.
Fig. (8): The percentages of correct ambiguities ($N_2$) in each signal strength for the observed baselines

- **The Reference Receiver**

Fig. (9): The percentages of correct $N_w$ in each signal strength on the signal frequency ($L_1$) for the observed baselines
**Fig. (10):** The percentages of correct ambiguities \( N_1 \) in each signal strength for the observed baselines

**Fig. (11):** The percentages of correct \( N_w \) in each signal strength on the signal frequency \( L_2 \) for the observed baselines
Chapter Five

Analysis & Results

Fig. (12): The percentages of correct ambiguities ($N_2$) results in each signal strength for the observed baselines.

5.2 The Elevation Angle

Examinations of the impact of the observations elevation angles on the KTH method have also been done, by finding out which elevation angle leads to correct fixed ambiguity. The results of fixed ambiguities differences divided also into groups, but based on their observations elevation angles (10 degrees range), starting with 5 degrees with maximum elevation angle 75 degree. The test done for the fixed ambiguities ($N$), and the wide-lane ambiguities ($N_w$), see Fig.(13) and Fig.(14) for more details see tables in Appendix B.
Fig. (13): The percentages of correct $N_\nu$ in each elevation angle group on the signal frequency ($L_1$) for the observed baselines

Fig. (14): The percentages of correct $N_1$ in each elevation angle group for the observed baselines
5.3 The GPS Satellites Azimuth

The azimuth can be defined as the direction given by the horizontal angle (0–360 degree) between the ellipsoidal meridian of the observer and the line measured in a clockwise direction from the north of the ellipsoidal meridian. By using the sky plot tools in GeG software one can shows in Fig.(15) the satellite paths as a function of elevation angle and azimuth for the reference receiver KIH₂.

![Polar sky plot for KIH₂ for six hour duration](image)

**Fig. (15):** Polar sky plot for KIH₂ for six hour duration

In Fig.(15) the satellites that have elevation angle between 60° to 90°, they appear only in the south part of the circle that cover this region of elevation angle and they are close to the azimuth angle 180°.

To detect the effects of the satellites azimuth on the ambiguity, groups of ambiguities differences are created based on the satellite azimuth. The azimuth is divided into groups with a range of 20 degrees; see Fig.(16) and Fig.(17), for more details see tables in appendix C.
**Fig. (16):** The percentages of correct $N_w$ in each group of azimuth on the signal frequency ($L_1$) for the observed baselines.

**Fig. (17):** The percentages of correct $N_1$ in each azimuth group for the observed baselines.
5.4 Average of Differences of Ambiguities

Furthermore, the influence of the signal strength on KTH method performance represented as the average of ambiguities cycles for the differences between the KTH method and GeG software vs the signal strength levels that were created in section (5.1). Graphs represent these relation shown in Fig.(5.13) to Fig.(5.17) for the fixed ambiguity differences for the rover and reference receiver on the frequency $L_1$ and $L_2$.

**Fig. (18):** Average of $N_1$ differences results by rover receiver
**Fig. (19):** Average of $N_2$ differences results by rover receiver

**Fig. (20):** Average of $N_1$ differences results by reference receiver
Fig. (21): Average of \( N_2 \) differences results by reference receiver

5.5 The Correlation Coefficient

By using the available tools in Excel an exponential trendline have been created by using Eq.(5.1) to calculate the least squares fit through points for the percentage of the correct ambiguities results with their relation to the signal strength in Sect. 5.1.

\[
Y = Ce^{bx}
\]  

(5.1)

Where \( Y \) is the vector of values of the dependent variable (percentage of correct ambiguities), \( x \) is the signal strength, \( C \) and \( b \) constant and \( e \) is the base of the natural logarithm.

Based on the exponential fit the square correlation coefficient \( r^2 \) between empirical and fitted value of \( Y \) (see Table 6), has been computed by using the formula (Helland 1987, p. 61-69):

\[
r^2 = 1 - \frac{ESS}{TSS}
\]  

(5.2)
where $ESS$ is the errors sum of square

$$ESS = \sum_{i=1}^{n} (Y_i - \bar{Y}_i)^2$$

and $TSS$ is the total sum of square

$$TSS = \sum_{i=1}^{n} (Y_i - \bar{Y})^2$$

where $\bar{Y}_i$ is the fitted value, $\bar{Y}_i$ is the mean value of $Y_i$ and $i = 1,2,3,...n$.

**Table (6):** Goodness of exponential fit ($r^2$) of the dependence between the signal strength and the percentage of correct ambiguities

<table>
<thead>
<tr>
<th>Baseline: KTH2 -</th>
<th>KBR1</th>
<th>KBR2</th>
<th>KBR4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Nw_{L1} - Rover$</td>
<td>0.434</td>
<td>0.869</td>
<td>0.900</td>
</tr>
<tr>
<td>$Nw_{L2} - Rover$</td>
<td>0.896</td>
<td>0.974</td>
<td>0.883</td>
</tr>
<tr>
<td>$N1 - Rover$</td>
<td>0.730</td>
<td>0.979</td>
<td>0.882</td>
</tr>
<tr>
<td>$N2 - Rover$</td>
<td>0.612</td>
<td>0.994</td>
<td>0.895</td>
</tr>
<tr>
<td>$Nw_{L1} - Reference$</td>
<td>0.533</td>
<td>0.854</td>
<td>0.948</td>
</tr>
<tr>
<td>$Nw_{L2} - Reference$</td>
<td>0.568</td>
<td>0.500</td>
<td>0.979</td>
</tr>
<tr>
<td>$N1 - Reference$</td>
<td>0.856</td>
<td>0.905</td>
<td>0.958</td>
</tr>
<tr>
<td>$N2 - Reference$</td>
<td>0.778</td>
<td>0.951</td>
<td>0.975</td>
</tr>
</tbody>
</table>

The square correlation coefficient $r^2$ based on the least squares fit for the dependence between the elevation angles and the percentage of correct ambiguities that were presented in Sect. 5.2 have been computed also by using Eq.(5.2), see Table (7).

**Table (7):** Goodness of exponential fit ($r^2$) of the dependence between the elevation angle and the percentage of correct ambiguities

<table>
<thead>
<tr>
<th>Baseline: KTH2 -</th>
<th>KBR1</th>
<th>KBR2</th>
<th>KBR4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Nw$</td>
<td>0.798</td>
<td>0.958</td>
<td>0.972</td>
</tr>
<tr>
<td>$N1$</td>
<td>0.901</td>
<td>0.988</td>
<td>0.974</td>
</tr>
</tbody>
</table>
Chapter Six

Conclusions & Recommendations

The signal propagates from the satellite transmitter to the ground receiver. Through this propagation signal can be weaken by different effects, which directly or indirectly affects the ambiguity resolution, and likewise the performance of the KTH method. This study finds the correlation between the signal strength, observation elevation angles and the ambiguities resolution. The effects of GPS satellite azimuth here, as matter of fact is more about the satellite geometry effect on the observation. Thus the most important elements in this study is the signal strength and it’s relation with the observation elevation angle.

6.1 Conclusions

- The effect of the signal quality on the ambiguity resolution appears in the different levels of the signal strength. In Table (6) and (7) one can see the direct correlation between the fixed ambiguity carried out by KTH method and the strength of the signal. The KTH method results of correct fixed ambiguities reach high percentages under the level of signal strength eight on the signal frequency $L_1$ by the rover receiver.

- The KTH method in baselines $KIH_2 - KBR_2$ and $KIH_2 - KBR_4$ achieved almost the same percentage of fixed ambiguities, which is between 74% and 86% in the levels of signal strength 7 and 8 with the rover receiver, and about 66% to 81% for the reference receiver in the levels of signal strength 5 and 6 on the frequency $L_1$ and $L_2$ respectively.
For the baseline $KTH_2 - KBR_1$ the KTH method obtained less percentage of success since the position of $KBR_1$ receiver was under the trees. The results shows percentage of correct ambiguities between 19% and 26% by the rover receiver by the signal strength 7 and 8, and between 20% and 42% with signal strength 5 and 6 on the frequency $L_1$ and $L_2$ respectively.

The KTH method results for correct wide-lane ambiguities ($N_w$) for the baseline $KTH_2 - KBR_1$ had the percentage of 31% for strength of signal 8 and 7 by the rover receiver, and about 22% to 47% in the levels of signal strength 5 and 6 respectively.

The observation elevation angle has impacted on the KTH method for determining the integer ambiguity resolution, see Fig.(5.13) and Fig.(5.14), as the best results for baseline $KTH_2 - KBR_1$ lies under the elevation angle range $(55 - 65)^\circ$ with a percentage of around 25%, and for the baseline $KTH_2 - KBR_2$ with percentage over 91%. The baseline $KTH_2 - KBR_4$ represent more than 89% success by range of elevation angel $(65 - 75)^\circ$.

The effect of satellite azimuth is depended on observation site, and since the observation have been done in the north of the equator (Stockholm), the most of the satellites that have the probability to have higher elevation angle will be located in the south, i.e. azimuth around 180 degree. From Fig.(5.16) and Fig.(5.17) one can see the results of GPS satellites azimuth for each baseline such as: $KTH_2 - KBR_1$ had percentage of correct results about 42% by range of $(90 - 110)^\circ$, baseline $KTH_2 - KBR_2$ had 54% under the range $(110 - 130)^\circ$, and for baseline $KTH_2 - KBR_4$ more than 94% under the range $(130 - 150)^\circ$.

The conclusion from the stated points above and Table (7) and (8) makes it very clear that there is a strong correlation between the performance of KTH method and the environments of observation as the case in this thesis for the three baselines. The degree of correlation is about 0.9 for
baseline KTH₂ - KBR₄, and as well for baseline KTH₂ - KBR₂ except for Nₜ result by the reference receiver on L₂ as shows a degree of 0.5 and that can be related to unsolved cycle slips. For baseline KTH₂ - KBR₄ the degree of correlation is between 0.9 and 0.6, except for Nₜ result is 0.4 by the rover receiver on L₄.

- Overall the quality of the KTH method performance in a good environment can be more than 85% and goes down to less than 30% for a bad environment. On the other hand, by analyzing the average of the correct ambiguities among the strength of the signal level, see Fig.(5.18) to Fig.(5.21), it comes close to zero cycles starting from the level of signal strength 5.

### 6.2 Recommendations

Through the practical work some difficulties and problems have been shown. If improvements can be made in the future, the KTH method can achieve higher percentages and success in testing than what we have now. Some improvements for further studies can be done as well for the use of KTH method in the future.

**Problems have to be avoided:**

- The choosing of the control software has to be done very carefully, for example most of the software packages do not show the exact method they used (black box). An example of this is Trimble Office Total Control software, as it appeared in this study, they did not use a double difference technique directly.
• In the ambiguities results by the KTH method it seems there are some cycle slips, which are un-detected by the GeG software. If these cycle slips solved, it could increase the percentages of correct ambiguities.

Future studies:

• The observation have to be done by using type of receiver antenna that enable after filtering the received signals, to have signals with a strong strength.

• To resolve the integer ambiguity by the KTH method, filtering of data has to be done, to insure that in processing, data with strongest signal strength among the observed data is in use. One can also set groups with possibility of known outcome accuracy.

• Since this study reaches agreements that the KTH method is affected by the GPS signal quality, further research improvements can be done to the KTH method.
References


### Appendix A

#### Table (8): The ambiguities \( N_{1} \) results related to the signal strength by the rover receiver

<table>
<thead>
<tr>
<th>Strength Of Signal</th>
<th>KTH2-KBR1</th>
<th>KTH2-KBR2</th>
<th>KTH2-KBR4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Obs.</td>
<td>Average in Cycle</td>
<td>Correct ( N_{1} )</td>
</tr>
<tr>
<td>Strength 1</td>
<td>21</td>
<td>-0.48</td>
<td>0</td>
</tr>
<tr>
<td>Strength 2</td>
<td>35</td>
<td>-4.51</td>
<td>1</td>
</tr>
<tr>
<td>Strength 3</td>
<td>53</td>
<td>-2.77</td>
<td>1</td>
</tr>
<tr>
<td>Strength 4</td>
<td>81</td>
<td>-4.72</td>
<td>4</td>
</tr>
<tr>
<td>Strength 5</td>
<td>348</td>
<td>-2.75</td>
<td>51</td>
</tr>
<tr>
<td>Strength 6</td>
<td>385</td>
<td>-1.98</td>
<td>76</td>
</tr>
<tr>
<td>Strength 7</td>
<td>75</td>
<td>-2.78</td>
<td>12</td>
</tr>
<tr>
<td>Strength 8</td>
<td>19</td>
<td>-2.21</td>
<td>5</td>
</tr>
<tr>
<td><strong>SUM</strong></td>
<td>1021</td>
<td>-</td>
<td>151</td>
</tr>
</tbody>
</table>

#### Table (9): The ambiguities \( N_{2} \) results related to the signal strength by the rover receiver

<table>
<thead>
<tr>
<th>Strength Of Signal</th>
<th>KTH2-KBR1</th>
<th>KTH2-KBR2</th>
<th>KTH2-KBR4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Obs.</td>
<td>Average in Cycle</td>
<td>Correct ( N_{2} )</td>
</tr>
<tr>
<td>Strength 3</td>
<td>2</td>
<td>-8</td>
<td>0</td>
</tr>
<tr>
<td>Strength 4</td>
<td>16</td>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>Strength 5</td>
<td>449</td>
<td>-2.23</td>
<td>42</td>
</tr>
<tr>
<td>Strength 6</td>
<td>513</td>
<td>-1.80</td>
<td>115</td>
</tr>
<tr>
<td>Strength 7</td>
<td>41</td>
<td>-1.49</td>
<td>8</td>
</tr>
<tr>
<td><strong>SUM</strong></td>
<td>1021</td>
<td>-</td>
<td>165</td>
</tr>
</tbody>
</table>

#### Table (10): The ambiguities \( N_{w} \) results related to the signal strength on the carrier \( L_{1} \) by the rover receiver

<table>
<thead>
<tr>
<th>Strength Of Signal</th>
<th>KTH2-KBR1</th>
<th>KTH2-KBR2</th>
<th>KTH2-KBR4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Correct ( N_{w,L1} )</td>
<td>Average in Cycle</td>
<td>%</td>
</tr>
<tr>
<td>Strength 1</td>
<td>21</td>
<td>-0.10</td>
<td>15.05</td>
</tr>
<tr>
<td>Strength 2</td>
<td>35</td>
<td>-1.63</td>
<td>17.14</td>
</tr>
<tr>
<td>Strength 3</td>
<td>53</td>
<td>-0.72</td>
<td>16.98</td>
</tr>
<tr>
<td>Strength 4</td>
<td>81</td>
<td>-1.12</td>
<td>11.11</td>
</tr>
<tr>
<td>Strength 5</td>
<td>348</td>
<td>-0.64</td>
<td>19.83</td>
</tr>
<tr>
<td>Strength 6</td>
<td>389</td>
<td>-0.29</td>
<td>22.37</td>
</tr>
<tr>
<td>Strength 7</td>
<td>75</td>
<td>-1.05</td>
<td>26.67</td>
</tr>
<tr>
<td>Strength 8</td>
<td>19</td>
<td>-0.89</td>
<td>31.58</td>
</tr>
<tr>
<td><strong>SUM</strong></td>
<td>1021</td>
<td>210</td>
<td>20.57</td>
</tr>
</tbody>
</table>
### Appendix (A)

**Table (11):** The ambiguities \( (N_{w}) \) results related to the signal strength on the carrier \( (L_2) \) by the rover receiver

<table>
<thead>
<tr>
<th>Strength Of Signal</th>
<th>KTH2-KBR1</th>
<th>KTH2-KBR2</th>
<th>KTH2-KBR4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Obs.</td>
<td>Correct N(w) (L_2)</td>
<td>Average in Cycle</td>
</tr>
<tr>
<td>Strength 3</td>
<td>2</td>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>Strength 4</td>
<td>16</td>
<td>1</td>
<td>-2.19</td>
</tr>
<tr>
<td>Strength 5</td>
<td>449</td>
<td>77</td>
<td>-0.83</td>
</tr>
<tr>
<td>Strength 6</td>
<td>513</td>
<td>119</td>
<td>-0.34</td>
</tr>
<tr>
<td>Strength 7</td>
<td>41</td>
<td>13</td>
<td>-0.95</td>
</tr>
<tr>
<td>SUM</td>
<td>1021</td>
<td>210</td>
<td>-</td>
</tr>
</tbody>
</table>

**Table (12):** The ambiguities \( (N_1) \) results related to the signal strength by the reference receiver

<table>
<thead>
<tr>
<th>Strength Of Signal</th>
<th>KTH2-KBR1</th>
<th>KTH2-KBR2</th>
<th>KTH2-KBR4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Obs.</td>
<td>Average in Cycle</td>
<td>Correct N(1)</td>
</tr>
<tr>
<td>Strength 1</td>
<td>134</td>
<td>-5.25</td>
<td>0</td>
</tr>
<tr>
<td>Strength 2</td>
<td>77</td>
<td>-4.01</td>
<td>4</td>
</tr>
<tr>
<td>Strength 3</td>
<td>210</td>
<td>-2.09</td>
<td>28</td>
</tr>
<tr>
<td>Strength 4</td>
<td>156</td>
<td>-4.04</td>
<td>27</td>
</tr>
<tr>
<td>Strength 5</td>
<td>444</td>
<td>-1.33</td>
<td>92</td>
</tr>
<tr>
<td>SUM</td>
<td>1021</td>
<td>-</td>
<td>151</td>
</tr>
</tbody>
</table>

**Table (13):** The ambiguities \( (N_2) \) results related to the signal strength by the reference receiver

<table>
<thead>
<tr>
<th>Strength Of Signal</th>
<th>KTH2-KBR1</th>
<th>KTH2-KBR2</th>
<th>KTH2-KBR4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Obs.</td>
<td>Average in Cycle</td>
<td>Correct N(2)</td>
</tr>
<tr>
<td>Strength 1</td>
<td>153</td>
<td>-5.25</td>
<td>3</td>
</tr>
<tr>
<td>Strength 2</td>
<td>45</td>
<td>-3.15</td>
<td>4</td>
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<td>Strength 3</td>
<td>123</td>
<td>-2.60</td>
<td>17</td>
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<td>Strength 4</td>
<td>133</td>
<td>-1.55</td>
<td>37</td>
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<td>Strength 5</td>
<td>459</td>
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<td>75</td>
</tr>
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<td>Strength 6</td>
<td>68</td>
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<td>29</td>
</tr>
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<td>-</td>
<td>165</td>
</tr>
</tbody>
</table>
Table (14): The ambiguities \( N_w \) results related to the signal strength on the carrier \( L_1 \) by the reference receiver

<table>
<thead>
<tr>
<th>Strength Of Signal</th>
<th>KTH2-KBR1</th>
<th>KTH2-KBR2</th>
<th>KTH2-KBR4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Obs.</td>
<td>Correct ( N_w _L1 )</td>
<td>Average in Cycle</td>
</tr>
<tr>
<td>Strength 1</td>
<td>134</td>
<td>20</td>
<td>-1.22</td>
</tr>
<tr>
<td>Strength 2</td>
<td>77</td>
<td>8</td>
<td>-0.23</td>
</tr>
<tr>
<td>Strength 3</td>
<td>210</td>
<td>45</td>
<td>-0.75</td>
</tr>
<tr>
<td>Strength 4</td>
<td>156</td>
<td>39</td>
<td>-0.97</td>
</tr>
<tr>
<td>Strength 5</td>
<td>444</td>
<td>98</td>
<td>-0.30</td>
</tr>
<tr>
<td>SUM</td>
<td>1021</td>
<td>210</td>
<td>-</td>
</tr>
</tbody>
</table>

Table (15): The ambiguities \( N_w \) results related to the signal strength on the carrier \( L_2 \) by the reference receiver

<table>
<thead>
<tr>
<th>Strength Of Signal</th>
<th>KTH2-KBR1</th>
<th>KTH2-KBR2</th>
<th>KTH2-KBR4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Obs.</td>
<td>Correct ( N_w _L2 )</td>
<td>Average in Cycle</td>
</tr>
<tr>
<td>Strength 1</td>
<td>153</td>
<td>21</td>
<td>-0.84</td>
</tr>
<tr>
<td>Strength 2</td>
<td>45</td>
<td>10</td>
<td>-0.69</td>
</tr>
<tr>
<td>Strength 3</td>
<td>123</td>
<td>24</td>
<td>-1.172</td>
</tr>
<tr>
<td>Strength 4</td>
<td>133</td>
<td>28</td>
<td>-0.14</td>
</tr>
<tr>
<td>Strength 5</td>
<td>499</td>
<td>95</td>
<td>-0.44</td>
</tr>
<tr>
<td>Strength 6</td>
<td>68</td>
<td>32</td>
<td>-0.22</td>
</tr>
<tr>
<td>SUM</td>
<td>1021</td>
<td>210</td>
<td>-</td>
</tr>
</tbody>
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### Appendix B

#### Table (16): The ambiguities ($N_a$) results related to the observations elevations angle on the carrier ($L_a$)

<table>
<thead>
<tr>
<th>Elevation Angle</th>
<th>KTH2-K8R1</th>
<th>KTH2-K8R2</th>
<th>KTH2-K8R4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>Correct</td>
<td>%</td>
</tr>
<tr>
<td>5 to 15 degree</td>
<td>53</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>15 to 25 degree</td>
<td>102</td>
<td>3</td>
<td>2.94</td>
</tr>
<tr>
<td>25 to 35 degree</td>
<td>186</td>
<td>13</td>
<td>9.56</td>
</tr>
<tr>
<td>35 to 45 degree</td>
<td>220</td>
<td>28</td>
<td>12.73</td>
</tr>
<tr>
<td>45 to 55 degree</td>
<td>497</td>
<td>78</td>
<td>19.65</td>
</tr>
<tr>
<td>55 to 65 degree</td>
<td>113</td>
<td>29</td>
<td>25.66</td>
</tr>
<tr>
<td>65 to 75 degree</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>SUM</td>
<td>1021</td>
<td>151</td>
<td>14.79</td>
</tr>
</tbody>
</table>

#### Table (17): The ambiguities ($N_w$) results related to the observations elevations angle on the carrier ($L_a$)

<table>
<thead>
<tr>
<th>Elevation Angle</th>
<th>KTH2-K8R1</th>
<th>KTH2-K8R2</th>
<th>KTH2-K8R4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Obs.</td>
<td>Correct Nw</td>
<td>%</td>
</tr>
<tr>
<td>5 to 15 degree</td>
<td>55</td>
<td>7</td>
<td>13.21</td>
</tr>
<tr>
<td>15 to 25 degree</td>
<td>102</td>
<td>14</td>
<td>13.73</td>
</tr>
<tr>
<td>25 to 35 degree</td>
<td>136</td>
<td>24</td>
<td>17.65</td>
</tr>
<tr>
<td>35 to 45 degree</td>
<td>220</td>
<td>51</td>
<td>23.18</td>
</tr>
<tr>
<td>45 to 55 degree</td>
<td>397</td>
<td>74</td>
<td>18.64</td>
</tr>
<tr>
<td>55 to 65 degree</td>
<td>113</td>
<td>40</td>
<td>35.40</td>
</tr>
<tr>
<td>65 to 75 degree</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>SUM</td>
<td>1021</td>
<td>210</td>
<td>20.57</td>
</tr>
</tbody>
</table>
Appendix C

Table (18): The ambiguities \( (N_1) \) results related to the GPS satellites azimuth on the carrier \( (L_1) \)

| Azimuth | KTH2-KBR1 | | KTH2-KBR2 | | KTH2-KBR4 | |
|---------|-----------|-----------|-----------|-----------|-----------|
|         | Total     | Correct   | %         | Total     | Correct   | %         | Total     | Correct   | %         |
|         | Obs.      | N1        |           | Obs.      | N1        |           | Obs.      | N1        |           |
| 10 to 30| 42        | 0         | 0         | 0         | 0         | -         | 0         | 0         | -         |
| 30 to 50| 11        | 0         | 0         | 120       | 10        | 8.33      | 112       | 26        | 23.21     |
| 50 to 70| 106       | 17        | 16.04     | 443       | 170       | 38.37     | 118       | 9         | 7.63      |
| 70 to 90| 63        | 18        | 28.57     | 635       | 315       | 49.61     | 190       | 99        | 50.77     |
| 90 to 110| 131      | 55        | 41.98     | 308       | 153       | 49.68     | 147       | 94        | 63.95     |
| 110 to 130| 49       | 6         | 12.24     | 58        | 31        | 53.45     | 63        | 55        | 87.30     |
| 130 to 150| 135      | 28        | 20.74     | 0         | 0         | -         | 75        | 71        | 94.67     |
| 150 to 170| 0        | -         | -         | 0         | 0         | -         | 271       | 220       | 81.18     |
| 170 to 190| 243      | 20        | 8.23      | 0         | 0         | -         | 367       | 213       | 58.04     |
| 190 to 210| 5        | 0         | 0         | 199       | 103       | 49.76     | 0         | 0         | -         |
| 210 to 230| 0        | -         | -         | 0         | 0         | -         | 98        | 10        | 10.20     |
| 230 to 250| 0        | -         | -         | 0         | 0         | -         | 169       | 16        | 9.47      |
| 250 to 270| 17       | 0         | 0         | 62        | 20        | 32.26     | 237       | 52        | 21.94     |
| 270 to 290| 122      | 4         | 3.28      | 148       | 47        | 31.76     | 135       | 19        | 14.07     |
| 290 to 310| 97       | 3         | 3.09      | 170       | 58        | 34.12     | 155       | 97        | 25.87     |
| 310 to 330| 0        | 0         | -         | 104       | 26        | 25        | 0         | 0         | -         |
| SUM     | 1021      | 151       | 14.79     | 2247      | 933       | 41.52     | 2337      | 1020      | 43.65     |

Table (19): The ambiguities \( (N_w) \) results related to the GPS satellites azimuth on the carrier \( (L_1) \)

| GPS Satellite Azimuth | KTH2-KBR1 | | KTH2-KBR2 | | KTH2-KBR4 | |
|-----------------------|-----------|-----------|-----------|-----------|-----------|
|                       | Total     | Correct   | %         | Total     | Correct   | %         | Total     | Correct   | %         |
|                       | Obs.      | Nw        |           | Obs.      | Nw        |           | Obs.      | Nw        |           |
| 10 to 30              | 42        | 5         | 11.90     | 0         | 0         | -         | 0         | 0         | -         |
| 30 to 50              | 11        | 2         | 18.18     | 120       | 13        | 10.83     | 112       | 26        | 23.21     |
| 50 to 70              | 106       | 27        | 25.47     | 443       | 170       | 38.37     | 118       | 12        | 10.17     |
| 70 to 90              | 63        | 2         | 3.17      | 635       | 316       | 49.76     | 390       | 199       | 51.03     |
| 90 to 110             | 131       | 54        | 41.22     | 308       | 155       | 50.32     | 147       | 94        | 63.95     |
| 110 to 130            | 49        | 10        | 20.41     | 58        | 32        | 55.17     | 63        | 55        | 87.30     |
| 130 to 150            | 135       | 31        | 22.96     | 0         | 0         | -         | 75        | 71        | 94.67     |
| 150 to 170            | 0         | -         | -         | 0         | 0         | -         | 271       | 220       | 81.18     |
| 170 to 190            | 243       | 45        | 18.52     | 0         | 0         | -         | 367       | 213       | 58.04     |
| 190 to 210            | 5         | 0         | 0         | 199       | 103       | 51.76     | 0         | 0         | -         |
| 210 to 230            | 0         | -         | -         | 0         | 0         | -         | 98        | 13        | 13.27     |
| 230 to 250            | 0         | -         | -         | 0         | 0         | -         | 169       | 16        | 9.47      |
| 250 to 270            | 17        | 4         | 23.93     | 62        | 20        | 32.26     | 237       | 53        | 19.41     |
| 270 to 290            | 122       | 17        | 19.93     | 148       | 47        | 31.76     | 135       | 19        | 14.07     |
| 290 to 310            | 97        | 13        | 13.40     | 170       | 58        | 34.12     | 155       | 37        | 23.87     |
| 310 to 330            | 0         | -         | -         | 104       | 27        | 25.96     | 0         | 0         | -         |
| SUM                   | 1021      | 210       | 20.57     | 2247      | 941       | 43.21     | 2337      | 1028      | 43.99     |


09-005  **Erik Olsson.** Exporting 3D Geoinformation from Baggis Database to CityGML. Supervisors: Peter Axelsson and Yifang Ban. April 2009.


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