Corrective Surface for GPS-levelling in Moldova

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Abstract

The main objective of this thesis is the construction of a corrective surface in the Moldova area for further conversion of the geodetic heights into normal heights. For this purpose a detailed analysis of the optimal combination of heterogeneous height data is presented, with particular emphasis on (i) modeling systematic errors and datum inconsistencies, (ii) separation of random errors and estimation of variance components for each height type, and (iii) practical considerations for modernizing vertical control systems. Although the theoretical relationship between geodetic, normal heights and height anomalies is simple in nature, its practical implementation has proven to be quite challenging due to numerous factors that cause discrepancies among the combined height data. In addition, variance component estimation is applied to the common adjustment of the heterogeneous heights. This leads to the connection between the proper modelling of systematic errors and datum inconsistencies with the estimated variance components. Ultimately, one of the main motivations for this work is the need to introduce modern tools and techniques, such as GPS/levelling, in establishing a vertical control. Therefore, part of this thesis is aimed at bringing to the forefront some of the key issues that affect the achievable accuracy level of GPS/levelling. Overall, the analysis of the optimal combination of the heterogeneous height data conducted herein provides valuable insight to be used for a variety of height related applications.
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1 Introduction

1.1. Thesis Objectives

This thesis focuses on the construction of a corrective surface in the Moldova area for further conversion of the geodetic heights into normal heights with accuracy less than 10 cm. Because appropriate gravimetry data are not available in the Moldova Republic, such a model, here called the MOLDGEO2005 solution, is based only on GPS/levelling data within Moldova and the modified satellite gravimetry based GRACE (EIGEN-CG03C) solution, in this area.

Also presented is the analysis of the optimal combination of height data, with particular emphasis on datum inconsistencies, systematic effects and data accuracy. Specifically, a vertical control network consisting of geodetic, normal and height anomaly data is investigated. The combination of these heterogeneous height data is complicated by a number of outstanding issues, including (i) modelling systematic errors and datum inconsistencies, (ii) separation of random errors and estimation of variance components for each height type, and (iii) practical considerations for modernizing vertical control systems.

The selection of the most appropriate corrector surface model for a particular mixed height data set in order to model the datum inconsistencies and the systematic effects is complicated and rather arbitrary as it depends on a number of variables such as data distribution, density and quality, which varies for each case. Therefore, we investigated the procedures for some assessing models.

In addition to a proper parametric model, the variance components used in the combined network adjustment of the geodetic, normal and height anomaly data must also be estimated. This is an important element for the reliable least squares adjustment of the geodetic data that is often neglected in practical height-related problems.

Ultimately, one of the main motivations for this thesis is the need to introduce modern tools and techniques in the establishment of a vertical control. The manipulation of equation (1.1) such that normal heights are obtained using geodetic height and height anomaly data is called GPS/levelling and is a procedure that is commonly used in practice and will undoubtedly dominate the future of vertical control.

Overall, the analysis of the optimal combination of heterogeneous height data conducted herein, with particular emphasis on datum inconsistencies, systematic effects and data accuracy, will provide valuable insight and practical results to be used for a variety of height-related applications.

1.2. Thesis Outline

The analysis and results of this thesis are presented in Chapters 2 through 6. An outline of the essential structure of this thesis is given below.

In Chapter 2, the background information regarding the height data types used in this thesis is presented. The discussion focuses on the main error sources affecting the
computation of quasigeoid, normal and geodetic heights. As well, the evaluation of EGM96 and GRACE models using GPS/levelling data is presented.

Chapter 3 addresses to reasons for combining the height data. Although the applications for the optimal combination of the heterogeneous height types are innumerable, a short-list of the most prevalent geodetic applications is discussed in this chapter. In particular, the concept of regional vertical datums and modernization issues are considered. Attention is also given to the process of GPS/levelling.

In Chapter 4, the combined least-squares adjustment scheme implemented throughout this work is described in detail. The formulation is provided for the case where absolute height data values are available, as in equation (1.1). Modelling systematic effects using a parametric corrector surface model and the role of the parametric model are also described in details.

In Chapter 5, the real data (GPS network and the corresponding normal heights at GPS points) scattered over Moldova used for creating the corrective surface are described and depicted. The numerical test to find the outliers and classical empirical approach for testing/assessing candidate parametric model performance over a vertical network of co-located GPS/levelling benchmarks, are described. Also here, a review of variance component estimation applied to geodetic applications is provided. Variance component estimation algorithm is scrutinized for use in the combined height network adjustment problem. Using the a priori uniform accuracy of the geodetic and normal heights as well as height anomalies we will estimate a-posteriori variance components for each type of height data that further will be used to get an ‘improved’ variance-covariance matrix for the each corresponding height type. And the corrective surface is described. This section essentially focuses on the discussion about the statistical results obtained for the height anomaly of control points and its values from quasigeoid surfaces created by different interpolation methods. Finally, the method chosen for creating the final result (i.e. the quasigeoid surface) is described.

Chapter 6 summarizes the main conclusions of the thesis. Finally, recommendations for future work are also provided.

1.3. Introduction to the study and scope of the thesis

The fitting of a combined gravity field model to a regional/local height reference datum is of practical importance for many applications. In fact, the most common use of the quasigeoid model is to transform GPS derived geodetic heights to normal heights when referring to the Moldovian Height Datum (Baltica77). According to Molodensky theory proposed in 1945, which show that the physical surface of the Earth can be determined from geodetic measurements alone, without using the density of the Earth’s crust, this can be achieved by applying the simple relationship between the three height types for each control point \(i\) as given by Heiskanen and Moritz (1967, p.292):

\[
(1.1) \quad h_i - H^*_i - \zeta_i = 0
\]
where, $h_i$ is geodetic height obtained from space-based systems such as GPS, $H_i^*$ is the normal height usually obtained from spirit levelling and $\zeta_i$ is the height anomaly obtained from a regional gravimetric geoid model or a global geopotential model, depending on the available data. The geometrical relationship between the triplet of height types is also illustrated in Figure 1.

![Figure 1: Relationship between geodetic, normal heights and height anomaly](image)

In practice, the application of equation (1.1) is more complicated due to numerous factors, which cause discrepancies when combining the different height data sets. Some of these factors include (i) random errors in the derived heights $h$, $H^*$, and $\zeta$ (ii) datum inconsistencies inherent among the height types each of which refers to a different reference surface, (iii) systematic effects and distortions in the height data caused by long wavelength quasigeoid errors, poorly modeled GPS errors (i.e. tropospheric effects), and over-constrained levelling network adjustments, (iv) assumptions and theoretical approximations made in processing observed data including neglecting sea surface topography effects or river discharge corrections for measured tide gauge values and incorrect normal height corrections and (v) instability of reference station monuments over time due to geodynamic effects, crustal motion and land subsidence, see Rummel and Teunissen (1989), Kearsley et al. (1993), and Kotsakis and Sideris (1999). The major part of these discrepancies is usually attributed to the systematic effects and datum inconsistencies, which can be described by a corrector surface model such that:

\[
0 = -h_i - H_i^* - \zeta_i - a^T x = 0
\]

where, the term $a^T x$ describes the corrector surface, $x$ is a vector of the unknown parameters and $a$ is the design matrix corresponding to the known coefficients of a pre-selected parametric model.

As evidenced from this brief discussion, the combination of the heterogeneous height data is complicated by a number of outstanding issues, including the optimal adjustment model, separation of errors for each height type (variance-covariance
component estimation) and optimal transformation models (corrector surfaces). The focus of this thesis will be on the latter, namely the $a_i^T x$ term in equation (1.2). In particular, the process of selecting and testing the corrector surface for modeling the discrepancies between the latest combined gravity field model (EIGEN-CG03C), and the Moldavian Height Datum (MHD) using GPS data, will be presented. Ultimately, a new vertical reference system, which utilizes heterogeneous height data from both satellite and land-based methods, should be established. However, for today’s needs, and as a preliminary step, a rigorous and efficient method for transforming heights between different reference surfaces is required.

The unknown parameters for a selected corrector surface model are obtained via a common least-squares adjustment of geodetic, normal and height anomaly data over a network of co-located GPS/levelling benchmarks. A key issue in this type of common adjustment is the separation of errors among each height type, which in turn allows for the improvement of the stochastic model for the observational noise through the estimation of variance components.

The area that will benefit from the implementation of variance component estimation (VCE) methods is the assessment of the a-posteriori covariance matrix for the height coordinates derived from GPS measurements. Also it provides a better weighting of heterogeneous data in a least squares adjustment. Furthermore, it will allow for the evaluation of the accuracy information provided for normal heights obtained from national/regional adjustments of conventional levelling data.

Finally, as this is among the first tests of its kind for the Moldova case, a detailed discussion of conclusions and more importantly insights for future work are presented at the end of the thesis.
2 Quasigeoid, Normal heights and Geodetic heights

The purpose of this chapter is to provide the necessary background information regarding the type of data and terminology used throughout this thesis. In particular, focus will be placed on describing the major error sources that affect the height anomalies, normal and geodetic heights.

2.1. Geoid and Quasigeoid

The geoid is the equipotential surface to which orthometric heights are referred, whereas the quasigeoid is the non-equipotential surface to which normal heights are referred. The geoid undulation \(N\) refers to the separation between the reference ellipsoid and the geoid measured along the ellipsoidal normal, whereas the height anomaly \(\zeta\) refers to the separation between the reference ellipsoid and the quasigeoid, also measured along the ellipsoidal normal. Correspondingly, the heights that refer to the geoid are orthometric heights \(H\) measured along the plumb line, whereas the heights that refer to the quasigeoid are normal heights \(H^*\) measured along the ellipsoid normal. These two reference surfaces and their corresponding height systems are shown schematically in Figure 2.

![Figure 2: The relations among geoid undulation N, orthometric height H, height anomaly \(\zeta\) and normal height \(H^*\).](image)

As well as the conceptual differences between the geoid and the quasigeoid outlined above, there are a number of theoretical and practical differences in their computation on land, but they are practically identical at sea. The principal difference stems from the
assumptions made concerning the treatment of the Earth’s topography during the solution of the geodetic boundary value problem.

Solving Laplace’s equation under certain boundary conditions in a spherical approximation yields the classical Stokes formula for the gravimetric determination of the geoid (Heiskanen and Moritz 1967, p.94). One problem with Stokes’s approach is that it requires the gravity observations be downward-continued from the Earth’s surface to the geoid. This requires knowledge of the bulk density distribution in the topography above the geoid. As this information is often unavailable, the formulae are simplified by assigning a reasonable constant density (typically, 2.670 kgm$^{-3}$) to the topographic masses, thus making the formulae more suitable for practical evaluation. Theoretically, however, the problem of unknown topographic density remains in Stokes’s solution to the geodetic boundary value problem.

As Molodensky et al. (1962) have shown, a slightly different geodetic boundary value problem may be formulated and solved at the Earth’s surface without Stokes’ hypothesis. Molodensky used two surfaces, called the telluroid and the quasigeoid, in which the concept of the geoid undulation is replaced by the height anomaly; see Figure 2. Plotting the height anomalies above the reference ellipsoid results in the quasigeoid surface that is identical to the geoid over the oceans, and a close approximation to the geoid over most land areas. However, exceptions do occur in areas of large Bouguer gravity anomalies and high topography, such as the Himalayas (for details see Sjöberg 1995 and Rapp 1997). The quasigeoid is not an equipotential surface of the Earth’s gravity field, and thus has no physical meaning (Heiskanen and Moritz 1967, p.294). Though in contrast to geoid determination, the quasigeoid can be determined somewhat more directly from surface gravity data without prior knowledge of the topographic bulk density.

Using the geometry shown in Figure 2, consider a point $P$ on the topography surface of the Earth, geodetic height $h$ above the reference ellipsoid and an orthometric height $H$ above the geoid. The orthometric height can be determined from spirit levelling measurements knowing the integral mean value of the Earth’s gravity ($\bar{g} = \frac{1}{H} \int_{0}^{H} g \cdot dH$) along the plumb line between $P$ and the geoid. However, knowledge of the sub-surface density is required to determine this. As this information is not routinely available, the Poincare-Prey gravity gradient is often used instead, which results in Helmert orthometric heights. Assuming that the ellipsoidal normal and plumb line are coincident between the point $P$ and the geoid, the geoid height is approximated by:

\begin{equation}
N \approx h - H
\end{equation}

Now consider a point $Q$ which has the same normal gravity potential $U_Q$ as the Earth’s gravity potential $W_P$ at the point $P$. The surface described by plotting all points $Q$ at a distance $\zeta$ below the Earth’s surface is called the telluroid, by Hirvonen (1960, 1961) and Molodensky et al. (1962) – see Figure 2. The height of $Q$ above the reference ellipsoid, measured along the ellipsoidal normal, is termed the normal height $H^*$ of $P$. The normal height can be computed from spirit levelling measurements using the integral mean of normal gravity ($\bar{\gamma} = \frac{1}{H^*} \int_{0}^{H^*} \gamma \cdot dH^*$) between the reference ellipsoid and the
telluroid. Thus, no prior knowledge of the topographic density distribution is required in
the determination of the height anomalies, only the mean value of normal gravity which
may be calculated analytically. The height anomaly is defined similarly to equation (2.1); see Heiskanen and Moritz (1967, p.292):

\[ \zeta = h - H^* \]  

(2.2)

Given the definitions above, the difference between the geoid height and the height
anomaly is identical to the difference between the normal and orthometric heights. Eliminating \( h \) from equations (2.1) and (2.2) gives:

\[ N - \zeta = H^* - H \cong C_2 \]  

(2.3)

where Heiskanen and Moritz, (1967) estimate the difference to be:

\[ C_2 \cong \frac{\Delta g_B}{\bar{\gamma}} H \]  

(2.4)

In equations (2.4) \( \Delta g_B \) is the Bouguer gravity anomaly at \( P \), whose computation also
involves an assumption of the topographic density, and \( \bar{\gamma} \) is the mean value of normal
gravity between the geoid and Earth’s surface. It can be estimated by equation 4-42 of
Heiskanen and Moritz (1967):

\[ \bar{\gamma} = \gamma \left[ 1 - (1 + f + m - 2f \sin^2 \phi) \frac{H}{a} + \frac{H^2}{a^2} \right] \]  

(2.5)

where \( \gamma \) is normal gravity on the surface of the reference ellipsoid, \( f \) is the geometrical
flattening of the reference ellipsoid, \( m \) is the Clairaut constant, \( a \) is the radius of the semi-
major axis, and \( \phi \) is the geodetic latitude of \( P \). The separation between the geoid and the
quasigeoid has also been investigated by Sjöberg (1995), who expresses it as a series of
the terrain height.

2.2. Normal heights

Height differences between points on the Earth’s surface have traditionally been obtained
through terrestrial levelling methods, such as spirit-levelling (and/or barometric levelling,
trigonometric levelling, etc).

Although costly and laborious, spirit-levelling is an inherently precise
measurement system whose procedural and instrumental requirements have evolved to
limit possible systematic errors. Associated random errors in leveling originate from
several sources, such as refractive scintillation or ‘heat waves’, refraction variation
between readings, vibrations of instrument due to wind blowing, and movement of rod or
non-verticality of rod caused by wind, terrain and unsteadiness of surveyor, to name a
few, see Gareau (1986) for details. These errors are generally dealt with through redundancy and minimized in the least-squares adjustment process (Vaniček et al. 1980).

However, it should be realized that national networks of vertical control established in this way involve large samples of measurements collected under inhomogeneous conditions, such as variable terrain, environments, and instruments, with different observers and over different durations. These results in a number of errors/corrections that must be made to the measurements, see Davis et al. (1981, pp.118-187) for details.

The problem with using only the elevation differences obtained from spirit-level for height-related applications is that the results are not unique as they depend on the path taken from one point to the other (due to non-parallelism of the equipotential surfaces). Thus, a number of different height systems can be defined, which use the measurements of vertical increments between equipotential surfaces along a path from spirit-leveling \((dn)\) and gravity measurements \((g)\), as given by:

\[
(2.6) \quad C_p = \int_{r_0}^{p} g \, dn
\]

where \(C_p\) is the geopotential number and represents the difference in potential between the constant value at point \(P_0\) at the geoid, \(W_0\), and the potential at the point, \(P\), on the surface, \(W_P\), as follows:

\[
(2.7) \quad C_p = W_0 - W_P
\]

All points have a unique geopotential number with respect to the geoid and it can be scaled by gravity in order to obtain a height coordinate with units of length, as we have become accustomed to using for describing heights \(H = \frac{C}{G}\). Depending on the type of ‘gravity’ value \(G\) used to scale the geopotential number, different types of heights can be derived. As normal heights are used in our numerical applications, we will define just this type of height. The normal heights were introduced by Molodensky in connection with his method of determining the physical surface of the Earth, see Heiskanen and Moritz (1967, Chapter 8). The normal height system is the basis of heights in many regions worldwide as is in Moldova. The normal height can be found with respect to the adopted ellipsoid via the geopotential number using the Molodensky condition:

\[
(2.8) \quad W_0 - W_P = U_0 - U_Q
\]

Consider a point \(P\) on the physical surface of the Earth. It has a certain potential \(W_P\) and also a certain normal potential \(U_P\), but in general \(W_P \neq U_P\). However, there is a certain point \(Q\) on the plumb line of \(P\), such that \(U_Q = W_P\); that is, the normal potential \(U\) at \(Q\) is equal to the actual potential \(W\) at \(P\). And \(U_Q\) is the normal potential of adopted ellipsoid \((U_0 = W_0)\). So, if for scaling the geopotential number we use the mean normal gravity
Along the ellipsoid normal, \( \vec{P} \), then we obtain normal heights denoted by \( H^*_P \), see Heiskanen and Moritz (1967, p.171):

\[
H^*_P = \frac{C_p}{\vec{P}}
\]

(2.9)

So, the normal height \( H^* \) of \( P \) is the geometric height of \( Q \) above the ellipsoid, and the distance \( PQ \) (see Figure 3) represents the height anomaly \( \zeta \).

![Diagram](image)

**Figure 3:** The normal height \( H^* \) and height anomaly \( \zeta \)

The normal height \( H^* \) of a ground point \( P \) is identical with the height above the ellipsoid, \( h \), of the corresponding telluroid point \( Q \). If the geopotential function \( W \) were equal to the normal potential function \( U \) at every point, then \( Q \) would coincide with \( P \), the telluroid would coincide with the physical surface of the earth, and the normal height of every point would be equal to its geometric height. Actually, however, \( W_p \neq U_p \); hence the difference:

\[
\zeta_p = h_p - H^*_p = h_p - h_Q
\]

(2.10)

is not zero. This explains the term “height anomaly” for \( \zeta \) (Heiskanen and Moritz 196, p.292). The development of modern GPS technology has led to a straightforward application of equation (2.10), which allows to ‘measure’ the height anomaly \( \zeta \) if \( h \) and \( H^* \) are known at the same point from GPS, precise levelling and gravity observations.

Equation (2.10) gives us today important information for the establishment of height reference systems on the whole and for the development of the European Vertical Reference System (EVRS), in particular.

Although levelling measurements are very precise (i.e., at the mm-level depending on the order or class of levelling), it is often the regional or national network adjustments of vertical control points that leads to the greatest source of (systematic) error. If the vertical datum (see Section 3.1) of a height network is based on fixing a single point (e.g., a tide gauge station) then the adjusted normal heights will contain a
constant bias over the entire network area. The situation is more complicated when an
over-constrained network adjustment is performed (i.e., fixing more than one tide gauge
station), which introduces distortions throughout the network.

2.3. Global Geopotential Model

By a global geopotential model (GGM), we mean a set of normalized spherical harmonic
coefficients $C_{nm}, S_{nm}$ ($n = 2, 3, 4, \ldots, n_{\text{max}}; m = 0, 1, 2, \ldots, n$) for the Earth’s gravitational
potential $V(r, \vec{\phi}, \lambda)$, together with several additional parameters ($GM$, is the product of
gravitational constant and the total mass of the Earth, $a$, semi-major axis of the reference
ellipsoid) [equation (2.12)]. Here $n$ is the degree and $m$ is the order of the harmonic
coefficients and $(r, \vec{\phi}, \lambda)$ are the spherical coordinates. The $n_{\text{max}}$ denotes the maximum
degree of the model. The higher $n_{\text{max}}$ is the more details the GGM contains about the
geoid and the gravity field of the Earth. A spherical harmonic series expansion [see
equation (2.12)] of the Earth’s gravitational potential $V(r, \vec{\phi}, \lambda)$ is suitable to describe the
global features of the Earth’s gravity field. Therefore, determination of GGMs essentially
is based on measurements of global characters, such as measurements from near-Earth
artificial satellites. The best method to determine high degree GGMs is to combine data
from satellite orbit perturbation with surface gravity measurements and satellite altimetry
data. During the last 30 years, a variety of GGMs, which express the Earth’s gravity field
and thus geoid heights in terms of harmonic basis functions, have been computed by
various groups.

In the next two sections we will be describe the EGM96 and EIGEN-CG03C
models, as they will be used in our numerical tests.

2.3.1. Earth Gravitational Model (EGM96)

The NASA Goddard Space Flight Center (GSFC), the National Imagery and Mapping
Agency (NIMA), and the Ohio State University (OSU) have collaborated to develop an
improved spherical harmonic model of the Earth’s gravitational potential to degree 360.
The new model, Earth Gravitational Model 1996 (EGM96) incorporates improved
surface gravity data, altimeter-derived anomalies from ERS-1 and from the GEOSAT
Geodetic Mission (GM), extensive satellite tracking data - including new data from
Satellite laser ranging (SLR), the Global Positioning System (GPS), NASA’s Tracking
and Data Relay Satellite System (TDRSS), the French DORIS system, and the US Navy
TRANET Doppler tracking system - as well as direct altimeter ranges from
TOPEX/POSEIDON (T/P), ERS-1, and GEOSAT. The final solution blends a low-
degree combination model to degree 70, a block-diagonal solution from degree 71 to 359,
and a quadrature solution at degree 360.

This model was used to compute geoid undulations accurate to better than one
meter (with the exception of areas void of dense and accurate surface gravity data) and
realize WGS84 as a true three-dimensional reference system. Additional results from the
EGM96 solution include models of the dynamic ocean topography to degree 20 from T/P
and ERS-1 together, and GEOSAT separately, and improved orbit determination for Earth-orbiting satellites; see Lemoine et al. (1998).

2.3.2. Combined Gravity Field Model EIGEN-CG03C

The gravity field combination model EIGEN-CG03C is an upgrade of EIGEN-CG01C. The model is based on the same CHAMP mission and surface: mean gravimetry and altimetry data in 0.5 x 0.5 degrees blocks, but takes into account almost twice as much GRACE mission data. Instead of 200 days now 376 days out of February to May 2003, July to December 2003 and February to July 2004 have been used. EIGEN-CG03C is complete to degree and order 360 in terms of spherical harmonic coefficients and resolves geoid and gravity anomaly wavelengths of 110 km. A special band-limited combination method has been applied in order to preserve the high accuracy from the satellite data in the lower frequency band of the geopotential and to form a smooth transition to the high frequency information coming from the surface data. Compared to pre-CHAMP/GRACE global high-resolution gravity models, the accuracy at 400 km wavelength could be improved by one order of magnitude to 3 cm and 0.4 mgal in terms of geoid heights and gravity anomalies, respectively. The overall accuracy of the full 360 model down to spatial features of 100 km is estimated to be 30 cm and 8 mgal, respectively. In general, the accuracy over the oceans is better than over the continents reflecting the quality of the available surface data; Flechtner (2005).

2.3.3. Height Anomaly Computed from Global Geopotential Model

The gravitational potential \( V \) [m^2 s^{-2}] of the Earth is given by a triple integral over the Earth. Let \( k \) denote the gravitational constant, let \( d\nu \) denote an element of volume, let \( \rho \) be the density of the volume element, and let \( l \) be the distance between the mass element \( \rho d\nu \) and the attracted point \( Q \), then the potential is given, Heiskanen and Moritz (1967, Chapter 1):

\[
V = k \iiint_{\text{Earth}} \frac{\rho}{l} d\nu
\]

The actual potential is described via geopotential coefficients \( \bar{C}_{nm} \) and \( \bar{S}_{nm} \) of degree \( n \) and order \( m \). They are coefficients in an orthonormal series expansion in \( \cos m\lambda \), \( \sin m\lambda \), and the associated Legendre polynomials \( P_{nm} \):

\[
V(r, \phi, \lambda) = \frac{GM}{R} \left( 1 + \sum_{n=2}^{\infty} \left( \frac{R}{r} \right)^{n+1} \sum_{m=0}^{n} \bar{P}_{nm}(\sin \phi) [\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda] \right)
\]

Respectively, \( GM \) the gravitational constant times mass of the Earth, \( R \) the Earth’s mean radius. The first term for \( n = 0 \) is nothing else than \( GM / R \). There is no term with \( n = 1 \)
if the origin is at the geocenter so the actual summation starts from $n = 2$. The attracted
point $Q$ has geodetic coordinates $(r, \phi, \lambda)$. Although $Q$ can be any point outside the geoid,
the present code is dedicated to small values of $H$. The series expansion in Legendre
polynomials is based on spherical coordinates $(r, \bar{\phi}, \lambda)$. The longitude $\lambda$ is the same in
both coordinate systems because of the rotational symmetry, while $\phi$ in general is
different from $\bar{\phi}$. The distance from $Q$ to the origin is denoted by $r$.

The geopotential coefficients $\overline{C}_{nm}$ and $\overline{S}_{nm}$ in equation (2.12) correspond to an
ellipsoid of revolution with semi-major axis $a = 6378136.46$ m and flattening $f = 1/298.25765$. These values correspond to a tide-free system.

Our goal is to compute the height anomaly $\zeta$ at $Q(\phi, \lambda, H^*)$. We subtract the
normal gravity potential $U + \Phi$ from the actual gravity potential $W = V + \Phi$, $\Phi$ being
the potential due to the centrifugal force, and get the anomalous potential $T$:

$$T = W - (U + \Phi) = (V + \Phi) - (U + \Phi) = V - U.$$  

The normal gravitational potential $U$ may as well be described by a series expansion in
associated Legendre polynomials. Because of the rotational symmetry there will be only
zonal terms, and because of the symmetry with respect to the equatorial plane there will
be only even zonal harmonics. The zonal harmonics of odd degree change sign for
negative latitudes and must be absent. Accordingly, the series has the form:

$$U = \frac{kM}{r} + \sum_{n=2}^{\infty} \overline{C}_{nm} \frac{P_{nm}(\sin \bar{\phi})}{r^{n+1}}.$$  \hspace{1cm} (2.13)

Next we have to determine the coefficients $\overline{C}_{nm}$. The reference values $\overline{C}_{nm}$ can be
computed from the closed expression:

$$J_{nm} = (-1)^{1+n/2} \frac{3e^n}{(n+1)(n+3)} \left(1 - \frac{n/2 + 5J_2}{2e^2}\right), \quad \text{for } n = 2, 4, 6, 8, 10 \text{ and } m = 0.$$  \hspace{1cm} (2.14)

where $e$ is the ellipsoid eccentricity, $J_{20}$ is the value implied by the defined flattening of
the reference ellipsoid. For $n = 2$ and $m = 0$ we recover the identity $J_{20} = J_{20}$ where $J_{20} =
108262.982126 \times 10^{-8}$ for WGS84 ellipsoid. For reasons of accuracy it is sufficient to
include five terms. The normalizing factors for $J_{nm}$ are $\sqrt{2n+1}$, where $m = 0$. So we get
the even zonal harmonics coefficients for degree $n = 2, 4, 6, 8, 10$ and order $m = 0$, by:

$$\overline{C}_{nm} = -\frac{J_{nm}}{\sqrt{2n+1}},$$  \hspace{1cm} (2.15)

The values obtained by equation (2.15) are given in Table 1:
Table 1: Even zonal harmonics coefficients of degree $n = 2, 4, 6, 8, 10$ and order $m = 0$ (see NIMA, 2004)

<table>
<thead>
<tr>
<th>coefficients</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{C}_{2,0}$</td>
<td>-0.484166774985 E-03</td>
</tr>
<tr>
<td>$\overline{C}_{4,0}$</td>
<td>0.790303733511 E-06</td>
</tr>
<tr>
<td>$\overline{C}_{6,0}$</td>
<td>-0.168724961151 E-08</td>
</tr>
<tr>
<td>$\overline{C}_{8,0}$</td>
<td>0.346052468394 E-11</td>
</tr>
<tr>
<td>$\overline{C}_{10,0}$</td>
<td>-0.265002225747 E-14</td>
</tr>
</tbody>
</table>

The normal gravity $\gamma$ is computed by:

$$\gamma(\phi) = \frac{\gamma_e \cos^2 \phi + (1-f)\gamma_p \sin^2 \phi}{\sqrt{\cos^2 \phi + (1-f)^2 \sin^2 \phi}}$$

where the normal gravity on the equator is $\gamma_e = 9.7803253359 \, \text{m/s}^2$, the normal gravity on the poles is $\gamma_p = 9.8321849378 \, \text{m/s}^2$ and the geometrical flattening $f = 1/298.257223563$. And the direct formula for computing $\gamma$ at $Q$ (see Figure 1), (Heiskanen and Moritz, 1967, p.293):

$$\gamma_H = \gamma(\phi) \left( 1 - 2 \left( 1 + f + m - 2 f \sin^2 \phi \right) \frac{H^*}{a} + 3 \left( \frac{H^*}{a} \right)^2 \right).$$

where semi-major axis is $a = 6378137$ meters, semi-minor axis is $b = 6356752.3142$ meters and $m = \frac{\omega^2 a^2 b}{GM} = 0.00344978650684$ (for WGS 84). Applying, Bruns’ formula to height anomaly $\zeta$, we have; see Heiskanen and Moritz (1967, p.293):

$$\zeta = \frac{T}{\gamma},$$

$T = W_p - U_p$ being the disturbing potential at ground level, and $\gamma$ being the normal gravity at the telluroid. Finally, the height anomaly $\zeta$ can be computed, see Heiskanen and Moritz (1967, p.107), by:

$$\zeta(r, \phi, \lambda) = \frac{k M}{R \gamma_H} \sum_{n=2}^{n_{\text{max}}} \left( \frac{R}{r} \right)^n \sum_{m=0}^{n} \overline{P}_{nm}(\cos \phi) \left[ C_{nm} \cos m \lambda + S_{nm} \sin m \lambda \right].$$
The height anomalies $\zeta$ above the ellipsoid may also be plotted. In this way we get a surface that is identical with the geoid over the oceans, because there $\zeta = N$, and is very close to the geoid anywhere else. This surface has been called the quasigeoid by Molodensky. However, the quasigeoid is not a level surface and has no physical meaning whatever. It must be considered as a concession to conventional conceptions that call for a geoidlike surface. From this point of view the normal height of a point is its elevation above the quasigeoid, just as the orthometric height is its elevation above the geoid, see Heiskanen and Moritz (1967, p.294).

2.3.4. RMS of GPS/levelling network versus Global Geopotential Models

The improvement of EIGEN-CG03C versus EGM96 is reflected in independent comparisons with geoid heights determined point-wise by GPS positioning and levelling, see Flechtner et al. (2005). Table 2 shows the results for EGM96 and CG03C using GPS/levelling data from the USA, Milbert (1998), Canada, Veronneau (2003); National Ressources Canada, GPS on BMs file, update February 2003 and Europe/Germany (Ihde et al. 2002).

Table 2: RMS differences of GPS/levelling and GGM (GRACE and EGM96). Unit: [cm]

<table>
<thead>
<tr>
<th>Gravity Model</th>
<th>USA (6169)</th>
<th>Canada (1930)</th>
<th>Europe (186)</th>
<th>Germany (675)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EIGEN-CG03C</td>
<td>43</td>
<td>35</td>
<td>38</td>
<td>20</td>
</tr>
<tr>
<td>EGM96</td>
<td>47</td>
<td>38</td>
<td>45</td>
<td>28</td>
</tr>
</tbody>
</table>

Table 3 shows the results for EGM96 and CG03C using 917 GPS/levelling points scattered over Moldova:

Table 3: RMS differences of GPS/levelling and GGMs for Moldova. Unit: [cm].

<table>
<thead>
<tr>
<th>Gravity Model</th>
<th>Moldova (917)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EIGEN-CG03C</td>
<td>23.3</td>
</tr>
<tr>
<td>EGM96</td>
<td>32.7</td>
</tr>
</tbody>
</table>

Root mean square (rms) difference of GPS/levelling and gravity field model derived heights anomaly differences, are given in cm, and the number of points are given in brackets.
The representations of the height anomalies derived from the combined gravity field model EIGEN-CG03C and the global geopotential model EGM96 over Moldova are depicted in Figure 4.

Figure 4: Height anomalies from GRACE (EIGEN-CG03C) and EGM96 global geopotential models over Moldova.

2.4. Geodetic heights

The Earth is close to an ellipsoid of revolution, known the as reference ellipsoid. The theoretical gravity field is chosen as that generated by a reference ellipsoid. While selecting the WGS 84 Ellipsoid and associated parameters, the original WGS 84 Development Committee decided to closely adhere to the approach used by the International Union of Geodesy and Geophysics (IUGG), when the latter established and adopted Geodetic Reference System 1980 (GRS 80). Accordingly, a geocentric ellipsoid of revolution was taken as the form for the WGS 84 Ellipsoid. The parameters selected to originally define the WGS 84 Ellipsoid were the semi-major axis \(a\), the Earth’s gravitational constant \(GM\), the normalized second degree zonal gravitational coefficient \(C_{2,0}\) and the angular velocity \(\omega\) of the Earth. These parameters are identical to those of the GRS 80 Ellipsoid with one minor exception. The form of the coefficient used for the second degree zonal is that of the original WGS 84 Earth Gravitational Model rather than the notation ‘\(J_2\)’ used with GRS 80 (see Table 4), see NIMA (2004) for details.

Table 4: Defining constants of the Geodetic Reference System 1980
In 1993, two efforts were initiated which resulted in significant refinements to these original defining parameters. The first refinement occurred when DMA recommended, based on a body of empirical evidence, a refined value for the GM parameter. In 1994, this improved GM parameter was recommended for use in all high-accuracy DoD orbit determination applications. The second refinement occurred when the joint NIMA/NASA Earth Gravitational Model 1996 (EGM96) project produced a new estimated dynamic value for the second degree zonal coefficient, for details, see NIMA (2004). A decision was made to retain the original WGS 84 Ellipsoid semi-major axis and flattening values (\(a = 6378137 \text{ m}, \text{ and } 1/f = 298.257223563\)). For this reason the four defining parameters were chosen to be: \(a, f, GM\) and \(\omega\). Further details regarding this decision are provided below. The reader should also note that the refined GM value is within 1\(\text{st}\) of the original (1987) GM value. Additionally there are now two distinct values for the \(C_{2,0}\) term. One dynamically derived \(C_{2,0}\) as part of the EGM96 and the other, geometric \(C_{2,0}\), implied by the defining parameters. Table 5 contains the revised defining parameters, see NIMA (2004).

**Table 5:** WGS 84 Four Defining Parameters

<table>
<thead>
<tr>
<th>Notation</th>
<th>Parameter</th>
<th>Unit</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>semi-major axis</td>
<td>m</td>
<td>6 378 137</td>
</tr>
<tr>
<td>(GM)</td>
<td>product of G and total mass M</td>
<td>m(^3)s(^{-2})</td>
<td>3 986 004.418(\times)10(^8)</td>
</tr>
<tr>
<td>(1/f)</td>
<td>reciprocal of flattening</td>
<td></td>
<td>298.257223563</td>
</tr>
<tr>
<td>(\omega)</td>
<td>angular velocity</td>
<td>s(^{-1})</td>
<td>0.729 211 51(\times)10(^{-4})</td>
</tr>
</tbody>
</table>

The WGS 84 Ellipsoid is identified as being a geocentric equipotential ellipsoid of revolution. An equipotential ellipsoid is simply an ellipsoid defined to be an equipotential surface, i.e., a surface on which the value of the gravity potential is the same everywhere. The WGS 84 ellipsoid of revolution is defined as an equipotential surface with a specific theoretical gravity potential (\(U\)). This theoretical gravity potential can be uniquely determined, independent of the density distribution within the ellipsoid, by using any system of four independent constants as the defining parameters of the ellipsoid. As noted earlier, these are the semi-major axis (\(a\)), the inverse of the flattening (\(1/f\)), the Earth’s angular velocity (\(\omega\)), and the Earth’s gravitational constant (\(GM\)). All other quantities,
which describing the normal field can be derived from these four defining constants and are called the derived quantities.

Because of its smooth well-defined surface, the ellipsoid offers a convenient reference surface for mathematical operations and is widely used for horizontal coordinates, Seeber (1993). The geodetic latitude $\phi$, and longitude $\lambda$, are defined in Figure 5, where it is assumed that the centre of the ellipsoid coincides with the Earth’s centre of mass, its minor axis is aligned with the Earth’s reference pole and the p-axis is the intersection of the meridian plane with the equatorial plane.

![Figure 5: Reference ellipsoid and geodetic coordinates ($\phi$, $\lambda$, $h$)](image)

The straight-line distance between a point $P$ on the surface of the Earth and its projection along the ellipsoidal normal onto the ellipsoid, denoted by $Q$, is the geodetic height $h$. The location of the point $P$ can also be defined in terms of Cartesian coordinates, $(x, y, z)$, which has greatly benefited from the advent of satellite-based methods, such as GPS. Using GPS (or another global navigation satellite system), three-dimensional coordinates of a satellite-signal receiver can be determined within the same reference frame used to determine the coordinates of the satellites. The curvilinear geodetic coordinates $(\phi, \lambda, h)$ offer an intuitive appeal that is lacking for Cartesian coordinates and are therefore preferred by users for describing locations on the surface of the Earth.

Using today’s available technology and techniques, geodetic heights can be obtained from a number of different systems, such as very long baseline interferometry (VLBI), satellite laser ranging (SLR), and navigation based systems such as DORIS, GPS, and GLONASS. Furthermore, satellite altimetry measurements are used to obtain geodetic heights over the oceans, which cover more than 70% of the Earth’s surface. Thus, although the most popular method in use today is GPS, the alternatives are set to broaden in the near future. This being said, all new global satellite-based navigation systems will benefit greatly from the experience gained by researchers and users working with GPS. In fact, many of the challenges and error sources that affect the quality of the positioning coordinates will still have to be dealt with. Therefore, it is appropriate to discuss some of the main error sources affecting the determination of geodetic heights.
using GPS, as it is the main tool used to obtain geodetic heights for all of the numerical tests used throughout this thesis.

Comprehensive overviews of the fundamental concepts, measuring, and processing procedures for GPS can be found in many textbooks such as Hofmann-Wellenhof et al. (1992), Parkinson and Spilker (1996a/b), and Kaplan (1996) and will not be dwelled on herein. The errors affecting GPS measurements originate from three sources, namely satellite errors, signal propagation errors and receiver errors (see Figure 6).

![Figure 6: Sources of errors for global navigation satellite systems](image)

All three types of error sources affect the quality of the estimated geodetic heights and the most significant will be discussed herein:

- **Orbital Errors**
At the satellite level, the most predominant source of error for geodetic height determination is the orbital errors. For short baselines, the orbit error is cancelled when differential processing is performed, however the effect is spatially correlated and therefore the level of cancellation/reduction is dictated by the baseline length. A conservative and perhaps even pessimistic estimate of the decorrelation of satellite orbit errors based on the baseline length is provided by the following linear relationship Seeber (1993, p. 297):

\[
\frac{\sigma_b}{b} \approx \frac{\sigma_\rho}{\rho}
\]

where \(\sigma_b\) is the baseline error for a baseline length \(b\). The satellite range is represented by \(\rho\) (approximately 22,000 km for GPS satellites) and used to compute the orbit error \(\sigma_\rho\). In general, the vertical coordinate is affected more than the horizontal coordinates because the largest orbit errors are in the along-track direction, which results in a tilting of the network. The best means to deal with this error source is to use precise ephemeris
information provided by the International GPS Service (IGS), which has a significantly lower $\sigma_\rho$ than the broadcast ephemeris information provided in the broadcast navigation message.

- **Tropospheric Delay Errors**

Atmospheric errors account for a large part of the error sources affecting the satellite signals as they propagate towards the receiver(s) located on the surface of the Earth. The signal travels through two parts of the atmosphere, namely the ionosphere and the troposphere. The troposphere ranges from 0 km to 40 km above the surface of the Earth and is considered a key deteriorating factor for height determination. Specifically, signals traveling through the troposphere suffer the effects of tropospheric attenuation, delay and short-term variations (scintillation). The magnitudes of these effects are a function of satellite elevation and atmospheric conditions such as temperature, pressure and relative humidity during signal propagation. Furthermore, the troposphere is a non-dispersive medium for GPS frequencies, which means that the tropospheric range errors are not frequency dependent and therefore cannot be cancelled through the use of dual-frequency measurements (unlike the ionospheric effects). The most damaging part is the relative tropospheric bias which is caused by errors in tropospheric refraction at one of the stations in a baseline configuration. The general estimate of the bias caused in geodetic height difference measurements, $\Delta h$, is given by Beutler et al. (1987):

\[
\Delta h = \frac{\Delta \rho_{\rho}}{\cos(z_{\text{max}})}
\]

where $\Delta \rho_{\rho}$ is the relative tropospheric zenith correction and $z_{\text{max}}$ is the zenith angle of the observation. Using this estimation, an error or unmodelled differential tropospheric delay of 1 cm in $\Delta \rho_{\rho}$ at a moderate satellite elevation angle of 20° ($z_{\text{max}}=70°$) yields an error of 3 cm in the estimated geodetic height difference. Two methods that can be applied to estimate tropospheric refraction include, modelling tropospheric parameters simultaneously with all other GPS parameters (clock, latitude, longitude, height, ambiguities), or independent modelling of the troposphere using water vapour radiometers and ground meteorological observations. Ultimately, the best means to deal with tropospheric effects for high precision height determination is by improving the measurements and models for water vapour content, see Dodson (1995).

- **Multipath**

Multipath is a signal propagation error, which occurs when a signal arrives at a receiver via multiple paths, Braasch (1996). It is caused by the reflection and diffraction of the transmitted signal by objects in the area surrounding the receiver antenna. In Elósegui et al. (1995), the magnitude of the multipath error on the vertical coordinate was estimated and found to be strongly dependent on the satellite elevation angle. For instance, a variation from 5° to 10° in elevation cut-off changed the estimates of the geodetic height from tens of millimetres to several centimetres (ibid.). From a processing point of view, the problem is juxtaposed as lowering the elevation cut-off (i.e., for VLBI and GPS measurements) helps to decorrelate the tropospheric and height parameters, but at the same time may cause an increase in multipath effects. Over the past decade, there have been numerous improvements to receiver and antenna technology (choke rings, ground
planes), which aid in mitigating the effects of multipath. Despite these technological advances, the best method for most GPS users to mitigate multipath effects is to simply avoid it by carefully selecting receiver station sites that are free of any reflective obstructions. Selecting a low multipath environment is an important consideration that must be adhered to when establishing permanent vertical control stations. Existing levelling stations may not be optimally located for such measures and therefore biases may exist in the geodetic heights of co-located GPS/levelling benchmarks.

- **Atmospheric and Ocean Loading**

For most combined height applications discussed in this thesis, a large network (e.g., of national scale) consisting of accurate geodetic height determinations is used. In such cases, it is important to consider the vertical motion of the Earth’s crust caused by differential loading effects of the atmosphere and ocean tides. In general, the deformation of the crust as a reaction to changing atmospheric pressure is at the level of 1 to 2 cm; see Van Dam et al. (1994). The larger displacement is due to ocean loading, which is more difficult to model and may cause height changes of more than 10 cm for stations situated near the coasts, Baker et al. (1995). This is important for GPS monitoring of tide gauge stations, which may be incorporated into vertical datum definitions (see Section 3.1).

The best means to deal with these effects are to apply corrections to the estimated heights based on global models (in conjunction with higher resolution local models, if they are available), which are designed to predict the response to loads. The accuracy of the global ocean load models may vary depending on the location, with more accurate predictions in the open oceans and degrading accuracy approaching the coastal areas. With GPS measurements, making observations over a 24-hour period averages out most of the error.

However, shorter occupation times may lead to significant biases in the estimated geodetic heights if appropriate corrections are not applied. It is important for users of vertical control stations to be aware of the type of ‘corrections’ that have been made to the supplied geodetic heights. Furthermore, when combining geodetic heights and height anomalies, it is imperative that both height anomalies and GPS-derived heights are reduced in a consistent manner, Poutanen et al. (1996).

- **Antenna Phase Centre Offsets**

At the receiver level, the antenna phase centre offsets are of great concern for accurate geodetic height estimates. GPS measurements are actually made with respect to the point in the antenna known as the phase centre, not the survey mark. Corrections must be applied to reduce the measurement to the unknown point. It has been shown that the antenna phase centre is not fixed and varies depending on the elevation of the satellite and also the frequency of the propagated signal. For the combined height networks used in this work, complications arise from the mixing of different antenna types, which may produce errors in the geodetic heights of up to 10 cm, Rothacher (2001). Estimated tropospheric parameters are also highly correlated with antenna phase centre patterns, which may be incorrectly interpreted in processing software, resulting in amplified errors, especially in the height component. Thus, it is important to use the same antenna make and model for network surveys in order to reduce the errors caused by antenna phase centre offsets. Although the mitigation of this error source seems simple compared to the complicated modelling of other error sources, this is a difficult task to manage, particularly for large networks.
3 Practical applications of data from combined heights

The optimal combination of geodetic, normal heights and height anomalies is well suited for a number of applications. This is exemplified by the simple geometrical relationship that exists between the triplet of heights, expressed in equation (1.1) and depicted in Figure 1. Traditional methods for establishing a vertical control, although precise, are very laborious, costly and impractical in harsh terrain and environmental conditions. On the other hand, geodetic heights can be efficiently and relatively inexpensively be established with dense coverage over land (i.e., using global navigation satellite systems) or over the oceans (i.e., using satellite altimetry), at a poorer accuracy level. The main problem with these techniques is that the heights refer to a fictitious reference ellipsoid approximating the true shape of the Earth and therefore do not embody any physical meaning. The link between geometrically-defined geodetic heights and heights with respect to a local vertical datum (i.e., normal heights) is provided by height anomalies.

Recognizing the inherent advantages and limitations of each type of height system, it is clear that a proper combination of the heterogeneous heights, with proper error analysis, will benefit in numerable applications. A number of important geodetic application areas that will benefit from the optimal combination of the heterogeneous height data have been identified, namely:

- modernizing regional vertical datums
- transforming between different types of height data

The following sections provide an overview of these important application areas.

3.1. Modernizing regional vertical datums

A vertical datum is a reference surface to which the vertical coordinate of points is referred. At a national level, some of the practical uses and benefits of a consistent regional vertical datum include, but are not limited to, the following, Zilkoski et al. (1992):

- accurate elevation models for flood mitigation
- accurate elevation models for environmental hazards
- enhanced aircraft safety and aircraft landing
- improved understanding of tectonic movement
- improved management of natural resources

Traditionally, geodesists have used three different types of vertical datums, according to Vaníček (1991), namely (a) geoid, (b) quasigeoid, and (c) reference ellipsoid. All of these reference surfaces can be defined either globally or regionally, such that they approximate the entire Earth’s surface or some specified region, respectively. With no official global vertical datum definition, most countries or regions today use regional vertical datums as a local reference height system. This has resulted in over 100 regional vertical datums being used all over the world, Pan and Sjöberg (1998). The datums vary
due to different types of definitions, different methods of realizations and the fact that they are based on local/regional data. A common approach for defining regional vertical datums is to average sea level observations over approximately 19 years, or more precisely, ~18.6 years, which corresponds to the longest tidal component period, Melchior (1978), Smith (1999), for one or more fundamental tide gauge. This average sea level value is known as mean sea level (MSL) and is used because it was assumed that the geoid and MSL coincided (more or less). This assumption is obviously false, as it is known today that the MSL and the geoid differ by approximately ± 2 meters, see Klees and van Gelderen (1997). Also, the geoid is by definition an equipotential surface, whereas MSL is not, due to numerous meteorological, hydrological, and oceanographic effects, Groten and Müller (1991). This discrepancy between MSL and the geoid is known as mean dynamic sea surface topography, hereinafter denoted by MSST. With the current demands for a cm-level accurate vertical datum, the discrepancy between the geoid and MSL cannot be ignored.

Figure 7 depicts a typical scenario for the establishment of a reference benchmark to define a regional vertical datum. The tide gauge records the instantaneous sea level height \( H_{\text{ISL}} \) and these values are averaged over a long term in order to obtain the mean value of the local sea level \( H_{\text{MSL}} \). The height of the tide gauge is also measured with respect to a reference benchmark that is situated on land a short distance from the tide gauge station. Then the height of the reference benchmark above mean sea level \( H_{BM} \) is computed by:

\[
H_{BM} = H_{\text{MSL}} + \Delta H_{BM-TG}
\]

Levelling begins from this benchmark and reference heights are accumulated by measuring height differences along levelling lines. The accuracy of the reference benchmark height derived in this manner is dependent on the precision of the height difference \( \Delta H_{BM-TG} \) and the value for mean sea level \( H_{\text{MSL}} \). If one assumes that the value for mean sea level is computed over a sufficiently long period of time which averages out all tidal period components and any higher frequency effects such as currents, then the accuracy depends on \( \Delta H_{BM-TG} \).

![Figure 7: Establishment of a reference benchmark height](image-url)
For highly accurate heights as those needed for a cm-level vertical datum, the tide gauges cannot be assumed to be vertically stable. It is well known that land motion at tide gauges is a source of systematic error, which causes distortion in the height network if it is not corrected for. Land motion at tide gauges and reference benchmarks may be caused by earthquakes or by erosion or other changes such as post-glacial rebound and land subsidence. The solution to this problem is to include an independent space-based geodetic technique such as GPS (or DORIS, GLONASS and in the future GALILEO) in order to estimate the land motion at these tide gauges. However, at this point in time, there are still too few measurements available at tide gauges to provide an accurate assessment of the global situation, Mitchum (2000).

As new methodologies and techniques evolve to the point where cm-level (and even subcm-level) accurate coordinates are needed, the distortions in traditionally-defined regional vertical networks are no longer acceptable. With this in mind, five main approaches have been identified by Vaníček (1991) for the realization of a “modern” regional vertical datum. These options are summarized below, with some additional remarks, see also Kearsley et al. (1993).

(i) Define the geoid by mean sea level as measured by a network of reference tide gauges situated along the coastlines of the country and fix the datum to zero at these stations. As stated previously, this approach will result in distorted heights as MSL is not an equipotential surface and it varies from the geoid on the order of a few meters. Also, by fixing the datum to zero at these tide gauge stations, one is assuming that the gauge measurements are errorless or any error inherent in the measurements is acceptable. This is also a boldly incorrect assumption.

(ii) Define the vertical datum by performing a free-network adjustment where only one point is held fixed. A correction factor (shift) is applied to the resulting heights from the adjustment so that the mean height of all tide gauges equals zero. This modified version of option (i) above relies heavily on the measurements from a single tide gauge, while ignoring the observations for MSL made at all other stations.

(iii) Use the best model available to estimate sea surface topography at the tide gauge stations and then adjust the network by holding MSL-MSST to zero for all tide gauges. This approach does eliminate most of the shortcomings identified in options (i) and (ii) above, however there are some practical limitations in terms of accuracy. Tide gauges are situated near coastal areas and even with the use of satellite altimetry, which has revolutionized sea surface observations and greatly improved SST models in open oceans, the performance in coastal areas is still quite poor. Global ocean circulation models derived from satellite altimetry data and hydrostatic models may reach accuracies of 2-3 cm in the open oceans, but the models fall apart in shallow coastal areas giving uncertainties on the order of tens of centimeters, Shum et al. (1997). Therefore, with significant problems still looming in the coastal regions, distortions will be evident in heights referred to a vertical datum that is defined with low accuracy SST models.

(iv) Define the vertical datum in the same manner as option (iii), but allow the reference tide gauges to ‘float’ in the adjustment by assigning them realistic a-priori weights (estimates of errors). This approach can incorporate all of the information for MSL and SST at the reference tide gauges. With improvements in models obtained from satellite altimetry and a better understanding of the process of tide gauge observations...
(e.g., reference benchmark stability, changes in position), estimates of the accuracy of the observations can be made.

(v) As in option (iv), but use estimates of orthometric heights from satellite-based geodetic heights and precise gravimetric geoidal heights. One of the main advantages of this approach is that it relates the regional vertical datum to a global vertical reference surface (since the satellite-derived heights are referred to a global reference ellipsoid).

This aids in the realization of an internationally accepted World Height System (WHS) or global vertical datum, see Colombo (1980) and Balasubramania (1994).

3.2. GPS/levelling

The inherent appeal of the seemingly simple linear geometrical relationship between the three height types is based on the premise that given any two of the height types, the third can be derived through simple manipulation of equation (1.1). There are several issues hidden within this statement that will be uncovered in the sequel (e.g., datum inconsistencies, systematic errors, data accuracy). The optimal combination of GPS-derived geodetic heights with gravimetrically-derived height anomalies for the determination of normal heights above mean sea level or more precisely with respect to a vertical geodetic datum is referred to as GPS/levelling. The process can be described as follows for the absolute case:

\[ H' = h - \zeta \]

This procedure has been the topic of several studies over the years, see for e.g., Engelis et al. (1985), Forsberg and Madsen (1990), and Sideris et al. (1992) and demonstrated to provide a viable alternative over conventional levelling methods for lower-order survey requirements. A major limitation of using GPS/levelling as a means for establishing heights with respect to a local vertical datum is that it is dependant on the achievable accuracy of the geodetic and height anomaly data. In practice, the relationship given by equation (3.2) is never fulfilled due to datum inconsistencies, systematic errors and distortions inherent among the triplet of height data, Jiang and Duquenne (1995), van Onselen (1997), Ollikainen (1997), Fotopoulos et al. (2001a). Thus, a more rigorous treatment for the integration of these different height types requires the incorporation of a parametric corrector surface model in equation (3.2). The role of such a model is to absorb the datum inconsistencies, any systematic errors and distortions that exist in the height data sets, Shrestha et al. (1993). More details are provided in the next chapters.

In practice, the GPS/levelling technique has become quite common and used often erroneously or with a poor understanding of the transformations between reference surfaces and systematic errors involved. As accuracy requirements increase, the incorrect application of equation (3.2) has more severe implications. Therefore, it is important to use a proper procedure for combining the heterogeneous height.
4 Combined Height Adjustment and Modelling of Systematic Effects

In this chapter, the algorithms and methodology used in the combined adjustment of the geodetic, normal heights and height anomaly data is presented. The first section describes the combined least-squares adjustment scheme for absolute height input values. The unknown parameters solved for in this adjustment are the ‘coefficients’ of some parametric model selected for dealing with the systematic errors and datum inconsistencies inherent among the heterogeneous height data. The main form of the parametric model is also provided. Finally, the procedure for assessing the parametric model performance is described.

4.1. General combined adjustment scheme

In this section, a description of the observation equations and mathematical models used for the multi-data adjustment of geodetic, normal heights and height anomalies is provided. The formulation herein forms the basis for all of the results presented in the remaining sections, with particular emphasis placed on the role of the systematic and random errors inherent among the heterogeneous height types. It should be recognized that there are a number of options available for combining these height data, see, e.g., Kearsley et al. (1993), Kotsakis and Sideris (1999), and Dinter et al. (2001). The algorithm used herein is an amalgamation of the procedures described in Kearsley et al. (1993), de Bruijne et al. (1997), Kotsakis and Sideris (1999), and others and it has been implemented and tested extensively using real data sets, see, e.g., Fotopoulos et al. (2001a,b). It was selected as the most appropriate adjustment scheme as it offers a practical solution to the problem given the data currently available.

4.1.1. Multi-data adjustment using absolute height data

In this section, the observation equation formulations are provided for the absolute case where the ‘observed’ input data to the adjustment are $h_i$, $H_i^*$ and $\zeta_i$ for a common set of co-located GPS/levelling benchmarks over a network area. Thus, the following discussion will focus on the typical scenario of a triplet of height information at each control point in the network where the observation equation for each station in the network is given, see Fotopoulos, (2003):

\[(4.1) \quad l_i = f_i + v_i^h - v_i^{H} - v_i^{\zeta} \]

where

\[(4.2) \quad l_i = h_i - H_i^* - \zeta_i \]
and \( i = 1, 2, \ldots, m \), where \( m \) is the number GPS/levelling points used in the combined adjustment. The \( v_h^i, v_{H^*}^i, v_\zeta^i \) terms are the random errors that correspond to \( h, H^*, \zeta \) at each GPS/levelling point. The \( f_i \) term in equation (4.1) refers to the total (combined) correction term for the systematic errors and datum inconsistencies in the multi-data test network and can be modeled according to a deterministic parametric form:

\[
f_i = a_i^T x
\]

In the absolute case, the covariance matrices are denoted by:

\[
E\{v_h v_h^T\} = C_h
\]

\[
E\{v_{H^*} v_{H^*}^T\} = C_{H^*}
\]

\[
E\{v_\zeta v_\zeta^T\} = C_\zeta
\]

where \( v_h, v_{H^*}, \) and \( v_\zeta \) are the vectors of \( m \times 1 \) dimension that contain the unknown random errors for each height type:

\[
v_i = [v_1^i, v_2^i, \ldots, v_m^i]^T \quad \text{for} \quad i = h, H^*, \zeta
\]

The general linear functional model used for the multi-data (combined) adjustment of the heterogeneous height data described above is given as follows:

\[
l = Ax + Bv
\]

\[
E\{v\} = 0
\]

\[
E\{vv^T\} = C_v
\]

where the \( m \times 1 \) vector of observations \( l \) is composed of the height ‘misclosure’ at each GPS/levelling benchmark as given in equation (4.2). The \( u \times m \) design matrix, \( A \), depends on the parametric model type (see Section 4.3), where \( u \) is the number of transformation parameters used in the model. \( B \) is the block-structured matrix denoted by:

\[
B = [I \quad -I \quad -I]
\]

where each \( I \) is an \( m \times m \) unit matrix. \( x \) is a \( u \times 1 \) vector containing the unknown parameters corresponding to the selected parametric model and \( v \) is a vector of random errors with zero mean [equation (4.9)], described by the formula:
where \( v(\cdot) \) is an \( m \times 1 \) vector of random errors for each of the \( h, H^*, \) and \( \zeta \) data types. The corresponding covariance matrix is described in general by equation (4.10).

Applying the least-squares minimization principle of:

\[
\begin{align*}
\min v^T P v &= v_h^T P_h v_h + v_{H^*}^T P_{H^*} v_{H^*} + v_{\zeta}^T P_{\zeta} v_{\zeta} = \min
\end{align*}
\]

where the block diagonal weight matrix \( P \) is:

\[
P = \begin{bmatrix}
P_h & 0 & 0 \\
0 & P_{H^*} & 0 \\
0 & 0 & P_{\zeta}
\end{bmatrix} = \begin{bmatrix}
C_h^{-1} & 0 & 0 \\
0 & C_{H^*}^{-1} & 0 \\
0 & 0 & C_{\zeta}^{-1}
\end{bmatrix}
\]

one can solve for the unknown parameters (i.e., coefficients) of the corrector surface model by:

\[
\hat{x} = [A^T (C_h + C_{H^*} + C_{\zeta})^{-1} A]^{-1} A^T (C_h + C_{H^*} + C_{\zeta})^{-1} l
\]

The combined adjusted residuals from the adjustment are given by:

\[
B\hat{v} = \hat{v}_h - \hat{v}_{H^*} - \hat{v}_{\zeta}
\]

where we can explicitly solve for the separate adjusted residuals, according to height data type, by applying the well known formulation, Mikhail (1976):

\[
\hat{v} = P^{-1} B^T (B P^{-1} B^T) (w - A\hat{x})
\]

where \( w = l \), and this is also shown in Kotsakis and Sideris (1999) as follows:

\[
\begin{align*}
\hat{v}_h &= C_h (C_h + C_{H^*} + C_{\zeta})^{-1} M l \\
\hat{v}_{H^*} &= C_{H^*} (C_h + C_{H^*} + C_{\zeta})^{-1} M l \\
\hat{v}_{\zeta} &= C_{\zeta} (C_h + C_{H^*} + C_{\zeta})^{-1} M l
\end{align*}
\]

where the \( M \) matrix is expressed by:

\[
M = I - A (A^T (C_h + C_{H^*} + C_{\zeta})^{-1} A)^{-1} A^T (C_h + C_{H^*} + C_{\zeta})^{-1}
\]
The accuracy of the corrector surface parameters can be computed by

\[
C_{\bar{\delta}} = \left[A^T \left(C_h + C_{H^r} + C_{\xi} \right)^{-1} A \right]^{-1} = \left[A^T C^{-1} A \right]^{-1}
\]

This formulation provides us with the interesting opportunity to evaluate the contribution of each of the height types through the evaluation of \(C_{\bar{\delta}}\), which can also be represented by the following expression:

\[
C_{\bar{\delta}} = \begin{bmatrix}
C_h & 0 & 0 \\
0 & C_{H^r} & 0 \\
0 & 0 & C_{\xi}
\end{bmatrix} = \begin{bmatrix}
\sigma^2_{hQ_h} & 0 & 0 \\
0 & \sigma^2_{H^rQ_{H^r}} & 0 \\
0 & 0 & \sigma^2_{\xiQ_{\xi}}
\end{bmatrix}
\]

The discussion of the stochastic model is given in Section 5.3.

**4.1.2. Note on an alternative formulation of the problem**

An alternative, yet equivalent formulation of the problem can be stated by replacing the functional model in equation (4.8) with (see Fotopoulos 2003):

\[
l = A x + v^*, \quad E\{v^*\} = 0
\]

where \(v^*\):

\[
v^* = B v = v_h - v_{H^r} - v_{\xi}
\]

It can easily be shown that the final equations corresponding to this alternative formulation are identical to those derived thus far. The unique perspective obtained by implementing this combined adjustment approach as described herein is embedded in two main areas; see Fotopoulos (2003):

- the evaluation of the contribution of the \(f_i = a_i^T x\) term, which refers to the total (combined) correction term for the systematic errors and datum inconsistencies in the multi-data test network, and
- the separation of residuals according to the height data types, which allows for the refinement of data covariance matrices.

Numerous studies have been conducted which focused on the first issue. Despite these efforts, a consistent approach for implementing and assessing the model performance has not been widely proposed or accepted. The concept of the multi-data one-dimensional adjustment should not be trivialized, as there are many sources of systematic and random errors involved that have to be dealt with properly in order to rigorously combine all of
the data and obtain meaningful and optimal (in the least squares sense) results. This problem will not only be addressed in the sequel from a theoretical point of view, but it will also be approached from the practical point of view, where real-world data limitations are taken into account.

4.2. Role of the parametric model

The main factors that cause discrepancies when combining the heterogeneous heights include the following (Schwarz et al. 1987, Rummel and Teunissen 1989; Kearsley et al. 1993):

- Random errors in the derived heights $h$, $H^*$, and $\zeta$

The covariance matrices for each of the height types are usually obtained from separate network adjustments of the individual height types. The main errors affecting the data were described in Chapter 2.

- Datum inconsistencies inherent among the height types

Each of the triplets of height data refers to a different reference surface. For instance, GPS-derived heights refer to a reference ellipsoid used to determine the satellite orbits. Normal heights, computed from levelling, refer to a local vertical datum, which is usually defined by fixing one or more tide-gauge stations (see discussion in Section 3.1).

- Systematic effects and distortions in the height data

These systematic effects have been described in Chapter 2 and are mainly caused by the long wavelength quasigeoid errors, which are usually attributed to the global geopotential model (i.e., EGM96, EIGEN-CG03C). Biases are also introduced into the gravimetric geoid model due to differences between data sources whose adopted reference systems are slightly different. In addition, systematic effects are also contained in the geodetic heights, which are a result of poorly modelled GPS errors, such as atmospheric refraction (especially tropospheric errors). Although spirit-levelled height differences are usually quite precise, the derived normal heights for a region or nation are usually the result of an over-constrained levelling network adjustment, which introduces distortions.

- Assumptions and theoretical approximations made in processing observed data

Common approximations include neglecting sea surface topography (SST) effects or river discharge corrections for measured tide gauge values, which results in a significant deviation of readings from mean sea level. Other factors include the use of approximations or inexact normal height corrections, Véronneau (2002). The computation of regional or continental quasigeoid models also suffers from insufficient approximations in the gravity field modelling method used, de Min (1990).

- Instability of reference station monuments over time

Temporal deviations of control station coordinates can be attributed to geodynamic effects such as post-glacial rebound, see, e.g., de Bruijne et al. (1997), crustal motion and land subsidence.

The combined effects of all these factors and others result in poorly estimated height values and, more importantly, inaccurate assessments, if any, of the results achievable by GPS/levelling.
Thus far, the burden of dealing with most of these factors (mainly the systematic errors and datum inconsistencies) has been designated to the use of a corrector surface model.

The role of the parametric model in the GPS/levelling problem gives the theoretical relationship among the three types of height data and the incorporation of an appropriate corrector surface model, the normal height for a new point (not belonging to the original multi-data network) is obtained as follows:

\[(4.26) \quad H^* = h - \zeta - a^T \hat{x} \]

The vertical reference system to which the computed value \(H^*\) for a new point refer is illustrated Figure 8, which provides an illustrative view of the various reference surfaces embedded in the different height data sets.

**Figure 8:** Illustrative view of GPS/levelling and the role of the corrector surface

In this figure, the points on the Earth’s surface represented by solid circles belong to the multi-data control network and the point denoted by a triangle is the ‘new’ point for which the normal height is to be computed via GPS/levelling technique. For the sake of this discussion, if one ignores the systematic effects, and concentrate on the datum inconsistencies, one can see from the figure that the role of the corrector surface is twofold. In general, the datum discrepancies occur between

(i) the local vertical datum and the quasigeoid model (both of which are supposed to represent different surfaces) and

(ii) the two ellipsoids to which the GPS measurements and height anomalies refer to. These discrepancies are typically not constant biases as depicted in the figure, but they may take on a more complicated form.
In order to obtain the normal height through GPS/levelling technique that refers to the local vertical datum for the new point, \( H_k \), a connection between the different height surfaces must be made. This connection is embedded in the corrector term \( f \) [equation (4.3)] and can take on many forms, depending on the model selection.

It should be cautioned, however, that the corrector surface model will provide a consistent connection between the heights derived from GPS/levelling technique and the official local vertical datum, only if the normal heights used in the multi-data adjustment also refer to the official local vertical datum.

It is evident that the parametric model plays a major role in the combined height adjustment process, which highlights the importance of the following issues:

(i) selecting the appropriate type of model
(ii) selecting the extent/form of the model
(iii) assessing the performance of the chosen model

These three issues will be reviewed in the following section. And all these issues will be investigated with relevance to practical problems using real data.

4.3. Modelling options

In practice, the various wavelength errors in the gravity solution may be approximated by different kinds of functions in order to fit the quasigeoid to a set of GPS/levelling points through an integrated least squares (LS) adjustment. Several models can be used ranging from a simple linear regression to more complicated seven parameter similarity transformation model, Kotsakis and Sideris (1999).

The choice of the parametric form of the corrector surface model is not a trivial task. In fact, the list of potential candidates for the ‘corrector’ surface is extensive. Arguably, the selection process is arbitrary unless some physical reasoning can be applied to the discrepancies between the GPS-derived height anomalies \( \zeta_{GPS} \), and the height anomalies from the combined gravity field model \( \zeta_{GRACE} \), which fulfills

\[
I_i = h_i - H_i^* - \zeta_i = a_i^T x + v_i
\]

(4.27)

\[
\Delta \zeta = \zeta_{GPS} - \zeta_{GRACE} = h_i - H_i^* - \zeta_i = a_i^T x + v_i
\]

(4.28)

where \( x \) is a \( n \times 1 \) vector of unknown parameters, and \( a_i \) is a \( n \times 1 \) vector of known coefficients, and \( v_i \) denotes a residual random noise term. The standard deviation of adjusted values for the residuals \( v_i \) traditionally is taken as the external indication of quasigeoid model absolute accuracy.

The parametric model \( a_i^T x \) is supposed to describe the systematic errors and datum inconsistencies inherent in the different height data sets. Its type varies in form and complexity depending on a number of factors. In the past, researchers have often utilized a simple tilted plane-fit model, which in several cases has satisfied accuracy requirements. However, as the achievable accuracy of GPS and quasigeoid heights improves, the use of such a simple model may not be sufficient. The problem is further
complicated because selecting the proper model type depends on the data distribution, density and quality, which varies for each case.

The family of models based on the general 7-parameter similarity datum shift transformation, with the simplified classic 4-parameter model, Heiskanen and Moritz (1967), Chapter 5, given by:

\[
(4.29) \quad a_i^T x = x_1 + x_2 \cos \phi_i \cos \lambda_i + x_3 \cos \phi_i \sin \lambda_i + x_4 \sin \phi_i
\]

and

\[
(4.30) \quad \Delta \zeta = \Delta a + \Delta Y_0 \cos \phi_i \cos \lambda_i + \Delta Y_0 \cos \phi_i \sin \lambda_i + \Delta Z_0 \sin \phi_i + \nu_i
\]

where \( \Delta X_0, \Delta Y_0, \) and \( \Delta Z_0, \) are the shift parameters between two ‘parallel’ datums and \( \Delta a \) are the changes in semi-major axis of the corresponding ellipsoids, \( \phi_i, \lambda_i \) are the latitude and longitude, respectively, of the GPS/levelling points. The full form of the design matrix would be given as follows:

\[
A_{m \times 4} = \begin{bmatrix}
1 & \cos \phi_1 \cos \lambda_1 & \cos \phi_1 \sin \lambda_1 & \sin \phi_1 \\
\vdots & \vdots & \vdots & \vdots \\
1 & \cos \phi_{m-1} \cos \lambda_{m-1} & \cos \phi_{m-1} \sin \lambda_{m-1} & \sin \phi_{m-1} \\
1 & \cos \phi_m \cos \lambda_m & \cos \phi_m \sin \lambda_m & \sin \phi_m
\end{bmatrix}
\]

An extended version of the above model is given with the inclusion of a fifth parameter; see e.g. Duquenne et al. (1995), as follows:

\[
(4.32) \quad a_i^T x = x_1 + x_2 \cos \phi_i \cos \lambda_i + x_3 \cos \phi_i \sin \lambda_i + x_4 \sin \phi_i + x_5 \sin^2 \phi_i
\]

and

\[
(4.33) \quad \Delta \zeta = \Delta a + \Delta Y_0 \cos \phi_i \cos \lambda_i + \Delta Y_0 \cos \phi_i \sin \lambda_i + \Delta Z_0 \sin \phi_i + a \Delta f \sin^2 \phi_i + \nu_i
\]

where \( \Delta X_0, \Delta Y_0, \) and \( \Delta Z_0, \) are the shift parameters between two ‘parallel’ datums and \( \Delta f, \Delta a \) are the changes in flattening and semi-major axis of the corresponding ellipsoids.

It should be noted that the parameters from such a ‘datum shift transformation’ do not represent the true datum shift parameters (translations, rotations and scale) because other long-wavelength errors inherent in the data (such as those in the geoid heights) will be interpreted as tilts and be absorbed by the parameters to some degree. Recently, a more complicated form of the differential similarity transformation model was developed and tested in the Canadian region and is given by Kotsakis et al. (2001):
\[(4.34)\]
\[a_i^T x = x_1 \cos \phi_i \cos \lambda_i + x_2 \cos \phi_i \sin \lambda_i + x_3 \sin \phi_i + x_4 \left( \frac{\sin \phi_i \cos \phi_i \sin \lambda_i}{W_i} \right) +
\]
\[x_5 \left( \frac{\sin \phi_i \cos \phi_i \cos \lambda_i}{W_i} \right) + x_6 \left( \frac{1 - f^2 \sin^2 \phi_i}{W_i} \right) + x_7 \left( \frac{\sin^2 \phi_i}{W_i} \right)\]

where \( W_i = \sqrt{1 - e^2 \sin^2 \phi_i} \), \( e^2 \) is the eccentricity and \( f \) is the flattening of the reference ellipsoid and \( \phi_i, \lambda_i \) are the horizontal geodetic coordinates of the network.
5 Practical application of GPS/levelling network

The Moldavian GPS network represents today ~ 1000 (971 are used in this work) GPS stations of 0th, 1st, 2nd, and 3rd order with geodetic coordinates and known normal heights at the same points obtained from old geometric levelling of 2nd, 3rd, and 4th order (97%). All normal heights are given in the Baltic 1977 height system, which is adopted in Moldova Republic. In Figure 9 the GPS/levelling network with normal heights in the Baltic77 height system is illustrated.

**Figure 9**: GPS network: 1st order sites, 2nd order sites and 3rd order sites.

5.1. Description of numerical tests

The additional comparisons of the observed height anomalies $\zeta^{GPS}$ of the 971 GPS/levelling points scattered over Moldova (see Figure 9) with the height anomalies at the same points obtained from the global geopotential model EGM96 (360,360) $\zeta^{EGM96}$...
and the combined gravity field model GRACE (EIGEN-CG03C) \(\zeta^{GRACE}\) were made for the detection of gross-errors (due to antenna height etc.). The residuals/observations were formed by:

\[
a_i^T x = \zeta^{GPS} - \zeta^{EGM96} = h_i - H_i^* - \zeta_i^{EGM96}
\]

and

\[
a_i^T x = \zeta^{GPS} - \zeta^{GRACE} = h_i - H_i^* - \zeta_i^{GRACE}
\]

To remove any outliers a routine 3RMS test was applied. The 3RMS test is applied to the residuals obtained from equations (5.1) and (5.2), for these residuals is calculated the RMS and the outlier points are found those points for which the residuals are higher then the 3 * RMS. After rejection of 19 outliers that was found by applying the 3RMS test, 35 GPS/levelling points distributed uniformly over the area was taken as control points for validation purpose. The remaining residuals (of 917 GPS/levelling points) were used in a least-squares adjustment to solve for the unknown parameters \(x\) of the pre-selected corrector surface.

The absolute differences between the combined gravity field model GRACE (EIGEN-CG03C) and the nation-wide set of GPS/levelling points vary from -0.738 meters to 0.751 m. The statistics for the absolute differences between the global geopotential models: GRACE (EIGEN-CG03C), EGM96, and GPS-derived values at the GPS/levelling points are given in Table 6.

<table>
<thead>
<tr>
<th>Table 6: Statistics of Residuals Before Fit from GRACE and EGM96. Unit: [m].</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GRACE (EIGEN-CG03C)</strong></td>
</tr>
<tr>
<td>Data</td>
</tr>
<tr>
<td>Original (936 points)</td>
</tr>
<tr>
<td>After 3RMS test (917 points)</td>
</tr>
<tr>
<td><strong>EGM96</strong></td>
</tr>
<tr>
<td>Original (936 points)</td>
</tr>
<tr>
<td>After 3RMS test (917 points)</td>
</tr>
</tbody>
</table>

The final set of 917 GPS/levelling stations with known geodetic coordinates and the normal heights \(H^*\) given in the Baltic 1977 height system are used for the numerical applications in the following sections. Accuracy of chosen \(H^*\) is estimated from 4 cm to 8 cm. Figure 10 illustrates the distribution of initial GPS/levelling data and the additional 35 GPS/levelling control points adopted below for the independent comparison (due to their exclusion from data processing).
Figure 10: GPS/levelling network and control points: ▲ - 1<sup>st</sup> order sites; ● - 2<sup>nd</sup> order sites; ■ - 3<sup>rd</sup> order sites; ★ - control points.

For a more clear illustration of distribution of the 35 GPS/levelling control points that are used for validation purpose are shown in the Figure 11.

Figure 11: 35 GPS/levelling control points
The statistics of differences for these 35 GPS/levelling control points before fitting the model obtained from the combined gravity field model GRACE (EIGN-CG03C) and global geopotential model EGM96 are given in the Table 7.

Table 7: The statistics of differences for 35 GPS/levelling control points before fitting. Unit: [m].

<table>
<thead>
<tr>
<th>35 pts</th>
<th>GRACE</th>
<th>EGM96</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>-0.060</td>
<td>-0.235</td>
</tr>
<tr>
<td>RMS</td>
<td>0.238</td>
<td>0.335</td>
</tr>
<tr>
<td>St.Dev.</td>
<td>0.230</td>
<td>0.238</td>
</tr>
<tr>
<td>MAX</td>
<td>0.463</td>
<td>0.251</td>
</tr>
<tr>
<td>MIN</td>
<td>-0.436</td>
<td>-0.775</td>
</tr>
</tbody>
</table>

Further more, the GRACE solution is transformed via datum shift parameters (described in details in the Section 4.3) to the Baltic 1977 system. This modified GRACE height anomalies surrounded the Moldova area, are used as additional – supported information to get a better results along the Moldavian border.

The four, five and seven parameters datum shift transformation models and their estimates are determined. The values of parameters of these models are presented in Table 8:

Table 8: Values of estimated transformation parameters

<table>
<thead>
<tr>
<th>4 Parameters Transformation GRACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>Δa</td>
</tr>
<tr>
<td>ΔX(m)</td>
</tr>
<tr>
<td>ΔY(m)</td>
</tr>
<tr>
<td>ΔZ(m)</td>
</tr>
<tr>
<td>$\hat{\sigma}_0 [m]$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5 Parameters Transformation GRACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>Δa</td>
</tr>
<tr>
<td>ΔX(m)</td>
</tr>
<tr>
<td>ΔY(m)</td>
</tr>
<tr>
<td>ΔZ(m)</td>
</tr>
<tr>
<td>aΔf</td>
</tr>
<tr>
<td>$\hat{\sigma}_0 [m]$</td>
</tr>
</tbody>
</table>
It should be noted that the parameters from such a ‘datum shift transformation’ do not represent the true datum shift parameters (translations, rotations and scale) because other long-wavelength errors inherent in the data (such as those in the height anomalies) will be interpreted as tilts and be absorbed by the parameters to some degree.

According to our numerical results for these three models, the seven parameter model gives the best fitting with minimum standard deviation 15.5 cm compared to four and five that gives 19.5 cm, 17.6 cm correspondingly. It can also be seen when compare the estimate parameters with their standard errors, that all estimates are significant.

### 5.2. Assessing the parametric model performance

In general, the process applied for selecting the best parametric model suffers from a high degree of arbitrariness in both choosing the model type and in assessing its performance. In our case we apply classic empirical approach, to the results of the combined least-squares adjustment of the geodetic-normal-anomaly heights. This test will be described below. It is assumed throughout the process that any gross errors/blunders have been detected and removed from the observational data in order for the results to be meaningful.

The most common method used in practice to assess the performance of the selected parametric model(s) is to compute the statistics for the adjusted residuals after the least squares fit. The adjusted residuals for each station in the network, \( \hat{v}_i \), are computed as follows:

\[
\hat{v}_i = h_i - H_i^* - \zeta_i - a_i^T \hat{x}
\]

In Figure 12, this classic empirical approach is illustrated.
The model that results in the smallest set of residuals is deemed to be the most appropriate (‘best’ fit). Note should be done here that the reduction in the average value to zero is imposed by the least-squares adjustment. In effect, these values give an assessment of the precision of the model as they indicate how well the data sets fit each other.

In Table 9 are summarized respectively the statistics of the differences between the height anomaly determined by GPS/levelling and quasigeoidal heights after fitting out the systematic biases using a four, five and seven parameters transformation, and for each model of parameters transformation the individual components of adjusted GPS, levelling and combined gravity field model residuals are given.

**Table 9:** Results of classical empirical test for the 917 GPS/levelling points used in the combined adjustment. Unit: [m].

<table>
<thead>
<tr>
<th>GRACE</th>
<th>4 Paramtrs Transf Model</th>
<th>5 Paramtrs Transf Model</th>
<th>7K Paramtrs Transf Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_h$</td>
<td>$V_H$</td>
<td>$V_\zeta$</td>
</tr>
<tr>
<td>Mean</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Max</td>
<td>0.094</td>
<td>0.135</td>
<td>0.375</td>
</tr>
<tr>
<td>Min</td>
<td>-0.101</td>
<td>-0.145</td>
<td>-0.404</td>
</tr>
<tr>
<td>RMS</td>
<td>0.030</td>
<td>0.044</td>
<td>0.121</td>
</tr>
<tr>
<td>RMS</td>
<td>0.195</td>
<td>0.176</td>
<td>0.155</td>
</tr>
</tbody>
</table>

The comparison of the statistical results of 917 GPS/levelling points used in the adjustment, before fitting the combined gravity field model with these 917 GPS/levelling points and after four, five and seven parameters model fitting is given in Table 10.
Table 10: Statistical results of 917 GPS/levelling points used in the adjustment before and after fitting. Unit: [m].

<table>
<thead>
<tr>
<th></th>
<th>GRACE</th>
<th>4 Tr.P.</th>
<th>5 Tr.P.</th>
<th>7 Tr.P.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.096</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>RMS</td>
<td>0.233</td>
<td>0.195</td>
<td>0.176</td>
<td>0.155</td>
</tr>
<tr>
<td>St.Dev.</td>
<td>0.213</td>
<td>0.195</td>
<td>0.176</td>
<td>0.155</td>
</tr>
<tr>
<td>Max</td>
<td>0.751</td>
<td>0.604</td>
<td>0.629</td>
<td>0.742</td>
</tr>
<tr>
<td>Min</td>
<td>-0.738</td>
<td>-0.650</td>
<td>-0.612</td>
<td>-0.601</td>
</tr>
</tbody>
</table>

The first statistics show that a good fit between the quasigeoid and GPS/levelling has been reached. This provided proof that the combined adjustment can optimally fit the combined gravity field model to the GPS/levelling points in the least squares sense. Therefore, this method is valid for testing the precision of the model, but it should not be interpreted as the accuracy or the prediction capability of the model. This fact, we can see from our statistical results for 917 GPS/levelling points used in the adjustment (see Table 10) and the statistical results for 35 GPS/levelling control points and GRACE model using corrective surface (four, five and seven parameters model) (see Table 11).

Figure 13 depicts a typical series containing the original height misclosures for the 35 GPS/levelling control points (not included in the adjustment), as computed from equation (4.2) and the residuals after the seven parameters model fit [equation (5.3)].

Figure 13: Example of height misclosures before and after seven parameters model fit

The statistical results for 35 GPS/levelling control points before fitting and after the four, five and seven parameters fitting are given in Table 11.
Table 11: Statistical results for 35 GPS/levelling control points before and after (four, five and seven parameters model) fitting. Unit: [m].

<table>
<thead>
<tr>
<th></th>
<th>GRACE</th>
<th>4 P M</th>
<th>5 P M</th>
<th>7 P M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.060</td>
<td>0.018</td>
<td>0.003</td>
<td>0.005</td>
</tr>
<tr>
<td>RMS</td>
<td>0.238</td>
<td>0.205</td>
<td>0.177</td>
<td>0.141</td>
</tr>
<tr>
<td>St.Dev.</td>
<td>0.230</td>
<td>0.204</td>
<td>0.177</td>
<td>0.141</td>
</tr>
<tr>
<td>MAX</td>
<td>0.463</td>
<td>0.436</td>
<td>0.397</td>
<td>0.304</td>
</tr>
<tr>
<td>MIN</td>
<td>-0.436</td>
<td>-0.523</td>
<td>-0.373</td>
<td>-0.326</td>
</tr>
</tbody>
</table>

The model performance was assessed based on a combination of measures of precision and accuracy. The latter is particularly important for applying the parametric models to independent/new points. In our case the models with the lowest RMS for the adjusted residuals were the most accurate. The best model on a national level was found to be the classic seven-parameter fit with an RMS of approximately 14 cm.

5.3. Variance Component Estimation

One of the key elements required for the least squares adjustment of geodetic data, in addition to proper modelling of systematic effects, is a realistic stochastic model for the observational noise. Although the effect of the data covariance matrix may not be readily evident in the solution of the unknown parameters, see Marana and Sanso (1996), for a detailed discussion, a poor covariance matrix may adversely affect decisions based upon statistical testing of hypotheses involving least-squares residuals and the estimated parameters. Additional reasons for using a correct covariance matrix for the observational noise include the examination of the relative magnitudes of the errors in observations due to different factors, the preservation of quality control and the facilitation of efficiently designed surveys, Rao and Kleffe (1988).

Often, in practice, the data covariance matrix is oversimplified or neglected entirely. In order to provide a better understanding of the inherent limitation of a simplified stochastic model, it is instructive to go through an example with one of the most common functional models used within the context of least-squares adjustment. The general Gauss-Markov model, which was described in Chapter 4, is repeated here for the sake of discussion:

\[
\begin{align*}
\mathbf{l} & = \mathbf{Ax} + \mathbf{v} \\
E\{\mathbf{v}\} & = 0
\end{align*}
\]

where, \( \mathbf{l} \) is an \( m \times 1 \) vector of observations, \( \mathbf{x} \) is a \( u \times 1 \) vector of unknown parameters, \( \mathbf{A} \) is an \( m \times u \) known design matrix that relates the unknown parameters with the observations and \( \mathbf{v} \) is a vector of random errors with zero mean:

\[
(5.5) \quad E\{\mathbf{v}\} = 0
\]

and a variance-covariance matrix given by:
\[ C_v = E[(l - Ax)(l - Ax)^T] = E[\nu \nu^T] = \sigma_0^2 Q_v \]

where \( Q_v \) is a known symmetric positive definite cofactor matrix, in our case this matrix will be replaced by the identity matrix, \( I \), see Kubic (1970), for more details, and \( \sigma_0^2 \) is an unknown variance factor. Such a simplified, in terms of computation, stochastic model is used extensively in many applications, including Sjöberg (1984), Caspary (1987) and Fotopoulos and Sideris (2003).

Additional simplifications can be made for uncorrelated sets of observations of one variance component each and is described by:

\[ C_v = diag[\sigma_i^2 I] = \begin{bmatrix} \sigma_1^2 I & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sigma_n^2 I \end{bmatrix} \]

This block diagonal form of stochastic model corresponds to vectors of observations that are commonly referred to as disjunctive, meaning that they are completely uncorrelated between groups. The a-posteriori variance factor can be estimated from the adjusted results according to the formula:

\[ \hat{\sigma}_0^2 = \frac{\hat{\nu}^T P \hat{\nu}}{m - u} \]

where \( m \), is the number of points, \( u \), is the number of transformation parameters, the adjusted least-squares residuals are denoted by the \( m \times 1 \) vector \( \hat{\nu} \), and \( P = C_v^{-1} \) is the data weight matrix.

This classic, and somewhat simplified approach, for the stochastic model is limiting as it allows for only one common variance factor of the CV matrix. This may be adequate for adjustments of observations of the same type and similar quality. However, it is not realistic for a variety of geodetic applications, where heterogeneous data types are involved. A more general treatment of the stochastic model includes the ability to estimate more than one variance and/or covariance components to improve the CV information, see Sjöberg (1984), as given by:

\[ \hat{\sigma}_h^2 = \frac{\hat{\nu}_h^T P_h \hat{\nu}_h}{(m - u) \ast (\mu / \mu_h)} \]

\[ \hat{\sigma}_{h'}^2 = \frac{\hat{\nu}_{h'}^T P_{h'} \hat{\nu}_{h'}}{(m - u) \ast (\mu / \mu_{h'})} \]

\[ \hat{\sigma}_\zeta^2 = \frac{\hat{\nu}_\zeta^T P_\zeta \hat{\nu}_\zeta}{(m - u) \ast (\mu / \mu_\zeta)} \]
where \( \mu = \sum_{i} \mu_{i} \), for \( i = h, H^*, \zeta \), and \( \mu_{i} \) is the number of points used in the combined adjustment. Given the estimated a-posteriori variance factors in equations (5.9)-(5.11), the data covariance matrix can finally be ‘improved’ by a simple scaling:

\[
C_{i}^{\text{improved}} = [\hat{\sigma}_{i}^{2} I] \quad \text{where} \quad i = h, H^*, \zeta
\]

So, for each estimated a-posteriori variance component in equations (5.9)-(5.11) we get an ‘improved’ variance-covariance matrix:

\[
C_{h}^{\text{improved}} = [\hat{\sigma}_{h}^{2} I]
\]

\[
C_{H^*}^{\text{improved}} = [\hat{\sigma}_{H^*}^{2} I]
\]

\[
C_{\zeta}^{\text{improved}} = [\hat{\sigma}_{\zeta}^{2} I]
\]

In our case, from 971 GPS/levelling points, after 19 rejections, and the other 35 GPS/levelling points excluded from the combined adjustment in order to estimate the real accuracy given by the comparison between the adjusted values and the known ones, the remaining 917 GPS/levelling points are used in combined adjustment. Moreover, and since the variance-covariance matrix of the GPS and levelling networks adjustments necessary for this kind of combined adjustment are not available, we have used the a priori uniform accuracy of the geodetic, normal heights fixed to 5 cm, 6 cm and for the combined gravity field model we have used a unit weight matrix. After the combined adjustment we get a posteriori values (the ‘improved’ variance-covariance matrix), the standard errors of each height type are given in Table 12.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model</th>
<th>4 Parameters Model</th>
<th>5 Parameters Model</th>
<th>7 Parameters Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{h} )</td>
<td>a priori value (m)</td>
<td>0.05</td>
<td>0.053</td>
<td>0.047</td>
</tr>
<tr>
<td>( \sigma_{H^*} )</td>
<td>a posteriori value (m)</td>
<td>0.06</td>
<td>0.075</td>
<td>0.068</td>
</tr>
<tr>
<td>( \sigma_{\zeta} )</td>
<td></td>
<td>1</td>
<td>0.210</td>
<td>0.190</td>
</tr>
</tbody>
</table>

5.4. GPS-quasigeoid Modelling
We can create a continuous surface from GPS/levelling data by some interpolation (or prediction) method, the choice of which is crucial for the result. A comprehensive compilation of different methods can be found in Watson (1992) and Burrough and McDonnell (1998). Some studies from recent years emphasising this fact can be mentioned, like Kotsakis and Sideris (1999) and Lee and Mezera (2000).

For creating a continuous surface, different interpolation techniques were tested using SURFER software (Golden Software, Inc.) Table 13 presents the statistics of differences between new quasigeoid surfaces by using different interpolation methods, versus 35 GPS/levelling control points. Among them the Minimum curvature, Natural Neighbour and Kriging give the minimum RMS values but the Kriging method also gives the lowest noise level (the difference between maximum and minimum values). Thus, we choose the Kriging method for creating continuous surface in this study.

Table 13: Results for the 35 GPS/levelling control points using the continuous surfaces (based on different interpolation techniques). Unit: [m].

<table>
<thead>
<tr>
<th>Interpolation method</th>
<th>Modified Shepard’s Method</th>
<th>Kriging</th>
<th>Natural Neighbor</th>
<th>Minimum Curvature</th>
<th>Nearest Neighbor</th>
<th>Inverse Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.046</td>
<td>-0.003</td>
<td>-0.004</td>
<td>0.002</td>
<td>-0.004</td>
<td>-0.001</td>
</tr>
<tr>
<td><strong>RMS</strong></td>
<td>0.241</td>
<td>0.052</td>
<td>0.056</td>
<td>0.058</td>
<td>0.072</td>
<td>0.073</td>
</tr>
<tr>
<td><strong>St.Dev.</strong></td>
<td>0.237</td>
<td>0.051</td>
<td>0.056</td>
<td>0.058</td>
<td>0.072</td>
<td>0.073</td>
</tr>
<tr>
<td><strong>Max</strong></td>
<td>1.314</td>
<td>0.088</td>
<td>0.128</td>
<td>0.148</td>
<td>0.175</td>
<td>0.143</td>
</tr>
<tr>
<td><strong>Min</strong></td>
<td>-0.181</td>
<td>-0.129</td>
<td>-0.133</td>
<td>-0.102</td>
<td>-0.230</td>
<td>-0.196</td>
</tr>
</tbody>
</table>

Kriging is a geostatistical approach to interpolate data based upon spatial variance and has proven useful and popular in many fields in geodesy as well, Matheron (1963). A thorough textbook on applied geostatistics is Isaaks and Srivastava (1989), on kriging in geographical information systems (GIS) in particular is Burrough and McDonnell (1998) and on kriging in general is Stein (1999). The reason that we choose Kriging method instead of least squares collocation, Moritz (1973), lies in the availability of software and as kriging has become an extremely important interpolation tool in GIS and, as such, has had give a lot of attention from scientists and software producers. Software like ArcInfo (ESRI Inc.) and Surfer (Golden Software, Inc.), for instance, have excellent modules dealing with kriging. Both kriging and least squares collocation are generalized estimation methods combining the behaviour of a systematic part and two random parts. A difference between kriging and least squares collocation lies in the treatment of the covariance structure of the random field, where in kriging semivariograms are used, Blais (1982, p.327), to estimate weights and in least squares collocation covariance functions, Moritz (1973, p.26).

In the combination of the combined gravity field model and corrective surface, it is necessary that they have the same grid size (in rows and columns). Table 14 and Figure 14 show the results of this comparison.
Table 14: Validation of the GRACE and new quasigeoid surface MOLDGEO2005, for 35 GPS/levelling control points. Unit: [m].

<table>
<thead>
<tr>
<th>Pts 35</th>
<th>GRACE</th>
<th>7 Parameters Model</th>
<th>Kriging method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.060</td>
<td>0.005</td>
<td>-0.003</td>
</tr>
<tr>
<td>RMS</td>
<td>0.238</td>
<td>0.141</td>
<td>0.052</td>
</tr>
<tr>
<td>St.Dev.</td>
<td>0.230</td>
<td>0.141</td>
<td>0.051</td>
</tr>
<tr>
<td>Max</td>
<td>0.463</td>
<td>0.304</td>
<td>0.088</td>
</tr>
<tr>
<td>Min</td>
<td>-0.436</td>
<td>-0.326</td>
<td>-0.129</td>
</tr>
</tbody>
</table>

The standard deviation of the discrepancies between the quasigeoid model and GPS/levelling height anomalies amounts to $\pm$ 5cm after fitting. This fact confirms the good fit reached in the test area between the Moldavian quasigeoid and GPS/levelling control points.

Figure 14: Differences between the height anomalies from GRACE and the new quasigeoid surface over Moldova. Contour interval: 0.1m.
Figure 15: A new quasigeoid surface over Moldova. Contour interval: 0.1 m.

From Table 14 it is clear that, using the seven parameter fitting to the GRACE residuals the RMS (root mean square) for the 35 GPS/levelling control points, decreases from 0.238 m before fitting to 0.141 m after seven parameters fitting, but creating the continuous surface using the Kriging interpolation the RMS reduces to 0.052m. This is because it is able to account for the detailed undulations in the levelling that are not sampled by GRACE alone.

However, a warning should be attached to both these methods of corrective surface fitting: the ‘corrective’ surfaces also model any errors present in the GPS and levelling data, thus sometimes giving over-optimistic error estimates.
6 Conclusions and Recommendations

In this final chapter the conclusions and recommendations that can be drawn from this thesis work are outlined. Our starting point was the seemingly simple geometrical relationship between geodetic, normal and quasigeoid heights shown in Sections 1.3 and 2.2. Recognizing the profound impact that satellite-based measurement systems have had on the practice of geodesy and surveying, the combination of these heterogeneous height types was discussed. The following discussion provides a summary of the key findings regarding main areas that were identified as outstanding issues, namely modelling systematic errors and datum inconsistencies, separation of random errors and estimation of variance components for each height type. It should be mentioned that numerical results for Moldova’s network are included in the main text and only the major points will be repeated here.

6.1. Conclusions

Several issues were also studied that provided insight into the practical problems encountered when implementing GPS/levelling. The incorporation of the parametric model is absolutely essential in the common height adjustment as it absorbs the datum inconsistencies and systematic errors in the original control network. The classical empirical approach and statistical measures of goodness of fit for testing/assessing candidate parametric model performance of a vertical network of co-located GPS/levelling benchmarks was described.

The use of combined GPS/levelling/quasigeoid networks definitely provides a very attractive evaluation scheme for the accuracy of the combined gravity field model. At the same time, GPS/levelling/quasigeoid networks constitute the skeleton of ‘common points’ in the attempt to find optimal transformation models between GPS and normal heights. These are two different problems, which, nevertheless, can be attacked simultaneously through a unified adjustment setting.

The outcome of this thesis is a quasigeoid model over Moldova which gives the possibility to substitute classic spirit leveling with GPS/quasigeoid in order to derive normal heights. The data we have used in this thesis are: the global geopotential model EGM96 and the combined gravity field model GRACE (EIGEN-CG03C) to derive height anomalies, one set of 917 normal heights, and, finally, GPS derived geodetic heights at the corresponding benchmarks. The comparisons between the used global geopotential models indicate the superiority of EIGEN-CG03C over EGM96, as for 917 GPS/levelling points the RMS difference from EIGEN-CG03C is 0.233 cm but from EGM96 it is 0.327 cm. Then the application of the gravimetric GRACE solution is possible with better accuracy in the regional/local scale after the additional transformation of GRACE to the GPS/levelling network. The use of a corrective surface to combine the gravity field model EIGEN-CG03C and GPS/levelling, significantly improved the determination of heights as observed from GPS in the Moldova area. Such an approach can account for any differences between the combined gravity field model and the vertical datum in any area. For the minimization of the residuals four, five and seven parameter transformation
models were used, successfully managing to absorb the datum inconsistencies between the height data and GPS, leveling, and long-wavelength quasigeoid, errors. The overall best agreement, ±14cm, between the combined gravity field model EIGEN-CG03C and GPS/leveling heights was achieved when we used the seven-parameter similarity transformation model. The use of the seven-parameter model slightly improves the residuals compared to the four-parameter one, with 6 cm. However, it should be noted that the approach can give too optimistic estimates of accuracy as it also absorbs any errors committed during the GPS or levelling surveys.

Another issue of this thesis is the variance component estimation (VCE) related to the common adjustment of the geodetic, normal height and height anomaly data types. The VCE results verified that the geodetic heights were the most accurate of the three height data types. Overly optimistic values for the variance factors result because correlations among heights of the same type are neglected (i.e., diagonal-only CV matrices). The model assessment procedure, described in Section 5.2, was used to verify the inadequacy of these models for absorbing the systematic errors and datum inconsistencies in the test network area. Therefore, the intrinsic connection between the systematic effects and datum inconsistencies and the VCE process (which presupposes the absence of biases and no systematic effects in the data) is evident. Variance component estimation has been used as a useful statistical tool for computing and testing the actual GGM noise level.

Furthermore, the results reached the ±5 cm level for the quasigeoid solution versus 35 GPS/levelling control points, by kriging interpolation method, indicating the advantages of this latest quasigeoid solution. So our final quasigeoid solution is created, by kriging interpolation method, using GPS/leveling data within Moldova area and the seven parameters modified EIGEN-CG03C solution surrounding the Moldova area, in order to improve the model along the country border. The comparison of GPS/levelling heights with the corresponding quasigeoid values showed a reasonable agreement with RMS of 5 cm.

As the derived quasigeoid model is precise enough to set GPS/levelling heights, we believe this quasigeoid model could meet the requirement of many potential users who would intend to convert GPS heights into their corresponding normal heights.

6.2. Recommendations for future work

The following is a list of some of the areas recommended for future work:

The adjusted residuals were separated according to height type and used to estimate a single variance component for each group. It may be worthwhile, especially in larger disconnected networks, to estimate more than one variance component for each height type. A practical problem encountered by ‘secondary’ users of height data is the lack of covariance information, the need of reliable information for the statistical behaviour of the geodetic, normal heights and GGM it’s of importance in the combined adjustment of heterogeneous data type.

The problem of statistical testing for various hypotheses regarding the a priori accuracy information and the modelling choices in GPS/levelling/quasigeoid networks needs to be addressed in more detail, especially, in view of the many different levels of
accuracy desired by GPS/levelling users. In this direction, the problem of optimization and design of GPS/levelling/quasigeoid networks is another important and interesting topic that certainly needs to be explored.

To achieve a sub-cm-level accuracy with the GPS/levelling data, high resolution gravity data are needed. Overall, more sophisticated methods, which will take into account the a-priori accuracies of the different height data, need to be employed and further investigated.

Temporal variations of height datums are very important and must be considered in view of the goal for a 1-cm accurate quasigeoid. This involves the incorporation of time variable geophysical, geodynamic and oceanographic models to account for processes such as sea level change, post-glacial rebound, plate subduction and plate movement, land subsidence, etc. in the quasigeoid model. Essentially we engage in a bootstrap procedure where data from different time periods is used to test the validity of these time variable models while this data is also integrated to improve the models themselves.

Given the current state of technologies with emerging global navigation satellite systems (i.e., GALILEO) and modernized existing systems (GPS) as well as new LEO missions for dedicated gravity field research (i.e., CHAMP, GRACE, GOCE), it is expected that a new era in geodesy is upon us offering many challenges and promising resolutions.
7 References


Sanchez L and Sandoval P (Eds.), Vertical Reference Systems Symposium, Cartagena, Colombia, Feb. 20-23, pp. 81-90.


Stein ML (1999) Interpolation of spatial data: some theory for kriging. Springer-Verlag, New York


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3001 Elisabeth Steger. Framställning av informationsmaterial över Sonfjällets nationalpark. 1986.
3003 Per Westberg. TRANSFER - A Program to Translate and Transfer Data Concerning graphic Elements from ASCII Files to Intergraph Design Files. 1987.
3008 Martin Lidberg. FRIHÖJD - ett datorprogram för höjdbestämning vid fri uppsättning. 1988. (LMV)
3039 Krzysztof Gajdamowicz. A Mobile Road Collecting System Based on GPS and Video Cameras. 1994.
3065 Thomas Rehders. Noggrannhetsstudier vid RTK-mätning och kvalitetsundersökning av GPS-mottagare.


3081 Tomas Larsson. Undersökning av beräkningsmetoder för GPS. TRITA-INFRA EX 03-025, Mars 2003. (Handledare: Asenjo och Sjöberg)


3085 Åsa Eriksson. Adapting the calibration method of a total station to laser scanner GS100. TRITA-INFRA EX 04-008. Februari 2004. (Handledare: Horemuz)


