



KTH Electrical Engineering

Analysis of Electricity Markets

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March 2011

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Chapter 1

Basic electricity market modelling

1.1 Demand model

The load is represented by three segments. At low prices the load is price independent on the level \hat{D} MW. When the price increases over the level c_D SEK/MWh, then the load is price sensitive. The price sensitivity is b , which means that if the price increases with x SEK/MWh from the level c_D , then the load decreases with xb MW. But there is also a minimum load, \underline{D} MW, and at that level the load will not decrease further. There is though a maximum price, \hat{c}_D SEK/MWh and it is assumed that the consumers prefer to be disconnected than to consume at a higher price, i.e., this is the ceiling of what the consumers are prepared to pay. This level can also be referred to as the Value Of Lost Load - VOLL. A certain load level D MW corresponds to the the power price λ_D SEK/MWh.

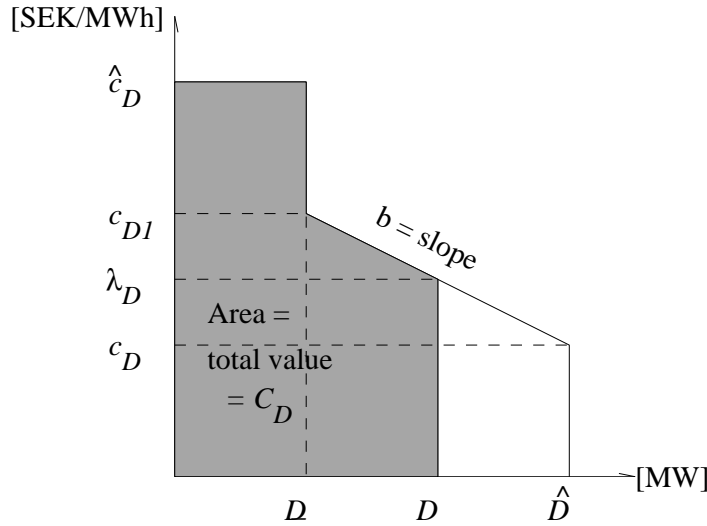


Figure 1.1. Calculation of total consumer value as a function of consumption level

The slope b can be expressed as:

$$b = \text{slope in MW}/[\text{SEK/MWh}] = \frac{\hat{D} - \underline{D}}{c_{D1} - c_D} = \frac{\hat{D} - D}{\lambda_D - c_D} \quad (1.1)$$

$$\Rightarrow c_{D1} = \frac{\hat{D} - \underline{D}}{b} + c_D \quad (1.2)$$

$$\lambda_D = \frac{\hat{D} - D}{b} + c_D \quad (1.3)$$

The consumer economic evaluation at a certain consumption level, $D \in [\underline{D}, \hat{D}]$, can now

be calculated as the shaded area in the figure:

$$\begin{aligned}
 C_D &= \hat{c}_D \underline{D} + (D - \underline{D}) \frac{c_{D1} + \lambda_D}{2} = \\
 &= \hat{c}_D \underline{D} + (D - \underline{D}) \left[\frac{\hat{D}}{b} - \frac{D + \underline{D}}{2b} + c_D \right] = \\
 &= k_{D0} + k_{D1}D + k_{D2}D^2
 \end{aligned} \tag{1.4}$$

where

$$k_{D0} = \underline{D} \left[\hat{c}_D - \frac{\hat{D}}{b} + \frac{\underline{D}}{2b} - c_D \right] \tag{1.5}$$

$$k_{D1} = \frac{\hat{D}}{b} + c_D \tag{1.6}$$

$$k_{D2} = -\frac{1}{2b} \tag{1.7}$$

With the assumption that all consumers have the same price, λ_D , the total cost for the consumers, C_{Dc} can be calculated as

$$C_{Dc} = \lambda_D D \tag{1.8}$$

The consumer surplus is the difference between the value of the consumption and the price for it. Figure 1.2 illustrates this.

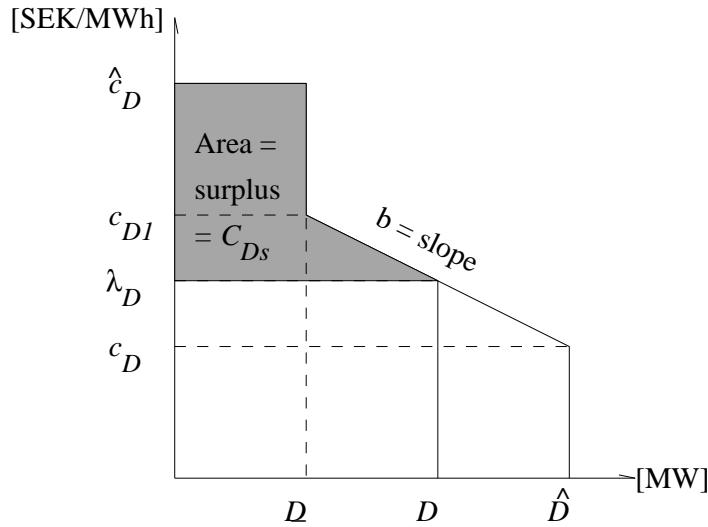


Figure 1.2. Calculation of total consumer surplus for a certain price

The consumer surplus, C_{Ds} for a certain price λ_D can be calculated as the difference between consumer value and consumer cost

$$\begin{aligned}
 C_{Ds} &= C_D - \lambda_D D = \\
 &= \begin{cases} \hat{c}_D \underline{D} + \frac{1}{2}(\hat{D} - \underline{D})(c_D + c_{D1}) - \lambda_D \hat{D} & \lambda_D \leq c_D \\ \underline{D}(\hat{c}_D - c_{D1}) + \frac{1}{2b}(D^2 - \underline{D}^2) & c_D \leq \lambda_D \leq c_{D1} \\ \underline{D}(\hat{c}_D - \lambda_D) & c_{D1} \leq \lambda_D \leq \hat{c}_D \end{cases}
 \end{aligned} \tag{1.9}$$

1.2 Production model

A production unit is represented with a production level, G_i , a maximum capacity \hat{G}_i , an operation cost level at minimum production, c_{Gi} , and a slope a reflecting changed cost at higher production level in the unit, c.f. figure 1.3.

The marginal cost, MC , (SEK/MWh), at the production level G_i can be calculated as

$$MC(G_i) = c_{Gi} + aG_i \quad (1.10)$$

The total cost for operation at level, G_i , i.e., the area below the curve can now be calculated as:

$$C_{Gi} = G_i \frac{c_{Gi} + MC(G_i)}{2} = c_{Gi}G_i + \frac{a}{2}G_i^2 \quad (1.11)$$

If there are several production units, then the total production cost, C_{Gtot} , is the sum of the costs in all units as

$$C_{Gtot} = \sum_{i \in I} \left[c_{Gi}G_i + \frac{a}{2}G_i^2 \right] \quad (1.12)$$

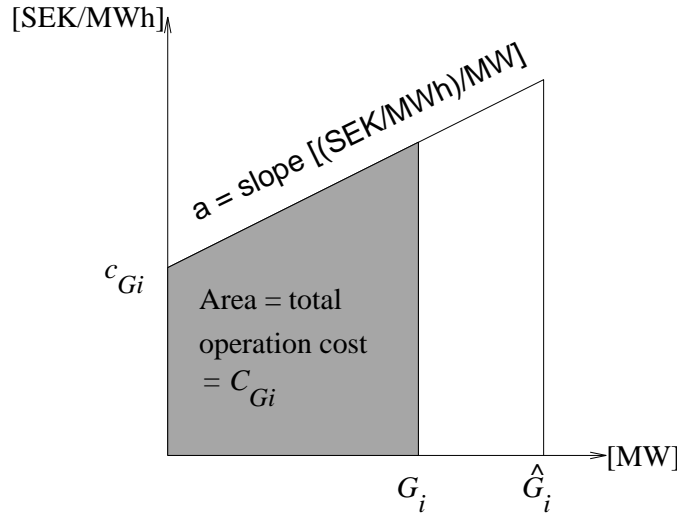


Figure 1.3. Calculation of total operation cost as a function of production level

With the assumption that all power is sold to the same price, λ_G , the total income for the producers, i.e., the total value of the production, C_{Gv} , can be calculated as

$$C_{Gv} = \lambda_G D = C_{Dc} \quad (1.13)$$

since there is the same price ($\lambda_G = \lambda_D$) and total production is equal to total load.

The producer surplus is the difference between the payment for the sold energy, C_{Gv} , and the operating cost. This is shown in figure 1.4.

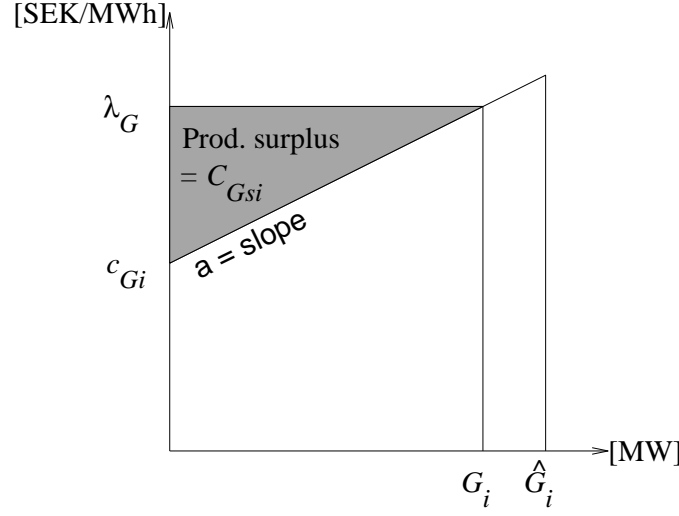


Figure 1.4. Calculation of total producer surplus as a function of production level and price

The producer surplus for unit i can now be calculated as the difference between the income and the operation cost, at the price level λ_G .

$$C_{Gsi} = \lambda_G G_i - C_{Gi} = \begin{cases} 0 & \lambda_G \leq c_{Gi} \\ \frac{a}{2} G_i^2 & c_{Gi} \leq \lambda_G \leq c_{Gi} + a \hat{G}_i \\ (\lambda_G - c_{Gi}) \hat{G}_i - \frac{a}{2} \hat{G}_i^2 & \lambda_D > c_{Gi} + a \hat{G}_i \end{cases} \quad (1.14)$$

1.3 The market function

With an assumption of a perfect market, the price should be on a level which maximizes the sum of consumer and producer surplus:

$$\max C_{Ds} + \sum_{i \in I} C_{Gsi} \quad (1.15)$$

Each unit has to produce within its limits:

$$0 \leq G_i \leq \hat{G}_i \quad (1.16)$$

and the total production has to be equal to the load:

$$\sum_{i \in I} G_i = D \quad (1.17)$$

With an assumption that the price is the same for the consumers and the producers ($\lambda = \lambda_G = \lambda_D$), then the total surplus can be calculated according to equations 1.9

and 1.14 as

$$\begin{aligned}
 C_{Ds} + C_{Gs} &= C_D - \lambda_D D + \sum_{i \in I} (\lambda_G G_i - C_{Gi}) = \\
 &= C_D - \sum_{i \in I} C_{Gi} + \lambda \underbrace{\left(\sum_{i \in I} G_i - D \right)}_{=0 \text{ (eq. 1.17)}} = C_D - \sum_{i \in I} C_{Gi} \quad (1.18)
 \end{aligned}$$

i.e. the total surplus is the total consumer value minus total production cost.

Example 1.1 Assume that there are two power stations located in one area. The data for the power stations are shown in the table. The load is 310 MW, independent of the

Unit	\hat{G}_{in}	c_{Gin}	a
i=1	150 MW	200 SEK/MWh	0,2 SEK/MWh/MW
i=2	250 MW	300 SEK/MWh	0,2 SEK/MWh/MW

Table 1.1. Data for example 1.1

price. Calculate the price, the production in each unit per hour, and total operation cost

Graphic solution: Draw the supply curve, i.e., the operation cost of the different units as a function of production, where the units are placed after each other according to increasing marginal operation cost. This is shown in figure 1.5. The load curve is then drawn as a vertical line.

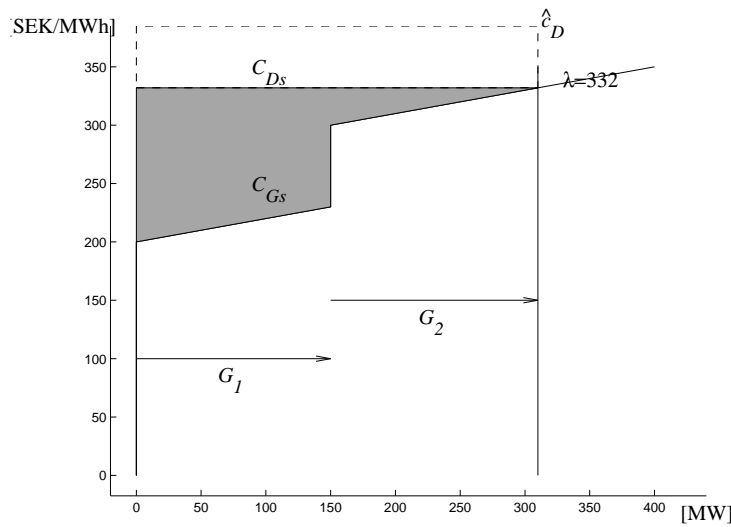


Figure 1.5. Example 1.1

The first unit will be fully used and the production in the second unit is then 310-150=160 MW. The price, λ , is set by the price cross between the marginal price of the last unit and the demand. If the price is higher than this level, then it is profitable for the producer to produce more. If the price on the other hand, is lower, then the operation costs of the last production will not be covered.

The price can be calculated with equation 1.10 as the marginal cost of the last unit when it produces so much so the total production covers the load.

$$\lambda = MC(G_2) = 300 + 0.2 \cdot 160 = 332 \text{ SEK/MWh}$$

The operation cost of each unit is the area below the curve. The not last used unit (only unit 1 in this example) can be identified in the figure and they produce as much as possible. The last unit produce up to the load level. The operation cost in the two units can be calculated according to equation 1.11 as

$$\begin{aligned} C_{G1} &= 200 \cdot 150 + \frac{0.2}{2} 150^2 = 32250 \text{ SEK/h} \\ C_{G2} &= 300 \cdot 160 + \frac{0.2}{2} 160^2 = 50560 \text{ SEK/h} \end{aligned}$$

The total operation cost is the sum of the production cost in the two units:

$$C_{Gtot} = \sum_{i \in I} C_{Gi} = C_{G1} + C_{G2} = 82810 \text{ SEK/h}$$

Solution using optimization: This method implies that the problem is formulated as an optimization problem according to equations 1.15-1.17. In this example the load is price independent, but a hypothetical maximal price which results in a formulation as:

$$\begin{aligned} \max Z &= C_{Ds} + C_{Gs} = D(\hat{c}_D - \lambda) + \sum_{i \in I} (\lambda G_i - C_{Gi}) = \\ &= D\hat{c}_D - \sum_{i \in I} C_{Gi} + \lambda \underbrace{\left(\sum_{i \in I} G_i - D \right)}_{=0} \\ &= D\hat{c}_D - \sum_{i=1}^2 \left[c_{Gi} G_i + \frac{a_i}{2} G_i^2 \right] \end{aligned}$$

when

$$\begin{aligned} 0 &\leq G_1 \leq 150 \\ 0 &\leq G_2 \leq 250 \\ G_1 + G_2 &= D \end{aligned}$$

With a price independent load as in this example, the maximization of consumer and producer surplus becomes the same as operation cost minimization. The result from the optimization is the values of G_1 and G_2 which maximizes the objective Z . The price level can then be calculated with eq. 1.10 applied to the last unit. The price level is the dual variable of the load balance constraint.

End of example 1.1

Example 1.2 Assume the same production system as above in example 1.1. The only difference here is that the load has a price sensitivity of $b=0.6$ when the price increases above 220 SEK/MWh. Calculate the price, the production in each unit per hour, and total operation cost.

General comment: First we study the supply and demand curves. They are shown in figure 1.6. The curves in the left graph in figure 1.6 are drawn by using the above data

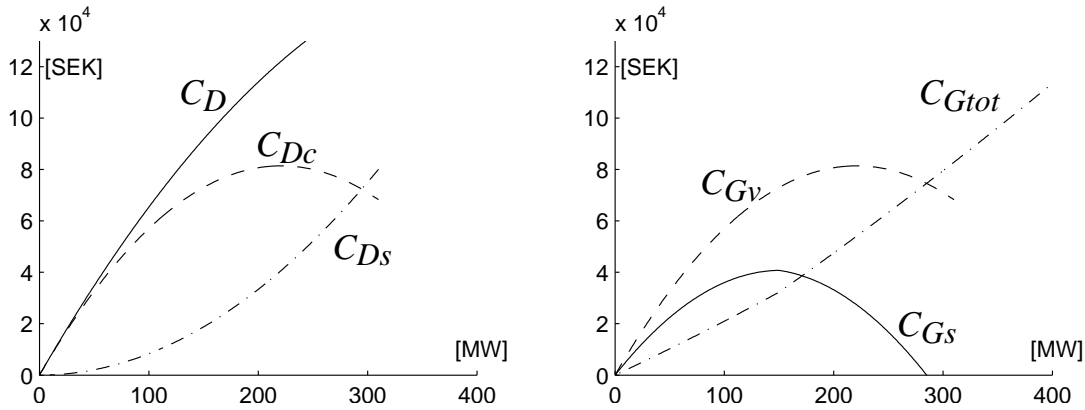


Figure 1.6. Example 1.2, demand (left), supply (right)

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in equations 1.4, 1.8 and 1.9 respectively.

$$\begin{aligned}
 C_D &= \left[\frac{\hat{D}}{b} + c_D \right] D - \frac{1}{2b} D^2 = \left[\frac{310}{0.6} + 220 \right] D - \frac{1}{1.2} D^2 \\
 C_{Dc} &= \lambda_D D = \left[\frac{\hat{D} - D}{b} + c_D \right] D = \left[\frac{310 - D}{0.6} + 220 \right] D \\
 C_{Ds} &= \frac{1}{2b} D^2 = \frac{1}{1.2} D^2
 \end{aligned}$$

As shown in the figure the consumer value increases continuously, but the consumer cost reaches a maximum and then it decreases. The decrease depends on the price sensitivity, i.e., a higher consumption only occurs when the price decreases, and a decreased price affects all consumption. The consumer surplus increases with increased consumption.

The curves in the the right graph in figure 1.6 are drawn by using the above data in

equations 1.12 and 1.13.

$$\begin{aligned}
C_{G_{tot}} &= \sum_{i \in I} \left[c_{Gi} G_i + \frac{a}{2} G_i^2 \right] = \\
&= \begin{cases} 200D + \frac{0.2}{2} D^2 & \text{if } D \leq 150 \text{ MW} \\ 32250 + 300(D - 150) + \frac{0.2}{2} (D - 150)^2 & \text{if } 150 \text{ MW} \leq D \leq 400 \text{ MW} \end{cases} = \\
&= \begin{cases} 200D + 0.1D^2 & \text{if } D \leq 150 \text{ MW} \\ -10500 + 270D + 0.1D^2 & \text{if } 150 \text{ MW} \leq D \leq 400 \text{ MW} \end{cases} \\
C_{G_v} &= \lambda_D D = \left[\frac{\hat{D} - D}{b} + c_D \right] D = \left[\frac{310}{0.6} + 220 \right] D - \frac{1}{0.6} D^2 \\
C_{G_s} &= C_{G_v} - C_{G_{tot}} = \\
&= \begin{cases} \left[\frac{310}{0.6} + 20 \right] D - \left[\frac{1}{0.6} + 0.1 \right] D^2 & \text{if } D \leq 150 \text{ MW} \\ -10500 + \left[\frac{310}{0.6} - 50 \right] D - \left[\frac{1}{0.6} + 0.1 \right] D^2 & \text{if } 150 \text{ MW} \leq D \leq 400 \text{ MW} \end{cases}
\end{aligned}$$

It must here be noted that it is assumed that the production units are used in such a way so the total production cost is minimized. This implies that up to the demand 150 MW, only unit 1 is used (since its operation is cheaper) and at higher load it is assumed that unit 1 is used at its full capacity and the rest is covered by unit 2.

As shown in the figure the producer cost increases continuously, while the producer value (=consumer cost) reaches a maximum and then it decreases. The producer surplus increases up to a certain point and then it decreases. In the figure it is unclear whether the second unit is used at the maximum. We first assume that it is used, and then the maximum surplus of the producer is obtained :

$$\begin{aligned}
\frac{dC_{G_s}}{dD} &= 0 = \left[\frac{310}{0.6} - 50 \right] - 2 \left[\frac{1}{0.6} + 0.1 \right] D \\
&\Rightarrow \\
D &= \left[\frac{310}{0.6} - 50 \right] / 2 / \left[\frac{1}{0.6} + 0.1 \right] = 132.08 \text{ MW}
\end{aligned}$$

This means that the second unit cannot be used since a load of 132.08 MW is covered by the first unit alone. This means that the maximum is found as:

$$\begin{aligned}
\frac{dC_{G_s}}{dD} &= 0 = \left[\frac{310}{0.6} + 20 \right] - 2 \left[\frac{1}{0.6} + 0.1 \right] D \\
&\Rightarrow \\
D &= \left[\frac{310}{0.6} + 20 \right] / 2 / \left[\frac{1}{0.6} + 0.1 \right] = 155.66 \text{ MW}
\end{aligned}$$

This is more than the first unit can produce by itself. Then the optimum must be exactly on the break point, i.e., when $D=150$ MW, the first unit produce on maximum capacity and the second unit is not started at all. This implies that if one profit maximizing producer can control the market, then the total load should be 150 MW corresponding to a power price (cost of marginal increase of consumption) of $\lambda = MC(G_2) = 300 + 0.2(150 - 150) = 300$ SEK/MWh. This is treated more in chapter 3.

Figure 1.7 shows the consumer and producer surplus curves as well as the sum of these,

$$\begin{aligned} Z &= C_{Ds} + C_{Gs} = [C_D - C_{Dc}] + [C_{Gv} - C_{Gtot}] \\ &= C_D - C_{Gtot} + \underbrace{(C_{Gv} - C_{Dc})}_{=0} = C_D - C_{Gtot} \end{aligned}$$

What is interesting here is that the total surplus, Z reaches its maximum at another load level compared to the load level that maximizes the producers surplus. The maximum point is here reached when

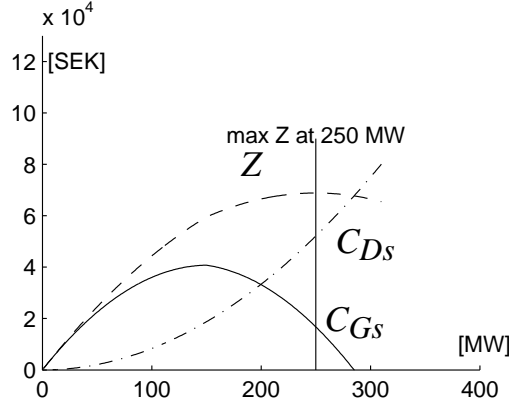


Figure 1.7. Example 1.2, supply and demand surplus

$$\begin{aligned} \frac{dZ}{dD} &= 0 = \frac{dC_D}{dD} - \frac{dC_{Gtot}}{dD} \\ \Rightarrow \\ \frac{dC_D}{dD} &= \frac{dC_{Gtot}}{dD} \end{aligned}$$

i.e. at a level when the marginal value of the consumption is equal to the marginal cost of the power production. This is, as shown in the figure, at a level when the second unit is used:

$$\begin{aligned} \frac{dC_D}{dD} &= \left[\frac{310}{0.6} + 220 \right] - \frac{2}{1.2}D \\ \frac{dC_{Gtot}}{dD} &= 300 + 0.2(D - 150) \\ \Rightarrow \\ D &= \left[\frac{310}{0.6} + 220 - 300 + 0.2 \cdot 150 \right] / \left[\frac{2}{1.2} + 0.2 \right] = 250 \text{ MW} \end{aligned}$$

This level is also shown in figure 1.7. As shown in the figure this level means a lower surplus for the producer compared to the level 132.08 MW, which gives the maximum surplus for the producer as shown above.

So the question is why the producer should produce 250 MW (total maximum surplus) instead of 150 MW when this gives a lower surplus for the producer? There are two possible explanations:

1. **Controlled market** This implies that there are some rules for the producer that defines that they must operate the system in such a way to maximize also the benefits for the consumers. It could be, e.g., that the producers are owned by the consumers, or some regulations that the producers have to follow.
2. **Perfect competition** This assumption is based on that there are many companies with many power plants that compete on a market. Assume, e.g., that a total production (= load) of 150 MW is considered. At this level the marginal production cost and marginal consumer value are

$$\begin{aligned}\frac{dC_{Gtot}}{dC} &= 300 + 0.2(150 - 150) = 300 \text{ SEK/MWh} \\ \frac{dC_D}{dC} &= \left[\frac{310}{0.6} + 220 \right] - \frac{2}{1.2}150 = 486 \text{ SEK/MWh}\end{aligned}$$

With the assumption of several producers this means that an increase of 1 MWh of production implies a possible profit of $486 - 300 = 186$ SEK. This means that it is profitable for a single producer to sell more, since there are consumers prepared to pay for this. This increase can go on until the marginal cost is equal to the marginal value for the consumers.

Graphic solution to problem: Draw the supply curve, i.e., the operation cost of the different units as a function of production, where the units are placed after each other according to increasing marginal operation cost. This is shown in figure 1.8. Then also draw the load curve including the load sensitivity. The sensitivity implies that the load decreases with 0.6 MW for each price increase of 1 SEK/MWh above the price level 220 SEK/MWh.

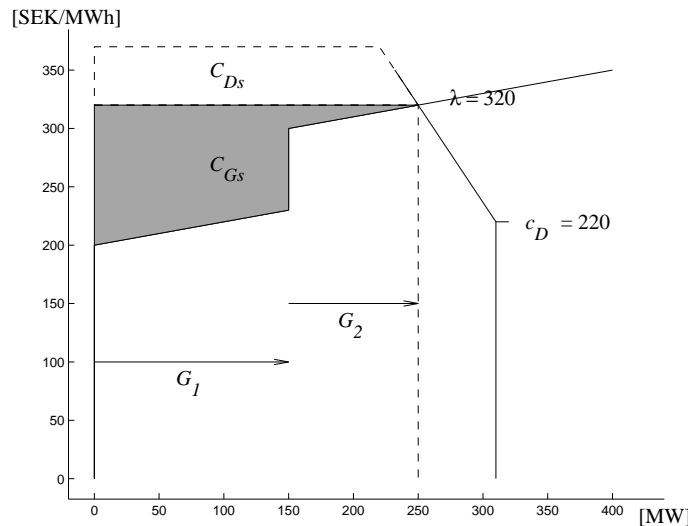


Figure 1.8. Example 1.2

In the cross point (production=demand=D, price = λ) between the demand curve and

the supply curve, the following expressions are valid:

$$\begin{aligned}\lambda &= 300 + 0.2 \cdot (D - 150) && \text{Supply curve, eq. ??} \\ \lambda &= \frac{310-D}{0.6} + 220 && \text{Demand curve, eq. 1.3}\end{aligned}$$

This forms a linear system of equations and the solution is the point $D = 250$ MW and $\lambda = 320$ SEK/MWh. This means that the marginal price, corresponding to the marginal cost in the last unit is 320 SEK/MWh. The production level in the second station is $G_2 = 250 - 150 = 100$ MW.

The operation cost in the two units can be calculated according to equation 1.11 as

$$\begin{aligned}C_{G1} &= 200 \cdot 150 + \frac{0.2}{2} 150^2 = 32250 \quad \text{SEK/h} \\ C_{G2} &= 300 \cdot 100 + \frac{0.2}{2} 100^2 = 31000 \quad \text{SEK/h}\end{aligned}$$

The total operation cost is the sum of the production in the two units:

$$C_{Gtot} = \sum_{i \in I_n} C_{Gin} = C_{G1} + C_{G2} = 63250 \quad \text{SEK/h}$$

Solution using optimization: This method implies that the problem is formulated as an optimization problem according to equations 1.15-1.17. In this example the load is price dependent. With an assumption that the slope continues to the Y-axis (i.e. $\underline{D} = 0$ in figure 1.1), the expression for the consumer value, c.f. equation 1.4, can be formulated as:

$$C_D = \left(\frac{\hat{D}}{b} + c_D \right) D - \frac{1}{2b} D^2 = \left(\frac{310}{0.6} + 220 \right) D - \frac{1}{2 \cdot 0.6} D^2$$

and the producer operation cost can be formulated as:

$$C_{Gtot} = \sum_{i=1}^2 \left(c_{Gi} G_i + \frac{0.2}{2} G_i^2 \right)$$

which results in a formulation as (c.f. eq. 1.18):

$$\begin{aligned}\max Z = C_{Ds} + C_{Gs} &= \left(\frac{310}{0.6} + 250 \right) D - \frac{1}{2 \cdot 0.6} D^2 - \sum_{i=1}^2 \left(c_{Gi} G_i + \frac{0.2}{2} G_i^2 \right) \\ &\text{when} \\ 0 \leq G_1 &\leq 150 \\ 0 \leq G_2 &\leq 250 \\ G_1 + G_2 &= D\end{aligned}$$

The free variables in this optimization problem are D , G_1 and G_2 . The result from the optimization include the values of these variables which maximizes the objective Z .

End of example 1.2

Unit	\hat{G}_{in}	c_{Gin}	a
i=1	100 MW	200 SEK/MWh	0,2 SEK/MWh/MW
i=2	150 MW	210 SEK/MWh	0,2 SEK/MWh/MW
i=3	150 MW	230 SEK/MWh	0,2 SEK/MWh/MW

Table 1.2. Data for example 1.3

Example 1.3 Assume that there are three power stations located in one area. The data for the power stations are shown in the table. The load at low prices is 350 MW, but when the price is higher than 200 SEK/MWh, then the load decreases with 3 MW for each SEK that the price increases. Calculate the price, the production in each unit per hour, and total operation cost.

Graphic solution: Draw the supply curve, i.e., the operation cost of the different units as a function of production, where the units are placed after each other according to increasing marginal operation cost. In this example there are several units which intersect. the method is then to find the corners in the piecewise linear curve. The method is the following:

1. Calculate the total production in all units for the price corresponding to zero and maximal production in each unit. This gives some points
2. Order the points according to increasing total production
3. Draw the points and the straight lines between the points

This method can now be applied. The production levels for the six points (2 production levels per unit) are shown in the table. To the right in the table the order of the points are shown, taken from the prices in the third column.

Unit	Production [MW]	price [SEK/MWh]	#1 [MW]	#2 [MW]	#3 [MW]	#1+2+3 [MW]	final order
#1	0	200	0	0	0	0	1
#1	100	220	100	50	0	150	3
#2	0	210	50	0	0	50	2
#2	150	240	100	150	50	300	5
#3	0	230	100	100	0	200	4
#3	150	260	100	150	150	400	6

Table 1.3. Intersecting point for example 1.3

This is shown in figure 1.9. There also the load curve can be drawn as a vertical line up to the price 200 SEK/MWh and then a curve with a slope.

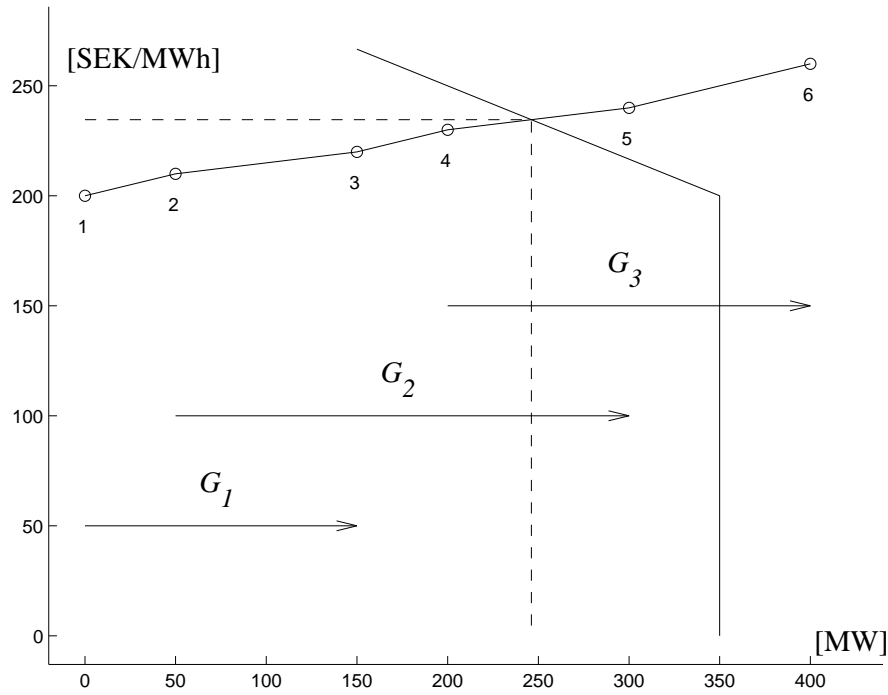


Figure 1.9. Example 1.3

The figure shows that between points 4 and 5 both stations #2 and #3 increase their generation when the price increases. Since both increase then the slope between these two points become $0.2/2=0.1$ SEK/MWh/MW. The intersection point can be identified as the cross between the load slope and the production point between point 4 and 5 in table 1.3. This means that in this point the production level and price can be calculated as:

$$\begin{aligned}
 \lambda &= 230 + 0.1(P - 200) && \text{production increases with unit \#2-\#3} \\
 P &= 350 - 3[\lambda - 200] && \text{demand curve} \\
 \Rightarrow \\
 \lambda &= 230 + 0.1(\lambda = 230 + 0.1(P - 200) - 200) \\
 \Rightarrow \lambda &= 305/1, 3 = 234.6154 \text{ SEK/MWh} \\
 \Rightarrow P &= 246.1538 \text{ MW}
 \end{aligned}$$

The production level of unit #1 is on maximum level since the marginal cost at installed capacity is lower than the price. For units #2 and #3 the price is equal to marginal cost at the production level, c.f. eq. 1.10. Using this information the production levels can be calculated as:

$$\begin{aligned}
 G_1 &= 100 \text{ MW} \\
 G_2 &= (MC_{G_2} - c_{G_2})/a_2 = (234.6154 - 210)/0.2 = 123.0769 \text{ MW} \\
 G_3 &= (MC_{G_3} - c_{G_3})/a_3 = (234.6154 - 230)/0.2 = 23.0769 \text{ MW}
 \end{aligned}$$

The operation cost in the three units can be calculated according to equation 1.11 as

$$\begin{aligned} C_{G1} &= 200 \cdot G_1 + \frac{0.2}{2} G_1^2 = 21000 \text{ SEK/h} \\ C_{G2} &= 210 \cdot G_2 + \frac{0.2}{2} G_2^2 = 27361 \text{ SEK/h} \\ C_{G3} &= 230 \cdot G_3 + \frac{0.2}{2} G_3^2 = 5361 \text{ SEK/h} \end{aligned}$$

The total operation cost is the sum of the production cost in the three units:

$$C_{Gtot} = \sum_{i \in I} C_{Gi} = C_{G1} + C_{G2} + C_{G3} = 53722 \text{ SEK/h}$$

Solution using optimization: This method implies that the problem is formulated as an optimization problem according to equations 1.15-1.17. In this example the load is price dependent. With an assumption that the slope continues to the Y-axis (i.e. $\underline{D} = 0$ in figure 1.1), the expression for the consumer value, c.f. equation 1.4, can be formulated as:

$$C_D = \left(\frac{\hat{D}}{b} + c_D \right) D - \frac{1}{2b} D^2 = \left(\frac{350}{3} + 200 \right) D - \frac{1}{2 \cdot 3} D^2$$

and the producer operation cost can be formulated as:

$$C_{Gtot} = \sum_{i=1}^3 \left(c_{Gi} G_i + \frac{0.2}{2} G_i^2 \right)$$

which results in a formulation as (c.f. eq. 1.18):

$$\max Z = C_{Ds} + C_{Gs} = \left(\frac{350}{3} + 200 \right) D - \frac{1}{2 \cdot 3} D^2 - \sum_{i=1}^3 \left(c_{Gi} G_i + \frac{0.2}{2} G_i^2 \right)$$

when

$$\begin{aligned} 0 &\leq G_1 \leq 100 \\ 0 &\leq G_2 \leq 150 \\ 0 &\leq G_3 \leq 150 \\ G_1 + G_2 + G_3 &= D \end{aligned}$$

The result from the optimization is the values of G_1 , G_2 and G_3 which maximizes the objective Z . The price level can then be calculated with eq. 1.11 applied to one of the units that does not produce on maximum level, i.e., #2 and #3. The price level is the dual variable of the load balance constraint.

End of example 1.3

1.4 Capacity deficit

If there is not enough power available in a certain area to cover the load, then it is necessary to disconnect consumers.

Example 1.4 Assume the same production system as above in example 1.2. The only difference here is that the load has a base level of 420 MW. Calculate the price, the production in each unit per hour, and total operation cost.

Solution to example 1.4 First we start to draw the supply and demand curves. This is shown in figure 1.10. As shown in the figure there is no price cross between the

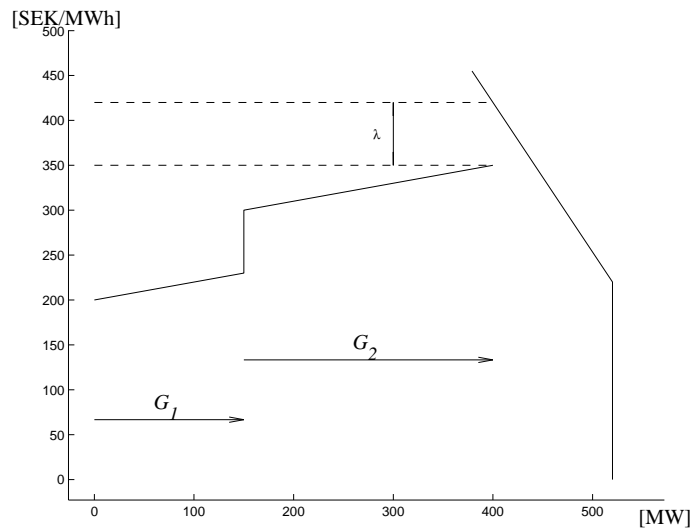


Figure 1.10. Example 1.4

supply and demand functions. The question is then what the price will be? First it must be noted that the consumption has to be on the same level, i.e., equal to maximal production ($= G_1 + G_2$ in the figure). The price can then formally be in the interval between the marginal cost of production and marginal value of the consumption. The most rational price is the one on marginal value of consumption. If the other extreme, marginal cost of production, is applied then there is no simple definition about which consumers that should not get what they are prepared to pay for, since the consumption at this price level is higher than the supply. It can though be noted that Nordpool (the Nordic European exchange) applies the price obtained from the supply bid curve if this situation would occur.

End of example 1.4

Chapter 2

Interaction between different markets

In the previous chapter there were consumers and producers and depending on the market structure, a price was formed. In a power system the price can change both over time and geographically. In many cases the price difference depends on limited capacity to transfer power between the two places/times with different prices. There are structurally two possibilities that markets can interact and that is between different time periods and geographically between different locations.

First assume in general that there are two markets with a possibility to trade between the two markets. In figure 2.1 an example of two markets, with one unit in each market, G_i is producing as much as the demand, D_i . The market balance for the two markets

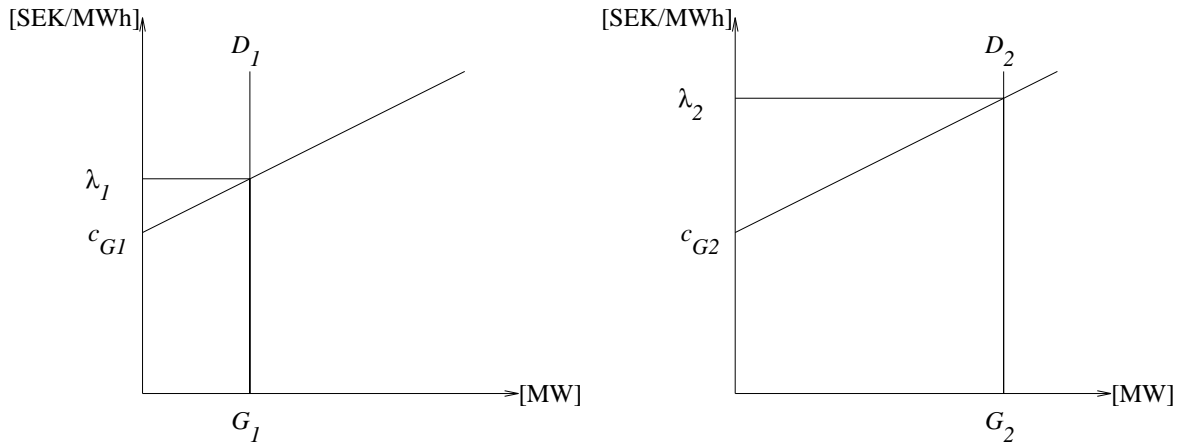


Figure 2.1. Example of two markets

can be formulated as:

$$\begin{aligned}\lambda_1 &= c_{G1} + a_1 G_1 = c_{G1} + a_1 D_1 \\ \lambda_2 &= c_{G2} + a_2 G_2 = c_{G2} + a_2 D_2\end{aligned}\tag{2.1}$$

Between these two markets it would be profitable to trade since the price in market 1, λ_1 , is lower than the price in market 2, λ_2 . If one assumes a rather small trading volume (P_{12} MWh/h from market 1 to 2) that will not affect the price, then the value of the trading (i.e. the marginal value), C_T is:

$$C_T = P_{12}(\lambda_1 - \lambda_2)\tag{2.2}$$

If we consider the trading as a changed demand in each market, the power balance in each market can be formulated as:

$$\begin{aligned}G_1 &= D_1 + P_{12} \\ G_2 &= D_2 - P_{12}\end{aligned}\tag{2.3}$$

The market balance in equation 2.3 can now be reformulated as:

$$\begin{aligned}\lambda_1 &= c_{G1} + a_1(D_1 + P_{12}) \\ \lambda_2 &= c_{G2} + a_2(D_2 - P_{12})\end{aligned}\quad (2.4)$$

Trading between the areas is profitable as long as there is a price difference between the markets and there is a possibility to trade. Now assume that there is no trading limit between the two markets in figure 2.1. If we assume that there is no cost for the trading (except for buying power on the low price market), and that the ones that have the possibility to trade will maximize their trading profit, then the trading will increase until

$$\lambda = \lambda_1 = \lambda_2 \quad (2.5)$$

By introducing equation 2.5 into 2.4, the resulting trading can be calculated as

$$c_{G1} + a_1(D_1 + P_{12}) = c_{G2} + a_2(D_2 - P_{12}) \quad (2.6)$$

$$\Rightarrow$$

$$P_{12} = \frac{1}{a_1 + a_2}(c_{G2} + a_2D_2 - c_{G1} - a_1D_1) \quad (2.7)$$

In figure 2.2 the resulting trade is shown. From market 1 point of view the price

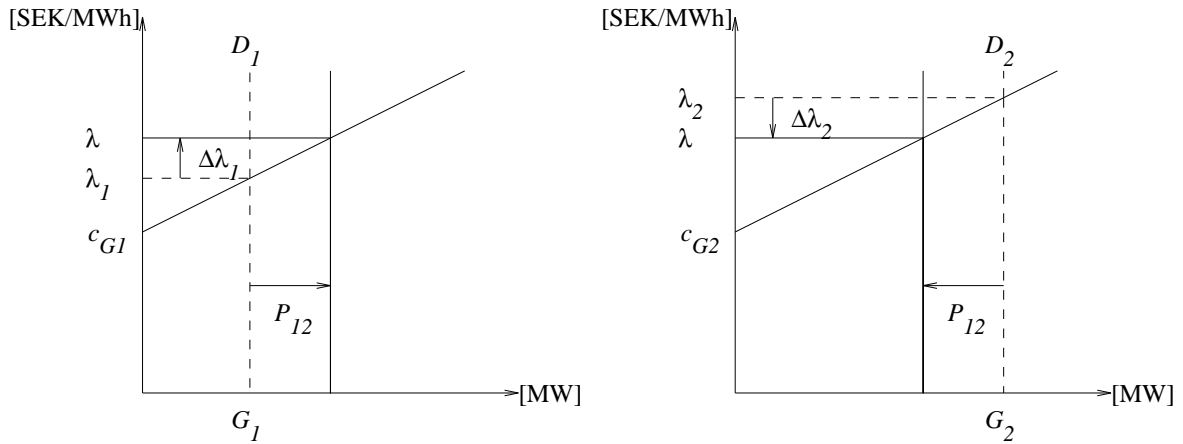


Figure 2.2. Full trading between two markets

increases because of the export. This means an increased cost for the consumers and on the same time an increased profit for the producers. In market 2 it is the opposite with a decreased price depending on the import. This means a decreased cost for the consumers and on the same time a decreased profit for the producers. It can be noticed that no limits on the trading is same as assuming one market with two units, G_1 and G_2 , and a total load of $D_1 + D_2$.

With a possibility to trade there is not only a consumer and producer surplus but also a "trading surplus" $= C_{Ts} = (\text{income of trading}) - (\text{cost of trading})$. Assume two markets

with trading P_{12} from market 1 to market 2. The total surplus is now

$$\begin{aligned}
C_s &= C_{Ds} + C_{Gs} + C_{Ts} = \underbrace{[C_{D1} - \lambda_1 D_1] + [C_{D2} - \lambda_2 D_2]}_{=C_{Ds}} + \\
&+ \underbrace{(\lambda_1 G_1 - C_{G1}) + (\lambda_2 G_2 - C_{G2})}_{=C_{Gs}} + \underbrace{P_{12}(\lambda_2 - \lambda_1)}_{=C_{Ts}} = \\
&= (C_{D1} + C_{D2}) - (C_{G1} + C_{G2}) + \\
&+ \lambda_1 \underbrace{(G_1 - D_1 - P_{12})}_{=0, \text{ eq. 2.3}} + \lambda_2 \underbrace{(G_2 - D_2 + P_{12})}_{=0, \text{ eq. 2.3}} = \\
&= C_D - \sum C_{Gi}
\end{aligned} \tag{2.8}$$

i.e. the total surplus is still the difference between total consumer value and total production cost.

2.1 Storage possibilities

One possible connection between two markets is when there is hydro power with storage capability. This means that water can be stored from one period to another future period.

Now assume that we have a power system consisting of both thermal power plants and hydro power plants. To cover a certain demand this means that the more hydro power (=water) that is used during one certain period, the less thermal power has to be used. In the example shown in figure 2.1 the trading possibility between period 1 and period 2 means that energy in the form of water is stored from period 1 to period 2. If more water is stored then more thermal power has to be used during period 1 to cover the load and less thermal power is used during period 2. This can also be formulated in such a way that if one MWh of water is stored from one period to the next, then this MWh can reduce the operation cost in that period with a certain amount. This is the so-called *water value*, i.e., the possibility for a certain amount of water to reduce the operation cost.

Example 2.1 Assume there are two periods, 1 and 2, and there are two units in the system with the following data: In period 1 the load is 250 MW and in period 2 it is

Unit	Type	\hat{G}_{in}	c_{Gin}	a
i=1	Hydro	200 MW	20 SEK/MWh	0 SEK/MWh/MW
i=2	Thermal	250 MW	300 SEK/MWh	0,2 SEK/MWh/MW

Table 2.1. Data for example 2.1

310 MW. Assume that the inflow to the hydro power plants corresponds to 200 MWh/h for the first period and 60 MWh/h in the second period, but water can be stored from

period 1 to period 2. Assume that both periods consists of 24 hours. Also assume that all water inflow is used during the studied two periods.

2.1a Calculate the production in all units and the period power prices when no water is stored between the periods.

2.1b Calculate the same as in 2.1a when the storage capacity is 1200 MWh.

2.1c Calculate the same as in 2.1b, but now assume a storage capacity of 3600 MWh.

2.1d Calculate the optimal storage level between the two periods.

Graphic solution to example 2.1a: Since no water will be stored between the two periods this means that the production capacity in the hydro power plant in the first period is $G_1(1) = 200$ MWh/h and in the second $G_1(2) = 60$ MWh/h. The same method as used in example 1.1 can now be used to calculate the power prices and the production in all plants. This means that $250-200= 50$ MWh/h is needed from unit #2 in period 1. The price (=marginal cost) is then:

$$\lambda_1 = c_{G2} + a_2 G_2 = c_{G2} + a_2 [D(1) - G_1(1)] = 300 + 0.2 \cdot 50 = 310 \text{ SEK/MWh}$$

In period 2 the power production in unit #2 becomes $310-60= 250$ MWh/h corresponding to a price (=marginal cost) of:

$$\lambda_2 = c_{G2} + a_2 [D(2) - G_1(2)] = 300 + 0.2 \cdot 250 = 350 \text{ SEK/MWh}$$

The result is shown in figure 2.3

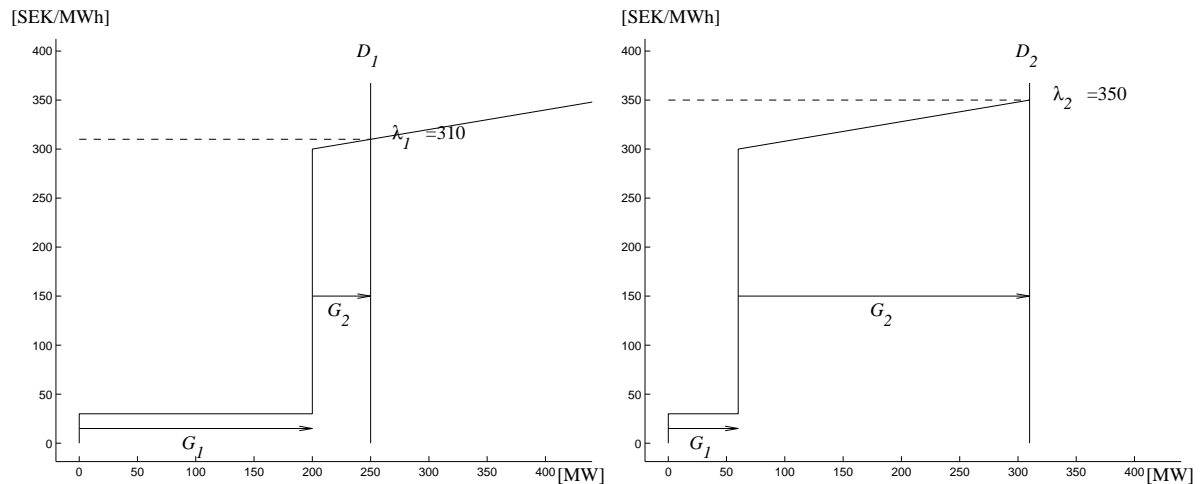


Figure 2.3. Example 2.1a, left=period 1, right=period 2

As shown in figure 2.3 the price is higher in period 2 than in period 1. This means that it is profitable to "buy" water in period 1 and "sell" it in period 2. The *water value* in

period 1 is the marginal cost in period 2 if there is room in the reservoir to store water from period 1 to period 2. This possibility will be treated in example 2.1b.

Solution using optimization As shown in eq. 2.8, the total surplus when there are trading possibilities is still the difference between consumer value and producer cost. Considering the balances in each area according to eq. 2.3, and with a hypothetical maximal price, \hat{c}_D , this result in an optimization formulation as:

$$\begin{aligned} \max Z = C_D - \sum C_{Gi} &= \hat{c}_D(D_1 + D_2) - \sum_{t=1}^2 \sum_{i=1}^2 \left(c_{Gi} G_i(t) + \frac{0.2}{2} G_i(t)^2 \right) \\ &\text{when} \\ 0 \leq G_1(t) &\leq 200 \\ 0 \leq G_2(t) &\leq 250 \\ G_1(t) + G_2(t) &= D_t \quad t \in [1, 2] \\ G_1(1) + P_{12} &= \text{inflow in period 1} = 200 \\ G_1(2) - P_{12} &= \text{inflow in period 2} = 60 \\ P_{12} &= 0 \end{aligned}$$

The free variables in this optimization problem are $G_1(t)$, $G_2(t)$, $t \in [1, 2]$ and P_{12} . The problem is formulated in such a way that it is easy to change the limits of P_{12} to, e.g., an interval instead which is the case in later examples. The result from the optimization includes the values of these variables which maximizes the objective Z . The total production cost ($= -Z$ if we neglect the consumer value) is in this example 101700 SEK for both units and both periods.

End of example 2.1a

Graphic solution to example 2.1b: A storage capacity of 1200 MWh means that the hydro power production can be decreased with 50 MWh/h during all 24 hours in period 1 and increased with 50 MWh/h during each hour in period 2, i.e., $P_{12} = 50$ MWh/h is stored from period 1 to period 2.

This means that $250 - (200 - 50) = 100$ MWh/h is needed from unit #2 in period 1. The price (=marginal cost) is then:

$$\lambda_1 = c_{G2} + a_2[D(1) - (G_{1-\text{inflow}}(1) - P_{12})] = 300 + 0.2 \cdot [250 - (200 - 50)] = 320 \text{ SEK/MWh}$$

In period 2 the power production in unit #2 becomes $310 - (60 + 50) = 200$ MWh/h corresponding to a price (=marginal cost) of:

$$\lambda_2 = c_{G2} + a_2[D(2) - (G_1(2) + P_{12})] = 300 + 0.2 \cdot 200 = 340 \text{ SEK/MWh}$$

The result is shown in figure 2.4. In this example the *water value* in period 1 is $= \lambda_1 = 320$ SEK/MWh since more water in period 1 can not be stored to period 2, but can lower the operation cost in period 1.

As shown in the figure 50 MWh/h less in period 1 increases the price while 50 MWh/h more in period 2 decreases the price.

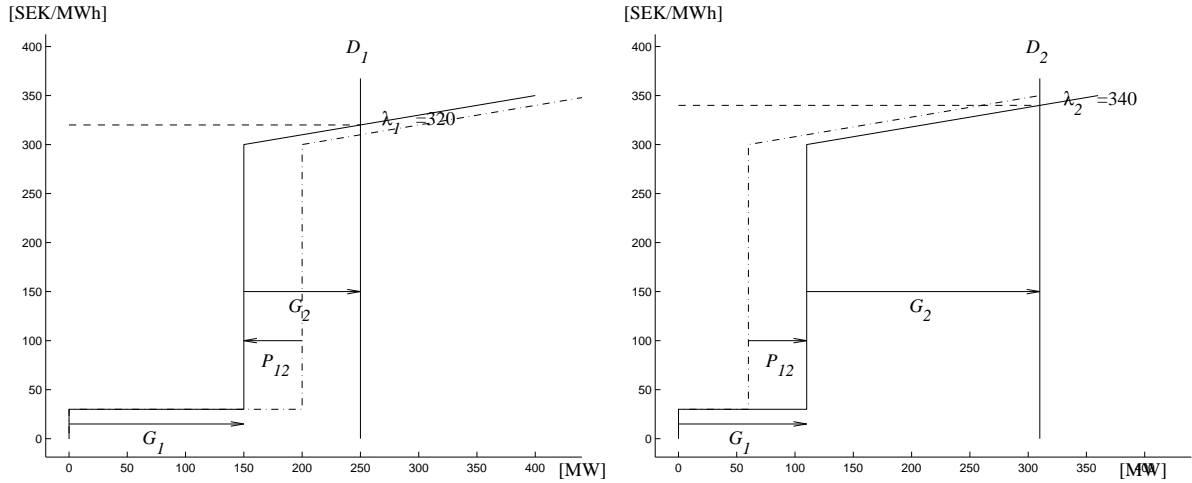


Figure 2.4. Example 2.1b

Solution using optimization We can now use the same formulation as in example 2.1a, but with the change that

$$P_{12} \leq 50$$

With this value the total operation cost becomes 100200 SEK. This means that the total operation cost of the system has decreased thanks to use of storage between period 1 and period 2. But it must also be noted that this include other consequences: The power price is higher for the consumers in period 1, but lower in period 2, and the one that store water from period 1 to period 2 earns money on this storage. But the total surplus in the system has increased since the production cost decreased and the consumer value remained the same.

End of example 2.1b

Graphic solution to example 2.1c: A storage capacity of 3600 MWh means that the hydro power production can be decreased with 150 MWh/h during all 24 hours in period 1 and increased with 150 MWh/h during each hour in period 2. But the question is here whether this storage level is optimal? It is only of economic interest to store water from one period to the next if the value of the water is higher in the second period. The amount of water to be stored should be increased until the marginal value is zero, i.e., there is no value in a further increase. The value of the water is equal to the marginal value of the last used thermal power plant. To store water from period 1 to period 2 means that the thermal power production is increased in period 1 and decreased in period 2. This is profitable if the thermal power production is more expensive in period 2 compared to period 1. When the marginal thermal power production has the same level in both period 1 and 2, then there is no extra value in a change of stored amount of water between period 1 and 2. For example 2.1b this can

be formulated as

$$\begin{aligned}
 \lambda_1 = 300 + 0.2 \cdot (\underbrace{250}_{D(1)} - \underbrace{200}_{G_{1-inflow}(1)} + P_{12}) &= 300 + 0.2 \cdot (\underbrace{310}_{D(2)} - \underbrace{60}_{G_{1-inflow}(2)} - P_{12}) = \lambda_2 \\
 250 - 200 + P_{12} &= 310 - 60 - P_{12} \\
 P_{12} &= 100
 \end{aligned}$$

This means that $P_{12} = 100$ MWh/h corresponding to 2400 MWh for a 24 hour period is the optimal storage level between period 1 and 2. With this storage, the power prices will be the same in both periods:

$$\lambda_1 = \lambda_2 = 300 + 0.2 \cdot 150 = 330 \text{ MWh/h}$$

With this amount of storage, the same method as used in example 1.1 can be used to calculate the power prices and the production in all plants. The result is shown in figure 2.5.

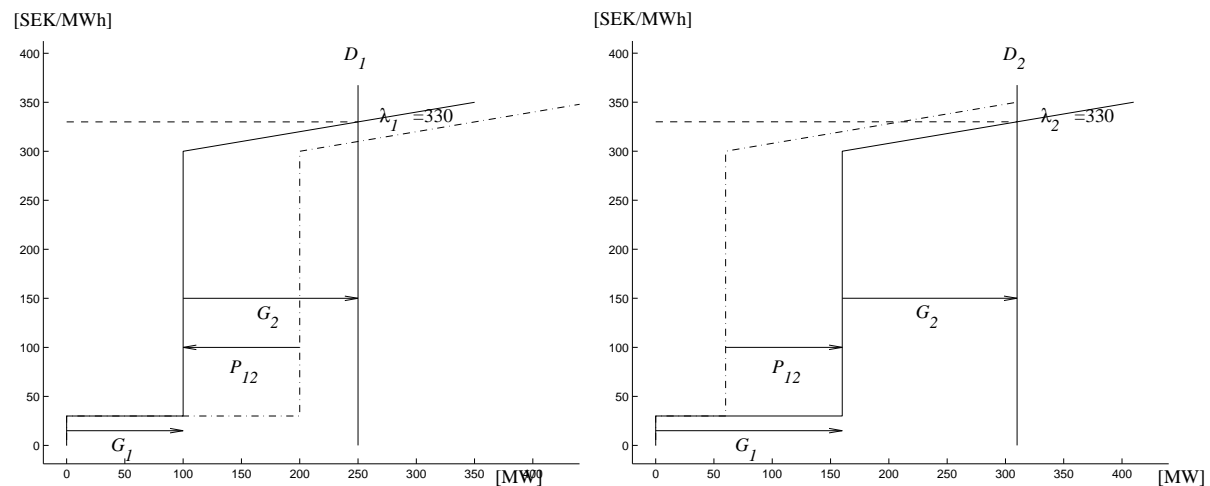


Figure 2.5. Example 2.1c

In this example the *water value* in period 1 is $\lambda_1 = \lambda_2 = 330$ SEK/MWh since more water in period 1 can be stored to period 2, and can lower the operation cost in both period 1 and period 2.

Solution using optimization We can also here use the same formulation as in example 2.1a, but with the change that

$$P_{12} \leq 150$$

With this value the total operation cost becomes 99700 SEK. This means that the total operation cost of the system has decreased even more thanks to use of storage between period 1 and period 2. But it must also be noted that this include other consequences: The power price is even higher for the consumers in period 1, but lower in period 2. But the one that store water from period 1 to period 2 does not earn money on this storage since the price is the same. The total surplus in the system has increased since

the production cost decreased and the consumer value remained the same.

End of example 2.1c

Solution to example 2.1d: In example 2.1c the storage level 3600 MWh was checked but the full level was not used, since a storage level higher than 2400 MWh = 100 MWh/h was not used. This means that the optimal storage level is 2400 MWh for the studied periods.

End of example 2.1d

2.2 Transmission limits

This section provides an introduction to congestion management and modeling in the case of transmission limits between two areas.

Introduction to congestion management

Congestion occurs when the transmission network is not sufficient to transfer electric power according to the market desire. A bottleneck is however not synonymous with supply outage or any other system failure. In a perfect market all energy is produced at the plants carrying the lowest production costs, but when congestion occurs more expensive power plant will be used in areas where the transmission is not sufficient to transfer energy produced in cheaper plants. The area with the higher price level will be called the upside of the bottleneck.

Congestion could lead to price differences in areas, which could jeopardize actor's financial undertakings. Another undesirable effect is an increased risk for the actors to use market power.

Congestion can be divided into two categories: *structural* and *temporary*. The temporary bottleneck is more random in location and is more unusual. An example is de-rating on a transmission during maintenance. The structural bottleneck depends on the transmission network and the location of production and consumption. A structural bottleneck occurs more regularly like for example during spring flood if there is congestion from an area with large amounts of hydro power plants. It is though, important to point out that a network without congestion is not equal to an optimal network. Large costs for extending the network must always be considered and the price of the "congestion free" network would be enormous. Even in a market opened for competition the actors will have to adopt both the rules associated with the commercial part and the ones set by physical limits. As mentioned before the electricity has very special physical properties like for example that it cannot be stored for delivery later. Another way to explain it that is not like the railroad system where trains can wait for other trains to pass, even if it could affect the timetables and income for actors. This will not cause a collapse in the network. When the energy is produced in the power plant the entire network must be able to carry it all the way down to the consumer in the same moment.

Risks of congestion

The risks can be divided into two areas sometimes inferring with each other: financial risks and risk of actors using market power. In an open market it is prerrequired that customers can chose supplier (in most cases the cheapest one) but when congestion occurs the market will be divided in several parts lowering competition. In the worst cases only one supplier might have such a big influence in an area giving him no incentive to keep the prices down. A too divided market will also affect the possibility to buy financial electricity contracts as futures, forwards and options since they normally hedge towards system price. Without the financial products the market competition will be even weaker. Among the actors there is also a concern about the lowered confidence in the open market.

Overview of congestion management methods

One of foundation of congestion management is that it should be economically efficient in the short run and provide correct incentives for network investment in the long run. The problem is to find a method, which at the same time provides the right long term signals for both market actors and system operators. In an ideal world, congestion management method will allow market players to make maximal use of the network and at the same time ensure system security. Today's methods can be divided and classified in many different ways; one suggested division is into non-market based and market based.

Non-market based

The non-market based methods are mostly found on the regulated markets but they can exist even on a deregulated market, which will be described further down in these paper. To the non-market based methods we can add *type of contract*, *first come - first serve*, *pro rata* and *channels*.

Type of contract can use the length of contract period as method of relieving congestion, i.e. contracts that is valid for a longer period get higher priority than a short term contract.

First come - first serve giving the capacity to the one who first to submit a bid to use it.

Pro rata gives the actors their share to total transmission capacity based on either their bids or the capacity in the direction they wish to transfer it.

Channels (or self owned transmissions) gives an actor the right to use a transmission with a guaranteed level of capacity.

Implicit auctioning (market splitting)

Implicit auction requires an organized power exchange on the high priced end (upside) of the congestion. The special thing about implicit auctioning is that both energy and the corresponding transmission between bidding areas are traded simultaneously and are coupled. If the available capacity is not enough to meet the required transmission, then a fee is added to every bid utilizing the transmission until enough bids are too high to be used. The income from this process will benefit the one that add the fees normally the system operator. Another way of describing this is when you have a more than fully booked airplane; the prices of the tickets are increased until enough passengers decide not to take that flight.

A special case of implicit auctioning is *Market splitting*. If applied internally in a market place there is a need for price indicators on both sides of the congestion, i.e. all bids submitted to the power exchange area specified to an area. If it after calculation of system price is not possible to perform the required transmission, a price is calculated on both sides of the transmission. The price will then be higher in one of the receiving end of a congested transmission; power is then bought from the cheaper area until the transmission capacity reaches its limit.

Explicit Auctioning

In *explicit auctioning*, energy flow and transmission are separated and the system operator can chose to set the whole or a part of the transmission capacity out on auction. Like many other auctions this is made in several ways, the contract can last from a day-today up to year-to-year basis giving a more dynamic exchange since the market is continuously changing. The price can be set so bidders pay what they bid or lowest acceptable bid can be used for all accepted bidders. This method should for its best performance be combined with the use-it-or-loose-it-principle that enables other market actors to use the capacity not used. The explicit auctioning method is a sometimes a good alternative for the interconnections as it does not require the same market structure on both sides of the transmission. The income from the auction will go to the operator/owner of the transmission.

Counter trading

Counter trading differs from the two previous methods as it can be used until and within the actual hour. This and the following method can be named a remedial methods since they are more of corrective characteristic. For using this method, there is a need for a working market for regulating power where the actors not only will submit their bids with price and amount but also must specify in which geographical area the power will be produced. When a transmission is congested the system operator uses this method to create a fictitious amount of additional extra capacity for the transmission by buying power (or paying for decreased consumption) in the deficit area and simultaneously buying it at the surplus area. Unlike the previous methods the system operator will

take the additional cost for congestion since it will pay a higher price for the energy bought in the deficit area than it is paid in the surplus area.

Re-dispatching

Re-dispatching assumes that there is a centralized unit e.g. the system operator, SO, who knows the marginal costs of all generators. When congestion occurs, the SO orders generators in the area where the flow is heading to start producing while generators in the other area are stopped. This will cost the SO since price is higher for the generators ordered to start than the ones system price are based on.

In figure 2.6 the congestion methods applied in Europe in 2006 are shown.

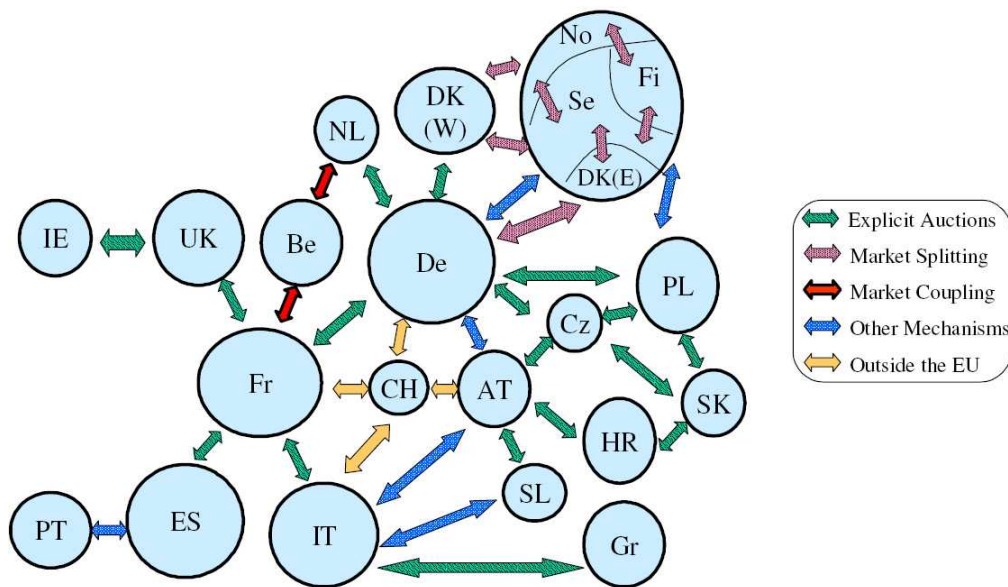


Figure 2.6. ETSO, Current Congestion Management Methods, Update 2006

Transmission limits between two areas

Assume now that there are two areas and there is a transmission possibility between these areas, and there is a limited capacity on the connection. In each area it is necessary that the total demand, D_n , and export to neighboring areas, P_{nk} is equal to the total production:

$$\sum_{i \in I_N} G_{in} = D_n + \sum_{k \in N} P_{nk} \quad \forall N \quad \text{and} \quad \forall t \quad (2.9)$$

Example 2.2 Assume there are two neighboring systems, A and B , where system A has the same production system as above in example 1.1. System B is nearly the same

as system A, the only difference is that the slope $a=0.3$ for both units. In system A the load is 250 MW and in system B the load is 310 MW.

2.2a Calculate the production in all units, the transmission between the system and the prices in both systems when the capacity of the line is 100 MW. Also assume two different transmission system operators in the different systems and assume their costs/benefits for the trading. Assume area pricing.

2.2b Calculate the same as in 2.2a when the capacity of the line is 40 MW.

2.2c Calculate the same as in 2.2b, but now assume counter buying.

2.2d Calculate the same as in 2.2a, but now assume that explicit auction is applied to manage the congestion. This means that there has first been an auction on how to use the transmission and in this case this resulted in a transmission of 40 MW from A to B.

2.2e Calculate the same as in 2.2d, but now assume that explicit auction is flexible in the operation phase in such a way that the real use of the interconnection can be changed if there is a market need for this.

Graphic solution to example 2.2a: First assume that there is no transmission line between the two systems. Then do as in example 1.1, i.e., draw the supply curves, where the units are placed after each other according to increasing marginal operation cost. This is shown in figure 2.7. In the figure also the load curve is drawn as a vertical line.

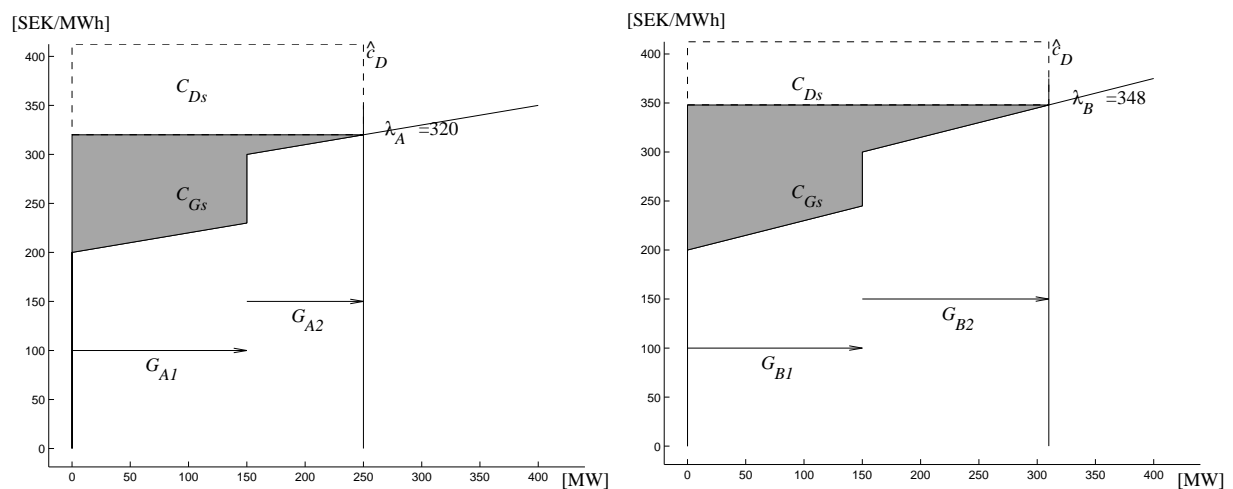


Figure 2.7. Example 2.2a, left=area A, right=area B

With an assumption of no transmission the price in area A and B can be calculated with equation as

$$\begin{aligned}\lambda_A &= c_A(G_2) = 300 + 0.2 \cdot 100 = 320 \text{ SEK/MWh} \\ \lambda_B &= c_B(G_2) = 300 + 0.3 \cdot 160 = 348 \text{ SEK/MWh}\end{aligned}$$

With a transmission line with the capacity 1 MW between region A and B it will be profitable to buy 1 MWh/h in area A for 320 SEK/MWh and sell it in area B for 348 SEK/h. The profit can be used to finance the cost of the transmission line. With an increased capacity on the line, more transmission will be profitable. For each MWh/h of increased transmission the price will increase in the low price area A and decrease in the high price area B. It is profitable to increase the transmission until the prices are equal. This happens when

$$\lambda_A = 300 + 0.2 \cdot (100 + P_{AB}) = 300 + 0.3 \cdot (160 - P_{AB}) = \lambda_B$$

The solution to this equation is $P_{AB} = 56$ MW, which results in prices of $\lambda_A = \lambda_B = 331.2$ SEK/MWh. In the data the maximum transmission is 100 MW, which means that this solution is allowed. From system A this means that the load increases with 56 MW and in system B the load decreases with 56 MW. The result is shown in figure 2.8. There is no income, nor any costs for the transmission system operators

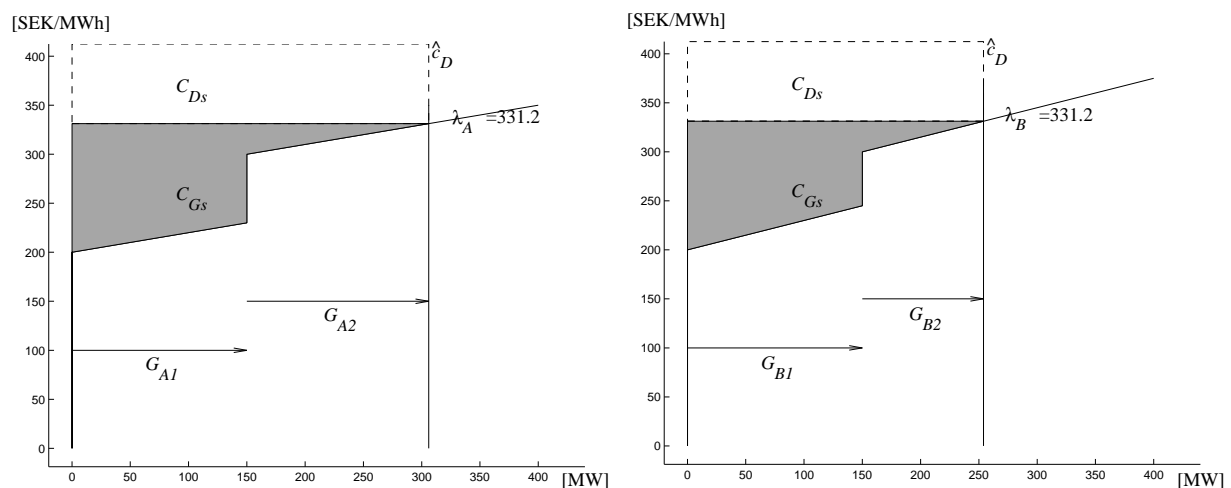


Figure 2.8. Example 2.2b

since the trade is now between two areas with the same price.

Solution using optimization The total surplus when there are trading possibilities is still the difference between consumer value and producer cost. Considering the balances in each area according to eq. 2.9, and with a hypothetical maximal price, \hat{c}_D , this result

in an optimization formulation as:

$$\begin{aligned}
\max Z = C_D - \sum C_{Gi} &= \hat{c}_D(D_1 + D_2) - \sum_{k=A}^B \sum_{i=1}^2 \left(c_{Gi} G_i(k) + \frac{a(i, k)}{2} G_i(k)^2 \right) \\
&\text{when} \\
0 \leq G_1(k) &\leq 150 \\
0 \leq G_2(k) &\leq 250 \\
G_1(A) + G_2(A) &= D_A + P_{AB} \\
G_1(B) + G_2(B) &= D_B - P_{AB} \\
P_{AB} &= 0
\end{aligned}$$

The free variables in this optimization problem are $G_1(k)$, $G_2(k)$, $k \in [A, B]$ and P_{AB} . The problem is formulated in such a way that it is easy to change the limits of P_{AB} to, e.g., an interval instead which is the case in later examples. By setting $P_{AB} = 0$, one obtains the result if figure 2.2a, where the total production cost ($= -Z$ if we neglect the consumer value) is 148465 SEK. $P_{AB} \leq 100$ gives the results in figure 2.2b, where the total production cost is 147681 SEK. The result from the optimization include the values of these variables which maximizes the objective Z . This shows that the total surplus increases (148465-147681=784 SEK/h) with a transmission possibility, but this also means that the consumer prices increases in area A and producers surplus decreases in area B.

End of example 2.2a

Graphic solution to example 2.2b : In example 2.2b it was shown that with no transmission between the areas the prices were different but with a transmission capacity larger than 56 MW the prices in the two areas became the same. With a capacity of 40 MW, the prices will not be the same. The prices can be calculated as

$$\begin{aligned}
\lambda_A &= 300 + 0.2 \cdot (100 + P_{AB}) = 300 + 0.2 \cdot (100 + 40) = 328 \text{ SEK/MWh} \\
\lambda_B &= 300 + 0.3 \cdot (160 - P_{AB}) = 300 + 0.3 \cdot (160 - 40) = 336 \text{ SEK/MWh}
\end{aligned}$$

From system A this means that the load increases with 40 MW and in system B the load decreases with 40 MW. The result is shown in figure 2.9. In this case the system operators make profits since they buy 40 MW at a price of 328 SEK/MWh and they sell it for 336 SEK/MWh. The total benefit becomes:

$$C_{TSO} = 40 \cdot (336 - 328) = 320 \text{ SEK/h}$$

Solution using optimization We can now use the same formulation as in example 2.2a, but with the change that

$$P_{12} \leq 40$$

With this value the total operation cost becomes 147745 SEK. This means that the total operation cost of the system is lower than with no transmission but higher compared to a transmission capacity of 56 MW.

End of example 2.2b

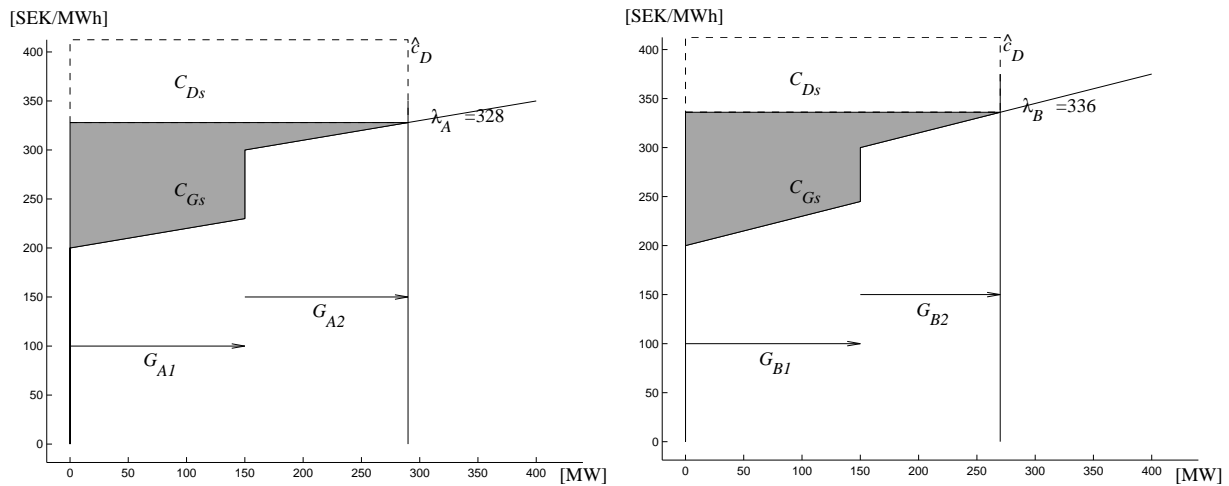


Figure 2.9. Example 2.2c

Graphic solution to example 2.2c Full counter buying between two areas means that for the price setting these areas are considered as one area, and the system operator then has to purchase power on one side and sell power on the other side in order to off-load the bottleneck. No transmission limits is the same as in figure 2.8. The price for both areas then becomes 331.2 SEK/MWh. But this assumes that it is possible to transmit 56 MW from area A to area B. In this example the limit is only 40 MW. This means that the system operator has to buy 16 MW on the high price side (= increase of generation in unit 2 from 140 to 156 MW) and sell 16 MW on the low price side (decrease in unit 2 from 120 to 104 MW). It must be noted the each MW has a different price depending on the marginal cost curves of the units. With an assumption that the purchase and selling is performed on marginal costs this means for the TSO:

$$\text{Purchase cost} = C_{TSO} = 16 \cdot 336 - 16 \cdot 328 = 128 \text{ SEK/h}$$

End of example 2.2c

Graphic solution to example 2.2d The explicit auction resulted in a transmission of 40 MW from A to B, i.e., $P_{AB} = 40$ MW. With this transmission the prices will become the same as in example 2.2b, i.e.

$$\begin{aligned} \lambda_A &= 328 \text{ SEK/MWh} \\ \lambda_B &= 336 \text{ SEK/MWh} \end{aligned}$$

We here assume that there is no flexibility of using the transmission capacity after the explicit auction is finished. This means in this case that although the price is lower in area A compared to B and there is still more capacity available on the interconnection, no more power will be transmitted. This is a consequence of the explicit auction. It must, however, be noted that there are administrative benefits of the explicit auction since each area can handle their own pricing since the trading with neighboring areas is already defined.

End of example 2.2d

Graphic solution to example 2.2e The explicit auction resulted in a transmission of 40 MW from A to B, i.e., $P_{AB} = 40$ MW. With this transmission the prices will become the same as in example 2.2d, i.e.

$$\begin{aligned}\lambda_A &= 328 \text{ SEK/MWh} \\ \lambda_B &= 336 \text{ SEK/MWh}\end{aligned}$$

We here assume that there is a flexibility in such a way that if there is a market need to change the interconnection operation, then this is performed. The price is now lower in A than in B, and there is room on the line which means that there is a market need to increase the transmission. The increase will continue until the prices are equal or if the transmission will be limited. In this case the transmission will increase up to a total of 56 MW as shown in example 2.2a.

The result of this flexible use was that the transmission increased with 16 MW (=56-40 MW). The total value (sum of consumer and producer surplus = changed total operation cost since the consumption level is not price sensitive) of this flexibility is the difference in operation cost = operation cost in example 2.2b - operation cost in example 2.2a = 147745-147681 = 64 SEK

End of example 2.2e

2.3 Transmission limits between several radial areas

"Radial" areas means that there are no loops between the areas. This means a comparatively small calculation extension from the previous section and this will be illustrated in examples below. But from practical handling point of view, a lot of possibilities can occur.

Example 2.3 Assume there are three neighboring systems, A, B and C where there are connections between A-B and B-C but no connection between A and C. In each system there is one $\hat{G}_A = \hat{G}_B = \hat{G}_C = 300$ MW unit with $c = 250$ SEK/MWh and $a = 0,3$ SEK/MWh/MW. In system A the load is $D_A = 70$ MW, in system B $D_B = 240$ MW, and in system C it is $D_C = 140$ MW. The system is shown in figure 2.10

2.3a Calculate the production in all units, the transmission between the systems and the prices in all systems when the capacity on the line is 100 MW. Assume area pricing.

2.3b Calculate the same as in 2.2a when the capacity of the line is 40 MW.

2.3c Calculate the same as in 2.2b, but now assume counter buying.

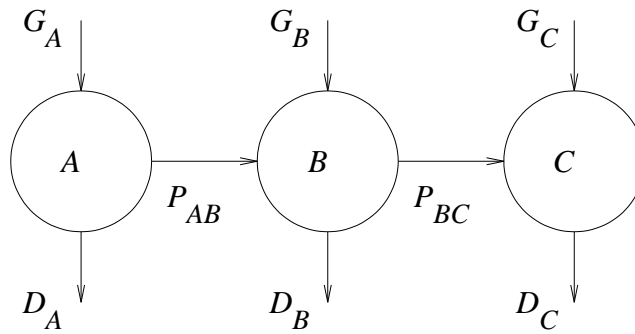


Figure 2.10. Example 2.3c

2.3d Assume line limits of 40 MW. Move 50 MW of load from B to C load so $D_B = D_C = 190$ MW. Now assume that there is one TSO for area A+B and one for area C. Assume counter trading within each TSO area and price areas between them. Calculate area prices, total production cost and costs/benefits for the TSO:s for the congestion handling.

2.3e The same system as the previous, but now the TSO in area A+B limits the export to area C in order to minimize their costs for counter trading. Calculate area prices, total production cost and costs/benefits for the TSO:s for the congestion handling.

For the analysis, start with the extremes, i.e., transmission limits = 0 MW or no transmission limits. Limits = 0 will result in local balance in each area, i.e.

$$\lambda_A = MC_A(G_A) = 250 + 0.3 \cdot D_A = 271 \text{ SEK/MWh}$$

$$\lambda_B = MC_B(G_B) = 250 + 0.3 \cdot D_B = 322 \text{ SEK/MWh}$$

$$\lambda_C = MC_C(G_C) = 250 + 0.3 \cdot D_C = 292 \text{ SEK/MWh}$$

Figure 2.11 shows this for the three areas.

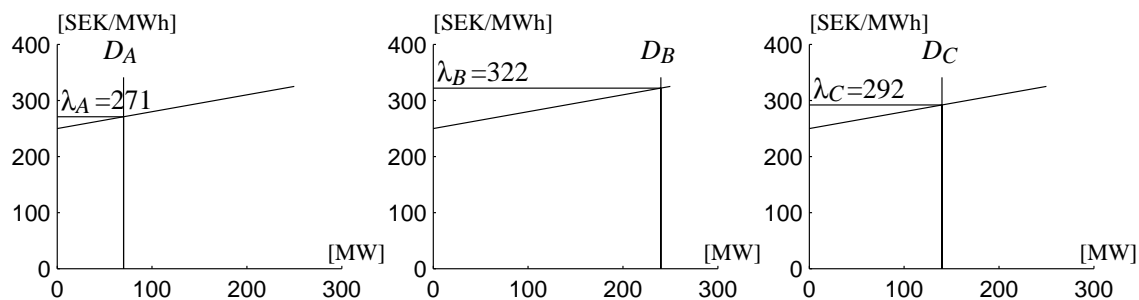


Figure 2.11. Example 2.3 Area prices with $P_{12} = P_{23} = 0$

Solution to example 2.3 With an assumption of full transmission between all three areas, this is the same as only one area with three units and a total load of $D = D_A + D_B + D_C =$

450 MW. Since there are three units with the same data, then these three units will share the production equally, i.e.

$$\begin{aligned}
 G_A = G_B = G_C &= \frac{450}{3} = 150 \text{ MW} \\
 \Rightarrow \\
 \lambda_A = \lambda_B = \lambda_C &= 250 + 0.3 \cdot 150 = 295 \text{ SEK/MWh} \\
 P_{AB} &= G_A - D_A = 150 - 70 = 80 \text{ MW} \\
 P_{BC} &= D_C - G_C = 140 - 150 = -10 \text{ MW}
 \end{aligned}$$

Solution using optimization The total surplus when there are trading possibilities is also here the difference between consumer value and producer cost. Considering the balances in each area according to eq. 2.9, and with a hypothetical maximal price, \hat{c}_D , this result in an optimization formulation as:

$$\begin{aligned}
 \max Z = C_D - \sum C_{Gi} &= \hat{c}_D(D_A + D_B + D_C) - \sum_{k=A}^C \left(c_{Gk} G_k + \frac{a_k}{2} G_k^2 \right) \\
 \text{when} \\
 0 \leq G_k &\leq 300, \quad k \in [A, C] \\
 G_A &= D_A + P_{AB} \\
 G_B &= D_B - P_{AB} + P_{BC} \\
 G_C &= D_C - P_{BC} \\
 \underline{P}_{AB} \leq P_{AB} &\leq \hat{P}_{AB} \\
 \underline{P}_{BC} \leq P_{BC} &\leq \hat{P}_{BC}
 \end{aligned}$$

The free variables in this optimization problem are G_k , $k \in [A, C]$, P_{AB} and P_{BC} . The upper and lower bounds for the transmission can be set to the same value if the consequences of a certain transmission is to be studied. By setting $P_{AB} = P_{BC} = 0$, one obtains the result if figure 2.11, where the total production cost ($= -Z$ if we neglect the consumer value) is 124815 SEK. $|P_{AB}| \leq 100$ and $|P_{BC}| \leq 100$ gives the same price in all areas ($= 295$ SEK/MWh) as shown above, where the total production cost is 122625 SEK. The result from the optimization include the values of these variables which maximizes the objective Z . This shows that the total surplus increases (124815-122625=2190 SEK/h) with this transmission possibility.

Solution to example 2.3a 100 MW transmission capacity means that the capacity is enough to make a one area system, since maximum 80 MW is needed for this. This means that the price in all areas is 295 SEK/MWh.

Solution to example 2.3b 40 MW transmission capacity is not enough to make a one area system. As shown in figure 2.11 the price is highest in area B, which means that this area will import 40 MW from area A. The optimum is 80 MW, but that is not possible. According to previous calculations, only 10 MW of transmission was needed between area B and C, so we start to assume that area B+C is one area, with one

price, and then we calculate the transmission afterwards to check if this is valid:

$$\begin{aligned}
\lambda_A &= c_A(G_A) = 250 + 0.3 \cdot (D_A + 40) = 283 \text{ SEK/MWh} \\
\lambda_{B+C} &= \text{equal split between } G_B \text{ and } G_C = \\
&= 250 + 0.3 \cdot \frac{D_B + D_C - 40}{2} = 301 \text{ SEK/MWh} \\
&\Rightarrow \\
G_B = G_C &= (D_B + D_C - 40)/2 = 170 \text{ MW} \\
P_{BC} &= D_C - G_C = 140 - 170 = -30 \text{ MW}
\end{aligned}$$

This means that the first assumption of one price area for systems B+C was valid since the transmission between the area with this assumption is within the limits. As shown in this example a division between area A and B+C occurs which results in different prices and a trade on the limit from the low price area to the high price area. Calculation of total production cost according to the optimization formulation give 122985 SEK which means that the benefit of the 40 MW transmission capacity compared to no capacity is $124815 - 122985 = 1830$ SEK/h, while the value of increasing the capacity of the line A-B from 40 to 80 MW is $122985 - 122625 = 360$ SEK/h. With area pricing and an assumption of that the TSO:s are splitting the trading value between each other, this means the benefits become:

$$C_{TSO-A} = C_{TSO-B} = 0.5 \cdot 40 \cdot (301 - 283) = 360 \text{ SEK/h}$$

Solution to example 2.3c Counter buying means that the price is first set as if there were no transmission limits. This means in this case that we will get the same prices as calculated previously, i.e., $\lambda_A = \lambda_B = \lambda_C = 295$ SEK/MWh. Then the TSO:s have to buy on one end of the congested line (A-B) and sell on the other side to release the congestion. With no transmission lines restrictions the market wants 80 MW but since only 40 MW is available, then the TSO:s have to buy 40 MW on one side and sell it on the other side. The prices for this will be the same as obtained in 2.3b. This means that the cost for the counter trading is

$$C_{TSO-A+B} = 40 \cdot (301 - 283) = 720 \text{ SEK/h}$$

An important issue here is that this calculation is based on that power is bought by the TSO in both area B and C and not only in area B where the congestion is.

Solution to example 2.3d We first assume that there are no congested lines. Since there are three units with the same data, then these three units will share the production equally (= sharing the total load which is the same as in example 2.3b), i.e.

$$\begin{aligned}
G_A = G_B = G_C &= \frac{450}{3} = 150 \text{ MW} \\
&\Rightarrow \\
\lambda_A = \lambda_B = \lambda_C &= 250 + 0.3 \cdot 150 = 295 \text{ SEK/MWh} \\
P_{AB} &= G_A - D_A = 150 - 70 = 80 \text{ MW} \\
P_{BC} &= D_C - G_C = 190 - 150 = 40 \text{ MW}
\end{aligned}$$

This means that TSO_{A+B} has to perform counter trading, but there is no area price difference. With the assumption that TSO_{A+B} also can use power from TSO_C for the needed counter trading (If this is cheaper and there is transmission possibilities), then the cost for the counter trading is

$$\begin{aligned}
 C(TSO_{A+B}) &= 40 \cdot \left[\underbrace{\left(250 + 0.3 \cdot \left[150 + \frac{40}{2}\right]\right)}_{\text{marginal cost for +40 MW in B+C}} - \underbrace{\left(250 + 0.3 \cdot [150 - 40]\right)}_{\text{marginal cost for -40 MW in A}} \right] = \\
 &= 40 \cdot 0.3 \cdot 60 = 720 \text{ SEK/h}
 \end{aligned}$$

So this is an increased cost for the actors in the market caused by the congested lines. But for the change of the total system surplus, one also have to consider the changed surplus of the producers since all power is sold to the marginal cost of the last produced MWh, while the marginal production cost is lower for the not-last produced MWh. The changed surplus is then the area corresponding to the triangle in figure 1.4. In area A the production is decreased and in area B+C increased due to the counter trade:

$$\begin{aligned}
 C_{Gs}(G_A) &= \underbrace{\left(250 \cdot 40 + \frac{0.3}{2}(150^2 - 110^2)\right)}_{\text{changed cost for -40 MW, eq. 1.11}} - 40 \cdot \underbrace{\left(250 + 0.3 \cdot [150 - 40]\right)}_{\text{marginal cost = paid level}} = \\
 &= \frac{0.3}{2}(150 - 110)(150 + 110) - 0.3 \cdot 40 \cdot 110 = \frac{0.3}{2} \cdot 40^2 = 240 \text{ SEK/h} \\
 C_{Gs}(G_B) &= \frac{0.3}{2} \cdot 20^2 = 60 \text{ SEK/h} \\
 C_{Gs}(G_C) &= \frac{0.3}{2} \cdot 20^2 = 60 \text{ SEK/h}
 \end{aligned}$$

which means that the total surplus has changed with $240+60+60-720=-360$ SEK/h. The surplus for the consumers has not changed because of the congestion since counter trade implies that there is still the same price for the consumers. Therefore it is enough to study the surplus of the producers and the traders.

Also this problem can be formulated with the optimization formula above. Without any restrictions the total production cost ($=-Z$) becomes 122625 SEK/h and with a 40 MW restriction it becomes 122985 SEK/h which means an increased cost (=decreased surplus) of $122985-122625=360$ SEK/h which is the same value as calculated previously.

Solution to example 2.3e As shown above the counter trade leads to a need for TSO_{A+B} to perform counter trading which leads to a cost for this company. One possibility that the TSO may have is to restrict the transmission to neighboring systems, i.e., TSO_C , if this decreases these costs. With the assumption that TSO_{A+B} forbids export to area C this will lead to the following situation, since G_C will supply D_C and counter trade

within system $A + B$ will lead to an equal split of load $D_A + D_B$ between G_A and G_B :

$$\begin{aligned}
 G_A = G_B &= \frac{70 + 190}{2} = 130 \text{ MW} \\
 G_C &= D_C = 190 \text{ MW} \\
 &\Rightarrow \\
 \lambda_A = \lambda_B &= 250 + 0.3 \cdot 130 = 289 \text{ SEK/MWh} \\
 \lambda_C &= 250 + 0.3 \cdot 307 \text{ SEK/MWh} \\
 P_{AB} &= G_A - D_A = 130 - 70 = 60 \text{ MW} \\
 P_{BC} &= D_C - G_C = 190 - 190 = 0 \text{ MW}
 \end{aligned}$$

As shown here this still leads to a need of counter trade, since the limit is 40 MW, but the optimum is 60 MW without restrictions. So TSO_{A+B} then has to counter buy 20 MW, i.e., an increased production in area B and a decreased production in area A . The cost for this becomes.

$$\begin{aligned}
 C(TSO_{A+B}) &= 20 \cdot \left[\underbrace{(250 + 0.3 \cdot [130 + 20])}_{\text{marginal cost for +20 MW in B}} - \underbrace{(250 + 0.3 \cdot [130 - 20])}_{\text{marginal cost for -20 MW in A}} \right] = \\
 &= 20 \cdot 0.3 \cdot 40 = 240 \text{ SEK/h}
 \end{aligned}$$

For the change of the total system surplus, one also have to consider the changed surplus of the producers. In area A the production is decreased while it is increased in area B due to the counter trade:

$$\begin{aligned}
 C_{Gs}(G_A) &= \underbrace{\left(250 \cdot 20 + \frac{0.3}{2} (150^2 - 130^2) \right)}_{\text{changed cost for -20 MW}} - 20 \cdot \underbrace{(250 + 0.3 \cdot [150 - 20])}_{\text{marginal cost = paid level}} = \\
 &= \frac{0.3}{2} \cdot 20^2 = 60 \text{ SEK/h} \\
 C_{Gs}(G_B) &= \frac{0.3}{2} \cdot 20^2 = 60 \text{ SEK/h}
 \end{aligned}$$

which means that the total surplus has changed with $60 + 60 - 240 = -120$ SEK/h caused by the counter trade. It can be noted that from the TSO:s point of view, limiting the transmission to area C decreases the cost for counter buying with $720 - 240 = 480$ SEK/h, so this strategy is profitable. But for the whole system, the operation cost increases with 120 SEK/h.

2.4 Transmission limits between several meshed areas

”Meshed” means that there are possible loops in the system. If this is the case, then it is essential to model how power flow through the loops. Since there are possible parallel paths between some nodes, and possible limits on some corridors, it is important to

study whether the corridor limits the possible transfer of power or if more power flows in a parallel path.

Figure 2.12 shows an example of such a system. In the figure there is a power system with three areas. Between the areas there are transmission lines with a possibility to transmit power. Each line has a parameter, the so-called *reactance*, denoted x . There are also other parameters on transmission lines, such as resistance and capacitance, but the most relevant one for transmission calculations is the reactance.

Below two different methods of how to handle these systems are described. In the *DC Power Flow method* the flows in parallel loops are modeled as the physical flows in interconnected AC grids. this means that the impact from line reactance is considered. In the *Multi-area method* it is assumed that flows in parallel loops can be controlled in order to use both loops at their maximum capacity. this means that with this method the line reactance is neglected.

2.4.1 DC Power Flow method

This method take into account the physical network parameters in order to consider possible loop flows. It is called **DC Power Flow**, since the equations are rather close to power flows in DC grids (although we here consider an AC grid). The power transmission on a transmission line can, if we neglect the resistance and capacitance, be calculated as:

$$P_{12} = \frac{U_1 U_2}{X_{12}} \sin(\delta_1 - \delta_2) \quad (2.10)$$

where

P_{12} = Power transmission on the line between node 1 and 2

U_1 and U_2 = Voltage levels in the two nodes

X_{12} = Line reactance between the two nodes

δ_1 and δ_2 = Voltage angles in the two nodes referred to some reference angle

It is then common to assume that the angle difference over a line is comparatively small and that the voltages are rather constant ($\approx 1 = 100\%$). This results in the approximative formula:

$$P_{12} = B_{12}(\delta_1 - \delta_2) \quad (2.11)$$

where

$B_{12} = 1/X_{12}$

The reactance X of a single line is proportional to the length of the line, i.e., if the line is double as long, then the reactance is double as high. For parallel lines the reactance is reduced, so two parallel lines have around half the reactance compared to only one line. These are approximative estimations and are based on the assumption that all lines have the same data.

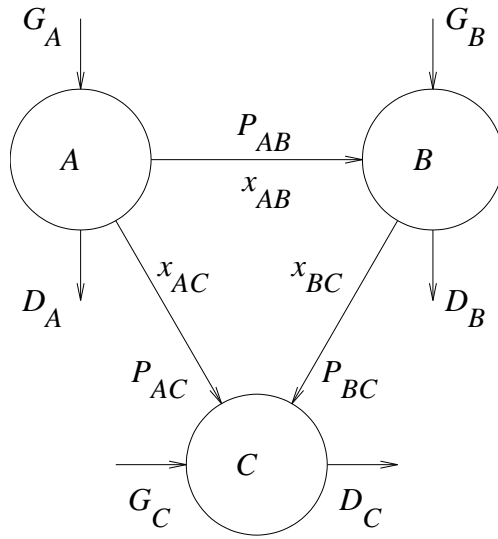


Figure 2.12. A meshed power system

With the above stated approximations (constant voltages, small angle differences, negligible capacitance and resistance) equation 2.11 becomes representative for all transmission lines such as the ones in figure 2.12. This means that a system of equations can be formulated for this system:

$$\mathbf{P}_{\text{trans}} = \begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} = \begin{bmatrix} B_{AB} & -B_{AB} & 0 \\ B_{AC} & 0 & -B_{AC} \\ 0 & B_{BC} & -B_{BC} \end{bmatrix} \begin{bmatrix} \delta_A \\ \delta_B \\ \delta_C \end{bmatrix} = \mathbf{B}' \boldsymbol{\Delta}' \quad (2.12)$$

It can though be noted that the \mathbf{B}' -matrix has not full rang, since the power transmission is related to the *angle differences* and not only the angles, c.f. eq. 2.11. The solution is to set one of the angles to zero, i.e., to set it as reference angle. If we in eq. 2.12 set $\delta_A = 0$, then that equation can be reformulated as

$$\mathbf{P}_{\text{trans}} = \begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} = \begin{bmatrix} -B_{AB} & 0 \\ 0 & -B_{AC} \\ B_{BC} & -B_{BC} \end{bmatrix} \begin{bmatrix} \delta_B \\ \delta_C \end{bmatrix} = \mathbf{B} \boldsymbol{\Delta} \quad (2.13)$$

In each node the net production is always the same as the net export, i.e.:

$$\mathbf{P}'_{\text{node}} = \begin{bmatrix} G_A - D_A \\ G_B - D_B \\ G_C - D_C \end{bmatrix} = \begin{bmatrix} P_{AB} + P_{AC} \\ -P_{AB} + P_{BC} \\ -P_{AC} - P_{BC} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} = \mathbf{C}' \mathbf{P}_{\text{trans}} \quad (2.14)$$

It can be noted that this equation does not have full rang either. There are no losses on the lines which means that the sum of all net productions in each not (= sum of all elements in the vector $\mathbf{P}'_{\text{node}}$) is equal to zero. If we take away the first row, then the

equation become:

$$\mathbf{P}_{\text{node}} = \begin{bmatrix} G_B - D_B \\ G_C - D_C \end{bmatrix} = \begin{bmatrix} -P_{AB} + P_{BC} \\ -P_{AC} - P_{BC} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} = \mathbf{C}\mathbf{P}_{\text{trans}} \quad (2.15)$$

The aim is now to reformulate these equations in order to get a relation between net production in the nodes and the transmission between the nodes, i.e.:

$$\mathbf{P}_{\text{trans}} = \mathbf{M}\mathbf{P}_{\text{node}} \quad (2.16)$$

The elements of the \mathbf{M} -matrix are the *Power Transfer Distribution Factors* - PTDF. They show the connection between the the changed net production in a certain node and the changed transmission on a certain line or in a certain transmission corridor:

$$PTDF_{ik} = \mathbf{M}_{ik} = \frac{\Delta \mathbf{P}_{\text{trans}-i}}{\Delta \mathbf{P}_{\text{node}-k}} = \frac{\text{changed transmission on line } i}{\text{changed net production in node } k} \quad (2.17)$$

It can be noted that PTDF is the general notation for the impact on the transmission from smaller *changes* in the production system. In reality the power system is a non-linear system which means that the calculation of the PTDF:s is based a linearized model of the system. Here the DC Power Flow method is used to model the relation between production and transmission and this method uses a linearized description for this relation, which means that the \mathbf{M} -matrix elements can be used directly as the PTDF:s.

By combining eqs. 2.13 and 2.15 we receive:

$$\mathbf{P}_{\text{node}} = \mathbf{C}\mathbf{B}\Delta \quad (2.18)$$

\Rightarrow

$$\Delta = (\mathbf{C}\mathbf{B})^{-1}\mathbf{P}_{\text{node}} \quad (2.19)$$

\Rightarrow

$$\mathbf{M} = \mathbf{B}(\mathbf{C}\mathbf{B})^{-1} \quad (2.20)$$

Example 2.4 Assume there are three neighboring systems, A , B and C where there are connections between A - B , B - C and A - C . In each system there is one $\hat{G}_A = \hat{G}_B = \hat{G}_C = 300$ MW unit with $c = 250$ SEK/MWh and $a = 0,3$ SEK/MWh/MW. In system A the load is $D_A = 70$ MW, in system B $D_B = 240$ MW, and in system C it is $D_C = 140$ MW. The system is shown in figure 2.12. The impedances are $x_{AB} = x_{BC} = 1$, and $x_{AC} = 2$ (i.e. this line is double as long).

2.4a Calculate the production in all units, the transmission between the systems and the prices in all systems when the capacity on the lines is 100 MW. Assume area pricing.

2.4b Calculate the same as in 2.4a, but now assume $x_{AB} = x_{BC} = 1$ and $x_{AC} = \infty$. This means in reality the same case as studied in example 2.3

2.4c Calculate the same as in 2.4a when the capacity of the line is 40 MW.

Start with an analysis concerning the coupling between net production in different nodes and the power transport on the interconnections. The matrices in eq. 2.20 are here

$$\begin{aligned}
 \mathbf{C} &= \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix} & \mathbf{B} &= \begin{bmatrix} -1 & 0 \\ 0 & -0.5 \\ 1 & -1 \end{bmatrix} \\
 \mathbf{CB} &= \begin{bmatrix} 2 & -1 \\ -1 & 1.5 \end{bmatrix} & \Rightarrow & (\mathbf{CB})^{-1} = \begin{bmatrix} 0.75 & 0.5 \\ 0.5 & 1 \end{bmatrix} \\
 \mathbf{B}(\mathbf{CB})^{-1} &= \begin{bmatrix} -0.75 & -0.5 \\ -0.25 & -0.5 \\ 0.25 & -0.5 \end{bmatrix} & \Rightarrow & \begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} = \begin{bmatrix} -0.75 & -0.5 \\ -0.25 & -0.5 \\ 0.25 & -0.5 \end{bmatrix} \begin{bmatrix} G_B - D_B \\ G_C - D_C \end{bmatrix}
 \end{aligned}$$

It is possible to check this result since there must be a balance in each node, as

$$\begin{aligned}
 P_{AB} + P_{AC} &= (-0.75 - 0.5)(G_B - D_B) + (-0.5 - 0.5)(G_C - D_C) = -(G_B - D_B) \\
 &\quad -(G_C - D_C) = G_A + D_A \quad \text{since total net production} = 0 \\
 -P_{AB} + P_{BC} &= (0.75 + 0.25)(G_B - D_B) + (0.5 - 0.5)(G_C - D_C) = G_B - D_B \\
 -P_{AC} - P_{BC} &= (-0.25 + 0.25)(G_B - D_B) + (0.5 + 0.5)(G_C - D_C) = G_C - D_C
 \end{aligned}$$

i.e., the node balance is fulfilled in each node.

Solution to example 2.4a With an assumption of full transmission between all three areas, this is the same as only one area with three units and a total load of $D = D_A + D_B + D_C = 450$ MW. Since there are three units with the same data, then these three units will share the production equally, i.e.

$$\begin{aligned}
 G_A = G_B = G_C &= \frac{450}{3} = 150 \text{ MW} \\
 &\Rightarrow \\
 \lambda_A = \lambda_B = \lambda_C &= 250 + 0.3 \cdot 150 = 295 \text{ SEK/MWh} \\
 \begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} &= \begin{bmatrix} -0.75 & -0.5 \\ -0.25 & -0.5 \\ 0.25 & -0.5 \end{bmatrix} \begin{bmatrix} 150 - 240 \\ 150 - 140 \end{bmatrix} = \begin{bmatrix} 62.5 \\ 17.5 \\ -27.5 \end{bmatrix} \text{ MW}
 \end{aligned}$$

This implies that the transmission is within the borders of the capacity for each line.

Solution using optimization The total surplus when there are trading possibilities is also here the difference between consumer value and producer cost. Considering the balances in each area according to eq. 2.9, and with a hypothetical maximal price, \hat{c}_D ,

this result in an optimization formulation as:

$$\begin{aligned}
\max Z = C_D - \sum C_{Gi} &= \hat{c}_D(D_A + D_B + D_C) - \sum_{k=A}^C \left(c_{Gk} G_k + \frac{a_k}{2} G_k^2 \right) \\
\text{when} \\
0 \leq G_k &\leq 250, \quad k \in [A, C] \\
P_{AB} + P_{AC} &= G_A - D_A \\
P_{AB} &= -0.75(G_B - D_B) - 0.5(G_C - D_C) \\
P_{AC} &= -0.25(G_B - D_B) - 0.5(G_C - D_C) \\
P_{BC} &= 0.25(G_B - D_B) - 0.5(G_C - D_C) \\
\underline{P}_{AB} \leq P_{AB} &\leq \hat{P}_{AB} \\
\underline{P}_{BC} \leq P_{BC} &\leq \hat{P}_{BC} \\
\underline{P}_{AC} \leq P_{AC} &\leq \hat{P}_{AC}
\end{aligned}$$

The free variables in this optimization problem are G_k , $k \in [A, C]$, P_{AB} , P_{AC} and P_{BC} . The upper and lower bounds for the transmission can be set to the same value if the consequences of a certain transmission is to be studied. Setting $|P_{AB}| \leq 100$, $|P_{AC}| \leq 100$ and $|P_{BC}| \leq 100$ gives the same price in all areas ($= 295$ SEK/MWh) as shown above, where the total production cost is 122625 SEK. The result from the optimization include the values of these variables which maximizes the objective Z .

Solution to example 2.4b As in the solution of example 2.4a, we first assume no limits on transmission between all three areas, i.e., one area with three units and a total load of $D = D_A + D_B + D_C = 450$ MW. Since there are three units with the same data, then these three units will share the production equally, i.e.

$$\begin{aligned}
G_A = G_B = G_C &= \frac{450}{3} = 150 \text{ MW} \\
\Rightarrow \\
\lambda_A = \lambda_B = \lambda_C &= 250 + 0.3 \cdot 150 = 295 \text{ SEK/MWh}
\end{aligned}$$

With $x_{AC} = \infty$, the **B**-matrix will change to

$$\begin{aligned}
\mathbf{B} = \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 1 & -1 \end{bmatrix} &\Rightarrow \mathbf{B}(\mathbf{CB})^{-1} = \begin{bmatrix} -1 & -1 \\ 0 & 0 \\ 0 & -1 \end{bmatrix} \\
&\Rightarrow \\
\begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} &= \begin{bmatrix} -1 & -1 \\ 0 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 150 - 240 \\ 150 - 140 \end{bmatrix} = \begin{bmatrix} 80 \\ 0 \\ -10 \end{bmatrix} \text{ MW}
\end{aligned}$$

This is the same result as from example 2.3. This means that the case with a radial system can be seen as a special case of the meshed system.

Solution to example 2.4c In this example the restrictions are set to 40 MW. If there are no restrictions, then the transmission on the line A-B is $P_{AB} = 62.5$ MW as shown

above in example 2.4a. The transmission on the other lines are below 40 MW in that example. This means that the flow on the line A-B now will be reduced to 40 MW. This means

$$\begin{aligned}
 P_{AB} &= -0.75(G_B - D_B) - 0.5(G_C - D_C) \\
 40 &= -0.75(G_B - 240) - 0.5(G_C - 140) \\
 &\Rightarrow \\
 G_C &= 420 - 1.5G_B
 \end{aligned}$$

Total production is equal to total consumption:

$$\begin{aligned}
 G_A + G_B + G_C &= D_A + D_B + D_C \\
 G_A + G_B + 420 - 1.5G_B &= 70 + 240 + 140 \\
 G_A &= 0.5G_B + 30
 \end{aligned}$$

The total operation cost can now be calculated as a function of G_B

$$\begin{aligned}
 C_{Gtot} &= \sum_{k=A}^C \left(c_{Gk} G_k + \frac{a_k}{2} G_k^2 \right) \\
 &= 250(G_A + G_B + G_C) + (0.5G_B + 30)^2 + G_B^2 + (420 - 1.5G_B)^2 \\
 &= 250 \cdot 450 + 3.5G_B^2 - 1230G_B + 177300
 \end{aligned}$$

Minimum cost is obtained when the derivative of total cost is zero

$$\begin{aligned}
 \frac{dC_{Gtot}}{dG_B} &= 0 = 7G_B - 1230 \\
 &\Rightarrow \\
 G_B &= 1230/7 = 175.71 \text{ MW} \\
 G_C &= 420 - 1.5G_B = 156.43 \text{ MW} \\
 G_A &= 0.5G_B + 30 = 117.86 \text{ MW} \\
 P_{AC} &= -0.25(G_B - D_B) - 0.5(G_C - D_C) = 7.86 \text{ MW} \\
 P_{BC} &= 0.25(G_B - D_B) - 0.5(G_C - D_C) = -24.29 \text{ MW}
 \end{aligned}$$

As shown in the result the transmission on the lines A-C and B-C is below the limit 40 MW. The prices in the areas now correspond to the marginal costs in the units in each area as:

$$\begin{aligned}
 \lambda_A &= 250 + 0.3 \cdot 117.86 = 285.36 \text{ SEK/MWh} \\
 \lambda_B &= 250 + 0.3 \cdot 175.71 = 302.71 \text{ SEK/MWh} \\
 \lambda_C &= 250 + 0.3 \cdot 156.43 = 296.93 \text{ SEK/MWh}
 \end{aligned}$$

These prices are the ones that will occur in each area if the congestions are handled with market splitting. If the counter buying method is used, then the system price

becomes 295 SEK/MWh as in example 2.4b. In that example the transmission on the lines became $P_{AB} = 80$ MW, $P_{AC} = 0$ MW and $P_{BC} = -10$ MW. To obtain the suitable balance in the system, then the *TSOs* in the areas have to trade between each other in order to change the unconstrained transmission to the suitable one.

$$\begin{aligned} P_{AB} &\Rightarrow \text{side A: buy 40-80 MW, side B: sell 40-80 MW} \\ P_{AC} &\Rightarrow \text{side A: buy 7.86-0 MW, side C: sell 7.86-0 MW} \\ P_{BC} &\Rightarrow \text{side B: buy -24.29+10 MW, side C: sell -24.29+10 MW} \end{aligned}$$

The trading results for the *TSO*:s in the different nodes are

$$\begin{aligned} TSO_A &\Rightarrow \text{buy in total } -40 + 7.86 = -32.14 \text{ MW} \\ TSO_B &\Rightarrow \text{buy in total } 40 - 14.29 = 25.71 \text{ MW} \\ TSO_C &\Rightarrow \text{buy in total } 14.29 - 7.86 = 6.43 \text{ MW} \end{aligned}$$

This problem can also be formulated as an optimization problem, as shown above. Here the upper and lower limits of the transmission are set to 40 MW and -40 MW respectively. The result then becomes as shown previously.

2.4.2 Multi-area method

This method is based on the assumption that the flows on the transmission lines between the different areas can be controlled in order to use the capacity as efficient as possible.

For the system in figure 2.12 this means that there has to be a load balance in each node as in equation 2.14. Also here the \mathbf{C}' -matrix has not full rang, since total production is equal to total load (= sum of all elements in the vector $\mathbf{P}'_{\text{node}}$. Neglecting the power balance in area *A* then results in (=eq. 2.15) as:

$$\mathbf{P}_{\text{node}} = \begin{bmatrix} G_B - D_B \\ G_C - D_C \end{bmatrix} = \begin{bmatrix} -P_{AB} + P_{BC} \\ -P_{AC} - P_{BC} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} = \mathbf{CP}_{\text{trans}} \quad (2.21)$$

So we still have to consider the area balance, but we neglect the coupling between area voltage angles and power flows on transmission lines.

If we first add a parameter denoted K (=total net transmission) on the left hand side, then eq. 2.21 becomes

$$\begin{bmatrix} K \\ G_B - D_B \\ G_C - D_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} \quad (2.22)$$

By inversion of the matrix in eq. 2.22 one get

$$\begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & -1 & -2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} K \\ G_B - D_B \\ G_C - D_C \end{bmatrix} \quad (2.23)$$

This is one way to show that the flows are not exactly defined when the generation is defined. There are an infinite number of possibilities (= different values of K) to fulfill the area balance requirements. But still there is not a full freedom to select K , since the transmission has to be within the limits.

Example 2.5 *This is the same example as 2.4, but here the multi-area modeling is used instead of the DC load flow. Assume there are three neighboring systems, A, B and C where there are connections between A-B, B-C and A-C. In each system there is one $\hat{G}_A = \hat{G}_B = \hat{G}_C = 300$ MW unit with $c = 250$ SEK/MWh and $a = 0,3$ SEK/MWh/MW. In system A the load is $D_A = 70$ MW, in system B $D_B = 240$ MW, and in system C it is $D_C = 140$ MW. The system is shown in figure 2.12.*

2.5a Calculate the production in all units, the transmission between the systems and the prices in all systems when the capacity on the lines is 100 MW. Assume area pricing.

2.5b Calculate the same as in 2.4a when the capacity of the line is 40 MW.

Solution to example 2.5a With an assumption of full transmission between all three areas, this is the same as only one area with three units and a total load of $D = D_A + D_B + D_C = 450$ MW. Since there are three units with the same data, then these three units will share the production equally, i.e.

$$\begin{aligned}
 G_A = G_B = G_C &= \frac{450}{3} = 150 \text{ MW} \\
 \Rightarrow \\
 \lambda_A = \lambda_B = \lambda_C &= 250 + 0.3 \cdot 150 = 295 \text{ SEK/MWh} \\
 \begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 1 \\ -1 & -1 & -2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} K \\ 150 - 240 \\ 150 - 140 \end{bmatrix} = (K = 0) \begin{bmatrix} 10 \\ 70 \\ -80 \end{bmatrix} \text{ MW}
 \end{aligned}$$

This implies that the transmission is within the borders of the capacity for each line. There are also other possible solutions (all fulfilling area balances and transmission limits)

$$\begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} = (K = 10) \begin{bmatrix} 20 \\ 60 \\ -70 \end{bmatrix} \quad (K = 30) \begin{bmatrix} 40 \\ 40 \\ -50 \end{bmatrix} \quad (K = -20) \begin{bmatrix} -10 \\ 90 \\ -100 \end{bmatrix}$$

Solution using optimization The total surplus when there are trading possibilities is also here the difference between consumer value and producer cost. Considering the balances in each area according to eq. 2.9, and with a hypothetical maximal price, \hat{c}_D ,

this result in an optimization formulation as:

$$\begin{aligned} \max Z = C_D - \sum C_{Gi} &= \hat{c}_D(D_A + D_B + D_C) - \sum_{k=A}^C \left(c_{Gk} G_k + \frac{a_k}{2} G_k^2 \right) \\ \text{when} & \\ 0 \leq G_k &\leq 300, \quad k \in [A, C] \\ G_A - D_A &= P_{AB} + P_{AC} \\ G_B - D_B &= -P_{AB} + P_{BC} \\ G_C - D_C &= -P_{AC} - P_{BC} \\ \underline{P}_{AB} \leq P_{AB} &\leq \hat{P}_{AB} \\ \underline{P}_{BC} \leq P_{BC} &\leq \hat{P}_{BC} \\ \underline{P}_{AC} \leq P_{AC} &\leq \hat{P}_{AC} \end{aligned}$$

The free variables in this optimization problem are G_k , $k \in [A, C]$, P_{AB} , P_{AC} and P_{BC} . The upper and lower bounds for the transmission can be set to the same value if the consequences of a certain transmission is to be studied. Setting $|P_{AB}| \leq 100$, $|P_{AC}| \leq 100$ and $|P_{BC}| \leq 100$ gives the same price in all areas (= 295 SEK/MWh) as shown above, where the total production cost is 122625 SEK. The result from the optimization include the values of these variables which maximizes the objective Z . This is though a so-called degenerated solution without a unique optimum since there are several transmission possibilities. The total production is though the same for all solutions, since the different possible optimal solutions have the same generation cost. Depending on how the solution program works for the optimization problem, one can get any of the possible solutions shown above, or any other solution that fulfills the restriction.

Solution to example 2.5b In this example the transmission limits are set to 40 MW. In example 2.5a, the net production in the three areas are

$$\begin{bmatrix} G_A - D_A \\ G_B - D_B \\ G_C - D_C \end{bmatrix} = \begin{bmatrix} 150 - 70 \\ 150 - 240 \\ 150 - 140 \end{bmatrix} = \begin{bmatrix} 80 \\ -90 \\ 10 \end{bmatrix}$$

With a restriction of 40 MW on each line, then the transmission to area B has to be reduced with 10 MW, if both lines should be exactly on their limits. This means that the production in area B has to be increased with 10 MW compared to the previous example, i.e. $G_B = 150 + 10 = 160$ MW. This will then lead to that $P_{AB} = 40$ MW and $P_{BC} = -40$ MW. The increased generation in area B then has to be compensated with a decreased production in area A and C keeping the transmission on line A-C within their restrictions. Since the marginal costs of G_A and G_C decreases with the same rate, the best way is to decrease the generation with 5 MW in each area. This leads to

$$\begin{aligned} G_A &= 150 - 5 = 145 \text{ MW} \\ G_C &= 150 - 5 = 145 \text{ MW} \\ P_{AC} &= G_A - D_A - P_{AB} = 145 - 70 - 40 = 35 \text{ MW} \end{aligned}$$

This means that the transmission on line A-C is below 40 MW, i.e., this is the optimal solution. If this had not been the case, then the production had to be distributed differently between area A and C . The area prices become

$$\begin{aligned}\lambda_B &= 250 + 0.3 \cdot 160 = 298 \text{ SEK/MWh} \\ \lambda_A = \lambda_C &= 250 + 0.3 \cdot 145 = 293.5 \text{ SEK/MWh}\end{aligned}$$

This problem can also be solved using the optimization formulation above. The result is then the same. The total production cost is 122647 SEK, which is an increase with 22 SEK depending on that the increased production cost for +10 MW in B is higher than the decreased cost of 10 MW less generation in area A and C .

It can be noted that equation 2.23 is based on node balances and will therefore be valid also here. But with the restrictions this means that it will only be valid for one value of K . The application here becomes:

$$\begin{bmatrix} P_{AB} \\ P_{AC} \\ P_{BC} \end{bmatrix} = \begin{bmatrix} 40 \\ 35 \\ -40 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & -1 & -2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} K = 35 \\ G_B - D_B = 160 - 240 \\ G_C - D_C = 145 - 140 \end{bmatrix}$$

Chapter 3

Market power

Market power means that a producer (or a consumer) could affect the market prices in such a significant way so it has an impact on the operation of the market.

3.1 Monopolistic behavior

Example 3.1 Assume a production system according to table 3.1. The base load is 310 MW and has a price sensitivity of $b=0.5$ when the price increases above 220 SEK/MWh. All units are owned by a profit maximizing monopoly.

Unit	\hat{G}_i	c_{Gi}	a
i=1	140 MW	100 SEK/MWh	0.4 SEK/MWh/MW
i=2	260 MW	192 SEK/MWh	0.4 SEK/MWh/MW

Table 3.1. Data for example 3.1

3.1a Calculate the power price and production when the price is set according to marginal cost.

3.1b Calculate the change in consumer and producer surplus when the monopolistic producer reduces its production to a total of 180 MW.

3.1c Calculate the production level (= total consumption), that maximizes the profit for the producer.

Solution to example 3.1a This is the same type of calculations as in example 1.2, it is just that the parameters are changed.

In the cross point (production=demand= D , price = marginal cost = λ) between the demand curve and the supply curve, the following expressions are valid:

$$\begin{aligned} \lambda &= 192 + 0.4 \cdot (D - 140) && \text{Supply curve, eq. ??} \\ \lambda &= \frac{310-D}{0.5} + 220 && \text{Demand curve, eq. 1.3} \end{aligned}$$

This forms a linear system of equations and the solution is the point $D = 293.33$ MW and $\lambda = 253.33$ SEK/MWh. This means that the price, corresponding to the marginal cost in the last unit is 253.33 SEK/MWh. The production level in the second station is $G_2 = 293.33 - 140 = 153.33$ MW. This price cross is shown in figure 3.1.

The operation cost in the two units can be calculated according to equation 1.11 as

$$\begin{aligned} C_{G1} &= 100 \cdot 140 + \frac{0.4}{2} 140^2 = 17920 \text{ SEK/h} \\ C_{G2} &= 192 \cdot 153.33 + \frac{0.4}{2} 153.33^2 = 34142 \text{ SEK/h} \end{aligned}$$

The total operation cost is the sum of the production in the two units:

$$C_{Gtot} = \sum_{i \in I_n} C_{Gin} = C_{G1} + C_{G2} = 52062 \text{ SEK/h}$$

Solution to example 3.1b In example 3.1a there was an assumption of perfect competition. This means that the price is set by the intersection between the marginal cost curves for production and the corresponding marginal value curve for consumption. This is shown in figure 3.1. In general terms one can say that if the producer here increases the price, then the producers profit increases initially, but the consumers will pay more, so their surplus decreases. It must though be noted that the consumption is price dependent.

Figure 3.1 shows that if the producer increases the price from 253.33 SEK/MWh to 480 SEK/MWh, then the load is reduced to

$$\begin{aligned} \lambda_D &= \frac{\hat{D} - D}{b} + c_D \\ \Rightarrow \\ D &= \hat{D} - b(\lambda_D - c_D) = 310 - 0.5(480 - 220) = 180 \text{ MW} \end{aligned}$$

since the consumers are assumed to be price sensitive. Compared to the case with the price 253.33 SEK/MWh, the producer will make a loss because of lowered consumption. The reduced income (= area B+C in figure 3.1) is then

$$B + C = \text{price} \cdot (\text{reduced volume}) = 253.33(293.33 - 180) = 28711 \text{ SEK}$$

But there is also a decreased cost since the producer will produce less depending on the reduced load. The reduced cost (= area B in figure 3.1) is

$$\begin{aligned} B &= \text{use eq. 1.11} = C_{G2}(253.33 - 140) - C_{G2}(180 - 140) = \\ &= (113.33 - 40) \left[192 + \frac{0.4}{2}(113.33 + 40) \right] = 26142 \text{ SEK} \end{aligned}$$

which implies that the decreased profit because of decreased consumption and cost for the producer (= area C in figure 3.1) is

$$C = (B + C) - B = 28711 - 26142 = 2569 \text{ SEK}$$

On the other hand, there is also an increased income since all the remaining consumers pay an increased price. This increased income for the producer (= area A in figure 3.1) is

$$A = (\text{price increase}) \cdot \text{volume} = (480 - 253.33)180 = 40800 \text{ SEK}$$

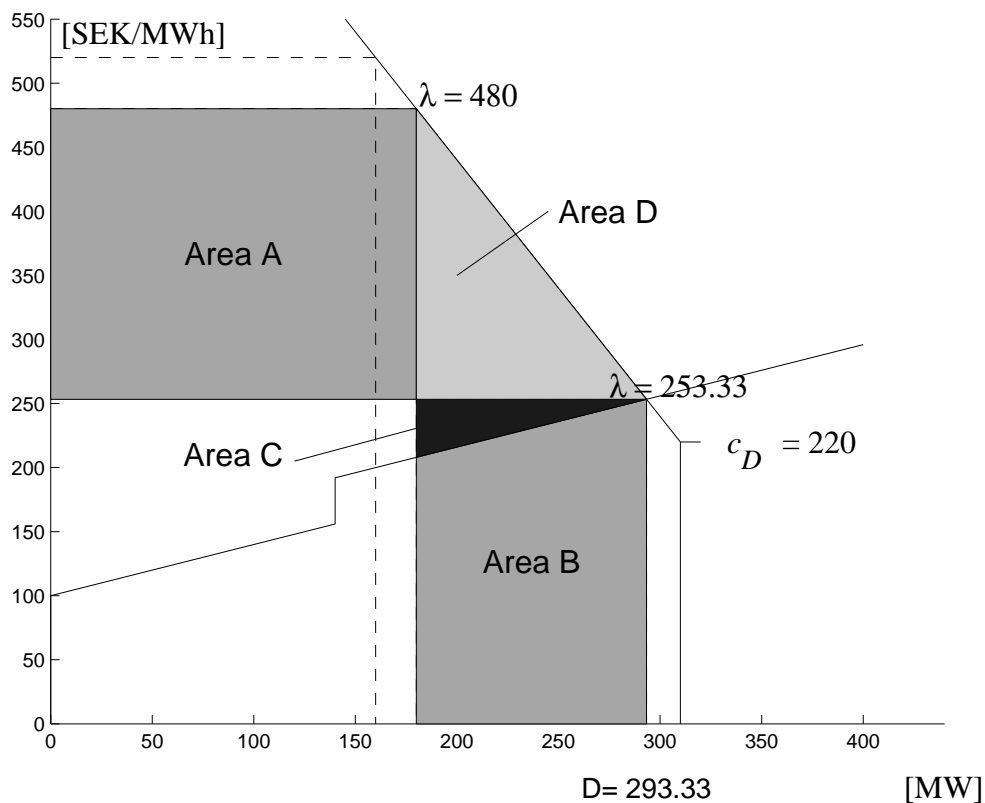


Figure 3.1. Demand and supply for two price levels in example 3.1b. The dashed line corresponds to the solution in example 3.1c

Since $A > C$, it is profitable for the producer to sell at the price 480 SEK/MWh instead of 253.33 SEK/MWh. It must though be noted that the consumer surplus is reduced, since they have to pay a higher price. As shown in eq. 1.9 the consumer surplus is the area between the consumer curve and the price level. This means that the changed consumer surplus is area $A+D$, i.e.

$$A + D = 40800 + \frac{1}{2}(480 - 253.33)(113.33 - 40) = 53644 \text{ SEK}$$

This means that the total surplus is reduced at price level 480 SEK/MWh compared to 253.33 SEK/MWh since $A + D > A - C$.

Solution to example 3.1c The total surplus for the producer as a function of demand has been calculated in example 1.2, but here the data are changed and the result becomes:

$$\begin{aligned}
C_{Gtot} &= \sum_{i \in I} \left[c_{Gi} G_i + \frac{a}{2} G_i^2 \right] = \\
&= \begin{cases} 100D + \frac{0.4}{2} D^2 & \text{if } D \leq 140 \text{ MW} \\ 17920 + 192(D - 140) + \frac{0.4}{2} (D - 140)^2 & \text{if } 140 \text{ MW} \leq D \leq 400 \text{ MW} \end{cases} = \\
&= \begin{cases} 100D + 0.2D^2 & \text{if } D \leq 140 \text{ MW} \\ -5040 + 136D + 0.2D^2 & \text{if } 140 \text{ MW} \leq D \leq 400 \text{ MW} \end{cases} \\
C_{Gv} &= \lambda_D D = \left[\frac{\hat{D} - D}{b} + c_D \right] D = \left[\frac{310}{0.5} + 220 \right] D - \frac{1}{0.5} D^2 \\
C_{Gs} &= C_{Gv} - C_{Gtot} = \\
&= \begin{cases} 740D - 2.2D^2 & \text{if } D \leq 140 \text{ MW} \\ 5040 + 704D - 2.2D^2 & \text{if } 140 \text{ MW} \leq D \leq 400 \text{ MW} \end{cases}
\end{aligned}$$

This curve together with consumer and total surplus are shown in figure 3.2.

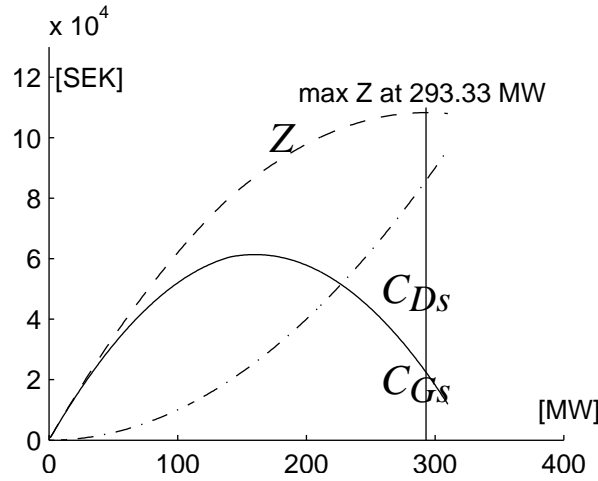


Figure 3.2. Example 3.1c, supply and demand surplus

With the assumption (also shown in the figure) that the maximum surplus of the producer is obtained when $D > 140$ MW, then this level is obtained when

$$\begin{aligned}
\frac{dC_{Gs}}{dD} &= 0 = 704 - 2.2 \cdot 2D \\
&\Rightarrow \\
D &= \frac{704}{4.4} = 160 \text{ MW}
\end{aligned}$$

This means that the maximum surplus of the producer is obtained at load level 160 MW, i.e., when the first unit produces on maximum level, and the second unit produces $160 - 140 = 20$ MW. The power price is then set by what the marginal consumers are

prepared to pay at this consumption level:

$$\lambda = \left[\frac{\hat{D} - D}{b} + c_D \right] = \left[\frac{310 - 160}{0.5} + 220 \right] = 520 \text{ SEK/MWh}$$

This solution is also shown as a dashed line in figure 3.1

Solution to example 3.1c using optimization The aim is here to maximize the benefit for the producer, considering how the demand is changing depending on the price:

$$\begin{aligned} \max Z = C_{Gv} - C_{Gtot} &= \left[\frac{\hat{D} - D}{b} + c_D \right] D - \sum_{k=A}^C \left(c_{Gk} G_k + \frac{a_k}{2} G_k^2 \right) \\ &\text{when} \\ \sum_{i \in I} G_i &= D \\ 0 \leq G_i &\leq \hat{G}_i, \quad i \in I \end{aligned}$$

The free variables in this optimization problem are G_i , $i \in I$ and D . This gives the same solution as above, i.e., $G_1 = 140$, $G_2 = 20$, $D = 160$.

End of example 3.1

Example 3.2 Assume the same production system as in example 3.1. The only difference here is that the profit maximizing monopoly is regulated to make a certain profit.

3.2a Assume that there are no fixed costs and that the required profit is 10 SEK/MWh. Calculate how to set the price.

3.3b Assume that the fixed costs are 25000 SEK/h and that the required profit is 10 SEK/MWh. Calculate how to set the price.

Solution to example 3.2a This means that the price should be set to (mean operation cost)+10 SEK/MWh. With the assumption that the marginal unit is unit 2, and that the load is decreased from its maximum level (310 MW) because of a high price

($\lambda > 220$ SEK/MWh) then this can be formulated as:

$$\begin{aligned}
\text{mean producer cost} + \text{profit} &= \text{consumer price} \\
&\Rightarrow \\
\frac{\text{total producer cost}}{\text{total production}} + \text{profit} &= \text{consumer price} \\
\frac{1}{D} \left\{ \sum_{i \in I} \left[c_{Gi} G_i + \frac{a}{2} G_i^2 \right] \right\} + C_{profit} &= \text{consumer price} \\
\frac{1}{D} \left\{ 100 \cdot 140 + \frac{0.4}{2} 140^2 + \right. &+ \\
+ 192 \cdot (D - 140) + \frac{0.4}{2} (D - 140)^2 \left. \right\} + 10 &= \text{consumer price} \\
-5040 + 136D + 0.2D^2 &= D[(\text{consumer price}) - 10]
\end{aligned}$$

There are now two possibilities: one is that we are on the price independent part of the load curve ($D = 310$ MW, $\lambda < 220$ SEK/MWh), and one that the load is decreased from its maximum level (310 MW) because of a high price ($\lambda > 220$ SEK/MWh). We first assume the first case, which results in:

$$\begin{aligned}
-5040 + 136 \cdot 310 + 0.2 \cdot 310^2 &= 310[\lambda - 10] \\
&\Rightarrow \\
\lambda = \frac{56340}{310} + 10 &= 191.74 \text{ SEK/MWh}
\end{aligned}$$

i.e. this solution is valid since $\lambda = 191.74 < 220$. This solution is shown in figure 3.3. As shown in the figure this method leads to a lower consumer price, but also that the marginal cost of the second unit is higher than the actual price. The marginal cost at the production level is

$$\frac{dC_{Gtot}}{dD} = c_{G2} + a_2 G_2 = 192 + 0.4 \cdot (310 - 140) = 260 \text{ SEK/MWh}$$

This means that although the last consumer at consumption level 310 MW is only prepared to pay 220 SEK/MWh, generation with the marginal cost 260 SEK/MWh is used.

Solution to example 3.2a using optimization Also this problem can be solved using an optimization formulation. The aim is now not to maximize the benefit, but instead to maximize the consumer surplus considering minimization of costs and the constraint

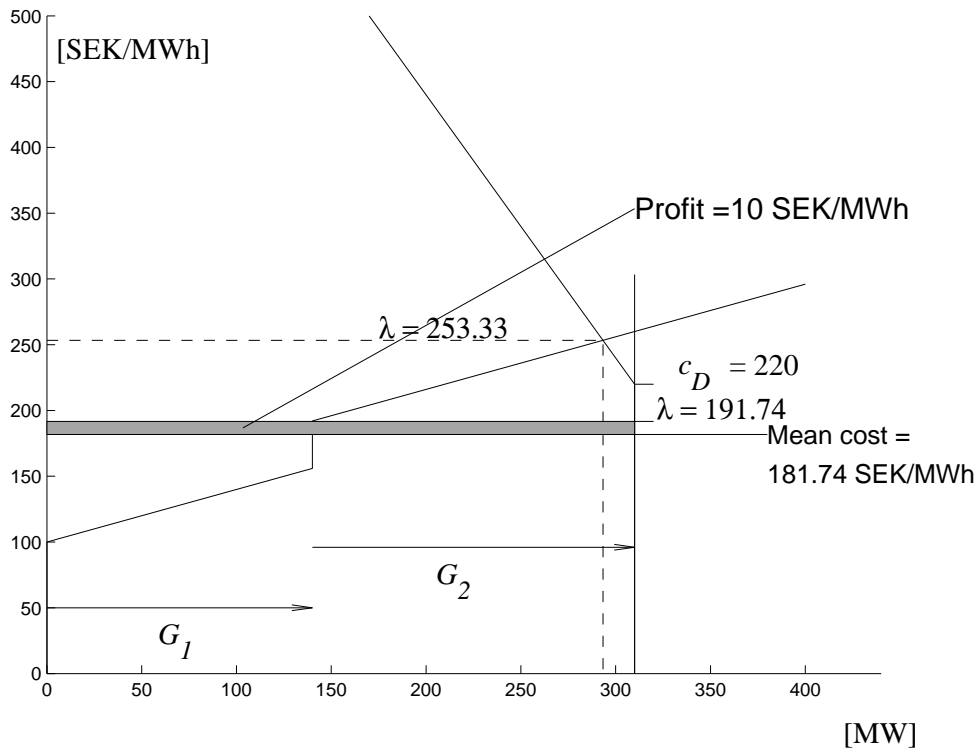


Figure 3.3. Example 3.2a,

of a predefined economic profit:

$$\begin{aligned} \max Z = C_{Ds} - \epsilon C_{G_{tot}} &= \left[\frac{\hat{D} - D}{b} + c_D \right] D - \epsilon \sum_{k=A}^C \left(c_{G_i} G_i + \frac{a_i}{2} G_i^2 \right) \\ \text{when} \\ \sum_{i \in I} G_i &= D \\ \lambda &= \frac{1}{D} \left\{ \sum_{i \in I} \left[c_{G_i} G_i + \frac{a_i}{2} G_i^2 \right] \right\} + C_{profit} + \frac{C_{fixed}}{D} \\ \lambda &\leq \frac{\hat{D} - D}{b} + c_D \\ 0 \leq G_i &\leq \hat{G}_i, \quad i \in I \\ D &\leq \hat{D} \end{aligned}$$

It can be noted that in this formulation it is important to value the consumer surplus higher than the production cost, otherwise one will get a solution with a lower consumption. For this certain problem a value of $\epsilon = 0.1$ gave the correct solution. It must be noted that it is still important to consider the production cost in the objective since otherwise it does not matter if one selects to run the units with low or high operation cost. This means that what is important is to set $\epsilon > 0$ but significantly lower than

1.0. The free variables in this optimization problem are G_i , $i \in I$, D and λ . Setting $C_{profit} = 10$ and $C_{fixed} = 0$ gives the same solution as above, i.e., $G_1 = 140$, $G_2 = 170$, $D = 310$ and $\lambda = 191.742$.

Solution to example 3.2b This means that the price should be set to (mean operation cost)+10+(fixed cost)/demand SEK/MWh. With the assumption that the marginal unit is unit 2, then this can be formulated as:

$$\begin{aligned}
 \text{mean producer cost} + \text{profit} + \frac{\text{fixed cost}}{\text{demand}} &= \text{consumer price} \\
 \Rightarrow \\
 \frac{\text{total producer cost}}{\text{total production}} + \text{profit} + \frac{\text{fixed cost}}{\text{demand}} &= \text{consumer price} \\
 \frac{1}{D} \left\{ \sum_{i \in I} \left[c_{G_i} G_i + \frac{a}{2} G_i^2 \right] \right\} + C_{profit} + \frac{C_{fixed}}{D} &= \text{consumer price} \\
 \frac{1}{D} \left\{ 100 \cdot 140 + \frac{0.4}{2} 140^2 \right\} + & \\
 + 192 \cdot (D - 140) + \frac{0.4}{2} (D - 140)^2 \Big\} + 10 + \frac{25000}{D} &= \text{consumer price} \\
 19960 + 146D + 0.2D^2 &= D(\text{consumer price})
 \end{aligned}$$

There are now two possibilities: one is that we are on the price independent part of the load curve ($D = 310$ MW, $\lambda < 220$ SEK/MWh, and one that the load is decreased from its maximum level (310 MW) because of a high price ($\lambda > 220$ SEK/MWh). We first assume the second case, which results in:

$$\begin{aligned}
 19960 + 146D + 0.2D^2 &= D \left[\frac{\hat{D} - D}{b} + c_D \right] = D \left[\frac{310 - D}{0.5} + 220 \right] \\
 19960 + 146D + 0.2D^2 &= D(620 - 2D + 220) \\
 19960 - 694D + 2.2D^2 &= 0 \\
 D &= 283.446 \text{ MW} \\
 \Rightarrow \\
 \lambda_D = \frac{310 - 283.446}{0.5} + 220 &= 273.11 \text{ SEK/MWh}
 \end{aligned}$$

i.e. this solution is valid since $\lambda = 273.11 > 220$. This solution is shown in figure 3.4. As shown in the figure this method leads to a higher consumer price, but also that the marginal cost of the second unit is lower than the actual price. The marginal cost at the production level is

$$\frac{dC_{G_{tot}}}{dD} = c_{G_2} + a_2 G_2 = 192 + 0.4 \cdot (283.446 - 140) = 249.38 \text{ SEK/MWh}$$

This means that more consumers could be supplied by more generation, since the marginal cost for more generation is lower than what consumers are prepared to pay.

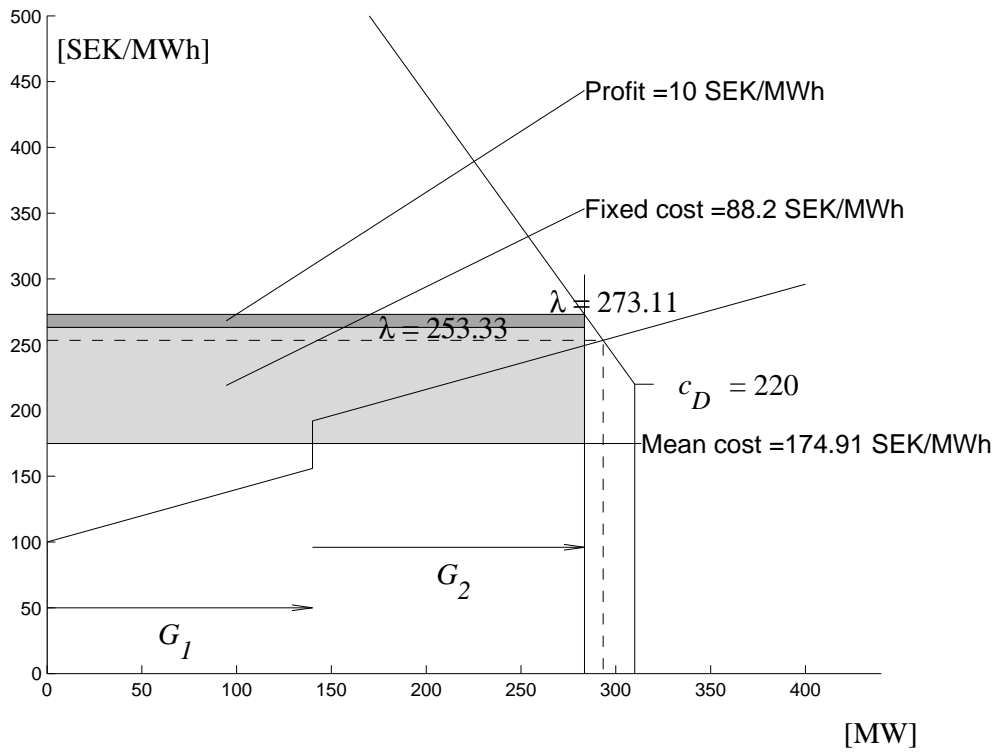


Figure 3.4. Example 3.2b,

But this would lead to a lower profit, since the price is reduced, and the assumption here is that the profit is regulated to a certain level in SEK/MWh.

Solution to example 3.2b using optimization The same optimization formulation as for example 3.2a can be used. The free variables in this optimization problem are G_i , $i \in I$, D and λ . Setting $C_{profit} = 10$ and $C_{fixed} = 25000$ gives the same solution as above, i.e., $G_1 = 140$, $G_2 = 143.45$, $D = 283.45$ and $\lambda = 273.11$.

End of example 3.2

Example 3.3 Assume the same production system as in example 3.1. The only difference here is that the profit maximizing company does not have a monopoly, it only controls (and receives the revenue from) the whole production in unit 2.

3.3a Calculate the change in consumer and producer surplus when the producer increases the price to 480 SEK/MWh.

3.3b Calculate the production level (= total consumption), that maximizes the profit for the producer.

Solution to example 3.3a Figure 3.5 describes the situation. The only difference compared to figure 3.1 is that area A is now divided into two subareas E and F where $A=E+F$.

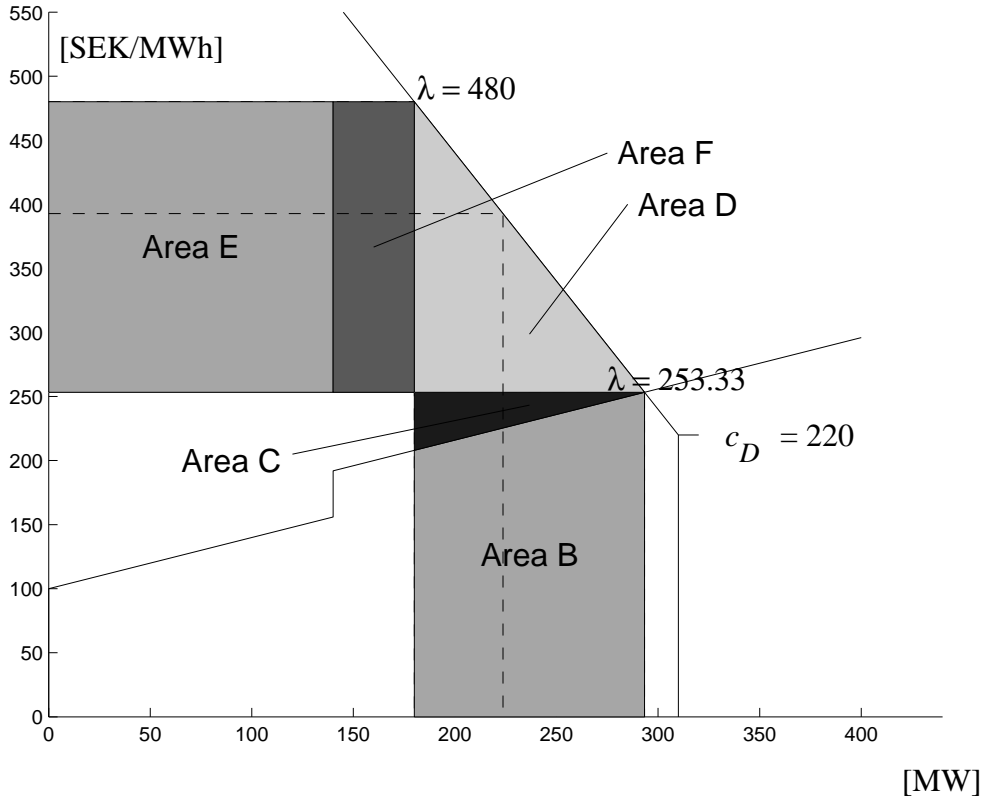


Figure 3.5. Demand and supply for two price levels in example 3.3. The dashed line corresponds to the solution in example 3.3b

With a price of 480 SEK/MWh, the consumption (c.f. eq. 1.3) becomes

$$D = \hat{D} - b(\lambda - c_D) = 310 - 0.5(480 - 220) = 180 \text{ MW}$$

The increased producer income because of higher prices for the remaining consumers is now only area F since area E is increased income for other producers:

$$F = (480 - 253.33)(180 - G_1) = 7466.67 \text{ SEK}$$

The decreased profit because of decreased consumption is the same as in example 3.1, i.e.

$$C = (B + C) - B = 0.5(293.33 - 180)(253.33 - [192 + 0.4 \cdot 40]) = 2569 \text{ SEK}$$

This means that also in this case it is profitable for the producer to increase the price since $F > C$. But the profit is lower. In addition to this it can be added that the strategic behavior for this owner of generator 2 is also profitable for the owners of generator 1, since the income for them increases (=area E). For the consumers the situation is the same as in example 3.1, i.e. their surplus decreases with $A + D = E + F + D$. This means that the total surplus also here decreases because of the strategic behavior.

Solution to example 3.3b The total surplus for the owner of unit 2 as a function of the demand (it is assumed that $D > 150$ MW, otherwise $C_{gtot} = C_{Gv} = 0$), can be calculated as:

$$\begin{aligned}
 C_{G2} &= \left[c_{G2}G_2 + \frac{a_2}{2}G_2^2 \right] = \\
 &= 192(D - 140) + \frac{0.4}{2}(D - 140)^2 \\
 C_{G2v} &= \lambda[D - G(1)] = \left[\frac{\hat{D} - D}{b} + c_D \right] [D - G(1)] = \\
 &= - \left[\frac{310}{0.5} + 220 \right] 140 + \left[\frac{310}{0.5} + 220 + \frac{140}{0.5} \right] D - \frac{1}{0.5}D^2 = \\
 &= -[620 + 220]140 + [620 + 220 + 280]D - 2D^2 \\
 C_{G2s} &= C_{G2v} - C_{G2}
 \end{aligned}$$

This curve together with consumer and total surplus are shown in figure 3.6

The maximum is obtained when

$$\begin{aligned}
 \frac{dC_{G2s}}{dD} &= 0 = \frac{dC_{G2v}}{dD} - \frac{dC_{G2}}{dD} \\
 &\Rightarrow \\
 192 + 0.4(D - 140) &= 1120 - 4D \\
 &\Rightarrow \\
 D &= 223.63 \text{ MW}
 \end{aligned}$$

This curve together with consumer and total surplus are shown in figure 3.6. The

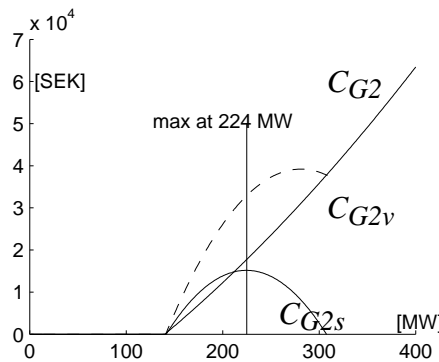


Figure 3.6. Demand and supply for generator two in example 3.3b

maximum surplus for the producer is obtained at load level 223.63 MW, i.e., when the first unit produces on maximum level, and the second unit produces $223.63 - 140 = 83.63$ MW. The power price is then set by what the marginal consumers are prepared to pay at this consumption level:

$$\lambda = \left[\frac{\hat{D} - D}{b} + c_D \right] = \left[\frac{310 - 223.63}{0.5} + 220 \right] = 392.74 \text{ SEK/MWh}$$

This solution is shown as a dashed line in figure 3.5. Compared to example 3.1 where the power price became 520 SEK/MW, it is shown here that a reduction of the monopoly (the company now only controls unit 2) leads to an improved competition and a lowered price. But it is still higher compared to a situation with full competition.

End of example 3.3

Example 3.4 Assume there are two periods, 1 and 2, and there are two units in the system with the following data: In period 1 the load is 250 MW and in period 2 it is

Unit	Type	\hat{G}_{in}	c_{Gin}	a
i=1	Hydro	200 MW	20 SEK/MWh	0 SEK/MWh/MW
i=2	Thermal	250 MW	300 SEK/MWh	0,2 SEK/MWh/MW

Table 3.2. Data for example 3.4

310 MW. Assume that the inflow to the hydro power plants corresponds to 200 MWh/h for the first period and 60 MWh/h in the second period, but water can be stored from period 1 to period 2. Assume that both periods consists of 24 hours. Also assume that all water inflow is used during the studied two periods. It is here assumed that there is full competition among the thermal power plants, but there is one owner of the hydro power system, and this owner maximizes its profit.

3.4a Calculate the production in all units and the period power prices when the storage capacity between the periods is 1200 MWh. This example is a parallel to example 2.1b but in that example perfect competition was assumed also for the hydro power part of the system.

3.4b Calculate the same as in 3.4a, but now assume a storage capacity of 3600 MWh. This example corresponds to example 2.1c where perfect competition was assumed.

Graphic solution to example 3.4: First assume that the owner of the hydro power plants selects to store no water from period 1 to 2. This then corresponds to example 2.1a. This means that the production capacity in the hydro power plant in the first period is $G_1(1) = 200$ MWh/h and in the second $G_1(2) = 60$ MWh/h. This means that $250-200= 50$ MWh/h is needed from unit #2 in period 1. The price (=marginal cost) is then:

$$\lambda_1 = c_{G2} + a_2 G_2 = c_{G2} + a_2 [D(1) - G_1(1)] = 300 + 0.2 \cdot 50 = 310 \text{ SEK/MWh}$$

In period 2 the power production in unit #2 becomes $310-60= 250$ MWh/h corresponding to a price (=marginal cost) of:

$$\lambda_2 = c_{G2} + a_2 [D(2) - G_1(2)] = 300 + 0.2 \cdot 250 = 350 \text{ SEK/MWh}$$

The total income for the hydro power producer with no storage now becomes:

$$C_{Dtot}(P_{12} = 0) = \lambda_1 G_1(1) + \lambda_2 G_1(2) = 310 \cdot 200 + 350 \cdot 60 = 83000 \text{ SEK/h}$$

For each MWh of hydro power that is moved from period 1 to period 2, there will be an increased value for this MWh, but at the same time the price will decrease in period 2 (lower income for all hydro power in that period) and increase in period 1 (increased income). With one stored MWh the following is changed: $G_1(1) = 199$ MWh/h and in the second $G_1(2) = 61$ MWh/h. This means that $250-199= 51$ MWh/h is needed from unit #2 in period 1. The price (=marginal cost) is then:

$$\lambda_1 = c_{G2} + a_2 G_2 = c_{G2} + a_2 [D(1) - G_1(1)] = 300 + 0.2 \cdot 51 = 310.2 \text{ SEK/MWh}$$

In period 2 the power production in unit #2 becomes $310-61= 249$ MWh/h corresponding to a price (=marginal cost) of:

$$\lambda_2 = c_{G2} + a_2 [D(2) - G_1(2)] = 300 + 0.2 \cdot 250 = 349.8 \text{ SEK/MWh}$$

The total income for the hydro power producer with 1 MWh of storage now becomes:

$$C_{Dtot}(P_{12} = 1) = \lambda_1 G_1(1) + \lambda_2 G_1(2) = 310.2 \cdot 199 + 349.8 \cdot 61 = 83068 \text{ SEK/h}$$

This is higher, so it is profitable to store some water. The question is then how much? First we formulate the income as a function of the amount of storage. This is then formulated as

$$\begin{aligned} C_{Dtot}(P_{12}) &= \lambda_1 G_1(1) + \lambda_2 G_1(2) = \\ &= (300 + 0.2[50 + P_{12}]) (200 - P_{12}) + (300 + 0.2[250 - P_{12}]) (60 + P_{12}) \end{aligned}$$

The optimum is obtained when the derivative of the income is zero, i.e.

$$\begin{aligned} \frac{dC_{Dtot}}{dP_{12}} &= 0 = 2(-0.2 - 0.2)P_{12} + (40 - 10 + 50 - 12) \\ &\Rightarrow P_{12} = 68/0.8 = 85 \text{ MWh/h} \end{aligned}$$

The income at this storage level becomes

$$C_{Dtot}(P_{12} = 85) = \lambda_1 G_1(1) + \lambda_2 G_1(2) = 327 \cdot 115 + 333 \cdot 145 = 85890 \text{ SEK/h}$$

The total income per hour for the hydro producer as a function of the amount of storage is shown in figure 3.7 together with the total surplus which was calculated in example 2.1. As shown in the figure the optimal storage for the profit maximizing hydro producer is not the same as the one that results in a maximal surplus in a system with perfect competition. This shows a possible consequence of market power.

Solution to example 3.4a and 3.4b:

In example 3.4a the storage contains 1200 MWh corresponding to 50 MWh/h. As shown in figure 3.7 the income for the producer increases up to 85 MWh/h so this means that the reservoir should be used up to its full capacity, i.e., 1200 MWh.

In example 3.4b the storage contains 3600 MWh corresponding to 150 MWh/h. As shown above the maximum income for the producer is at a storage of 85 MWh/h so

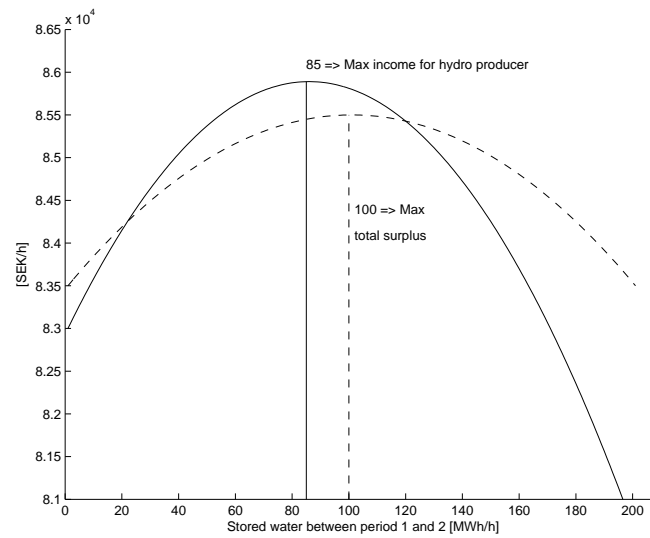


Figure 3.7. Example 3.4, Hydro producer income: $C_{Dtot}(P_{12})$ and total surplus

this means that the reservoir should be used at this level corresponding to $24 \cdot 85 = 2040$ MWh.

Solution using optimization The objective for the producer is here to maximize its profit considering that the prices on the market are affected by the production level. This leads to an optimization formulation as:

$$\begin{aligned}
 \max Z &= \lambda_1 G_1(1) + \lambda_2 G_1(2) \\
 \text{when} \\
 \lambda_1 &= c_{G2} + a_2 G_2(1) \\
 \lambda_2 &= c_{G2} + a_2 G_2(2) \\
 0 &\leq G_1(t) \leq 200 \\
 0 &\leq G_2(t) \leq 250 \\
 G_1(t) + G_2(t) &= D_t \quad t \in [1, 2] \\
 G_1(1) + P_{12} &= \text{inflow in period 1} = 200 \\
 G_1(2) - P_{12} &= \text{inflow in period 2} = 60 \\
 0 &\leq P_{12} \leq 50: \text{ example 3.4a, or } 150: \text{ example 3.4b}
 \end{aligned}$$

The free variables in this optimization problem are $G_1(t)$, $G_2(t)$, $t \in [1, 2]$ and P_{12} . The result from the optimization includes the values of these variables which maximizes the objective Z .

End of example 3.4a

3.2 Strategic behavior

Strategic behavior means that one actor adjust how it acts on the market, considering how other actors act on the market.

Example 3.5 Assume the same total production system as in example 3.1. The only difference here is that there are two profit maximizing companies which form a cartel and agree on production levels and prices. The ownership of the production system is divided between the two actors as:

Owner	Unit	\hat{G}_{in}	c_{Gin}	a
O1	i=1	70 MW	100 SEK/MWh	0.8 SEK/MWh/MW
	i=2	130 MW	192 SEK/MWh	0.8 SEK/MWh/MW
O2	i=3	70 MW	100 SEK/MWh	0.8 SEK/MWh/MW
	i=4	130 MW	192 SEK/MWh	0.8 SEK/MWh/MW

Table 3.3. Data for example 3.5

With this data the cost of total production at a certain level will be exactly the same as previously, if the producers are assumed to be price takers, i.e., if the cheapest units are started first, then the marginal cost will increase in the same way as in, e.g., example 3.1. Assume the same load as in example 3.1, i.e., the load is constant = 310 MW as long as the price is lower than 220 SEK/MWh and then it decreases with 0.5 MW for each price increase step of 1 SEK/MWh.

3.5a Calculate the production level (= total consumption), that maximizes the profit for the producers.

3.5b Calculate the changed income for producer O1 if they increases its production with 20 MW, i.e., cheats on the cartel agreement. Producer O2 is assumed to have the same production level as in example 3.5a.

3.5c Calculate the production level that maximizes the surplus of producer O1, i.e., optimal cheating on the cartel agreement. Producer O2 is assumed to have the same production level as in example 3.5a.

Solution to example 3.5a Since the two owners cooperate concerning the prices, then the optimum is at the same point as in example 3.1c, i.e., the two units with lower marginal costs, #1 and #3, operate on full capacity. This leads to a total production = demand = 160 MW, corresponding to full production = 70 MW in units #1 and #3 and 10 MW of production in units #2 and #4. The price level becomes

$$\lambda_{Da} = \frac{D_m - D}{b} + c_D = \frac{310 - 160}{0.5} + 220 = 520 \text{ SEK/MWh}$$

This solution is found as the dashed line in figure 3.8.

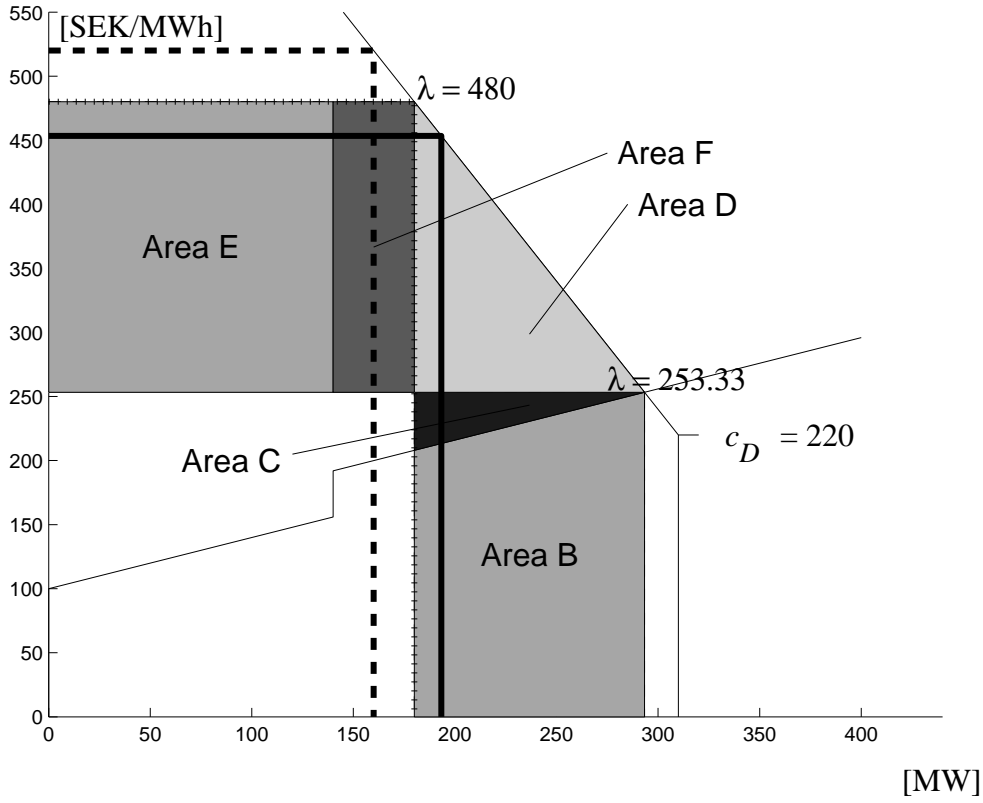


Figure 3.8. Demand and supply for two price levels in example 3.5. The dashed line corresponds to the solution in example 3.5a, the dotted line example 3.5b, and the straight line example 3.5c

Solution to example 3.5b An increase of 20 MW of production (which is sold so also the demand increases with 20 MW, i.e., $D=160+20=180$ MW) corresponds to a change of the price level to

$$\lambda_{Db} = \frac{310 - 180}{0.5} + 220 = 480 \text{ SEK/MWh}$$

This solution is found as the dotted line in figure 3.8. The changed income (=changed surplus) from the still operating 70 MW in unit #1 for its owner O1 is

$$\Delta C_{Gs1} = 70(\lambda_{Db} - \lambda_{Da}) = -2800 \text{ SEK}$$

i.e., the profit in this unit is decreased because of the increased production in O1:s unit #2. The changed production cost in unit #2 becomes

$$\begin{aligned} \Delta C_{2-} &= \left[192 \cdot 30 + \frac{0.8}{2} 30^2 \right] - \left[192 \cdot 10 + \frac{0.8}{2} 10^2 \right] \\ &= 192 \cdot 20 + 0.4 \cdot (30^2 - 10^2) = 4160 \text{ SEK} \end{aligned}$$

This means increased operation costs since the unit increases its production from 10 to 30 MW. The increased income from the extra production is

$$\Delta C_{2+} = 20 \cdot 480 = 9600 \text{ SEK}$$

The net result becomes

$$\Delta C_{Gs1} - \Delta C_2 + \Delta C_{12+} = 2640 \text{ SEK}$$

This means that it is profitable for owner O1 to cheat on the cartel agreement, and increase its production with 20 MW in unit 2.

Solution to example 3.5c Here it is assumed that owner O2 produces 70 MW in unit #3 and 10 MW in unit #4 according to the solution in example 3.5a. The rest of the load is then covered by the units of owner O1, i.e., unit 1 and 2. The revenue, C_{Gv-O1} , total production cost, C_{G-O1} and total surplus, C_{Gs-O1} depending on total production level ($= D_{O1} = D - 80$) for owner O1 is

$$\begin{aligned} C_{Gv-O1} &= \lambda_D [D - 80] = \left[\frac{\hat{D} - D}{b} + c_D \right] [D - 80] = \\ &= \left[\frac{310}{0.5} + 220 + \frac{80}{0.5} \right] D - \frac{1}{0.5} D^2 = 1000D - 2D^2 \\ C_{G-O1} &= \sum_{i=1}^2 \left[c_{Gi} G_i + \frac{a_i}{2} G_i^2 \right] = 100 \cdot 70 + \frac{0.8}{2} 70^2 \\ &+ 192(D - 150) + \frac{0.8}{2} (D - 150)^2 = \\ &= -10840 + 72D + 0.4D^2 \\ C_{Gs-O1} &= C_{Gv-O1} - C_{G-O1} \end{aligned}$$

The maximum level is obtained when

$$\begin{aligned} \frac{dC_{Gs-O1}}{dD} &= 0 = \frac{dC_{Gv-O1}}{dD} - \frac{dC_{G-O1}}{dD} \\ &\Rightarrow \\ 1000 - 4D &= 72 + 0.8D \\ &\Rightarrow \\ D &= 193.33 \text{ MW} \\ &\Rightarrow \\ G_2 &= D - 150 = 43.33 \text{ MW} \\ \lambda_D &= \frac{D_m - D}{b} + c_D = \frac{310 - 150}{0.5} + 220 = 453.33 \text{ SEK/MWh} \end{aligned}$$

This solution is found as the straight line in figure 3.8.

End of example 3.5

Example 3.6 Assume the same load, owners and production system as in example 3.5.

3.6a In example 3.5c, owner O1 cheated on the cartel and increased its production to increase the surplus. Now assume that owner O2 considers this, i.e. the solution of example 3.5c, and maximizes its production level based on this. Calculate the production level ($=$ total consumption), that maximizes the profit for owner O2.

3.6b Assume that owner O1 considers the strategic behavior of owner O2 in example 3.6a. Calculate the production level, that maximizes the profit for owner O1.

3.6c Assume that owner O2 considers the strategic behavior of owner O1 in example 3.6b. Calculate the production level, that maximizes the profit for owner O2.

Solution to example 3.6a The result from example 3.5c was that owner O1 produces 70 MW in unit #1 and 43.33 MW in unit. The rest of the load is then covered by the units of owner O2, i.e., unit #3 and #4. The revenue, C_{Gv-O2} , total production cost, C_{G-O2} and total surplus, C_{Gs-O2} depending on total production level ($= D_{O2} = D - (70 + 43.33)$) for owner O2 is

$$\begin{aligned}
 C_{Gv-O2} &= \lambda_D [D - 113.33] = \left[\frac{\hat{D} - D}{b} + c_D \right] [D - 113.33] = \\
 &= \left[\frac{310}{0.5} + 220 + \frac{113.33}{0.5} \right] D - \frac{1}{0.5} D^2 = 1066.67D - 2D^2 \\
 C_{G-O2} &= \sum_{i=3}^4 \left[c_{Gi} G_i + \frac{a_i}{2} G_i^2 \right] = 100 \cdot 70 + \frac{0.8}{2} 70^2 \\
 &+ 192(D - 183.33) + \frac{0.8}{2} (D - 183.33)^2 = \\
 &= -12796 + 45.33D + 0.4D^2 \\
 C_{Gs-O2} &= C_{Gv-O2} - C_{G-O2}
 \end{aligned}$$

The maximum level is obtained when

$$\begin{aligned}
 \frac{dC_{Gs-O2}}{dD} &= 0 = \frac{dC_{Gv-O2}}{dD} - \frac{dC_{G-O2}}{dD} \\
 \Rightarrow \\
 1066.67 - 4D &= 45.33 + 0.8D \\
 \Rightarrow \\
 D &= 212.78 \text{ MW} \\
 \Rightarrow \\
 G_4 &= D - 113.33 - 70 = 29.44 \text{ MW} \\
 \lambda_D &= \frac{D_m - D}{b} + c_D = \frac{310 - 183.33}{0.5} + 220 = 414.44 \text{ SEK/MWh}
 \end{aligned}$$

This solution is shown as a dashed line in figure 3.9.

Solution to example 3.6b The result from example 3.6a was that owner O2 produces 70 MW in unit #3 and 29.44 MW in unit #4. The rest of the load is then covered by the units of owner O1. The revenue, total production cost, and total surplus, depending

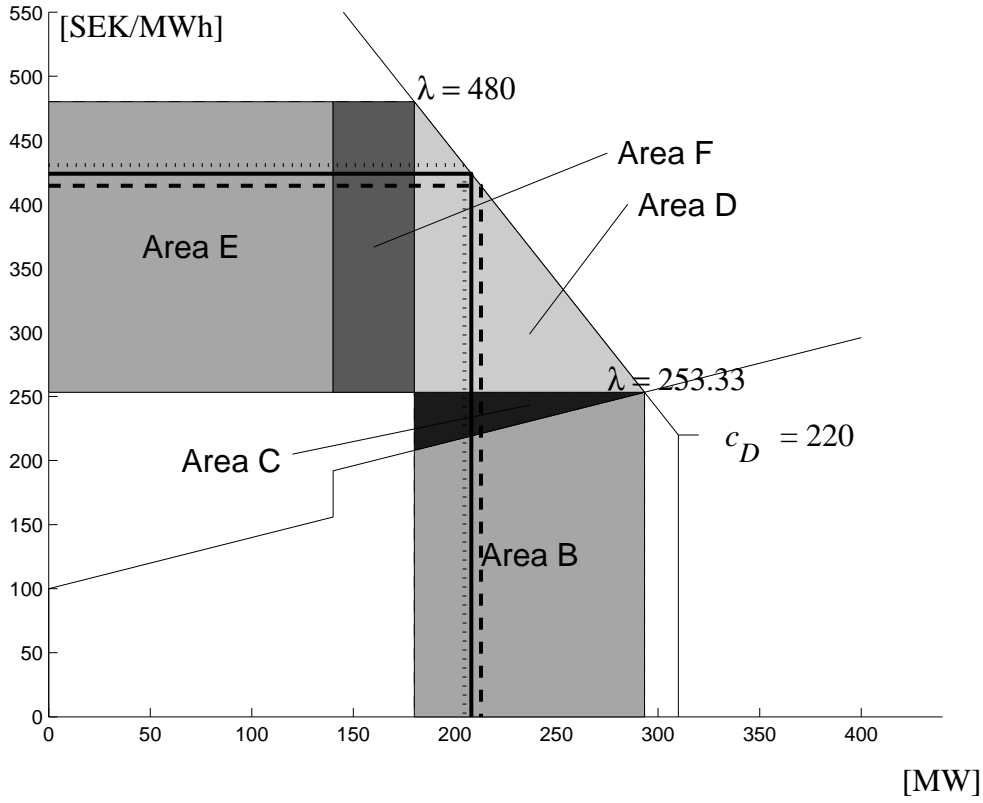


Figure 3.9. Demand and supply for two price levels in example 3.6. The dashed line corresponds to the solution in example 3.6a, the dotted line example 3.6b, and the straight line example 3.6c

on total production level ($= D_{O1} = D - (70 + 29.44)$) for owner O1 is

$$\begin{aligned}
 C_{Gv-O1} &= \lambda_D [D - 99.44] = \left[\frac{\hat{D} - D}{b} + c_D \right] [D - 99.44] = \\
 &= \left[\frac{310}{0.5} + 220 + \frac{99.44}{0.5} \right] D - \frac{1}{0.5} D^2 = 1038.89D - 2D^2 \\
 C_{G-O1} &= \sum_{i=1}^2 \left[c_{Gi} G_i + \frac{a_i}{2} G_i^2 \right] = 100 \cdot 70 + \frac{0.8}{2} 70^2 \\
 &+ 192(D - 183.33) + \frac{0.8}{2} (D - 183.33)^2 = \\
 &= -12089 + 56.44D + 0.4D^2 \\
 C_{Gs-O1} &= C_{Gv-O1} - C_{G-O1}
 \end{aligned}$$

The maximum level is obtained when

$$\begin{aligned}
\frac{dC_{Gs-O1}}{dD} &= 0 = \frac{dC_{Gv-O1}}{dD} - \frac{dC_{G-O1}}{dD} \\
&\Rightarrow \\
1038.89 - 4D &= 56.44 + 0.8D \\
&\Rightarrow \\
D &= 204.67 \text{ MW} \\
&\Rightarrow \\
G_2 &= D - 99.44 - 70 = 35.23 \text{ MW} \\
\lambda_D &= \frac{D_m - D}{b} + c_D = \frac{310 - 169.44}{0.5} + 220 = 430.65 \text{ SEK/MWh}
\end{aligned}$$

This solution is shown as a dotted line in figure 3.9.

Solution to example 3.6c The result from example 3.6b was that owner O1 produces 70 MW in unit #1 and 35.23 MW in unit #2. The rest of the load is then covered by the units of owner O2. The revenue, total production cost, and total surplus, depending on total production level ($= D_{O2} = D - (70 + 35.23)$) for owner O2 is

$$\begin{aligned}
C_{Gv-O2} &= \lambda_D [D - 105.23] = \left[\frac{\hat{D} - D}{b} + c_D \right] [D - 105.23] = \\
&= \left[\frac{310}{0.5} + 220 + \frac{105.23}{0.5} \right] D - \frac{1}{0.5} D^2 = 1050.46D - 2D^2 \\
C_{G-O2} &= \sum_{i=1}^2 \left[c_{Gi} G_i + \frac{a_i}{2} G_i^2 \right] = 100 \cdot 70 + \frac{0.8}{2} 70^2 \\
&+ 192(D - 175.23) + \frac{0.8}{2} (D - 175.23)^2 = \\
&= -12402.02 + 51.82D + 0.4D^2 \\
C_{Gs-O2} &= C_{Gv-O2} - C_{G-O2}
\end{aligned}$$

The maximum level is obtained when

$$\begin{aligned}
\frac{dC_{Gs-O2}}{dD} &= 0 = \frac{dC_{Gv-O2}}{dD} - \frac{dC_{G-O2}}{dD} \\
&\Rightarrow \\
1050.46 - 4D &= 51.82 + 0.8D \\
&\Rightarrow \\
D &= 208.05 \text{ MW} \\
&\Rightarrow \\
G_2 &= D - 105.31 - 70 = 32.82 \text{ MW} \\
\lambda_D &= \frac{D_m - D}{b} + c_D = \frac{310 - 175.31}{0.5} + 220 = 423.90 \text{ SEK/MWh}
\end{aligned}$$

This solution is shown as a straight line in figure 3.9.

End of example 3.6

Example 3.7 Assume the same load, owners and production system as in examples 3.5 and 3.6. Assume that owners O1 and O2 continues to consider the other ones action until an equilibrium is obtained. This is the so-called Cournot equilibrium. This equilibrium means that none of the players gains anything extra by changing their bids.

Solution to example 3.7 In the equilibrium point there is no gain in changing the bids for any of the actors. First we therefore have to formulate the benefits and costs for the two players in order to evaluate the marginal costs and benefits. We first make the general statement that the equilibrium will occur on a level when generators G_1 and G_3 produce on their maximum (=70 MW each). The marginal cost of these units are the same as the marginal cost of the other 2 units of the same owners. This means that (from strategic point of view) it does not matter whether owners 1 and/or 2 changes their production in the first loaded units (G_1 and G_3) or the secondly loaded units (G_2 and G_4). But here we assume that the "strategic behavior" will occur in units G_2 and G_4 . The value and benefit (and corresponding marginal values) of owner 1 and 2 therefore become:

$$\begin{aligned}
C_{Gv-O1} &= \lambda_D[G_1 + G_3] = \left[\frac{\hat{D} - D}{b} + c_D \right] [G_1 + G_2] \\
C_{G-O1} &= \sum_{i=1}^2 \left[c_{Gi}G_i + \frac{a_i}{2}G_i^2 \right] \\
C_{Gv-O2} &= \lambda_D[G_3 + G_4] = \left[\frac{\hat{D} - D}{b} + c_D \right] [G_3 + G_4] \\
C_{G-O2} &= \sum_{i=3}^4 \left[c_{Gi}G_i + \frac{a_i}{2}G_i^2 \right] \\
D &= \sum_{i=1}^4 G_i \\
&\Rightarrow \\
\frac{dC_{Gv-O1}}{dG_2} &= \left[\frac{\hat{D} - D}{b} + c_D \right] - \frac{1}{b}[G_1 + G_2] \\
\frac{dC_{G-O1}}{dG_2} &= c_{G2} + a_2G_2 \\
\frac{dC_{Gv-O2}}{dG_4} &= \left[\frac{\hat{D} - D}{b} + c_D \right] - \frac{1}{b}[G_3 + G_4] \\
\frac{dC_{G-O2}}{dG_4} &= c_{G4} + a_4G_4
\end{aligned}$$

The equilibrium means that the marginal cost must be equal to the marginal benefit for both actors at the same time as the total production is equal to the load, i.e.

$$\left\{ \begin{array}{l} \frac{dC_{Gs-O1}}{dD} = 0 = \frac{dC_{Gv-O1}}{dD} - \frac{dC_{G-O1}}{dD} \\ \frac{dC_{Gs-O2}}{dD} = 0 = \frac{dC_{Gv-O2}}{dD} - \frac{dC_{G-O2}}{dD} \\ \sum_{i=1}^4 G_i = D \end{array} \right. \Rightarrow \left\{ \begin{array}{l} \left[\frac{\hat{D} - D}{b} + c_D \right] - \frac{1}{b}[G_1 + G_2] = c_{G2} + a_2 G_2 \\ \left[\frac{\hat{D} - D}{b} + c_D \right] - \frac{1}{b}[G_3 + G_4] = c_{G4} + a_4 G_4 \\ \sum_{i=1}^4 G_i = D \end{array} \right.$$

This forms a linear system of equations with three equations and three unknown variables: G_2 , G_4 and D . The solution of this is $G_2 = G_4 = 33.53$ MW, $D = 207.06$ MW and $\lambda_D = 425.88$ SEK/MWh

End of example 3.7

From previous examples the following results can be collected. Also the results from full competition are included. For all these example $G_1 = G_3 = 70$ MW.

Example	Behavior	λ [SEK/MWh]	D [MWh]	G_2 [MWh]	G_4 [MWh]
3.5a	Monopoly = cartel	520	180	10	10
3.5c	O1 cheating cartel	453.33	193.33	43.33	10
3.6a	O2 cheating cartel	414.44	212.78	43.33	29.44
3.6b	O1 cheating cartel	430.65	204.67	35.23	29.44
3.6c	O2 cheating cartel	423.90	208.05	35.23	32.82
3.7	Cournot	425.88	207.06	33.53	33.53
3.1a	Full competition	253.33	293.33	76.67	76.67

Table 3.4. Results for examples with different competition levels

As shown in table 3.4 there will be different price levels depending on the competition level. The highest price is obtained when there is a monopoly and the lowest price is at full competition. A working cartel leads to the same price as the monopoly, while strategic behavior among companies that know (and use the knowledge) that they can have an influence on the market price leads to a price which is in between the monopoly price and the price obtained at full competition.

Chapter 4

Environmental restrictions

In economic theory there is a term called "externality". This means: When the activity of one entity (a person or a firm) directly affects the welfare of another in a way that is not transmitted by market prices, that effect is called an **externality** (because one entity directly affects the welfare of another entity that is "external" to it). Environmental problems caused by power production is an example of an "externality", since there are other persons (and animals etc) living now and in the future which are affected by power production emission, while the power production formally is trading only between the producer and the consumer.

In this section several types of handling of "externality" will be treated and analyzed. These are

- **Fixed price** This means that low emission units will be paid a higher fixed price than the normal price in order to reduce the emissions.
- **Subsidies and Taxes** This means that subsidies are provided for low emission units and/or taxes on high emission units.
- **Certificates** This put a constraint that total low emission production must be $>$ certain level.
- **Emission rights** This method puts a constraint that total high emission production must be $<$ certain level.
- **Electricity disclosure** This means formally that each consumer receives a notation in the electricity bill about the origin of its power production. The consumer then has a possibility to choose low emission production if he/she wants.
- **Feebate system** This is an internal trading system between all power production where companies with low emissions will be compensated by companies with higher emissions (an economic zero-sum game to reduce emissions).

Other possible methods to reduce emissions are, e.g., to forbid power plants with too high emissions, to only allow units that reduce their emissions with a certain amount during a certain time, or to subsidize the investment in low-emission plants. These methods are though not treated here, since they will only have an impact on the power price in such a way that some units are not in the system or the capital cost changes.

We first start with a base case scenario where there are no restrictions on the emissions. Here we only treat CO_2 emissions but the same method can be used for, e.g., SO_X (Sulphur) or NO_X (Nitrogen) emissions.

Example 4.1 Assume a power system according to table 4.1 and a total, price independent, load of 490 MW. Assume perfect competition and no considerations of externalities.

Unit	Type	\hat{G}_{in}	c_{Gin}	a	kgCO ₂ /MWh
i=1	Coal	200 MW	100 SEK/MWh	0.1 SEK/MWh/MW	1000
i=2	bio	200 MW	150 SEK/MWh	0.1 SEK/MWh/MW	0
i=3	nat-gas	200 MW	200 SEK/MWh	0.1 SEK/MWh/MW	440
i=4	bio	100 MW	250 SEK/MWh	0.1 SEK/MWh/MW	0
Emissions from burning coal		CO ₂ emission when burning coal is slightly different for different coal types but is approximately 95 gCO ₂ /MJ of burn coal \Rightarrow 342 kgCO ₂ /MWh. With an assumed efficiency in a coal condensing plant (only generation of electricity) of 34.2 % \Rightarrow 1000 kgCO ₂ /MWh-el			
Emissions from burning natural gas		consists mainly of methan, CH ₄ so burning of gas results in less CO ₂ since energy is also obtained from burned hydrogen resulting in emission of water. CO ₂ emissions when burning natural gas is approximately 55 gCO ₂ /MJ of burned gas \Rightarrow 198 kgCO ₂ /MWh. With an assumed efficiency in a combined cycle gas plant (only generation of electricity) of 45 % \Rightarrow 440 kgCO ₂ /MWh-el			

Table 4.1. Data for example 4.1

Calculate production, production cost, economic surplus and CO₂ emissions in each unit as well as consumer price.

Solution to example 4.1 Figure 4.1 shows the supply and demand curves. The two low cost units #1 and #2 will be used on maximum capacity ($G_1 = 200$ MW, $G_2 = 200$ MW), while 90 MW is needed from the third cheapest unit, unit #3 ($G_3 = 90$ MW), to meet the load. The fourth most expensive unit is not needed. This means that the price will be

$$\lambda = c_{G3} + a_3 \cdot G_3 = 200 + 0.1 \cdot 900 = 209 \text{ SEK/MWh}$$

The production cost in each unit becomes:

$$\begin{aligned} C_{G1} &= c_{G1}G_1 + \frac{a_1}{2}G_1^2 = 100 \cdot 200 + \frac{0.1}{2}200^2 = 22000 \text{ SEK} \\ C_{G2} &= 150 \cdot 200 + \frac{0.1}{2}200^2 = 32000 \text{ SEK} \\ C_{G3} &= 200 \cdot 90 + \frac{0.1}{2}90^2 = 18405 \text{ SEK} \\ C_{G4} &= 0 \text{ SEK} \end{aligned}$$

The total operation cost becomes 72405 SEK. The economic surplus in each unit is the

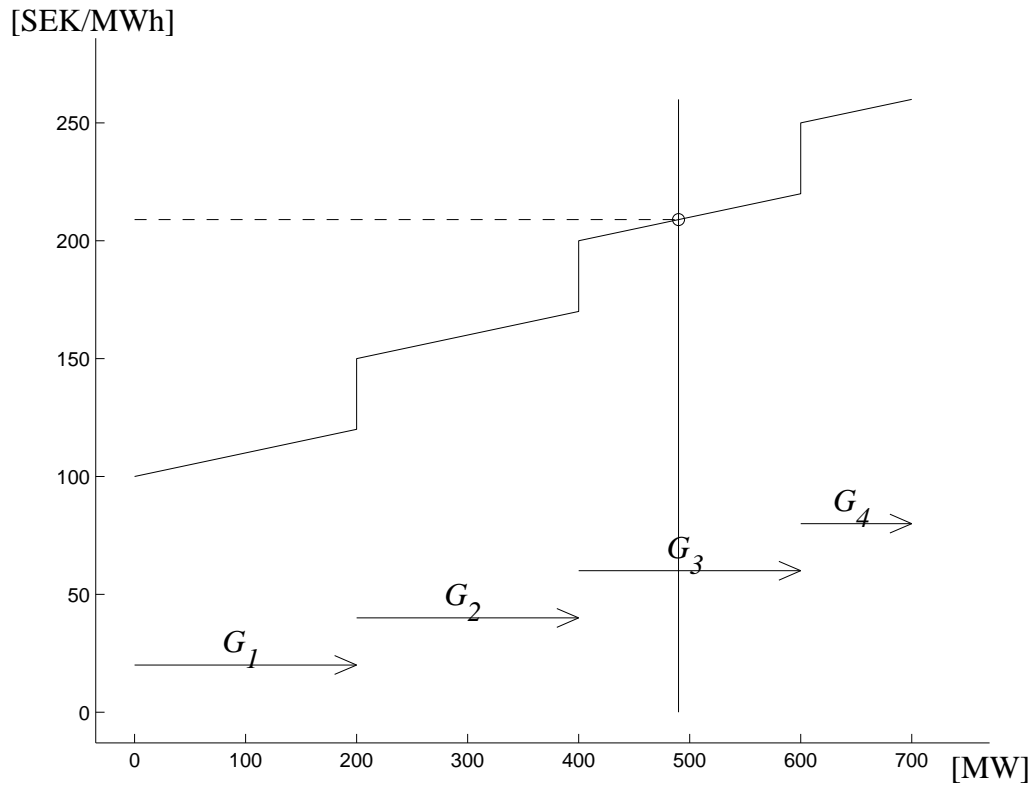


Figure 4.1. Demand and supply for example 4.1

revenue from income minus cost of production

$$C_{s1} = \lambda G_1 - C_{G1} = 209 \cdot 200 - 22000 = 19800 \text{ SEK}$$

$$C_{s2} = 209 \cdot 200 - 32000 = 9800 \text{ SEK}$$

$$C_{s3} = 209 \cdot 90 - 18405 = 405 \text{ SEK}$$

$$C_{s4} = 0 \text{ SEK}$$

The emissions become:

$$CO_{21} = \mu_1 G_1 = 1000 \cdot 200 = 200000 \text{ kg } CO_2$$

$$CO_{22} = \mu_2 G_2 = 0 \cdot 200 = 0 \text{ kg } CO_2$$

$$CO_{23} = \mu_3 G_3 = 440 \cdot 90 = 39600 \text{ kg } CO_2$$

$$CO_{24} = \mu_4 G_4 = 0 \cdot 0 = 0 \text{ kg } CO_2$$

Total CO_2 emissions become $200000 + 39600 = 239600$ kg.

End of example 4.1

4.1 Fixed price

Example 4.2 Assume a power system according to table 4.1 and a total, price independent, load of 490 MW. Assume that in order to reduce the CO_2 emissions a constant

price program is introduced, which means that all power plants that do not emit CO_2 will get a constant price. In this example this means that the bio fueled unit 2 and 4 will get this market independent feed in tariff. The feed in tariff is set as low as possible but so high so the CO_2 emissions are minimized.

4.2a Calculate the lowest needed price in order to get all CO_2 -free units producing on maximum level.

4.2b Calculate the extra cost for the increased price to the CO_2 free generation (the fixed price is higher than market price) when it is paid by the tax payers, so it is not included in the electricity market.

4.2c Calculate the extra cost for the CO_2 free generation when it is paid with an increase of the consumer price.

Solution to example 4.2a In order to minimize the CO_2 emissions it is necessary that both CO_2 free units produce at their maximum level. In order to get unit 4 to do this it is necessary to offer a price on the marginal cost at maximum level, i.e.

$$\lambda_{fee} = c_{G4} + a_4 \cdot \hat{G}_4 = 250 + 0.1 \cdot 100 = 260 \text{ SEK/MWh}$$

With this feed-in tariff 300 MW will come from unit 2+4 and 190 MW will be needed from low price unit 1.

End of example 4.2a

Solution to example 4.2b Figure 4.2 shows the supply and demand curves.

The two CO_2 -free units 2 and 4 will be used on maximum capacity ($G_2 = 200$ MW, $G_4 = 100$ MW), while 190 MW is needed from the cheapest unit, unit 1 ($G_1 = 190$ MW), to meet the load. The third unit is not needed. There will then be two prices, one for the CO_2 -free units, λ_{fee} , and one for the consumers and unit 1, λ_D

$$\lambda_D = c_{G1} + a_1 G_1 = 100 + 0.1 \cdot 190 = 119 \text{ SEK/MWh}$$

The production cost in each unit becomes:

$$C_{G1} = c_{G1} G_1 + \frac{a_1}{2} G_1^2 = 100 \cdot 190 + \frac{0.1}{2} 190^2 = 20805 \text{ SEK}$$

$$C_{G2} = 150 \cdot 200 + \frac{0.1}{2} 200^2 = 32000 \text{ SEK}$$

$$C_{G3} = 0 \text{ SEK}$$

$$C_{G4} = 250 \cdot 100 + \frac{0.1}{2} 100^2 = 25500 \text{ SEK}$$

This implies that the total operation cost is 78305 SEK, which is an increase with 5900 SEK. The economic surplus in each unit is revenue from income minus cost of production

$$C_{s1} = \lambda_D G_1 - C_{G1} = 119 \cdot 190 - 20805 = 1805 \text{ SEK}$$

$$C_{s2} = \lambda_{fee} G_2 - C_{G2} = 260 \cdot 200 - 32000 = 20000 \text{ SEK}$$

$$C_{s3} = 0 \text{ SEK}$$

$$C_{s4} = \lambda_{fee} G_4 - C_{G4} = 260 \cdot 100 - 25500 = 500 \text{ SEK}$$

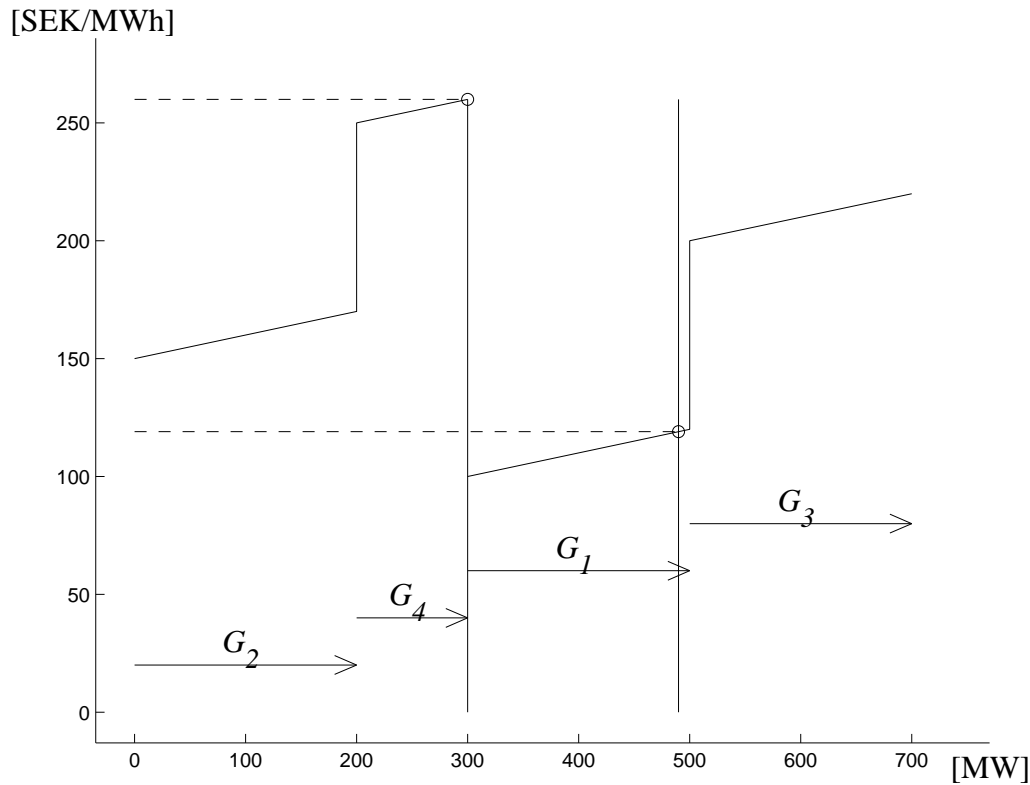


Figure 4.2. Demand and supply for example 4.2

The emissions become:

$$\begin{aligned}
 CO2_1 &= \mu_1 G_1 = 1000 \cdot 190 = 190000 \text{ kg } CO_2 \\
 CO2_2 &= \mu_2 G_2 = 0 \text{ kg } CO_2 \\
 CO2_3 &= \mu_3 G_3 = 0 \text{ kg } CO_2 \\
 CO2_4 &= \mu_4 G_4 = 0 \text{ kg } CO_2
 \end{aligned}$$

This means that the total emissions is 190000 kg CO_2 which is a decrease of 49600 kg CO_2 . The cost for this decrease is $5900/49600 = 0.119$ SEK/kg CO_2 .

End of example 4.2b

Solution to example 4.2c In this example the consumers are not price sensitive. This means that if the price goes up they still consume the same amount of power. This means in this example that if the consumers have to pay the cost for fixed price program, then they will still consume on the same level, i.e., 490 MW. This implies that the production, production cost, emissions and power price will be the same as in example 4.2b. The only different is that the consumer price will increase. The increase (for each MWh of consumption) can be calculated as:

$$\Delta\lambda_D = (\lambda_{fee} - \lambda_D) \frac{G_2 + G_4}{D} = (260 - 119) \frac{200 + 100}{490} = 86.33 \text{ SEK/MWh}$$

This implies that the consumer price becomes $\lambda_D + \Delta\lambda_D = 119 + 86.33 = 205.33$ SEK/MWh.

4.1.1 Comments concerning introduction of a fixed price system

By comparing the results of example 4.1 and 4.2 some conclusions can be drawn. These are, of course, valid for these examples, but some of these also includes some general issues concerning the introduction of a fixed price system.

We first start with the comments when the extra cost of the fixed price is paid through the tax, i.e., example 4.2b.

- The introduction of the fixed price system results in a decreased cost for the consumers purchase of electricity. This depends on that there is a (forced) increase in the supply (300 MW of market price independent production).
- But the cost of this has to be paid through taxes in some way, so the question is then how this affects the taxes for the consumers and/or the economy of the country, if, e.g., the companies have to pay increased taxes that finance the fixed price system, and/or if there is an increased taxes on private persons in the country.
- The surplus of the involved companies with units with CO_2 emissions decreases (from 20205 SEK to 1805 SEK) which of course is a significant decrease. If there is one company that only owns these units it is rational for the companies who owns unit 1 and 3 (if they only owns these units) to not promote the introduction of a fixed price (if they only view the company's profit. It must though be noted that these companies also have to consider the risk of being negatively treated in media if they are not positive to systems that reduce the CO_2 emissions.

Now to comments concerning example 4.2c, where the extra cost for the whole system ($=\lambda_{fee} - \lambda_D$) is paid by the consumers.

- The price for the consumers decrease also if the consumers have to pay for the feed-in tariff system. In example 4.1 the price became 209 SEK/MWh, while it in example 4.2c became 205.33 SEK/MWh. The reason is that there is an increased supply (modifies the price that is set by the marginal cost), but the increased cost is paid in a mean cost manner.
- The surplus of the involved companies with units with CO_2 emissions decreases (of course since this is the aim of the system).
- Since the price decreases (compared to example 4.1) this implies that the total surplus for the whole generation industry will decrease since the feed in tariff also result in an increased cost.

4.2 Subsidies and taxes

Subsidies or taxes are often introduced in order to compensate for externalities. Examples are subsidies to environmentally friendly power production or taxes on emissions. In both cases the external effects concerning the environment is not directly integrated in the production cost in the power plants but in this way they are internalized. The subsidy or the tax can be added both on the investment side, or on the operation cost. Concerning emission tax it is more natural to have it on the emissions, i.e., on the operation (and not on the investment). Another application is to have subsidies/taxes on the consumption to compensate for externalities. Consumption subsidies can be motivated by, e.g., that from the society point of view also low income households should have the right to cheap electricity, while taxes can be an incentive for households or industry to use the electricity more efficient. A general difference between taxes/subsidies on production or consumption side is that they imply different possibilities to set different levels on subsidies on different producer groups (e.g. environmental friendly sources) or consumer groups (different taxes on industry/households).

Example 4.3 *Assume a power system according to table 4.1 and a total, price independent, load of 490 MW. Assume that in order to reduce the CO₂ emissions a CO₂ tax is introduced and/or the low emission units are subsidized with a certain amount per produced kWh. In this example this means that the bio fueled unit #2 and #4 will get this subsidy.*

Both the CO₂ tax and the subsidy are set as low as possible but so high so the production in CO₂ free generation is maximized.

Calculate production, production cost, economic surplus and CO₂ emission in each unit and consumer price for the three cases:

4.3a Introduce a CO₂ tax that is as low as possible but still maximizes the production in the CO₂ free units.

4.3b Introduce a CO₂ tax that is as low as possible and at the same time minimizes the CO₂ emissions.

4.3c Introduce a subsidy for the CO₂ free units that is as low as possible but still maximizes the production in these

Solution to example 4.3a To maximize the production in CO₂ free units means that the marginal cost for maximum production in the high cost bio-fuelled unit #4 can not be higher than the (marginal cost + CO₂-tax) in the last produced MWh in CO₂-emitting units.

When unit #4 produces on its maximum, the marginal cost is

$$c_{G4} + a_4 \cdot \hat{G}_4 = 250 + 0.1 \cdot 100 = 260 \text{ SEK/MWh}$$

This means that the power price will be on this level, i.e.

$$\lambda = 260 \text{ SEK/MWh}$$

At this price and production level the total needed power production is CO_2 emitting units is $490 - (200 + 100) = 190$ MW to be divided between unit #1 and #3. First assume that this power is produced in the coal fired unit #1. Then the marginal cost is

$$c_{G1} + a_1 \cdot G_1 = 100 + 0.1 \cdot 190 = 119 \text{ SEK/MWh}$$

which means that the needed CO_2 tax is $260 - 119 = 141$ SEK/MWh corresponding to $141/1000 = 0.141$ SEK/kg CO_2 . With this CO_2 tax the marginal cost for the gas fired unit #3 becomes

$$c_{G3} + 440 \cdot 0.141 + a_3 \cdot \hat{G}_3 = 262 + 0.1 \cdot G_3 \text{ SEK/MWh}$$

which means that for any level of production the marginal cost will be higher than for unit #1. The conclusion is then that $G_1 = 190$ MW, $G_3 = 0$ MW and $C_{CO_2-tax} = 0.141$ SEK/kg CO_2 . Figure 4.3 shows the supply and demand curves, with this CO_2 tax.

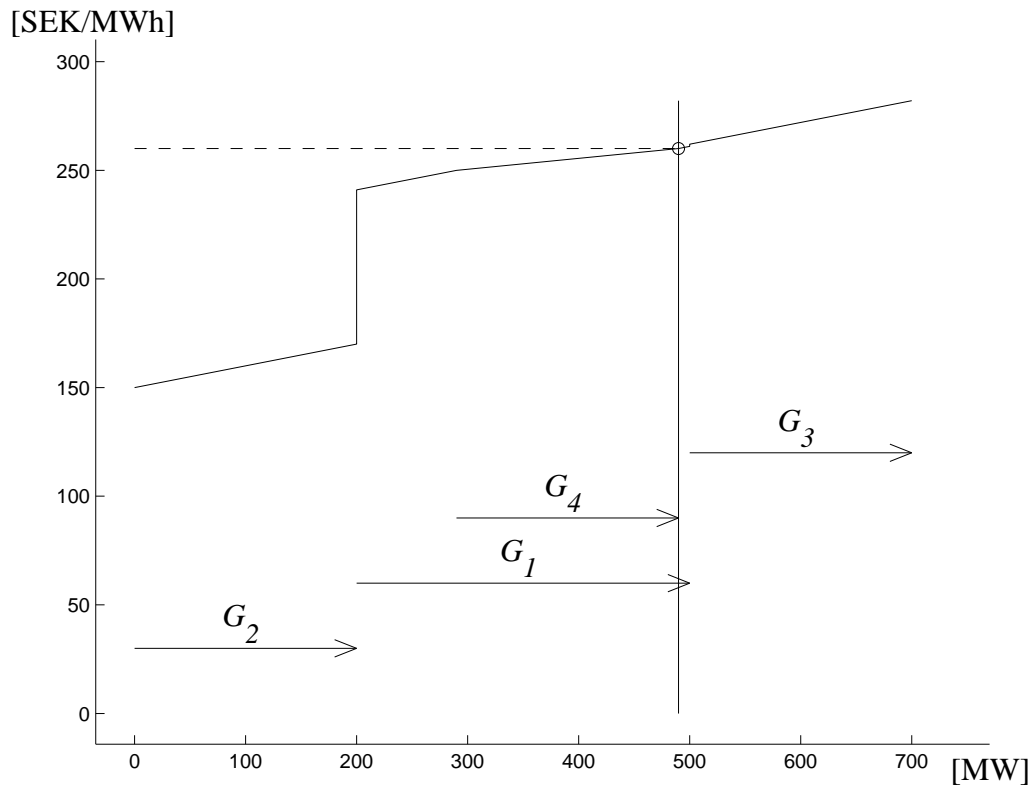


Figure 4.3. Demand and supply for example 4.3a. CO_2 tax = 0.141 SEK/kg CO_2 .

The production cost in each unit becomes:

$$\begin{aligned}
 C_{G1} &= (c_{G1} + 1000C_{CO2-tax})G_1 + \frac{a_1}{2}G_1^2 = \\
 &= (100 + 1000 \cdot 0.141) \cdot 190 + \frac{0.1}{2}190^2 = 47595 \text{ SEK} \\
 C_{G2} &= 150 \cdot 200 + \frac{0.1}{2}200^2 = 32000 \text{ SEK} \\
 C_{G3} &= 0 \text{ SEK} \\
 C_{G4} &= 250 \cdot 100 + \frac{0.1}{2}100^2 = 25500 \text{ SEK}
 \end{aligned}$$

This implies that the total operation cost is 105095 SEK (including cost for CO_2 tax), which is an increase with 32690 SEK. The increase consists of fuel cost increase of 5900 SEK and total CO_2 tax cost of 26790 SEK. The economic surplus in each unit is revenue from income minus cost of production:

$$\begin{aligned}
 C_{s1} &= \lambda_D G_1 - C_{G1} = 260 \cdot 190 - 47595 = 1805 \text{ SEK} \\
 C_{s2} &= \lambda_D G_2 - C_{G2} = 260 \cdot 200 - 32000 = 20000 \text{ SEK} \\
 C_{s3} &= 0 \text{ SEK} \\
 C_{s4} &= \lambda_D G_4 - C_{G4} = 260 \cdot 100 - 25500 = 500 \text{ SEK}
 \end{aligned}$$

The emissions become:

$$\begin{aligned}
 CO2_1 &= \mu_1 G_1 = 1000 \cdot 190 = 190000 \text{ kg } CO_2 \\
 CO2_2 &= \mu_2 G_2 = 0 \text{ kg } CO_2 \\
 CO2_3 &= \mu_3 G_3 = 0 \text{ kg } CO_2 \\
 CO2_4 &= \mu_4 G_4 = 0 \text{ kg } CO_2
 \end{aligned}$$

End of example 4.3a

Solution to example 4.3b To also minimize the CO_2 emissions means that the high emission coal unit has to be replaced with the natural gas fired unit which has higher fuel costs but lower emissions. To make this happen the CO_2 tax has to be on a level so $G_3 = 190$ MW and $G_1 = 0$ MW. This means that on these production levels the marginal cost (including CO_2 tax) has to be on the same level for these two units:

$$\begin{aligned}
 c_{G3} + 440C_{CO2-tax} + a_3 \cdot 190 &= c_{G1} + 1000C_{CO2-tax} + a_1 \cdot 0 \\
 200 + 440C_{CO2-tax} + 19 &= 100 + 1000C_{CO2-tax} \\
 C_{CO2-tax} &= \frac{200 + 19 - 100}{1000 - 440} = 0.2125 \text{ SEK/kg } CO_2
 \end{aligned}$$

Figure 4.4 shows the supply and demand curves, with this CO_2 tax.

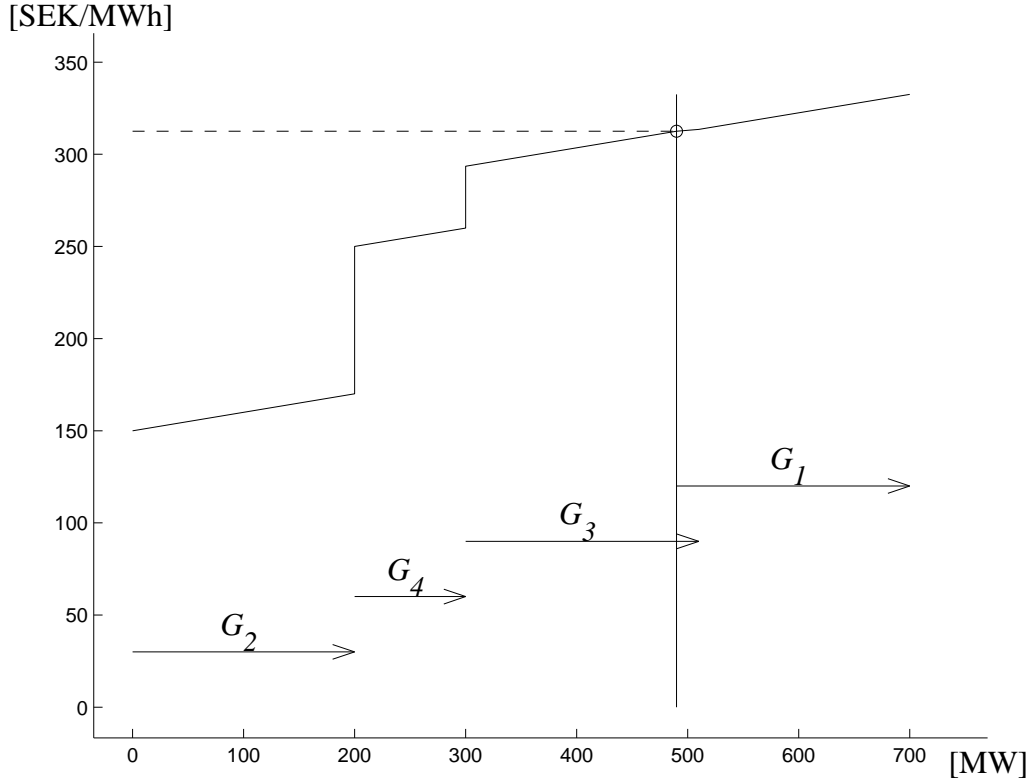


Figure 4.4. Demand and supply for example 4.3b. CO_2 tax = 0.2125 SEK/kg CO_2 .

The production cost in each unit becomes:

$$\begin{aligned}
 C_{G1} &= 0 \\
 C_{G2} &= 150 \cdot 200 + \frac{0.1}{2} 200^2 = 32000 \text{ SEK} \\
 C_{G3} &= (c_{G3} + 440C_{CO_2\text{-tax}})G_3 + \frac{a_3}{2} G_3^2 = \\
 &= (200 + 440 \cdot 0.2125) \cdot 190 + \frac{0.1}{2} 190^2 = 57570 \text{ SEK} \\
 C_{G4} &= 250 \cdot 100 + \frac{0.1}{2} 100^2 = 25500 \text{ SEK}
 \end{aligned}$$

This implies that the total operation cost is 115070 SEK (including cost for CO_2 tax), which is an increase with 42665 SEK compared to the reference case in example 4.1. The increase consists of fuel cost increase of 24900 SEK and total CO_2 tax cost of 17765 SEK. The power price is now changed since it is the marginal cost (including CO_2 tax) that sets the price. The new price become:

$$\lambda_D = c_{G3} + 440C_{CO_2\text{-tax}} + a_3 \cdot 190 = 312.5 \text{ SEK/MWh}$$

The economic surplus in each unit is revenue from income minus cost of production:

$$\begin{aligned}
 C_{s1} &= 0 \\
 C_{s2} &= \lambda_D G_2 - C_{G2} = 312.5 \cdot 200 - 32000 = 30500 \text{ SEK} \\
 C_{s3} &= \lambda_D G_1 - C_{G1} = 312.5 \cdot 190 - 57570 = 1805 \text{ SEK} \\
 C_{s4} &= \lambda_D G_4 - C_{G4} = 312.5 \cdot 100 - 25500 = 5750 \text{ SEK}
 \end{aligned}$$

The emissions become:

$$\begin{aligned}
 CO2_1 &= \mu_1 G_1 = 0 \text{ kg } CO_2 \\
 CO2_2 &= \mu_2 G_2 = 0 \text{ kg } CO_2 \\
 CO2_3 &= \mu_3 G_3 = 440 \cdot 190 = 83600 \text{ kg } CO_2 \\
 CO2_4 &= \mu_4 G_4 = 0 \text{ kg } CO_2
 \end{aligned}$$

End of example 4.3b

Solution to example 4.3c The idea here is to maximize the production in the CO_2 free units by the introduction of a subsidy to these units. With a load of 490 MW and an installed capacity in the CO_2 free units of 300 MW, this means that 190 MW is needed in CO_2 emitting units. Since there is no economic change of these units compared to the base case, this means that this 190 MW will be produced in the cheapest units, i.e., unit #1. The marginal cost is then

$$c_{G1} + a_1 \cdot G_1 = 100 + 0.1 \cdot 190 = 119 \text{ SEK/MWh}$$

Then the power price has to be on this level, otherwise this unit will produce more, i.e.

$$\lambda = 119 \text{ SEK/MWh}$$

In order to make the other units to produce on its maximum, then the marginal cost (including subsidy) of the most expensive CO_2 free unit (= unit #4) has to be on this level

$$\begin{aligned}
 c_{G4} - C_{CO2-subsidy} + a_4 \cdot \hat{G}_4 &= 250 - C_{CO2-subsidy} + 0.1 \cdot 100 = 119 \text{ SEK/MWh} \\
 \Rightarrow \\
 C_{CO2-subsidy} &= 250 + 10 - 119 = 141 \text{ SEK/MWh}
 \end{aligned}$$

It can be noted that this is the same level as the CO_2 tax in example 4.3a. The difference here is that the tax (that increases the marginal cost corresponding to an increased power price) is replaced with a subsidy which lowers the marginal cost and power price.

Figure 4.5 shows the supply and demand curves, with this subsidy of CO_2 free units.

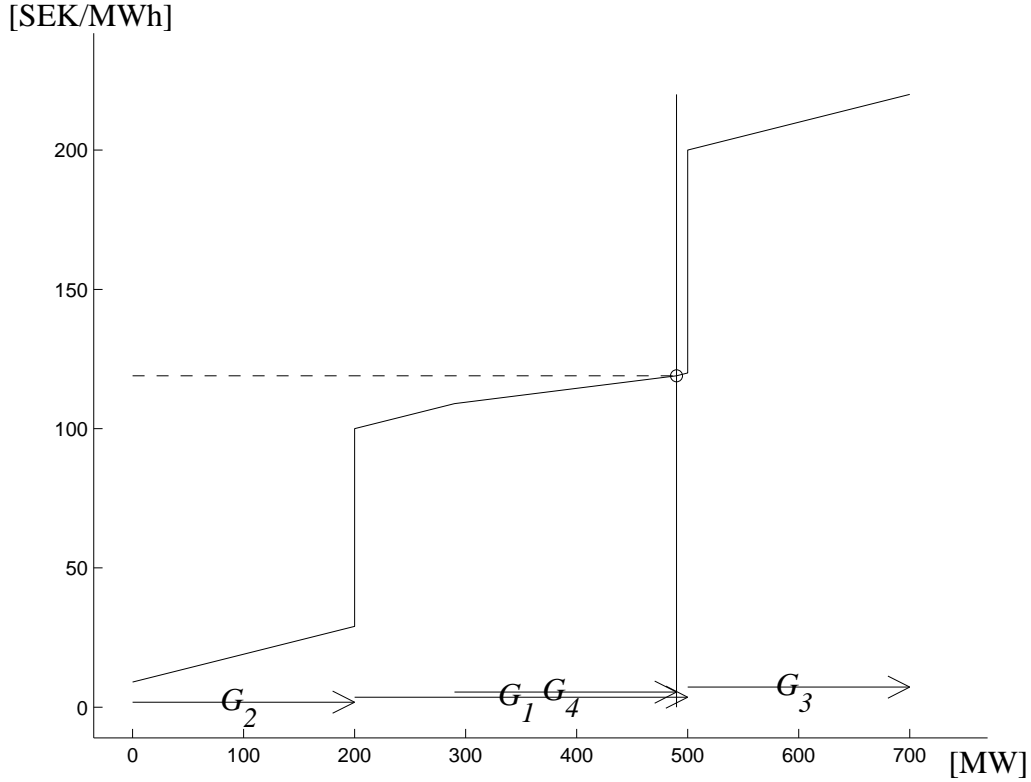


Figure 4.5. Demand and supply for example 4.3c. Subsidy of CO_2 free units = 141 SEK/MWh.

The production cost in each unit becomes:

$$\begin{aligned}
 C_{G1} &= c_{G1}G_1 + \frac{a_1}{2}G_1^2 = \\
 &= 100 \cdot 190 + \frac{0.1}{2}190^2 = 20805 \text{ SEK} \\
 C_{G2} &= (150 - 141) \cdot 200 + \frac{0.1}{2}200^2 = 3800 \text{ SEK} \\
 C_{G3} &= 0 \text{ SEK} \\
 C_{G4} &= (250 - 141) \cdot 100 + \frac{0.1}{2}100^2 = 11400 \text{ SEK}
 \end{aligned}$$

This implies that the total operation cost is 36005 SEK (including subsidy), which is a decrease with 36400 SEK compared to the reference case in example 4.1. The decreased operation cost consists of a subsidy of $141(200+100)=42300$ SEK, and fuel cost increase of 5900 SEK (same as in example 4.3a). The economic surplus in each unit is revenue from income minus cost of production:

$$\begin{aligned}
 C_{s1} &= \lambda G_1 - C_{G1} = 119 \cdot 190 - 20805 = 1805 \text{ SEK} \\
 C_{s2} &= \lambda_D G_2 - C_{G2} = 119 \cdot 200 - 3800 = 20000 \text{ SEK} \\
 C_{s3} &= 0 \text{ SEK} \\
 C_{s4} &= \lambda_D G_4 - C_{G4} = 119 \cdot 100 - 11400 = 500 \text{ SEK}
 \end{aligned}$$

It can be noted that this is the same surplus per unit as in example 4.3a. The emissions become:

$$\begin{aligned} CO2_1 &= \mu_1 G_1 = 1000 \cdot 190 = 190000 \text{ kg } CO_2 \\ CO2_2 &= \mu_2 G_2 = 0 \text{ kg } CO_2 \\ CO2_3 &= \mu_3 G_3 = 0 \text{ kg } CO_2 \\ CO2_4 &= \mu_4 G_4 = 0 \text{ kg } CO_2 \end{aligned}$$

which are also the same as in example 4.3a).

End of example 4.3c

4.3 Certificates

A certificate system implies that there is parallel system separate from the power system where so-called *electricity certificates* are traded. The idea is to create competition between different forms of *certified* energy production. Electricity producers receive a certificate for each MWh of *certified* electricity that they produce. These certificates are then sold to electricity users, who are obliged to purchase certificates equivalent to a certain proportion of their electricity use. In this way, producers of *certified* electricity will receive additional revenue, above the price for the actual electricity. It is possible to trade in the certificates, and they form a market of their own, separate from the actual physical market of electricity.

Example 4.4 Assume a power system according to table 4.1 and a total, price independent, load of 490 MW. Assume that in order to reduce the CO_2 emissions a certificate system is introduced. In this example this means that the bio fueled unit #2 and #4 will receive these certificates.

Calculate production, production cost, economic surplus and CO_2 emission in each unit, certificate price and consumer price for the two cases:

- 4.4a Assume that there is a target that the consumers have to get 60% of their electricity from CO_2 free sources. To obtain this a certificate system is introduced, where unit #2 and #4 receive one certificate for each produced MWh.
- 4.4b Assume that unit #2 does not exist but it is an option for the future. This means the the power price + certificate price has to be at least equal to operation cost + investment cost in this unit. The same data as in example 4.4a are valid and the investment cost for unit #2 is 105 SEK/MWh.
- 4.4c Assume that there are two one hour periods where the load is 250 MW in the first period and 350 MW in the second period. The requirement for the retailers is that their consumers for the whole period must get 50 % of their electricity from CO_2 free units. This means that one allow banking of certificates, i.e., they are

produced in one period and used in another period. Calculate the production and certificate price. In this example a_2 is changed to 0.3.

Solution to example 4.4a The requirement is that the consumers should get 60 % of their consumption from CO_2 free sources. This corresponds to $0.6 \cdot 490 = 294$ MW from unit #2 and #4, and $0.4 \cdot 490 = 196$ MW from unit #1 and #3. The corresponding marginal costs are

$$\begin{aligned} c_{G1} + a_1 \cdot G_1 &= 100 + 0.1 \cdot 196 = 119.6 \text{ SEK/MWh} \\ c_{G4} + a_4 \cdot G_4 &= 250 + 0.1 \cdot (294 - 200) = 259.4 \text{ SEK/MWh} \end{aligned}$$

To make unit #4 to produce on this level, then the certificate price has to be

$$\lambda_{cert} = 259.4 - 119.6 = 139.8 \text{ SEK}$$

With this certificate price the marginal cost is equal for unit #1 and #4, which is necessary since none of these units produce on their installed capacity. The resulting power price is then

$$\lambda_P = 119.6 \text{ SEK/MWh}$$

The consumers will on their electricity bill pay for the power price and the total cost of the certificates, i.e.

$$\lambda_D = \lambda_P + \frac{\lambda_{cert} \cdot 0.6 \cdot 490}{490} = \lambda_P + 0.6 \cdot \lambda_{cert} = 203.48 \text{ SEK/MWh}$$

This means that the consumers will pay a lower price (203.48 SEK/MWh) than the total revenue per MWh to the owners of the certified units (259.4 SEK/MWh). Figure 4.6 shows the supply and demand curves, with the certificate included as a decrease in operation cost for unit #2 and #4. The production cost (including revenue from received certificates) in each unit becomes:

$$\begin{aligned} C_{G1} &= c_{G1}G_1 + \frac{a_1}{2}G_1^2 = \\ &= 100 \cdot 196 + \frac{0.1}{2}196^2 = 21520.8 \text{ SEK} \\ C_{G2} &= (150 - 139.8) \cdot 200 + \frac{0.1}{2}200^2 = 4040 \text{ SEK} \\ C_{G3} &= 0 \text{ SEK} \\ C_{G4} &= (250 - 139.8) \cdot 94 + \frac{0.1}{2}94^2 = 10800.6 \text{ SEK} \end{aligned}$$

This implies that the total operation cost is 36361.4 SEK (including certificates), which is a decrease with 36043.6 SEK compared to the reference case in example 4.1. The decreased operation cost consists of an income from sold certificates of $139.8(200+94)=41101.2$ SEK, and fuel cost increase of 5057.6 SEK. The economic surplus in each unit

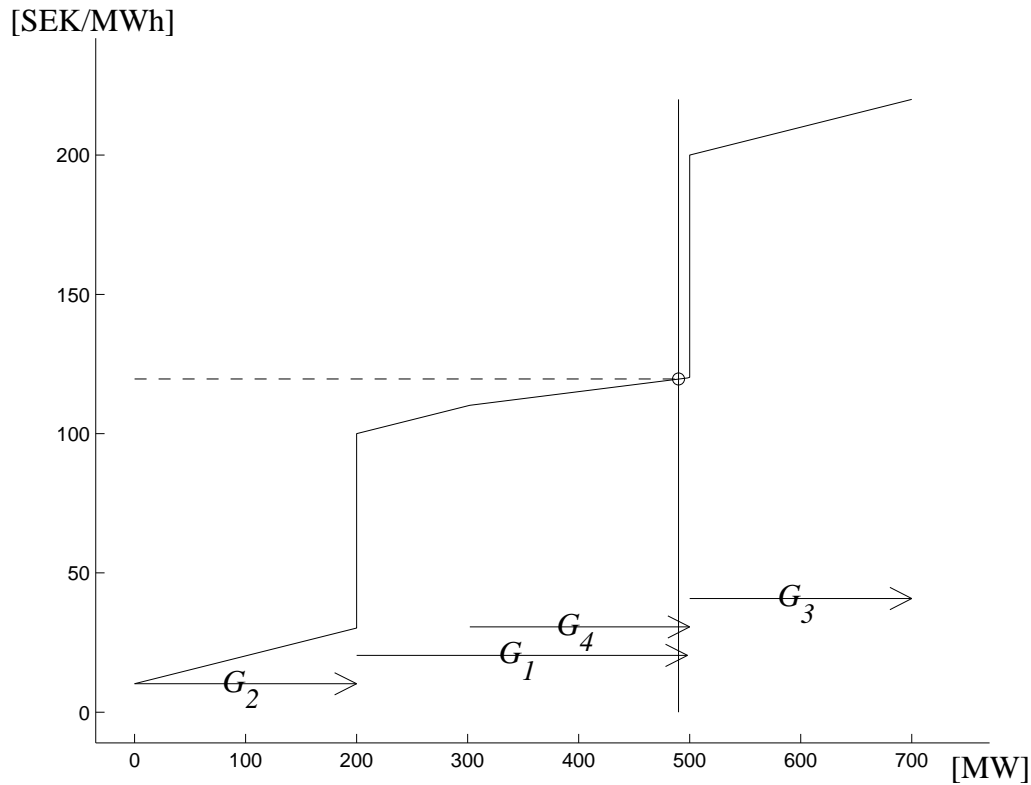


Figure 4.6. Demand and supply for example 4.4a. Certificate price = 139.8 SEK.

is revenue from income minus cost of production:

$$C_{s1} = \lambda_P G_1 - C_{G1} = 119.6 \cdot 196 - 21520.8 = 1920.8 \text{ SEK}$$

$$C_{s2} = \lambda_P G_2 - C_{G2} = 119.6 \cdot 200 - 4040 = 19880 \text{ SEK}$$

$$C_{s3} = 0 \text{ SEK}$$

$$C_{s4} = \lambda_P G_4 - C_{G4} = 119.6 \cdot 94 - 10800.6 = 441.8 \text{ SEK}$$

The emissions become:

$$CO2_1 = \mu_1 G_1 = 1000 \cdot 196 = 196000 \text{ kg } CO_2$$

$$CO2_2 = \mu_2 G_2 = 0 \text{ kg } CO_2$$

$$CO2_3 = \mu_3 G_3 = 0 \text{ kg } CO_2$$

$$CO2_4 = \mu_4 G_4 = 0 \text{ kg } CO_2$$

End of example 4.4a

Solution to example 4.4b Also in this example the requirement is that the consumers should get 60 % of their consumption from CO_2 free sources. This still corresponds to 294 MW from unit #2 and #4, and 196 MW from unit #1 and #3. The corresponding

marginal costs are as in example 4.4a:

$$\begin{aligned} c_{G1} + a_1 \cdot G_1 &= 119.6 \text{ SEK/MWh} \\ c_{G4} + a_4 \cdot G_4 &= 259.4 \text{ SEK/MWh} \end{aligned}$$

To make unit #4 to produce on this level, then the certificate price has to be

$$\lambda_{cert} = 259.4 - 119.6 = 139.8 \text{ SEK}$$

The question is then if this is enough to cover the total cost for unit #2 so this unit will be built? As calculated in example 4.4a, the operation cost and surplus of unit #2 are:

$$\begin{aligned} C_{G2} &= (150 - 139.8) \cdot 200 + \frac{0.1}{2} 200^2 = 4040 \text{ SEK} \\ C_{s2} &= \lambda_P G_2 - C_{G2} = 119.6 \cdot 200 - 4040 = 19880 \text{ SEK} \end{aligned}$$

The surplus corresponds to $19880/200 = 99.4 \text{ SEK/MW}$. Since the fixed cost is 105 SEK/MWh unit #2 will not be built. To make it profitable the certificate price has to increase with at least $105 - 99.4 = 5.6 \text{ SEK/MWh}$. At this certificate price, though, unit #4 will have a lower marginal cost (since also this unit will get higher revenue from certificates) than unit #1. Thereby unit #4 will increase its production and unit #1 will decrease its production. But still 190 MW is needed from unit #1 to cover the load of 490 MW. The marginal cost of unit #1 is then equal to the power price:

$$\lambda_P = c_{G1} + a_1 \cdot 190 = 119 \text{ SEK/MWh}$$

At this power price the operation cost and surplus of unit #2 are:

$$\begin{aligned} C_{G2} &= (150 - \lambda_{cert-b}) \cdot 200 + \frac{0.1}{2} 200^2 = 32000 - \lambda_{cert-b} 200 \text{ SEK} \\ C_{s2} &= \lambda_P G_2 - C_{G2} = 119 \cdot 200 - (32000 - \lambda_{cert-b} 200) = \lambda_{cert-b} 200 - 8200 \text{ SEK} \end{aligned}$$

The lowest needed certificate price to not make a loss in unit #2, i.e., operation surplus is equal to fixed cost, is

$$\begin{aligned} \lambda_{cert-b} 200 - 8200 &= 105 \cdot 200 \\ \Rightarrow \\ \lambda_{cert-b} &= \frac{105 \cdot 200 + 8200}{200} = 146 \text{ SEK/MWh} \end{aligned}$$

Figure 4.7 shows the supply and demand curves, with this certificate price included as it lowers the operation cost.. It can be noted that with this certificate price, unit #2 will be built but the certificate price is so high that there will be more CO_2 free generation than needed (300 MW instead of required 294). If one lowers the certificate price, then unit 2 will not be constructed at all.

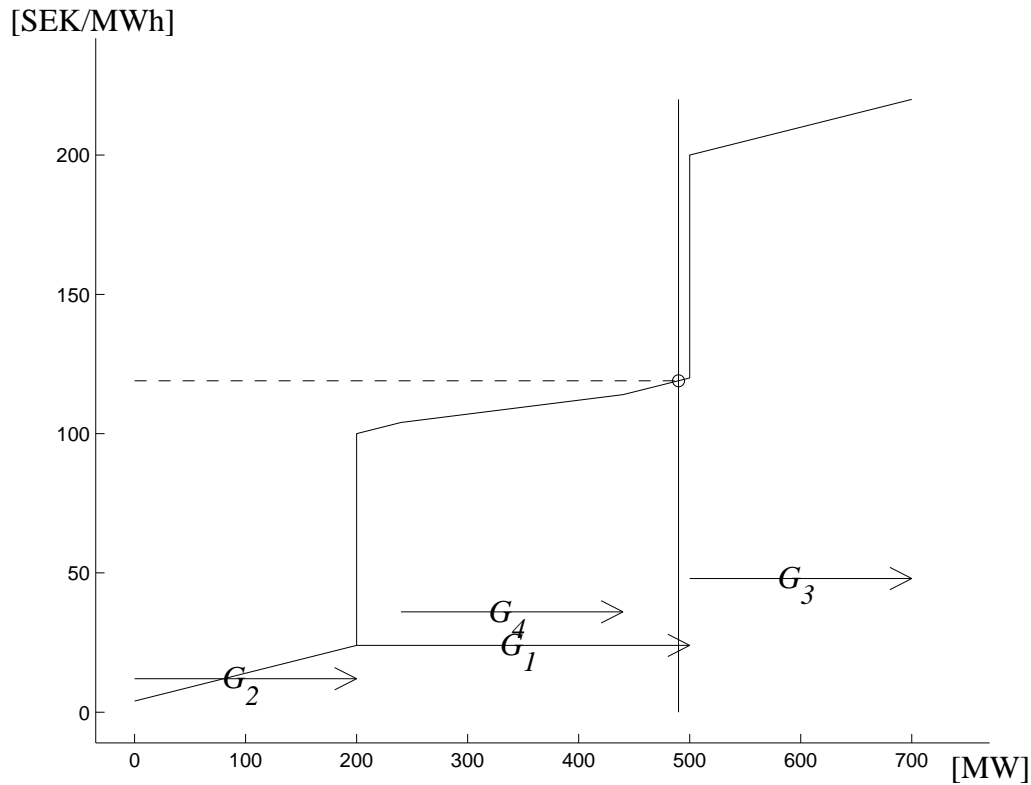


Figure 4.7. Demand and supply for example 4.4b. Certificate price = 146 SEK.

End of example 4.4b

Solution to example 4.4c The total load for the two periods is $250+350=600$ MWh. The requirement is that the total CO_2 free generation should be 50 % of this, i.e., $0.5 \cdot 600 = 300$ MWh. If only low cost production is required (or obtained by market competition), then CO_2 emitting unit #1 will produce full power in both periods ($200+200=400$ MWh) while CO_2 free unit #2 will produce the rest, i.e., $50+150=200$ MWh. This means that certificates are needed to increase the revenue for unit #2, so these units produce more and unit #1 less. In this example units #3-4 are not needed.

In each period the marginal cost of unit #1 should be equal to the marginal cost (including revenue from certificate) of unit #2. Otherwise it should be profitable to move production from one of the units to the other. The certificate price will be the same in the two periods, since there is no restrictions of how much certificates that can be storee between the periods. This can be formulated as

$$\begin{aligned} c_{G1} + a_1 \cdot G_1(1) &= c_{G2} + a_2 \cdot G_2(1) - C_{cert} \\ c_{G1} + a_1 \cdot G_1(2) &= c_{G2} + a_2 \cdot G_2(2) - C_{cert} \end{aligned}$$

In addition to this there must be load-production balance in each period and the total production in unit #2 is given,

$$G_1(1) + G_2(1) = 250$$

$$G_1(2) + G_2(2) = 350$$

$$G_2(1) + G_2(2) = 300$$

This forms a linear system of equation as

$$100 + 0.1 \cdot G_1(1) = 150 + 0.3 \cdot G_2(1) - C_{cert}$$

$$100 + 0.1 \cdot G_1(2) = 150 + 0.3 \cdot G_2(2) - C_{cert}$$

$$G_1(1) + G_2(1) = 250$$

$$G_1(2) + G_2(2) = 350$$

$$G_2(1) + G_2(2) = 300$$

or

$$\begin{bmatrix} 0.1 & 0 & -0.3 & 0 & 1 \\ 0 & 0.1 & 0 & -0.3 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} G_1(1) \\ G_1(2) \\ G_2(1) \\ G_2(2) \\ C_{cert} \end{bmatrix} = \begin{bmatrix} 50 \\ 50 \\ 250 \\ 350 \\ 300 \end{bmatrix}$$

The solution to this is $G_1(1) = 112.5$ MW, $G_1(2) = 187.5$ MW, $G_2(1) = 137.5$ MW, $G_2(2) = 162.5$ MW, $C_{cert} = 80$ SEK/MWh. Figure 4.8 shows the supply and demand curves for both periods, with this certificate price for both periods included as it lowers the operation cost for unit #2.

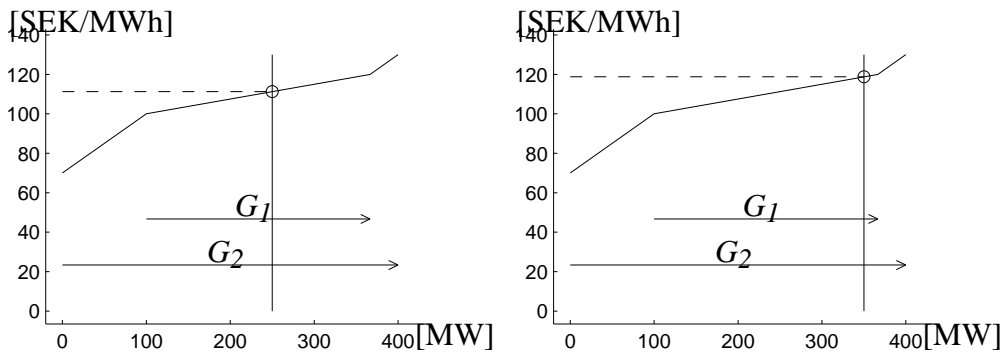


Figure 4.8. Demand and supply for period 1 (left) and period 2 (right) in example 4.4c. Certificate price = 80 SEK.

This result means that in period 1 137.5 certificates are obtained while only $0.5 \cdot 250 = 125$ certificates are used. The consequence is that $137.5 - 125 = 12.5$ certificates are *banked* from period 1 to period 2. It can be noted that if the amount of *banked* certificates is known, then it is trivial to calculate the power production in all units and periods.

The power price during the two periods can be calculated from the fact that the units do not produce on their limits, i.e., the power price is equal to marginal cost:

$$\begin{aligned}\lambda_1 &= c_{G1} + a_1 \cdot G_1(1) = 100 + 0.1 \cdot 112.5 = 111.25 \text{ SEK/MWh} \\ \lambda_2 &= c_{G1} + a_1 \cdot G_1(2) = 100 + 0.1 \cdot 187.5 = 118.75 \text{ SEK/MWh}\end{aligned}$$

As shown here the certificate price is constant, but the power price varies.

This problem can also be formulated as an optimization problem. This formulation is more general compared to the previous one. The limitation of the previous one is that it becomes more complicated if some units produce on their limit since the marginal costs are then not equal, as assumed in this case. The optimization formulation becomes:

$$\begin{aligned}\max Z = C_D - \sum C_{Gi} &= \hat{C}_D - \sum_{t=1}^2 \sum_{i=1}^2 \left(c_{Gi} G_i(t) + \frac{a_i}{2} G_i(t)^2 \right) \\ \text{when} & \\ 0 \leq G_1(t) &\leq 200 \\ 0 \leq G_2(t) &\leq 200 \\ G_1(t) + G_2(t) &= D_t \quad t \in [1, 2] \\ G_2(1) + G_2(2) &= 300\end{aligned}$$

The free variables in this optimization problem are $G_1(t)$, $G_2(t)$, $t \in [1, 2]$. The certificate price is formally the dual variable of the last constraint, i.e., how much does the value of the objective change if one changes the CO_2 free requirement with one MWh.

End of example 4.4c

4.4 Emission rights

The EU Emissions Trading Scheme (ETS) is governed by the Emissions Trading Directive (2003/87/EC). Initially, trading only covers emissions of one greenhouse gas - carbon dioxide - from energy installations and certain energy-intensive industrial sectors.

The European Commission views trading in carbon dioxide emission allowances as an important way of achieving the EU's Kyoto commitment to reduce emissions. The objective is to create an efficient European market in greenhouse gas emission allowances with the least possible diminution of economic development and employment within the EU.

Example 4.5 Assume a power system according to table 4.1 and a total, price independent, load of 490 MW. Assume that in order to reduce the CO_2 emissions an emission right system is introduced. In this example this means that the bio fueled unit #1 and #3 will receive these emission rights. There is also a trading system of CO_2 allowances. In this market it is possible for owners of units #1 and #3 to buy rights to emit CO_2 or to sell their own surplus.

- 4.5a Assume that an emission right system is introduced, where unit #1 has received allowances of 150 ton CO_2 and unit #3 has received allowances of 50 ton CO_2 . The current and expected market price of allowances is 60 SEK/ton, CO_2 . Calculate the power price.
- 4.5b Assume the same unit data as in example 4.5a but now the current and expected market price of allowances is 220 SEK/ton, CO_2 . Calculate the power price.
- 4.5c Assume the same unit data as in example 4.5a. It is though assumed that the studied system has an impact on the market price of allowances. The basic price at no trading is 60 SEK/ton CO_2 . For each sold allowance (=1 ton of CO_2), the price decreases with 0.5 SEK and for each bought allowance it increases with 0.1 SEK. Calculate the power price.
- 4.5d Assume the same unit data as in example 4.5a. But now there is a closed system where no trading of allowances is allowed with external players. Calculate the price of power and allowances.

Solution to example 4.5a The available allowances can be sold to the market or bought from the market. This means that producing electric power implies a cost for the owners of unit #1 and #3 since this means that they can not sell as many allowances. The extra cost is

$$C_{CO_2-1} = \frac{(1000 \text{ kg}CO_2/\text{MWh}) \cdot (60 \text{ SEK/ton})}{1000 \text{ kg/ton}} = 60 \text{ SEK/MWh}$$

$$C_{CO_2-3} = 440 \cdot 60/1000 = 26.4 \text{ SEK/MWh}$$

This means that the marginal cost for unit #1 and #3 become

$$MC(G_1) = c_{G1} + C_{CO_2-1} + a_1 G_1 = 160 + 0.1 G_1 \text{ SEK/MWh}$$

$$MC(G_3) = c_{G3} + C_{CO_2-3} + a_3 G_3 = 226.4 + 0.1 G_3 \text{ SEK/MWh}$$

Figure 4.9 shows the supply and demand curves, with these marginal costs included for unit #1 and #3.

It can be noted that with this price of allowances, unit #1 and #2 will be used on their full while the rest, $490 - 200 - 200 = 90$ MW, is produced in unit #3. This implies that the following amounts of allowances are used:

$$\text{used allowances in unit \#1} = \frac{(200 \text{ MWh}) \cdot (1000 \text{ kg}CO_2/\text{MWh})}{1000 \text{ kg/ton}} = 200 \text{ ton}CO_2$$

$$\text{used allowances in unit \#3} = \frac{90 \cdot 440}{1000} = 39.6 \text{ ton}CO_2$$

this implies that unit #1 has to buy $200 - 150 = 50$ allowances from the market, while unit #3 can sell $50 - 39.6 = 10.4$ allowances (or store them for future use). The power price is set by the marginal cost in unit #3

$$\lambda = c_{G3} + C_{CO_2-3} + a_3 G_3 = 226.4 + 0.1 \cdot 90 = 235.4 \text{ SEK/MWh}$$

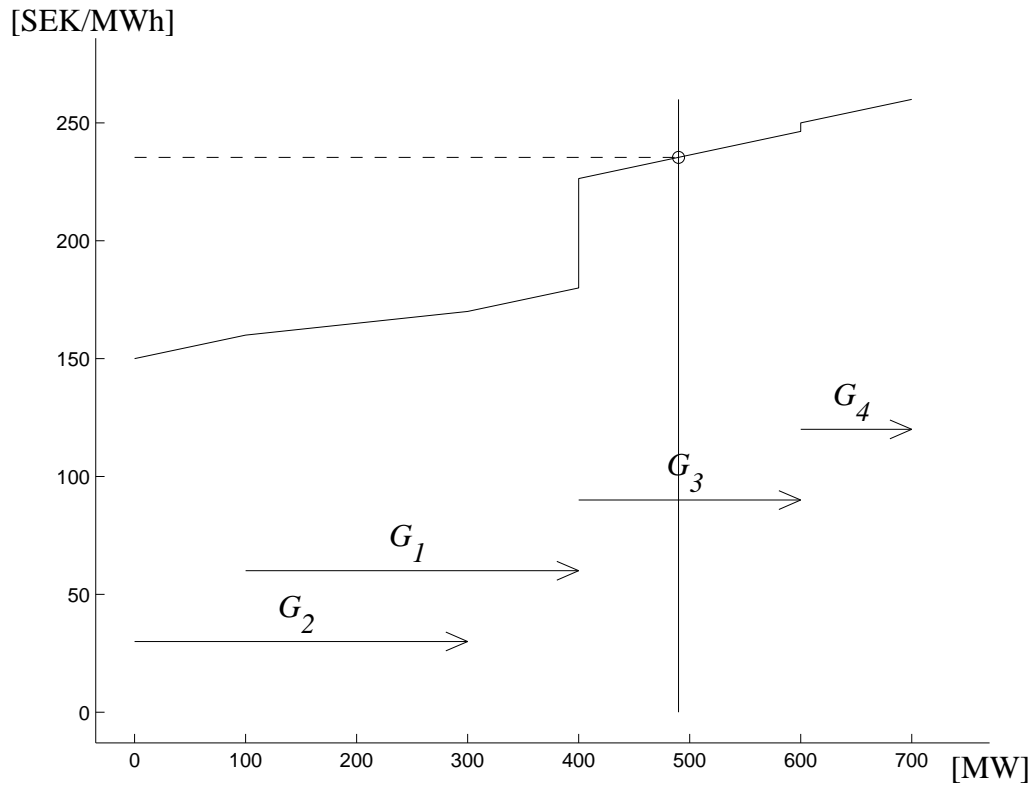


Figure 4.9. Demand and supply for example 4.5a. CO_2 allowances price = 60 SEK/ton CO_2 .

End of example 4.5a

Solution to example 4.5b Also in this example producing electric power implies a cost for the owners of unit #1 and #3 because of the price on the allowances. The extra cost is

$$C_{CO_2-1} = \frac{(1000 \text{ kg}CO_2/\text{MWh}) \cdot (220 \text{ SEK/ton})}{1000 \text{ kg/ton}} = 220 \text{ SEK/MWh}$$

$$C_{CO_2-3} = 440 \cdot 220/1000 = 96.8 \text{ SEK/MWh}$$

The marginal cost for unit #1 and #3 become

$$MC(G_1) = c_{G1} + C_{CO_2-1} + a_1 G_1 = 320 + 0.1 G_1 \text{ SEK/MWh}$$

$$MC(G_3) = c_{G3} + C_{CO_2-3} + a_3 G_3 = 296.8 + 0.1 G_3 \text{ SEK/MWh}$$

Figure 4.10 shows the supply and demand curves, with these costs.

It can be noted that with this price of allowances, unit #1 will now have a so high marginal cost so it will not be used. But unit #3 is still needed the production in this unit is $490-200-100=190$ MW. This implies that the following amounts of allowances

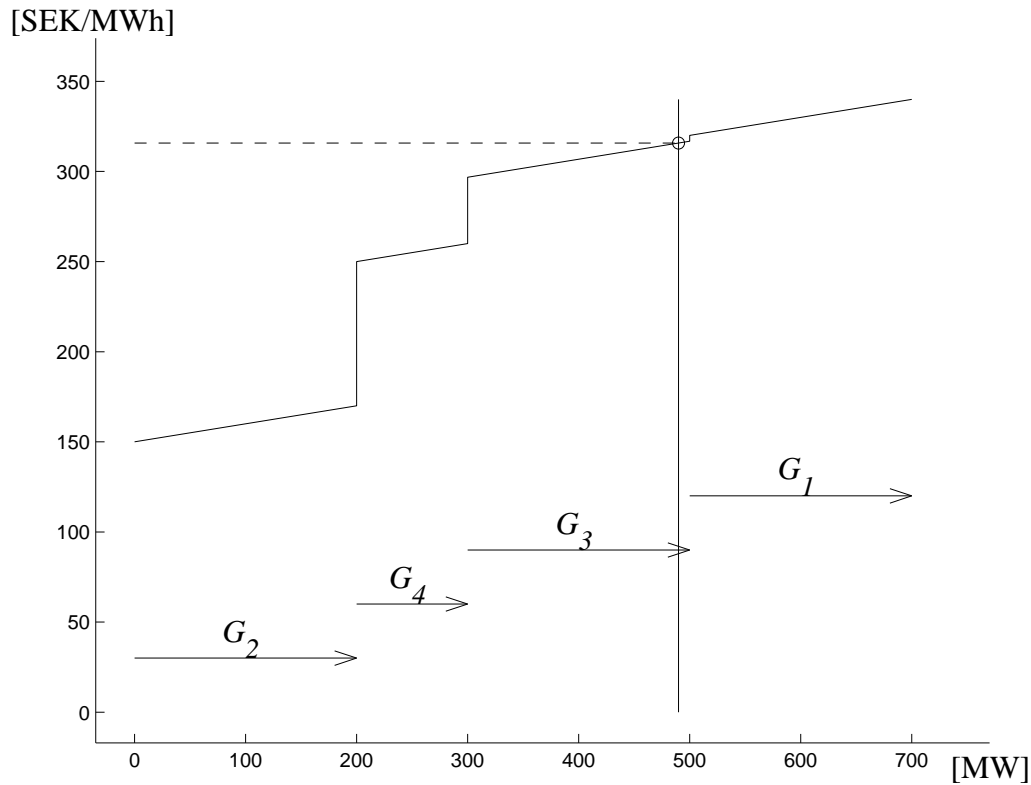


Figure 4.10. Demand and supply for example 4.5b. CO_2 allowances price = 220 SEK/ton CO_2 .

are used:

$$\begin{aligned} \text{used allowances in unit \#1} &= 0 \text{ ton}CO_2 \\ \text{used allowances in unit \#3} &= \frac{190 \cdot 440}{1000} = 83.6 \text{ ton}CO_2 \end{aligned}$$

This implies that unit #3 has to buy $83.6 - 50 = 33.6$ allowances from the market, while unit #1 can sell $150 - 0 = 150$ allowances (or store them for future use). The power price is set by the marginal cost in unit #3

$$\lambda = c_{G3} + C_{CO_2-3} + a_3 G_3 = 296.8 + 0.1 \cdot 190 = 315.8 \text{ SEK/MWh}$$

End of example 4.5b

Solution to example 4.5c Start the calculations with the assumption that the power production is equal to the solution of example 4.5a. In that example the price of allowances was 60 SEK/ton- CO_2 . The result was shown in figure 4.9 and resulted in that unit #1 has to buy $200 - 150 = 50$ allowances from the market, while unit #3 can sell $50 - 39.6 = 10.4$ allowances (or store them for future use). If we assume that unit #3 sells their allowances to the market, then the total trade with the market (outside the

studied system) is the net of $50-10.4=39.6$ bought allowances. This means that the price of these allowances becomes:

$$\lambda_{CO_2} = 60 + 0.1 \cdot 39.6 = 63.96 \text{ SEK/ton } CO_2$$

The marginal cost for unit #1 and #3 become

$$\begin{aligned} C_{CO_2-1} &= 1000 \cdot 63.96/1000 = 63.96 \text{ SEK/MWh} \\ C_{CO_2-3} &= 440 \cdot 63.96/1000 = 28.1424 \text{ SEK/MWh} \\ \Rightarrow \\ MC(G_1) &= c_{G1} + C_{CO_2-1} + a_1 G_1 = 163.96 + 0.1 G_1 \text{ SEK/MWh} \\ MC(G_3) &= c_{G3} + C_{CO_2-3} + a_3 G_3 = 228.1424 + 0.1 G_3 \text{ SEK/MWh} \end{aligned}$$

Figure 4.11 shows the supply and demand curves, with these costs.

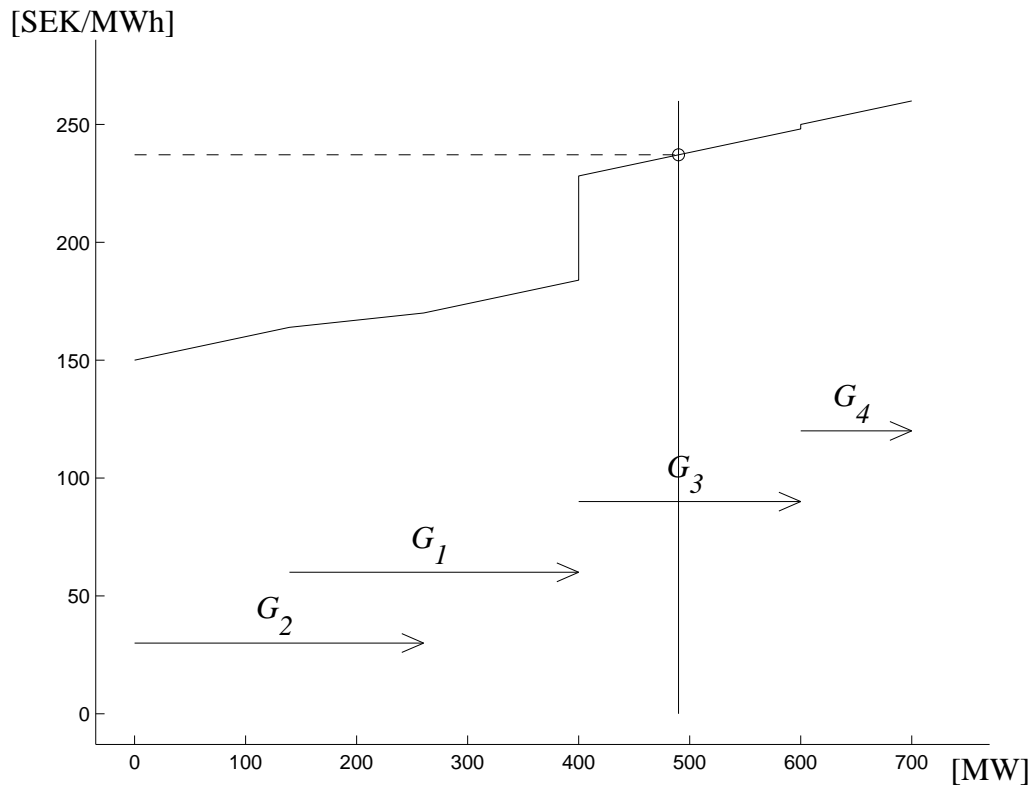


Figure 4.11. Demand and supply for example 4.5c. CO_2 allowances price = 63.96 SEK/ton CO_2 .

The power price is set by the marginal cost in unit #3

$$\lambda = c_{G3} + C_{CO_2-3} + a_3 G_3 = 228.1424 + 0.1 \cdot 90 = 237.1424 \text{ SEK/MWh}$$

With the parameter values in this example, the calculations became rather simple. But for other parameter values, the price change caused by the trading of allowances may

change the merit order of the plants. To obtain the operation of the system, this can be formulated as an optimization problem:

$$\begin{aligned} \max Z = C_D - \sum C_{Gi} &= \hat{C}_D - \sum_{i=1}^4 \left(c_{Gi} G_i + \frac{a_i}{2} G_i^2 \right) \\ \text{when} & \\ 0 \leq G_1 &\leq 200 \\ 0 \leq G_2 &\leq 200 \\ 0 \leq G_3 &\leq 200 \\ 0 \leq G_4 &\leq 100 \\ c_{G1} &= 100 + \lambda_{CO_2} \\ c_{G3} &= 200 + 0.44\lambda_{CO_2} \\ \lambda_{CO_2} &= 60 - 0.1([150 - G_1] + [50 - 0.440G_3]) \\ \sum_{i=1}^4 G_i &= 490 \end{aligned}$$

The free variables in this optimization problem are G_i , $i \in [1, 4]$, c_{G1} , c_{G3} , and λ_{CO_2} .

End of example 4.5c

Solution to example 4.5d Start the calculations with the assumption that the power production is equal to the solution of example 4.5a. In that example the price of allowances was 60 SEK/ton- CO_2 . The result was that unit #1 has to buy $200-150=50$ allowances from the market, while unit #3 can sell $50-39.6=10.4$ allowances (or store them for future use). This means a net import of 39.6 allowances which is not allowed in this example since there is a closed system without trade with external players.

We start to assume that there has to be a reduction in *either* unit #1 or unit #3 (not in both units) in order to reduce the emissions corresponding to 39.6 allowances.

Reduction in unit #1 means

- Reduce power production with 39.6 MW (corresponding to 39.6 allowances). Operation cost decrease is

$$C(G_1 = 200) - C(G_1 = 200 - 39.6) = c_{G1}39.6 + a_1(200^2 - 160.4^2) = 5387.184 \text{ SEK}$$

- Increase production in unit #4 with 39.6 MW. Operation cost increase is

$$C(G_4 = 39.6) - C(G_4 = 0) = c_{G4}39.6 + a_439.6^2 = 10056.816 \text{ SEK}$$

- Total cost increase to reduce emissions by replacing energy production in unit #1 with unit #4 = $10056.816 - 5387.184 = 4669.632$ SEK

Reduction in unit #3 means

- Reduce power production with $39.6/0.440=90$ MW (corresponding to 39.6 allowances). Operation cost decrease is

$$C(G_3 = 90) - C(G_3 = 90 - 90 = 0) = c_{G3}90 + a_3(90^2) = 18810 \text{ SEK}$$

- Increase production in unit #4 with 90 MW. Operation cost increase is

$$C(G_4 = 90) - C(G_4 = 0) = c_{G4}90 + a_490^2 = 23310 \text{ SEK}$$

- Total cost increase to reduce emissions by replacing energy production in unit #3 with unit #4 = $23310 - 18810 = 450$ SEK

This means that the cheapest way to fulfill the requirements of no emission trading with external traders is to produce 200 MW in unit #1, 200 MW in unit #2 and 90 MW in unit #4. The price of the emission rights is thereby set in such a way that the marginal cost of unit #4 must be equal to the marginal cost (including cost of emissions) for unit #3, i.e.

$$\begin{aligned} c_{G4} + a_4G_4 &= c_{G3} + a_3G_3 + \lambda_{CO2} \frac{440}{1000} \\ \Rightarrow \\ \lambda_{CO2} &= \frac{1000}{440} [c_{G4} + a_4G_4 - c_{G3} - a_3G_3] \\ &= \frac{1000}{440} [250 + 0.1 \cdot 90 - 200 - 0] = 134.09 \text{ SEK/ ton-CO2} \end{aligned}$$

This means that the marginal cost of unit #1 (=operation cost + cost of allowances that partly have to be bought from unit #3) then becomes:

$$c_{G1} + a_1G_1 + \lambda_{CO2} \frac{1000}{1000} = 234.09 + 0.1 \cdot G_1$$

Figure 4.12 shows the supply and demand curves, with these costs.

The power price is set by the marginal cost in unit #4

$$\lambda = c_{G4} + a_4G_4 = 250 + 0.1 \cdot 90 = 259 \text{ SEK/MWh}$$

To obtain the operation of the system a suitable way is to formulate the problem as an optimization problem:

$$\begin{aligned} \max Z = C_D - \sum C_{Gi} &= \hat{C}_D - \sum_{i=1}^4 \left(c_{Gi}G_i + \frac{a_i}{2}G_i^2 \right) \\ &\text{when} \\ 0 \leq G_1 &\leq 200 \\ 0 \leq G_2 &\leq 200 \\ 0 \leq G_3 &\leq 200 \\ 0 \leq G_4 &\leq 100 \\ \sum_{i=1}^4 G_i &= 490 \\ G_1 + 0.440G_3 &\leq 50 + 150 \end{aligned}$$

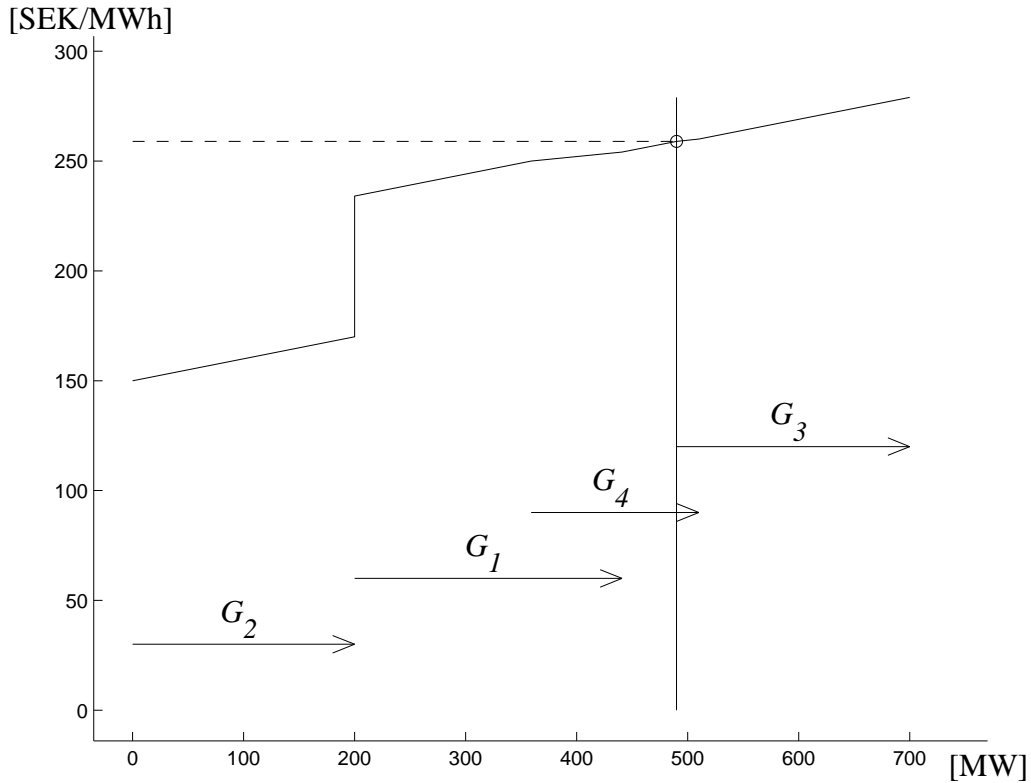


Figure 4.12. Demand and supply for example 4.5d. CO_2 allowances price = 134.09 SEK/ton CO_2 .

The free variables in this optimization problem are G_i , $i \in [1, 4]$. λ_{CO_2} is the dual variable of the last constraint, while the power price, λ , is the dual variable of the second last constraint.

End of example 4.5d

4.5 Electricity disclosure

In a regulated market with no competition there was perhaps little reason to provide this information since consumers did not have a choice about their electricity product. They could not switch supplier unless they moved to another region. Today this is less and less the case. As the electricity markets of Europe and many parts of the world open, consumers are given a choice about who they buy their power from and even what product they buy. In such a deregulated, competitive market consumers need to be able to distinguish between the products they are being offered. Electricity disclosure helps them do this, by making it mandatory for all electricity suppliers to label their products with a list of ‘ingredients’ (the supply mix) and their nutritional value (the environmental effects).

Electricity disclosure started in the USA with the State of California’s Power Content Label being one of the first mandatory labels, applied from 1998. Today more than

21 States (all those that have liberalized markets + three non-liberalized) have similar requirements, although each state may have its own label design and information requirements. There is no standard disclosure system in the US as each State has jurisdiction over its electricity industry.

The disclosure momentum in Europe is also picking up. Austria was the first country to pass a law on disclosure, and labels on consumer bills are being applied from 2002.

There are two basic types of how this system can be introduced. One is a *physical trade system* where the trade during each trading period, e.g. an hour, is directly reflected in the electricity bill per consumer. This means that each type of production corresponds to a certain electricity market, but depending on the structure of consumers requirements, power can with some restrictions be traded between the different markets. If there are three types of consumers who want coal, natural gas and bio electricity, then there will be three separate markets. But if there are some consumers who want CO_2 -free electricity, but the other ones are passive and do not care (they get "grey" electricity), then CO_2 -free electricity can be sold to the grey market but not vice versa.

An alternative is to introduce a parallel system: Guarantee of Origin, a GoO system. This means that the electricity price is formed according to the standard way (as in example 4.1), but for each MWh produced in a certain power plant, a certificate is obtained. There is then a parallel market for each type of certificates (in the example coal, natural gas and bio certificates). For each certificate a price is formed based on supply and demand of each type of certificate. One can see the certificate system described in section 4.3 as a special case of this. In section 4.3 there were only certificates for one part of the production sources.

Example 4.6 *Assume a power system according to table 4.1 and a total, price independent, load of 490 MW. Assume that the consumers in the system are interested to know (and also influence) the origin of their electricity production and because of that an electricity disclosure system is introduced so each consumer gets information on their bill about the origin of their electricity.*

4.6a *Assume first that the system is operated as in example 4.1. Calculate the percentage of each power source that is stated on each consumers bill with the assumption that no consumer require any specific type of electricity.*

4.6b *Assume the same as in example 4.6a but now 30 % of the consumers require CO_2 -free electricity. Calculate the power price and the percentage of each power source that is stated on the consumers bills. There are now two types of consumers: "Green" consumers, who require CO_2 -free electricity, and the passive consumers which get the rest, also denoted "grey" electricity. Assume that the CO_2 -free and grey electricity products are traded on different markets, where CO_2 -free electricity can be sold to the grey market but not vice versa.*

4.6c *Assume the same as in example 4.6b but now 50 % of the consumers require CO_2 -free electricity. Calculate the power price and the percentage of each power*

source that is stated on the consumers bills for the two consumer groups. Assume two markets: One for each type of consumer with a possible trade between the two markets.

4.6d Assume the same as in example 4.6b where now 50 % of the consumers require CO_2 -free electricity. But now the disclosure system is organized as one electricity market but a certificate system is introduced to finance the consumer requested rescheduling of the units.

Solution to example 4.6a The result from example 4.1 was that coal fueled unit #1 produced 200 MWh/h, bio fueled unit #2 produced 200 MWh/h, while natural gas fueled unit #3 produced 90 MW. This implies that each consumer will get a bill stating

$$\begin{aligned}\text{Electricity from coal} &= 200/490 = 40.8\% \\ \text{Electricity from bio} &= 200/490 = 40.8\% \\ \text{Electricity from natural gas} &= 90/490 = 18.4\%\end{aligned}$$

End of example 4.6a

Solution to example 4.6b The result from example 4.6a shown that the scheduling of units do not have to change since there is already 40.8 % of all electric production from bio. This means that the consumers with requirements of bio energy on their bill will get:

$$\begin{aligned}\text{Total Electricity requirement} &= 0.3 \cdot 490 = 147 \text{ MWh/h} \\ \text{Electricity from bio} &= 100\%\end{aligned}$$

The other consumers will get the remaining power, i.e.

$$\begin{aligned}\text{Total Electricity requirement} &= 0.7 \cdot 490 = 343 \text{ MWh/h} \\ \text{Electricity from coal} &= 200/343 = 58.3\% \\ \text{Electricity from bio} &= (200 - 147)/343 = 15.5\% \\ \text{Electricity from natural gas} &= 90/343 = 26.2\%\end{aligned}$$

It can be noted that one can see this as two markets, where one of the markets only handles bio-energy. This market buys 200 MWh/h of bio energy, sells 147 MWh/h to the bio-consumers and 53 MWh/h to the grey consumers. The price will be equal on the two markets since the price on the bio-energy market is set by the grey consumers which are ready to pay as much as the marginal cost of the last produced MWh in the natural gas fired unit. This means that the price for all consumers becomes the same as in example 4.1, 209 SEK/MWh.

End of example 4.6b

Solution to example 4.6c Here the requirements from the consumers who want CO_2 -free generation is 50 %, corresponding to $0.5 \cdot 490 = 245$ MWh/h which is higher than the result obtained in example 4.1. This means that the scheduling of the units has to be

changed in order to fulfill consumers requirements. The solution is then that 45 MW is moved from the most expensive CO_2 -emitting unit (=unit #3) to the cheapest extra CO_2 -free production (= unit #4). The result is unit #1: 200 MW, unit #2: 200 MW, unit #3: 45 MW, unit #4: 45 MW. This means that the consumers with requirements of bio energy on their bill will get:

$$\begin{aligned}\text{Total Electricity requirement} &= 0.5 \cdot 490 = 245 \text{ MWh/h} \\ \text{Electricity from bio} &= 100\% \\ \text{Price} = \text{Marginal cost} &= \lambda = c_{G4} + a_4 \cdot 45 = 254.5 \text{ SEK/MWh}\end{aligned}$$

The consumers without requirements of bio energy will get:

$$\begin{aligned}\text{Total Electricity requirement} &= 0.5 \cdot 490 = 245 \text{ MWh/h} \\ \text{Electricity from bio} &= 0\% \\ \text{Price} = \text{Marginal cost} &= \lambda = c_{G3} + a_3 \cdot 45 = 204.5 \text{ SEK/MWh}\end{aligned}$$

In this example the result is two separate markets without any trading between them. The result is thereby two different prices, one for each type of product. Figure 4.13 shows the supply and demand curves for the two separate markets.

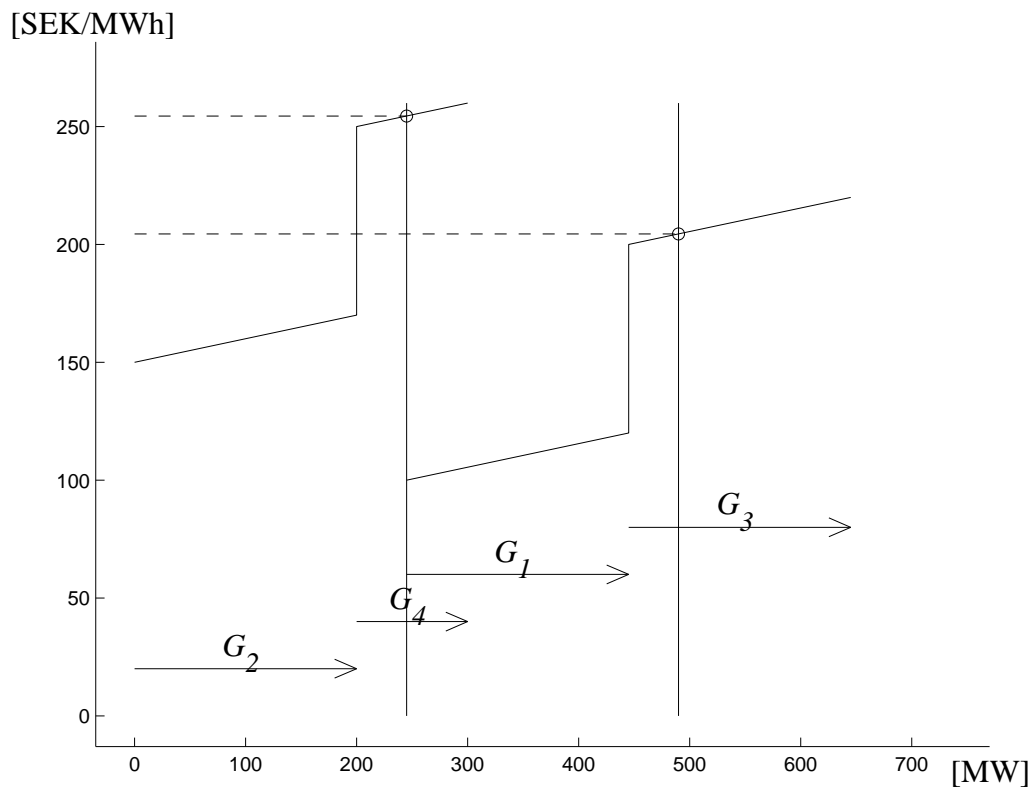


Figure 4.13. Demand and supply curves for example 4.6c. 50 % of all consumers require CO_2 -free generation.

Solution to example 4.6d Here the requirements from the consumers who want CO_2 -free generation is 50 %, the same as in example 4.6c. This means that the unit scheduling will be the same, i.e., unit #1: 200 MW, unit #2: 200 MW, unit #3: 45 MW, unit #4: 45 MW. The difference here, compared to example 4.6c, is that a certificate system is introduced to finance the consumer requested rescheduling of the units.

The requested change, compared to the basic scheduling in example 4.1 is that 45 MW is moved from the most expensive CO_2 -emitting unit (=unit #3) to the cheapest extra CO_2 -free production (= unit #4). The operation cost in the last MWh of unit #4 is

$$c_{G_4} + a_4 G_4 = 250 + 0.1 \cdot 45 = 254.5 \text{ SEK/MWh}$$

while the operation cost of the last replaced MWh in unit #3 is

$$c_{G_3} + a_3 G_3 = 200 + 0.1 \cdot 45 = 204.5 \text{ SEK/MWh}$$

This implies that unit #4 has to get certificates corresponding to the difference between its operation cost (254.5 SEK/MWh) and the cheapest competitor (204.5 SEK/MWh) in order to produce. This means that the certificate price has to be $254.5 - 204.5 = 50$ SEK/MWh. It must here be noted that the consumers here require " CO_2 -free generation" which means that unit #2 will also get these certificates (although they do not need it to operate). Figure 4.14 shows the supply and demand curves, with this certificate price included as a way to lower the operation costs.

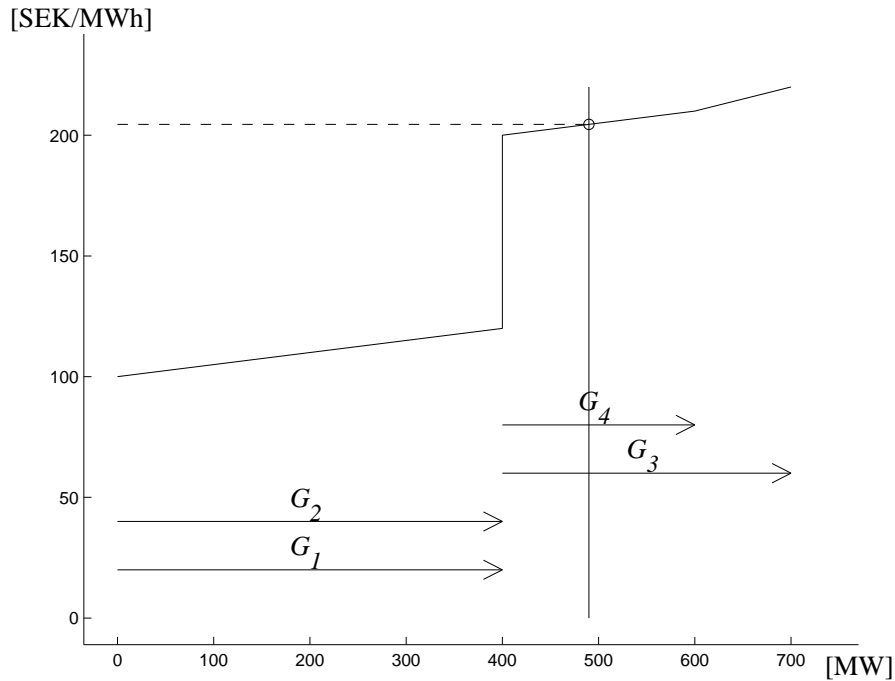


Figure 4.14. Demand and supply for example 4.6d. Certificates for CO_2 -free generation cost = 50 SEK/MWh.

The consumers with requirements of bio energy on their bill will get:

$$\begin{aligned}
 \text{Electricity from bio} &= 100\% \\
 \text{Electricity price} &= \text{Marginal cost} = \lambda = \\
 &= c_{G3} + a_3 \cdot 45 = 204.5 \text{ SEK/MWh} \\
 \text{Certificate price} &= 50 \text{ SEK/MWh} \\
 \text{Total price} &= 204.5 + 50 = 254.5 \text{ SEK/MWh}
 \end{aligned}$$

The consumers without requirements of bio energy will get:

$$\begin{aligned}
 \text{Electricity from bio} &= 0\% \\
 \text{Electricity price} &= \text{Total price} = \lambda = 204.5 \text{ SEK/MWh}
 \end{aligned}$$

This means that the prices for the two groups are exactly the same as in example 4.6c.

End of example 4.6d

4.6 Feebate system

"Feebate" is a portmanteau of "fee" and "rebate". In general, a feebate program is a (usually, self-financing) system of government imposed surcharges (fees) and refunds (rebates) that are used to shift market purchasing preferences toward an economically, socially or politically desired goal. Originally coined in the 1990s, feebate programs have typically been used to shift buying habits in the transportation and energy sectors.

One example is the Swedish system for reduction of NOX charges to reduces emissions of nitrogen oxides. The Environmental Charge for Emissions of Nitrogen Oxides from Energy Production Act came into effect on 1 January 1992. The Act stipulates that the charge is payable on emissions of nitrogen oxides (NOX) from boilers, stationary combustion engines and gas turbines having a measured useful output of at least 25 gigawatt hours a year (the limit between 1992 and 1995 was 50 GWh). Throughout the period the charge has been SEK 40 per kilo nitrogen oxides emitted, calculated as nitrogen dioxide (NO₂). The total environmental charge paid into the system is repaid to those liable to the charge in proportion to each production unit's share of total useful energy production. Hence, the idea is to reward emissions that are low in relation to energy production (i.e., low specific emissions).

Example 4.7 Assume a power system according to table 4.1 and a total, price independent, load of 490 MW. Assume that a feebate system is introduced in order to reduce the CO₂ emissions. This implies that a fee for CO₂ emissions is introduced and the sum of the payments is then paid back to owners of the units in relation to the energy production that caused the emissions.

4.7a Assume first a CO₂ fee of 0.08 SEK/kgCO₂. Calculate the power production in each unit and the consumer price.

4.7b Assume the same as in example 4.7a but now the fee is increased to 0.17 SEK/kgCO₂.

4.7c Assume the same as in example 4.7a but now the fee is increased to 0.3 SEK/kgCO₂.

4.7c Now assume the same as in example 4.7c but now all units are included in the system, i.e., also the units that do not emit any CO₂ at all.

Solution to example 4.7a The result from example 4.1 was that coal fueled unit #1 produced 200 MWh/h, bio fueled unit #2 produced 200 MWh/h, while natural gas fueled unit #3 produced 90 MW. First assume here that the production will be the same in all units. This means that units #1 and #3 will be charged for their emissions but then all money goes back to them in relation to their energy production.

$$CO_2\text{-fee in unit \#1} = 200 \cdot 1000 \cdot 0.08 = 16000 \text{ SEK}$$

$$CO_2\text{-fee in unit \#3} = 90 \cdot 440 \cdot 0.08 = 3168 \text{ SEK}$$

This means a total income of 16000+3168=19168 SEK corresponding to 19168/(200+90) = 66.1 SEK/MWh. This means that the constant part of the marginal cost for units #1 and #3 becomes:

$$c_{G1} = 100 + 0.08 \cdot 1000 - 66.1 = 113.9 \text{ SEK/MWh}$$

$$c_{G3} = 200 + 0.08 \cdot 440 - 66.1 = 169.1 \text{ SEK/MWh}$$

The assumption was that there will be the same production in all units compared to the base case. At the load level 490 MW unit #3 then should produce 90 MW. At this production level the marginal cost is now 169.1 + 90 · 0.1 = 178.1 SEK/MWh, which is still lower than the lowest marginal cost in unit #4 (250 SEK/MWh). It is also higher than the highest cost in unit #2 (150 + 200 · 0.1 = 170 SEK/MWh). This means that the price, set by the marginal cost becomes 178.1 SEK/MWh. The supply and demand curves are shown in figure 4.15. This example shows that the units in the feebate system that emit more than the mean value (coal unit #1) will get higher costs, while the units emitting less than the mean value (gas unit #3) will get a lower cost.

End of example 4.7a

Solution to example 4.7b It is not trivial to solve this problem in a simple way. One approach is an iterative procedure where one assumes a rebate level, calculate the marginal cost and then the generation in each unit, and then update the rebate level until an acceptable convergence is obtained. The details will though not be presented here, but the result will be analyzed. The result is that the rebate is 127.73 SEK/MWh which leads to that the basic marginal costs in the CO₂ emitting units become:

$$c_{G1} = 100 + 0.17 \cdot 1000 - 127.73 = 142.27 \text{ SEK/MWh}$$

$$c_{G3} = 200 + 0.17 \cdot 440 - 127.73 = 147.07 \text{ SEK/MWh}$$

What is now seen is that the marginal costs in units #1, #2 and #3 are rather close. The coal unit price increased, while the gas fired unit price decreased. The supply

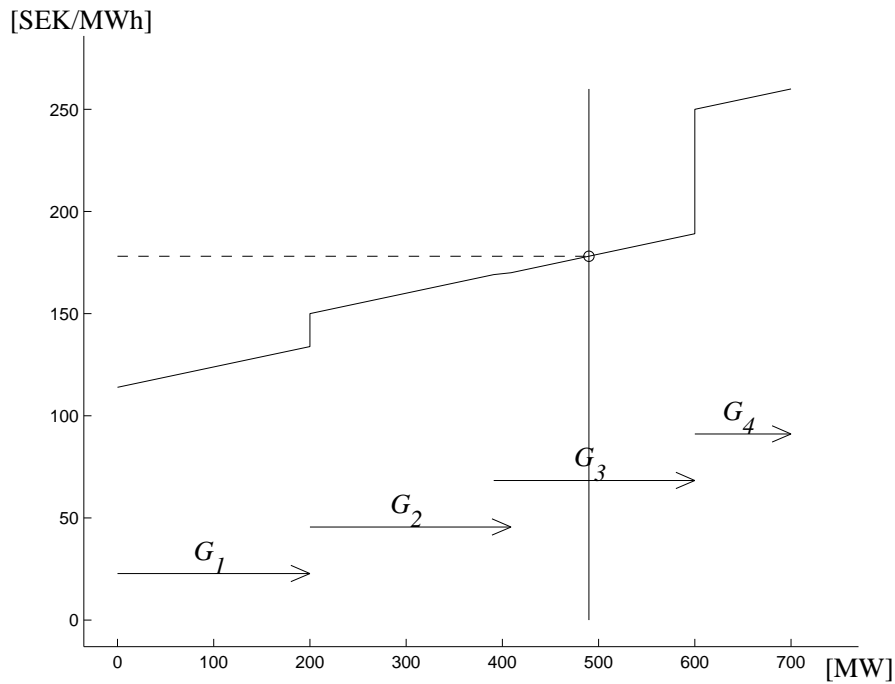


Figure 4.15. Demand and supply for example 4.7a. Fee-bate system with CO_2 -fee = 0.08 SEK/kg CO_2 . Rebate to CO_2 -emitting units.

and demand curves are shown in figure 4.16. The production in the different units are $G_1 = 200$ MW, $G_2 = 130.34$ MW, $G_3 = 159.66$ MW and $G_4 = 0$ MW. The marginal cost, i.e., the power price is 163.03 SEK/MWh.

This example shows that the units in the feebate system that emit more than the mean value (coal unit #1) will get higher costs, while the units emitting less than the mean value (gas unit #3) will get a lower cost. In this example it is also shown that the total emissions increased since the gas power plant now got a lower cost than the bio fuelled plant #2. This depends on that the system leads to that power plants that emit some CO_2 receives the rebate while this is not the case for units without any emissions at all.

End of example 4.7b

Solution to example 4.7c The same approach to solve the problem as in example 4.7b will be used, but also here only the results will be analyzed. The result is that the rebate is 184.14 SEK/MWh which leads to that the basic marginal costs in the CO_2 emitting units become:

$$\begin{aligned} c_{G1} &= 100 + 0.3 \cdot 1000 - 184.14 = 215.86 \text{ SEK/MWh} \\ c_{G3} &= 200 + 0.3 \cdot 440 - 184.14 = 147.86 \text{ SEK/MWh} \end{aligned}$$

What is now seen is that the marginal costs in high emitting units #1 increases significantly, while the gas fired unit price decreased to around the same level as in example 4.7b. The supply and demand curves are shown in figure 4.17. The production in the different units are $G_1 = 90$ MW, $G_2 = 200$ MW, $G_3 = 200$ MW and $G_4 = 0$ MW. The

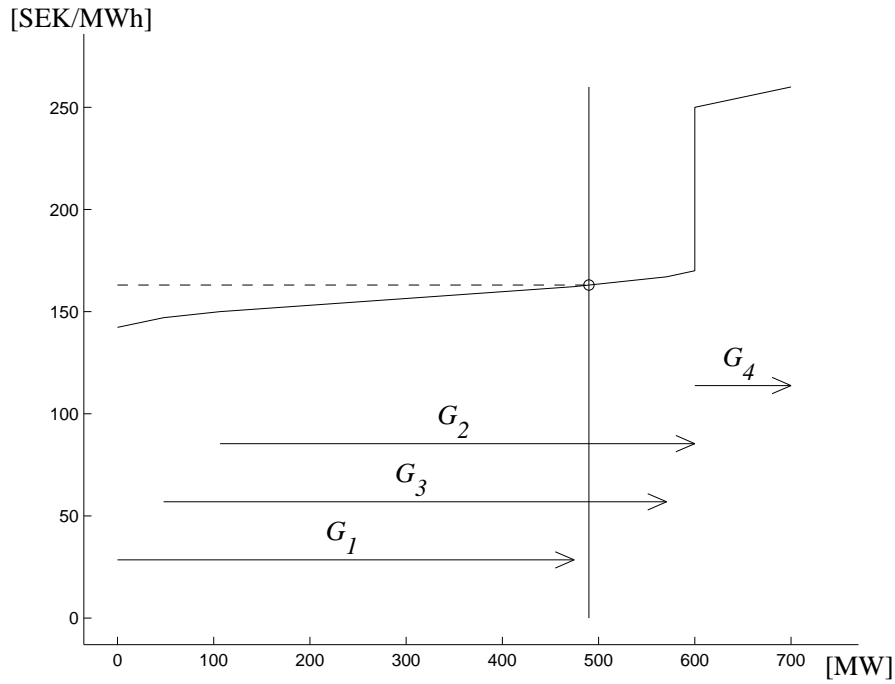


Figure 4.16. Demand and supply for example 4.7b. Fee-bate system with CO_2 -fee = 0.17 SEK/kg CO_2 . Rebate to CO_2 -emitting units.

marginal cost, i.e., the power price is 224.86 SEK/MWh.

This example shows that now the emission cost became so high that coal unit #1 will be so expensive that the production will decrease. The production decrease leads to that less money will be paid for the CO_2 -fee which means that comparatively less money will be paid to low emitting units. It must be noted that if the coal-fired power plant does not produce anything, then there is no money to be paid to the gas-fired units since it now produces on a "system mean emission level for CO_2 emitting plants.

End of example 4.7c

Solution to example 4.7d The same approach to solve the problem as in example 4.7b-c will be used, but also here only the results will be analyzed. The result is that the rebate is 51.18 SEK/MWh which is lower than in example 4.7d since the fees paid in have to be distributed on more plants. This leads to that the basic marginal costs in all power plants (not only the CO_2 emitting ones) become:

$$\begin{aligned} c_{G1} &= 100 + 0.3 \cdot 1000 - 51.18 = 348.82 \text{ SEK/MWh} \\ c_{G2} &= 150 + 0.3 \cdot 0 - 51.18 = 98.82 \text{ SEK/MWh} \\ c_{G3} &= 200 + 0.3 \cdot 440 - 51.18 = 280.82 \text{ SEK/MWh} \\ c_{G4} &= 250 + 0.3 \cdot 0 - 51.18 = 198.82 \text{ SEK/MWh} \end{aligned}$$

What is now seen is that the marginal costs in high emitting unit #1 increased and the marginal cost in the CO_2 free units decreased so much that unit #1 became the most expensive unit. The supply and demand curves are shown in figure 4.18. The

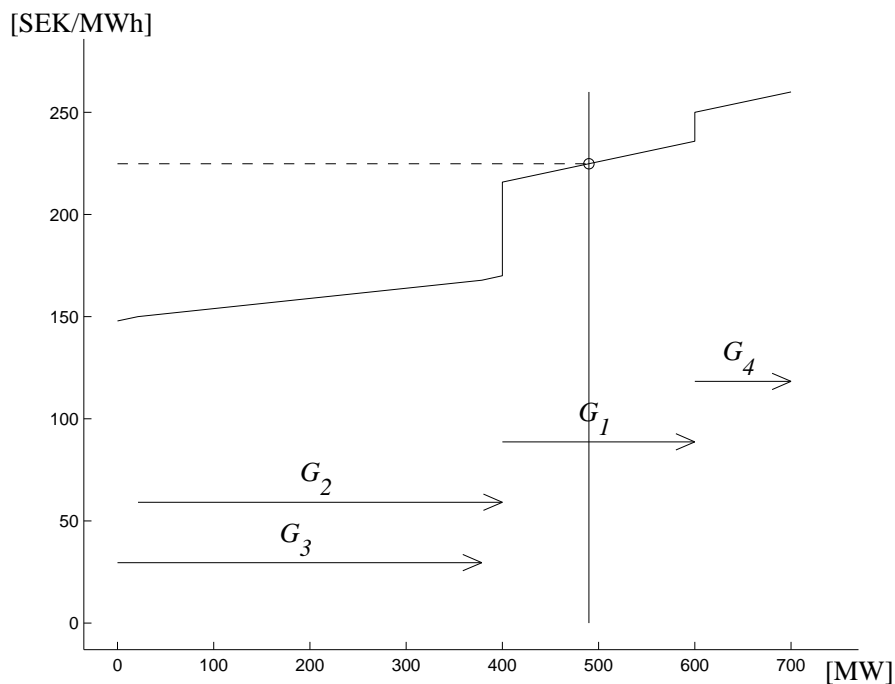


Figure 4.17. Demand and supply for example 4.7c. Fee-bate system with CO_2 -fee = 0.3 SEK/kg CO_2 . Rebate to CO_2 -emitting units.

production in the different units are $G_1 = 0$ MW, $G_2 = 200$ MW, $G_3 = 190$ MW and $G_4 = 100$ MW. The marginal cost, i.e., the power price is 299.82 SEK/MWh.

This example shows that now the emission cost became so high that coal unit #1 will be so expensive that its production will be replaced totally with the other units. This means that the CO_2 emissions decrease significantly.

End of example 4.7d

Some main results for example 4.7 are summarized in table 4.2.

Example	Rebate receivers	CO_2 fee [SEK/kg CO_2]	Rebate [SEK/MWh]	λ [SEK/MWh]	CO_2 -emissions kg
4.1	-	-	-	209	239600
4.7a	unit #1+#3	0.08	66.10	178.10	239600
4.7b	unit #1+#3	0.17	127.73	163.03	270250
4.7c	unit #1+#3	0.3	184.14	224.86	178000
4.7d	all	0.3	51.18	299.82	83600

Table 4.2. Results for example 4.7 with fee-bate system for CO_2 emissions

The table shows that it is possible to reduce the amount of emissions, but a wrongly designed system can in reality increase the emissions, example 4.7b. But it must though be noted that also in example 4.7b, the costs for the high emitting unit #1 increases which reduces the long term interest to invest in this type of power plants. The power

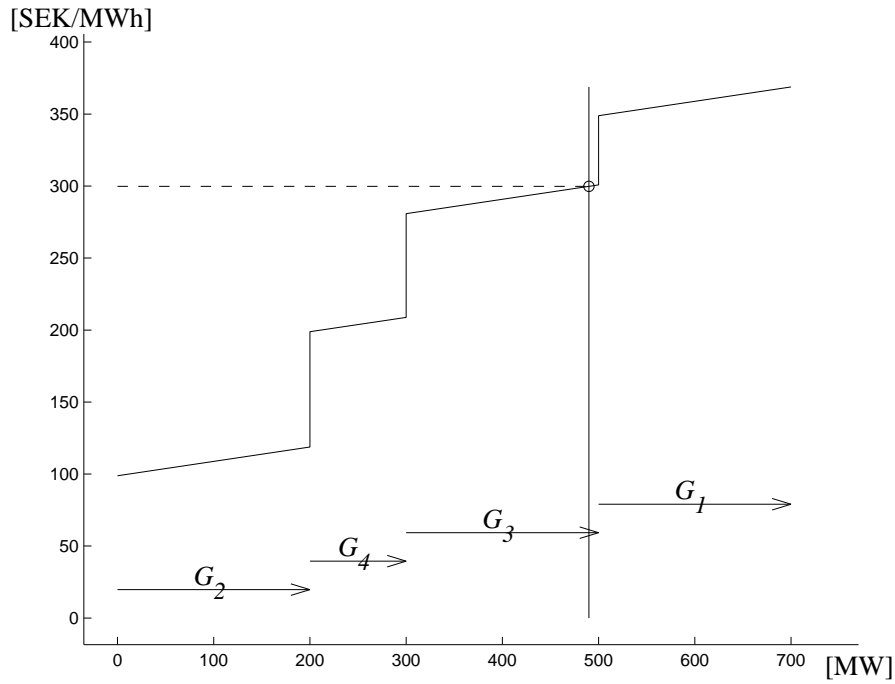


Figure 4.18. Demand and supply for example 4.7d. Fee-bate system with CO_2 -fee = 0.3 SEK/kg CO_2 . Rebate to all units.

prices increases in the system when the emission costs are designed in such a way that it has a significant impact on the total emissions.

It can be noted that a fee-bate system from economic point of view is not trivial. The common approach is to maximize the total surplus in the system and then the result concerning prices and generation is obtained from this solution. But in a feebate system the net-cost for the system is zero since the total fee paid to the system is the same as the total rebate paid back. This means that if one maximizes the total surplus in the system, then the power production result will for all examples 4.7a - 4.7d be the same as in the reference example 4.1. This is natural since the solution of example 4.1 minimizes the total operation cost (i.e., maximizes the total surplus). A feebate system is based on the assumption that each unit is scheduled individually and the reaction from competitors is not considered strategic, i.e., each actor considers the CO_2 fee and the rebate as given parameters. In reality a changed production in one unit will affect both the emissions and the rebate. This will be further developed in future editions of this compendium.

Chapter 5

Planning and operation for an efficient production-load balance

A power system always has to have a balance between production and consumption, i.e., for a power system consisting of i nodes

$$\sum_i G_i = \sum_i D_i + \text{total transmission losses} \quad (5.1)$$

This equation is valid independent of the length of the period, i.e., if G_i and D_i are measured as MWh/h, MWs/s or MW. The aim of the market is to keep the continuous balance between production and consumption in an efficient and reliable way. It can be noted that at the "strike price", the production is equal to the consumption. First the general structure of operation planning will be presented.

The planning (i.e. before a specific hour) and operation (i.e. within the specific hour) of a power system can be illustrated as in figure 5.1. The overall aim of any market

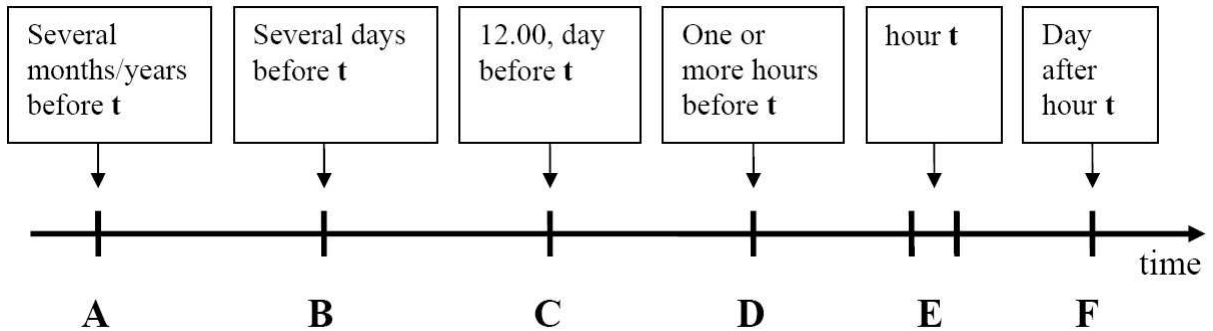


Figure 5.1. Planning and operation of a power system.

design is to obtain a reliable power supply and an efficient use of available resources.

A: Seasonal/yearly planning

Physical aim: The aim of this planning is in a system with larger amounts of hydro power to decide how much water (in reservoirs and inflow) that should be stored for future use. Too much water stored for months ahead, or next year could lead to future spillage, while too little stored water could lead to lack of energy and capacity which could cause water deficit in the future. In a thermal power system the seasonal planning includes scheduling and coordination of unit maintenance.

Market process: In a hydro power system the tool for optimal storage is the water value ,c.f. section 2.1, which is an evaluation of the water in different reservoirs for

different time periods. This value becomes often rather flat, since different values in different periods is an indication that it is profitable to change the storage plan. Each hydro power company makes its own estimation of its water value and the result is used internally for planning and externally for trading. For thermal unit maintenance the aim is normally to schedule maintenance to low load periods.

B: Weekly planning

Physical aim: The aim of this planning in a hydro system is to decide how the available water for the week in the specific region (available from inflow and storage where the use for the storage is planned in stage A) should be distributed between the days of the week. In a thermal power system there may be some planning concerning amount of staff needed for operation of units needed to supply forecasted load.

Market process: The tool for optimal storage in a hydro system is also here the water value, which is estimated for each actor on the market.

C: Daily planning - Day-ahead market

Physical aim: The aim of this planning is to decide for each plant how it should be operated, i.e., how much power that should be produced in each power plant during each hour in the coming day. In a hydro power system the available water for the whole day is a result from the weekly planning. In the planning the consumption level, production in other sources and transmission limits have to be considered. In a thermal power system there are two parts: the *unit commitment*, which includes which units that have to be on line for production and/or reserve keeping, and the *economic scheduling* which includes the planned production level in each unit.

Market process: In this description it is assumed that the day is divided into whole hours. this is the case in, e.g., the Nordic power market, but there may also be other time steps as, e.g., half hours. There are four stages here:

- C1 First the transmission limits for each hour and each bottleneck is decided. These limits are calculated by the TSO:s. It is based on an assumed production and consumption level in each area or each node together with a dimensioning rule including the N-1 criterion (= the system has to stand an outage even if the largest unit or the heaviest transmission line fails) and margins considering uncertainties. The margins have to consider that the limits considers MW limits, but in stage 3, these limits are used as hourly limits considering MWh/h. The difference is illustrated in figure 5.2. The figure shows that the limit concerning MWh/h has to be lower than the MW limit if the transmission changes during the hour.
- C2 The second stage in a liberalized market with several produceres is the production/consumption bidding process where the aim is to put bids to the spot market (Nordpool in the Nordic market) concerning how much power in total that could

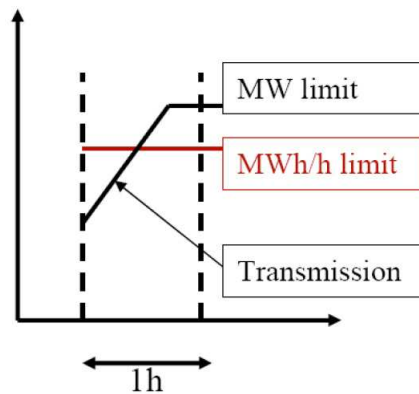


Figure 5.2. Transmission limits during an hour.

be produced during each hour in each company at which price. The bids concerning each hour have to be delivered the day before the real trading day. This is the *day-ahead market*: in Nordic system the bids have to be delivered before noon a certain day for the whole next day.

- C3 The third stage is the calculation of which bids that should be accepted considering transmission limits between bidding areas (Nordic system is based on areas), or limits between nodes where a node-based system (based on DC load flow) is used. This means that as many cheap bids as possible from stage 2 are accepted regarding the transmission limits from stage 1. This calculation is made by the exchange (Nordpool in the Nordic market), and the system price (strike price neglecting all congestions) and area prices are set by the strike price.
- C4 The fourth stage is the operation planning which is performed individually by the owners of the different production sources after the result of the bids. This means that the accepted amount of production bids for each hour have to be met with produced power in some way (combination of own production in each company and bilateral trade with other companies). This stage includes production planning with the physical aim as above.

D: Intra day market

Physical aim: To get an economical operation of the companies power plants for each specific hour during the day. This means that the hydro power stations should be operated only if the production can not be performed cheaper in a competing power plant. Here better forecasts for each hour can be used, since the decisions are taken closer to the hour.

Market process: It is performed after the spot market for each hour has been closed. This includes in the Nordic power market the Elbas Market and other bilateral trade. The main aim is to cover forecast errors concerning load levels, wind power production levels, production costs and/or outages for each hour. Bids are on the Elbas market

distributed electronically and accepted when they are found economical. There are the same stages here as concerning the daily market:

- D1 The transmission limits for each bottleneck are set by the TSO:s. This means that they can be updated compared to what is previously set for the daily market.
- D2 Bids concerning production in a certain area can be delivered.
- D3 For each geographical area only bids that can be used in that area (considering transmission limits and prescheduled trading) can be accepted.
- D4 When a bid is accepted, both the seller and buyer have to consider this in their planning.

E: Hourly operation

Physical aim: During each hour the system balance between production and consumption has to be kept continuously in an economical and reliable way, considering production and transmission limits.

Market process: The continuous balance is kept with primary control (frequency controlled power plants that keep enough margins for this) which is distributed between different power plants in different regions. Used primary reserves are offloaded using secondary reserves which in the market means accepted bids to the regulating market. These bids can also be used for offloading of bottlenecks (counter trading) in order to keep enough margins also on these bottlenecks. There may also be an automatic system (Automatic Generation Control - AGC) which changes the production in some selected units in order to keep the balance and offload units activated by the primary control. It is the Transmission System Operators, the TSO:s that is responsible for AGC settings and performs the counter trading and accept bids to the regulating market. The stages are:

- E1 The transmission limits for each bottleneck are set by the TSO:s. Now the MW limits (not MWh/h) are considered.
- E2 Bids concerning production increase and decrease in each area or in each node are delivered before each hour.
- E3 When bids are needed (for total system needs and/or for offloading of bottlenecks, i.e. counter trading) they are accepted by the TSO:s. If an AGC system is active, then some power plants are activated automatically.
- E4 When a bid is accepted the company that put the bid has to activate it directly, or to be more specific, start the procedure of activation. It may take several minutes until the full bid is activated.

F: Post trading

Aim: The hour has now passed which means that the physical balance has already been kept, as described for step E above. But what happened in step E was that the TSO, e.g., bought power from one producer to supply the system since some consumers that perhaps had a contract with another producer increased their consumption. The aim of the post trading is to arrange economical balance so it matches the contracts on physical delivery on the market.

Market process: The contracts for each actor must have been reported to the TSO. All actors connected to the grid must have a contract concerning the economical responsibility for the production and/or consumption. This is denoted *balance responsibility*. If, e.g., retailer R_1 has a contract to deliver all power to consumer D_1 (i.e. retailer R_1 is balance responsible for consumer D_1) then this information has to be reported to the TSO. Then retailer R_1 can write contracts concerning production with producer P_1 and P_2 . The aim of steps A-D for retailer R_1 is then to try to buy the expected consumption of consumer D_1 for hour h from producer P_1 and P_2 or some other producers. These contracts then have to be reported to the TSO. The basic market tool, the "balance responsibility", means that there is an **economic** incentive for the market actors to keep an hourly balance between production and consumption. The **physical** intra-hour balance is kept by the system operators, the TSO:s. From this point of view an exchange can be considered as a trader, who purchase and sell power.

Example 5.1 Assume a system with 2 retailers R , 2 producers P and 2 loads D . Only the physical actors, producers and consumers, have to have a registered balance responsibility for their connection. Here the retailers have taken over this for their respective consumers, while the producers have this for themselves. The estimated loads for hour h are $D_1 = 200$ MW and $D_2 = 300$ MW, while the reported contracts between the different actors are: R_1 has a contract to deliver all power to consumer D_1 and R_1 buys the expected consumption from the producers; 100 MW from P_1 and 100 MW from P_2 . R_2 has a contract to deliver all power to consumer D_2 and the contracts with the producers are: 150 MW from P_1 and 150 MW from P_2 . In addition to this P_1 has sold 80 MW to P_2 during hour h . This means that P_1 has reported a contracted production of $100+150-80=170$ MW, while P_2 reported $100+150+80=330$ MW. This means that the planning seems OK.

During hour h the loads became $D_1=210$ MWh and $D_2=280$ MWh. Producer P_1 got an outage and could only produce 150 MWh. This means that for the whole hour there will be an up regulation need (i.e. difference between planned production and real need) of $\Delta D_1 + \Delta D_2 + \Delta P_1 = 10 - 20 + 20 = 10$ MW. It is here assumed that this extra need is bought from producer P_2 , who produced this in addition to the contracted production.

Calculate the needed post trading for this hour.

Solution to example 5.1 The post trading is valid for the balance responsible actors. The post trading is then between them and the TSO. The trading refers to the difference between reported plans and real outcome.

Retailer R_1 got a load of 210 MWh (D_1) and has a contracted purchase of 200 MWh. This means that R_1 will purchase 10 MWh from the TSO.

Retailer R_2 got a load of 280 MWh (D_2) and has a contracted purchase of 300 MWh. This means that R_2 will sell 20 MWh to the TSO.

Producer P_1 produced 150 MWh but has a contract to sell 170 MWh. This means that P_1 will purchase 20 MWh from the TSO.

Producer P_2 produced the contracted 330 MWh and in addition to this the up regulation of 10 MWh that is sold to the TSO.

The TSO :s net purchase becomes $-10+20+-20+10=0$ MWh

The production during an hour is always as large as the consumption during that hour. This means that the role of the TSO is to move money between the different actors since if someone has produced too little, then it is always someone else that has produced too much. The net trading (in MWh) is always zero for the TSO. But the economical result for the TSO is not necessary zero, since it depends on the pricing for the imbalances. It can be mentioned that the price for up regulation is normally more expensive than the power price for that hour. For retailer R_1 , who had to purchase 10 MWh from the TSO, and the cost for this is the cost that the TSO had to pay for the up-regulation, it had probably been economically better to have a contract for the correct load. This means that the system of post trading leads to an *economical* incentive for each balance responsible actor to keep the balance.

Example 5.2 Assume a power system with two 1000 MW power plants in two areas, 1 and 2. The cost data for the power plants are

$$MC_1 = 150 + 0.02 \cdot G_1 \text{ SEK/MWh}$$

$$MC_2 = 150 + 0.02 \cdot G_2 \text{ SEK/MWh}$$

The structure of the system is shown in figure 5.3.

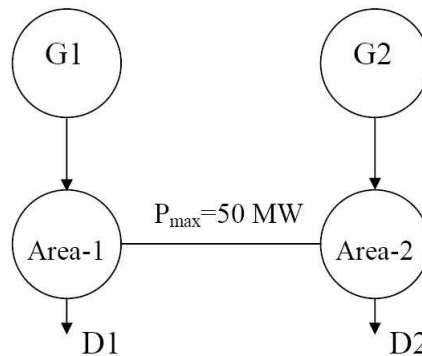


Figure 5.3. Layout of power system in example 5.2

The two loads D_1 and D_2 for a 24 hour period are shown in figure 5.4. There are values for each minute, i.e., to be more exact, MWminutes/minute.

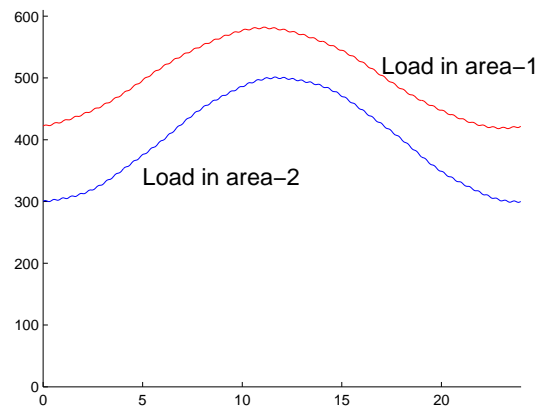


Figure 5.4. The loads D_1 and D_2 per minute for a 24 hour period in example 5.2

- 5.2a Assume that there is a perfect forecast of the loads according to figure 5.4, but in the day ahead trading only hourly mean values, MWh/h, are treated. Calculate the hourly mean values for the loads.
- 5.2b Start with the hourly load figures from example 5.2a. Assume that there is perfect competition and there is a production and transmission system as in figure 5.3. Calculate the scheduled hourly production and transmission from this information.
- 5.2c Assume that the difference between the hourly mean values of demand, generation and transmission compared to the corresponding minute mean values are traded on the regulating market. Determine the needed trading on the regulation market for hour 4-7.
- 5.2d Determine the needed trading on the regulation market for hour 8-11.
- 5.2e Assume now that there is an AGC (Automatic Generation Control) system present. The aim of this AGC system is to keep the scheduled transmission (the prescheduled hourly transmission) and control the production in G1 and G2 to keep the balance and the transmission. Study hour 8-11 and compare the results.

Solution to example 5.2a For each hour the amount of MWh/h can be calculated by just summing up all MWminutes/minute for each hour and divide the result with 60. The result is shown in figure 5.5.

End of example 5.2a

Solution to example 5.2b For each hour the minimum cost schedule becomes the solution of the following optimization problem (i.e., minimize total production cost considering

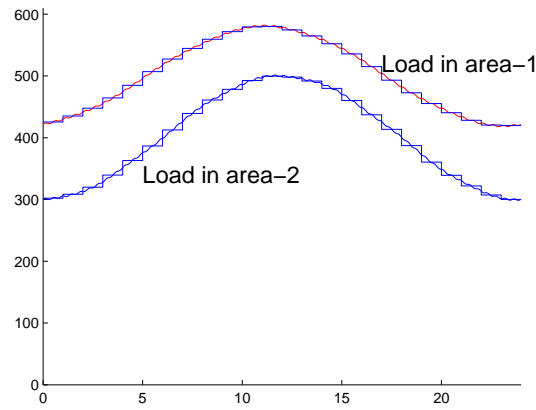


Figure 5.5. The loads D_1 and D_2 per hour for a 24 hour period in example 5.2

power balance in each area and production and transmission limits):

$$\begin{aligned} \min Z &= \sum_{k=1}^2 \left(150G(k) + \frac{0.02}{2}G(k)^2 \right) \\ \text{when} \\ 0 &\leq G(k) \leq 1000 \\ G(1) &= D_1 - P_{21} \\ G(2) &= D_2 + P_{21} \\ P_{21} &\leq 50 \end{aligned}$$

In this case the problem can be solved comparatively easy. If it is possible, then the least cost solution is to divide the production equally between unit 1 and 2, since both systems have the same marginal cost as function of production level. This is possible when the load difference between area 1 and 2 is less than two times transmission capacity, i.e., less than $2 \cdot 50 = 100$ MW. If the load difference is higher, then the transmission is set to the capacity 50 MW and the production in each area is set to cover the difference between load and inter-area transmission. The result is shown in figure 5.6. As shown in the figure the transmission line is fully used during hour 1-8 and 21-24 when the load difference is larger than 100 MW. During these hours the generation in area 1 and 2 are different. It is only when the line is not fully used, hour 9-20, that the production can be equally split between area 1 and 2.

End of example 5.2b

Solution to example 5.2c During hour 4-7 the transmission line is fully used, and this means that the balancing within each hour has to be performed in each area. Figure 5.7 shows the load = needed supply during hour 4-7. The production/transmission is planned according to the hourly mean values, while the real production has to follow the real load. The deviation within the hours is shown in figure 5.8. This means that within each hour both up- and down- regulation have to be performed. Lets study the first hour in figure 5.7. The hourly mean load is higher than the real load in the

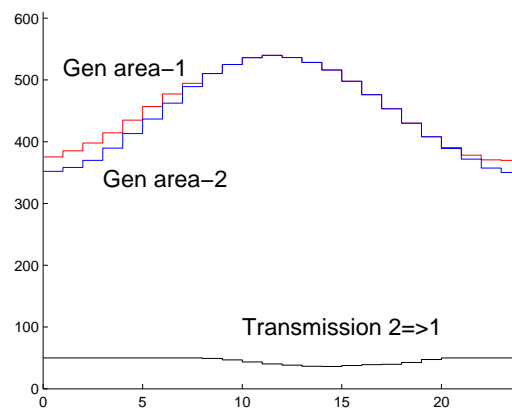


Figure 5.6. The planned generation and transmission per hour for a 24 hour period in example 5.2

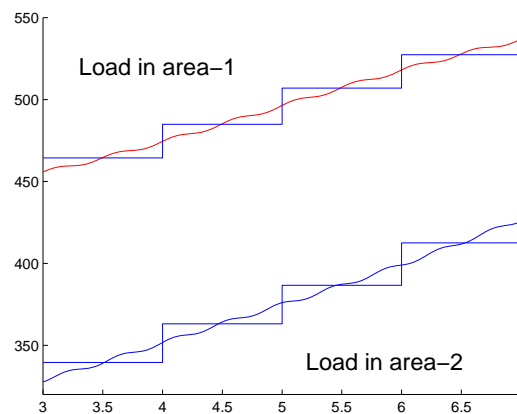


Figure 5.7. The hourly and minute supply need in both areas for hour 4-7 in example. 5.2

beginning of this hour and lower at the end. The production is scheduled according to the mean load. This means that there has to be down regulation in the first part of the hour and up- regulation in the latter part. This means that the needs in the regulation market are shown in figure 5.8. Negative values correspond to a need of down regulation while positive values means up-regulation.

End of example 5.2c

Solution to example 5.2d During hour 8-11 the transmission line is fully used, except for the first 70 minutes. This means that the balancing, except for the first 70 minutes, can be performed in the plant with the lowest cost, independent of its location. By this follows that the transmission on the line will change in order to use the cheapest power production for balancing. Figure 5.9 shows the load = needed supply during hour 8-11 and the transmission between the areas. The resulting production change is shown in figure 5.10

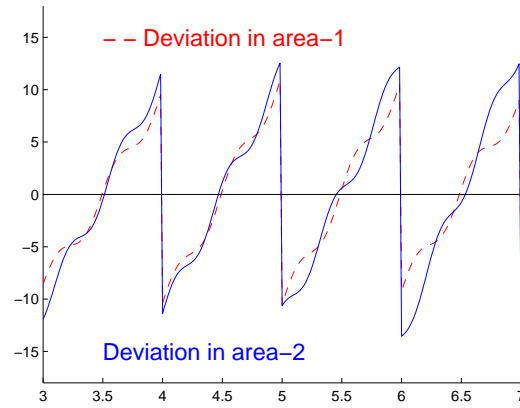


Figure 5.8. The intra hourly deviation in both areas for hour 4-7 in example. 5.2

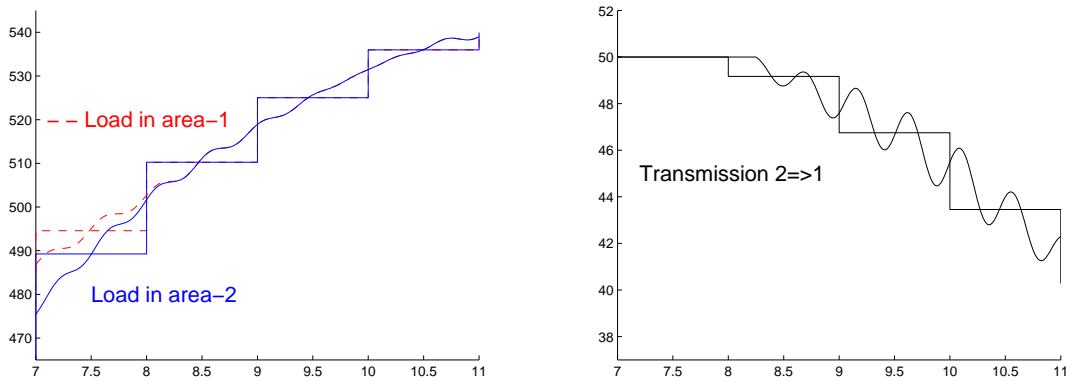


Figure 5.9. The hourly and minute supply need in both areas for hour 8-11 (left) and transmission (right) in example 5.2

End of example 5.2d

Solution to example 5.2e The impact of the AGC system is that fluctuating transmission, as in figure 5.9 is not allowed. All load deviations within each hour and area instead have to be balanced within that area. Figure 5.11 shows the load deviations within each area for hour 8-11. The AGC system causes these variations to be covered within each area, i.e., the load variations = the generation variation. Compare figures 5.11 and 5.10-right. then one can see that the AGC system caused more regulation in the generators. The reason is that the two loads D1 and D2 do not vary in exactly the same way. This means that the total regulation need in the whole system is lower than the sum of the regulation needs in each area.

End of example 5.2e

Comments to example 5.2 In the example the coupling between the hourly scheduling and the intra-hour balancing, where the deviation is treated by the regulating market. In this example a perfect forecast has been assume, which leads to that the hourly

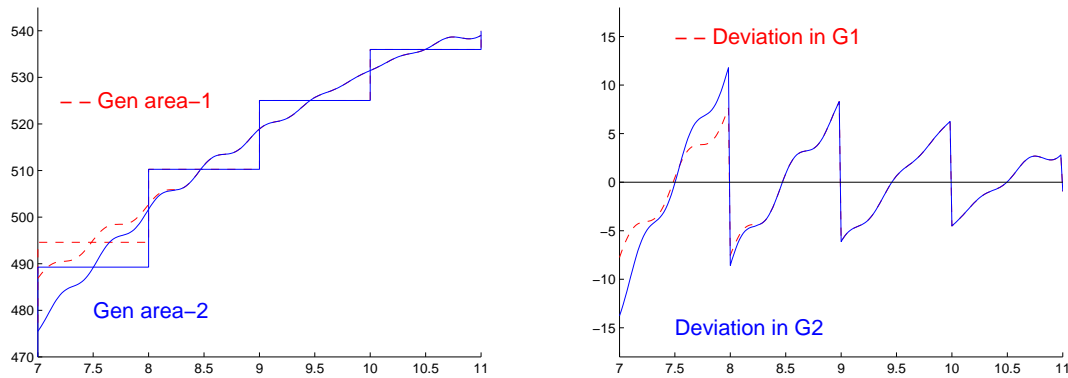


Figure 5.10. The hourly and minute generation in both areas for hour 8-11 (left) and deviation (right) in example. 5.2

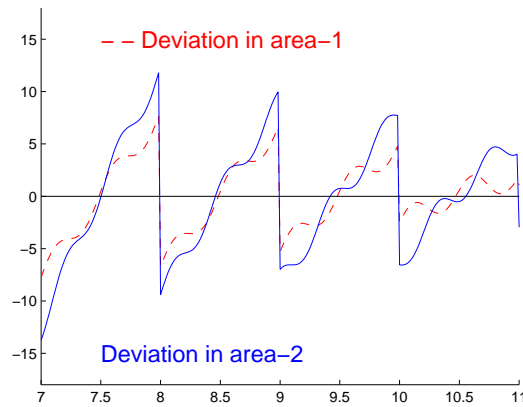


Figure 5.11. The intra hourly deviation in both areas for hour 8-11 in example. 5.2

scheduling is based on the real mean value. The consequence of this is that there will always be a need of both up- and down regulation within each hour. In reality the forecasts of the hourly energy consumption (= hourly mean power) is normally so good, which leads to that the most common consequence is that it during each hour is either up- or down regulation, not both. Concerning AGC system it can be noted that this system can increase the needed regulation in power plants (compare figures 5.11 and 5.10-right) but on the other hand it may be a large challenge to organize a market so the cheapest regulation power is always used.

Chapter 6

Investments

6.1 Concerning the role of subsidizing rarely used units

The aim of this section is to analyze the connection between risk of capacity deficit, the power price and the function of the market. The section will show that there are three central variables: The amount of subsidized power, SP , the risk of capacity deficit, $LOLP$ and the maximal accepted price, λ_{max} . For a certain system one of these can be calculated from the other two.

Assume that the question is what an "acceptable" supply reliability is? This question is then directly related to the amount of subsidized power and the maximal accepted price.

First assume a simplified power system consisting of

1. One area, i.e., no bottlenecks
2. A known distribution of future power consumption available as a duration curve
3. An assumption that the consumption is price independent
4. The power stations are assumed to be 100 % reliable, i.e., the installed capacity is always available
5. The power production only consists of a certain type of gas turbines
6. If a power station can get their costs covered, then it is built.

The structure below is that first an analysis is based on these assumptions and then the assumptions are commented. The analysis is illustrated with an example with the following data:

- The future power consumption is assumed to be Gaussian distributed (duration function $\phi(x)$) with mean value $m_x = 20000$ MW and a standard deviation of $\sigma_x = 2500$ MW. A practical consequence of this assumption is that there are very small (but >0) probabilities of extremely high load levels.
- The cost of a gas turbine is assumed to $\alpha_G = 300$ kSEK/MW,year and $c_G = 0.5$ kSEK/MWh

Figure 6.1 shows the load duration curve, $F(x)$, and the needed price in order to cover the total costs for investments in gas turbines.

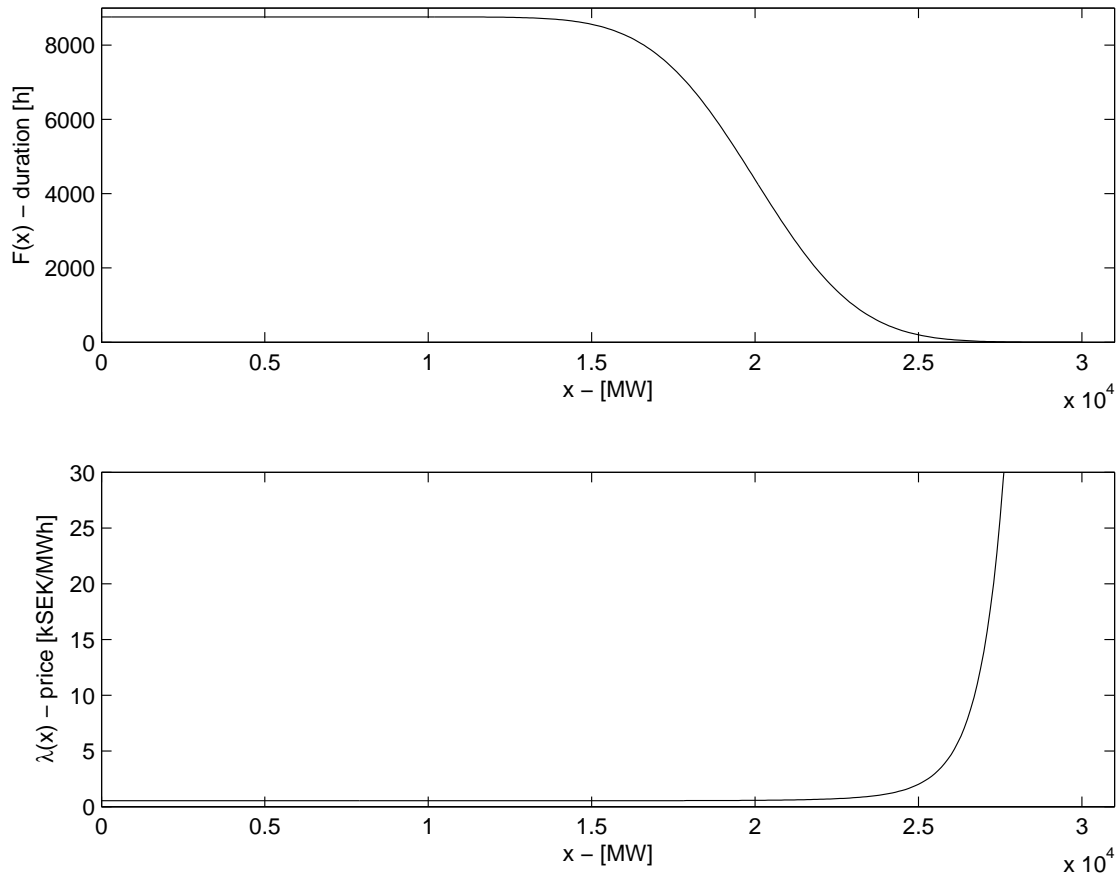


Figure 6.1. A: Duration curve for the consumption (upper figure), B: Needed price level for profitable investment in gas turbine (below).

The load duration curve, $F(x)$, is defined as

$$F(x) = \text{Number of hours per year that the load is } \geq x \quad (6.1)$$

$$= 8760 \left[1 - \phi \left(\frac{x - m_x}{\sigma_x} \right) \right] \quad (6.2)$$

It can be noted that, e.g., the level 0.5 hours per year in reality means, perhaps, 2 hours every 4:th year, since the power consumption varies between different years. The needed price corresponds to an energy price that covers the total cost of the power plant (including capital costs) at a utilization time that the power consumption has at this level. The needed price level, $\lambda(x)$ can be calculated as

$$\lambda(x) = c_G + \frac{\alpha_G}{F(x)} \quad (6.3)$$

Table 6.1 shows some of the values from figure 6.1

In figure 6.2 some new variables are introduced and the same data as in figure 6.1 are shown but for the interval when the load is larger than 25000 MW, i.e., high load situations. The three new variables are :

x - load level MW	$F(x)$ - duration h/year	$\lambda(x)$ - needed price kSEK/MWh
>25500	121.8	> 3.0
>26000	71.8	>4.7
>26500	40.8	>7.8
>27000	22.4	>13.9
>27500	11.8	>25.9
>28000	6.0	>50.3
>28500	3.0	>102.1
>29000	1.4	>215.7
>29500	0.6	>473.9
>30000	0.3	>1081.8

Table 6.1. Utilization time and needed price depending on load level in figure 6.1

$$\begin{aligned}
P &= \text{Installed capacity} \\
\lambda_{max} &= \text{Maximal accepted price level} \\
M &= \text{Load level corresponding to } \lambda_{max} \\
R &= P - M = \text{reserve capacity that has to be subsidized}
\end{aligned}$$

It can be noted that $F(P)$ means the probability that the load is larger than the installed capacity which is the same as the risk of capacity deficit. This is normally denoted $LOLP$ = Loss Of Load Probability:

$$LOLP(P) = \text{Loss Of Load Probability at installed capacity} = P = \quad (6.4)$$

$$= F(P) = \text{Number of hours per year that the load is } \geq P \quad (6.5)$$

Relevant questions are now:

- A How much installed capacity, P , is needed? This is the same question as: What reliability level, $LOLP = F(P)$, is required?
- B How high prices, λ_{max} can be accepted? This is the same question as to ask how much power, R that has to be subsidized. This is shown below.

A: If one has $P=29000$ MW installed, then there will be deficit of power during 1.4 h/year ($P \Rightarrow LOLP$). If one accepts an LOLP during 1.4 hours/year then it is necessary to install 29000 MW ($LOLP \Rightarrow P$). The lower the risk, the more power has to be installed. This example is shown in figure 6.2.

Now assume that one in an analysis have come to the conclusion that 29000 MW of capacity is enough, i.e., one accepts a risk a capacity deficit with a mean value of 1.4 h/year.

If one assumes that all power stations should only be paid with the current power price, then the price has to be $\lambda(29000) = 215.7$ kSEK/MWh at this consumption level in order to make it profitable to invest in a gas turbine that is only used 1.4 h/year. (Not shown in the figure since it is outside the window).

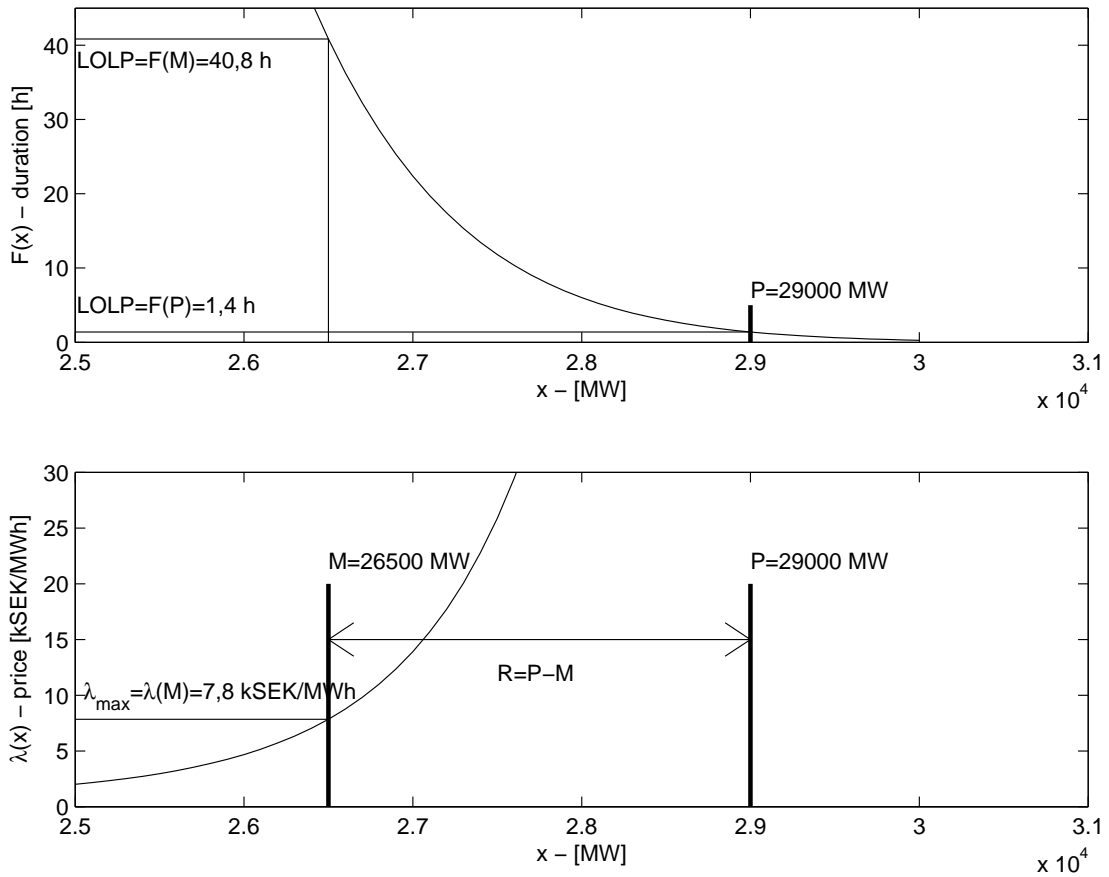


Figure 6.2. A: Duration curve for the consumption (upper figure), B: Needed price level for profitable investment in gas turbine (below).

6.1.1 Maximum price

Now assume that the society considers that there are too large problems if one accepts a price larger than λ_{max} . If this is the case, then only M MW will be installed since power stations with lower utilization time will not be profitable. Figure 6.2B shows a combination of $\lambda_{max} = 7.8 \text{ kSEK/MWh}$ which corresponds to $M = 26500$ MW. More power will not be installed since the total cost (investment + operation cost) is higher than the profit. If the risk of capacity, $LOLP = F(M)$ is considered too high ($F(26500) = 40.8$ h/year), then investments in more power stations in some way have to be subsidized. with an assumption, as in figure 6.2, that $LOLP = 1.4$ h/year is acceptable, then this will require that $R = P - M = 2500$ MW has to be subsidized.

B: If a higher price than 7.8 kSEK/MWh ($\lambda_{max} = 7.8$) is not accepted, then this implies that one have to subsidize $R = P - M = 29000 - 26500 = 2500$ MW (26500 and 7.8 kSEK/MWh are on the same row in table 6.1). This means that λ_{max} and $LOLP \Rightarrow R$. If one, on the other hand has decided how much reserve power one can subsidize, then it is possible to estimate which price one has to accept R and $LOLP \Rightarrow \lambda_{max}$. The higher the price, the lower amount of reserve power that has to be subsidized. Table 6.2 shows required amount of reserve power as a function of accepted price.

Accepted price λ_{max} [kSEK/MWh]	Needed amount of reserve power [MW]
3.0	3500
4.7	3000
7.8	2500
13.9	2000
25.9	1500
50.3	1000
102.1	500
215.7	0

Table 6.2. Needed amount of reserve power for different accepted power prices at an accepted LOLP of 1.4 h/year

There are some principal consequences of this:

1. If one has a maximum price, λ_{max} , but no subsidized power stations, $R = 0$, it is then possible to estimate the resulting LOLP. In the here studied example $\lambda_{max} = 7.8$ kSEK/MWh \Rightarrow $LOLP = 40.8$ h/year. ($\lambda_{max}, R = 0 \Rightarrow LOLP$)
2. If one sets a maximum price, λ_{max} , and a certain amount of reserve power, R , then it is possible to estimate the resulting risk of capacity deficit. With $\lambda_{max} = 7.8$ kSEK/MWh and $R = 2500$ MW the load up to 26500 MW will be covered by the market since the costs are covered, while the load between 26500 MW and 29000 MW will be covered by subsidized plants. Higher loads can not be covered, $LOLP = 1.4$ h/year. ($\lambda_{max}, R \Rightarrow LOLP$)
3. If one assumes a maximum price, λ_{max} , accepts the concept of subsidized power plants and assumes an acceptable risk of capacity deficit, it is then possible to estimate how much reserve capacity that has to be subsidized. With $\lambda_{max} = 7.8$ kSEK/MWh and $LOLP = 1.4$ h/year then it is needed to subsidize $R = 2500$ MW of reserve power. ($\lambda_{max}, LOLP \Rightarrow R$) It can be noted that if the reserve power is bid into the market at a lower price than λ_{max} , then a larger amount of reserve power is needed to keep down the $LOLP$ to 1.4 h/year, see below.
4. If one assumes a certain amount of reserve power, R and a given risk of capacity deficit, it is then possible to estimate the price that the reserve power has to be bided into the market in order not to replace market financed power stations. In the example presented here this means that with $R = 2500$ MW of reserve power and $LOLP = 1.4$ h/year it is possible to bid in this power to the market at the price $\lambda_{max} = 7.8$ kSEK/MWh ($LOLP, R \Rightarrow \lambda_{max}$)

As shown in this analysis there are 3 central variables: The amount of subsidized reserve power, R , risk of capacity deficit, $LOLP$ and the maximum accepted price λ_{max} . One of these can be estimated based on information on the two other ones.

6.1.2 On bid price for reserve power

Assume now that there is no formal maximum price, but there is a decision on which price that the subsidized reserve power will use when it is bid into the market. An example of this is shown in figure 6.3. In the figure there are two types of reserve

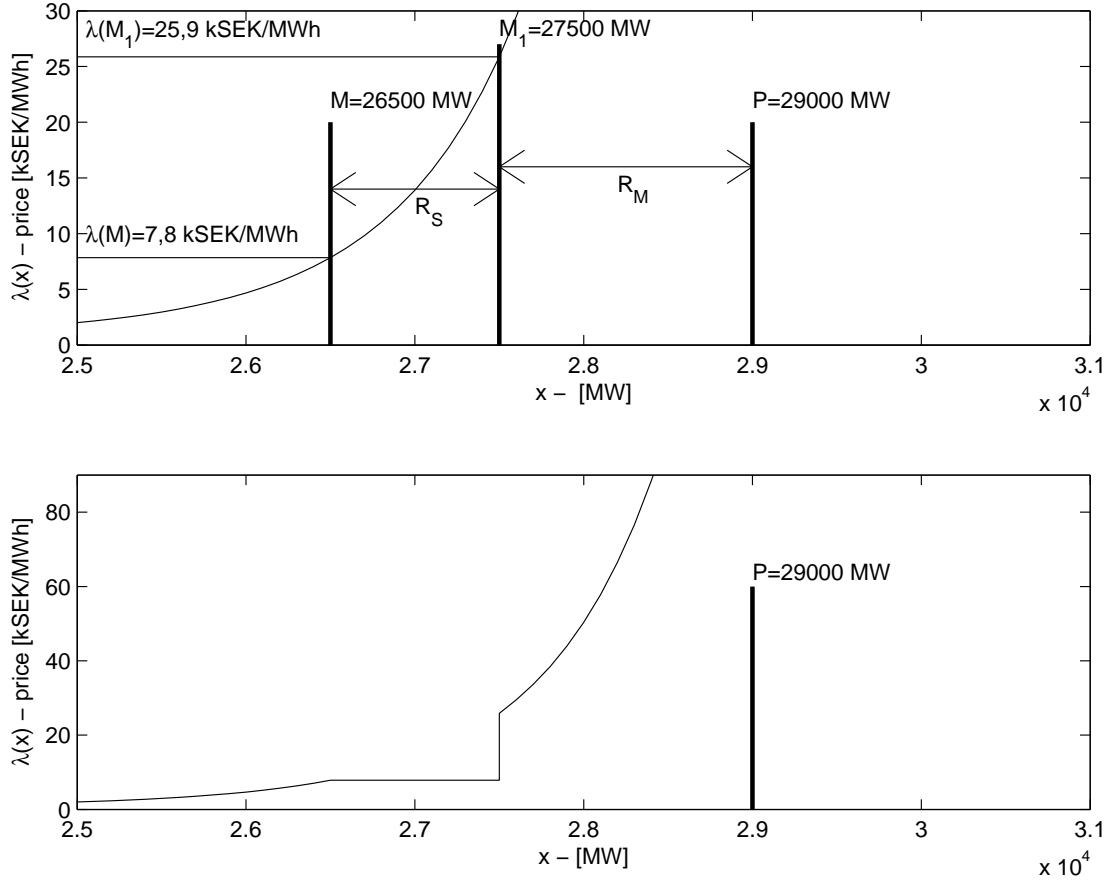


Figure 6.3. $\lambda(M) = 7.8$ kSEK/MWh. A: Needed price for a market financed gas turbine (upper figure), B: Actual marginal price to the market at different load levels (below).

power:

$$R_S = \text{Subsidized reserve power} \quad (6.6)$$

$$R_M = \text{Other reserve power financed by the market} \quad (6.7)$$

$$(6.8)$$

For the example in figure 6.3 the following is valid:

1. The subsidized reserve power ($R_S = 1000$ MW) is bid into the market at a price of 7.8 kSEK/MWh.
2. This means that it is only when this is not enough that more reserve power is needed. This occurs at the consumption level $26500 + 1000 = 27500$ MW.

3. For load level above 27500 MW the reserve power plants have to be financed by the market, since the subsidized reserve power plants are already used. This corresponds to that the first extra MW that comes in at load level 27500 MW has to get a price of $\lambda(M_1 = M + R_S) = 25.9$ kSEK/MWh. For more power even higher prices are needed.
4. How much not subsidized reserve power that is needed depend on accepted price level or required reliability level (corresponding to a certain LOLP level). In the example in figure 6.3 market finance reserve power of $R_M = P - R_S - M = 1500$ MW. This corresponds to an LOLP of 1.4 h/year and an accepted price of 215.6 kSEK/MWh.

A comment is that in this example there is a need to accept the price 215.7 kSEK/MWh in order to get a LOLP of 1.4 h/year. If one instead had set the price of subsidized reserve power to 50.3 kSEK/MWh ($\lambda(P - R_S) = \lambda(28000) = 50.3$ kSEK/MWh according to table 6.1), then the maximal price instead becomes 50.3 SEK/MWh (and not 215.7 kSEK/MWh) for an accepted LOLP of 1.4 h/year. This case is illustrated in figure 6.4. The conclusion is that pricing of subsidized reserve power is essential, since

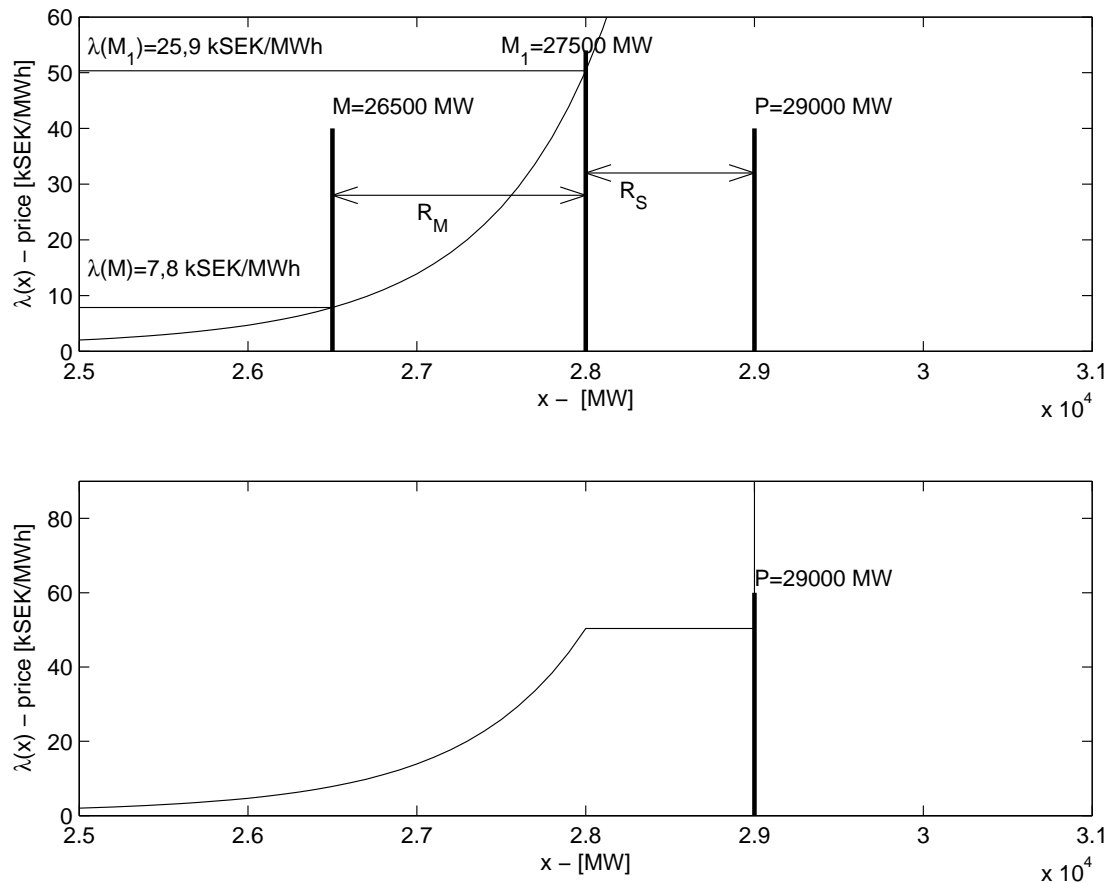


Figure 6.4. A: Needed price for a market financed gas turbine (upper figure),
B: Actual marginal price to the market at different load levels (below).

subsidized power on the market will compete with fully market financed power plants.

If subsidized plants lower the price on the market, this will reduce the interest of the market to invest in market financed power plants. This can lead to very high prices and/or a high risk of capacity deficit and/or requirements of more subsidized reserve power plants.

6.1.3 Comments on assumptions in illustrative example

Here are some comments concerning the basic assumptions which were presented in the beginning of the section:

1. *One area, i.e., no bottlenecks:* This analysis can be made for a whole system including bottlenecks. But that means that there will be different price levels (as in section 2.2) and different risks of capacity deficit in different areas, since the load and trading capacity varies.
2. *A known distribution of future power consumption available as a duration curve*
The result is of course very dependent on the structure of this curve. If this curve is wrong then the results are wrong. The principal problem is that if one overestimate the duration curve, then one overestimate the interest of the market to construct power plants with low utilization times, which has an impact on the risk of capacity deficit.
3. *An assumption that the consumption is price independent:* Price dependent load can be modeled as a production source, where the load decrease at a certain price instead is a corresponding production at the same price.
4. *The power stations are assumed to be 100 % reliable, i.e., the installed capacity is always available:* It is possible to include the outages in the duration curve by the use of the theory of Equivalent Load Duration Curve, *ELDC*.
5. *The power production only consists of a certain type of gas turbines:* In a real power system there are of course a lot of types of power plants with different operation and investment costs. This has though no principal impact on the price curve $\lambda(x)$. At high utilization times (x is small) high investment costs per MW is not the dominant problem since there are many MWh that can share the investment cost. But at low utilization times (x is large) low investment costs are essential since the investment costs can only be distributed to comparatively few produced MWh.
6. *If a power station can get their costs covered, then it is built.:* This is probably not true since the future always is uncertain and there has to be an expected profit (including risk premium) before an investment decision is taken. This can in principal be included by raising the costs of new power plants.