Optimal Speed Trajectory for a Heavy Duty Truck Under Varying Requirements*

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Abstract— **The optimal speed trajectory for a heavy duty truck is calculated by using the Pontryagin's maximum principle. The truck motion depends on controllable tractive and braking forces and external forces such as air and rolling resistance and road slope. The solution is subject to restrictions such as maximum power and position dependent speed restrictions. The intended application is driving in environments with varying requirements on the velocity due to e.g. legal limits and traffic. In order to limit the vehicle to a speed trajectory that follows the normal traffic flow, data from real truck operation have been analysed and used for setting upper and lower boundaries for the decelerations. To evaluate the solution, simulations have been performed on a segment of a road normally used as a distribution test cycle. Three different policies were compared where the solution adopts to free optimization, optimization following traffic flow and finally cruise control using look-ahead control. Results from the simulations show that fuel consumption and trip time can be reduced simultaneously while following the traffic flow.**

I. INTRODUCTION

Manufacturers of heavy vehicles are constantly striving to reduce fuel consumption. The first reason is to reduce CO_2 emissions. The total road freight transportation in the European Union amounts to 1 700 billion tonne-kilometers annually [1], which corresponds to the emission of 100 million tonnes CO_2 equivalent [2]. Thus, each percent reduction of fuel consumption gives, on a European scale, a 1 million tonne reduction of CO_2 emissions. The second reason to reduce fuel consumption is to lower the cost for the customers. About one third of the total cost for a typical European long haulage company consists of fuel [3], and hence fuel consumption reduction will significantly reduce their cost. Reducing fuel consumption can be done through improvements in hardware as well as software. This paper focuses on the concept of look-ahead cruise controllers, which utilizes future road topography to control the vehicle speed. Today, look-ahead controllers are commercially available by most major truck manufacturers, and are reported to give fuel savings of up to three percent [4]. These controllers are mostly designed for individual vehicles driving at highways, without considering any interaction with surrounding traffic. However, there are several constraining factors that could be considered to improve these systems further, especially when driving outside the highway. The main focus of this paper is varying speed limits. Of course there are legal speed limits to obey but there are also more dynamic constraints that limit the speed profile of the vehicle, often imposed by the relation to other vehicles. Examples are other trucks that could be joined for platooning benefits [5], surrounding (car) traffic, intersections, junctions and ramps. Traffic signals and variable speed limits are other examples. This paper addresses how to find the most energy efficient way of driving in such dynamic conditions. This is illustrated in Fig. 1. A truck is subject to resistive forces, road topography and speed constraints from e.g. road curvature and speed limits. Within this setting, the objective in this paper is to minimize fuel consumption. The problem of finding the energy optimal speed trajectory for a given path with varying topography and speed limits is posed as an optimal control problem, see [6] for an overview of these methods in automotive applications. Moreover, it is shown that this problem can be solved using Pontryagin's maximum principle. There are various examples of calculations of an optimal speed trajectory using Pontryagin's maximum principle for the application of rail bounded trains, see e.g. [7], [8]. For heavy duty vehicles, the results concern mostly highway driving. In [9] a 3,5% fuel reduction for a heavy duty truck on a highway is achieved with a dynamic programming algorithm utilizing look-ahead information about topography. A nonlinear MPC using a mixed integer approach for combined gear selection and speed control for a heavy duty truck is presented in [10]. However, neither of these two papers treats limitations on how the decelerations are allowed to be performed, which will be addressed in this paper.

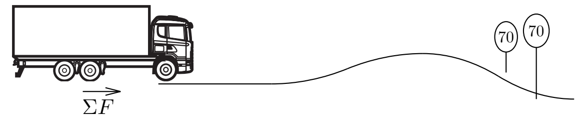

Fig. 1: A truck is driving in an environment containing road elevation and varying speed restrictions while being influenced by the resulting force ΣF.

The main contribution of this paper is the use of driving statistics collected from vehicles in live operation in order to set boundary conditions for the velocity. From this data, a statistical description of driving patterns is extracted, by identifying the magnitude of deceleration during speed changes. These deceleration values are then used as limitations in the optimization problem. This corresponds to forcing the vehicle to adapt to an accepted driving behaviour, and within these limits minimize the energy consumption.

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II. PROBLEM FORMULATION

The objective of this paper is to solve an optimization problem that can be summarized as

$$\begin{array}{ll} \underset{\text{traction, braking}}{\textbf{minimize}} & \text{fuel consumption, trip time} \\ \textbf{subject to} & \text{vehicle dynamics,} \\ & \text{speed restrictions,} \\ & \text{traffic flow,} \\ & \text{road elevation.} \end{array} \quad (1)$$

Modelling the vehicle as a point mass, the differential equation for the motion is given by:

$$\frac{dK(s)}{ds} = F_{in}(s) + F_b(s) + F_w(K(s)) + F_g(s), \quad (2)$$

where s is the position and $K(s) = v^2/2$ is the kinetic energy per mass unit given by the velocity v. The right hand side consists of forces per mass unit such that $F_{in}(s)$ is the tractive force, $F_b(s)$ is the braking force, $F_w(K(s))$ is the sum of the resistive forces originating from air and roll resistance and $F_g(s)$ is the force resulting from changes in the potential energy. The position s is chosen as the independent variable since road slope and speed restrictions are dependent on position rather than time. The vehicle has a given initial and final kinetic energy denoted K_0 and K_f respectively: The tractive and braking forces are restricted by velocity dependent limits:

$$0 \leq F_{in}(s) \leq F_{max}(K(s)), \quad (3a)$$

$$F_{bmax}(K(s)) \leq F_b(s) \leq 0. \quad (3b)$$

The vehicle is modelled to have constant maximum tractive and braking power, P_{max} and P_{bmax} respectively. The maximum tractive and braking forces F_{max} and F_{bmax} are then given by the relation $F = P(2K(s))^{-1/2}$. The resistive force F_w is assumed to be nonpositive and have negative first derivative:

$$F_w(K(s)) \leq 0, \quad (4a)$$

$$F'_w(K(s)) < 0. \quad (4b)$$

Varying speed restrictions are imposed over the area by an upper bound for the kinetic energy $K_u(s)$ such that

$$h_u(K, s) \triangleq K_u(s) - K(s) \geq 0 \quad (5)$$

and a lower bound for the kinetic energy $K_l(s)$ such that

$$h_l(K, s) \triangleq K(s) - K_l(s) \geq 0. \quad (6)$$

The main objective in this paper is to find a fuel optimal solution. However, if the fuel consumption would be the only entity to minimize, the solution would be to drive as slowly as possible since the energy losses increase with higher velocity because of (4b). In order to also take the trip time in the solution into consideration, a power loss constant in time P_c is added to the fuel consumption. A natural interpretation of this loss is that it represents constant power losses in the vehicle, such as fuel to keep the engine running. Adding this loss, the objective is to minimize the total sum of input energy:

$$\min_{F_{in}, F_b} \int_0^S (F_{in}(s) + \frac{P_c}{\sqrt{2K(s)}})ds. \quad (7)$$

III. OPTIMAL CONTROL

A. Pontryagin's maximum principle

Equation (7) will be minimized by applying the Pontryagin's maximum principle as done in [8], which will result in an Hamiltonian equation to maximize and a couple of differential equations to be solved for over the interval. More specifically, the principle states [11] that given a minimization problem

$$\min_{u(s)} \int_0^S f_0(s, x(s), u(s))ds \quad (8)$$

subject to

$$\frac{dx(s)}{ds} = f(s, x(s), u(s)), \quad (9)$$

the optimal solution is given by pointwise maximization of the Hamiltonian H given by

$$H(s, x, u, \psi) = -f_0(s, x, u) + \psi^T f(s, x, u). \quad (10)$$

The variable ψ is the adjoint-state variable with derivative

$$\frac{d\psi}{ds} = -\frac{\partial H}{\partial x} \quad (11)$$

having $\psi(0)$ and $\psi(S)$ free given that the values on $x(0)$ and $x(S)$ are fixed.

B. Analysis of the Hamiltonian

In order to apply the maximum principle to the problem formulated in section II, let $\psi(s)$ be the adjoint-state variable corresponding to the state variable $K(s)$. The Hamiltonian is then given by:

$$H = F_{in}[\psi - 1] + F_b\psi - \frac{P_c}{\sqrt{2K}} + F_w(K(s))\psi + F_g(s)\psi. \quad (12)$$

When solving the maximization problem with state constraints, one method [12] is to form the Lagrangian L given by:

$$L = H + \mu_u(s)h_u(K, s) + \mu_l(s)h_l(K, s), \quad (13)$$

where $\mu_u(s)$ and $\mu_l(s)$ are non-negative Lagrange multipliers for the upper and lower bounds on the kinetic energy respectively. For the Lagrange multipliers, the conditions

$$\mu_u(s)h_u(K, s) = 0 \quad (14)$$

and

$$\mu_l(s)h_l(K, s) = 0 \quad (15)$$

are satisfied everywhere. From (14) it can be seen that $\mu_u = 0$ when $K < K_u$ and $\mu_u \geq 0$ when $K = K_u$. Similarly, from (15) it can be seen that $\mu_l = 0$ when $K > K_l$ and $\mu_u \geq 0$ when $K = K_l$. These conditions are known as the complementary slackness conditions.

The differential equation for the adjoint-state variable is given by taking the negative derivative of the Lagrangian with respect to the state variable, in this case the kinetic energy. Doing so gives:

$$\begin{aligned} \frac{d\psi}{ds} &= [1-\psi]\frac{\partial F_{in}}{\partial K} - \psi\frac{\partial F_b}{\partial K} - \frac{P_c}{(2K)^{3/2}} \\ &\quad - \psi F_w' + \mu_u - \mu_l. \end{aligned} \tag{16}$$

The Pontryagin's maximum principle states that the optimal control is received by maximizing the Hamiltonian (12). Since the only variables that can be controlled are F_{in} and F_b, the optimal control can be found directly from the Hamiltonian.

Full power: If $\psi(s) > 1$ then $F_{in}(s) = F_{max}$ and $F_b(s) = 0$, called the full power regime, since maximum tractive force will maximize the Hamiltonian.

Partial power: If $\psi(s) = 1$ then $0 \leq F_{in}(s) \leq F_{max}$ and $F_b(s) = 0$, called the partial power regime. The optimal control is not given directly by the Hamiltonian here.

Coasting: If $0 < \psi(s) < 1$ then $F_{in}(s) = 0$ and $F_b(s) = 0$, called the coasting regime, since both the tractive and the braking force should be equal to zero in order to maximize the Hamiltonian.

Partial braking: If $\psi(s) = 0$ then $F_{in}(s) = 0$ and $F_{bmax} \leq F_b(s) \leq 0$, called the partial braking regime. The optimal control is not given directly by the Hamiltonian here.

Full braking: If $\psi(s) < 0$ then $F_{in}(s) = 0$ and $F_b(s) = F_{bmax}$, called the full braking regime, since maximum braking force will maximize the Hamiltonian.

In the partial power regimes, where $\psi(s) \equiv 1$ and in partial braking regimes, where $\psi(s) \equiv 0$, the optimal control cannot be found directly from the Hamiltonian but must be found by other means.

C. Partial power

In the partial power regimes, where $\psi(s) \equiv 1$, (16) gives:

$$\frac{P_c}{(2K)^{3/2}} + F_w'(K(s)) - \mu_u(s) + \mu_l(s) = 0. \tag{17}$$

There are possibly three different scenarios in which this can occur, depending on if the vehicle is driving at any of the two speed limits or not. Considering the case when the vehicle has a kinetic energy between the upper and lower bounds, (17) becomes:

$$\frac{P_c}{(2K)^{3/2}} + F_w'(K(s)) = 0 \tag{18}$$

which only has one solution $K = K_s$ where the constant K_s is called the stabilization energy that will depend on how P_c and F_w are chosen. To see that there is only one solution, one can note that both of the two terms in (18) are monotonically decreasing in K. The left hand side of (18) can thus only be zero for one specific value of K. Considering the case when the vehicle is keeping a kinetic energy equal to the upper bound, then $h_u(K, s) = 0$ and $\mu_u(s) > 0$ by (14). Equation (17) becomes:

$$\frac{P_c}{(2K)^{3/2}} + F_w'(K(s)) - \mu_u(s) = 0 \tag{19}$$

which has a solution $K \leq K_s$. Using the same argumentation when driving at the lower bound, equation (17) has a solution $K \geq K_s$.

Since the vehicle is limited by a maximum tractive force, it might not be able to keep the constant kinetic energy K_s or follow the upper or lower bound for the kinetic energy. Whether or not partial power can occur at a specific position depends on how steep the uphills are and how rapid the changes in the speed limits are. The conditions for the different types of partial power are:

- $K(s) = K_s$ can occur when $K_l \leq K(s) \leq K_u$ and $0 \leq -F_g(s) - F_w(K(s)) \leq F_{max}$.
- $K(s) = K_l$ can occur when $K_s \leq K_l$ and $0 \leq -F_g(s) - F_w(K(s)) + \frac{dK_l}{ds} \leq F_{max}$.
- $K(s) = K_u$ can occur when $K_u \leq K_s$ and $0 \leq -F_g(s) - F_w(K(s)) + \frac{dK_u}{ds} \leq F_{max}$.

D. Partial braking

In partial braking regimes where $\psi(s) \equiv 0$, (16) becomes:

$$\frac{P_c}{(2K)^{3/2}} - \mu_u(s) + \mu_l(s) = 0. \tag{20}$$

There are possibly three different scenarios in which this can happen. Either the vehicle is driving at the upper speed limit, at the lower speed limit or at a speed somewhere in between. For each of these cases, (20) becomes:

$$\frac{P_c}{(2K)^{3/2}} - \mu_u(s) = 0, \tag{21a}$$

$$\frac{P_c}{(2K)^{3/2}} = 0, \tag{21b}$$

$$\frac{P_c}{(2K)^{3/2}} + \mu_l(s) = 0. \tag{21c}$$

For (21a), there is no restrictions on the solution and for (21b) and (21c) there is no solution. Partial braking can thus only occur in situations when the vehicle is driving at the upper speed limit $K(s) = K_u(s)$ and, because of the limited braking force, when $F_{bmax} \leq -F_w(K(s)) - F_g(s) + \frac{dK_u}{ds} \leq 0$.

E. Linking of intervals

As discussed in section III, the optimal control can in situations when $\psi(s) = 1$ or $\psi(s) = 0$ not be found directly from the Hamiltonian. In some special cases, typically for shorter distances, $\psi(0)$ can be chosen such that (2) and (16) can be integrated over the full interval without $\psi(s)$ ever being constantly equal to one or zero on any subinterval and satisfying the boundary conditions for K. In these cases the optimal solution is found by a shooting method in order to find a value on $\psi(0)$ that will solve for (2) and (16) over the full interval. For most problem formulations consisting of longer driving distances however, there might be sections where the optimal solution is for the vehicle to keep a constant kinetic energy. Finding the optimal solution will consist of linking all or some of these sections by finding the point of entry and exit by integrating (2) and (16) given the boundary conditions for $\psi(s)$ and $K(s)$ on each interval.

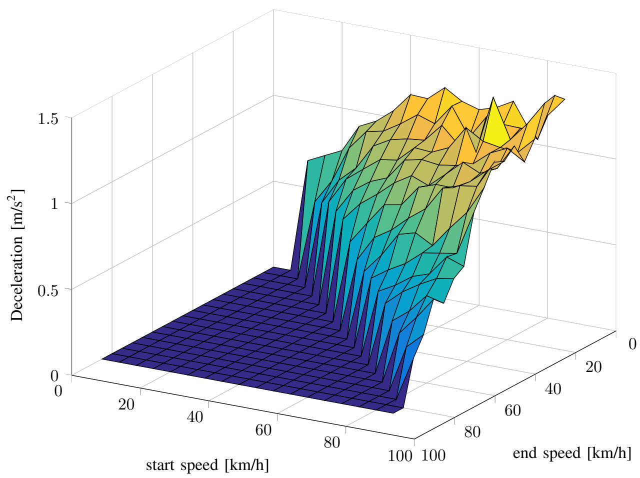

Fig. 2: Mean of rate of deceleration as a function of start and end speed.

F. *Jumping conditions for* ψ

The Lagrange multipliers $\mu_u(s)$ and $\mu_l(s)$ are not continuous function but can jump from 0 to some positive value when the velocity reaches the corresponding limit. From this fact, it follows that (16) neither is continuous at these points. As discussed in [12] the adjoint-state variable may have a discontinuity at these points given by:

$$\psi(s^-) = \psi(s^+) + \eta(s) h_K(s), \tag{22}$$

where $\eta(s) \geq 0$, $h(s) \geq 0$, $\eta(s)h(s) = 0$, s^- is the right-hand side limit of the point of reaching or leaving the state constraint, s^+ is the left-hand side limit and $h_K(s)$ is the derivative of a state constraint such as (5) and (6) with respect to the state variable, in this case K. By inserting (5) and (6) into (22), it can be concluded that ψ can make a positive jump when reaching or leaving the upper speed limit and a negative jump when reaching or leaving the lower speed limit. These facts will be used during linking of the intervals.

IV. TRAFFIC BEHAVIOUR ANALYSIS

A. *Data analysis*

In order to make the received solution realistic from a traffic perspective, an analysis of real traffic has been performed. At positions where the maximum allowed speed varies, the trajectory received from the optimization problem in this paper can possibly differ a lot from what is a normal and acceptable way of driving. From simulations without a lower bound on the velocity, it was found that when the vehicle approached an area where the maximum allowed velocity was decreasing, the optimal solution was to coast until the lower speed was attained. This might be optimal from a fuel perspective, but would probably not be accepted neither by the driver nor the surrounding traffic. With this in mind, the allowed speed of the vehicle during decelerations is limited by *bounding speed limits* which are imposed in order to refrain the vehicle from deviating too much from what can be considered as normal. In order to limit unrealistic long sections of coasting, a lower bounding speed limit for the velocity was imposed in these situations. Since it may be dangerous for the vehicle to decelerate too quickly, an upper bounding speed limit was also imposed. The bounding speed limits should be distinguished from the speed restrictions since they are imposed on the problem formulation to adopt the vehicle to the traffic flow while the speed restrictions are imposed to make the vehicle follow the law. In order to set the bounding limits, traffic data from real operation were analysed with the aim at finding usable statistics regarding what a normal way of decelerating is.

The data used to derive these statistics come from a heavy duty distribution truck in the UK. It was selected from a database of several vehicles since its velocity was varying between many different values during operation. The velocity signal was collected from a driving distance of $39\,000$ km and sampled at 20 Hz. The feature selected for analysis was the mean and standard deviation of the decelerations. A deceleration was defined as the distance between a local maxima and a local minima of the velocity. Since the velocity signal contained some high frequency noise, it was filtered with a low pass filter with cut-off frequency 1 Hz. Since the magnitude of the decelerations depends on the start and the end speed, the range of speed which was 0-90 km/h was divided into bins of 5 km/h. By doing so, decelerations from e.g. 70-75 km/h to 30-35 km/h were analysed separately. Since a deceleration might start or end with a very long transient not really belonging to the intended lowering of the speed, the top and bottom 10% of the change in speed was not considered to be part of the deceleration. Since the vehicle was often keeping an almost constant speed around the limit of any of the bins, a deceleration from one bin to an adjacent one was not considered as a deceleration. From a total number of $20\,160$ decelerations, the resulting mean value can be seen in Fig. 2. The most common deceleration was from 85-90 km/h to 75-80 km/h which occurred 596 times and the least frequent was from 80-85 km/h to 10-15 km/h which occurred 16 times.

B. *Bounding speed limits based on traffic data*

In areas where the upper speed limit is set to a constant value, the lower bounding limit is also set to a constant value. The difference between these limits is set such that a vehicle could travel at a reference speed at a value between these two and deviate ± 5 km/h from the reference speed. The lower limit is thus set 10 km/h below the upper limit. In areas where the upper speed limit decreases, the traffic data will be used in order to restrict how this deceleration is allowed to occur by imposing a lower bounding limit. The main idea is that the rate of deceleration from one speed to another should be limited by the mean rate of deceleration plus/minus one standard deviation. These bounding limits are implemented by starting at the point where the new upper speed limit starts and then draw two lines backward from this point until they meet the corresponding previous speed limit. The slope of the two lines will then be given by the mean and standard deviation of this particular change in speed,

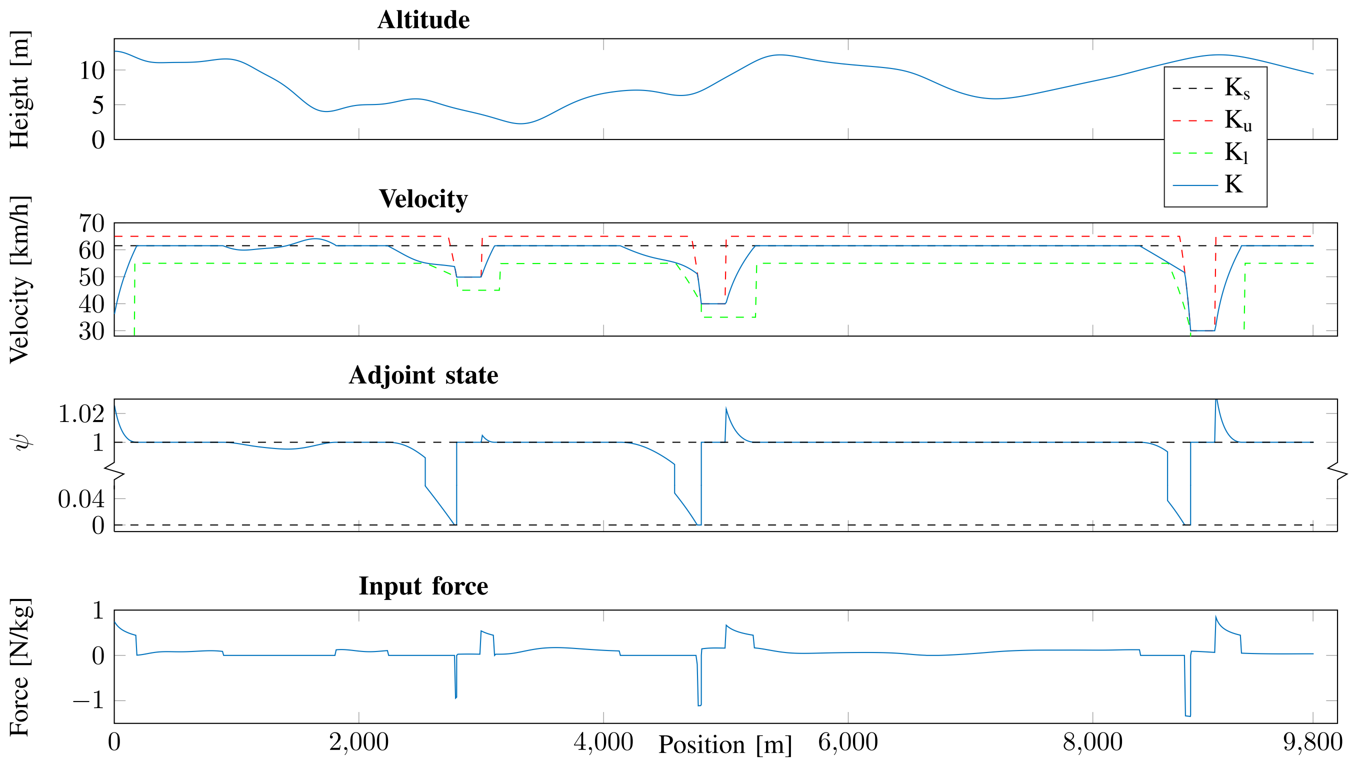

Fig. 3: Result of a simulation that uses bounding speed limits showing altitude, velocity, adjoint-state variable and tractive force. The kinetic energies are converted to the unit km/h. The plot showing the adjoint-state variable has a discontinuity for readability.

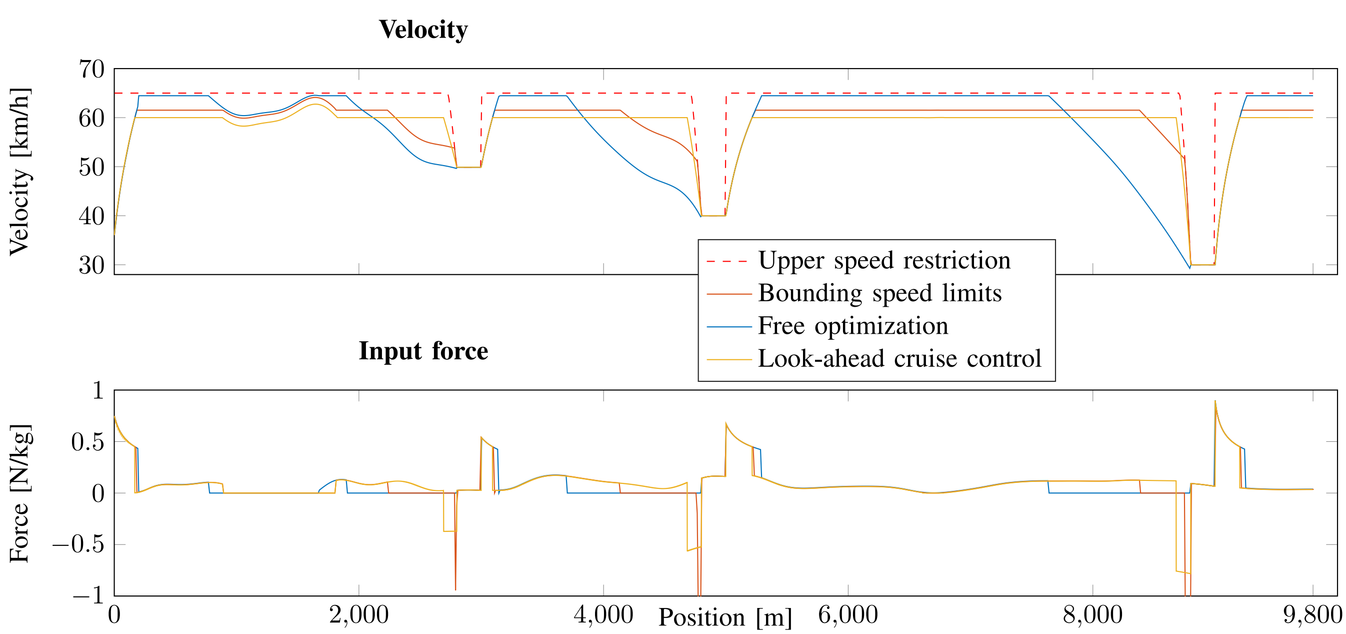

Fig. 4: Resulting velocity and tractive force from three different simulations using bounding speed limits, look-ahead cruise-control and free optimization.

TABLE I: Resulting energy consumption and trip time for the three different simulations.

Case	Energy(J/kg)	Time(s)
Bounding speed limits	925	615
Free optimization	808	635
Look-ahead cruise control	999	621

taken from the statistics that was shown in Fig. 2. In areas where the upper speed limit increases, there is no bounding speed limits that put restrictions on the rate of acceleration. With the problem formulation in this paper, the consequence will be that all accelerations are done with maximum power. Since a heavy duty truck usually accelerates much slower than other traffic, there is no need to restrict this behaviour by imposing bounding speed limits.

V. SIMULATION RESULTS

A. General characteristics of the solution

The final algorithm for finding the optimal trajectory by applying Pontryagin's maximum principle was performed as discussed in section III. Simulations were performed on real road data from a segment between the Swedish cities Södertälje and Järna, which is a typical distribution cycle. From the 96 km road, a segment of 10 km height profile was used, while the speed restriction was set manually in order to impose different amplitude in the changes of it. The truck was modelled to have a maximum power of 300 kW and a total weight of 40 000 kg. The upper speed limit was set to 65 km/h and the lower speed limit at 55 km/h. At three different positions, the upper speed limit was set to 50, 40 and 30 km/h for 200 m. During these short episodes of lowered speed limits, the vehicle was not allowed to deviate from the reference speed. To compare the result from using the methods in this paper, the simulation was run three times under different restrictions. In the first simulation, shown in Fig. 3, the vehicle follows the bounding speed limits derived from the deceleration statistics and the four graphs show altitude, speed, adjoint-state variable and input force. A few interesting sections of the solution can be noted. At around 1 km, the optimal control is for the vehicle to coast and the shooting algorithm makes sure that ψ reaches/leaves the value 1 at the same position as K reaches/leaves K_s. At the three areas of lowered speed restriction, the vehicle maximizes the coasting distance by barely touching the lower bounding speed limit before reaching the upper bounding speed limit, where partial braking starts.

B. Comparing strategies

In order to be able to evaluate the performance of the solution, the simulation was performed two additional times under different conditions. In each of the simulations, the bounding speed limits were set differently. The second simulation was performed with the speed restrictions but without any bounding speed limits. In the third simulation, the vehicle adopts to a simple cruise control strategy, keeping 60 km/h where it is allowed and decelerates to obey the speed limits by following the mean deceleration for that specific speed change. However, since look-ahead control using topography is today commonly used, the vehicle is still allowed to use this technique during areas of constant speed limits. This is done by finding areas of coasting and full power as discussed in section III. A comparison between the three simulations can be seen in Fig. 4 and the results are summarized in Table I. As can be seen, the simulation following the bounding speed limits has lower fuel consumption as well as lower trip time. The simulation using free optimization has the lowest fuel consumption, but also the greatest trip time. As can be seen in Fig. 4, the three different solutions all keep different speed on the areas where the upper speed limit is 65 km/h. This is due to the fact that P_c in (7) is set differently in each case which is done in order to get more comparable trip times.

VI. CONCLUSION

The main conclusion is that there is a great amount of fuel that can be saved by coasting as much as possible when the speed needs to be lowered. Raising the speed on areas where constant speed is held can be done to compensate for the loss in time during deceleration. This method can be performed by the vehicle and still follow what can be considered a normal way of driving by forcing the speed trajectory of the vehicle to be within the bounding speed limits.

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