Wind tunnel blockage corrections for wind turbine measurements

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Abstract:

Wind-tunnel measurements are an important step during the wind-turbine design process. The goal of wind-tunnel tests is to estimate the operational performance of the wind turbine, for example by measuring the power and thrust coefficients. Depending on the sizes of both the wind turbine and the test section, the effect of blockage can be substantial. Correction schemes for the power and thrust coefficients have been proposed in the literature, but for high blockage and highly loaded rotors these correction schemes become less accurate.

A new method is proposed here to calculate the effect a cylindrical wind-tunnel test section has on the performance of the wind turbine. The wind turbine is modeled with a simplified vortex model. Using vortices of constant circulation to model the wake vortices, the performance characteristics are estimated. The test section is modeled with a panel method, adapted for this specific situation. It uses irrotational axisymmetric source panels to enforce the solid-wall boundary condition. Combining both models in an iterative scheme allows for the simulation of the effect of the presence of the test-section walls on wind turbines performance.

Based on the proposed wind-tunnel model, a more general empirical correlation scheme is proposed to estimate the performance characteristics of a wind turbine operating under unconfined conditions by correcting the performance measured in the confined wind-tunnel configuration. The proposed correction scheme performs better than the existing correction schemes, including cases with high blockage and highly loaded rotors.

Descriptors:
Wind tunnel, wind turbine, numerical model, measurements, blockage, empirical correction scheme
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CHAPTER 1

Introduction

1.1. Wind turbine essentials

In general, a wind turbine is a mechanical device that extracts kinetic energy from a flowing stream of air and converts it into mechanical energy (Hansen 2008), that is later usually converted to electricity. There are many different types of wind turbines, but the most common wind-turbine type is the Horizontal-Axis Wind Turbine (HAWT). In this configuration the rotational axis of the wind turbine is oriented horizontally. As shown in Figure 1.1, the enveloping power extracting stream-tube expands due to the extraction of kinetic energy. Extracting kinetic energy from a moving incompressible airflow reduces the velocity and the power extracting stream tube must expand, due to mass conservation. The presence of the wind turbine and the power extraction is already felt upstream of the turbine. This results in a decreased velocity upstream of the turbine blades, compared to the incoming free-stream velocity (Medici et al. 2011). Furthermore, due to the rotation of the wind turbine, an additional rotational velocity component is generated in the wake (Hansen 2008).

1.2. Wind energy industry

According to the IPCC (2007), the increased anthropogenic emissions of greenhouse gases (e.g. CO$_2$ and CH$_4$), partly due to the energy sector, has lead to a substantial effect on the climate balance. In the face of climate change due to the enhanced greenhouse effect, the interest in finding viable technologies for exploiting renewable-energy resources for the production of electricity has grown substantially. One of the possibilities to avoid greenhouse gas emissions is to generate electricity from widely available wind resources. One industry that has had many successes in developing economically viable technologies is the wind-energy industry.

Investments in the wind-energy industry are increasing strongly (GWEC 2012). Figure 1.2 shows the global annual newly installed wind capacity and it is clear from it that the wind-energy industry is growing fast worldwide. Consequently the global cumulative wind capacity is also increasing and more and more wind turbines are becoming operational to produce electricity (Figure 1.3). However, there are big discrepancies between geographical regions:
1. INTRODUCTION

While Europe, North America and Asia are investing heavily in wind energy, Latin America, Africa and the Pacific area are lagging behind. During 2012 in Europe alone, investments in wind energy ranged between 12.9G€ and 17G€, accounting for almost 26% of the new installed electricity capacity in Europe.

Due to the increased investments in the wind energy sector and the better energy conversion technologies, investors start to expect a higher return. This means that total costs in the overall design and manufacturing process must decrease. The need for better and more accurate methods in the design process are therefore of paramount importance. Testing of wind turbines is one part of this design process and methods used in the evaluation process of wind turbines can be improved.
1.3. Wind tunnel testing of wind turbines

The estimation of the performance of wind turbines can be done in several ways. One possible way is to evaluate the wind turbine using the BEM theory (Burton et al. 2011). Here, the General Momentum Theory is combined with the Blade Element Method. The blade is represented as a sequence of individual airfoil segments, combined into a wind turbine blade. Another method often used in the wind industry is numerical analysis of the fluid using CFD programs (Vermeer et al. 2003). Here the Navier-Stokes equations are solved using a discretization of the flow field. This is often computationally heavy to perform and therefore very expensive, especially if accuracy is pursued. When one needs to evaluate the performance of an existing wind turbine, real-time data can be used. However, in order to have statistically significant data, it is necessary to have data available spanning a long time period, often up to one or two years. The other option is to evaluate a scaled-down model of the wind turbine and to do the measurements in an enclosed and controlled environment, i.e. a wind tunnel (Pope & Rae 1999).

Measuring the performance of a wind turbine in a wind tunnel is a crucial step during the design process of wind turbines (Manwell & McGowan 2009). Once the preliminary design of a wind turbine is done, experiments are performed to assess the actual behavior of the wind turbine and to verify whether or not the wind turbine operates as designed. In order to control the operating conditions during the experiments, an enclosed environment is recreated. During these wind-tunnel experiments, several wind-turbine operating conditions can be investigated under different controlled environmental conditions (freestream velocity, velocity gradient, turbulence, etc.). These test are performed on a scaled model of the designed wind turbine. The scaling down must be performed ideally in such a way that air flow characteristics (e.g. Reynolds-number, tip-speed ratio and Mach-number) remain the same for the scaled down model. This leads sometimes to impossible scenarios (e.g. supersonic flows or too high rotational speed of the rotor). Due to the weak dependence
on the Reynolds number (Adaramola & Krogstad 2011), only matching the tip-speed ratio is pursued during wind-tunnel measurements for wind turbines.

The wind-tunnel test section can be either a closed (enclosed by walls) or an open (no walls present) section (Pope & Rae 1999). Open test sections have less interference from the limited wind-tunnel size, because the air is not bounded by the solid walls. Therefore, the impact of wall effects on the measurements is limited. According to Glauert (1936), open test sections can handle wind turbines up to 60-70% of the test-section area. In a closed section on the other hand, the presence of walls alters the flow field around the wind turbine inside the test section compared to the flow field around a wind turbine in unconfined conditions. The size of the wind turbines must therefore be reduced more for measurements in closed test sections, given the same wind-tunnel size.

The presence of walls in closed test sections has an impact on the flow field around the wind turbine (Chen & Liou 2011). The wake behind the wind turbine is not allowed to expand in a similar way as it would do in unconfined conditions. The flow between the wall and the wake will also alter its characteristics. In unconfined conditions, this flow will have the same properties as the incoming free-stream flow. In the wind tunnel, the flow is funneled in an area with a decreasing cross-section size. This will accelerate the flow and is clearly not the same as in the unconfined case. This accelerating flow will have an impact on the performance of the wind turbine during measurements and this phenomenon is called blockage. Blockage correction factors are often used to correct for these mismatches. However, for highly loaded rotors, these corrections are often not accurate enough (Werle 2009).

1.4. Why are improved blockage corrections necessary?

Blockage correction schemes have been introduced in the past. One of the first ones who proposed a blockage correction scheme was Glauert (1936) and his scheme was initially derived for propellers. It was later realized that it could also be used for wind turbines, although it has severe limitations for wind turbines operating under high blockage ratios and heavily loaded conditions. Many years later Mikkelsen & Sørensen (2002) made successful modifications to this theory to avoid these limitations. Up to today, these blockage correction schemes have been widely used.

Other blockage correction schemes have been proposed, but are rarely adopted. Maskell (1963) developed a blockage correction scheme for bluff bodies in closed wind tunnels. This theory was the first to address blockage corrections for non-streamlined bodies, but it is neither designed for (nor easily adapted to) wind-turbine measurements. Ashill & Keating (1988) extended the work from Maskell who created a correction scheme which does not require modeling of flows in the tunnel. They used a two-component method in which the knowledge of two components of flow velocity at the outer boundary must be known. This method was also not derived with powered propeller or power
1.5. OBJECTIVES OF THE THESIS

extracting wind turbines and blockage effects in mind. Hensel (1951) proposed a method, based on the method used by Thom (1943), in which point and line doublets are used to model bodies of revolution, finite straight wings, and finite swept wings. Hackett et al. (1979) build off of Hensel's work: Here source and sink elements are used instead of doublets. During experiments, wall pressures were measured along the centerline of the tunnel walls or the roof, and on the floor when necessary. These indicative pressures were then used to determine source or sink strengths, spans and locations on the tunnel centerline which define a body outline to build up a model equivalent to the test model and its wake. These methods are also not specifically adapted to wind-turbine specific situations, but it is interesting to point out the use of point and line sources or doublets to model solid bodies. Finally, Fitzgerald (2007) proposed a model where propellers and the wind tunnel are modeled with source and sink elements or doublet elements: This model is specifically adapted to powered propellers and can be extended to wind turbines. Because of the crude propeller model, based on a single doublet element, it is not adopted as a general approach to determine the effect of blockage for a generic wind turbine during wind-tunnel measurements.

The correlation between the measured performance data from wind-tunnel measurements and the measured performance data of unconfined operating wind turbines are often not accurate enough. The blockage corrections that have been used specifically in wind-turbine measurements are derived based on the Actuator Disk Theory (Glauert 1936). This is however a crude theory based on assumptions that highly simplify the flow (Werle 2010) and neglect effects that might have an impact on the performance of the measured wind turbine. This results in discrepancies between the corrected measured wind-tunnel data and the data from the unconfined operating wind turbines (Werle 2009).

The availability of a new, fast and accurate numerical method for calculating the near-wake structure and the performance of a wind turbine (Segalini & Alfredsson 2013), provides the opportunity to build a numerical model of the wind turbine in a wind tunnel by extending the model with test-section walls. It is indeed possible to simulate the effect of the test-section walls on the near-wake structure and the performance of the wind turbine. The results can be compared with the near-wake structure and the performance of the same wind turbine operating under unconfined conditions, resulting in a relation between the two situations.

1.5. Objectives of the thesis

The main objectives of the thesis work are twofold. The first main objective is to create a fast numerical model, based on the Simplified Vortex Method (Segalini & Alfredsson 2013), where the effect of the presence of test-section walls is simulated. This vortex method is modeled assuming irrotational flow
outside the vortex filaments characterizing the wake. The presence of the test-
section walls will therefore be modeled using potential flow characteristics. The
walls will be modeled using source panels, enforcing the numerical boundary
condition that represents the solid wind-tunnel wall at a certain location in
space. The main goal is to be able to compare performance data of a wind
turbine operating in unconfined conditions with performance data of the same
wind turbine operating in a closed test-section configuration. Both cases need
to be calculated from the model.

The second main objective is to postulate a new empirical correlation
scheme to account for blockage during wind-tunnel measurements of any generic
wind turbine. Based on the proposed simulation model, several simulations will
be calculated on a dedicated machine. The results of these simulations will be
used to derive a new empirical correlation scheme where the least possible
amount of measurement data is needed to estimate the performance of the
same wind turbine in unconfined operating conditions.

To obtain these main objectives the thesis is built up in several chapters,
each describing an important aspect of the content. In chapter 2 an overview of
the theoretical background and the most important existing blockage correction
schemes is provided. In chapter 3 the implementation of the proposed wind-
tunnel model is described together with an example of the calculation process.
In chapter 4 a general blockage correction scheme is proposed and validated,
based on data collected from numerous simulation cases of the wind-tunnel
model.
CHAPTER 2

Theoretical background

2.1. Governing equations

The Navier-Stokes equations can describe the behavior of several fluids (including air and water). These equations are strongly coupled and non-linear in nature and therefore very difficult and expensive to solve. Depending on the specific fluid and flow type, assumptions can be made so that the equations can be simplified, making them solvable or providing physical insight.

For a steady-state, incompressible, inviscid and irrotational flow some simplifications can be made. Steady-state means that all time-dependent terms vanish \( \frac{\partial}{\partial t} = 0 \). The incompressibility of the fluid results in a constant fluid density \( \rho = \text{constant} \) while the inviscid character of the fluid makes all viscosity-related forces negligible. Irrotationality of the fluid implies that the fluid does not rotate on a macroscopic level, due to shear stresses \( \nabla \times \mathbf{u} = 0 \). Since gravity and other body forces are expected to be negligible, these terms are can also be omitted. These simplifications give rise to the Euler equations in integral form

\[
\int_{\Sigma} (\mathbf{u} \cdot \mathbf{n}) d\Sigma = 0, \tag{2.1}
\]
\[
\int_{\Sigma} u_i (\mathbf{u} \cdot \mathbf{n}) d\Sigma = -\int_{\Sigma} p d\Sigma, \tag{2.2}
\]

while the differential form can be expressed as

\[
\nabla \cdot \mathbf{u} = 0, \tag{2.3}
\]
\[
\mathbf{u} \cdot \nabla \mathbf{u} = -\frac{\nabla p}{\rho}. \tag{2.4}
\]

Following Anderson (2007), the following identity is valid for any scalar function \( \Phi \)

\[
\nabla \times (\nabla \Phi) = 0. \tag{2.5}
\]

Therefore, in the case of irrotational flows, the velocity can be expressed as the gradient of that scalar function \( \Phi \)

\[
\mathbf{u} = \nabla \Phi. \tag{2.6}
\]
2. THEORETICAL BACKGROUND

This scalar function is called the velocity potential. Using equation 2.6, the continuity equation (eq. 2.3) can be rewritten as

\[ \nabla \cdot \nabla \Phi = \nabla^2 \Phi = 0. \tag{2.7} \]

This equation is called the Laplace equation for steady state, incompressible, inviscid and irrotational flow. Based on the solution of the velocity potential of the Laplace equation, the velocity field of a steady state, incompressible, inviscid and irrotational flow can be calculated.

Note that the Laplace equation is a linear differential equation. This implies that, if \( \{\Phi_1, \Phi_2, ..., \Phi_n\} \) is a set of \( n \) solutions of the Laplace equation, then

\[ \Phi = \sum_{k=1}^{n} c_k \Phi_k, \tag{2.8} \]

with \( c_k \) arbitrary constants, is also a solution of the Laplace equation, namely

\[ \nabla^2 \Phi = \sum_{k=1}^{n} c_k \nabla^2 \Phi_k = 0. \tag{2.9} \]

The linearity of the Laplace equation is a very important property. After obtaining elementary solutions for either the velocity or velocity potential, the solution that satisfies the boundary condition can be reduced to searching for the right linear combination and the solving of a set of algebraic equations.

However, solving the Laplace equation tells nothing about the pressures. This can be done by applying the Bernoulli equation

\[ \frac{u^2}{2} + \frac{p}{\rho} = \text{const.} \tag{2.10} \]

Based on the velocities, calculated from the solution of the Laplace equation, the corresponding pressure can be indeed calculated.

2.2. The panel method

2.2.1. Point source

One possible solution of equation 2.7 is a point source/sink element. The potential of a point-source element, placed in the origin of a spherical coordinate system can be expressed as

\[ \Phi(r) = -\frac{\sigma}{4\pi r}, \tag{2.11} \]

where \( \Phi \) is the velocity potential, \( r \) the distance from the point of interest to the source element and \( \sigma \) the point-source strength expressing a certain amount of volumetric flow. The velocity field, accompanying this velocity potential is purely radial and can be found by taking the derivative in the radial direction of this velocity potential, as

\[ v = \nabla \Phi = -\frac{\sigma}{4\pi} \nabla \left( \frac{1}{r} \right) = \frac{\sigma}{4\pi} \frac{r}{r^3}, \tag{2.12} \]
or in spherical velocity components
\[ (v_r, v_\theta, v_\phi) = \left( \frac{\sigma}{4\pi r^2}, 0, 0 \right). \] (2.13)

The velocity field decreases indeed at a rate of \( r^{-2} \) and has a singularity at the position of the source. If \( \sigma > 0 \), a source is represented and if \( \sigma < 0 \) a sink is represented.

If the source element is not located at the origin but at a point \( r_0 \), the expressions for the velocity potential and the velocity components need to be changed. In the general form they become
\[ \Phi = -\frac{\sigma}{4\pi |r - r_0|}, \quad v = \frac{\sigma}{4\pi |r - r_0|^2} r. \] (2.14)

This basic point-source element principle can be extended to line, surface and volume source elements with variable source-strength distributions (Katz & Plotkin 1991). The expression of the velocity potential for a line, surface and volume in Cartesian form are, respectively
\[ \Phi(x, y, z) = -\frac{1}{4\pi} \int_{l} \frac{\sigma(x_0, y_0, z_0)dl}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}}, \] (2.15)
\[ \Phi(x, y, z) = -\frac{1}{4\pi} \int_{S} \frac{\sigma(x_0, y_0, z_0)dS}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}}, \] (2.16)
\[ \Phi(x, y, z) = -\frac{1}{4\pi} \int_{V} \frac{\sigma(x_0, y_0, z_0)dV}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}}. \] (2.17)

2.2.2. Ring-shaped source panel

Given a cylindrical tunnel shape and an axisymmetric distribution of the radial induced velocity, a particular source panel can be used. This panel is ring-shaped and has a constant source-strength distribution along its entire surface.

Because of the axialsymmetry of the ring, it is possible to reduce the number of velocity components. For the specific case, only two induced velocities need to be calculated, i.e. an axial induced velocity and a radial induced velocity. For every point in space, the reference frame of the ring can be oriented so that there will only be a radial and axial velocity component. This can be done because the ring has an axisymmetric source distribution.

The derivation of the analytical expressions for these velocities is performed in Appendix C. The parameters used to calculate the axial and radial induced velocities are the source-strength distribution, \( \sigma \), the geometry of the ring (i.e. its radius \( R \) and the axial length \( 2\delta \)), the axial location, \( z \), of the point in space and the radial location, \( \rho \), of the point in space. This is shown in Figure 2.1.

The velocity potential can be calculated from
\[ \Phi = -\frac{\sigma R}{4\pi} \int_{0}^{2\pi} \left[ \sinh^{-1} \left( \frac{\delta - z}{G} \right) - \sinh^{-1} \left( \frac{-\delta - z}{G} \right) \right] d\theta. \] (2.18)
2. THEORETICAL BACKGROUND

The axial velocity can be calculated by

\[ u_y = \frac{\sigma R}{\pi} [F_1 - F_2] \]  \hfill (2.19)

The radial velocity can be calculated using the following formula

\[ u_\rho = \frac{\sigma R}{2\pi \rho} \left[ (\delta - z)F_1 + (\delta + z)F_2 + \frac{\rho^2 - R^2}{(R + \rho)^2} [(\delta - z)F_3 + (\delta + z)F_4] \right] \]  \hfill (2.20)

where

\[ F_1 = \frac{1}{\sqrt{(R+\rho)^2 + (\delta - z)^2}} \tilde{K} \left( \sqrt{\frac{4R\rho}{(R+\rho)^2 + (\delta - z)^2}} \right), \]  \hfill (2.21)

\[ F_2 = \frac{1}{\sqrt{(R+\rho)^2 + (\delta + z)^2}} \tilde{K} \left( \sqrt{\frac{4R\rho}{(R+\rho)^2 + (\delta + z)^2}} \right), \]  \hfill (2.22)

\[ F_3 = \frac{1}{\sqrt{(R+\rho)^2 + (\delta - z)^2}} \Pi \left( \frac{4R\rho}{(R+\rho)^2}, \frac{4R\rho}{(R+\rho)^2 + (\delta - z)^2} \right), \]  \hfill (2.23)

\[ F_4 = \frac{1}{\sqrt{(R+\rho)^2 + (\delta + z)^2}} \Pi \left( \frac{4R\rho}{(R+\rho)^2}, \frac{4R\rho}{(R+\rho)^2 + (\delta + z)^2} \right), \]  \hfill (2.24)

with \( \tilde{K}(\alpha) \) the complete elliptic integral of the first kind with argument \( \alpha \) and \( \Pi(\alpha, \beta) \) the complete elliptic integral of the third kind with arguments \( \alpha \) and \( \beta \). The values of \( \tilde{K}(\alpha) \) and \( \Pi(\alpha, \beta) \) can be found in tables or in integrated software functions.

The resulting velocity field is shown in Figure 2.2. It is clear that the velocity field is symmetric, both axially and radially. Close to the ring panel,
2.2. THE PANEL METHOD

the radial induced velocity is greatest and is pointed inwards. Near the ring axis the radial induced velocity decreases to zero, due to symmetry. The axial velocity increases initially when the point of interest moves away from the ring while, after some distance, it decreases again.

2.2.3. Wall modeling

To model a solid wall, the wall boundary condition must be satisfied. This condition states that the normal velocity at the wall location should be zero. Due to the linear characteristic of the potential flow, the total velocity \( v_{\text{tot}}(x, y, z) \) at some point \( P \equiv (x, y, z) \) is the sum of all induced velocities, caused by perturbations in the flow. These induced velocities can be divided into two main groups. The first represents the induced velocities of all source panels. The second one represents all other external induced velocities.

In order to enforce the wall boundary conditions, the source strength of each individual source must be calculated. To do this for \( n \) sources, exactly \( n \) control points at the wall are needed. In these control points the boundary condition of zero normal velocity must be enforced. In all other points of the wall, the boundary condition cannot be enforced. Hence, the more panels representing the wall are used (more points in which the wall condition is enforced) and the better the solid wall is modeled. For each control point \( P \),
given a constant source-strength distribution $\sigma$, the boundary condition can be written as
\[
\sum_{i=1}^{n} \sigma_i \vec{V}_i(P) \cdot \vec{n}(P) + \vec{V}_{\text{ext}}(P) \cdot \vec{n}(P) = 0,
\]
with $n$ the total number of sources, $\sigma_i$ the source strength of source $i$, $\vec{V}_i(P)$ the velocity component induced by the source $i$ with source strength $\sigma = 1$ in point $P$, $\vec{n}(P)$ the normal direction at point $P$ and $\vec{V}_{\text{ext}}(P)$ the external induced velocity at point $P$.

This equation is valid for all $n$ control points, which leads to $n$ independent equations in $n$ variables ($\sigma_i$) and can be written in matrix form as
\[
V_{\text{pan}} \Sigma = -V_{\text{ext}},
\]
where the matrix $\Sigma$ is the unknown source distributions. This set of equations can be solved and leads to $\sigma$-values for all sources that will enforce the solid wall boundary condition.

2.3. The vortex model

2.3.1. Vortex quantities
The vorticity vector can be calculated from the curl of the velocity field as
\[
\xi = \nabla \times \vec{u}.
\]
If $\xi = 0$ at every point in the fluid, the flow is called irrotational, to indicate that the motion of each fluid particle through space is a pure translation. If $\xi \neq 0$ at some points in the fluid, the flow is called rotational and the motion of the fluid has a finite angular velocity in these points. Vortex lines are field lines that are parallel to the vorticity vector and are described by
\[
\xi \times d\ell = 0.
\]
Vortex lines that pass through a closed curve in space form a vortex tube. When the cross-sectional area of such a vortex tube is infinitesimally small, it is called a vortex filament. For inviscid flows, it can be proven that the circulation around a vortex filament is constant
\[
\Gamma_{\text{vf}} = \int \xi dS = \text{constant}
\]
Based on this property, Helmholtz derived his vortex theorems:
1. The strength of a vortex filament is constant along its length.
2. A vortex filament cannot start or end in a fluid. It must form a closed path or extend to infinity.
3. The fluid that forms a vortex filament continues to form a vortex filament and the strength of the vortex filament remains constant as the filament moves about.
that are equivalent to Kelvin’s theorem, that simply states that the circulation around any closed curve moving with the fluid remains constant with time. This theorem basically states that if a closed contour is observed at one instant and that closed contour is followed over time, the circulation over the two locations of this contour are equal.

### 2.3.2. Biot-Savart’s law

The velocity field may be expressed as the curl of a vector field \( \mathbf{B} \)

\[
\mathbf{u} = \nabla \times \mathbf{B},
\]

and \( \mathbf{B} \) can be selected in such a way that

\[
\nabla \cdot \mathbf{B} = 0.
\]

The vorticity can be rewritten in terms of this vector field, after some vector manipulation this becomes

\[
\xi = \nabla \times \mathbf{u} = \nabla \times (\nabla \times \mathbf{B}) = \nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} = -\nabla^2 \mathbf{B}.
\]

Using Green’s theorem, the vector field \( \mathbf{B} \) in a point \( \mathbf{P} \), at a distance \( r \) from the origin and at a distance \( |\mathbf{r}_0 - \mathbf{r}_1| \) from the generic vorticity source, is

\[
\mathbf{B}(r_0) = \frac{1}{4\pi} \int_V \frac{\xi(r_1)}{|\mathbf{r}_0 - \mathbf{r}_1|} dV,
\]

and the induced velocity is

\[
\mathbf{u}(r_0) = \frac{1}{4\pi} \int_V \nabla \times \frac{\xi(r_1)}{|\mathbf{r}_0 - \mathbf{r}_1|} dV.
\]

When the vorticity source is only located in a vortex filament with a cross section area \( dS \) selected so that it is normal to \( \xi \) and with the direction of the filament

\[
dl = \frac{\xi}{\xi} dl,
\]

and outside this vortex filament there is no vorticity, given the fact the circulation is constant for a vortex filament

\[
\Gamma = \xi dS,
\]

the induced velocity can be rewritten after executing the curl operation and for an infinitesimal volume \( dV = dS dl \), resulting in

\[
\mathbf{u}(r_0) = \frac{\Gamma}{4\pi} \int \frac{dl \times (\mathbf{r}_0 - \mathbf{r}_1)}{|\mathbf{r}_0 - \mathbf{r}_1|^3}.
\]

This equation is called the Biot-Savart law for inviscid flows. Given the local vorticity distribution of a vortex filament, Equation 2.37 allows for the calculation of the induced velocity in a point in space due to the presence of that vortex filament.
2.3.3. **Simplified vortex model for wind turbine wakes**

The vortex model is a simplified model based on vortices to simulate inviscid wind turbine wakes, originally proposed by Joukowsky (1912) and extended by Segalini & Alfredsson (2013). The model assumes a constant circulation along the blade, from blade tip to blade root, as shown in Figure 2.3. This approximation means that the modeled wind turbine operates at maximum power production condition for a given set of operating parameters. The model calculates the tip-vortex path iteratively, allowing for a wake expansion or contraction. Based on the tip-vortex path, the induced velocity in the rest of the domain can be calculated.

2.3.4. **Description**

The wind turbine is modeled with \( N_b \) line vortices with a constant circulation \( \Gamma^* \) along the blade location. The line vortex represents a vortex tube with a constant local radius \( \delta^* \). According to Helmholtz’s theorem, vortex lines cannot begin or end inside the domain. They must either form a closed loop or extend to infinity. The bound vortex at the blade must therefore also extend to infinity, since it does not form a closed loop. The bound vortex will extend from the blade root to infinity in a straight line while the bound vortex from the blade tip will extend to infinity as a helical line.

To account for the wind-turbine rotation, a rotating reference frame is introduced, according to Figure 2.3. The \( x \)-axis is aligned with the first blade,
2.3. THE VORTEX MODEL

the z-axis is oriented normal to the rotating plane of the turbine and the y-axis is chosen normal to both the x- and z-axis so a right-handed axis frame is created. The origin of the reference frame is chosen to be located at the wind-turbine hub. The free-stream velocity is oriented along the z-axis so that \( U_\infty = U_\infty^* e_3 \) (with \( e_3 = [0, 0, 1] \)). The angular velocity of the rotor is also aligned with the z-axis, \( \Omega = \Omega^* e_3 \)

The dimensional quantities are scaled with the density \( \rho^* \), the free-stream velocity \( U^*_\infty \) and the rotor radius \( R^* \) in order to make them dimensionless, \( \lambda = \frac{\Omega^* R^*}{U^*_\infty}, \Gamma = \frac{\Gamma^*}{U^*_\infty R^*}, \delta = \frac{\delta^*}{R^*}, \mathbf{x} = \frac{\mathbf{x}^*}{R^*} \)

with \( \lambda \) the tip-speed ratio and \( \mathbf{x} \) the dimensionless vector \([x, y, z] \).

The root-vortex path is assumed to be straight, described by \( \gamma_{\text{root}} = z e_3 \), while the tip-vortex path is assumed to be helix shaped with a variable helix radius and pitch angle. For the first blade, this vortex path can be described by

\[
\gamma_{\text{tip}} = r_W(z) \Psi + z e_3 ,
\]

with
\[
\Psi = [\cos(\phi_W(z)), \sin(\phi_W(z)), 0] ,
\]
\[
\tilde{\Psi} = [-\sin(\phi_W(z)), \cos(\phi_W(z)), 0] .
\]

These are orthogonal unitary vectors. Equations 2.39, 2.40 and 2.41 allow for the description of the path of the tip vortex in a rotating reference frame. Due to flow symmetry, the tip-vortex paths of the other blades have the same radial function \( r_W(z) \). The angular distribution function \( \phi_W(z) \) is shifted for the blade, according to the blade number \( i \)

\[
\phi_{W,i}(z) = \phi_{W,1}(z) + \frac{2\pi(i - 1)}{N_b} .
\]

The blade is modeled as a line vortex, described mathematically by \( \gamma_{\text{blade},i} = \mu[\cos(\alpha_i), \sin(\alpha_i), 0] \) where \( \alpha_i = \phi_{W,i}(0) \) and \( \mu = (x^2 + y^2)^{1/2} \leq 1 \) is the normalized radial location at the rotor disk.

To compute the induced velocity due to the presence of the vortex lines, the Biot-Savart law is applied (section 2.3.2). At this point, the tip vortex radial function \( r_W(z) \) and angular function \( \phi_W(z) \) are still unknown. These can be determined by means of the Helmholtz theorems. The velocity of the tip vortex system can be derived by adding the two contributions: the free-stream contribution \( U_\infty^* e_3 \) and the velocity induced by the vortex system itself \( \mathbf{u}_f \).

In a rotating reference frame the streamlines are steady state, the following kinematic equation holds

\[
[u_f(\gamma_{\text{tip}}) + \mathbf{u}_{\text{ext}} - \lambda e_3 \times \gamma_{\text{tip}}] \times \frac{d\gamma_{\text{tip}}}{dz} = 0 ,
\]

with \( \mathbf{u}_f \) the free-stream velocity, \( \mathbf{u}_{\text{ext}} \) the external velocity, \( \lambda \) the tip-speed ratio, \( \gamma_{\text{tip}} \) the tip vortex path, and \( \frac{d\gamma_{\text{tip}}}{dz} \) the derivative of the tip vortex path with respect to the axial coordinate.
with the first two terms expressing the normalized vortex contribution and an externally induced velocity (normally only the free-stream velocity), respectively, and the third term expresses the change to a rotating reference frame. This equation states that the vortex path is a streamline of the flow field. Given

\[
\frac{d\gamma_{\text{tip}}}{dz} = \frac{dr_W}{dz} \Psi + r_W \frac{d\phi_W}{dz} \Psi + e_3, \quad (2.44)
\]

this equation can be rearranged to obtain two ordinary differential equations for \(r_W(z)\) and \(\phi_W(z)\)

\[
\frac{dr_W}{dz} = - \left( \left( u_f + u_{\text{ext}} \right) \times e_3 \right) \cdot \tilde{\Psi} + \frac{\left( u_f \times \Psi \right) \cdot \Psi}{1 + \left( u_f \times \Psi \right) \cdot \Psi}, \quad (2.45)
\]

\[
\frac{r_W d\phi_W}{dz} = \lambda r_W - \frac{\left( \left( u_f + u_{\text{ext}} \right) \times e_3 \right) \cdot \left( u_f \times \Psi \right) \cdot \Psi - 1}{\left( u_f \times \Psi \right) \cdot \Psi - 1}. \quad (2.46)
\]

Since solving this set of equations needs information about \(u_f(\gamma_{\text{tip}})\) itself, an iterative scheme is needed.

### 2.3.5. Wind turbine performance parameters

The thrust and power coefficient can be calculated by integration over the normalized blade length \(\mu\) of the following expressions

\[
dC_T = \frac{2 \lambda N_b \Gamma}{\pi} \mu (1 + a_b(\mu)) d\mu, \quad (2.47)
\]

\[
dC_P = \frac{2 \lambda N_b \Gamma}{\pi} \mu (1 - a_b(\mu)) d\mu, \quad (2.48)
\]

with \(a_b^\prime\) the tangential induction factor at the blade and \(a_b\) the axial induction factor at the blade. These induction factors can be calculated by using the projected induced velocity at the blade in the direction of interest, i.e.

\[
a_b(\mu) = 1 - \frac{u_{f,z}(\mu)}{U_\infty^*}, \quad (2.49)
\]

\[
a_b^\prime(\mu) = \frac{u_{f,\theta}(\mu)}{\lambda \mu U_\infty^*}, \quad (2.50)
\]

where \(u_{f,z}\) is the induced velocity from the turbine (namely from the tip vortex, blade vortex and root vortex) in the axial direction, \(u_{f,\theta}\) the induced velocity from the turbine in the azimuthal direction and \(U_\infty^*\) the magnitude of the free-stream velocity.

### 2.4. Glauert’s wind tunnel interference model

Glauert’s wind tunnel interference model is based on a similar approach as the actuator disk theory (Glauert 1936). It also uses a simplified model of a wind turbine in a one-dimensional flow. The wind tunnel is represented as an ideal porous disk extracting power from the flow. The wind tunnel and wind
turbine configuration is shown in Figure 2.4. During the analysis, the following assumptions are made:

- The flow is in steady state, radially homogeneous, incompressible and irrotational.
- There is no frictional drag associated with the turbine disk.
- The actuator disk represents a turbine with an infinite number of blades.
- There is a uniform distributed thrust force acting on the disk.
- The flow field in the wake is non-rotating.

Now consider a control volume around the wind turbine and the wind tunnel as shown in Figure 2.4. The power extracting stream-tube around the wind turbine acts as a boundary for two smaller control volumes: the stream-tube control volume ($CV_1$) and the wind tunnel control volume ($CV_2$).

At the inlet, the flow enters the control volume with velocity $U_\infty$ and pressure $p_\infty$. The cross-area of the stream-tube control volume at the inlet is $A_\infty$ and the total cross-area of the whole control volume is $C$. The fluid flows through the disk with a cross-area $A_d$ at a velocity $U_d$, exerting a force $T$ on the turbine and extracting a power $P$ from the flow. The pressure just upstream of the disk is $p_u$ and the pressure just downstream of the disk is $p_d$. At the outlet, the flow inside the stream-tube control volume leaves the control volume at a lower velocity $U_W$ and a pressure $p_W$. The cross-sectional area of the stream-tube control volume at the outlet is $A_W$ and the total cross-sectional area of the whole control volume is $C$. The flow inside the wind-tunnel control volume leaves the control volume with a velocity $U_2$ and a pressure $p_2$. In the far-wake region, this pressure must be equal to the pressure in the stream-tube control volume, thus $p_2 = p_W$ but might be different from the free-stream pressure $p_\infty$.

For the quantitative analysis of the Glauert wind-tunnel interference model, the thrust force acting on the wind turbine, the power the wind turbine extracts from the flow and the effect of the rotor operation on the local flow field are of interest. The full derivation is shown in Appendix A.
The thrust force can be calculated in two ways. The first way is to determine it based on mass and momentum balance. This approach leads to the expression

\[ T = \rho A_d U_\infty^2 \alpha (1 - \beta) \left[ 1 + \frac{\epsilon \alpha}{2 \beta^2} (1 + \beta) \right] + O(\epsilon^2), \tag{2.51} \]

where \( \alpha = U_d / U_\infty \), \( \beta = U_W / U_\infty \) and \( \epsilon = A_d / C \). This expression consists of two main terms, neglecting the higher order terms of \( \epsilon \). The first term is independent of \( \epsilon \) and can thus be regarded as the unconfined thrust force. The second term depends linearly on \( \epsilon \) and for higher blockage ratios \( \epsilon \), this term will become more important.

The second way to determine the thrust force is based on the pressure across the actuator disk. This approach leads to a different expression

\[ T = \frac{\rho A_d U_\infty^2}{2} \left( 1 - \beta \right) \left( 1 + \beta + 2 \epsilon \alpha / \beta \right) + O(\epsilon^2), \tag{2.52} \]

This expression also consists of two main terms. Again, the first term is independent of \( \epsilon \) and can thus be regarded as the unconfined thrust force. The second term depends linearly on \( \epsilon \) and for higher blockage ratios \( \epsilon \), this term will become more important.

Equations 2.51 and 2.52 can both be used to determine the thrust force, but they still depend on two unknowns, i.e. \( \alpha \) and \( \beta \) that are related to each other. It is however possible to relate \( \alpha \), \( \epsilon \) and \( \beta \) in order to eliminate \( \beta \) from the thrust equation. The expression of \( \beta \) in terms of \( \alpha \) becomes

\[ \beta = (2 \alpha - 1) + 2 \epsilon \alpha (1 - \alpha) \frac{(1 - \alpha)^2}{(2 \alpha - 1)^2} + O(\epsilon^2). \tag{2.53} \]

Using this expression for \( \beta \) a new equation for the thrust yields

\[ T = 2 \rho A_d U_\infty^3 \alpha (1 - \alpha) \left[ 1 + \epsilon \frac{\alpha}{2 \alpha - 1} \right] + O(\epsilon^2). \tag{2.54} \]

This result can be found from both previous expressions for the thrust force. The expression for the extracted power thus becomes

\[ P = U_d \, T = 2 \rho A_d U_\infty^3 \alpha^2 (1 - \alpha) \left[ 1 + \epsilon \frac{\alpha}{2 \alpha - 1} \right] + O(\epsilon^2). \tag{2.55} \]

The thrust and power coefficients, generally defined as

\[ C_T = \frac{2T}{\rho A_d U_\infty^2}, \tag{2.56} \]

\[ C_P = \frac{2P}{\rho A_d U_\infty^3}, \tag{2.57} \]
2.4. GLAUERT’S WIND TUNNEL INTERFERENCE MODEL

These equations state that the impact of the tunnel presence, having a blockage ratio $\epsilon$, can be estimated as the unconfined coefficient plus an extra factor depending on the blockage ratio $\epsilon$.

2.4.1. The equivalent free-stream velocity

The determination of an equivalent free-stream velocity is a standard way of determining the impact of wind tunnel wall interference. The equivalent free-stream velocity will give the same thrust and disk velocity (and thus power) in unconfined conditions as the wind turbine in confined wind-tunnel conditions. The equivalent free-stream velocity $U_\infty'$ must therefore satisfy the condition

$$T = 2\rho A_d U_d (U_\infty' - U_d).$$  \hspace{1cm} (2.60)

$U_\infty'$ is related to $U_\infty$, since the thrust must be the same to equation 2.54 (Appendix A). This yields

$$\frac{U_\infty'}{U_\infty} = 1 + \epsilon \frac{\alpha(1 - \alpha)}{2\alpha - 1} + O(\epsilon^2),$$ \hspace{1cm} (2.61)

or based on the thrust coefficient, in unconfined conditions normally defined as $C_T = 4\alpha(1 - \alpha)$,

$$\frac{U_\infty'}{U_\infty} = 1 + \epsilon \frac{C_T}{4\sqrt{1 - C_T}} + O(\epsilon^2).$$ \hspace{1cm} (2.62)

It can be directly pointed out that for measured thrust coefficients approaching 1, the equivalent free-stream velocity approaches a singularity as $C_T \rightarrow 1$. For measured thrust coefficients higher than 1, this approach becomes useless, as visible in Figure 2.5. This is the main drawback of the Glauert model and this situation needs to be avoided when applying Glauert’s method. This usually results in small wind-turbine models in large wind tunnels.

With the equivalent free-stream velocity now calculated, it is possible to calculate the corrected thrust and power coefficients. They can be calculated by

$$C'_T = C_T \left(\frac{U_\infty}{U_\infty'}\right)^2,$$ \hspace{1cm} (2.63)

$$C'_P = C_P \left(\frac{U_\infty}{U_\infty'}\right)^3.$$ \hspace{1cm} (2.64)
Correcting the thrust and power coefficients can thus be done by simply rescaling them with a second or third power of the inverse of the equivalent free-stream velocity. This results in a simple correction scheme, easily implemented once the measured values and the wind-tunnel geometry is known.

2.5. Mikkelsen & Sørensen blockage model

Mikkelsen & Sørensen (2002) also derived a wind-tunnel blockage model based on the actuator disk theory, similar to the Glauert wall interference model. The analysis starts with the formulation of a set of five equations derived from the control volume analysis. These equations are two mass conservation equations, an expression for the thrust force, an equation for the pressure difference in the power extracting stream-tube and the global momentum balance on the whole tunnel configuration.

\[ U_W A_W = U_d A_d, \quad (2.65) \]
\[ U_2(C - A_W) = U_\infty C - U_d A_d, \quad (2.66) \]
\[ T = \frac{\rho A_d}{2}(U_W^2 - U_2^2), \quad (2.67) \]
\[ \Delta p = p_W - p_\infty = \frac{\rho}{2}(U_W^2 - U_2^2), \quad (2.68) \]
\[ T - \Delta p C = \rho U_W A_W(U_W - U_\infty) - \rho U_2(C - A_W)(U_\infty - U_2). \quad (2.69) \]
These equations represent a set of 5 equations in 5 variables. Using the dimensionless parameters this set of equations can be rewritten as:

\[ \beta \sigma = \alpha, \]  
(2.70) \[ \gamma (1 - \epsilon \sigma) = 1 - \epsilon \alpha, \]  
(2.71) \[ C_T = \beta^2 - \gamma^2, \]  
(2.72) \[ C_p = 1 - \gamma^2, \]  
(2.73) \[ \epsilon C_T - C_p = 2\beta \sigma \epsilon (\beta - 1) - 2\gamma (1 - \epsilon \sigma)(1 - \gamma). \]  
(2.74)

In the Glauert approach, these equations were assumed to behave linearly for small blockage ratios \( \epsilon \). Mikkelsen & Sørensen (2002) did not assume this and solved this set of equations without linearization. They reported the solution in an explicit expression for \( \alpha \) in terms of \( \epsilon \) and \( \sigma \) (the wake expansion coefficient):

\[ \alpha = \frac{\sigma (\epsilon \sigma^2 - 1)}{\sigma \epsilon (3 \sigma - 2) - 2 \sigma + 1}, \]  
(2.75) and the thrust and power coefficients become

\[ C_T = \left( \frac{\alpha}{\sigma} \right)^2 - \left( \frac{1 - \epsilon \alpha}{1 - \epsilon \sigma} \right)^2, \]  
(2.76) \[ C_p = \alpha C_T = \alpha \left[ \left( \frac{\alpha}{\sigma} \right)^2 - \left( \frac{1 - \epsilon \alpha}{1 - \epsilon \sigma} \right)^2 \right]. \]  
(2.77)

The equivalent free-stream velocity, based on the same conditions as the Glauert approach, can be found by equalizing the thrust coefficient to the equivalent free-stream thrust coefficient

\[ C_T = 4 \alpha \left( \frac{U'}{U}\right), \]  
(2.78) resulting in

\[ \frac{U'}{U} = \alpha - \frac{1}{4} \frac{C_T}{\alpha}. \]  
(2.79) The calculation of the equivalent free-stream is thus performed based on 3 parameters that can be measured in wind-tunnel experiments. The \( \alpha \)-value depends on both the wake expansion coefficient \( \sigma \) and the blockage ratio \( \epsilon \). The third parameter is the measured thrust coefficient. It is directly clear that there is no singularity when the thrust coefficient goes to 1 or exceeds it. This effect can be seen in Figure 2.6. It is the main advantage of this approach, even though it requires one extra parameter to be measured, i.e. the wake expansion coefficient.

To estimate the corrected thrust and power coefficients, equation 2.79 can be applied in the expressions for the corrected thrust and power coefficients stated in the Glauert model (Eq. 2.63 and 2.64).
2. THEORETICAL BACKGROUND

Figure 2.6: Results of the Mikkelsen & Sørensen correction on the corrected free-stream velocity. **Solid line**: Blockage ratio of 1%. **Dashed line**: Blockage ratio of 9%. **Dashed-dotted line**: Blockage ratio of 16%. **Dotted line**: Blockage ratio of 25%.
CHAPTER 3

Wind tunnel model for wind turbines

3.1. General model concept

The general concept of the wind tunnel model for wind turbines is based on the simplified vortex model (see section 2.3) to represent the wind turbine and a panel method (see section 2.2) to model the wind-tunnel walls.

Since the simplified vortex model operates in a rotating reference frame, the test section is chosen to be cylindrical. To enforce the wall boundary condition, the method proposed in section 2.2.3 is used, where $\vec{V}_{\text{ext}}$ consists in general of three main contributions. The first contribution is the induced velocity from the wind turbine. The second contribution is the free-stream velocity impact, normal to the wall location. Since this velocity is always located along the wind tunnel and wind turbine axis, the free-stream velocity is parallel to the wall and no normal component is present. The third contribution is the velocity induced by the rotating reference frame. The latter two contributions do not have a component normal to the wind-tunnel wall for a cylindrical test section.

The wind tunnel will introduce an additional externally induced velocity term in equation 2.43. This means that

$$u_{\text{ext}} = u_t(\gamma_{\text{tip}}) + U_{\infty} e_3,$$

where

$$u_t(\gamma_{\text{tip}}) = \sum_{j=1}^{\text{panels}} (\sigma_j U_j(\gamma_{\text{tip}})).$$

This is the induced velocity that all panels exert on the wind-turbine wake.

Since the boundary condition at the wall is of zero normal velocity, the induced velocity from the wind turbine on the test-section walls needs to be neutralized by the sources. The sources now also induce a certain velocity field throughout the fluid which is felt by the wind-turbine wake too. This newly induced velocity will have an impact on the location and shape of the wake and thus the induced velocity by the wind turbine on the test-section walls. In turn, the source strength distribution has to be adapted to the new wind turbine induced velocity. It is clear that this requires an iterative scheme. The general iterative scheme, used to solve this problem, is represented in Figure 3.1. The main steps in this iterative scheme are
1. **Initialization**: In the initialization step the wind-tunnel geometry is set up, the wind-turbine operating parameters are determined and an initial wake geometry is initialized. This initial wake geometry is taken to be the wake geometry of a wind turbine operating under unconfined conditions. The initial source strength of the individual panels is set to be zero.

2. **Wind-turbine impact**: The wind-turbine wake is calculated, based on the wind-turbine operating parameters, the source-strength distribution of the source panels and its induced velocity. When the wind-turbine wake is calculated, the induced velocity in the test-section wall control points is calculated.

3. **Wind-tunnel impact**: The source-strength distribution of the test-section source panels is calculated, based on the induced velocity from the wind turbine and the wind-tunnel geometry and the source-panel discretization.

4. **Check convergence**: The convergence criterion used to assess the convergence of the iterative scheme is based on the source-strength distribution. If the relative difference of the absolute value between the previous and the current source-strength distribution is smaller than a certain threshold, the iterative scheme is assumed to be converged. If this is not the case, the iteration is started again with the newly calculated source-strength distribution and newly calculated wake geometry as initial values.

5. **Generate final results**: After the process is converged, the final results are calculated and stored for further processing.
Table 3.1: Ratio of the radial velocity variation magnitude to the mean radial velocity magnitude at the tunnel wall at the wind turbine location.

\[
\frac{\bar{U}_{rad}}{\bar{U}_{rad}} (%) \quad 10R \quad 8R \quad 6R \quad 5R \quad 4R \quad 3R \quad 2R
\]

<table>
<thead>
<tr>
<th></th>
<th>1 Blade</th>
<th>0.34</th>
<th>0.31</th>
<th>0.33</th>
<th>0.38</th>
<th>0.48</th>
<th>0.72</th>
<th>1.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 Blades</td>
<td>0.001</td>
<td>0.002</td>
<td>0.004</td>
<td>0.007</td>
<td>0.015</td>
<td>0.042</td>
<td>0.166</td>
<td></td>
</tr>
</tbody>
</table>

3.2. Tunnel geometry

Initially it was proposed to build up the tunnel geometry from quadrilateral source panels (see Appendix B) covering the test-section wall. When the variation of the local radial velocity is small compared to its average, the induced radial velocity can be assumed to be axisymmetric and the circumferential variation of the source-strength distribution is also axisymmetric. When this is the case, ring-shaped sources can be used (section 2.2.2).

Figure 3.2 shows the average of the radial induced velocity for a one bladed turbine operating in unconfined conditions in a test section with a radius of 3 times the wind-turbine radius. This means no effect of a wind-tunnel wall presence is taken into account yet. In the graph, the azimuthal variation is shown with error-bars. At the wind-turbine location, the variation is expected to be the greatest. With a ratio of standard deviation to average of about 0.72%, the effect of variation in the circumferential direction can be neglected. This configuration is the worst case scenario and from Table 3.1 it is clear that the ratio of standard deviation to mean radial induced velocity decreases fast for larger wind tunnels and for wind turbines with increasing number of blades. This table also shows the effect of variation for an extreme blockage ratio ($R = 2 \rightarrow \epsilon = 25\%$). Even for this configuration a maximum variation of about 2% is observed and this is still negligible.

The effect of the wind tunnel presence on the asymmetry is difficult to estimate. It is however assumed that the ratio of the radial velocity variation magnitude to the mean radial velocity magnitude will remain nearly of the same order of magnitude.

When the panels represent the wall, i.e. the panel center points are the same as the wall boundary control points, the self-induced velocity of a ring-shaped panel becomes undefined. The problem lies in the complete integral of the first kind. Its value goes to infinity if the argument goes to 1. This can be seen in Figure 3.3. This is clearly the case in equation 2.21 and 2.22 when $\rho \rightarrow R$ and $y \rightarrow 0$. The argument of the complete elliptic integral is not equal to 1, but it goes towards 1. This can have negative numerical effects and needs to be avoided.
3. WIND TUNNEL MODEL FOR WIND TURBINES

To avoid this problem, the ring panels are placed at a distance farther away from the control points location, so $\rho < R$. This means that the radius of the control points needs to be at the same radius as the actual modeled tunnel, while the radius of the panels needs to be bigger.

In order to have a smooth transition from a crude mesh to a refined mesh, a linear axial refinement scheme is proposed. This allows the ring length to vary constantly with a small ring length in the neighborhood of the wind turbine. This is done because the biggest difference in induced velocity is expected in that neighborhood. A final tunnel geometry is shown in Figure 3.4 and clearly shows the ring-shaped source panels with an axial refinement in the location of the wind turbine.

3.3. Simulation process

The simulation process is designed so that the power and thrust coefficients of a wind turbine operating with or without wind-tunnel walls can be compared. Therefore, both cases are calculated with the same turbine operating parameters.
3.3. SIMULATION PROCESS

Figure 3.3: Values for the complete elliptic integral of the first kind.

Figure 3.4: Wind tunnel geometry of the final model.
3. WIND TUNNEL MODEL FOR WIND TURBINES

The variable input parameters can be subdivided into two main groups: turbine-input parameters and tunnel-input parameters. The turbine-input parameters are the number of blades $N_b$, the tip-speed-ratio $\lambda$ and the circulation $\Gamma$. The tunnel-input parameter is the test-section radius $R$.

The output parameters can also be subdivided into two main groups: turbine-output parameters and tunnel-output parameters. The tunnel-output parameters are the tunnel discretization, the source strength of the panels $\sigma(z)$ and the mass flow at the tunnel inlet $\dot{m}_{in}$. The turbine-output parameters are the wake shape $\gamma_{tip}$ and its components (i.e. $r_W$ and $\phi_W$), the thrust coefficient defined with the free-stream velocity $U_e$, the power coefficient defined with the free-stream velocity $U_e$ and the mass flow through the turbine disk area $\dot{m}_{disk}$.

3.3.1. Flow field characteristics

One very important characteristic of this wind-tunnel model is the difference between the free-stream velocity, here depicted as $U_e$, and the velocity at the inlet of the tunnel, written as $U_\infty$. Due to the presence of the wind turbine, the flow decelerates inside the test section ($U_\infty < U_e$). This is shown schematically in Figure 3.5. The flow velocity at the tunnel inlet can therefore also be regarded as the inlet velocity for an actual wind-tunnel test.

This effect of the numerical model has a consequence on the definition of the thrust and power coefficients. This is done based on the free-stream velocity $U_e$, resulting in the thrust coefficient $C^T_{T,R}$ and the power coefficient $C^P_{T,R}$. By calculating the mass flow at the tunnel inlet $\dot{m}_{in}$, the flow velocity $U_\infty$ can be estimated since $\dot{m}_{in} = \pi R^2 U_\infty \rho$. This allows for a second definition of thrust and power coefficients based on the tunnel inlet velocity. These values would represent the measured thrust and power coefficients ($C^\infty_{T,R}$ and $C^\infty_{P,R}$). These coefficients can be related to each other by

$$C^T_{T,R} = (\dot{m}_R)^2 C^\infty_{T,R} \tag{3.3}$$
$$C^P_{T,R} = (\dot{m}_R)^3 C^\infty_{P,R} \tag{3.4}$$

with $\dot{m}_R$ the mass-flow-ratio. This is the ratio of mass flowing through the same cross section ($A = \pi R^2$) of the confined case ($\dot{m}_{in}$) to unconfined case ($\dot{m}_e$):

$$\dot{m}_R = \frac{\dot{m}_{in}}{\dot{m}_e} = \frac{\rho A U_\infty}{\rho A U_e} = \frac{U_\infty}{U_e} < 1 \tag{3.5}$$

Three main types of coefficients are thus calculated from the numerical simulation model: the coefficients calculated in the unconfined condition case, the coefficients calculated in the confined case scaled with $U_e$ (calculated) and the coefficients calculated in the confined case scaled with $U_\infty$ (measured). An overview of the the power and thrust coefficients are given in Table 3.2.
3.4. Results of the wind tunnel model

The example case input parameters are the following:

- Number of blades $N_b = 3$
- Tip-speed-ratio $\lambda = 5$
- Circulation $\Gamma = 0.1862$
- Wind tunnel radius $R = 3$

This case represents a common wind-turbine type (3 blades) operating at maximum power production conditions (maximum circulation) and at an average tip-speed ratio in a wind tunnel with a high blockage ratio.

3.4.1. Source strength distribution

Figure 3.6 shows the radial induced velocity from wind turbine on the test-section wall location in the final iteration. It is clear that the strongest radial induced velocity occurs in the vicinity of the wind turbine, both before and behind. This can obviously be expected for the upstream flow, since the presence of the wind turbine will divert the incoming axial flow in the radial direction. This effect is stronger close to the wind turbine. In the area just behind the
wind turbine, the wake is expected to grow and the flow will divert further until the wake has settled.

To counteract the radial induced velocity from the wind turbine and enforce the wall boundary condition, the source-strength distribution needs to have a distribution as shown in Figure 3.7. As expected, the highest source strengths occur in the vicinity of the wind turbine. The resulting radial induced velocity by the wind tunnel walls is shown in Figure 3.6. It must be the exact negative of the radial induced velocity from the wind turbine in order to meet the wind tunnel wall boundary condition. The radial net effect on the wind tunnel wall location is therefore 0.

The mass flow, shown in Figure 3.8, is nearly constant (±0.4%) along the tunnel length, clearly demonstrating the quality of the present wall modeling. The mass flow inside the tunnel is $\dot{m}_\infty = 26.88$ while the mass flow through the same cross section of the unconfined case yields $\dot{m}_e = 28.27$. This results in a mass-flow ratio of $\dot{m}_R = 0.951$.

### 3.4.2. Wake structure

To investigate the wake structure, two wake characteristics are investigated: the wake radius and the wake pitch angle.
3.4. RESULTS OF THE WIND TUNNEL MODEL

Figure 3.7: Axial distribution of the source strength for the different source panels in the discretized wind tunnel wall.

It is clear from Figure 3.9 that the radius of the wake decreases due to the presence of the wind tunnel. This can be expected, since the flow that usually flows around the turbine now gets funneled between the wake and the wind tunnel walls. It is also clear that the maximum wake radius is reached faster in the confined case.

Figure 3.10 shows the pitch angle of the wake helix. For the confined case the wake pitch angle is larger than the one for the unconfined case. This clearly indicates the elongation of the wake vortex structure. Also, it must be pointed out that the wake pitch angles also settles faster in the confined case than in the unconfined case.

3.4.3. Thrust and power coefficients

As mentioned before, the numerical model’s output consists of several thrust and power coefficients. All these coefficients are listed in Table 3.2. The values of these thrust and power coefficients for the example case are shown in Table 3.3.

The first observation is that the unconfined thrust coefficient and confined calculated thrust coefficient are the same, while the respective power coefficients are not. Their values are however closer than the unconfined and confined measured power coefficients.
Another observation is that the confined measured power coefficient exceeds the Betz-limit (i.e. $C_P = 0.593$). For a wind turbine operating under unconfined conditions this is impossible, but for the same wind turbine operating within a confinement this is possible and is mainly due to the presence of the wind-tunnel walls and the funneling of the air stream through the wind turbine, therefore increasing its power output.

Table 3.3: Overview of the different thrust and power coefficients calculated in the example model.

<table>
<thead>
<tr>
<th></th>
<th>Unconfined</th>
<th>Confined measured</th>
<th>Confined calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_T$</td>
<td>0.9387</td>
<td>1.0380</td>
<td>0.9387</td>
</tr>
<tr>
<td>$C_P$</td>
<td>0.5631</td>
<td>0.6727</td>
<td>0.5786</td>
</tr>
</tbody>
</table>
3.4. RESULTS OF THE WIND TUNNEL MODEL

Figure 3.9: Tip vortex radius in the wake structure. *Solid line*: Unconfined case. *Dashed line*: Confined case.

Figure 3.10: Tip vortex pitch angle in the wake structure. *Solid line*: Unconfined case. *Dashed line*: Confined case.
CHAPTER 4

Blockage correction derivation

The goal of the blockage correction derivation chapter is to find empirical correlations between the measured performance characteristics of a generic wind turbine under confined and unconfined conditions. In order to obtain these correlations more simulated cases need to be investigated. Given the model described in chapter 3, which allows for the fast calculation of a single case, it is possible to simulate many cases in an acceptable time period.

4.1. Simulation cases

The input parameters of the simulation cases are the number of blades \( N_b \), the tip-speed ratio \( \lambda \), the circulation \( \Gamma \) and the test-section radius \( R_t \). The \( N_b \) is chosen to be 1, 3 and 7. The tip-speed ratio \( \lambda \) is investigated between 3 and 8. These represent the minimum and maximum values of the typical rotation speed of a working rotor. Lower values are almost never reached when wind turbines are in operation and higher tip-speed ratios are usually avoided. The \( \Gamma \)-variable ranges between 10\% and 100\% of the maximum values of the circulation \( \Gamma_{\text{max}} = (8\pi)/(9\lambda N_b) \) (Segalini & Alfredsson 2013). The test-section radius \( R_t \) ranges between 10 and 2. These values have been chosen to represent a range between a small blockage ratio \( (R_t = 10 \rightarrow \epsilon = 1\%) \) and an unusually large blockage ratio \( (R_t = 2 \rightarrow \epsilon = 25\%) \).

4.2. Needed correlations

In order to determine a practical correction, it is preferred to use as few measured performance characteristics from the confined case as possible to determine the unconfined performance characteristics. The thrust coefficient, the power coefficient and the blockage ratio are important measured characteristics. The mass-flow ratio is a needed characteristic which is not measurable and needs to be correlated with the measurable characteristics. The thrust and power coefficient of the atmospheric operating wind turbine are the main performance characteristics that need to be estimated.

4.2.1. Correlation of the mass-flow-ratio

In the numerical model, the mass-flow-ratio is calculated as one of the model outputs. However, in wind-tunnel measurements this value is unknown and
cannot be measured. Therefore a correlation between measurable characteristics, such as the thrust and power coefficients, and the mass-flow-ratio is needed.

Based on the expected effect both the blockage ratio $\epsilon$ and the measured thrust coefficient $C_{T,R}^\infty$ will have on the mass-flow-ratio, the following correlation is proposed:

$$\dot{m}_{R,est} = 1 - f(\epsilon, C_{T,R}^\infty),$$

(4.1)

where the function $f$ will have two natural limits:

$$\lim_{\epsilon \to 0} f(\epsilon, C_{T,R}^\infty) = 0,$$

(4.2)

$$\lim_{C_{T,R}^\infty \to 0} f(\epsilon, C_{T,R}^\infty) = 0.$$

(4.3)

The first limit states that as the blockage ratio becomes smaller (and thus the tunnel size becomes bigger) the effect of the wall presence decreases and the measured wind turbine will act more like a wind turbine operating in unconfined conditions. The second limit states that, for a small thrust coefficient (and thus a small effect of the wind turbine on the flow) the effect on the mass-flow ratio decreases since a small thrust force does not decelerate the flow too much.

The function $f$ is still unknown. Based on the data it is clear that the function should be a smooth function in both variables. As shown in Figure 4.2, it is proposed to use a Taylor expansion of the function $f$ in the two variables, given the two natural limits, of the form

$$f(\epsilon, C_{T,R}^\infty) \cong \epsilon C_{T,R}^\infty \left(\alpha_1 + \alpha_2 \epsilon + \alpha_3 C_{T,R}^\infty + \alpha_4 \epsilon^2 + \alpha_5 \epsilon C_{T,R}^\infty + \alpha_6 (C_{T,R}^\infty)^2\right).$$

(4.4)

By performing the least squares method (LSM) to determine these coefficients, the general expression for the function $f$ is fitted to the data. For every set of data, calculated in the simulation cases, $\dot{m}_{R,est}$, $C_{T,R}^\infty$ and $\epsilon$ are known. It can be expected that higher order expansions will have a better agreement with the data than lower order expansions, but the higher order expansions need more correlating coefficients.

The values of the Taylor expansion are shown in Table 4.1. Comparing the calculated mass-flow-ratio from the simulation cases with the estimated mass-flow-ratio, according to the proposed correlation scheme results in a very good agreement between the two. Figure 4.2 shows the comparison between the calculated mass-flow-ratio and estimated mass-flow-ratio as function of the measured thrust coefficient for given the blockage ratios.

### 4.2.2. Correlation of the thrust coefficient

The thrust coefficient calculated in the numerical model $C_{T,R}^e$ and the thrust coefficient measured in the wind tunnel $C_{T,R}^\infty$ are related to the mass-flow-ratio $\dot{m}_{R}$, as stated in section 3.3.1. This is due to the velocity used in the definitions
Table 4.1: Empirical coefficients of the Taylor expansion for the estimated mass-flow-ratio.

<table>
<thead>
<tr>
<th>(a_1)</th>
<th>(a_2)</th>
<th>(a_3)</th>
<th>(a_4)</th>
<th>(a_5)</th>
<th>(a_6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.192</td>
<td>-0.519</td>
<td>0.393</td>
<td>3.029</td>
<td>-1.395</td>
<td>0.010</td>
</tr>
</tbody>
</table>

Figure 4.1: Comparison between the simulated mass-flow-ratio \(\dot{m}_R\) \text{(calculated)} and the estimated mass-flow-ratio \(\dot{m}_R\) \text{(estimated)} according to the proposed correlation scheme, with a margin of error of 0.5%.

of these thrust coefficients. The relation is

\[
C_{T,R}^e = (\dot{m}_R)^2 C_{T,\infty}^\infty. \tag{4.5}
\]

As already mentioned in section 3.3.1, the thrust coefficient of the unconfined situation and the calculated thrust coefficient from the confined situation for the example case are the same. Figure 4.3 shows in a more general way the ratio of the calculated thrust coefficient to the thrust coefficient of the unconfined situation for all the simulated cases. It is clear that the values of this ratio are close to 1 and any difference is negligible (0.02%). It can therefore be assumed that the following relation holds

\[
C_{T,\infty}^e \cong C_{T,R}^e. \tag{4.6}
\]
Combining equation 4.6 and equation 4.5 leads to a correlation between the confined measured thrust coefficient $C_{T,R}^\infty$ and the unconfined thrust coefficient $C_{T,\infty}$, namely

$$C_{T,\infty} = (\dot{m}_R)^2 C_{T,R}^\infty, \quad (4.7)$$

where the mass-flow-ratio used in this correlation is the estimated mass-flow-ratio correlation derived in section 4.2.1.

In Figure 4.4 the comparison between the calculated thrust coefficient under unconfined condition and the corrected thrust coefficient is shown. The green dots show the values of the thrust coefficient as would be measured in a wind tunnel, the black dots show the values of the measured thrust coefficient corrected with the previously proposed mass-flow-ratio correlation and the newly proposed thrust coefficient correlation. It is clear that the correction scheme reduces the over-estimation of the confined measured thrust coefficient to values close to the thrust coefficient under unconfined conditions.

### 4.2.3. Correlation of the power coefficient

The goal of this section is to relate the confined measured power coefficient $(C_{P,R}^\infty)$ to the power coefficient under unconfined conditions $(C_{P,\infty})$. The power coefficient calculated in the numerical model $C_{P,R}$ and the power coefficient measured in the wind tunnel $C_{P,R}^\infty$ are related to the mass-flow-ratio $\dot{m}_R$, as
### 4.2. Needed Correlations

Figure 4.3: Ratio of the calculated thrust coefficient in confined conditions $C_{T,R}$ to the calculated thrust coefficient in unconfined conditions $C_{T,\infty}$ as function of the mass-flow-ratio $\dot{m}_R$ for different blockage ratios $\epsilon$.

Figure 4.4: Comparison of the calculated thrust coefficient in unconfined conditions with the confined measured thrust coefficient (green) and the corrected thrust coefficient (black), based on the proposed correlation.
4. BLOCKAGE CORRECTION DERIVATION

stated in section 3.3.1. This is due to the velocity used in the definitions of these thrust coefficients. The relation is

\[ C_{eP,R} = (\dot{m}_R)^3 C_{\infty P,R} \]  \hspace{1cm} (4.8)

In contrast to the thrust coefficient, the calculated power coefficient from the confined situation and the power coefficient from the unconfined situation are not the same. The values of the calculated power coefficient are closer to the power coefficient under unconfined conditions than to the confined measured power coefficient. Figure 4.6 shows the ratio of the calculated power coefficient in confined conditions \( C_{eP,R} \) to the calculated power coefficient in unconfined conditions \( C_{eP,\infty} \) as function of the mass-flow-ratio \( \dot{m}_R \) for different blockage ratios \( \epsilon \). It is clear from this graph that for higher blockage ratios there is a significant difference between the values of the calculated power coefficient and the power coefficient under unconfined conditions, which should not be neglected. It’s however clear that the effect seems to scale with the blockage ratio \( \epsilon \). Therefore the following general correlation is proposed

\[ \frac{C_{eP,R}}{C_{eP,\infty}} = 1 + \epsilon g (\epsilon, \dot{m}_R) \]  \hspace{1cm} (4.9)

where the function \( g \) is an empirical function. With the relation between the calculated power coefficient and the measured power coefficient (equation 4.8) known, the expression for the correlation between the measured power coefficient and the power coefficient under unconfined condition becomes

\[ C_{eP,\infty} = \frac{(\dot{m}_R)^3 C_{\infty P,R}}{1 + \epsilon g (\epsilon, \dot{m}_R)} \]  \hspace{1cm} (4.10)

This expression has a very important difference with respect to the existing blockage correction schemes. In the Glauert and Mikkelsen & Sørensen schemes, where the equivalent free-stream velocity scaled by the wind tunnel inlet velocity is the inverse of the mass-flow-ratio, the measured power coefficient is scaled with the third power of the mass-flow-ratio to obtain the power coefficient in unconfined conditions. The additional effect, corrected by the denominator in equation 4.9, is neglected despite the fact that it has an impact of 2% – 5% and should not be neglected.

At this point, the empirical function \( g \) is not yet known and must be derived based on the data calculated in the simulation cases. By introducing the variable

\[ \phi = (1 - \dot{m}_R)\epsilon^{-0.7772} \]  \hspace{1cm} (4.11)

the correlation function \( f \) is fitted to be the exponent of a third order polynomial in the scaled variable \( \phi \) and can be written as

\[ g (\epsilon, \dot{m}_R) = \exp \left( \beta_3 \phi^3 + \beta_2 \phi^2 + \beta_1 \phi + \beta_0 \right) \]  \hspace{1cm} (4.12)
4.3. VALIDATION OF THE PROPOSED CORRECTION SCHEME

Validation of this newly proposed correction scheme is performed by applying the existing Glauert and Mikkelsen & Sørensen correction schemes and the proposed correction scheme to the calculated measured thrust and power coefficient. Based on the simulation data, where both the measured thrust and power coefficients and the thrust and power coefficients under unconfined conditions are available, the correction schemes are applied and compared. The comparison results are shown in Figure 4.8 for a low blockage ratio of 1%, in Figure 4.9 for a high blockage ratio of 11% and Figure 4.10 for an extreme blockage ratio of 25%. In these figures the percentual difference between the corrected and the calculated thrust or power coefficient are shown in function of the measured thrust or power coefficients. The percentual difference in thrust

The four empirical constants are \( \beta_3 = 34.43; \beta_2 = -52.95; \beta_1 = 47.87; \beta_0 = -21.37 \) and the fitting is shown in Figure 4.5. The final result of the correlation function, using equation 4.11 and equation 4.12, are shown in Figure 4.6.

In Figure 4.7 the comparison between the calculated power coefficient under unconfined condition and the corrected power coefficient is shown. It is clear that the correction scheme reduces the over-estimation of the measured power coefficient significantly. The final results of the correction show values close to the power coefficient under unconfined conditions, with a slight over-estimation of maximum 3%. This over-estimation occurs in situation of very high blockage ratios \( (\epsilon \geq 3) \) with a high loading \( (C_{T,R}^{\infty} \rightarrow 1) \).
Coefficient can be defined as
\[ \delta_{C_{e,T,\infty}} = \frac{C_{e,T,\infty}^{(corrected)}}{C_{e,T,\infty}^{(calculated)}} - 1. \] (4.13)

In a similar way the percentual difference in power coefficient can be defined.

### 4.3.1. Results for a low blockage ratio

Figure 4.8 shows the results of the comparison between the blockage correction schemes for a low blockage ratio (i.e. \( \epsilon = 1\% \)). For the thrust coefficient the Glauert model seems to be correcting the measured thrust coefficients very well. The maximum error of this blockage correction scheme, at high turbine loading, is limited to an under-estimation of about 0.5%. The Mikkelsen & Sørensen scheme corrects the measured thrust coefficient less well, with an under-estimation of about 2%. The proposed model seems to correct the measured thrust coefficient better, with a slight over-estimation of about 0.05%.

For the power coefficient the Glauert model seems to be correcting the measured power coefficients also very well. The maximum error of this blockage correction scheme, at high turbine loading, is limited to an under-estimation of about 0.5%. The Mikkelsen & Sørensen scheme corrects the measured power
4.3. Validation of the Proposed Correction Scheme

Figure 4.7: Comparison of the calculated power coefficient under unconfined conditions with the measured thrust coefficient (green) and the corrected power coefficient (black), based on the proposed correlation.

coefficient less well, with an under-estimation of about 3%. The proposed model seems to correct the measured thrust coefficient best, with a slight over-estimation of about 0.05%.

For low blockage ratios, the proposed model seems to be the one that predicts the thrust and power coefficient of a turbine operating under unconfined conditions best.

4.3.2. Results for a high blockage ratio

Figure 4.9 shows the results of the comparison between the blockage correction schemes for a high blockage ratio (i.e. $\epsilon = 11\%$). For the thrust coefficient the Mikkelsen & Sørensen correction scheme seems to be more accurate than the Glauert model for high turbine loading. For low turbine loading, the Glauert model seems to be more accurate than the Mikkelsen & Sørensen scheme. The maximum error of the Mikkelsen & Sørensen scheme is about 4%, while the error of the Glauert scheme already approaches 10%. Both the Glauert and Mikkelsen & Sørensen correction schemes seem to under-estimate the performance of the actual wind turbine significantly. Note that for measured thrust coefficients higher than 1, the Glauert model is not able to calculate a correction, since the equivalent free-stream velocity reaches a singularity at $C_T = 1$. 
4. BLOCKAGE CORRECTION DERIVATION

The proposed correction scheme seems to correct the measured thrust coefficient better with a slight over-estimation. The maximum error (ca. 0.1%) is very small compared to the other correction schemes.

For the power coefficient the Mikkelsen & Sørensen correction scheme seems to be more accurate than the Glauert model for high turbine loading. For low turbine loading, the Glauert model seems to be more accurate than the Mikkelsen & Sørensen scheme. The maximum error of the Mikkelsen & Sørensen scheme is about 4%, while the error of the Glauert scheme already approaches 20%. Both the Glauert and Mikkelsen & Sørensen correction schemes seem to under-estimating the performance of the actual wind turbine significantly. The proposed correction scheme seems to correct the measure thrust coefficient best with a slight over-estimation. The maximum error (ca. 0.5%) is very small compared to the other correction schemes.

For high blockage ratios, the proposed model seems again to be the one that predicts the thrust and power coefficient of a turbine operating under unconfined conditions best.

4.3.3. Results for extreme blockage ratio

Figure 4.10 shows the results of the comparison between the blockage correction schemes for a extreme blockage ratio (i.e. $\epsilon = 25\%$). For the thrust coefficient it is clear that the Glauert and Mikkelsen & Sørensen correction schemes become less and less useful. The proposed model on the other hand corrects the measured thrust coefficient very well, even for extreme blockage ratios. The maximum error is estimated at only 1%, much smaller than the other correction schemes.
Figure 4.9: Comparison of the thrust (left) and power (right) coefficient error of the different blockage correction schemes for a blockage ratio of 11%.

For the power coefficient there is a similar issue. It seems that the Glauert and Mikkelsen & Sørensen correction schemes have trouble correcting the values, while the proposed correction scheme seems to correct the measured power coefficient very well. Its maximum error is limited to about a 2% over-estimation and is clearly much lower than the other correction schemes.

Even for extreme blockage ratios, where the other correction schemes have trouble predicting the thrust and power coefficient, the proposed model seems again to be the one that predicts the thrust and power coefficient of a turbine operating under unconfined conditions best.
4. BLOCKAGE CORRECTION DERIVATION

Figure 4.10: Comparison of the thrust (left) and power (right) coefficient error of the different blockage correction schemes for a blockage ratio of 25%.
CHAPTER 5

Summary and conclusions

After introducing the necessary theoretical background, the Glauert wind tunnel interference model and Mikkelsen & Sørensen blockage correction schemes are described. Together with their derivation, their use and limitations are pointed out. The Glauert correction schemes is an easy and straightforward correction scheme, widely used in wind-tunnel measurements. It has a big disadvantage when the turbine operates in small wind tunnels and when the rotors are highly loaded, where a singularity is reached for thrust coefficient $C_T \approx 1$, thus not allowing for accurate calculations in these conditions. The Mikkelsen & Sørensen correction scheme overcomes this problem and it is able to breach this singularity. However, in this correction scheme one more measured variable is needed, requiring an extra measurement to be done.

Based on the availability of a fast simplified vortex model for modeling the wind turbine, a wind-tunnel model for wind turbines is proposed. Based on the properties of the simplified vortex model, a cylindrical test section is created. For a wide range of blockage ratios, the radial induced velocity from the wind turbine was investigated. The variation of the circumferential distribution of the radial induced velocity was found to be very small, even for the worst possible scenario investigated. The induced velocity from the wind turbine was thus assumed to be axisymmetric, allowing for the modeling of the wind-tunnel wall with ring-shaped source panels.

The numerical model calculates both a case in unconfined conditions and confined wind-tunnel conditions. The unconfined case calculates the wake shape and the performance parameters (i.e. thrust and power coefficient) of the wind turbine under unconfined conditions. In the confined wind-tunnel case, the wake shape and the performance parameters are calculated based on the same dimensionless unconfined operating conditions. Due to the presence of the wind turbine and the enclosure of the wind tunnel, the velocity in the inlet area of the wind tunnel is lower than the unconfined free-stream velocity. This velocity is calculated and used to rescale the performance parameters. These rescaled performance parameters are the same as one would measure in the wind tunnel with that inlet velocity. The performance parameters and wake shapes, from the unconfined and confined cases, can be compared. It was observed that in the wind-tunnel case the wake shrinks in size and elongates.
Since the simplified vortex model was already validated and used for determining the unconfined performance parameters, a general approach for validating the numerical model is proposed. By deriving a general blockage correction scheme from many simulation cases and applying this correction scheme and the existing Glauert and Mikkelsen & Sørensen schemes to the calculated confined data, a comparison between their results and the calculated unconfined data is proposed.

Based on the proposed wind-tunnel model and multiple simulation cases, a blockage correction scheme is thus derived. This process leads to the derivation of three empirical correlation functions where the performance parameters of the wind turbine operating under unconfined conditions can be estimated only from performance parameters measured in wind tunnel tests of the same wind turbine. Two of these correlations are needed for the thrust and power coefficients. The third correlation is needed to estimate the mass-flow ratio, a quantity that cannot be measured in a wind-tunnel test, but is a surrogate of the equivalent free-stream velocity. The correction schemes for the mass-flow ratio and the thrust coefficient are very accurate. The correction scheme for the power coefficient is accurate with a small error of ca. 3% for very extreme cases, which are almost never reached in wind-tunnel tests.

The newly proposed correlation scheme is compared with the Glauert wind tunnel interference model and with the Mikkelsen & Sørensen model to validate it. Here, all models were applied to the measured performance parameters from the simulations and compared to the performance parameters of the wind turbines in unconfined conditions. The newly proposed model was found to correct these values much better than the Glauert or Mikkelsen models.
APPENDIX A

Glauert model for small blockage ratios

Consider a control volume around the wind turbine and the wind tunnel as shown in Figure A.1. The power extracting stream-tube around the wind turbine acts as a boundary for two smaller control volumes: the stream-tube control volume \( CV_1 \) and the wind tunnel control volume \( CV_2 \).

At the inlet, the flow enters the control volume with a velocity \( U_\infty \) at a pressure \( p_\infty \). The cross-area of the stream-tube control volume at the inlet is \( A_\infty \) and the total cross-area of the whole control volume is \( C \). The fluid flows through the disk with a cross-area \( A_d \) at a velocity \( U_d \), exerts a force \( T \) on the turbine and extracts a power \( P \) from the flow. The pressure just upstream of the disk is \( p_u \) and the pressure just downstream of the disk is \( p_d \). At the outlet, the flow inside the stream-tube control volume leaves the control volume at a lower velocity \( U_W \) and a pressure \( p_W \). The cross-area of the stream-tube control volume at the outlet is \( A_W \) and the total cross-area of the whole control volume is \( C \). The flow inside the wind tunnel control volume leaves the control volume with a velocity \( U_2 \) at a pressure \( p_2 \). In the far-wake region, this pressure must be equal to the pressure in the stream-tube control volume, thus \( p_2 = p_W \).

A.1. Dimensionless parameters

The dimensionless parameters used in this analysis are:

- The axial induction factor:
  \[ a = 1 - \frac{U_d}{U_\infty} \quad (A.1) \]

- The \( \alpha \)-factor:
  \[ \alpha = \frac{U_d}{U_\infty} \quad (A.2) \]

- The wake expansion factor:
  \[ \sigma = \frac{A_W}{A_d} = \frac{U_d}{U_W} \quad (A.3) \]

- The wake velocity factor:
  \[ \beta = \frac{U_W}{U_\infty} \quad (A.4) \]

- The blockage ratio:
  \[ \epsilon = \frac{A_d}{C} \quad (A.5) \]

- The free-stream acceleration factor:
  \[ \gamma = \frac{U_d}{U_\infty} = \frac{1 - \epsilon \alpha}{1 - \epsilon \alpha / \beta} \quad (A.6) \]
The power coefficient:
\[ C_P = \frac{P}{1/2\rho A d U_\infty^3} \]  
(A.7)

The thrust coefficient:
\[ C_T = \frac{T}{1/2\rho A d U_\infty^2} \]  
(A.8)

A.2. Derivation of the thrust force

A.2.1. Thrust force based on mass and momentum balance

For both control volumes, both mass and momentum must be conserved. The mass conservation for both control volumes leads to
\[ CV_1 : \quad \dot{m}_1 = \rho A_\infty U_\infty = \rho A_d U_d = \rho A_W U_W \]  
(A.9)
\[ CV_2 : \quad \dot{m}_2 = \rho (C - A_\infty) U_\infty = \rho (C - A_W) U_W \]  
(A.10)

while momentum conservation in the axial direction leads to
\[ CV_1 : \quad \dot{m}_1 (U_W - U_\infty) = -T + X + p_\infty A_\infty - p_W A_\infty \]  
(A.11)
\[ CV_2 : \quad \dot{m}_2 (U_2 - U_\infty) = -X + p_\infty (C - A_\infty) - p_W (C - A_W) \]  
(A.12)

X is eliminated by summing A.11 and A.12, leading to
\[ X = p_\infty (C - A_\infty) - p_W (C - A_W) - \dot{m}_2 (U_2 - U_\infty) \]  
(A.13)

Using equations A.10, A.13 and A.11, an expression for the thrust can be derived.
\[ T = C(p_\infty - p_W) - \rho U_\infty (C - A_\infty)(U_2 - U_\infty) + \rho A_d U_d (U_\infty - U_W) \]  
(A.14)

Eliminating \((p_\infty - p_W)\) can be done with help from Bernoulli’s equation, applied to the free-stream control volume
\[ \frac{p_\infty}{\rho} + \frac{U_\infty^2}{2} = \frac{p_W}{\rho} + \frac{U_W^2}{2} \]  
(A.15)

leading to
\[ T = \rho C \left( U_2^2 - U_\infty^2 \right) - \rho U_\infty (C - A_\infty)(U_2 - U_\infty) + \rho A_d U_d (U_\infty - U_W) \]  
(A.16)
The derivation of the thrust force based on mass and momentum balance, in terms of the dimensionless parameters, is based on the following derivation:

\[ T = \frac{\rho C}{2} \left( U_2^2 - U_\infty^2 \right) - \rho U_\infty (C - A_\infty)(U_2 - U_\infty) + \rho A_d U_d (U_\infty - U_W) \]

\[ = \rho A_d U_\infty^2 \left[ \frac{U_2}{2A_d} - \frac{U_2^2 - U_\infty^2}{U_\infty} \right] - \frac{C - A_\infty}{A_d} \left( U_2 - U_\infty \right) + \frac{U_d}{U_\infty} \left( U_\infty - U_W \right) \]

\[ = \rho A_d U_\infty^2 \left[ \frac{(1 - \alpha) - (1 - \alpha/\beta)}{(1 - \alpha/\beta)} \right] \left[ \frac{1}{2} \left( \frac{1 - \alpha}{1 - \alpha/\beta} + \frac{1}{\epsilon} \right) \right] + \alpha(1 - \beta) \]

For \( \epsilon \ll 1 \), the expression can be simplified. Using the Taylor expansion

\[ [(1 + \beta) - 2\alpha][1 - \alpha/\beta]^{-2} = [(1 + \beta) - 2\alpha] \cdot [1 + 2\epsilon \alpha/\beta + O(\epsilon^2)] \]

\[ = 1 + \beta + 2\epsilon \alpha/\beta + O(\epsilon^2) \]

A.2.2. Thrust force based on the pressure across the actuator disk

Another expression for the thrust force can be based on the pressure difference across the disk area:

\[ T = A_d (p_a - p_d) \]  

(A.21)

The Bernoulli equation is only valid from the far upstream to just in front of the disk and from just behind the disk to far downstream. It is not valid when crossing the disk. Using Bernoulli’s equation in the stream-tube upstream and downstream of the actuator disk,

\[ \frac{p_a}{\rho} + \frac{U_d^2}{2} = \frac{p_\infty}{\rho} + \frac{U_\infty^2}{2} \]  

(A.22)

\[ \frac{p_d}{\rho} + \frac{U_W^2}{2} = \frac{p_\infty}{\rho} + \frac{U_W^2}{2} \]  

(A.23)

Using equations A.15, A.22 and A.23, the expression for the thrust (equation A.21) becomes

\[ T = \frac{\rho A_d}{2} (U_2^2 - U_W^2) \]  

(A.24)
The derivation of the thrust force based on the pressure across the actuator disk, in terms of the dimensionless parameters, is based on the following derivation:

\[
T = \frac{\rho A_d}{2} (U_\infty^2 - U_k^2)
\]

\[
= \frac{\rho A_d U_\infty^2}{2} \left[ \frac{U_k^2}{U_\infty^2} - U_k^2 \right]
\]

\[
= \frac{\rho A_d U_\infty^2}{2} \left[ \left( \frac{1 - \alpha}{1 - \epsilon \alpha / \beta} \right)^2 - \beta^2 \right]
\]

\[
= \frac{\rho A_d U_\infty^2}{2} \left[ \frac{(1 - \alpha)^2 - \beta^2 (1 - \epsilon \alpha / \beta)^2}{(1 - \epsilon \alpha / \beta)^2} \right]
\]

\[
= \frac{\rho A_d U_\infty^2}{2} \left[ \frac{(1 - \beta^2) - 2\alpha (1 - \beta)}{(1 - \epsilon \alpha / \beta)^2} \right]
\]

\[
= \frac{\rho A_d U_\infty^2}{2} (1 - \beta) \left[ \frac{(1 + \beta) - 2\alpha}{(1 - \epsilon \alpha / \beta)^2} \right]
\]

(A.25)

For \( \epsilon \ll 1 \), the expression can be simplified. Using the Taylor expansion

\[
[(1 + \beta) - 2\alpha(1 - \epsilon \alpha / \beta) = [(1 + \beta) - 2\alpha] \cdot [1 + 2\epsilon \alpha / \beta + O(\epsilon^2)]
\]

(A.26)

and neglecting higher-order terms of \( \epsilon \), the expression for the thrust can be reduced to

\[
T = \frac{\rho A_d U_\infty^2}{2} (1 - \beta) (1 + \beta + 2\epsilon \alpha / \beta) + O(\epsilon^2)
\]

(A.28)

**A.2.3. Elimination of wake velocity factor**

It’s possible to eliminate \( \beta \), assuming \( \beta \) scales linearly with \( \epsilon \):

\[
\beta = \beta_0 + \epsilon \beta_1
\]

(A.29)

Since the thrust is the same, equations A.20 and A.28 must give the same results. Based on these two equations, the elimination of \( \beta \) goes as follows:

\[
\rho A_d U_\infty^2 \alpha (1 - \beta) \left[ 1 + \frac{\epsilon \alpha}{2 \beta_0^2} (1 + \beta) \right] + O(\epsilon^2) = \frac{\rho A_d U_\infty^2}{2} (1 - \beta) (1 + \beta + 2\epsilon \alpha / \beta) + O(\epsilon^2)
\]

(A.30)

\[
2\alpha \left[ 1 + \frac{\epsilon \alpha}{2 \beta_0^2} (1 + \beta_0) \right] + O(\epsilon^2) = 1 + \beta_0 + \epsilon \beta_1 + 2\epsilon \alpha / \beta_0 + O(\epsilon^2)
\]

(A.31)

\[
2\alpha + \epsilon \left[ \frac{\alpha^2}{\beta_0^2} + \frac{\alpha^2}{\beta_0^2} \right] + O(\epsilon^2) = 1 + \beta_0 + \epsilon (\beta_1 + s\alpha / \beta_0) + O(\epsilon^2)
\]

(A.32)

To determine \( \beta_0 \) and \( \beta_1 \), one can see that the first order terms must be equal and the second order terms must be equal. The first order equality leads to:

\[
2\alpha = 1 + \beta_0
\]

(A.33)

\[
\beta_0 = 2\alpha - 1
\]

(A.34)
Using the expression for $\beta_0$, leads to an expression for $\beta_1$:

\[
\frac{\alpha^2}{\beta_0^2} + \frac{\alpha^2}{\beta_0} = \beta_1 + \frac{2\alpha}{\beta_0},
\]

(A.35)

\[
\beta_1 = \frac{\alpha^2}{\beta_0^2} + \frac{\alpha^2}{\beta_0} - \frac{2\alpha}{\beta_0}
\]

(A.36)

\[
= \frac{\alpha}{\beta_0} \left( \frac{\alpha + \alpha \beta_0 - 2\beta_0}{\beta_0} \right)
\]

(A.37)

\[
= \frac{\alpha (1 - \alpha)^2}{(2\alpha - 1)^2}
\]

(A.38)

Finally, this leads to a result for $\beta$:

\[
\beta = \beta_0 + \epsilon \beta_1
\]

(A.39)

\[
\beta = (2\alpha - 1) + 2\alpha \left( \frac{1 - \alpha}{(2\alpha - 1)^2} \right) + O(\epsilon^2)
\]

(A.40)

### A.2.4. Thrust force

Using the expression for $\beta$, the thrust coefficients can be written in terms of only $\alpha$ and $\epsilon$. The elimination of $\beta$ from equation A.20 goes as follows:

\[
T = \rho A_d U_{\infty}^2 \alpha (1 - \beta) \left[ 1 + \frac{\epsilon \alpha}{2\beta \epsilon} (1 + \beta) \right] + O(\epsilon^2)
\]

(A.41)

\[
= \rho A_d U_{\infty}^2 \alpha (1 - 2\alpha + 1 - 2\alpha \frac{(1 - \alpha)^2}{(2\alpha - 1)^2}) \left[ 1 + \frac{\epsilon \alpha}{2(2\alpha - 1)^2} (1 + 2\alpha - 1) \right] + O(\epsilon^2)
\]

(A.42)

\[
= 2\rho A_d U_{\infty}^2 \alpha (1 - \alpha) \left[ 1 + \frac{\epsilon \alpha^2 - \alpha (1 - \alpha)}{(2\alpha - 1)^2} \right] + O(\epsilon^2)
\]

(A.43)

\[
= 2\rho A_d U_{\infty}^2 \alpha (1 - \alpha) \left[ 1 + \epsilon \frac{\alpha^2 - \alpha (1 - \alpha)}{2\alpha - 1} \right] + O(\epsilon^2)
\]

(A.44)

The elimination of $\beta$ from equation A.28 goes as follows:

\[
T = \frac{\rho A_d U_{\infty}^2}{2} (1 - \beta) (1 + \beta + 2\epsilon \alpha / \beta) + O(\epsilon^2)
\]

(A.46)

\[
= \frac{\rho A_d U_{\infty}^2}{2} (1 - 2\alpha + 1 - 2\alpha \frac{(1 - \alpha)^2}{(2\alpha - 1)^2}) \left[ 1 + 2\alpha - 1 + 2\alpha \frac{(1 - \alpha)^2}{(2\alpha - 1)^2} + \frac{2\epsilon \alpha}{2\alpha - 1} \right] + O(\epsilon^2)
\]

(A.47)

\[
= \rho A_d U_{\infty}^2 \alpha (1 - \alpha) \left[ 1 + \epsilon \frac{(1 - \alpha)^2 + (2\alpha - 1)}{(2\alpha - 1)^2} \right] + O(\epsilon^2)
\]

(A.48)

\[
= 2\rho A_d U_{\infty}^2 \alpha (1 - \alpha) \left[ 1 + \epsilon \frac{(1 - \alpha)^2 + (2\alpha - 1) - \alpha (1 - \alpha)}{(2\alpha - 1)^2} \right] + O(\epsilon^2)
\]

(A.49)

\[
= 2\rho A_d U_{\infty}^2 \alpha (1 - \alpha) \left[ 1 + \epsilon \frac{\alpha}{2\alpha - 1} \right] + O(\epsilon^2)
\]

(A.50)
which is the same equation as equation A.45. The thrust coefficient becomes

\[
C_T = \frac{T}{1/2 \rho A_d U_\infty^2}
\]  
(A.51)

\[
C_T = 4\alpha(1 - \alpha) \left[ 1 + \epsilon \frac{\alpha}{2\alpha - 1} \right] + O(\epsilon^2)
\]  
(A.52)

### A.3. Derivation of the extracted power

The power extracted from the flow is given by

\[
P = U_d T = U_\infty \alpha T
\]  
(A.53)

\[
P = 2\rho A_d U_\infty^3 \alpha^2(1 - \alpha) \left[ 1 + \epsilon \frac{\alpha}{2\alpha - 1} \right] + O(\epsilon^2)
\]  
(A.54)

The power coefficient becomes

\[
C_P = \frac{P}{1/2 \rho A_d U_\infty^4}
\]  
(A.55)

\[
C_P = 4\alpha^2(1 - \alpha) \left[ 1 + \epsilon \frac{\alpha}{2\alpha - 1} \right] + O(\epsilon^2)
\]  
(A.56)

### A.4. Relating to the wake expansion factor

Starting from the equations A.20 and A.28, given the relation

\[
\beta = \frac{\alpha}{\sigma}
\]  
(A.57)

an expression can be derived to write the \(\alpha\)-factor in terms of \(\sigma\), disregarding the \(O(\epsilon^2)\)-terms:

\[
2\alpha \left[ 1 + \frac{\epsilon\alpha}{2\beta}(1 + \beta) \right] = 1 + \beta + \frac{2\epsilon\alpha}{\beta}
\]  
(A.58)

\[
2\alpha + \frac{\epsilon\alpha^2}{\beta^2} + \frac{\epsilon\alpha^2}{\beta} = 1 + \beta + \frac{2\epsilon\alpha}{\beta}
\]  
(A.59)

\[
2\alpha + \epsilon\sigma^2 + \epsilon\alpha = 1 + \alpha/\sigma + 2\epsilon\sigma
\]  
(A.60)

\[
\alpha(2 + \epsilon\sigma - 1/\sigma) = 1 + 2\epsilon\sigma - \epsilon\sigma^2
\]  
(A.61)

leading to

\[
\alpha = \frac{\sigma(1 + \epsilon\sigma(2 - \sigma))}{(2\sigma - 1) + \epsilon\sigma^2}
\]  
(A.63)

Alternatively, the relation between \(\sigma\) and \(\alpha\) becomes

\[
a = 1 - \alpha
\]  
(A.64)

\[
a = \frac{(\sigma - 1)(1 + \epsilon\sigma^2)}{(2\sigma - 1) + \epsilon\sigma^2}
\]  
(A.65)
A.5. Equivalent free-stream velocity

The free-stream velocity must satisfy the condition

\[ T = 2\rho d U_d (U'_\infty - U_d) \quad (A.66) \]

Combined with the equation for thrust

\[ T = 2\rho d U^2_\infty \alpha (1 - \alpha) \left[ 1 + \epsilon \frac{\alpha}{2\alpha - 1} \right] + O(\epsilon^2) \quad (A.67) \]

the ratio of the equivalent free-stream velocity to the wind tunnel free-stream velocity can be derived:

\[ \frac{2\rho d U_d (U'_\infty - U_d)}{U'_\infty U_\infty} = 2\rho d U^2_\infty \alpha (1 - \alpha) \left[ 1 + \epsilon \frac{\alpha}{2\alpha - 1} \right] + O(\epsilon^2) \quad (A.68) \]

\[ \frac{U_d}{U_\infty} \left( \frac{U'_\infty}{U_\infty} - \frac{U_d}{U_\infty} \right) = \alpha (1 - \alpha) \left[ 1 + \epsilon \frac{\alpha}{2\alpha - 1} \right] + O(\epsilon^2) \quad (A.69) \]

\[ \alpha \left( \frac{U'_\infty}{U_\infty} - \alpha \right) = \alpha (1 - \alpha) \left[ 1 + \epsilon \frac{\alpha}{2\alpha - 1} \right] + O(\epsilon^2) \quad (A.70) \]

\[ \frac{U'_\infty}{U_\infty} = 1 + \frac{\alpha (1 - \alpha)}{2\alpha - 1} + O(\epsilon^2) \quad (A.71) \]

or based on the expression for \( C_T = 4\alpha (1 - \alpha) \),

\[ \frac{U'_\infty}{U_\infty} = 1 + \frac{C_T}{4\sqrt{1 - C_T}} + O(\epsilon^2) \quad (A.72) \]
A. GLAUERT MODEL FOR SMALL BLOCKAGE RATIOS
APPENDIX B

Quadrilateral source panel expressions

A quadrilateral source panel is a flat source distribution surface with four corners. In general, the four corners are described in the xy-plane, while the z-axis is oriented perpendicular to the panel. The corner point are thus given as \((x_1, y_1, 0), \ldots, (x_4, y_4, 0)\). Following the results obtained by Hess & Smith (1962), the expressions for the velocity potential and the velocity components, based on the results of the integration process and expressed in the local reference frame of the panel, can be subdivided in far field equations and near field equations.

The near-field equations on the other hand are much more complex. The expression for the velocity potential can be written as

\[
\Phi(x, y, z) = -\frac{\sigma}{4\pi} \left\{ \begin{array}{l}
(x - x_1)(y_2 - y_1) - ((y - y_1)(x_2 - x_1)) \ln r_1 + r_2 + d_{12} \\
+ (x - x_2)(y_3 - y_2) - ((y - y_2)(x_3 - x_2)) \ln r_2 + r_3 + d_{23} \\
+ (x - x_3)(y_4 - y_3) - ((y - y_3)(x_4 - x_3)) \ln r_3 + r_4 + d_{34} \\
+ (x - x_4)(y_1 - y_4) - ((y - y_4)(x_1 - x_4)) \ln r_4 + r_1 + d_{41} \\
\end{array} \right\} \\
+ |z| \left[ \tan^{-1} \left( \frac{m_{12}e_1 - h_1}{2r_1} \right) - \tan^{-1} \left( \frac{m_{12}e_2 - h_2}{2r_2} \right) \\
+ \tan^{-1} \left( \frac{m_{23}e_2 - h_2}{2r_2} \right) - \tan^{-1} \left( \frac{m_{23}e_3 - h_3}{2r_3} \right) \\
+ \tan^{-1} \left( \frac{m_{34}e_3 - h_3}{2r_3} \right) - \tan^{-1} \left( \frac{m_{34}e_4 - h_4}{2r_4} \right) \\
+ \tan^{-1} \left( \frac{m_{41}e_4 - h_4}{2r_4} \right) - \tan^{-1} \left( \frac{m_{41}e_1 - h_1}{2r_1} \right) \right] \right. \right. (B.1)
\]

with

\[
d_{ij} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \tag{B.2}
\]

\[
m_{ij} = \frac{y_j - y_i}{x_j - x_i} \tag{B.3}
\]

\[
r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + z^2} \tag{B.4}
\]

\[
e_i = (x - x_i)^2 + z^2 \tag{B.5}
\]

\[
h_i = (x - x_i)(y - y_i) \tag{B.6}
\]
The velocity components, derived from the velocity-potential equation, are

\[ u(x, y, z) = \frac{\sigma}{4\pi} \left[ \frac{y_1 - y_4}{d_{12}} \ln \frac{r_1 + r_2 + d_{12}}{r_1 + r_2 - d_{12}} + \frac{y_1 - y_2}{d_{23}} \ln \frac{r_2 + r_3 + d_{23}}{r_2 + r_3 - d_{23}} ight. \\
+ \frac{y_1 - y_3}{d_{31}} \ln \frac{r_3 + r_4 + d_{31}}{r_3 + r_4 - d_{31}} + \frac{y_4 - y_1}{d_{41}} \ln \frac{r_1 + r_4 + d_{41}}{r_1 + r_4 - d_{41}} \right] \quad (B.7) \]

\[ v(x, y, z) = \frac{\sigma}{4\pi} \left[ \frac{x_2 - x_1}{d_{12}} \ln \frac{r_1 + r_2 + d_{12}}{r_1 + r_2 - d_{12}} + \frac{x_2 - x_3}{d_{23}} \ln \frac{r_2 + r_3 + d_{23}}{r_2 + r_3 - d_{23}} ight. \\
+ \frac{x_2 - x_4}{d_{34}} \ln \frac{r_3 + r_4 + d_{34}}{r_3 + r_4 - d_{34}} + \frac{x_4 - x_1}{d_{41}} \ln \frac{r_1 + r_4 + d_{41}}{r_1 + r_4 - d_{41}} \right] \quad (B.8) \]

\[ w(x, y, z) = \frac{\sigma}{4\pi} \left[ \tan^{-1} \left( \frac{m_{12}e_1 - h_1}{zr_1} \right) - \tan^{-1} \left( \frac{m_{12}e_2 - h_2}{zr_2} \right) \right. \\
+ \tan^{-1} \left( \frac{m_{23}e_2 - h_2}{zr_2} \right) - \tan^{-1} \left( \frac{m_{23}e_3 - h_3}{zr_3} \right) \right. \\
+ \tan^{-1} \left( \frac{m_{34}e_3 - h_3}{zr_3} \right) - \tan^{-1} \left( \frac{m_{34}e_4 - h_4}{zr_4} \right) \right. \\
+ \tan^{-1} \left( \frac{m_{41}e_4 - h_4}{zr_4} \right) - \tan^{-1} \left( \frac{m_{41}e_1 - h_1}{zr_1} \right) \right] \quad (B.9) \]

Figure B.1 shows the vector field representation based on the near field equations.
The far-field equations are used when the point of interest is located far away from the source panel, i.e. normally when the distance is 3 to 5 times larger than the average panel diameter/diagonal. This will improve the computational efficiency. In this case the source panel with area $A$ is perceived as a point source. This results in the following equations

\[ \Phi(x, y, z) = -\frac{\sigma A}{4\pi \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}} \]  \hspace{1cm} (B.10)

\[ u(x, y, z) = \frac{\sigma A(x-x_0)}{4\pi \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}} \]  \hspace{1cm} (B.11)

\[ v(x, y, z) = \frac{\sigma A(y-y_0)}{4\pi \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}} \]  \hspace{1cm} (B.12)

\[ w(x, y, z) = \frac{\sigma A(z-z_0)}{4\pi \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}} \]  \hspace{1cm} (B.13)
B. QUADRILATERAL SOURCE PANEL EXPRESSIONS
APPENDIX C

Ring-shaped source panel expressions

C.1. Velocity potential expression

The starting point for deriving the expression of the velocity potential is the general expression for a source/sink panel with a constant source/sink strength distribution in Cartesian coordinates:

\[ \Phi = -\frac{\sigma}{4\pi} \int_S \frac{dS}{\sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}} \]  

(C.1)

Any point \( P(x_0, y_0, z_0) \) on the surface of the ring, with a constant radius \( R \) and an axial distance \( \eta \), can be described by \( S = (R\cos\theta, \eta, R\sin\theta) \). This also results in \( dS = Rd\eta d\theta \). The ring has a length ranging from \((-\delta, \delta)\). Since the ring is axisymmetric, any point in space can be described by only an axial distance \( y \) and a radial distance \( \rho \). This results in a new expression for the velocity potential

\[ \Phi = -\frac{\sigma R}{4\pi} \int_0^{2\pi} d\theta \int_{-\delta}^{\delta} \frac{d\eta}{\sqrt{(\rho - R\cos\theta)^2 + (y - \eta)^2 + (R\sin\theta)^2}} \]  

(C.2)

with

\[ G^2 = R^2 + \rho^2 - 2R\rho \cos\theta \]  

(C.3)

and introducing the substitution

\[ (y - \eta) = G \sinh(\psi) \]  

(C.4)

with

\[ \int_a^b \frac{d\nu}{\sqrt{G^2 + \nu^2}} = \int_{\sinh^{-1}(b/G)}^{\sinh^{-1}(a/G)} d\psi \]  

(C.5)

\[ = \sinh^{-1} \left( \frac{b}{G} \right) - \sinh^{-1} \left( \frac{a}{G} \right) \]  

(C.6)

the final expression for the velocity potential becomes

\[ \Phi = -\frac{\sigma R}{4\pi} \int_0^{2\pi} \left[ \sinh^{-1} \left( \frac{\delta - y}{G} \right) - \sinh^{-1} \left( \frac{-\delta - y}{G} \right) \right] d\theta \]  

(C.7)

C.2. Axial velocity expression

For deriving the axial velocity, the derivative of the velocity potential must be taken in the axial direction

\[ u_y = \frac{\partial \Phi}{\partial y} \]  

(C.8)
C. RING-SHAPED SOURCE PANEL EXPRESSIONS

With

\[ d \left( \sin^{-1}(x) \right) = \frac{dx}{\sqrt{1 + x^2}} \]  

(C.9)

this becomes

\[ u_y = \frac{\sigma R}{4\pi} \int_0^{2\pi} \left[ \frac{1}{\sqrt{G^2 + (\delta + y)^2}} - \frac{1}{\sqrt{G^2 + (\delta - y)^2}} \right] d\theta \]  

(C.10)

Using the following transformations

\[ \cos(\theta) = 2\cos^2 \left( \frac{\theta}{2} \right) - 1 \]  

(C.11)

\[ \beta = \frac{\theta}{2}, \; d\beta = \frac{d\theta}{2} \]  

(C.12)

\[ G^2 + \Delta^2 = R^2 + \rho^2 + \Delta^2 - 2R\rho \cos \theta \]

\[ = \left[ (R + \rho)^2 + \Delta^2 \right] \left[ 1 - \frac{4R\rho}{(R + \rho)^2 + \Delta^2} \cos^2(\beta) \right] \]  

(C.13)

with \( \Delta = (\delta - y) \) or \( \Delta = (\delta + y) \) and with the definition of a complete elliptic integral of the first kind

\[ \tilde{K}(\alpha) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - \alpha^2\cos^2(\theta)}} \]

\[ = \frac{1}{2} \int_0^{\pi} \frac{d\theta}{\sqrt{1 - \alpha^2\cos^2(\theta)}} \]  

(C.14)

the final expression for the axial velocity induced becomes

\[ u_y = \frac{\sigma R}{\pi} \left[ \frac{1}{\sqrt{(R + \rho)^2 + (\delta - y)^2}} \tilde{K} \left( \sqrt{\frac{4R\rho}{(R + \rho)^2 + (\delta - y)^2}} \right) - \frac{1}{\sqrt{(R + \rho)^2 + (\delta + y)^2}} \tilde{K} \left( \sqrt{\frac{4R\rho}{(R + \rho)^2 + (\delta + y)^2}} \right) \right] \]  

(C.15)

C.3. Radial velocity expression

For deriving the radial velocity, the derivative of the velocity potential must be taken in the radial direction

\[ u_\rho = \frac{\partial \Phi}{\partial \rho} \]  

(C.16)

With

\[ d \left( \sin^{-1}(x) \right) = \frac{dx}{\sqrt{1 + x^2}} \]  

(C.17)

this becomes

\[ u_\rho = \frac{\sigma R}{4\pi} \int_0^{2\pi} \rho - R\cos(\theta) \left[ \frac{\delta - y}{\sqrt{G^2 + (\delta - y)^2}} + \frac{\delta + y}{\sqrt{G^2 + (\delta + y)^2}} \right] d\theta \]  

(C.18)
C.3. RADIAL VELOCITY EXPRESSION

Using the following transformations

\[ \cos(\theta) = 2\cos^2\left(\frac{\theta}{2}\right) - 1 \]  
(C.19)

\[ \beta = \frac{\theta}{2}, \quad d\beta = \frac{d\theta}{2} \]  
(C.20)

\[ G^2 + \Delta^2 = R^2 + \rho^2 + \Delta^2 - 2R\rho \cos \theta \]

\[ \frac{\rho - R\cos(\theta)}{G^2} = \frac{1}{2\rho} \left( \frac{\rho^2 - 2\rho R \cos \theta + R^2}{G^2} + \frac{\rho^2 - R^2}{G^2} \right) \]

\[ \frac{\rho - R\cos(\theta)}{G^2} = \frac{1}{2\rho} \left( 1 + \frac{\rho^2 - R^2}{G^2} \right) \]  
(C.22)

with \( \Delta = (\delta - y) \) or \( \Delta = (\delta + y) \), the expression reduces to

\[ u_\rho = \frac{\sigma R}{2\pi \rho} \int_0^{\pi/2} \left( 1 + \frac{\rho^2 - R^2}{G^2} \right) \left[ \frac{\delta - y}{\sqrt{(R + \rho)^2 + (\delta - y)^2}} \sqrt{1 - \frac{4R \rho}{(R + \rho)^2 + (\delta - y)^2} \sin^2(\beta)} \right] \]

\[ + \frac{\delta + y}{\sqrt{(R + \rho)^2 + (\delta + y)^2}} \sqrt{1 - \frac{4R \rho}{(R + \rho)^2 + (\delta + y)^2} \sin^2(\beta)} \]  
(C.23)

With the definition of a complete elliptic integral of the first kind

\[ \tilde{K}(\alpha) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - \alpha^2 \sin^2(\theta)}} \]  
(C.24)

and the definition of a complete elliptic integral of the third kind

\[ \Pi(\alpha, \beta) = \int_0^{\pi/2} \frac{d\theta}{(1 - \alpha^2 \sin^2(\theta)) \sqrt{1 - \beta^2 \sin^2(\theta)}} \]  
(C.25)

the final expression for the radial velocity can be written as

\[ u_\rho = \frac{\sigma R}{2\pi \rho} \left\{ \frac{\delta - y}{\sqrt{(R + \rho)^2 + (\delta - y)^2}} \tilde{K} \sqrt{1 - \frac{4R \rho}{(R + \rho)^2 + (\delta - y)^2}} \right. \]

\[ + \frac{\delta + y}{\sqrt{(R + \rho)^2 + (\delta + y)^2}} \tilde{K} \sqrt{1 - \frac{4R \rho}{(R + \rho)^2 + (\delta + y)^2}} \]

\[ + \frac{\rho^2 - R^2}{(R + \rho)^2} \sqrt{(R + \rho)^2 + (\delta - y)^2} \Pi \left( \frac{4R \rho}{(R + \rho)^2 + (\delta - y)^2}, \frac{4R \rho}{(R + \rho)^2 + (\delta - y)^2} \right) \]

\[ + \frac{\delta + y}{\sqrt{(R + \rho)^2 + (\delta + y)^2}} \Pi \left( \frac{4R \rho}{(R + \rho)^2 + (\delta + y)^2}, \frac{4R \rho}{(R + \rho)^2 + (\delta + y)^2} \right) \} \]  
(C.26)
C. RING-SHAPED SOURCE PANEL EXPRESSIONS
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