Licentiate Thesis

Effects of Dark Matter in Astrophysical Systems

Stefan Clementz

Theoretical Particle Physics, Department of Physics, School of Engineering Sciences, Royal Institute of Technology, SE-106 91 Stockholm, Sweden

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Abstract

When studying astrophysical structures with sizes ranging from dwarf galaxies to galaxy clusters, it becomes clear that there are vast amounts of unobservable gravitating mass. A compelling hypothesis is that this missing mass, which we call dark matter, consists of elementary particles that can be described in the same manner as those of the standard model of particle physics. This thesis is dedicated to the study of particle dark matter in astrophysical systems.

The solar composition problem refers to the current mismatch between theoretical predictions and observations of the solar convection zone depth and sound speed profile. It has been shown that heat transfer by dark matter in the Sun may cool the solar core and alleviate the problem. We discuss solar capture of a self-interacting Dirac fermion dark matter candidate and show that, even though particles and antiparticles annihilate, the abundance of such a particle may be large enough to influence solar physics.

Two venues for observing dark matter are through direct and indirect detection methods. Direct detection experiments aim to measure recoiling atoms in a dark matter-target nuclei interaction while indirect detection methods aim to observe other signals that dark matter may give rise to, an example being the particles that are produced as dark matter annihilates. We combine the two for inelastic dark matter, where a small mass splitting separates two dark matter particles and scattering takes one into the other. The scattering kinematics is affected by the mass splitting, which in turn affects direct detection and solar capture rates. We also discuss the information contained in a direct detection signal and how it can be used to infer a minimal solar capture rate of dark matter.

When comparing simulated dark matter halos with collisionless dark matter with dark matter halos inferred from observations, problems appear in the smallest structures. A proposed solution is self-interacting dark matter with long range forces. As the simplest models are under severe constraints, we study self-interactions in a model of inelastic dark matter.

key words: dark matter, self-interactions, solar capture, helioseismology, inelastic dark matter, direct detection, indirect detection.
Sammanfattning

När man studerar astrofysikaliska strukturer med storlekar allt ifrån dvärggalaxer till galaxkluster visar det sig finnas mycket stora mängder icke-observerbar gravi-
tationell massa. En lockande hypotes kallade mörka materia består av elementar-
partiklar som kan beskrivas på samma sätt som partiklarna i standardmodellen. I
denna avhandling har effekter av mörk materia i astrofysikaliska system studerats.

Problemet med solens sannansättning syftar på den dåliga överenstämmelsen
mellan teoretiska föresägelser och observationer av djupet på solens konvektionszon
och ljudhastighetsprofil. Det har visats att mörk materia kan leda värme effektivt i
solen, vilket sänker temperaturen i solens kärna och därmed lindrar problemet. Vi
diskuterar infångning av en själv-växelverkande Dirac-fermion och visar att, även
om partiklar och antipartiklar annihilerar så kan infångningen vara stor nog att
påverka solfysiken.

Två olika sätt att observera mörk materia är genom direkt och indirekt de-
tektion. Direkt detektion innebär att försöka mäta de rekyler som uppstår i en
kollision mellan mörk materia och en atom medan man med indirekt detektion
försöker mäta andra signaler som mörk materia ger upphov till, till exempel de
partiklar som uppstår när mörk materia annihilerar. Vi kombinerar de två i fal-
let med inelastisk mörk materia, en modell med två mörk materia-partiklar vars
massor skiljer sig väldigt lite och spridning omvandlar det ena tillståndet till det
andra. Detta påverkar spridningskinematiken vilket i sin tur påverkar spridnings-
hastigheten i experiment för direkt detektion och infångningshastigheten i solen.
Vi diskuterar även vilken information som kan tas fram ur en signal och hur denna
can användas för att bestämma en undre gränspunkt på solens infångningshastighet av
mörk materia.

När man jämför simulerade mörk materia-halos där mörk materia är kollisionslös
med de som beräknas från observationer uppstår problem i de minsta strukturerna.
En lösning som föreslagits är självväxelvärkande mörk materia med lång räckvidd.
De enklaste modellerna är väldigt begränsade så vi studerar självväxelvärkningar
i en modell med inelastisk mörk materia.

key words: mörk materia, självväxelverkan, solinfångning, helioseismologi, in-
elastisk mörk materia, direkt detektion, indirekt detektion.
Preface

This thesis is the result of my research at the Department of Theoretical Physics (now Department of Physics) from December 2014 to April 2017. The first part of the thesis presents a summary of the evidence that supports the existence of dark matter and lists different possible candidates. It also contains a description of how particle dark matter may interact with ordinary matter as well as with itself and what consequences this may have on experimental results and the evolution of astrophysical bodies. The second part contains the three papers that my research has resulted in.

List of papers

The scientific papers included in this thesis are:

1. Paper [1] (I)
   M. Blennow and S. Clementz
   *Asymmetric capture of Dirac dark matter by the Sun*
   JCAP **1508**, 036 (2015)
arXiv:1504.05813

   M. Blennow, S. Clementz and J. Herrero-Garcia
   *Pinning down inelastic dark matter in the Sun and in direct detection*
   JCAP **1604**, 004 (2016)
arXiv:1512.03317

   M. Blennow, S. Clementz and J. Herrero-Garcia
   *Self-interacting inelastic dark matter: A viable solution to the small scale structure problems*
arXiv:1612.06681
The thesis author’s contribution to the papers

I participated in the scientific work as well as in the writing of all papers included in this thesis. I am also the corresponding author for all three papers.

1. The paper was based on the work that I did as part of my M.Sc. thesis with some improvements. I performed all numerical computations, constructed all figures except fig. 1 and wrote most of the paper.

2. I performed all numerical computations, constructed all figures and wrote large parts of the paper.

3. I developed the mathematical tools to find a stable numerical solution to calculate the scattering cross sections, performed all numerical computations except those of sec. 2.1, constructed all figures except fig. 1 and wrote most of the paper.
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Part I

Introduction and background material
Chapter 1

Introduction

Throughout time, mankind has always been concerned with understanding how nature works. Physicists have, with the help of mathematics, arrived at two very successful theories to describe the world. The theory of general relativity that describes the macroscopic world and the standard model of particle physics (SM) that describes the building blocks of all that we know of. The theory of general relativity has so far passed every experimental test while the standard model appears to be flawed in various aspects. It postulates that neutrinos are massless while neutrino oscillation experiments show that they do in fact have mass. It also fails in generating the asymmetry of baryons and antibaryons in the early Universe.

When we observe astrophysical objects with sizes ranging from dwarf galaxies to the entire Universe, a troublesome problem appears. Describing gravity with the general theory of relativity, stellar objects that are gravitationally bound to galaxies as well as galaxies that are gravitationally bound to galaxy clusters appear to be more strongly bound than they should be unless there is a large amount of mass that we cannot observe. The first observation of missing mass is widely attributed to Fritz Zwicky who, in 1933, observed that the velocity dispersion of galaxies within the Coma cluster was too high to be explained by the amount of visible matter [4]. This observation was disregarded for several decades before the subject was considered again after new, rigid observations verified the claims that mass was missing, this time due to stars in galaxies swirling around the center at too large velocities, a nice historical account of which can be found in ref. [5]. Nowadays, a wealth of experimental data supports the existence of missing mass.

One popular solution to the missing mass problem is to assume that it is made up of at least one new type of particle that can be described using the framework of particle physics. In this way, not only can one describe the missing mass with these so called dark matter (DM) particles but one can also incorporate them in the solution of the other problems of the SM. If the shortcomings of the SM can be explained by unknown elementary particles, we have strong reasons to expect
these new particles to interact with those of the SM. This expectation has spawned a variety of different techniques to look for DM particles.

This thesis deals with the phenomenology of DM particle physics. Currently, direct detection (DD) methods attempt to measure the recoils produced when atoms in underground experiments scatter against DM particles originating from the DM halo around the Milky Way (MW). Indirect detection (ID) experiments attempt to detect DM by searching for unexpected features in astrophysical systems that can be explained by DM particles, or the particles that are produced as DM annihilates. DM self-interactions may also be responsible for solving problems in DM halos that surround galaxies.

1.1 Outline

This thesis is organized as follows: In chapter 2, evidence for DM is presented together with a list of common particle physics models and a brief mentioning of alternatives. Chapter 3 contains an in-depth discussion regarding DM interactions with SM particles through effective operators or mixing. We also discuss how to calculate amplitudes and relate quantum mechanical potentials by comparing amplitudes calculated from quantum mechanics and Feynman diagrams in quantum field theory. Finally, we discuss a particular model of inelastic DM. Chapter 4 concerns the properties of DM halos such as the DM density and velocity distribution profiles. Also described are the problems that arise in smaller DM halos and how DM self-interactions can solve them. In chapter 5, we review DD and ID experimental efforts and discuss how a signal in a DD experiment can be used to make predictions for ID experiments. Chapter 6 concludes the thesis.
Chapter 2

Dark matter

2.1 Observational evidence for dark matter

There is now an overwhelming amount of observational evidence for DM from various independent observations [5–7]. The history of how it became one of the biggest problems of modern physics is interesting and involves mainly four different observations.

2.1.1 Galaxy clusters

The first observational piece of evidence comes from the Coma cluster in 1933 by Fritz Zwicky [4]. He was realized that the velocity measurement of galaxies within the cluster could be used along with the virial theorem to estimate the mass of the entire galaxy cluster. The virial theorem states that the time average over the total gravitational potential energy $U$ is related to the time average over the total kinetic energy $T$ by

$$-2\langle T \rangle = \langle U \rangle.$$  \hfill (2.1)

The kinetic energy can be estimated by taking the mean of squared velocities for a sample of galaxies $\overline{v^2}$ for the velocity and the total mass of the cluster $M_{\text{tot}}$ for the mass. The average gravitational potential energy can be estimated using some radius $R$ that is representative of the distance between two galaxies. This leads to the relation

$$M_{\text{tot}}\overline{v^2} = \frac{1}{2}G\frac{M_{\text{tot}}^2}{R},$$  \hfill (2.2)

from which $M_{\text{tot}}$ can be found. Zwicky’s estimate for the mass came out to be several hundred times larger than the estimates produced by observing luminous matter. A few years later, another study performed on the Virgo cluster showed similar results [8]. Others were skeptical of the results and the general opinion was that these systems were simply not understood well enough and attention was mainly focused on other subjects for over thirty years.
2.1.2 Galactic rotation curves

The next huge piece of evidence for DM comes from galactic rotation curves, which tell us how fast stars and interstellar gas of a galaxy rotates around its center [5–7]. With Newtonian mechanics, one can easily deduce that the tangential velocity of stars will be

\[ v(r) \sim \sqrt{G M(r)/r}. \tag{2.3} \]

where \( G \) is the gravitational constant and \( M(r) \) is the mass enclosed at radius \( r \). Intuitively, this tells us that the velocity of stars should decay as \( r^{-1/2} \) in the regions where \( r \) is so large that most of the galaxy’s mass is contained within. With the advent of radio telescopes, galactic rotation curves could be derived from measurements of the 21 cm line of hydrogen. Time and time again, what was observed were rotation curves that would tend to go to as \( v(r) \to \text{const} \). This could only be the case if the mass contained within the radius \( r \) satisfies \( M(r) \propto r^\gamma \). This makes no sense when taking into account that most of the visible mass of a galaxy is contained in the central bulge. The only conclusion that can be drawn from this is that a large quantity of unseen mass is occupying a much larger volume than the one containing most of the luminous matter or that the theory of gravity is wrong.

2.1.3 Gravitational lensing

A third piece of evidence comes from gravitational lensing [9, 10]. Einstein made the prediction using his general theory of relativity that massive objects would deflect light twice as much as the prediction from Newtonian mechanics. The idea of weak gravitational lensing is simple, the information is hidden in the statistics of an image of many galaxies hidden behind a large structure like a galaxy cluster. The light is lensed when passing through the galaxy cluster, which has the effect of magnifying the background galaxies as well as stretching them. These effects can help deduce the mass distribution of the galaxy cluster. However, performing the measurements is very difficult since galaxies tend to be viewed from an angle where they are not circular. The shear from the gravitational lens is about one percent of the effect of the observed ellipticity of the galaxy. The reduction of the shear noise requires a large sample of background galaxies to be measured to average out the shape noise.

In fact, one of the most convincing evidence for DM comes from gravitational lensing of the galaxy cluster 1E0657-558, commonly called the Bullet Cluster [11]. A galaxy cluster contains not only galaxies but a very large amount of intergalactic gas. The galaxies and gas behave differently in a collision between two clusters. The galaxies will behave like collisionless particles while the gas will slow down from friction. This naturally leads to a separation between the gas and galaxies after the collision, which is precisely what is seen in the Bullet Cluster. It is a fact that about one percent of a galaxy cluster mass is in the form of stars making up galaxies, 5 – 15 percent in the form of interstellar gas and the rest in the form of DM [12–14]. It was deduced by the means of weak lensing that the mass of each
cluster traced the galaxies and not the gas thus giving proof that the majority of mass in the system must be in the form of DM.

2.1.4 Cosmic microwave background

A fourth piece of evidence comes from the cosmic microwave background (CMB). When the Universe was young, particles existed in chemical and thermal equilibrium in a hot plasma. As it expanded, heavier particles seized to be produced effectively and decayed or annihilated away. After some time, only free protons, helium nuclei, electrons, photons and neutrinos still existed until eventually the temperature was low enough for the electrons to become bound to form atoms. The scattering rate of photons on free electrons decreased as the free electron density decreased until the photons stopped interacting at all. These photons are the ones that make up the CMB. The CMB spectrum is essentially invariant regardless of the direction in which it is measured and its temperature is about 2.725 K. What is really interesting are the fluctuations in the CMB spectra that are found below the mK range. These fluctuations tell us that the Universe was not perfectly homogeneous at the time when the photons decoupled. The fluctuations seen in the CMB can be decomposed into large and small scale fluctuations.

The large scale fluctuations are rather easy to understand. The Universe was matter dominated at the time of last scattering. Since the matter was not uniformly distributed across the Universe, mass was dragged towards places that already had an overabundance of mass by gravitation, which deepened the wells in the gravitational potentials in the Universe. A photon we see coming from a place with a large amount of mass, i.e. from a gravitational well, would have to climb out of it, resulting in the photon being redshifted. The photons coming from these regions would therefore have a colder temperature than those in the surrounding region.

The small scale fluctuations are a bit more subtle, but more informative. Since DM decouples from the SM soup early, it starts clumping up at earlier times, which will create net movement of the baryons towards the DM clumps. This will in turn increase the pressure in the gravitational well. At some point, the pressure will become large enough for the fluid to start expanding outwards. The expansion will keep going until the pressure can no longer drive it but gravitational contraction takes place and the cycle starts over. These are so-called baryonic acoustic oscillations. As photon decoupling occurs, depending on whether the fluid was in a contracting or expanding phase, the photons from such regions will be redshifted or blueshifted. The small scale fluctuations also tells us that DM was cold (non-relativistic). If DM was hot, their large velocities would allow them to escape small density perturbations, effectively erasing small scale fluctuations. Since these are observed in the data, DM must have been cold enough to become trapped in these shallow gravitational wells.

The CMB has been measured to a remarkable precision by the Planck satellite [15]. The evidence for DM within the observed CMB requires one to assume ΛCDM as the underlying cosmology. The space-time distance between two points in
ΛCDM is assumed to be described by the Friedmann-Lemaître-Robertson-Walker metric [16]

\[ ds^2 = dt^2 - a(t)^2 \left[ \frac{1}{1 - kr^2} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right], \]  

(2.4)

where \( k \) describes the curvature of space and the scale factor \( a(t) \) parametrizes the expansion of the Universe. The Friedman equations tell us that the expansion of the Universe, i.e., the behaviour of \( a(t) \), depends on the content of DM, baryonic matter, the curvature \( k \) and dark energy which traditionally enters as a cosmological constant \( \Lambda \) in the Einstein field equation [16, 17]. As briefly discussed above, the observed CMB spectrum can be used to derive these quantities. The results from the Planck collaboration indicate that within ΛCDM cosmology, there is roughly five times as much DM in the Universe as there is baryonic matter [18].

### 2.2 Dark matter candidates

There is a large number of different DM candidates [19–21]. A very common assumption regarding DM is that it is an elementary particle that is part of some dark sector that interacts very weakly with the SM particles. The reason to believe that this is the case is because we know that the SM is flawed. A nice fix to the problems of the SM is to introduce new particles that can also explain the observed amount of DM. Simply put, particle DM can kill at least two birds with one stone. That said, many particle DM models do not aim to solve the SM problems but attempt to address only the existence of DM and astrophysical observations. There are many plausible DM candidates but it is interesting to list the more popular models.

### 2.2.1 MACHOs and black holes

An early hypothesis was that DM is simply made up of baryonic objects that would be difficult to observe since their emitted light signal is so weak. The common name for these objects are MAssive Compact Halo Objects (MACHOs). For example, after a neutron star is formed, it will radiate away energy, which lowers its temperature. If no source is available to heat the neutron star, it will be practically unobservable since its emitted light signal is so weak. Other viable objects could, for example, be brown and old white dwarf stars. Gravitational microlensing has been used to search for MACHOs. By observing some tens of millions of stars for 7 years, ref. [22] found that the DM content of the MW is primarily made of something different from MACHOs, at least for objects in the mass range \( 10^{-7} M_\odot \lesssim M \lesssim 15 M_\odot \). This was later confirmed in the higher end of the mass range, \( 0.1 M_\odot \lesssim M \lesssim 20 M_\odot \) by ref. [23]. Thus, it seems like the majority of DM in galaxies are not made up of baryonic matter, which strengthens the support for DM being non-baryonic.
2.2. Dark matter candidates

Black holes are extreme astrophysical objects that form when very massive objects are forced to occupy very small regions in space. The very simple relation between the Schwarzschild radius of a black hole, $r_{\text{BH}}$, and its mass $M$ reads

$$r_{\text{BH}} = \frac{2GM}{c^2}.$$

(2.5)

A fascinating and easily grasped fact is that if the entire Earth was compressed into a black hole, its resulting radius would be about 9 mm, not much larger than the size of a peanut. We know of a large number of black holes, one of which is located in the center of our own galaxy. The production mechanism for black holes today is extremely violent supernova explosions, where the quantum pressure of the Pauli principle in a neutron star is overpowered by the gravitational attraction of a giant star. However, in the early Universe, primordial black holes within a wide range of masses may have been created during or after the period of inflation and make up the DM [24]. Much effort has gone into studying these as DM candidates.

2.2.2 Particle dark matter

There is no shortage of elementary particle candidates for DM and it is at least interesting to understand what makes them important in particle physics. The following is a list of proposed DM candidates that are theoretically well motivated.

Neutrinos

The SM does not even need to be modified to find an interesting DM candidate as it was early noted that neutrinos made up the DM. At a first glance, they make great candidates since they interact only by the exchange of $W$ and $Z$ bosons. However, their very low mass leads to two problems. Results from the Planck satellite tells us that $\sum \nu m_\nu < 0.23$ eV [18]. If neutrinos were truly the DM, they would be hot DM [16], which is incompatible with the results from the CMB. Moreover, their small mass indicate that the neutrino density in the Universe is simply too small to make up a significant portion of the total DM density. The three standard light neutrinos partaking in the SM are thus not the bulk of DM.

The caveat here is that the three neutrinos of the SM are left-handed. The mass terms of fermions in the Lagrangian of the SM couples the left- and the right-chiral fields of the fermion as

$$\mathcal{L}_{\text{mass}} = -m \bar{\Psi} \Psi = -m \left( \bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \right).$$

(2.6)

Since the SM does not contain any right-handed neutrinos, the neutrinos of the SM are massless but we know from neutrino oscillations that the three mass eigenstates of neutrinos have different masses. The simple addition of right handed neutrinos can solve this problem by allowing for simple Dirac masses to be generated by the Higgs mechanism [25]. Since the right handed neutrinos interact extremely weakly
with the SM (it is a singlet under all three groups), they can be the DM. Additional neutrino species and right-handed neutrinos could also be responsible for creating the matter-antimatter asymmetry in the universe through leptogenesis although it appears that sterile neutrinos cannot explain both at the same time [25].

**Axions**

There is a gauge invariant term that is usually not explicitly written down in the SM Lagrangian,

\[ \bar{\theta} \frac{\alpha_a}{8\pi} G^a_{\mu \nu} \tilde{G}^{a, \mu \nu}. \] (2.7)

The appearance of this term in the Lagrangian is due to two causes. The first is the vacuum structure of QCD and the second is the Adler-Bell-Jackiw anomaly appearing from chiral transformations of the QCD fields to diagonalise the mass matrix in the electroweak sector. The Lagrangian term leads to CP violation in the strong sector. A direct physical consequence of this is that the neutron gains an electric dipole moment, which is not seen experimentally. Precise measurements of this attribute of the neutron has been used to place the limit \( |\bar{\theta}| < 10^{-10}. \) This is an example of extreme fine-tuning. A possible solution to the problem is to promote \( \bar{\theta} \) to a dynamical field and imposing a global \( U(1)_{PQ} \) symmetry on the SM Lagrangian that is subsequently broken, which automatically drives \( \bar{\theta} \) to an extremely small number [26].

The consequence is that the axion arises as the Nambu-Goldstone boson when the \( U(1)_{PQ} \) symmetry breaks [27, 28]. The original axion was very quickly ruled out but generalizations led to axionic DM candidates that interact very weakly with SM particles. The mass of the axion is heavily constrained from astrophysical bounds resulting in a very light axion. Nevertheless, the production rate of these particles could be very large in the early Universe and they could therefore be a DM candidate [29].

**Supersymmetry**

In supersymmetric (SUSY) models, supersymmetric partners for every particle of the SM are added, where all of these particles are exactly identical to their SM counterpart besides their spin which differs by a half [30]. That is, all fermions get bosonic counterparts and all bosons get fermionic counterparts. The scalars corresponding to the quarks and leptons are called squarks and sleptons respectively, while the fermions corresponding to the Higgs, photon, Z-boson etc., are called Higgsino, photino, Zino and so on.

A good theoretical motivation for SUSY is to solve to the hierarchy problem. The Higgs mechanism is responsible for breaking the \( SU(2) \times U(1) \) gauge group of the SM electroweak sector to the \( U(1) \) group of quantum electrodynamics (QED) [31–33]. This mechanism requires the Higgs field which, upon taking a vacuum expectation value, gives rise to a neutral scalar particle. This is the Higgs
2.2. Dark matter candidates

The problem with the Higgs boson is that its couplings to other particles are proportional to the fermion masses. There is no symmetry protecting the Higgs mass and since we expect new physics at some scale much larger than the electroweak scale, $\Lambda_{EW}$ $\sim$ 100 GeV, the mass of the Higgs boson would get very large radiative corrections, placing it at a much larger value than the observed value at $m_h$ $\sim$ 125 GeV [36]. Taking SUSY into account, every loop contribution to the Higgs mass would be canceled by the contribution from the SUSY partner. This protection of the Higgs mass would then keep the value down to what we observe as long as the SUSY particles are not much heavier than their SM counterparts. In general, this would imply that the masses of the SUSY particles lie in the TeV range [37].

Another interesting fact in the SM is the running of the coupling constants. As the energy scale increases, the strength of the $U(1)$ gauge coupling increases while the $SU(2)$ and $SU(3)$ gauge couplings decrease. With SUSY particles added, it is possible to have a unification of all three couplings at a single energy scale. This could then signal that a larger gauge group was broken at this energy scale. Much like $SU(2) \times U(1)$ was broken to $U(1)$ of QED by the Higgs mechanism, $SO(10)$ can be broken down into the SM gauge group. This is the general idea behind Grand Unified Theories, commonly abbreviated as GUTs [38].

Within SUSY, the lightest neutralino (of which there are 4), a linear combination of the SUSY partners of the Higgs, the hypercharge and third $SU(2)$ vector boson, is a prime DM candidate provided that R-parity is a good symmetry of the theory. R-parity takes the value $-1$ for SUSY particles and $+1$ for ordinary particles. If R-parity is violated, the neutralino can decay into SM particles and there can be lepton and baryon number violations allowing for proton decay on which there are extremely stringent limits [39]. Thus, R-parity violating SUSY is very constrained. SUSY dark matter has been extensively studied as DM, see e.g. ref. [40].

Kaluza-Klein dark matter

Kaluza-Klein (KK) DM arises in models with universal extra dimensions (UEDs) through which the SM particles propagate [41]. These UEDs are curled up in the sense that their geometry can be described by, e.g., a circle with a very small circumference. If they are sufficiently small in size, very high energies are required to probe them, which hides them from the four-dimensional space-time that we are familiar with.

In KK theories, fields will depend on the extra dimensions as $\psi(x, x_5, ...)$ where $x$ is the usual 4 dimensional event vector. The action in a KK model is naturally generalised to an $N$-dimensional integral of the Lagrangian with the higher-dimensional fields. The interesting physics arises when integrating out the extra dimensions. The resulting effective action describes a 4-dimensional theory where every field in the $N$-dimensional theory gives rise to infinitely many particles with ever increasing masses. Every such set of fields appearing in the effective Lagrangian is called a
KK tower corresponding to the original higher-dimensional field. The SM particles appear at the lowest level of some of the towers.

As in the case of SUSY, there can be a conserved quantity called KK parity that arises as a remnant of momentum conservation in the extra dimensions. If KK parity is conserved, the lightest KK particle in one of the towers could be a DM candidate.

**Asymmetric dark matter**

In order to produce the baryon asymmetry of the Universe, the three Sakharov conditions need to be fulfilled [42]. These are:

1. Baryon number violation
2. C and CP violation
3. Interactions outside of thermal equilibrium

These conditions are in principle already satisfied by the electroweak sector of the SM since the weak force is both C and CP violating, baryon number violation occurs through sphalerons [43], while out of thermal equilibrium interactions occur during the electroweak phase transition. However, they are not strong enough to produce the large asymmetry that is observed. The main idea behind asymmetric dark matter is that the overabundance of matter over antimatter is connected to a similar asymmetry in the dark sector [44]. Some asymmetry then needs to develop in one of the two sectors or both at the same time. Any such initial asymmetry is then transferred to the other sector by interactions such that the SM gets an abundance of baryons over antibaryons, while the dark sector gets an abundance of DM over antiDM.

An interesting curiosity regarding asymmetric DM is the fact that the observed abundances of DM, $\Omega_{\text{DM}}$, and baryons, $\Omega_{\text{B}}$, satisfy the approximate relationship [18],

$$\Omega_{\text{DM}} \simeq 5\Omega_{\text{B}}.$$  \hspace{1cm} (2.8)

If the number density of DM and SM particles are similar, which is the case in many asymmetric DM models [44], the relationship between the DM and baryon abundances implies a similar relationship between the DM and baryon (proton) masses, i.e., $m_{\text{DM}} \sim 5m_p$. This is of interest since, as will be discussed in sec. 5, there are signals in direct detection experiments that fit DM masses at this order of magnitude as well as a possibility to alleviate the solar composition problem.
Chapter 3

Dark matter interactions

Before moving on to discuss how DM interactions may affect DM halos and give rise to signatures in DD and ID experiments, we will discuss how these interactions may be modelled. This chapter will start with a discussion of the effective operator approach followed by mentioning hidden sectors that connects DM to the SM sector through Higgs or kinetic mixing operators. Next, quantum mechanical scattering theory is described, which is necessary for paper III of this thesis. This is followed by reviewing how to extract potentials that can be used to calculate cross sections using the framework of quantum mechanics rather than quantum field theory. Next comes a discussion of how DM interacts with atoms which is crucial to understand when placing bounds on scattering cross sections from DD data. Other processes such as solar capture of DM from the halo also relies crucially on our understanding of the DM-nucleus scattering. Finally, a special case of an inelastic DM model that is considered in papers II and III of this thesis is presented.

3.1 Dark matter effective interactions

A very common way to model interactions of DM with the fermions of the SM is to use effective operators. In theories with mediators that live at some energy scale $\Lambda$ that is much larger than those that are involved in typical interactions in experiments, in particular those that occur in DD experiments as well as in DM halos, the heavy mediators can be integrated out to form effective operators that describe the DM interactions. This is not a far-fetched assumption as we do not see these mediators in high-energy colliders.

It is straight forward to write down interaction terms for scalar $\phi$ and vector $X_\mu$ DM fields as they can interact directly with the quarks and leptons of the SM. In principle, one only has to imagine Lorentz-invariant operators of mass dimension
Chapter 3. Dark matter interactions

5 or higher to construct interaction terms. An example of a direct interaction of a scalar DM particle $\phi$ and a SM fermion field $f$ is given by the operator

$$\mathcal{L}_{\text{scalar}, \text{int}} = \frac{c}{\Lambda} \phi \bar{f} f,$$  \hfill (3.1)

and an example with vector DM is

$$\mathcal{L}_{\text{vector}, \text{int}} = \frac{c}{\Lambda} X_{\mu} X^{\mu} \bar{f} f$$  \hfill (3.2)

where $c = g g_f$ is an effective coupling constant, $g$ is the coupling at the DM-mediator vertex and $g_f$ is the corresponding coupling of the fermion-mediator vertex.

The general structure of operators coupling DM fermions $\chi$ and SM fermions is of the form

$$\mathcal{L}_{\text{fermion}, \text{int}} = \frac{c}{\Lambda^2} (\bar{\chi} \Gamma \chi)(\bar{f} \Gamma' f),$$  \hfill (3.3)

where the Dirac field bilinears $\Gamma$ and $\Gamma'$ must carry the same number of Lorentz indices and the choices available are $\Gamma = 1, \gamma^5, \gamma^\mu, \gamma^\mu\gamma^5, \Sigma_{\mu\nu}$. These operators and a similar set of operators for real/complex scalar dark matter are listed in, e.g., ref [45].

All effective operators listed above are non-renormalizable since the effective coupling constant $c/\Lambda^n$ has dimension $1$/mass$^n$.

3.2 Hidden sectors

There are a couple of very simple extensions of the SM that allow for renormalizable interactions between the dark and SM sectors. The dark photon is a well studied example [46] as well as the Higgs portal [47–50]. These models are often considered in the context of hidden sectors, which are hidden in the sense that they may exhibit a vast complexity while remaining virtually invisible to us except for their very weak interactions through some field in the dark sector that couples only to the SM Higgs or through mixing between the neutral SM vector bosons and dark vector bosons. The strength of the mixing is set by the mixing parameter $\epsilon$.

3.2.1 Higgs portal

The general idea behind the Higgs portal is very simple. Denoting the Higgs field by $H$, the term $H^\dagger H$ is a singlet under the SM gauge group. If a real or complex scalar field $\phi$ exists in addition to the SM particles, the Lagrangian can contain the term

$$\mathcal{L}_{\text{Higgs portal}} = c H^\dagger H \phi^\dagger \phi.$$  \hfill (3.4)

This term provides interactions between the particles in the hidden sector that the $\phi$ couples to and SM particles that the Higgs couples to. In the very minimal case,
\(\phi\) is a DM candidate itself without the presence of an extended dark sector as long as the Lagrangian is invariant under a \(\phi \rightarrow -\phi\) transformation \[51, 52\]. The other possibility is that \(\phi\) is a heavy scalar in the dark sector in which case the dark sector has to contain at least one lighter particle that can be a DM candidate. The field \(\phi\) can also be a dark gauge boson.

### 3.2.2 Dark photons

In models with dark photons, interactions between the dark and SM sectors occurs through kinetic mixing between the SM photon and a dark photon \(A'\) with mass \(m_{A'}\). We consider the Lagrangian

\[
\mathcal{L}_{\text{QED mixing}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} - \frac{\epsilon}{2} F_{\mu\nu} F'^{\mu\nu} + \frac{1}{2} m_{A'}^2 A'_{\mu} A'^{\mu},
\]

(3.5)

where \(F_{\mu\nu}\) is the field strength of the photon and \(F'_{\mu\nu}\) is the field strength of the dark photon. Making a redefinition of the photon field \(A_\mu \rightarrow A_\mu - \epsilon A'_{\mu}\) gives the Lagrangian

\[
\mathcal{L}_{\text{QED mixing}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} \frac{\epsilon}{2} + \frac{1}{2} m_{A'}^2 A'_{\mu} A'^{\mu}.
\]

(3.6)

The field strength of the photon field is appropriately normalized but not the field strength of the dark photon. Making the redefinition \(A'_{\mu} \rightarrow A'_{\mu} / \sqrt{1 - \epsilon^2}\) fixes the normalization at the cost of redefining the dark photon mass,

\[
\mathcal{L}_{\text{QED mixing}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} \frac{\epsilon}{2} \left(\frac{m_{A'}}{\sqrt{1 - \epsilon^2}}\right)^2 A'_{\mu} A'^{\mu}.
\]

(3.7)

With this redefinition of the photon field, any field in the SM that couples to the electromagnetic field will couple to the dark photon since

\[
\mathcal{L}_{\text{QED, int}} = -\epsilon \bar{\psi} \gamma^\mu \psi A_\mu - \frac{\epsilon \epsilon}{\sqrt{1 - \epsilon^2}} \bar{\psi} \gamma^\mu \psi A'_\mu.
\]

(3.8)

The kinetic mixing between the dark and SM photons as derived above is only valid in the limit where the dark photon mass is very small in comparison to the Z-bosons mass. Since we really want a theory that is gauge invariant with respect to the electroweak force, \(A'_{\mu}\) mixes with the hypercharge field \(B_{\mu}\) (with field strength \(B_{\mu\nu}\)) of the electroweak sector rather than the QED photon. The Lagrangian after EW symmetry breaking becomes

\[
\mathcal{L}_{\text{EW mixing}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} - \frac{\epsilon c_W}{2} F'_{\mu\nu} F'^{\mu\nu}
\]

\[
+ \frac{\epsilon s_W}{2} F'_{\mu\nu} Z^{\mu\nu} + \frac{1}{2} m_Z^2 Z_{\mu} Z^{\mu} + \frac{1}{2} m_{A'}^2 A'_{\mu} A'^{\mu},
\]

(3.9)

where we used that \(B_\mu = -s_W Z_\mu + c_W A_\mu\), \(s_W\) and \(c_W\) are the sine and cosine of the Weinberg angle, \(Z_\mu\) is the Z boson field and \(m_Z\) is its mass. In this case,
diagonalizing the Lagrangian is rather messy but possible \cite{53–55}. The SM photon can be decoupled by the same procedure as in the previous case except that now $A_\mu \to A_\mu - e c_W A'_\mu$, which again induces couplings of the charged SM particles to $A'$. The mess appears when trying to get rid of the kinetic mixing between $Z$ and $A'$, and normalizing their kinetic terms while keeping the mass eigenstates diagonal. The redefinition of $Z$ will induce couplings between the SM fields and the dark photon that depend on $m_{A'}$, $m_Z$ and $\epsilon$. It turns out that, in the limit where $m_{A'} \ll m_Z$, the mixing between the dark photon and the $Z$ is proportional to $m_{A'}^2 / m_Z^2$ \cite{55}. In this case, the SM particles interact with the dark sector through photon mixing and any interaction picked up from $Z$ mixing is highly suppressed.

There can also be mass-mixing between the dark photon and the SM $Z$-boson \cite{56, 57}. The Lagrangians for these models are similar to the above, but mixing occurs through the fields themselves,

$$
\mathcal{L} = -\frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} - \frac{1}{2} m_Z^2 Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{2} m_{A'}^2 A'_\mu A'^{\mu} - \delta m^2 A'_\mu Z^\mu. \quad (3.10)
$$

Again, the redefinitions of the $Z$ induces couplings between the SM and the dark photon.

### 3.3 Cross sections

#### 3.3.1 Non-relativistic scattering theory

Consider the scattering between two particles with masses $m_1$, $m_2$ and velocities $\mathbf{v}_1$, $\mathbf{v}_2$, that are located at $\mathbf{x}_1$ and $\mathbf{x}_2$ respectively. The wave function $\Psi(\mathbf{x}_1, \mathbf{x}_2)$ that describes the scattering process satisfy the Schrödinger equation

$$
\left[ -\nabla_x^2 / 2m_1 - \nabla_y^2 / 2m_2 + V(\mathbf{x}_1 - \mathbf{x}_2) \right] \Psi(\mathbf{x}_1, \mathbf{x}_2) = E \Psi(\mathbf{x}_1, \mathbf{x}_2). \quad (3.11)
$$

where $V(\mathbf{x}_1 - \mathbf{x}_2)$ is a potential that describes their interaction. Making the change of variables

$$
\mathbf{x} = \mathbf{x}_1 - \mathbf{x}_2, \quad \mu = \frac{m_1 \mathbf{x}_1 + m_2 \mathbf{x}_2}{m_1 + m_2} \quad (3.12)
$$

brings eq. (3.11) into the form

$$
\left[ -\nabla^2_\mu / 2(m_1 + m_2) - \nabla^2_\mathbf{x} / 2\mu + V(\mathbf{x}) \right] \Psi(\mathbf{x}, \mu) = E \Psi(\mathbf{x}, \mu), \quad (3.13)
$$

where $\mu = m_1 m_2 / (m_1 + m_2)$ is the reduced mass of the two particles. Working in a coordinate system where the total momentum is zero, $\mu$ is constant and thus $\Psi(\mathbf{x}, \mu) = \psi(\mathbf{x})$. The scattering wave function is now governed by the equation

$$
\left[ -\nabla^2_\mathbf{x} / 2\mu + V(\mathbf{x}) \right] \psi(\mathbf{x}) = E_{c.o.m} \psi(\mathbf{x}). \quad (3.14)
$$
3.3. Cross sections

This wave function is identical to that of a single particle with mass $\mu$ and velocity $v = v_1 - v_2$ so that the momentum and center of mass energy is $k = \mu v$ and $E_{\text{c.o.m.}} = k^2/2\mu$. Up to a normalization constant, the asymptotic behaviour of the wave function that solves eq. (3.14) is given by [58]

$$\psi(x) = e^{ik \cdot x} + \frac{e^{ikr}}{r} f(\theta). \quad (3.15)$$

The scattering amplitude is given by $f(\theta)$ where $\theta$ is the angle between the incoming and outgoing momenta and the differential cross section is finally given by

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2. \quad (3.16)$$

When inelastic scattering can occur, the Schrödinger equation has to describe the evolution between several different particle states. Scattering is inelastic when the momentum of the outgoing state is different from the incoming one. This may happen when atomic excitation is possible in a collision, or, as we shall see, in scattering processes with models in which DM is inelastic [59]. This can be represented by promoting $\psi(x_1, x_2)$ to an $N \times 1$ vector and the potential to an $N \times N$ matrix. In component form, the wave function in eq. (3.15) is generalised to

$$\psi_n(x) = c_n e^{ik_n \cdot x} + \frac{e^{ik_n r}}{r} f_n(\theta) \quad (3.17)$$

where $c_n = 1$ if the $n$th state is scattering and the $f_n(\theta)$ are the amplitudes of the outgoing states.

### 3.3.2 Amplitudes

The amplitude $f(\theta)$ can be calculated in various ways. In the first order Born approximation [58], the amplitude is given by

$$f(\theta) = -\frac{\mu}{2\pi} \int e^{-i(k_f - k_i) \cdot r'} V(r') d^3 r' = -\frac{\mu}{2\pi} \tilde{V}(k_f - k_i), \quad (3.18)$$

where $k_f$ is the momentum of the particle after the collision. The difference of the two, $q = k_i - k_f$, is the momentum transfer. The Born approximation works well if the potential is weak, and only slightly deforms the incoming wave function. The direct analogue of this in quantum field theory is that the tree level diagram provides the dominant contribution to the amplitude.

However, when the potential describing the interaction is attractive and long ranged, the coupling constant as well as the DM mass are large and the particles involved in a collision have low relative velocities, the Born approximation fails due to the formation of quasi bound states. Diagrammatically, the amplitude picks up large contributions from diagrams in which there are exchanges of multiple mediator
particles, see fig 3.1. Luckily, there are methods to avoid calculating infinitely many Feynman diagrams with an ever increasing number of loops.

One such method is the partial wave analysis in which the wave function is decomposed in spherical harmonics, which is well described in, e.g., ref [58]. Assuming that the potential is spherically symmetric and aligning the coordinate system in a way so that \( \mathbf{k} = k\hat{z} \), the expansion is

\[
\psi(\mathbf{x}) = \sum_l (2l + 1) P_l(\cos(\theta)) R_l(r). \tag{3.19}
\]

The asymptotic wave function in eq. (3.15) becomes

\[
\psi(r, \theta) = \sum_l (2l + 1) P_l(\cos(\theta)) \left( \frac{e^{ikr} - (-1)^l e^{-ikr}}{2ikr} + \frac{f_l}{r} e^{ikr} \right), \tag{3.20}
\]

and the Schrödinger equation in eq. (3.14)

\[
\left[ \frac{1}{r^2} \partial_r (r \partial_r) - \frac{l(l + 1)}{r^2} + k^2 - 2\mu V(r) \right] R_l(r) = 0. \tag{3.21}
\]

The Schrödinger equation has to be solved for each \( l \) and mapped onto the corresponding term of eq. (3.20) to determine the partial amplitude \( f_l \). Once all individual \( f_l \) are known, the full amplitude is given by

\[
f(\theta) = \sum_l (2l + 1) P_l(\cos(\theta)) f_l. \tag{3.22}
\]

The generalization of this procedure to the case of multiple scattering channels is straight forward. However, if the energy in the collision is small resulting in closed channels, the numerical solution will diverge. This problem was only approximately solved for the general problem setting of paper III in which we present a significantly improved numerical solution.
3.3. Cross sections

3.3.3 Quantum field theory and non-relativistic potentials

Given a Lagrangian density, it is not trivial to find the non-relativistic potential that would describe the interaction. We can derive the effective non-relativistic potential by requiring that the $S$-matrix element is the same regardless of whether it is calculated from the first order Born-approximation in quantum mechanics or from quantum field theory.

The example we take is the case of $n$ fermion fields interacting with each other by exchange of a massive vector boson,

$$
\mathcal{L}_{\text{int}} = -\frac{1}{4}(F_{\mu\nu})^2 - \frac{1}{2}m_A^2 A_\mu A^\mu + \bar{\psi}_i (i\gamma^\mu \partial_\mu - m_i) \psi_i - g_i \bar{\psi}_i \gamma^\mu \psi_i A_\mu. \quad (3.23)
$$

At tree level, only the $t$-channel diagram in fig. 3.2 contributes to the matrix element for scattering between two particles where $i \neq j$. In the case that $i = j$, the particles are indistinguishable and the $u$-channel diagram contributes as well. In the unitarity gauge, the amplitude is

$$
i\mathcal{M} = -g_i g_j \bar{u}_i^{s'}(p_3) \gamma^\mu u_i^s(p_1) \frac{-i}{q^2 - m_A^2} (g_{\mu\nu} - \frac{q_\mu q_\nu}{m_A^2}) \bar{u}_j^{k'}(p_4) \gamma^\mu u_j^k(p_2), \quad (3.24)
$$

where $s, s', k, k'$ indicate spins of particles and $p_1$ to $p_4$ are the four-momenta of the particles as indicated in the Feynman diagram. Assuming that we work in the center of mass frame, the four-momenta are given by

$$
p_1 = (E_1, p_1) \simeq (m_1, p_1), \quad p_3 = (E_1, p_3) \simeq (m_1, p_3),
$$

$$
p_2 = (E_2, -p_1) \simeq (m_2, -p_1), \quad p_4 = (E_2, -p_3) \simeq (m_2, -p_3), \quad (3.25)
$$

where we used that $E_i \simeq m_i$ in the non-relativistic limit. Moreover, defining the spinors as in ref. [60], we find

$$
\bar{u}_i^{s'}(p_3) \gamma^\mu u_i^s(p_1) \simeq 2m_i \delta^{s,s'} \delta_\mu^0. \quad (3.26)
$$
The last identification to make is that in the center of mass frame, the momentum $q$ carried by the vector boson is

$$q = p_1 - p_3 \simeq (0, p_1 - p_3).$$  \hfill (3.27)

All plugged in, the matrix element becomes

$$iM = -g_ig_j \frac{i}{(p_3 - p_1)^2 + m_A^2} 4m_i m_j \delta^{s,s'} \delta^{k,k'}$$  \hfill (3.28)

We can identify the S-matrix element as

$$\langle p_3, p_4 | iT | p_1, p_2 \rangle = (2\pi)^4 \delta^4(p_3 + p_4 - p_1 - p_2)iM$$  \hfill (3.29)

In the language of quantum mechanics, the S-matrix element in the first order Born-approximation is given by

$$\langle p_3 | iT | p_1 \rangle = -2\pi i \delta(E_{p_3} - E_{p_1}) \tilde{V}(p_3 - p_1) \delta^{s,s'} \delta^{k,k'},$$  \hfill (3.30)

where $\tilde{V}$ is the Fourier transform of $V(x)$. Although it is not obvious, one can identify that the Fourier transform of the potential will be given by

$$\tilde{V}(p_3 - p_1) = -g_ig_j \frac{1}{(p_3 - p_1)^2 + m_A^2}$$  \hfill (3.31)

The reason that the masses do not enter in the potential is that we normalize particle states differently in quantum field theory and quantum mechanics:

$$\text{QM : } | p, s \rangle = a_p^* | 0 \rangle, \quad \text{QFT : } | p, s \rangle = \sqrt{2E_p} a_p^* | 0 \rangle,$$  \hfill (3.32)

which eliminates the factor $4m_i m_j$. There is also a difference in the definition of the amplitudes. It is straightforward to check that the differential cross section found by plugging $\tilde{V}$ into eq. (3.18) and subsequently using eq. (3.16) agrees with the cross section from QFT. Finally, the potential in position space is found by performing an inverse transform of eq. (3.31), which gives

$$V(r) = \frac{g_ig_j}{4\pi} e^{-m_A r}.$$  \hfill (3.33)

We can make further sense of this by considering the case in which $g_1 = g_2 = e$, where $e$ is the electromagnetic charge, and $m_A \to 0$. We then have that the potential reduces to

$$V(r) = \frac{\alpha}{r}.$$  \hfill (3.34)

What we have found is that interactions between, for example, an electron and a muon can, unsurprisingly, be modelled by a repulsive Coulomb potential. The Coulomb potential as well as the Yukawa potential of eq. (3.33) where $m_A$ is small are both examples of long range forces. These will be central in the discussion...
regarding self-interacting DM in halos in section 4.3 as they can give rise to large scattering cross sections in the non-relativistic limit.

As described in numerous textbooks on the subject of non-relativistic scattering, the situation is a little bit trickier when we deal with indistinguishable particles [58, 61–63]. In addition to the t-channel diagram of fig. 3.2, the amplitude picks up a negative contribution from the u-channel diagram as well. On the other hand, since we deal with fermions, the quantum mechanical spatial wave function must be symmetric or antisymmetric depending on whether the incoming particles are described by a triplet or a singlet spin state. This implies that the amplitude will contain two parts

\[ f(\theta) \rightarrow f(\theta) \pm f(\pi - \theta). \] (3.35)

where + indicates an incoming triplet and − a singlet state. For identical fermions such as electrons, this takes care of the spin structures of both the t- and the u-channel diagram as well as their relative minus sign.

### 3.3.4 Dark matter-nucleus scattering

When dark matter collides with nuclei, calculating the cross section as above is not sufficient. When the momentum transfer in a collision is large enough, roughly when the wave-length \( \frac{h}{q} \) is smaller than the atomic nucleus radius, scattering does not take place between DM and the entire nucleus, but rather a part of the atom. It is then important to understand the inner structure of the nucleus. Usually, this dependence is factored into a form factor \( F(q) \) along with the zero momentum transfer cross section \( \sigma_0 \) which is the total cross section in the limit where no momentum is exchanged. The recoil that a nucleus picks up in a collision in terms of the momentum transfer will be \( E_R = \frac{q^2}{2m_N} \) where \( m_N \) is the mass of the nucleus. It is conventional to write down the differential cross section (not to be confused with the different differential cross section \( d\sigma/d\Omega \)) as a function of the recoil energy [40],

\[ \frac{d\sigma}{dE_R} = \frac{m_N}{2\mu^2v^2} \sigma_0 |F(E_R)|^2. \] (3.36)

The reason for the denominator taking this form is that the total cross section at zero momentum (energy) transfer will be

\[ \int_0^{2\mu^2v^2/m_N} \frac{d\sigma}{dE_R} \bigg|_{E_R=0} dE_R = \sigma_0. \] (3.37)

where the integral is over all possible recoil energies in the collision. All this tells us is that if we set the form factor equal to one, we recover \( \sigma_0 \).
As DM scatters with atoms, the scattering cross section will be spin-dependent or spin-independent. The following operator example $\mathcal{L}_{SI}$ yields a spin-independent cross section while $\mathcal{L}_{SD}$ yields a spin-dependent contribution:

$$\mathcal{L}_{SI} = \frac{c_q}{\Lambda^2} \bar{\chi} \gamma^\mu \chi \gamma_\mu q,$$

$$\mathcal{L}_{SD} = \frac{d_q}{\Lambda^2} \bar{\chi} \gamma^\mu \gamma^5 \chi \gamma_\mu \gamma^5 q.$$  \hspace{1cm} (3.38)

The spin-dependence of the latter is seen in the non-relativistic limit when the fields are expanded in terms of their spinors. An example that gives rise to the former type of operator is the kinetic mixing discussed in sec. 3.2.2. In a theory where DM particles couple to a dark gauge boson that mixes kinetically with the SM photon, DM-nucleus scattering arises naturally with a cross section proportional to the mixing parameter squared.

We can gain understanding of why the form factor is necessary by considering DM-nucleus scattering through $\mathcal{L}_{SI}$. The amplitude to calculate will be of the form

$$\mathcal{M} \sim \langle \chi_f, N | \mathcal{L}_{SI} | \chi_i, N \rangle.$$  \hspace{1cm} (3.39)

The state $|N\rangle$ of the nucleus is a complicated object consisting of valence and sea quarks as well as gluons. When the quark operators in $\mathcal{L}_{SI}$ act on $|N\rangle$, only the valence quarks will contribute to the matrix element [45, 64]. We can then write down the matrix element in terms of the number of nucleons $A$ and protons $Z$ as

$$i \mathcal{M}(q = 0) = Z f_p + (A - Z) f_n,$$  \hspace{1cm} (3.40)

where $f_p = (2c_u + c_d)/\Lambda^2$ and $f_n = (c_u + 2c_d)/\Lambda^2$ are DM-proton and DM-neutron couplings. If $\mu_{\chi N}$ is the reduced mass of the DM-nucleus system and $\chi$ is a Dirac fermion, then $\sigma_0$ is given by

$$\sigma_0 = \frac{\mu_{\chi N}^2}{\pi} |Z f_p + (A - Z) f_n|^2 = \sigma_{\chi p} \left( \frac{\mu_{\chi N}}{\mu_{\chi p}} \right)^2 |Z + (A - Z) \kappa|^2.$$  \hspace{1cm} (3.41)

The last step defines the DM-proton cross section $\sigma_{\chi p}$ as $\sigma_0$ when $m_N = m_p$ and $Z = A = 1$. Note that $\kappa = f_n/f_p$ is defined as the isospin-violating factor that takes into account that DM may couple more ($\kappa > 1$) or less ($\kappa < 1$) strongly to neutrons than to protons. Other operators yielding spin-independent cross sections have the same form apart from other definitions of the coupling constants to protons and neutrons. In calculating the amplitude, the interaction between the DM and the quark constituents of the nucleons in the atom will take place at a specific location. However, the nucleons are smeared out all over the nucleus. This is precisely what the form factor is defined to take into account. It is a reasonable assumption that the mass density $\rho(r)$ describes how the particles are distributed inside the nucleus. According to how we define amplitudes in terms of the Fourier transform of the potential, we have that the form factor is given by

$$F(q) = \frac{1}{m_N} \int \rho(r) e^{iq \cdot r} d^3r.$$  \hspace{1cm} (3.42)

The factor of $1/m_N$ properly normalizes the density since the integral of $\rho(r)$ over the entire nucleus gives the mass of the nucleus.
3.3. Cross sections

A popular, very simple form factor is the exponential \([65, 66]\)
\[ F(E_R) = e^{-E_R/2E_0}, \]  
(3.43)
where \(E_0 = \frac{3\hbar^2}{2m_N R^2}\) is a characteristic energy that depends on the nucleon mass and its radius \(R\). Another commonly used form factor is the Helm form factor \([67–69]\) that assumes a density profile
\[ \rho(r') \sim \int \rho_0(r') \rho_1(r - r') d^3r', \]
where \(\rho_0\) is a constant hard sphere density and \(\rho_1 \sim \exp(-(r/s)^2)\), where \(s\) controls the surface thickness of the nucleus. The form factor for this model is
\[ F(q^2) = 3 \frac{j_1(qR)}{qR} e^{-(qs)^2/2}, \]  
(3.44)
where \(j_1(x)\) is a spherical bessel functions of the first kind. Using shell model calculations, ref. \([70]\) showed that the Helm form factor is very accurate at smaller momentum transfers, but generally overpredicts the magnitude of the form factor and dislocates its minima at larger momentum transfers.

As usual when spin is involved, spin-dependent scattering is not as intuitive as the spin-independent case shown above. Calculating the cross section in this case requires knowledge of nuclear physics as the quarks that DM couples to will contribute to the spin of the nucleon, which in turn contributes to the spin of the nucleus. Quoting the results, the cross section is given by \([40, 68, 71]\)
\[ \frac{d\sigma}{dq^2} = \frac{8}{\pi v^2} \frac{J + 1}{J} G_F^2 \left[ a_p \langle S_p \rangle + a_n \langle S_n \rangle \right]^2 \frac{S(q)}{S(0)}. \]  
(3.45)
Here, the constant \(J\) is the total angular momentum of the nucleus and \(G_F\) is the Fermi constant. The quantity \(\langle S_p \rangle\) is the expectation value for the spin of the nucleus that is carried by all protons and \(\langle S_n \rangle\) is the corresponding quantity for the neutrons. The constants \(a_p\) and \(a_n\) are defined as
\[ a_{p,n} = \sum_{u,d,s} \frac{d_q}{\sqrt{2}G_F \Lambda^2} \Delta q^{(p,n)}, \]  
(3.46)
where \(\Delta q^{(p,n)}\) represents the fraction of spin of each proton or neutron that is carried by quarks of type \(q\). Finally, we identify the form factor
\[ |F(q)|^2 = S(q)/S(0). \]  
(3.47)
where
\[ S(q) = (a_p + a_n)^2 S_{00}(q) + (a_p - a_n)^2 S_{11}(q) + (a_p + a_n)(a_p - a_n) S_{01}(q). \]  
(3.48)
The form factors \(S_{ij}(q)\) can then be calculated from nuclear shell model calculations. Assuming that particles in scattering processes are non-relativistic, Lagrangian interaction terms such as the one presented in eq. (3.38) can be described by 3-dimensional Hamiltonian interaction terms invariant under Galilean transformations rather than Lorentz transformations. Using accurate numerical shell model
calculations, one can extract form factors for a variety of interesting operators as done in, e.g., refs. [72, 73]. The results from this procedure are more theoretically robust than the approximations in eqs. (3.43) and (3.44) since one can attain specific form factors for any particular isotope. This can be done for both spin-independent and spin-dependent operators.

### 3.4 Inelastic dark matter

We will now discuss a specific model for inelastic DM particles. In general, inelastic DM models contain at least two DM particles with masses that fulfil

\[ m_2 - m_1 = \delta, \quad |\delta| \ll m_2, m_1. \]  

(3.49)

There are many inelastic DM models of varying complexity [74–76]. We will look at a simple example that illuminates the important points of a general inelastic DM model and then comment briefly on effective theories. The Lagrangian for our example is given by

\[ \mathcal{L} = -\frac{1}{4}(F'_{\mu\nu})^2 - \frac{1}{2}m_A A'_\mu A'^\mu \bar{\psi} (i\gamma^\mu \partial_\mu - M) \psi - g\bar{\psi}\gamma^\mu \psi A'_\mu, \]  

(3.50)

where \( \psi \) is a Dirac fermion interacting via a massive gauge boson \( A' \). Suppose that we add a Majorana-type mass term to the Lagrangian so that all terms contributing to the mass of the fermion are

\[ \mathcal{L}_{mass} = -M\bar{\psi}\psi - \frac{\delta}{4} (\bar{\psi}^c \psi + \bar{\psi} \psi^c), \]  

(3.51)

where \( M \gg \delta \). We can define the fermion field in terms of two left-handed spinors \( \eta \) and \( \xi \) such that \( \psi^T = (\eta^T \ i\xi \ \sigma^2) \). In this notation, the mass terms of the Lagrangian can be written as

\[ \mathcal{L}_{mass} = -\frac{i}{2} (\eta^T \sigma^2 \ \xi^T \sigma^2) \left( \begin{array}{c} \delta^2 \\ \frac{\delta}{M} \\ \frac{\delta}{2} \end{array} \right) \left( \begin{array}{c} \eta \\ \xi \end{array} \right) + h.c. \]  

(3.52)

The kinetic part of the Lagrangian in this notation becomes

\[ \mathcal{L}_{kinetic} = \bar{\psi}i\gamma^\mu \partial_\mu \psi = i\eta^T \sigma^2 \partial_\mu \eta + i\xi^T \sigma^2 \partial_\mu \xi \]  

(3.53)

where \( \bar{\sigma}^\mu = (1, \sigma) \). This is exactly the kinetic structure of Majorana particles. To diagonalize the mass matrix, the fields \( \varphi_1 \) and \( \varphi_2 \) are defined by the matrix \( U \)

\[ \begin{pmatrix} \eta \\ \xi \end{pmatrix} = U \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}. \]  

(3.54)

In order to find two distinct Majorana particles, we require that \( U \) does not mix \( \varphi_1 \) and \( \varphi_2 \) in the kinetic terms in eq. (3.53). This is ensured by \( U \) being unitary. The mass matrix is diagonalized if, in addition to unitarity, \( U \) satisfies

\[ U^T \begin{pmatrix} \frac{\delta}{2} & M \\ M & \frac{\delta}{2} \end{pmatrix} U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}. \]  

(3.55)
3.4. Inelastic dark matter

By imposing these constraints on a general complex valued matrix, one finds that $U$ has the form

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} i & 1 \\ -i & 1 \end{pmatrix}.$$ (3.56)

The kinetic and mass terms of the Lagrangian in terms of $\varphi_1$ and $\varphi_2$ are now given by

$$\mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{mass}} = \sum_{i=1,2} i\varphi_i^\dagger \partial^\mu \varphi_i - \frac{i}{2} m_i (\varphi_i^T \sigma^2 \varphi_i - \varphi_i^T \sigma^2 \varphi_i^*) ,$$ (3.57)

where $m_1 = M - \delta/2$ and $m_2 = M + \delta/2$ and we identify the mass splitting as $\delta$.

Finally, we can define two four-component fields $\chi_i^T = (\varphi_i^T \ i\varphi_i^T \sigma^2)$ and find that the full Lagrangian can be written on the form

$$\mathcal{L} = -\frac{1}{4} (F_{\mu
u}^\prime)^2 - \frac{1}{2} m_{A'} A'_\mu A'^\mu + \sum_{i=1,2} \frac{1}{2} \chi_i (i\gamma^\mu \partial_\mu - m_i) \chi_i + \mathcal{L}_{\text{int}} .$$ (3.58)

The true reason why these models are called inelastic is apparent when analyzing the interaction term,

$$\mathcal{L}_{\text{int}} = \frac{ig}{2} \chi_2 \gamma^\mu \chi_1 A'_\mu - \frac{ig}{2} \chi_1 \gamma^\mu \chi_2 A'_\mu .$$ (3.59)

This tells us that in any interaction vertex, there will be a $\chi_1$ and a $\chi_2$ connecting to the $A'$ field, but no vertex with two $\chi_1$ or $\chi_2$. Hence, a $\chi_1$ particle scattering into a $\chi_2$ particle is a tree level process while a $\chi_1$ or $\chi_2$ remaining the same after scattering can occur only through loop diagrams, as can be seen in fig. 3.3.

The models are thus inelastic in the sense that in non-relativistic collisions with nucleons, a significant amount of kinetic energy will be absorbed or released because of the change in mass of the incoming and outgoing DM particle. It should also be noted that this is a direct consequence of our choice of Majorana masses. If the left-handed and right-handed fields in $\psi$ picks up Majorana masses of different magnitudes, there will be terms connecting two $\chi_1$ or two $\chi_2$ fields directly to the $A'$ [75]. These terms will however be suppressed by a factor of order $O(\delta/m_i)$ so that
the species conversion cross section will be dominant as long as it is kinematically allowed.

The zero-momentum scattering cross section in the inelastic case is slightly modified as compared to the elastic cross section, $\sigma_0$. As a consequence of the outgoing state having a different momentum, it will pick up a phase-space factor that depends on the relative velocity of the two colliding particles $v$ and the reduced masses of the DM and target particle $A$ as

$$\sigma_{0,\text{inel}} = \sqrt{1 - \frac{2\delta}{\mu_{XA}v^2}} \sigma_0.$$  \hspace{1cm} (3.60)

One can also remain agnostic as to what the underlying physics at higher energies is and construct inelastic DM scenarios with effective interaction operators by starting with the effective operator of eq. (3.3) and then apply the breaking of the Dirac fermion into two Majorana fermions. The resulting scattering cross section with $\Gamma = \gamma^\mu$, $\Gamma' = \gamma_\mu$ yields a spin-independent cross section. Unlike the the elastic DM case, the only effective operator that gives rise to spin-dependent cross sections is [77]

$$\mathcal{L}_{\text{int}} = \frac{c}{\Lambda^2} \bar{\psi} \Sigma^{\mu\nu} \psi \bar{f} \Sigma_{\mu\nu} f,$$  \hspace{1cm} (3.61)

where again $\Lambda$ is the energy scale at which the effective theory breaks down and $\Sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$. 

...
Chapter 4

Dark matter halos

4.1 Dark matter halos and their galaxies

To understand how galaxies formed in the early Universe requires an understanding of how the DM halos that hosts them formed. Due to the nonlinear density perturbations at the scales where structures form, numerical simulations provide the best tools to study how these structures evolved over time. As the Universe expands, the regions with higher densities of DM will expand a little slower than the regions with a less than average densities of DM. Gravitational “collapse” occurs when the overdense regions become about twice as dense as the universal average, at which point they stop expanding [78]. A consequence of this is that the smaller halos of galactic sizes are more dense than the halos of galaxy clusters, since the galaxy sized halos collapsed at an earlier time. As the baryonic content within these halos collides, excites and radiates away energy, it will contract towards the parts of the halo where the gravitational potential well is the deepest. As more and more baryonic matter accumulates and continues to cool, star formation is initiated inside the large clouds of gas. DM halos will not collapse to the same extent because they lack an effective cooling mechanism. DM halos thus tend to be much more extended in size than the galaxies they host [79, 80].

We know that most, if not all, larger galaxies contain a black hole at their centers. When these black holes feed on matter that falls into them, large amounts of gravitational binding energy will be released. This radiative pressure will not affect stars but will have large effects on the gas, possibly to the extent that gas is driven beyond the escape velocity of smaller galaxies [81]. Furthermore, supernovae are fairly energetic events that occur on a regular basis in galaxies. When these go off, a large amount of energy is released, which in turn injects a large amount of heat into the halo. Even though numerical simulations of the evolution of DM halos have reached very impressive levels, it is difficult to incorporate these effects. Typically, $N$-body simulations where the number of particles are in the billions
are employed. However, the total mass of a galaxy the size of our own is roughly $10^{12} M_\odot$, which is an insanely large number for an $N$-body simulation. Since this implies that masses of the particles in the billion body simulations have masses thousands of times larger than the Sun, this explains the difficulty of fully resolving DM halos. As we will discuss in sec. 4.2, the supernova and black hole feedback effects described above may help to resolve problems that occur in smaller halos.

### 4.1.1 Dark matter halo profiles

When analysing the effects of particle DM in DM halos, it is often necessary to make assumptions regarding the DM density profile and its velocity distribution. These will naturally depend on the details of collapse, such as asymmetries in the collapsing cloud, feedback effects that induce time variation in the gravitational potential and so on. Nevertheless, there are a few spherically symmetric density profiles that are usually considered. A very common profile to consider is the Navarro-Frenk-White (NFW) profile \[82\]

$$\rho(r) = \frac{\rho_{\text{crit}} \delta_c}{(r/r_s)(1 + r/r_s)^2},$$  \hspace{1cm} (4.1)

where $r_s = r_{200}/c$ is a characteristic radius, $\delta_c$ is a characteristic overdensity and $\rho_{\text{crit}} = 3H_0^2/8\pi G$, $H_0$ is the current Hubble constant. The constant $c$ is determined by requiring that the average density of the halo within the radius $r = r_{200}$ is $200\rho_{\text{crit}}$.

Another notable profile is the Einasto profile \[83\]

$$\rho(r) = \rho_s \exp \left(-c \left(\frac{r}{r_s}\right)^{1/\alpha} - 1 \right),$$  \hspace{1cm} (4.2)

where $r_s$ is the radius in which half of the total halo mass is contained, $\rho_s$ is the density at this radius and $\alpha$ is a constant that describes the logarithmic slope, $d\ln \rho/d\ln r \propto r^{1/\alpha}$ and $c$ is a number to be fit.

These profiles have both been used in the context of finding out what the local dark matter density $\rho_\chi$ is, local in this case referring to the dark matter density in the MW at our radius \[84\]. Actually, there are many studies that attempt to determine $\rho_\chi$ under various assumptions and most of these find values of $\rho_\chi$ towards the higher end of the range $0.3 - 0.4$ GeV/cm$^3$ \[85–88\].

### 4.1.2 Galactic velocity distribution

Another very important property of the galactic DM halo is the velocity distribution of the particles in it. Often, when needed to be specified, a Maxwell–Boltzmann distribution is assumed

$$f(\vec{v}) = \frac{n_\chi}{(2\pi/3)^{3/2}\sigma^3} \exp \left(-\frac{3}{2} \frac{\vec{v}^2}{\sigma^2} \right),$$  \hspace{1cm} (4.3)
where $n_\chi = \rho_\chi/m_\chi$ is the local number density of DM particles, $\sigma = \sqrt{3/2}v_c$ is the velocity dispersion and $v_c$ is the tangential velocity at the relevant radius (i.e., $v_c = 220$ km/s at the Sun’s distance from the galactic center). This distribution is derived under the assumption that $\rho(r) \propto r^{-2}$ and that the halo is characterized by a fixed temperature $T$. The physical motivation for this type of profile is that the total mass within the radius $r$ is proportional to $r$ and the predicted rotation curves will be constant, which is precisely what is observed. The velocity distribution is often cut off at the escape velocity of the galaxy (at the radius of the Sun, $v_{esc} \approx 530$ km/s, although this value has rather large errors [89]). This cut-off also leads to a different normalization factor. Velocity distributions that are extracted from simulations of DM halos of similar size to the MW shows that the standard halo model yields a reasonable description [90–92].

Although the model shown above is widely used, it should be noted that there is a formula, called the Eddington equation, that can be used to derive the velocity distribution for any given isotropic spherical halo density profile $\rho$,

$$f(\mathcal{E}) = \frac{1}{\sqrt{8\pi^2}} \left[ \int_0^{\mathcal{E}} \frac{d\Psi}{\sqrt{\mathcal{E} - \Psi}} \frac{d^2 \rho}{d\Psi^2} + \frac{1}{\sqrt{\mathcal{E}}} \left( \frac{d\rho}{d\Psi} \right)_{\Psi=0} \right], \quad (4.4)$$

where $\Psi(r) = -\phi(r) + \phi(\infty)$ and $\mathcal{E} = -E_{kin} + \Psi(r)$ and $\phi(r)$ is the gravitational potential.

Since the Sun moves through the galactic halo, the velocity distribution in the Solar rest frame will be shifted by a Galilean transformation with the solar velocity $v_\odot$, $f_{Sun}(\vec{v}) = f(\vec{v} + \vec{v}_\odot)$. The speed distribution as observed at the location of the Sun by an observer moving with the Sun will then be given by integrating out the angular dependence of the velocity distribution and the result is

$$f_{Sun}(v) = \frac{3n_\chi v}{2\sqrt{\pi^2}v^2} \left[ \exp \left( -\frac{3}{2} \left( \frac{v - v_\odot}{\sigma} \right)^2 \right) - \exp \left( -\frac{3}{2} \left( \frac{v + v_\odot}{\sigma} \right)^2 \right) \right]. \quad (4.5)$$

Going even further, the velocity distribution as observed by someone on the Earth is found by making second Galilean transformation of the velocity distribution with the velocity of the earth, $f_{Earth}(v) = f(\vec{v} + \vec{v}_\odot + \vec{v}_e(t))$ where $\vec{v}_e(t)$ is the velocity of the Earth in the solar frame. Even though $v_e \ll v_\odot$, this will give rise to a time dependence with a one year period in the velocity distribution [66, 93–95].

### 4.2 Problems in small scale structures

Although $\Lambda$CDM very successfully predicts the behaviour of larger structures such as cluster and large galaxy halos, it makes some predictions that are in tension with observations. The greatest discrepancies are found in DM halos that host dwarf galaxies, hence they are referred to as the small scale structure problems.
4.2.1 Cusp-vs-core

The first problem is related to the mismatch between observed and numerically constructed DM density profiles of halos that contain dwarf and low-surface brightness galaxies. The density profiles from $N$-body simulations behave as $r^{-\alpha}$ where $\alpha \gtrsim 1$ at small radii. In other words, these profiles are expected to be cuspy towards their center [82, 96–102]. Indeed, the NFW profile in eq. (4.1) can be seen to have this behaviour at small radii. However, when the predicted profiles are compared to the observed dwarf galaxy profiles, a problem arises as these appear to be cored [103–116], i.e., their radial density profile tends to a constant value close to the halo center. This mismatch between predicted cusps vs observed cores is called the cusp-vs-core problem. The problem is not exclusive to halos hosting dwarf galaxies but there are also indications of cores, or at least shallow cusps, with $\alpha < 1$ in galaxy clusters [117–119].

4.2.2 Missing satellites

The second problem is related to the number of predicted subhalos that are large enough to host dwarf galaxies. In ΛCDM, the smallest halos with the largest densities collapse first and as these begin to merge into larger halos, tidal forces will act on the remaining subhalos and strip them of DM. However, simulations show that there will be a significant number of massive subhalos that survive the gravitational interactions to this day [120, 121]. These subhalos are then expected to host dwarf galaxies which are nowhere to be seen. This is called the missing satellites problem. In principle, this may not even be an issue. After all, we are talking about DM halos, not galaxies, which are very difficult to detect if star formation never took place. Maybe they are there but we are just not seeing them because they never effectively formed stars?

4.2.3 Too big to fail

The “too big to fail” problem as formulated in refs. [122, 123] arises when looking at the actual size of the missing satellites. Basically, the extremely well resolved $N$-body simulation Aquarius [102] predicts at least ten subhalos with a maximum of the tangential velocity $V_{\text{max}} > 25$ km/s, while the brightest dwarf spheroidals of the MW are all approximately in the range $10 < V_{\text{max}} < 25$ km/s. Why did the heaviest halos fail in their star formation when they should be the most resilient to any known effects that could inhibit their star-formation – they should be too big to fail. One could argue that the Milky Way is just one halo and that it is a statistical fluke that its largest subhalos failed. However, the same observation has been made in the dwarf spheroidals of the Andromeda Galaxy [124] as well as in isolated halos hosting field galaxies [125].
4.3 Self-interacting dark matter in halos

The problems described in the previous section can be solved by lowering the central density of the DM halo. The too big to fail problem will be solved due to the removal of DM from the core, as this will decrease the gravitational potential that in turn decreases the tangential velocities of contained stars. As was briefly mentioned, baryonic physics such as supernova feedback might over time transform a cuspy halo into a cored one, see, e.g., refs. [126–132]. A more exotic solution is that self-interactions among DM particles could do the job [133, 134]. The magnitude of the cross section required to solve these problems is very large and there are various bounds on the self-scattering cross section that we will review before discussing how a solution to the small scale structure problems can be found using DM interactions.

4.3.1 Bounds on self-scattering cross sections

A first way of placing bounds on the self-scattering cross section was mentioned in sec. 2.1.3 and revolves around the observation of colliding galaxy clusters. In the collision of two clusters, DM particles in each cluster will scatter off each other during their passage. This will evaporate the halos as particles may become gravitationally unbound to both halos if they scatter at angles close to $\pi/2$. Another effect, perhaps more important, is the slow-down of the dark matter halos when particles scatter and lose momentum. As DM slows down, the mass density of the DM halos are located behind the collisionless galaxies that never lost their momentum through collisions. Since observations show that the DM clouds and galaxies are generally centered around the same points, one can place an upper bound on the self-scattering cross sections. These phenomena were studied numerically in ref. [135], that found that while the peaks of the DM distributions and galaxies trace each other, the DM halo mass centroids deform in different ways, depending on the nature of the scattering cross section, inducing a separation of the mass centroids. Observations of colliding clusters and reconstructions of their mass densities have been used to place bounds on the self-scattering cross section [136–138].

A second method to place bounds is that self-interactions of DM particles bound to cluster halos will affect the general structure of the halo. Specifically, inside some radius $r_c$ where each DM particle scatters on average at least once over the lifetime of the halo, the overall structure of the halo should become spherically symmetric over time. One study estimating these effects set very strong bounds [139]. Later, numerical studies have shown that the bounds are significantly less stringent than what was first thought [140].

The third method is based on the evaporation of DM subhalos that host galaxies in cluster halos. The DM particles bound to the cluster halo will on average have a much larger velocity than those bound to the subhalo. As pointed out in ref. [141], DM particles bound to the cluster will scatter against DM particles inside the subhalos, the result of which will almost certainly be the loss of a DM particle in the subhalo. As mass is lost by the subhalo, the galaxy it contains will experience a
shallower gravitational potential, which will alter its structure. One can then place an upper bound on the self-scattering cross section through the requirement that the mass lost over the lifetime of the halo is small so that the galaxies are not too severely affected.

The strongest constraint on the self-scattering cross section $\sigma_{\chi\chi}$ from colliding cluster observations is given by [138]

$$\frac{\sigma_{\chi\chi}}{m_\chi} < 0.47 \text{ cm}^2/\text{g}.$$  \hspace{1cm} (4.6)

The reported constraints from cluster shapes and halo evaporation are of similar magnitude but these are less rigid.

### 4.3.2 Solving the small scale structure problems with self-interacting dark matter

Having discussed bounds on DM self-scattering cross sections, we can now discuss how self-interactions may have a positive effect on small DM subhalos. Since the older DM simulations that predicted cuspy profiles all neglected the effect that self-scattering may have on halos, it was pointed out that the answer might lie in their inclusion into the halo evolution [133, 134]. The idea is that when a subhalo forms, its central DM region is occupied by a large collection of DM particles with little kinetic energy that gives the halo a cuspy profile. If DM self-interacts, scattering will take place between the DM particles that are strongly bound in the cuspy region and DM particles that have gained a fairly large amount of kinetic energy from falling into the center of the halo. In the collision, the particle that was initially trapped in the cuspy region will gain energy and move on a trajectory that takes it further out from the halo center. The overall effect will be the gradual transformation of a cuspy halo into a cored one as heat is transferred from the outer parts into the center of the halo.

Numerical simulations that include the effects of self-interactions that give rise to self-scattering cross sections show that cores can form in halos in this manner [142–147]. The general consensus is that the small scale structure problems can be solved by DM self-scattering with the cross sections around [148–151]

$$\frac{\sigma_{\chi\chi}}{m_\chi} \sim 1 \text{ cm}^2/\text{g},$$  \hspace{1cm} (4.7)

which puts it at odds with the constraints discussed in sec. 4.3.1. This makes it difficult for constant self-scattering cross sections to induce appropriately sized cores in small halos.

Keeping in mind that the average DM particle velocity in clusters is $v_{\text{clusters}} \sim \mathcal{O}(1000 \text{ km/s})$ while in dwarves $v_{\text{dwarves}} \sim \mathcal{O}(10 \text{ km/s})$, it was proposed in ref. [152] that the mediator particle in the self-scattering process could be a light scalar or vector boson that in the framework of quantum mechanics can be described by a
Yukawa-potential as in eq. (3.33). In this scenario, an appropriate mix of coupling constants, mediator and DM masses can yield a velocity dependent self-scattering cross section that is large at the low velocities in dwarves, leading to the creation of cores, while being small enough at the large velocities in clusters to avoid the constraints. This mix of parameters usually imply that the scattering process is non-perturbative and partial wave analysis must be used to find the scattering cross section. It has been shown in numerical simulations that these models do create cores in small scale objects as well as reduce the amount of sub-structure of a halo [153] in this framework. Several studies on this topic have followed with various DM models containing light mediators [150, 154–158].

Models where a single DM particle interacts via a vector boson that couples to the SM through mixing are under very strong constraints from DD experiments [159, 160]. This is a problem since the mediator will exist in a large abundance in the thermal bath of the early Universe, which requires them to be unstable or they will make up a large fraction of the DM or even overclose the Universe by themselves [159]. Assuming that the mediator is unstable, the element abundances produced in the Big Bang nucleosynthesis may not match the observed values unless the mediator decays quickly enough [159]. In sec. 3.2, it was seen that the interaction term between a dark photon and SM charged particles is proportional to the mixing, which means that both the DM-nucleus scattering cross section and mediator decay will be suppressed by the mixing squared. In order to describe interactions by mixing with a mediator that decays fast enough, one must somehow get around the strong DD constraints to relax the constraint on the mixing parameter.

The evasion of DD constraints is a natural feature in inelastic DM models such as the one presented in sec. 3.4. In paper III of this thesis, we considered the two-state inelastic DM scenario with a light mediator and we showed that the self-scattering cross section for the inelastic models in sec. 3.4 can indeed reproduce the behaviour necessary to solve small scale structure problems while avoiding the bounds from cluster sized halos.
5.1 Direct detection experiments

The idea of DD was proposed shortly after it was discovered that DM may consist of weakly interacting particles [161]. The idea is that as the Earth plows through the Milky Way halo, a large flux of DM particles will pass through it. As long as DM interacts with SM particles, a collision will occur every now and then, which gives the recoiling nucleus a sizeable recoil energy. A wealth of DD experiments employ various methods to look for these recoiling atoms. The DM particles that these experiments attempt to detect are generally in the GeV-TeV range, which implies that the recoil energies of the atoms are typically in the 1-100 keV range. The threshold energies $E_{\text{thr}}$, i.e., the smallest recoil energies that DD experiments can measure are generally in the lowest end of this range. For this reason, DD experiments have very little sensitivity to DM lighter than a few GeV, which is why bounds on such particles are very weak.

The differential rate in a DD experiment that consists of a single type of target nuclei at which a DM particle with velocity $v$ interacts and gives rise to a particular recoil energy $E_R$ will be given by [69]

$$R(E_R, t) = \frac{1}{m_A} \int_{v_m}^{\infty} d^3 v v f_{\text{det}}(\vec{v}, t) \frac{d\sigma(v, E_R)}{dE_R}$$ (5.1)

and is measured in counts/kg/day/keV. In the case where the DD experiment consist of several targets, the total differential rate is the sum of the differential rates on each target multiplied by the fractional detector mass of that particular target. The mass of the target particle enters as $m_A$. The velocity distribution of DM particles in the halo enters as $f_{\text{det}}(\vec{v}, t)$, which is expressed in the rest frame of the experiment. As described in Sec. 4.1.2, the possible time dependence of the
velocity distribution comes from Earth’s revolution around the Sun. The differential cross section \(d\sigma(v,E_R)/dE_R\) is given in eq. (3.36). The velocity \(v_m\) that enters in the lower limit of the integral is the smallest velocity that can give rise to a recoil energy \(E_R\). Kinematics tells us that it is given by

\[
v_m = \sqrt{\frac{m_A E_R}{2\mu_{\chi A}^2}},
\]

(5.2)

where \(\mu_{\chi A} = m_\chi m_A/(m_\chi + m_A)\) is the reduced mass of the DM-target nuclei system.

The differential rate for scattering with a spin-independent cross section can be found by plugging eqs. (3.36) and (3.41) into (5.1)

\[
\mathcal{R}(E_R, t) = A_{\text{eff}}^2 \tilde{\eta}(E_R) |F_A(E_R)|^2,
\]

(5.3)

where the effective number of nucleons \(A_{\text{eff}} = [Z + (A - Z)\kappa]\) has been defined and \(\tilde{\eta}(v_m)\) is given by

\[
\tilde{\eta}(v_m) = C \int_{v_m}^{\infty} v \tilde{f}_{\text{det}}(v, t) dv,
\]

(5.4)

The speed distribution that enters in \(\tilde{\eta}(v_m)\) is the velocity distribution integrated over the angles

\[
\int f(\vec{v}, t) d^3 v = \frac{\rho_\chi}{m_\chi} \int v^2 \tilde{f}(v, t) dv,
\]

(5.5)

where \(\tilde{f}(v)\) is renormalized since the number density of DM is explicitly written out. It is important to note that \(\tilde{\eta}(v_m)\) is a function of the recoil energy since \(v_m = v_m(E_R)\). The spin-dependent case can be treated in the same way with the appropriate form factors and different constants multiplying the differential rate.

### 5.1.1 Direct detection bounds and conflicting results

The absence of a signal in a DD experiment can be translated into an upper bound on \(\sigma_{\chi p}\) using eq. (5.3) as long as a velocity distribution, local background density, DM mass and nuclear form factors are known. The standard assumptions are to use the Helm form factor, which is given in eq. (3.44), the standard halo model in eq. (4.5) replacing the velocity \(v_\odot\) with the averaged earth velocity during the time of data taking, and a local background density \(\rho_\chi = 0.3\ \text{GeV/cm}^3\). In the spin-independent case, it is also usually assumed that DM couples in the same way to protons and neutrons, i.e., \(A_{\text{eff}} = A\) is the number of nucleons in the target atom.

There have been a large number of DD experiments that produced bounds on the DM scattering cross section and there is no point in mentioning all of them. However, notably strong bounds on the spin-independent cross section come from
from PANDAX-II [162] and LUX [163], both of which employ xenon as their target. Due to the large abundance of isotopes Xe\(^{129}\) and Xe\(^{131}\) that have non-zero angular momentum in these experiments, they can also provide limits on the spin-dependent scattering cross section [164, 165]. SuperCDMS and CDMSlite, using a germanium target have also produced strong limits on the spin-independent cross section, in particular in the lower DM mass range [166, 167]. Another notable experiment that has provided strong bounds on the spin-dependent cross section is PICO-2L [168]. Generally, the spin-independent cross section bounds are orders of magnitudes stronger than the spin-dependent one. This is largely due to the \(A^2\) amplification of the spin-independent cross section.

Although most experiments claim no evidence for a signal and place bounds on the DM scattering cross section, the picture is complicated by experiments that have reported a signal that can be interpreted as a result of DM scattering inside the detector. DAMA/NaI [169] and subsequently DAMA/LIBRA [170] have shown a strong signal that can be interpreted as evidence for DM scattering for over a decade. The signal they report seeing is an annual modulation, as mentioned in Sec. 4.1.2. Not only the two DAMA experiments but also CoGeNT reported a signal that could be due to DM [171]. However, the cross section and DM mass that would give rise to the signals in DAMA and CoGeNT do not match. Simple models with elastic scattering in each experiment are also completely ruled out by many experiments including LUX, PANDAX-II and SuperCDMS [162, 163, 166]. These conflicting reports have led to many more or less exotic ideas to explain why certain experiments might find a signal while others do not.

### 5.1.2 Inelastic dark matter signatures and solving the direct detection discrepancies

Before the xenon experiments set their strong limits, only CDMS ruled out the signal seen in DAMA. Inelastic DM, already described in sec. 3.4, was originally proposed to reconcile the two experiments by DM scattering from a lower mass state \(\chi\) into a slightly more massive state \(\chi^*\) [172]. The effect of the mass-splitting \(\delta\) is that the kinematics in the collision become significantly altered if it is comparable in size to the typical kinetic energy of the DM particles in the halo.

The lowest velocity to give rise to a particular recoil energy when scattering is inelastic will no longer be given by eq. (5.2), but the expression is instead [173]

\[
v_m = \left| \sqrt{\frac{m_A E_R}{2 \mu_{\chi A}^2}} + \frac{\delta}{\sqrt{2 m_A E_R}} \right|.
\]  

(5.6)

This has a very important implication, \(v_m\) is no longer a monotonically increasing function in \(E_R\).

When \(\delta > 0\), kinetic energy is lost in the collision in order to produce the heavier state, i.e., the scattering process is endothermic. The minimum velocity \((v_m)^{\text{endo}}_{\text{min}}\)
to give rise to any recoils and the corresponding recoil energy $E_{\text{min}}$ are in this case given by

$$\left(v_m\right)_{\text{endo}} = \sqrt{\frac{2\delta}{\mu_A}} \quad \text{and} \quad E_{\text{min}} = \frac{\mu_A \delta}{m_A}.$$  \hspace{1cm} (5.7)

The key point of this model is that if $\delta$ is large enough, only the particles in the halo with the largest velocities can scatter and give rise to DD signals. For a fixed $\delta$, increasing $m_A$ will decrease the minimum DM velocity to give rise to a given recoil. Taking advantage of the fact that the DM scattering rate can be highly suppressed on lighter nuclei, the signal in the DAMA experiment could be explained by scattering of the iodine atoms while CDMS would not see a signal due to germanium being too light for the majority of DM particles to scatter against.

Another interesting feature is the non-zero minimum value of $\left(v_m\right)_{\text{endo}}$, which has a significant impact on the observed differential rate as seen in an experiment. This traces back to the velocity integral $\bar{\eta}(v_m)$ that enters in eq. (5.3). Since the integrand in $\bar{\eta}(v_m)$ is a positive function, its value will be maximized when $v_m = \left(v_m\right)_{\text{endo}}$. In the elastic case, $\left(v_m\right)_{\text{endo}} = 0$ and consequently both the velocity integral and the differential rate are maximal at the lowest recoil energy that the experiment is sensitive to. If endothermic scattering is taking place, the maximum of the differential rate will be shifted towards $E_{\text{min}}$ due to the larger flux of particles that can interact.

Moreover, by measuring a differential rate, one can derive $\bar{\eta}(E_R)$ using eq. (5.3). If a maximum is found at some recoil energy $E_{\text{min}}$ that lies inside the region of measured recoil energies, eq. (5.7) provides a link between $\delta$ and $m_\chi$ through $E_{\text{min}}$. A signal in a second experiment with a different type of target particle can then potentially be used to determine both the DM mass and mass-splitting [2]. This is not possible if $m_\chi \gg m_A$ in both experiments as they will both observe maxima at $E_{\text{min}} = \delta$ but at least $\delta$ would be known.

The stronger constraints from DD experiments leave little room for the simple inelastic DM model to solve the conflicting DD problem [174, 175]. This fact has led some to propose more or less elaborate models such as isospin-dependent DM [176], exothermic DM or combinations of the two [177–181]. Isospin violation takes advantage of the fact that $A_{\text{eff}}$ can become significantly reduced depending on the value of $\kappa$. This reduction occurs in xenon and germanium if $\kappa = -0.7$ and $-0.8$, respectively. In exothermic DM models, it is assumed that a significant fraction of the halo consists of the higher mass state that scatters into the lower one. The minimum velocity and corresponding recoil energy are given by

$$\left(v_m\right)_{\text{exo}} = 0 \quad \text{and} \quad E_{\text{min}} = -\frac{\mu_A \delta}{m_A}.$$  \hspace{1cm} (5.8)

in the exothermic case. Interestingly, $E_{\text{min}}$ is the same regardless of whether the model is endothermic or exothermic. It has been shown that exothermic scattering can open up a parameter space in which DM can explain or alleviate the tension between different DD observations.
5.1.3 Halo-independent methods

If a differential rate is measured in a DD experiment, it has been shown that the information contained in the signal can be used to predict the rate in other DD experiments or to extract other information, such as placing a lower bound on the DM mass [182, 183].

The crucial observation lies in the parameters of the differential rate. Isolating \( \tilde{\eta}(v_m) \) yields

\[
\tilde{\eta}(v_m) = \frac{1}{A_{\text{eff}}^2} \frac{\mathcal{R}(E_R)}{|F_A(E_R)|^2},
\]

where all of the DM physics is contained on the left-hand side of the expression while all detector and target physics is on the right-hand side, except the atomic target mass entering in \( v_m \). Since all experiments see the same velocity distribution, experiments probing energy ranges that correspond to the same range in \( v_m \) will observe the same \( \tilde{\eta} \) in this velocity range. Thus, given a rate in one experiment, one can predict the rate in another.

Another piece of information within a measured differential rate is found by taking the derivative of eq. (5.9) with respect to \( v = v_m \). This will essentially extract the speed distribution at \( v = v_m \) weighted by the constant \( C \),

\[
C \tilde{f}_{\text{det,extr}} (v) = - \frac{d\tilde{\eta}(v)}{dv} = - \frac{1}{v A_{\text{eff}}^2} \frac{d}{dv} \left( \frac{\mathcal{R}(E_R)}{|F_A(E_R)|^2} \right),
\]

where \( E_R \) is now to be considered a function of \( v \). The lowest velocity one can probe by this method will be set by the threshold energy \( E_{\text{thr}} \) of the experiment, i.e., \( C \tilde{f}_{\text{det,extr}} \) will be known for velocities \( v > v_m(E_{\text{thr}}) \).

In the elastic case, the analysis above is straightforward because \( v_m \) is an increasing function of \( E_R \). In the inelastic case, every \( v_m \) corresponds to two different recoil energies \( E_{\pm} \) where \( E_+ > E_{\text{min}} \) and \( E_- < E_{\text{min}} \). In order to separate the inelastic case from the elastic one, it is necessary for \( E_{\text{min}} \) to fall into the energy interval measured by the experiment, which allows for techniques that determine whether inelastic scattering is taking place or not [184]. The derived \( \tilde{\eta}(E_R) \) will now have two regions in \( E_R \), one for larger and one for smaller recoil energies than \( E_{\text{min}} \), in which \( \tilde{\eta}(E_R) \) evaluates to the same range of numbers. One can then use either of these intervals with eq. (5.10) to gain insight into the velocity distribution and DM parameters.

5.2 Indirect detection

A complementary way of looking for DM is to observe annihilation products such as neutrinos and photons from DM annihilation or decay taking place inside astrophysical objects such as the Sun, the MW and its surrounding dwarf galaxies. These detection techniques are often referred to as indirect detection techniques [185]. The
accumulation of DM in stars may also affect their evolution and their asteroseismology in detectable ways. The focus will lie on the latter as paper I and II of this thesis are both focused on solar effects but we will first present a short discussion of other indirect detection methods.

5.2.1 Dark matter annihilation in halos

When DM annihilates and SM particles are produced, the final products may be any type of stable SM particles with a flux that depends on the particles that are initially created. Indirect detection of DM then implies the observation of these particles if they cannot be explained by any known astrophysical sources. The generated flux will be the largest from nearby halos with a large dark matter density, such as the MW galactic center and the dwarf galaxies that surrounds it.

The most interesting scenario is if DM annihilate directly into $\gamma\gamma$, $Z\gamma$ or $h\gamma$. If this is the case, one would see a large flux of photons within a narrow interval in the energy spectrum. The observation of such a line would hint towards the DM mass as the observed photon energy would be directly related by very simple kinematics. There have been several reports of observed photon lines in the keV to TeV ranges but they have generally been accounted for by astrophysical sources or disappeared when more data has become available or more refined analyses of the data have been performed [186].

There have been other interesting signals from observations of charged particles. For example, there has been a large fraction of high energy positrons reported by the experiments AMS [187] and PAMELA [188]. These results call for some unknown source of positrons, which has been proposed to be annihilating DM but the excess could also be due to nearby pulsars [185]. Other very interesting signals are antiprotons and also antideuterons and antihelium [185]. An observation of either would be very interesting because of the extremely low astrophysical background.

Searches for the neutrino component of DM annihilation in the MW have also been performed in the large neutrino telescopes Super-K [189], ANTARES [190] and Icecube [191]. Notably, constraints on the spin-dependent scattering cross section from neutrino observatories are very strong.

5.2.2 The Sun as a dark matter experiment

As the Sun plows through the halo on its orbit in the MW, DM particles can scatter against solar nuclei. As DM particles fall into the gravitational potential of the Sun, their velocity will be at least equal to the escape velocity of the Sun, which at the solar radius is $v_{\text{esc}}(R_\odot) \sim 620$ km/s. If they scatter while traversing the Sun, they will lose energy and become gravitationally bound if their velocity $v$ is reduced to less than that of the local escape velocity, i.e., $v < v_{\text{esc}}(r)$ where $r$ is the radial distance to the solar center. Any particle captured in this fashion will necessarily traverse the Sun again and subsequent scatterings will bring them to rest within the solar core. If DM annihilation is inefficient, such as in the case of asymmetric
5.2. Indirect detection

DM, the accumulated amount of DM may be large enough to impact the behaviour of the Sun.

The change in behaviour is due to the thermal conductivity of DM [192–195]. Captured DM can transfer heat extremely effectively if it scatters often enough to pick up a lot of energy from high velocity solar particles in the solar core while still being able to travel a significant distance into the outer regions of the Sun before depositing this energy. The net result of the heat transfer by DM is a reduced temperature in the solar core, a direct consequence of which is a significantly reduced neutrino production. This led to DM cooling of the solar core to be proposed as a solution to the solar neutrino problem, which was an observed discrepancy where the predicted flux of solar neutrinos was three times larger than the flux that was observed in detectors [193, 195, 196]. This was later explained in terms of electron-neutrinos oscillating into $\mu$ and $\tau$ neutrinos as experiments became capable of detecting neutral current interactions and DM fell out of favor as a possible explanation [197].

However, there has been a revived interest in DM being captured in copious amounts. The agreement between helioseismological predictions and observations were outstanding until observations required a reduction of the abundance of the heavier elements in the Sun [198]. This introduced a problem when comparing solar simulations to observational data as the predicted sound speed and the depth of the convective zone do not match observations [199, 200]. Heat transfer by captured DM in the Sun has been proposed as a possible solution to this problem. There is in fact a narrow window in DM mass of around 3 GeV in which DM can bring theoretical predictions into agreement with helioseismology, even though the ratio of the number of DM particles to the number of baryons in the Sun is less than one in ten billion [201, 202]. However, this requires a large scattering cross section on nuclei that depends on the momentum transfer as well as DM being asymmetric so that no annihilation takes place.

If DM is captured by the Sun and annihilates, it will not produce notable effects on helioseismology. It may however produce SM particles, which in turn decay to neutrinos with much larger energies than those coming from fusion processes. The flux of neutrinos from DM annihilations in the Sun at a distance $d$, which is large enough to consider the DM distribution as a point-source of neutrinos, will be given by

$$\frac{d\phi}{dE} = \frac{\Gamma_{\text{ann}}}{4\pi d^2} \sum_f \text{Br}_f \frac{dN_f}{dE}. \quad (5.11)$$

The annihilation rate enters as $\Gamma_{\text{ann}}$ and the sum is over all possible initial states $f$ with branching ratios $\text{Br}_f$ while $dN_f/dE$ is the spectrum of neutrinos that arise as the initial states decay. The denominator takes into account that we observe the flux per area which at $d$ is given by $4\pi d^2$.

As will be seen shortly, it is possible to place a bound on the scattering cross section of DM on nuclei by expressing the annihilation rate in terms of the solar capture rate which can be calculated for a specified DM mass and scattering cross...
section. Under the assumption that all annihilation takes place through a specific channel, one can then calculate the neutrino flux using eq. (5.11) [203, 204]. In this way, upper bounds on the scattering cross section can be placed for an assumed DM mass since the observed neutrino flux would be ruled out. This has been the assumption for many studies in neutrino telescopes to place bounds on the scattering cross sections [205–208].

5.2.3 Solar capture and annihilation of DM

The rate at which DM is captured by the Sun, or any other massive body given that newtonian dynamics is sufficient to describe its gravitational field, is given by a standard calculation [209, 210]. When a DM particle is captured, it will continue to scatter on solar nuclei until it eventually reaches equilibrium with the thermal background. This is the process of thermalization which brings particles into a state described by a number distribution \( n_\chi(r) \). DM particles that have already been captured can also become targets for incoming DM particles to be captured on. Captured DM particles may also be ejected if the recoil energy they absorb by an incoming DM particle is large enough. In addition, high-velocity nuclei will give rise to evaporation of captured DM particles. An understanding of these concepts is necessary before the captured amount of DM can be determined.

Solar capture

The capture rate on element \( i \) will, using notation common in literature, be given by

\[
\frac{dC_i}{dV} = \int_0^\infty dv f(v) w \Omega_i(r, v) \tag{5.12}
\]

where \( dV \) is a volume element, \( r \) is the distance from the solar center and \( v \) is the velocity of the DM particles described by the speed distribution \( f(v) \) far away where the gravitational potential of the Sun is negligible. The function \( w = w(r, v) \) is the velocity of the DM particle after falling into the gravitational potential and is given by \( w^2 = v^2 + v_{\text{esc}}(r)^2 \) due to energy conservation. The factor \( \Omega_i \) is given by

\[
\Omega_i(r, v) = \frac{\rho_i(r)}{m_i} w \int_{E_m(r, v)}^{E_{\text{max}}(r, v)} \frac{d\sigma(w)}{dE_R} dE_R \tag{5.13}
\]

where \( \rho_i(r) \) is the mass density of the target nucleon \( i \) and the differential cross section \( d\sigma/dE_R \) under consideration is written on the form of eq. (3.36). The integral limits are the largest possible recoil \( E_{\text{max}}(r, v) \) and the smallest recoil \( E_m(r, v) \) that leads to capture

\[
E_{\text{max}}(r, w) = \frac{2\mu_{\chi_i}^2}{m_i} w^2, \quad E_m(v) = \frac{1}{2} m_\chi v^2, \tag{5.14}
\]

where \( \mu_{\chi_i} \) is the reduced mass of the DM mass and the nucleon mass \( m_i \). The maximum and minimum energies above also define the velocities \( v \) that should be
5.2. Indirect detection

integrated over. The largest possible recoil energy for large $v$ will eventually fall below the minimal recoil energy necessary for a particle to be captured. Thus, $E_{\text{max}} = E_m$ sets the largest velocity that allows capture $v_{\text{cross}}$. The limits are thus given by

$$v_{\text{lower}} = 0, \quad v_{i\text{cross}}^i(r) = \sqrt{\frac{4m_\chi m_i}{m_\chi - m_i}} v_{\text{esc}}(r).$$

(5.15)

The capture rate from spin-independent scattering will then be given by summing over all elements $i$ that contribute to capture

$$C_{\text{Sun}} = 4\pi \sum_i A_i^2 \int_0^{R_\odot} dr \frac{r^2}{4\pi} \rho_i(r) \int_{v_{\text{lower}}}^{v_{i\text{cross}}^i(r)} dv C(v) \int_{E_m(r,v)}^{E_{\text{max}}(r,v)} \left| F(E_R) \right|^2 dE_R.$$

(5.16)

where eqs. (3.36), (5.5) have been used. The form factor in eq. (3.43) is usually considered due to its simplicity, which allows for an analytic computation of the integral in recoil energies.

In the case that capture is due to spin-dependent interactions, it is in most cases a good approximation to neglect capture on all elements except for hydrogen in which case the form factor is set to unity [73]. This is motivated by the fact that the abundance of nuclei with spin is very small compared to the hydrogen abundance, which is not compensated for by a large amplification due to a $A_i^2$ factor as in the spin-independent case.

The capture of inelastic DM, in the case of endothermic scattering, by the Sun has been studied [211–213] and it is also the model under consideration in paper II, where we also looked at the capture due to exothermic interactions. In these cases, the capture rate is, surprisingly, given by the same expression as in the elastic case, i.e., eq. (5.16). The only difference lies in the scattering kinematics which alters $E_{\text{max}}$, $E_m$, $v_m$ and $v_{i\text{cross}}^i$ in the velocity and recoil integrals, as these will generally depend on the mass splitting $\delta$.

**Self-capture and ejection**

If DM has large self-interaction cross sections, as can be motivated by the small-scale structure problems, self-scattering of incoming DM particles onto ones that are already captured may become relevant [214]. Self-capture can be treated in essentially the same way as the case where DM particles are captured by nuclei except $A$ and $|F(E_R)|$ are set to one, and $m_i \to m_\chi$. Another quantity that is needed for self-capture is the distribution $n_\chi(r)$, which describes already captured DM particles inside the Sun. The self-capture rate is then given by

$$C_{\text{self}} N = n_\chi \sigma_{\chi\chi} \int_0^{R_\odot} dr 4\pi r^2 n_\chi(r) \int_0^{v_{\text{esc}}(r)} dv C(v) \left( v_{\text{esc}}^2 - v^2 \right).$$

(5.17)

The constant self-scattering cross section enters as $\sigma_{\chi\chi}$ and $N$ on the left-hand side is the total number of captured DM particles. On the right-hand side, $N$ is
incorporated in \( n_\chi(r) \), which is normalized so that

\[
N = \int_0^{R_\odot} n_\chi(\vec{r}) d^3 r. \tag{5.18}
\]

The upper limit in the velocity integral in eq. (5.17) is due to the fact that, if \( v \geq v_{\text{esc}} \), the recoil energy must be so large that the target escapes.

Self-scattering can also lead to the capture of the incoming particle while ejecting the target particle with the rate

\[
C_{\text{eject}} N = n_\chi \sigma_{\chi\bar{\chi}} \int_0^{R_\odot} dr \ 4\pi r^2 n_\chi(r) \int_0^\infty dv v \bar{f}(v)v^2. \tag{5.19}
\]

For \( v < v_{\text{esc}} \), the incoming particle will always be captured if the target is ejected.

In paper I of this thesis, we studied self-capture of a Dirac fermion with equal proportions of DM and antiDM in the halo. The mass of both particles are the same and \( \sigma_{\chi\bar{\chi}} \) will be their self-scattering cross section, which we assumed is constant. In this case, an incoming DM particle ejecting an antiDM particle will increase the number of DM particles by one and decrease the number of antiDM particles by one. When \( v > v_{\text{esc}} \), an incoming particle may also escape while ejecting a target, which is unlikely because DM is localized in the center of the Sun. Since the escape velocity of the Sun in the solar core is much larger than the local escape velocity of the MW, there will be no such particles in the DM halo.

**Thermalization**

When DM has scattered, it must re-scatter a number of times before it can, together with the other captured DM particles, be collectively described by an equilibrium number density \( n_\chi(r) \). This density depends on the magnitude of the scattering cross section.

If the scattering cross section is very small, there may not even be a proper distribution to ascribe the DM particles to. This is because they have not had sufficient time to lose enough energy to thermalize. One can conservatively estimate that the minimum cross section has to be larger than \( \sim 10^{-50} - 10^{-48} \text{ cm}^2 \) for thermalization to occur [212]. This is generally the studied case since a smaller cross section leads to capture rates that are so small that weak scale DM has no interesting impact on the Sun.

If thermalization does occur, there will be two limiting cases. If the cross section is small enough for the average DM particle to travel large distances between collisions, the DM particles will be well described by an isothermal Boltzmann distribution [215]

\[
n_{\text{ISO}}(\vec{r}) \propto e^{-m_\chi \phi(\vec{r})/k_B T}, \tag{5.20}
\]

where \( \phi(r) \) is the solar gravitational potential and \( T \) is the temperature of the distribution. A very common assumption is that the density surrounding the thermalized DM cloud is equal to the solar core density \( \rho_c \) and that the temperature of
the distribution is equal to the solar core temperature \( T_c \). The number distribution is then given by [1]

\[
n_{\text{ISO}}(\vec{r}) = \frac{N}{\pi^{3/2} r_x^{3/2}} e^{-|\vec{r}|^2/r_x^2}
\]

where \( r_x^2 = 3 k_B T_c / 2 \pi G m_\chi \rho_c \). The assumptions may be justified by the exponential decay beyond \( r = r_x \) and \( r_x = 0.05 R_\odot \) for a 5 GeV DM particle, which implies that the vast majority of the DM particles are contained within the inner core of the Sun.

If, on the other hand, the cross section is large enough, the distribution will be determined by the local temperature at all radii. The case of local thermal equilibrium was investigated for simple isotropic cross sections like those considered here in refs. [195, 216], and also in the generalized cases where the scattering cross section has a velocity and momentum transfer dependence [217].

In the case of inelastic DM, thermalization is not obviously going to occur. The mass splitting is roughly equal in size to the kinetic energy of DM particles in the DM halo if the mass splitting for GeV-TeV scale DM is of the order keV-MeV. Since the temperature in the solar core is roughly one keV, all inelastic DM particles that have been captured will at some point become locked in the lower mass state since upscattering requires extremely high velocity nuclei to scatter against. The number of such high-velocity particles are then exponentially suppressed since the Boltzmann distribution that describe them has such a small temperature. In order for inelastic DM to thermalize, they must have some smaller elastic scattering component, which can be realized as mentioned in sec. 3.4.

### Evaporation

The high-velocity nuclei in the solar core will give rise to another phenomenon for light DM particles. The highly energetic atoms in the solar core may transfer enough energy to DM particles to leave them gravitationally unbound. This phenomena is called evaporation and has been studied both analytically and numerically [215, 218–220]. The general result is that, almost independent of the scattering cross section, DM particles with masses \( m_\chi \lesssim 4 \) GeV will evaporate rather than annihilate at a rate that is directly proportional to the number of captured particles, \( \Gamma_{\text{evap}} = C_{\text{evap}} N \).

### Annihilation

The annihilation rate \( \Gamma_{\text{ann}} \) of DM is straight forwardly calculated by

\[
\Gamma_{\text{ann}} = \frac{\langle \sigma_{\text{ann}} v \rangle}{2} \int n(x)^2 d^3x = \frac{1}{2} C_{\text{ann}} N^2,
\]

where \( \langle \sigma_{\text{ann}} v \rangle \) is the thermally averaged annihilation cross section. The factor of a half enters because there are, in the large \( N \) limit, \( N^2 / 2 \) distinct pairs to annihilate.
Number evolution of captured DM

Given all of the above, the number of DM particles $N$ as a function of time $t$ will be determined by the differential equation

$$\dot{N}_\chi = C_{\text{Sun}} + (C_{\text{Self}} - C_{\text{evap}}) N_\chi - C_{\text{ann}} N^2_\chi. \quad (5.23)$$

Note that the factor of $1/2$ is missing in the annihilation term since each annihilation event annihilates two particles.

Considering a DM particle without self-interactions and a large enough mass for evaporation to be neglected, i.e., $C_{\text{self}} = C_{\text{evap}} = 0$, the solution to eq. (5.23) is

$$N(t) = \sqrt{\frac{C_{\text{Sun}}}{C_{\text{ann}}}} \tanh(t/\tau_{\text{eq}}), \quad (5.24)$$

where $\tau_{\text{eq}} = 1/\sqrt{C_{\text{Sun}}C_{\text{ann}}}$. When $t > \tau_{\text{eq}}$, annihilation and capture is in equilibrium, which implies that $\dot{N} = 0$ and thus

$$\Gamma_{\text{ann}} = \frac{1}{2} C_{\text{Sun}}. \quad (5.25)$$

By inserting this relation into eq. (5.11), it is possible to compute the neutrino flux independent of the DM annihilation cross section.

In the case of multi-component DM, the number evolution of each species will be found by coupled differential equations. In these cases, self-interactions implies that the change of captured DM particle numbers will include contributions from ejections and that there may be co-annihilations among different types of captured DM species.

The main point of paper I was to show that, in a scenario where the galactic abundance of DM and antiDM is the same but the capture rate in the Sun of each species is different, the amount of DM captured may be of similar magnitude as in the asymmetric DM case even though annihilation is taking place. This work shows that there are other options than asymmetric DM models that may help to resolve the solar composition problem.

5.2.4 A lower bound on the solar capture rate of dark matter

An interesting feature of the solar capture rate appears when analyzing the integral in velocity. Just as in the direct detection rate, the factor $C\tilde{f}(v)$ appears in the solar capture rate. If a DD experiment measures a signal that is strong enough for $C\tilde{f}_{\text{det}}(v)$ to be determined, one can directly plug the known information into the
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solar capture rate [221]. Using eq. (5.10), a lower bound on the solar capture rate of DM can be computed as

\[
C_{\text{Sun}} \geq 4\pi \sum_i A_i^2 \int_0^{R_{\odot}} dr r^2 \rho_i(r) \int_{v_{\text{thr}}}^{v_{\text{cross}}(r)} dv \left( -\frac{d\eta(v)}{dv} \right) \int_{E_{\text{m}}(r,v)}^{E_{\text{max}}(r,v)} |F(E_R)|^2 dE_R.
\] (5.26)

This will be a lower bound due to the lower integration limit in velocity, which is now \(v_{\text{thr}}\). The reason for this is the fact that the entire velocity space cannot be known from DD experiments. As discussed in sec. 5.1.3, the lowest possible recoil energy that an experiment can probe sets a lower limit on the velocity integral. As was also mentioned, this velocity is given by plugging the experiments threshold energy into eq. (5.2), \(v_{\text{thr}} = v_m(E_{\text{thr}})\).

Most important to point out is that the lower bound is independent of the halo velocity distribution, local DM distribution and the DM scattering cross section, which eliminates some of the uncertainties that goes into calculating the capture rate. A word of caution is that it is assumed that the detector and the Sun observe the same velocity distribution. The velocity distribution is fundamentally different due to the movement of the Earth relative to the Sun, which implies that the assumption breaks down at small velocities. The only free parameter is the DM mass. One could also question the assumption one makes that the velocity distribution in the solar frame has been the same over its entire lifetime.

The main purpose of paper II was to extend the above analysis to models where DM is scattering inelastically. In this case, eq. (5.26) still holds but \(v_{\text{thr}}\) will be modified. Now, \(v_{\text{thr}}\) is replaced by \((v_m)_{\text{end}}^{\text{endo}}\) or \((v_m)_{\text{exo}}^{\text{exo}}\) depending on whether endothermic or exothermic scattering is taking place. The only free parameter in this case will also be the DM mass since we choose to relate the mass splitting to the DM mass using that the observed minimum relates the two according to eqs. (5.7) and (5.8).
Chapter 6

Summary and conclusions

In this part of the thesis, the framework for the papers of Part II has been set. We briefly reviewed various observations that support the existence of dark matter and a number of particle candidates as well as briefly mentioned some alternatives. We discussed some of the interactions that can take place between particle dark matter and standard model particles as well as self-interactions amongst DM particles. We also introduced a simple example model of inelastic dark matter. Some properties of dark matter halos were mentioned as well as the clash between predictions and observations in smaller structures and how they can be solved with dark matter physics. Finally, we reviewed some experimental efforts that attempt to detect dark matter and how the information contained in such a signal can be used to make predictions. We also mentioned what parts in this introduction that the three papers of part II elaborated upon.

In paper I, we discuss the solar capture processes of a model in which dark matter is as Dirac fermion with equal components of particles and antiparticles. There are two interesting scenarios for a large number of captured particle to accumulate in this case. We showed that if the capture rates of DM and its antiparticle are different on solar nuclei, the accumulated number may be as large as in the case of asymmetric DM, which implies that these models may also be considered when attempting to solve the solar composition problem. We also showed that the asymmetry due to stochastic fluctuations being boosted into a permanent asymmetry by self-capture is not likely to have produced a large asymmetry.

In paper II, we discuss how to predict the solar capture rate of dark matter given a rate in a direct detection experiment has been measured. In particular, we extend the analysis of elastic dark matter models to the case in which dark matter scatters inelastically. We also studied the capture of dark matter through exothermic scattering. We found that if a significant part of the halo consist of the higher mass state that down-scatters in an experiment, one can predict the full capture rate of dark matter by the Sun. We also showed that the standard halo model in combination with capture by dark matter scattering exothermically will
place very strong bounds on the branching ratio to various initial states due to the absence of a signal in neutrino telescopes. We also found that signals in two different experiments may uniquely determine both the dark matter masses and the mass splitting. We also investigated the effects of isospin-violation.

In paper III, we studied the self-scattering cross section of a simple inelastic dark matter model given that the mediator particle is very light with respect to the dark matter particle and the energy that is transferred in collisions. We showed that the self-scattering cross section in this case may be large enough to solve the small scale structure problems while simultaneously avoiding bounds from larger structures. We also estimated the relative abundance of the two states after its production has ceased and the down-scattering rate becomes negligible.
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Part II

Scientific papers
Paper I
M. Blennow and S. Clementz
Asymmetric capture of Dirac dark matter by the Sun
JCAP 1508, 036 (2015)
arXiv:1504.05813
Paper II
M. Blennow, S. Clementz and J. Herrero-Garcia

Pinning down inelastic dark matter in the Sun and in direct detection
JCAP 1604, 004 (2016)
arXiv:1512.03317
Paper III
M. Blennow, S. Clementz and J. Herrero-Garcia

Self-interacting inelastic dark matter: A viable solution to the small scale structure problems

arXiv:1612.06681