“Evaluating the Impact of Waiting Time Uncertainty on Passengers’ Decisions”

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“Evaluating the Impact of Waiting Time Uncertainty on Passengers’ Decisions”

by

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Abstract

Service reliability is one of the main factors influencing public transport level of service and, thus, passengers’ satisfaction. Public transport services are subject to various sources of uncertainty related to traffic conditions, public transport operations and passenger demand. Passengers are able to form their perception of trip attributes and service reliability through accumulating experiences of repetitive travel choices. Perceived service reliability can be improved either by increasing the ground-truth service reliability (e.g. introduce exclusive bus lanes, control strategies etc.) or by providing real time information (RTI) to passengers. However, RTI prediction schemes might not be perfectly accurate and thus, passengers might be able to account for the reliability of the provided information as well.

The learning mechanism of individuals becomes, as a result, an important component in Dynamic Transit Assignment Models (DTAM) which enables accounting for how perceived reliability of service and the provided information evolves, through iterative network loading.

This thesis provided the modeling framework for passengers’ perception of reliability and its effects on decision making with respect to path choice. Within-day effect is represented through the incorporation of scheduling constraints, while passengers’ learning mechanism accounts for updates in their expectations and the perceived level of information credibility in the day-to-day context.

The proposed model was applied to Stockholm’s rapid transit network which was simulated in BusMezzo, an agent-based public transport assignment model. The application used the real-world timetables, vehicle schedules and RTI prediction scheme. Passengers’ learning function was analysed under various specifications which corresponded to different levels of adaptation.

The results highlight the importance of capturing service uncertainty and the credibility associated with alternative information sources, while they stress the need for empirical estimation and validation of the proposed model. This study also provides the framework for future evaluation of measures which aim to improve service reliability.

Key words: perceived reliability, uncertainty, waiting time, day-to-day learning, information, credibility, TAM
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1 Introduction

1.1 Background

Service reliability is one of the main factors influencing public transport level of service and, thus, passengers’ satisfaction. Public transport services are subject to various sources of uncertainty related to traffic conditions, public transport operations (dispatch times, dwell times etc.) and passenger demand (e.g. random arrival at stop). For example, traffic congestion might lead to transit service disturbance (late bus arrival at stop) which, in turn, might entail a higher load of passengers waiting at stops. These passengers will experience longer waiting times and some of them might be denied boarding, while both the delay effect and the increase in passengers’ loads at stops might be propagated to the remaining downstream trip (if the bus does not have time to catch up with the scheduled timetable). Passengers’ waiting time at stops is thus a random variable subject to day-to-day variations and the interaction between vehicle and passenger stochastic arrival processes.

Perceived service reliability can be improved either by increasing the ground-truth service reliability (e.g. introduce exclusive bus lanes, control strategies etc.) or by providing real time information (RTI) to passengers. The dissemination of RTI concerning predicted vehicle arrival times has the potential to help passengers adjust their decisions during the trip by updating their expectations, and also to reduce the uncertainty related to these expectations. For example, a passenger at the stop of the above example could switch directly to another line or stop, if he/she receives information of a late bus arrival, while without this information he/she would probably consider to update his/her choice only after a lot of time has passed. It is therefore important to account for passengers’ response to information in path choice modeling and transit demand estimation and forecasting.

At the same time, the development of behavioural research has revealed the need to account for more realistic traveler responses under uncertainty, in cases where the decision is made in a repetitive (day-to-day) context. Learning mechanisms, which represent the way passengers update their expectations based on the experiences of the previous days, have been introduced in several models which account for day-to-day dynamics [Nuzzolo et al., 2001; Whaba and Shalaby, 2005], while some approaches have also considered passengers’ ability to reconstruct the distribution of
experienced travel attributes and thus derive the reliability associated with the information source [Miscio 2012].

Moreover, a lot of effort has been put to reconcile the efficiency of Expected Utility Theory, in model applications, with the advantages of Prospect Theory’s [Kahneman and Tversky, 1979] behavioural proximity, in order to provide more realistic but also computationally attractive models. Especially in departure time choice context, people are conventionally assumed to plan their trips according to a desired arrival time, which can be used as their reference point. In this case scheduling constraints are taken into account during the decision making, with respect to this reference point (early or late arrivals and probability of arriving late), and capture the impact of travel time uncertainty on passengers choices [Noland and Polak, 2002; Ettema and Timmermans, 2006].

While many of the previous studies modeled and evaluated the impacts of RTI on passengers’ decisions compared with static information, they did not account for its potential to reduce uncertainty by improving passengers’ anticipations [Coppola and Rosati, 2009; Cats et al., 2011]. Furthermore, it is conventionally assumed that passengers perceive RTI as perfectly credible and, thus, it dominates over other sources of information, if it is available when passengers evaluate their alternative choices.

In other words, RTI has so far been the solely and fully reliable information source when available, without having explored (a) how accurately it describes the actual distributions of the predicted time components; (b) if users are able to capture this accuracy, especially if the decision is made in a repetitive (day-to-day) context. As a result, passengers’ perception of uncertainty under the impact of information has not been sufficiently addressed when it comes to DTAM. Such an approach would require the explicit modeling of how passengers’ anticipations are formed and evolve with the interaction of all sources of information on one hand (static; real-time; and experience: e.g. boarding denial), and the role of their learning mechanism in day-to-day dynamics, which can lift the assumption of RTI full credibility, on the other.

1.2 Objectives

Given the background, the main objective of this study is to investigate passengers’ potential to perceive the uncertainty related to any information source, given their ability to accumulate past experience, and its deviation from their expectations, in their memory. Passengers’ choices would then reflect the impact of uncertainty on network’s performance and could provide an important base for evaluating alternative measures for reducing uncertainty.

This study focuses on waiting time and the relevant information and its objective is pursued to be accomplished through the implementation of a more realistic Path Choice model developed in two interdependent directions:
First, by implementing the explicit modeling of passengers’ anticipations with respect to the waiting time at stop, integrating all information sources and the associated uncertainty, in combination with scheduling constraints at the time of decision making. Second, by accounting for an explicitly formulated learning mechanism which will allow passengers update these anticipations and their credibility in a day-to-day (repetitive) context through accumulated experience.

The ultimate goal of such an approach is to accommodate the evaluation of measures which aim to improve transit reliability, through different scenario combinations between service regularity and RTI availability, in order to capture the effect of information in passengers’ anticipations in comparison to their experiences.

### 1.3 Thesis Outline

The remainder of the thesis is organized as follows: Chapter 2 provides a literature review with respect to information provision and the uncertainty associated with it. Moreover, it presents an overview of behavioural research related to decision making under risk, as well as different modeling approaches which aim to capture uncertainty under different decision contexts and assumptions. The motivation of this study is also further articulated in this chapter.

Chapter 3 presents the methodological approach to the formulation of the Preferred Arrival Time Path Choice Model, by providing the argumentation and the role of each of its components. It also presents the implementation requirements and stresses the limitations and assumptions for its application in BusMezzo, a public transport agent-based simulation model for Dynamic Transit Assignment. The case study is described in Chapter 4 where Stockholm’s backbone transit network and its characteristics are introduced, together with a complete description of the sensitivity analysis’ design with respect to the model components. Technical details of the simulation process are also discussed in this chapter. The results and their analysis are demonstrated in Chapter 5, which is arranged with respect to the processes and measures that are under investigation in a first level, and the alternative variations of sensitivity analysis in a second level. Both aggregate and disaggregate analyses have been conducted in this chapter and a comparison among scenarios is provided.

The drawn conclusions are highlighted in Chapter 6, where the purpose and the main results are recapitalized. The added value of this study is reflected through the main outcomes of the case study, while limitations and further research suggestions are outlined. Finally, Appendices include supplementary graphs for the analysis of Chapter 5.
2 Literature Review

2.1 Service reliability and passengers’ response

Service reliability is one of the main factors which determine network’s performance and passengers’ satisfaction. Abkowitz et al. (1978) define service reliability as “the invariability of service attributes that influence the decisions of travellers and transit providers”. As a result, reliability can be examined from the perspective of both service users and providers.

A classification based on each side’s interest is also provided by Ceder [2007], who summarizes reliability attributes with respect to passengers’ concern (waiting time, boarding time, seat availability, in-vehicle time, total travel time, transfer time, missed connections etc.), to agency’s concern (regularity, load-counts distribution, punctuality, missed trips, breakdowns etc.) as well as exogenous attributes such as traffic delays, accidents, and weather conditions which are associated with the agency.

These attributes are of course interrelated, since they are the outcome of demand and supply interaction. For example, headway distribution and waiting times are two sides of the same coin, as the former determines the latter. Any inconsistency to scheduled headways leads to passengers’ experience of irregular service (e.g. long waiting times) and also to demand fluctuations (e.g. higher load counts at stop). In this study, the focus is put on the experience of variability from the demand side, and how this is perceived by transit users.

Empirical findings suggest that reliability is important among other trip attributes and that is included in passengers’ evaluation of transit trips [Abkowitz et al, 1978; TCRP, 2003; Ceder, 2007]. Ceder [2007] provides the empirical findings of Balcombe et al. [2004] that improvement of reliability of the service is twice as important as to increase the frequency of the service. Its quantitative incorporation in demand modeling could thus account for a more realistic representation of travellers’ decision process.

Literature offers an abundance of studies which have employed reliability measures in demand models in order to quantify its effect on passengers’ decisions, usually by using the statistical properties of the assumed distributions of the examined attributes. The main approaches are based on the quantification of travel time through
the mean-variance model [Black and Towris, 1993] or the scheduling delay model [Noland and Small, 1995; Noland et al., 1998; Small et al., 1999] while most of them focus on car traffic related choices, mainly regarding route and departure time. Both approaches assume passengers being rational utility maximizers when it comes to the evaluation and selection of their alternatives.

The mean variance model is generally described by the function:

\[ U = \bar{T} + \lambda V(T) \]  \hspace{1cm} (2.1)

Where the utility \( U \) is a function of the mean, \( \bar{T} \), and the variance, \( V(T) \), of the travel time \( T \) [ reviewed by Li et al., 2010].

Respectively, the schedule-delay model is defined as follows:

\[ E(U) = aE(T) + \beta E(SDE) + \gamma E(SDL) + \varnothing P_L \]  \hspace{1cm} (2.2)

Where the expected utility \( E(U) \) depends on mean values of travel time \( (T) \), Early Arrival (SDE), Late Arrival (SDL) and the probability of late arrival \( P_L \), and it can be explicitly calculated given the assumption of travel time distribution and an exogenously defined Preferred Arrival Time (PAT) for the passengers, which also defines SDE and SDL [Noland and Small, 1995].

Noland and Polak [2002] compare results of the two approaches which were applied in the context of departure time choice. The importance of the lateness probability as an explanatory variable as well as the conclusion that variability per se does not need to be added when scheduling effects are included in the model, were two important outcomes of the review. The latter can be considered particularly interesting when one considers the existence of alternative variability measures as mentioned by other studies, e.g. distribution’s range, deviation from mode, etc. [Bogers, 2009; Tilahun and Levinson, 2010; Zerguini et al, 2011].

Tilahun and Levinson [2010] also stress the weakness of the standard deviation as an overall reliability measure due to lack of differentiation between early and late arrivals. They compare such a model with two alternative formulations: (a) the possibility of arriving late or early coupled with travelers’ usual experience, using the mode (the most frequently encountered travel time) to position their preference on a certain route; and (b) the use of extreme values of (long) travel time in addition to the traveler’s frequent experience, using the right range from the median to the extreme value of late arrival, and the aggregate probability of being late more than 5 minutes from usual. Reliability ratios of the standard deviation model, however, give different values to the ones mentioned in Noland and Polak’s review [2002].

Li et al [2010] in their review on “willingness to pay on travel time reliability” mention that the empirical estimates of reliability ratio vary significant in literature. The authors suggest that the model which describes reliability best is the one based on the mode paradigm since it includes both frequency and experienced times together.
Bogers [2009] suggests that, concerning variability measure choice, it is its relation to the expected value which is important, as well as the type and the formulation of the problem which define the measure selection.

Noland [1999] applied the schedule-delay framework in a route choice problem by varying the degree of travel time variation, as described by Noland et al [2002]. This study yielded a shift in departure time and route choice, which suggest that scheduling effects need to be taken into account when it comes to route choice modeling. In the field of public transport, Bates et al. [2001] applied an extended version of the schedule-delay model for the UK railway service, where extra disutility associated with unreliability per se appeared, due to passengers’ inability to fully adjust their (discrete) departure time options, compared to the availability of continuous car traffic departure times.

The above-mentioned studies have been based on Expected Utility Theory (EUT) and, until today, the main approach when it comes to application of decision models is the maximization of utility or maximization of expected utility when a choice involves uncertainty [Chorus et al, 2006]. However, EUT has been criticized for its inability to predict realistic behaviours in decision making under uncertainty. Kahneman and Tversky [1979] introduced an alternative account of individual decision-making under risk, aiming to accommodate the cases of decision making where expected utility theory invalidates as a descriptive model: Prospect Theory (PT) tries to capture more realistically behavioral aspects of individuals’ decision-making process and is developed under controlled laboratory experiments. It assumes that prospects (or lotteries) are evaluated in a two-step process: the editing phase, where prospects are coded as gains and losses with respect to some neutral reference point, and the evaluation phase, where the prospect of higher value is chosen.

This reference dependence, as described by Li and Hensher [2011], constitutes the main difference between PT and EUT and allows for different value functions for gains and losses relative to the reference point, which might be influenced by the presentation of the prospect [Kahneman and Tversky, 1979; Tversky and Kahneman, 1992; Avineri, 2004; Li and Hensher, 2011], rather than a utility function over the final wealth. Moreover, behavioural evidence suggests that losses loom larger than gains, which is captured by the slope ratios and the kink at the reference point in the value function. Li and Hensher [2011] have reviewed Prospect Theory contributions in departure time choices, route choice and other applications under travel time variability and stress the lack of empirical estimates of the Prospect Theoretic Parameters for value and probability weighting functions. Only few of the researchers have estimated them through experiments (the authors mention Michea and Polak, 2006 and Schwanen and Ettema, 2009) while the rest have used previous studies’ estimates (e.g. Tversky and Kahneman’s, 1992). Moreover, the selective use of Prospect Theory Components, even by omitting the probability weighting function, and the issues related to the reference point, as described by the authors, address the need for further research in the area of PT application.
However, the incorporation of the notion of the reference point through the PAT and the demarcation between the early and late arrival, in the schedule delay model, has put EUT closer to PT providing a more realistic frame to describe passengers’ decision making process.

2.2 Learning through Experience and Information

Including the notion of reliability in trip choice contexts implies passengers’ repetitive exposure to the alternative modes or routes so that they are able to form their own estimate of service performance. Indeed humans have the ability to store their experiences in their memory, and, thus, form their expectations according to their accumulated experience. Learning mechanisms have been extensively studied through the development of cognitive science, and aim to represent this ability of learning through experience. Reinforcement Learning theory suggests that individuals are learning by the interaction with the environment that they are in, rather than just by imitating or being shown by others. In that context, individuals are not assumed to have a complete knowledge of this environment and, as a result, they learn through trial-and-error and (delayed) reward of subsequent situations [Sutton and Barto, 1998].

In a transit network this would practically imply a user who is iteratively trying out alternative choices, e.g. alternative paths, in order to identify what is the least costly one (in terms of general cost). After a few iterations, the user would have an estimate of this cost and its components (travel times, fares, preferences etc.), which every new experience would update and, ideally, he/she would be able to construct the full distribution of each component after a sufficient number of days.

However this does not imply passengers’ full knowledge of the network and the reliability of its services, but rather the accumulation of their perceived experiences, which might be, however, different from the measured [Watkins et al., 2011].

Static Information, such as planned headways and timetables, is provided to passengers in order to enable the latter to be aware of the transport services and plan their trips accordingly, while it reflects the average conditions of the transport systems (e.g. average travel times between stops). However, this information would be accurate and perfectly reliable only in cases where the service runs without any deviation from schedule, which is not realistic, especially in congested urban networks with high frequency of service lines.

The development of Intelligent Transport System (ITS) technologies over the last years, including the use of Automatic Vehicle Location (AVL) and Automatic Passenger Counting (APC) data, have paved the ground for Advanced Traveler Information Systems (ATIS) and the provision of Real Time Information (RTI). RTI is deployed as a measure to reduce passengers’ uncertainty with concern to, among others, vehicle arrival times and trip times. Empirical studies have shown the effect of RTI on passengers’ perceptions update and have demonstrated a decrease in
Information provision has also been found to expedite passengers’ learning when it comes to repetitive choices. Ben-Elia and Shiftan [2010] conducted a laboratory example of a route-choice problem, in a repetitive context, where RTI about the range of travel times is provided, by incorporating behavioural decision theories in an Expected Utility econometric model. Their work reflected the effect of learning in both short- and long-run decision-making, and also in correlation with information provision. Without information, the passengers base their decisions mostly on recent outcomes, while in the long-run learning was also based on exploration. On the contrary, in case of RTI provision of variability, in the form of a plus/minus error added to the prediction, decisions were made based both on short and long-run learning and the RTI was a core element in the memorization process and hastened the learning process. With respect to the sensitivity towards variability, the most common attitude without information was risk aversion, whilst with RTI risk-seeking was observed.

The above studies suggest the importance of information and its related reliability in the decision making, regarding anticipations based either on experience or RTI, especially in repetitive context which is the base of DTAM. The following section reviews some important attempts to incorporate these findings in Dynamic Path-choice Modeling.

### 2.3 Dynamic Transit Assignment Modeling

Path choice modeling is the core component of a Transit Assignment Model (TAM). The latter is the last step of the four-step demand forecasting process, following trip generation, trip distribution, and mode choice. Path choice models aim to capture passengers’ perceptions and travel behaviour so that realistic path shares are predicted. Dynamic Traffic or Transit Assignment (DTA) models describe the supply and demand interaction in a transport network under time evolution.

Early assignment models were based on heuristic rules for the estimation of the shortest path in a transport network [Dial, 1967; LeClerq, 1972] and provided the foundations for the development of the frequency-based approach. In this approach, passengers select a subset of attractive routes in order to minimize the expected total trip time, usually expressed through the sum of access, waiting and travel time [Spiess and Florian, 1989], for which they board the first arriving vehicle (since they do not know the schedule). The strategy for selecting these attractive routes is denoted in literature as “hyperpath”, from the original work from Nguyen and Pallotino [1988]. These models have lately been expanded to include capacity constraints, boarding probabilities and en-route decisions [Kurauchi et al., 2003; Schmöcker et al., 2010], but they lack the ability to account for dynamic transport operations and the variation in users’ perceptions and preferences.
Schedule-based models, on the other hand, are based on passengers’ awareness of the timetable of specific vehicle runs (instead of line-level frequencies) and employ time-dependent utility functions for the evaluation of the path alternatives. The utility values are usually treated as random variables and the decision process is underlied by the maximum utility probability [Ceder, 2007; Cats, 2011].

Hickman and Wilson [1995] presented a within-day dynamic assignment model that accounts for travel time time-dependency and variation in transit services. It uses a deterministic path choice model that can incorporate RTI in bus arrivals at stops, in order to investigate the effect of RTI in travel time savings. Concerning transit path shares, Nuzzolo et al. [2001], by modifying the models of Nuzzolo and Russo [1998] as described by the authors, developed the “doubly dynamic schedule-based transit assignment model” defining explicitly the day-to-day and with-in day dynamics applied both in regular and irregular services to reflect the effect of passengers’ learning and updating mechanisms with and without information. Their application suggested the importance of the day-to-day dynamics in on-board load level changes, due to both service irregularity and congestion.

Rochau et al. [2010] have tried to reconcile the notion of “hyperpath”, as the strategy to select attractive paths, and the scheduled-based approach in a dynamic model which accounts for reliability in the sense of boarding probability. Information availability in different levels is also taken into account. Reliability here is however, treated as a deterministic term, since the approach considers probabilities as exogenous variables.

Cats [2011] provides a review of frequency-based and scheduled-based models as the conventional TAM, which use the static equilibrium assignment method and he highlights their limitations to account for stochastic network conditions and the effect of information.

Peeta and Ziliaskopoulos [2001] discuss the advantages of simulation-based models as a more convenient way to represent the complex processes, and their interaction within a transport network, realistically and with computational robustness. Information availability and multiple user classes, which can account for a more detailed demand and supply operations modeling, are some of the advantages simulation-based models can contribute in DTA.

Wahba and Shalaby [2005] stress the limitations of current dynamic transit assignment models with respect to travel time uncertainty effect on departure time and the integration of new information and experiences into passengers’ cognitive model. The authors suggest a multi-agent learning based approach to account for passengers’ accumulated experience and how it affects trip choices. RTI is assumed to be interpreted as a recent experience with its corresponding reliability and it is combined with previous experience when added to the passengers’ memory. Miscio [2012] takes this approach one step further and defines the reliability coefficients, for each type of
information that takes part in the decision making, in his framework of agent-based, day-to-day learning in transit dynamic modeling. Bogers [2009], in his day-to-day learning (car traffic) route choice model, also accommodates the distinction between the Expectation and the RTI reliability, indicating a trade-off between the corresponding beta parameters in the Utility function. His model accounts also for the salience of the observations and its effect on both habitual and non-habitual routes.

Concerning irregularity, Cats et al. [2011] try to examine the relationship between the RTI levels and path shares under different service disruption scenarios. The author provides a review of today’s state-of-the-art simulation-based Transit Assignment Models stressing the lack of dynamic representation of both supply and demand interaction. The authors further present an agent-based transit simulation model, based on Cats [2011]. This work consists of the dynamic modeling of both transit operations and passenger decisions where the dynamic transit path choice is based on the anticipations of the passengers and accounts for passengers’ adaptive behaviour during the trip, according to the level of available information. Information prediction can be based on dynamic prediction schemes based on real-time data. As a result, the level of prediction accuracy is not predefined or assumed, but based on the prediction method and the extent to which it incorporates the traffic and transit dynamics. Its application on the investigation of savings in all travel time components is also important for the significance of RTI level (stop-, cluster- and network-level) in the planning process.

Table 2.1 presents the significant contributions in the field of TAM and it illustrates the gradual developments made throughout the years. Incorporation of capacity constraints and information provision are increasingly addressed in the most recent applications while the need to account for dynamic prediction schemes instead of instantaneous (perfectly accurate) predictions with respect to RTI is concerned. This also stresses the investigation of RTI accuracy in real-world and how it is perceived by passengers, which can in turn be modeled. The shift from static assignment models to time-dependent supply and demand representations is also apparent while some recent approaches account also for stochasticity in supply operations and passengers’ arrival at stops, providing a more realistic representation of demand and supply dynamics.

However, the adaptive nature of decision makers expressed through the ability to take en-route decisions has only recently been incorporated. Concerning passengers’ learning process, its implementation in day-to-day dynamics has been included in a few models, but it still lacks the connection with passengers’ own formulation of perceived service performance.

As a result, there are a lot of limitations and assumptions which can be examined to be refined for the more appropriate representation of demand. Path choice utility
Table 2.1: Summary of contributions concerning the reviewed studies in TAM

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x: denotes implementation
D=Deterministic
S=Stochastic
models are, in the majority of applications, based on expectations (mean values) of the relevant trip attributes. Moreover, both information’s uncertainty and credibility level are neglected and estimations are based on the highest level of information, i.e. RTI when it is available. RTI potential to reduce the uncertainty associated with this expectation. The extent of uncertainty reduction achieved by RTI provision clearly depends on its prediction accuracy and perceived credibility.

Ettema and Timmermans [2006] set the base to this direction in their effort to estimate the costs of travel time uncertainty together with the benefits from RTI. The authors use expected utility theory while taking into account the trade-off between the probability of a delay (or early arrival) against the consequences of the delay (or early arrival). The authors based their framework on the work of Nolland and Small [1995] described above. In order to capture uncertainty and the response to information, they introduced the following additional variables, for a departure time \( t \) in car traffic context: first, the true travel time, as it can be measured on the road over a large number of days - and thus its true distribution can be defined; then, the perceived travel time, which is the perception the traveller might have of the true distribution of travel times; third, the predicted travel time, which is the travel time prediction for a certain departure time \( t \) on a particular day, and it is assumed to differ from the true travel time by a term \( m \), described by a mean and variance; and finally, the perceived travel time prediction, which is the assessment of the predicted travel time. The explicit calculations of misperception cost and information benefits can be derived from the assumption of the travel time distribution.

Further research in this direction, extended to several trip decisions in public transport context, and combined with passengers’ ability to learn in an iterative decisions context, would provide a more thorough insight on the assessment of the measures which aim to improve reliability in transit operations and facilitate more efficient planning strategies.

Figure 2.1 illustrates the framework in which the above concepts are combined in order to account for passengers’ perceptions, formulated by both their experiences (of service reliability) and information provision. This framework also accounts for the day-to-day learning which in turn determines the credibility level of all information sources. These notions will be applied in a dynamic and disaggregate setting into a public transport system, described in the following chapters.
Figure 2.1: Passengers’ response to service reliability and information
3 Methodology

3.1 Model Formulation

The purpose of this section is to suggest a dynamic modeling approach which captures the impact of uncertainty on transit path choices. Passengers’ adaptation and the integration of information regarding travel components (e.g. trip times, crowding) are conceptualized as within-day and day-to-day learning processes.

Information provision is instrumental in reducing passengers’ uncertainty with respect to remaining trip components, such as waiting times or travel times. It has evolved over the years from static means (e.g. timetables) to real-time provision (Information display at stops, Variable Message Signs etc.). The uncertainty related to information, from the perspective of passengers’ perception, is based on two factors: first, the reliability of information provision and second, the accuracy compared to the actual experience. The former is related to the generation of the information (i.e. prediction scheme) while the latter is derived by the difference between the provided value and the actual experience after the choice has been made.

In a repetitive choice context, this deviation can affect passengers’ expectations, and thus trip decisions, under the assumption that experience plays a role in the formation of individual’s expectations. In technical terms, this implies that individuals’ have two inter-linked cognitive features: (1) Memory formation - record information concerning their experienced travel attributes; (2) Learning process – incorporating new information into the formulation of a credibility indicator associated with each type of information. For example, a passenger who habitually boards a certain bus line from a certain stop, for which he knows the headway ($H_1$) will be able to construct the distribution of his/her experienced waiting times (Figure 3.2), and thus update his expectation ($WT_1 > H_1/2$). In case the stop provides RTI about the waiting time, he might also have observed that his/her actual waiting time is consistently different from the RTI display upon arrival at stop. In other words, a distribution of projected waiting times would be constructed, corresponding to the one of the experienced waiting times. As a result, the passenger might update the credibility attributed to this RTI source which will in turn determine his/her future anticipation.
However, even nowadays, there might be cases where the passenger has no information at all about the travel attributes related to his/her decision. Even if in advanced urban transit networks this is rarely the case, this might be a user characteristic (e.g. no internet access, physical disability etc.). Nevertheless, if the user performs the same decision a lot of times, he/she will be probably able to form an expectation for the travel attribute just by experience. Consequently, experience can be considered an additional information source, which, similarly to provisioned information, can be assigned its own credibility by the passengers according to repetitive choices’ outcome.

As a result, one could conclude that there are three types of information sources which play a role in passengers’ travel decisions: Static Information, Real Time Information and Experience (EXP) when it comes to repetitive decision making. Static information describes the pre-trip information available to passengers. This information can include the network topology, provided lines at certain stops, and timetables. It is conventionally considered to be passengers’ Prior Knowledge (PK).

Static Information cannot be treated with respect to variability, since it has always the same value for a certain trip attribute, for example when it comes to waiting time, it is a function of the line’s headway, and is hence uniform across the population. In contrast, Experience and RTI are both subject to day-to-day variability due to the probabilistic path choice and the stochastic nature of transit supply dynamics (e.g. traffic conditions).

In order to illustrate these concepts, let us denote a trip related attribute, $t \in T$, where $T$ is a set of trip-related attributes such as waiting time, in-vehicle time, crowdedness, number of transfers, fare, etc.

Passengers’ expectations from each information source are then denoted by $t_{n,l,s}^{\exp(\lambda)}$, where the superscript $\exp$ indicates the expectation with regard to each information source $\lambda \in \{PK, RTI, EXP\}$. $n$, $l$ and $s$ are passenger-, line- and stop-specific indices, respectively. After the decision has been made, each passenger has
experienced a certain value of the relative trip attribute, which is denoted by $t^{\text{act}}_{n,l,s}$ where the superscript $\text{act}$ indicates the actual (experienced) value.

The deviation between the expected and the experienced values can then lead passengers to estimate the credibility levels of each information source (denoted by $\alpha_{n,l,s}$) which in turn form the anticipations for the following decision, denoted by $t^{\text{ant}}_{n,l,s}$, where the superscript $\text{ant}$ indicates the anticipated value (Figure 3.1).

It should be noted that the perceptual errors may result in a discrepancy between actual travel experience and perceived travel experience. Previous studies offer empirical evidence that such discrepancies may exist [Avineri, 2004; Dziekan and Kottenhoff, 2007]. However, for the sake of this study the terms “experienced” refers to remembered as well as actual travel experience.

The proposed dynamic path choice model includes four modules: (i) the incorporation of experienced (trip-related values) in the decision making as an additional information source; (ii) the information credibility coefficients formed by the daily update of passengers’ memory by this experience; (iii) the adaptive users’ behaviour, as reflected through the continuous evaluation of the available choices and (iv) the scheduling effect which becomes particularly important with the definition of the lateness probability which captures the within day variability of travel attributes. These modules will be explicitly illustrated in the sections 3.1.1 and 3.1.2.

At this point, the definition of the term “day” has to be given in order the reader to be able to understand the context of the within-day and day-to-day dynamics. The term here actually describes each iteration in the simulation environment, and it is used in order to conventionally account for passengers daily decisions when it comes to trip choices. Note that Traffic or Transit Assignment Models usually simulate a certain time period during the day, this usually being the morning or afternoon peak hours, in order to capture the maximum demand that occurs within one day. In an iterative context, where the simulation environment accommodates also updates in passengers’ strategies, consecutive iterations of the demand dynamics can be assumed to mimic the day-to-day adaptive behaviour of passengers in real world. For example how passengers adjust their departure time choice for the home-to-work trip every morning according to the experienced traffic conditions of the previous days.
3.1.1 Day-to-day Dynamics

Passenger’s expectation from experience (EXP) will, of course, vary with respect to the service’s line and stop combination, while expectations among different passengers might not be the same, even if they regard the same service combination, due to personal variations in previous experiences and/or memory formulation. This expectation with respect to experience ($t^{exp(EXP)}_{n,l,s}$) is assumed to be formed by the repetitive experiences of the passengers, every time they visit a stop and choose a certain line which serves his destination. These experiences are stored cumulatively in their memory. The underlying learning function of this process can be defined as:

$$t^{exp(EXP)}_{n,l,s}(d + 1) = \left(1 - \kappa_n^{exp(EXP)}\right) * t^{exp(EXP)}_{n,l,s}(d) + \kappa_n^{EXP} * t^{act}_{l,s,n}(d) \quad (3.1)$$

$$0 \leq \kappa_n^{EXP} \leq 1$$

Where:

- $d$ is the day (nr. of iteration) passengers’ choice and experience take place
- $\kappa_n^{EXP}$ is the weight each additional experience has on passenger’s expectation and can be defined as a function of the day $d$.

Notice the notations of expectation (exp) and experience (EXP), which differentiate only with respect to the case of the letters (lower-case and upper-case respectively).

Assuming an infinite amount of accumulated experience, the passenger would be able to construct the exact distribution of experienced (actual) values of the respective
travel attribute. This would determine his/her knowledge level with respect to uncertainty. For example, a passenger with many stored days in memory would have an expected value, derived from Equation 3.1, closer to the expected experienced value of the actual distribution, and, at the same time, he/she would have a sufficient amount of observations of how this expectation deviated from the actual experience. In other words, assuming a distribution of actual waiting times with a range from 5 min to 10 min and mean waiting time equal to 8.5 min, a passenger with sufficient amount of days stored in his memory would be able to estimate an expectation close to 8.5 min but he would also be aware that his waiting time can reach the duration of 10 minutes.

Experience can, thereby, refine passengers expectations based on Prior Knowledge (headways are the same over the days) on one hand, and on the other hand can define the uncertainty related to this expectation.

This is also one of the main motivations for Real Time Information provision which aims to help passengers shift their expectations closer to the actual values of the service performance and also reduce the uncertainty of the passengers concerning these expectations. This means that even if a passenger is perfectly aware of the actual distribution of the waiting times, in the above mentioned example, RTI should be instrumental in making him/her aware where he/she stands “today” in this distribution (from 5 to 10 minutes).

In current Transit Assignment applications passengers are assumed to incorporate RTI into their decision mechanisms as fully credible [Cats, 2011]. However, since there is not enough evidence of to what extend RTI predicts accurate information, it can be assumed that passengers are able to perceive possible deviations between projected and experienced values, and thus, construct the respective distributions for each projected value. Remember that this model formulation does not account for errors in passengers’ perceptions, thus there is no difference between what passengers perceive as experience and the actual values.

Consequently, passengers’ accumulated experience conduces also to the awareness of RTI projection deviation from experience and, assuming sufficient memory, to the formation of the distribution of this deviation. RTI projected values during the day can be considered passengers expectations with respect to this information source and the uncertainty related to that can be captured by the deviation of the experienced value after the choice has been made.

The expectation-experience gap described above, for all information sources, can be captured in passengers’ perception through the day-to-day update of their memory by explicitly accounting for the deviation value. This update defines how the credibility coefficient evolves by recalculating $\alpha_{n,t,s}$, at the outset of each day:

$$\alpha_{n,t,s}(d + 1) = \left(1 - \kappa \alpha^d\right) \alpha_{n,t,s}(d) + \kappa \alpha^d \left(\delta^d(d)\right)^\nu$$ (3.2)
Where:

- $\lambda$, is the information source, $\lambda = \{EXP, RTI, PK\}$
- $\delta$ is the absolute deviation effect, defined as follows:

$$
\delta^\lambda(d) = \left( \frac{t_{n,t,s}^{exp}(\lambda)(d)}{t_{n,t,s}^{act}(d)} - 1 \right) + 1 \quad (3.3)
$$

- $\kappa^a$ is the weight each additional deviation has on passenger’s credibility level. This parameter reflects the recency effect, as in Equation 3.1, and can be a function of the day $d$, in order to accommodate two concepts: (1) the concept of decay as the diminishing weight each added day has in the overall evolution process of passengers’ perception, and (2) the recency effect on passengers’ learning mechanism, which reflects the weight a new added experience has, compared to the estimation of the previous day. Both concepts are described by Bogers, [2009].

- $\nu$ reflects the salience effect on passenger’s memory. In other words, whether a large deviation would have a smaller or a higher impact on passengers’ expectations. There have been arguments concerning what describes best individuals’ behaviour [Arentze and Timmermans 2003, Bogers 2009] while Bogers [2009] suggest that this can depend on individuals’ habitual choices. In this case, habitual and non-habitual passengers can be disaggregated into different groups, according to the amount of days in their memory, and different salience values can be applied.

Note that the learning function is defined as individual ($n$) specific, which allows for variations in the formulation of both the expectations and the credibility coefficients across the population (Equations 3.1 and 3.2). These could include differences in trip frequencies, cognitive mechanisms, etc.

Finally, since the credibility coefficients reflect the weight each expectation has in the overall anticipation, it is their relative value, with respect to the remaining information sources available, which matters. As such, their absolute value can be normalized accordingly at each decision point.

### 3.1.2 Within-day Dynamics

This section describes the within-day dynamics of passengers’ anticipation and how they are incorporated in the decision model. The adaptive nature of the passengers, when it comes to decision making, suggests that they can update their anticipation according to the provided information from the time they decide to make a trip (conventionally the beginning of a day) to the time they arrive at a boarding stop and finally to the actual time of boarding. For example, a passenger might be denied boarding due to capacity constraints. This could be accommodated by an updating
loop of the anticipated waiting time in case of boarding decision which could in turn result to a different path choice (or even a different stop to continue the trip). Moreover, a share of the passengers might have access to RTI through a personal mobile device (e.g. smartphone) and thereof can have instantaneous access to the information regarding the whole network.

These anticipations are then incorporated in the decision model, which represents how the passenger evaluates them - or in other words, what is the weight he/she assigns to each attribute’s value and the respective trade-offs. The trip choice which is studied here is made with respect to path alternatives. As such, during the trip passengers evaluate the path alternatives, through an adaptive decision model, for the remaining downstream trip.

The Utility path choice model adopted in this study incorporates scheduling considerations in line with the study by Ettema and Timmermans [2006] who based their model on the schedule-delay model of Noland and Small [1995] in the context of trip departure time choice.

It is hence assumed that each traveller $n$ has a preferred arrival time ($pat$) at the destination. The notion of the preferred arrival is introduced in order to capture passengers’ response to uncertainty around the arrival time at destination. Since trip scheduling is a result of other (scheduled) activities, passengers might be elastic or less elastic to fluctuations in arrival times to their destinations, in correspondence to how punctual they have to be to their schedule. In morning peak hours for example, where most trips serve work purposes, passengers’ preferred arrival time would be expected to be coordinated to the job start time. A strict job start time would, in turn, entail an inelastic attitude to delays.

Given passengers’ preferred arrival time, their anticipations for the travel time would determine their departure time from their origin. In a day-to-day context, where passengers can store their past experiences and update their strategies, departure time choice could be updated according to travel time anticipations, so that the passenger makes sure that arrives on time. This is why scheduling constraints are widely used within the frame of departure time choice. However, since travel time is a mode- and path- specific characteristic, the scheduling constraints can be included within any trip decision context, and even more efficiently, in combined decision context, where the passenger can adjust through the learning process, departure time, mode and path choices.

The deterministic part of the utility function of individual $n$ for a path alternative $i$ takes, thus, the following form:

$$V_{i,n} = \Gamma_{n} * T_{i,n}^{ant} + \beta_{n}^{sde} * sde_{i,n}^{ant} + \beta_{n}^{sdi} * sdl_{i,n}^{ant} + \beta_{n}^{late} * p_{i,n}^{late} \quad (3.4)$$

where:
- \( \Gamma_n \) is the vector of path attributes’ parameters
- \( T_{i,n}^{\text{ant}} \) is the vector of anticipated values of the corresponding attributes,
- \( sde_{i,n}^{\text{ant}} \) and \( sdl_{i,n}^{\text{ant}} \) are the anticipated early and late schedule delays respectively, defined as follows:

\[
sde_{i,n}^{\text{ant}} = \max((pat_n - at_{i,n}^{\text{ant}}),0) \quad (3.5)
\]
\[
sdl_{i,n}^{\text{ant}} = \max((at_{i,n}^{\text{ant}} - pat_n),0) \quad (3.6)
\]

where \( at_{i,n}^{\text{ant}} \) is the anticipated arrival time at destination which is defined by adding the anticipated travel time, \( tt_{i,n}^{\text{ant}} \), to time instance \( \tau \) when the decision-making takes place:

\[
at_{i,n}^{\text{ant}} = \tau + t_{i,n}^{\text{ant}} \quad (3.7)
\]

- \( p_{i,n}^{\text{late}} \) is the probability of late arrival at destination, independently of what the anticipated value of Equations 3.5 and 3.6 is (Figure 3.2), defined as:

\[
p_{i,n}^{\text{late}} = p(pat_n < at_{i,n}^{\text{ant}}) \quad (3.8)
\]

- \( \beta_n^{sde} \), \( \beta_n^{adt} \), \( \beta_n^{plate} \) are the corresponding scheduling coefficients. The scheduling constraints have been widely used in approaches which aim to model travelers’ departure time choice. In these cases, the estimated values are in line with the Prospect Theory which suggests that individuals evaluate losses higher than gains in decisions under risk [Kahneman and Tversky, 1979]. This determines the relation between the absolute values of the relevant coefficients: \( |\beta_n^{adt}| > |\beta_n^{sde}| \), while both have negative signs. The parameter of the travel time lies between those two showing that passengers prefer to travel than to arrive late, but prefer to arrive early than to travel. The parameter of the lateness probability is also negative indicating the negative effect uncertainty has on passengers’ preferences in within-day dynamics.

Concerning vector \( T_{i,n}^{\text{ant}} \), its elements include the anticipated values for all applicable trip attributes. These anticipations are formed by integrating all the above mentioned information sources:

\[
t_{i,n}^{\text{ant}} = \sum_{\lambda} \alpha_{n,i,s} \cdot \exp(\lambda) \quad (3.9)
\]

\( \lambda = PK, RTI, EXP \)

This formulation allows each type of information to be treated separately, in the sense that if some of the information sources is not available on day \( d \), passengers’ anticipations are still formed by the combination of those available. For example, if RTI is not available when making a boarding decision, passenger will form his/her
anticipation according to the expectations formed only by experience (EXP) and Prior-Knowledge (PK). In this case the credibility coefficients will have to be normalized as it has already been mentioned in the previous section.

Note that this normalization does not affect the relative value of each credibility coefficient, which is the important concept during the information integration, but only their absolute values.

The overall anticipated value for each trip attribute \( t_{i,n}^{\text{ant}} \) for the path \( i \) for all remaining path-relevant stops, \( s \), to the destination is:

\[
t_{i,n}^{\text{ant}} = \sum_s c_{n,s}^{\text{ant}} \quad (3.10)
\]

\( t_{n,s}^{\text{ant}} \) is, thus, a function of the anticipated values of each line \( l \), of the relevant set of lines which serve the destination at every stop of the remaining downstream trip. The rule of the minimum anticipated value until the arrival of the first arriving service of each relevant set of lines, \( L_r \), can be adopted here. But it should be noted that passenger’s anticipation is time-dependent and dynamic, as new en-route experiences or information may become available and influence passenger’s perception. Hence, the anticipation upon making a decision at time \( \tau \) is formulated as follows:

\[
t_{n,s}^{\text{ant}}(\tau) = \min_{l \in L_r} t_{n,l,s}^{\text{ant}}(\tau) = \min_{l \in L_r} \left( \sum_{l} \alpha_{n,l,s} \ast c_{n,l,s}^{\text{ant}}(\tau) \right) \quad (3.11)
\]

Concerning the scheduling effect, anticipated early and late schedule delays are calculated according to the anticipated arrival time, \( a_{i,n}^{\text{ant}} \), which is the sum of the time of decision making \( (\tau) \) to the anticipated remaining travel time, \( t_{i,n}^{\text{ant}} \). The preferred arrival time, \( pat_n \), can be exogenously defined for each passenger, assuming a departure time \( t_{i,n}^{\text{dep}} \), from origin stop.

The anticipated travel time, \( t_{i,n}^{\text{ant}} \), can be formed as the sum of the anticipated values of the trip components of vector \( T \) which form it (waiting time, in-vehicle time, walking time etc.), which are derived from the expected attributes’ values weighted by the credibility coefficients as it has been described above, for every time \( \tau \) where a decision takes place.

The estimation of the probability of arriving late, given the \( pat_n \) (Figure 3.2), is given by the integral of the total anticipated travel time distribution, \( t_{i,n}^{\text{ant}}(\tau) \), and, as such, it captures the variability in within-day dynamics:

\[
p_{i,n}^{\text{late}} = p(pat_n < a_{i,n}^{\text{ant}}(\tau)) = p(\tau_{i,n}^{\text{ant}}(\tau) - pat_n > 0) =
\]

\[
= \int_{pat_n}^{\infty} (at_{i,n}^{\text{ant}}(\tau)) \, d\tau = \int_{pat_n}^{\infty} \tau + (tt_{i,n}^{\text{ant}}(\tau)) \, d\tau \quad (3.12)
\]
Equation 3.12 can be explicitly calculated if the characteristics of the total travel time distribution are known.

![Figure 3.3: Probability of late arrival](image)

3.2 Implementation

3.2.1 Requirements

The model formulation is designed with the potential to capture the network dynamics and, as such, to represent the path choice decision context as realistically as possible. The implementation of the model would, therefore, require a simulation environment which accounts for the dynamic representation of both supply and demand operations. More specifically, the model requires a disaggregate demand representation so that each traveler is a unique entity with its own characteristics (preferences and behaviour) which lead to disaggregate trip choices throughout the simulation process. Passengers’ adaptive nature in within-day dynamics requires the stochastic representation of demand processes, such as arrival at stops and decision making, i.e. passengers can update their anticipations according to their en-route experiences, which, among others, include RTI provision, boarding denials, extreme delays etc. As a result, the key sources of service uncertainty have to be captured (traffic conditions, dwell times, transit dynamics, etc.) and RTI generation and dissemination has to be accommodated during the simulation period.

3.2.2 Agent-based Simulation

Agent-based models represent the simultaneous operations and interactions of multiple agents, in an attempt to mimic and predict the behavior of complex systems. They are used in multiple applications, among which is the traffic assignment problem. Simulation models advantages include the realistic representation of traffic conditions and the classification of the users (agents) which aims to capture differences with respect to preferences, decision mechanisms and other
characteristics, across travelers’ population. As a result, such a platform would be appropriate to satisfy the requirements for the model implementation, mentioned above.

3.2.2.1 BusMezzo

The implementation of the model was embedded in a public transport agent-based simulation model, BusMezzo, which represents both demand and supply operations dynamically. Passengers’ generation is a stochastic process which is based on time dependent OD matrices. Passengers’ arrival at stop is a stochastic process which follows a Poisson distribution. Their progress through the network follows a two-stage decision process: first passengers decide whether to stay, board or alight at each stop they cross throughout their trip, and then, provided that this stop serves their remaining trip, they evaluate the alternative paths, which serve their destination. This structure in passengers’ decision making process is important because it satisfies the adaptive nature of the individuals at each stage of their trip, i.e. passengers do not make their choices from a pre-defined set of path alternatives.

Moreover, passengers’ perception is formed by their PK, their accumulated travel experience (if applicable) and RTI, if available. Their distribution to vehicles, which are also represented as agents, is, thus, a result of consequent trip choices which depend on travelers’ perception of the trip components.

The different sources of public transport operations uncertainty including traffic conditions, vehicle capacities, dwell times, vehicle schedules and service disruptions are also modeled explicitly and facilitate different scenario tests. Finally the generation and dissemination of RTI is implemented dynamically since BusMezzo can accommodate different prediction schemes [Cats, 2011].

3.2.3 Model Specification

In a previous application of BusMezzo, Cats [2011] estimated a path-choice model for an urban transport network. The path choice utility model included the following travel components (Equation 3.13): Total anticipated waiting time ($t_{w, n}^{\text{ant}}$), total anticipated in-vehicle time ($t_{v, n}^{\text{ant}}$), total anticipated walking/connection time ($t_{c, n}^{\text{ant}}$) and number of transfers ($t_{\text{trans}}$).

$$V_{ln} = \beta_{n}^{\text{wait}} * t_{w, n}^{\text{ant}} + \beta_{n}^{\text{int}} * t_{v, n}^{\text{ant}} + \beta_{n}^{\text{walk}} * t_{c, n}^{\text{ant}} + \beta_{n}^{\text{trans}} * t_{\text{trans}}$$  (3.13)

Equation 3.13, thus, represents the first part of the Utility path-choice model presented in Equation 3.4 of the previous section. Concerning the scheduling coefficients ($\beta_{n}^{\text{ade}}$, $\beta_{n}^{\text{adi}}$, $\beta_{n}^{\text{late}}$), the required data collection and their estimation lies beyond the scope of this thesis. Hence, two options were available: either they would not be included in this application at all, or estimated values from previous case studies would be used.
For the latter case, the amount of originally estimated values available from the literature review is limited when it comes to schedule constraints’ applications with a significant range of values, while most estimates refer to car traffic context. Moreover, BusMezzo performs single simulation runs which emulate the within-day network dynamics. Day-to-day learning mechanism is not accommodated, which also sets the departure time choice not applicable at this stage.

The abovementioned reasons, together with the importance of the sensitivity analysis of the remaining modules (learning function parameters and credibility coefficients base values) of the formulated model led to the decision not to incorporate the scheduling effect at all, in order to avoid increased complexity in the interpretation of the results. The impact of uncertainty is therefore studied in the context of path choice decisions while disregarding its potential impact on trip departure time choice.

At this point, few implementation assumptions were made. First, a very important step was to define the aggregation level with respect to passenger choices identification over the days. The most detailed form of the model proposes each trip related attribute (and its anticipated value) to be passenger-, stop- and line- specific. However, since so far it is impossible to keep track of every passenger beyond single simulation replications in BusMezzo, the Origin-Destination (OD) pair aggregation level was selected to account for passengers’ classification and tracing over the days. The possible implications of this assumption are discussed further below in this report.

Moreover, even though the model formulation can accommodate uncertainties associated with various trip-related components, the focus in this study will be on the uncertainty related to the waiting time information. As a result, the within-day and day-to-day dynamics had to be specified as follows:

### 3.2.3.1 Day-to-day Dynamics

Since the credibility coefficients are now defined for only one trip attribute, this being the waiting time, their formulation in Equation 3.2 is based only on the information regarding the latter. As a result, the deviation parameter for each information source of Equation 3.3 is now formed:

$$
\delta^{i}(d) = \left| \frac{wt_{i,s,n}^{exp}(\lambda)}{wt_{i,s,n}^{act}(d)} - 1 \right| + 1 \quad (3.14)
$$

Where $wt_{i,s,n}^{exp}(\lambda)$ is the waiting time expectation from each information source and $wt_{i,s,n}^{act}$ is the actual waiting time at each stop.

The credibility coefficients, with respect to the information regarding waiting time, are formed as follows:
\[ a^\lambda_{i,s,OD}(d) = \left( 1 - \kappa a^\lambda \right) * a^\lambda_{i,s,OD}(d) + \kappa a^\lambda * \left( \delta^\lambda(d) \right)^\nu \] (3.15)

Where \( a^\text{RTI}_{i,s,OD}(d) \) is the average credibility coefficient over all passengers within the same OD pair, \( n \in N_{OD} \):

\[ a^\lambda_{i,s,OD}(d) = \frac{\sum_{n \in N_{OD}} a^\lambda_{i,s,n}(d)}{\sum n} \] (3.16)

\[ 0 \leq \kappa a^\lambda = f(d) \leq 1 \] (3.17)

For the normalization of alphas a relative (to their sum) weighted factor has to be calculated:

\[ w^\lambda_{i,s,OD} = \frac{\alpha^\lambda_{i,s,OD}}{\alpha^\text{PK}_{i,s,OD} + \alpha^\text{RTI}_{i,s,OD} + \alpha^\text{EXP}_{i,s,OD}} \] (3.18)

\[ \lambda = \text{PK, RTI, EXP} \]

and multiplied to each one of them so that:

\[ \sum_{\lambda} \left( w^\lambda_{i,s,OD} * a^\lambda_{i,s,OD} \right) = 1 \] (3.19)

Similarly, passengers’ expected waiting time, due to experience, at the beginning of the following day is also formed by their experience at the end of each decision day:

\[ w_{t,s,n}^\text{EXP}(d) + 1 = \left( 1 - \kappa_{OD}^\text{EXP} \right) * w_{t,s,OD}^\text{EXP}(d) + \kappa_{OD}^\text{EXP} * w_{t,s,n}(d) \] (3.20)

\[ w_{t,s,OD}^\text{EXP}(d) = \frac{\sum_{n \in N_{OD}} w_{t,s,n}^\text{EXP}(d)}{\sum n} \] (3.21)

\[ 0 \leq \kappa_{OD}^\text{EXP} = f(d) \leq 1 \] (3.22)

Note that the terms \( \kappa_{OD}^\text{EXP} \), \( \kappa a^\lambda \), \( \nu \) are expected to play an important role in how passengers decisions evolve over the day-to-day assignment process. It is hence their sensitivity analysis that will offer insights on how passengers’ perception and behaviour are affected by uncertainty, as it has been explained in the model formulation section. Variations among passengers can be accommodated through different \( \kappa \) values, which would in turn imply different learning patterns. A more “sensitive” learner would have a higher \( \kappa \) value than a less sensitive learner for example (and would “jump” easier among alternatives according to recent outcomes).

Similarly, variations in \( \nu \) values can accommodate different patterns when it comes to high deviations between expectations and experiences in populations. For example a passenger with a lot of stored days in his memory might give less weight in a high
deviation if this is unusual relatively to what he has experienced so far. This would be accounted by a $\nu > 1$ value for this individual.

### 3.2.3.2 Within-day Dynamics

The overall anticipated waiting time, $w_{t,x,n}^{\text{ant}}$, is formed from all expected values in accordance with Equation 3.9:

$$w_{t,x,n}^{\text{ant}} = \sum_{\lambda} w_{t,x,n}^{\text{ant}(\lambda)} = \sum_{\lambda} \alpha_{t,x,OD}^{1} \cdot w_{t,x,n}^{\text{exp}(\lambda)}, n \in N_{OD} \quad (3.23)$$

where:

- $w_{t,x,n}^{\text{exp}(PK)}$ is the static information user has access to (e.g. timetable),
- $w_{t,x,n}^{\text{exp}(RTI)}$ is the aggregate expectation by experience, as the outcome of previous days, for all the passengers of the same OD pair,
- $w_{t,x,n}^{\text{exp}(RTI)}$ is the provided information at any time $\tau$ the passenger evaluates the existing path alternatives.

Note that in within-day dynamics passengers are represented as agents, so only their expectation from (previous days’) experience has to be aggregated in the OD level.

Equation 3.24 is then incorporated in the Utility function (Equation 3.13), summed over all possible stop-line combinations of the remaining downstream trip, for the evaluation of each path alternative, in line with the Equations 3.10 and 3.11.

Note that the stop-line-OD specification implies that passengers’ anticipations and experiences are actually stored in this level, and, as such, they are “global” in the sense that are appointed to any passenger that uses the same combination, as an input for the simulation of the decision making of the following day. This assumption does not, however, violate the learning process of individuals, which is anyway based on repetitive stop-line choices, but it does limit the variation within the population, since it is this same -with respect to passenger specific characteristics- passenger who uses the several stop-line combinations.

### 3.2.3.3 Convergence

The day-to-day dynamics represent the learning process of individuals. As such there should be some point where users’ anticipation will get close to the experienced waiting time. That would be the point where the system will reach convergence and
the relevant measures of all modules will be stabilized and become available for analysis.

\[
\left| \frac{WT_{ant,OD}(d)}{WT_{act,OD}(d)} - 1 \right| \leq \theta \quad (3.25)
\]

where \( \theta \) is the threshold value for convergence of the anticipated waiting time into the experienced waiting time.

Note that for the convergence criterion it is the anticipated waiting time at the beginning of the day or, more appropriately in terms of actual calculation, at passengers’ arrival time at stop, which is compared with the experienced waiting time at the end of the day, and not the dynamic anticipation that a passenger forms within-day from the arrival time at stop until the boarding decision.

For the sake of the calculation of this convergence ratio, the overall anticipation upon arrival at stop had to be estimated with the RTI value, if available, projected at that time (as generated by BusMezzo under a certain prediction scheme).

3.2.3.4 Day-to-day and within-day interaction

Day-to-day learning is not accommodated in BusMezzo at its current implementation. As a result, the day-to-day dynamics had to be implemented externally in a way that would allow the update of the relevant components and their incorporation into the simulation runs for the emulation of the within-day dynamics. For the purpose of this interaction, a MATLAB\textsuperscript{8} script was written in order to process the outputs of within-day dynamics of BusMezzo, update the appropriate values (expectation by experience and credibility coefficients) as described in the equations mentioned in the section above, and, finally, “feed” them back to BusMezzo for the simulation of the “following” day, until convergence is reached.

More specifically, within-day disaggregate passengers’ assignment results were stored at each Stop-Line combination level, within a certain Origin-Destination pair, hereinafter symbolized with the ODSL (Origin-Destination-Stop-Line) acronym. This includes passengers’ PK, RTI projection upon arrival at stop and experienced waiting time at the end of the simulation period, which are stored in an output file generated by BusMezzo at the end of each simulation run. These data are then processed through the MATLAB\textsuperscript{8} script in order the expectations (by experience) of the following day to be calculated and the alpha coefficients to be updated, at the ODSL aggregation level. The input and output files for the simulation in BusMezzo are presented in Figure 3.3.

Since BusMezzo is a stochastic simulation model, each day has to be replicated more than once, for statistical inference. The results of each replication have therefore to be stored and averaged in order to calculate the network measures of each day. These measures were mainly distinguished according to the level of aggregation. As a
result, at the end of each simulation period the ODSL-specific measures (such as the credibility coefficients) were calculated as a weighted average according to the passenger loads associated with different ODSL combinations. Then the results were
averaged over the number of replications in order to get the system values for a given day.

With respect to the passenger-specific measures (e.g. anticipated and experienced waiting times), the values were simply averaged over the total assigned passengers at every day replication and then averaged over the number of replications.

The iterative assignment process is performed by using the following steps:

1. Initialize base values for credibility coefficients \( \alpha_\text{1,5,OD} \), to be used at day=1 and for every first occurrence of each ODSL combination
2. Set \( k = f(d) \), for all \( \kappa^{RTI}, \kappa^{PKE}, \kappa^{EXP} \)
3. Simulate within-day dynamics in BusMezzo
4. Store experienced waiting time, Prior Knowledge and RTI projection upon arrival at stop for all \( n \) simulated passengers at all stops within each OD pair
5. Calculate the overall anticipation for the current day, for the convergence criterion:
   a. \( \alpha^{PK}_{\text{1,5,OD}} \cdot \alpha^{EXP}_{\text{1,5,OD}} \cdot \alpha^{RTI}_{\text{1,5,OD}} \cdot \alpha^{\text{ACT}}_{\text{1,5,OD}} \)
   b. Average for each ODSL combination
6. Update each credibility coefficient \( \alpha_\text{1,5,OD} \) according to the expected and experienced waiting time \( \alpha^{\text{ACT}}_{\text{1,5,OD}} \), by:
   a. \( \alpha_\text{1,5,OD}^{n+1} = \left(1 - \kappa^{\alpha^2}\right) \cdot \alpha_\text{1,5,OD}^n + \kappa^{\alpha^2} \cdot \left( \delta_\text{1,5,OD}^n \right) \), for all \( n \)
   b. Average for each ODSL combination
   c. Normalize so that \( \sum_1 \alpha_\text{1,5,OD} = 1 \)
7. Update passengers’ waiting time anticipations based on experience
   a. \( \alpha^{\text{EXP}}_{\text{1,5,OD}}(d + 1) = \left(1 - \kappa^{\text{EXP}}_{\text{n}}\right) \cdot \alpha^{\text{EXP}}_{\text{1,5,OD}}(d) + \kappa^{\text{EXP}}_{\text{n}} \cdot \alpha^{\text{ACT}}_{\text{1,5,OD}}(d) \), for all \( n \)
   b. Average within each ODSL combination
8. Check the overall convergence criterion \( \left| \frac{\alpha^{\text{ACT}}_{\text{1,5,OD}}(d)}{\alpha^{\text{EXP}}_{\text{1,5,OD}}(d)} - 1 \right| \leq \theta \):
   a. If true Stop simulation
   b. If false go to Step 9
9. Update history file with the average anticipations, experiences and credibility coefficients by day, Increase day and Go to step 3.

Main algorithm loop for each replication:

Require: network, OD matrices

Require: parameters file and alternative scenario specifications (base \(a\), RTI level, \(k\), \(v\), \(\theta\))

while \(\left| \left( \frac{wt_{s,t,OD}^{(d-1)}}{wt_{s,t,OD}^{(d-1)}} \right) - 1 \right| \geq \theta\) do

Perform within-day Transit Assignment to generate passengers’ stop-line choices \(i_{s,t,n}(\alpha^1, wt^{exp(PK,RTI)}, wt^{act})\)

for \(n = 1, ..., N\) do

Update waiting time anticipations due to experience \(wt_{s,t,n}^{exp(EXP)}\)

Update credibility coefficient \(\alpha_{s,t,\lambda}^1\)

end for

Calculate \(wt_{s,t,OD}^{exp(EXP)}\) and \(\alpha_{s,t,OD}^1\) for each of the unique ODSL combinations

Normalize credibility coefficients: \(\sum_A \alpha_{s,t,OD}^1 = 1\)

Calculate system’s \(\alpha^3, wt^{exp(PK)}, wt^{act}\)

Update history file

Increase \(d\)

end while
4 Case Study

4.1 Network Description

The day-to-day simulation assignment model was applied to the Stockholm's rapid transit system. The network readily available for this study consists of seven Metro lines (10-11; 13-14; 17-19), four high-demand trunk bus lines (1-4) and a LRT line (Tvärbanan, line 22). The network is represented in Figure 4.1. The application uses the real-world timetables, vehicle schedules and walking distances between platforms and stops. The three different transit modes have different vehicle types, operating speeds, travel time variability and are operated with different holding control strategies and dwell time functions [Cats, 2011].

The stop-to-stop OD matrices have been calculated by a proportional trip distribution procedure using data that have been collected from Metro entrances, passenger counts at LRT stations and transfer points and Blue bus automatic counts [Cats, 2011]. Moreover, since the peak-period headway of each of the transit lines is in the range of 5 to 10 minutes, all travelers are assumed to arrive randomly at stops, with their arrival derived from a Poisson distribution.

Passengers are assumed to have prior knowledge of planned headways and timetable in-vehicle time for all lines that serve the transit network. Moreover, in this case study, all passengers are assumed to have full and instantaneous access to Real Time Information for the whole transport network (e.g. through smartphones). Passengers are therefore able to evaluate the credibility of RTI information for the total remaining downstream trip through repetitive encounters. Passengers’ accumulated Experience is also incorporated in their anticipations during the decision making process.
The morning peak period, 6:00-9:00, is simulated. However, in order to allow a warm-up period for the transit service supply and sufficient time for travellers to reach their destination, passengers are generated between 07:30-08:30 am. At each simulation run the number of passengers is approximately 150000. Their choice-set consists of 99270 path alternatives while approximately 9000 ODSL combinations are experienced by the passengers’ population at each simulation run.

4.2 Scenario Description

The main purpose this case study is to demonstrate the capabilities of the proposed model and investigate how the model performs under various learning model parameters. More specifically, the alternative scenarios were designed to test the effects of different values of the following terms:

- Initial value of the RTI credibility coefficient (c.c.), $\alpha_{RTI}^{base}$
- Recency (and also decay) learning term, $\kappa$
- Salience learning function parameter, $\nu$
In current DTA models, RTI is perceived as 100% credible. The aim of this work is to explore the impact of passengers' ability to assign credibility values to each one of the information source, which are expected to converge after a sufficient number of experienced days. As a result, the initial value of the credibility coefficient is expected to affect the learning period but not its outcome. The different $\alpha_{RTI}^{base}$ values which are tested are 0, 0.5 and 1.0. At this point it has to be mentioned that the base value for the experience (EXP), as information source, is zero (since there is no experienced day stored in the memory) and thus the credibility coefficient for PK is calculated: $\alpha_{PK}^{base} = 1 - \alpha_{RTI}^{base}$.

The $\alpha$ formulation, reflects the weight each new experience has on the experienced values on the following day. In the case of the credibility coefficients this experience refers to the deviation between the expected value and the actual value of waiting time of each information source (Equations 3.14 and 3.15).

However, the same formulation was also used for the recency parameter of expectation from experience, $\kappa^{\text{EXP}}$, for the corresponding update of the expected waiting time, since the learning process refers to the same passenger. Finally, the recency term is applied uniformly across the population in every scenario.

The following formulations were tested and applied for the learning process (Equations 3.15 and 3.20):

- $\kappa_1 = 0.5$
- $\kappa_2 = 1/\sqrt{(d + 1)}$
- $\kappa_3 = 1/(d + 1)$
- $\kappa_4 = 1/(d + 1)^2$

where $d$ is the serial number of the latest day (number of iterations). The recency effect follows the curves of Figure 4.2. These alternative functions allow for variations in the weight each new experience has, compared to the accumulated experience so far in passengers' memory. This is decreasing in the last three cases, $\kappa_2 - \kappa_4$, accommodating users' decreasing response in recent outcomes, but with different rates. As a result, users with $\kappa = \kappa_2$ are more sensitive to recent outcomes compared to users with $\kappa = \kappa_3$ and $\kappa = \kappa_4$ in ascending order, and as a result, they are expected to switch more easily among alternatives. The passengers of the first case, where $\kappa_1 = 0.5$, are assigning the last experience the same constant value, which remains above the other three alternative formulation after day 4 (Figure 4.2). These users will fluctuate even more among path alternatives during the remaining of the learning period. As it has been mentioned in the Model Formulation part, recency term formulation as a function of the day accommodates also the decay effect which reflects the diminishing (or increasing) weight of each added experience compared to the average (not accumulated) weight of each day so far. This effect is illustrated here by the equation:
\[ \text{decay}(d) = \kappa \frac{1 - \kappa}{d - 1} \quad (4.1) \]

and is presented in Figure 4.3.

**Figure 4.2**: Recency parameter evolution for various term specifications

**Figure 4.3**: Decay effect under various formulations of the recency parameter
Passengers with \( \kappa = \kappa_1 \) and \( \kappa = \kappa_2 \) assign higher weight in the last added day, compared to the average weight of the previous days. However in the first case this weight has an increasing rate, resulting from the term’s constant value through the learning period, while in the second it decreases reflecting the amplification effect of the square root in the denominator of the ratio of the term formulation. The opposite effect is observed when the recency term is inversely proportional to the square of the number of the stored days (\( \kappa = \kappa_4 \)), where the diminishing effect of the square increases the weight of the last observed day even though it remains lower than the average weight of the previously stored days. Recency term being a simple inverse proportional to the number of days induces no decay effect (\( \kappa = \kappa_3 \)). This analysis aimed to show the contradiction in the two effects that might occur within the same formulation.

In this case study, each of these formulations has been used uniformly among the population. However, in future applications their combination could be incorporated in order to accommodate variations among the population within the same simulation experiment. The same applies, of course, for all the terms described in this section.

The last parameter to be examined in the case study, was the salience parameter \( v \), which reflects the sensitivity of the passengers to the high deviations when it comes to the credibility coefficients. The outcomes of these parameters will indicate how different assumptions about this sensitivity affect passenger decisions and ultimately network performance. The case study values were 0.5, 1 and 2. Remember that the salience parameter is a power on the deviation from experience, \( \delta \), and as such it strengthens or weakens its impact on passengers’ perception, according to the given value (\( v > 1 \) and \( v < 1 \) respectively).

In total, eight combinations of these terms were analyzed. Table 4.1 summarizes the Scenario design.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>( \alpha_{\text{RTI base}} )</th>
<th>( \kappa )</th>
<th>( v )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1K3V3</td>
<td>0</td>
<td>1/(d + 1)</td>
<td>2</td>
</tr>
<tr>
<td>A2K3V3</td>
<td>0.5</td>
<td>1/(d + 1)</td>
<td>2</td>
</tr>
<tr>
<td>A3K3V3</td>
<td>1</td>
<td>1/(d + 1)</td>
<td>2</td>
</tr>
<tr>
<td>A2K1V3</td>
<td>0.5</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>A2K2V3</td>
<td>0.5</td>
<td>1/( \sqrt{(d + 1)} )</td>
<td>2</td>
</tr>
<tr>
<td>A2K4V3</td>
<td>0.5</td>
<td>1/(d + 1)^2</td>
<td>2</td>
</tr>
<tr>
<td>A2K4V2</td>
<td>0.5</td>
<td>1/(d + 1)^2</td>
<td>1</td>
</tr>
<tr>
<td>A2K4V1</td>
<td>0.5</td>
<td>1/(d + 1)^2</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 4.1: Alternative scenarios
Note that the values which have been tested, represent the diminishing sensitivity of the last observed experience as days pass and user becomes more experienced, apart from $\kappa=0.5$ which reflects the equal weight between the accumulated and the new experience (Figure 4.2).

4.3 Replications

BusMezzo includes several interrelated stochastic processes, both in the demand and supply side (passengers arrival time, travel times, dwell times etc). As a result, multiple replications are required in order to provide statistically robust results.

The total waiting time experienced by the passengers is an important measure and it is the outcome of the combination of all these stochastic processes in the system. Given this measure, the number of required replications is calculated by using Equation 4.2 [Burghout, 2004; Dowling et al., 2004].

\[N(m) = \left(\frac{\omega(m)\varepsilon m^{-1/2 - \alpha}}{\mu(m)\varepsilon}\right)^2\]  \hspace{1cm} (4.2)

where

- \(N(m)\) = number of replications required given m initial simulation runs;
- \(\bar{X}(m)\) and \(S(m)\) = estimated mean and standard deviation from a sample of m simulation runs, respectively;
- \(\varepsilon\) = allowable percentage error of estimate \(\bar{X}(m)\) of \(\mu\);
- \(\alpha\) = level of significance.

An initial set of 10 repetitions was tested and yielded \(\varepsilon < 0.3\), something which guarantees statistical robustness.

Each scenario was simulated for 23 days resulting with day-to-day learning outputs for 24 iterations, regardless of the convergence criterion. All the reported outputs refer to the average measured over 10 simulation runs for each day.
5 Results and Analysis

5.1 Learning function

5.1.1 Expected Waiting time from Experience

The time-related trip attributes and the way their anticipations and experienced values evolve over the learning process are the core outputs of this study. More specifically, in a first exploration, it is the waiting time expectation by Experience which is examined, and then the overall anticipation over the day and how they both evolve during the learning period. Expectations from PK or RTI are not expected to evolve over time, since PK is fixed, not subject to any variation, while RTI is expected to fluctuate due to supply side’s stochasticity, but around the same levels over the days. Being formed by these three components, the overall anticipated waiting time is thus expected to evolve towards the experienced waiting time.

This section describes the evolution of the waiting time expectation formed by Experience, while the overall anticipation will be analysed in Section 5.1.3, following the analysis of the credibility coefficients in Section 5.1.2 which are a part of its formulation.

Remember that the expectation from Experience is only a function of the recency parameter (\( \lambda \)), which is in turn a function of the day, and the actual waiting time experienced in each ODSL combination (Equation 3.20). Since departure time choice is not accommodated in this application, the overall experienced waiting time across the network is at the same level for every alternative scenario. This also holds for the overall RTI projection and of course for the expectation of the PK which is anyway static (half the service headway).

As a result, variations in this expectation are expected only in the four scenarios A2K1-K4V3, where the recency function varies, and can be compared with the network PK and RTI expectations as well as with the network experienced waiting time over the days.

Figure 5.1 summarizes this comparison between the four scenarios. Scenarios A2K2V3 and A2K1V3 demonstrate similar trend by converging relatively fast to the experienced waiting time. This can be explained from the high recency parameter values for both scenarios. In fact, this term is higher for A2K2V3 (Figure 4.2) until
day 4, then it gets lower. However, since in both scenarios the expectation reaches the experienced waiting time around that day, their evolution during the following days is quite similar.

On the other hand, A2K3V3 demonstrates a slower convergence towards the experienced waiting time, but it seems to stabilize in a just slightly lower level by the day 23. However, since the learning period is set by default to 23 days, the exact convergence time cannot be determined. The A2K4V3 expectation converges to a significantly lower value and demonstrates a rather slow learning process due to the recency parameter formulation as a function of the day ($\kappa^4 = 1/(d + 1)^2$). The deviation from the experienced waiting time is even higher in this case.

![Figure 5.1](image)

**Figure 5.1:** Evolution of waiting time expectation by experience - recency term variation.

This behavioural interpretation of Figure 5.1 lies in the notion of the learning process and its formulation. Passengers who assign higher weight to the last experiences (higher $\kappa$ term), update their expectations faster, closer to the experienced values, than the ones who assign lower weight. Fluctuations in experience are, thus, followed by fluctuations in expectations and might result in a more flexible behaviour in decision making. On the other hand, passengers who are less sensitive to the new experiences (lower $\kappa$ term) need more time to update their expectations closer to their
experience, thus they demonstrate slower learning patterns. As a result, their anticipations are expected to evolve in a smoother pattern which underlies a more stable decision making process.

Note that these curves follow the trend of the recency parameter functions in Figure 4.2, as expected. Experienced, RTI and PK waiting times demonstrated similar patterns among the three scenarios and were hence arbitrarily selected for illustration purposes. Figure 5.2 presents the variation among the population for these expectations.

![Figure 5.2: Evolution of the variation in distributions of the waiting time expectations by experience - recency term variation](image)

As expected from the above analysis, this variation follows the experienced waiting time variation for the higher $\kappa$ term values (A2K1V3 and A2K2V3) while it remains lower where the learning mechanism is lower, reflecting the low impact of recent experiences in passengers expectations.

The evolution of these distributions over the days is also presented in Figures 5.3 and 5.4 for the scenarios A2K2V3 and A2K4V3, which demonstrate the biggest differences. In the first case, it is obvious that the distribution spreads over the
population, following the curves of standard deviation illustrated in Figure 5.2, and also changes shape. It is apparent that on the last day of the learning process there have been formed two “peaks”, one between 60-90sec and the other between 150-165 sec. This distribution follows the distribution of the experienced waiting times over the ODSL combinations, as expected, which also demonstrated these peaks. Taking into account that the headway of the services is 5-10min thus, and we refer to Stop-Line combinations, the first peak suggests a significant part of the population experiencing waiting times related to joint headways (in a not completely unreliable service). The distributions of A2K1V3 and A2K3V3 follow the same pattern.

![Figure 5.3](image)

On the other hand, in the case where the learning mechanism fails to provide sufficient approximation (A2K4V3, \(\kappa_4 = 1/(d + 1)^2\)) of the expectation to the experience, the variation across the population is smaller since the learning function underweights the experience and thus, yields an underestimated expectation on each simulated day.

For the scenarios A1K3V3 and A3K3V3 only the initial value of RTI credibility coefficient changes, which does not affect the formulation of passengers’ expectation from Experience. As a result, their expectation curve is similar to the A2K3V3 in Figure 5.1, as they share the same recency parameter function. Respectively, the curve for the scenarios A2K4V1 and A2K4V2 is similar to the curve of A2K4V3 in the same graph. The distributions of the waiting time expectation from experience of these scenarios are respectively similar to the one presented in Figures 5.3 and 5.4.
5.1.2 Credibility Coefficients

The credibility coefficients are the main component of the suggested model. Their evolution under the different scenarios indicates how meaningful their incorporation in passengers’ perception is, in the sense of shaping passengers’ perception about the different information sources. Since they are formed as a function of their base values, and the recency and salience terms values, they are expected to evolve in different ways under the three scenario groups.

Their analysis has also been conducted in two levels: first, in the network level, the evolution of the overall values, expressed through the weighted average of their distribution in the whole network, has been examined and compared among the various scenarios. Then, it is the distribution of each coefficient among the population which is examined day over day. This distribution will reflect the variation over the network. Since the base values are unique for the whole population, and thus the standard deviation for the first day is zero, it is expected that the distribution will become wider during the learning period.

5.1.2.1 Credibility coefficients under various initial values of RTI credibility

The following graphs (Figures 5.5 - 5.7) illustrate the alpha values convergence for the whole network, under different initial values of RTI credibility coefficient ($\alpha_{\text{base}}$). The salience parameter ($v$) is set equal to 2 for these scenarios, indicating high sensitivity of passengers in deviations between expectations and experiences, but on the other hand, the recency term function might counteract this effect. Convergence is achieved around day 4 for all of them, despite the extreme difference in initial values. It is also evident that the three scenarios show no significant difference with respect to
the network value for the last simulated day. Figure 5.8 summarizes the final credibility coefficient values of the last day.

**Figure 5.5**: Network coefficients over the days – aRTIbase=0.

**Figure 5.6**: Network coefficients over the days – aRTIbase=0.5.

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Figure 5.7: Network coefficients over the days – aRTI_{base}=1.

Figure 5.8: Network coefficients under different initial values for RTI coefficient.

It is apparent that the overall network credibility for all information sources converges fast and smoothly towards the same value, which is approximately one third of their sum. This apparent trend in the three scenarios raises the question whether the credibility coefficients for all information sources tend to converge to these values after a sufficient amount of learning days. Hence it stresses the
importance of further investigation in the distribution of the credibility coefficients among all ODSL combinations, before the above statement is concluded.

Interestingly, this investigation revealed high variation among the different ODSL combinations. As expected, all coefficients’ distribution spread immediately after the first day. In particular, during the first four days the distributions changes shape by moving massively from the edges towards the center of its range (0 - 1), with this effect being quite less intense for the scenario A2K3V3 (alphaRTIbase = 0.5) as expected, and at the same time the observations spread gradually from the center to the edges. Note that the observations do not tend to concentrate around 0.33 as the weighted average indicates, but instead they tend to spread more and more to the edges as the days pass. The detailed distributions of all coefficients of all scenarios over the simulation days can be found in the Appendices of this report.

Figure 5.9 illustrates the distribution as it is formed for the anticipated values of the day 24. It is evident that the three scenarios have similar results, while there are some important differences with respect to the three types of information.

The distribution of the credibility coefficient of Experience is quite more spread, compared to the ones of RTI and PK. This probably reflects the higher share of ODSL combinations which acquire a relatively higher credibility to information by Experience. This result can indicate a relatively lower gap between passengers’ expectations of the waiting time formed by Experience and actual waiting time, than expectations formed by PK and RTI.

At the same time, the distributions of RTI and PK credibility coefficients are also positively skewed but rather mesokurtic. PK credibility coefficient distribution has, however, more observations concentrated in the right side of its mean, opposite to RTI’s which in turn has a bigger tail. However, in all three scenarios all the distributions tend to spread among the ODSL combinations (see the detailed distributions in Appendices - Figures A.2 and A.3).
Figure 5.9: Anticipated credibility coefficients for day24, under various initial values of RTI coefficient.
5.1.2.2 Credibility coefficients under various formulations of recency term

The next set of scenarios (A2K1-K4V3), is designed in order to differentiate with respect to recency parameter functions (under constant alphaRTIbase=0.5 and v=2). Since the recency parameter formulation is critical for the learning process, for both the expected waiting time from experience and the credibility coefficients, it is expected that the different values are going to affect the distributions of the credibility coefficients.

More specifically, the diminishing sensitivity in the last observed experience is more intense in A2K4V3 followed by A2K3V3, A2K2V3 and A2K1V3 in descending order (Figure 4.2). This means that the adaptivity of the users (expressed through the experience stored in the ODSL combinations they visit) is higher in A2K1V3, followed by A2K2V3, A2K3V3 and A2K4V3 in descending order, as days pass. This raises the interest on how these differences affect the credibility coefficients and passengers’ anticipation of waiting time in the long run.

In terms of convergence time, the results are similar to the previous scenarios, as all coefficients converge around day four and five with the only exception being Scenario A2K4V3 ($\kappa = 1/(d + 1)^2$), in which the diminishing effect is the most intense. In this case, the credibility coefficient of RTI seems to converge fast, around day 4, while for EXP and PK coefficients convergence seems to take place between day 7 and 8. This scenario also demonstrates the clearest difference between the coefficient of PK and the ones of EXP and RTI, with the two latter being almost equal and a bit higher than the former (Figure 5.10).

![Figure 5.10](image-url): Network coefficients over the days – A2K4V3
Figure 5.11 summarizes the weighted coefficient of every scenario. Again, the overall network values seem to be at the same levels, at least without any significant difference that one can argue over. A more thorough investigation is thus required for the distribution of the coefficients over all ODSL combinations.

From these distributions, it can be seen that A2K1V3 and A2K2V3 coefficients show similar trend with fast spreading and stabilization in the final form of their distribution the coefficient of Experience. The shift of the average value towards 0.3 is also evident, as well as the gradual observation of values with extreme high or low coefficients values, with the range of the distributions covering almost the full scale between 0 and 1 from the first days of the learning period. The detailed distributions can be found in Appendices (Figures A.4 and A.5) while Figure 5.12 summarizes the final form of all coefficients in all scenarios. There are slight differences in the scenarios that converge faster (A2K1V3 and A2K2V3) while all the scenarios demonstrate the variation in passengers’ waiting time experiences, and its effect on the respective expectation and its credibility. This is reflected by the higher variation in the credibility coefficient of EXP compared to the ones of RTI and PK.

The mean values of these distributions are around the same levels (0.33) indicating that there is no dominant Information source in the overall network. However, the differences in the concentration of the mass of the observations indicate the differences in reliability perceptions over the population. For example, the distributions of PK coefficients tend to be normalized around their mean, while the ones of RTI coefficients tend to concentrate the mass of their observations on the left of the mean, but with longer right tails, indicating a higher deviation in passengers’ perceptions and a relatively low “trust” in RTI as an information source. As a matter of fact, the standard deviations of these distributions vary from 0.118 to 0.143 for RTI
coefficient, from 0.15 to 0.183 for EXP coefficient and from 0.111 to 0.137 for PK coefficient.

Figure 5.12: Anticipated credibility coefficients for day 24, under various recency term functions.
These results also form an argument against the speculation that all coefficients will converge towards one third of their sum in the long-run learning process. Moreover, at this point the learning function has to be considered in combination with the evolution of the waiting time expectation by experience analysed in section 5.1.1. A2K2V3 and A2K1V3, with the most adaptive learning functions, converge faster to the actual experience and A1-A3K3V3 follow a bit slower. On the contrary, A2K4V3, with the slowest learning function, appears quite slower and seems to require a lot more days so that the expectation reaches the levels of experience. Since all these scenarios have the same salience parameter value, the same levels of convergence can be assumed when their credibility coefficients reach their final values. As a result, the distributions of the two “slower” scenarios (A2K3V3 and A2K4V3) can be considered as an “early” stage of their final stabilization. Learning mechanisms variations across population, which yield this kind of differences in passengers’ expectations and perceptions of reliability, are thus important to be considered in future applications.

5.1.2.3 Credibility coefficients under various values of salience term

The last variation, with respect to the salience parameter, \( \nu \), was applied to the scenario A2K4V3, which demonstrated the slowest learning mechanism, in order to have a clearer evolution of the relative effects. The corresponding set of scenarios (A2K4V1-3) is summarized in Figure 5.16, while Figures 5.13-5.15 show the network coefficients’ evolution over the days for each scenario.

The differences in the convergence period and the average values are slight in these scenarios. However, it is interesting that the scenario with the lowest salience value (A2K4V1, \( \nu = 0.5 \)) yields the identical evolution of the credibility coefficients of RTI and PK. The fact that their initial values are equal and the learning function underestimates (\( \nu < 1 \)) the deviation parameter, \( \delta \), reflecting an inelastic user behaviour, could explain this mitigation of the differences between these two information sources in passengers perception. This mitigation is also apparent in the smooth curves (no fluctuations) of the credibility coefficients (Figure 5.13).

Moreover, for the scenarios A2K4V2 and A2K4V3, as the salience term increases (from 1 to 2 respectively), the differences in the overall credibility levels becomes clearer (Figures 5.14 and 5.15) but the values at the end of the learning period remain in the same levels (Figure 5.16).

However, comparing A2K4V1 and A2K4V3 as the extreme scenarios for \( \nu \) value (0.5 and 2 respectively) it is interesting that there is a trade-off between the credibility coefficients of Experience and PK, while the RTI coefficient stabilizes in the same levels. In A2K4V1, where intense deviations of anticipation and experience are weakened, the Experience coefficient remains lower than the one of RTI and of PK, which are in the same level. The same happens for the PK coefficient in A2K4V3 due to the magnification of the deviations. This can be justified by that the learning period
is not long enough for the certain recency parameter function \( \kappa_4 = \frac{1}{(d + 1)^2} \) in order the distributions to evolve in the final form, which should resemble the ones of the scenarios that have reached convergence.

**Figure 5.13**: Network coefficients over the days – A2K4V1

**Figure 5.14**: Network coefficients over the days – A2K4V2
Taking a look at the detailed distributions of the coefficients (Appendices – Figure A.6) one can see the fast change the first 3 days (where the recency function’s value is higher than in the following days) but very slow evolution later on. This is also in line with the evolution of the waiting time expectation from experience.
Figure 5.17 summarizes the anticipated coefficient values for day 24. It is obvious that the standard deviation of the coefficients becomes lower as the salience parameter becomes lower. However, experience (EXP) still seems to demonstrate the highest variation in all scenarios.
Figure 5.17: Anticipated credibility coefficients for day=24, under various salience term values
5.1.2.4 Disaggregate analysis of credibility coefficients

From the distributions of the credibility coefficients it is clear that there is no overall convergence to a certain value, as the network values appeared to imply. In order to illustrate the variation among the ODLS combinations a certain combination has been chosen and the evolution of its credibility coefficients is illustrated in the following figures (5.18 and 5.19), over the different scenarios.

For this ODSL, it is the experience (EXP) which is considered more credible. However, its credibility value varies among scenarios. It is also visible that in scenario A2K1V3, where each new experience shares the 50% of the anticipation, the curves show great fluctuation, which reflects the sensitivity to changes, as expected. Also, in scenario A2K4V1, which demonstrates the highest insensitivity to changes, the curves are quite smoother while the final values remain significantly lower.

It is also interesting to notice that the curves of RTI and PK coefficients have the same trend in all the scenarios for this ODSL combination. A more careful look to other combinations of the sample, showed that this does not hold in every case (as also the detailed distributions of these values show), but might be related to the specific reliability conditions with respect to the topology of the stop. For example, a stop which is at the beginning of a lines’ route, might be provided with RTI that corresponds to historical data (e.g. if the vehicle has not started its trip yet) and is therefore equivalent to a static headway (PK). An unreliable service of this stop would then make both RTI and PK relatively unreliable, while a sufficient amount of days would probably make experience the most reliable information source, depending also on the characteristics of the actual waiting time distribution.

Further examination in the sample of the ODSL combinations would be required in order to draw conclusions about different trends in the evolution of the credibility coefficients and the factors which determine them. At this stage a simple analysis by type of mode has been conducted and illustrated in Figure 5.20, for the scenario A2K3V3.

While the distribution of the rail coefficients is quite similar to the distribution of the full ODSL set (Figure 5.9), significant differences are illustrated for the bus lines. The significant concentration of the PK coefficients between 0.1 and 0.4 reflects the overall unreliable nature of the bus lines. The corresponding range for RTI is somewhat more extended to higher values, but it still reflects that RTI fails to capture the actual waiting times for passengers at bus stops. For that reason, passengers seem to trust more their expectations formed by their experiences, which are in turn concentrated in a more extended range which includes higher values of credibility and demonstrated a higher average as well, almost equal to the sum of the remaining two.
Figure 5.18: Sample ODSL coefficients’ evolution under the different scenarios I (A1-A3K3V3, A2K1V3)
Figure 5.19: Sample ODSL coefficients’ evolution under the different scenarios II (A2K3-K4V3, A2K4V1-V2)
These results are in line with the study of Cats and Loutos [2013] which reflects both the unreliability of the schedule (PK) but also the failure of the currently used algorithm to provide accurate information to passengers. The authors present the explicit formulation the current method’s stops, which is based both on static and real-time data, the latter only in case the next arriving vehicle has already departed.

The overall lower reliability of buses relatively to the rail modes is mainly explained by the various uncertainty factors which are not relevant to the rail modes. Such examples are the interaction with car traffic, the drivers’ behaviour, boarding delays or delays at intersections, etc. However, since the rail share is approximately ten times higher than the bus share, as it will be shown further in this chapter, the distribution of the rail coefficients dominates in the overall set, affecting also the average network values, as it has been shown already in the previous sections.

**Figure 5.20**: Distribution of credibility coefficients by type of mode (bus vs. rail)

### 5.1.3 Overall anticipation

The overall anticipation, defined as the sum of the different expectations weighted by the credibility coefficients (Equation 3.23) is analyzed in this section. Note that since the overall RTI and PK values are stable over the days, the anticipation is expected to follow the trend of the expectation from experience, since all scenarios demonstrate almost the same values for the credibility coefficients, with exceptions only during the first days before convergence. The analysis is split according to the scenario categorization.
5.1.3.1 Overall anticipation under various initial values of RTI credibility

For the different base values of the RTI credibility coefficients (A1-A3K3V3), the evolution of the three anticipations is quite similar over the learning period (Figure 5.21), apart from day 1 (due to the initial values of the credibility coefficients of RTI). This was expected since on day 2 the credibility coefficients have already shifted towards their final values (Figures 5.5-5.7) in all three scenarios, while the learning function, and thus the expectation from Experience, is the same for all three scenarios.

![Graph](image)

**Figure 5.21**: Experience - Anticipation deviation under various initial RTI coefficients

Moreover, in these scenarios, both the deviation and its standard deviation are decreasing, something which reflects evolution towards convergence, over the whole network. The average deviation and its standard deviation are in similar levels for all of them as well (Figure 5.22).
5.1.3.2 Overall anticipation under various formulations of recency term

In the case of the recency parameter function variation, the results are expected to mainly follow the different trends in the evolution of the expectation from Experience, since the credibility coefficients for the whole network have stabilized around the same values for all scenarios. Indeed A2K1V3 and A2K2V3, with the most adaptive learning functions at the beginning of the learning period, yield the fastest convergence to their final value, compared to A2K3V3 which converges in bit slower manner (day 18) and to A2K4V3 which demonstrates a continuous decreasing trend, indicating that the learning period is not sufficient to reach convergence. The deviation between anticipation and experience in this scenario varies significantly over these four scenarios A2K1V3 and A2K2V3 (Figure 5.23) which represent the most adaptive users converge in higher values (around 40sec and 32sec respectively), followed by A2K3V3 which also reaches convergence but in lower levels (approx. 26sec).

The standard deviations of the distribution of these values follow the same trend, which reflects the variation across the population. For example, a population of “sensitive” users ($\kappa = 0.5$) are expected to differentiate more according to their different experiences, thus follow the variation of their experiences. As a result, A2K1V3 is the scenario with the highest variation among these ones which have reached convergence (Figure 5.24).
Conclusions about A2K4V3 cannot be safely drawn. Its convergence criterion might have been fulfilled but this does not imply that passengers’ learning mechanism has converged. This is because the ratio used in the criterion (Equation 3.25) is affected by the underestimation of the deviations due to the learning function formulation. As a result, it is speculated that this scenario would also reach a lower deviation between anticipation and experience with a lower standard deviation as well, after a sufficient number of learning days. However, its variation at this stage reflects the higher range of discrepancies between anticipations and experiences across the population, compared to the scenarios that demonstrate better convergence.

Finally note that the initial values of these curves (day 1) are for all four scenarios the same, since the share the same RTI credibility coefficient base value, thus passengers have the same anticipations.

Figure 5.23: Experience - Anticipation deviation under various recency functions
Figure 5.24: Experience - Anticipation deviation under various recency functions

5.1.3.3 Overall anticipation under various values of salience term

Finally, the variation in salience term, which is applied to the scenario with the least adaptive learning function \((\kappa_4 = 1/(d + 1)^2)\), demonstrates similar behaviour for the three alternative values as none of them reaches convergence. However, the difference in passengers’ behaviour, represented by the two contradictory values \((\nu_1 = 0.5 < 1 \text{ and } \nu_3 = 2 > 1)\) which underestimate and overestimate respectively its value in the learning function, is clear in the evolution of the deviation between anticipations and experience. A2K4V1 is quite slower than A2K4V3 in approximating the experienced values, while A2K4V2 demonstrates an intermediate behaviour, more similar to A2K4V1 (Figure 5.25). The variation in this overall “anticipation-experience” gap, reflected in the standard deviation of the distributions of these gaps, indicates the dispersion of these values in the population (Figure 5.26). The least adaptive the users are the higher the range of discrepancies across the population gets.
Figure 5.25: Experience - Anticipation deviation under various salience terms

Figure 5.26: Experience - Anticipation deviation under various salience terms
5.1.3.4 Disaggregate analysis of waiting time anticipation

In order to illustrate more comprehensively the decrease in the difference between passengers’ anticipations and experiences a certain line and its respective stops throughout its route are used. Metro green line 18 with direction from South to North is used for illustration. The following graph presents how the anticipated waiting time and the experienced waiting time differ at the beginning of the learning process and at the end of it. On the first day the w.t. anticipation (blue bars) is only based on RTI while at the end of the learning period it is a combination of all information sources. In the latter case, also the expectation formed by experience is presented (red bars). It is evident that the expectation formed by experience has gradually updated passengers’ anticipation. It should be noted that in some other cases, a decrease in the experienced waiting times has also been observed, even though this is not the case for this certain line, where the experienced waiting time seems to remain stable (green bars).

The end of the learning period yields also the decrease in the standard deviation of this “anticipation – experience gap”, since the discrepancy between the two values reaches the same levels for most of the Stop-Line pairs.

![Figure 5.27: Evolution of experience-anticipation deviation in Line 18 SN](image)

5.1.4 Concluding notes on the learning process

There are two main points to be stressed at the end of the passengers’ anticipation analysis:

(1) As presented in the Methodology part of this report, passenger’ anticipation is defined as the weighted sum of RTI projection, PK and expectation from
experience. PK is static and does not evolve over the days. Moreover, due to the so-called waiting time paradox [Avineri, 2004], PK will always underestimate passengers waiting time in unreliable services. On the other hand, there is no reason for RTI to systematically underestimate or overestimate the projected information. However, the empirical study of Cats and Loutos [2013] provides evidence that the prediction scheme of Storstockholms Lokaltrafik (SL), the public transport authority of Stockholm County transit service, which is implemented in BusMezzo in this case study, yields a systematic underestimation of the network waiting time. This underestimation is also reflected in this application, together with PK underestimation and is illustrated in Figure 5.1. This justifies the constantly positive deviation between the anticipations and experiences of passengers, even for the scenarios which have reached convergence, as it has been presented in this section. This deviation, however, decreases significantly over the days, first by the expectation formed by experience which gradually approximates the actual network value, and, then, by the formulation of the credibility coefficients which adjust all information expectations into an overall anticipation according to the credibility of the former (Equation 3.23). The decrease depends on how fast or slow the individuals adjust their perceptions closer to their experience (reflected in the variations of the learning function – recency term).

(2) It has been clear from all the graphs which present credibility coefficients and also from the evolution of expectation from experience from all scenarios, that the critical period for passengers to adjust their perceptions are the first 4-5 days where the recency term is relatively “high” compared to the remaining of the learning period (Figure 4.2). This means that scenarios which have reached convergence, or are close to reach convergence (A1-A3K3V3, A2K1-K3V3) will manage to demonstrate the effects on passengers’ perceptions and anticipations in their full extend, whereas the ones that are far from approximating the actual system performance within that period (A2K4V1-V3), will more likely need a quite longer learning period (which is not accommodated in this case study) to converge. The importance of the above is stressed when one considers a mixed population with respect to the learning characteristics which is going to yield a combination of the above analysed results.

5.2 Impact on Passengers’ decisions

So far the analysis of the results has focused on the sensitivity of the expectations and credibility coefficients under the several parameter variations. However, the purpose of this study is also to investigate the impact of these variations in passengers’ decisions. For that purpose, three measures have been selected for analysis: the number of transfers per passenger and the mode share between rail and bus, as well as the total travel time, at the end of the simulation of the learning period. All the scenarios have been compared with the “base” one of No-Learning mechanism and no-RTI provision (NL_noRTI), in order to assess the benefit for
accounting for both RTI provision and information uncertainty in passengers’ decision.

5.2.1 Number of transfers and mode shares

Figure 5.29 summarizes the number of transfers for all scenarios. It is clear that all of them demonstrate a slight increase in the average number of transfers per passenger. However, this is not a significant change but it indicates different assignment results, which may account for behavioural changes in passengers’ response to waiting time uncertainty and information credibility.

Following the number of transfers, it would be interesting to see how the passengers switch from one mode to the other (rail vs. bus) over the different scenarios. The base scenario (NLnoRTI) has been used for the comparison. Table 5.1 provides the shares in the base case and indicates their marginal changes in absolute passenger numbers.

![number of transfers per passenger](https://via.placeholder.com/150)

**Figure 5.28:** Average no. transfers per scenario

The change in percentages is not significant, also because the rail already holds roughly 90%. However, absolute passengers’ indicate significant load of passengers to be served during the peak hour.

**Table 5.1:** Modal Split in NoLearning-noRTI Base Scenario

<table>
<thead>
<tr>
<th>Mode</th>
<th>Loads NLnoRTI</th>
<th>Share</th>
<th>1% shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus</td>
<td>13056 pass</td>
<td>8.88%</td>
<td>131 pass</td>
</tr>
</tbody>
</table>
The net changes in absolute number of passengers of that switch from bus to rail are illustrated in Figure 5.29. It is interesting that all scenarios demonstrate positive deviations. This means that in all simulated cases there was a decrease in bus shares, in favor to the rail modes. This observation, combined with the distribution of the coefficients by mode (Figure 5.20), indicate that passengers switched to rail lines, as they are more reliable.

Note, however, that the average numbers of transfers, together with the shift from bus to rail are network specific effects. Application of the model to any other urban network might demonstrate different results. For this purpose, a more rigid spatial analysis of the under examination network could be the purpose of a future study, which would reflect interrelated processes when it comes to areas of service and transfer nodes.

Figure 5.29: Net passengers’ shift from bus to rail compared to NL_noRTI scenario

5.2.2 Total travel time per passenger

The total travel time is also investigated in this study, as the sum of all trip components. Despite the differences in passengers’ choices, the average travel time per passenger remains around the same levels over the different scenarios, with even lower changes than in the waiting time evolution (where maximum decrease reaches 3.2% in a scenario that has not, however, reached convergence – A2K4V3).

Figure 5.30 presents the average (per passenger) total travel time and its standard deviation per scenario on the last day of the learning period, compared to the base scenario of no RTI provision and no Learning (NL-noRTI). The percentage change in total travel time is always lower than 0.6%. Moreover, no conclusion can be drawn for whether there is a trend of increase or decrease in total travel time, with the incorporation of the learning mechanism, as the scenarios that have reached convergence (A1-A3K3V3, A2K1-K3V3) do not follow the same pattern. In
previous application of BusMezzo it has been shown that the RTI provision in the
network level (through smartphones) decreases passengers’ waiting time by 4.2%
and the total travel time by 2.5% for the whole network, compared to a base
scenario of no RTI provision. The decrease was even higher for both cases in the
inner city network [Cats, 2011]. In that application RTI is incorporated in
passengers’ decision process as fully credible. However, the outcomes of the
learning process of this case study are not in favor of an argument that RTI
provision decreases significantly passengers waiting time and total travel time, with
respect to the overall network performance.

As a result, it seems that we come up with a paradox here, with respect to
passengers’ perception and their behaviour: From one hand, when passengers
perceive RTI projection as fully credible and their anticipation adopts this projected
value, despite the fact that it underestimates their experience, they come up with
choices which indeed decrease their experienced waiting time and total trip time.
On the other hand, when their anticipations are formed by the integration of all
Information sources, and seem to capture more realistically the actual experienced
values, passengers’ choices lead to less significant changes in their experiences.

Figure 5.30: Network travel time changes across the alternative scenarios

However, it should be noted that these are the aggregate network results which
present the overall performance metrics. The overall neutral benefit of accounting
for information credibility does not exclude certain Stop-Line combinations or OD
pairs from being particularly benefited by this formulation. As previous sections of
this analysis have already demonstrated, a disaggregate investigation of these effects might reveal significant impacts which might vary across the network. This also depends on the topology of Stockholm’s network and its dynamics [see also Cats and Jernelius, 2013]. As a result, a spatial investigation would be required in order safe conclusions to be drawn.
6 Conclusions

6.1 Discussion of Results and Study Contributions

The analysis presented in Chapter 5 of this report investigated two elements that have been introduced into BusMezzo, a Dynamic Transit Model. First, incorporating Experience as a distinct Information source and second, introducing the credibility coefficients in order to capture uncertainty in day-to-day dynamics. Both elements were based on a learning mechanism which was demonstrated and assessed under different scenarios.

The investigation of this application was conducted in two dimensions. (1) The sensitivity of networks’ expectations, formulated through Experience, credibility coefficients and overall anticipation with respect to the waiting time was examined. This analysis is based on the variation of the components of the learning function and is illustrated in Section 5.1 of this report. (2) The changes in passengers’ choices, which the alternative scenarios yielded at the end of the learning period (last simulated day), through BusMezzo, were compared to the base case of no learning mechanism and no RTI.

The analysis of the credibility coefficients (Section 5.1.2) yielded two interesting patterns. The first pattern is concerned with the variation within the population (ODSL combinations) with respect to the credibility of information, as it is reflected through the distribution of the credibility coefficients over the days. Especially with respect to experience, there seems to be a full range of credibility values which reflects the variation across the population, with respect to the gap between their expectations (from experience) and experiences (Figures 5.9 and 5.12).

The second pattern concerns the overall values of the coefficients that merge in some average which does not imply full credibility to any of the information, in contradiction to iterative assignments which are based only on accumulated experience on one hand, and the ones that assume that RTI is perceived fully credible, on the other. Interestingly, these values stabilized in the same level for the three coefficients (around 0.3 for each of them when computed across the population, see Figures 5.5-5.17), which is something that cannot be fully explained when it comes to the overall network performance. However, the detailed
distributions especially in the scenarios that seem to have reached convergence (A2K1V3, A2K2V3 and A2K3V3), indicates that this does not tend to be a uniform value, not even in the long run, thus it rejects the hypothesis that all information sources would share the same credibility levels for an average passenger with sufficient accumulated experience. At this point, it has to be mentioned that for all scenarios, during these 23 days, the overlap level of the simulated ODSL combinations set is higher than 99% between the iterations, meaning that each combination is repeated almost every day within the simulation period, thus it “gains” sufficient experience for its analysis and the conclusions that this yields.

The disaggregate analysis of random ODSL combinations has also confirmed the above counterargument while it has illustrated the robust application of the model in the alternative scenarios, by revealing the trends that were expected a-priori for each formulation, i.e. passengers’ sensitivity and degree of adaptation according to variations in the learning mechanism (Figures 5.18 and 5.19).

Moreover, the categorization of the coefficients according to the mode (bus vs. rail) suggests the different levels of reliability and also the dominance of experience as information source in the case of the buses, where the service is less reliable and RTI prediction scheme fails to provide with accurate information (Figure 5.20). However, further investigation is required in order to examine whether the aggregate result of the average credibility values might be explained by values that counteract each other.

The values of the credibility coefficients define, in turn, the overall anticipation of the waiting time which seems to be more realistic in the sense that it approximates better the experienced waiting time in all examined scenarios. The reader is referred to Figures 5.21-5.26 where the gradual decrease in the gap between the anticipation and experience is illustrated together with the decrease in its variation. This gap is reduced (compared to the cases where RTI or/and PK information are the only Information sources, even with shared credibility) through the experience gained from the learning process. In fact, the underestimation of waiting times diminishes through the learning process down from 107 sec to approximately 26 sec in the most promising scenario (A2K3V3- Figure 5.22). At the same time it reflects the weakness of the RTI to fully determine passengers’ anticipation, due to the suggested level of its credibility (significantly lower than 100%). This was one of the objectives of this study and its effect on passengers’ choices is reflected through the changes in number of transfers and the mode shares.

The above discussion yields some important conclusions that highlight the added value of this work in the field of DTA:

(1) The role of experience in a Dynamic Transit Assignment context, where both demand and supply are represented dynamically and thus enable to account for the adaptive behaviour of the passenger. Learning is thus achieved in both the
within-day and the day-to-day context. With respect to the within-day learning, it is the dissemination of RTI as well as the en-route experiences, as a result of the demand and supply interaction, that suggest the update in passengers’ anticipations and thus, the update in their choices. This feature was already accommodated in BusMezzo. Therefore, the contribution in within-day dynamics lies, at this stage, on incorporation the credibility coefficients which weight now the expected values of each information source and lift the assumption that passengers incorporate RTI as fully credible in their decision mechanism, in contradiction to any other DTA model or package used by researchers and practitioners so far. These credibility coefficients’ values are the outcome of the day-to-day learning, together with the expectation formed by experience which also plays a significant role during decision making, as it has been presented already in Chapter 5. Passengers’ day-to-day accumulated experience has thus a double role in TAM:

- Passengers are able to derive expectations from the accumulated experienced values of waiting times, which is modeled as a Markov process update mechanism (Equation 3.1). These expectations thus update their Prior Knowledge at the beginning of their trip. The accumulated deviations of these expectations from the actual values quantify passengers’ perception of waiting time uncertainty in the day-to-day dynamics.

- The potential of the modeling approach of the latter, which has been used for the definition of the notion of the credibility coefficients, provides the possibility to account for the uncertainty of all information sources, including RTI. The model hence integrates into passengers’ perception their perception concerning RTI credibility as formed based on their experience. RTI is thus being assessed with respect to how well it is able to predict the actual waiting time experienced by the passengers, i.e. its instrumental role of informing passengers where they stand today with respect to the distribution of waiting times. This contribution with respect to the role of experience and learning becomes particularly important given the fact that BusMezzo simulator generates RTI based on a commonly used prediction scheme which is also used by SL at the time this report is written. In this application, the simulated results also match empirical evidence [Cats and Loutos, 2013] that this scheme systematically underestimates the actual waiting time at stop. It is thereof assumed that passengers are able to perceive this deviation, or any possible deviations produced by different prediction schemes. This work provides the modeling components necessary for the evaluation of alternative prediction schemes and various RTI provision levels, with respect to their impact on passenger experience.

(2) The modeling framework of this approach, as described in Section 3.1, allows the incorporation of various trip attributes. Each passenger might construct his/her own appreciation of on-board crowding by accumulating experience, which can then be contradicted to any information they receive during their trip. As a
result, RTI and alternative scenarios with respect to its dissemination can be evaluated on the basis of their impact under different levels of passengers’ experience. Agent-based simulation environments are ideal for accounting for variations in passengers’ experiences, perceptions and preferences.

(3) Taking into account that we live in the world of information and instantaneous use of data (GPS and AVL, APC etc.) one can argue that, sooner or later, passengers’ decisions in urban transit networks will be based exclusively on information provided en-route. This study provides a methodological and modeling framework for the evaluation of the role of this information, promoting the importance of effective RTI in an operational level into the investment decisions of the strategic level. All the above mentioned advantages that this case study provides suggest its novelty within the context of ITS, as well as significant contributions to an existing agent-based DTA simulator, BusMezzo, for future applications.

(4) The suggested model provides the explicit formulation of the components which represent the evolution of passengers’ perception of reliability over iterative choices and, as a result, it paves the ground for future applications which aim to evaluate alternative measures of increasing service reliability, either ground-truth reliability improvements or information provision.

6.2 Limitations and Future Research

The above contributions undoubtedly reflect the added value of this study in the DTA research. However, model implementation was subject to several limitations which required simplified modeling assumptions. These limitations are outlined below along with suggestions for future research that could address these issues and further improve the proposed model.

The model scheduling constraint components – namely, the early and late arrivals and the lateness probability - as formulated in Section 3.1, were excluded because of two factors: (1) the limitations with respect to the beta parameters of these variables and; (2) the current implementation of BusMezzo, which does not accommodate the departure time choice in within-day dynamics (the reader is also referred to Section 3.2). However, its inclusion will presumably have substantial implications on the results obtained for the alternative scenarios at the end of the learning period. More specifically, in the evaluation of the network performance, there is no significant difference observed in the evolution of the network waiting times and total travel times (see also Figure 5.30), which might had been yielded otherwise. Note that scheduling constraints have been a core element of the Preferred Arrival Time Path Choice Model presented in Section 3.1 with a special interest on the lateness probability as a within-day variability measure to affect passengers’ decisions (Figure 3.2).

Moreover, the “memory level” of ODSL combinations had to be specified instead of the agent (passenger)-level for the accumulation of passengers’
experiences, due to random passenger generation process in BusMezzo. This assumption hinders the variations across the population with respect to the learning mechanism and passengers’ anticipations and as such, it is necessary that is refined in future applications in order to account for a more realistic implementation of passengers’ demand. This case study investigated the learning mechanism under a uniform update process in every scenario, in order to reveal the basic trends of each component, for the sake of a preliminary sensitivity analysis. However, different “learning prototypes” could be applied across the population in order to capture a more representative effect of the reality. Despite the fact that this did not prevent this specific case study from yielding meaningful results, its combination with the departure time choice implementation in BusMezzo would definitely lead to a refined and more rigorous Transit Assignment Model. Regarding a more comprehensive sensitivity analysis, RTI provision level and reliability scenarios should also be further examined in order to account for information credibility potential impact under various scenarios of service performance.

The above describes mostly the limitations and assumptions of the model application and suggests relevant future refinements in order to realize the complete modeling framework. However, a very fundamental assumption was made also at the level of the model formulation. The latter implicitly implies that there is no deviation between passengers’ perceptions and their experiences. In other words, the model formulation assumes that passengers are able to store in their memory the exact actual values of the attributes they have experienced, e.g. the exact waiting times. Ettema and Timmermans [2006] based their model on two types of gaps: (1) the inconsistency between the actual/measured (from field data) travel time and the predicted travel and; (2) the inconsistency between the perceived travel time and the perceived predicted travel time. The potential second gap was not addressed in this study.

Given this, an error term with specific distribution characteristics could be incorporated in passengers’ expectation formulation for each information source. Even though the model formulation does not exclude such error incorporation, it should be mentioned that this would be particularly meaningful in the case this would systematically overestimate or underestimate the actual values, in order to be able to account for potential derived “gains” or “losses”. For example, in the case RTI projection is underestimated by passengers’ perception at a certain level $m$, and the actual waiting time is underestimated at some other level $n$, the benefits of RTI in decreasing passengers’ uncertainty would be a function of the distribution of the difference of the two terms, $m-n$.

Empirical studies on the effect of RTI on passengers’ perceptions are based on questionnaires addressing passengers’ experienced waiting times, before and after the installation of RTI equipment. The reduction of perceived waiting times and the increase in perceived service reliability, even if the latter is actually decreased, are two important outcomes of such studies, together with stress reduction and increase
in passengers’ satisfaction [TCRP, 2003; Dziekan and Kottenhoff, 2007]. Additional studies in this direction would provide estimates of the $m$ and $n$ error terms described above, for a full scale application with respect to passengers’ perceptions.

The employment of empirical studies constitutes also the most important step in order the parameters of the model to be estimated. This applies for both the utility function (Equation 3.4) which describes passengers’ evaluation process of alternatives, specifically for the calibration of the scheduling constraints, and also for the learning function estimation (Equations 3.1 and 3.2). Variations across individuals would also shed light on the learning prototypes mentioned before, which could be applied over a population.

Revealed Preference (RP) studies suggest the most accurate way to capture passenger’s preferences as well as decision making ‘rules’ since they are based on actual observations on the field rather than hypothetical choice-sets (Stated Preference studies) which might entail some systematic bias in individuals’ response [Wardman, 1988]. In order to decipher the learning mechanism of the individuals with respect to the formulation and the evolution of their anticipations, RP data should be collected with respect to passengers’ path choices (e.g. through personal tracking devices). Moreover, the participants will have to be able to provide their own estimates of the trip attributes with in connection with the provided information at any time of the trip. Their perceptions of the experienced values of their trip attributes and the respective deviation from the schedule or projected information should also be collected at the end of each trip, as well as their anticipations for following trips.

In this way, data for the variables of Equation 3.1, namely the expectation by Experience and the actual (or perceived actual) would be collected for calibration and estimation of the learning function, as well as estimates for the credibility coefficients could be derived based on passengers’ anticipations during the trip, used for the Equation 3.2. In this way, both passengers’ preferences and their perception formulation would be revealed while the data could lead to the model estimation, something which suggests an immediate requirement for future applications.
List of References


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Appendices

Complementary graphs for the Section 5.1

Figure A. 1: Waiting time expectation by Experience under various initial values of RTI coefficients
Figure A.2: Credibility coefficients distribution for (A1-A3)K3V3 (I)
Figure A. 3: Credibility coefficients distribution for (A1-A3)K3V3 (II)
Figure A.4: Credibility coefficients distribution for A2K1,K2,K4V3
Figure A.5: Credibility coefficients distribution for A2K1,K2,K4V3 (II)
Figure A. 6: Credibility coefficients distribution for A2K4V2,V1