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Distribution Locational Marginal Pricing by Convexified ACOPF and Hierarchical Dispatch

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Abstract—This paper proposes a hierarchical economic dispatch (HED) mechanism for computing distribution locational marginal prices (DLMPs). The HED mechanism involves three levels: The top level is the national (regional) transmission network, the middle level is the distribution network, while the lowest level reflects local embedded networks or microgrids. Each network operator communicates its generalized bid functions (GBFs) to the next higher level of the hierarchy. The GBFs approximate the true cost function of a network by a series of affine functions. The concept of Benders cuts are employed in simulating the GBFs. The AC optimal power flow (ACOPF) is convexified and then used for dispatching generators and calculating GBFs and DLMPs. The proposed convexification is based on the second order cone reformulation. A sequential optimization algorithm is developed to tighten the proposed second order cone relaxation of ACOPF. The properties of the sequential tightness algorithm are discussed and proved. The HED is implemented in the GAMS grid computing platform. The GBFs and DLMPs are calculated for the modified IEEE 342 node low voltage test system. The numerical results show the utility of the proposed HED and GBF in implementing DLMP.

Index Terms—Convexified ACOPF, Hierarchical economic dispatch, Generalized bid function, DLMP.

NOMENCLATURE

Sets:

N Nodes.

L Lines.

Parameters:

A_{nl}, B_{nl} Node to line incidence matrix.

x_l, r_l Reactance and resistance.

g_n, b_n Shunt conductance and susceptance.

K_l Squared power capacity of line l .

p_n^{min}, p_n^{max} Lower and upper bound of p_n .

q_n^{min}, q_n^{max} Lower and upper bound of q_n .

p_{d_n}, q_{d_n} Active and reactive power demand.

Variables:

p_n, q_n Active and reactive power generation.

p_{s_l}, q_{s_l} Active and reactive power injection.

p_{o_l}, q_{o_l} Active and reactive power loss.

v_n, V_n Voltage magnitude and voltage square.

v_{s_l}, v_{r_l} Sending and receiving end voltage.

V_{s_l}, V_{r_l} Sending and receiving end voltage square.

θ_l Voltage phase angle difference.

$\theta_{s_l}, \theta_{r_l}$ Sending end and receiving end phase angle.

I. INTRODUCTION

SMART GRID advocates envisage a future in which small customers are responsive to local market conditions with devices that reduce electricity consumption at times of high prices and increase consumption at times of low prices. The increasing penetration of devices capable of responding to market prices is increasing the need for, and the utility of, improved distribution pricing signals. Efficient distribution pricing signals reflect both losses and congestion on the distribution network. Such prices vary across both time and locations and reflect the short-run marginal cost of the transportation of electricity from one point on the distribution network to another.

In recent years there has been rapidly increasing interest in locational marginal pricing of distribution networks, especially to facilitate integration of distributed energy resources [1]. The benefit of distribution locational marginal prices (DLMPs) for charging management of electric vehicles is discussed in [2]. References [3], [4] discuss boosting of demand-side responses using DLMPs. Reference [5] demonstrates how DLMPs can alleviate congestion caused by high penetration of electric vehicles (EV) and heat pumps (HP). Reference [6] proposes to integrate DLMPs and optimization in controlling future distribution networks where electronic devices are enabled to receive control signals generated from DLMP.

Given the very large number of nodes in typical distribution networks, the complexity involved in calculating DLMPs must be properly addressed. Decentralized dispatch is an attractive solution in smart grids [7]–[9]. To address the complexity, a decentralized optimal power flow (OPF) calculated by a Lagrangian-based decomposition procedure is proposed in [10]. Reference [11] reduces high-voltage radial distribution networks to simple networks by feeder reduction techniques. Reference [12] proposes the decentralized economic dispatch for smart grids using the concept of self-organizing dynamic agents. A distributed multi-agent paradigm is proposed in [13] to calculate DLMPs.

Reference [13] reports the DLMPs in a 12-bus distribution network calculated by both direct current OPF (DCOPF) and alternating current OPF (ACOPF). The results show that DLMPs from ACOPF are higher than the ones from DCOPF (this is partly because marginal loss costs are included in the ACOPF approach). The DLMP difference between DCOPF and ACOPF is larger in congestion cases (DLMP of bus 1 is 78.33\$/MWh by DCOPF but 149.99\$/MWh by ACOPF). On the other hand, reference [2] calculates DLMPs by DCOPF because of the ACOPF complexity in distribution networks

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with a large number of nodes.

Limitations on the ability to share network information is another issue in calculating DLMPs. The importance of coordination between the distribution and transmission network layer is discussed in [14] and [15]. This issue is significant for the operation of a power system with large-scale integration of distributed energy resources.

We can distinguish two main challenges in implementing DLMPs: (1) Computational complexity: The DCOPF assumptions are not often valid in distribution grids with high resistance to reactance (R/X) ratio [16]. On the other hand, the ACOPF in distribution networks with a large number of nodes might not be computationally tractable. This demands an OPF formulation which is accurate enough and at the same time which can be solved efficiently for distribution networks with a large number of nodes. (2) Sharing of network information: the correct calculation of DLMPs needs full information of the whole transmission, distribution and local network. Sharing of detailed network information between different network layers might not be feasible or practical. In this paper, these two challenges of implementing DLMPs are addressed.

For the computation complexity challenge, we propose a convexified ACOPF based on second order cone relaxation (SOC-ACOPF) [17]–[19]. The tightness of the employed relaxation is enforced by a sequential tightness algorithm. Though Semi-Definite Programming (SDP) based ACOPF ([20]–[22]) is also one approach to solve ACOPF, the computational limits of SDP are shown in [23]. Efficient algorithms for solving SDP-based ACOPF model remains to be found [20]. SDP relaxations are exact only for limited types of problems [24]–[26]. Even for a 2-bus 1-generator system, SDP-based ACOPF can be infeasible and inexact [27]. [28] formulate the load flow problem of radial distribution network as conic programming by defining new variables. Our formulations do not need to define these new variables. [29] make use of polynomial optimization problem (POP) based ACOPF and then relaxed the POP model by second order cone programming. A hierarchy of SOCP problems is solved to obtain the bounds of ACOPF. The constraints in the hierarchy [29] are increasing during the solution process. On the contrast, the number of the constraints in the sequential tightness algorithm proposed in our paper does not grow in the solution process. By replacing the positive semidefinite condition with its necessary SOCP constraints, reference [30] formulate a mixed SDP/SOCP approach to improve the computational performance of the moment relaxations of ACOPF. But the numerical computation results from [30] show that the mixed SDP/SOCP are still not as fast as pure SOCP based ACOPF compared with our results. [31] use current square variables to formulate the SOCP relaxation of ACOPF and then recover voltage phase angle from solutions of other variables. In our SOC-ACOPF formulation, we explicitly include voltage phase angle variables. Thus the voltage phase angle solutions can be obtained directly.

To address the issue of sharing information between different network layers, the concept of GBF is proposed. The convexified ACOPF and the GBF are placed in a proposed HED mechanism. We prove that if the GBFs are communicated, the proposed HED achieves results very close to the global

economic dispatch. We demonstrate the proposed solutions to implementing DLMP by numerical simulations using a GAMS model. The rest of this paper is organized as follows. Section II explains the convex SOC-ACOPF model and sequential tightness algorithm. Two theorems are presented in this sections to show important properties of SOC-ACOPF. Section III introduces the concept of HED mechanism and GBF. Section IV present the numerical results and discussions. The convergence of HED mechanism is proved numerically. As the communication requirement between network operators, the capacity of GBF is shown to be small. Section V concludes the advantages of the proposed SOC-ACOPF approach, HED mechanism and GBF.

II. CONVEXIFIED ACOPF

A. Second order cone and convex envelope

The original nonconvex ACOPF (based on branch flow model [17], [19]) is formulated in optimization problem (1).

$$\text{Minimize}_{\Omega} f(p_n, q_n, p_{o_l}, q_{o_l}) \quad (1a)$$

subject to

$$p_n - p_{d_n} = \sum_l (A_{nl} p_{s_l} - B_{nl} p_{o_l}) + g_n V_n, \forall n \in N \quad (1b)$$

$$q_n - q_{d_n} = \sum_l (A_{nl} q_{s_l} - B_{nl} q_{o_l}) - b_n V_n, \forall n \in N \quad (1c)$$

$$p_{o_l} = \frac{p_{s_l}^2 + q_{s_l}^2}{V_{s_l}} r_l, \forall l \in L \quad (1d)$$

$$p_{o_l} x_l = q_{o_l} r_l, \forall l \in L \quad (1e)$$

$$V_{s_l} - V_{r_l} = 2r_l p_{s_l} + 2x_l q_{s_l} - r_l p_{o_l} - x_l q_{o_l}, \forall l \in L \quad (1f)$$

$$v_{s_l} v_{r_l} \sin \theta_l = x_l p_{s_l} - r_l q_{s_l}, \forall l \in L \quad (1g)$$

$$p_{s_l}^2 + q_{s_l}^2 \leq K_l, \forall l \in L \quad (1h)$$

$$V_n = v_n^2, \forall n \in N \quad (1i)$$

$$v_n^{\min} \leq v_n \leq v_n^{\max}, \forall n \in N \quad (1j)$$

$$\theta_l^{\min} \leq \theta_l \leq \theta_l^{\max}, \forall l \in L \quad (1k)$$

$$p_n^{\min} \leq p_n \leq p_n^{\max}, \forall n \in N \quad (1l)$$

$$q_n^{\min} \leq q_n \leq q_n^{\max}, \forall n \in N \quad (1m)$$

Where $\Omega = \{p_n, q_n, p_{s_l}, q_{s_l}, p_{o_l}, q_{o_l}, V_n, v_n, \theta_l\} \in \Re$ is the set of decision variables. Equations (1b) and (1c) represent the active and reactive power balance. A_{nl} and B_{nl} are the incidence matrices of the network with $A_{nl} = 1$, $B_{nl} = 0$ if n is sending end of branch l , and $A_{nl} = -1$, $B_{nl} = 1$ if n is receiving end of branch l . Equations (1d)–(1e) represent active power and reactive power loss. Equations (1f)–(1g) are derived by taking the magnitude and phase angle of voltage drop phasor along line l respectively [17], [19]. Constraints (1j)–(1m) are bounds for voltage magnitude, voltage phase angle difference, active power generation and reactive power generation. Constraints (1d), (1g) and (1i) are nonconvex constraints in the optimization problem (1). Using second order cones [19], constraint (1d) can be relaxed to the rotated second order cone constraint in (2).

$$p_{o_l} \geq \frac{p_{s_l}^2 + q_{s_l}^2}{V_{s_l}} r_l, \forall l \in L \quad (2)$$

The nonconvex constraint (1g) can be written as $z_{h_l} = x_l p_{s_l} - r_l q_{s_l}$ where $z_{h_l} = v_{m_l} z_{\theta_l}$, $v_{m_l} = v_{s_l} v_{r_l}$ and $z_{\theta_l} = \sin \theta_l$. The nonconvex terms $v_{m_l} z_{\theta_l}$, $v_{s_l} v_{r_l}$ and $\sin \theta_l$ can be replaced by their convex hulls. We assume $v_{m_l}^{min}$, $z_{\theta_l}^{min}$, $v_{s_l}^{min}$, $v_{r_l}^{min}$ are lower bounds and $v_{m_l}^{max}$, $z_{\theta_l}^{max}$, $v_{s_l}^{max}$, $v_{r_l}^{max}$ are upper bounds for the corresponding variables. The convex hull of the nonconvex terms are derived in (3) which are linear and convex. Equations (3i)-(3j) are valid for $0 < \theta_l^{max} < \frac{\pi}{2}$ [32]. Bounds of the variables can be determined *a priori*. The convex hull of $v_{s_l} v_{r_l} \sin \theta_l$ is expressed by three parts in (3). The first part include constraints (3a)-(3d). These constraints are the convex hull of $z_{h_l} = v_{m_l} z_{\theta_l}$. The second part include constraints (3e)-(3h). These constraints are the convex hull of $v_{m_l} = v_{s_l} v_{r_l}$. The third parts include (3i)-(3j). These constraints are the convex hull of $z_{\theta_l} = \sin \theta_l$.

$$z_{h_l} \geq v_{m_l}^{min} z_{\theta_l} + z_{\theta_l}^{min} v_{m_l} - v_{m_l}^{min} z_{\theta_l}^{min}, \forall l \in L \quad (3a)$$

$$z_{h_l} \leq v_{m_l}^{max} z_{\theta_l} + z_{\theta_l}^{max} v_{m_l} - v_{m_l}^{max} z_{\theta_l}^{max}, \forall l \in L \quad (3b)$$

$$z_{h_l} \leq v_{m_l}^{min} z_{\theta_l} + z_{\theta_l}^{max} v_{m_l} - v_{m_l}^{min} z_{\theta_l}^{max}, \forall l \in L \quad (3c)$$

$$z_{h_l} \leq v_{m_l}^{max} z_{\theta_l} + z_{\theta_l}^{min} v_{m_l} - v_{m_l}^{max} z_{\theta_l}^{min}, \forall l \in L \quad (3d)$$

$$v_{m_l} \geq v_{s_l}^{min} v_{r_l} + v_{r_l}^{min} v_{s_l} - v_{s_l}^{min} v_{r_l}^{min}, \forall l \in L \quad (3e)$$

$$v_{m_l} \geq v_{s_l}^{max} v_{r_l} + v_{r_l}^{max} v_{s_l} - v_{s_l}^{max} v_{r_l}^{max}, \forall l \in L \quad (3f)$$

$$v_{m_l} \leq v_{s_l}^{min} v_{r_l} + v_{r_l}^{max} v_{s_l} - v_{s_l}^{min} v_{r_l}^{max}, \forall l \in L \quad (3g)$$

$$v_{m_l} \leq v_{s_l}^{max} v_{r_l} + v_{r_l}^{min} v_{s_l} - v_{s_l}^{max} v_{r_l}^{min}, \forall l \in L \quad (3h)$$

$$z_{\theta_l} \geq \cos\left(\frac{\theta_l^{max}}{2}\right)\left(\theta_l + \frac{\theta_l^{max}}{2}\right) - \sin\left(\frac{\theta_l^{max}}{2}\right), \forall l \in L \quad (3i)$$

$$z_{\theta_l} \leq \cos\left(\frac{\theta_l^{max}}{2}\right)\left(\theta_l - \frac{\theta_l^{max}}{2}\right) + \sin\left(\frac{\theta_l^{max}}{2}\right), \forall l \in L \quad (3j)$$

In a similar way the convex hull of constraint (1i) is derived in (4).

$$V_n \geq v_n^2, \forall n \in N \quad (4a)$$

$$V_n \leq (v_n^{max} + v_n^{min})v_n - v_n^{max}v_n^{min}, \forall n \in N \quad (4b)$$

Using (2), (3) and (4), the optimization problem (1) can now be reformulated as a second order cone (SOC) program. This proposed SOC-ACOPF represented by [(1a)-(1c),(1e)-(1f),(1j)-(1m)] can be solved efficiently using interior point method (IPM) [33].

B. Sequential tightness algorithm

The conic relaxations in (2) are not guaranteed to be tight. To deal with this problem, we propose a sequential algorithm to improve the tightness. This sequential algorithm is based on Theorem 1 and Theorem 2 below:

Theorem 1

Assume that:

(1). the objective function of nonconvex ACOPF (1) and the proposed SOC-ACOPF is convex;

(2). nonconvex ACOPF (1) has exactly one global optimal solution $(p_{s_l}^*, q_{s_l}^*, p_{o_l}^*)$;

(3). non-exact optimal solution (if exists) of SOC-ACOPF $p_{o_l} > p_{o_l}^*$.

Then, constraint $p_{o_l} \leq p_{o_l}^*$ guarantees the tightness of constraint (2).

Proof. Firstly, we consider the case that SOC-ACOPF is exact (the relaxations in (2)-(4) are tight). The proof of theorem 1 is based on theorem 3 in reference [18] which proves that second order conic optimal power flow (SOPF) has at most one optimal solution when SOPF is exact for a radial network. Because our SOC-ACOPF model has one more constraint (1g) (to make our model valid for both radial and meshed networks) than the SOPF model described in reference [18], we are actually reducing the feasible region of SOPF in reference [18]. This means that either we keep the unique optimal solution in the feasible region or we exclude the optimal solution. For both cases, the conclusion that there is at most one global optimal solution of SOC-ACOPF when SOC-ACOPF is exact is still valid. Assume the exact optimal solution of SOC-ACOPF is $p_{o_l}^e$. If $p_{o_l}^e \neq p_{o_l}^*$, then $p_{o_l}^e$ must be the global optimal solution of the nonconvex ACOPF since $f^e \leq f^*$. Where f^e and f^* are the value of objective function for SOC-ACOPF and the nonconvex ACOPF respectively. This contradicts our assumption that there is exactly one global optimal solution of the nonconvex ACOPF. So $p_{o_l}^e \neq p_{o_l}^*$ is not valid. Constraint $p_{o_l} \leq p_{o_l}^*$ is feasible for SOC-ACOPF when it is exact.

When SOC-ACOPF is not exact, assume $(p_{s_l}', q_{s_l}', p_{o_l}', V_{s_l}')$ is the optimal solution of SOC-ACOPF after we have put the constraint $p_{o_l} \leq p_{o_l}^*$:

$$p_{o_l}^* \geq p_{o_l}' \geq \frac{p_{s_l}'^2 + q_{s_l}'^2}{V_{s_l}'} r_l \quad (5)$$

If $p_{o_l}' \neq \frac{p_{s_l}'^2 + q_{s_l}'^2}{V_{s_l}'} r_l$, then:

$$p_{o_l}' > p_{o_l}^* \quad (6)$$

(6) contradicts (5). So $p_{o_l}' \neq \frac{p_{s_l}'^2 + q_{s_l}'^2}{V_{s_l}'} r_l$ is not valid. Constraint (2) must be tight. $\square$

Theorem 1 shows that as long as we have a good estimation of $p_{o_l}^*$, then we can tighten constraint (2). The problem is that it is difficult to estimate $p_{o_l}^*$ before solving nonconvex ACOPF. We propose the following theorem to design the tightness algorithm.

Theorem 2

If the assumptions in theorem 1 hold and the decreasing sequence $p_{o_l,i} \leq p_{o_l,i-1}$ converges to $p_{o_l}^*$ which is the optimal solution of nonconvex ACOPF, then sequential constraints $p_{o_l} \leq p_{o_l,i}$ guarantee the tightness of constraint (2).

Proof. The proof of theorem 2 is based on theorem 1 and L'Hopital's rule. We denote here the sequential optimization problem constrained by $p_{o_l} \leq p_{o_l,i}$ as SOC-ACOPF- i where i is the index for the sequence. Because SOC-ACOPF is assumed to be feasible for $p_{o_l} \leq p_{o_l}^*$ and $p_{o_l,i} \geq p_{o_l}^*$, SOC-ACOPF- i is always feasible. If the solution of SOC-ACOPF- i is denoted as $(p_{s_l,i}^*, q_{s_l,i}^*, p_{o_l,i}^*, V_{s_l,i}^*)$, we have:

$$p_{o_l,i} \geq p_{o_l,i}^* \geq \frac{p_{s_l,i}^{*2} + q_{s_l,i}^{*2}}{V_{s_l,i}^*} r_l \quad (7)$$

From theorem 1, $p_{o_l} \leq p_{o_l}^*$ guarantees the tightness of equation (2):

$$\lim_{i \rightarrow +\infty} \frac{p_{s_l,i}^{*2} + q_{s_l,i}^{*2}}{V_{s_l,i}^*} r_l = p_{o_l}^* \quad (8)$$

And we have assumed in theorem 2 that sequence $p_{o_l,i} \leq p_{o_l,i-1}$ converges to $p_{o_l}^*$:

$$\lim_{i \rightarrow +\infty} p_{o_l,i} = p_{o_l}^* \quad (9)$$

According to L'Hopital's rule, (7)-(9) imply:

$$\lim_{i \rightarrow +\infty} p_{o_l,i}^* = p_{o_l}^* \quad (10)$$

In other words, the solutions of sequential optimization SOC-ACOPF- i converge to where equation (2) is tight. $\square$

Theorem 2 suggests we can find optimal solution of SOC-ACOPF by solving sequential SOC-ACOPF- i . The challenge is how to design or calculate the proper sequence $p_{o_l,i}$ converging to $p_{o_l}^*$. Here we propose to begin with a rather rough estimation of $p_{o_l}^*$ and then iteratively improve the estimation quality. The numerical results show that the proposed tightness algorithm can make the relaxation in constraint (2) tight.

Given (1h), the initial estimation $K_{o_l,0}$ is simply calculated by (11):

$$K_{o_l,0} = \frac{K_l}{V_{s_l}^{min}} r_l \quad (11)$$

Where $V_{s_l}^{min}$ is the lower bound of V_{s_l} . The sequential SOC-ACOPF- i is then constrained by (12):

$$p_{o_l} \leq K_{o_l,i}, \forall l \in L \quad (12)$$

Where $K_{o_l,i}, i \in \{0, 1, 2, \dots, M\}$ is the i^{th} estimation of $p_{o_l}^*$. The tightness algorithm works by iteratively reducing the upper bounds $K_{o_l,i}$ of power loss constraints that violate tightness criterion ε . If $p_{o_l} - \frac{p_{s_l,i}^2 + q_{s_l,i}^2}{V_{s_l,i}} r_l > \varepsilon$, $K_{o_l,i+1} = \alpha_l K_{o_l,i}$, where $0 < \alpha_l < 1$ is the decreasing parameter. The proposed sequential tightness algorithm is summarized in algorithm 1. This algorithm is illustrated in Fig. 1 where X^* denotes the

specified a priori. The original nonconvex feasible region of ACOPF is plotted with a solid line in Fig. 1. The convex feasible region of proposed SOC-ACOPF is plotted with a dashed line in Fig. 1. We will show the performance of this algorithm in Section 3 of this paper. It is worth to mention that the proposed sequential tightness algorithm is only tight for the relaxation in (2). Because there are additional relaxations we have introduced in (3a)-(4b), the final solutions of the proposed SOC-ACOPF model are generally not tight for the constraint (1g) in ACOPF. Using interior point method to solve the proposed SOC-ACOPF model in polynomial time does not violate the NP-hardness proof [34], [35] of ACOPF because the proposed SOC-ACOPF is still a relaxed model of ACOPF.

Fig. 1. The conceptual diagram of the proposed sequential tightness algorithm

III. THE HIERARCHICAL ECONOMIC DISPATCH

To define hierarchical economic dispatch (HED), we should firstly explain the traditional centralized economic dispatch (CED). We assume here centralized economic dispatch as one system operator dispatching all generations in transmission, distribution and local networks. To fulfill this dispatch task, the system operator should obtain all network information about his responsible area. Mathematically, CED is to solve ACOPF problem (1) or SOC-ACOPF [(1a)-(1c),(1e)-(1f),(1j)-(1m)]. The CED is a very large scale optimization problem considering the enormous nodes, lines and DERs. To release the complexity of CED, we propose the HED mechanism in this paper. HED actually decompose CED by Benders decomposition. We show that Benders cuts in this proposed hierarchical economic dispatch have specific economic meanings in the defined GBF.

A. The hierarchical economic dispatch mechanism

We propose a three-level dispatch mechanism with each network operator responsible for its own network. The network layers are connected through tie lines. At the third level, local network or microgrid operators (LNOs) carry out their own dispatch considering local network constraints. LNOs communicate dispatch results through the proposed GBFs to the second level of hierarchy. At the second level, the distribution network operators (DNOs) run another optimization problem taking into account the submitted GBFs from all connected LNOs and the second-level network constraints. The results of the second level optimization is packed in the form of GBFs and submitted to the first level of hierarchy. At the first level of hierarchy, transmission network operator (TNO) solves the dispatch problem of transmission network taking into account

Algorithm 1: Sequential tightness algorithm

Input: **ACOPF Parameters**;
Output: **Tight Optimal Solution of SOC-ACOPF** ;
Initialization ;
 $K_{o_l,0} = \frac{K_l}{V_{s_l}^{min}} r_l$;
do
 Solve SOC-ACOPF (1a)-(1c), (1e)-(1f), (1j)-(1m),
 (2), (3), (4) with constraint (12) ;
 $\varepsilon_l = p_{o_l} - \frac{p_{s_l,i}^2 + q_{s_l,i}^2}{V_{s_l,i}} r_l$;
 if $\varepsilon_l > \varepsilon$ **then**
 $i = i + 1$;
 $K_{o_l,i} = \alpha_l K_{o_l,i-1}$;
 $\bar{\varepsilon}_l = \max_l \{\varepsilon_l\}$;
 while $\bar{\varepsilon}_l > \varepsilon$ **and** $i < M$;

final solution. M denotes the maximum number of iterations

the submitted GBFs from DNOs. Once the top level of the hierarchy is completed, the dispatch results are determined and the resulting nodal prices can be computed and communicated back down to the hierarchy. This is illustrated in Fig. 2.

Fig. 2. The conceptual diagram of the proposed HED mechanism

B. Generalized bid function

The concept of GBF is proposed to avoid communicating detailed network and regional bid information between dispatch levels. The HED problem for LNOs contingent on the total power generation is set out in (13).

$$F_k(P_{k,j}^{sum}) = \text{Minimize} \sum_{n \in N_k} f(p_n, q_n) \quad (13a)$$

$$\text{subject to } (1b), (1c), (1e), (1f), (1j) - (1m) \\ (2), (3), (4), (12)$$

$$\sum_{n \in N_k} p_n = P_{k,j}^{sum} : (\alpha_{k,j}) \quad (13b)$$

Where F_k is the cost of local network dispatch as a function of its total power generation $P_{k,j}^{sum}$. $k \in K$ is the index of the LNO. $f(p_n, q_n)$ is the generation cost of the local DGs. N_k is the set of local nodes. $j \in J_k$ is the index of GBFs. If we use $S(P_{k,j}^{sum})$ to denote the feasible region of (p_n, q_n) as a function of $P_{k,j}^{sum}$. The optimization value function $F_k(P_{k,j}^{sum})$ is convex because: (1). $f(p_n, q_n)$ is jointly convex on $\{(p_n, q_n, P_{k,j}^{sum}) | (p_n, q_n) \in S(P_{k,j}^{sum}), P_{k,j}^{sum} \in \mathbb{R}^+\}$; (2). $S(P_{k,j}^{sum})$ is convex on $\mathbb{R}^+$; and (3). $\mathbb{R}^+$ is convex (see proposition 2.1 in reference [36]). F_k can be approximated from below by a set of affine functions as in (14) shown by Fig. 3.

$$F_k(P_{k,j}^{sum}) \geq \hat{F}_{k,j} + \hat{\alpha}_{k,j} \left(\sum_{n \in N_k} p_n - \hat{P}_{k,j}^{sum} \right) \\ \forall j \in J_k, \forall k \in K \quad (14)$$

We define GBF as the set of parameters of the affine approximator (14). These parameters are communicated through set $LNO_k = \{(\hat{F}_{k,j}, \hat{\alpha}_{k,j}, \hat{P}_{k,j}^{sum}) : j \in J_k\}$ to the DNO. Once all

Fig. 3. Approximation of a convex cost function by affine functions

LNO_k sets are communicated, the DNO solves the following dispatch problem (15).

$$G_v(P_{v,j}^{sum}) = \text{Minimize} \sum_{n \in N_v} f(p_n, q_n) + \sum_{k \in K} F_k \quad (15a)$$

$$\text{subject to } (1b), (1c), (1e), (1f), (1j) - (1m)$$

$$(2), (3), (4), (12), (14)$$

$$\sum_{n \in N_v} p_n = P_{v,j}^{sum} : (\alpha_{v,j}) \quad (15b)$$

Same as F_k , G_v is also a convex optimization value function based on proposition 2.1 in reference [36]. G_v can be approximated from below by affine functions (16b). $v \in V$ is the index of the DNO. The parameters of these affine functions are communicated through set $DNO_v = \{(\hat{G}_{v,j}, \hat{\alpha}_{v,j}, \hat{P}_{v,j}^{sum}) : j \in J_v\}$ to TNO. TNO solves the optimization problem (16).

$$\text{Minimize} \sum_{n \in N_t} f_t(p_n, q_n) + \sum_{v \in V} G_v \quad (16a)$$

$$\text{subject to } (1b), (1c), (1e), (1f), (1j) - (1m) \\ (2), (3), (4), (12)$$

$$G_v(P_{v,j}^{sum}) \geq \hat{G}_{v,j} + \hat{\alpha}_{v,j} \left(\sum_{n \in N_v} p_n - \hat{P}_{v,j}^{sum} \right) \\ \forall j \in J_v, \forall v \in V \quad (16b)$$

Once optimization problem (16) is solved, TNO finds the nodal prices. It also communicates the total power generation requirement $\hat{P}_{v,j}^{sum}$ to DNOs ($\hat{j}$ is the index of final GBF for which the HED converges). Given $\hat{P}_{v,j}^{sum}$, DNOs find the nodal prices for their networks. The DNO also communicates $\hat{P}_{k,j}^{sum}$ to the connected LNOs. LNOs then calculate the nodal prices for their local networks. The underlying mathematical structure behind the HED mechanism is the Benders decomposition approach. The proposed HED relaxes the decomposition of economic dispatch by taking $P_{k(v),j}^{sum}$ as the complicating variables instead of the tie-line power flows p_{sl}, q_{sl} . This leads to fewer GBFs when multiple tie-lines exist. In this way, DNOs or LNOs do not need to submit GBFs for each tie-line (F_k and G_v are not functions of tie-line power flows). Constraints (14) and (16b) are actually Benders cuts for aggregated tie lines. If the problem is convex, it is proved that optimal solution can be found within finite iterations [37].

Algorithm 2: Hierarchical Economic Dispatch Mechanism

Initialization;
TNO solve transmission network dispatch (16);
TNO broadcast $\hat{P}_{v,j}^{sum}$ to DNOs;
DNOs solve distribution network dispatch (15);
DNOs broadcast $\hat{P}_{v,j}^{sum}$ to LNOs;
DNOs submit DNO_k to TNO;
LNOs solve local network dispatch (14);
LNOs submit LNO_k to DNO;

IV. NUMERICAL RESULTS AND DISCUSSION

The proposed SOC-ACOPF and its tightness algorithm are examined through several case studies. Then the proposed HED is simulated. All the models are coded in GAMS. MATPOWER and LINDOGLOBAL are used for comparison. Simulations are run on a PC with Intel i7-2760QM 2.4 GHz CPU and 8 GB of RAM.

A. The performance of proposed SOC-ACOPF

The results of SOC-ACOPF with sequential tightness algorithm are reported in Table I and Table II. The relaxation gap of constraints (2) is calculated as:

$$Gp_{o_l} = \max_l \left\{ p_{o_l} - \frac{p_{s_l}^2 + q_{s_l}^2}{V_{s_l}} r_l \right\} \quad (17)$$

The stopping criteria for the SOC-ACOPF iteration is $Gp_{o_l} < 10^{-9}$. In all reported cases, the Gp_{o_l} indicator is less than 10^{-9} . The optimality gap of the proposed SOC-ACOPF is measured in $Gp_f = \frac{f^* - f}{f^*} \times 100\%$. In the calculation of Gp_f , f^* is the global solution of nonconvex ACOPF calculated by LINDOGLOBAL and f is the solution of proposed SOC-ACOPF found by MOSEK solver.

TABLE I
THE PERFORMANCE OF SEQUENTIAL TIGHTNESS ALGORITHM:
OBJECTIVE SOLUTION

Case	Objective [\$]			Gp_{o_l}	Gp_f [%]
	SOC	matpower	Global		
IEEE14	8076.99	8081.53	8078.80	0	0.02
IEEE57	41673.08	41737.79	41698.64	0	0.06
IEEE118	129619.60	129660.70	129626.45	0	0.01
IEEE300	718109.18	719725.11	719459.62	0	0.18

TABLE II
THE PERFORMANCE OF SEQUENTIAL TIGHTNESS ALGORITHM:
COMPUTATION TIME

Case	CPU time [s]		
	SOC	matpower	Global
IEEE14	0.10	0.11	0.20
IEEE57	0.19	0.12	2.31
IEEE118	0.33	0.30	27.1
IEEE300	3.10	0.40	257.68

As we can see from Table I, the results from SOC-ACOPF is very close to the global solutions obtained by LINDOGLOBAL.

B. Convergence of hierarchical economic dispatch

The IEEE 342 node network [38] is modified here to illustrate the operation of the proposed HED. This test case has transmission network (nodes P1-P4, P7-P8), distribution network (nodes P5-P6, P9-P390) and local networks (nodes S193-S240 in the eight spot 277/480V networks). The local networks are connected to the distribution network by transformers denoted as X3, X4 to X22 in Fig. 6. One 50 MW generator is located in the transmission network. Each 13.2 kV distribution feeder is equipped with one 7.5 MW distributed generator (DG). We distribute 48 DGs among all nodes of the local networks (one 3 MW DG at each node). To simulate congestion in the distribution network and local networks, we increase the load levels in all the local networks by four times and then reduce the tie-line transformer capacity (X10-X18 is reduced to 3 MVA and X21-X22 is reduced to 4 MVA). We assume all generators are dispatchable. To accelerate computation, we design a GAMS grid computing structure to assign the dispatch task of each network to different threads as demonstrated in Fig. 5.

Fig. 4. The hierarchy of the modified IEEE 342 node test system

Fig. 5. The GAMS grid computing structure of hierarchical economic dispatch

Fig. 6. Local low voltage networks

The dispatch results of HED are listed in Table III. We denote the case of no congestion as the base case in this paper. CED results denoted as 'C' are also listed. The HED results (denoted as 'H') converge to the solution very close to centralized dispatch dispatch cost. All cases converge within three iterations shown in Fig. 7. Because of approximations used in HED mechanism, the final cost of HED is a bit different from centralized dispatch (the difference is within 1% after three iterations). The CPU time for computation is within 1.8 second. If the LNOs and DNO submit their GBFs (i.e. parameters of three affine approximators in one package) to the next higher hierarchy, the HED converges in one iteration. It is worth to mention that Benders decomposition is a way to build GBFs in HED. HED does not work in an iterative way.

Fig. 7. The convergence of the proposed HED mechanism

C. Distribution locational marginal price

The nodal prices are shown in Fig. 8. All nodal prices are very close to each other in the base case. The only differences in the nodal prices are due to the marginal cost of energy loss. This can be clearly observed from the small price spikes at the ending nodes (S193, S198, S203, S210, S217, S224, S231 and S236) of each local network. The congested distribution transformers are indicated in the legend of Fig. 8. When congestion happens, the nodes located in the local network have higher prices. The consumers with price-responsive load can response to these higher prices. This can be observed by comparing consumer payment and DG income when congestion happens with the ones in the base case. We plot the total payment of consumers in the local networks in Fig. 9. The payment difference as compared to the base-case payments is also shown in Fig. 9. The congestion management potential of DLMP can be further demonstrated by the income increase of DGs when congestion happens. This is shown in Fig. 10. When congestion happens, the increased nodal prices give local DGs strong incentive to produce more energy.

D. The generalized bid function

The GBFs are listed in Tables IV-VIII. We sum the $\hat{F}_{k,j}$ and $\hat{G}_{v,j}$ parameters in the GBFs for all networks and list the

Fig. 8. The nodal prices in local networks

Fig. 9. The total payment of consumers in local networks

Fig. 10. The total income of DGs in local networks

TABLE III
THE RESULTS OF THE ACTIVE POWER DISPATCH FROM HED MECHANISM, C: CENTRALIZED DISPATCH, H: HED

Network	Generator	Base Case [MW]		x10-x11-x12 [MW]		x13-x14-x15 [MW]		x16-x17-x18 [MW]		x21-x22 [MW]	
		C	H	C	H	C	H	C	H	C	H
Transmission	1	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00
Distribution	2	7.50	7.50	7.50	7.50	7.50	7.50	7.50	7.50	7.50	7.50
	3	7.50	7.50	7.50	7.50	7.50	7.50	7.50	7.50	7.50	7.50
	4	7.50	7.50	7.50	7.50	7.50	7.50	7.50	7.50	7.50	7.50
	5	7.50	7.50	7.50	7.50	7.50	7.50	7.50	7.50	7.50	7.50
	6	7.50	7.50	7.50	7.50	7.50	7.50	7.50	7.50	7.50	7.50
	7	7.50	7.50	7.50	7.50	7.50	7.50	7.50	7.50	7.50	7.50
	8	7.50	7.50	7.50	7.50	7.50	7.50	7.50	7.50	7.50	7.50
	9	5.93	5.96	0.99	0.94	0.14	0.15	0.10	0.10	0.05	0.04
Local 4	29	0.00	0.00	2.89	3.00	0.00	0.00	0.00	0.00	0.00	0.00
	30	0.00	0.00	1.40	2.07	0.00	0.00	0.00	0.00	0.00	0.00
Local 5	36	0.00	0.00	0.00	0.00	2.44	3.00	0.00	0.00	0.00	0.00
	37	0.00	0.00	0.00	0.00	0.17	2.86	0.00	0.00	0.00	0.00
	38	0.00	0.00	0.00	0.00	0.18	0.00	0.00	0.00	0.00	0.00
Local 6	43	0.00	0.00	0.00	0.00	0.00	0.00	2.44	3.00	0.00	0.00
	44	0.00	0.00	0.00	0.00	0.00	0.00	2.87	2.97	0.00	0.00
	45	0.00	0.00	0.00	0.00	0.00	0.00	0.65	0.00	0.00	0.00
Local 8	53	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.98	3.00
	54	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.00	3.00
	55	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.13
Total Cost [€]		731.67	732.25	757.91	755.09	766.93	762.38	769.15	766.96	769.15	776.38

accumulated results in the last row of the Tables IV-VIII. As in Tables III-VII, the LNOs submit their calculated GBFs to the DNO. Accordingly, the DNO takes into account submitted GBFs from LNOs and prepares its GBFs to be submitted to the TNO. The TNO calculates the dispatch instructions and nodal prices will be communicated back down to the hierarchy. Note that LNOs only need to submit their GBFs to the DNO. Bids from the 48 local DGs are not required to be submitted to the DNO. In other words, with GBFs, the network layers would not need to share their detailed network information between each others. Table IV - Table VIII demonstrate the communication burden of HED is small.

TABLE IV
THE GBFs IN BASE CASE

Network	$j = 1$		$j = 2$		$j = 3$	
	p_{sum}	α	p_{sum}	α	p_{sum}	α
DNO	21.60	8.00	58.46	18.00	58.46	18.00
LNO1	7.20	24.00	0.00	20.00	0.00	19.98
LNO2	6.64	24.50	0.00	20.50	0.00	20.48
LNO3	10.17	26.50	0.00	21.00	0.00	20.98
LNO4	10.67	27.00	0.00	21.50	0.00	21.47
LNO5	10.86	27.50	0.00	22.00	0.00	21.98
LNO6	10.88	28.00	0.00	22.50	0.00	22.48
LNO7	6.25	27.00	0.00	23.00	0.00	22.95
LNO8	8.37	27.50	0.00	23.50	0.00	23.45
Cost [€]	1825.28		632.25		632.25	

V. CONCLUSION

This paper proposes solutions to practical challenges with implementing locational marginal prices in distribution networks. The challenges considered here include (1) the computational complexity of nodal prices in AC optimal power flow and (2) the network information which potentially must be communicated. We propose a SOC-ACOPF to calculate nodal prices in a distribution network. The proposed SOC-ACOPF can be solved efficiently to global optimality while it has more

TABLE V
THE GBFs IN CASE OF X10-X11-X12 CONGESTION

Network	$j = 1$		$j = 2$		$j = 3$	
	p_{sum}	α	p_{sum}	α	p_{sum}	α
DNO	20.32	8.00	53.44	18.00	53.44	18.00
LNO1	7.10	24.00	0.00	19.98	0.00	20.00
LNO2	6.55	24.50	0.00	20.48	0.00	20.49
LNO3	10.00	26.50	0.00	20.98	0.00	20.99
LNO4	13.77	27.50	5.07	23.50	5.07	23.50
LNO5	10.69	27.50	0.00	21.98	0.00	21.99
LNO6	10.71	28.00	0.00	22.48	0.00	22.49
LNO7	6.16	27.00	0.00	22.98	0.00	22.98
LNO8	8.28	27.50	0.00	23.48	0.00	23.48
Cost [€]	1876.40		655.09		655.09	

TABLE VI
THE GBFs IN CASE OF X13-X14-X15 CONGESTION

Network	$j = 1$		$j = 2$		$j = 3$	
	p_{sum}	α	p_{sum}	α	p_{sum}	α
DNO	20.03	8.00	52.65	18.00	52.65	18.00
LNO1	7.08	24.00	0.00	20.00	0.00	20.00
LNO2	6.53	24.50	0.00	20.50	0.00	20.50
LNO3	9.97	26.50	0.00	21.00	0.00	21.00
LNO4	10.47	27.00	0.00	21.50	0.00	21.50
LNO5	14.58	28.00	5.86	24.00	5.86	24.00
LNO6	10.68	28.00	0.00	22.50	0.00	22.49
LNO7	6.14	27.00	0.00	23.00	0.00	22.99
LNO8	8.27	27.50	0.00	23.50	0.00	23.49
Cost [€]	1888.70		662.38		662.38	

accurate results than DCOPF. The tightness of the relaxation from the power loss constraints in the SOC-ACOPF can be guaranteed by the proposed sequential tightness algorithm. The issue of network information sharing is addressed through the proposed concept of GBF. The HED mechanism is also proposed to decompose the dispatch task of multiple connected networks. A three-level network is considered. At the third level, LNOs communicate the dispatch cost of their networks through the proposed GBFs. At the second level, DNO runs another optimization considering its own network conditions

TABLE VII
THE GBFs IN CASE OF X16-X17-X18 CONGESTION

Network	$j = 1$		$j = 2$		$j = 3$	
	p_{sum}	α	p_{sum}	α	p_{sum}	α
DNO	20.03	8.00	52.60	18.00	0.00	18.00
LNO1	7.08	24.00	0.00	20.00	0.00	20.00
LNO2	6.53	24.50	0.00	20.50	0.00	20.50
LNO3	9.97	26.50	0.00	21.00	0.00	21.00
LNO4	10.47	27.00	0.00	21.50	0.00	21.49
LNO5	10.66	27.50	0.00	22.00	5.97	21.99
LNO6	14.66	28.50	5.97	24.50	0.00	24.50
LNO7	6.14	27.00	0.00	23.00	0.00	22.99
LNO8	8.27	27.50	0.00	23.50	0.00	23.49
Cost [€]	1892.15		666.96		666.96	

TABLE VIII
THE GBFs IN CASE OF X21-X22 CONGESTION

Network	$j = 1$		$j = 2$		$j = 3$	
	p_{sum}	α	p_{sum}	α	p_{sum}	α
DNO	20.00	8.00	52.54	18.00	52.54	18.00
LNO1	7.08	24.00	0.00	19.96	0.00	20.00
LNO2	6.53	24.50	0.00	20.46	0.00	20.49
LNO3	9.96	26.50	0.00	20.97	0.00	20.99
LNO4	10.47	27.00	0.00	21.47	0.00	21.49
LNO5	10.65	27.50	0.00	21.94	0.00	21.99
LNO6	10.67	28.00	0.00	22.44	0.00	22.49
LNO7	6.14	27.00	0.00	22.95	0.00	22.99
LNO8	12.45	31.50	6.13	27.50	6.13	27.50
Cost [€]	1900.90		676.38		676.38	

and submitted GBFs of the LNOs. The results of this optimization are packed in the form of GBFs and communicated to the first level of hierarchy i.e. TNO. Once the optimization problem of the first level of hierarchy is solved, the dispatch results and nodal prices are communicated back down to the hierarchy. The convergence of HED mechanism is guaranteed by the convexity of the SOC-ACOPF. The simulation results show the utility of proposed mechanism for implementing nodal pricing in distribution and local networks. The proposed mechanism is tested in a stationary environment. Studying the impact of uncertainty is a possible future extension of this work.

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