Long term chaotic attitude behaviour on highly eccentric orbits

INTEGRAL Case Study

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**Acronyms**

ATOPS  ATitude and Orbit Propagator System  
CoM  Centre of Mass  
DoF  Degree of Freedom  
ECI  Earth Centred Inertial  
EOL  End Of Life  
EPS  Electrical Power System  
ESA  European Space Agency  
GEO  Geostationary Orbit  
GTE  Global Truncation Error  
HEO  High Elliptical Orbit  
INTEGRAL  INTErnational Gamma-Ray Astrophysics Laboratory  
LEO  Low Earth Orbit  
LTE  Local Truncation Error  
ORBGEN  ORBit GENerator  
RK  Runge-Kutta  
SCARAB  Spacecraft Atmospheric Re-Entry and Aerothermal Break-Up  
SDO  Space Debris Office  

**Notations**

### Variables

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<td>Earth oblateness coefficient</td>
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<td>Right ascension of the ascending node</td>
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Abstract

The main issues discussed in this paper are related to the refinement of the on-ground casualty risk computation for the specific case of INTERnational Gamma-Ray Astrophysics Laboratory (INTEGRAL). The current approach, unable to predict the spacecraft attitude motion, assumes random tumbling motion as initial condition to simulate the fragmentation process. The wide experience in break-up analysis, acquired after years of practice with simulation, identified attitude to be one of the major drivers of uncertainty. The Space Debris Office (SDO) demanded a specific research in the field of the long-term propagation applied to the attitude motion and INTEGRAL offered the perfect test bench to conduct a preliminary study in this direction. In particular, observing whether environmental torques were able to trigger stable attitude motion, maintainable till re-entry, was considered to be the major challenge. The propagation of coupled orbital−attitude motion for a random attitude configuration represents only one side of the coin. Indeed, chaos theory analysis constituted the other. The use of the Poincaré map in a non-canonical way managed to bring evidence for constrained motion in the angular rate motion of INTEGRAL, under gravity perturbations. Such results allowed to conduct further investigation on the overall attitude motion and estimate that the attitude configuration at the re-entry appears as precession about the maximum axis of inertia, in the majority of the cases.

Keywords: long-term propagation, attitude, stability, Poincaré map, momentum sphere.
1. Introduction

In 1993, INTErnational Gamma-Ray Astrophysics Laboratory (INTEGRAL) was selected as observatory−type mission dedicated to fine spectroscopy and imaging of space. Capturing the highly energetic gamma rays in the GeV range, INTEGRAL has allowed studying deep space processes, such as supernovae and hypernovae explosion, and matter acceleration in the neighbourhood of black holes [1].

INTEGRAL hosts an Imager, a Spectrometer, an X-Ray Monitor and an Optical Transient Camera which with their two tonnes in weight constitute the heaviest scientific payload ever placed in orbit by the European Space Agency (ESA). The four instruments, carefully chosen to complement each other, are co-aligned to observe the same region of the sky [2] [3]. Such configuration defines the body frame orientation with the x-axis pointing in the direction of the target observed by the instruments, with z-axis oriented normally to solar-panels and y-axis defined by right-hand rule (fig. 1).

Launched from the Baikonur Cosmodrome in Kazakhstan by a Proton rocket on the 17th of October 2002, INTEGRAL was injected into a highly eccentric orbit. Final tuning brought it to a perigee altitude of 48000 km and 72 h period ensuring 100% observation time, minimal radiation dose and instruments interference and full coverage with three ground station [2] [3].

1.1. Re-entry

Towards the end of 2012, in order to meet the requirements set by the ESA’s Space Debris Mitigation Policy on limiting the release of debris, the casualty risk on ground and the long-term presence of spacecraft in the protected orbital regions Low Earth Orbit (LEO) and Geostationary Orbit (GEO), a full simulation campaign was conducted in order to study the End Of Life (EOL) scenario and evaluate the available options [5] [6].

Preliminary analysis [7] revealed that, although INTEGRAL would have experienced large variation in perigee and apogee altitude, due to Sun and Moon third body perturbations, no natural re-entry would have occurred within 200 years (fig. 2).

![Figure 1: INTEGRAL body frame orientation represented for SCARAB Model [4].](image1)

![Figure 2: Evolution of perigee altitude for INTEGRAL for 200 year [7].](image2)

Drifting into LEO for some time periods and crossing the GEO ring roughly every decade, INTEGRAL was representing an unacceptable risk (fig. 3)[7].
Thus, a full campaign of investigations on orbit manoeuvres was conducted by ESA to understand INTEGRAL’s long-term evolution in term of orbital lifetime, re-entry epoch, and re-entry location.

With the delta-v available in 2012, a direct re-entry manoeuvre was infeasible at that time but may be marginally possible in 2020 and 2028 when the satellite will experience a phase of lower perigee altitudes. Further analysis revealed this idea to be unfeasible due to delta-v restrictions required for nominal operations but raised the question whether an earlier manoeuvre would have led to a different long-term evolution of the perigee altitude and ultimately to re-entry [7].

Among the various scenarios evaluated (direct and delayed re-entry, Moon transfer, super-GEO, under-GEO and super-LEO graveyard), all but the first and the last appeared to exceed the available delta-v budget. A single burn consuming all the available delta-v resulted in re-entry: in 2020 for a very early apogee manoeuvre performed in the first half of 2013 (fig. 4) and around 2029 for a large perigee manoeuvre performed before July 2017 (fig. 4).

A final investigation was performed for manoeuvres that would have not required complete depletion of the available fuel: such as Moon resonances and partial or split manoeuvre. No significant effect was registered for the former to induce the spacecraft re-entry given the fact that the available delta-v allowed to reach resonances ranging between 12:1 to 7:1.

On the contrary, partial manoeuvres highlighted a very different behaviour: 25 m/s of delta-v in 2015 would induce re-entry in the 2028/9 allowing up to 8 more years of nominal operations in terms of fuel consumption, re-entry could be obtained with a manoeuvre of 35 m/s in 2017 allowing up to 4 more years of operations, down to 15 m/s re-entry would be achieved but not before the 2100 and smaller delta-v would have not led to re-entry [7].

The possibility of an overall increase of the mission duration made the partial manoeuvre extremely attractive and the 25 m/s in 2015 was selected as the first option.

The manoeuver consisted of three major burns plus a touch-up for final tuning so it was designed to meet the coverage requirements and to execute the firing at the perigee. A post-
manoeuver was finally performed to induce a long-term repeated pattern of 3 orbits every 8 days.

In the nominal scenario, re-entry was obtained on the 27th of February 2029 (fig. 5).

1.2. Break-up

The final step to verify the compliancy with requirements set by the Space Debris Mitigation Policy consisted in studying the fragmentation process upon re-entry, determining fragments surviving to ground impact, and assessing the on-ground casualty risk computed as the total probability of the fragments to hit densely populated area weighted with the impacting area and energy, assuming an uncontrolled re-entry [6].

The Spacecraft Atmospheric Re-Entry and Aerothermal Break-Up (SCARAB) software package was used to study the aero-thermal break-up process for the detailed model of INTEGRAL [9].

Considering the location of the perigee in Southern Hemisphere and that the Northern Hemisphere hosts around 90% of the human population, the most desirable scenario from an on-ground risk point of view consists of an entry in the deeper layers of the atmosphere with a steep slope, no afore-circularization, and breakup occurring close to the perigee. With fragmentation occurring over the South Pole only a limited number of fragments reach the Northern Latitudes, by escaping the atmosphere after break-up as having a high ballistic coefficient and making another full orbital revolution before re-entering. In this way, the on-ground risk is extremely contained.

Preliminary analysis on the INTEGRAL final orbit demonstrated an insensitivity of the orbital elements to solar radiation pressures and air density variations with marginal influence on the actual re-entry altitude. The dependence of the fragmentation process on the altitude was studied showing that for perigee altitudes of around 70 km the number, mass and casualty area, defined as the impacting region on ground, of surviving fragments have a minimum, a lower perigee limits not only the spread of fragments along the ground track but at the same time...
the ones reaching the Northern Hemisphere[9].

For the whole set of simulations, the on-ground risks never exceeded $3 \cdot 10^{-5}$ as the re-entry occur in the South over sparsely populated latitudes. The risk is mostly driven by fragments impacting at Northern latitudes[9].

2. Study purpose

In the previous studies, the motion of INTEGRAL was investigated exclusively in its three Degree of Freedom (DoF), propagating simply its position and velocity disregarding completely its attitude and using average values.

Nevertheless, in order to refine the on-ground risks assessment, the attitude motion cannot be neglected since it directly affects the fragmentation process. In fact, depending on the disposition at re-entry, the ablation can affect different parts of the spacecraft producing fragments of different sizes, identifying the attitude as one of the major drivers of uncertainty in current high fidelity break-up risk estimation models [10].

Dealing with attitude uncertainty in risk estimation processes, the nominal or random tumbling state is often assumed [6]. However, for a certain type of orbit, a strong coupling between attitude and orbit has been identified as a driver for chaos in the attitude motion[11].

Among the several candidates, High Elliptical Orbit (HEO)s represent the perfect case study, as the gravity field alone is sufficient to trigger chaotic behaviour under certain conditions. These theories, focused on deriving the conditions for chaos from the Hamiltonian approach, did not address methodologies for indicating chaos in a practical case.

The work of this thesis proceed from the results of previous studies, trying to identify in the chaoticity, induced by HEO regions of constrained stability, if existent, for INTEGRAL’s attitude motion. The outcomes can be further used as basis for realistic simulations of break-up [12].

With this goal in mind, it has been decided to focus on analysing the attitude induced by the environment on the INTEGRAL spacecraft after EOL, under the assumption that passivation operations would deplete all fuel and leave the spacecraft without residual motion.

The work was split into two big tasks: the propagation of the INTEGRAL’s orbit and attitude from a given initial state till the time of re-entry, after the development and the validation of a new propagator and the stability analysis of the afore-generated data [13].

3. Methodology

Predicting the orbit of a satellite is a physical problem addressed to a mathematical one: the initial conditions and the object dynamics, carrying the physics of the problem, are fed to an algorithm that estimates the new position and velocity of the satellite by iterative approximation process.

3.1. INTEGRAL’s initial value problem

The motion of a finite dimension object in space can be considered as the product of the translation of its Centre of Mass (CoM) and the rotation of the object itself about it. Constraining the motion of a satellite in six DoF requires a system of twelve independent equations with twelve independent variables, generally given in group of three in term of position, velocity, angular rate and attitude.

The motion INTEGRAL falls exactly in the latter case. Its orbital-attitude prediction requires:

• an initial condition which fixes the position of the spacecraft in space and time;
• a set of ordinary differential equations which describes the system dynamics for the considered ambient space.

3.1.1 Initial conditions

INTEGRAL’s initial state is described by its two components: the orbital state and the attitude state. The former is epoch-defined and thus maintained fix for all the simulations starting from the same starting date. The latter is selected in such a way that the stability of the system can be studied and will be described afterwards.

In this context, the 1st of February 2016 was selected as initial epoch and the operational orbit provided Cartesian position and velocity derived from the j2000 orbital elements, here reported: \(a=81121.45\ \text{km}, e=0.81, i=52.13\ \text{deg}, \Omega=210.99\ \text{deg}, \omega_p=259.31\ \text{deg}, \nu=231.43\ \text{deg}.

3.1.2 Frames of reference

Before dealing with the physical laws that describe the motion of INTEGRAL in six DoF, it is prudent to describe the different coordinate systems. Having clear in mind the frame in which the variables are derived allows to correctly deal with variables whose evolution is influenced by another defined into a different coordinate system.

It is common practice to represent the orbital motion of a satellite in the Earth Centred Inertial (ECI) reference frame. Centered in the Earth, the ECI orientation is defined by the vernal equinox and by Earth North pole that fix respectively x-axis and z-axis in space (fig. 7). ECI allows to describe the motion of the spacecraft and the relevant perturbations in a simple and clean way.

![Figure 7: The orientation of the ECI and of the orbital frames.](image)

Taking into account the attitude motion, the simplification of considering it on top of the orbital one requires a switch from one coordinate system to another. The attitude of the spacecraft is determined as the angular difference between the inertial frame, centred this time in the CoM of the spacecraft, and body frame \(B\), afore-defined for the case of INTEGRAL (fig. 8).

The angular rate is defined as velocity that induces the body frame to rotate from the body frame \(B_i\) to \(B_{i+dt}\) during the time \(dt\).

![Figure 8: The orientation of the body frame with the respect of the inertial frame centred in the CoM of the satellite.](image)
In the end, although not needed for propagation purpose, the orbital frame is important for understanding the overall movement of spacecraft on-orbit and it is described in the following.

Determining the instantaneous orientation, the orbital frame is uniquely defined by $O_z$ and $O_x$ axes, respectively oriented in the direction of the centre of the Earth, opposite to the position vector $r$, and tangent to the trajectory, or collinear to the velocity vector $v$ with $O_y$ obtained by the right hand rule (fig. 7).

For HEO, such disposition does not define an ordinate frame since the perpendicularity between $r$ and $v$ is not globally guaranteed. Thus some manipulations are needed: although not always orthogonal, position and velocity define the orbital plane and their cross product defines the orientation of $O_y$. Repeating the procedure for $O_y$ and either $r$ or $v$ fix $O_z$ and $O_x$ axes allowing to obtain an orthogonal orbital frame. The choice of one system or the other should be in principle equivalent, although it is common practice to keep the system aligned with the position vector.

3.2. Dynamical systems

Spacecrafts in HEO are influenced by the combined effects of gravity, aerodynamic force, solar pressure and in addition the luni-solar perturbations. Considering the specific case represented by INTEGRAL’s orbit and in the context of this study, some assumptions can be made, allowing to simplify the dynamics of the physical problem.

First of all, given the high perigee altitude, the effect of the aerodynamic force is negligible until the actual re-entry. Moreover, INTEGRAL is assumed to enter the atmosphere with a steep path without circularization, allowing to ignore the aerodynamic effect until fragmentation occurs, that represents the stopping condition for the propagation.

Secondly, as previous analysis showed, the effect of the solar pressure on the orbit appeared to be negligible, as varying the reflection coefficient of the surface of the spacecraft, an ignorable variation of $\pm 4$ hours in the re-entry date is observed (fig. 6). In the evaluation attitude motion, its omission is theoretically incorrect. Although, it is justified as its effect, dominant only at the apogee under particular moon phase condition, acts practically as secondary perturbation. Moreover, in the context of this study investigating the chaotic motion induced by high elliptical orbit, solar radiation pressure is likely to add further disturbances to a potential chaotic system. Thus, for a better understanding and a greater extensibility of the study to similar cases, solar pressure has been disregarded.

Considering gravity as the unique force acting on INTEGRAL’s orbit and attitude, the ordinary differential equations expressing the evolution in time of position, velocity, angular rate and attitude, in quaternion, are described as follows:

\[
\dot{r}_s = v_s (1)
\]

\[
\dot{v}_s = -\frac{\mu E r_s}{\|r_s\|^3} (1 + J_2) - \frac{\mu S}{\|r_s\|^3} (r_s + F r_s) + \frac{\mu M}{\|r_s\|^3} (r_s + F M)
\]

\[
\dot{\omega}_s = \Gamma^{-1} (\hat{\omega} I \omega_s + \frac{3 \mu E}{\|r_s\|^5} r_s^B I r_s^B + \frac{3 \mu S}{\|r_s\|^5} r_s^SS I r_s^SS + \frac{3 \mu M}{\|r_s\|^5} r_s^SM I r_s^SM)
\]

\[
\dot{\beta}_s = \frac{1}{2} \hat{\beta}_s \omega_s
\]

The rate of change of the velocity and of the angular rate, $\dot{v}_s$ and $\dot{\omega}_s$ respectively, require further explanation. Derived from the classical $n$-body formulations:

\[
\dot{V}_s = \sum_{j=1}^{N-1} -\frac{\mu_j (R_s - R_j)}{\|R_s - R_j\|^3}
\]
and rewritten in the frame of the strongest gravitational source:

\[ \dot{\mathbf{v}}_s = -\frac{\mu_E \mathbf{r}}{||\mathbf{r}_s||^3} + \sum_{j=1}^{N-1} \mu_j \left( \frac{\mathbf{r}_s - \mathbf{r}_j}{||\mathbf{r}_s - \mathbf{r}_j||^3} + \frac{\mathbf{r}_j}{||\mathbf{r}_j||^3} \right) \]  

(6)

the total acceleration consists of two terms: the gravitational acceleration of the Earth, further extended to the \( J_2 \) component to consider the significant effect of the Earth's oblateness, and the direct and indirect components of the third body perturbations. In the approximation of \( r_s << r_j \), the perturbation can be written in the exact scalar formulation without loss of precision:

\[ \frac{\mathbf{r}_{sj}}{||\mathbf{r}_{sj}||^3} + \frac{\mathbf{r}_s}{||\mathbf{r}_s||^3} = \frac{1}{||\mathbf{r}_{sj}||^3} \left[ \mathbf{r}_s - r_j \left( 1 - \frac{||\mathbf{r}_{sj}||^3}{||\mathbf{r}_j||^3} \right) \right] = \frac{1}{||\mathbf{r}_{sj}||^3} \left( \mathbf{r}_s + r_j F \right) \]

(7)

where \( F \) is the exact scalar function:

\[ F = q \left[ \frac{3 + 3q + q^2}{1 + (1+q^2)^2} q = \frac{\mathbf{r}_s (\mathbf{r}_s - 2\mathbf{r}_j)}{||\mathbf{r}_j||^2} \right] \]

(8)

The angular acceleration is directly derived by the conservation of the angular momentum:

\[ \dot{\mathbf{H}} = \frac{d}{dt} \left( \mathbf{H}^B \times \mathbf{H}^B \right) \]

(9)

\[ \tau^B = I^B \dot{\omega}^B + \dot{I}^B \omega^B \]

(10)

With the inertia tensor \( I \) and the angular rate \( \omega \) already defined in the body frame of the spacecraft and the external torque \( \tau \) limited to the gravity gradient ones, the angular acceleration \( \dot{\omega} \) is given in eq.(3).

The rotation from the inertial to the body frame was achieved through a rotation corresponding to the quaternion, implemented via the Hamilton product:

\[ \mathbf{r}^B = \beta^* \mathbf{r} \]

(11)

With \( \beta^* \) representing the conjugate quaternion.

The last consideration is dedicated to the position of the Sun and the Moon. Despite available in the form of very precise ephemeris, it is generally convenient to have the possibility to compute them with some less precise analytical technique. We report the one offered by the Astronomical Almanac (1992:C24), presenting an accuracy of 0.01 deg [14].

### 3.3. ATOPS

To propagate the state of INTEGRAL, a dedicated numerical integrator was constructed from scratch: the ATtidue and Orbit Propagator System (ATOPS). Initially written in Matlab, ATOPS was translated into Python programming language in order to ensure a wider use in the SDO available computing infrastructure.

ATOPS consists of three modules: bFunc.py, RungeKutta.py and prop_*_3body.py.

The bFunc.py contains the basic functions required by ATOPS to run, i.e. planCoo implements the algorithm to compute the position of Sun and Moon. The greatest achievement is the implementation of the numba library that allows to achieve performance comparable to C, C++ and Fortran by compiling pure Python routines to native machine instructions [15].

RungeKutta.py, as suggested by the name, implements the Runge-Kutta (RK) Fehlbergh 7(8) routine. Technically speaking, it is an adaptive step size method that, in principle, allows to obtain a fix tolerance integration.

The 7(8) order was chosen after a quick test campaign, which compared the results offered by this pair with ones obtained by selecting a lower and an upper pairs as sparring partners. In particular, RK 4(5) resulted in being unpractical for the orbital/attitude propagation: in the ATOPS configuration, un-explainable peaks were registered at perigee [16]. Selecting lower tolerances and different propagator step...
size could have been helpful but would have induced an increase of computation time. Similar behaviour was obtained for RK 7(8) and RK Verner 8(9), although at the time of comparing their performances, it clearly appeared that the increase of accuracy obtained by selecting a higher order pair was accompanied by a parallel reduction of speed. In several studies, Fehlbergh 7(8) routine ensured on one hand enough propagation speed and on the other hand a good containment of Local Truncation Error (LTE) [17].

The peculiarity of RungeKutta.py is the multi-tolerances implementation. In fact, the user has the possibility to define different tolerances for the different variables involved. Once the LTE is computed, it is split into the different components according to how the tolerances were assigned to state variables. Compared with the maximum error allowed, the state is accepted only in the case of compliance for all the defined tolerances. Otherwise, it is rejected and re-calculated for the new step size. With the given errors, the relative step sizes are computed and the minimum one is taken so that the optimal step size is determined based on the greater LTE.

Moreover, RungeKutta.py offers the user the opportunity to select variables to be normalized, practical when dealing with variables such as the quaternion. For the prop module, two different versions are available depending on whether the positions of third body perturbation sources are analytically computed or externally provided with the propagation module assuming the name of prop_int_3body.py and prop_ext_3body.py respectively.

Each version contains in addition to the ordinary differential equations, which are problem specific, two different propagation routines: a simple time integration, for which the state is displayed for the selected output step size, and a more advanced integration. The latter, in particular, is developed to study the stability of the problem and consists in a fixed true anomaly integration. In principle, the propagation remains basically a fixed time integration, however the state to be stored is selected not based on the time but on the true anomaly. To do that, the data of the previous fixed time integration are taken as reference and processed to extract the true anomaly. Compared with the one of reference, the time before the encounter of the reference true anomaly is saved, together with an indication of the maximum propagator step size. The step size is computed from a starting one that ensures the state to be in the range of the true anomaly of reference and decreased proportionally with the perigee altitude:

\[
h_t = \left( \frac{H}{H_{ref}} \right)^4 h_{ref}
\]

The choice of the fourth exponential has no physical relations and the law has no general validity but allows to maintain an efficient speed and to meet the reference target with a suitable margin.

At the time before the encounter, the new maximum propagator step size is passed and the state with true anomaly closer to the reference one is stored. The scale of the propagator step size has shown to effectively induce a maximum deviation from the reference of 0.25 deg in the worst case represented by the perigee.

To sum up, the main steps are outlined in the flowchart shown in fig. 9 where the user provides the initial and final epoch (Ts) and the initial state (\(\mu_0\)), stored in the ATOPS configuration dictionary. The simple time integration (state_at_time) takes place and from its result the true anomaly time is generated according to the aforementioned strategy. Fed to the state_at_true_anomaly, the fixed true anomaly integration can be performed (fig. 9).

In the end, a stop condition on the perigee altitude for the re-entry prevents the propagator
3.4. ATOPS configuration for INTEGRAL

Propagator step size and tolerance are problem specific quantities that have to be accurately defined for each numerical integrator in order to obtain feasible results. ATOPS is not an exception to this and has been tuned for the specific case of INTEGRAL.

The angular rate is the main constraint of the step size. 100 s for the time propagation ensures a good mapping of the attitude motion. Although for fixed true anomaly integration, the step size has to be reduced to 20 s close to the perigee allowing to obtain the state for a true anomaly close to the reference one with 0.25 deg of maximum deviation.

For the selection of the tolerance, a sensitivity analysis was performed. Simulations with tolerances ranging from $10^{-6}$ to $10^{-30}$ were run and compared in term of the re-entry date. As soon as the forecast converged, the lowest tolerance with stable re-entry date was chosen. A tolerance of $10^{-9}$ provided the first repetitions and was selected for the orbital propagation. For the attitude and the angular rate, it was not possible to run a sensitivity analysis and a value of $10^{-6}$ was selected based on the experience of previous attitude simulation.

As expected, the higher precision obtained by selecting higher tolerances is paid at high cost of computational time. If the re-entry date does not change, there are no reasons to select higher tolerances. Indeed, considering the worst case of LTE equals to the provided tolerance, the Global Truncation Error (GTE) after roughly thirteen years of propagation exceeds the magnitude of certain variables, e.g. the quaternion. In fact, the purpose of this study does not consist in a propagation not affected of errors but in the correct implementation of the system dynamics. In this way, the qualitative behaviour of the motion is ensured, thus allowing the study through statistical analysis of those variables which cannot be purified from the error.

4. Validation process

The possibility to deduce a specific behaviour in INTEGRAL attitude motion through ATOPS simulations required the propagation system to undergo a full-scale validation process. With the objective to demonstrate that ATOPS's results are in line with the previous simulations, the validation phase is a two-way process that intend to study the qualitative behaviour of INTEGRAL orbital and attitude motion and compare it with the reference one provided respectively by ORBit GENerator (ORBGEN), SDO validated propagator, and by the SOHO Failure Scenario.[18].

4.1. ORBGEN

ORBGEN is one of the main validated orbital propagator at disposal of SDO that was used in the selection and validation campaign for the manoeuvre disposal and provided the nominal re-entry scenario.

ORBGEN, differently from ATOPS, considered the full environmental perturbations: aerodynamic drag and solar radiation pressure
Figure 10: The different evolution of perigee altitude and re-entry date for ORBGEN (in black) and ATOPS (in green).

were modelled and applied considering a reference area derived for the case of INTEGRAL tumbling motion. The different physical model together with the implementation of a different RK pair, inducing a different GTE, allows only a qualitatively comparison between ORBGEN and ATOPS results.

Despite the different re-entry dates, roughly 12 days in difference, the behaviour of the perigee decay was similar, demonstrating the validity of the system dynamics implemented in ATOPS (fig. 10).

4.2. SOHO failure scenarios

The SOHO failure scenario[18] investigate failure scenarios due to a power drop resulting from uncontrolled tumbling and at rest condition in different orbital configurations.

Understanding whether INTEGRAL would have fallen in some steady state and could have passively re-oriented itself in a favourable Sun pointing attitude was the key objective. In particular, two worst cases were evaluated[18]:

- an undefined malfunction induces spin at high rate about x-axis, associated to a power drop due to missed-immediate-recover and insufficient illumination condition;
- INTEGRAL falls in an attitude configuration of no-exposure to Sun associated to discharge during the eclipse.

In the first case, the choice of x-axis as rotation axis for the spacecraft was wisely selected: indeed, starting from an operational configuration with the z-axis pointed towards the Sun, the transition induces INTEGRAL to re-orient and reach stable rotation about the principal axis of inertia, largely off-pointed from the Sun (fig. 11).

The latter represents the worst case spinning scenario since the tumbling does not allow re-charge of the Electrical Power System (EPS) via INTEGRAL’s solar panels[18]. The small difference among the principal moment of inertia makes flat spin transition incredibly slow, variable between 4000 and 16000 s, and induces final rate about the principal axis slightly lower than the initial one[18].

From the transition, two simulations were run for the satellite spinning respectively at 24 and 1 deg/s about the principal axis of inertia, showing that in both of the cases the environmental forces and torques were not producing any evident effect, with the spacecraft maintaining its overall motion[18].

The constant attitude about x- and y-principal axes, maintained along one orbit...
revolution, shows that the environmental torque does not induce a significant effect on INTEGRAL attitude motion (fig. 12).

Figure 12: INTEGRAL attitude with the respect of x- and y- principal axis of inertia for 1 deg/s spin [18].

ATOPS showed similar results by displaying directly the invariability of angular rate to the external torque: the angular rate remains constant in the principal inertia, body and inertial frames, demonstrating that no rotation occurs among the three coordinate systems (fig. 13).

Figure 13: Angular rate three-days evolution in principal inertia, body, inertial and orbital frames for INTEGRAL spinning at 1 deg/s in SOHO failure spinning scenario.

For the non-spinning satellite, the SOHO Failure Scenario [18] highlighted that INTEGRAL experiences negligible aerodynamic torque, while gravity gradient torque dominating at the perigee passage and solar pressure torque becoming relevant at the apogee.

However, the non-spinning simulations does not take into account the influence of the third body perturbations and thus cannot be used to validate ATOPS. Nevertheless, the spinning scenario offers a good test bench and ATOPS results appear to be in line with the SOHO failure scenario of a spinning satellite [18].

5. Stability analysis

Highly perturbed systems, in which errors cannot be separated from the results and play a significant role on the outcome, require a statistical-type study. In particular chaos theory has demonstrated to be extremely powerful in investigating the sensitivity to initial conditions of deterministic systems, whose evolution in time is uniquely defined by their dynamics.

When small differences in the initial conditions yield to a completely divergent response, technically defined recurrent aperiodic, the system is characterized by deterministic chaotic behaviour [11]. Chaos theory is “the qualitative study of unstable aperiodic motion in deterministic nonlinear systems” [19].

The simplest way to discriminate between chaotic and stable motion, highlighting respectively the most distinctive features, is through the phase plane representation. Portraying the position of the object in function of its velocity, the behaviour of the motion can be assessed from some key features appearing in the trajectory evolution [11]: single point structures reveal equilibrium, the recurrent nature of periodic motion is displayed by trajectories that close on themselves forming circular structure, open regular curves indicate quasi-periodic motion and chaos takes the rest with open, not regular and repeating geometry.
The global aperiodicity of chaos does not tell anything about its local behaviour that can tend to an unstable periodic state that never re-appears in the future, in a continuous switch from one to another [11].

The ability of the phase plane representation to distinguish between periodic and chaotic motion decreases significantly for long-term simulations, thus different techniques have been developed in order to solve the problem [11]. Among them, the Poincaré map is particularly fitting for studying the stability of the system presented in the context of this work.

5.1. Poincaré maps

The capability to assess the global stability is achieved by evaluating the dynamical system behaviour at a certain condition. Translated into a plane, the behaviour of the system is represented at intersection with the system trajectory. Technically speaking, the \( n \)-dimensional curves intercept the \( n \)-1 dimensional hypersurface, the so-called Poincaré section, allowing to assess the system stability by observing the distribution of the intersections (fig. 14).

In particular periodicity, resulting from closed overlapping trajectories, appears in a single point on the Poincaré map. Quasi-periodic trajectories arise in the form of nested closed concentric curves, the better-known islands, or manifolds densely covered by parallel open regular curves.

Periodicity and quasi-periodicity are commonly defined as integrable. In contrast, ergodic trajectories present an essentially random motion. In conclusion, if the motion cannot be defined neither completely integral nor completely ergodic, then it is chaotic [20].

The Poincaré map most remarkable advantage consists in the hyper-plane, that slicing the orbit at fix conditions, avoid the ambiguity encountered by the phase plane. Although the stability assessment is valid only at the cut, overall global stability can be hypothesised under certain circumstance between two close intersections.

In the context of this study, attitude chaotic motion induced by high elliptical orbit can be assessed by searching for patterns in the Poincaré map created for the angular rate long-term propagated from initial conditions distributed over the quaternions hyper-sphere. The initial angular rate was kept zero considering the fact that basically any spinning configuration appeared to be insensitive to environmental torques.

5.2. Random quaternion distribution

The attitude is randomly disturbed over the hyper-sphere by picking three points, ranging
from 0 to 1 and computing the unit quaternion as follows:

\[
\beta = \begin{bmatrix}
\sqrt{1 - X_0 \cos(2\pi x_2)} \\
\sqrt{1 - X_0 \sin(2\pi x_1)} \\
\sqrt{X_0 \cos(2\pi x_1)} \\
\sqrt{X_0 \sin(2\pi x_2)}
\end{bmatrix}
\tag{13}
\]

The algorithm takes a random point on the sphere and converts it into the form of the quaternion. To do that, each point on the sphere can be obtained by two following rotations: one orienting \(z\) in an arbitrary rotation and one about \(z\) itself. In the quaternion nomenclature:

\[
R_z = [a, 0, 0, z], \quad R_{xy} = [w, x, y, 0]
\]

\[
R = R_{xy} R_z = [aw, ax + zy, ay - zx, zw]
\tag{14}
\]

Eq.(12) is derived by distributing \(Z\) between 1 and -1 and \(X, Y\) over a circle of radius \(\sqrt{1 - Z^2}\), substituting \(w\) with \(\sqrt{X_0}\) and multiplying \(x\) and \(y\) by \(\sqrt{1 - X_0}\) according to the uniform spherical distribution. The second and third components are just the explicit cosine and sine angle difference. The distribution is built in a way that takes care of already taking rid of the quaternion conjugate [22].

5.3. Sampling and rate of convergence

Randomly generating the initial conditions requires to find a measure of the number of samples necessary to obtain a consistent dataset to evaluate statistically the stability of the system. The Monte Carlo rate of convergence came to help. Considering the stochastic problem represented by:

\[
f = E(X)
\tag{15}
\]

where \(X\) is a uniformly distributed random variable. Then "for the law of large numbers", the \(n\) amount of \(X\) copies that ensures the convergence of \(f_n\) to \(f\), for \(E(X) < \infty\) can be estimated by the mean over \(n\) [23]:

\[
\frac{1}{n} (X_1 + \cdots + X_n) = E(X_1)
\tag{16}
\]

In particular, for the specific case of INTEGRAL, the rate of convergence was implemented studying the magnitude of the angular rate at the re-entry, showing that already for 400-500 samples, stable mean was obtained (fig. 15).

Nevertheless, the rate of convergence gives just an indication of minimum number of samples. Quite often, more samples are needed in order to display the desired output quantity.

5.4. Angular rate/momentum map

In order to understand which structures can be expected for the motion of INTEGRAL with the afore initial conditions, a reference case study of a gravity gradient stabilized satellite was selected. In particular, the Poincaré map showed similar structures for different moment of inertia configurations, respectively \(I_{yy} > I_{xx} > I_{zz}\) and \(I_{xx} > I_{yy} > I_{zz}\).

Poincaré maps succeeded in displaying a quasi-periodic behaviour of the angular momentum represented through the aforementioned islands and open regular curves (fig. 16) [24].

With this scenario taken as reference,
the attitude behaviour of INTEGRAL was captured at the intersection of its orbit with the hyper-surface fixed at the perigee of a specific epoch. Despite the large number of simulations, the intersection on the Poincaré section, equal to the number of the simulations, appeared to be not enough to highlight any structures. Thus it has been decided to extend the limitation of a specific epoch of the hyper-surface to first three close dates.

The choice of gathering together three perigee passes derives from the necessity of constructing a map with similar level of torques. However, negligible changes were observed in the Poincaré representation, which required a further extension to a full year.

From a theoretical point of view, this choice does not seem to violate any principle: indeed, the Poincaré section remains fixed in the space represented by true anomaly variable and although multiple points for the same initial condition are collected, they are not enough to form structures. Moreover, the choice of year-representation seems to be practical allowing to evaluate if the gravitational torques, acting alone on the spacecraft, trigger stability on INTEGRAL attitude motion and if a trend can be observed with the proceeding of the years.

Gathering a whole year of perigee passes make structures appear in 2024 (fig. 17) and are maintained till re-entry (fig. 18). In particular, from island and manifold structures, quasi-periodic motion can be observed.

An increase of chaos is observed in the last year with the manifold disappearing and the nicely circular trajectories evolving in a doughnut structure. Such behaviour can be probably explained due to the rapid decay of the perigee experienced in the last year.

Although considering that perigee passes for one year is in principle wrong, given the difference torques experienced along the orbit, the presence of structures is encouraging, a stimulus for further research.
6. Quality description attitude motion

The final goal of this study consisted in providing a quality description of the overall attitude motion of INTEGRAL.

Several time-simulations were run with reduced output step size to 600 s. A larger data file was generated for answering to the specific request of mapping the attitude motion while avoiding under-sampling. After 100 simulations, the spacecraft does not exceed the maximum angular rate level of 0.25 deg/s (fig. 19).

The chosen step size allows to record the attitude at least every half rotation ensuring that between two following states storing, the spacecraft did not go through a full rotation. This would have caused a loss of information leading to the impossibility to provide a complete picture of the attitude motion.

Ensured of the validity of the data, the analysis of the motion was conducted studying the evolution of attitude motion on the momentum sphere. In particular from the definition of the kinetic rotational energy, the Poinso's ellipsoid is generated by assigning to the semi-axes in the x-, y- and z-directions the values of the kinetic energy computed for the maximum, intermediate and minimum inertia respectively.

\[
K_R = \frac{1}{2} \mathbf{\omega}^T \mathbf{I} \mathbf{\omega} = \frac{H_2^2}{2I_1} + \frac{H_2^2}{2I_2} + \frac{H_3^2}{2I_3} (17)
\]

The intersection between the energy ellipsoid and the sphere constructed with radius equal to the magnitude of the angular momentum of the spacecraft identifies the possible angular rate experienced by the satellite. In particular three different motions can be observed (fig. 20):

- precession about \( H_3 \);
- precession about \( H_1 \);
- tumbling motion.

In torque-free motion, given the conservation of the angular momentum and of the total energy, precession about \( H_1 \) and \( H_3 \) any axis is stable, meaning that a spacecraft moving on any closed curve keeps on rotating on that curve. The application of a constant perturbation to the attitude motion induces a deflation of the energy ellipsoid that brings the system to seek its minimum energy state, represented by a flat spin about the maximum axis of inertia.

In the representation of the momentum sphere for the case of INTEGRAL a variable behaviour was detected. In particular starting from the same condition of zero angular rate,
Figure 20: Precession and tumbling motion representation over the momentum sphere[25].

Figure 21: Evolution of the gravity gradient and 3rd body perturbation torques.

Figure 22: The connection between the gravity gradient torque and the perigee altitude decay.

The motion experienced by the satellite is a direct consequence of the attitude and of the position of the spacecraft to the gravitational sources which contribute to angular acceleration of the spacecraft. The theoretical postulation of this behaviour was done in [11] and is confirmed here in a concrete setting.

Over the years, while third body perturbations remain fairly constant, the gravity gradient torque increases and peaks between February and August of 2021, then diminishing to peak again at re-entry (fig. 21).

The sinusoidal growth of the gravity gradient torque, which constitutes the major contribution to INTEGRAL angular acceleration, corresponds to the perigee decay, which explains its overall behaviour (fig. 22). The increasing/decreasing energy of the system, associated with the effect of the torque inducing a variation in the opposite/same direction of the angular rate induces an inflation/deflation of the energy ellipsoid that can be observed with a movement of the angular rate lines towards $H_1/H_3$ respectively (fig. 23).

Moreover, the effect of the gravity gradient torque on the attitude motion is trivial to be deduced from the orientation of the torque and spacecraft angular rate. However, the effect of the environmental torques in inducing a change in the angular rate is inversely proportional to the higher angular rate, itself. This behaviour is observed in a delay of the line shifting. Such behaviour is consistent with the SOHO Failure Scenario showing that a spin of 1 deg/s prevents the environmental torque to induce any significant effect.

In the end, over 100 cases were evaluated in the last year before the re-entry. 70% of
those showed precession about the maximum principal axis of inertia, 20% showes tumbling motion and only 10% precession about the minimum principal axis of inertia.

7. Conclusion and future work

In this study, the application of the Poincaré map to the results obtained for the long-term propagation of INTEGRAL’s orbital-attitude revealed the presence of structures that can be reconnected to quasi-periodic angular rates. The extension of the Poincaré section from single to multiple epochs, although quantitative in nature, is an expedient way to evaluate the stability with a reduced number of simulations when using the similarity in torque magnitude and stability of the motion over a few perigee passes. Moreover, the use of a chaos indicator, such as Lyapunov characteristic number could provide interesting information on the time of stabilization as is in use for orbital dynamics problem. However, they cannot be readily exported to the long term attitude motion problem, as the dependency of the linearized motion, i.e. the Jacobian, depends on the definition of metric to be used to define what is close in phase space (to add to the problem, in the case of attitude on a closed manifold).

From the confirmation of constrained motion, a qualitative description of INTEGRAL attitude motion is performed through the use of the momentum sphere. An explanation of the movement of the angular rate line is offered and its relation to the gravity gradient torque, despite restricted, is given. Further research can include extra perturbations, such as solar radiation pressure, and indicate how it influences fragmentation behaviour and associated on-ground casualty risk.

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