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This is the accepted version of a paper published in *IEEE Transactions on Wireless Communications*. This paper has been peer-reviewed but does not include the final publisher proof-corrections or journal pagination.

Citation for the original published paper (version of record):

B. da Silva Jr., J M., Fodor, G., Fischione, C. (2017)
Fast-Lipschitz Power Control and User-Frequency Assignment in Full-Duplex Cellular Networks
IEEE Transactions on Wireless Communications, 16(10): 6672-6687
<https://doi.org/10.1109/TWC.2017.2728062>

Access to the published version may require subscription.

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Fast-Lipschitz Power Control and User-Frequency Assignment in Full-Duplex Cellular Networks

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Abstract—In cellular networks, the three-node full-duplex transmission mode has the potential to increase spectral efficiency without requiring full-duplex capability of users. Consequently, three-node full-duplex in cellular networks must deal with self-interference and user-to-user interference, which can be managed by power control and user-frequency assignment techniques. This paper investigates the problem of maximizing the sum spectral efficiency by jointly determining the transmit powers in a distributed fashion, and assigning users to frequency channels. The problem is formulated as a mixed-integer nonlinear problem, which is shown to be non-deterministic polynomial-time hard. We investigate a close-to-optimal solution approach by dividing the joint problem into a power control problem and an assignment problem. The power control problem is solved by Fast-Lipschitz optimization, while a greedy solution with guaranteed performance is developed for the assignment problem. Numerical results indicate that compared to the half-duplex mode, both spectral and energy efficiencies of the system are increased by the proposed algorithm. Moreover, results show that the power control and assignment solutions have important, but opposite roles in scenarios with low or high self-interference cancellation. When the self-interference cancellation is high, user-frequency assignment is more important than power control, while power control is essential at low self-interference cancellation.

I. INTRODUCTION

In order to meet the need for explosive data volumes and data rates, wireless network operators seek to enhance the spectral efficiency in lower-frequency bands [1], and to exploit higher-frequency bands such as the millimeter waves. The research and standardization communities are currently studying physical layer technologies, including massive MIMO systems, spectrum sharing in mmWave networks, new waveforms, non-orthogonal multiple access technologies, and full-duplex communications [2], [3].

In-band full-duplex (FD) transceivers are expected to improve the attainable spectral efficiency of traditional wireless networks operating with half-duplex (HD) transceivers by a factor of two [3]. Due to recent advancements in mitigating the inherent self-interference (SI) by means of passive suppression, analog and/or digital cancellation, in-band FD technology is quickly approaching the phase of commercial deployments in low-power wireless networks [4].

José Mairton B. da Silva Jr. was supported by CNPq, a Brazilian research-support agency, and by The Lars Hierta Memorial Foundation. G. Fodor was partially sponsored by the joint Wireless@KTH-Ericsson project Nano-Machines to MIMO Wearables and Smart Phone Communications (Naomi). The simulations were performed on resources provided by the Swedish National Infrastructure for Computing (SNIC) at PDC Centre for High Performance Computing (PDC-HPC).

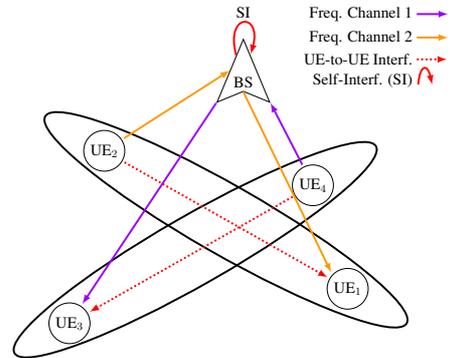


Figure 1. A cellular network employing three node FD with two UEs pairs. The base station selects pairs of UEs, represented by the ellipses, and jointly schedules them for FD transmission in the UL and DL. To mitigate UE-to-UE interference (red dotted line), it is advantageous to assign DL/UL users for FD transmission in the same frequency that are far apart, such as UE_1 - UE_2 and UE_3 - UE_4 .

Although in-band FD promises to instantly double the data capacity of existing technology, its deployment in wireless local area and cellular networks is challenging due to the large number of legacy devices and wireless access points. A viable introduction of FD technology in cellular networks is offered by three-node full-duplex (TNFD) deployments, in which only the wireless access points or base stations (BSs), typically equipped with multiple antennas, implement FD transceivers to support the simultaneous downlink (DL) and uplink (UL) communication with two distinct user equipments (UEs) on the same frequency channel [5]. In TNFD networks, the inherently present UE-to-UE interference may become the performance bottleneck, especially as the capability of FD transceivers to suppress SI improves.

To understand TNFD operations, consider the TNFD network with two UEs pairs in Figure 1. Unlike when using HD transmissions, a TNFD network must deal with both SI and UE-to-UE interference, indicated by the red dotted lines between UE_1 - UE_2 and UE_3 - UE_4 . The UE-to-UE interference power depends on the UEs locations and propagation environments, as well as the UE transmission powers. Coordination mechanisms are required in order to mitigate the negative effects of interference on the spectral efficiency of the system [6]. The two most important mechanisms are UE pair-frequency assignment and power control [5]–[8]; together, these determine which UEs transmit simultaneously on the frequency channels and with which powers the UE and the BS will transmit. Therefore, it is crucial to understand the trade-offs between UL and DL performance of TNFD systems in the

design of efficient and fair medium access control protocols, and also coordination mechanisms that help to realize the FD potential even for legacy UEs.

In this paper, we focus on the problem of joint power control and user-frequency channel assignment, assuming TNFD transmissions and a frequency selective wireless environment. Specifically, we investigate the fundamental problem of sum spectral efficiency maximization in a single-cell system. We formulate this problem as a mixed integer nonlinear optimization, which we call the *joint assignment and spectral efficiency maximization* (JASEM) problem. The decision variables are arranged in the assignment matrix, each element of which is a binary variable that indicates the association of an UL user to a DL user and a frequency channel. We show that the JASEM problem is a non-deterministic polynomial-time (NP)-hard problem, implying that no optimal solution in polynomial time can be obtained.

To find a close-to-optimal solution to JASEM, we decompose it into two parts that correspond to the power control problem and user-frequency channel assignment problem, respectively. It turns out that the assignment problem is also NP-hard. To solve it, we propose a greedy algorithm that has a guaranteed performance with respect to the optimal solution. To solve the power control problem, we develop a novel algorithm that is especially suited for TNFD networks. Our proposed power control algorithm is distributed and sets the signal-to-interference-plus-noise ratio (SINR) targets at each receiver such that the achieved sum rate is close to optimal. The sum rate maximization problem is non-convex, so we propose using the Fast-Lipschitz (FL) optimization [9] to solve it in a distributed and fast manner. A key component of the proposed power control algorithm is that it transforms the problem of finding the transmit power levels into a sequence of feasibility checking sub-problems, where the sum of the spectral efficiency targets is optimized while maintaining the feasibility of the power vector. The second element of the proposed algorithm consists of setting the transmit powers that minimize the sum power consumption and achieve the SINR targets. An important characteristic of the power control algorithm is that it operates in a distributed manner, taking advantage of the TNFD setup, where each transmit-receive pair sets its own transmit power levels based on locally available information typically transmitted by the legacy BSs in order to facilitate mobility. Although we investigate the single-cell case, the proposed power control algorithm can also be used in multi-cell systems, provided that inter-cell interference measurements are available.

The analytical and numerical results show that the optimality gap between an exhaustive solution of JASEM and the proposed joint solution is small. Our proposed solution outperforms existing methods in the literature, such as random assignment without power control, and the use of HD mode in interference-limited scenarios. With the proposed distributed power control solution, we obtain gains in terms of energy saving, in addition to the spectral efficiency gains in interference-limited scenarios. Our overall finding is that

smart assignment solutions are important when the scenario is interference-limited, whereas smart power control solutions are more important in SI-limited scenarios.

The remainder of the paper is organized as follows. Section II discusses relevant and closely related works. Section III presents the system model and main parameters, followed by the problem formulation and the notation used throughout the paper. Section IV analyses the power control problem. Using matrix analysis, we arrive at a sequence of feasibility checking sub-problems to maximize the sum spectral efficiency. Since the problem is not convex, we use the framework of FL optimization in Section V to derive a distributed solution to approximate the optimal SINR targets in a distributed manner, and use these targets to obtain the transmit powers, also in a distributed manner. In Section VI, we study the user-frequency channel assignment problem, for which we propose an approximate greedy solution. We show its guaranteed performance and, for benchmarking purposes, also propose a natural alternative solution to the assignment problem using the Hungarian algorithm for bipartite matching, which can be understood as a generalization to frequency selective fading environments of an heuristic in [10]. Section VII presents numerical results and compares the performance of the proposed joint solution with alternative assignment and power allocation schemes in UE-to-UE interference limited and SI-limited scenarios.

II. RELATED WORKS

The impact of FD radios on cellular systems has been analysed recently in [6], [11], [12], which provide valuable insights into the design aspects and performance of such systems in terms of rate and energy performance. However, the problem of distributed joint power and user-frequency channel allocation in TNFD networks has not been addressed in these works.

Power allocation and user assignment have been analysed in [10], [13], [14]. In [13], the authors have used heuristics to assign UL and DL users to frequency channels in an ultra-dense network. In a similar manner, in [10] the authors have addressed the scheduling (corresponding to our user pairing) and power allocation for small-cell networks, where the heuristic solutions of the scheduling and power allocation algorithm are evaluated for scenarios with Rayleigh fading. However, the authors in [10] do not take into account the frequency channel assignment along with user pairing, i.e., the users are paired with each other without specific assignment to frequency channels, thus representing a flat fading environment. The authors in [14] have analysed user-subcarrier assignment via matching theory. However, none of the works mentioned above have investigated a distributed power control and user-frequency channel assignment. We are interested in distributed solutions to offload the operational burden imposed by a large number of diverse devices on the BS. In addition, we propose a benchmarking solution that can be understood as a generalization to frequency selective fading environments of an heuristic in [10], named therein as Algorithm 2, and herein as modified Hungarian algorithm.

Other papers have developed distributed algorithms that are applicable in TNFD networks [7], [8], [15]. The authors of [15] have tackled the problem of the UE-to-UE interference from an information theoretic perspective, without relating to resource allocation and power control. In [7], the authors have proposed a distributed power control for general wireless networks using approximation techniques. The proposed framework has not taken into account the specific aspects of TNFD in cellular networks, and thus the resulting performance are suboptimal. We have studied a weighted sum spectral efficiency maximization problem in our previous work [8]. In that paper, we have designed an auction theory based distributed algorithm to the assignment problem without taking into account frequency-selective fading, and assuming centralized power allocation. Notice that frequency selective channel introduces major technical challenges compared to [8] due to its variation between channels for UL and DL UEs, specially for distributed solution mechanisms that require information exchange between users.

A typical and natural objective of many physical layer procedures designed for TNFD cellular networks is to maximize the sum spectral efficiency [16]–[20]. The authors in [16] have considered a joint subcarrier and power allocation problem, without taking into account the UE-to-UE interference. In [17], the authors have also considered a joint subcarrier and power allocation, but now with UE-to-UE interference and the assumptions that users can share several subcarriers. The authors have proposed a centralized solution for the joint problem, and considered a penalty factor along with a transformation of the binary variables into continuous variables. Similarly, the authors of [18] have taken into account the UE-to-UE interference, which has led to a formulation of the assignment problem as a 3D-matching problem. The authors have used a centralized water-filling solution to sub-optimally solve the power allocation problem. The work reported in [19] have considered the application of TNFD transmission mode in a cognitive femto-cell scenario with bidirectional transmissions from UEs, and have developed sum-rate optimal resource allocation and power control algorithms. The authors in [20] have considered scheduling and power allocation in multi-cell systems, and have proposed a distributed solution for the user selection and used geometric programming to solve the power allocation problem. However, these works have not considered a distributed solution approach for the power control problem, have not addressed the optimal user-frequency assignment problem, and have not provided energy saving gains.

Fast-Lipschitz (FL) optimization is a recently proposed framework to solve distributed optimization problems over networks by using fixed point iterations [9], [21], [22]. The main characteristic of a FL problem is that the optimal solution is found using fixed point iterations over the constraints, which are known for the fast convergence [23]. In addition, the structure of FL optimization does not rely on convexity or standard interference functions. Differently from dual decomposition methods [24] and alternating direction method of multipliers (ADMM) [25], FL optimization does

not require convexity of the problem, and – provided that some qualifying conditions are met–, it ensures an optimal solution for non-convex problems in a fast and distributed manner. Due to this fast and general solution approach, FL optimization has been used to solve general power control problems to minimize the sum power with different quality of service constraints [22]. However, FL optimization has not been used in TNFD networks, and the fundamental problem of sum spectral efficiency maximization has not been addressed before.

In the light of this survey of related literature, the main contributions of this paper are as follows:

- The problem formulation of JASEM and the proposed joint distributed power control using FL optimization and greedy assignment solution are new. The joint formulation to maximize the sum spectral efficiency is a natural objective in TNFD networks, and appears as a complex optimization for which no known methods are available.
- Due to the complexity of JASEM, we split the solution into two parts: power control problem and user-frequency assignment. For power control, we use the FL optimization framework to develop a distributed SINR setting close to the optimal and a power control solution that minimizes the sum power in the system. We propose a greedy solution to approximate the NP-hard problem of assigning UL and DL users to frequency channels.
- We evaluate the proposed joint solution by a realistic system simulator and gain insights that help design TNFD cellular networks operating in scenarios with both high and low SI capabilities.

III. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We consider a single-cell cellular system in which the BS is FD capable, whereas the UEs served by the BS are only HD capable, as illustrated by Figure 1. In Figure 1, the BS is subject to SI and the UEs transmitting in the UL (UE₂ and UE₄) cause UE-to-UE interference to co-scheduled UEs receiving in the DL, that is to UE₁ and UE₃ respectively. We assume that within a large number of served users a smaller subset is selected by an appropriate scheduler for data transmission in the UL and reception in the DL, which is supported by current implementation of Long Term Evolution (LTE) systems [26, Section III]. The number of UEs in the UL and DL are denoted by I and J , respectively, which are constrained by the total number of frequency channels in the system F , i.e., $I \leq F$ and $J \leq F$. Accordingly, the sets of UL and DL users are denoted by $\mathcal{I} = \{1, \dots, I\}$ and $\mathcal{J} = \{1, \dots, J\}$ respectively, and the set of frequency channels is denoted by $\mathcal{F} = \{1, \dots, F\}$.

We assume a frequency selective environment, such that the composite channel gains depend on the frequency and consist of small- and large-scale fading. Let G_{ibf} denote the channel gain between transmitter UE i and the BS on frequency channel f , G_{bjf} denote the channel gain between the BS and the receiving UE j on frequency channel f , and G_{ijf} denote the interfering channel gain between the UL transmitter UE i

and the DL receiver UE j on frequency channel f . The channel state information (CSI) is assumed known, which is also in accordance with some works in the full-duplex literature [6], [16], [18], [19]. To take into account the residual SI power that leaks to the receiver, we define β as the SI cancellation coefficient, such that the SI power at the receiver of the BS is βP_j^d when the transmit power is P_j^d .

The vector of transmit powers in the UL for all the UEs is denoted by $\mathbf{p}^u = [P_1^u \dots P_I^u]^T$, whereas the vector of DL transmit powers by the BS is denoted by $\mathbf{p}^d = [P_1^d \dots P_J^d]^T$. As illustrated in Figure 1, the UE-to-UE interference depends heavily on the geometry of the co-scheduled UL and DL users, i.e., the *pairing* of UL and DL users, which also depends on the frequency channels assigned to UL and DL users. Therefore, UE pairing and frequency channel allocation are key functions of the system. Accordingly, we define the assignment matrix, $\mathbf{X} \in \{0, 1\}^{I \times J \times F}$, such that

$$x_{ijf} = \begin{cases} 1, & \text{if UE}_i \text{ is paired with UE}_j \text{ on frequency } f, \\ 0, & \text{otherwise.} \end{cases}$$

The SINR at the BS of transmitting user i and the SINR at the receiving user j of the BS, both assigned to frequency channel f , are given by

$$\gamma_{if}^u = \frac{P_i^u G_{ibf}}{\sigma^2 + \sum_{j=1}^J x_{ijf} P_j^d \beta}, \quad \gamma_{jf}^d = \frac{P_j^d G_{bjf}}{\sigma^2 + \sum_{i=1}^I x_{ijf} P_i^u G_{ijf}}, \quad (1)$$

respectively, where x_{ijf} in the denominator of γ_{if}^u accounts for the SI at the BS, whereas x_{ijf} in the denominator of γ_{jf}^d accounts for the UE-to-UE interference caused by UE $_i$ to UE $_j$ on frequency f , and σ^2 is the noise power.

Thus, the achievable spectral efficiency by each user is given by the Shannon equation (in bits/s/Hz) for the UL and DL as

$$C_i^u = \sum_{f=1}^F C_{if}^u = \sum_{f=1}^F \log_2(1 + \gamma_{if}^u), \quad (2a)$$

$$C_j^d = \sum_{f=1}^F C_{jf}^d = \sum_{f=1}^F \log_2(1 + \gamma_{jf}^d), \quad (2b)$$

where the sums are taken along the frequency dimension because at most one SINR is non-zero for every UL/DL user.

B. Problem Formulation

Our goal is to devise the pairing and assignment of UEs in the UL and DL to frequency channels, that maximize the sum spectral efficiency over all users. Specifically, we formulate the joint assignment and spectral efficiency maximization (JASEM) problem as

$$\underset{\mathbf{X}, \mathbf{p}^u, \mathbf{p}^d}{\text{maximize}} \quad \sum_{i=1}^I C_i^u + \sum_{j=1}^J C_j^d \quad (3a)$$

$$\text{subject to} \quad P_i^u \leq P_{\max}^u, \quad \forall i, \quad (3b)$$

$$P_j^d \leq P_{\max}^d, \quad \forall j, \quad (3c)$$

$$\sum_{j=1}^J \sum_{f=1}^F x_{ijf} \leq 1, \quad \forall i, \quad (3d)$$

$$\sum_{i=1}^I \sum_{f=1}^F x_{ijf} \leq 1, \quad \forall j, \quad (3e)$$

$$\sum_{i=1}^I \sum_{j=1}^J x_{ijf} \leq 1, \quad \forall f, \quad (3f)$$

$$x_{ijf} \in \{0, 1\}, \quad \forall i, j, f. \quad (3g)$$

The main optimization variables are \mathbf{p}^u , \mathbf{p}^d and \mathbf{X} . Constraints (3b) and (3c) limit the transmit powers per-user and per-channel DL power constraint, whereas constraints (3d)-(3f) assure that at most one UE in the DL can share the frequency resource with a UE in the UL and vice-versa. Note that if $I \neq J$, some users cannot be paired and will transmit in HD mode if there are frequency channels available. Thus, a frequency channel can be used at most by a pair of users in the UL and DL. Multiple user pairs in the same frequency channel is possible with multi-user multiple input multiple output (MU-MIMO) or non-orthogonal multiple access (NOMA), but these approaches are not in the scope of this paper.

Problem (3) belongs to the category of mixed integer nonlinear programming (MINLP), which is known for its high complexity and computational intractability. In addition, problem (3) belongs to the category of 3-D nonlinear assignment problems, where the problem with linear objective function is known to be NP-hard [27, Section 10.2]. Therefore, to arrive at a close-to-optimal solution to problem (3), we will split it into two parts corresponding to power control and frequency channel assignment problems. For power control, we propose to use Fast-Lipschitz optimization [9], [21] to develop a distributed SINR target setting solution. With the SINR targets from Fast-Lipschitz optimization, we find the power vectors that minimize the sum power consumption in a distributed fashion. As for the frequency channel assignment, it is known to be an Axial 3-Dimensional Assignment Problem (3-DAP) [27, Section 10.2], and it is NP-Hard, which motivates us to propose a greedy solution with guaranteed performance. We use the proposed greedy solution for the frequency assignment along with the distributed power control from Fast-Lipschitz optimization. We remark that the problem formulation (3) is original, and the solution approach we develop in the following is original as well.

C. Notation

Vectors and matrices are denoted by bold lower and upper case letters, respectively. We denote by \mathbf{I} the identity matrix, and by $\mathbf{0}$ a vector or matrix where all elements are zero.

The gradient of a function $\mathbf{f}(\mathbf{x})$ is defined as the transpose of the Jacobian matrix, i.e., $[\nabla \mathbf{f}(\mathbf{x})]_{ij} = \partial f_j(\mathbf{x}) / \partial x_i$, whereas $\nabla_i \mathbf{f}(\mathbf{x})$ denotes the i th row of $\nabla \mathbf{f}(\mathbf{x})$. Note that $\nabla \mathbf{f}(\mathbf{x})^k = (\nabla \mathbf{f}(\mathbf{x}))^k$, which is not to be confused with the k th derivative. The spectral radius is denoted $\rho(\cdot)$. Vector norms are denoted $\|\cdot\|$ and matrix norms are denoted $\|\|\cdot\|\|$. Unless specified $\|\|\cdot\|\|$ and $\|\cdot\|$ denote arbitrary norms. We denote by $\|\|\mathbf{A}\|\|_{\infty} = \max_i \sum_j |A_{ij}|$ the norm induced by the ℓ_{∞} vector norm, where $\|\mathbf{x}\|_{\ell_{\infty}} = \max_i |x_i|$. These matrix norm definitions are coherent with [28]. All inequalities in this paper are intended *element-wise*, i.e., $\mathbf{A} \geq \mathbf{B}$ means $A_{ij} \geq B_{ij}$ for all i, j .

IV. POWER CONTROL ANALYSIS FOR JASEM

To develop a close-to-optimal solution of problem (3), we split the problem into two parts corresponding to power control and frequency channel assignment problems. We first concentrate on the power allocation problem, and assume the

assignment is already performed, whereas in Section VI we provide an approximate solution and a benchmark solution to the assignment problem. To this end, let us assume N pairs of UL and DL users, where $N = I = J$, and that a specific pair $n \triangleq (k, l)$ contains one UL and DL UEs sharing a frequency channel. In this case, the sum spectral efficiency maximization is formulated as:

$$\underset{\mathbf{p}^u, \mathbf{p}^d}{\text{maximize}} \quad \sum_{i=1}^I C_i^u + \sum_{j=1}^J C_j^d, \quad (4a)$$

$$\text{subject to} \quad \mathbf{p}^u, \mathbf{p}^d \in \mathcal{P}, \quad (4b)$$

where \mathcal{P} is the set in which the power allocation constraints (3b)-(3c) are fulfilled. From results of [29], we know that the optimal transmit power allocation will have either P_i^u or P_j^d equal to zero or, P_{\max}^u or P_{\max}^d , given that i and j share a frequency channel and form a pair. However, future cellular networks are expected to move from a fully centralized to a more distributed architecture [30], offloading the computation and orchestration burden imposed by a large number of diverse devices on the BS. With this objective in mind, we wish a distributed power control solution such that the UL and DL users can autonomously set their transmit powers and decrease the burden on the BS.

First, we rewrite problem (4) in a shortened form by defining $\mathbf{p} = [\mathbf{p}^u \mathbf{p}^d]^T$ and $K \triangleq I + J$. Since the frequencies have already been assigned to users, we omit the index f from the channel gains, i.e., G_{ib} denotes the channel gain between UL user i and the BS, whereas G_{bj} denotes the channel gain for DL user j and the BS. Accordingly, we define the vector $\mathbf{G}_b = [\sigma^2/G_{1b}^u, \dots, \sigma^2/G_{Ib}^u, \sigma^2/G_{b1}^d, \dots, \sigma^2/G_{bJ}^d]^T$ as the noise power divided by the interesting channel gains for UL and DL users, respectively; and the matrix containing the interfering channel gains \mathbf{F} of size $K \times K$ whose terms are defined as:

$$F_{lk} = \begin{cases} 0, & \text{if } x_{kl} = 0, \\ \beta/G_{kb}^u, & \text{if } k \leq I \text{ and } I < l \leq K, \\ G_{lk}/G_{bk}^d, & \text{if } I < k \leq K \text{ and } l \leq I. \end{cases} \quad (5)$$

Note that the index $k \leq I$ indicates a UL user, while $I < k \leq K$ indicates a DL user. We can now reformulate the SINR in Eq. (1) for the k th user sharing the frequency channel with the l th user in pair $n = (k, l)$ as

$$\gamma_k(\mathbf{p}) = \frac{P_k}{G_{bk} + (\mathbf{F}\mathbf{p})_k} = \frac{P_k}{G_{bk} + P_l F_{lk}}, \quad (6)$$

where P_k is the transmit power of user k , G_{bk} is the k th element of the column vector \mathbf{G}_b , and the expression $(\mathbf{F}\mathbf{p})_k$ stands for the k th element of the column vector $\mathbf{F}\mathbf{p}$, which results in a single term in the denominator of (6), representing the interference caused by user l to user k . Using this representation, we write problem (4) in a joint formulation of UL and DL users as:

$$\underset{\mathbf{p}}{\text{maximize}} \quad \sum_{k=1}^K \alpha_k \log(1 + \gamma_k(\mathbf{p})) \quad (7a)$$

$$\text{subject to} \quad P_k \leq P_{\max}^{(k)}, \forall k, \quad (7b)$$

where $P_{\max}^{(k)}$ is equal to P_{\max}^u or P_{\max}^d if k is a UL or DL user, respectively; and α_k converts natural logarithm into logarithm in base 2, i.e., $\alpha_k = 1/\log(2)$.

A. Problem Transformation

As a first step to solving problem (7), we consider the standard equivalent hypograph [24, Sec. 3.1.7] form of problem (7), in which one new variable and one new constraint are included:

$$\underset{\mathbf{p}, \mathbf{t}}{\text{maximize}} \quad \sum_{k=1}^K t_k \quad (8a)$$

$$\text{subject to} \quad t_k \leq \alpha_k \log(1 + \gamma_k(\mathbf{p})), \forall k, \quad (8b)$$

$$P_k \leq P_{\max}^{(k)}, \forall k, \quad (8c)$$

where the new variables $t_k > 0$ can be interpreted as spectral efficiency targets. Expanding the constraint (8b), we get

$$\alpha_k \log(1 + \gamma_k(\mathbf{p})) \geq t_k,$$

$$\frac{P_k}{G_{bk} + P_l F_{lk}} \geq (\exp(t_k/\alpha_k) - 1),$$

$$P_k \geq (\exp(t_k/\alpha_k) - 1)(G_{bk} + P_l F_{lk}),$$

$$\mathbf{p} \geq \mathbf{T}(\mathbf{G}_b + \mathbf{F}\mathbf{p}),$$

$$(\mathbf{I}_K - \mathbf{T}\mathbf{F})\mathbf{p} \geq \mathbf{T}\mathbf{G}_b, \quad (9)$$

where \mathbf{T} denotes a diagonal matrix of size $K \times K$ with entries $(\exp(t_k/\alpha_k) - 1)$. Notice that each diagonal entry $(\exp(t_k/\alpha_k) - 1)$ of matrix \mathbf{T} can be related to an adaptive SINR target γ_k^{tgt} , i.e., $\gamma_k^{\text{tgt}} = \exp(t_k/\alpha_k) - 1$. In the next section, we will transform problem (8) into a sequence of *feasibility checking sub-problems*, by means of which the sum of the spectral efficiency targets can be optimized while maintaining the feasibility of the power vector with respect to constraints (8c) and (9).

The feasibility of the power vector and its relation to the SINR targets are established by the following useful lemma.

Lemma 1. Consider problem (8). The diagonal matrix \mathbf{T} of adaptive SINR targets is feasible for problem (8) if and only if

$$\rho(\mathbf{T}\mathbf{F}) < 1. \quad (10)$$

Proof. From inequality (9), we notice that $\mathbf{T}\mathbf{F} > \mathbf{0}$ and has a special structure, with non-negative entries only in the position of the interfering user. Thus, from the Perron-Frobenius Theorem [31], there exists a non-negative right eigenvector \mathbf{p} of $\mathbf{T}\mathbf{F}$. Furthermore, we also know that $(\mathbf{I} - \mathbf{T}\mathbf{F})^{-1}$ is non-negative, which implies that the power vector that fulfils inequality (9) at equality is uniquely evaluated as

$$\mathbf{p} = (\mathbf{I} - \mathbf{T}\mathbf{F})^{-1} \mathbf{T}\mathbf{G}_b. \quad (11)$$

Therefore, we can use eq. (11) to find the optimal powers for the given SINR targets. ■

From the SINR Eq. (6), it is clear that there is exactly one interfering user per frequency channel, which allows to decouple the UL and DL UEs sharing the same frequency channel in problem (8). Specifically, let us assume that for pair $n = (k, l)$ we have the matrices $\mathbf{F}^{(n)}$ and $\mathbf{T}^{(n)}$ composed of the entries related to the k th and l th users, respectively. Thus, the matrices $\mathbf{F}^{(n)}$, $\mathbf{T}^{(n)}$ and $\mathbf{T}^{(n)}\mathbf{F}^{(n)}$ can be shown to have the form of

$$\mathbf{F}^{(n)} = \begin{bmatrix} 0 & F_{lk} \\ F_{kl} & 0 \end{bmatrix}, \mathbf{T}^{(n)} = \begin{bmatrix} \gamma_k^{\text{tgt}} & 0 \\ 0 & \gamma_l^{\text{tgt}} \end{bmatrix},$$

$$\mathbf{T}^{(n)}\mathbf{F}^{(n)} = \begin{bmatrix} 0 & \gamma_k^{\text{tgt}} F_{lk} \\ \gamma_l^{\text{tgt}} F_{kl} & 0 \end{bmatrix}.$$

Therefore, given the simple structure of the $\mathbf{T}^{(n)}\mathbf{F}^{(n)}$, we can evaluate its eigenvalues as

$$\lambda^{(n)} = \pm \sqrt{\gamma_k^{\text{tgt}} \gamma_l^{\text{tgt}} F_{kl} F_{lk}}. \quad (12)$$

In addition to Lemma 1, it will be useful to formulate an upper bound on the product of SINR targets

$$\gamma_k^{\text{tgt}} \gamma_l^{\text{tgt}} < \frac{1}{F_{kl} F_{lk}} = \frac{G_{kb}^u G_{bl}^d}{\beta G_{lk}}, \quad (13a)$$

$$\gamma_k^{\text{tgt}} \gamma_l^{\text{tgt}} < \frac{G_{kb}^u G_{bl}^d}{\beta G_{lk}}. \quad (13b)$$

Rewriting inequality (13b) as a function of the initial spectral efficiency targets t_k and t_l , we have the following inequality:

$$\begin{aligned} & \left(\exp\left(\frac{t_k}{\alpha_k}\right) - 1 \right) \left(\exp\left(\frac{t_l}{\alpha_l}\right) - 1 \right) < \frac{G_{kb}^u G_{bl}^d}{\beta G_{lk}}, \\ & \exp\left(\frac{t_k}{\alpha_k} + \frac{t_l}{\alpha_l}\right) - \exp\left(\frac{t_k}{\alpha_k}\right) - \exp\left(\frac{t_l}{\alpha_l}\right) < \frac{G_{kb}^u G_{bl}^d}{\beta G_{lk}} - 1. \end{aligned} \quad (14)$$

In order to transform the strict inequality in (14) into a nonstrict inequality, we introduce a predefined parameter $\epsilon < 1$ such that

$$\exp\left(\frac{t_k}{\alpha_k} + \frac{t_l}{\alpha_l}\right) - \exp\left(\frac{t_k}{\alpha_k}\right) - \exp\left(\frac{t_l}{\alpha_l}\right) \leq \epsilon \left(\frac{G_{kb}^u G_{bl}^d}{\beta G_{lk}} - 1 \right). \quad (15)$$

With inequality (15) we can assure that the spectral efficiency targets fulfil Lemma 1 if (15) holds. Therefore, we can use inequality (15) to rewrite problem (8) as a function of the spectral efficiency targets t_k and t_l as:

$$\begin{aligned} & \underset{t_k, t_l}{\text{maximize}} && t_k + t_l \\ & \text{subject to} && (15), \end{aligned} \quad (16a)$$

$$t_k, t_l \geq 0. \quad (16b)$$

Notice that inequality (15) is not convex. Thus, problem (16) is not concave and there is no guarantee that a local optimal value is also globally optimal. To solve this problem in a distributed fashion, we resort to FL optimization [9].

V. FAST-LIPSCHITZ SINR TARGET UPDATES AND DISTRIBUTED POWER CONTROL

Since problem (16) is not convex and we are interested in a distributed solution, we use FL optimization to solve the SINR setting problem. The main advantages of this framework is that the objective function or constraints are not required to be convex, and the optimal solution is found using fixed-point iterations, which are known for fast convergence. For clarity, the framework of FL optimization is summarized in Appendix A. Using this framework, we reformulate problem (16) using the relaxation of the FL form for problems with fewer constraints than variables, which is given in [21, Section V.B]. Consider a partitioned variable $\mathbf{x} = [t_k \ t_l]^T \in \mathcal{X} \subset \mathbb{R}^2$ and the following optimization problem:

$$\underset{\mathbf{x}}{\text{maximize}} \quad f_0(\mathbf{x}) \quad (17a)$$

$$\text{subject to} \quad t_k \leq f_{t_k}(\mathbf{x}), \quad (17b)$$

$$\mathbf{x} \in \mathcal{X}, \quad (17c)$$

where the functions $f_0(\mathbf{x})$, $f_{t_k}(\mathbf{x})$ and the set \mathcal{X} are defined as

$$f_0(\mathbf{x}) = t_k + t_l, \quad (18a)$$

$$f_{t_k}(\mathbf{x}) = t_k - \gamma h(\mathbf{x}), \quad (18b)$$

$$\mathcal{X} = \left\{ \mathbf{x} : \begin{cases} t_k \in \mathcal{X}_k = \{t_k : a_k \leq t_k \leq b_k\} \\ t_l \in \mathcal{X}_l = \{t_l : a_l \leq t_l \leq b_l\} \end{cases} \right\},$$

where for clarity and for the sake of easy of presentation in the sequel, we introduce the functions $h(\mathbf{x})$, $u(\mathbf{x})$, $v(\mathbf{x})$ as:

$$\begin{aligned} h(\mathbf{x}) &\triangleq \exp\left(\frac{t_k}{\alpha_k}\right) \left(\exp\left(\frac{t_l}{\alpha_l}\right) - 1 \right) - \exp\left(\frac{t_l}{\alpha_l}\right) + \\ &\quad \epsilon \left(1 - \frac{G_{kb}^u G_{bl}^d}{\beta G_{lk}} \right), \end{aligned} \quad (19a)$$

$$u(\mathbf{x}) \triangleq \frac{1}{\alpha_k} \exp\left(\frac{t_k}{\alpha_k}\right) \left(\exp\left(\frac{t_l}{\alpha_l}\right) - 1 \right), \quad (19b)$$

$$v(\mathbf{x}) \triangleq \frac{1}{\alpha_l} \exp\left(\frac{t_l}{\alpha_l}\right) \left(\exp\left(\frac{t_k}{\alpha_k}\right) - 1 \right). \quad (19c)$$

Note that γ in Eq. (18b) is the positive parameter used in [9, Section II.B] to transform constraint (15) into function $f_{t_k}(\mathbf{x})$, and that the upper bounds b_k, b_l can be understood as the maximum achievable spectral efficiency without interference. Notice that problems (16) and (17) are equivalent, which means that the optimal solution to problem (17) is also optimal to problem (16). With this reformulation, we now state the following lemma to establish that problem (17) is Fast-Lipschitz.

Lemma 2. Consider optimization problem (17). If $v(\mathbf{x}) < u(\mathbf{x})$, and the parameter γ is constrained as

$$\frac{1}{2u(\mathbf{x})} < \gamma < \frac{1}{u(\mathbf{x})}, \quad (20)$$

then problem (17) is Fast-Lipschitz.

Proof. See Appendix B. ■

Remark 1. Problem (17) being Fast-Lipschitz is important because it ensures that there is a unique Pareto optimal solution (see Definition 3 in Appendix A). Thus, the Fast-Lipschitz optimization framework allows us to solve a non-convex problem at optimality, provided that the problem is Fast-Lipschitz.

If the conditions of Lemma 2 are fulfilled, then we can find t_k using fixed point iterations. Using the proof in Appendix B, we fix t_l and find the optimal t_k from fixed point iterations $t_k^{(n+1)} = f_{t_k}(\mathbf{x})^{(n)}$. Thus, we need to keep updating t_l in order to obtain the solution that maximizes problem (17). Section V.A shows a decentralized algorithm that uses the FL solution to the power control problem and can be executed by the UL and DL UEs in a distributed fashion. Therefore, using Algorithms 1 and 2 we find the Pareto optimal points for t_k and t_l , thus solve at optimality the non-convex problem (16).

A. Distributed SINR Target Updates using Fast-Lipschitz

Algorithm 1 comprises the steps that are necessary to find the optimal t_k, t_l of the FL optimization. The input variables of Algorithm 1 are the channel gains G_{kb}^u , G_{bl}^d , G_{lk} , and the SI cancellation term β . To acquire the channel gains, the BS transmits reference signals in the DL and receives them in UL, on which these signals are standardized by 3rd Generation Partnership Project (3GPP) [32], [33]. For the DL channel, the BS can use the reported received signal strength indicator

Algorithm 1 FL Optimization to find t_k

```
1: Input:  $G_{kb}^u, G_{bl}^d, G_{lk}, \beta, \alpha_k, \alpha_l, t_k^{(0)}, a_k, b_k, t_l, \delta_p, \delta_a$ 
2: Evaluate  $u(\mathbf{x}), v(\mathbf{x}), h(\mathbf{x})$  from Eqs. (19)
3: Set  $\gamma^{(0)} = \frac{3}{4u(\mathbf{x})}$  and evaluate  $f_{t_k}(\mathbf{x})^{(0)}$ 
4: Set  $n = 0$ 
5: while  $|t_k^{(n)} - t_k^{(n-1)}| < \delta_p$  do
6:    $n \leftarrow n + 1$ 
7:   Update  $t_k^{(n)} = \max \left\{ \min \left\{ f_{t_k}(\mathbf{x})^{(n-1)}, b_k \right\}, a_k \right\}$ 
8:   Update  $u(\mathbf{x}), v(\mathbf{x}), h(\mathbf{x})$  from Eqs. (19)
9:   Set  $\gamma^{(n)} = \frac{3}{4u(\mathbf{x})}$  and evaluate  $f_{t_k}(\mathbf{x})^{(n)}$ 
10:  if  $|t_k^{(n)} - t_k^{(n-1)}| = |t_k^{(n-1)} - t_k^{(n-2)}|$  and  $|h(\mathbf{x})| > \delta_a$  then
11:    Stop the algorithm and set  $\zeta = 1$ 
12:  end if
13: end while
14: Output:  $t_k, \zeta$ 
```

(RSSI) or the reference signal receive power (RSRP) to estimate or measure the quality of the channel [32, Section 5.1.8]. For the UL channel, the users can transmit specific reference signals, such as the sounding reference signal (SRS), or the demodulation reference signal (DMRS) to enable the BS to acquire CSI. As for the interference channel between UL and DL users, the recently standardized measurement signals for device-to-device (D2D) communications can be used, such as the sidelink transmission and reception reference signals [34]. The algorithm also needs the initial values for t_k , denoted as $t_k^{(0)}$, the upper and lower limits of t_k , denoted by a_k and b_k , the initial point for t_l , and some precision targets for convergence denoted by δ_p, δ_a . The precision targets can be predefined and broadcast by the BS, whereas the lower bounds are zero and the upper bounds can be sent via measurements to each user based on the channel gains and signal to noise ratio (SNR). The initial value for $t_k^{(0)}$ is decided based on the t_l given as input.

Initially, $u(\mathbf{x}), v(\mathbf{x}), h(\mathbf{x})$ are evaluated. Next, γ is set as the midpoint of the interval defined in Eq. (20) and the function $f_{t_k}(\mathbf{x})$ according to Eq. (18b) is evaluated (lines 2 and 3). Then, the algorithm updates t_k using the fixed point iterations and ensuring that $a_k \leq t_k^{(n)} \leq b_k$ (line 7). In the next step, $u(\mathbf{x}), v(\mathbf{x}), h(\mathbf{x}), \gamma$ and $f_{t_k}(\mathbf{x})$ are updated. The algorithm iterates until $|t_k^{(n)} - t_k^{(n-1)}| < \delta_p$ (line 5), which implies that the result δ_p is close to the fixed point.

As another stopping criterion, if $t_k^{(n)}$ has reached the upper/lower bound twice, the algorithm stops (line 10). This happens when the first stopping criterion cannot be fulfilled while keeping $t_k^{(n)}$ within the predefined bounds. We define this situation as an infeasibility. Note that the algorithm also enforces $|h(\mathbf{x})| > \delta_a$, so that the bound can be reached if $|h(\mathbf{x})|$ is small enough (from Eq. (15) it should be zero). Accordingly, $\zeta = 1$ is set to highlight that the initial value given to t_l does not lead to a feasible t_k . As outputs, the algorithm has the variable t_k and the infeasibility check ζ . From now on, we will refer to the outputs as $t_k|_{t_l}$ and $\zeta|_{t_l}$ to highlight that the outputs were evaluated with the input t_l .

Next, a unidimensional search algorithm is used to update t_l . Notice that the objective function has only one maximum, because problem (17) is Fast-Lipschitz and has a unique Pareto

Algorithm 2 Golden Search to find t_l

```
1: Input:  $G_{kb}^u, G_{bl}^d, G_{lk}, \beta, \alpha_k, \alpha_l, a_k, b_k, a_l, b_l, \delta_g$ 
2: Define  $\phi \triangleq (-1 + \sqrt{5})/2$ 
3: Evaluate  $t_{l_1} = a_l\phi + (1 - \phi)b_l$  and  $t_{l_2} = (1 - \phi)a_l + \phi b_l$ 
4: Evaluate  $t_k|_{t_{l_1}}$  and  $t_k|_{t_{l_2}}$  from Algorithm 1
5: while  $t_{l_2} - t_{l_1} < \delta_g$  do
6:   if  $(1 - \zeta|_{t_{l_1}})(t_k|_{t_{l_1}} + t_{l_1}) \geq (1 - \zeta|_{t_{l_2}})(t_k|_{t_{l_2}} + t_{l_2})$  then
7:     Update intervals:  $b_l \leftarrow t_{l_2}, t_{l_2} \leftarrow t_{l_1}$ 
8:     Update  $t_k$  and  $t_l$ :  $t_k \leftarrow t_k|_{t_{l_1}}$  and  $t_l \leftarrow t_{l_1}$ 
9:     Update the evaluated value for  $t_k|_{t_{l_2}}$ :  $t_k|_{t_{l_2}} \leftarrow t_k|_{t_{l_1}}$ 
10:    Update the new point to search:  $t_{l_1} = a_l\phi + (1 - \phi)b_l$ 
11:    Evaluate  $t_k|_{t_{l_1}}$  from the new  $t_{l_1}$  using Algorithm 1
12:   else
13:     Update intervals:  $a_l \leftarrow t_{l_1}, t_{l_1} \leftarrow t_{l_2}$ 
14:     Update  $t_k$  and  $t_l$ :  $t_k \leftarrow t_k|_{t_{l_2}}$  and  $t_l \leftarrow t_{l_2}$ 
15:     Update the evaluated value for  $t_k|_{t_{l_1}}$ :  $t_k|_{t_{l_1}} \leftarrow t_k|_{t_{l_2}}$ 
16:     Update the new point to search:  $t_{l_2} = a_l\phi + (1 - \phi)b_l$ 
17:     Evaluate  $t_k|_{t_{l_2}}$  from the new  $t_{l_2}$  using Algorithm 1
18:   end if
19: end while
20: Output:  $t_l, t_k$ 
```

optimal solution [9]. To find this unique maximum, we propose to use the Golden Section search [35], because it does not rely on finding derivatives and requires only one new computation at every iteration as opposed to the widely used bisection method (see Algorithm 2).

In this algorithm the golden ratio ϕ is used as the constant reduction factor of the interval. Subsequently, the two points t_{l_1}, t_{l_2} are defined followed by evaluating t_k using the FL optimization in Algorithm 1 (lines 3-4). Then, the algorithm checks which part of the interval gives the higher sum $t_k + t_l$, if it is the lower part of the interval containing t_{l_1} (see line 6), namely $[a_l, t_{l_2}]$, or the upper part containing t_{l_2} (see line 12), namely $[t_{l_1}, b_l]$. Notice that the feasibility of the solution is also checked, on which the algorithm considers ζ in the sum.

Depending on the interval (lower or upper), the algorithm updates the upper/lower bound and the respective values of t_k (see lines 7-9 and 13-15). With this, we update the current value of t_k and t_l (see lines 8-14). In the following, t_{l_1} (t_{l_2} in the respective upper interval) is updated and the algorithm finds t_k for the new given value of t_l (see lines 10-11 and 16-17). The algorithm stops once the desired precision δ_g is achieved.

The communication complexity is an important measure of a distributed algorithm [36], and for the proposed Fast-Lipschitz distributed SINR updates it can be accounted as follows. User k in the inner loop requires the input information (see line 1 of Algorithm 1), which accounts for 12 messages. Similarly, user l requires as input information 11 messages (see line 1 of Algorithm 2), but only 3 specific messages to user l from the BS, inputs on a_l, b_l and δ_g . Subsequently, user k reports 2 messages to user l in the outer loop after convergence, whereas user l reports the updates of a_k, b_k, t_l and t_k to user k , which accounts for 4 more messages. This process continues until convergence of the outer loop (see line 5 of Algorithm 2), or until the maximum number of iterations allowed is reached, defined herein as N_{\max} . Thus, for each pair at most $6N_{\max} + 15$ messages between user k and l are required, which in total sum up to $N(6N_{\max} + 15)$, where

N is the total number of pairs. Therefore, the communication complexity is $O(N)$.

An important aspect of the Golden Search solution in Algorithm 2 is splitting the interval such that the solution used for t_k remains feasible. To this end, the algorithm must verify that the solution is feasible by considering ζ in the sum. However, we need to incorporate into the algorithm that when the infeasibility happens, it removes the branch of t_l that leads to infeasibility. To this end, the following Lemma 3 will be useful (see line 6).

Lemma 3. Let t_l, t_k be given by Algorithm 2. Suppose that such t_l and t_k make Algorithm 1 infeasible. Then, the Golden Search solution cuts the branch that resulted in t_l and leads to the feasibility direction, which is the other branch of the search.

Proof. See Appendix C. ■

Therefore, we can now solve at optimality the SINR setting problem (16). We find in a distributed manner t_k and t_l from problem (16), which actually correspond to the *feasible* minimum required spectral efficiency for UL and DL users. The objective in problem (16) is to maximize the sum of both spectral efficiency targets, with constraint (15) to explicitly require that the spectral efficiencies are feasible. Because of these distributed spectral efficiency (or SINR) targets, an explicit fixed minimum requirement is not necessary. In the following subsection, we use the SINR targets obtained as the solution of the FL optimization to evaluate the power vector in a distributed manner.

B. Distributed Power Control

Using the optimal SINR targets obtained as the solution to problem (16), we can now develop a distributed power control algorithm based on the definition of standard interference functions [37], which for clarity is recalled below.

Definition 1. A function $\mathcal{I}(\mathbf{p}) : \mathbb{R}^n \rightarrow \mathbb{R}$ is a standard interference function if for all vector $\mathbf{p} \geq 0$ the following properties are satisfied:

- Positivity: $\mathcal{I}(\mathbf{p}) > 0$ for all \mathbf{p} ;
- Monotonicity: If $\mathbf{p} \geq \mathbf{p}'$, then $\mathcal{I}(\mathbf{p}) > \mathcal{I}(\mathbf{p}')$;
- Scalability: For all scalar $\alpha > 1$, $\alpha\mathcal{I}(\mathbf{p}) > \mathcal{I}(\alpha\mathbf{p})$.

Notice that Eq. (9) can be written as an inequality based on an appropriate interference function $\mathcal{I}^W(\mathbf{p})$, as follows:

$$P_k \geq \mathcal{I}_k^W(\mathbf{p}) = \gamma_k^{\text{gt}}(G_{bk} + F_{lk}P_l), \quad (21)$$

$$P_l \geq \mathcal{I}_l^W(\mathbf{p}) = \gamma_l^{\text{gt}}(G_{bl} + F_{kl}P_k). \quad (22)$$

Using this form, it is straightforward to determine the power control update iterations that minimize the sum power and achieve the predefined SINR targets. The following lemma shows that with given feasible SINR targets, the transmit powers for the pair of UL-DL users sharing a frequency channel can be determined in a distributed manner while minimizing the sum power.

Lemma 4. Consider a feasible set of SINR targets for users k and l sharing a frequency channel. The power vector that

minimizes the sum transmit power of the two users while attaining the SINR targets is found in a distributed manner as:

$$P_k(n+1) = \min \left\{ P_{\max}^{(k)}, \mathcal{I}_k^W(\mathbf{p}(n)) \right\}, \quad (23a)$$

$$P_l(n+1) = \min \left\{ P_{\max}^{(l)}, \mathcal{I}_l^W(\mathbf{p}(n)) \right\}, \quad (23b)$$

where n denotes the iteration counter.

Proof. From [37], it can be proved that the power vector found from Eqs. (23) denotes a sequence of monotone decreasing feasible power vectors that converge to a unique fixed point \mathbf{p}^* [37, Lemma 1], which minimizes the sum power that attain the SINR targets. If the SINR targets lead to a power higher than the maximum in (23), the proposed algorithm projects this power onto the set \mathcal{P} defined by the $P_k \in [0, P_{\max}]$. ■

From the results of [29] we know that either user k or l or both transmit at maximum power. Therefore, in practice, $P_k(0) = P_{\max}^{(k)}$ and $P_l(0) = P_{\max}^{(l)}$ is typically a good starting point.

VI. ASSIGNMENT SOLUTIONS FOR JASEM

With the distributed solution to the powers, we are now interested in a solution for the frequency channel assignment. In this section we assume a set of fixed powers – such as the maximum power allocation (P_{\max}^u and P_{\max}^d) – for both UL and DL transmissions. Recall that the optimal powers will have either P_i^u or P_j^d equal to zero, or P_{\max}^u or P_{\max}^d , or both with P_{\max}^u and P_{\max}^d , which motivates us to consider both users transmitting at maximum power. Since we cannot provide the optimal powers without knowing the assignment, we reasonably assume fixed powers and later optimize the powers using what we developed in sections IV-V. With this assumption, we have the following combinatorial problem in binary variables \mathbf{X} :

$$\underset{\mathbf{X}}{\text{maximize}} \quad \sum_{i=1}^I \sum_{j=1}^J \sum_{f=1}^F s_{ijf} x_{ijf} \quad (24a)$$

$$\text{subject to} \quad \sum_{j=1}^J \sum_{f=1}^F x_{ijf} = 1, \quad \forall i, \quad (24b)$$

$$\sum_{i=1}^I \sum_{f=1}^F x_{ijf} = 1, \quad \forall j, \quad (24c)$$

$$\sum_{i=1}^I \sum_{j=1}^J x_{ijf} = 1, \quad \forall f, \quad (24d)$$

$$x_{ijf} \in \{0, 1\}, \quad \forall i, j, f, \quad (24e)$$

where the matrix $\mathbf{S} = [s_{ijf}] \in \mathbb{R}^{I \times J \times F}$ represents the sum of UL and DL spectral efficiencies on frequency resource f , i.e., $s_{ijf} = C_{if}^u + C_{jf}^d$. Furthermore, equalities (24b)-(24d) follow from the inequality terms (3d)-(3f), where it is now required to assign a frequency channel to every UL and DL user.

The assignment problem (24) has $(n!)^2$ possible assignments and is a well known NP-hard problem named Axial 3-DAP [27, Section 10.2]. Thus, no optimal solution in polynomial time can be obtained, unless P = NP. Nevertheless, in the following subsections we propose two solutions: a greedy solution with guaranteed performance, based on our previous work [38] (Section VI.A), and another solution based on a modification of the Hungarian algorithm, especially adopted

Algorithm 3 Greedy Solution for the Axial 3-DAP

1: Input: $G_{ibf}, G_{bjf}, G_{ijf}, \beta, P_{\max}^u, P_{\max}^d$
2: Evaluate $s_{ijf} = C_{if}^u + C_{jf}^d$ for every i, j, f
3: $\mathcal{I}_a = \mathcal{J}_a = \mathcal{F}_a = \emptyset \leftarrow$ Sets of assigned users and resources are initially empty
4: **while** $|\mathcal{I}_a| \neq I$ **and** $|\mathcal{J}_a| \neq J$ **do**
5: $s_{\max} = \max s_{ijf}, \forall i, j, f \leftarrow$ Sort in descending order s_{ijf}
6: $(i^*, j^*, f^*) = \arg \max s_{\max} \leftarrow$ Select the pair and frequency with maximum sum spectral efficiency
7: $x_{i^*j^*f^*} = 1 \leftarrow$ Assign the frequency channel f^* to pair (i^*, j^*)
8: $\mathcal{I}_a = \mathcal{I}_a \cup \{i^*\}, \mathcal{J}_a = \mathcal{J}_a \cup \{j^*\}, \mathcal{F}_a = \mathcal{F}_a \cup \{f^*\} \leftarrow$ Update the set of assigned resources
9: Remove the already assigned users and frequency (i^*, j^*, f^*) from s_{ijf}
10: **end while**
11: **Output:** \mathbf{X}

to the Axial 3-DAP problem (Section VI.B). We provide two solutions to compare our approximate solution to Axial 3-DAP, the greedy solution, with an heuristic that splits the 3-DAP into two 2-D assignment problems, which can be understood as a generalization to frequency selective fading environments of an heuristic in [10].

A. Greedy Solution for the Axial 3-DAP

Differently from other works that reduce the assignment problem (24) to two dimensions [19], we aim at jointly assigning UE pairs to frequencies. A related work developed some other heuristics applicable to problem (24) but it relies on meta-heuristics using graph theory, and some simplifications of the cost s_{ijf} [27, Section 10.2.5]. This approach cannot be used in our case, because it either requires unacceptable simplifications or is computationally too complex. In our previous work [38], we proposed a greedy solution to jointly solve an Axial 3-DAP and a power allocation problem aiming at maximizing the minimum spectral efficiency of a full-duplex cellular system. To solve the assignment problem, we use the approach from our previous work [38]. The Axial 3-DAP can be understood as the maximization of a function over a constraint set that is the intersection of 3 matroids [27], which allows us to have performance guarantees in terms of sum spectral efficiency. More precisely, let us consider \mathbf{X}^* and \mathbf{X}_G as the optimal and the greedy solution found by our greedy algorithm, and let the objective function, the sum spectral efficiency, be $c(\mathbf{X}^*)$ and $c(\mathbf{X}_G)$, respectively. From Hausmann et al. [39, Corollary 4], any greedy solution aiming to solve the maximization or minimization of a real-valued weight function over a constraint set defined as the intersection of k matroids yields a performance guarantee of at least $1/k$, i.e., $c(\mathbf{X}_G)/c(\mathbf{X}^*) \geq 1/k$. Therefore, our proposed greedy solution has a performance guarantee of at least $1/3$. Actually, as the numerical results will show, the optimality gap is lower than the guaranteed.

Algorithm 3 comprises the steps of the greedy solution to solve the Axial 3-DAP in (24). Initially, the s_{ijf} for all pairs and frequency channels are evaluated. Next, we define the sets that will contain the assigned users in UL, DL, and the frequency channels (see lines 2-3). Subsequently, the algorithm sorts the matrix \mathbf{S} in descending order and selects the UE pair and frequency channel that achieves the highest s_{ijf} (see

Algorithm 4 Solution of JASEM problem (3)

1: Solve the assignment problem using the greedy solution in Algorithm 3
2: Find in a distributed manner t_k and t_l using Algorithm 1-2
3: Find in a distributed manner \mathbf{p}_u and \mathbf{p}_d from t_k and t_l
4: **Output:** $\mathbf{X}, \mathbf{p}_u, \mathbf{p}_d$

lines 5-7). With this, the assignment matrix \mathbf{X} and the sets of UL and DL users, as well as the set of frequency channels already used are updated (see lines 7-8). Finally, the assigned users and frequency channels from the matrix \mathbf{S} are removed. The loop is continued until all users have been assigned to a frequency channel (see line 9). Notice that the computationally demanding effort is sorting the matrix \mathbf{S} , which has worst-case complexity $O(n \log(n))$, where n is the dimension of the vector to be sorted. In our case, the matrix \mathbf{S} is of dimension $I \times J \times F$, and since $I = F = J$, the worst-case complexity is $O(F^3 \log F)$.

B. Solution Approach Based on A Modified Hungarian Algorithm

Since problem (24) is NP-hard, we propose to develop an approximate solution based on a transformation of the Axial 3-DAP into two 2-D assignment problems: one between the UL users and the frequency channels, and another between the DL users and the frequency channels. The idea here is to first assign the UL users to frequency channels such that the sum spectral efficiency for all UL users is maximized. We then use the information of the already assigned UL users to pair the DL users to frequency channels such that the sum spectral efficiency of all DL users is maximized. Both assignments can be solved optimally through the Hungarian algorithm [27, Section 3.2]. This solution approach can be understood as a generalization of Algorithm 2 in [10], on which the authors first try to assign a channel to the UL user with highest channel gain, and then use this information to assign the channel that achieves the highest SINR in the DL. Since the authors in [10] do not assume a frequency selective fading environment, a generalization to frequency selective fading environments is necessary, and a possibility is using the proposed modified Hungarian algorithm. We propose this algorithm in order to benchmark against our proposed greedy solution, and also to show that splitting 3-DAP into 2-D assignment problems does not, in fact, lead to a good performance in terms of sum spectral efficiency.

C. Summary

The solution of the initial JASEM problem (3) can be summarized as follows in Algorithm 4. The algorithm initially solves the assignment problem of UL and DL users to frequency channels using the greedy solution in Algorithm 3. With the assignment, it uses a distributed solution provided in Algorithm 2 to find the close-to-optimal SINRs of UL and DL users. Finally, the algorithm uses the distributed power control iterations of (23) to set the power vectors of UL and DL users.

The assignment solution in Algorithm 3 provides an approximation to the assignment problem (24), while Algorithm 2 provides a close-to-optimal solution to the power

Table I
SIMULATION PARAMETERS

Parameter	Value
Cell radius	100 m
Number of UL UEs [$I = J$]	[4 5 25]
Monte Carlo iterations	400
Carrier frequency	2.5 GHz
System bandwidth	5 MHz
Number of freq. channels [F]	[4 5 25]
LOS path-loss model	$34.96 + 22.7 \log_{10}(d)$
NLOS path-loss model	$33.36 + 38.35 \log_{10}(d)$
LOS probability	$\min(18/d, 1)(1 - \exp(-d/36)) + \exp(-d/36)$
Shadowing st. dev. LOS	3 dB
Shadowing st. dev. NLOS	4 dB
Thermal noise power [σ^2]	-116.4 dBm/channel
Average user speed	3 km/h
User antenna height	1.5 m
BS antenna height	10 m
SI cancelling level [β]	[-70 - 110] dB
UE max power [P_{\max}^u]	24 dBm
BS max power [P_{\max}^d]	24 dBm/channel

control problem (16). Thus, by using both algorithms we are providing a suboptimal solution to the JASEM problem (3). However, as the numerical results in Section VII show, the proposed solution method provides a close-to-optimal solution to JASEM.

VII. NUMERICAL RESULTS AND DISCUSSION

We consider a single cell system operating in an urban micro environment assuming 2.5 GHz carrier frequency and a system bandwidth of 5 MHz [40]–[44]. For this bandwidth in an LTE system [41], the maximum number of frequency channels is $F = 25$, which corresponds to the number of available frequency channel blocks. The total number of served UEs varies between $I + J = 8 \dots 50$, where we assume that after UE pairing the number of UE transmitting in UL (I) is equal to the number of UE receiving in DL (J). The total transmit power budget is 38 dBm [44, Table 6.0A], and the transmitting power of the BS is 24 dBm per frequency channel [43, Table 4.5], due to the bandwidth of 5 MHz and the 25 frequency channels considered. The parameters of this system are set according to Table I.

To evaluate the performance of the proposed assignment and power control solution in this environment, we use the Rudimentary Network Emulator (RUNE) as a basic platform for system simulations [45] and extend it to FD cellular networks. With the RUNE FD simulation tool, the environment specified in Table I can be readily generated. Below we report the results obtained by Monte Carlo simulations using either an exhaustive search algorithm to solve problem (3) or the proposed solution for assignment and power control.

Section VII.A analyses the optimality gap between the initially proposed sum rate maximization problem in (3), named P-OPT, our greedy solution for the Axial 3-DAP with FL optimization for the power control, named G-FLIP, and the Hungarian assignment solution with FL optimization for the power control, termed H-FLIP.

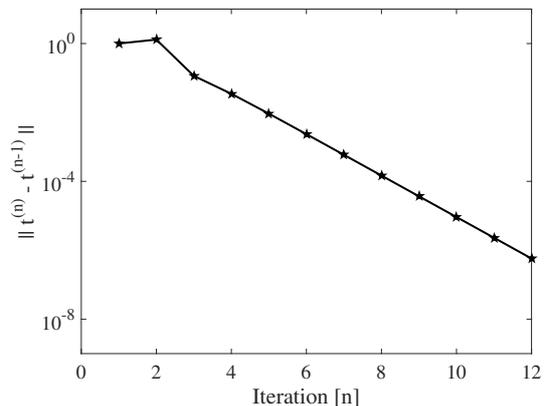


Figure 2. Convergence of the FL power control algorithm 1. Notice the solution converges in approximately 12 iterations with an accuracy of 10^{-6} .

In the following we compare the performance of these algorithms in two distinct scenarios: interference limited scenario and SI limited scenario. In Section VII.B, we assume the scenario is interference limited, i.e., the SI cancellation level β is good (low β) such that the UE-to-UE interference is the limiting factor. Conversely, in Section VII.C we consider an SI limited scenario, i.e., the SI cancellation is poor (high β) such that the performance limiting factor is the SI.

Sections VII.B–VII.C compare the performance of G-FLIP with four other algorithms:

- 1) Assignment according to the greedy solution but with Equal Power Allocation (EPA), named G-EPA;
- 2) Random assignment with EPA, named R-EPA;
- 3) Random assignment but with the proposed FL power control, named R-FLIP;
- 4) Half-Duplex, named HD.

Notice that in the HD solution we need to assign UL and DL users to frequency channels, for which we use the Hungarian assignment, which in this situation is optimal.

A. Analysis of Optimality Gap

To evaluate the optimality gap between the proposed G-FLIP, H-FLIP and P-OPT, we evaluate the objective function of problem (3). Recall from Section III.B that P-OPT is NP-hard, thus we solve it by exhaustive search for scenarios with a small number of users and frequency channels. Specifically, we consider a small system with reduced number of UL and DL users and frequency channels. Moreover, we consider a SI cancelling level of -110 dB, i.e., with $\beta = -110$ dB.

In Figure 2 we show the convergence of the fixed-point iterations $t_k^{(n+1)} = f_{t_k}(\mathbf{x})^{(n)}$ for a pair of UL-DL users. As expected, the solution converges fast, in approximately 12 iterations, and with an accuracy of 10^{-6} .

With this, we can now evaluate the performance of the proposed G-FLIP and H-FLIP with respect to P-OPT. Figure 3 shows the CDF of the sum spectral efficiency for all users, which is the objective function of problem (3). As expected, P-OPT yields the highest sum spectral efficiency, followed by G-FLIP and H-FLIP. Recall that P-OPT solves the NP-hard problem (3), and an optimality gap is expected. In addition, H-FLIP results the lowest sum spectral efficiency, which shows

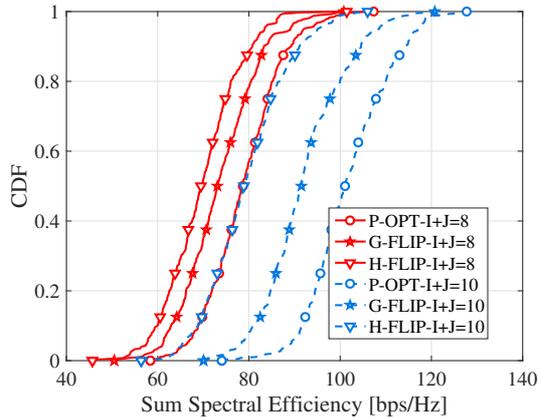


Figure 3. CDF of the sum spectral efficiency with reduced number of users. The proposed G-FLIP achieves a performance close to the optimal P-OPT and a better than H-FLIP. Notice that H-FLIP has the lowest sum spectral efficiency regardless of the number of users.

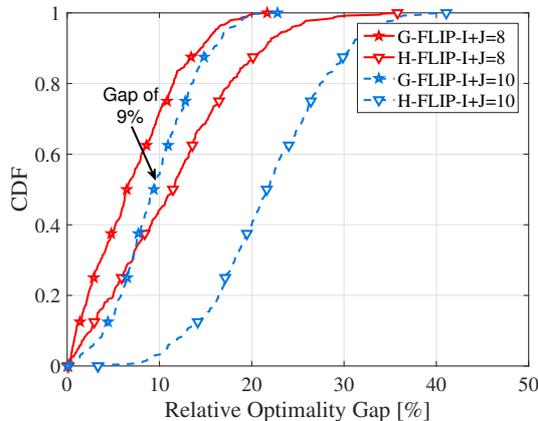


Figure 4. CDF of the relative optimality gap between P-OPT and the proposed G-FLIP and H-FLIP. The relative gap slowly increases with the number of users for G-FLIP. Conversely, for H-FLIP the relative gap almost doubles when increasing the number of users.

that greedily assigning all UL and DL users to frequency channels is better than doing the assignment in separated phases for UL and DL users. Accordingly, it shows that the proposed solution greedy assignment solution outperforms a generalization of a scheduling algorithm in [10].

To understand the relative optimality gap between the algorithms, Figure 4 shows the relative optimality gap between P-OPT and G-FLIP and H-FLIP. As the number of users increases, the optimality gap between P-OPT and G-FLIP increases from approximately 6.5% to 9% at the 50-th percentile, i.e., the gap remains small as the number of users increases in the system. However, for H-FLIP, the relative gap increases more when the number of users increases, it grows from approximately 11.5% to 21.5%. This result suggests that G-FLIP is more suited to systems with a large number of users than H-FLIP.

B. Analysis for Interference-limited Regime

In this subsection we study the performance of G-FLIP and compare it with G-EPA, R-EPA, R-FLIP and HD in terms of sum spectral efficiency and total power consumption. Here we assume a very high SI cancellation level of -110 dB, which

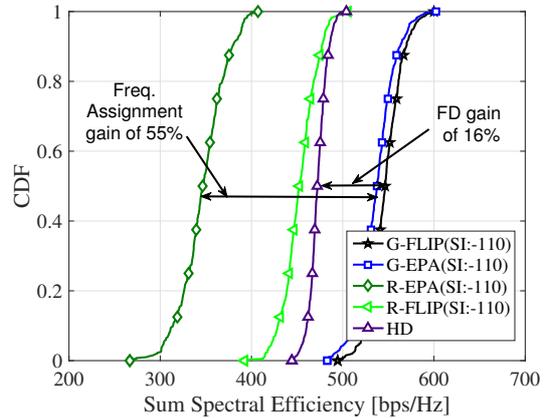


Figure 5. CDF of the sum spectral efficiency with $\beta = -110$ dB, i.e., the system is limited by the UE-to-UE interference. We notice that G-FLIP has a relative gain of approximately 16% with respect to HD systems. In addition, most of the gains can be achieved by a smart frequency assignment rather than a smart power control.

represents an interference-limited situation. Our objective is to evaluate the impact of frequency channel allocation and power control on the sum spectral efficiency and system-wide power consumption and, ultimately, to demonstrate the gains of the proposed algorithms.

Figure 5 shows the CDF of the sum spectral efficiency. The proposed G-FLIP algorithm has a gain of approximately 16% at the 50-th percentile with respect to HD systems, and recall that this HD solution has the optimal assignment that could be hardly used in a real system. Notice that R-FLIP performs close to HD, with a relative difference of approximately 4% at the 50-th percentile. R-EPA achieves the lowest sum spectral efficiency among all algorithms, and with the usage of a smart power control such as the FL, the performance improves in approximately 30.6%. This is shown as a power control (PC) gain in Figure 5. By comparing G-FLIP and G-EPA, notice that the PC relative gain is small, approximately 1.6%. The large gains come from the usage of the greedy frequency assignment, as can be observed by comparing G-EPA with R-EPA, the relative gain is approximately 55%. This happens because the impact of the assignment is large in the user-to-interference channel, since the path-loss between interfering users may vary more than the path-loss for each user and their frequency channels. The power control also acts directly on the received interference at the DL, but a higher variation in the interference term may happen due to poor assignment than poor power allocation. Therefore, for interference-limited scenarios, most of the gains can be achieved by a smart frequency assignment, and the power control has little impact on the sum spectral efficiency.

Figure 6 shows the CDF of the total transmit power. Notice that G-EPA, R-EPA and HD are represented by a straight line, which is a result of using a fixed power allocation at maximum value. By comparing G-FLIP with any algorithm using EPA, we can observe a relative energy saving of approximately 48%. In addition, R-FLIP has a lower power consumption than G-FLIP. This is because the greedy assignment is able to transmit with a higher power on better (i.e. higher gain)

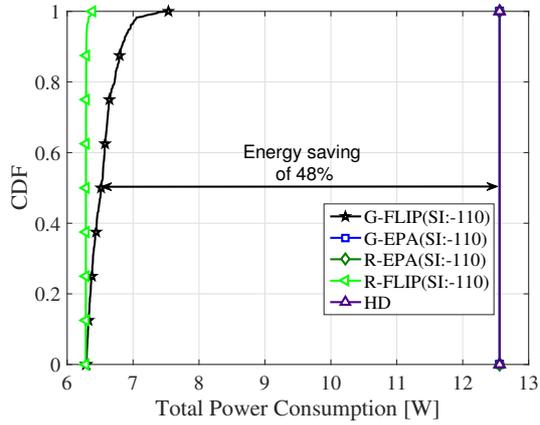


Figure 6. CDF of the total power consumption with a system limited by the UE-to-UE interference. We notice that using FL power control solution we have approximately 48 % of energy saving gains.

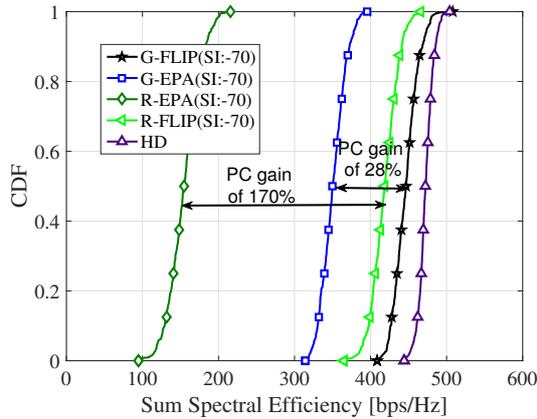


Figure 7. CDF of the sum spectral efficiency with $\beta = -70$ dB, i.e., the system is limited by the SI. We notice a performance degradation for full-duplex communications, but the relative difference between G-FLIP and HD is only 5 %. Now, most of the gains can be achieved by a smart power control instead of a smart frequency assignment algorithm.

channels.

The high energy saving gains with FL power control are expected. With the SINR targets obtained by FL optimization, the distributed power control algorithm by [37] can be used to set the powers that reach the target and minimize the sum power. Recall that due to the strict inequality in Eq. (14), we are close to (but do not necessarily reach) the optimal SINRs that maximizes the sum spectral efficiency. Thus, our solution with FL power control realizes a system whose spectral efficiency is close to optimal but with low power consumption.

C. Analysis for SI-limited Regime

In an opposite manner, in subsection VII.C we analyse a system limited by the SI, on which the SI cancelling level is -70 dB. Similar to subsection VII.B, the performance of the proposed algorithm G-FLIP is compared to G-EPA, R-EPA, R-FLIP and HD.

Figure 7 shows the sum spectral efficiency, and as expected, the performance has degraded for all algorithms with full-duplex communications. The proposed G-FLIP achieves the best performance among the algorithms with full-duplex, and

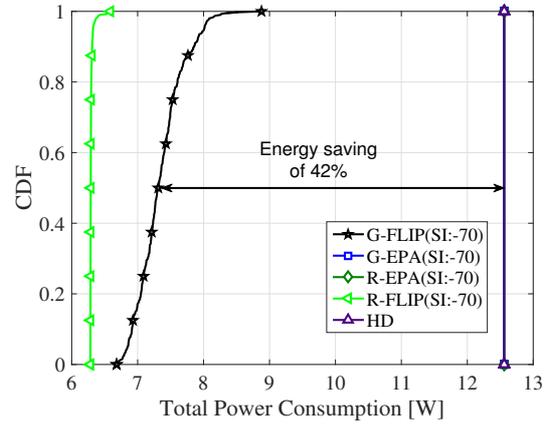


Figure 8. CDF of the total power consumption when the system is limited by the SI. The proposed solution G-FLIP provides a relative energy saving of approximately 42 % with respect to HD and all other algorithms transmitting with EPA.

the relative difference to HD is small, approximately 5 % at the 50-th percentile. Notice that the PC gain between G-FLIP and G-EPA is high, approximately 28 % at the 50-th percentile. Differently from Figure 5, now most of the gains come from power control, and this is shown by the relative difference between R-EPA and R-FLIP, with a relative gain of approximately 170.5 % at the 50-th percentile. In addition, R-FLIP outperforms G-EPA, which shows that a smart power control outperforms a smart frequency assignment in the SI-limited regime. The reason for this effect is that although the frequency channel assignment finds a “good” pair (i.e. well separated and therefore appropriate for co-scheduling), the large impact of the residual SI is not dealt with. To reduce this effect, power control is advantageously used, since it directly acts on the received SI power. Therefore, power control, instead of the frequency assignment, has the important role for the SI-limited regime.

Figure 8 shows the total power consumption, and as before, we have a high energy saving when using FL power control. For G-FLIP, an energy saving gain of approximately 42 % in the 50-th percentile can be observed, which is smaller than the gain in Figure 6. The reason for this result is that the BS transmits at a higher power for many users to compensate for the high SI.

Overall, when the scenario is limited by the SI the power control is more important than the frequency assignment, and with the proposed G-FLIP we have a small relative difference of 5 % to HD but transmitting at lower power, with an energy gain of approximately 42 %.

VIII. CONCLUSION

In this paper we considered the joint assignment and spectral efficiency maximization problem in FD cellular networks. Our objective was to maximize the sum spectral efficiency in a distributed manner while optimizing the assignment of users to frequency channels. This problem was posed as a mixed integer nonlinear optimization, called JASEM, which was shown to be NP-hard. To solve JASEM, we split it into two subproblems, power control and frequency channel

assignment. For the power control, we proposed a novel distributed SINR setting solution based on Fast-Lipschitz optimization. The user-frequency assignment problem was also NP-hard, and we proposed a greedy solution with guaranteed performance in terms of sum spectral efficiency. Using such an optimization, we provided a distributed SINR setting power control algorithm that can be used with our greedy solution to provide a close-to-optimal solution for the initial JASEM problem.

The numerical results showed that our solution approach improved the spectral efficiency of the users with in an interference-limited regime, achieved higher spectral efficiency than HD transmissions, and showed large energy saving gains. The results also indicated that a smart assignment solution achieves more gains in interference-limited regime than smart power control solution, whereas this situation is reversed in SI-limited regime.

In future works, we intend to analyse the impact of assignment and power control in multi-cell FD systems in terms of sum spectral efficiency and fairness, while taking into account practical requirements on the user quality of service class identifier. In addition, analysis of FD systems with MU-MIMO or NOMA to maximize the fairness is also envisioned.

APPENDIX A FAST-LIPSCHITZ OPTIMIZATION

For the sake of self containment and the need for introducing a preliminary result, we now give a brief formal definition of FL problems. For a thorough discussion of FL properties we refer the reader to [9], [21].

Definition 2. A problem is said to be on *FL form* if it can be written as

$$\begin{aligned} & \underset{\mathbf{x}}{\text{maximize}} && f_0(\mathbf{x}) \\ & \text{subject to} && x_i \leq f_i(\mathbf{x}) \quad \forall i \in \mathcal{A} \\ & && x_i = f_i(\mathbf{x}) \quad \forall i \in \mathcal{B}, \end{aligned} \quad (25)$$

where

- $f_0(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is a differentiable scalar ($m = 1$) or vector valued ($m \geq 2$) function.
- \mathcal{A} and \mathcal{B} are complementary subsets of $\{1, \dots, n\}$.
- For all i , $f_i : \mathbb{R}^n \rightarrow \mathbb{R}$ is a differentiable function.

From the individual constraint functions we form the vector valued function $\mathbf{f} : \mathbb{R}^n \rightarrow \mathbb{R}^n$ as $\mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}) \ \dots \ f_n(\mathbf{x})]^T$.

Remark 2. We restrict our attention to a *bounding box* $\mathcal{D} = \{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{a} \leq \mathbf{x} \leq \mathbf{b}\}$. We assume \mathcal{D} contains all candidates for optimality and that \mathbf{f} maps \mathcal{D} into \mathcal{D} , $\mathbf{f} : \mathcal{D} \rightarrow \mathcal{D}$. This box arises naturally in practice, since any real-world decision variable, such as transmitting power, must be bounded.

Definition 3. A problem is said to be *Fast-Lipschitz* when it can be written on FL form and admits a unique Pareto optimal solution \mathbf{x}^* , defined as the unique solution to the system of equations

$$\mathbf{x}^* = \mathbf{f}(\mathbf{x}^*).$$

A problem written on FL form is not automatically Fast-Lipschitz. There are several qualifying conditions that, when

General Qualifying Condition	
GQC	<p>(GQC.a) $\nabla f_0(\mathbf{x}) \geq \mathbf{0}$ with non-zero rows</p> <p>(GQC.b) $\ \nabla f(\mathbf{x})\ < 1$</p> <p>There exists a $k \in \{1, 2, \dots\} \cup \infty$ such that</p> <p>(GQC.c) when $k < \infty$, then $\nabla f(\mathbf{x})^k \geq \mathbf{0}$</p> <p>(GQC.d) when $k > 1$, then $\left\ \sum_{l=1}^{k-1} \nabla f(\mathbf{x})^l \right\ _\infty < \mathbf{q}(\mathbf{x})$, where $\mathbf{q}(\mathbf{x}) \triangleq \min_j \frac{\min_i [\nabla f_0(\mathbf{x})]_{ij}}{\max_i [\nabla f_0(\mathbf{x})]_{ij}}$</p>
Qualifying Condition 3	
Q₃	<p>(Q_{3.a}) $\nabla f_0(\mathbf{x}) > \mathbf{0}$</p> <p>(Q_{3.b}) $\ \nabla f(\mathbf{x})\ _\infty < \frac{\mathbf{q}(\mathbf{x})}{1 + \mathbf{q}(\mathbf{x})}$</p>

Table 2
FAST-LIPSCHITZ QUALIFYING CONDITIONS FROM [21]. **Q₃** IMPLIES THE GENERAL CONDITION (**GQC**), BUT (**Q₃**) IS MUCH EASIER TO USE FROM AN ANALYTICAL AND COMPUTATIONAL POINT OF VIEW COMPARED TO (**GQC**).

fulfilled, guarantee that problem (25) is Fast-Lipschitz (see Table 2). Note that only at least one condition in Table 2 needs to be satisfied. The special condition (**GQC**) is implied by (**Q₃**) and thus (**GQC**) is more general in the sense that if (**Q₃**) holds, then (**GQC**) holds. However, this implies also that (**GQC**) can be more conservative than (**Q₃**).

Note that we only show the (**Q₃**) in this paper. There are already six known qualifying conditions in FL optimization [21]. We did not report the other conditions in this paper because they are not used here and therefore are not included in this appendix.

APPENDIX B PROOF OF LEMMA 2

In the formulation of problem (17), there are no constraints for variable t_l . Nevertheless, we can enforce $t_l \in \mathcal{X}_l$ twice and get the equivalent problem

$$\underset{\mathbf{x}}{\text{maximize}} \quad f_0(\mathbf{x}) \quad (26a)$$

$$\text{subject to} \quad t_k \leq f_{t_k}(\mathbf{x}), \quad (26b)$$

$$t_l \leq f_{t_l}(\mathbf{x}) = b_l, \quad (26c)$$

$$\mathbf{x} \in \mathcal{X}. \quad (26d)$$

This problem is Fast-Lipschitz if

$$\nabla \mathbf{f}(\mathbf{x}) = \begin{bmatrix} \nabla_{t_k} f_{t_k}(\mathbf{x}) & \nabla_{t_k} f_{t_l}(\mathbf{x}) \\ \nabla_{t_l} f_{t_k}(\mathbf{x}) & \nabla_{t_l} f_{t_l}(\mathbf{x}) \end{bmatrix} \text{ and } \nabla f_0(\mathbf{x}) = \begin{bmatrix} \nabla_{t_k} f_0(\mathbf{x}) \\ \nabla_{t_l} f_0(\mathbf{x}) \end{bmatrix}$$

fulfils the condition (**GQC**) of Table 2. Since $f_{t_l}(\mathbf{x}) = b_l$ is constant, and $f_0(\mathbf{x}) = t_k + t_l$, $\nabla \mathbf{f}(\mathbf{x})$ and $\nabla f_0(\mathbf{x})$ simplify to

$$\nabla \mathbf{f}(\mathbf{x}) = \begin{bmatrix} \nabla_{t_k} f_{t_k}(\mathbf{x}) & 0 \\ \nabla_{t_l} f_{t_k}(\mathbf{x}) & 0 \end{bmatrix} \text{ and } \nabla f_0(\mathbf{x}) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Following the steps in [21], we can consider problem (17) with a fixed $t_l \in \mathcal{X}_l$ and with t_k as the only variable, and then rewrite as

$$\underset{\mathbf{x}}{\text{maximize}} \quad f_{0|t_l}(t_k) \quad (27a)$$

$$\text{subject to} \quad t_k \leq f_{t_k|t_l}(t_k), \quad (27b)$$

$$t_k \in \mathcal{X}_k. \quad (27c)$$

For clarity, we refer to problem (27) as *subproblem (27)*. We use Proposition 20 in [21] to prove that problem (17) is Fast-Lipschitz. To this end, we need to prove the following:

(a) Subproblem (27) fulfils the **(GQC)** for all $t_l \in \mathcal{X}_l$;

(b.iii) It holds that for all $\mathbf{x} \in \mathcal{X}$

$$\frac{\|\nabla_{t_l} f_{t_k}(\mathbf{x})\|_\infty}{1 - \|\nabla_{t_k} f_{t_k}(\mathbf{x})\|_\infty} < \frac{\delta_{t_l}(\mathbf{x})}{\Delta_{t_k}(\mathbf{x})}, \quad (28)$$

where $\delta_{t_l}(\mathbf{x}) = \nabla_{t_l} f_0(\mathbf{x})$ and $\Delta_{t_k}(\mathbf{x}) = \nabla_{t_k} f_0(\mathbf{x})$.

To prove (a) we need to check **(GQC)**, and since the qualifying condition **(Q₃)** implies **(GQC)**, we use **(Q₃)** to subproblem (27). This is equivalent to check the following two sub-conditions:

(Q_{3.a}) $\nabla_{t_k} f_{0|t_l}(t_k) > 0$;

(Q_{3.b}) $\|\nabla_{t_k} f_{t_k|t_l}(t_k)\|_\infty < \frac{q(t_k)}{1+q(t_k)}$.

The gradients $\nabla_{t_k} f_{0|t_l}(t_k)$, $\nabla_{t_k} f_{t_k|t_l}(t_k)$, and the function $q(t_k)$ are given by

$$\begin{aligned} \nabla_{t_k} f_{0|t_l}(t_k) &= 1, \\ \nabla_{t_k} f_{t_k|t_l}(t_k) &= 1 - \gamma u(\mathbf{x}), \\ q(t_k) &= 1. \end{aligned}$$

Since $\nabla_{t_k} f_{t_k|t_l}(t_k)$ is scalar, we have the following for **(Q_{3.b})**:

$$\|\nabla_{t_k} f_{t_k|t_l}(t_k)\|_\infty = |1 - \gamma u(\mathbf{x})|.$$

Notice that **(Q_{3.a})** is fulfilled because $\nabla_{t_k} f_{0|t_l}(t_k) = 1 > 0$.

Let us now analyse **(Q_{3.b})**:

$$|1 - \gamma u(\mathbf{x})| < \frac{1}{2}. \quad (30)$$

Note that we can have $(1 - \gamma u(\mathbf{x})) > 0$, which implies the following bound on γ :

$$\frac{1}{2u(\mathbf{x})} < \gamma < \frac{1}{u(\mathbf{x})}. \quad (31)$$

As for the case when $(1 - \gamma u(\mathbf{x})) < 0$, we have the bound on γ as:

$$\frac{1}{u(\mathbf{x})} < \gamma < \frac{3}{2u(\mathbf{x})}. \quad (32)$$

To prove (b.iii), we need to first derive the gradients $\nabla_{t_l} f_{t_k}(\mathbf{x})$, $\nabla_{t_k} f_{t_k}(\mathbf{x})$, and also $\delta_{t_l}(\mathbf{x})$ and $\Delta_{t_k}(\mathbf{x})$, which are given by

$$\nabla_{t_l} f_{t_k}(\mathbf{x}) = -\gamma v(\mathbf{x}), \quad (33a)$$

$$\nabla_{t_k} f_{t_k}(\mathbf{x}) = 1 - \gamma u(\mathbf{x}), \quad (33b)$$

$$\delta_{t_l}(\mathbf{x}) = 1, \quad (33c)$$

$$\Delta_{t_k}(\mathbf{x}) = 1. \quad (33d)$$

Since $\nabla_{t_l} f_{t_k}(\mathbf{x})$ and $\nabla_{t_k} f_{t_k}(\mathbf{x})$ are scalars, we have the following for (b.iii):

$$\|\nabla_{t_l} f_{t_k}(\mathbf{x})\|_\infty = |-\gamma v(\mathbf{x})|,$$

$$\|\nabla_{t_k} f_{t_k}(\mathbf{x})\|_\infty = |1 - \gamma u(\mathbf{x})|.$$

Let us now analyse inequality (28):

$$\frac{\gamma v(\mathbf{x})}{1 - |1 - \gamma u(\mathbf{x})|} < 1, \quad (35a)$$

$$|1 - \gamma u(\mathbf{x})| < 1 - \gamma v(\mathbf{x}), \quad (35b)$$

which implies the following inequalities:

- $1 - \gamma u(\mathbf{x}) > 0$ implies that $1 - \gamma u(\mathbf{x}) < 1 - \gamma v(\mathbf{x})$, and then

$$\gamma(v(\mathbf{x}) - u(\mathbf{x})) < 0. \quad (36)$$

Since $\gamma > 0$, inequality (36) is fulfilled if

$$v(\mathbf{x}) < u(\mathbf{x}), \text{ and if} \quad (37)$$

$$\gamma < \frac{1}{u(\mathbf{x})}. \quad (38)$$

- $1 - \gamma u(\mathbf{x}) < 0$ implies that $-1 + \gamma u(\mathbf{x}) < 1 - \gamma v(\mathbf{x})$, and then

$$\gamma(v(\mathbf{x}) + u(\mathbf{x})) < 2,$$

$$\frac{1}{u(\mathbf{x})} < \gamma < \frac{2}{v(\mathbf{x}) + u(\mathbf{x})}.$$

Note that we also require that $v(\mathbf{x}) < u(\mathbf{x})$ when $1 - \gamma u(\mathbf{x}) < 0$, because otherwise we would have $2/(v(\mathbf{x}) + u(\mathbf{x})) < 1/(u(\mathbf{x}))$, which violates the bound on γ defined above.

Based on the requirements from (a) and (b.iii), we have inequalities on γ that allow us to use fixed point iterations to solve problem (17). To rely only on $u(\mathbf{x})$, we choose the case when $1 - \gamma u(\mathbf{x}) > 0$, which results in the following requirement:

$$\frac{1}{2u(\mathbf{x})} < \gamma < \frac{1}{u(\mathbf{x})},$$

and $v(\mathbf{x}) < u(\mathbf{x})$. If for such conditions on t_k and t_l there is no γ that satisfies inequality (20), then problem (17) is not Fast-Lipschitz.

APPENDIX C

PROOF OF LEMMA 3

From Algorithm 1, the infeasibility of t_k occurs when the upper or lower bound for t_k is reached and $|h(\mathbf{x})|$ is not small enough (see line 10). From the expression of $f_{t_k}(\mathbf{x})$ in Eq. (18b), we can take its second derivatives with respect to t_k and t_l :

$$\frac{\partial^2 f_{t_k}(\mathbf{x})}{\partial t_k^2} = -\frac{\gamma}{\alpha_k} u(\mathbf{x}), \quad (40a)$$

$$\frac{\partial^2 f_{t_k}(\mathbf{x})}{\partial t_l^2} = \frac{\gamma}{\alpha_l} v(\mathbf{x}), \quad (40b)$$

$$\frac{\partial^2 f_{t_k}(\mathbf{x})}{\partial t_k \partial t_l} = -\frac{\gamma}{\alpha_l \alpha_k} \exp\left(\frac{t_k}{\alpha_k}\right) \exp\left(\frac{t_l}{\alpha_l}\right). \quad (40c)$$

If t_k hits the upper bound and $|h(\mathbf{x})|$ is not small enough, we should increase t_l , because it increases $f_{t_k}(\mathbf{x})$ since the function is monotonically non-decreasing in t_l from Eq. (40b). Then, this will cause a decrease in t_k , because the function is monotonically non-increasing in t_k from Eq. (40a). Now, if t_k hits the lower bound and $|h(\mathbf{x})|$ is not small enough, the behaviour is the opposite, and we should decrease t_l , which also decreases $f_{t_k}(\mathbf{x})$ and later increases t_k .

Therefore, if for a given t_l we face later an infeasibility on t_k , we should cut the branch we used and go for the other direction in the search.

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