



**KTH Electrical Engineering**

# Test Charge Response of a Dusty Plasma with Grain Size Distribution and Charging Dynamics

Muhammad Shafiq



Doctoral Thesis

Space and Plasma Physics  
School of Electrical Engineering  
Royal Institute of Technology

Stockholm 2006

## **Test Charge Response of a Dusty Plasma with Grain Size Distribution and Charging Dynamics**

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### **Abstract**

This doctoral thesis reports analytical and numerical results for the electrostatic response of a dusty plasma to a moving test charge. Two important physical aspects of dusty plasmas, namely grain size distribution and grain charging dynamics were taken into account. In the first case, a dusty plasma in thermal equilibrium and with a distribution of grain sizes is considered. A size distribution is assumed which decreases exponentially with the grain mass for large sizes and gives a simple smooth reduction for small sizes. The electrostatic response to a slowly moving test charge, using a second order approximation is found and the effects of collisions are also investigated. It turns out that for this particular size distribution, there is a remarkably simple result that the resulting effective distribution for the electrostatic response is a kappa (generalized Lorentzian) distribution. In the second case, we present an analytical model for the shielding of a slowly moving test charge in a dusty plasma with dynamical grain charging for cases both with and without the collision effects. The response potential is treated as a power series in test charge velocity. Analytical expressions for the response potential are found up to second order in test charge velocity. The first-order dynamical charging term is shown to be the consequence of the delay in the shielding due to the dynamics of the charging process. It is concluded that the dynamical charging of the grains in a dusty plasma enhances the shielding of a test charge. To clarify the physics, a separate study is made where the charging is approximated by using a time delay. The resulting potential shows the delayed shielding effect explicitly. The terms in the potential that depend on the charging dynamics involve a spatial shift given by the test charge velocity and the charging time. The wake potential of a fast moving test charge in the case of grain charging dynamics was also found. It was observed that the grain charging dynamics leads to a spatial damping and a phase shift in the potential response. Finally, combining these two physical aspects, generalized results for the electrostatic potential were found incorporating the terms from both grain size distribution and grain charging dynamics. The generalized results contain the previous work where these two effects were studied separately and which can now be found as special limiting cases. This kind of work has relevance both in space and astrophysical plasmas.

**Keywords:** Dusty plasmas, Complex plasmas, Grain size distribution, Grain charging dynamics, Lorentzian distribution, Kappa distribution, Test charge response, Delayed shielding, Energy loss, Drag force, Wake field.

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# List of papers

## List of papers included in the thesis

The thesis is based on the papers listed below. In the introduction part they are referred to as P-1 – P-6.

1. M. Shafiq and M. A. Raadu, Test charge response of a dusty plasma with a grain size distribution, *Physics Letters A* **305**, 79 (2002).
2. M. A. Raadu and M. Shafiq, Shielding of a slowly moving test charge in a dusty plasma with dynamical grain charging, *Phys. Plasmas* **10**, 3484, (2003).
3. M. Shafiq and M. A. Raadu, Delayed shielding of a slowly moving test charge in a dusty plasma with dynamical grain charging, *IEEE Trans. Plasma Sci.* **32**, 627 (2004).
4. M. Shafiq and M. A. Raadu, Energy loss of test charges in a dusty plasma in the presence of dynamical grain charging, in *New Vistas in Dusty Plasmas, Proceedings of the Fourth International Conference on the Physics of Dusty Plasmas (ICPDP-4)*, Orleans, France, 13-17 June 2005, Ed. L. Boufendi, M. Mikikian and P. K. Shukla, Conference Proceedings Series, American Institute of Physics, **799**, 490-493, (2005).
5. M. A. Raadu and M. Shafiq, Test charge response for a dusty plasma with both grain size distribution and dynamical charging, under review in *Physics of Plasmas*.
6. M. Shafiq and M. A. Raadu, Effect of grain charging dynamics on the wake potential of a moving test charge in a dusty plasma, under review in *Physics of Plasmas*.

## List of papers not included in the thesis

7. M. Shafiq and M. A. Raadu, Potential due to a slowly moving test charge in dusty plasma, Transactions of the Royal Institute of Technology, TRITA-ALP-2001-04, 8 pp., (2001).
8. M. Shafiq and M. A. Raadu, Electrostatic response of a dusty plasma with a grain size distribution to a moving test charge, in Dusty Plasmas in the New Millenium, Proc. of the Third International Conference on the Physics of Dusty Plasmas (ICPDP-2002), Durban, South Africa, 20-24 May 2002, Ed. R. Bharuthram, M. A. Hellberg, P. K. Shukla, and F. Verheest, Conference Proceedings Series, American Institute of Physics, **649**, 422-425, (2002).
9. M. A. Raadu and M. Shafiq, Effect of grain charging dynamics on the response of a dusty plasma to a moving test charge, in Dusty Plasmas in the New Millenium, Proc. of the Third International Conference on the Physics of Dusty Plasmas (ICPDP-2002), Durban, South Africa, 20-24 May 2002, Ed. R. Bharuthram, M. A. Hellberg, P. K. Shukla, and F. Verheest, Conference Proceedings Series, American Institute of Physics, **649**, 430-433, (2002).
10. M. Shafiq and M. A. Raadu, Effect of a grain size distribution on the response of a dusty plasma to a moving test charge, in Proc. URSI XXVII General Assembly, Maastricht, the Netherlands, 17-24 August 2002 (CD-ROM), 128 (paper 501 HGE2.P.2), (2002).
11. A. M. Mirza, M. Shafiq, M. A. Raadu, and K. Khan, Chaotic behavior of electron-positron dusty magnetoplasma with equilibrium flows, in Dusty Plasmas in the New Millenium, Proc. of the Third International Conference on the Physics of Dusty Plasmas (ICPDP-2002), Durban, South Africa, 20-24 May 2002, Ed. R. Bharuthram, M. A. Hellberg, P. K. Shukla, and F. Verheest, Conference Proceedings Series, American Institute of Physics, **649**, 426-429, 2002.
12. M. A. Raadu and M. Shafiq, Potential of a moving test charge in a dusty plasma in the presence of grain size distribution and grain charging dynamics, in Proc of 12th Int. Congress on Plasma Physics, 25-29 October 2004, Hyper Article on Line, <http://hal.ccsd.cnrs.fr/ccsd-00003135/en/>, 10 pp., 2004.

13. M. Shafiq and M. A. Raadu, Wake potential of a moving test charge in a dusty plasma with dynamical grain charging, in Proc. 31st EPS Conf. on Plasma Phys., London, 28 June - 2 July 2004, ECA Vol. **28G**, P-4.074 (4 pp.), 2004.
14. M. A. Raadu and M. Shafiq, Solitary waves in dusty plasmas with charge fluctuations, in New Vistas in Dusty Plasmas, Proceedings of the Fourth International Conference on the Physics of Dusty Plasmas (ICPDP-4), Orleans, France, 13-17 June 2005, Ed. L. Boufendi, M. Mikikian and P. K. Shukla, Conference Proceedings Series, American Institute of Physics, **799**, 486-489, (2005).
15. M. A. Raadu and M. Shafiq, Wake potential of a test charge using the stationary phase method, in Proc. 32nd EPS Conf. on Plasma Phys., Tarragona, Spain, 27 June - 1 July 2005, ECA Vol. **29C**, P-5.119 (4 pp.), 2005.
16. M. A. Raadu and M. Shafiq, Test charge response for a dusty plasma with size distribution and charging dynamics at in Proc. 13th International Congress on Plasma Physics (ICPP), Kiev, Ukraine, 22-26 May (4 pp.), 2006.



# Contributions to international conferences

1. M. Shafiq (M. Shafiq and M. A. Raadu), Potential of a moving test charge in a dusty plasma including dynamical grain charging at 11th International workshop on the physics of dusty plasmas, 28 June-1 July, 2006, Williamsburg, Virginia, USA (contributed oral).
2. M. Shafiq (M. A. Raadu and M. Shafiq), Test charge response for a dusty plasma with size distribution and charging dynamics at 13th International Congress on Plasma Physics (ICPP), 22-26 May 2006, Kiev, Ukraine (contributed poster).
3. M. Shafiq (M. A. Raadu and M. Shafiq), Wake potential of a test charge using the stationary phase method at 32nd EPS Conference on Plasma Physics, 27 June-1 July, 2005, Tarragona, Spain (contributed poster).
4. M. Shafiq (M. Shafiq and M. A. Raadu), Energy loss of test charges in a dusty plasma in the presence of dynamical grain charging at ICPDP-4, 13-17 June 2005, Orleans, France (contributed poster).
5. M. Shafiq (M. A. Raadu and M. Shafiq), Solitary waves in dusty plasmas with charge fluctuations at ICPDP-4, 13-17 June 2005, Orleans, France (contributed poster).
6. M. Shafiq (M. A. Raadu and M. Shafiq), Potential of a moving test charge in a dusty plasma in the presence of grain size distribution and grain charging dynamics at 12th ICPP, 25-29 October 2004, Nice, France (contributed poster).
7. M. Shafiq (M. Shafiq and M. A. Raadu), Plasma wake potential for a moving test charge in a multi-component dusty plasma at Workshop on Theoretical Plasma Physics, 5-16 July 2004, Trieste, Italy (contributed poster).

8. M. Shafiq (M. Shafiq and M. A. Raadu), Wake potential of a moving test charge in a dusty plasma with dynamical grain charging at 31st EPS Conference on Plasma Physics, 28 June-2 July, 2004, London, United Kingdom (contributed poster).
9. M. Shafiq (M. Shafiq and M. A. Raadu), Delayed shielding of a test charge in a dusty plasma with grain charging dynamics at International Topical Conference on Plasma Physics, Complex Plasmas in the New Millennium, 8 - 12 September 2003, Santorini, Greece (contributed oral).
10. M. Shafiq (M. Shafiq M. A. Raadu), Effect of a grain size distribution on the response of a dusty plasma to a moving test charge at URSI XXVII General Assembly, 17 - 24 August 2002, Maastricht, the Netherlands (contributed poster).
11. M. Shafiq (A. M. Mirza, M. Shafiq, M. A. Raadu and K. Khan), Chaotic behavior of electron-positron dusty magneto-plasma with equilibrium flows at 3rd International Conference on the Physics of Dusty Plasmas ICPDP-2002, 20-24 May 2002, Durban, South Africa (contributed poster).
12. M. Shafiq (M. Shafiq and M. A. Raadu), Electrostatic response of a dusty plasma with a grain size distribution to a moving test charge at 3rd International Conference on the Physics of Dusty Plasmas ICPDP-2002, 20-24 May 2002, Durban, South Africa (contributed poster).
13. M. Shafiq (M. A. Raadu and M. Shafiq), Effect of grain charging dynamics on the response of a dusty plasma to a moving test charge at 3rd International Conference on the Physics of Dusty Plasmas ICPDP-2002, 20-24 May 2002, Durban, South Africa (contributed poster).
14. M. Shafiq (M. Shafiq and M. A. Raadu), Electrostatic response of a dusty plasma with a grain size distribution to a moving test charge at 36th Nordic Plasma and Gas Discharge Symposium, M/S Polarlys, Hurltigruten, 23-26 February 2002, Norway (contributed oral).
15. M. Shafiq (M. Shafiq and M. A. Raadu), Electrostatic potential of a slowly moving test charge in multicomponent dusty plasma at 5th European Workshop on Dusty and Colloidal Plasmas, 23-25 August 2001, Potsdam, Germany (contributed poster).

# List of Figures

1.1	Noctilucent clouds (NLC). . . . .	14
2.1	Comparison of Coulomb potential ( $\phi_C$ ) and Debye potential ( $\phi_D$ ) for $\lambda_D = 1$ and $q_t = 1$ . . . . .	26
3.1	Charging of dust particle . . . . .	30
3.2	Only ions with an impact parameter smaller than the limit $b_c$ can contribute to the ion current on the dust grain. The other ones ( $b > b_c$ ) are deflected in the electrostatic potential of the particle. . . . .	33
3.3	Charging of dust particles in a dusty plasma takes place mainly by (i) collection of plasma particles (left), (ii) photoemission (centre) and (iii) secondary electron emission (right). . . . .	36
3.4	Schematic diagram of the device used to disperse dust into the plasma column [52]. . . . .	37
3.5	Langmuir probe characteristics obtained under identical conditions, except for the absence (upper plot) and the presence (lower plot) of kaolin dust [52]. . . . .	38
4.1	Size distribution $h(a)$ plotted against grain size $a$ for different $\beta$ and corresponding $\alpha$ values . . . . .	45





# Contents

<b>Acknowledgments</b>	<b>v</b>
<b>List of papers</b>	<b>ix</b>
<b>Contributions to international conferences</b>	<b>xiii</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Background . . . . .	1
1.2 Dusty plasmas . . . . .	2
1.3 Basic dusty plasma characteristics . . . . .	5
1.3.1 Debye length . . . . .	5
1.3.2 Macroscopic neutrality . . . . .	6
1.3.3 Intergrain spacing . . . . .	6
1.3.4 Coulomb coupling parameter . . . . .	7
1.3.5 Characteristic frequencies . . . . .	7
1.4 Dusty plasmas in space and laboratory . . . . .	8
1.4.1 Dusty plasma in space . . . . .	9
1.4.2 Dusty plasma in laboratory . . . . .	14
1.5 Layout of the thesis . . . . .	16
<b>2 Theoretical background</b>	<b>19</b>
2.1 Introduction . . . . .	19
2.2 Plasma as a dielectric medium . . . . .	21
2.3 Test charge approach . . . . .	22
2.4 Calculation of the dielectric constant . . . . .	23
2.5 Plasma dispersion function . . . . .	25
2.6 Debye shielding . . . . .	25
<b>3 Dust grain charging</b>	<b>29</b>
3.1 Introduction . . . . .	29
3.2 Different dust charging mechanisms . . . . .	31

3.2.1	Collection of plasma particles . . . . .	32
3.2.2	Secondary electron emission . . . . .	34
3.2.3	Photoemission . . . . .	35
3.2.4	Other charging process . . . . .	36
3.3	Dust charging in the laboratory . . . . .	36
<b>4</b>	<b>Dust grain size distribution</b>	<b>41</b>
4.1	Introduction . . . . .	41
4.2	Lorentzian or kappa distribution . . . . .	41
4.3	Kinetic description of the dusty plasma with dust size distribution . . . . .	42
4.4	Effective dust distribution function . . . . .	43
<b>5</b>	<b>Plasma wake potential</b>	<b>47</b>
5.1	Introduction . . . . .	47
5.2	Dust-acoustic and dust-ion-acoustic waves . . . . .	48
5.3	Wake potential . . . . .	49
5.3.1	Wake potential for dust-acoustic (DA) waves . . . . .	49
5.3.2	Wake potential for dust-ion-acoustic (DIA) waves . . . . .	50
5.4	Experimental verification of wake potential . . . . .	51
<b>6</b>	<b>Summaries of the papers</b>	<b>53</b>
6.1	Paper 1 . . . . .	54
6.2	Paper 2 . . . . .	55
6.3	Paper 3 . . . . .	56
6.4	Paper 4 . . . . .	57
6.5	Paper 5 . . . . .	57
6.6	Paper 6 . . . . .	58
	<b>Bibliography</b>	<b>61</b>

# Chapter 1

## Introduction

*This chapter is devoted to an introduction to dusty plasma. A general overview of its basic characteristics and its presence in the laboratory and space is presented.*

### 1.1 Background

Plasma is found almost everywhere throughout the universe. Its appearance varies from diffuse interstellar clouds through stellar coronas to dense interiors of stars. They all have one thing in common: the matter is (partially) ionized. Also here on earth one can witness many kinds of plasma phenomena, both natural and man-made. Natural plasma like lightning and fire are common to everybody; besides, many artificially generated plasmas, like street lamps and neon lights are familiar even to non-physicists.

The history of plasma physics goes back to Faraday's study of the "dark discharges". In the beginning, many prominent physicists and researchers were engaged in research on discharges in gases and the first systematic studies on the particles of ionized gases were undertaken by Langmuir and Tonks in the 1920's. Gradually plasma research spread in other directions, of which three were particularly significant. First, the development of radio led to the discovery of the ionosphere, the natural "plasma roof" above the atmosphere, which bounces back radio waves and sometimes absorbs them. Starting with the study of the propagation of radio waves in the ionosphere, a wide variety of plasma waves were identified, in general spreading differently along magnetic field lines than perpendicular to them. Second, astrophysicists recognized that much of the universe consisted of plasmas, and that understanding astrophysical processes required a better grasp on plasma physics. This was particularly true for the Sun, whose intensely magnetic sunspots

produced many intricate plasma phenomena (e.g. solar flares). The pioneer in this field was Hannes Alfvén, who around 1940 developed the theory of magnetohydrodynamics, or MHD, in which plasma is treated essentially as a conducting fluid. Finally, fusion research for the possible source of energy in future. The Sun releases its energy by combining hydrogen nuclei to form helium, but this thermonuclear fusion process needs enormous temperatures and pressures, like those found at the center of the Sun. However, since gas at such a temperature becomes a plasma, the idea arose to hold it trapped inside a magnetic field, without it actually touching any material walls. The effort to produce such “controlled thermonuclear fusion” started in the early 1950s and has grown into a great international undertaking, with thousands of scientists and huge, sophisticated machines. Recently a fusion experiment managed to extract as much fusion energy as was invested in the plasma, but we are still a long way from commercial use of such energy.

In recent years, a lot of research work has been done in plasma physics due to its several technological applications, such as direct conversion of heat into electricity by magnetohydrodynamic means, the propulsion of space vehicles, and the development of new electronic devices [1]. Plasmas are mainly characterized by two parameters namely number density  $n$  and thermal energy  $KT_e$  and plasma applications cover an extremely wide range of  $n$  and  $KT_e$ .

## 1.2 Dusty plasmas

It appears that much of the solid part of universe is in the form of dust. Therefore, plasmas and dust are two main ingredients of the universe. The interplay between both has opened up a new and fascinating research domain, that of dusty plasmas. A dusty plasma (more recently called complex plasma in analogy with complex fluids e.g. colloidal suspensions) is loosely defined as a normal electron-ion plasma with an additional charged component of micron- or submicron-sized particulates. The past fifteen years have seen tremendous growth in the field of dusty or complex plasma physics. Dusty plasmas are an ubiquitous component of space, found in protostellar clouds, protoplanetary disks, cometary tails, and in the planetary rings. As a result, the physics underlying the manner in which dust grains interact with one another and their plasma/gas environment is a field of research having broad implications on our understanding of the development of astrophysical systems. Dusty plasmas are also found in various laboratory devices and industrial processes.

The physics of dusty plasmas has recently been studied intensively be-

cause of its importance for a number of applications in space and laboratory plasmas [2, 3]. The presence of dust particles (grains) changes the plasma parameters and affects the collective processes in such plasma systems. In particular, the charged dust grains can effectively collect electrons and ions from the background plasma. There has recently been considerable interest in new wave modes, electrostatic potential and energy loss of charged projectiles in dusty plasmas and this is currently a rapidly growing area in dusty plasma physics [4]. Numerous unique types of electrostatic and electromagnetic waves exist in dusty plasmas because of the dust charging process.

For typical space and laboratory plasmas, the condition  $a \ll \lambda_D, d$  is fulfilled, where  $a$ ,  $\lambda_D$ ,  $d$  are respectively the grain radius, Debye shielding distance and the average intergrain distance. In the case where  $\lambda_D < d$ , the dusty plasma can be considered as a collection of isolated screened grains (which may be referred to as dust-in-plasma) whereas in the case where  $d < \lambda_D$ , the charged dust also participates in the screening process and therefore in the collective behavior of the ensemble (a true dusty plasma). The term “dust-laden plasma” may be used to include both cases.

New discoveries are continuing to fuel excitement about the field of dusty plasmas. Recent topics that have generated a great deal of interest in the community include observations of Mach cones [5] and solitons [6], plasma crystals [7] and the formation of voids and void-like regions [8] (areas in a dusty plasma in which no dust is present) in a variety of experiments in dusty plasmas. At the same time, because of the large size of the dust particles, gravitational effects can play a significant role in determining the properties of dusty plasmas. Microgravity investigations of dusty plasmas using sounding rockets flights and parabolic flights have begun recently [9]. Also a series of experiments performed onboard the International Space Station (ISS) with the “PKE-Nefedov Laboratory” show rich possibilities to study basic phenomena (e.g., waves, 3D crystals, phase transitions, fluid behavior, coagulation, etc.) under microgravity conditions [10]. Future ISS experiments are planned which will use the recently proposed International Microgravity Plasma Facility as a dedicated complex plasma microgravity experiment. This clearly suggests a bright future for the field.

The grains of a dusty plasma also have a range of sizes and shapes, and the charge can fluctuate in time, leading to highly complex behavior. When such particles are present in a plasma, they become charged (usually negatively because of the greater mobility of electrons) and thus contribute to the plasma dynamics as a separate species. These particles have much larger masses and usually larger charges than those of the plasma ions, but much smaller charge-to-mass ratios. This so-called dusty plasma contains a highly diverse range of collective modes of oscillation, unstable behavior, and lin-

ear and nonlinear waves. Over the years, dusty plasmas have been studied mostly as a theoretical construct, due to a lack of direct observations of dust in space. In the past two decades, however, the situation has changed dramatically: First Voyager mission's observations revealed new planetary rings and complicated structure ("spokes") in the rings themselves; more recently, the Galileo and Ulysses missions made measurements of dust streams from Jupiter, and rocket flights collected data through noctilucent clouds high in Earth's atmosphere [11].

The presence of the charged microparticles in the plasma can lead to significant modification of the bulk plasma parameters such as the charge distribution and potential profile. This also introduces new physical processes into the system, e.g., effects associated with particle charge variation, etc. Microparticles can change properties of usual acoustic waves in a plasma (e.g., dust-ion acoustic wave) [12]. Furthermore, the presence of heavy charged particles gives rise to new potential structures that explain the energy loss mechanism of charged projectiles in dusty plasma [13]. Thus the potential behavior in complex plasmas can be completely different from that in usual two component electron-ion plasmas.

Dusty plasmas are an attractive area for plasma research for a number of reasons:

- The dust acquires an electrical charge and thus is subject to electromagnetic as well as gravitational forces.
- The charged dust particles participate in the collective plasma processes.
- The macroscopic dust component can be visualized easily, allowing direct observation at the most fundamental (kinetic) level for the first time ever.
- The plasma movement is slowed down, and therefore the physics is observable in slow motion.
- Finally, liquid and crystalline plasma states can be investigated.

The basics of dusty plasma dynamics and the way in which these unique states of matter form has massive implications in the advancements of science. Additionally the part played by dusty plasmas in the birth of stars and galaxies is something that could help us understand the birth of the earth and possibly even life itself; if a substance is present throughout the universe it seems pretty important that we understand it. Dust particles in space carry information about their birth at a remote site in space and time. By

space measurements we learn where dust comes from. From its state and composition we obtain knowledge about the environment in and the processes by which the dust was formed and was altered on its way to us.

It is clear that dusty plasma physics is a vast field and therefore one has to limit himself to a certain category of dusty plasmas. Most of the previous work on dusty plasmas has not studied dust size distribution and dust charging dynamics extensively and has assumed constant charge and fixed size of the dust grains. In this work, we present results for the electrostatic potential for the cases of the dust size distribution and dust charging dynamics for a test charge moving through a multicomponent dusty plasma and the effect of delayed shielding due to dynamical charging of the dust on the electrostatic potential is discussed. The wake field for a fast moving test charge in the absence and presence of dynamical charging is also treated. Finally, the combined effects of both grain size distribution and charging dynamics are investigated.

## 1.3 Basic dusty plasma characteristics

To understand the basic dusty plasma principles properly, it seems useful to re-examine some basic characteristics such as Debye length, macroscopic neutrality, intergrain spacing, Coulomb coupling parameter and characteristic frequencies, etc. In the following few sections, these basic characteristics and various notations are elaborated.

### 1.3.1 Debye length

The Debye length is an important physical parameter in a plasma: it provides the distance over which the influence of the electric field of an individual charged particle is felt by other charged particles (such as ions) inside the plasma. The charged particles actually rearrange themselves in order to shield all electrostatic fields within a Debye distance. In dusty plasmas, the Debye length can be defined as follows [4, 14]:

$$\lambda_D = \frac{\lambda_{De} \cdot \lambda_{Di}}{\sqrt{\lambda_{De}^2 + \lambda_{Di}^2}} \quad (1.1)$$

where  $\lambda_{De} = \sqrt{\frac{KT_e}{4\pi n_e e^2}}$  and  $\lambda_{Di} = \sqrt{\frac{KT_i}{4\pi n_i e^2}}$  are the Debye lengths associated to electrons and ions respectively, and  $T_e$ ,  $n_e$ ,  $T_i$  and  $n_i$  are the electron and ion mean temperatures and densities.

Dust particles of radius  $a$ , separated by a given distance  $d$ , can only be considered as individual isolated grains if the criterion  $a \ll \lambda_D \ll d$  is met, i.e. if the physical dimensions of the plasma are large enough for the shielding to take place. If so, considering that the electron's mobility is higher than that of the ions, shielding is primarily performed by electrons and equation (1.1) becomes:

$$\lambda_D \simeq \lambda_{De} = \sqrt{\frac{KT_e}{4\pi n_e e^2}}$$

For typical working conditions in an argon RF capacitively-coupled plasma, where  $T_e = 3eV$  and  $n_e = 10^{15} m^{-3}$ ,  $\lambda_D$  is in the order of a few hundreds of micrometers.

### 1.3.2 Macroscopic neutrality

Dusty plasmas are characterized as a low-temperature ionized gas whose constituents are electrons, ions and micron-sized dust particulates. The presence of dust particles (grains) changes the plasma parameters and affects the collective processes in such plasma systems. In particular, the charged dust grains can effectively collect electrons and ions from the background plasma. Thus in the state of equilibrium, the electron and ion densities are determined by the neutrality condition which is given by

$$en_{i0} - en_{e0} + q_d n_{d0} = 0 \quad (1.2)$$

where  $n_{e,i,d}$  is the concentration of plasma electrons, singly charged positive ions (with charge  $e$ ) and dust particles respectively,  $e$  is the magnitude of electron charge and  $q_d = Z_d e (-Z_d e)$  is the amount of charge present on the dust grain surface when the grains are positively (negatively) charged with  $Z_d$  being the number of charges residing on the surface of dust grain. Note that the charge of the dust particle can vary significantly depending on plasma parameters. In studying the basic physics of dusty plasmas, the third term on the left in equation (1.2) carries very interesting implications. The presence of the dust particles in the plasma allows the formation of electric fields within the plasma, alters the local plasma potential profile, modifies the transport of particles in the plasma, modifies certain types of ion plasma waves, and introduces new dust plasma wave modes.

### 1.3.3 Intergrain spacing

We consider a multi-component dusty plasma composed of electrons, positively charged ions, and extremely massive charged dust grains, with a neutral background. The dust grain radius  $a$  is usually much smaller than the



dusty plasma Debye length  $\lambda_D$ . When the intergrain spacing  $d$  is much smaller than  $\lambda_D$ , the charged dust particulates can be treated as massive point particles similar to multiply charged negative (or positive) ions in a multi-species plasma. Note that for dusty plasmas  $d < \lambda_D$  and the dust particles can be considered as massive point particles where the effect of neighboring particles can be significant. For dust-in-plasma  $d > \lambda_D$ , and in this situation the dust particles are completely isolated from their neighbors..

### 1.3.4 Coulomb coupling parameter

Charged dust grains can be either weakly or strongly correlated depending on the strength of the Coulomb coupling parameter

$$\Gamma_d = \frac{q_d^2}{d(KT_d)} \exp\left(-\frac{d}{\lambda_D}\right) \quad (1.3)$$

where  $d = (3/4\pi n_{d0})^{1/3}$  is the intergrain spacing,  $n_{d0}$  is the initial dust density,  $q_d$  is the dust charge,  $KT_d$  is the dust thermal energy. When  $\Gamma_d \gg 1$ , the dusty plasma is strongly coupled and this condition is met in several laboratory dusty plasmas, such as dust plasma crystals [15, 16]. A dusty plasma is considered as weakly coupled as long as  $\Gamma_d \ll 1$ . Previous works [17, 18] investigated the dust-acoustic instability produced in this regime using a one-dimensional simulation model assuming constant dust charge.

### 1.3.5 Characteristic frequencies

Just as the usual electron-ion plasma, a dusty plasma is also macroscopically neutral. The disturbance of such a plasma from its equilibrium position can result in collective particle motions and hence can lead to the built up of an internal space charge field. This internal space charge field in turn will then try to restore the original space charge neutrality but due to inertia, there will be a collective oscillation of plasma particles around their equilibrium positions. These collective oscillations are characterized by the frequency of oscillation  $\omega_p$  known as the plasma frequency. In order to derive an expression for the plasma frequency, the continuity equation, momentum equation and Poisson's equation for the electrons, ions and dust particles shall be taken into account. Linearization and simplification of these equations results in the following expression for the plasma frequency summed over species  $z$ :

$$\omega_p^2 = \sum_z \frac{4\pi n_{z0} q_z^2}{m_z} = \sum_z \omega_{pz}^2 \quad (1.4)$$

where  $z = e$  for electrons,  $i$  for ions and  $d$  for the dust. This can be understood in the following fashion. For the purpose of understanding, let us focus our attention to the electron motion only. In this case, the internal space charge field (which is built up when the plasma particles are displaced from their equilibrium position) will try to attract electrons back to their equilibrium positions but due to inertia, the electrons will overshoot and move to the other extreme and will again be pulled back by the space charge field of opposite polarity. In this way, the electrons will continuously oscillate around their original positions with a frequency  $\omega_{pe}$  called the electron plasma frequency. Similarly ions and dust particles will oscillate around their equilibrium positions with ion plasma frequency  $\omega_{pi}$  and dust plasma frequency  $\omega_{pd}$  respectively. Since the frequency of such oscillations depends on mass, charge and number density of plasma particles, it is different for the electrons, ions and dust particles. The electrons oscillate around ions with the electron plasma frequency  $\omega_{pe}$ , ions oscillate around the charged dust grains with the ion plasma frequency  $\omega_{pi}$  and dust particles oscillate around their equilibrium positions with the dust plasma frequency  $\omega_{pd}$ .

In a partially or low ionized plasma when there is a significant number of neutral particles present inside the plasma, we encounter frequent collisions between plasma particles (electrons, ions and dust grains) and neutrals. These collisions can be characterized by collision frequencies. There are three types of collision frequencies known as electron-neutral collision frequency  $\nu_{en}$ , ion-neutral collision frequency  $\nu_{in}$  and dust-neutral collision frequency  $\nu_{dn}$  respectively. The general expression for collision frequency  $\nu_{zn}$  for scattering of plasma species  $z$  by the neutrals can be written as

$$\nu_{zn} = n_n \sigma_z^n V_{Tz}$$

where  $n_n$  is the number density of neutral particles,  $\sigma_z^n$  is the scattering cross section and  $V_{Tz} = \sqrt{\frac{kT_z}{m_z}}$  is the thermal speed of the species  $z$ .

## 1.4 Dusty plasmas in space and laboratory

Dusty plasmas are important in technological applications and in astrophysical situations. They can be formed under laboratory conditions (e.g., plasma processing reactors, laboratory experiments, rocket exhaust and fusion experiments) with sizes of the dust cloud of a few millimeters to astrophysical systems (e.g., planetary rings, comet tails, nebula, interstellar medium and noctilucent clouds) with an enormous range of sizes. In the following, we shall discuss briefly about the presence of dusty plasmas in space and laboratory.

### 1.4.1 Dusty plasma in space

We now focus our attention on dusty plasmas in space where they occur in a wide variety of environments. Dust is present in such diverse objects as interstellar clouds, solar system, cometary tails, planetary rings and noctilucent clouds etc. Since charged dust grains are common in low earth orbit and in the interplanetary medium, the presence of this charged material can cause both physical damage and electrical problems for spacecraft. Charged dust particles in space plasmas may help to explain the formation of the planets, planetary rings, comet tails and nebulae. Given the variety of arenas in which a dusty plasma may play a role, it is important to understand the physical properties of this plasma system in space.

The space between the stars is filled with cosmic dust and elements like hydrogen and helium which make up the “interstellar medium”. The interstellar medium is mainly made of hydrogen atoms. The density of hydrogen in interstellar space is on average about 1 atom per cubic centimeter. In the extremes, as low as 0.1 atom per cubic centimeter has been found in the space between the spiral arms and as high as 1000 atoms per cubic centimeter are known to exist near the galactic core.

The interstellar medium also contains cosmic dust. These particles are much bigger than hydrogen atoms. However, there are far fewer particles of cosmic dust than there are hydrogen atoms in the same volume of space. It is estimated that cosmic dust is 1000 times less common than hydrogen atoms in the interstellar medium. The dust grains in the interstellar or circumstellar clouds are dielectric (ices, silicates, etc.) and metallic (graphite, magnetite, amorphous carbons etc.).

Our solar system which consists of the sun and all the objects that orbit around it, is also believed to contain a large amount of dust grains and the origins of these dust grains in the solar system could possibly be, for example, micrometeoroids, space debris, man-made pollution and lunar ejecta, etc. In the following, few sections are presented very briefly some important characteristic of the dust particles and their plasma environments in a number of different regions of our solar system, namely interplanetary space, comets, planetary rings, and earth’s atmosphere.

#### **Interplanetary space**

The space between the planets is far from empty. It contains electromagnetic radiation (photons), hot plasma (electrons, protons and other ions), the solar wind, cosmic rays, microscopic interplanetary dust particles, and magnetic fields (primarily the Sun’s). Interplanetary dust particles (or IDPs

in common usage) are extremely small (0.001 *cm* in diameter) grains or particles and are among the most primitive materials in the solar system. The earth's surface is constantly being rained upon by interplanetary dust particles (IDP's), from a few to several hundred micrometers in diameter. The mass distribution of this dust flux peaks at around 200  $\mu m$  [19]. This dust is thought to be derived from collisions of asteroidal material and from comets [20]. Typical parameters of dust-laden plasmas in interplanetary space are  $n_e \simeq 5 \text{ cm}^{-3}$ ,  $T_e \simeq 10^5 K$ ,  $n_d \simeq 10^{-12} \text{ cm}^{-3}$ ,  $a = 2 - 10 \mu m$  and  $a/\lambda_D \simeq 5$  [4].

### Comets

A comet is basically a ball of ice and dust that looks like a star with a tail. Some comets do not have tails, looking like round spots of light. Most comets have three parts: a nucleus, a head (coma), and a tail.

Our solar system began as a vast cloud of gas and dust. Several billion years ago, this cloud slowly rotated around our very young sun and particles within the cloud collided with one another. During this time some objects (Comets) were obliterated by these collisions, while others grew in size and were to later become the planets.

Dubbed “dirty snowballs” by some astronomers, comets are composed of dust grains, chunks of dirt, and ice. Apart from water, cometary ice also contains carbon dioxide, ammonia, and methane. Much of the ice remains frozen, preserving the dust trapped inside since the birth of the solar system.

As a comet nears the Sun, its icy layers begin to evaporate, releasing dust and dirt to form a curved dust tail. The liberated gases glow as they become heated, producing the coma, a luminous envelope surrounding the solid core (nucleus). Ionized by the Sun's radiation and pushed backwards by the solar wind, the heated gas trails behind and also glows, forming a second, straight tail. When approaching closer to the Sun, a comet's nucleus heats up. As gas and dust molecules previously bound inside the nucleus are released into the coma, they emit radiation.

Throughout the early period, comets probably filled the solar system. Their collisions with the early planets played a major part in the growth and evolution of each planet. Over the years, comets actually became rarer within our solar system. They no longer fill the skies as they did 4 billion years ago, and today a prominent naked-eye comet can be expected only about once a decade. Astronomers with powerful telescopes can see many more comets, but even in this case it is still rare for as many as 15 or 20 comets to be detectable in the sky at any one time.

Occasionally, comets have been known to experience “outbursts” when

their brightnesses increase dramatically within a short period of time. This can be due to a fresh eruption of new material from a comet's surface, or sometimes this occurs when the nucleus splits into two or more pieces, exposing previously hidden sections of its material to the sun's heat for the first time. While most of the planets' orbits are near-circles, the orbits of most comets are extremely elongated ellipses. Some "short-period" comets take up to 200 years to orbit the Sun. Other "long-period" comets exhibit far longer orbital periods, lasting up to hundreds of thousands, even a million years.

Scientists believe that comets bombarded the young earth, perhaps seeding it not only with the water (which enabled life to begin), but also with organic compounds that would later evolve into living matter. Of particular interest to dusty plasma researchers studying comets is to study the composition and properties of dust grains which can help to understand the development of solar system. The data from the Vega and Giotto spacecrafts have provided much information about the cometary dust particles. A detailed analysis of this data showed that a significant amount of small grains were found to exist in cometary environment obeying a size distribution fitted by a power law given by the relation  $n(a) \simeq a^{-s}$ , with  $s$  being equal to 3.3 for Vega and 4.1 for Giotto [4].

### Planetary rings

We shall now discuss the presence of dust particles in planetary rings, locations where the occurrence of charged dust was first suspected since the completion of successful Voyager missions. In our solar system, planetary rings are found around all the giant planets (Jupiter, Saturn, Uranus and Neptune), showing spectacular variety. Some of these like, the A, B and C rings of Saturn and the eleven narrow Uranian rings are optically thick and composed primarily of large bodies (1 *cm* to 10 *m*). Every other ring system has been found to contain a large population of micron-sized dust. Below we provide a brief description for the understanding of the origin of dust particles in planetary rings.

**Jupiter** Jupiter is the fifth planet from the Sun and by far the largest within our solar system. A number of probes have had visited Jupiter. Pioneer 10 flew past Jupiter in December 1973, followed by Pioneer 11 exactly one year later. They provided important new data about Jupiter's magnetosphere, and took some low resolution photographs of the planet. Later on Voyager 1 flew by Jupiter in March 1979 followed by Voyager 2 in July of the same year. The Voyagers vastly improved our understanding of the Galilean

moons and discovered Jupiter's rings. They also took the close up images of the planet's atmosphere. Recently, in 2000, the Cassini probe, en route to Saturn, flew by Jupiter and provided some of the highest-resolution images ever made of the planet.

Jupiter has a faint planetary ring system composed of smoke-like dust particles knocked off from its moons by meteor impacts. The main ring is made of dust from the satellites Adrastea and Metis. Two wide gossamer rings encircle the main ring, originating from satellites Thebe and Amalthea. There is also an extremely tenuous and distant outer ring that circles Jupiter backwards. Its origin is uncertain, but this outer ring might be made of captured interplanetary dust. The Halo ring consists of charged particles that are being pulled into the planet. It is believed that the moon's low gravity releases dust and particles from their atmospheres when struck by meteorites. Another possibility is that dusty ring material may also have been leftover from comets that crashed into Jupiter. The grain size distribution in Jupiter ring system is known to obey a power law which is given by [4]

$$n_d(a) da = Ca^{-\beta} da$$

with radius range  $[a_{\min}, a_{\max}]$ . In the above relation,  $n_d(a) da$  denotes the differential number density of the grains per unit volume with radius between  $a$  and  $a + da$ ,  $\beta$  is the power law index and  $C$  is a normalization constant.

**Saturn** Saturn is the sixth planet from the Sun. It is a gas giant, the second-largest planet in the solar system after Jupiter. Saturn has large rings made mainly out of ice and space debris (dust). A number of probes have also visited Saturn. It was first visited by Pioneer 11 in 1979 and the following two years by Voyager I and Voyager II. The Cassini-Huygens orbiter and probe has arrived to study Saturn and its moon Titan.

Saturn is probably best known for its planetary rings, which make it one of the most visually remarkable objects in the solar system. The three rings were first observed by Galileo Galilei in 1610. Later on they keep on increasing in number and are given letter names in order of their discovery. The Whole known Ring System consists of D, C, B rings, Cassini Division, A, F, G and E rings. In addition, there are some small intermediate rings. They extend from 6,630 *km* to 120,700 *km* above Saturn's equator, and are composed of silica rock, iron oxide, and ice (dust) particles. Radial, spoke-like features in the broad B-ring were also found by the Voyagers. The features are believed to be composed of fine, dust-size particles. It is likely that the overall ring composition is not too different from that of the various icy satellites [21].

**Uranus** Uranus is the seventh planet from the Sun. It is a gas giant, the third largest by diameter and fourth largest by mass. Uranus is composed primarily of rock and various ices. NASA's Voyager II is the only spacecraft to have visited the planet. Launched in 1977, Voyager made its closest approach to Uranus on January 24, 1986 before continuing on its journey to Neptune.

Uranus has a faint planetary ring system, composed of dark particulate matter up to 10 meters in diameter. This ring system was discovered in March 1977 by the Voyager II. The planet is encircled by eleven major rings believed to be formed by debris from broken up space objects. Uranus' rings are distinctly different from those at Jupiter and Saturn. The outermost epsilon ring is composed mostly of ice boulders several feet across. A very tenuous distribution of fine dust also seems to be spread throughout the ring system.

**Neptune** Neptune is the eighth planet from the sun, and the outermost gas giant in our solar system. It is fourth largest by diameter and third largest by mass. Neptune has been visited by only one spacecraft, Voyager II, which flew by the planet on August 25, 1989.

By the 1980's, scientists found that Jupiter, Saturn, and Uranus all have rings. Astronomers wondered if Neptune had rings too. From Earth, no rings could be seen around Neptune. When the space probe Voyager II studied Neptune in 1989, it spotted ring arcs that the astronomers had suspected. A ring arc is a partial ring. As the probe got closer, astronomers could see that the arcs were actually part of one complete ring. Voyager II found that Neptune had four rings in all. The two main rings are very bright and quite narrow. They appear to be made up of fine dust and small particles thought to have been made by tiny meteorites smashing into Neptune's moons. The mass and size distribution of these dust grains is not yet known and further studies are required to determine mass and size distributions more accurately. The outermost ring contains very bright clumps of particles. Neptune's other two rings are a lot wider but do not have any brightness to them. The ring closest to Neptune is about 1,100 miles wide. The other faint ring is about 3,600 miles wide.

### **Earth's atmosphere**

The closest example of naturally occurring dusty plasmas in the earth's environment are the Noctilucent clouds (NLC) which are formed in the arctic mesosphere (approximately 80 – 110 *km* altitude) in the summer. This is the coldest place in the earth's environment, where the summer temperatures drop to 100 *K*, compared to a normal winter temperature above 200



Figure 1.1: Noctilucent clouds (NLC).

*K.* The term noctilucent comes from Latin and means luminous at night and therefore, these clouds are seen at night. They occur in the summer at about  $85\text{ km}$  altitude because the rising arctic air is strongly cooled by expansion, and is in fact colder than in the winter.

Since the noctilucent clouds occur at such a great height, they remain in sunlight and continue to glow long after sunset. These clouds are visible only when illuminated by sunlight from below the horizon while the ground and the lower layers of the atmosphere are in the earth's shadow; otherwise they are too faint to be seen. Photographs have revealed wind speeds of up to  $650\text{ kph}$  associated with noctilucent clouds. The cloud particles are ice. They are the highest clouds in the earth's atmosphere and the altitude of these clouds puts them in the ionosphere (by comparison, most 'terrestrial' clouds occur below  $10\text{ km}$  altitude), where there are free electrons created by solar wind particles. The electrons can attach to the ice particles thus creating a "dusty" plasma.

### 1.4.2 Dusty plasma in laboratory

A very brief introduction to the presence of dusty plasmas in laboratory is provided in the following. In Chapter 3, we shall also briefly discuss an experiment explaining the charging of dust particles under the laboratory condition.



Dusty plasma research involves the study of the interactions between charged dust particles and plasma in which the dust particles are suspended. The presence of dusty plasmas in space as discussed above can be considered as a starting point for the understanding of the laboratory dusty plasmas in the sense that there are two main features which differentiate laboratory dusty plasmas from space and astrophysical plasmas. Firstly laboratory discharge have geometric boundaries whose properties like structure, composition, temperature, conductivity etc. can have an influence on the formation and transport of the dust grains. Secondly, the external circuit which maintains the dusty plasma, imposes varying boundary conditions on the dusty discharge. In the following we shall discuss briefly the presence of dusty plasmas in laboratory devices, particularly in direct current (dc) and radio-frequency (rf) discharges, plasma processing reactors and fusion plasma devices.

Historically, many of the first dusty plasma investigations were performed using rf glow plasma. In these systems, the suspended microparticles often form regular two-dimensional lattice structures called plasma crystals. Experiments on Mach cones, void formation, and strongly coupled phenomena have all been primarily performed using rf glow discharge plasma sources.

In dc glow discharge dusty plasma experiments, the regular 2-D crystalline structures are often not observed. The suspended microparticles generally remain in a more fluid-like state. Experiments have focused on e.g., dust acoustic waves and dust ion-acoustic waves, vortices, and three dimensional particle transport.

The main aim of early dusty plasma investigations was obtaining a good control of contamination in plasma-processing reactors, either by eliminating dust particles from the gas phase or by preventing them from getting into contact with the surface. This task has almost been accomplished, and the knowledge gained in the course of elaborate studies can now be utilized in new research directions. Application of macroscopic grains is one of the recent developments in the material science. The common use of low-pressure plasma processing reactors and the easy availability of the laser light scattering diagnostics showed that many of these discharges produced and trapped large quantities of macroscopic dust grains. These dust particles in the plasma are not anymore considered as unwanted pollutants. Contrariwise, at present they have turned into production goods. Low-pressure plasmas have the unique property of dust trapping, so that the position and residence time of particles in the reactor can be controlled. This offers numerous opportunities of particle processing, like surface modification (coating, etching), bulk modification (melting, crystallization), and many others. Particles generated in low-pressure discharges have typically very well-defined size and

shape. Scanning electron microscopes (SEMs) of the dust using a low-energy probe reveal narrow size distributions.

Significant amounts of small dust particles with sizes between a few nanometers and a few  $10 \mu m$  are found at the bottom of many fusion devices. Though it is not a major problem today, dust is considered a problem that could arise in future long pulse fusion devices. This is primarily due to its radioactivity and due to its very high chemical reactivity. These dust particles are heavier than the hydrogen isotopes which are the fuel in the fusion devices. Though some mechanisms leading to the formation of dust in tokamaks such as desorption, sputtering and evaporation have been identified, their relative importance is not yet adequately understood. Very little is known about the transport and fate of dust particles, e.g. whether they interact repetitively with the fusion plasma. Practically no experience exists about the behavior of radioactive particles in fusion devices. Charging due to radioactive decay can also give dust particles particular properties. Recently, the dust particles were collected from the TEXTOR-94 which is a medium sized tokamak [22]. It is not known yet how quickly dust particles can be transported in the tokamak interior.

## 1.5 Layout of the thesis

The previous sections of this chapter contains an introduction to dusty plasmas and some basic dusty plasma principles such as Debye length, macroscopic neutrality, Coulomb coupling parameter and characteristic frequency. A general overview of the presence of dusty plasmas in space and laboratory is also presented. The next chapter is devoted to the development of the theoretical background for understanding the dielectric theory and its role in the calculation of the electrostatic potential. A brief introduction of the test charge approach and the Debye screening of the test charge in the plasma and theoretical calculation of the dielectric constant is also given. Chapter 3 is devoted to the discussion of different charging models for dust particles in dusty plasmas. Both theoretical models (e.g. orbit motion limited theory, secondary electron emission, photoemission etc.) as well as the laboratory studies of charging processes is discussed. The grains of a dusty plasma also have a range of sizes and shapes leading to highly complex behavior. Chapter 4 contains the theoretical background which is helpful in finding the effect of dust size distribution on the electrostatic potential of a slowly moving test charge in a dusty plasma. Chapter 5 contains the results for the wake potential of moving test charge in dusty plasma. Preliminary results from the previous studies are presented and experimental verification

of wakefields were discussed. Finally, Chapter 6 contains a summary of the published papers included in the present thesis (P1-P6).



# Chapter 2

## Theoretical background

*In this chapter, we develop a theoretical background for the calculation of electrostatic potential by using the test charge approach.*

### 2.1 Introduction

Dusty plasmas have become a topic of great interest because they provide an excellent tool for exploring many of the fundamental assumptions used in plasma physics. The study of dusty plasma is important for at least four different scientific communities of plasma research namely astrophysics and space science, semiconductor manufacturing, basic plasma physics research and physics of atmosphere. Astrophysicist and space theorists were the first who took interest in the study of dusty plasmas because the places like planetary rings, comet tails and nebulae in the universe are full of dusty plasmas. Interest sparked by the space craft such as the Voyagers, which revealed the presence of the dust particles immersed in planetary magnetospheric plasmas. The dust particles were seen by Mie scattering of sunlight and detected electrically by their impact upon the space craft. In astrophysical context, the dust motion and dynamics is crucial to understanding the peculiar phenomena observed in planetary dust rings, in the interstellar dust clouds and in cometary tails. The big boost in the increase of the interest in dusty plasmas came from a completely different field - the semiconductor industry. Dust appeared to be a critical issue in the development of microelectronics. In the last few years industrial plasma processing researchers have discovered that particles suspended in plasmas are a major cause of costly wafer contamination during semiconductor manufacturing. Submicron particles deposited on the surface can obscure device regions, cause voids and dislocations, and reduce the adhesion of thin films. As the semiconductor technology aims

at smaller structures and thinner films, elimination of even smallest dust particles has become an urgent issue. Initially it was considered that contamination originates from external sources, but soon it was discovered that the plasma itself can create dust particles. It was found that the plasmas used for deposition and etching can grow particles in the gas phase, or they can release them from vacuum vessel walls. The particles become charged in the plasmas, and they levitate above the wafers, until the critical moment comes when the plasma is switched off and they fall on the substrate. Much effort has been made to understand particle growth, charging of the dust, levitation and transport in order to devise schemes aimed at avoiding contamination. For basic plasma physics researchers, dusty plasma offers a wide and unexplored territory, such as waves, instabilities and strongly coupled plasma. The presence of dust particles modifies the collective behavior of the plasmas, sometimes by only depleting the electrons, while at low phase velocity, the dust response leads to new collective electric fields, and new waves and instabilities. Dusty plasma systems represent a new frontier in applied physics and modern technology. They exhibit new and unusual behavior, and provide a possibility for modified or entirely new collective modes of oscillations and instabilities, as well as coherent nonlinear structures. Unlike usual plasmas, dusty plasmas can be easily condensed to form crystalline structures, providing new tools to study waves and oscillations in strongly coupled systems. Another fundamental field, in which the dust presence has an important role, is the physics of the atmosphere. Dust appears as aerosols in the atmosphere or as essential constituent of the noctilucent clouds. Due to environmental awareness, this novel interest is rising. Recognizing the great overlap in the physics of these different areas, and the rapid growth of interest in dusty plasmas, motivates us to study the dusty plasma.

A dielectric formulation of the electrostatic potential and energy loss was studied by Pines and Bohm [23], Sitenko [24], and other workers for the case of dilute electron-ion plasmas. Neufeld and Ritchie [25] studied the interaction of charged particles with a plasma in early 1950s. They calculated the potential distribution as well as the energy loss of a test charge in an electron plasma, considering a fixed background of ions. This work was further extended by incorporating the dynamics of ions [26, 27]. Arista and Ponce [28] evaluated the potential using a simplified model to describe long-(short-) wavelength collective-(individual-) excitations for an electron gas. Later, the electrostatic potential and energy loss of a pair of charges in correlated motion was calculated through a degenerate electron gas by using Lindhard's dielectric function [29]. Recently, Otani and Bhattacharjee [30] studied the Debye shielding and particle correlation for strongly correlated dusty plasmas. They used numerical simulations by considering different

numbers of dust particles in electron and ion plasmas, and found liquid-like short range correlation functions for two particles averaged over the total number of the dust particles. Later [31], the long-range correlation function for dusty plasma was developed and the numerical results for a broad range of dust densities and plasma temperatures were obtained. The correlations induced by the dust for the electron and ion densities as well as for the dust charges were investigated, and the results exhibited a long-range correlation. More Recently, M. H. Nasim et. al. [32] calculated the potential and energy loss of a single test charge particle in a multi-component dusty plasma using dielectric theory both analytically and numerically by ignoring the dust charge fluctuation effects and by using the Vlasov-Poisson model. They assumed the electron and ion number densities to be Boltzmann type while the dynamics of dust was considered to be governed by the Vlasov equation. This work was further extended to two projectiles [33] moving very close to each other in a weakly coupled dusty plasma. A system is said to be weakly coupled as long as its Coulomb potential energy is smaller than its thermal energy. The analytical and numerical results for the slowing down of two heavy projectile ions passing through a multi-component dusty plasma were presented. Within the linear dielectric approach, the electrostatic potential and the energy loss of these projectiles were computed for different values of  $K_D$  (the normalized effective wave number) and  $R$  (the separation between the two projectiles) retaining two-ion-correlation effects. An enhancement in the energy loss was observed. These results were compared with that of a single projectile case.

In this thesis, we have studied the electrostatic potential of a test projectile in the dusty plasma by using the linearized Vlasov theory [34, 35, 36]. We have presented a simple derivation of the electrostatic potential for the cases with and without dust size distribution and with/without the presence of dynamical grain charging. The effect of delayed shielding due to dynamical grain charging of the dust to the electrostatic potential is discussed. The separate results for the effects of grain size distribution and grain charging dynamics are presented. Later on, these two effects namely grain size distribution and grain charging dynamics are considered at the same time. The results for the wake field potential produced by a fast moving test charge in a dusty plasma are also presented.

## 2.2 Plasma as a dielectric medium

The plasma has been defined as a statistical ensemble of mobile charged particles; these charged particles move randomly in the system, interact with each

other through their own electromagnetic forces, and respond to the electromagnetic disturbance which may be applied from external sources. A plasma is therefore inherently capable of sustaining rich classes of electromagnetic phenomena.

A proper description of such electromagnetic phenomena may be obtained if we know the macroscopic plasma response to a given electromagnetic disturbance. The function which characterizes these responses is known as plasma response function. All the macroscopic properties of the plasma (as a medium) are contained in these response functions. If the external disturbance to the plasma is a time dependent electric field  $\mathbf{E}$ , then the response function appears in the form of the displacement  $\mathbf{D}$ . The relationship between the displacement and field given by the dielectric tensor as  $\mathbf{D} = \varepsilon \mathbf{E}$ . The dielectric tensor  $\varepsilon$  is complete in the sense that it contains all the information about the linear electromagnetic properties of the plasma. It is sometimes quite useful to single out certain element of this tensor and construct another response function known as the dielectric response function. This dielectric response function describes essentially the longitudinal properties of the plasma, and effectively describe the collective behavior of the plasma.

### 2.3 Test charge approach

The method has proved to be a very powerful tool to calculate the electrostatic potential as well as the energy loss of some particle considered as “test particle” in dusty plasma. It is useful to calculate the potential due to a point test charge moving with a uniform speed through a dusty plasma, which in the absence of the test charge is uniform and field free. The charge density of the test charge is represented by

$$\rho_t = q_t \delta(\mathbf{X} - \mathbf{V}_t t) \quad (2.1)$$

where  $q_t$  is the test charge and  $\mathbf{V}_t$  is the test charge velocity. This test charge can either be one of the plasma particle or a particle from the external beam, singled out as a test particle. If the test particle is at rest and the background plasma is Maxwellian, the potential  $\phi$  is a shielded potential. The test charge at rest polarizes the plasma, and acquires a shielding cloud. This shielding cloud is made up of an excess of charge having the opposite sign from  $q_t$  and a deficiency of the charge having like sign. This shielding of charge is known as Debye screening. The plasma can be thought of as built up of a collection of these dressed test particles.



The potential field around a test charge particle located at the origin ( $\mathbf{X} = 0$ ); in vacuum, is given as

$$\phi_c(\mathbf{X}) = \frac{qt}{X} \quad (2.2)$$

In the plasma the spatial distribution of charged particles is effected by the presence of such a potential field and deviates from a uniform distribution. The space-charge field so induced around the point charge in turn produces an extra potential field, which should be added to the original potential  $\phi_c(\mathbf{X})$ ; a new effective potential  $\phi(\mathbf{X})$  is thus constructed as a summation of these two. In the next section we calculate this dielectric response function (dielectric constant), using a kinetic theory treatment of the plasma.

## 2.4 Calculation of the dielectric constant

We calculate the dielectric response function of plasma within a kinetic-theoretical description for an unmagnetized plasma ( $B = 0$ ). For this purpose, we employ the Boltzmann-Vlasov equation coupled with Poisson's equation.

Linearized forms of the Vlasov and Poisson's equations for collisionless and field free plasma are given by the relations

$$(\partial_t + \mathbf{V} \cdot \nabla) f_{\alpha 1} - \frac{q_\alpha}{m_\alpha} \nabla \phi_1 \cdot \nabla_v f_{\alpha 0} = 0 \quad (2.3)$$

$$-\nabla^2 \phi_1 = 4\pi \sum_\alpha q_\alpha \int f_{\alpha 1}(\mathbf{X}, \mathbf{V}, t) d\mathbf{V} + 4\pi \rho_{ext} \quad (2.4)$$

The dielectric response function can be calculated by using the standard integral transform technique. The integral transform (Space-Time Fourier transform) can be defined as

$$\Psi(\mathbf{K}, \omega) = \int d\mathbf{X} \int dt \Psi(\mathbf{X}, t) \exp[-i(\mathbf{K} \cdot \mathbf{X} - \omega t)]$$

and the inverse transform as

$$\Psi(\mathbf{X}, t) = \frac{1}{(2\pi)^4} \int d\mathbf{K} \int d\omega \Psi(\mathbf{K}, \omega) \exp[i(\mathbf{K} \cdot \mathbf{X} - \omega t)]$$

Taking the Space-Time Fourier transform of equations (2.3) and (2.4), yield

$$f_{\alpha 1}(\mathbf{K}, \mathbf{V}, \omega) = -\frac{q_\alpha}{m_\alpha} \frac{\mathbf{K} \cdot \nabla_{\mathbf{v}} f_{\alpha 0}}{(\omega - \mathbf{K} \cdot \mathbf{V})} \phi_1(K, \omega) \quad (2.5)$$

$$K^2 \phi_1(K, \omega) = 4\pi \sum_{\alpha} q_\alpha n_{\alpha 0} \int f_{\alpha 1}(\mathbf{K}, \mathbf{V}, \omega) d\mathbf{V} + 4\pi \tilde{\rho}_{ext} \quad (2.6)$$

where  $\mathbf{K}$  is the wave vector,  $\omega$  is the wave frequency and  $\tilde{\rho}_{ext}$  is the Fourier transform of  $\rho_{ext}$ . Here we assume that the electric field perturbation is turned on adiabatically i.e.  $\text{Im}(\omega) > 0$ . This ensures that for  $t = -\infty$  the perturbation is zero and that we have a causal response [1]. Substituting the value of  $f_{\alpha 1}(\mathbf{K}, \mathbf{V}, \omega)$  into Equation (2.6) and solving for the potential, we get

$$\phi_1(K, \omega) = \frac{4\pi \tilde{\rho}_{ext}}{K^2 D(K, \omega)} \quad (2.7)$$

where  $D(K, \omega)$  is the dielectric response function [1], which plays a very important role to describe the behavior of the plasma and is written as

$$D(K, \omega) = 1 + \sum_{\alpha} \frac{\omega_{p\alpha}^2}{K^2} \int d\mathbf{V} \frac{\mathbf{K} \cdot \nabla_{\mathbf{v}} f_{\alpha 0}(\mathbf{V})}{(\omega - \mathbf{K} \cdot \mathbf{V})}$$

where  $\omega_{p\alpha}^2 = \frac{4\pi n_{\alpha 0} q_\alpha^2}{m_\alpha}$  is the plasma frequency of the  $\alpha$ -species.

If the plasma is in thermodynamic equilibrium, then with the aid of Maxwellian velocity distribution we calculate this response function. If the system is isotropic, we may choose the wave vector along the  $V_x$ -axis. Then, the dielectric response function is

$$D(\mathbf{K}, \omega) = 1 + \frac{1}{\sqrt{2\pi}} \sum_{\alpha} \sqrt{\frac{m_\alpha}{T_\alpha}} \frac{K_{d\alpha}^2}{K^2} \int_{-\infty}^{\infty} dV_x \frac{KV_x}{KV_x - \omega} \exp\left(-\frac{m_\alpha V_x^2}{2T_\alpha}\right)$$

still assuming that  $\text{Im}(\omega) > 0$ . The dielectric response function can be defined for all frequencies using analytical continuation [1]. We now write this as

$$\begin{aligned} D(\mathbf{K}, \omega) &= 1 + \sum_{\alpha} \frac{K_{d\alpha}^2}{K^2} W(Z_\alpha) \\ &= 1 + \sum_{\alpha} \chi_\alpha(K, \omega) \end{aligned} \quad (2.8)$$

where  $K_{D\alpha} = \omega_{p\alpha}/V_{t\alpha}$  is the Debye wave number and is the inverse of Debye length  $\lambda_{D\alpha} = \sqrt{\frac{T_\alpha}{4\pi n q_\alpha^2}}$ ,  $V_{t\alpha} = \sqrt{T_\alpha/m_\alpha}$  is the thermal velocity,

$Z_\alpha = \omega / (KV_{t\alpha})$ ,  $\chi_\alpha(K, \omega)$  is the susceptibility of a plasma species and  $W(Z)$  is the well known plasma dispersion function [1] explained in the following section.

## 2.5 Plasma dispersion function

The plasma dispersion function mentioned above is defined for  $\text{Im}(Z) > 0$  as

$$W(Z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx \frac{x}{x-Z} \exp\left(-\frac{x^2}{2}\right)$$

for  $\text{Im}(Z) = 0$ , we may write the plasma dispersion function [37]

$$W(Z) = 1 + Z \exp(-Z^2/2) \left[ i\sqrt{\frac{\pi}{2}} - \int_0^Z dy \exp(y^2/2) \right]$$

For the purpose of mathematical analysis, it is useful to find series expansion of  $W(Z)$  for small and large arguments.

For  $|Z| < 1$ ,  $W(Z)$  can be expressed in a convergent series as

$$W(Z) = i\sqrt{\frac{\pi}{2}} Z \exp(-Z^2/2) + 1 - Z^2 + \frac{Z^4}{3} - \dots + \frac{(-1)^{n+1} Z^{2n+2}}{(2n+1)!!} \quad (2.9)$$

where

$$(2n+1)!! = (2n+1)(2n-1)(2n-3)\dots(3)(1)$$

For  $|Z| > 1$ ,  $W(Z)$  can be expressed in an asymptotic series as

$$W(Z) = i\sqrt{\frac{\pi}{2}} Z \exp(-Z^2/2) - \frac{1}{Z^2} - \frac{3}{Z^4} - \dots + \frac{(2n-1)!!}{Z^{2n}} \quad (2.10)$$

## 2.6 Debye shielding

As mentioned earlier the dielectric response function provides a direct measure of the extent to which external test charge is screened by the induced space charge in the plasma. The screening is dynamic in the sense that it depends on the frequency variable as well as the wave vector. So, when the test charge is moving through a plasma with velocity  $V_t$  then the space-charge density associated with it is represented by equation (2.1), so that its Fourier components are

$$\tilde{\rho}_{ext}(\mathbf{K}, \omega) = 2\pi q_t \delta(\omega - \mathbf{K} \cdot \mathbf{V}_t)$$

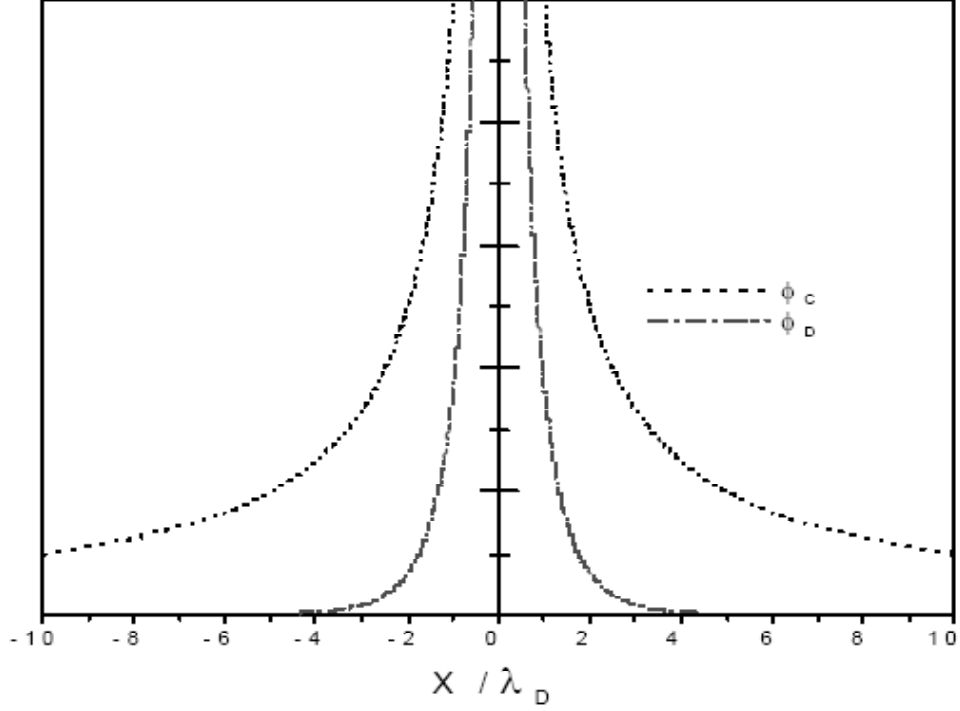


Figure 2.1: Comparison of Coulomb potential ( $\phi_C$ ) and Debye potential ( $\phi_D$ ) for  $\lambda_D = 1$  and  $q_t = 1$ .

using this value of space charge density in equation (2.7), the perturbed electrostatic potential will become

$$\phi_1(\mathbf{K}, \omega) = \frac{8 \pi^2 q_t (\omega - \mathbf{K} \cdot \mathbf{V}_t)}{K^2 D(K, \omega)} \quad (2.11)$$

carrying out the inverse transformation, and performing the  $\omega$  integration, we obtain the following result

$$\phi_1(\mathbf{X}, t) = \frac{q_t}{2 \pi^2} \int d\mathbf{K} \frac{\exp(i\mathbf{K} \cdot \mathbf{r})}{K^2 D(K, \mathbf{K} \cdot \mathbf{V}_t)} \quad (2.12)$$

where  $\mathbf{r} = \mathbf{X} - \mathbf{V}_t t$ .

The dynamic screening of the test charge for slow (fast) test charge velocity were calculated [38]. However, for a particular case in which the test charge is assumed to be stationary at the origin (i.e.,  $\mathbf{V}_t = 0$ ) immersed in a steady-state electron plasma, the dielectric constant described by the Equation (2.8), will reduce to

$$D(K, \omega) = 1 + \frac{1}{K^2 \lambda_D^2}$$

substituting this value of dielectric constant into Equation (2.12), we will get,

$$\phi_1(\mathbf{X}) = \frac{q_t}{2\pi^2} \int d\mathbf{K} \frac{\exp(i\mathbf{K} \cdot \mathbf{X})}{K^2 + \lambda_D^{-2}} \quad (2.13)$$

performing three-dimensional  $K$ -integration, we obtain the following result for  $\phi_1(\mathbf{X})$

$$\phi_1(\mathbf{X}) \equiv \phi_D(\mathbf{X}) = \frac{q_t}{X} \exp\left(-\frac{X}{\lambda_D}\right)$$

this potential is called Debye-Hückel potential.

The Debye-Hückel potential along with Coulomb potential is shown in figure (2.1). The space-charge distribution induced in the plasma is determined not only from the bare Coulomb potential  $\phi_C(X)$ ; but also from the effective potential in a self-consistent fashion. The calculations along these lines were originally carried out by Debye and Hückel [39] in 1923 in connection with the theory of screening in a strong electrolyte. For distances much smaller than the Debye length, the effective potential is essentially equivalent to the bare Coulomb potential, while for distances larger than the Debye length, the potential field decrease exponentially. The potential field around a test point charge is effectively screened out by the induced space-charge field in the one component plasma (OCP). Here, we extend these calculations to a multi-component dusty plasma containing electrons, ions and charged dust particles.



# Chapter 3

## Dust grain charging

*This chapter deals with the charging of dust grains in a dusty plasma. Different theories for the charging are presented. The results of an experiment on dust charging are also presented.*

### 3.1 Introduction

Dusty plasmas are low-temperature ionized gases that contain dust particles in the size range of  $nm$  to  $\mu m$ . Sometimes these particles can be considered as wanted particles and sometimes as unwanted, depending on the type of application. Any dust particle generated/immersed in the plasma tends to acquire an electric charge and hence respond to electric forces. Primarily, a dust particle immersed in plasma acquires electric charge by collecting electrons and ions from the background plasma and sometimes by emitting electrons. An important feature of a dusty plasma is the grain charge fluctuation [40]. Thus the electric charge on the dust particles is a time-dependent quantity and has to be analyzed as a dynamic variable. Calculation of charge on a particle is the starting point of every theory of dusty plasmas, and it is important to study the charging of particles, because it is necessary in determining or predicting most of the plasma properties.

Particles normally tend to acquire a negative charge. This negative charge can reach levels of more than 10,000 elementary charges, depending on the particle radius and the plasma density. Hence, such heavily charged objects will be strongly affected by electric fields. They can be suspended against gravity by the electric fields in wall sheaths. The Coulomb interaction is a factor that completely rules the particle kinetics. It determines particle velocities, spatial distributions in the plasma, their lifetimes and also the final size of the particles before they are expelled out of the plasma due to

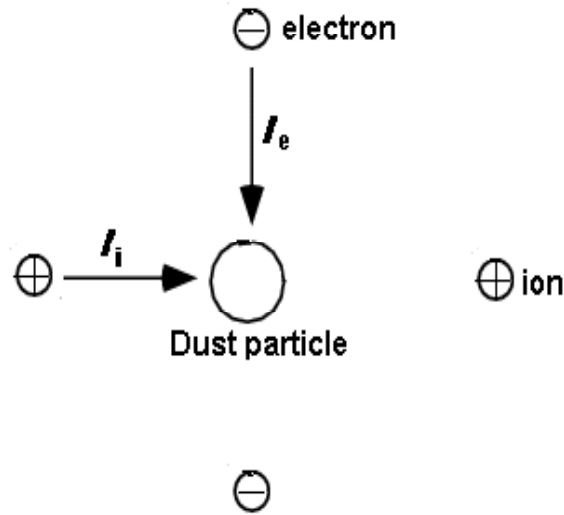


Figure 3.1: Charging of dust particle

forces acting on them. A good control over dust trapping and/or growth in a discharge can be achieved only with a detailed understanding of the Coulomb interactions, for which the knowledge of particle charge is a must.

The particle charge however is a dynamic quantity, as already noted. This does not allow for the generalization of results and separate measurements have to be performed at different operating conditions. The electron and ion interactions at the particle surface depend on the electron affinity of the particle, also the processes of secondary electron emission, photo emission etc. depend on the particle material. Hence, the final state depends not only on the plasma parameters but also on the chemical composition of the dust grains.

There are several processes which cause the charge on a grain, calculation of the equilibrium charge on a grain can become quite complex if all processes are included. For grains in a plasma with a temperature  $T_e$  for electrons (mass  $m_e$ ) and  $T_i$  for ions (mass  $m_i$ ) the fact that the flux of electrons have a thermal velocity  $v_{te}$  which is larger than that of the heavier ions which have a smaller thermal velocity  $v_{ti}$  will make the grain charge  $Q$  and its surface potential  $\phi_s$  negative. The impinging plasma particles onto a dust grain can also induce electrons and ion currents to the grains surface. These currents are functions of the surface potential of a dust grain and the sum of the currents due to different charging processes determines the charging rate  $\frac{dQ}{dt}$  of the grain. A particle with zero charge that is immersed in a plasma will gradually charge up by collecting the electron and ion currents according to



the relation [41]

$$\frac{dQ}{dt} = I_e + I_i$$

and charge equilibrium occurs when  $I_e + I_i = 0$  or  $\phi_s = -2.51kT/e$  for a basic model of an electron-ion plasma. Figure (3.1) shows a simple charging process on a dust particle. The charge itself is related to the surface potential by  $Q = C\phi_s$ , where  $C$  is the capacitance of a grain in a plasma [42]. Absorption of plasma ions by the dust grain leads to a positive charging, tending to raise the surface potential. The grain surface potential affects the currents of the electrons and the ions. The surface potential is often negative repelling electrons and attracting ions. The electron current is then reduced and the ion current enhanced. Charging of micron-sized grains can amount to several thousand electron charges, for masses of million to billion proton masses. Since grain charges are determined by the plasma potentials and can fluctuate, and dust comes in all sizes, in an almost continuous range from macromolecules to rock fragments, no wonder that dusty plasma research is a wide open field.

Due to the electric charge attained by the dust particles in the plasma, they can interact with the plasma electric field or with the electrons and ion flows as well as with each other via electric forces. The study of dusty plasma cannot be carried out without a detailed analysis of the plasma and of these interactions.

Here, we shall consider the phenomenon of charging of these dust particles and the effect it has on study and analysis of plasmas in general. Different theories that exist for charging and determination of floating potential of the dust grains are studied and their advantages/disadvantages are discussed.

## 3.2 Different dust charging mechanisms

The dust charge is an important property of dust grains since it affects the particle dynamics. The main point in dealing with the study of dusty plasmas is the understanding of the charging of dust grains once they are immersed/introduced into the ordinary electron-ion plasmas. There are many elementary processes like the collection of plasma particles, photoemission and thermionic emission etc. which can lead to the charging of grains. These processes could be quite complex and mainly depend on the environment around the dust grain itself. Assuming that the particles that are formed in the plasma from monomers or externally added are initially neutral, then there exist different mechanisms for charging. They are:

### 3.2.1 Collection of plasma particles

Let us suppose that we introduce the neutral dust particles in a simple electron-ion plasma. What happens is that the plasma particles (electrons and ions) start impinging on the surface of these newly introduced dust particles and some of them get collected on the dust grain surface. As we know that the electrons possess much more higher thermal velocities as compared to ions and therefore, the electrons will reach the dust grain surface much earlier than the ions thus making the dust grain charge and its surface potential negative. On the other hand, the collection of positively charged ions tend to make the dust grain charge and its surface potential positive. These electron and ion currents are also effected by the grain surface potential. When the grain surface potential is negative, it starts repelling the electrons and attracting the ions. In other words, the current carried by the electrons is reduced while the current carried by the ions is increased and vice versa.

In the following, we shall calculate the charging current  $I_e$  (for the electrons) and  $I_i$  (for the ions) to the dust grain by using the orbital motion limited theory.

#### The basic model: orbit-limited theory

The simplest model that predicts the charge of particles in plasma is based on orbit-limited probe theory. A brief review of the common orbit-limited theory of charge collection is presented. The classical Orbit Motion Limited theory developed by Mott-Smith and Langmuir [43] in 1926 represented a reference in the probe diagnostic analysis for several decades. In fact, this was the starting point for a new field of research: charging of small particles in plasmas. This theory describes the way how the study of the currents collected by a small probe ( $a \ll \lambda_D$ [44, 45]) leads to information on the plasma parameters. We consider a finite-sized neutral dust particle immersed in an unmagnetized plasma whose constituents are electrons and ions. This particle is like floating probe in the plasma, collect charge carriers and attain a certain charge, negative or positive, depending on the dominant charging mechanism. The charge on dust particle can arise for example, either by collecting charge carriers (electrons and ions) from the plasma or sometimes by emitting electrons. In a plasma in which emission processes are unimportant, the equilibrium charge is negative because the flux of electrons to an uncharged surface is high relative to that of ions. On the other hand, when electron emission is significant, the equilibrium charge is positive.

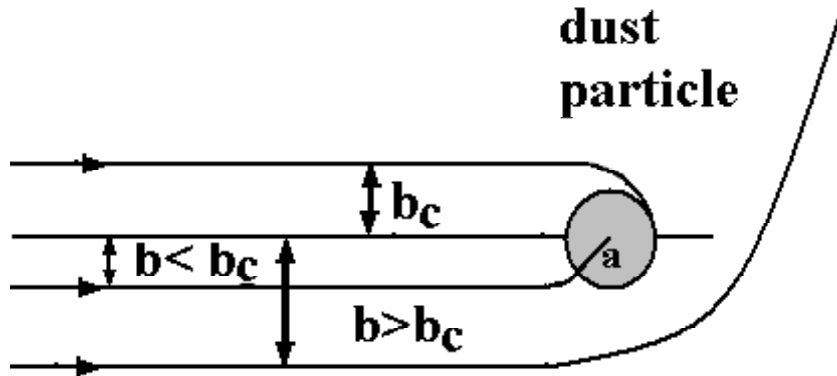


Figure 3.2: Only ions with an impact parameter smaller than the limit  $b_c$  can contribute to the ion current on the dust grain. The other ones ( $b > b_c$ ) are deflected in the electrostatic potential of the particle.

**Ions and electron currents** According to the OML theory, the ions collide with the particle when they have an impact parameter smaller than a well-defined value  $b_c$  (see figure (3.2)). If the impact parameter is larger than  $b_c$ , the ions are deflected in the electrostatic field of the particle but are not collected on the particle surface. The above mentioned limit  $b_c$  corresponds to the case where the ions impact to the particle tangentially and depends on the magnitude of the attractive central potential. This gives the name Orbital Motion Limited (OML) theory.

When integrating over the contribution of all the ions which hit the particle, the ion current is obtained as

$$I_i = en_i a^2 \sqrt{\frac{8kT_i}{m_i}} \left( 1 - \frac{e\phi_f}{kT_i} \right) \quad (3.1)$$

where  $\sigma_i = b_c^2 = a^2 \left( 1 - \frac{e\phi_f}{kT_i} \right)$  is the ion collection area of the particle,  $\sqrt{\frac{kT_i}{m_i}}$  is the ion thermal velocity, and  $\phi_f$  is the particle floating potential. The integration has been carried out under the assumption of a Maxwellian ion distribution. It can be observed that  $b_c^2 > a^2$ , since  $\phi_f < 0$ , thus the ion collection area of a negative particle is larger than its geometric cross-section area.

For the electrons, the calculations are similar, with the only difference that the electron collection cross section is  $\sigma_e = b_{c,e}^2 = a^2 \left( 1 + \frac{e\phi_f}{kT_e} \right)$ . The

electron current is

$$I_e = -\pi en_e a^2 \sqrt{\frac{8kT_e}{\pi m_e}} \exp\left(\frac{e\phi_f}{kT_e}\right) \quad (3.2)$$

It was considered here that only electrons and ions with higher thermal energies  $kT_{e,i} > e\phi_f$  are collected. The results of the OML theory have been proved to be valid when employed in the case of small particles ( $a \ll \lambda_D$ [44, 45]). For essentially all conditions of interest in connection with dusty plasma, the grain size is small enough to justify the use of OML theory for quantitative calculations of the ion current to the grain and thus of the grain charge and floating potential.

### Limits of validity for OML theory

The orbital-motion-limited (OML) theory has been widely used to calculate the ion response to a charged grain immersed in plasma. The validity of OML theory is limited to the ideal case when the electrons and ions are collisionless, and when the density and velocity distributions of charge carriers are isotropic. It is also assumed that the potential near the grain has a suitable form [44]. The condition of collisionless ions and electrons is not fulfilled in typical experimental situations due to the short collision mean free paths of the charge carriers. For example in Helium, at pressures at around 100 Pa, the mean free path for ions  $\lambda_{mfp}$  is of the same order, or smaller than the Debye length. On the other hand, when the ions and electron drifts are present in the plasma, then the populations of the charge carriers are no longer Maxwellian when the drift velocity is comparable to thermal velocity. The OML theory is shown to become exact in the limit of small grain size, and to be very accurate in calculating ion current to the grain for typical conditions pertinent to dusty plasma.

### 3.2.2 Secondary electron emission

The effect of electrons or ions bombarding grains with high energies can lead to an ionization of the grain material and ejection of electrons from the grain surface. This process is called secondary electron emission. The emission of secondary electrons can be caused by both electrons and ions. The flux of secondary electrons depends on the energy of the plasma electrons/ions. The number of secondary electrons also depends on the material properties of the dust grains.

### Electron impact

In case of electrons, secondary electron emission is a phenomenon that occurs when electrons impact on a dust grain surface with sufficient energy to ‘knock’ additional electrons from the surface of that grain. This electron outflow can be considered as positive current onto the particle [46, 47]. Generally, one electron gives rise to several secondary electrons. When the electron strikes the dust grain surface, several things can happen: An electron may be scattered or reflected by the dust grain surface without penetration. It may be stopped and stick at the grain surface. If the electron penetrates the dust grain surface, it may excite other electrons which may then be emitted at the surface. Some electrons may pass through the grain and leave with a little loss of energy. For plastic materials similar to the particles used in most of the experiments, high electron and ion energies are necessary for producing secondaries, and thus the contribution of the secondary electron emission to the particle charging is small (negligible).

### Ion impact

The electron secondary emission can also be produced by ion bombardment [48, 49, 50]. When the low energy ions (less than 1  $KeV$ ) are incident on a dust grain surface, these ions may become neutralized by the electrons tunneling from within the grain across the potential barrier. The energy released can then excite other electrons leading to surface emission.

### 3.2.3 Photoemission

The absorption of solar ultraviolet radiation by the dust particle can release photoelectrons from the dust surface and hence can lead to positive charging current on the dust grain surface. Photoelectrons can be emitted from the surface of a dust grain when there is an incoming flux of photons with energy  $h\nu$  ( $h$  is Planck’s constant and  $\nu$  is the photon frequency) exceeding the photoelectric work function of the dust grain ( $W_f$ ). The emission of photoelectrons depends on (i) the frequency (energy) of the incident photons, (ii) the dust grains surface area, (iii) the material properties of dust grain i. e. its photoemission efficiency and (iv) on the grain surface potential, which may, if positive, recapture a fraction of the photoelectrons. It is worth mentioning here that various metals have photoelectric work function  $W_f < 5 eV$ .

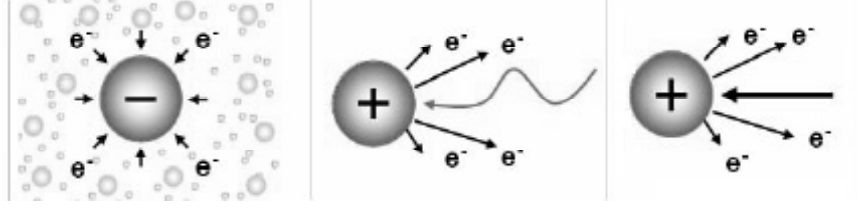


Figure 3.3: Charging of dust particles in a dusty plasma takes place mainly by (i) collection of plasma particles (left), (ii) photoemission (centre) and (iii) secondary electron emission (right).

### 3.2.4 Other charging process

Apart from the above mentioned processes which represent the main source of electric charge on the dust particles, a few other processes namely thermionic emission, field emission, radioactivity and impact ionization can be important, under circumstances which are mostly relevant under the extreme conditions especially for dusty plasmas in space and will not be discussed here.

## 3.3 Dust charging in the laboratory

In the previous section, we have discussed various charging mechanisms for charging of dust particles in a dusty plasma. The discussions were mainly based on theoretical investigations. In this section, we confine ourselves to a laboratory experiment for the measurement of the dust grain charge in dusty plasma. In the experimental studies of dusty plasmas, two interrelated lines of research have been followed. One is related with the experimental verification of dust charging processes, and the other with observing wave modes due to charged dust. Our concern is with the processes related to the experimental explanation/verification of the dust charging processes. Some parallel experimental studies have focused on the charging processes [Walch et. al. [51] and Barken et. al. [52]]. Since we are interested mainly in the studies where dust particles take part in the collective behavior (non-isolated dust grains), we shall confine ourselves to the results of an experiment by Barken et. al. [52] for the case of non-isolated dust grains ( $a \ll d < \lambda_D$ ) where  $a$  is the dust grain radius,  $d$  the average intergrain distance and  $\lambda_D$  is the plasma Debye length.

It is well known that any dust particle generated/immersed in a plasma tend to acquire an electric charge (negative or positive depending on the dominant charging mechanism) and hence will respond to the electric forces.

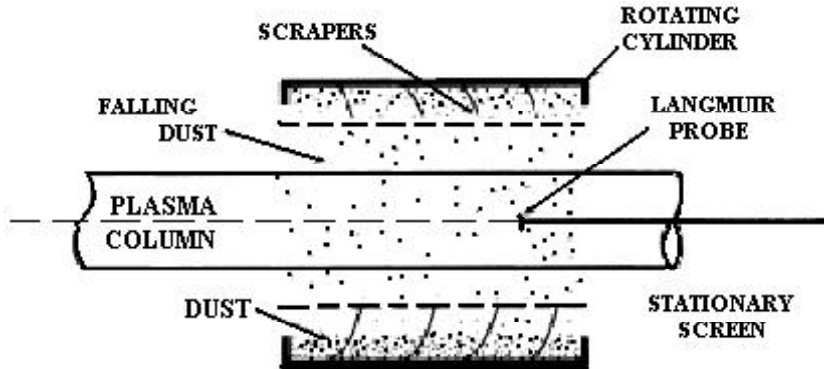


Figure 3.4: Schematic diagram of the device used to disperse dust into the plasma column [52].

In order to understand dusty plasma formation and their effects in regions in space and earth's atmosphere, it seems essential to understand the charging mechanisms of dust grains. Barken and his coworkers [52] carried out an experiment for studying the dust charging mechanism in 1994. Their experimental setup consisted of a plasma column, a rotating cylinder and a stationary screen as shown in Figure (3.4). The plasma column used was 4 cm in diameter and 80 cm long and some portion of this plasma column was surrounded by a dust dispenser. Dust dispenser was composed of a rotating cylinder and a stationary screen (to limit the dispensed dust grains size). Potassium plasma was used in this experiment. By the surface ionization of potassium atoms from an atomic beam oven on a hot tantalum plate ( $\sim 2500K$ ), Barken and his coworkers were able to produce a fully ionized magnetized plasma ( $B \leq 4 \times 10^3 G$ ). The plasma parameters used for this experiment were  $T_e = T_i = 0.2 eV$  and plasma densities were in the range  $\sim 10^6 - 10^{10} cm^{-3}$ .

The material used for the dust grains was hydrated aluminium silicate. The dust grains were in various sizes and shapes in the beginning and stationary screen was used to limit the dust grains size to less than  $100 \mu m$ . An electron microscope was used to study the sizes and shapes of the grains. By analyzing the samples of dust grains from within the chamber and of the photographs taken with the electron microscope, the researchers were able to find the average dust grain size and dust number density to be  $r_d \simeq 5 \mu m$  and  $n_d \simeq 5 \times 10^4 cm^{-3}$  respectively. Almost 90% of the dust grains were found to lie in the size range  $1 - 15 \mu m$ .

In this experiment, the dust grains were considered to be negatively charged (they become negatively charged by the collection of fast moving

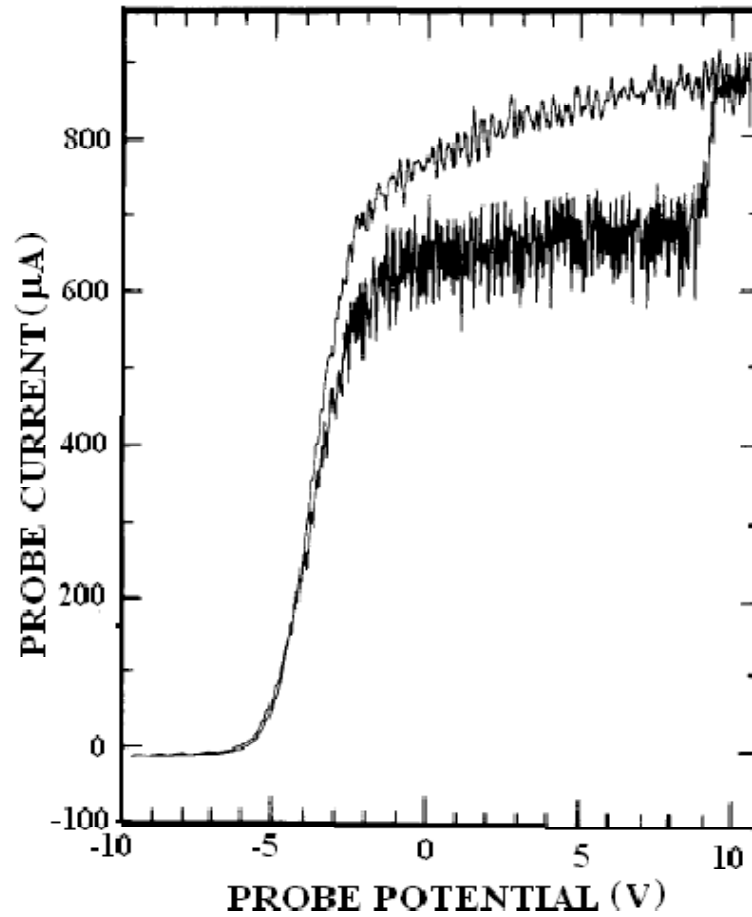


Figure 3.5: Langmuir probe characteristics obtained under identical conditions, except for the absence (upper plot) and the presence (lower plot) of kaolin dust [52].



electrons). In order to understand how the negative charge is distributed between electrons and negatively charged dust grains in the plasma chamber, a Langmuir probe was used. The experimental data for Langmuir probe was obtained and plotted under the identical conditions except for i) the absence of dust and ii) presence of dust. The upper curve in figure (3.5) shows the Langmuir probe characteristics in the absence of dust while the lower curve shows the characteristics when the dust is present. It can easily be observed from this figure (3.5) that the electron saturation current in the absence of dust (upper curve) is larger than the case when the dust is present (lower curve). The explanation was that when the dust particles are present, some electrons becomes attached to the dust grain surface. These electrons which are collected on the dust grain surface have very low energy and therefore, were not collected by the Langmuir probe. Hence a significant decrease in electron current was observed in the case when dust is present as compared to the case when dust is not present.



# Chapter 4

## Dust grain size distribution

*This chapter contains the theoretical background which is helpful in finding the effect of dust size distribution to the electrostatic potential of a slowly moving test charge in a dusty plasma.*

### 4.1 Introduction

It is well known fact that the dust grains in space come in all sizes, in an almost continuous spectrum ranging from macromolecules to rock fragments and asteroids. This is much less the case for the laboratory experiments, where the range of the dust particles used is controlled within tight limits. While many papers has tried to account in some way for charge fluctuations, comparatively little has been done in other facets of dust grains, like their distribution in mass and size. In the absence of fully self-consistent kinetic theories, we will consider charged dust as continuously distributed in size range, using a power law for the number of particles of smaller sizes and an exponential decrease for large masses. In the present case, this distribution leads us to an equivalent to the kappa (Lorentzian) distribution of monosized particles. Before proceeding further, it seems reasonable to discuss Lorentzian or Kappa distribution as our results contained in P-1 mainly concern with this distribution.

### 4.2 Lorentzian or kappa distribution

The Lorentzian (or Kappa) distribution has been widely used for conventional plasmas (not for a dust component). In many physical situations, both in the laboratory and in space, velocity distributions are not Maxwellian [53]. Most

space plasmas are observed to have velocity distribution functions exhibiting non-Maxwellian suprathermal tails. Shocks and turbulent fields provide acceleration mechanisms that may lead to distributions of a power-law form. The distribution function decreases generally as a power law of the velocity  $v$  instead of exponentially [54], and have been found useful in modeling both laboratory and space plasmas. A useful function to model such plasmas is the generalized Lorentzian (or kappa) distribution [55] and its simple one dimensional form is given by the following relation:

$$f_{k1D}(v) = \frac{(\kappa\pi\theta^2)^{-\frac{1}{2}} \gamma(\kappa)}{\gamma(\kappa - 1/2)} \left\{ 1 + \frac{v^2}{\kappa\theta^2} \right\}^{-\kappa} \quad (4.1)$$

This generalized Lorentzian function is very convenient to model observed velocity distributions [56], since it is quasi-Maxwellian at low and thermal energies, while its non-thermal tail decreases as a power-law at high energies, as generally observed in space plasmas. For typical space plasmas,  $\kappa$  generally lies in the range 2 – 6.

This ‘‘Kappa’’ distribution tends to a Maxwellian for  $\kappa \rightarrow \infty$  since

$$\left\{ 1 + \frac{v^2}{\kappa\theta^2} \right\}^{-\kappa} = \exp\left(-\frac{v^2}{\theta^2}\right) \quad (4.2)$$

for  $\kappa \rightarrow \infty$ . The kappa distribution holds as versatile tool for the kinetic modeling of waves and instabilities in space plasmas. The Maxwellian distribution is, in fact, simply a special case of the kappa distribution, i.e., the kappa distribution reduces to Maxwellian in the limit as  $\kappa$  tends to infinity.

### 4.3 Kinetic description of the dusty plasma with dust size distribution

The presence/addition of a charged dust component to a conventional plasma makes the physics more complex. It modifies conventional plasma behavior and leads to the appearance of the new wave modes. A dusty plasma can be modeled as a multifluid system including the dust as a negatively charged fluid. It has gradually become clear that the dust can be a very active component with, sometimes, a dramatic effect on its surroundings and it is very important to study the properties of single or collections of charged dust particles and their effects on the environment in which they occur. Although in most papers, concerning dusty plasmas, the dust grains are considered as mono-sized because of the treatability reasons, a real dusty plasma consists of grains of different sizes. A basic modification to conventional plasma theory

is that the dust grains can be expected to have a distribution of sizes [57, 58]. It is then natural to introduce an extended distribution function  $f(\mathbf{x}, \mathbf{v}, a)$  including the distribution over the particle size  $a$ . The density distribution with respect to size,  $h(a)$ , is then

$$h(a) \equiv \int f(\mathbf{x}, \mathbf{v}, a) d^3v$$

It is further assumed that there is a unique charge  $q(a)$  for a given radius, the evolution of the distribution function  $f(\mathbf{x}, \mathbf{v}, a)$  is given by linearized Vlasov equation[59]

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} + \frac{q(a)}{m(a)} E \frac{\partial f_0(v, a)}{\partial v} = 0$$

The charge  $q(a)$  is determined by the charging and discharging processes acting on the dust grains. All perturbations are assumed to depend on a single space coordinate  $x$  and corresponding velocity  $v$ . With a standard Landau analysis [60], applying a Fourier transform in space and Laplace transform in time and solving the initial value problem leads that the dust responds to electrostatic perturbation as if it had an effective zeroth-order distribution function[61]:

$$f_{d0}^{eff}(v) \equiv \frac{n_d}{\omega_{pd}^2} \int_0^{\infty} \frac{q(a)^2}{\varepsilon_0 m(a)} f_0(v, a) da \quad (4.3)$$

Integrating over  $v$ , it follows that  $\omega_{pd}^2$  is given by

$$\omega_{pd}^2 \equiv \int_0^{\infty} \frac{q(a)^2}{\varepsilon_0 m(a)} h(a) da$$

This give the dust plasma frequency in the presence of dust size distribution.

## 4.4 Effective dust distribution function

Particle distribution functions with power law forms are frequently observed in space plasmas throughout the solar system. In practice, they are often modeled using Kappa distributions (defined below). The existence of such non-Maxwellian distribution functions can have profound effects on wave

propagation and transport processes in these plasmas; effects that are of significant interest in space physics. The kinetic theory of such processes depends in an essential way on the plasma dielectric function and, therefore, on the plasma dispersion function.

It is clear that the form of the size distribution function  $h(a)$  must depend on the complex combination of processes leading to the production of the plasma dust component. In order to have a specific example of the size distribution function  $h(a)$ , the following form will be assumed:

$$h(a) da = An_d a^\beta \exp(-\alpha^3 a^3) da$$

where

$$A = 3\alpha^{\beta+1}/\Gamma([\beta+1]/3)$$

and the equilibrium density of all the dust grains is given by

$$n_d = \int_0^\infty h(a) da$$

Here  $n_d$  is the total number density of the dust particles. The exponential dependence on the cube of the radius (i.e., mass) has been used in astrophysics, for example, in the early work of Oort and van de Hulst [62]. The power law is introduced here to give a convenient adjustable reduction for small sizes (In what follows, it will appear that the present choice is particularly fortunate, leading to simple explicit results). The size distribution function  $h(a)$  plotted against grain size  $a$  is shown in Figure (4.1). It can be observed that as  $\beta \rightarrow \infty$ , the distribution function approaches a delta function for monosized particles.

A fully realistic choice of the distribution over velocity for a given grain size would also require a detailed analysis of collision and relaxation processes. The suitable simple first choice could be thermal distribution for example a Maxwellian distribution with a unique temperature, independent of the particle size [59]. Assuming now a dust temperature  $T$ , the velocity distribution function  $f(v, a)$  for one velocity component is

$$f_0(v, a) = h(a) \left\{ \frac{m(a)}{2\pi kT} \right\}^{\frac{1}{2}} \exp\left(-\frac{m(a)v^2}{2kT}\right) dv da \quad (4.4)$$

where mass  $m(a)$  follows directly from the material density of the dust grain and its volume:

$$m(a) = \frac{4}{3}\pi\rho a^3$$



Comparing equation (4.5) with equation (4.1), this can easily be recognized to be a kappa distribution also known as the generalized Lorentzian distribution with  $\kappa = (1/2 + \beta/3)$ . Due to the equivalence established here for the linear electrostatic response of a dust component, many of the results from previous studies of the kappa distribution can immediately be applicable to the present case.



# Chapter 5

## Plasma wake potential

*This chapter contains a brief introduction to the wake potential of a moving test charge in dusty plasma. A brief overview of the problem is presented including the experimental evidence of the existence of wakefields in plasma.*

### 5.1 Introduction

Dusty plasmas consist of heavy highly charged dust particles immersed in a plasma environment. The massive and highly charged dust species introduces a number of new and interesting phenomena into plasma physics. In particular, there has been a great deal of interest in the study of wake fields in plasma due to their applications in many phenomena e.g., in the acceleration of particles [63, 64] and in the formation of dust particles into regular crystalline structures in dusty plasmas [65, 66, 67, 68].

The theory of wake potentials [69, 70, 71] has got much attention in recent years. A number of mechanisms have been proposed to explain the attractive wake field responsible for crystal formation.

Vladimirov and Nambu [69] first showed that the collective interaction of the static dust particulate with low frequency oscillations in the ion flow in a dusty plasma can provide the attractive oscillatory wake potential along the ion flow direction. Vladimirov and Oshihara [70] extended this theory to consider periodic structures along and perpendicular to the ion flow direction. Later on, Oshihara and Vladimirov [71] showed that charged dust grains in a sheath region can attract each other when they were trapped in the potential minima behind the dust particle in the trailing wake cone due to the collective effect of ion acoustic waves. Such an attraction between highly charged dust particles overcomes the Coulomb repulsion and may explain the observed crystal formation in a plasma.

Shukla and Rao [72] explained the wake potential formation in terms of the resonant interaction of the drifting grains and the extremely low frequency dust-acoustic waves in an unmagnetized dusty plasma. They argued that the resonant interaction of the slowly moving or static grains with the extremely low frequency dust-acoustic wave involving the dust grains would be more effective in forming the wake potential.

Recently, M. H. Nasim [73] studied wake-field excitations in a multi-component dusty plasma by using the fluid as well as the Vlasov-Poisson model. It was shown that the ion-wake field may be oscillatory ( $V_t < C_{DA}$ ), constant ( $V_t \simeq C_{DA}$ ), or exponentially decreasing ( $V_t > C_{DA}$ ), depending upon the test charge velocity  $V_t$  relative to the dust acoustic speed  $C_{DA}$ . More recently, S. Ali [74] has studied attractive wake field formation due to an array of dipolar projectiles in a multicomponent dusty plasma for modified dust acoustic waves and using Laplace Fourier technique, presented analytical and numerical results for the wake field potential due to a cluster of  $N^2$  projectiles.

## 5.2 Dust-acoustic and dust-ion-acoustic waves

It is well known that an unmagnetized dusty plasma supports low frequency electrostatic dust-acoustic [75] and dust-ion-acoustic waves [76] which are two novel potential disturbances when charged dust grains are added in an electron-ion plasma. In the low phase velocity (in comparison with the electron and ion thermal velocities) dust-acoustic waves, the restoring force comes from the pressure of the inertialess electrons and ions, while the inertia is provided by the dust mass. The time and space scales on which the dust-acoustic wave (DAW) arises are completely new and were absent in the normal electron-ion plasma. On the other hand, the dust-ion-acoustic wave (DIAW) is comparatively a fast wave with its phase velocity lying between electron and ion thermal velocities. The dust-ion-acoustic wave (DIAW) frequency is much larger than the dust plasma frequency and hence stationary dust grains do not respond to the dust-ion-acoustic wave. In the case of dust-ion-acoustic wave, the restoring force comes from the pressure of inertialess electrons, whereas the ion mass provides the inertia. Due to the unequal number of electrons and ions in dusty plasmas, the phase velocity of dust-ion-acoustic wave (DIAW) is larger than the usual ion-acoustic velocity  $C_s = \sqrt{T_e/m_i}$  [77] (where  $T_e$  is the electron temperature and  $m_i$  is the ion mass. Both the DIAW and the DAWs have been observed experimentally [78, 79, 80].

## 5.3 Wake potential

In order to calculate the potential of a test charge including the wakefield effects in a collisionless and unmagnetized dusty plasma, we consider two types of electrostatic responses and ignore the Landau damping in both cases. The general expression for the electrostatic potential of a test particle with charge  $q_t$  moving with velocity  $V_t$  is [35]

$$\phi = \frac{q_t}{2\pi^2} \int \frac{\delta(\omega - \mathbf{K} \cdot \mathbf{V}_t) \exp i[\mathbf{K} \cdot \mathbf{X} - \omega t]}{K^2 D(K, \omega)} d\mathbf{K} d\omega \quad (5.1)$$

where  $D(K, \omega)$  is the dielectric constant. If we take a frame of reference moving with the test charge, the potential is then independent of time so that we change the coordinates by putting  $\mathbf{r} = \mathbf{X} - \mathbf{V}_t t$ .

### 5.3.1 Wake potential for dust-acoustic (DA) waves

In the first case, we assume the presence of low-frequency dust-acoustic (DA) waves having the dielectric constant of the form

$$D(\mathbf{K}, \omega) = 1 + \frac{1}{K^2 \lambda_D^2} - \frac{\omega_{pd}^2}{\omega^2} \quad (5.2)$$

taking the inverse of this dielectric constant and inserting it into equation (5.1), we get

$$\phi = \phi_D + \phi_{W1}$$

where  $\phi_D = (q_t/r) \exp(-r/\lambda_D)$  is the well known Debye-Hückel potential, whereas  $\phi_W$  is the wake potential given by

$$\phi_{W1} = \frac{q_t}{2\pi^2} \int \frac{\lambda_D^2 \omega_{da}^2 \delta(\omega - \mathbf{K} \cdot \mathbf{V}_t) \exp[i \mathbf{K} \cdot \mathbf{r}]}{(1 + K^2 \lambda_D^2)(\omega^2 - \omega_{da}^2)} d\mathbf{K} d\omega \quad (5.3)$$

with  $\omega_{da} = K \lambda_D \omega_{pd} / (1 + K^2 \lambda_D^2)^{1/2}$  is the DA waves frequency. It can easily be observed from the above equation (5.3) that the potential changes its sign depending upon whether  $\omega$  is larger or smaller than  $\omega_{da}$ . If  $\omega$  is close to  $\omega_{da}$ , there appears a strong resonant interaction between waves and the test particle and when the latter moves with a velocity larger than the phase velocity of the DA wave, the potential behind the test particle appears to be a wake potential. The solution of the above equation (5.3) for the long wavelength (as compared to Debye length) DA waves is [81] with  $K_{\perp} \rho < 1$ , using cylindrical coordinates  $\rho$  and  $Z$  (parallel to  $\mathbf{V}_t$ ) is

$$\phi_{W1}(\rho = 0, Z) = \frac{2q_t}{|Z - V_t t|} \cos\left(\frac{|Z - V_t t|}{L_{da}}\right) \quad (5.4)$$

where  $L_{da} = \lambda_D (V_t^2 - \lambda_D^2 \omega_{pd}^2)^{1/2} / \lambda_D \omega_{pd}$  is the effective length giving an upper limit to the integral in the  $K_Z$  direction. It can be observed from above equation (5.4), that the wake potential is attractive for  $\cos\left(\frac{|Z - V_t t|}{L_{da}}\right) < 0$ .

### 5.3.2 Wake potential for dust-ion-acoustic (DIA) waves

In this case, we assume the presence of dust-ion-acoustic (DIA) waves for which the dielectric constant is

$$D(\mathbf{K}, \omega) = 1 + \frac{1}{K^2 \lambda_{De}^2} - \frac{\omega_{pi}^2}{(\omega - \mathbf{K} \cdot \mathbf{v}_{i0})^2} \quad (5.5)$$

where  $\mathbf{v}_{i0}$  is the ions mean velocity near the plasma sheath.

Taking the inverse of this dielectric constant and inserting it into equation (5.1), we get

$$\phi = \phi_D + \phi_{W2}$$

where  $\phi_D$  is the well known Debye-Hückel potential as defined previously and in the above section, whereas  $\phi_{W2}$  is the wake potential given by

$$\phi_{W2} = \frac{q_t}{2\pi^2} \int \frac{\lambda_{De}^2 \omega_{di}^2 \delta(\omega - \mathbf{K} \cdot \mathbf{V}_t) \exp[i\mathbf{K} \cdot \mathbf{r}]}{(1 + K^2 \lambda_{De}^2) [(\omega - \mathbf{K} \cdot \mathbf{v}_{i0})^2 - \omega_{di}^2]} d\mathbf{K} d\omega \quad (5.6)$$

with  $\omega_{di} = KC_s / (1 + K^2 \lambda_{De}^2)^{1/2}$  is the DIA wave frequency and  $C_s = \sqrt{T_e/m_i}$  is the ion-acoustic speed. It can easily be observed from the above equation (5.6) that the potential changes its sign depending upon whether  $\omega - \mathbf{K} \cdot \mathbf{v}_{i0}$  is larger or smaller than  $\omega_{di}$ . If  $\omega - \mathbf{K} \cdot \mathbf{v}_{i0}$  is close to  $\omega_{di}$ , there appears a strong resonant interaction between waves and the test particle and when the latter moves with a velocity larger than the phase velocity of the DA wave, the potential behind the test particle appears to be a wake potential. The solution of the above equation (5.6), using  $L_{di}$  to give a cut-off to the integral in the  $K_Z$  direction, is [72]

$$\phi_{W1}(\rho = 0, Z) = \frac{q_t}{|Z|} \cos\left(\frac{|Z|}{L_{di}}\right) \quad (5.7)$$

where  $L_{di} = \lambda_{De} |M^2 - 1|^{1/2}$ ,  $M = u_{i0}/C_s$  and for a plasma with streaming ions  $v_{i0} = \widehat{Z}u_{i0}$ . It can be observed from above equation (5.4), that the wake potential is attractive for  $\cos\left(\frac{|Z|}{L_{di}}\right) < 0$ .

## 5.4 Experimental verification of wake potential

In order to confirm the theoretical prediction of wakefields [72, 81] and attractive negative force (responsible for the formation of dust crystals and dust molecules) between the dust grains, Takahashi et al. [82] carried out an experiment by using the optical manipulation technique method (in this method the dust particles are moved by radiation pressure from laser light) in a simple hexagonal crystal. In the experiment, a rf plasma was created by providing current to the electrode at 13.56 MHz. The plasma chamber was filled by methane gas diluted with Argon gas at a pressure of 87 Pa. Carbon particles of nano-meter size were used as seeds for particles which after 30 minutes growth at a rf power of 2 W appeared to be in spherical form with a diameter of  $5.4 \mu m$  and the equilibrium position of these particles was near the plasma-sheath boundary. As a result of illumination of these particles by an Ar ion laser (wave length 488 nm), a simple hexagonal Coulomb crystal was found and was observed by a video camera which was used to capture the scatted light.

Now in order to manipulate the particles, a laser with power density larger than Ar ion laser is to be used. For this purpose, a semiconductor laser with a wavelength of 690 nm was used. Upon the passage of this semiconductor laser light through the transparent particles, it was observed that the top particles in vertical rows in dust crystal moved along the direction of the light propagation. It was also observed that the upper particles causes an attractive force on lower ones but lower ones could not cause a force on the upper particles. Assuming that there are particles in ion flows of the pre-sheath or sheath region, the author argues that the dust attraction in the experiment is due to the wake potential. This experiment demonstrated the formation of dust Coulomb crystals due to the wake potential in a plasma with a finite ion flow and showed that a dust particulate located in the upstream of ion flows creates an attractive force on another dust particulate in the lower part. It was found that many dust particulates are strongly correlated in the vertical direction along the ion flow. The grains are found to link together and arrange themselves into a crystalline structure.

Later on, Melzer et al. [83], performed an experiment on the formation of two-particle dust molecules by taking two particles of different masses. It also confirmed the previous theoretical predictions [72, 81] and experimental results [82].



# Chapter 6

## Summaries of the papers

The Doctoral thesis presented here is based on a collection of six published papers (P-1 to P-6). In this thesis work, theoretical studies of the potential perturbations caused by a test charge in dusty plasmas have been carried out by focusing attention principally on dust size distribution and dust charging dynamics. The main strands of investigation presented are:

- Test charge response of a dusty plasma with a grain size distribution,
- Shielding of a slowly moving test charge in dusty plasma with dynamical grain charging
- Delayed shielding of a slowly moving test charge in dusty plasma due to dynamical grain charging
- Drag force on a test charge in dusty plasma in the presence of grain charging dynamics using Maxwell stress tensor
- Test charge response of a dusty plasma in the presence of both grain size distribution and grain charging dynamics and
- Plasma wake potential for a moving test charge in dusty plasma for both with and without grain charging dynamics

In our calculations we have treated the dust grains both for the case of a constant size and with a distribution over size. Moreover, the effects of dust-neutral collisions for both the case of dust size distribution as well as that of dust charging dynamics is also incorporated.

The summaries of these papers are given in the following sections. Discussions and conclusions are presented for each paper in the corresponding section.

## 6.1 Paper 1

It is well known that the dust grains in a dusty plasma come in all sizes ranging from nanometer to micrometer sizes. In natural plasmas or manufactured plasmas contaminated with dust, a distribution of grain sizes is to be expected. The form of the grain size distribution strongly influences the linear dielectric response of a dusty plasma. In paper one (P-1), the test charge response of a dusty plasma with a grain size distribution was studied and analytical results for the electrostatic response to a slowly moving test charge, using a second order approximation were found. The effects of collision were also taken into account.

A dust size distribution  $h(a)$  over the grain radius  $a$  of the form  $h(a) = An_d a^\beta \exp(-\alpha^3 a^3)$  was assumed. This gives an exponential decrease with grain mass for large sizes and gives a simple smooth reduction for small sizes. It was shown that this particular choice for the size distribution leads to an effective distribution known as kappa distribution (generalized Lorentzian distribution) given by the expression [Eq. (5) in P-1]

$$f_{d0}^{eff}(v) = Cn_d \left[ \frac{3kT\alpha^3}{2\pi\rho} \right]^{(\beta+1)/3} (v^2 + 3\alpha^3 kT/2\pi\rho)^{-(5/6+\beta/3)}$$

where  $C$  is a constant and  $\kappa = \frac{2\beta+5}{6}$  (as discussed in paper 5 that this value of  $\kappa$  is incorrect and should be replaced by  $\kappa = \frac{2\beta+3}{6}$ ). The expression for the electrostatic potential was found and expressed in the form of strength functions. It was also found that the expressions for these strength functions are in turn functions of two parameters namely  $A(\beta)$  and  $B(\beta)$  and an effective Debye wave number given by the relation  $K_{eff} = \sqrt{K_{De}^2 + K_{Di}^2 + K_D^2}$ , where  $\beta$  is the power law index of the size distribution for small radii and is related to the kappa distribution by the relation  $\kappa = \frac{2\beta+5}{6}$ . It was shown that the effect of different choices for the size distribution are contained in the parameters  $A(\beta)$  and  $B(\beta)$  as well as in the effective Debye wave number  $K_{eff}$ . The effective Debye wave number  $K_{eff}$  was shown to depend through  $K_D^2 \left( K_D^2 = \int_0^\infty \frac{q(a)^2}{\varepsilon_0 kT} h(a) da \right)$  on the form of the size distribution which applies for a general choice of the grain size distribution function  $h(a)$ . The structure of the strength functions also depends on the power law index  $\beta$  of the size distribution through the parameters  $A(\beta)$  and  $B(\beta)$  and strength functions are shown to contain a significant term  $A^2(\beta) K_D^2 - B(\beta) K_{eff}^2$ . The analysis leads to a weak dependence on the power law index  $\beta$  but it becomes significant when  $K_D \rightarrow K_{eff}$ .

Taking into consideration the low frequency nature of the plasma response, collisions can be expected to modify the plasma response. Generaliz-



ing the standard analysis by including a distribution over dust grain size, the dielectric response function  $D(\mathbf{K}, \omega)$  for a Krook collision model and BGK model were found and shown to be functions of grain size distribution with the introduction of a size dependent collision frequency  $\nu_a = \nu_0 S(a)$ , where  $\nu_0$  is constant and  $S(a)$  is the shape function. It was noted that both cases reduce to the collisionless case if the newly introduced collision frequency  $\nu_a$  is assumed to be zero. The asymptotic result for the collisional response linearly proportional to  $\mathbf{V}_t$  was found to vary as  $\nu_0 r^{-2}$ , in agreement with the previous results [84]. The corresponding collisionless response was found to be  $r^{-3}$ , and it was concluded that the relative importance of the collisional term increases with distance.

## 6.2 Paper 2

In this paper (P-2), the shielding of a slowly moving test charge in dusty plasma with dynamical grain charging is taken into account and it was found that the dynamical charging of grains in a dusty plasma enhances the shielding of test charges. The dielectric constant for a dusty plasma was found to be modified by an extra term if the effects of grain charge fluctuation were taken into account. The response potential was found as a power series (in  $\mathbf{V}_t$ ) and the analytical expressions were found up to second order in test charge velocity.

It was shown that the dielectric response function for a dusty plasma is modified by an extra term if the effects of grain charge fluctuations are included. The expression for the electrostatic potential up to second order in test charge velocity for this case was found and expressed in terms of strength functions. These results clearly showed the effect of charging dynamics on the dusty plasma response to a moving test charge. An increase in the effective Debye wave number was noted which leads to an enhancement in the shielding of the test charge. The strength functions were found to contain groups of terms with exponential shielding factor  $\exp(-K_1 r)$ . The strength function proportional to first order in test charge velocity was found to be [Eq. (30) in P-2]

$$g_{11}(r) = \left[ \frac{\pi^2 K_{dch}^2}{\nu_0} \exp(-K_1 r) - \frac{\sqrt{2}\pi^{3/2} K_d^2}{K_1^2 r V_{td}} \left\{ 1 + \frac{\Phi_1(K_1 r)}{2} - \frac{(1 + K_1^2 r^2) \Psi(K_1 r)}{2K_1 r} \right\} \right]$$

However, extra unshielded terms due to dynamical charging appeared in the strength functions proportional to second order in test charge velocity.

These terms were expressed as a combination of algebraic functions of the radial distance  $r$  and the newly defined functions  $\Phi(K_1 r)$  and  $\Psi(K_1 r)$ . A comparison of the present analyses in the absence of dynamical charging ( $K_{dch} = 0$ ) was made with the earlier results [85] and were found to be in agreement after some corrections to previous work were made. It was argued that the response due to charging dynamics could be interpreted as the effect of shielding delayed by the finite charging time of the dust grains. This effect was considered in detail in paper 3 ( P-3) explained in the following section.

The effects of collisions were also taken into account and an analysis was made using the Bhattanagar-Gross-Krook (BGK) model. The dielectric response function  $D(\mathbf{K}, \omega)$ , for a BGK model was found and shown to be function of both plasma dispersion function  $W(\xi)$  and a function  $G(\xi)$ , given by the relation

$$G(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\exp(-t^2/2)}{t - \xi} dt$$

Both  $W(\xi)$  and  $G(\xi)$  depend on the form of the equilibrium distribution function (here Maxwellian) and are related in general as  $W(\xi) = -G'(\xi)$ . The asymptotic result for the collisional response linearly proportional to  $\mathbf{V}_t$  was found to vary as  $\nu_0 r^{-2}$  which is in agreement with previous results [84]. The collisionless response was noted to be asymptotically proportional to  $r^{-3}$  leading us to conclusion that the relative importance of the collisional term increases with distance.

### 6.3 Paper 3

It is well known that the dynamical charging of dust grains in a dusty plasma modifies both the plasma dielectric response function and the nature of the electrostatic wave modes. In this paper (P-3), we have presented analytical results for the electrostatic potential due to a test particle moving with a velocity  $V_t$  in a multi-component dusty plasma using a plasma dielectric that takes into account the dynamical charging of the dust grains. The dynamics of the charging was approximated using a time delay response to the potential and the resulting potential was shown to contain the delayed shielding effects explicitly. The expression for the electrostatic potential was separated into two parts,  $\phi_1$  being independent of the dynamical charging and  $\phi_{ch}$  including the dynamical charging effects. The analytical expression for the potential  $\phi_{ch}$  was found to be [Eq. (15) in P-3]

$$\phi_{ch} \approx \phi_R(r) + \phi_s(r) + \frac{q_t K_{dch}^2 V_t \cos \lambda}{8\pi \varepsilon_0 \nu} \left\{ 1 - \frac{K_{dch}^2 K_1 r}{2K_1^2} \right\} \exp(-K_1 r)$$

If the time delay due to dynamical grain charging was considered to be negligible ( $\nu^{-1} \approx 0$ ) only the first two terms,  $\phi_R(r)$  and  $\phi_s(r)$  giving the instantaneous response would be significant. These two terms arise from the enhanced shielding due to grain charge fluctuations. The remaining terms came from the finite grain charging time and leads to the spacial shift of the potential response. This gave a clearer interpretation of the effects of dynamical grain charging on the potential response: the modifications are due to a combination of enhanced shielding and a finite grain charging time.

## 6.4 Paper 4

In the test particle case, an important force is the drag force. A test particle includes a polarization cloud that follows it as it moves through a dusty plasma. This polarization cloud exerts a drag on the test charge. As a result of this drag, the test particle can emit plasma waves and electromagnetic radiation. The drag force acting on the charged dust grains can also cause an instability of electrostatic waves in dusty plasmas. In the present work (P-4), the energy loss (drag force) of a test charge projectile passing through a dusty plasma in the presence of dynamical grain charging up to first order in test charge velocity with two different methods was studied. In the first case, the drag force was calculated with the help of Maxwell stress tensor. The electric force was written in terms of the Maxwell stress tensor for a sphere around the test charge. For a sphere with radius tending to zero the force was found to be just that on the test charge. For a finite radius, forces on the plasma were also included which made it possible to see how the force on the test charge was balanced by the force on the plasma. However, this method fails for the zero radius. In the second case a direct approach was used and the drag force was found from a simple physical model. The general analytical results were presented and compared with the previous results.

## 6.5 Paper 5

It is well known that the form of the grain size distribution strongly influences the linear dielectric response of a dusty plasma. For a class of size distributions and a thermal velocity distribution, there is an equivalence to a Lorentzian distribution of mono-sized particles. On the other hand, the dynamical charging of grains in a dusty plasma leads to an enhanced time dependent shielding of a test charge. In this paper (P-5) the combined effect of both grain size distribution and dynamical grain charging on the

response to a slowly moving test charge are analyzed. The dynamical charging contribution to the plasma dielectric has a complicated dependence on the parameters for the size distribution and on the charging rate. However this dependence can be expressed in terms of known functions. Series expansions were used to derive the potential response to the test charge. Previous work on grain size distribution [P-1] was extended by taking into account the effect of charging dynamics. The analytical as well as numerical results for the response potential for a test charge moving through a multicomponent dusty plasma were presented and it was found that the previous results can be retrieved by choosing appropriate values for different parameters.

An important point to be mentioned here is about the derivation and the final form of kappa distribution or generalized Lorentzian distribution. In deriving the expression for kappa distribution [Equation (5) in P1], a factor  $q(a)^2/\varepsilon_0 m(a)$  was mistakenly dropped. While retaining this factor, a correct expression for kappa distribution with the spectral index  $\kappa = \frac{2\beta+3}{6}$  was found in the present case. The weighing factor  $q(a)^2/\varepsilon_0 m(a)$  appearing in equation (5) [equation (4) in P1] also leads to an effective dust density  $n_{eff}$  which is different from the actual dust density  $n_d$  and is given by the expression

$$n_{eff} = \int_{-\infty}^{\infty} C n_d \left[ \frac{3kT\alpha^3}{2\pi\rho} \right]^{\frac{\beta}{3}} \left( v^2 + \frac{3kT\alpha^3}{2\pi\rho} \right)^{-(1/2+\beta/3)} dv = n_d \frac{\Gamma(\beta/3)}{\Gamma(\beta/3 + 1/3)}$$

## 6.6 Paper 6

The potential response of a dusty plasma to a moving test charge strongly depends on the speed of the test charge. For a test charge moving with a high velocity as compared to the dust-acoustic speed, a distinctive wake-field is produced trailing behind the position of the test charge. In this communication (P-6), we have presented the dusty plasma response to a fast moving test charge, when dispersion effects are small and the dust behaves as a cold plasma component. The effects of dynamical grain charging are included, and the cases with and without these effects are analyzed and compared. In the first case, we presented results for the wake potential of a test charge in the absence of grain charging dynamics. In the second case, the wake-field excited by a test charge moving with a relatively higher speed through a dusty plasma was calculated in the presence of grain charging dynamics. In the second case, grain charging dynamics lead us to a spatial damping and a phase shift in the potential response. Maximum response was found on the wake cone with apex angle determined by the ratio between the

dust acoustic velocity and the test charge velocity. The structure of the wake field was found to be stretching in the direction of the test charge velocity when it increases. The physics of the plasma was represented by the choice of the dielectric function while all grains were considered to be of same size.



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