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1. INTRODUCTION

TONAL NOISE emitted from duct systems, e.g. from air-conditioners [1] or turbochargers of car engines [2], has a strong impact on perceived noise annoyance. The literature shows a rich variety of studies aiming to design and enhance compact silencers for various applications. Traditionally, silencers and mufflers are designed either for the low frequency range (plane waves) or for high frequencies with a large number of propagating modes. Typical applications with low frequencies (or long wavelengths compared to the duct radius) are intake and exhaust systems in automobiles whereas for instance the HVAC (heating, ventilation and air conditioning) systems of power plants exhibit high frequencies. For both these extreme cases a large number of publications exist and a reference work on the subject is Ref. [3]. The intermediate and mid- frequency range, however, where the plane wave and a low number of acoustic modes propagate, is much less treated in the literature on silencers. The importance of this frequency range for e.g. medium sized HVAC systems as found in aircraft was highlighted in the recent EU-funded project IDEALVENT (“Integrated Design of Optimal Ventilation Systems for Low Cabin and Ramp Noise”) [1,4]. As part of the project, measurements and numerical computations uncovered the strong contribution of higher order acoustic modes to the noise emitted by such systems [5,6].

The work presented in this paper, aims at finding new solutions for controlling sound in such applications with a low number of dominant modes. The mitigation strategies are based on micro-perforated plates (MPPs) placed along the propagation path of the sound waves creating surfaces with acoustic resistance. The MPPs are used in two configurations; the first configuration aims at damping the plane wave mode, based on the work of Kabral and Åbom [7], and is designed to realize the so called Cremer impedance at a given target frequency. By adopting a wall impedance for optimal attenuation which was firstly described by Cremer [8] for rectangular ducts and later derived for circular ducts by Tester [9], extremely high damping can be achieved with very compact silencers as shown in Ref. [10]. The Cremer silencer is realized with locally reacting chambers and micro-perforated cover plates. At a frequency where the filter impedance matches the Cremer impedance, optimal damping is guaranteed for the plane wave. The drawback, however, is that the design can only realize the optimum Cremer impedance at one single frequency [7]. Furthermore, the impedance is optimized to damp the plane wave whereas other modes experience less damping and can be strongly reflected at the silencer inlet.

In systems of larger duct dimensions, e.g. HVAC systems in aircraft, higher order acoustic modes can be present and spinning modes may be triggered particularly strongly, for instance in rotating machinery [5,6]. For such cases an additional silencer or filter stage is required to damp the higher order spinning modes. Here an idea based on so called ‘modal filters’ originally proposed by Allam and Åbom [11] will be tested. A modal filter consists of MPPs that are inserted perpendicularly to the duct cross sections (parallel to the duct axis and the flow), which affects spinning modes only. To date this design has not been investigated in the literature and the paper at hand aims at describing its basic behavior in terms of filter length, plate position, and plate impedance by solving the Helmholtz equation (for cases without flow) with internal impedance boundary conditions. The Helmholtz equation is solved numerically using the Finite Element Method (FEM) for varying internal boundary conditions. The effect of different plate impedances and plate positions is tested. For model validation, a filter prototype consisting of four plates is built and its acoustic properties are measured.

The performance of silencers is commonly described by the Transmission Loss or Insertion Loss [3]; those values, however, do not represent the actual absorption inside the damper. Therefore the Power Loss is computed, which requires the decomposition of measured and computed acoustic pressure fields into their complex modal amplitudes. To this end a modal decomposition technique is used which was presented in detail in earlier work [12]. The decomposed data allows for attributing the Power Loss inside the silencer to single modes, which is useful for the design process as well as for gaining knowledge of the damping mechanisms. As shown in this paper, the proposed modal filter is completely ‘invisible’ for the plane wave mode, which predestines its application in a two-filter stage, e.g. in combination with a Cremer silencer.

The paper has the following structure. First, the multi-port model is introduced (section 2 A), that describes the properties of the acoustic silencer with a scattering matrix containing reflection and transmission coefficients. This scattering matrix can be used, to compute the Transmission Loss, the Power Loss and the Reflection Loss, as described in section 2 B. Next, the two silencer types are introduced; the Cremer silencer in section 2 C and the modal filter in section 2 E. A numerical model to evaluate different properties of the modal filter is set up and validated in section 3 B and using this model, the influence of the filter length, the plate position and the plate resistance is investigated in section 3 C. Finally, measurements are performed on a combination of two filter prototypes and the Transmission Loss and Power Loss is computed in section 3 D.

2. THEORY

A. Mode Matching Method and Scattering Determination

Figure 1: Sketch of the measurement setup to determine the scattering matrix S of the silencer [12]. Multiple external loads and sensors on the inlet and outlet are used to create independent sound fields and decompose them into complex mode amplitudes.

To characterize the acoustic properties of a silencer, most importantly the transmission of sound between its inlet and outlet has to be determined. For efficient silencers this transmission will be very low. However, knowing only the sound transmission does not allow for conclusions about the true absorption inside the silencer, as fractions of the noise may be reflected at the silencers inlet. A complete study of a silencer should therefore not only include sound transmission but also inlet reflections. A passive multi-port model in the form of a scattering matrix as described in Ref. [13] contains a complete set of transmission and reflection coefficients that fully describes the acoustic properties of the silencer. The scattering matrix is determined with the multiple source (or multiple excitation) method described in Ref. [12] corresponding to the two-source method for the plane wave range [13–15]. This procedure is independent of reflections from the boundaries of the test setup and eliminates the need for reflection free terminations. This is an advantage compared to traditional testing of silencers as it is described in various standards that often requires reflection-free terminations which can be difficult to realize especially for low frequencies.

Following the terminology in Ref. [12] a scattering matrix S can be defined

$$\mathbf{p}_+ = S \mathbf{p}_-, \quad (1)$$

where $\mathbf{p}_\pm = [\mathbf{p}_{a\pm} \ \mathbf{p}_{b\pm}]^T$ are vectors of complex mode amplitudes propagating upstream and downstream at the inlet ‘a’ and the outlet ‘b’ of the silencer as shown in Figure 1 and the superscript T denotes the transpose. The scattering matrix contains reflection matrices $\mathbf{R}$ at the reference cross sections (‘a’ and ‘b’) and transmission matrices $\mathbf{T}$ between those reference cross sections for all modes

$$S = \begin{bmatrix} \mathbf{R}_a & \mathbf{T}_{ba} \\ \mathbf{T}_{ab} & \mathbf{R}_b \end{bmatrix}. \quad (2)$$

All components of the scattering matrix are related to complex pressure amplitudes of the acoustic duct eigenmodes which have to be extracted from sampled sound fields using special post-processing. Determining the scattering can be a challenging task, particularly when several modes are propagating. For efficient silencers the differences in the pressure fluctuations at the inlet and the outlet are deliberately large which puts demands on the measurement systems in terms of measurement range and accuracy.

The multi-load and mode-matching procedures used in the experiments and numerical computations of this paper are described in detail in Ref. [12]. Within the scope of the present work, only ducts with circular cross sections are considered. The shapes of the induct modes can be described as

$$\Psi_{mn}(\varphi, r) = \exp(\mathbf{i} m \varphi) J_m(\kappa_{mn} r), \quad (3)$$

where the mode shape Ψ_{mn} at a polar coordinate (φ, r) is the product of the circular mode shape $\exp(\mathbf{i} m \varphi)$ and the radial mode shape $J_m(\kappa_{mn} r)$, with J_m being the Bessel function of first kind and order m , $\mathbf{i} = \sqrt{-1}$ being the complex unit, and κ_{mn} being the radial wavenumber or the n -th root of the radial derivative of J_m at the duct wall. The modes in this paper are denoted as (m, n) -modes, where m is the azimuthal mode order, n is the radial mode order, and the

(0,0)-mode is the plane wave mode. The phase velocity of the modes is given by a wave number, which for a quiescent media is

$$k_{mn} = \sqrt{k^2 - \kappa_{mn}^2}, \quad (4)$$

with $k = \frac{\omega}{c}$ being the wave number of the plane wave mode. The wave-numbers for the experiments in this paper are additionally modified to account for thermo-viscous damping at the duct walls, based on the theory in Ref. [16].

B. Measurement of Power Dissipation in Silencers

The literature suggests different measures for the acoustic Power Loss in silencers. All approaches relate an incident acoustic power P_i to an emitted acoustic power P_o and express the Power Losses PL as

$$PL = 10 \log \left(\frac{P_i}{P_o} \right). \quad (5)$$

Common characteristics of a silencer are the Insertion Loss IL and the Transmission Loss TL [3]. The Insertion Loss relates the acoustic power at the inlet and the outlet side of the silencer under working conditions, i.e. without anechoic rig terminations. In contrast, the Transmission Loss is usually measured on special test rigs under anechoic conditions. For a certain mode it is represented by the transmission coefficients in the scattering matrix in Eq. (2)

$$TL_{ab} = -10 \log(|T_{ab}|^2). \quad (6)$$

Note that this formula assumes ducts with the same cross-section and fluid properties on both sides. The Transmission Loss is a useful measure, as it relates the acoustic power at the inlet and the outlet of the silencer. However, it does not account for reflections at the silencer's inlet. As a consequence, strongly reflective (and hence low transmissive) systems have high Transmission Losses, even though only little power is actually absorbed inside the silencer. Kabral and Åbom [7] defined a Dissipation Loss, which subtracts reflections from the incident acoustic power in order to correct the Transmission Loss and to show the actual power dissipation inside the silencer. The normalized incident acoustic power is then defined as $P_i = 1 - |R_a|^2$ and the emitted power is $P_o = |T_{ab}|^2$. Hence, the dissipative Transmission Loss is

$$TL_{dis,ab} = 10 \log \left(\frac{1 - |R_a|^2}{|T_{ab}|^2} \right), \quad (7)$$

and the ratio of transmitted and dissipated acoustic power is referred to as the reflective Transmission Loss, such that

$$TL_{ab} = TL_{dis,ab} + TL_{ref,ab}, \quad TL_{ref,ab} = -10 \log(1 - |R_a|^2). \quad (8)$$

In this work, the reflected acoustic power at the filter inlet is accounted for as emitted acoustic power rather than being subtracted from the incident power. The normalized incident power is hence $P_i = 1$ and the emitted power is $P_o = T_{ab}^2 + R_a^2$. Inserting this into Eq. (5) results in the Power Loss based on the scattered sound

$$PL_{ab} = -10 \log(T_{ab}^2 + R_a^2). \quad (9)$$

Eq. (9) is valid for a single mode, but can easily be generalized to account for the scattering between modes by introducing a Power Loss vector

$$PL = -10 \log(\text{diag}(\tilde{\mathbf{S}}\mathbf{S})), \quad (10)$$

where $\tilde{\mathbf{S}}$ is the complex conjugated and transpose of the scattering matrix $\mathbf{S}$ and the logarithm is applied element wise on a vector containing the diagonal elements of $\tilde{\mathbf{S}}\mathbf{S}$. The vector PL contains the Power Losses for the different modes from inlet to outlet and vice versa. In the same way a Transmission Loss vector can be defined

$$TL = -10 \log(\text{diag}(\tilde{\mathbf{T}}\mathbf{T})), \quad \mathbf{T} = \begin{bmatrix} \mathbf{T}_{ab} & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_{ba} \end{bmatrix}. \quad (11)$$

The matrix notation allows investigation of the damping in the acoustics modes separately. If scattering between different modes and cases with flow are considered, the exergy scattering matrix defined in Ref. [17] must be used instead of the conventional scattering matrix.

C. Noise Mitigation Strategies for Midrange Frequency Sound

The noise mitigation strategies presented in this paper target frequencies for which a low number of higher order induct modes are propagating. They are based on perforated plates that are used as acoustic resistances placed at positions where high acoustic velocity perturbations are expected.

Figure 2: a) Mode shapes of the first four circular duct modes in ascending order of their cut-on frequencies computed with Eq. (3). Shading depicts the pressure maxima. b) Possible positions for resistive plates which would result in high acoustic velocity normal to the surface (dashed lines).

In Figure 2 a), the mode shapes of the pressure of the first four circular duct modes in ascending order of their cut-on frequencies are plotted. The position of the velocity maxima changes with the mode order and so does, by implication, the position of the resistive plates too (Figure 2 b)). In this work, silencer designs are presented that are specific for single modes and which can be used in a cascade to realize good dissipation in applications where multi-modal sound is propagating. As this design has not been investigated in the literature, the focus of this paper is on the properties of the so called ‘modal filter’ which is introduced in more detail in section 2 E.

For the plane wave mode, a new type of dissipative silencer proposed by Kabral et al. [7,10] with cavity structures covered by perforated plates is suggested, using optimized wall impedance (Cremer wall impedance) for a certain target frequency (section 2 D).

D. Compact Dissipative Silencers with a Cremer Wall Impedance

Kabral and Åbom used an expansion chamber muffler with a micro-perforated cover tube to design silencers with optimal wall impedance (Cremer impedance, [8]) that reduces tonal noise of compressors in turbochargers [7],[10]. In their design strategy, cavity structures are fitted with a micro-perforated cover sheet, so that the wall impedance equals the Cremer value at one target frequency. The Cremer impedance for the plane wave in a circular wave guide is given by Tester [9]

$$Z_{Cc} = (0.88 - 0.38i) \frac{kr}{\pi(1+M)^2}, \quad (12)$$

where r is the duct radius and M is the Mach number of the background plug flow.

In Figure 3, a sketch of a Cremer silencer and a photo of a prototype are presented. The geometry was designed corresponding to the theory in Ref. [10] in order to get peak damping at a Helmholtz number of $He=3.7$ for a duct of radius $r=0.075$ m. The frequency and the duct dimensions were linked to the application with noise control on an axial compressor of an HVAC system as part of the European project ‘IDEALVENT’ [1,5,6]. The design is very compact with a 13% increase in outer diameter and a filter length of only one duct diameter (1D) but (as shown in section 3 D of this paper) the setup is very efficient for the plane wave at the target frequency.

Figure 3: a) Sketch of the Cremer silencer where wall cavities are covered with micro-perforated sheets. b) Photo of the prototype.

E. Modal filter

Åbom and Allam first mentioned the idea of using micro-perforated plates perpendicular to the duct cross sections (in axial direction) in order to filter higher mode components from the acoustic field [11]. Until now, no detailed analysis and test of this concept has been published. The essential idea is to place an acoustic resistance in the form of a perforated plate in the velocity maxima of a given spinning duct mode and to use the thermo-viscous losses of oscillating air in the holes of the MPPs in order to reduce the acoustic energy. As some modes will be affected to a greater extent than others depending on the number of inserted plates and the plate position, such a silencer is denoted as a ‘modal filter’. For the investigations in this paper, the plates are placed in a star-shaped manner, where a single plate divides the cross section in two parts, two plates divide the cross section in four parts and so forth.

In Figure 2 b), positions for resistant plates in the velocity maxima are already presented for the (1,0)-mode and (2,0)-mode. Such configurations, however, do not cause sufficient damping. The reason being that spinning modes always appear in complex conjugated pairs, i.e. the (m,n) mode is the (+m,n) mode plus the (-m,n) mode. Those degenerate states may be written in trigonometrical form corresponding to Eq. (3)

$$\Psi_{\pm mn}(\varphi, r) = J_m(\kappa_{mn} r) (\cos(\varphi m) \pm i \sin(\varphi m)) . \quad (13)$$

As the cosine has its maxima in the minima of sine and vice versa, a filter with the form presented in Figure 2b) can only attenuate parts of the spinning mode and leaves a non-spinning part. A physical interpretation may be the superposition of the degenerate states which creates standing wave patterns inside the cross section as the imaginary part (spinning part) of the mode vanishes for $\Psi_{+mn} + \Psi_{-mn}$. A pair of degenerate modes with equal energy content effectively does not spin and velocity nodes are established at the plate position. On the one hand, this creates energy dissipation, as parts of the propagating mode have to be transformed into its degenerate correspondence. But once this state is reached, the plates located in the velocity nodes are ineffective and do not further dissipate energy. This state is stable as it trivially fulfils the impedance boundary condition for the Helmholtz equation at the plates and no attenuation in axial direction is experienced. In order to reach such an equilibrium state, a fraction of the spinning mode has to be decelerated and accelerated in the opposite direction; the rotational energy from the acceleration stays within the propagating modes whereas the energy needed for the deceleration is dissipated. This is ideally half of the original energy. In other words, the maximum Power Loss for such a filter is 3dB regardless of the plate impedance or the filter length.

In order to sufficiently dampen both of the degenerate states, a principle has to be employed that uses MPPs as acoustic resistances (dashed lines) in both the velocity nodes and the velocity maxima (visualized in Figure 4). A modal filter of this design is able to counteract velocity nodes at the plates as it adds resistance to both parts of the spinning modes. It should be mentioned, however, that in Figure 2b), the mode shapes are assumed to be uninfluenced by the MPPs; this is not true as the internal boundary condition is not trivial anymore and a boundary value problem has to be solved in order to correct the mode shapes. The paper in hand does not aim at finding an analytical solution for such cases but gives a general overview of the acoustic behavior based on numerical solutions. Therefore only cases without flow are considered, which limits the results to applications with low Mach-number flow, e.g. HVAC systems where the Mach-number is usually smaller than 0.1.

Figure 4: a) MPPs symmetrically placed in the pressure minima and maxima of the (1,0)-mode and (2,0)-mode. b) A sketch of a filter of length L inside a duct with two plates with a separation angle ϕ_p between the plates, which is sufficient to attenuate the (1,0)-mode.

The MPPs used in this paper are commercially available plates of the type Acustimet™ [18] with slit-shaped perforations. Their normalized impedance Z_{MPP} can be estimated applying Maa's perforation model of oscillating air in short tubes [19] to slit-shaped geometries as done by Allard and Atalla [20]

$$Z_{MPP} = r + i x_m. \quad (14)$$

Here, r is the resistance and x_m is the mass reactance of the plate and they can be described as

$$r = \operatorname{Re} \left(i \frac{\omega}{\sigma c} \left[1 - \frac{\tanh(k\sqrt{i})}{k\sqrt{i}} \right]^{-1} t_p \right) + r_e + r_{non-lin} \quad (15)$$

$$x_m = \operatorname{Im} \left(i \frac{\omega}{\sigma c} \left[1 - \frac{\tanh(k\sqrt{i})}{k\sqrt{i}} \right]^{-1} t_p \right) + x_e, \quad (16)$$

where t_p is the thickness of the plate with slits of the width a and the perforation ratio σ ,

$k = a \sqrt{\frac{\omega}{4\nu}}$ is a dimensionless shear wave number, and ν is the kinematic viscosity. The first term in the resistance and mass reactance accounts for the viscous losses inside the perforation, r_e is the viscous end correction due to surface resistance, x_e is the mass end correction, and $r_{non-lin}$ are the non-linear losses due to (possibly) high sound intensities in the small holes. More detailed descriptions of the terms in Eqs. (15) and (16) can be found in Ref. [21]. A sketch of a slit-shaped plate is shown in Figure 5 together with an example plot showing MPP impedance.

Figure 5: a) Sketch of a slit-shaped MPP with the thickness t_p , the perforation rate σ and the slit width a . b) Impedance of a slit-shaped MPP as real part (resistance, dashed line) and imaginary (reactance, solid line) part ($a=0.15\text{mm}$, $t_p=1\text{mm}$, $\sigma=0.05$). At f_0 , the real and imaginary part become equal.

The real and imaginary part of the impedance was computed using Eqs. (15) and (16) (neglecting non-linear terms) for a plate with a thickness of $t_p=1\text{mm}$, a slit width $a=0.15\text{mm}$, and a perforation ratio $\sigma=0.05$, which represents typical plate dimensions used for this work. One observation from Figure 5 a) is the almost constant resistance, whereas the graph of the reactance has a linear slope. This fits well with the measurements in Ref. [21] for Acustimet™. As a consequence, a simplified impedance model that describes the properties of Acustimet™ with only two geometry dependent constants is suggested here

$$Z_{MPP} = r_0(1 + i \frac{f}{f_0}). \quad (17)$$

The crossover frequency f_0 is the frequency where the reactance equals the crossover resistance r_0 , whereas for frequencies above f_0 , the reactance becomes the dominant part of the impedance. In most parts of this paper, only resistant plates are investigated, meaning that the frequencies considered are much smaller than the crossover frequency and the imaginary part of Z_{MPP} is negligible. Such plates, where the imaginary part of the impedance can be neglected, will be denoted here as ‘ideal MPPs’. From a technical point of view, ideal MPPs could only be realized with very high crossover frequencies. This means that the (dominant) internal reactance of the holes $Im\left(i\left[1 - \frac{\tanh(k\sqrt{t})}{k\sqrt{t}}\right]^{-1}\right)$, which is the slope of the reactance curve in Figure 5, has to be reduced. If the internal reactance is related to the inertia of an oscillating air column in a perforation hole, it stands to reason that it scales directly with the perforation volume, and (as seen in Eq. (16)) indirectly with the perforation ratio, i.e. $x_m \sim a \frac{t_p}{\sigma}$. Thus, narrower slits, thinner plates, and higher perforation ratios give a lower reactance. The resistance, however, is found to be rather constant being approximately the $\omega = 0$ limit of $r(\omega)$. It is hence dominated by the resistance on the potential flow through the perforation holes caused by the thermo-viscous losses in the boundary layers at the active surface of the holes. Therefore the resistance scales linearly with the active surface, i.e. indirectly with the width of the slits and the perforation ratio σ , and directly with the thickness of the plate t_p and $r \sim \frac{1}{a} \frac{t_p}{\sigma}$. The ‘ideal MPP’ is hence a thin plate with narrow slits and a high perforation ratio. It follows from the above that the crossover frequency scales with $\frac{1}{a^2}$ independently of the plate thickness and the perforation ratio.

Conceptually, modal filters consist of resistant plates placed in all velocity maxima of the dominant acoustic modes. Each additional plate however creates a surface that causes flow resistance and increases the pressure drop between the inlet and the outlet of the silencer. This pressure drop scales indirectly with the hydraulic diameter D_H that is four times the ratio of the perfused cross-sectional area A_D and the wetted surface S_D

$$D_H = 4 \frac{A_D}{S_D}. \quad (18)$$

The pressure drop ΔP over a filter of the length L is

$$\Delta P = 4 k_{surf} \frac{L}{D_H} \rho_0 U^2, \quad (19)$$

with the dimensionless surface roughness k_{surf} , the fluid density ρ_0 and the squared (mean) velocity U^2 .

For a straight pipe, D_H simply equals the duct diameter. For the modal filter with a number of N plates, however, the wetted surface increases, namely by the surface of the (two sided) plates, i.e. $S_D = D(\pi + 2N)$. Assuming thin plates, the perfused area remains constant and the hydraulic diameter for the modal filter is $D_M = \frac{\pi D}{(\pi + 2N)}$. Hence, the relative ratio of pressure drops is

$$\frac{\Delta p_M - \Delta p_D}{\Delta p_D} = \frac{k_{MPP}}{k_{duct}} \left(\frac{D}{D_M} \right) - 1 = \frac{k_{MPP}}{k_{duct}} \frac{(\pi + 2N)}{\pi} - 1. \quad (20)$$

For smooth MPPs ($k_{MPP} = k_{duct}$) the pressure drop increases by $\frac{2N}{\pi}$ which is around 64% per plate compared to an empty piece of duct with the same dimensions. This apparent drawback is counteracted by the advantage of a fully integrated design without additional external space requirements. It is worth mentioning, that the general pressure drop in a piece of smooth pipe is very low and the additional pressure drop caused by a compact modal filter even with many plates may not change the overall performance of the system significantly.

F. Scaling

The quantities in this paper are scaled as follows; the acoustic frequencies are scaled to the Helmholtz number

$$He = \frac{D}{2} k, \quad (21)$$

with k being the wave number of the plane wave. The normalized cut-on frequencies f' of the higher order modes are $f'_{1,0} = 1.841$, $f'_{2,0} = 3.054$, and $f'_{0,1} = 3.832$.

As demonstrated later in this paper, for filters longer than around $2D$, the transmission loss is linear to the filter length and therefore the results in this paper are normalized to the filter length in duct diameters

$$TL^* = TL \frac{D}{L} [dB]. \quad (22)$$

3. OBJECTIVES AND RESULTS

A. Objectives of the Investigation

In the paper at hand, a modal filter was investigated numerically to gain a general understanding of the important parameters for the filter design. In a first step, a numerical model was set up and validated with measurements. Second, investigations were conducted on the validated model; namely, a modal filter with two plates was used to compute the damping of the (1,0)-mode as a function of the angle between the plates, the impedance of the plates, and the length of the filter. Lastly, a damper was designed to reduce the tonal noise of an axial compressor. The damper was a two-stage silencer, which first attenuates the spinning modes using the modal filter and secondly the plane wave mode with a Cremer silencer.

B. Numerical Modelling

The Helmholtz equation in the frequency domain with internal impedance boundary conditions was solved numerically with the finite element method (FEM) using the commercial Solver Comsol Multi-Physics®. Therefore, the MPPs of the modal filter were defined as impedance normal boundary conditions at internal (volume-less) surfaces, i.e. the thickness of the plates was neglected. The channel walls were sound-hard boundaries ($\nabla p = 0|_{wall}$). The impedance Z_{MPP} of the plates was computed using Eq. (17) with the crossover frequency $f_0 = 2.1 \text{ kHz}$ and the resistance $r_0 = 1.35$.

The computational domain consisted of the four parts presented in Figure 6, namely: (1) perfectly matched layers at the inlet and outlet in order to reduce boundary reflections, (2) a source section applying a volume force in the shape of the excited mode, (3) a decomposition zone at the inlet and outlet of the filter, where the pressure fluctuations were sampled and decomposed into acoustic modes, and (4) the modal filter consisting of two plates in a star shape.

Figure 6: Computational domains for the filter computations using FEM.

The computations were run on a grid consisting of hexametrical elements with quadratic element shape functions designed to resolve the highest frequency (3000 Hz, $He=4.11$) with five elements per wavelength. In the source region a volume force in the shape of the acoustic modes (as presented earlier in Figure 2) was applied that excited single modes separately. The problem was considered to be symmetric, i.e. that the degenerated modes experience similar damping and the filter inlet/outlet is exchangeable.

Figure 7: Prototype of the modal splitter. a) Star-shaped MPPs with b) slit-shaped perforation and c) mounted filter.

In this paper, the filter prototype depicted in Figure 7 was used for model validation. It was originally built to reduce the noise emitted by an axial compressor and consisted of four micro-perforated plates of similar impedance in a star-shaped assembly with constant angle (45°) between the plates (Figure 7a)). MPPs of the type Acustimet™ (Figure 7b)) were used. The modal filter (Figure 7c)) had a length of one duct diameter ($L=D=150$ mm). The filter segment was mounted to the flow-rig facilities at The Marcus Wallenberg Laboratory for Sound and Vibration Research (MWL) at the Royal Institute of Technology (KTH), Sweden. The scattering matrix was determined using the measurement and post-processing procedures described and evaluated in Ref. [12], from which the Transmission Loss could be computed (Eq. (10)).

The plates' reactance and resistance values provided by the supplier were inaccurate and the formulas for rectangular slits did not work well for the complex slit geometry of Acustimet™. The crossover frequency and resistance in Eq. (17) was therefore fitted to the results for the plane wave from the experiments for a Helmholtz number of 3.75. (the frequency was arbitrarily chosen). The fitted values were $r_0 = 1.35$ and $f_0 = 2.1$ kHz. The computed modal transmission loss is presented in Figure 8 (solid line with dots) and compared to the experimental data (solid line). The computational results and the measurements showed a very good agreement for all frequencies and the deviation was found to be less than 0.5 dB for all computed frequencies in the (1,0)-mode and (2,0)-mode. This suggests that the numerical model and the simplified impedance formula are valid and that it correctly covered the acoustics. Higher deviations of around 1dB were only found for the (0,1) mode, that should have been unaffected by the filter. The geometrical discontinuity in the prototype at the connection of the plates, however, could result in small attenuation of higher order radial modes.

Figure 8: Comparison of measurements and FEM computations; the solid lines show the measurement and the solid lines with dots show the FEM results.

C. Analysis of Filter Properties

The properties of different modal filters were tested numerically. In a first test, the influence of the filter length on the transmission loss was investigated in order to find a sufficient numerical setup for which dissipation effects are dominating over inlet and outlet reflections (see Eq. (8)). Second, the effect of modal filters with different numbers of plates on the (1,0)-mode and the (2,0)-mode were tested. The scattering for filters with one, two, and four plates was computed up to the cut-on of the first radial mode. Third, on a filter with two plates, the plate position for optimal damping was investigated.

All those tests were computed assuming ideal MPPs (plates without acoustic reactance). Introducing reactance to the impedance boundary condition would be necessary to design filters for applications with high frequencies compared to the size of the perforation holes. This paper does not aim to give a complete guidance for such applications; however, the scattering of a modal filter consisting of two plates of 'real MPPs' was computed to investigate changes in the plate position for optimal transmission loss.

Filter length. The transmission loss for different filter lengths ($L=0.1D$ to $5D$) for filters with two perpendicular plates ($\phi_p = 90^\circ$) was computed for the (1,0)-mode for different resistances at a Helmholtz number of 2.0 which was close after the cut-on frequency of the (1,0)-mode. In Figure 9, the Transmission Loss per duct diameter is plotted over the filter length in duct diameters. A constant value was observed for filters longer than $1D$ for $r_0 = 0.5$ and $r_0 = 1$, and $2D$ for $r_0 = 5$. For shorter filters, oscillations in the Transmission Loss were detected, resulting from the superposition of reflections from the filter inlet and outlet. This finding corresponded to the behavior known from established silencers [3]. For the subsequent computations in this paper, filter lengths of $4D$ were chosen to guarantee linear filter behavior.

Figure 9: Damping for the (1,0) spinning mode over filter length for ideal MPPs of different resistances at $He=2.0$. Filter configuration as in Figure 4 b) with $\phi_p=90^\circ$.

Number of plates. To underline the arguments in section 2 E, the scattering matrix of filters with one, two, and four plates of length $4D$ was computed for an ideal MPP with a resistance of 1.5. The resistance was chosen to be in the range of optimal resistances as shown later in this section. From the scattering matrix, the Transmission Loss and the Power Loss were computed (as shown in Figure 10).

Figure 10: Transmission loss and Power loss of a modal filter of length $4D$ with one plate (grey line), two plates (black line), and four plates (dashed line) in a star-shape. No modal scattering between the (1,0)-mode and (2,0)-mode occurred.

Close to the cut-on frequency a high difference between PL and TL was found due to inlet reflections, whereas for higher Helmholtz numbers the solutions converged. Even though the Transmission Loss was reasonably high especially close to the cut-on frequency, the Power Loss was low. Values between 15 – 20 dB were computed, which corresponded to 4-5 dB attenuation per duct diameter.

For setups with less plates than velocity minima and maxima in the modes, i.e. a single plate for the (1,0)-mode and between one and two plates for the (2,0)-mode, the filter attenuated the spinning mode by 3dB which emphasizes the arguments in section 2 E.

In Figure 11 the velocity profile at the duct surface is shown for the (1,0)-mode entering a filter with a single plate at a Helmholtz number of 3.7. Once the mode reaches the filter, the mode shape transforms to reach a standing wave pattern which creates velocity minima at the plate.

Figure 11: Plot of the spinning (1,0)-mode entering a modal filter with a single resistant plate ($r_0=1.5$) at a Helmholtz number of 3.4. Exchanging energy with its degenerate mode, the spinning mode shape transforms into a standing (not rotating) wave pattern with pressure nodes along the MPP.

Plate position. The Transmission Loss in a filter with two ideal MPPs with a length $L=4D$ and ϕ_p between 15° and 90° was computed in 1° steps for plates of different reactance and for different acoustic frequencies for the (1,0)- and the (2,0)-mode. Figure 12 shows the results in the form of the attenuated power due to Transmission Loss, normalized to the maximum attenuation for each case. In Figure 12 a) the (1,0)-mode is presented, which had the highest Transmission Loss for a plate-separation of 90° independent of the plate resistance and the acoustic frequency. For this separation, the plate geometry perfectly broke the mode symmetry with plates in all velocity minima and maxima. For the (2,0)-mode (Figure 12 b)), however, the optimal position changed with the plate resistance and acoustic frequency as the number of plates was not sufficient to break the mode symmetry of both degenerate states as argued in section 2 E.

It stands to reason that the findings for the (1,0)-mode are applicable to any higher circular mode due to the shared (though higher order) symmetry in the mode-shapes. The plate position for the highest acoustic Transmission Loss for a spinning (m,n)-mode is

$$\phi_{opt} = \frac{\pi}{m}, \quad (23)$$

if a number of $2m$ ideal MPPs are used. As shown previously in Figure 10, a filter designed for a certain mode is also effective for modes of lower order. The number of plates should therefore be chosen to break the symmetry of the propagating mode of highest order. If the number of available plates is smaller than $2m_{max}$, the plate position has to be optimized based on the expected mode content of the acoustic field.

Figure 12: Optimal plate positioning for different resistance and frequency levels for two-plate filters (as in Figure 4). Results for the spinning (1,0) and the (2,0)-mode for a filter of length $L=4D$.

Plate resistance. For filters with ideal MPPs the optimal resistance in terms of highest Power Loss along the plates is of great interest for the filter design. To evaluate possible maxima within a reasonable range of resistances, the Transmission Loss was computed for a filter of two plates with a separation angle of 90° for resistances between 0.1 and 5 for the (1,0)-mode and with four plates and 45° separation for the (2,0)-mode. The results are shown in Figure 13. The plot shows the Transmission Loss for different frequencies corresponding to Helmholtz numbers between 0.7 and 4.1 for the (1,0)-mode (solid lines) and the (2,0)-mode (dashed lines). Smooth curves with broad maxima for all frequencies and for both modes were found. The value of the peak decreases with ascending frequency and is shifted to higher resistances, but could be located for r_0 between 1 and 2. Especially for higher frequencies, the curves flattened out so that (small) deviations of the chosen resistance from the optimal resistance have little influence on the damping. For a value of $r_0 = 1$, the impedance equals $Z_{MPP} = \rho c$, which is the characteristic acoustic impedance for the plane wave mode. The highest Transmission Loss was found for frequencies close to the cut-on frequency with values of around 8 dB per duct diameter filter length. For higher frequencies, the damping decreased to values of around 5-6 dB per duct diameter.

Figure 13: Optimal resistance for ideal plates. Power Loss over reactance for the (1,0)-mode (solid line) and the (2,0)-mode (dashed line) for different frequencies ($He < 4.1$).

Power Loss along ‘real’ MPPs. In case of none-ideal (‘real’) MPPs, the acoustic properties of the filter become more complex. In the scope of this paper, a ‘real’ MPP has been investigated for different ratios f_0/r_0 at different frequencies. The study was not meant to be a complete analysis, but was aimed at showing the possible influences of the parameters, focusing on the optimal plate position. To simplify the investigation, either the reactance or the resistance was set to a fixed value and the other value was altered. The results are presented in Figure 14. Figure 14 a) shows the normalized attenuated power due to Transmission Loss for fixed $r_0 = 1.5$ (which was a maximum found for the ideal MPP) and varying f_0 and the right hand graph shows the results for fixed $f_0 = 1000 \text{ Hz}$ (which was a reasonable value for MPPs of the Acustimet™ type) and varying r_0 (always for a Helmholtz number of $He = 2.0$). The main finding of this analysis was the optimal relative angle between the plates, which compared to the ‘ideal’ MPP was now dependent on the impedance (even for the (1,0)-mode). This was due to changed mode shapes inside the filter compared to the solutions for the Helmholtz equation without internal boundary conditions and hence different positions for the velocity maxima of the degenerated states. For higher f_0 the solution converged towards the solution for ‘ideal MPPs’, but decreasing f_0 shifted the optimum to lower plate angles (Figure 14 b)). However, the slope of the curves was found to be flat for increasing angles and even for very low values of $f_0 = 500 \text{ Hz}$ the decay in Transmission Loss from the optimum position (45°) to the symmetric solution (90°) was less than 20% (which corresponds to less than 1dB divergence). Similar findings were observed for the resistance in Figure 14 b). A high $r_0 = 5$ combined with $f_0 = 1000 \text{ Hz}$ considerably shifted the optimum plate position, but the general decay of Transmission Loss to the symmetric solution was low (again lower than 1dB). As a consequence of this finding, in applications where extensive numerical optimizations are inappropriate or accurate MPP properties are not available, the symmetric plate position as suggested in Eq. (23) is still a very reasonable choice.

Figure 14: Power-loss over angle between the plates for different plate impedances. a) Fixed $r_0=1.5$. b) Fixed $f_0=1000$ Hz.

D. Combination of Modal filter and Cremer silencer

The properties of the modal filter in Figure 7 were tested in combination with a Cremer silencer that was optimized to reduce the tonal noise of an axial compressor. The modal content of the compressor noise can be seen in Figure 15 as the amplitudes of the modal source cross-spectra [5]. A very strong peak was measured for the blade passing frequency (BPF) of the rotor blades at $He=3.7$ (around 2700 Hz), which was pronounced in all modes. A two-stage filter consisting of a Cremer silencer (to damp the plane wave mode) and a modal filter (to damp the spinning modes) was designed.

The Cremer silencer was designed based on the properties described in section 2 C and had peak damping at the compressor's BPF. The highest propagating circumferential mode was the (2,0)-mode and following the considerations in section 3 C, the modal filter consisted of four plates with an angle of 45° between all the plates (see also Figure 7). The MPP data for the filter was fitted to the experimental data ($r_0=1.35$ and $f_0=2.1$ kHz).

The scattering matrix was measured for the single filters as well as for the filter combination and the Transmission Loss and Power Loss was computed using Eq. (10) and Eq. (11). The results are presented in Figure 16 (Transmission Loss in a) and Power Loss in b)). In the plane wave range, the Cremer silencer (dotted line) started to be effective for $He \approx 3.4$ and had a Transmission Loss peak around the BPF of the compressor (16dB), but dropped strongly at the cut-on frequency of the (0,1) mode ($f'_{0,1} = 3.832$). Furthermore, the Cremer silencer gave good Transmission Loss for the (1,0)-mode (15dB peak) and for the (2,0)-mode (15 dB peak).

Figure 15: Source auto-spectrum $|G|$ of an axial compressor for aircraft HVAC systems, decomposed into the excited modes [5].

Compared to that, the modal filter (dashed line) had much lower Transmission Loss for the (1,0)-mode, but reasonable Transmission Loss for the (2,0)-mode. The filter combination improved the Transmission Loss for the spinning modes slightly for the (1,0)-mode and strongly for the (2,0)-mode.

The plot of the Power Loss (Figure 16 b)) shows the acoustic power that was actually absorbed inside the silencer and the two silencer stages. The Power Loss for the plane wave mode was around 4dB lower at the peak than the Transmission Loss due to reflected acoustic power at the filter inlet. It should be noted, that if the reflections are small, i.e., for a dominantly dissipative system, the Transmission Loss and the Power Loss will be almost equal.

Figure 16: a) Transmission Loss and b) Power Loss of the modal splitter, the Cremer silencer, and the combination of both.

The clearest effect of the modal filter could be found for the (2,0)-mode, for which the power absorption of the combined silencer increased significantly compared to the single silencer stages. The reflective Transmission Loss (TL – PL, Eq. (8)) for the Cremer silencer was found to be between 5-10 dB for the (2,0)-mode indicating that large portions of the acoustic energy were only reflected and not absorbed. Prefixing the modal filter reduced the reflected power in the (2,0)-mode and created a distinct peak in the power loss at the target frequency of the Cremer silencer. It stands to reason, that the acoustic power that is reflected at the inlet of the Cremer silencer has to pass the modal filter twice which increases the effective surface of the modal filter stage. The total Power Loss of the silencer can then be higher than the sum of the power losses in the single stages.

4. DISCUSSION AND CONCLUSION

A novel silencer type that reduces the acoustic power of higher order duct modes has been investigated. The mode shapes of the different eigenmodes in Eq. (3) were used to set up resistant plates in the velocity maxima of the acoustic field in order to absorb acoustic energy. This approach assumes steady mode shapes even though technically the plates change the eigenvalue problem that has initially led to the acoustic modes. However, deriving eigensolutions for cases with internal boundary conditions is challenging as the Helmholtz equation does not decouple for such cases. Instead of searching for analytical solutions, a numerical approach was chosen in order to investigate the basic properties of the modal filters.

The plane wave mode and the higher order radial modes were approached with a new type of dissipative silencer, using periodic cavity structures applied to the duct walls and covered with MPPs. In section 2 D of this paper, such a silencer was designed, using the optimal wall impedance (Cremer Impedance) in order to reach high attenuation with very compact dampers. However, the resultant performance from a measurement of a prototype in Figure 16 at the peak frequency was much smaller than the Transmission Losses that were realized in Ref. [10]. Reasons here could be insufficient knowledge about the actual impedance of the MPPs used as well as inaccuracies during the manufacturing of the prototype.

Strictly speaking, even this conventional design of the Cremer silencer can be denoted as a modal filter, as it uses the radial symmetry of the plane wave to particularly affect this certain mode. However, the word ‘modal filter’ in this paper was rather connected to a special design, where the acoustic impedances were placed inside the duct cross sections, see Figure 4 and Figure 7. The study was focused on filters containing plates with negligible mass reactance. Such plates can for instance be thin micro-perforated panels with small holes and a high perforation rate, as described in section 2 E.

Furthermore, only cases without flow were studied, which limits the results to applications with low Mach-numbers as found e.g. in HVAC systems. As a fully developed flow will not change the symmetry in the cross sections, the findings in this paper concerning the optimal plate positions will remain, even for applications with higher Mach-numbers. The optimal acoustic plate resistance, however, will have to be investigated separately for such cases.

The study showed, that a sufficient filter design to stop spinning modes consists of two resistant plates per propagating mode placed in a symmetric star shape. The transmission loss was highest close to the cut-on frequency, which was however, mainly due to strong inlet reflections at the filter inlet. That was demonstrated by plotting the Power Loss inside the filter in Figure 10. The Power Loss takes inlet reflections and the scattering of energy between the modes into account. Close to the cut-on frequency, the Power Loss of a filter with two plates or four plates was low, and it increased with frequency, converging towards the Transmission Loss meaning that the filter became purely dissipative for frequencies further above the cut-on. The losses for those frequencies were between 4-5 dB per duct diameter. This value is low compared to conventional silencers, especially compared to silencers using the Cremer wall impedance were TL of 60 dB per duct diameter are realizable at a target frequency [7]. However, the losses achieved by modal filters are broadband which predestines them for applications with strong flow noise. Furthermore, long filters with high damping may be realized due to the integrated design. It needs to be mentioned here that the surface of the plates causes additional flow-resistance that creates a pressure drop along the filter. Computing its hydraulic diameter, this increase in pressure drop could be calculated to be around 64% per inserted plate (Eq. (20)), assuming smooth plates. The smoothness of the plates relies on the manufacturing process and the true pressure drop might be higher, especially for Acustimet™ that was used for the experiments in this paper. However, the pressure drop in empty pipes is generally very low compared to e.g. other components of HVAC systems, which should make the additional resistance manageable. Restrictions of the application of a modal filter are then only systems with strongly swirling flow, e.g. positioned close behind rotating machinery.

By assuming an ideal MPP with a negligible mass reactance the optimum configuration (angle between plates) and resistance value is investigated, see Figure 12 and Figure 13. It is concluded that the optimum for damping a spinning mode of order N is $2N$ MPP plates equally spread out. The resistance should lie in the range of 1-2 times the plane characteristic impedance. The effect of dropping the assumption of a negligible mass reactance is investigated in Figure 14. The optimal (symmetric) plate positions for real MPPs are still a reasonable choice for applications with considerably high mass reactance.

The filters designed for different modes can be used in cascades in order to realize good damping in cases where a reasonably low amount of modes is propagating. It was shown, that a filter designed for a certain spinning mode not only affects lower modes, but also higher modes. For higher modes, the maximum damping is 3dB, see Figure 10 and Figure 11. Those modes transform energy into their degenerate state until an equilibrium state with velocity nodes at the plates is reached. A single resistant plate added to the duct cross section could be seen as a modal diffuser, which distributes modal energy equally to the degenerate states of each circumferential mode. A cascade of two silencers was tested experimentally. A modal filter designed to sufficiently damp the (2,0)-mode was prefixed to a Cremer silencer designed to reduce the BPF of an axial compressor. Both filters were compact with a size of one duct diameter. It could be shown, that especially the Power Loss was improved with this filter combination, with a strong peak for the (2,0)-mode at the compressor's BPF.

5. ACKNOWLEDGEMENT

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