Model Predictive Control Oriented Experiment Design for System Identification: A Graph Theoretical Approach

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Abstract

We present a new approach to Model Predictive Control (MPC) oriented experiment design for the identification of systems operating in closed-loop. The method considers the design of an experiment by minimizing the experimental cost, subject to probabilistic bounds on the input and output signals due to physical limitations of actuators, and quality constraints on the identified model. The excitation is done by intentionally adding a disturbance to the loop. We then design the external excitation to achieve the minimum experimental effort while we are also taking care of the tracking performance of MPC. The stability of the closed-loop system is guaranteed by employing robust MPC during the experiment. The problem is then defined as an optimization problem. However, the aforementioned constraints result in a non-convex optimization which is relaxed by using results from graph theory. The proposed technique is evaluated through a numerical example showing that it is an attractive alternative for closed-loop experiment design.

Keywords: Closed-loop identification, Optimal Input design, System identification, Model predictive control, Constrained systems.

1. Introduction

1.1. Motivations and objectives

MPC has become very important for controlling constrained multivariable processes in industry. As in any other model-based controller, the MPC performance highly depends on the quality of the model being used. However, modeling and identification are known to be very demanding in terms of time and resources. On the other hand, due to unavoidable changes in the dynamics of the process over time, for example because of raw material variations or equipment wear, the model has to be updated regularly. Therefore, there is an emerging demand for more efficient data driven modeling and model updating techniques (Patwardhan and Gopaluni, 2014).

A primary step in any modeling approach based on experimental data is to monitor the behaviour of the system and collect data. The collected data can highly affect the efficiency of the system identification. This concept has led to the growth of the topic of experiment design for system identification, see Goodwin and Payne (1977). Identification experiments can be done in either open or closed-loop conditions. However, in many practical applications, systems can only operate under closed-loop settings due to stability issues, production restrictions, economic considerations or inherent feedback mechanisms which necessitate designing and performing system identification experiments under closed-loop settings. A well known problem in closed-loop identification is the conflict between the identification and control requirements, i.e., while more exciting inputs can increase the quality of the identified model, the control performance will be affected by the presence of exciting inputs.

The closed-loop experiment design problem can be translated into the design of an additive external excitation. The existing literature on closed-loop experiment design for linear systems is quite rich, see Hildebrand and Gevers (2003);

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Jansson (2004); Hjalmarsson and Jansson (2008); Hildebrand and Solari (2009) and the references therein. However, many of these methods have difficulty in dealing with nonlinear and implicit feedback such as MPC. In Ebadat et al. (2014b) a graph theoretical approach is explored, but a known controller is assumed and indirect identification is employed.

This paper considers the problem of closed-loop experiment design for MPC. The main objective of this paper is to design closed-loop identification experiment for MPC, in which the generated data contain enough information for updating the model. The updated model should result in a good control performance when being used by MPC.

1.2. Related work

The existing literature on closed-loop experiment design for MPC is quite exhaustive. A large portion of the literature trying to find a balance between the identification and control requirements in the MPC by adding a new constraint to the MPC optimization problem, which can assure persistent excitation of the controlled input signal, see for example Genceli and Nikolaou (1996); Aggelogiannaki et al. (2004); Marafioti (2010); Larsson et al. (2013). In Heirung et al. (2015), an approach to online experiment design for MPC is proposed, which is based on certainty equivalence. This method also requires integrating new constraints to MPC. However, the main difference with previous approaches is that the proposed method by Heirung et al. (2015) tries to reduce the parameter uncertainties at the same time. More research related to experiment design for MPC can be found in Shouche et al. (1998, 2002).

There are a few challenges in the implementation of the existing methods: a) the obtained optimization problem that needs to be solved at each iteration of MPC is nonconvex; b) in the presence of process and measurement noises the input and output constraints which arise for safety or practical limitations should be imposed probabilistically and this cannot be handled by the existing methods.

1.3. Statement of contributions

In this paper, which is an extension of the idea developed by Ebadat et al. (2014b) to more general controllers with implicit control law, we present a new approach for MPC oriented experiment design in the presence of probabilistic constraints on input and output signals. The idea is to design the external excitation achieving the minimum experimental effort, while we are also taking care of the tracking performance of MPC. We add a constraint on the quality of the estimated model in terms of the Fisher information matrix (Goodwin and Payne, 1977). The objective is to obtain an exciting enough input signal guaranteeing that the estimated model is in the set of models that satisfy the desired control specifications with a given probability, i.e., we follow the idea of application oriented input design, see Bombois et al. (2006); Hjalmarsson (2009); Gevers and Ljung (1986).

Adding an external excitation can improve the information content of the closed-loop system, however, since the system is operating in closed-loop it may endanger the stability of the closed-loop system. One contribution of this paper is that we consider the external excitation as a bounded disturbance. We thus employ robust output feedback MPC which can assure the stability of the system under bounded input disturbances (Mayne et al., 2000).

On the other hand, the resulting optimization program arising from the input design formulation is non-convex. A convex approximation of the problem can be found by extending the results in Valenzuela et al. (2015), where the problem is defined as finding the optimal probability density function (pdf) for the external excitation instead. The probability distribution of the external excitation is then characterized as the convex combination of the measures describing the set of n -dimensional distributions of stationary Markov processes of a given order. The resulting problem is convex in the decision variables.

Moreover, the above mentioned optimization problem requires to know the control law in order to evaluate and predict the cost and constraints for different external excitations. For online controllers such as MPC the control law is not known in advance and instead it is an implicit and nonlinear function of the observed state. This difficulty is also handled by evaluating MPC for each of measures defining the convex hull of the set of n -dimensional distributions of stationary Markov processes of a given order.

1.4. Structure of the manuscript

This paper is organized as follows: In Section 2 the problem is defined and the existing challenges are described. Section 3 discusses the application of a graph theoretical approach in finding the convex approximation and solving the problem. Section 4 contains some numerical results and evaluates the effectiveness of the proposed approach.

Notation: $\mathbb{R}$ stands for the real set and $\mathbb{R}^{n \times m}$ is the set of real $n \times m$ matrices. The expected value and the probability measure associated with a given random variable are denoted by $\mathbb{E}\{\cdot\}$, and $\mathbb{P}\{\cdot\}$ respectively. Sometimes a subscript is added to $\mathbb{E}$ and $\mathbb{P}$ to clarify the random variables involved.

2. Problem definition

The problem considered in this paper is the following. Assume that the goal is to identify a discrete-time and Linear Time-Invariant (LTI) system, which is described in state space form as

$$\mathcal{S}: \begin{aligned} x_{t+1} &= A_0 x_t + B_0 u_t, \\ y_t &= C_0 x_t + v_t, \end{aligned} \quad (1)$$

where $x_t \in \mathbb{R}^{n_x}$ is the state, $u_t \in \mathbb{R}^{n_u}$ is the input and $y_t \in \mathbb{R}^{n_y}$ is the output. The signal $v_t \in \mathbb{R}^{n_y}$ corresponds to measurement noise where $\{v_t\}$ is a white noise process with a bounded support¹, zero mean and pdf p_v . The schematic representation of system $\mathcal{S}$ in (1) is shown in Figure 1.

Figure 1: Schematic representation of system (1), where $G_0(q) = C_0(qI - A_0)^{-1}B_0$ is the transfer function of the noiseless system and q is time shift operator.

As a first step to identify a model for (1), we define a model class $\mathcal{M}$, which is parametrized by the vector $\theta \in \mathbb{R}^{n_\theta}$ such that the system $\mathcal{S}$ is identifiable (Ljung, 1999). The model set considered here is given by

$$\mathcal{M}(\theta): \begin{aligned} x_{t+1} &= A(\theta)x_t + B(\theta)u_t, \\ y_t &= C(\theta)x_t + v_t, \end{aligned} \quad (2)$$

where $\{v_t\}$ is the white noise process introduced in (1). It is assumed that the model (2) coincides with system (1) when $\theta = \theta_o$, i.e., there is no undermodeling (Ljung, 1999). We call θ_o the true parameter vector.

The system $\mathcal{S}$ in (1) is operating in closed-loop with a controller designed based on the best available estimation of θ_o (denoted by $\hat{\theta}$) such that the closed-loop system is having acceptable performance. The closed-loop setting is shown in Figure 2, where y_t^d is the desired output. In the following, we assume that the system $\mathcal{S}$ in (1) is being controlled by MPC, and the resulting closed-loop system is asymptotically stable.

Figure 2: Block diagram of the closed-loop system.

¹This assumption is necessary in most of practical applications due to the limitations in the actuators.

However, due to changes in the process dynamics over time, at some point the plant-model mismatch increases such that the closed-loop performance is not satisfactory. In order to detect the performance deterioration, the degradation in the control performance that comes from the mismatch between the model and the system is quantified with the definition of an application cost-function, which is a scalar measure of the performance deterioration. We denote such a cost by $V_{\text{app}}(\theta)$. By employing the application cost-function, we can define the set of acceptable parameters from a control perspective as

$$\Theta(\gamma) \triangleq \left\{ \theta \in \mathbb{R}^{n_\theta} \mid V_{\text{app}}(\theta) \leq \frac{1}{\gamma} \right\}. \quad (3)$$

This set will be referred to as the *application set*, see Ebadat (2015) for more elaborated description of the application cost. The parameter $\gamma \geq 0$ is a user-defined constant which imposes an upper bound on the performance degradation. If the performance violates its upper bound, a model update is required.

In order to update the model a system identification experiment should be performed to collect data. The data needs to be collected in such a way that we can retrieve the desired closed-loop performance for the updated model with prescribed probability $\alpha \in (0, 1)$. Since the system is working in closed-loop, the identification experiment is performed by adding an external excitation to increase the information content in the collected data, as shown in Figure 3. On the other hand, the external excitation deteriorates the closed-loop performance and may cause instability issues for the closed-loop system. Thus, the external excitation should be designed in such a way that the performance degradation is kept within prescribed limits while the obtained relative information is maximized.

Figure 3: The schematic representation of the closed-loop system with external excitation.

If the true system is described by (1), then the closed-loop system with external excitation can be written as:

$$\begin{aligned} x_{t+1} &= A(\theta_o)x_t + B(\theta_o)u_t^c + B(\theta_o)r_t, \\ y_t &= C(\theta_o)x_t + v_t, \end{aligned} \quad (4)$$

where u_t^c is the control signal and r_t is the external excitation.

The objective is to design an experiment for the closed-loop system (4), that generates N samples of the external excitation r_t , to be used for the identification of the unknown parameters θ in (2). To this end, we consider the experiment design problem discussed next.

2.1. Experimental cost

Since the system is operating in closed-loop, we need to preserve the output of the plant y_t close to y_t^d during the identification experiment. Hence, we choose to minimize the following experimental cost:

$$J = \mathbb{E} \left\{ \sum_{t=1}^N \left\| y_t - y_t^d \right\|_{R_y}^2 + \left\| \Delta u_t \right\|_{R_u}^2 \right\}, \quad (5)$$

where

$$u_t = u_t^c + r_t, \quad (6)$$

$$\Delta u_t = u_t - u_{t-1}, \quad (7)$$

and $R_y \in \mathbb{R}^{n_y \times n_y}$ and $R_u \in \mathbb{R}^{n_u \times n_u}$ are positive definite weighting matrices. The first term in (5) penalizes the deviations from the desired output, i.e., the control cost, while the second term is responsible for minimizing the energy of the input changes. We note that the expected value in (5) is with respect to the stochastic processes $\{r_t\}$ and $\{v_t\}$.

2.2. Input and output constraints

In practical applications, it is common to have bounds on the maximal input and output amplitudes allowed by the process. These constraints appear due to physical limitations and/or to preserve the system close to a safe operating point. However, since both the input and output of the system contain a stochastic component which cannot be measured, these bounds cannot be forced in a deterministic sense. Thus, we consider instead the following probabilistic constraints during the identification process:

$$\begin{aligned}\mathbb{P}\{y_t \in \mathbb{Y}\} &> 1 - \varepsilon_y, \quad t = 1, \dots, N, \\ \mathbb{P}\{u_t \in \mathbb{U}\} &> 1 - \varepsilon_u, \quad t = 1, \dots, N,\end{aligned}\tag{8}$$

where ε_u and ε_y are positive scalars defining the desired probability of being in the safe bounds for the input and output signals, respectively.

2.3. Experiment design constraint

In addition to the previous constraints, we require that the updated (or newly) designed controller based on the estimated parameters can guarantee an acceptable control performance, i.e., the estimated parameters should lie in the application set with high probability:

$$\mathbb{P}\{\hat{\theta}_N \in \Theta(\gamma)\} \geq \alpha.\tag{9}$$

This is the main idea in application oriented experiment design (Hjalmarsson, 2009). Similar ideas have also been used in least costly identification and identification for control (Bombois et al., 2006; Hjalmarsson, 2009; Gevers and Ljung, 1986).

The constraint (9) is in general non-convex. Thus, to further simplify the problem we will use its *ellipsoidal approximation* which results in the following Linear Matrix Inequality (LMI), see (Ebadat et al., 2014a)

$$\frac{1}{\chi_{\alpha}^2(n_{\theta})} \mathcal{J}_F(\theta_o) \succeq \frac{\gamma}{2} \nabla_{\theta}^2 V_{\text{app}}(\theta_o),\tag{10}$$

where $\chi_{\alpha}^2(n_{\theta})$ is the α -percentile of the χ^2 -distribution with n_{θ} degrees of freedom, $\mathcal{J}_F(\theta_o)$ is the Fisher information matrix which quantifies the information regarding the unknown parameters in the observations of the output (Goodwin and Payne, 1977), and $\nabla_{\theta}^2 V_{\text{app}}(\theta_o)$ is the second order derivative of $V_{\text{app}}(\theta)$ with respect to θ evaluated at $\theta = \theta_o$. We refer to (10) as the *experiment design constraint*.

Finally, the closed-loop experiment design we are interested in solving can be formulated as

Problem 1 Design $\{r_t^{\text{opt}}\}_{t=1}^N$ as the solution of

$$\begin{aligned}\underset{\{r_t\}_{t=1}^N}{\text{minimize}} \quad & \mathbb{E} \left\{ \sum_{t=1}^N \|y_t - y_t^d\|_{R_y}^2 + \|\Delta u_t\|_{R_u}^2 \right\}, \\ \text{subject to} \quad & x_{t+1} = A(\theta_o)x_t + B(\theta_o)u_t, \\ & y_t = C(\theta_o)x_t + v_t, \quad t = 1, \dots, N, \\ & u_t = u_t^c + r_t, \quad t = 1, \dots, N, \\ & \mathbb{P}\{y_t \in \mathbb{Y}\} > 1 - \varepsilon_y, \quad t = 1, \dots, N, \\ & \mathbb{P}\{u_t \in \mathbb{U}\} > 1 - \varepsilon_u, \quad t = 0, \dots, N, \\ & \frac{1}{\chi_{\alpha}^2(n_{\theta})} \mathcal{J}_F(\theta_o) \succeq \frac{\gamma}{2} \nabla_{\theta}^2 V_{\text{app}}(\theta_o),\end{aligned}\tag{11}$$

where u_t is the input to the system and u_t^c is the controller output and is assumed to be given by the controller. Note that Problem 1 has a very similar structure as Model Predictive Control, however, they are not necessarily the same since we are not considering a receding horizon approach in this problem.

There are, however, several challenges in solving Problem 1:

a) For disturbance free linear systems, it is possible to guarantee the stability of the closed-loop system with MPC by combining a stable estimator and a stable state feedback controller. However, this is not guaranteed in the presence

of state and output disturbances. Therefore, adding external excitation can endanger the stability of the closed-loop system. In this paper, we consider the external excitation as a bounded disturbance and employ robust output feedback MPC, which can guarantee the stability of the system under bounded input disturbances, see Mayne et al. (2006).

b) In order to evaluate the cost-function and constraints for the closed-loop system, the control input needs to be known in advance. For a general offline controller, e.g., output feedback compensator, the control law is known. for an online controller such as MPC, the control law is known but usually an implicit function of the current state estimate. Hence, the control input is not usually known in advance for MPC. This issue is also addressed in this paper by extending the method proposed in Valenzuela et al. (2015).

c) Finally, the optimization problem (11) is in general non-convex due to the possible nonlinearity of the controller and the probabilistic constraints, and is difficult to solve explicitly. Moreover, evaluating the probability constraints and expected values are also challenging problems. These issues are relaxed by employing the method introduced in Valenzuela et al. (2015) for closed-loop and constrained system identification.

In the following section we will briefly describe the convex approximation of Problem 1.

3. Convex approximation of the experiment design problem via graph theory

To find a convex approximation of Problem 1, we first assume that r_t belongs to an alphabet $\mathcal{C}$ with $n_{\mathcal{C}}$ elements. We restrict $r_{1:N}^2$ to be a realization of a Markov process with stationary probability mass function (pmf)³ $p(r_{1:n_m})$, where $n_m \leq N$ is given, and p is restricted to set of n_m -dimensional distributions of stationary Markov processes of a given order. Based on these assumptions, it is possible to use the technique proposed in Valenzuela et al. (2015) to obtain the optimal excitation $r_{1:N}^{\text{opt}}$ by designing p . Building on Valenzuela et al. (2015), we can express p as the convex combination of the pmfs that correspond to the extreme points of the convex set

$$\mathcal{P}_{\mathcal{C}} := \left\{ p : \mathcal{C}^{n_m} \rightarrow \mathbb{R} \left| p(\mathbf{x}) \geq 0, \forall \mathbf{x} \in \mathcal{C}^{n_m}; \sum_{\mathbf{x} \in \mathcal{C}^{n_m}} p(\mathbf{x}) = 1; \sum_{v \in \mathcal{C}} p(v, \mathbf{z}) = \sum_{v \in \mathcal{C}} p(\mathbf{z}, v), \forall \mathbf{z} \in \mathcal{C}^{n_m-1} \right. \right\}, \quad (12)$$

which are found using graph theoretical tools. If we denote by $\{p_j\}_{j=1}^{n_{\mathcal{V}}}$ such pmfs, we can express every $p \in \mathcal{P}_{\mathcal{C}}$ as

$$p = \sum_{j=1}^{n_{\mathcal{V}}} \beta_j p_j, \quad (13)$$

where $\beta_j \geq 0$ for all $j \in \{1, \dots, n_{\mathcal{V}}\}$, and $\sum_{j=1}^{n_{\mathcal{V}}} \beta_j = 1$.

Since any pmf $p \in \mathcal{P}_{\mathcal{C}}$ can be written as (13), we can find the optimal pmf p^{opt} in two steps: (i) evaluate the cost-function and constraints for every pmf p_j , and (ii) find the optimal weights $\{\beta_j^{\text{opt}}\}_{j=1}^{n_{\mathcal{V}}}$ by minimizing the cost-function subject to the the input, output and quality constraints. Given $\{\beta_j^{\text{opt}}\}_{j=1}^{n_{\mathcal{V}}}$, the optimal pmf p^{opt} is then

$$p^{\text{opt}} = \sum_{j=1}^{n_{\mathcal{V}}} \beta_j^{\text{opt}} p_j. \quad (14)$$

3.1. Evaluation of cost-function and constraints for each p_j

To estimate the cost-function and constraints values in (11) for each p_j , we sample $r_{1:T}^j$ from p_j , and $v_{1:T}$ from its pdf p_v (with T sufficiently large), and then we simulate the closed-loop system with robust MPC for that specific realization and obtain $u_{1:T}^j$ and $y_{1:T}^j$ for $j \in \{1, \dots, n_{\mathcal{V}}\}$. Based on $u_{1:T}^j$ and $y_{1:T}^j$, we approximate the cost-functions and the constraints for each pmf p_j as follows:

Cost-function: The expected value $\mathbb{E}_{v_t, r_t^j} \{\cdot\}$ associated with p_j is approximated using Monte-Carlo simulations as

$$\mathbb{E}_{e_t, r_t^j} \left\{ \|y_t^j - y_t^d\|_Q^2 + \|\Delta u_t^j\|_R^2 \right\} \approx \frac{1}{T} \sum_{t=1}^T \left\{ \|y_t^j - y_t^d\|_Q^2 + \|\Delta u_t^j\|_R^2 \right\}. \quad (15)$$

²We use this notation to show the first N samples of signal r_t .

³A probability mass function is a probability measure whose support is a set with finite cardinality.

In the following, we denote by J_j the Monte-Carlo approximation (15) for $r_{1:T}^j$.

Amplitude constraints: The probability can be approximated for each pmf, p_j as

$$\begin{aligned}\mathbb{P}_{e_t, r_t^j} \{y_t^j \in \mathbb{Y}\} &\approx \frac{1}{T} \sum_{t=1}^T \mathbf{1}_{y_t^j \in \mathbb{Y}}, \\ \mathbb{P}_{e_t, r_t^j} \{u_t^j \in \mathbb{U}\} &\approx \frac{1}{T} \sum_{t=1}^T \mathbf{1}_{u_t^j \in \mathbb{U}},\end{aligned}$$

where $\mathbf{1}_X = 1$ if X is true, and 0 otherwise.

Experiment design constraint: To integrate the experiment design constraint, we need to compute the Fisher information matrix in (10) for each basis input $r_{1:T}^j$. This can be obtained by using the techniques in Spall (2008, 2000).

3.2. Finding the optimal weights

Based on the approximations of the cost-function and the constraints, we finally solve the following problem:

$$\begin{aligned}& \underset{\{\beta_j\}_{j=1}^{n_{\mathcal{V}}} \in \mathbb{R}^{n_{\mathcal{V}}}}{\text{minimize}} && \sum_{j=1}^{n_{\mathcal{V}}} \beta_j J_j \\& \text{subject to} && \sum_{j=1}^{n_{\mathcal{V}}} \beta_j \mathbb{P}_{e_t, r_t^j} \{u_t^j \in \mathbb{U}\} > 1 - \varepsilon_u, \\& && \sum_{j=1}^{n_{\mathcal{V}}} \beta_j \mathbb{P}_{e_t, r_t^j} \{y_t^j \in \mathbb{Y}\} > 1 - \varepsilon_y, \\& && \frac{1}{\chi_{\alpha}^2(n_{\theta})} \sum_{j=1}^{n_{\mathcal{V}}} \beta_j \mathfrak{I}_F^j \succeq \frac{\gamma}{2} \nabla_{\theta}^2 V_{\text{app}}(\theta_o), \\& && \sum_{j=1}^{n_{\mathcal{V}}} \beta_j = 1, \\& && \beta_j \geq 0, \text{ for all } j \in \{1, \dots, n_{\mathcal{V}}\}.\end{aligned} \tag{16}$$

As we can see from the optimization problem (16), the design of the optimal pmf p^{opt} only requires finding the weights $\{\beta_j\}_{j=1}^{n_{\mathcal{V}}}$. The problem (16) is a convex problem which can be directly formulated as an semidefinite problem with LMIs as constraints and thus can be solved using convex optimization tools.

Once the optimal pmf is computed according to (14), we can obtain a realization $r_{1:N}^{\text{opt}}$ by running a Markov chain with stationary pmf p^{opt} . See Valenzuela et al. (2015) for more details.

Remark 1 The optimization problem (16) is independent of the controller structure, and hence the approach presented in this section can be also applied when the controller is known. Thus, the technique introduced in this article extends the results in Ebadat et al. (2014b), where the controller law is assumed known, and an indirect method is employed to solve the identification problem.

Remark 2 The optimized values in (16) are an approximation of the values obtained in the system by applying $r_{1:N}^{\text{opt}}$. This approximation is in the sense that the difference between the optimized and the real values goes to zero as $n_m \rightarrow \infty$. This asymptotic result in n_m is expected since the system usually takes a convolution over an infinite number of terms, and hence a distribution over an infinite number of terms is needed to fully describe its behavior. We refer to Valenzuela et al. (2016) for more details on the convergence of the method.

4. Numerical example

We consider a linearized model of two interconnected water tanks. A pump with input u_t is connected to the upper tank. There is a hole in the bottom of the upper tank with free flow to the lower tank. The lower tank also has a hole

in the bottom part with free flow out of the tank and the lower tank level is considered as the output, y_t . A linearized output error model of the coupled tanks is given by

$$\begin{aligned} x_{t+1} &= \begin{bmatrix} \theta_3 & \theta_4 \\ 0 & 1 \end{bmatrix} x_t + \begin{bmatrix} 4.5 \\ 0 \end{bmatrix} u_t, \\ y_t &= [\theta_1 \quad \theta_2] x_t + v_t, \end{aligned} \quad (17)$$

where the measurement noise v_t is white with uniform distribution over $[-0.1, 0.1]$ and $\theta = [\theta_1, \theta_2, \theta_3, \theta_4]$ are the model parameters.

The model parameter values describing the true system are

$$\theta_o = [0.12 \quad 0.059 \quad 0.74 \quad -0.14]^T.$$

It is assumed that the system is being controlled by an MPC where the goal is to preserve the water level in the lower tank close to a reference value. Therefore, the MPC cost-function is given by

$$J_{\text{MPC}} = \frac{1}{2} \left(Q_y \sum_{t=1}^{N_y} |y_t - y_t^d|^2 + Q_u \sum_{t=1}^{N_u} |u_t|^2 + \|x_{N_y+1}\|_{Q_f}^2 \right), \quad (18)$$

with $Q_y = 1$, $Q_u = 0.01$ and $Q_f = 0.001$.

The input signal is required to satisfy $|u_t| \leq 2$, the states are constrained to $-25 \leq x_t \leq 20$, and there is no constraint imposed on the output.

The MPC is tuned based on an initial model. To proceed, we assume that we need a re-identification experiment to improve the control performance. The application cost is chosen as

$$V_{\text{app}}(\theta) = \frac{1}{N} \sum_{t=1}^N |y_t(\theta) - y_t(\theta_o)|^2, \quad (19)$$

over a step response of the system with MPC as feedback controller. The signals $y_t(\theta)$ and $y_t(\theta_o)$ are the closed-loop outputs when the controller is tuned based on the parameters θ and θ_o , respectively. The application cost-function measures the deviation in the closed-loop output that comes from not using the true parameters.

We design an external excitation $\{r_t\}_{t=1}^N$ where $N = 10000$, using the proposed approach in this paper for the re-identification step. During the re-identification, a robust MPC is used to guarantee the stability of the closed-loop system (see Appendix A for more details about robust MPC).

In order to find the optimal external excitation, we design $\{r_t\}_{t=1}^N$ following Section 3, where $n_m = 2$ and $r_t \in \mathcal{C} = \{-0.1, -0.05, 0, 0.05, 0.1\}$. We then solve the optimization problem (16) with the following settings:

$$\varepsilon_y = \varepsilon_u = 0.1, \quad \gamma = 2000, \quad Q = 0, \quad R = 1, \quad \alpha = 0.98.$$

The Hessian of the application function can be computed using either numerical methods, see D'Errico (2007), or by Monte-Carlo methods, see Ebadat et al. (2014a). The sets $\mathbb{U}$ and $\mathbb{Y}$ are given by

$$\begin{aligned} \mathbb{U} &= \{u_t \in \mathbb{R} \mid |u_t - u_t^r| \leq 0.15\}, \\ \mathbb{Y} &= \{y_t \in \mathbb{R} \mid |y_t - y_t^d| \leq 0.9\}, \end{aligned} \quad (20)$$

where u_t^r is the steady state control input before adding the external excitation.

The optimal obtained external excitation is employed to perform 100 Monte-Carlo simulations. The input-output data is then used to estimate θ . A random binary sequence with values ± 0.1 is also used to excite the system and the results are compared to the obtained optimal excitation.

A part of the input ($u_t - u_t^r$) and output ($y_t - y_t^d$) signals for both cases is shown in Figures 4 and 5, respectively. It can be seen that for the optimal external excitation the input and output lie mostly within the white area, which means that input and output constraints are satisfied with high probability, while for the random binary excitation the input and output signals violate the constraints frequently. The results for the average satisfaction of probability over the

Excitation signal	$\mathbb{P}\{u_t \in \mathcal{U}\}$	$\mathbb{P}\{y_t \in \mathcal{Y}\}$
random binary signal	61.72%	75.22%
r_t^{opt}	95.25%	98.91%

Table 1: Average fulfilment probability of the input and output signals over the Monte-Carlo simulations for the generated realization of the obtained optimal pdf and the prbs.

Figure 4: Part of the input signals for one of the Monte-Carlo simulations for both optimal excitation and the random binary excitation.

Figure 5: Part of the output signals for one of the Monte-Carlo simulations for both optimal excitation and the random binary excitation.

Monte-Carlo simulations are summarized in Table 1 for both cases. It is observed that for the optimal excitation the input and output constraints are fulfilled with the required probability (90%), however, this is not true for the random

Figure 6: The evaluation of the application cost for different Monte-Carlo simulations for (Top) optimal external excitation and (Bottom) random binary signal.

binary excitation.

In order to check if the estimated parameters are acceptable from a control point of view, i.e., that they are lying inside the application set, the application cost-function is computed for all the estimations. The results are shown in Figure 6. It is clear from Figure 6 that for the obtained estimates, the application function is greater than the one obtained with random binary signal. This means that the random binary signal is better in satisfying the quality constraint, i.e., the identification requirements. However, From Figures 4 and 5 it can be concluded that the cost of satisfying the identification requirements with higher accuracy is violating the control constraints and thus the random binary signal excites the system more than required, resulting in a performance degradation from a control perspective.

5. Conclusion

A new approach is proposed to design optimal input signal for closed-loop system identification with an MPC controller in use. The method adds a bounded external disturbance on the controlled input to generate an exciting enough input signal for the system. The optimal external excitation is then defined as the solution of a constrained optimization problem. The cost-function in the optimization problem is penalizing any deviation of the closed-loop output from its desired value while trying to minimize the input energy. The input and output constraints are imposing probabilistic bounds on the input and output signals, which are unavoidable in industrial processes. At the same time it is guaranteed that enough information is obtained from the experiment by adding an application oriented experiment design constraint. A graph theoretical approach is then employed to find a convex approximation and solve the problem. The proposed approach is then evaluated on a numerical example. The results shows that the optimal input is less exciting compared to a binary white noise with the same but still exciting enough to identify acceptable model from control perspective.

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Appendix A. Robust output feedback Model Predictive Control

MPC is a control strategy where a model is used to predict the system behaviour for different control inputs and finding the optimal control action. What makes MPC different from other optimal controllers is that it uses the receding horizon principle. This means that at each sample time the optimization problem in MPC is solved online

and an optimal input trajectory is obtained. However, only the first element of the obtained optimal input vector is applied to the system.

For disturbance free linear systems, one can guarantee the stability of the closed-loop system with MPC by combining a stable estimator and a stable state feedback controller. However, the stability of the resulting closed-loop system is not necessarily guaranteed when state and output disturbances are present. To guarantee a stable closed-loop system, robust controller design methods are employed in MPC. One idea is to use the tube-based robust output feedback MPC proposed in Mayne et al. (2006), which is briefly described here.

We consider the system

$$\begin{aligned} x_{t+1} &= A_0 x_t + B_0 u_t + w_t, \\ y_t &= C_0 x_t + v_t, \end{aligned} \quad (\text{A.1})$$

where w_t is the state disturbance. The system is subject to bounded state and output disturbances

$$w_t \in \mathbb{W}, \quad v_t \in \mathbb{V}, \quad (\text{A.2})$$

where the sets $\mathbb{W}$ and $\mathbb{V}$ are compact, convex and have the origin as their interior. We also assume that the system is subject to input and state constraints, i.e.,

$$u_t \in \mathbb{U}, \quad x_t \in \mathbb{X}. \quad (\text{A.3})$$

To estimate the state from output observations, a state estimator is required. One can use the following Luenberger observer:

$$\begin{aligned} \hat{x}_{t+1} &= A_0 \hat{x}_t + B_0 u_t + L(y_t - \hat{y}_t), \\ \hat{y}_t &= C_0 \hat{x}_t, \end{aligned} \quad (\text{A.4})$$

where $\hat{x}_t$ is the current state of estimator and $\hat{y}_t$ is the observer output. The matrix L is chosen such that the spectral radius of the matrix $A_L \triangleq A_0 - LC_0$ is strictly less than 1, i.e. $\rho(A_L) < 1$.

The state estimation error defined by

$$\tilde{x}_t \triangleq x_t - \hat{x}_t, \quad (\text{A.5})$$

satisfies

$$\tilde{x}_{t+1} = A_L \tilde{x}_t + \tilde{\delta}_t, \quad (\text{A.6})$$

where $\tilde{\delta}_t \triangleq w_t - Lv_t$. The disturbance signal $\tilde{\delta}_t$ lies in the set $\tilde{\Delta}$, given by

$$\tilde{\Delta} \triangleq \mathbb{W} \oplus (-L\mathbb{V}). \quad (\text{A.7})$$

Since $\rho(A_L) < 1$, it is then possible to find a positively invariant set for the system (A.6) (Rakovic et al., 2005). If we denote the invariant set by $\tilde{S}$, then

$$A_L \tilde{S} \oplus \tilde{\Delta} \subseteq \tilde{S}, \quad (\text{A.8})$$

which states that if $\tilde{x}_0 \in \tilde{S}$, then $\tilde{x}_t \in \tilde{S}$ for all $t > 0$. This result enables us to bound the true states, x_t , by the estimated states, $\hat{x}_t$, and the invariant set $\tilde{S}$, by the following proposition (Mayne et al., 2006):

Proposition 1 If the initial observer state and system state are chosen such that $\tilde{x}_0 \in \tilde{S}$, then for all disturbance sequences $\{w_t\}$ and $\{v_t\}$, x_t satisfies $x_t \in \hat{x}_t \oplus \tilde{S}$ for all $t > 0$.

If we remove the disturbances w_t and v_t from (A.1), we obtain the nominal system

$$\begin{aligned}\bar{x}_{t+1} &= A_0\bar{x}_t + B_0\bar{u}_t, \\ \bar{y}_t &= C_0\bar{x}_t,\end{aligned}\tag{A.9}$$

where $\bar{x}_t \in \mathbb{R}^{n_x}$ is the nominal state and $\bar{u}_t \in \mathbb{R}^{n_u}$ is the input to the nominal system. The nominal system can be employed to cancel out the disturbances by forcing the observer trajectory to be close to the nominal trajectory. This motivates the following choice for the control action u_t

$$u_t = \bar{u}_t + Ke_t,\tag{A.10}$$

where e_t is the error between the nominal state and the state estimate, i.e.,

$$e_t \triangleq \hat{x}_t - \bar{x}_t.\tag{A.11}$$

The signal e_t in turn satisfies

$$e_{t+1} = A_K e_t + \bar{\delta}_t,\tag{A.12}$$

where $\bar{\delta}_t \triangleq LC\tilde{x}_t + Lv_t$ and $A_K = A_0 + B_0K$. The matrix K is chosen such that $\rho(A_K) < 1$.

Provided that the assumptions in Proposition 1 are satisfied, a set $\bar{\Delta}$ can be defined such that $\bar{\delta}_t \in \bar{\Delta}$, for all $t > 0$, i.e.:

$$\bar{\Delta} \triangleq LC\tilde{S} \oplus LV.\tag{A.13}$$

Due to the fact that $\rho(A_K) < 1$, there exists a compact convex set, denoted by $\bar{S}$, which is positively invariant for the system (A.12) and the constraint set $\bar{\Delta}$. The invariant set satisfies

$$A_K\bar{S} \oplus \bar{\Delta} \subseteq \bar{S},\tag{A.14}$$

and it can be obtained using the techniques in Rakovic et al. (2005). Having $\bar{S}$, the following result is obtained (Mayne et al., 2006):

Proposition 2 If the initial observer and reference system state are chosen such that $e_0 \in \bar{S}$, then $\hat{x}_t \in \bar{x}_t \oplus \bar{S}$ for all sequences $\{w_t\}_{t>0}$ and $\{v_t\}_{t>0}$ and for all $t > 0$.

Combining the definitions (A.11) and (A.5), we can write

$$x_t = \bar{x}_t + e_t + \tilde{x}_t.\tag{A.15}$$

Based on the Propositions 1 and 2 it is possible to bound e_t and $\tilde{x}_t$, thus the constraints on the actual state, x_t , and control action, u_t , are fulfilled by appropriate choice of $\bar{x}_0$ and $\{\bar{u}_t\}_{t>0}$.

If the sets $\mathbb{W}$ and $\mathbb{V}$ are sufficiently small, we can assume that the robust positively invariant sets $\tilde{S}$ and $\bar{S}$ satisfy

$$S \triangleq \tilde{S} \oplus \bar{S} \subset \mathbb{X}, \quad K\bar{S} \subset \mathbb{U}.$$

Finally, the required ingredients for robust output feedback MPC are summarized in the following theorem by Mayne et al. (2006):

Theorem 1 Suppose that the initial true state, the observer and the nominal system states satisfy

$$\begin{aligned} x_0 \in \mathbb{X}, \quad \hat{x}_0 \in \mathbb{X}, \quad \bar{x}_0 \in \mathbb{X}, \\ \tilde{x}_0 \in \tilde{\mathcal{S}}, \quad e_0 \in \tilde{\mathcal{S}}, \end{aligned}$$

then

$$x_0 \in \bar{x}_0 + S. \quad (\text{A.16})$$

Moreover, if $\bar{x}_t \in \mathbb{X} \ominus S$ and $\bar{u}_t \in \mathbb{U} \ominus K\tilde{\mathcal{S}}$ for all $t > 0$, then

$$x_t \in \mathbb{X}, \quad u_t \in \mathbb{U},$$

for all $t > 0$.

From Theorem 1, we see that the nominal system states and input are subject to the following tighter constraints

$$(\bar{x}_t, \bar{u}_t) \in \bar{\mathbb{X}} \times \bar{\mathbb{U}}, \quad \bar{\mathbb{X}} \triangleq \mathbb{X} \ominus S, \quad \bar{\mathbb{U}} \triangleq \mathbb{U} \ominus K\tilde{\mathcal{S}}, \quad (\text{A.17})$$

and that the initial and final nominal states must satisfy:

$$\hat{x}_t \in \bar{x}_t \oplus \tilde{\mathcal{S}}, \quad \bar{x}_{N_y} \in \mathbb{X}_f, \quad (\text{A.18})$$

where $\mathbb{X}_f$ is the terminal constraint set for the nominal system (A.9). Building on (A.17) and (A.18), the set of admissible control sequences for the nominal system starting from $\bar{x}_t$, will be given by

$$\bar{\mathcal{U}}(\bar{x}_t) = \{ \bar{\mathbf{u}} \mid \bar{\mathbf{u}} \in \bar{\mathbb{U}}, \quad \bar{\mathbf{x}} \in \bar{\mathbb{X}}, \quad \bar{x}_{N_y} \in \mathbb{X}_f \}, \quad (\text{A.19})$$

where

$$\begin{aligned} \bar{\mathbf{x}} &= \{ \bar{x}_t, \bar{x}_{t+1}, \dots, \bar{x}_{t+N_y-1} \}, \\ \bar{\mathbf{u}} &= \{ \bar{u}_t, \bar{u}_{t+1}, \dots, \bar{u}_{t+N_u-1} \}. \end{aligned}$$

Here N_u and N_y are the control and prediction horizons, respectively. Thus, the optimization problem that needs to be solved at time t in the robust MPC is

$$(\bar{x}_t^*, \bar{\mathbf{u}}^*) = \arg \min_{\bar{x}_t, \bar{\mathbf{u}}} \{ J_{\text{MPC}}(\bar{\mathbf{x}}, \bar{\mathbf{u}}) \mid \bar{\mathbf{u}} \in \bar{\mathcal{U}}(\bar{x}_t), \quad \hat{x}_t \in \bar{x}_t \oplus \tilde{\mathcal{S}} \}, \quad (\text{A.20})$$

where $\hat{x}_t$ is the observer state at time t and J_{MPC} is the MPC cost function. A common cost function used in MPC is given by

$$J_{\text{MPC}}(\bar{\mathbf{x}}, \bar{\mathbf{u}}) = \bar{\mathbf{x}}^T Q_x \bar{\mathbf{x}} + \bar{\mathbf{u}}^T Q_u \bar{\mathbf{u}} + \bar{x}_{N_y}^T Q_f \bar{x}_{N_y}, \quad (\text{A.21})$$

where $Q_x \in \mathbb{R}^{(n_x(N_y-1)) \times (n_x(N_y-1))}$, $Q_u \in \mathbb{R}^{(n_u(N_u-1)) \times (n_u(N_u-1))}$ and $Q_f \in \mathbb{R}^{n_x \times n_x}$ are user defined positive definite matrices. The last term in the MPC cost function (A.21) is terminal cost, denoted by $V_f(x_{N_y})$. The terminal cost $V_f(x_{N_y})$ and the final states should also satisfy the general stabilizing conditions. We refer to Mayne et al. (2000) for more details.

Finally, the optimal control law for MPC at time t is Rakovic et al. (2005):

$$u_t^c = \bar{u}_t^* + K(\hat{x}_t - \bar{x}_t^*). \quad (\text{A.22})$$

Remark 3 The robust MPC formulation can be easily used for reference tracking by small modifications of the cost function and the constraints. We refer to Alvarado et al. (2007) for more details.