Master Thesis

Trajectory Optimization for Aircraft Evasive Maneuvering

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Thesis work performed at aeronautics company Avioniq

by

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Stockholm 2017
1. ABSTRACT

The aim of this work has been to identify hidden parameter value patterns during evasive maneuvering for a typical jet fighter. The work has created a performance model for a fighter aircraft and this model has then been combined with a missile model to simulate an enemy attack. By doing different kinds of simulations with a certain amount of predetermined scenarios, different outcomes could be evaluated when making small changes in the maneuvers during each specific scenario. The span of parameters that conducts a flying airplane’s trajectory is vast and the evaluation of different decisions that is up on the table for a pilot in a given situation might give new insights when optimizing tactical air fighting scenarios.

After evaluating different scenarios with different input values in form of different turn and climb angles etc, it was clear that small changes resulted in vast differences regarding the outcome, when being chased by the missile. By analyzing the results, it can be concluded that there are underlying patterns regarding controllable parameter values when the airplane tries to get rid of the chasing missile. For example; one section in this work describes that by keeping a straight flight path for a certain amount of seconds after a specified value of turn angle - results in survival of the attack. Keeping level flight for too many seconds however has a lethal outcome. The results seem also to follow a continuous - non-randomized - pattern. This type of detailed analysis could be used to help a pilot to optimize the performance of the maneuver.

NOMENCLATURE

\[ T \]  Thrust

\[ V \]  Speed

\[ \alpha \]  Angle of attack

\[ \alpha_0 \]  Zero lift offset angle

\[ \gamma \]  Elevation angle

\[ \phi \]  Roll angle

\[ L \]  Lift force

\[ C_L \]  Lift coefficient

\[ C_{Di} \]  Zero lift drag with area

\[ \rho \]  Atmospheric density

\[ EBK \]  After-burner, switched on or off.

\[ \psi \]  Horizontal velocity direction angle

\[ x \]  North-South axis position

\[ y \]  West-East axis position

\[ b \]  Mass change, i.e. fuel burn

\[ h \]  Altitude

\[ D \]  Aerodynamic drag

\[ Q_{dyn} \]  Dynamic pressure

\[ C_{Lo} \]  Lift slope

\[ \eta \]  Induced drag coefficient

\[ S_{ref} \]  Reference area
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2. INTRODUCTION

Always when it comes to the optimization of the chance of survival, man succeeds to call out the most innovative thinking and to develop the sharpest technology. Military vehicles have constantly pushed the current mechanics to its limits and the battlefield has been a key factor in the evolution of new heights of engineering. In a conflict as a whole, the advantage to the part that entrenches the initiative in the air cannot be exaggerated. By dominating the skies, a fierce grip seizes the weaker part in most aspects. The aim of this work has been to investigate underlying controllable parameter value patterns that most probable will be evaluated more extensively by computers in the future. The outcome of a scenario in an aerial environment is a combination of the conductance from combat direction centers and the pilots’ performances. Regarding the maneuvers in detail, the outcome very much builds on the intuition and experience of the conducting personnel; in this case the pilot’s conception of his or her surroundings with the assistance from computers.

The aviation industry is constantly seeking new approaches to improve the performance of existing airplanes. Not the least in the military aviation sector there is a constant striving to reach new levels regarding maneuverability and performance in all aspects. One way of improving fighting ability is to develop brand new vehicles or enhancing existing ones, which however is a very costly business. Another - more economic - way of improving the cogency in the aerial arena is to optimize already existing flying tactics, by evaluating and enhancing flight paths and trajectories.

The objective of this work is to evaluate if there exist any symmetric underlying parameter value structure for how different flight maneuvers should be executed in order to optimize the chance of survival for a typical combat jet in the event of an attack from a hostile missile.

It is of course always up to the pilot to decide what kind of maneuvers that are about to be carried out in each specific situation. The more experience skills the pilot possesses - hopefully the more optimal decision he or she will make in different situations. Studies similar to this work have been performed in the past, for example Near-Optimal Missile Avoidance Trajectories via Receding Horizon Control (2007), by Karelahti, Virtanen, Raivio. However, the development in the offensive weapons’ arena is constantly developing more sophisticated ways of tracking its enemy. Consequently, research grows as well concerning evaluation of how different types of evasive maneuvers could be enhanced to stack up against an attack.

This work will first construct a performance model for a jet fighter and then combine this with an existing missile performance model. The aim is further on to try to evaluate how a specific scenario would have turned out if acted upon by the pilot in many different ways and by doing so; an optimal choice could in theory be recommended by a computer to the pilot in the beginning of an event. When performing evasive maneuvers in order to avoid the missile, for example one specific turn could lead to survival of the attack by a missile, if performed in a certain amount of turn angle degrees. By trying different angles repeatedly on the exact same scenario and by verifying the closest distance to the missile, the optimal choice could be recommended to the pilot in detail. In that way it should be possible to calculate the exact amount of recommended turn angle degrees and thrust percentage etc, in order to optimize the chance of survival in the case of an attack.

Regarding the management of flying a typical civilian passenger plane for example, the autopilot manages most of the flying and flight path planning. In the case of an extraordinary event however, that causes one or more systems on the airplane to break down, the steering and overall conducting is suddenly handed over to the pilot without any further ado. The future in commercial – and of course in military - aviation will most likely put greater demands on the computerized assistance optimization for pilots. Research in this area have during recent years taken place, on how to enhance artifical intelligence concerning evasive maneuvering for jet fighters, for example in Genetic Fuzzy based Artificial Intelligence for Unmanned Combat Aerial Vehicle Control in Simulated Air Combat Missions (2016) by Ernest, Carrol, Schumacher, Clark, Cohen, Lee. [1]
3. OBJECTIVE

The tasks in this work are to be described as:

- Be able to generate trajectories for a simulated airplane that performs realistic maneuvers allowed by typical fighter aircraft parameters. The purpose with this is to make it possible to use these trajectories to simulate a target.
- To be able to evaluate different outcomes of the same simulated scenario, based on different maneuvers for the target airplane and in that way be able to choose the best way for the targeted airplane to avoid simulated attacks.

To be able to fulfill the objectives stated above, this work will be carried out in the following way:

- Establish what kind of parameters that will have to be taken into account to make the simulation useful and realistic.
- Set up equations of motions and introduce parameters needed.
- Separate the different maneuvers and make it more or less possible to easily switch the order of the maneuvers.
- Set up different maneuver scenarios by giving control variables in the form of G-forces (changing angle of attack), roll angle and thrust percentage, respectively.
- Evaluate variables position and speed final values respectively when maneuvers have been carried out and to establish whether or not a hit with a chasing missile is likely to occur.

When discussing attack and defensive tactics for Air-to-Air missiles the concept of BVR – Beyond Visual Range – is often used. Missiles have such a high speed and can travel over such vast distances that it is possible to attack an enemy aircraft without needing the pilot to have it in visual range.

When trying to avoid an enemy missile, it is crucial to affect the missile’s trajectory in such a way that its chances of a hit are minimized. This is done by minimizing the missile’s range and at the same time try to keep such a short distance that the fighter can still remain in the combat surroundings without fleeing the field entirely. [3] By switching altitudes and performing turns that the missile has to follow, the induced drag is increased for the missile - which unlike the jet fighter it is chasing - does not have a fuel tank that can sustain the missile in air for a long time. [4]

The missile does have a higher speed compared to the aircraft that it is chasing. However, this could be used against it when performing tight turns, as performing a drastic turn at a higher speed means a higher load factor, resulting in more increased drag. [5] An optimal time for launching missiles is when being at a high altitude with low density and a high speed, which together maximizes the range of the missile.

The density shifts with altitude and therefore the choice of the altitude is crucial when predicting how fast an airplane for example could perform a turn or any other maneuver. [6] In general, a missile is faster than an airplane but has less capability of maneuvering due to its relatively small rudders and it also has less fuel capacity. The atmospheric density does not change linearly with altitude but exponentially, i.e. a chasing missile’s aerodynamic drag changes not so much when the airplane changes the altitude from 15000 to 12000 meters of altitude. However, when changing the altitude from 4000 to 1000 meters, the missile is exposed to a substantial amount of aerodynamic drag, causing a lot of kinetic energy loss, even though the maneuvering capacity is better the higher the density. [3]
4. CHOICE OF INPUT DATA

There are types of airplanes that are hard to compare structural wise, but nevertheless an airplane like the Swedish J35 Draken, which has served as model parameter data in this work, is a good example of an airplane that has unclassified data and a classic jet fighter structure. The design does change to some extent when new airplanes and new versions of airplanes are developed as well as the different parts that build up the airplane. However the changes are not revolutionary in a way that makes simulation of certain older airplanes taken out of duty during the recent decades useless, when conducting investigations concerning aeronautical overall behaviour regarding jet fighters.

There are different versions of most jet fighters and the exact specifications do change to some extent whether the airplane has a clean structure or if it carries missiles or other cargo, that affect the mass and aerodynamic drag. The airplane that serves as model data in this work is a clean J35 Draken.

Unclassified data for parameter values for the J35 Draken at different altitudes and Mach numbers layed the foundation of the simulations, as the equations implementing them can be used for any aircraft one chooses to investigate.

After being released and locked onto its target, the missile cruises in the direction of the target in different ways depending on the type of the missile, while making small adjustments in all three dimensions as the target switches its translational position over time. Some missiles have fuel for a long time and can adjust the thrust along the way, which means that it is harder to get rid of, as it is chasing the airplane for a longer time before it loses too much kinetic energy. Many missiles however, have a limited amount of fuel that is burned in the beginning after it has been released from the enemy attacking airplane and from there cruises through the atmosphere with rudder adjustments. The typical structure of a typical missile allows a higher speed than the target, but the relatively small rudders makes it less maneuverable than an airplane.

Avioniq provided a performance model for a missile, which combined with this work’s model completed the possibility of simulating flight for a J35 Draken, when being attacked by an Air-to-Air missile. There is a great variety of different BVRAAM:s (Beyond Visual Range Air-to-Air Missile) and depending on the type, the behaviour and guidance algorithm differ. Therefore, the results are of course different when simulating a missile trying to hit an evading airplane, depending on what type of weapon system one is simulating. The new Meteor missile is an example of a missile with a continuously variable thrust, for example. [7] The missile type used during the simulations in this work has been a medium range air-to-air missile with augmented proportional navigation parameterized with fictive values.

\[ \text{Fig. 4.1: The Meteor is an example of a missile that can be controlled with different levels of thrust throughout its cruise towards a target.} \]
5. J35 DRAKEN OVERVIEW

In the late 1940s - in the dawn of the era of the atomic bomb threat from the east – Sweden needed to develop a fighter that was able to stop atomic bombers on altitudes higher than 10 km that flew faster than the speed of sound. The J35 Draken was the result of that process - a fighter that flew for the first time in 1955 and has been enhanced and upgraded 15 times during the almost half a century long active duty period that it has been in service, ready to defend Swedish borders. Three of the most famous Swedish fighters – J35 Draken, JA37 Viggen and JAS 39 Gripen – all were complete new projects and have formed the backbone of the Swedish airforce since the 1950s. The J35 is for example well known for its double delta wing configuration, structural wise.

The J35 Draken was taken out of service in 1998 and the airplane itself is manned by one pilot, has a maximum speed of Mach 2 and its length, wingspan and height is 15,34 m, 9,42 m and 3,89 meter respectively. It has a wing area of 50 m², its empty fuel and load mass is 8,25 tons and its maximum take-off weight is 17,6 tons. [8] The J35 Draken was during its active duty armed with a 30 mm automatic m/55 canon, 75 mm rockets and robots. In total 644 Draken were built and about one hundred of them were sold to Finland, Denmark and Austria. [9]
6. PERFORMANCE ANALYSIS

The program MatLab [10], developed by the company MathWorks, has been used for the simulations and calculations in this work. The nonlinear equation system (6.1)-(6.7) are solved and iterated for every timestep with new values over time, which leads to a trustworthy simulation with the most important parameters. However, in the programming code the angles in the equations are from time to time modified to some extent to act properly in all attitudes, to avoid potential mathematical singularities. As most of this work’s focus is on the outcome of the airplane’s maneuvers in the atmosphere, the internal control systems are not being handled in the work, besides shifting roll- and pitch angle, over time during the simulations. A model is nothing more than a model and to describe the world to its completeness, countless numbers of variables have to be taken into account. However, to describe an unerring flight path (6.1)-(6.7) are looked upon as fulfilling in this work. Further on, sonic booms and other aerodynamic effects for supersonic speeds are not taken into account, as well as sideslip and other unsteady aerodynamical effects like wind etc. In other words, a clean flight is taken for granted. Overall, the forces acting on the airplane could be divided into three categories; aerodynamic, thrust and gravitational forces, respectively. The attitude control variables used when investigating the fighter aircraft’s movements and maneuvers are basically the roll and the pitch values. [11] These together constitute basically all maneuvers executed by a typical jet fighter, as rudder movement is more or less never used on modern fighter aircrafts.

The control variables when simulating are the thrust percentage factor (100% with EBK activated mostly), the roll angle and the number of G-forces (i.e. load factor in this case) respectively. [12] This work focuses on modelling flight in three spatial dimensions. [13] [14]

The change of speed, described with the equation

\[ m\dot{V} = T\cos(\alpha + \epsilon) - D - mgsin(\gamma) \] (6.1)

depends on the thrust, aerodynamical drag and three different attitude angles. The change of climb angle which is solved in

\[ mV\dot{\gamma} = T\sin(\alpha + \epsilon)\cos(\phi) + L\cos(\phi) - mgsin(\gamma) \] (6.2)

depends on several parameters. The left hand side of (6.2) has the value zero in equilibrium when performing level flight, for example. Further on, the derivative of the angle between the horizontal axis with respect to time is solved by using

\[ mV\dot{\psi}\cos(\gamma) = T\sin(\alpha + \epsilon)\sin(\phi) + L\sin(\phi) \] (6.3)

which in other words describes in what direction the horizontal component of the speed vector is directed towards. [15] When this value is zero, the airplane is flying straight along the North-South axis with increasing positive values in the Figures. This value depends of course on the roll angle of the airplane and if the airplane wings are horizontally straight, this value is zero. The change of altitude with respect to time is determined by

\[ \dot{h} = V\sin(\gamma) \] (6.4)

which only is a function of the speed and the angle of climb. The change of the mass of the aircraft is a function of the fuel burn, which leads to the equation

\[ \dot{m} = -b. \] (6.5)

Jet fighters today have the capability of releasing external stores when needed and of course the mass changes when the airplane fires its ammunition effects the mass, however ammunition mass use is not taken into account in this work. Regarding the change of position along the x- and y-axis (North-South and West-East axis in this work’s illustrations, respectively) over time, the values are given by solving the equations

\[ \dot{x} = V\cos(\gamma)\cos(\psi) \] (6.6)

\[ \dot{y} = V\cos(\gamma)\sin(\psi) \] (6.7)
which depend of course on the speed as well as on the climb and turn angle, respectively in (6.6)-(6.7). The function values are retrieved from:

\[ Q_{dyn} = 0.5 \rho V^2 \]

(6.8)

where the dynamical pressure is what the aircraft structure is exposed to during flight, which depends on the speed of the aircraft as well as on the density of the atmosphere. The lift coefficient of the airplane, calculated from

\[ C_L = C_{L\alpha}(\alpha - \alpha_0) \]

(6.9)

is a function of the angle of attack and two coefficients, that are obtained from airplane data. The aerodynamical drag, obtained from

\[ D = (C_{D0} + S_{ref}\eta C_L^2)Q_{dyn}, \]

(6.10)

is acting on the opposite direction of the speed direction of the airplane depends on the dynamical pressure, as well as on coefficients and the reference area that changes, depending on what airplane one is examining. The lift force, defined as

\[ L = Q_{dyn}S_{ref}C_L, \]

(6.11)

has its vector direction perpendicular to the speed direction \[ \text{[16]} \] of the airplane, straight upwards from the main wings in each moment. The value of this parameter is a function of the dynamical pressure, the reference area as well as the lift coefficient of the airplane. The number of current G:s along the vertical airplane axis (roll angle and consequently the gravity force vector taken into account) is retrieved from

\[ G \rightarrow \text{Lift force / Weight} \]

(6.12)

which basically is to the lift force divided by the weight of the airplane.

Fig. 6.1: The relationships between the parameters needed.
7. RESULTS

The simulations were divided into two separate major resulting parts; one that handled the roll away maneuver when flying towards an hostile missile and from there rolling away into a direction away from the threat. In this sequence, a subtask was to simulate a zero G dive towards the ground during the roll away maneuver and to introduce a formula for pull-up as late as possible, without crashing to the ground. This is interesting as a missile loses much more of its kinetic energy at low altitudes due to the thick density. Another roll away maneuver simulated was to initiate maximum G possible every timestep during the maneuver, in order to turn around as fast possible. The other part handled the evasive maneuvers after the roll away maneuver when being chased by the enemy missile, approaching from behind. This part can be investigated in countless forms due to the unlimited amount of combinations of maneuver sequences that could be set up. A number of probable interesting maneuvers that was thought to be hard for a potential enemy missile to follow were set up and tested with different control variable values:

- Predicting future possible flight paths if maximum G was to be initiated at every timestep, in order to be able to prevent crash and certain altitudes.
- Roll away from missile – with maximum time in dive in zero G straight towards ground.
- Roll away from missile – with continuously maximum G load.
- Different evasive maneuver sequences, when initial heading is away from threat.

7.1 Future flight prediction

Below a visualization of the resulting translational positions, depending on what G-force that is initiated. The blue crosses show the final destinations, respectively, that correspond to certain values of G:s.

Fig. 7.1: Potential view of future position from that could support the pilot when deciding the next move.
When focusing on a specific moment during flight, the pilot has the option to perform turns in any direction. The pilot can choose to initiate between 1 and 7 G:s (7 G:s is said to be the limit for the J35 Draken, regarding airplane structure. An angle of attack of 15° is stated as a limit, in order to avoid stalling) and to perform this in a chosen roll angle. Depending on the value of the turn’s G-force and the roll angle respectively, the airplane ends up in a new position a few seconds later if this chosen G-force was to be held with this roll angle. In these Fig. 7.1-7.4, the airplane always reaches its desired destination by initiating positive values on the pitch’s angle of attack. Fig. 7.1 and Fig. 7.3 plots every final destination after a number of seconds of future flight prediction from a specific actual position, showing G-turn values between 1-7 in roll angles 0 – 360° with 30° jumps.

A more or less symmetric scheme is obtained and the different options for the pilot in the given situation have pros and cons if choosing each travel path, respectively. This 3D-view of future positioning might give useful feedback to the pilot, if properly visualized, when deciding the amount of G-forces that are about to be initiated. In this specific example in Fig. 7.2, the blue crosses for each G value respectively are on similar positions as in Fig. 7.1, plotted with the speed value if that path was to be chosen. Continuing the same flight path leads to no change in speed, i.e. 100 %. If the airplane instead would choose to perform a 7 G elevation with 0° roll angle, the airplane would a few seconds later only fly with 68% of its initial speed. If instead during level flight a 5 G turn with a 30° roll angle was suddenly to be initiated and held during a few seconds, the airplane would due to the increased drag etc lose 15% of its speed.

Fig. 7.2: Window of future possible positions, visualized from the direction of speed, i.e. the airplane travels along the North-South axis into the paper.

When predicting future flying and implementing acceptable G-force conditions (max 7 G and 15° pitch angle, respectively, for J35 Draken) and a condition of not losing 15% speed or more as in this case; a window of future possible maneuvers resulting positions could be set up for different roll angle. This is visualized with the red boarder in Fig. 7.2, which could be assisting a pilot when he or she evaluates the next move. In other words, all the choices of final future destinations shown in Fig. 7.1 are not always possible, depending on the structure and speed conditions of the airplane model one is examining.
In Fig. 7.3, the simulation visualized in Fig. 7.1 is seen from the side of the airplane, which is indicated by the blue arrow to the left in the illustration. The harder turns – in any direction – the shorter the airplane of course travels in the horizontal plane, as more kinetic energy is invested in gaining altitude or redirecting the airplane downwards. Of course, depending on the amount of time into the future one is predicting, the distance between every future translational position changes.

![Graph of future flight positions](image)

**Fig. 7.3:** Future flight positions simulated in 3D, same case as in Fig. 7.1, visualized from the side of the airplane.

In Fig. 7.4, the same case as in Fig. 7.3 is visualized, this time with a chosen future flight path illustrated. The numbers indicate the amount of G:s that are needed to reach the specific destinations, respectively, indicated with blue crosses. The red line shows the future flight trajectory to the final destination if the pilot decides at the actual moment to take a 4 G turn with a $60^\circ$ rolling angle and keep it for a few seconds.

Again, the controllable parameters in this example are only the thrust percentage, roll angle and the angle of attack, respectively. The future translational positions depend on a variety of parameters, described earlier, for example the actual speed of the airplane as well as the density on the current altitude. By simulating these future trajectories, it becomes possible to predict the performance ability at a future position, if a certain maneuver at that future moment was to be wished to be initiated. By performing this kind of predictions, some movements could be prohibited, if those would for example lead to a severe loss of speed.
In Fig. 7.5 and 7.6 the actual flight path of the airplane is chosen to be a rather assertive gain in altitude, followed by a steep dive, without any turns in the horizontal plane. The flight trajectory is chosen in this example in order to illustrate what happens with the maneuvering freedom when the airplane drops and gains in speed, respectively. The blue arrows illustrate the positions of the airplane’s position throughout the sequence.

By plotting a window of possible future destinations every timestep during the flight, a tunnel of possible maneuvering is formed around the airplane along its path over time, which can be seen in Fig. 7.5 and 7.6. The red borders are similar to the ones illustrated in Fig. 7.2, but with maximum G as acceptable limit instead of acceptable speed loss. When the airplane gains altitude heavily, the speed decreases rapidly and therefore the maneuverability decreases. The density decreases as well with altitude, which also leads to less maneuverability for the airplane. The tunnel size is therefore smaller at the top of the elevation in Fig. 7.5, as the airplane’s speed and the aerodynamic density has diminished at those positions, which leads to a narrower range of possible turns in any direction. Further on, when the airplane redirects its speed directions and starts diving towards the ground, potential energy is switched to kinetic energy.

When diving towards ground, the density increases as well, which allows a higher load factor when performing maneuvers. This increase in speed during the steep dive and the increase in density enlarge the window of possible maneuvers, as a higher G-force could be initiated with a higher speed, more momentum and a steadily increasing atmospheric density.

In other words; every red circle shows the maximum deviation from predetermined flight path in all directions, if the airplane (blue arrows) a few seconds earlier would have chosen to initiate an angle of attack corresponding to maximum G in any roll angle. By having a greater circle the possible movement for the airplane is increased. This fact is of course also better in a sense of predictability for the chasing missile. When having the option to perform hard turns quickly, it is harder for the enemy - a chasing airplane or missile - to foresee the future flight path.
**Fig. 7.5:** Airplane trajectory starting with rapid elevation, followed by a climb and ending with a dive.

**Fig. 7.6:** Possible future turn maneuvering tunnel visualized over time; the airplane follows a predetermined flight path.
7.2 Roll away maneuver, maximum dive time

In Fig. 7.7 and 7.8 a simulation is visualized of initial heading towards threat (towards increasing positive positions values along the North-South axis, missile not visualized in illustrations) with immediate $165^\circ$ roll away from level flight with dive towards ground. The trajectory starts at 13000 meters of altitude at the coordinate 0 on both the North-South and West-East axis, respectively. Blue arrows indicate the actual position of the airplane continuously at every timestep, red lines indicate future predicted path if max-G was to be initiated one second later at every actual position, respectively. The airplane continues zero-G-drop towards ground until crash with ground is predicted, i.e. when a red line hits the ground for the first time. When crash is predicted - maximum G-pull-up is initiated, with one second to spare, in order to avoid crash until level flight is reached and forward on kept, close to sea level altitude.

The result of this maneuver was a hit by the missile. This was due to the fact that the airplane turns around from the threat in this maneuver and the missile shortcuts through the airplane’s trajectory. The missile gains higher speed due to the airplane’s dive which is not compensated by the thicker aerodynamic density and the missile strikes the airplane with a very high speed, approaching from above. However, even though this maneuver resulted in a hit by the missile, it might be an interesting maneuver if the target initially is further away from the airplane. In that case, this tactic would force the missile to endure maximum density during its flight.

This concept does exist with today’s technology to prevent fighter pilots from crashing to the ground if not being able to temporarily conduct the airplane, due to G-force induced loss of consciousness (abbreviated as G-LOC) after heavy G turns or similar reasons. Fig. 7.8 shows the same simulation as in Fig. 7.7, illustrating the maneuver from the side, visualizing the continuous potential future flight paths if maximum G was to be initiated.
In this roll away maneuver, the initial heading is, like in the previous section, towards the missile and increasing positive position values along the North-South axis. The initial level flight towards the threat is followed by an immediate 165° roll away with continuously maximum G-force that is allowed by current density and airplane structure, throughout the maneuver. When roll away has been carried out and elevation angle is zero, the roll angle is reset to 0° and level flight directly away from the threat is kept.

The advantage with performing a maximum G roll away compared to the maximum dive in Fig. 7.7 and 7.8 is that the airplane turns around and redirects its speed in the opposite direction of the threat as fast as possible. The downside with this tactic is that the airplane is still acting on a relatively high altitude, which leads to only a small density increase for the chasing missile. Further on, the maximum G cannot be as high during the maneuver on a higher altitude, as the density there is less dense.

This is however favourable for the airplane as a higher speed could be obtained at a higher altitude but this is the case for the chasing missile as well. On higher altitudes with less density; it is also harder to perform harder turns. However, even though the density is less on higher altitudes, the airplane can due to its structure perform harder turns compared to the chasing missile.

This maneuver also resulted in a hit by the chasing missile. As the airplane decreases its altitude during the roll away and the missile approaches rapidly due to the fact that the airplane is turning around; the missile hits with a fast speed, approaching from above.
Fig. 7.9: Roll away maneuver with continuously maximum G until opposite direction of flight is obtained, when level flight is initiated.

Fig. 7.10: Same scenario as in Fig. 7.9, visualized from the side.
7.4 Evasive maneuver sequence - repeating turns

In this sequence, each left and right repeating turn is executed with a roll angle corresponding to approximately 4 G until desired horizontal turn angle is reached. If model was to be examined with airplane going straight along North-South axis without turning, this would result in a hit with the chasing missile. Navy blue trajectory indicates turning to the other side immediately when 30° horizontal turn angle is reached (not to be confused with roll angle). Red, yellow, purple, light blue and green trajectories indicate keeping straight flight during 4, 8, 12, 16 and 20 seconds after 30° turn angles are reached, respectively. No version of this turn angle leads to a hit. This is thanks to the turns that the missile to some extent is forced to imitate that leads to losing too much kinetic energy before reaching the airplane.

Tab. 7.1: Evaluation of survival chance for each trajectory in Fig. 7.11, when turning left and right 30° while keeping flight at the same altitude with a chasing missile approaching from behind (not visualized).

<table>
<thead>
<tr>
<th>Shortest distance to target</th>
<th>Straight flight time after each turn</th>
<th>Turn angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1445 meters</td>
<td>Zero seconds, navy blue path in Fig. 7.11,7.12.</td>
<td>30°</td>
</tr>
<tr>
<td>1367 meters</td>
<td>4 seconds, red path in Fig. 7.11,7.12.</td>
<td>30°</td>
</tr>
<tr>
<td>1017 meters</td>
<td>8 seconds, yellow path in Fig. 7.11,7.12.</td>
<td>30°</td>
</tr>
<tr>
<td>747 meters</td>
<td>12 seconds, purple path in Fig. 7.11,7.12.</td>
<td>30°</td>
</tr>
<tr>
<td>427 meters</td>
<td>16 seconds, light blue path in Fig. 7.11,7.12.</td>
<td>30°</td>
</tr>
<tr>
<td>359 meters</td>
<td>20 seconds, green path in Fig. 7.11,7.12.</td>
<td>30°</td>
</tr>
</tbody>
</table>

Fig. 7.11: Simulation of different versions of 30° turns.
Fig. 7.12: Same scenario as in Fig. 7.11, visualized from above.

Tab. 7.2: Evaluation of Fig. 7.13, with corresponding trajectories as in Fig. 7.11 and 7.12 but instead with 45° turns. No hit was concluded in this case neither, even though some margins were not exaggeratedly large.

<table>
<thead>
<tr>
<th>Shortest distance to target</th>
<th>Straight flight time after each turn</th>
<th>Turn angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1758 meters</td>
<td>Zero seconds, navy blue path in Fig. 7.13.</td>
<td>45°</td>
</tr>
<tr>
<td>1325 meters</td>
<td>4 seconds, red path in Fig. 7.13.</td>
<td>45°</td>
</tr>
<tr>
<td>920 meters</td>
<td>8 seconds, yellow path in Fig. 7.13.</td>
<td>45°</td>
</tr>
<tr>
<td>624 meters</td>
<td>12 seconds, purple path in Fig. 7.13.</td>
<td>45°</td>
</tr>
<tr>
<td>208 meters</td>
<td>16 seconds, light blue path in Fig. 7.13.</td>
<td>45°</td>
</tr>
<tr>
<td>32 meters</td>
<td>20 seconds, green path in Fig. 7.13.</td>
<td>45°</td>
</tr>
</tbody>
</table>

As the turn angle is larger in the case illustrated in Fig. 7.13, 45° instead of 30°, the airplane moves slower in North-South direction, but the chasing missile needs to match the trajectory with larger rudder adjustments when following. Notice the increase in West-East direction as a larger deviative turn angle from North-South axis is performed. This decrease in overall speed in the North-South direction leads to a faster catch up by the chasing missile. When evaluating the differences between Tab. 7.1 and Tab. 7.2, the margins to a hit are overall decreased when the turn angle is increased.
Fig. 7.13: The same flight pattern as illustrated in Fig. 7.11 and 7.12 but instead with 45° horizontal path turns (not to be confused with roll angle).

As can be read in Tab. 7.3; Straight flight paths (purple, light blue and green flight paths) between 12-20 seconds with 60° turn angle would - unlike the 30° and 45° turns cases - have lethal outcome, theoretically according to the tests. By instead following the navy blue, red or yellow trajectory leads to surviving the attack, according to the results. This type of detailed information - where small changes matter - might be useful assistance for a pilot in a highly stressful scenario like this. The distance margins to the missile for 4-8 seconds of straight flight after each turn were also smaller than for 30° and 45° turns, respectively.

Tab. 7.3: Simulation of corresponding trajectories as in Fig. 7.11, 7.12 and 7.13 but with an even larger turn angle, 60°.

<table>
<thead>
<tr>
<th>Shortest distance to target</th>
<th>Straight flight time after each turn</th>
<th>Turn angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1614 meters</td>
<td>Zero seconds, navy blue path in Fig. 7.14.</td>
<td>60°</td>
</tr>
<tr>
<td>916 meters</td>
<td>4 seconds, red path in Fig. 7.14.</td>
<td>60°</td>
</tr>
<tr>
<td>385 meters</td>
<td>8 seconds, yellow path in Fig. 7.14.</td>
<td>60°</td>
</tr>
<tr>
<td>Missile hits airplane.</td>
<td>12 seconds, purple path in Fig. 7.14.</td>
<td>60°</td>
</tr>
<tr>
<td>Missile hits airplane.</td>
<td>16 seconds, light blue path in Fig. 7.14.</td>
<td>60°</td>
</tr>
<tr>
<td>Missile hits airplane.</td>
<td>20 seconds, green path in Fig. 7.14.</td>
<td>60°</td>
</tr>
</tbody>
</table>

In Fig. 7.14; a visualization of the case whose values are summarized in Tab. 7.3. As the overall speed in North-South direction is even more decreased with a larger turn angle, 60°, the missile catches up with its target even faster and actually hits it in some cases.
When dealing with these - in terms of time - relatively short sequences with high speeds, every second has a big impact on the overall trajectory. As could be read in Tab. 7.4 - the tighter turns the better concerning this specific scenario, although a completely straight flight path without any turns leads to a hit by the missile. By executing straight level flight for a specific number of seconds after every turn - if the turn exceeds a specific degree value - decides if the outcome is survival or being shot down, respectively.

When summarizing Tab. 7.1, 7.2 and 7.3 - a clear pattern regarding straight flight time, turn angles and closest distance to target can easily be highlighted, if one was about to support the pilot with a recommendation of exactly how to execute the flight path.

**Tab. 7.4: Summary of Tab. 7.1, 7.2 and 7.3.**

| Shortest distance to target, 30° turn angle | Shortest distance to target, 45° turn angle | Shortest distance to target, 60° turn angle | Straight flight time  
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1445 meters</td>
<td>1758 meters</td>
<td>1614 meters</td>
<td>Zero seconds</td>
</tr>
<tr>
<td>1367 meters</td>
<td>1325 meters</td>
<td>916 meters</td>
<td>4 seconds</td>
</tr>
<tr>
<td>1017 meters</td>
<td>920 meters</td>
<td>385 meters</td>
<td>8 seconds</td>
</tr>
<tr>
<td>747 meters</td>
<td>624 meters</td>
<td>Missile hits airplane.</td>
<td>12 seconds</td>
</tr>
<tr>
<td>427 meters</td>
<td>208 meters</td>
<td>Missile hits airplane.</td>
<td>16 seconds</td>
</tr>
<tr>
<td>359 meters</td>
<td>32 meters</td>
<td>Missile hits airplane.</td>
<td>20 seconds</td>
</tr>
</tbody>
</table>
7.5 Evasive maneuver sequence - repeating elevations and dives

Flight trajectories in this scenario consist of repeating elevations and dives of ±3 G (gravitational force taking into consideration), where each dive or elevation stops when elevation angle has reached a certain value with an initial altitude of 8000 meters. These maneuvers force the chasing missile to adjust its straight forward up and down over time, even though different kinds of missiles respond differently to a changed flight path. By doing these maneuvers, more aerodynamic drag is obtained by both the airplane and missile, compared to if the flight path would have been straight level flight, which in these simulations would have resulted in a hit.

Tab. 7.5: Flight maneuvers with threat approaching from behind along the North-South axis (missile not in illustrations).

<table>
<thead>
<tr>
<th>Shortest distance to target</th>
<th>Elevation angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Missile hits airplane.</td>
<td>0°</td>
</tr>
<tr>
<td>2200 meters</td>
<td>±20°, blue trajectory in Fig. 7.15, 7.16.</td>
</tr>
<tr>
<td>4300 meters</td>
<td>±30°, green trajectory in Fig. 7.15, 7.16.</td>
</tr>
<tr>
<td>4800 meters</td>
<td>±40°, red trajectory in Fig. 7.15, 7.16.</td>
</tr>
</tbody>
</table>

Fig. 7.15: Visualization of flight paths in Tab. 7.5, seen from the side of the airplane’s trajectories.

Unlike the previous evasive maneuver in section 7.4, the airplane changes its direction of speed and starts to elevate directly after the dive and vice versa. It does never fly with a constant angle of attack. Fig. 7.16 shows the same simulation as Fig. 7.15 from a different point of view. The increased margin to a hit with an increased angle of attack is interesting due to the fact that an increased angle leads to an overall slower pace in the North-South direction. This means that the missile catches up with its target faster if it would hold a straight flight path, which it however does not with this type of missile model. Regardless, more aggressive elevations and dives seem to affect the missiles responding trajectory to a severe extent.
In Fig. 7.17 and 7.18; a different kind of maneuver sequence is shown, only simulated in one version. The trajectory starts at 3000 meters, at value zero on the North-South and West-East axis, respectively. After performing a series of maneuvers, the simulation ends at the sharp elevation some 25 000 meters into the North-South axis. After the initial elevation a $130^\circ$ 2 G roll, with loss of altitude as a result, is initiated. This is followed by a level flight right turn. Finally, a 3 G turn $40^\circ$ left roll turn is performed which results in a heavy gain of altitude.

Results show a hit by the missile, from several different missile initial altitude values. The kinetic energy that the missile loses when it chases the initial elevation performed by the airplane does not make up for the decreased distance between the missile and the airplane. When gaining altitude heavily in the beginning at low altitudes, the missile further on gets to fly in decreased pressure, when maneuver sequence starts with an elevation by the chased airplane. At low initial altitudes, complicated maneuvers seem to not have the time to get desired result, as the missile most probably attacks from a higher altitude.

This initial maneuver could be compared to the evasive maneuver sequence described in section 7.7, where the airplane starts with a steep diving instead of an elevation. It seems to be as the priority is to first pick up speed and at the same time travel in the opposite direction of the missile.
Fig. 7.17: Simulation of a nonsymmetric maneuver with steep dives, elevations and sharp turns.

Fig. 7.18: Flight path in Fig. 7.17, visualized from the rear.
7.7 Evasive maneuver sequence - Dive, turn, elevation

Elevation with three different final altitudes are simulated in this scenario and visualized in Fig. 7.19 and 7.20. During the initial dive, whether or not crash with ground is likely to occur is evaluated at every timestep (red crosses in Fig. 7.19 and 7.20). When crash is predicted if dive was to be continued - a fast catch up is initiated. This evasive maneuver sequence is interesting as both a determined dive to maximum density level as well as a heavy elevation are parts of the sequence, which should be hard for a chasing missile to endure during the right circumstances. At the same time, the airplane mostly travels in a direction away from the threat.

Tab. 7.6: Simulation of initial dive away from the threat, followed by a left turn, ending with three different types of elevation altitudes, marked red in Fig. 7.19 and 7.20.

<table>
<thead>
<tr>
<th>Shortest distance to target</th>
<th>Final elevation ending altitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>3300 meters</td>
<td>2000 meters</td>
</tr>
<tr>
<td>3300 meters</td>
<td>3000 meters</td>
</tr>
<tr>
<td>3300 meters</td>
<td>4000 meters</td>
</tr>
</tbody>
</table>

Fig. 7.19: Evasive maneuver sequence with dive, turn and three different types of elevations.

The steep initial dive by the followed by left turn on low altitude with thick density makes the missile give up before the elevations are performed. The result was therefore survival in all three versions of this evasive maneuver, shown in Fig. 7.19 and 7.20. This could be compared to the maneuver described in section 7.6, which started with an elevation and hits by the missile.
7.8 Evasive maneuver sequence - Different turn radii

In this sequence, the airplane initially follows a path directed away from the chasing missile. It starts with diving to a low altitude with ground crash evaluations (red crosses in Fig. 7.21 and 7.22), when pull-up is initiated at a few hundred meters of altitude. After the dive, three different types of left turns with different radii are performed, ending with a 2 G altitude climb to approximately 8000 meters after the 180° turn is reached, followed by level flight. During the left turns, three different types of roll angles are tested, which combined with the condition of flight on same altitude throughout the turn determines the angle of attack needed. In other words, the trajectory ends with flying into the direction at which the threat arrived from initially. By doing this maneuver sequence, the idea is to make the enemy missile a lot of drag at low altitude and at the still time turn back and reapproach the scene of battle.

Tab. 7.7: Simulation of initial dive away from the threat, followed by different types of turns around, ending with elevation.

<table>
<thead>
<tr>
<th>Shortest distance to target</th>
<th>Roll angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Missile hits airplane.</td>
<td>0°, not in Fig. 7.21, 7.22</td>
</tr>
<tr>
<td>3700 meters.</td>
<td>40°, light blue path in Fig. 7.21, 7.22.</td>
</tr>
<tr>
<td>2400 meters</td>
<td>60°, black path in Fig. 7.21, 7.22.</td>
</tr>
<tr>
<td>Missile hits airplane.</td>
<td>70°, green path in Fig. 7.21, 7.22.</td>
</tr>
</tbody>
</table>

The sharp 70° (green path) turn leads to being shot down by the missile. This is due to the fact that the airplane turns around too fast and is more or less head on with the missile at the end of the turn. The two other turns with larger turn radii on the other hand take enough time to perform to force the missile time to catch up and follow in behind the airplane during the turns. By doing so, the missile’s travel path is maximized, while performing a drag-inducing turn with at a high atmospheric density.
Fig. 7.21: The different versions of evasive maneuver sequence with different turn radii.

A larger turn radius in this simulation diminishes the missile’s chances to take shortcuts and it has to endure the high density that the airplane has dived down into. In other words - if being able to match the time reference so that the airplane performs the turns $70^\circ$ when the missile is behind closely enough without being hit - the outcome is fortunate.

Fig. 7.22: The exact same simulation as visualized in Fig. 7.21 but with view angle from the rear.
This work has created a performance model for a typical jet fighter and has combined this with a performance model of a medium range air to air missile with augmented proportional navigation parameterized with fictive values. By doing so, it has been possible to determine that patterns regarding optimal flight trajectory can be elucidated, when adjusting control variables in a number of different flight maneuvers.

By analyzing the results, one can notice that - whether or not the airplane is shot down by the chasing missile - is strongly correlated to small differences in control variables values. The results seem not to be randomized but for a given scenario it seems possible to find the critical parameter values for when the outcome changes. It also seems to be that the smallest distance to the threat in some scenarios follows a continuous function. When evaluating Fig. 7.12, 7.13 and 7.14 in Tab. 7.4 for example, one can see clearly that harder turns ought to be recommended, as these turns without drifting to the sides when completed do increase the chance of survival. On the contrary when repeating elevations and dives instead, visualized in Fig. 7.15; a better choice seems to be fairly large dive and elevating angle, respectively. Further on, when examining the sequence described in section 7.8; performing a too sharp turn leads to letting the missile shortcut to the target, without having time to fall in directly behind the airplane, as was the case at the start of that specific scenario.

Larger or smaller angles - each specific scenario seems to have an optimal parameter value and the simulations in this work seem to point towards a non-randomized underlying pattern. Of course the results differ depending on the type of guidance algorithm that is steering the actual missile one is examining during the simulations. The differences could be vast between different missiles and therefore the optimal tactic for an airplane trying to evade the missile is different from time to time. There is nevertheless no reason to believe why similar result patterns would not be obtained when performing studies on other types of missiles, as well.

These types of situations are of course extremely stressful to the pilots, which demands a strong focused mentality on solving the situation. Keeping a straight flight path for a few seconds too long every turn, for example, might not be an obvious difference for the conducting personnel in the given moment, when a great span of parameters is under the lens during severe time pressure. Further on, the exact position of the hostile missile might not be known. Computer performed evaluations of this sort might assist the pilot’s intuitive feeling for how to act in detail, when being attacked and a great amounts of actions must be carried out correctly in a short amount of time with minor acceptable margins.

The simulation control variable values in this work are set intuitively. Regarding the maneuver sequences with different versions in this work, only a small variety of angles and flight time alternatives for every scenario are examined and compared. Whether or not an algorithm exists for predicting the optimal choice directly, is up for future investigation in this subject. What can be concluded for sure is that optimizing controllable parameters in detail by probable existing underlying patterns could increase the overall fighting potential substantially for a jet fighter. Instead of being shot down by the missile - a small pattern foreseen parameter change could result in surviving the attack and a retake of the initiative of the fight with a chance to strike back.
ACKNOWLEDGEMENTS

The work behind this paper has been performed on behalf of the Swedish aeronautics company Avioniq. Thanks to the guidance from computer engineer CTO Martin Ohlsson and CEO Mikael Grev, former fighter pilot in the Swedish Airforce, my conception of military air fighting tactics has increased. I want to thank Avioniq particularly for giving me an understanding regarding maneuvering methods and how to make simulations over such situations more realistic. By implementing Avioniq’s missile model’s parameter values with the aircraft performance model that constitutes much of this work, I have realized the possibilities with combining models governed by laws of physics. It has been an exciting lesson how to use this method to simulate a potential realistic event and further on how to seek improvements in that scenario.

This master thesis has been approved by the Department of Aeronautics at the Royal Institute of Technology in Stockholm, Sweden.

BIBLIOGRAPHY


[3] In discussion with Avioniq.


