Generic 6-DOF Added Mass Formulation for Arbitrary Underwater Vehicles based on Existing Semi-Empirical Methods

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Abstract

The KTH Maritime Robotics Lab is developing a simulation framework for experimental autonomous underwater vehicles in MATLAB and Simulink. This project has developed a formulation for added mass of the vehicle, to be implemented in this simulation framework. The requirements of the solution is that it should require low computational power, be a general formulation applicable on arbitrarily shaped vehicles and be verified against literature. Different existing methods and formulations for primitive bodies have been investigated, and combining these methods has resulted in a simplified but adequate method for calculating the added mass of arbitrarily shaped hulls and control surfaces, that is easy to implement in the existing simulation framework. The method has been verified by calculating added mass coefficients for two existing vehicles, and comparing the values to the coefficients already calculated for the vehicles in question. Some limitations have been identified, such as the interaction effects between components of the vehicle not being taken into account. To determine the extent of the errors due to this simplification and to fully validate and verify the model, future work in the form of CFD calculations or experiments on added mass measurements need to be conducted. There is also an uncertainty in the calculation of the coupled coefficients $m_{26}$ and $m_{35}$, and results on these coefficients should be handled with care.
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Nomenclature

\( \alpha_0 \)  Constant of the relative proportions of a spheroid [-]
\( \Delta \)  Frequency ratio wet/dry [-]
\( \rho \)  Density of water \([kg/m^3]\]
\( a_{\text{fin}} \)  Maximum height above the centerline of the vehicle fins [m]
\( a_{ij} \)  The 2-dimensional added mass in the \( i \)th direction caused by an acceleration in the \( j \)th direction \([kg/m]\]
\( AR \)  Aspect ratio [-]
\( b \)  Span of control surface [m]
\( c \)  Chord of control surface [m]
\( e \)  Eccentricity of meridian elliptical section of spheroid [-]
\( k_1 \)  Lamb’s k-factor [-]
\( k_{\text{rot}} \)  Coefficient of added moment of inertia
\( M_a \)  Added mass matrix
\( m_a \)  Added mass [kg]
\( m_{ij} \)  The added mass in the \( i \)th direction caused by an acceleration in the \( j \)th direction [kg]
\( m_s \)  Structural mass [kg]
\( r \)  Radius of the circular cylinder [m]
\( R(x) \)  Radius of vehicle hull [m]
\( r_1 \)  Half the length of spheroid [m]
\( r_2 \)  Radius of spheroid [m]
\( S \)  Surface area \([m^2]\)
\( t \) Thickness of control surface [m]

\( V \) Volume \([m^3]\)

\( x_{cb} \) Distance center of the plate - the center of buoyancy of plate in x-direction [m]

\( y_{cb} \) Distance center of the plate - the center of buoyancy of plate in y-direction [m]
Chapter 1

Introduction

Autonomous underwater vehicles, also called unmanned underwater vehicles, are unmanned and untethered submersible vehicles. Due to the versatility, they are becoming increasingly popular and can be used in a wide range of applications from ocean-based research, ocean floor mapping, Antarctic exploration to ocean farming and harvesting. With the elimination of humans, the vehicle can reach unexplored places that have been previously inaccessible. The construction of AUVs has however been limited by several aspects that are extra notable when working underwater such as battery capacity, communication, navigation, visibility etc. The increase in popularity and use presents a higher demand and new challenges to overcome these limits. Future AUVs need to be more maneuverable as well as have an increased range in terms of distance, depth and speed to achieve higher precision and efficiency.

The KTH Maritime Robotics Lab is involved in the development of such AUVs and is currently developing a simulation framework. It is built in the programming language Simulink and includes dynamic modeling of arbitrary shaped AUVs together with various user interfaces. Simulations and modeling of AUVs provides several advantages; a good understanding of the forces and moments acting on the vehicle is crucial for a successful design and can provide a solid base on which to build the whole design process, and a model can determine the value of design parameters and validate design choices. This way, mistakes can be avoided when the actual construction is begun since they were identified at an earlier stage.

One of the many aspects in modeling underwater vehicles (or any vehicles moving in a fluid) is the one of added mass. When a body accelerates in a fluid, the fluid around the body is disturbed and accelerated which requires additional force than that required to move the body in vacuum. In vacuum, this phenomenon would not exist since there would be no particles to move. This increase in inertia can be explained as the energy required to establish the field of flow around the body when it is moving in any direction or, simpler put, the energy required to move the fluid out of the way of the moving body. The required energy acts as a pressure on the hull, and since pressure acts normal to the
surface, the geometry of the body is of great importance (Perrault et al., 2002). This phenomenon has been known since as early as 1836 (Gracey, 1941), and since then a considerable amount of research has been done on the subject.

There are several ways to undertake this problem. Most methods, in particular the older ones, are semi-empirical ones. An AUV can be divided into hull, control planes and propeller. For hulls, an approximation of a spheroid is often made for underwater bodies, and a method developed by Horace Lamb is used (Imlay, 1961). It is a semi-empirical method that depends on the ratio of the length and width of the spheroid. This is a very commonly used model (Doherty, 2011; Imlay, 1961; Humphreys & Watkinson, 1978; Lee et al., 2011; Prestero, 2001) since a spheroid is a good approximation of many slender body underwater vehicles. Another common method is the slender body theory (Newman, 1977), which is widely used in all fields of naval architecture. This requires the assumption of a slender body and is generally a good method since it offers the possibility to work with a more accurate shape without the need of excessive simplification. However, this method is not enough in itself since it does not provide a method to calculate the axial added mass (along the direction in which the vehicle is slender). To get a method that covers all aspects of the added mass, i.e. a method that can calculate the coefficients in all directions, one can combine several established methods and get complete expressions for the added mass of an arbitrarily shaped body.

Regarding the control surfaces, such as rudders and elevators, a common approximation is the one of a flat plate. In most cases, underwater vehicles are fairly small, and thus also have small control surfaces. The added mass contributions of said control surfaces to the complete vehicle are consequently only a fraction of those of the hull. Therefore, the errors incurred from approximations of the control surfaces will be less significant than those of the hull. Still, the approximation of a flat plate is fairly accurate and used widely even for bigger wings such as those of airplanes, presented in a report from NACA (National Advisory Committee for Aeronautics) by Gracey (1941). An example of the use of flat plate approximation in an underwater context, and a slightly newer source, is found in Humphreys & Watkinson (1978). That example also includes a comparison of the analytically derived values with experimental data, showing quite low errors, thus proving the method as a reliable one. The percent difference error is low (under 12%) in all but four coefficients.

The above described examples of methods are all based on potential flow theory and regression analysis. With the development of more and more advanced computers and computer models, methods such as CFD and boundary element methods (panel methods) are getting increasingly more common. Although these methods are very accurate, they require more computational power and are quite complicated. In an extensive simulation program, it is of importance to use simpler models to make the model less demanding and require less computational power, especially when the shape of the hull is as simple as AUV hulls often are.

Regarding propellers, geometry is more complicated due to non-existent symmetry around
any plane. The boundary element method can give very accurate values for such complicated shapes (Ghassemi & Yari, 2011), however it is not optimal to use such a complicated and time-consuming method for such a small part of a simulation model. In this simulation framework, all thrusters are simply modeled as thrust and torque as a function of rpm and propeller characteristics based on precalculations or measured propeller performance. As a result of this simplification, only two categories of shapes will have to be investigated: the hull and a wing-like control surface.

As mentioned, when implementing this phenomenon of added mass into a simulation framework, one has to consider several aspects. The theory needs to provide enough accuracy to yield acceptable results, but still not use overly complicated approaches which will slow down the simulation. It is also important to be able to use the model for any AUV, so the theory needs to be applicable to arbitrary shapes. Existing added mass calculations in models are generally very specific and targeted on a specific vehicle or shape, often very briefly explained and rarely the focus of the report, but rather a small section in a bigger whole. The challenge therefore lies in combining existing theories and models into a broader and more general model. Keeping this in mind, the implementation of the theory in Simulink will provide a reliable and enhanced formulation for the added mass section of the simulation model that will work on a wide range of differently shaped AUVs, leading to an increased performance in developed vehicles.

### 1.1 Problem Statement

The simulation framework as it is today is lacking an adequate formulation for the added mass coefficients of general bodies of hulls and control surfaces. The current formulation is very simplified and does not take the influence of the control surfaces into account. Another important aspect is that if the coefficients are calculated simultaneously as the model runs, a lot of computational power will be required. It would be beneficial to find a solution providing pre-calculations of the coefficients for an arbitrary body of hull and control surface, from which values can be easily extracted and interpolated to fit the current modeling situation.

To be able to verify the model, existing values are required for comparison with computed values resulting from the simulation. These values will be found in literature on similarly shaped bodies to be able to give an accurate comparison.

Requirements of the solution:

- Low computational power requirement/quick simulations
- General formulation applicable on arbitrary shapes of hull and control surfaces
- Verified against experiments
1.2 Aim of thesis

The aim of this thesis is to provide an added mass formulation that is integrated into the existing Simulink AUV simulation framework. This includes analysis of existing and established added-mass formulations for primitive bodies and the combination of these, to provide a simplified model for an arbitrary shaped body and control surface. The goal is to calculate the coefficients for the arbitrary shape, to store these for future extractions when they are needed in the simulation.

1.3 Notation and coordinate system

To describe the motion of the AUV, it is modeled in a six degree of freedom system (6DOF). This includes the linear motions surge, sway and heave and the angular motions roll, pitch and yaw. The notation used for these motions are according to SNAME (1950) and are presented in Table 1.1.

<table>
<thead>
<tr>
<th>Degree of Freedom</th>
<th>Motion</th>
<th>Forces and moments</th>
<th>Linear and angular velocities</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Surge (motion in the x-direction)</td>
<td>X</td>
<td>u</td>
</tr>
<tr>
<td>2</td>
<td>Sway (motion in the y-direction)</td>
<td>Y</td>
<td>v</td>
</tr>
<tr>
<td>3</td>
<td>Heave (motion in the z-direction)</td>
<td>Z</td>
<td>w</td>
</tr>
<tr>
<td>4</td>
<td>Roll (rotation around the x-axis)</td>
<td>K</td>
<td>p</td>
</tr>
<tr>
<td>5</td>
<td>Pitch (rotation around the y-axis)</td>
<td>M</td>
<td>q</td>
</tr>
<tr>
<td>6</td>
<td>Yaw (rotation around the z-axis)</td>
<td>N</td>
<td>r</td>
</tr>
</tbody>
</table>

This thesis only concerns the coefficients of added mass in a local body-fixed coordinate system with origin in the vehicles center of buoyancy. The transformation from local to global coordinate system, regarding additional forces is done in the Simulink simulation program.

The local coordinate systems are placed as presented in Figure 1.1 and 1.2.

1.4 Assumptions and limitations

To facilitate calculations, some assumptions have been made in the development of the model. These are listed below.

1. Deeply submerged body. The body is assumed to be completely submerged and away from the surface, seabed and any other surfaces underwater. When the body is deeply submerged, the added mass coefficients can be considered constant.
Figure 1.1 – The local coordinate system for the hull. Origin in center of buoyancy.

Figure 1.2 – The local coordinate system for the control surfaces. Origin in center of buoyancy.

2. No waves or currents. No waves, currents or other disturbances are taken into account.

3. Inviscid (frictionless) fluid and no circulation. This enables the use of potential flow theory.

4. The vehicle is rotationally symmetric around the $x$-axis. The cross sections in the XY and XZ planes are identical.

5. The components of the vehicle are treated in isolation. This means effects from interaction between parts are neglected.
Added mass is the pressure induced forces and moments due to fluid accelerating with an accelerating body. When a body is accelerating in any direction, it has to accelerate the surrounding fluid as it is moving through it which requires additional forces and moments than if the vehicle was moving in vacuum. The \textit{added mass} is a hypothetical volume of fluid accelerating with the same acceleration as the body. It is important to note that there is no distinct mass of fluid moving with a specific acceleration; it is just a convenient way of describing the additional forces and moments. In reality, all fluid particles surrounding the submerged body will move with different acceleration. For example, a fluid particle one meter away from the hull will not have the same acceleration as a particle right next to the vehicle, but to facilitate calculation, a finite volume is assumed.

The fluid forces due to added mass are given by:

$$\begin{bmatrix} X_{am} \\ Y_{am} \\ Z_{am} \\ K_{am} \\ M_{am} \\ N_{am} \end{bmatrix} = -M_a \frac{d}{dt} \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix}$$

(2.1)

Where $M_a$ is the \textit{added mass matrix} of a body in six degrees of freedom, and is defined as:
Another common notation, and the one that will be used in this report, for these coefficients is:

\[
M_a = \begin{bmatrix}
X_{\ddot{u}} & X_{\ddot{v}} & X_{\ddot{p}} & X_{\ddot{q}} & X_{\ddot{r}} \\
Y_{\ddot{u}} & Y_{\ddot{v}} & Y_{\ddot{p}} & Y_{\ddot{q}} & Y_{\ddot{r}} \\
Z_{\ddot{u}} & Z_{\ddot{v}} & Z_{\ddot{p}} & Z_{\ddot{q}} & Z_{\ddot{r}} \\
K_{\ddot{u}} & K_{\ddot{v}} & K_{\ddot{p}} & K_{\ddot{q}} & K_{\ddot{r}} \\
M_{\ddot{u}} & M_{\ddot{v}} & M_{\ddot{p}} & M_{\ddot{q}} & M_{\ddot{r}} \\
N_{\ddot{u}} & N_{\ddot{v}} & N_{\ddot{p}} & N_{\ddot{q}} & N_{\ddot{r}}
\end{bmatrix}
\]  
(2.2)

These coefficients represent the forces in six different degrees of freedom due to acceleration in each combination of degrees of freedom. For example, a force in the X-direction (1) due to an acceleration in the y-direction (2), \(\ddot{v}\), is represented by the term \(X_{\ddot{v}} = m_{12} = M_{a,12}\). More generally, a force in the \(i\)th direction due to an acceleration in the \(j\)th direction (see Table 1.1 for notation) is represented by \(M_{a,ij}\). The diagonal elements of the matrix are the primary coefficients, relating movement in one direction to the force or moment in that same direction. The non-diagonal coefficients are the coupled or secondary coefficients. All added mass coefficients depend entirely on the geometry of the vehicle, together with the density of the surrounding fluid.

### 2.1 Reduction of the added mass matrix

The added mass matrix contains 36 coefficients, and in a real fluid all of them would be distinct. However, with the assumption of ideal fluid according to assumption 3 in Section 1.4, the constants are symmetric with respect to the diagonal of the matrix: (Imlay, 1961; Newman, 1977).

\[
M_a = M_a^T
\]  
(2.4)

or
\[ M_{a,ij} = M_{a,ji} \]  \hspace{1cm} (2.5)

i.e., \( m_{14} = m_{41} \). This reduces the number of individual coefficients to 21.

In addition, symmetry of the body will reduce this number even further. If a body is symmetric in all three planes (XY, XZ, YZ), only the six coefficients on the diagonal are non-zero, and there is no coupling between different degrees of freedom. An example of this is a spheroid, or an ellipsoid of revolution. This is the most general body where analytical results are available for comparison (Newman, 1977).

However, a completely symmetric body is not realistic as a representation of an AUV. Adding fins and/or non-symmetry of the body around the XY or YZ plane will increase the number of individual coefficients, although XZ-symmetry can usually be assumed.

### 2.2 Added mass coefficients by slender body theory

A slender body is a body whose characteristic length in the longitudinal direction (x) is considerably larger than the body’s characteristic length in the other two directions (y and z). In other words, the slenderness ratio \( d/L \) is small. Additionally, the variations in y and z-direction are small along the x-axis. Considering a slender body, slender body theory can be applied to calculate the added mass. This means that we consider the body as a longitudinal stack of thin slices; each considered a two-dimensional section with an easily calculated added mass, as illustrated in Figure 2.1. The effects are then integrated along the longitudinal axis (x) to approximate the total added mass for the whole body.

To start with, the hull of the vehicle will be considered, that is the control planes will be disregarded for now. Therefore, rotational symmetry around the x-axis will be assumed, i.e. port-starboard and top-bottom symmetry. This will reduce the added mass matrix significantly and leave the following coefficients:

\[
M_{a,\text{body}} = \begin{bmatrix}
m_{11} & 0 & 0 & 0 & 0 & 0 \\
0 & m_{22} & 0 & 0 & m_{26} & 0 \\
0 & 0 & m_{33} & 0 & m_{35} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & m_{53} & 0 & m_{55} & 0 \\
0 & m_{62} & 0 & 0 & 0 & m_{66}
\end{bmatrix} \hspace{1cm} (2.6)
\]

In addition to this, the following simplifications can also be made:
\[ m_{22} = m_{33} \\
m_{55} = m_{66} \] (2.7)

### 2.2.1 2-dimensional coefficients

The added mass coefficients for various simple two-dimensional shapes are tabulated in Newman (1977, p. 145). The simplest shape is a circular cylinder (Figure 2.2, for which the 2D-added mass coefficients are presented below. The 2D-coefficients will be written as \( a_{ij} \):

\[ a_{22} = \pi pr^2 \\
a_{33} = \pi pr^2 \\
a_{44} = 0, \] (2.8)

where \( r \) is the radius of the circular cylinder.

### 2.2.2 3-dimensional coefficients

The translational and rotational 3D coefficients are determined by integrating over the length \( L \) of the body according to Lewis (1989, p. 56). Origin lies in the center of buoyancy of the body as stated in Section 1.3:
Figure 2.2 – The coordinate system of a 2D-slice of a slender body. The coordinate system is matching that of local coordinate system of the three-dimensional body.

\[
m_{22} = \int_L a_{22} dx \\
m_{33} = \int_L a_{33} dx
\]

\[
m_{44} = \int_L a_{44} dx \\
m_{55} = \int_L x^2 a_{33} dx \\
m_{66} = \int_L x^2 a_{22} dx,
\]

where \(a_{44}\) is zero for a circular cross section in an inviscous fluid, as stated in Equation 2.8. The coupled coefficients are calculated as

\[
m_{26} = \int_L x a_{22} dx \\
m_{35} = -\int_L x a_{33} dx.
\]

If the body is fore-aft symmetric around the origin in the coordinate system defined in Section 1.3, these coefficients will be zero, since \(\int_L x dx = 0\). This is consistent with the reduction of the added mass matrix due to symmetry discussed in Section 2.1.
2.3 Axial added mass by Lamb’s k-factors

Slender body theory sums up all the added mass coefficients for the thin 2D-sections perpendicular to the x-axis. Due to this, the method does not provide a way to calculate the axial added mass in the direction of the axis along which the integration is made, i.e. the x-direction. For this purpose, another method is presented below.

The added mass of a prolate spheroid (an ellipsoid of revolution, see Figure 2.3) can be calculated using Lamb’s k-factors (Imlay, 1961). The expression for the axial added mass coefficient $m_{11}$ is:

$$m_{11} = -k_1 \frac{4}{3} \pi \rho r_1 r_2^2$$  \hspace{1cm} (2.12)

where $r_1$ is half the length of the spheroid, $r_2$ is the radius, and $k_1$ is one of Lamb’s k-factors and is dependent on the ratio between the length and width of the spheroid:

$$k_1 = \frac{\alpha_0}{2 - \alpha_0},$$  \hspace{1cm} (2.13)

where $\alpha_0$ is a constant describing the relative proportions of the spheroid. This constant is defined as

$$\alpha_0 = \frac{2(1 - e^2)}{e^2} \left(\frac{1}{2} \ln \frac{1+e}{1-e} - e\right),$$  \hspace{1cm} (2.14)

where $e$ is the eccentricity of the meridian elliptical section:

$$e = 1 - \frac{r_2^2}{r_1^2}.$$  \hspace{1cm} (2.15)
In Equation 2.12 it is shown that the k-factor can be seen as a ratio between the added mass and the mass of the displaced fluid. This mass is the "virtual" finite volume described in the beginning of this chapter.

2.4 Added mass coefficients for a flat plate

As mentioned earlier, the approximation of wings and control surfaces as flat plates is fairly common and is used in both the aeronautical and the maritime sector. Humphreys & Watkinson (1978) shows an example of using it for underwater vehicles, while Gracey (1941) shows a use in aeronautics. In the following section, the theory of calculating added mass for flat plates will be presented.

2.4.1 Translational added mass

The only non-zero translational added mass for a flat plate is the added mass generated from movement perpendicular to the plane; movement in the plane of the plate will not generate any forces since the plate has no thickness. The coordinate system used is the same as the local coordinate system for a wing/control surface described in Section 1.3.

\[ m_{33} = k_{trans} \frac{\pi \rho c^2 b^3}{4}, \]

(2.16)

where \( c \) is the chord (or short side) of the control surface, and \( b \) is the span (or the long side). \( k_{trans} \) is a factor accounting for the 3D effects that occur due to the plate’s finite length:

\[ k_{trans} = \frac{1}{\sqrt{1 + \frac{1}{AR}}} \]

(2.17)

The \( k_{trans} \) coefficient is also presented in Figure 2.4 and is a function of the aspect ratio of the plate:

\[ AR = \frac{b^2}{S}, \]

(2.18)

where \( S \) is the surface area of the plate.
2.4.2 Rotational added mass

The expression for the added inertia in rotation is based on Gracey (1941) and Malvestuto & Gale (1947). The rotation around the x-axis, with origin in the center of the plate, generates an added inertia $m_{44}$ according to:

$$m_{44} = k_{rot} \frac{\pi \rho c^2 b^3}{48}.$$  \hfill (2.19)

The added inertia due to a pitching moment rotation around the y-axis, is expressed as:

$$m_{55} = k_{rot} \frac{\pi \rho b^2 c^3}{48}.$$  \hfill (2.20)
In Equation 2.20, the aspect ratio used in determining $k_{rot}$ from Figure 2.5 is instead $\frac{1}{AR}$ (Malvestuto & Gale, 1947).

To account for a center of buoyancy not coinciding with the center of the plate, a correction has to be made (Gracey, 1941):

\[
m_{44} = k_{rot} \frac{\pi \rho c^2 b^3}{48} + y_{cb}^2 k_{trans} \frac{\pi \rho c^2 b}{4} \tag{2.21}
\]

\[
m_{55} = k_{rot} \frac{\pi \rho b^2 c^3}{48} + x_{cb}^2 k_{trans} \frac{\pi \rho c^2 b}{4}, \tag{2.22}
\]

where $y_{cb}$ and $x_{cb}$ are the distances from the geometric center of the plate to the center of buoyancy of the plate in y and x-direction.

\[\text{Aspect ratio of plate, } A = \frac{b^2}{s}\]

**Figure 2.5** – Coefficient of additional moment of inertia, (Gracey, 1941)
Chapter 3

Theory implementation

This chapter will describe how the theory presented in the previous chapter will be used.

3.1 The component build-up method

The added mass coefficients for the entire vehicle will be calculated separately for each component. This method is called the body build-up technique or the component build-up method, illustrated in Figure 3.1, and has several advantages. Firstly, this method mostly provides a way to analytically determine the coefficients fairly easy without excessive computational effort. Secondly, the method is not limited to a small region around the equilibrium of the vehicle, which some methods struggle with. Third, treating the parts separately makes it possible to utilize the symmetry of each individual part: the whole vehicle will most likely not be symmetric including the control planes, but each component will very likely be locally symmetric. Since symmetry reduces the number of individual added mass coefficients, this will simplify calculations. Finally, and most important in this context, is that it enables calculation of coefficients for an arbitrarily shaped AUV. Many methods require the shape of the vehicle as an initial input, which is not desirable in this case (Perrault et al., 2002).

However, this method has some drawbacks as well. The most obvious one is that the interaction between components is not taken into account. For example, when looking at a single control plane in isolation, the water is able to flow around all edges of the plane. When attached to a hull, however, the water will only be able to flow around three edges, since the fourth one will be attached to the hull. Effects like this generate quite a complication and will be discussed in the following sections. Another aspect that is not taken into account is physical constraints on the coefficients, e.g. the stall angle of the control planes. These constraints will have to be controlled and accounted for in the computer model, in the lift calculations.
As mentioned above, the propeller will be simplified as a combination of small control surfaces. As a result of this, only two kinds of shapes will have to be taken into account: the hull and an arbitrary wing-like control surface. All thrusters are modeled as thrust and torque as a function of rpm and propeller characteristics based on precalculations or measured propeller performance. The reason for treating these two shapes separately is that they require different methods, but similar shapes like the rudders and elevators, can be approximated in the same way. The hull, however, needs a different approach. In this chapter, the application of the theories presented will be applied on a vehicle.
3.2 Vehicle hull

There are different ways to approximate the shape of a hull to facilitate calculating the added mass. Many of the earlier works use the approximation with a spheroid, and use a semi-empirical method presented in for example Malvestuto & Gale (1947). This is a very simple and straight-forward method; however the hull bodies in this simulation program will most often be of a Myring-type shape (Myring, 1981). The spheroid approximation will therefore not be sufficiently similar. In Figure 3.2, it can be observed that the largest difference between a Myring-type shape and a spheroid lies in the ends of the body. Hence, at a rotational movement, the calculated added mass will show fairly different values and generate rather large errors. In equation 2.10 and 2.11 it can be seen that the further away from the center of buoyancy a slice is, the larger its influence on the added mass. Large differences in the ends of the AUV would therefore create rather significant errors. Of course all Myring shapes do not look the same; some are more similar to a spheroid. But using the strip method, as described in 2.2, is more inclusive and more accurate for a larger number of differently shaped vehicles.

![Figure 3.2 – Comparison of Myring-shape (black) with approximated spheroid (red).](image)

For a complete model, the two theories will be combined. The axial added mass coefficient for translational acceleration along the x-axis will be calculated using an approximation of a spheroid according to the theory presented in Section 2.3. For the rest of the coefficients, slender body theory will be used according to Section 2.2.

To approximate the hull with a spheroid, the method of equivalent ellipsoid is used (Korotkin, 2009). The diameter and volume of the approximated spheroid are assumed to be the same as the diameter and volume of the actual hull, and therefore the equivalent length of the approximated spheroid can be determined according to:

\[
V_{\text{hull}} = V_{\text{spheroid}} = \frac{4\pi}{3} r_1 r_2^2 \rightarrow r_1 = \frac{3V_{\text{hull}}}{4\pi r_2^2},
\]

(3.1)

where \( r_1 \) is half the length of the spheroid, and \( r_2 \) is the radius, here taken the same as the radius of the hull.
For many AUVs, the center of gravity can be moved to change the behaviour of the vehicle in the water. For the added mass, the center of gravity will be assumed to coincide with the center of buoyancy of the hull. In the longitudinal direction, this is usually a good approximation, since a level trim at zero speed is desirable. The vertical distance between the two points, on the other hand, is usually not zero, to generate a righting moment to keep the vehicle upright. This distance is however usually fairly small, and will be disregarded. For vehicles with a very large vertical cg-cb separation it might be of interest to investigate this issue further in the future.

3.2.1 Significance of $m_{11}$

For such a slender body as the AUV hull, the axial coefficient $m_{11}$ will not be highly significant. Typically, for a body with a slenderness ratio of over 8, this coefficient contributes less than 3 % to the mass term (Humphreys & Watkinson, 1978). Moving forward along the x-axis will not require as large a mass of fluid to move as for transversal movement along the y- or z-axis, since the cross section perpendicular to the flow will be much smaller. Hence, the errors arising from approximating the AUV as a spheroid will be less significant.

3.3 Control surfaces

When modeling a control surface, it is not possible to calculate the added mass with the strip method used for the hull of the vehicle since a control surface is not always slender. This is especially true for small AUVs with small control surfaces. It would be desirable to be able to calculate the added mass for an arbitrary cross section and then integrate over the length of the wing, but without a slender body approximation this is not possible since the 3D-effect when going from two-dimensional to three-dimensional coefficients will have a large impact on non-slender bodies.

Therefore, the control surfaces of the vehicle will be approximated differently. For the translational added mass, an arbitrary control surface is approximated as a block circumscribing the control surface according to Figure 3.3. In lack of a more exact model, the added mass in one direction will be calculated as a flat plate with dimensions of the side of the box perpendicular to that direction. This will give the added mass expressions:
\begin{align}
m_{11} &= k_{\text{trans}} \frac{\pi \rho t^2 b^3}{4} \\
m_{22} &= k_{\text{trans}} \frac{\pi \rho t^2 c^3}{4} \\
m_{33} &= k_{\text{trans}} \frac{\pi \rho c^2 b^3}{4}.
\end{align}

(3.2)

Figure 3.3 – Approximation of a wing for the translational added mass coefficients.

For the added moment of inertia (forces created by rotational movement), an arbitrary control surface on the vessel is modeled as a flat plate. This is a reasonable simplification to make, since the thickness of the control surface generally is much smaller than the width and length. It is also assumed that the center of gravity, center of buoyancy and center of rotation all coincide. In reality, this is not always the case, but since control planes of most AUVs are small in relation to the rest of the vehicle, the differences between the points are usually small, thus justifying the simplification.

If the center of buoyancy does not coincide with the geometric center of the plate, the non-diagonal elements of the added mass matrix will not be zero, as assumed before. For example, a translational movement in \( z \)-direction will generate a moment around the \( y \)-axis, corresponding to the coefficient \( m_{33} \). However, with such small dimensions as the control surfaces, these values are sufficiently small in relation to the primary coefficients to be negligible (Malvestuto & Gale, 1947). Only primary coefficients will be calculated for the control planes.
3.4 Complete method

In this section, the complete method is summarized and compiled. For the hull, the coefficients are determined with

\[
m_{11,\text{hull}} = -k_1 \frac{4}{3} \pi \rho r_1 r_2^2
\]
\[
m_{22,\text{hull}} = \int_L \pi \rho r(x)^2 dx
\]
\[
m_{33,\text{hull}} = \int_L \pi \rho r(x)^2 dx
\]
\[
m_{44,\text{hull}} = 0
\]
\[
m_{55,\text{hull}} = \int_L \pi \rho x^2 r(x)^2 dx
\]
\[
m_{66,\text{hull}} = \int_L \pi \rho x^2 r(x)^2 dx
\]

and

\[
m_{26,\text{hull}} = \int_L \pi \rho x r(x)^2 dx
\]
\[
m_{35,\text{hull}} = -\int_L \pi \rho x r(x)^2 dx.
\]

The method for all wings, fins, rudders and elevators are as follow:

\[
m_{11,\text{fin}} = k_{\text{trans}} \frac{\pi \rho t^2 b^3}{4}
\]
\[
m_{22,\text{fin}} = k_{\text{trans}} \frac{\pi \rho t^2 c^3}{4}
\]
\[
m_{33,\text{fin}} = k_{\text{trans}} \frac{\pi \rho c^3 b^3}{4}
\]
\[
m_{44,\text{fin}} = k_{\text{rot}} \frac{\pi \rho c^2 b^3}{48} + y_{cb} m_{33,\text{fin}}
\]
\[
m_{55,\text{fin}} = k_{\text{rot}} \frac{\pi \rho b^2 c^3}{48} + x_{cb} m_{33,\text{fin}}
\]
\[
m_{66,\text{fin}} = 0.
\]

where \( c \) is the chord of the control surface, \( b \) is the span and \( t \) is the thickness. The coupled coefficients for control surfaces are disregarded. See Chapter 2 for more details.
Chapter 4

Model implementation

In this chapter, it is explained how the theory presented in previous chapters is used in the actual simulation model.

4.1 Current model

The current model is divided into two steps:

1. Pre-calculations
2. Running of the simulation

The pre-calculations consists of a matlab script that is run as a first step. It calculates properties of the components such as lift and drag coefficients for different conditions (velocity, angle of attack, etc.) and stores them in a mat-file. From this file, properties can be extracted during the simulation as they are needed. This requires a smaller computational effort than if the coefficients were to be calculated during each step of the simulation. The segment of the pre-calculations that regards the added mass is explained below.

4.1.1 Calculation of added mass coefficients

The model aims to calculate the added mass coefficients for two categories of bodies: a hull and a control surface. The different steps in the process are presented in Figure 4.1 and explained further below.
1. Input: load CAD-file. The model is based on CAD-files of stl-type which are loaded into matlab. This is done by a function that reads an STL file in binary format into vertex and face-matrices in three dimensions.

2. Transform coordinate system. The standard local coordinate system of the model is to have origin located in the geometric center on the back of the component. To be able to calculate the added mass coefficients correctly, the local coordinate system is transferred to a system with origin in the center of buoyancy.

3. Approximate with simpler shape. This step represents Chapter 3. Approximations and simplifications are made to be able to calculate the added mass coefficients of...
an arbitrary body.

4. **Determine coefficient.** The actual calculations presented in Chapter 2 and 3 are made.

Step 1-4 are done for each component of the vehicle, isolated and considered in a local coordinate system, in the pre-calculations. After this, the simulation is done with a complete vehicle (Step 5 and 6).

### 4.1.2 Step size/slice width significance

In step one in Figure 4.1, the STL-file is read into Matlab in the form of vertices. STL is a file format that describes a triangulated surface by the unit normal and vertices of the triangles. This data is then transformed, using the matlab function `convexhull`, into a simple shape function in the form of radius as a function of the length of the vehicle, \( R(x) \). This function, however, might have large gaps where the radius does not change (for example the cylindrical part of the hull, where the radius is constant). This will generate very large "slices" when using the slender body theory. In Figure 4.2, two cases are illustrated. The upper one, with only a few slices, some very wide, will produce errors for the coefficients where the 2-dimensional slices are multiplied with the distance to the slice from origin; this is done for all rotational and coupled coefficients (Equation 2.10 and 2.11). When the slices are smaller (the lower case in Figure 4.2) close to infinitesimal slices, the value of the coefficient will be more accurate. This is done by resampling the vectors \( x \) and \( r \) for \( R(x) \) with linear interpolation in matlab `interp1`.

![Figure 4.2](image-url) – Two different divisions of slices for an example vehicle.
To be able to decide the minimum required number of slices, the convergence is investigated. The value of the coefficient is calculated with decreasing interval (increasing number of slices), starting with a very large step and successively halving it. Since the exact value is unknown, the percent change in the value is calculated as:

\[ d_{n+1} = 100 \cdot \frac{u_{n+1} - u_n}{u_n} \]  \hspace{1cm} (4.1)

where \( u \) is the value of the coefficient and \( n \) is the iteration number. For example, if starting at using 10 slices and doubling the number for every iteration, the percentage difference is calculated as:

\[ d_{40} = 100 \cdot \frac{u_{40} - u_{20}}{u_{20}} \]
\[ d_{80} = 100 \cdot \frac{u_{80} - u_{40}}{u_{40}} \]
\[ \ldots \]
\[ d_{5120} = 100 \cdot \frac{u_{5120} - u_{2560}}{u_{2560}} \]  \hspace{1cm} (4.2)

5120 representing halving of the step 8 times. When the difference in a step is less than 1%, it is considered converged. In Figure 4.3 below, an example of the convergence is presented. The left side graphs show the value of the coefficients when using different number of slices, and the right graphs showing the percentage decrease at each step. In this example, the convergence of the coefficient values occurs at a step number of 5000 steps. The convergence will be discussed in further detail in the following chapter, Chapter 5.
4.1.3 Addition to current model

The concrete additions to the current model are summarized in Appendix A.
Chapter 5

Verification of model

Verification of models is important due to several reasons. The assumptions and approximations made can be justified in theory, but it is still very difficult to know how much they actually influence the end result. To determine this, a specified experiment is required where you can isolate the different approximations and results to trace how much a specific variable affects the result. This can be done through model experiments.

Determining the added mass and inertia of a body via experiments can be difficult and there are a few different methods including planar motion mechanism (PMM) test, rotating arm test, circular motion test (CMT) and variations of these (Lee et al., 2011; Phillips, 2010). Many of experiments are based on frequency measurement, as described below. To verify the model without conducting tailored experiments is difficult since there are several methods available for calculating added mass, and the results of different methods can be varying. Therefore, as a first step in verification of the model, comparison with existing experiments and values is performed. First, a frequency-based added mass experiment on plates is used for comparison, in Section 5.1. Secondly, values of added mass for existing AUVs are used in comparison with values acquired from the model for similar vehicles, in Section 5.2.

5.1 Plate modeling verification

One method of determining the added mass of a body is to measure the eigenfrequency of the body dry and fully wetted, and from that derive the added mass. This has been done by Stenius et al. (2015) for rectangular plates of different materials such as metals as well as composites. Using forced vibrations, the eigenfrequencies are determined for plates of different dimensions clamped along one short side according to Figure 5.1. The added mass is determined according to the equation:
Figure 5.1 – Left: mounting of the plates in the experiment. Right: The experimental setup.

\[
\Delta = \sqrt{\frac{m_s}{m_s + m_a}}
\]  \hspace{1cm} (5.1)

where \( \Delta \) is the ratio between wet and dry eigenfrequency, \( m_s \) the structural mass and \( m_a \) the added mass from the surrounding medium.

However, the experiment in Stenius et al. (2015) is using plates of different stiffness and density to investigate how the added mass is affected by these variables. As a result, plates with the exact same plate dimensions show different values of added mass, which would not be possible in the model calculations since the value only depends on the dimensions of the plate and the density of the water. This leads to the possibility that the movement of the plates in the experiment might be influenced by the density and stiffness of the plate, since plates with the same dimensions showing different added mass values have different stiffness. In the experiment, Since the experiment is more aimed at measuring added mass for plates of different materials than different dimensions, the result may be a bit misleading and not isolated enough with regards to the variable of interest (the dimensions of the plate).

An experiment like this could be of use when verifying the results of the model. However, the plates used have to be short and have a high stiffness, to get accurate results. The experiment setup is a good base for future work.

## 5.2 Comparison with existing AUVs

To get a general picture of the accuracy of the coefficients, a comparison with existing AUVs and their added mass coefficients is performed. This will not provide the most exact verification, but an indication if something is highly inaccurate, which coefficients
are more accurate than others and what coefficients need focus for improvement. Exact values are difficult to specify for most AUVs, but below is a comparison of two AUVs with different properties and geometry.

Since the two vehicles in the verification below have the same setup (four fins in the aft), the total coefficients for the vehicles are calculated in the same way:

\[
\begin{align*}
    m_{11} &= m_{11,\text{hull}} + 4m_{11,\text{fin}} \\
    m_{22} &= m_{22,\text{hull}} + 2m_{33,\text{fin}} \\
    m_{33} &= m_{33,\text{hull}} + 2m_{33,\text{fin}} \\
    m_{44} &= m_{44,\text{hull}} + 4(m_{44,\text{fin}} + x_r^2 m_{33,\text{fin}}) \\
    m_{55} &= m_{55,\text{hull}} + 2(m_{55,\text{fin}} + x_cs^2 m_{33,\text{fin}}) \\
    m_{66} &= m_{66,\text{hull}} + 2(m_{55,\text{fin}} + x_cs^2 m_{33,\text{fin}}) \\
    m_{26} &= m_{26,\text{hull}} - 2x_cs m_{33,\text{fin}} \\
    m_{35} &= m_{35,\text{hull}} + 2x_cs m_{33,\text{fin}}
\end{align*}
\]

(5.2)

where \(x_r\) is the distance from the centerline of the hull to the centerpoint of the fins and \(x_cs\) is the longitudinal distance from the center of buoyancy of the vehicle to the centerpoint of the fins. See Figure 1.1 and 1.2 for the coefficients’ directions.

### 5.2.1 REMUS

The REMUS AUV is developed at the Woods Hole Oceanographic Institute and is a small, low cost underwater vehicle for a range of different oceanographic applications. (Prestero, 2001) It has a cylindrical shape with a conical end as shown in Figure 5.2. It also includes four wings at the tail. The sonar in the fore of the AUV is disregarded in the added mass calculations.

The coefficient \(m_{11}\) is approximated by Prestero in the same way as presented in this report (by assuming a spheroid-shaped hull), but the approximated length of the spheroid is the vehicle length, and not based on the vehicle volume, leading to some differences in values.
The rest of the coefficients are determined by integrating expressions for a 2-dimensional section over the length of the vehicle, the same method used in this report. A significant difference, however, is that the fins are included in the integration by using another 2-dimensional cross section. In Equation 2.8, the expression for a circular cylinder is presented. To include the fins however, other cross sections have been used. For the crossflow added mass ($m_{22}, m_{33}, m_{55}, m_{66}, m_{26}, m_{35}$), a circle with fins as shown in Figure 5.3 and Equation 5.3 is used:

$$a_{22,f} = a_{33,f} = \pi \rho \left( a_{fin}^2 - R(x)^2 + \frac{R(x)^4}{a_{fin}^4} \right)$$  \hspace{1cm} (5.3)

Where $a_{fin}$ is the maximum height above the centreline of the vehicle fins and $R(x)$ is the radius of the hull over the length of the vehicle. The rolling added mass ($m_{44}$) is only caused by the fins, the hull itself is assumed to have no contribution to this coefficient. Therefore, the cross section of a cross is used as presented in Figure 5.3 and Equation 5.4:

$$a_{44,cross} = \frac{2}{\pi} \rho a_{fin}^4$$  \hspace{1cm} (5.4)

The expressions are then integrated according to Equations 2.9 to 2.11, but with different integration limits.
Convergence of the step number for the Remus AUV (Prestero, 2001)

The convergence of the step size of Remus was investigated according to Section 4.1.2. The coefficients $m_{55}, m_{66}, m_{26}$ and $m_{35}$ are most affected by the step size and are therefore used as a base in determining it. Additionally, $m_{55} = m_{66}$ and $m_{26} = -m_{35}$. The result is presented in Figure 5.4.

![Figure 5.4](image)

**Figure 5.4** – Convergence of coefficient values for different step sizes for Remus.

As can be seen in Figure 5.4, the coupled coefficients $m_{26}$ and $m_{35}$ are most dependent on the step size, while $m_{55}$ and $m_{66}$ do not show a very large error even when using very large slices. For the Remus vehicle, a step number of 5000 steps is required to get a convergence difference of less than 1%.

Coefficient values

The results of the verification with the AUV Remus are presented in Table 5.1 and the values for the added mass are based on Prestero (2001) and Prestero (2002).

All primary coefficients except the rotational coefficient $m_{44}$ are similar. However, the rest of the coefficients have fairly large differences. The difference in $m_{44}$ is around a factor 10, and was investigated by trying to replicate the value stated in Prestero (2001). This investigation is presented below.
### Table 5.1 – Comparison of calculated added mass for the REMUS vehicle and added mass values from the report Prestero (2001).

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Added mass coeffs. from Prestero (2001)</th>
<th>Computed added mass</th>
<th>Unit</th>
<th>Percentage discrepancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>m11</td>
<td>0.93</td>
<td>0.92</td>
<td>kg</td>
<td>0.8 %</td>
</tr>
<tr>
<td>m22</td>
<td>35.5</td>
<td>31.5</td>
<td>kg</td>
<td>11.3 %</td>
</tr>
<tr>
<td>m33</td>
<td>35.5</td>
<td>31.5</td>
<td>kg</td>
<td>11.3 %</td>
</tr>
<tr>
<td>m44</td>
<td>0.0704</td>
<td>0.0093</td>
<td>kg·m²/rad</td>
<td>86.8 %</td>
</tr>
<tr>
<td>m55</td>
<td>4.88</td>
<td>3.46</td>
<td>kg·m²/rad</td>
<td>29.1 %</td>
</tr>
<tr>
<td>m66</td>
<td>4.88</td>
<td>3.46</td>
<td>kg·m²/rad</td>
<td>29.1 %</td>
</tr>
<tr>
<td>m26</td>
<td>-1.93</td>
<td>0.19</td>
<td>kg·m</td>
<td>109.9 %</td>
</tr>
<tr>
<td>m35</td>
<td>1.93</td>
<td>-0.19</td>
<td>kg·m</td>
<td>109.9 %</td>
</tr>
</tbody>
</table>

### Analysing $m_{44}$ in comparison to Prestero results

The rotational added mass coefficient around the x-axis, $m_{44}$, is only dependent on the fins of the vehicle, since the hull is rotationally symmetric and does not generate any added mass if viscosity is neglected. When looking at the coefficient in Prestero (2001), $m_{44}$ is calculated according to Equation 2.10 and 5.4 combined into the expression:

$$m_{44} = \int_{x_{fin1}}^{x_{fin2}} \frac{2}{\pi} \rho a_{fin}^4 dx.$$  \hspace{1cm} (5.5)

When calculating the value of the coefficient with the integration limits and values in Table 5.2, the result is

$$m_{44} = 0.0092 \text{ kgm}^2/\text{rad}.$$ \hspace{1cm} (5.6)

### Table 5.2 – Values used when calculating the coefficient $m_{44}$ in Prestero (2001).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value (Prestero, 2001)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density $\rho$</td>
<td>$1030\text{kg/m}^3$</td>
</tr>
<tr>
<td>Fin height above centerline $a_{fin}$</td>
<td>0.1172m</td>
</tr>
<tr>
<td>Aft end of fin section $x_{fin}$</td>
<td>$-0.685m$</td>
</tr>
<tr>
<td>Forward end of fin section $x_{fin2}$</td>
<td>$-0.611m$</td>
</tr>
</tbody>
</table>

and not 0.0704, as stated in Prestero. This suggests that there is an error in Prestero (2001) regarding fins. When comparing the computed value of $m_{44}$ to the value in Equation 5.6, instead of the value from the Prestero (2001) report in Table 5.1, the percentage discrepancy is only 1.8% instead of 87%, which suggest the method in this report is correct.
Analysing $m_{26}$ and $m_{35}$ in comparison to Prestero results

Regarding the coupled coefficients $m_{26}$ and $m_{35}$, it can be seen that there is a significant difference, and that the values even have opposed signs. This might have several explanations.

1. Small differences in shape and/or center of buoyancy/origin of coordinate system. Origin is supposedly the same in both methods, but moving the center of buoyancy 7 cm gives the same values as stated by Prestero, indicating that very small changes in shape and/or center of buoyancy can give significant differences in results.

2. The calculation of the coefficient for the control surfaces in this report gives a value too small. As a result of using the component build-up method, all components are treated in isolation thus disregarding all interaction effects. As explained earlier, when looking at a fin, treating it without any regard to the interaction with other components will mean that the calculated coefficient assumes the fluid can flow around all edges of the wing, when in reality, it can only flow around three edges since one is attached to the hull. This is a problem for all coefficients, and the extent of the error can only be determined by conducting experiments tailored to this specific problem. Naturally, using a method where hull and fins are treated together, like the method in Prestero presented in 5.2.1, would give a more accurate result, but this would present difficulties when treating an arbitrary vehicle.

To investigate this, the total coefficients are calculated with a larger control surface contribution $m_{33,\text{fin}}$. To determine a reasonable value, some reverse calculations can be done to determine a new value for the added mass for only a fin, $m_{33,\text{fin}}$:

$$m_{33,\text{fin}} = \frac{(m_{33,\text{remus}} - m_{33,\text{hull}})}{2},$$

where $m_{33,\text{hull}}$ is the $m_{33}$ for only the hull, calculated with the model, and $m_{33,\text{remus}}$ is the value for the coefficient stated in Prestero (2001). From this, a value of $m_{33,\text{fin}} = 2.35\text{kg}$ is acquired, instead of $m_{33,\text{fin}} = 0.34\text{kg}$ which is the value resulting from the method presented in this report, i.e. approximating a fin with a flat plate. This is a significant difference, and using the new larger value in Equation 5.2, instead of the one calculated with the model, affects all coefficients except $m_{11}$ and gives the values in Table 5.3.

As can be seen, these values are very similar. This is an indication that there is a difference in how the contribution from the control surfaces is included in the complete model. Since the value of $m_{44}$ could not be reproduced from Prestero (2001), as stated above, it is possible that the error in calculating $m_{44}$ is persisting in calculation of the other coefficients as well in Prestero (2001). It is also possible that the interaction effects that are disregarded in the model in this report, or other effects such as antennas or sensors that are possibly included but not stated in Prestero (2001), are very significant, and to determine this, further investigation and experiments need to be carried out.
Table 5.3 – Added mass coefficient with different coefficient for the fins (derived from Prestero)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Added mass coeffs. from Prestero (2001)</th>
<th>Computed added mass</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>m11</td>
<td>0.93</td>
<td>0.92</td>
<td>kg</td>
</tr>
<tr>
<td>m22</td>
<td>35.5</td>
<td>35.5</td>
<td>kg</td>
</tr>
<tr>
<td>m33</td>
<td>35.5</td>
<td>35.5</td>
<td>kg</td>
</tr>
<tr>
<td>m44</td>
<td>0.0704</td>
<td>0.059</td>
<td>kg · m²/rad</td>
</tr>
<tr>
<td>m55</td>
<td>4.88</td>
<td>4.42</td>
<td>kg · m²/rad</td>
</tr>
<tr>
<td>m66</td>
<td>4.88</td>
<td>4.42</td>
<td>kg · m²/rad</td>
</tr>
<tr>
<td>m26</td>
<td>-1.93</td>
<td>-2.24</td>
<td>kg · m</td>
</tr>
<tr>
<td>m35</td>
<td>1.93</td>
<td>2.24</td>
<td>kg · m</td>
</tr>
</tbody>
</table>

5.2.2 LILLEN

*Lillen* is a small AUV at Saab Underwater Systems (Hedmo, 2008). As seen in Figure 5.5 and 5.6, the hull is very slender and fairly small. The shape of the AUV is approximated based on the report Hedmo (2008), and differences and errors might occur due to the lack of an exact drawing of the vehicle in the report. The values of the added mass coefficient are only stated in an appendix without explanation of how they were acquired, which makes tracing of differences and errors difficult. In addition to the hull shown in Figure 5.5, four fins of the dimensions 5x5 cm are added in the back of the vehicle.

![Figure 5.5 – Lillen AUV.](image)

Convergence of the step number of for the Lillen AUV (Hedmo, 2008)

The convergence for Lillen (Hedmo, 2008) is presented in Figure 5.7. The size of the slices mostly affects the coefficients $m_{55}$ and $m_{66}$, as for the Remus AUV. However, for Lillen it converges faster, and only 2500 slices is required for a percental convergence of
Figure 5.6 – LILLEN AUV. Photo from Hedmo (2008).

less than 1%, as compared to 5000 for the Remus.

Figure 5.7 – Convergence of coefficient values for different step sizes for Lillen.

Coefficient values

The result is presented in Table 5.4. Firstly, there is a fairly large difference in the axial coefficient $m_{11}$. Looking at Figure 5.8, one can see that the spheroid approximated for
calculating the added mass coefficient $m_{11}$ is not very similar to the actual shape of the vehicle. *Lillen* has a very cylindrical hull, generating a larger $m_{11}$ than the one of the more slender spheroid. $m_{11}$, when calculated as presented in this report, is most accurate for spheroid-similar hulls.

![Figure 5.8 – LILLEN and an equivalent spheroid.](image_url)

**Table 5.4 – Comparison of calculated added mass for Lillen and added mass values from Hedmo (2008).**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Added mass coeff. from Hedmo (2008)</th>
<th>Computed added mass</th>
<th>Unit</th>
<th>Percentage discrepancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{11}$</td>
<td>0.47</td>
<td>0.22</td>
<td>kg</td>
<td>53.8</td>
</tr>
<tr>
<td>$m_{22}$</td>
<td>13</td>
<td>15.5</td>
<td>kg</td>
<td>-19.1</td>
</tr>
<tr>
<td>$m_{33}$</td>
<td>13</td>
<td>15.5</td>
<td>kg</td>
<td>-19.1</td>
</tr>
<tr>
<td>$m_{44}$</td>
<td>0.00064</td>
<td>0.00044</td>
<td>$kg \cdot m^2/rad$</td>
<td>31.4</td>
</tr>
<tr>
<td>$m_{55}$</td>
<td>1.4</td>
<td>1.97</td>
<td>$kg \cdot m^2/rad$</td>
<td>-41</td>
</tr>
<tr>
<td>$m_{66}$</td>
<td>1.4</td>
<td>1.97</td>
<td>$kg \cdot m^2/rad$</td>
<td>-41</td>
</tr>
<tr>
<td>cross</td>
<td>0.28</td>
<td>-</td>
<td>$kg \cdot m$</td>
<td></td>
</tr>
<tr>
<td>$m_{26}$</td>
<td>-</td>
<td>-0.69</td>
<td>$kg \cdot m$</td>
<td>-146.7</td>
</tr>
<tr>
<td>$m_{35}$</td>
<td>-</td>
<td>0.69</td>
<td>$kg \cdot m$</td>
<td>-146.7</td>
</tr>
</tbody>
</table>

For the coupled coefficients $m_{26}$ and $m_{35}$, Hedmo (2008) only gives one value called *cross*. As stated in Equation 2.11, the coupled coefficients $m_{26}$ and $m_{35}$ have the same value but opposed signs (negative and positive) for rotationally symmetric hulls, but which coefficient is negative in this case is not stated in the report. As can be seen in Table 5.4, these coupled coefficient have a very high percentage discrepancy. As stated in Section 5.2.1, small variations in the hull shape and location of center of buoyancy will have large influence on these coefficients. Since the shape of Lillen was approximated simply from rough pictures and descriptions of the vehicle (Hedmo, 2008), this is very likely the cause of the large difference in the computed value and the value acquired from Hedmo (2008). When the geometry of the vehicle is more exactly known, the coupled coefficients will have a more accurate value. Nonetheless, the coupled coefficients are
very sensitive and results on these coefficients should be handled with caution.

5.3 Significance of added mass coefficients for the control surfaces

The control surfaces of most AUVs are small in comparison to the rest of the hull. As a result, the contribution from the control surfaces to the total added mass coefficients is small as well. This is presented in Table 5.5. The only coefficients where the fin contribution accounts for more than 10% are $m_{44}$, where the hull does not contribute to the added mass (in an inviscous fluid), and the coupled coefficients $m_{26}$ and $m_{35}$. However, looking at the coefficient values in Table 5.1 and 5.4, these coefficients are considerably smaller than the rest of the coefficients (except $m_{11}$), leading to the errors arising from possible faults in the fin coefficient calculations being small as well.

Table 5.5 – Contribution from the control surfaces to the total added mass coefficients of the vehicle used for verification.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Remus control surface</th>
<th>Lillen control surface</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>contribution [%]</td>
<td>contribution [%]</td>
</tr>
<tr>
<td>m11</td>
<td>3.1</td>
<td>6.8</td>
</tr>
<tr>
<td>m22</td>
<td>2.2</td>
<td>0.93</td>
</tr>
<tr>
<td>m33</td>
<td>2.2</td>
<td>0.93</td>
</tr>
<tr>
<td>m44</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>m55</td>
<td>8.1</td>
<td>3.6</td>
</tr>
<tr>
<td>m66</td>
<td>8.1</td>
<td>3.6</td>
</tr>
<tr>
<td>m26</td>
<td>229</td>
<td>15</td>
</tr>
<tr>
<td>m35</td>
<td>229</td>
<td>15</td>
</tr>
</tbody>
</table>
Chapter 6

Discussion

Throughout the process, the requirements presented in the problem statement in Section 1.1 have been considered and used as a base for decisions. Treating all components separately does, as stated above, entail some errors and unaccounted effects. However, the structure of the simulation program together with the requirement of a general formulation applicable on arbitrary vehicles make this simplification difficult to avoid.

6.1 Accuracy of model

Deciding the accuracy of a model without any known exact solutions is a very difficult task. The optimal solution would be to conduct an experiment, isolating the measurement of the added mass forces alone under different conditions and for different vehicles. Additionally, the use of more advanced numerical methods such as CFD can give a good understanding of the added mass for more complicated shapes and the interaction between components. However, this is not in the scope of this report, so other approaches have to be made to get an indication of any large errors, and point to future improvements.

Comparing values acquired from the model with coefficients for existing AUVs has generated varying results. There are some errors and differences, but overall the values are in the right order of magnitude. When comparing to values acquired from unknown or different models based on different assumptions, exact convergence cannot be expected.

6.2 Control surface coefficients

When initially looking at the results from the comparison with Remus, there is an indication that the values of the added mass coefficients for the fins (from the model in
this report) are inaccurate. However, since the value of one coefficient, $m_{44}$, in Prestero (2001) could not be reproduced with the exact method stated to have been used, but instead a much smaller value was obtained, it is likely that there is an underlying error in Prestero’s calculations. Looking at the comparison with Lilien (Hedmo, 2008) for the coefficient $m_{44}$, it can be seen that the value is here larger than the given value, further strengthening this notion. Furthermore, potential errors in the control surface coefficients will be small as a consequence of the control surfaces being small, as presented in Section 5.3.

### 6.3 The component build-up method

One notable aspect of using the component build-up method, as mentioned before, is that no interaction effects are taken into account. Considering a vehicle with four fins in the aft, rotating around the longitudinal axis, only the fins will contribute to the flow around the vehicle. However, looking at only four fins rotating around the same axis with the same distance to the axis of rotation, will not give the same result. In the second scenario, the fluid is free to flow in between the axis of rotation and the fins, which is not the case if there was a hull in the way. Scenario one and two are illustrated in Figure 6.1, together with a third one that will generate yet another result. This phenomenon will most likely affect all coefficients in some way, but to determine the exact extent of this error, experiments tailored to this specific situation or advanced numerical calculations such as CFD need to be conducted.

![Figure 6.1](#)

**Figure 6.1** – Different cases when calculating the coefficient for rolling motion. Vehicle seen from the front.

One solution to get around the issues with the component build-up method could be to use the same method as Prestero (presented in Section 5.2.1). This however requires that the complete geometry of the vehicle is known when determining the coefficient. As the model is constructed today, it is based on the concept of being able to add arbitrary components together in a simple way. This is done in Step 5 (see Figure 4.1) of the model, i.e. after determining the coefficients. Determining the added mass coefficient based on the complete geometry of the vehicle would require a re-structure of the model.
Another alternative to the component build-up method is the panel method (the boundary element method applied on fluid dynamics). Using the panel method could be a good alternative, but since the shape of the hull is always a slender body of mostly rotational symmetry, a panel method would require an excessive amount of work for a small increase in accuracy. Circular cylinders are simple shapes, meaning it is easier to use the slender body method. For the control surfaces, it could be a good approach; however that would entail a significant amount of work for just a small part of the model. The coefficients from the wings and the axial coefficient for the hull (which is where a panel method would have improved the accuracy of the result) are generally much smaller than the rest of the total coefficients for the vehicle, hence making the increase in accuracy small as well. Instead, energy was focused on other matters, such as improving the code and the implementation in the simulation program.
Chapter 7

Conclusion

In conclusion, the method developed in this thesis will provide an improved way of determining the added mass of arbitrarily shaped AUVs, compared to the current formulation. Using semi-empirical methods for simplified shapes has contributed towards fulfilling the requirement of using low computational power. The component build-up method and treating every component isolated fulfills the requirement of obtaining a rough estimate of the added mass for arbitrarily shaped AUVs. The third initial objective was to verify the model against experiments in literature. However, after more research it was discovered that experimental results on purely the added mass were not found to be available. Therefore, the model was instead verified against available added mass coefficient values for existing vehicles. The results from this comparison are slightly ambiguous, but satisfactory enough and a good base for future improvements. The coupled coefficients $m_{26}$ and $m_{35}$ are showing differences when compared to given values for existing vehicles, and results on these coefficients should be handled with caution until the exact extent of the error is determined through further work.

7.1 Future work

The most beneficial focus for future work is to conduct experiments tailored to added mass measurements to further determine the accuracy of the model. Experiments like this will clarify what parts of the model lack accuracy and should be prioritised. Focusing the experiments on the control surface formulation would clear up any uncertainties regarding the value of those coefficients discussed above. The formulation for the control surfaces is based on a fairly general simplification (flat plate approximation) and further investigation of alternatives to this method could entail several benefits.

The issue on the fins addressed in Section 6.3 can be attended. Either, a way to work around the problem can be developed, or focus can be put on determining the extent of
the errors arising from the component build-up method, and subsequently compensate for this error in the existing model.

One of the limitations of the model is that it is only applicable on rotationally symmetric hulls, i.e. hulls that have the same cross section in the XY and the XZ plane. A modification to the model that could include other hulls as well would broaden the applicability of the model to a wider range of hull shapes, or ensure higher precision for non-rotationally symmetric hulls.
References


Myring, D. F. 1981. *A theoretical study of the effects of body shape and mach number*
on the drag of bodies of revolution in subcritical axisymmetric flow. Tech. rept. Royal Aircraft Establishment.


Appendix A

Additions to the model

This appendix lists the additions that needs to be made to the existing model to accommodate the added mass calculations.

A.1 Files to add to .../mfunctions/

- calcHullshape.m
- calcBodyAddedMass.m
- calcWingAddedMass.m

A.2 Additions to calcBodyHydromech.m

Right before component parameter display:

```matlab
% ADDED MASS SECTION
[x,r] = calcHullshape(vertecies,1000);
[m11,m22,m33,m44,m55,m66,m26,m35]= calcBodyAddedMass(x,r,dens,iVol);
M_a = diag([m11 m22 m33 m44 m55 m66]);
M_a(2,6) = m26;
M_a(3,5) = m35;
M_a = M_a + triu(M_a,1)'; % Make M_a symmetric
```

and at cmp assignment:
A.3 Additions to `calcWingHydromech.m`

After hydrostatics and before hydrodynamics:

```matlab
% ADDED MASS
% L = chord
% W = span
% H = thickness

cb_ma = abs(iCB) - [L/2 0 0]; % Distance from middle of plate to CB.
[m11,m22,m33,m44,m55,m66,k] = calcWingAddedMass(W,L,dens,cb_ma);
m_vec = [m11 m22 m33 m44 m55 m66];
M_a = diag(m_vec,0);
```