Impact of fuel assembly bowing on the power density distribution and its monitoring in Siemens/KWU-PWR

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Abstract

During the lifetime of fuel assemblies, irradiation and fluid mechanical forces can cause a permanent deformation in lateral direction, called bowing. This geometric change affects the local moderator-fuel ratio which in turn affects the local power density. As the fuel rods are the second safety barrier within the barrier concept, it has to be ensured that the power density actuation limits are not systematically exceeded.

It is currently being discussed, how fuel properties like enrichment, plutonium content and burn-up quantitatively affect the local power increase. For selected gap locations, the bow model of the two-group diffusion code SIMULATE-3 (S3) has been verified with the Monte Carlo code MCNP-5 (MC5). The impact of gaps between different fuel types was investigated with S3 by performing simulations in a full core model of a generic four-loop Siemens/KWU-PWR. Based on the S3 results, four semi-empirical functionals were derived that make it possible to calculate the maximal relative power increase in the gap-near assemblies from the spectral indices and the coolant densities. The results were finally discussed in terms of their impact on the core monitoring.

For most gap sizes, a good accordance between the MC5 and the S3 results of the maximal relative power increase near the gap could be observed. However, the power increase of the inner pin rows showed a different behaviour for both codes.

It was found that the relative power increase raises with raising plutonium content and with raising coolant density of the assemblies directly adjacent to the large water gap. The largest increase of the hot-spot factor $F_q$ was 38 % for the maximal gap size of 15 additional water gaps between two uranium oxide assemblies.

The power peak induced by gaps that occur in the vicinity of an in-core measurement position can be detected if the gaps are large enough. Gaps between assemblies without instrumentation remain entirely undetected.

The current margins would cover up to eleven additional water gaps for the most unfavourable gap position within the investigated core configuration.
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Chapter 1

Introduction and Motivation

In the mid-nineties an increasing amount of fuel assemblies with permanent lateral deformation was registered in the Swedish nuclear power plant Ringhals [1]. As a consequence, manufacturers and operators have worked on developing theoretical models to explicitly simulate the impact of non-nominal water gaps. The Swedish company Studsvik Scandpower has performed a modification to their neutron transport and diffusion code package CASMO-4 and SIMULATE-3 that made it possible to introduce non-nominal gaps into the reactor core lattice [2]. Since approximately the year 2000, significant amount of bowed assemblies were also registered in German pressurized water reactors (PWR) [3]. The consequences were problems in handling assemblies during refuelling outage, increased control rod insertion times, as well as fretting corrosion on spacers. Following an investigation of the Reactor Safety Commission (RSK) and the fuel assembly manufacturers, the bowing could be reduced by design changes of the assembly skeletons [3].

Deviations from the nominal gap size, especially gap widening, can cause a large increase of the local power density in the vicinity of such gap [4]. As the fuel rods contain more than 99 % of the radioactive inventory of a nuclear power plant, their mechanical integrity is a major part of the barrier concept and their integrity has to be ensured under all operating conditions.

In German PWRs of the Vor-Konvoi and Konvoi class, the power density is limited by the global and the local power density limitation system. The global core limitation system, that consists of ex-core detectors located inside the biological concrete shield, is used among others during start-up operations and for the measurement of the neutron flux level during load operations [5]. However, it is not possible to resolve the in-core power distribution from the ex-core measurements. The radial positions of the ex-core detectors can be seen in Figure 1.1.

The main task of the local limitation system is to limit the peak power density $\dot{q}'$ [6]. It consists of two separate measuring systems, the Aeroball Measuring System (AMS) and the Power Density Detector System (PDDS). The AMS is used as the
reference measuring system and provides highly resolved three dimensional data of the power distribution inside the core. It is operated on demand, about once every 14 days, to calibrate the PDDS which continuously monitors the power density in the core [7].

The 28 measuring lances of the AMS are mounted inside empty control rod guiding tubes in the third outer pin row of the corresponding assemblies. Their radial positions are displayed in Figure 1.1. If a measurement is to be taken, small steel balls containing vanadium, that are usually located outside the core, are pneumatically transported from above into the core where they form a closed column over the entire active core height [6]. A schematic view of one measuring lance can be seen in Figure 1.2.

![Core Map with Measuring Positions](image)

**Figure 1.1:** Core map with measuring positions of the in-core and ex-core measurement systems of a German PWR and the boundaries of the surveillance zones (according to [8] and [9])

During their irradiation in the neutron field, radioactive vanadium-52 is formed in the balls by neutron absorption [6]. After an activation time of some minutes, the balls are transported to the instrumentation room. This room is equipped with
rows of $\gamma$-detectors, one row serves for the measurement of one ball column \cite{7}. There are 32 detectors in each row which split the ball columns into 32 discrete axial sections with a corresponding average activation value. From these values, the average relative power density $q^S$ in the sections of the assemblies containing a measuring position can be determined. Using an interpolation and extrapolation algorithm, the average power density in the sections of all assemblies $\bar{q}$ can be determined \cite{10}.

$$\bar{q} = A \cdot q^S$$

The coupling matrix $A$ has to be calculated individually for every specific core. With $\bar{q}$ and analytical factors, it is possible to calculate the hot-spot factor $F_q$ which gives the relation between the core-wide peak power density $\hat{q}'$ and the average power density $\bar{q}'$.

$$F_q = \frac{\hat{q}'}{\bar{q}'}$$

To obtain absolute power density values, an additional energy balance around the core has to be formulated. Usually this is done by measuring the core inlet and outlet coolant temperatures.

For the continuous power density monitoring, at each of the eight measuring positions of the PDDS, six fixed self-powered neutron detectors are installed in axial direction. The entire core is divided into surveillance zones and every detector of the PDDS is assigned to exactly one surveillance zone \cite{9}. Using the measurements of the AMS and the interpolation and extrapolation algorithm, the detectors of the PDDS are calibrated in such a way that they show the peak power density of their corresponding surveillance zone.
The peak power density is the main variable that is to be controlled by the PDDS. If the value deviates from its setpoint and exceeds a certain margin, the limitation systems will activate staggered countermeasures of increasing intensity with increasing deviation. In case the countermeasures remain unsuccessful in bringing back the controlled variable to the setpoint and the value has approached the response level, the reactor protection system initiates protective actions. The safety limit is the value of the linear power density for which it is verified that just no cladding damages occur, it takes the value of 532 W/cm$^3$. The response level is set to 475 W/cm$^3$, so that for normal specified operation, considering all uncertainties in the measurement process, the calculation and the material design, the safety limit is never exceeded. Figure 1.3 schematically shows the evolution of a controlled variable that deviates from the setpoint and is controlled by staggered countermeasures of the limitation and protection systems.

![Figure 1.3: Schematic of staggered limits and controlled variable](image)

The margin between the actuation limit and the safety limit is 12 % [3] which accounts for 8 % measurement uncertainty and another 4 % for effects that might not have been considered during the design verification process. It is therefore the question if there are possible locations of large water gaps that can lead to an undetected peak power increase that is not covered by the actual margin. Practically, this would result in an increase of the measurement uncertainty and in the reduction of the response level to a smaller value which would limit the operational flexibility of the power plant.

Investigations performed by Studsvik [4] and Anderson [1] have shown that large water gaps can lead to an increase of the hot-spot factor of more than 30 % for the maximal possible gap size of the corresponding assembly row.
However the reports did not provide a detailed explanation what fuel types were used for the analysis and what impact different fuel type combinations have on the power increase.

To clarify this questions, it was the goal of this thesis to investigate how the fuel properties and thermal-hydraulic conditions affect the relative power increase in the vicinity of large water gaps and to asses their detectability and the consequences on the core monitoring.
Chapter 2

Physical Background

2.1 Reasons for bowing and bowing shapes

Fuel assemblies are made up of five main components. The control rod guiding tubes are welded together with the spacer grids and the assembly foot and head, this is called the skeleton. In contrast to this more or less stiff structure, the fuel rods are only wedged by small metal plates which act like springs inside the spacer grids. To keep the assemblies pushed down against the upwards streaming coolant, so called hold-down springs are installed on the assembly head.

The forces the assemblies are exposed to during the reactor operation can be divided into external and internal forces. All forces induced by the coolant, such as buoyancy, friction forces and flow resistance, can be defined as forces that impact on the assembly from the outside. However, there are also forces that result from the internal tensions of the skeleton. Such tensions in the control rod guiding tubes have two reasons, the first is the uneven swell induced by the neutron fluence and the second one is an uneven thermal expansion during operation. Over a certain amount of time, all these factors lead to a deformation of the assemblies.

The most common bowing shapes are the C-shape and the S-shape, but also more exotic shapes can be found, such as P-shape and B-shape (sometimes also denoted as W-shape). Figure 2.1 gives a qualitative impression of the different bowing shapes. In this context it is worth to note that some reactors characteristically show a higher abundance of the S-shape while in most reactors the C-shape is the one that is observed most frequently [1]. In addition to the pure lateral deformation, torsion and bowing over the assembly corners have also been observed.

A major problem is that the in-core bowing during the operation of the reactor can not be directly detected, it can only be measured on unloaded assemblies. Because of the absence of fluid-mechanical forces and the thermal expansion, the bowing outside the core might differ significantly from the bowing under operating conditions. Codes to calculate the in-core bow on basis of the measured ex-core bow are still under development and are not fully validated yet.
2.2 Parameters affecting power density change

2.2.1 Moderator-fuel ratio

The impact of a non-nominal water gap can be explained by analysing the behaviour of the four factor formula for an undermoderated system like a PWR. An increase of the moderator-fuel ratio mainly affects the resonance escape probability $p$ and the thermal utilisation factor $f$. The resonance escape probability is increased because the neutrons suffer more collisions in the moderator and less collisions in the resonance area of U-238, so more neutrons escape the resonance capture and reach thermal energy and can cause new fissions. At the same time, the thermal utilisation factor $f$ slightly decreases because more neutrons are absorbed in the moderator. The fast fission factor $\varepsilon$ is slightly decreased because the fraction of fast neutrons decreases in the vicinity of the gap, due to the better moderation. As fast neutrons contribute only about 3% of the fission heat, this effect is of minor importance. The reproduction factor $\eta$ can be regarded as constant. When all the effects on $k_\infty$ are summed up, the multiplication factor in the vicinity of a widened gap increases.

If the dependence of $k_\infty$ is plotted over the ratio of moderator volume $V_{\text{H}_2\text{O}}$ to fuel volume $V_{\text{UO}_2}$ for a generic uranium oxide fuel (UOX), a graph like in Figure 2.2 can be obtained. The maximum of $k_\infty$ represents the point of optimal moderation.
beyond this point, the absorption of neutrons in the water prevails over the higher resonance escape probability. It can be seen that this point depends on the boron concentration for a fixed fuel radius and that the maximum is shifted to lower $V_{\text{H}_2\text{O}}/V_{\text{UO}_2}$-ratios for higher boron concentrations. A gap widening raises the local $V_{\text{H}_2\text{O}}/V_{\text{UO}_2}$-ratio and consequently $k_\infty$ increases, the narrowing of a gap leads to the inverse effect.

Figure 2.2: Dependence of $k_\infty$ from the moderator-fuel ratio for different fuel pin radii and boron concentrations [13]

**Fuel type and burn-up**

The appearance of the curve of $k_\infty$ depends on the initial fuel composition and the burn-up. Especially mixed oxide (MOX) and UOX assemblies differ in their behaviour to react on deviations of the moderator-fuel ratio. Inside and in the direct vicinity of a MOX assembly, the neutron spectrum is harder than in an assembly with lower plutonium content. There are two reasons for this effect. First, the fissile plutonium isotopes have a higher average fission neutron energy, second, the even neutron-number nuclides Pu-240 and Pu-242 have large capture resonances around 1 eV and 2 eV that absorb most of the neutrons before they can become thermal. In Figure 2.3 $k_\infty$ is displayed for an exemplary UOX and MOX fuel, $V_{\text{Br}}$ stands for the fuel volume. Increasing the plutonium content shifts the entire curve to the right because it needs a stronger moderation to compensate the absorption of thermal neutrons by even neutron-number nuclides. For a PWR, the $V_{\text{H}_2\text{O}}/V_{\text{Br}}$-ratio in the assembly lattice is between 1.6 and 2.1, depending on the assembly design.
If now a gap is introduced between two assemblies of the same fuel type, it can be seen from the graph that the power increase is larger for MOX fuel than it would be for UOX fuel. Furthermore it can be concluded that for a small increase of the $V_{\text{H}_{2}\text{O}}/V_{\text{Br}}$-ratio, the power increases almost linearly.

The behaviour of a pair of one MOX and one UOX assembly requires a more detailed reflection. Here it has to be considered that the spectra of the MOX and the UOX assembly overlap in the gap and in the first outer pin rows, so the effect of the gap can not be regarded separate for each assembly. If the first pin rows of the assemblies are imagined as a mixture out of MOX fuel and UOX fuel, the behaviour of such pair can be explained.

![Figure 2.3: Dependency of $k_{\infty}$ from the moderator-fuel ratio for UOX and MOX fuel](image)

Compared to a MOX-MOX pair, the power increase in the MOX assembly of a MOX-UOX pair will be flatter due to the lower average plutonium content, however the increase in UOX will be steeper than in an UOX-UOX pair. Plutonium is a decay product of neptunium that is produced by consecutive neutron capture and decay of U-238. Consequently, the plutonium content and thereby the fraction of Pu-240 and Pu-242 in an UOX assembly continuously grows with its burn-up. At the end of its service life time an UOX assembly contains approximately 1 wt% plutonium, compared to 5 wt% of a fresh MOX assembly. The characteristic of such UOX assembly will therefore slightly approach that of a MOX assembly with increasing burn-up.
2.2. PARAMETERS AFFECTING POWER DENSITY CHANGE

Geometric thermal-hydraulic feedbacks

The considerations that have been made so far are correct in a static model, when no feedback on the thermal-hydraulic flow conditions and no feedback between power and fuel temperature is considered. In reality, the geometric changes affect flow cross sections and so the flow velocity distribution between the assemblies. The widening of a gap causes an increased coolant flow through the gap so the improved heat transfer reduces the coolant and fuel temperatures in comparison to a constant flow distribution. A local power peak causes higher fuel temperatures which enhances the Doppler feedback that compensates a part of the reactivity gain from the gap.
Chapter 3

Description of the used program codes

3.1 MCNP-5

The Monte Carlo Neutron Particle Transport code MCNP-5 (MC5) has been developed at the Oak Ridge National Laboratory (ORNL) since the 1950s. It is applicable to various very complex problems such as the calculation of radiation shielding, criticality problems in fuel storages or power reactors and the determination of activation products in reactor core components.

Because of the capability of a Monte Carlo code to in principle represent the physics in an exact manner, where the error of the result is only depending on the number of simulated neutron histories, MC5 is widely used as a benchmark for other neutronic codes.

The basic principle of every Monte Carlo program is to keep track of every simulated particle from its birth by a fission to its absorption or its leakage out of the system. For every collision the neutrons suffer, there is a certain probability that they are absorbed or scattered, if they are absorbed, their history is killed. Absorbed neutrons can be captured or they can cause new fissions and thereby start a new neutron generation. The probability of the two possibilities is determined by the fission and capture cross sections of the corresponding nuclides. If the neutrons are scattered, the code calculates their new energy and their new flight directions. A larger number of simulated neutron histories leads to a higher statistical precision of the results, however a high precision does not guaranty a high accuracy. The precision alone does not contain any information if the result has converged to the correct physical solution.

To make sure that no results are collected until the fission source has converged, a number of inactive cycles has to be run before the active cycles. Usually the convergence of the effective multiplication factor $k_{\text{eff}}$ and the Shannon entropy are used as a criterion for the source convergence.

The number of cycles that should be skipped is depending on the so called dominance ratio, defined as the ratio of the fundamental mode eigenvalue and the first higher eigenvalue. For large power reactors the dominance ratio is usually very close
to unity and therefore a lot of cycles need to be skipped until the higher eigenmodes
have disappeared. By simulating only a part of the geometry, for example one quar-
ter of a full reactor core, it is possible to decrease the dominance ratio. However,
it still needs some user experience to decide how many cycles to skip, to avoid that
a bias is introduced in the results or that too much computation time is wasted if
too many cycles are skipped.

3.2 CMS package

The CMS code package is developed by Studsvik Scandpower and contains codes
for various purposes such as analysis of transients, burn-up distribution, criticality
problems and the calculation of nuclide inventories of spent nuclear fuel.

3.2.1 CASMO-5

CASMO-5 (C5) is a two-dimensional neutron and gamma transport code for PWR
and BWR lattices. The basic idea of any neutron transport code is to pre-compute
cross sections and flux distribution on a fine-mesh scale, usually on pin cell level,
with a large number of energy groups. C5 solves the one-dimensional transport
equation on this level using 586 energy groups.
Based on the pre-computed cross section data and flux distribution, a two-dimensional
calculation is performed on assembly lattice level with reflective boundary condi-
tions.
As it is not possible to solve the transport equation in its most general form, codes
like C5 apply some simplifications to the model, such as the discretisation of direc-
tions and discretisation of the energy spectrum.
To create the few-group cross section libraries for the diffusion code S3, usually a
C5 burn-up calculation with branches to different boron concentrations, moderator
densities and gap sizes is performed for each type of fuel. While the earlier versions
of CASMO were restricted to single assembly calculation and 5×5 multi assembly
calculations, C5 is capable of also simulating full core setups.

3.2.2 SIMULATE-3

SIMULATE-3 (S3) is a three-dimensional two-group nodal code that can be applied
to PWR and BWR problems such as pin-power reconstruction, burn-up calculations
or criticality searches. The core geometry is split up into parallelepipeds, the so
called nodes. The assemblies of a PWR are split into a number of axial nodes and
four planar nodes.
The pre-computed cross sections from C5 are homogenized within each node, intra-
nodal varying cross sections that occur because of the burn-up distribution, different
enrichment zones or burnable poisons, are modelled by a special algorithm.
S3 solves the neutron diffusion equation for each node in a coupled mode between
the neutronic and the thermal-hydraulic model. This model is based on a simple heat balance around the reactor core where the coolant inlet temperature and flow are considered to be known. It is assumed that the heat produced within each node is deposited in the coolant of this node and no horizontal cross flow is modelled, the pressure drop over the core is neglected. It is further assumed that the coolant stays monophasic in the entire core. The major advantages of a nodal diffusion code are the extremely short execution time with only very small drawbacks in the accuracy of the solution in comparison with a neutron transport code.

3.2.3 SNF

SNF is used to calculate isotopic compositions of spent nuclear fuel. For this purpose the code uses the irradiation history of the fuel assemblies from a S3 burn-up calculation. The code tracks the neutron fluence the assemblies have been exposed to and thereof calculates the concentration of activation products and fission products. In total more than 300 nuclides can be tracked.

3.3 Python

Since the creation of the MC5 input files, especially for the bowed configurations, required some thousands of cells and universes, it was necessary to automatize this process. This was done by implementing a python program with which the MC5 input files were written for different gap sizes and only required minor manual adaptation of the cell structure for the different bow configurations. Moreover, all data post-processing wasn done with python 2.7 as well.
Chapter 4

Calculation methods

In order to investigate how the relative power increase depends on the gap size and the fuel properties, the power density distribution of a reference core configuration with nominal water gaps has to be compared to configurations with non-nominal water gaps.

4.1 Calculation of the relative power change with MC5

For the calculation of the power density distribution, fission heating tallies have to be introduced in the fuel regions. A very convenient way is to create a superimposed mesh tally with the FMESHn card, which is decoupled from the actual geometry and therefore does not require additional surface definitions for tally splitting. In MC5, the mesh tallies can only be used together with the cell averaged flux tally (F4) making it necessary to introduce a tally multiplier card (FMn) to retrieve the fission heating.

There are two different tallies in MC5 that can be used to calculate cell wise energy depositions, depending on how the gamma-ray heating is treated. While the fission heating tally (F7) deposits the gamma rays locally, the F6 tally deposits them elsewhere, that is why the result from the F7 tally is larger.

The F7 tally can be duplicated from the F4 flux tally by using the multipliers for total fission cross section and Q-value [14]. If the material number on the FMn card is set to zero, the code automatically applies the fuel composition of the cell in which the particle is tracked [15].

MC5 tally results are always normalized to the number of starting particles and not to time, therefore the result from the F7 tally is given in units of energy deposition per gram fuel. This value is directly proportional to the average linear power density of a cell. If the total number of simulated neutron histories and the material properties remain constant, the relative power change between a bowed and an unbowed configuration can be calculated by the cell-wise difference of the F7 tally results.
The relative power change \((RPC)\) of cell \(i\) can then be calculated as:

\[
RPC(i) = \frac{E_{\text{bow}}^f(i) - E_{\text{ref}}^f(i)}{E_{\text{ref}}^f(i)}
\]

\(E^f(i)\) : fission heat deposition in cell \(i\) [MeV/g]

### 4.2 Calculation of the relative power change with S3

To retrieve individual pin-powers, the pin-power reconstruction option has to be activated in S3. If this is done, the program prints the relative pin-power densities in terms of the core average power density for every node to the output file. The total core power is fixed by the user, so the average power density does not change. Therefore the relative power increase can be calculated equivalently as in MC5.
Chapter 5

Bow modelling

5.1 Realisation of bowing in MC5

The MC5 code practically does not have any build-in limitations to design a geometry. This leaves a lot of freedom to the user, however it is the user’s responsibility to take care of geometry inconsistencies.

To help increasing the usability, the code provides input cards which make it easier to design geometries which are often subject of criticality analysis, such as reactor cores.

Very powerful tools are repeated structures, used in combination with universe based cell definitions. Repeated structures can be used to create equidistant grids of universes which can be assigned to cells containing other universes.

The assemblies were divided into 32 equidistant axial cells. Each cell contained a repeated structure representing the assembly lattice with three additional pin cell rows containing only water. As MC5 treats repeated structures as infinite in all directions, the relevant area has to be cut out by separate surfaces, forming the so called window. The bowing was modelled as a unidirectional planar displacement of the cells and the shift of the window surfaces, so that the cells contained the fuel pin lattice and half of the water gap to the surrounding assemblies. The bow amplitude of an assembly is depending on the bow of its row neighbours. Going from the largest to the smallest bow amplitude in an assembly row, the maximal displacement within assembly $F + 1$ can be calculated as:

$$v_{\text{max}}^{F+1} = v_{\text{max}}^F - d_{\text{nom}}$$

Here $d_{\text{nom}}$ denotes the nominal gap size. The notation follows the schematic picture in Figure 5.1. As an example, two bowed assemblies between two unbowed assemblies are displayed with their cell structure and their deflection curve, the half water gaps also belonging to each cell are not shown for the reason of clarity.

The largest displacement $v_{\text{max}}^F$ of the row is equivalent to the delta gap size of the large water gap that is to be simulated, so it has to be fixed by the user. In this work only integer multiples of the nominal gap size have been simulated.
With the maximal bow amplitude of each assembly, the displacement of the cells above and beneath the core mid height can be calculated for the corresponding assembly as:

\[ v_i = v_{\text{max}} \cos \left( \pi \left( \frac{1}{N} (i - 1) + \frac{1}{2N} - \frac{1}{2} \right) \right) \quad i = 1 \ldots N \]

Here \( N \) is the number of axial cells and \( i \) is the cell index starting from the bottom of the core. The displacement \( v_i \) represents the value of the cosine function in the vertical cell mid. The remaining water gap between the displaced cells of assembly \( F \) and assembly \( F + 1 \) is the difference of the displacements plus the nominal gap size.

\[ d_i^F = v_i^F - v_i^{F+1} + d_{\text{nom}} \]

![Figure 5.1: Axial cells and notation to implement bowed assemblies in the MC5 model](image)

The mesh grid to collect the fission heat deposition in every pin cell is completely independent from the simulated geometry. Therefore it had to be adapted to the displaced cells as well, otherwise it would have been possible that the mesh grid cuts through a fuel pin and the results are collected in the wrong position.
5.2 Realisation of bowing in S3

S3 is customized to model large power reactors and offers a lot of built-in functions to create the desired core geometry. Gap sizes can be entered separately for x- and y-direction for every gap position, either as absolute values or as delta to the nominal gap.

The bowing shape can be modelled as a simple displacement of the assemblies in lateral direction or as cosine shape with the maximal deformation in the middle of the core height, also plane-wise independent displacements can be entered [2]. In Figure 5.2 an example card to enter a non-nominal water gap in y-direction is shown. The nominal gap size between the assemblies and between the outer row of assemblies and the core baffle is 0.12 cm. As a consequence, there is one more gap in y-direction than there are assemblies in a row. If a gap size equal to three additional water gaps is to be entered, the size of the next three gaps in the bow direction is set to zero. The widened water gap then has a width of four nominal water gaps in total.

![Figure 5.2: S3 card to enter the maximal gap sizes in y-direction, the nominal gap size is 0.12 cm](image)

To create the cross section libraries for S3, combined C5 burn-up and branch calculations for various gap sizes have to be performed for all fuel types first. The bowing itself is not simulated by the code, the adaptation to the new geometry is done by varying the macroscopic cross sections. The effect of a non-nominal water gap on the power distribution is just the result of the macroscopic cross section deviation. S3 does not adapt the flow conditions due to the changed flow cross sections.
Chapter 6

S3 core model

All calculations of this work were based on a generic S3 core model of a German 4-loop PWR (Vor-Konvoi/Konvoi), designed by Siemens/KWU. The core contained 193 fuel assemblies with an active height of 390 cm. Only the active zone of the core was filled by the assemblies, beneath and above the active core, the axial reflectors were located. So the assemblies can be regarded as a bundle of zircaloy fuel rods, without any foot or head structures. Each assembly was split into 32 axial and four planar nodes, which contained the assembly lattice and half of the surrounding water gap.

Figure 6.1: Assembly types with different initial enrichment and plutonium content
Regarding their initial composition, the fuel assemblies can be separated into three different types: uranium oxide (UOX), uranium-gadolinium (UGD) and mixed oxide (MOX) assemblies. The fresh UOX assemblies had a homogeneous U-235 enrichment of 4.4 wt% and 20 water filled control rod guiding tubes. In the UGD assemblies, eight fuel pins where doped with 7 wt% gadolinium oxide (Gd$_2$O$_3$) as burnable poison between a core height of 31 cm and 349 cm. The U-235 enrichment of the pins not containing gadolinium was the same as in the UOX assemblies. The rods of the UGD assemblies containing gadolinium are depicted in Figure 6.1.

The central assembly in the core was an UGD assembly that had an initial enrichment of 3.9 wt% U-235 and a gadolinium content of 7 wt% in the same rods as the other UGD assemblies.

The outer pin row of the MOX assemblies had a lower plutonium content than the inner pins, additionally the MOX assemblies contained four water filled fuel rods on the inside of the assembly. The layout of the MOX assemblies with the zones of different initial plutonium content can be seen on the right side of Figure 6.1. The nominal water gap between the assemblies, that is defined by the distance between the outer side of two adjacent spacer grids, was 1.2 mm, the spacer grids themselves were not included in the model. The amount of fission energy that escapes the fuel rods in form of gamma radiation and is deposited in the coolant was 3 %.

Some important data of the core geometry and the assemblies are listed in Table 6.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>active core height</td>
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<tr>
<td>outer fuel rod diameter</td>
<td>10.75 mm</td>
</tr>
<tr>
<td>inner fuel rod diameter</td>
<td>9.3 mm</td>
</tr>
<tr>
<td>nominal water gap</td>
<td>1.2 mm</td>
</tr>
<tr>
<td>assembly pitch (spacers + water gap)</td>
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<td>pin pitch</td>
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<td>assembly lattice</td>
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<td>core inlet temperature</td>
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<td>boron concentration</td>
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</tr>
<tr>
<td>boron composition</td>
<td>27.5% B-10, 72.5% B-11</td>
</tr>
<tr>
<td>gamma smearing</td>
<td>3%</td>
</tr>
</tbody>
</table>

Table 6.1: Overview of the most important parameters of the S3 core model

Nodal codes might encounter problems in correctly reconstructing the neutron flux distribution in the vicinity of the reflector because of the large flux gradients at the edge of the core. Therefore the reflector region was modelled by a mixture of materials which has proven itself by excessive user experience. The compositions of the reflectors can be found in Appendix A.
Since the goal was to perform the calculations on the basis of an equilibrium burn-up distribution, five consecutive burn-up calculations corresponding to five fuel cycles have been performed using a generic core loading pattern. In Appendix B the core loading patterns for each cycle can be found. Figure 6.2 displays the core loading in terms of the fuel types and residence times of each fuel assembly. The core state 144 hours after the start of the fifth fuel cycle was defined as the reference state. After this time, the samarium, promethium and xenon concentrations have reached an equilibrium.

Figure 6.2: Core loading pattern applied in this work, with fuel types and assembly residence times
Chapter 7

MC5 core model

The MC5 model was implemented by making use of the universe based cell structuring and repeated structures. The lowest universe level were the pin cells in the assembly lattice. Each rectangular pin cell was built up out of the cylindrical fuel column surrounded by the cladding which in turn was surrounded by the coolant. This universe was then multiplied in the planar directions to create an assembly lattice which represented the next higher universe. The pin cells containing water filled rods were defined separately.

MC5 generally interprets a repeated structure as infinite in all spatial directions, therefore the correct area has to be cut out by additional surfaces. These surfaces were defined in such a way so that the resulting cell contained the assembly lattice and half of the water gap to the surrounding assemblies. The geometric layout of the assembly lattices was identical to the one of the S3 model.

The core lattice was represented by a planar rectangular grid with a cell size of $23 \times 23$ cm. This grid covered the core as well as the core baffle and the reflector region, this can be seen in the horizontal cross section of Figure 7.1a. Each assembly was represented by an individual universe and was placed at the corresponding position in the repeated structure of the core lattice. The core baffle was assembled out of cells that contained steel on the side that faced the reactor core and borated water in the cells on the other side. The cells of the reflector were filled with borated water only.

In axial direction, the fuel column and the coolant of each assembly were divided into 32 axial cells with individual material properties. In the vertical cross section of Figure 7.1b, the 32 axial cells of each assembly are indicated by different colors. The entire core lattice, that represented the highest universe, was filled in a cylinder with a radius of 298 cm.

Only one quarter of the S3 core was implemented in MC5, including the axis spanning assemblies. This was done by limiting the geometry in negative x and y direction with two reflective surfaces. Above and beneath the core, axial reflectors of one meter thickness were placed.
The water density in the entire reflector region was uniformly set to 0.74 g/cm$^3$, the boron concentration of 807 ppm was the same as inside the core. A major difference to the S3 model was that the fuel composition within each cell was homogenized and that no intranodal cross section variation was implemented, consequently all fuel pins within one axial cell contained the same fuel. The compositions were retrieved from a SNF calculation in the reference state that has been defined in Chapter 6. SNF generally only provides nodal averaged compositions. A table listing all the nuclides that were used in the MC5 model can be found in Appendix C. The average fuel temperature was uniformly set to the average fuel temperature of S3 corresponding to 900 K, the cladding temperature was set to 600 K. All cross sections were taken from the ENDF/B-VII.0 library. In total numbers the MC5 model consisted of 10306 cells and 1800 different material definitions which resulted in an input file that was as long as 400,000 lines. Table 7.1 lists the most important data of the model. The chemical composition of
the cladding and the steel baffle can be found in Table A.4 of Appendix A where BOX represents the cladding and CRS the steel composition.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>active core height</td>
<td>390 cm</td>
</tr>
<tr>
<td>thickness of axial reflectors</td>
<td>100 cm</td>
</tr>
<tr>
<td>outer core cylinder diameter</td>
<td>298 cm</td>
</tr>
<tr>
<td>boron concentration in core and reflector</td>
<td>807 ppm</td>
</tr>
<tr>
<td>boron composition</td>
<td>27.5% B-10, 72.5% B-11</td>
</tr>
<tr>
<td>water density of reflector</td>
<td>0.74 g/cm³</td>
</tr>
<tr>
<td>steel baffle thickness</td>
<td>4.1 cm</td>
</tr>
<tr>
<td>baffle density</td>
<td>7.8 g/cm³</td>
</tr>
<tr>
<td>cladding material</td>
<td>Zircaloy-4</td>
</tr>
<tr>
<td>cladding density</td>
<td>5 g/cm³</td>
</tr>
<tr>
<td>MOX fuel density</td>
<td>10.141 g/cm³</td>
</tr>
<tr>
<td>UOX fuel density</td>
<td>10.175 g/cm³</td>
</tr>
</tbody>
</table>

Table 7.1: Overview over the most important parameters of the MC5 core model
Chapter 8

Results

8.1 Validation of S3 with MC5

The first calculation that was performed was the reference configuration. In total 2500 active and 600 inactive cycles were simulated with a neutron batch size of 150k which resulted in a total number of 3.75E+8 simulated neutron histories. The converged source was used as initial source for all subsequent bowed configurations. For these calculations another 100 cycles were skipped, the number of active cycles however remained constant. Due to the high effort and computational time only a few gaps were verified with MC5 calculations.

In the following sections the locations of the large gaps are denoted as an alphabetical code, the first digit denotes the vertical assembly row, the second digit denotes the horizontal assembly row of the bowed assembly according to Figure 6.2. The last digit indicates the location of the large gap with respect to the bowed assembly, the sides are denoted with the cardinal directions.

The fuel type combination names are structured in such a way that the first named fuel type is always the one from which a special quantity is retrieved. The second is its neighbour assembly on the other side of the large water gap.

The gap positions that were simulated with MC5 were M-12-S, J-9-S and K-13-S for which the final $k_{eff}$ values and the standard deviations for the different gap sizes can be found in Appendix D.
8.1.1 Maximal relative power increase

Figure 8.1 shows the maximal relative power increase for gap J-9-S between two MOX-2 assemblies in the middle of the core height. The graph displays the S3 result and the results of the MC5 calculations with the associated relative errors for two, six, ten and fourteen additional water gaps.

![Graph showing maximal relative power increase](image)

Figure 8.1: Maximal relative power increase of MC5 and S3 for MOX-2 (J-9-S)

It can be observed that the points for two and six additional gaps are in good accordance. While MC5 gives a larger relative power increase for ten additional gaps than S3, the MC5 result for 14 additional gaps is smaller than the one of S3. The dashed lines display the interval of the relative power change for which the results can be regarded as verified by the MC5 calculations. For the gap J-9-S the interval has a width of 18 % absolute. The relative error of the relative power change is the sum of the relative error of the reference configuration and the relative error of the bowed configuration. For all displayed points, the total relative error was 5 %.

In Figure 8.2 and Figure 8.3 the maximal relative power change in UGD-1 and UOX-1 is displayed for the gap M-12-S in dependence of the gap size.

The relative power increase for UGD-1/UOX-1 obtained from MC5 is larger than the one of S3 for gap sizes smaller than ten additional water gaps. For a gap size of ten additional gaps, the results show a very good accordance. For UOX-1/UGD-1, a similar behaviour can be observed, here the results for six and ten additional gaps coincide very good, while for two and four additional gaps a larger deviation can be noticed. The total relative errors were between 4 % and 4.3 %.
For the assemblies UGD-1 and UOX-1 the verified interval width is 10 % and 6 % which is a much smaller value than for MOX-2/MOX-2. The major reason is that the initial plutonium content in the MOX assemblies varied between the inner and the outer pin rows, while the UOX and UGD assemblies had a homogeneous initial enrichment. Consequently the homogenization of the fuel composition in the MC5 model causes a larger deviation from the S3 results for assemblies that have a larger intranodal cross section variation.

A larger deviation for smaller gap sizes between the MC5 and S3 results can also be observed for gap K-13-S with the fuel combination UOX-1/UOX-3 respectively UOX-3/UOX-1 that are shown in Figure 8.4 and Figure 8.5. The relative errors for the displayed points were between 4.1 % and 4.4 %.

Both graphs of the MC5 results show an evolution of the relative power change that can be described as root shape, in contrast to the S3 results that shows a clear linear dependency of the gap size. Also in this case, the major reason of the differences between the MC5 and the S3 results is the different treatment of the fuel composition homogenisation.

In consequence the intervals are very large, both having a width of 27 % absolute. The deviations between the maximal relative power change obtained from MC5 and S3 for the different gap positions and gap sizes are listed in Table 8.1.
Figure 8.3: Maximal relative power increase of MC5 and S3 for UOX-1 (M-12-S)

Table 8.1: Absolute deviations between MC5 and S3 in %
8.1. VALIDATION OF S3 WITH MC5

Figure 8.4: Maximal relative power increase of MC5 and S3 for UOX-1 (K-13-S)

Figure 8.5: Maximal relative power increase of MC5 and S3 for UOX-3 (K-13-S)
8.1.2 Power increase in the inner pin rows

Figure 8.6 shows the maximal relative power change of the five outer pin rows for 10 additional water gaps for the gap M-12-S in the assembly UGD-1.

![Figure 8.6: Maximal relative power increase over the first five pin rows for UGD-1](image)

It can be seen that the relative power change becomes smaller with increasing distance from the large water gap. The results of MC5 and S3 show a good accordance for the first and the second pin row. From the third pin row on, the relative power increase of MC5 is larger than the one of S3.

The relative error of the relative power increase was around 4.1 % for all displayed data points.

The decline in assembly UOX-3 for the gap K-13-S shows a similar behaviour as can be seen in Figure 8.7. Also here, the increase predicted by MC5 is larger than the one resulting from S3.

The deviation between the MC5 and the S3 results becomes especially apparent when the maximal relative power change in the third pin row in dependence of the gap size change is analysed. For the fuel type combination UOX-3/UOX-1 it is displayed in Figure 8.8.

The power increase shows nearly no dependency with respect to the gap size change and is more or less constant. For the other investigated gap positions an identical behaviour could be found.

One possible reason is the influence of statistics on the MC5 solutions. Due to the large geometry, the number of fission events per axial fuel pin cell is comparably small.
Consequently, a very small change of the fission density in a cell might cause a seemingly large change of the power density. In areas with a high power increase, as in the vicinity of the gap, these fluctuations are hidden. However in regions with lower fission density they might show in a statistical noise with a gap size independent behaviour.

The only two practicable possibilities to decrease the influence of such fluctuations would be to either decrease the size of the simulated geometry, or to increase the number of simulated neutron histories.

For these reasons, the MC5 model can be regarded as valid only in the first pin row near the large water gap. As the S3 bow model has been verified by Studsvik \[2\] also for the inner pin rows, it was justified to perform the main parameter analysis with S3.
Figure 8.8: Maximal relative power increase over the third pin row for UOX-3 in dependence of the gap size.
8.2 S3 parameter analysis

In this section it will be presented how the fuel properties and the moderator density quantitatively influence the relative power increase in the vicinity of a large water gap.

8.2.1 Axial power increase

The assembly bow was modelled as a cosine bow shape with the maximal bow in the middle of the core height. To investigate the axial distribution of the relative power change, the fuel combinations MOX-2/MOX-2 (J-9-S), UOX-2/MOX-2 (J-9-E) and UGD-1/UOX-1 (M-12-S) were selected. Figure 8.9 shows the maximal relative power change in the corresponding axial node in dependence of the core height for a gap size of 12 additional gaps. The node numbering starts at the bottom of the core. For better comparability, the nodal values have been normalized to their corresponding maximum. As can be seen, all fuel combinations have the maximal relative power change in node 16, node 17 has an only slightly lower value. Although these nodes represent the middle of the core height, the shapes of the curves are not symmetric.

![Figure 8.9: Maximal relative power increase of each node over the core height](image)

If the average of the normalized values are calculated for the upper core half and the lower core half, a deviation of the two values can be observed.
The values of Table 8.2 indicate that the relative power increase is stronger in the lower core half than in the upper core half. Furthermore this effect is strongest for the combination UGD-1/UOX-1 and weakest for MOX-2/MOX-2, the value of UOX-2/MOX-2 lies in between.

To understand the reason for the asymmetry, it is necessary to analyse the evolution of the coolant density in axial direction. While flowing through the reactor core from the bottom up, the coolant temperature increases and the coolant expands, leading to a density reduction.

<table>
<thead>
<tr>
<th></th>
<th>MOX-2</th>
<th>UOX-2</th>
<th>UGD-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>upper core half</td>
<td>0.3079</td>
<td>0.2789</td>
<td>0.2418</td>
</tr>
<tr>
<td>lower core half</td>
<td>0.3088</td>
<td>0.2822</td>
<td>0.2528</td>
</tr>
<tr>
<td>upper - lower</td>
<td>-0.0009</td>
<td>-0.0033</td>
<td>-0.011</td>
</tr>
</tbody>
</table>

Table 8.2: Average relative power changes in % for upper and lower core half

Figure 8.10 shows the node-wise averaged coolant density in dependence of the axial height for the neighbour assemblies of the three investigated fuels. The higher power of the fresh fuel assemblies UGD-1/UOX-1 causes a larger increase of the coolant temperature and therefore a stronger density reduction compared to the other two combinations.

Figure 8.10: Node-wise coolant density in neighbour assemblies over the core height

A higher moderator density in the neighbour assembly causes a larger increase of the moderator-fuel ratio and therefore a larger power increase which becomes
clear by recalling Figure 2.3. A more detailed analysis about the influence of the coolant density of the assembly and its neighbour will be presented in section 8.3.

8.2.2 MOX fuel

For the investigation of the MOX fuel behaviour, gap size changes between three different combinations of assembly types have been investigated. Those were the combinations MOX-2/UOX-2 (J-9-E), MOX-2/MOX-2 (J-9-S) and MOX-4/MOX-1 (K-14-S). The gap size was increased from the nominal size in steps of two additional gaps (= 0.24 cm) up to the maximal possible gap size in the assembly row. Figure 8.11 shows the maximal relative power change $RPC_{\text{max}}$ of the different fuel combinations. On the x-axis, the gap size in numbers of additional gaps is displayed, for example two additional gaps are equal to a total gap size of three nominal gaps (= 0.36 cm).

![Figure 8.11: Maximal relative power change in MOX fuel in dependence of the gap size](image)

The highest relative power increase shows for the combination MOX-2/MOX-2 with over 60 % for the maximal possible gap size, this combination also shows the strongest gradient.

As can be seen, the gradient of the combination MOX-4/MOX-1 is weaker than the one of MOX-2/MOX-2, the maximal power increase is 58 %. The combination MOX-2/UOX-2 shows the weakest gradient and with 55 % it also shows the smallest maximal power increase. Most notably is that the relative power increase depends linearly on the gap size for all fuel combinations.
This can be explained by recalling the graph of Figure 2.3 which shows $k_\infty$ in dependence of the moderator-fuel ratio. As long as the deviations of the moderator-fuel ratio are small, the change of $k_\infty$ can be regarded as linear.

The strength of the gradient can be correlated to two quantities. The most important one is the plutonium content of the two assemblies. Especially the concentration of the isotopes Pu-240 and Pu-242 has a high influence on the neutron spectrum due to their high capture resonances. The second one is the coolant density of the regarded assembly, because it determines the initial moderator-fuel ratio on the $k_\infty$ curve.

With increasing plutonium content, the $k_\infty$ curve is shifted to higher moderator-fuel ratios, as the curve becomes steeper to the left, the gradient of the curve in the relevant moderator-fuel ratio area becomes steeper.

Consequentially this causes the gradient of the relative power increase to become stronger for higher plutonium contents in the two assemblies. It is important to note that not only the plutonium content of the assembly from which $RPC_{max}$ is retrieved, but also the plutonium content of the neighbour assembly is of high relevance. This can be observed for the gradients of MOX-2/MOX-2 for which the gradient is stronger than for MOX-2/UOX-2 because UOX-2 contains much less plutonium than MOX-2.

The influence of the coolant density on the relative power increase can be observed by comparing the gradients of MOX-4/MOX-1 with the one of MOX-2/MOX-2. Although the plutonium content of MOX-1 is higher than in MOX-2, the relative power increase in MOX-4 is lower than in MOX-2. The reason is that the coolant density in MOX-4 is about 4.3 % higher in the core mid plane. The higher coolant density and lower total plutonium content shifts the $k_\infty$ curve to the left. As the curve becomes flatter to the right side, the corresponding $k_\infty$ increase is smaller, moreover the coolant density of the neighbour assembly MOX-1 is lower than for the other combinations leading to a poorer moderation.

Table 8.3 summarizes the data of the three fuel combinations. The first column contains the plutonium content of the assembly and its neighbour. The second column shows the coolant density of the assembly and its neighbour in the core mid height and the third column contains the gradients of the relative power increase per gap of the corresponding assembly.

<table>
<thead>
<tr>
<th>Assembly</th>
<th>Pu tot [%]</th>
<th>Pu-240+ Pu-242 [%]</th>
<th>coolant density [g/cm$^3$]</th>
<th>power gradient [%/gap]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOX-2/UOX-2</td>
<td>4.009 / 0.566</td>
<td>1.418 / 0.088</td>
<td>0.699 / 0.695</td>
<td>3.65</td>
</tr>
<tr>
<td>MOX-2/MOX-2</td>
<td>4.009 / 4.008</td>
<td>1.418 / 1.418</td>
<td>0.704 / 0.699</td>
<td>4.10</td>
</tr>
<tr>
<td>MOX-4/MOX-1</td>
<td>3.317 / 4.449</td>
<td>1.439 / 1.480</td>
<td>0.707 / 0.696</td>
<td>3.88</td>
</tr>
</tbody>
</table>

Table 8.3: Special assembly properties for MOX fuel combinations
8.2.3 UOX and UGD fuel

The investigation of the UOX and UGD fuel behaviour was performed on the fuel type combinations UOX-1/UGD-1 (M-12-S), UGD-1/UOX-1 (M-12-S), UOX-2/MOX-2 (J-9-E) and UGD-1/MOX-1 (N-9-S). Again the graph in Figure 8.12 shows the maximal relative power increase in dependence of the gap size.

![Figure 8.12: Maximal relative power change in UOX fuel in dependence of the gap size](image)

Both fuels, UGD and UOX show a linear dependence between the relative power increase and the gap size. The combination with the strongest gradient is UGD-1/MOX-1 followed by UOX-2/MOX-2 that shows the largest maximal power increase of 58 %. Also here, the impact of the plutonium content of the neighbour assemblies stands out, the two combinations of fresh UGD and UOX which do not contain any significant amount plutonium have a much lower gradient than the two other fuels that are adjacent to MOX assemblies.

The slightly larger gradient of the UGD in comparison with the UOX fuel is a result of the coolant density difference between the two assemblies. The coolant densities and the power gradient.

Table 8.4 shows the plutonium contents of the different fuel combinations, the coolant densities and the power gradient.

It can be summarized that the relative power increase is governed by the same phenomena that also occur in MOX fuel.

Here it is important to note that the fuel properties of the neighbour assembly do not only have a high importance, but that their importance is even higher than the properties of the assembly from which $RPC_{\text{max}}$ is retrieved.

This can be observed by comparing the gradients of the combinations.
MOX-2/UOX-2 and UOX-2/MOX-2. Although the plutonium content in MOX-2 is much higher than in UOX-2, the gradient of the relative power increase of UOX-2 is stronger.

<table>
<thead>
<tr>
<th></th>
<th>Pu tot [%]</th>
<th>Pu-240+ Pu-242 [%]</th>
<th>coolant density [g/cm³]</th>
<th>power gradient [%/gap]</th>
</tr>
</thead>
<tbody>
<tr>
<td>UOX-1/UGD-1</td>
<td>0.011 / 0.013</td>
<td>0.00 / 0.00</td>
<td>0.692 / 0.689</td>
<td>3.21</td>
</tr>
<tr>
<td>UGD-1/UOX-1</td>
<td>0.013 / 0.011</td>
<td>0.00 / 0.00</td>
<td>0.689 / 0.692</td>
<td>3.31</td>
</tr>
<tr>
<td>UOX-2/MOX-2</td>
<td>0.566 / 4.009</td>
<td>0.088 / 1.418</td>
<td>0.695 / 0.699</td>
<td>3.80</td>
</tr>
<tr>
<td>UGD-1/MOX-1</td>
<td>0.012 / 4.449</td>
<td>0.00 / 1.418</td>
<td>0.691 / 0.697</td>
<td>3.89</td>
</tr>
</tbody>
</table>

Table 8.4: Special assembly properties for UOX and UGD fuel combinations

8.2.4 Power increase in the inner pin rows

The measuring lances of the AMS are located in the third outer pin row of the assemblies, inside empty control rod guiding tubes. It is therefore of interest to investigate the power change in the vicinity of those locations. The investigations were performed for the measuring positions in the assemblies N-9 (MOX-1/UGD-1), K-14 (UOX-1/MOX-4), J-9 (MOX-2/MOX-2) and J-13 (UGD-3/UGD-1) because they have shown a relatively high power increase and therefore have been of special interest for the core monitoring.

Figure 8.13 shows the maximal relative power change in the third outer pin row for the assemblies named before. In contrast to the maximal relative power change in the first pin row, the strongest increase in the third row does not occur for the combination with the highest plutonium content which would be MOX-2/MOX-2, but for UOX-1/MOX-4. The increase of MOX-2/MOX-2 is even weaker than for the pair UGD-3/UGD-1, where UGD-1 contains nearly no plutonium at all. The combination MOX-1/UGD-1 shows the weakest increase of all combinations. The decline of the local power increase in dependence of the distance from the gap can be clearly seen in Figure 8.14.

Figure 8.13 shows the maximal relative power change in the third outer pin row for the assemblies named before. In contrast to the maximal relative power change in the first pin row, the strongest increase in the third row does not occur for the combination with the highest plutonium content which would be MOX-2/MOX-2, but for UOX-1/MOX-4. The increase of MOX-2/MOX-2 is even weaker than for the pair UGD-3/UGD-1, where UGD-1 contains nearly no plutonium at all.

While MOX-2/MOX-2 shows the largest increase in the first pin row, already in the second pin row it has the third strongest increase and has become the weakest from row four on. From the graphs it can be concluded that the high plutonium content in the MOX-2 assembly weakens the spreading of the local power peak from the gap vicinity into the assembly. To explain this effect one has to take into account that neutrons coming from the vicinity of the gap first have to penetrate the outer pin rows before they can reach the pins inside the assembly. All plutonium isotopes have a higher absorption cross section in the thermal energy area than U-235. Consequently in fuel containing a significant amount of plutonium, a large
fraction of the thermal neutrons from the vicinity of the gap will be absorbed in the first two pins rows while the inner pin rows are shadowed.

Figure 8.13: Maximal relative power change in the third outer pin row

This means that with raising plutonium content the shadowing becomes stronger and that the decline of the power increase is steeper with respect to the distance to the gap. For the combination UOX-1/MOX-4, it can be noted that this effect is inverted. UOX-1 does practically not contain any plutonium, so the shadowing becomes negligible. Moreover the fast neutrons emitted by the MOX-1 can easily penetrate the first pin rows, due to the smaller cross sections in the fast energy area. Because of the continuous neutron capture in U-238, the plutonium content in uranium fuel raises with the burn-up. This causes the UGD-3 assembly to show a similar behaviour as the MOX-2/MOX-2 combination, however the shadowing effect is smaller due to the lower plutonium content. With the shadowing effect, it also can be explained why the combinations UGD-3/UGD-1, MOX-2/MOX-2 and MOX-1/UGD-1 show a non-linear behaviour in Figure 8.13. The raise of the thermal flux due to the increased gap size is mostly compensated by the captures in plutonium isotopes of the first two pin rows. For the clarification of the question if the occurrence of a gap can be detected by the AMS, it is important to know what power change can be expected in the vicinity of the measuring position. Figure 8.15 displays the average relative power change and the variance of the relative power increase within the third pin row for the same fuel combinations as before, example wise for a gap size of ten additional gaps.
Figure 8.14: Maximal relative power increase in row 1 to 5 for 10 additional water gaps

The graph illustrates once again that the relative power change in the third pin row increases with decreasing plutonium content of the regarded assembly. Another important difference between the fuel type combination is that the spread of the relative power change within the third pin row increases with increasing plutonium content.

The values of maximal absolute negative spread $s^-$ and positive spread $s^+$ from the average power change within the third pin row are listed in Table 8.5 for the different fuel type combinations.

<table>
<thead>
<tr>
<th>Fuel Type Combination</th>
<th>$RPC$ [%]</th>
<th>$s^-$</th>
<th>$s^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOX-1/UGD-1</td>
<td>3.724</td>
<td>0.276</td>
<td>0.459</td>
</tr>
<tr>
<td>MOX-2/MOX-2</td>
<td>5.032</td>
<td>0.187</td>
<td>0.301</td>
</tr>
<tr>
<td>UGD-3/UGD-1</td>
<td>6.074</td>
<td>0.131</td>
<td>0.123</td>
</tr>
<tr>
<td>UOX-1/MOX-4</td>
<td>8.835</td>
<td>0.0949</td>
<td>0.107</td>
</tr>
</tbody>
</table>

Table 8.5: Average relative power change and maximal negative and positive spread within the third outer pin row
Figure 8.15: Average relative power increase with minima and maxima in the third outer pin row
8.3 Semi-empirical function for the relative power increase

8.3.1 Spectral weighting

In the previous sections before it has been demonstrated that the relative power increase of the considered assembly is correlated to the plutonium content of the neighbour assembly and the coolant density. The goal was to derive a parameter that comprises all fuel properties and the coolant density, so that it is possible to calculate the maximal relative power increase in dependence of this single parameter. For this purpose the so called spectral index $S$ is introduced here, it is defined as the ratio of fast to thermal flux:

$$S = \frac{\Phi_f}{\Phi_{th}}$$

It is a spatially continuous function and defined at all points of the core, however it is common practice to work with assembly averaged spectral indices. Figure 8.16 shows the distribution of the assembly averaged spectral indices in the core quarter, obtained from a calculation in the reference state.

![Figure 8.16: Assembly averaged spectral indices of the core quarter](image)

When analysing the distribution of the spectral index, two major parameters have to be taken into account. The first one is the plutonium content that significantly increases the spectral index which can be observed for the fresh MOX assemblies at the positions marked with dark red. The second parameter is the coolant density because it is directly proportional to the moderator-fuel ratio. In terms of the spectral index this means that a higher...
coolant density, like in the assemblies on the edge of the core, leads to a smaller spectral index due to the better moderation.

In the following section the averaged spectral index of the assembly from which a certain value is retrieved is denoted with $S_A$ while the average spectral index of its neighbour, on the other side of the gap, is denoted as $S_N$.

If the first few pin rows near the gap are regarded, it is evident that the spectra of both assemblies will overlap in this zone and that the real neutron spectrum in the vicinity of the gap has to be an average between the two individual spectra.

This leads to the idea to describe the spectrum in the vicinity of the gap as a linear superposition of the two individual assembly spectra.

This superposition can be described by the spectral index of the assembly itself $S_A$ and the index of its neighbour $S_N$ by linearly weighting them with the spectral weights $x_A$ and $x_N$:

$$x_A \cdot S_A + x_N \cdot S_N$$

The spectral weight can be interpreted as the importance of the assembly properties and the properties of its neighbour. In total 16 different gap positions were simulated giving two values of the maximal power change $RPC_{\text{max}}$ for each simulation, as there are always two adjacent assemblies.

If the equation for the superposition is written on the left side and the $RPC_{\text{max}}$ values are written on the right side of the equation for every simulated combination, one obtains a system of linear equations.

$$Ax = b$$

Here $A$ is the matrix of the known spectral indices $S_A$ and $S_N$, the right side $b$ is the vector of corresponding $RPC_{\text{max}}$ values and $x = (x_A, x_N)$ is the vector with the two unknown spectral weights. Now it has to be determined how the two spectra have to be weighted so that the correlation between the $RPC_{\text{max}}$ values and the spectral indices becomes strongest.

This leads to the linear regression problem, where the vector $x^*$ has to be found so that the residuum of the right side of the equation becomes minimal.

$$||Ax^* - b||_2 = \min ||Ax - b||_2$$

Here the suffix 2 denotes the Euclidean norm. This method was applied to the dataset of $RPC_{\text{max}}$ values for every analysed gap size, the resulting spectral weights are listed in Table 8.6 for different the gap sizes.

From the numerical values of the table it can be seen that the spectrum of the neighbour assembly has a larger effect on $RPC_{\text{max}}$ than the spectrum of the assembly from which $RPC_{\text{max}}$ is retrieved. Although this parametrisation is a result of a pure numerical algorithm, it is in accordance with what has been found out in section 8.2 and is also physically reasonable. The fuel properties with respect to the power effect of a gap can be regarded as an importance function which decreases continuously with increasing distance from the gap. This means that the fuel properties of pins lying further away from the gap have a smaller influence on the power increase in the vicinity of the gap than pins lying closer to the gap.
Table 8.6: Calculated spectral weights for different gap sizes

<table>
<thead>
<tr>
<th>spectral weights</th>
<th>additional water gaps</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>$x_A$</td>
<td>0.458</td>
</tr>
<tr>
<td>$x_N$</td>
<td>0.542</td>
</tr>
</tbody>
</table>

Regarding the first pin row near the gap, it is therefore the first pin row of the neighbour assembly which has the second highest importance. The pins of the assembly itself lie further away from the gap and have a slightly lower importance. The fact that the spectral weights are nearly constant for all gap sizes can be explained by the linearity of the relative power increase that has been found in section 8.2. As long as the increase is linear, each line of the right side $b$ of the regression problem can be described as the multiplication of an individual constant and the gap size. But as the gap size is the same for all data points it is only a multiplicative constant, which generally does not change the solution of a linear regression.

The superposition of the two spectra in the vicinity of the gap can be interpreted as a weighted average spectral index $S_w$:

$$S_w = x_A \cdot S_A + x_N \cdot S_N$$

This interpretation can be used to obtain a simple functional for the correlation between the assembly spectra and the resulting maximal relative power increase. Figure 8.17 shows the data points and the linear regression for a gap size of two additional water gaps. From the graph it can be concluded that the relative power change increases with increasing $S_w$, so the harder the spectrum in the vicinity of the gap is, the higher is the resulting relative power increase. This is in accordance with the theory, that a higher plutonium content and a lower coolant density lead to a steeper increase of $k_\infty$.

The result of the regression for six and ten additional water gaps is shown in Figure 8.18 and Figure 8.19. When the linear equations in Figure 8.18 to Figure 8.20 are compared, it can be noticed that the gradient of the relative power increases with the gap sizes. It is helpful to recall the shape of the curves of $k_\infty$ in dependence of the moderator-fuel ratio of Figure 2.3.

Here, increasing the plutonium content in the fuel means to shift the curve to right side, where every value on the $S_w$-axis represents a separate curve. Consequently, the steepest part of the curve is shifted into the area of the relevant moderator-fuel ratio.

If one now imagines a certain increase of the moderator-fuel ratio from the design point, it is obvious that the increase of $k_\infty$ will be larger the more the curve is shifted to the right which is equivalent to an increase of $S_w$. 
For some of the gap positions, the gap size was limited to less than 14 additional gaps by the corresponding assembly row, this is why the total number of data points used for the regression is smaller for this gap size. It also explains why the spectral weights deviate more from the average value than for the other gap sizes. The smaller number of data points increases the statistical weight of the points not lying close to the regression line and therefore influence the regression more. Finally, the slope and the additive constant can be described as functions of the gap size. By analysing the evolution of the values in dependence of the gap size, a linear function was chosen for this purpose, so that the maximal relative power increase can be written as:

\[ \text{RPC}_{\text{max}} = m \bar{S}_w + c = (a_1 s + a_2) \bar{S}_w + b_1 s + b_2 \]

Here \( s \) denotes the number of additional water gaps. The parameters were determined by performing a linear regression on the slope values and the additive constants of the functionals presented above. The maximal relative power increase can then be calculated as:

\[ \text{RPC}_{\text{max}} = (0.0872 s + 0.0205) \cdot \bar{S}_w + 2.442 s - 0.536 \]

Table 8.7 shows the relative deviations between the parameters displayed in the graphs and the corresponding functional values of the linear regression.
Table 8.7: Deviations from the linear regression for slope and additive constant, given in %

<table>
<thead>
<tr>
<th>additional water gaps</th>
<th>2</th>
<th>6</th>
<th>10</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>-3.1</td>
<td>1.16</td>
<td>0.61</td>
<td>-0.42</td>
</tr>
<tr>
<td>c</td>
<td>4.15</td>
<td>-1.39</td>
<td>-0.66</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Figure 8.18: $RPC_{max}$ in dependence of $\bar{S}_w$ for six additional water gaps

$RPC = 0.55 \bar{S}_w + 13.919$
8.3. SEMI-EMPIRICAL FUNCTION FOR THE RELATIVE POWER INCREASE

Figure 8.19: \( RPC_{\text{max}} \) in dependence of \( \overline{S}_w \) for ten additional water gaps

\[ RPC = 0.898 \overline{S}_w + 23.727 \]

Figure 8.20: \( RPC_{\text{max}} \) in dependence of \( \overline{S}_w \) for fourteen additional water gaps

\[ RPC = 1.236 \overline{S}_w + 33.814 \]
Improved weighting functions

Most of the data points of the graphs shown previously lie very close to the regression line. However especially the two points at $S_w = 9$ and $S_w = 11$ show a quite large deviation from the predicted value. The first data point is the result of a gap at K-14-S, the second one of H-14-S.

Table 8.8 shows the average absolute deviation $\overline{D}$ between simulation results and regression. Also the the maximal absolute positive deviation $D^{+}_{\text{max}}$ and the maximal absolute negative deviation $D^{-}_{\text{max}}$ is listed. The average absolute deviation was calculated as:

$$\overline{D} = \frac{1}{n} \sum_{i=1}^{n} |y_i - f(x_i)|$$

In the equation, $y_i$ are the results from the simulation and $f(x_i)$ are the function values of the regression.

<table>
<thead>
<tr>
<th>deviations</th>
<th>2</th>
<th>6</th>
<th>10</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{D}$</td>
<td>0.191</td>
<td>0.503</td>
<td>0.852</td>
<td>1.414</td>
</tr>
<tr>
<td>$D^{+}_{\text{max}}$</td>
<td>0.770</td>
<td>2.299</td>
<td>4.157</td>
<td>5.77</td>
</tr>
<tr>
<td>$D^{-}_{\text{max}}$</td>
<td>-0.365</td>
<td>-1.774</td>
<td>-3.014</td>
<td>-4.447</td>
</tr>
</tbody>
</table>

Table 8.8: Average absolute and maximal absolute deviation of the data points from the functionals given in %

The maximal positive deviation is the most critical quantity here, as the regression underestimates the value of the simulation by more than 5% for 14 additional water gaps.

By analysing the points at $S_w = 9$ and $S_w = 11$, it could be found that the point with the largest positive deviation was retrieved from assembly K-14 whose neighbour K-15 had a 5% higher coolant densities in the core mid plane. The largest negative deviation was retrieved from assembly H-15 showing a coolant density that was nearly 5% higher than the one of it neighbour H-14.

From this observation, the conclusion could be drawn that assemblies who’s neighbours have a much higher coolant density show a larger power increase than predicted by the regression function. On the other hand the power increase in assemblies with the higher density was lower than predicted.

This led to the conclusion that the weighted average spectral index alone is not sufficient to describe the relative power increase and that the coolant density has to be treated in a separate step. Therefore the regression method had to be improved to reduce the maximal positive deviation.

With the same logic as for the spectral weights, a new regression was performed on the weighted average spectral index $\overline{S}_w$, the coolant density of the assembly $\rho_A$ and
the coolant density of its neighbour $\rho_N$ in the core mid plane and the corresponding $RPC_{\text{max}}$ values.

Again the linear system of equations

$$Ax = b,$$

where every line of the matrix $A$ is defined as

$$a = (S_w, \rho_A, \rho_N),$$

had to be solved with respect to $x$.

Table 8.9 shows the average absolute deviation and the maximal absolute deviation of the data points from the improved regression functions.

<table>
<thead>
<tr>
<th>deviations</th>
<th>2</th>
<th>6</th>
<th>10</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{D}$</td>
<td>0.147</td>
<td>0.378</td>
<td>0.52</td>
<td>0.691</td>
</tr>
<tr>
<td>$D_{\text{max}}^+$</td>
<td>0.288</td>
<td>0.633</td>
<td>1.817</td>
<td>2.052</td>
</tr>
<tr>
<td>$D_{\text{max}}^-$</td>
<td>-0.304</td>
<td>-0.98</td>
<td>-1.59</td>
<td>-1.165</td>
</tr>
</tbody>
</table>

Table 8.9: Average absolute and maximal absolute deviation of the data points from the improved functionals in %

As can be seen, the improved algorithm provides much smaller average deviations and also a reduction of the maximal deviations. Especially the maximal positive deviations were the regression underestimates the simulation results could be decreased significantly.

Table 8.10 lists the weights of the improved regression, $p_S$ denotes the weight of the weighted average spectral index, $p_A$ is the coolant density weight of the assembly and $p_N$ the coolant density weight of the neighbour assembly. As it was mentioned before, the contribution of the coolant density of the assembly itself is negative, while the contribution of its neighbour is positive. The reason is that a higher coolant density in the assembly itself leads to a smaller moderator-fuel ratio and, so that the $k_{\infty}$ curve is shifted to left and the gradient in the relevant area becomes flatter. At the same time, the higher moderator density inside the neighbour assembly leads to a better moderation.

In order to calculate the maximal relative power increase for a special combination of fuel assemblies, the assembly averaged spectral indices and the coolant densities in the core mid plane have to be determined once. With the spectral weights of Table 8.6 the weighted average spectral indices can be calculated.

The new parameter $K$ can then be calculated with the parameters given in Table 8.10 in the following way:

$$K = p_S S_w + p_A \rho_A + p_N \rho_N$$
The linear functions of Figure 8.21 to Figure 8.24 show $\text{RPC}_{\text{max}}$ in dependence of $K$. As for the first set of functionals, it is convenient to describe the slope and the additive constant of the improved functionals in terms of the gap size. In contrast to the initial set of functionals, the additive constant of the improved functionals did not show a linear behaviour, so it was described by a third degree polynomial. The maximal relative power increase then becomes:

$$\text{RPC}_{\text{max}} = (3.579 s - 0.818) \cdot K - 0.0004 s^3 + 0.00633 s^2 - 0.00835 s + 0.0257$$

Again $s$ denotes the number of additional water gaps.

Table 8.11 shows the relative deviations between the parameters displayed in the graphs and the corresponding functional values of the linear regression. The deviation between four data points and a third degree polynomial that was constructed from four points is always zero.

Table 8.10: New weighting factors for the improved functionals

<table>
<thead>
<tr>
<th>additional water gaps</th>
<th>2</th>
<th>6</th>
<th>10</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>optimized weights</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_s$</td>
<td>0.00188</td>
<td>0.00546</td>
<td>0.0089</td>
<td>0.0212</td>
</tr>
<tr>
<td>$p_A$</td>
<td>-0.0825</td>
<td>-0.215</td>
<td>-0.3754</td>
<td>-0.654</td>
</tr>
<tr>
<td>$p_N$</td>
<td>0.147</td>
<td>0.415</td>
<td>0.715</td>
<td>1.138</td>
</tr>
</tbody>
</table>

Table 8.11: Deviations from the linear regression for slope, in %

<table>
<thead>
<tr>
<th>additional water gaps</th>
<th>2</th>
<th>6</th>
<th>10</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>-4.71</td>
<td>1.47</td>
<td>0.82</td>
<td>-0.59</td>
</tr>
</tbody>
</table>
Figure 8.21: $RPC_{max}$ in dependence of the new parameter $K$ for two additional water gaps

Figure 8.22: $RPC_{max}$ in dependence of the new parameter $K$ for six additional water gaps
Figure 8.23: $\text{RPC}_{\text{max}}$ in dependence of the new parameter $K$ for ten additional water gaps

\[ \text{RPC} = 34.685K + 0.175 \]

Figure 8.24: $\text{RPC}_{\text{max}}$ in dependence of the new parameter $K$ for fourteen additional water gaps

\[ \text{RPC} = 49.581K + 0.0515 \]
8.4 Impact on $F_q$

The detectors of the PDDS are calibrated with the peak power density value of their corresponding surveillance zone, obtained from measurements of the AMS and the interpolation and extrapolation algorithm of the plant process computer. It therefore has to be investigated if the occurrence of non-nominal water gaps leads to a change of the hot-spot factor $F_q$, which consequently would cause a non-conservative calibration of the PDDS detectors.

There are two possibilities for a gap positions to have a large impact on $F_q$. Either the adjacent assemblies have a high plutonium content which gives reason to expect a high relative power increase, or the power density in the reference state is already very close to the peak power density.

Figure 8.25 shows the distribution of the linear power in the reference state in middle of the core height, normalised to the core-wide peak power density.

![Figure 8.25: Relative linear power distribution and location of the hot-spot in the middle of the core height for the reference configuration](image)

The arrows indicate the maximal values in the corresponding surveillance zone and the location of the hot-spot factor where the peak power density is 404 W/cm. The fresh assemblies of the second outer row show the highest relative power densities. Due to the high burn-up and the decrease of the neutron flux at the edge
of the core, the outer assembly row shows the lowest relative power density in the core. The power production inside the core is more or less evenly distributed, which is the result of the loading pattern that is optimised for this purpose.

For the analysis of $F_q$, six critical gap positions were identified that showed a very high relative power increase or a very high initial power density, those were the gap positions with the corresponding fuel type combinations listed in Table 8.12.

<table>
<thead>
<tr>
<th>gap position</th>
<th>fuel type combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>M-12-S</td>
<td>UGD-1/UOX-1</td>
</tr>
<tr>
<td>K-13-S</td>
<td>UOX-1/UOX-3</td>
</tr>
<tr>
<td>J-9-S</td>
<td>MOX-2/MOX-2</td>
</tr>
<tr>
<td>J-9-E</td>
<td>MOX-2/UOX-2</td>
</tr>
<tr>
<td>N-9-S</td>
<td>MOX-1/UGD-1</td>
</tr>
<tr>
<td>H-13-S</td>
<td>UGD-1/UGD-3</td>
</tr>
</tbody>
</table>

Table 8.12: Analysed gap positions in terms of $F_q$

In Figure 8.26, the ratio between the hot-spot factor in the bowed configuration $F_{q\text{bow}}$ and in the reference state $F_{q\text{ref}}$, in dependence of the gap size at the different gap positions, is displayed. The maximal possible gap size in the assembly rows varies between 11 and 15 additional water gaps.

The combination UGD-1/UGD-3 shows the largest increase of 38 %, followed by UGD-1/UOX-1 with 35 % and UOX-1/UOX-3 with an increase of 34 %.

While combinations including MOX assemblies showed a very high relative power increase of more than 60 %, they play a minor role for the increase of $F_q$, the largest value is reached by MOX-2/UOX-2 and MOX-1/UGD-1 with 21 %. The combination MOX-2/MOX-2 that showed the highest relative power increase is the combination that leads to the smallest increase of $F_q$ of the analysed combinations.

To become leading in terms of $F_q$, the power increase at the position of the gap has to be so large that the $F_q$ value of the reference state is exceeded. So for every gap position there is a gap size threshold $s_{\text{min}}$ that has to be reached first. $s_{\text{min}}$ is depending on the ratio between the reference peak power density and the power density at the gap position in the reference state and is consequently a function of the core loading pattern and the operating conditions.

The combination UOX-1/UGD-1 shows the smallest threshold of all analysed combinations, here already a gap size increase of two additional gaps causes a raise of $F_q$. For UOX-1/UOX-3 and MOX-1/UGD-1, a gap size increase of at least four additional gaps is required so that these locations becomes leading in terms of $F_q$.

Eight, respectively fourteen additional water gaps are required for the fuel type combinations MOX-2/UOX-2 and MOX-2/MOX-2.
8.4. IMPACT ON $F_q$

As soon as the gap size threshold is reached, the location of $F_q$ will jump from the location in the reference state to the first pin row near the large water gap. For even larger gaps, the increase of $F_q$ is governed by the linear function of the corresponding fuel type combination. Applying the improved semi-empirical function for $RPC_{\text{max}}$ that has been derived in the previous section, the increase of the hot-spot factor can be written as:

$$F_q(K, s_{\text{min}}, s) = F_q(s_{\text{min}}) + F_{q\text{ref}} \cdot RPC_{\text{max}}(K, s - s_{\text{min}})$$

$s$ and $s_{\text{min}}$ are to be entered in numbers of additional water gaps. The calculation of the gap size threshold $s_{\text{min}}$ is performed by finding the first even number of additional gap sizes for which the reference hot-spot factor $F_{q\text{ref}}$ is exceeded:

$$F_q(s_{\text{min}}) = \frac{q'}{\bar{q}'} \cdot RPC_{\text{max}}(K, s_{\text{min}}) > F_{q\text{ref}}$$

Here $q'/\bar{q}'$ denotes the ratio between linear power density in the vicinity of the gap in the reference state and the average linear power density. The values of the hot-spot factor increase that were found here are in the same order of magnitude as the results from an analysis that has been performed for assembly bowing in the NPP Ringhals [1].

Figure 8.26: Increase of the hot-spot factor $F_q$ for different gap positions
Chapter 9

Relevance for the core monitoring

The major question is, if and to what extend, the local power increase induced by assembly bowing can be detected by the in-core measuring system. To assess this question it has to be distinguished between gaps that occur in the direct vicinity of a measuring position and gaps that occur between assemblies with no measuring position.

In the previous section it has been shown that the hot-spot factor $F_q$, that is directly proportional to the peak power density, raises by more than 38 % for the gap position H-13-S and the maximal possible gap size. Neither of the adjacent assemblies is equipped with an AMS detector, it is therefore impossible to detect the power peak induced by this gap.

Also the peak power increase induced by gap M-12-S which can be as large as 35 % stays undetected for the same reason.

In case the gap occurs in the direct vicinity of a measuring position it is possible to detect the resulting power peak under certain conditions.

Areva indicates the accuracy of the AMS neutron flux measurement of the EPR-PWR with 1 % [5], here a conservative relative error of 3 % was assumed. This means that changes of the neutron flux at the measuring positions, that are within the boundaries of ±3 %, can not be reliably detected. To determine the power increase in the first pin row, assuming an increase of 3 % at a measuring position in the third pin row, the results of [subsection 8.2.4] have to be recalled. There, the magnitude and the spread of the relative power increase in the third outer pin row of several assemblies that are equipped with an AMS measuring lance was presented.

It was found that a larger plutonium content generally leads to a smaller relative power increase but to a larger spread within the third outer pin row.

Assuming the maximal spread between the maximal power increase and the power increase in the vicinity of the AMS measuring position, an estimation of the maximal undetected power increase in the third pin row can be given. [Table 9.1] displays the maximal spread plus the AMS measurement error of 3 % for the investigated assemblies with measuring positions.
As long as the maximal relative power change in the third outer pin row is smaller than the displayed values, it is not possible to reliably detect the power peak induced by the gap. If these detectability thresholds are displayed as horizontal lines, as it is done in Figure 9.1, it can be seen that for each fuel type combination a specific number of additional water gaps remains undetected. Because of the larger spread and the smaller increase in the third pin row, the number of undetected gaps is larger in MOX fuel than in UOX or UGD fuel.

<table>
<thead>
<tr>
<th>gap position</th>
<th>fuel type combination</th>
<th>( RPC_{\text{max}} - RPC_{\text{min}} + 3% )</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-9-S</td>
<td>MOX-1/UGD-1</td>
<td>5.736</td>
</tr>
<tr>
<td>J-9-S</td>
<td>MOX-2/MOX-2</td>
<td>5.456</td>
</tr>
<tr>
<td>K-13-S</td>
<td>UGD-3/UGD-1</td>
<td>4.545</td>
</tr>
<tr>
<td>K-13-S</td>
<td>UOX-1/MOX-4</td>
<td>4.803</td>
</tr>
</tbody>
</table>

Table 9.1: Maximal spread of the relative power change in the third outer pin row plus AMS measuring error

Figure 9.1: Maximal relative power increase in the third pin row and detectability thresholds
Table 9.2 lists the maximal gap size that remains undetected as well as the resulting relative increase of the hot-spot factor for the analysed gap positions in the vicinity of a measuring position. As can be seen, even if large gaps occur in the vicinity of a measuring position, the undetected peak power increase can be as large as 22%.

<table>
<thead>
<tr>
<th>gap position</th>
<th>fuel combination</th>
<th>undetected additional gaps</th>
<th>undetected $\Delta F_q [%]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-9-S</td>
<td>MOX-1/UGD-1</td>
<td>11</td>
<td>+22</td>
</tr>
<tr>
<td>J-9-S</td>
<td>MOX-2/MOX-2</td>
<td>8</td>
<td>+4</td>
</tr>
<tr>
<td>K-13-S</td>
<td>UGD-3/UGD-1</td>
<td>7</td>
<td>+15</td>
</tr>
<tr>
<td>K-13-S</td>
<td>UOX-1/MOX-4</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 9.2: Maximal gap sizes that remain undetected and resulting hot-spot factor increase

The peak power density in the reference state was 404 W/cm, leaving a margin of 17.5% to the actuation limit of 475 W/cm. With the margin of 12% to the safety limit, a peak power rise of 29.5% could be permitted without exceeding the safety limit. For the most unfavourable gap position M-12-S this margin covers up to 11 additional gaps, all other investigated gap positions would lead to a larger number of gaps that is covered by the margin. However, as presented above, the peak power increase can only be detected in very few cases, most of the power peaks induced by large gaps will remain undetected. The power density values can be regarded as a randomly distributed quantity around the measured values, that can be for example described with a Gaussian distribution. As was explained, the current measurement uncertainty for the peak power density is 8% plus a margin of 4% to account for additional effects that might not have been considered during the design verification process. To assess if the uncertainty is increased by the assembly bow, the incidence of all possible gap positions needs to be evaluated and weighted with the resulting peak power increase. It would then be possible to see if the additional contribution of the gaps leads to a larger variance of the peak power values.
Chapter 10

Conclusion

In this thesis the impact of large water gaps on the power density change in the adjacent assemblies was investigated with respect to the fuel composition and the thermal-hydraulic conditions.

It has been shown that the plutonium content is the major parameter that determines the magnitude of the relative power increase. The plutonium content of the neighbour assembly thereby appeared to be of greater importance than the one of the assembly from which the power increase was retrieved.

In general, a higher plutonium content in both assemblies caused a larger relative power increase. While a higher coolant density in the assembly itself caused a smaller increase, a higher coolant density in the neighbour assembly led to a larger increase.

In terms of the increase of the hot-spot factor, a combination of fresh UGD and UOX assemblies appeared to be most critical. The power peak induced by gaps that do not occur in the direct vicinity of a measuring position of the in-core measurement system can not be detected at all.

If the gaps occur between assemblies from which one is equipped with a detector, the power peak can be detected if the gap is large enough.

The results of the MC5 calculations have widely confirmed the relative power increases obtained from the S3 calculations. However, an entire verification was not possible as the fuel composition of the MC5 model was homogenized within each of the axial cells, without any adaptation to the cross section variation. This was due to that SNF only provides nodal averages of the fuel compositions. An improved model could make use of the new code version SIMULATE-5 that provides pin-wise nuclide composition data, so that a completely equivalent MC5 model could be designed. Moreover, the MC5 calculations could be performed using a larger number of neutron histories in order to decrease the statistical noise in the results.

One important fact has to be kept in mind for the assessment of the quantitative power increase. All methods that were applied in this thesis were based on static thermal-hydraulic conditions and constant temperatures without any adaptation of the flow conditions or fuel temperatures.
In reality, a higher local power automatically causes higher fuel temperatures which lead to a Doppler feedback that compensates a part of the reactivity gain due to the gap.

A larger gap also enhances the coolant flow through the gap. Due to better heat transfer at the cladding surface, the temperature is lower, consequently the cladding might accommodate higher power peaks than indicated by the static analysis. Further investigations should therefore include subchannel analysis to properly represent the coolant flow in the assembly lattice and the gap.
Appendix

A SIMULATE-3 material definitions

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Curium
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## D Final $k_{\text{eff}}$ values

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Table A5: Final $k_{\text{eff}}$ values and standard deviations for the configurations of different gap size
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