



# Comparison of Punching Shear Design Provisions for Flat Slabs

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# Abstract

A new generation of EN 1992-1-1 (2004) also known as Eurocode 2 is under development and currently there is a set of proposed provisions regarding section 6.4 about punching shear, PT1prEN 1992-1-1(2017). It was of interest to compare the proposal with the current punching shear design provisions.

The aim of this master thesis was to compare the punching shear resistance obtained in accordance with both design codes. Furthermore the effect of some parameters on the resistance was to be compared. It was also of interest to evaluate the user-friendliness of the proposal.

In order to meet the aim, a case study of a real flat slab with drop panels was performed together with a parametric study of a pure fictive flat slab. The parametric study was performed for inner, edge and corner columns in the cases prestressed, without and with shear reinforcement.

It was concluded that the distance  $a_v$  from the column axis to the contra flexural location has a big influence on the punching shear resistance. The factor  $d_{dg}$  considering concrete type and aggregate properties also has a big impact on the resistance. The simplified estimation of  $a_v$  according to 6.4.3(2) in PT1prEN 1992-1-1 (2017) may be inaccurate in some cases.

The length  $b_0$  of the control perimeter has a larger effect on the resistance in EN 1992-1-1 (2004) than in PT1prEN 1992-1-1 (2017).

In PT1prEN 1992-1-1 (2017), studs located outside the second row has no impact on the resistance.

The tensioning force in a prestressed flat slab has a larger influence on the resistance in PT1prEN 1992-1-1 (2017) than in EN 1992-1-1 (2004). Furthermore, the reinforcement ratio is increased by the tendons, and thus affect the resistance in PT1prEN 1992-1-1 (2017).

Clearer provisions for the definition of the support strip  $b_s$  for corners and ends of walls are needed in PT1prEN 1992-1-1 (2017).

It may be questionable if the reduction of the perimeter for a large supported area in accordance with 6.4.2(4) in PT1prEN 1992-1-1 (2017) underestimates the resistance

in some cases.

Considering the work-load with PT1prEN 1992-1-1 (2017), more parameters are included. However, they may not require that much effort to obtain.

Keywords: Punching shear, resistance, concrete, flat slab, design provisions, Eurocode 2, case study, parametric study, shear reinforcement, prestressed

# Sammanfattning

En ny generation av EN 1992-1-1 (2004) också känd som Eurokod 2 är under utveckling och för nuvarande existerar ett förslag på bestämmelser gällande avsnitt 6.4 om genomstansning, PT1prEN 1992-1-1 (2017). Det var av intresse att jämföra förslaget med de nuvarande bestämmelserna gällande genomstansning.

Syftet med examensarbetet var att jämföra den beräknade genomstansningsbärförmågorna för de två dimensioneringsbestämmelserna. Vidare skulle påverkan för några av parametrarna på bärförmågorna jämföras. Det var också önskvärt att bedöma användarvänligheten för förslaget.

För att uppfylla målet gjordes en fallstudie på ett verkligt pelardäck med förstärkningsplattor samt en parameterstudie på ett påhittat pelardäck. Parameterstudien utfördes för inner-, kant- och hörnpelare i fallen förspänt, utan och med skjuvarmering.

Slutsatsen var att avståndet  $a_v$  från pelaraxeln till inflexionslinjen har en stor påverkan på bärförmågan. Faktorn  $d_{dg}$  som tar hänsyn till betongtyp och fraktionsegenskaper har också stor inverkan på bärförmågan. Den förenklade ansättningen av  $a_v$  enligt 6.4.3(2) i PT1prEN 1992-1-1 (2017) skulle kunna vara för avvikande från den verkliga längden i vissa fall.

Kontrollperiferins längd  $b_0$  har en större inverkan på bärförmågan enligt EN 1992-1-1 (2004) än PT1prEN 1992-1-1 (2017).

Halfenankare placerade utanför den andra raden har ingen inverkan på bärförmågan enligt PT1prEN 1992-1-1 (2017).

Spännkraften i ett förspänt pelardäck har större inverkan på bärförmågan enligt PT1prEN 1992-1-1 (2017) än i EN 1992-1-1 (2004). Dessutom ökas armeringsinnehållet av spännvåjrarna, vilket påverkar bärförmågan i PT1prEN 1992-1-1 (2017).

En tydlig definition av  $b_s$  krävs i fallen med hörn och ändar på väggar i PT1prEN 1992-1-1 (2017).

Det kan ifrågasättas om reduktionen av kontrollperiferin för ett stort stödjande område enligt 6.4.2(4) i PT1prEN 1992-1-1 (2017) underskattar bärförmågan i vissa fall.

Vad gäller arbetsbördan gällande PT1prEN 1992-1-1 (2017), ingår fler parametrar. Dock kräver dessa inte så mycket arbetsmöda att ta fram.

Nyckelord: Genomstansning, bärförmåga, betong, pelardäck, dimensioneringsbestäm-  
melser, Eurokod 2, fallstudie, parameterstudie, tvärkraftsarmering, förspänt

# Preface

During our master thesis many people have been a big help. First off, we would like to thank our supervisor Mikael Hallgren. The topic of our master thesis was discussed with Mikael and in the end we chose to compare the current Eurocode 2 for punching shear resistance with a new proposal that is going to be realised around 2020. Mikael is involved in the process with the new Eurocode and is also very engaged in this subject. He has been a big inspiration and a key person for us. He has always been there for us and made time to discuss our work during the whole period. The majority of the information was provided by him. We really appreciated his engagement in us and our thesis.

A researcher that deserves our deepest gratitude is Miguel Fernández Ruiz. He offered a lot of his tight schedule to help us sort out uncertainties about the proposed provisions.

During this journey Tyréns has offered us space at their office, there for we would like to direct our gratitude towards Tyréns. The staff at Tyréns has also been a big help, by sharing their knowledge with us and bringing a good spirit. An extra big thank you to Banipal Adam that has discussed ideas with us and also helped us with the programs.

We would also like to thank Karl Graah-Hagelbäck that also was involved in finding an interesting subject to write about and also helped us to find a real world case to study. In the end it was Peter Törnblom that suggested a case that we could work with, a big thank you to him.

An additional gratitude is directed to our examiner Anders Ansell for taking time to correct our master thesis and for being a great support during the years at KTH.

Last but not least, we would like to give a big thanks to our friends and families that have been there for us and supported us throughout all these years at KTH and during our master thesis period.

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# Nomenclature

## Uppercase letters

$A_c$	Area of concrete	$[mm^2]$
$A_{sw}$	Area of one perimeter of shear reinforcement around the column	$[mm^2]$
$B$	Column width	$[mm]$
$C_{Rd,c}$	Parameter/Factor	$[-]$
$D$	Diameter of the circular column	$[mm]$
$E_s$	Modulus of elasticity of flexural reinforcement	$[GPa]$
$L_{max}$	Maximum span length	$[mm]$
$L_{min}$	Minimum span length	$[mm]$
$L_x$	Span length in x-direction	$[mm]$
$L_y$	Span length in y-direction	$[mm]$
$M_{Ed}$	Bending moment	$[Nm]$
$N_{Ed,y}$	Normal forces in y-direction	$[N]$
$N_{Ed,z}$	Normal forces in z-direction	$[N]$
$N_x$	Number of bars within the support strip in x-direction	$[-]$
$N_y$	Numbers of bars within the support strip in y-direction	$[-]$
$V$	Acting shear force	$[N]$
$V_{Ed}$	Design shear force	$[N]$
$V_{flex}$	Shear force associated to flexural capacity	$[N]$
$V_{Rc}$	Punching shear resistance	$[N]$
$V_{Rd,c}$	Design punching shear resistance	$[N]$
$V_{Rd,max}$	Maximum punching shear resistance	$[N]$
$V_{Rd,s}$	Contribution from shear reinforcement to punching shear resistance	$[N]$
$W_1$	Distribution of shear	$[mm]$

**Lowercase  
letters**

$a_v$	Location in the reinforcement directions where the radial bending moment is zero	[mm]
$b_0$	Control perimeter	[mm]
$b_{0, out}$	Control perimeter at which shear reinforcement is not required	[mm]
$b_b$	Diameter of a circle with the same surface area as in the region inside the control perimeter	[mm]
$b_y$	Length of the control perimeter in y-direction	[mm]
$b_z$	Length of the control perimeter in z-direction	[mm]
$b_s$	Width of support strip	[mm]
$c_1$	Column length parallel to the eccentricity of the load	[mm]
$c_2$	Column length perpendicular to the eccentricity of the load	[mm]
$c$	Diameter of a circular column	[mm]
$cc_x$	Spacing between the reinforcement bars in the x-direction	[mm]
$cc_y$	Spacing between the reinforcement bars in the y-direction	[mm]
$d$	Effective depth	[mm]
$d_g$	Maximum aggregate size	[mm]
$d_{g,0}$	Standard aggregate size	[mm]
$d_{dg}$	Coefficient taking account of concrete type and its aggregate properties	[mm]
$d_{eff}$	Effective depth	[mm]
$d_H$	Effective depth	[mm]
$d_l$	Length increment of the perimeter	[mm]
$d_v$	Shear-resisting effective depth	[mm]
$d_{v, out}$	Outer shear-resisting effective depth	[mm]
$d_y$	Effective depth of reinforcement in y-direction	[mm]
$d_z$	Effective depth of reinforcement in z-direction	[mm]
$e$	Distance of $d_l$ from the axis about which the moment acts / eccentricity	[mm]
$e_b$	Eccentricity of the resultant of shear forces with respect to the centroid of the control perimeter	[mm]
$e_p$	Eccentricity of the normal forces related to the centre of gravity of the section at control section in the x- and y-direction	[mm]
$e_{par}$	Eccentricity parallel to the slab edge	[mm]
$e_y$	Eccentricities $\frac{M_{Ed}}{V_{Ed}}$ along y-axis	[mm]
$e_z$	Eccentricities $\frac{M_{Ed}}{V_{Ed}}$ along z-axis	[mm]

$f_{cd}$	Concrete compressive strength	[MPa]
$f_{ck}$	Characteristic compressive cylinder strength of concrete at 28 days	[MPa]
$f_{ywd}$	Yield strength of the shear reinforcement	[MPa]
$f_{ywd,ef}$	Effective design strength of the punching shear reinforcement	[MPa]
$h_H$	Depth of enlarge column head	[mm]
$l_H$	Distance from the column face to the edge of the column head	[mm]
$k$	Parameter/Factor	[-]
$k_1$	Parameter/Factor	[-]
$k_b$	Shear gradient enhancement factor	[-]
$k_m$	Parameter/Factor	[-]
$p$	Column head width	[mm]
$r_{cont}$	Distance from centroid to the control section	[mm]
$r_s$	Distance between column axis to line of contraflexure bending moment	[mm]
$s_r$	Radial spacing of the perimeters of shear reinforcement	[mm]
$s_t$	Average tangential spacing of perimeters of shear reinforcement measured at the control perimeter	[mm]
$u_0$	Column perimeter	[mm]
$u_1$	Basic control perimeter	[mm]
$u_{1*}$	Reduced control perimeter	[mm]
$u_i$	Length of the control perimeter	[mm]
$v_{ed}$	Maximum shear stress	[Pa]
$v_{min}$	Minimum punching shear resistance	[Pa]
$v_{Rd,c}$	Design value of the punching shear resistance of slabs without shear reinforcement	[Pa]
$v_{Rd,cs}$	Design value of the punching shear resistance of slabs with shear reinforcement	[Pa]
$v_{Rd,max}$	Maximum punching shear resistance	[Pa]

## Greek symbols

$\alpha$	Angle between the shear reinforcement and the plane of the slab	[-]
$\beta$	Parameter accounting for concentrations of the shear forces	[-]
$\eta_c$	Factor corresponding to the concrete contribution	[-]
$\eta_s$	Factor corresponding to the shear reinforcement contribution	[-]
$\eta_{sys}$	Parameter accounts for the performance of punching shear reinforcement systems	[-]
$\gamma_c$	Partial factor for concrete	[-]
$\mu$	Shear gradient enhancement factor	[-]
$\nu$	Strength reduction factor for concrete crack in shear	[-]
$\phi$	Diameter of a tension reinforcement bar	[-]
$\psi$	Rotation of the slab	[-]
$\rho_l$	The bonded flexural reinforcement ratios	[-]
$\rho_w$	Transverse reinforcement ratio	[-]
$\sigma_{cp}$	Normal concrete stresses	[-]
$\sigma_d$	Average normal stress in the x- and y- direction over the width of the support strip $b_s$	[Pa]
$\tau_{Ed}$	Average acting shear stress over a cross section	[Pa]
$\tau_{Rd,c}$	Shear stress resistance of members without shear reinforcement	[Pa]
$\tau_{Rd,cs}$	Shear stress resistance of members with shear reinforcement	[Pa]
$\tau_{Rd,max}$	Maximum punching shear resistance per unit	[Pa]

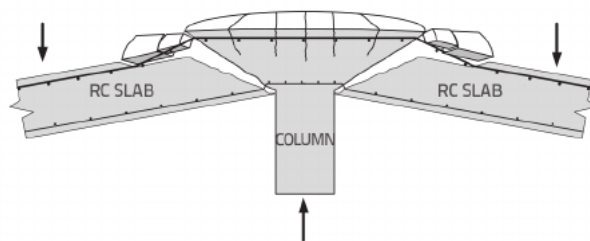


# Chapter 1

## Introduction

Concrete slabs supported by columns with capitals were introduced in the United States and Europe at the beginning of the 20th century (Muttoni, 2008). It was not until the 1950's that flat slabs without capitals became common. Flat slabs enable better usage of room height and installations compared to slabs supported by beams. Many offices and car-park buildings use this type of support system today. There is however one prominent drawback with flat slabs. Since the contact surfaces between the slab and the columns are generally small, high stresses are concentrated to these connections. If the stresses reach a certain limit the slab might fail in a mode called punching shear.

The first signs of cracking become observable above the column on the upper slab surface when the concrete tensile strength is reached (Hallgren, 1996). This causes cracking from the column outwards in a radial direction. As the load increases, also tangential cracks appear around the column. At failure, an inclined inner shear crack breaks through the slab. At this point, the column punches out a conical shaped slab portion bounded by this crack. This is known as punching shear and it is usually a brittle and sudden type of failure, see figure 1.



**Figure 1.1** Punching shear failure in a flat slab (Hallgren, 1996)

A flat slab structure known to partially collapse due to punching shear is Piper's row car park in Wolverhampton (Wood, 2017). Studies show that several factors, one of which deterioration of the top floor slab, may have led to the punching shear failure. The failure at one column increased the loads onto the adjacent eight columns leading to a progressive collapse of the slab. Luckily no one was injured.

This example stresses the necessity in designing flat slabs in a way that sufficient punching shear resistance is reached.

## 1.1 Problem description

In 1975 the Commission of the European Community started a program in the field of construction to facilitate trading and harmonisation of technical specifications (SS-EN 1990). Part of this program was to establish a set of technical rules, Eurocodes, for the design of structures meant to initially be an alternative to, and ultimately replace the national rules in the member states. In 1989 the preparation and the publication of the Eurocodes was transferred to the European Committee for Standardization in order to make them European Standards. In Sweden the earlier national codes were replaced by the Eurocode system, comprising ten standards and their respective parts, as of January 2011. These are to be followed since within the field of structural design of load bearing structures and their components.

What concerns punching shear, provisions for how to design concrete components are currently given in section 6.4 of Eurocode 2 (2004). The formulation for estimating the punching shear resistance is based on the results of experimental tests (SCA, 2010a). These however, do not reflect reality very well and consequently the formulation is not as accurate as desired. This is being considered in the ongoing development of the second generation of Eurocode 2 which will be released around 2020. Currently, only a set of proposed provisions exist given in section 6.4 in PT1prEN 1992-1-1 (2017). The formulae in these provisions have been derived on the basis of a consistent mechanical approach rather than an empirical approach as in EN 1992-1-1 (2004).

Now it is of interest to compare the proposed punching shear design provisions with the current provisions in Eurocode 2, section 6.4. It is important to stress that the proposed provisions are not confirmed as the new punching shear design provisions to be included in the second generation of Eurocode 2. It may be though and considering the range of load bearing structures currently being designed in accordance with Eurocode 2 all over Europe, it will have a huge effect on future structural safety.

## 1.2 Aim

The aim of the thesis was to apply the proposed and current design provisions to flat slab structures and compare the punching shear resistance. Furthermore, the effect of each parameter on the resistance was to be compared. The aim was also to evaluate the user-friendliness of the proposed design provisions in the sense of computational effort and obstacles encountered.

## 1.3 Scope and delimitations

The outline of the thesis mainly includes a theory part and a result part. The theory part gives a description of the phenomenon punching shear. This is followed by a presentation of each set of provisions. Ultimately the theory behind the finite element method is presented. After the theory part, the procedure for a case study and a parametric study is given. The result part is divided in a case study section and a parametric study section. The thesis is finally tied together in a discussion followed by conclusions.

The thesis investigated many aspects of the design codes, however some delimitations existed and are given here.

- The thesis only treated flat slabs
- The case study did not investigate a pure flat slab, rather one with drop panels and without shear reinforcement which was usual in Sweden before.
- In the case study, only the inner columns were analysed.
- The flat slab in the case study was not shear reinforced or prestressed.
- The case study did not include edge or corner columns, these were treated in the parametric study instead.
- The study did not investigate concrete types other than normal weight concrete
- The load combination performed in the case study was limited to loading of the four quadrants adjacent to the column and only for four columns.
- The load combination did not include point loads, only distributed loads.
- No deeper study of the eccentricity factor  $\beta$  was included.
- Openings and inserts, as instructed in 6.4.2(4) in PT1prEN 1992-1-1 (2017) were not treated in this thesis.
- Both the measured and the simplified distance  $a_v$  according to 6.4.3(2) in PT1prEN 1992-1-1 (2017) were treated, but only the simplified values were used in the calculations.
- Compressive membrane actions as described in 6.4.3(5) were not treated.
- The shear reinforced flat slab in the parametric study only included studs.



# Chapter 2

## Method

### 2.1 Literature Study

In order to increase knowledge and understanding about the current and the proposed provisions information about punching shear, the current provisions, the proposed provisions and background documents was gathered and studied.

#### 2.1.1 Punching Shear

The information sources that were used to gather information about punching shear were books and articles that were borrowed from the library or gathered from search databases provided by KTH. The sources that were studied were theories from different researchers.

#### 2.1.2 Current Design Provisions

The current design provisions in section 6.4 in Eurocode 2 (2004) were studied in detail in order to be able to perform punching shear calculations. Further knowledge has been gathered from SCA (2010a), commentaries and calculation examples.

#### 2.1.3 Proposed Design Provisions

Background information of the proposed provisions PT1prEN 1992-1-1 (2017) was given by Mikael Hallgren who is involved in creating the new provisions. This document consists of derivations of the equations that are available in the proposed provision. Some other documents that are mentioned in the background document were also analysed to be able to understand the origin of the equations. To sort out some questions about the proposed provisions that appeared, a meeting with researcher Miguel Fernández Ruiz was held. He explained uncertainties connected

to the proposed provisions and the background document.

### 2.1.4 Finite Element Analysis

The proposed provisions account for the bending moments in the slab when determining the locations of contra flexure. When determining the design shear force, the eccentricity factor  $\beta$  was calculated from the bending moment transferred between the slab and the column. In order to find the bending moments and to perform the load combinations, the flat slabs were modelled in FEM Design 16 Plate. AutoCAD was used to create a 2D-template with the contours as reference for the models.

### 2.1.5 Mathcad

The calculations of the punching shear resistance according to Eurocode 2 (2004) and PT1prEN 1992-1-1 (2017) were performed using Mathcad. It provided a way to create a template that simplified the calculation procedure.

## 2.2 Case Study

A case study was performed on a real world flat slab to obtain realistic parameters. The case study was about the second floor of Hästen 21. Drawings and structural design calculations were provided from Tyréns digital archive.

## 2.3 Parametric Study

A parametric study was performed in order to analyse the effect on the punching shear resistance from different parameters. This study was performed on a fictive flat slab inspired by calculation example D in SCA (2010b). A code was written in Matlab 2013 accounting for all provisions in section 6.4 in Eurocode 2 (2004) and PT1prEN 1992-1-1 (2017). The code was used for plotting the graphs showing the effect from the individual parameters.

# Chapter 3

## Theory

### 3.1 Punching shear models

Many theories about punching shear originates from what has been observed and measured in experiments. However, far from all researchers agree on the same mechanisms behind the failure mode. Many researchers have over the years tried to develop models that reflect the structural behaviour at punching shear failure. Consequently, numerous different theories exist. Some of the models are presented below. These are from the works of Kinnunen and Nylander, Broms, Hallgren and Muttoni.

#### 3.1.1 Model by Kinnunen and Nylander

Kinnunen and Nylander (1960) performed a series of punching shear tests on circular concrete slabs without shear reinforcement. All slabs were symmetrically supported on a column and loaded along the circumference with a distributed load. They observed that the conical shear crack and the radial cracks formed concrete segments that rotated like rigid bodies. With the observations as a basis, Kinnunen and Nylander developed a model for estimating the failure load at punching, see figure 3.1. Geometrically, the model comprises one of the slab segments bounded by the shear crack and two radial cracks with the angle  $\Delta\phi$  in-between. It is assumed that the segment rotates around the centre of rotation positioned in the root of the shear crack.

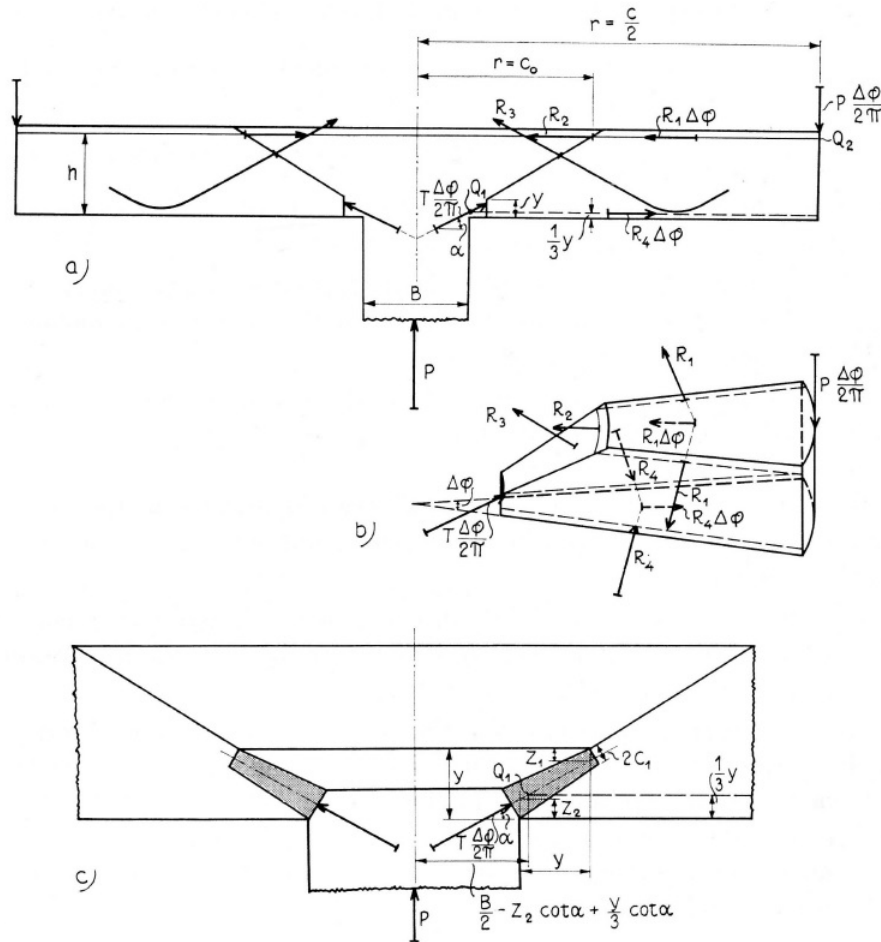
If the total load applied along the slab circumference is denoted  $P$ , the fraction of the load which acts upon the slab segment is  $P\frac{\Delta\phi}{2\pi}$ . The resultant from the forces in the tangential reinforcement is denoted  $R_4$  and in the radial reinforcement  $R_2$ . The neutral plane coincides with the centre of rotation and below this plane the concrete is exposed to compressive strains. The resultant to the compressive tangential forces is denoted  $R_4$ . The model assumes that the slab segment is carried by a compressed conical shell between the column perimeter and the root of the shear crack with the resultant  $T\frac{\Delta\phi}{2\pi}$ .

The mentioned resultants are proportional to the angle of rotation  $\psi$  up until yielding of the reinforcement. The model is based on the assumption that the segment is in equilibrium. Kinnunen and Nylander formulated the failure criterion that failure occurs when the tangential concrete strain at the bottom of the slab reaches a characteristic value. The tangential concrete strain is inversely proportional to the distance from the centre of the slab and the critical value at the distance  $y$  from the column is

$$\varepsilon_{ct} = 0.0035 \left( 1 - 0.22 \frac{B}{h} \right) \quad (3.1)$$

when the ratio between the width of the column and the effective depth  $\frac{B}{h} \leq 2$ . If the ratio is larger than 2 the critical tangential strain is given by

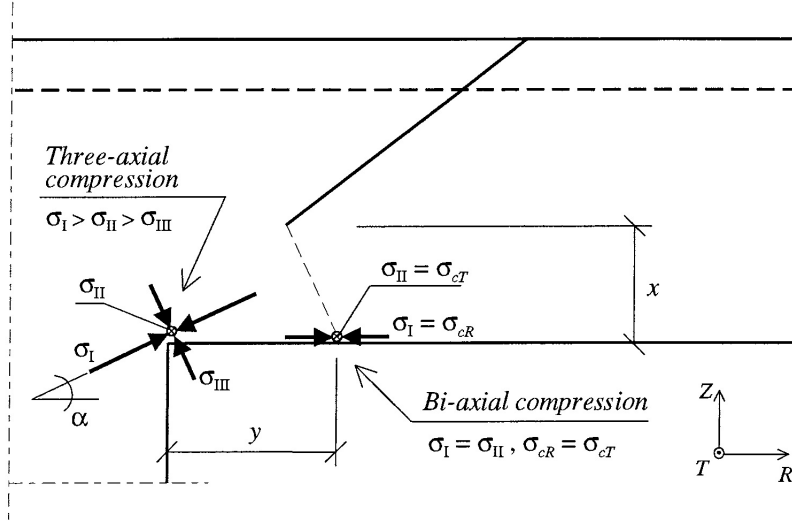
$$\varepsilon_{ct} = 0.0019 \quad (3.2)$$



**Figure 3.1** Punching shear model by Kinnunen and Nylander (1960)







**Figure 3.3** The three-axial and the bi-axial state in the model by Hallgren (1996)

In agreement with this theory Hallgren assumes that a horizontal crack opens in the tangential compressive zone immediately before failure and that  $\varepsilon_{cTu} = -\varepsilon_{cZu}$ . The crack causes the radial stress  $\sigma_{cR}$  in the bi-axial compressive zone to decrease. This in turn causes the principal stress  $\sigma_{III}$  to decrease in the three-axial compressive zone. When the three-axial state becomes unstable, the shear crack splits the compressed wedge and punching shear failure occurs.

Based on fracture mechanics Hallgren formulates the ultimate tangential strain as

$$\varepsilon_{ctu} = \frac{3.6 \cdot G_F^\infty}{x \cdot f_{ct}} \left( 1 + \frac{13 \cdot d_a}{x} \right)^{-1/2} \quad (3.3)$$

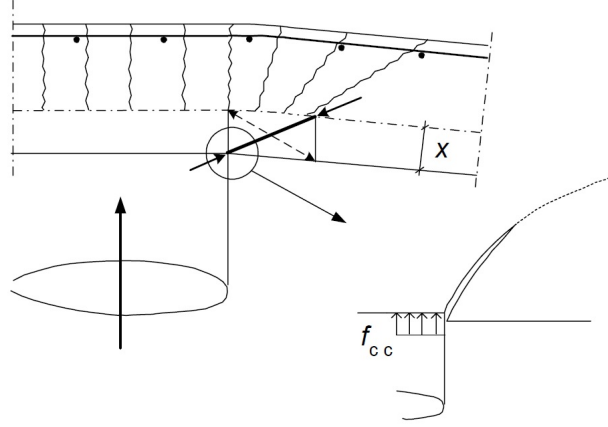
where  $G_F^\infty$  is the fracture energy for an infinitely large structural size,  $f_{ct}$  is the tensile concrete strength and  $d_a$  is the maximum aggregate size. The size-effect is considered through the depth  $x$  of the compression zone.

### 3.1.3 Model by Broms

Broms (2005) also modified the model by Kinnunen and Nylander (1960) to account for size effect and concrete brittleness. Broms assumes that the load is transferred to the column through an internal column capital. For flat slabs, failure generally occurs when a critical tangential concrete strain is reached in the slab at the column perimeter.

Because of the global curvature of the slab, the support reaction is concentrated along the circumference of the column. At a critical tangential strain, the support

reaction and the strain causes an almost vertical crack to open at the intersection between the slab and the column. This crack forces a flatter angle of the inclined compression strut. In the compression zone, tensile strains perpendicular to the inclined shear crack due to shear deformation leads to the collapse of the internal column capital. This triggers punching shear, see figure 3.4.



**Figure 3.4** Punching shear model according to Broms (2005)

The ultimate tangential concrete strain is given as

$$\varepsilon_{cpu} = 0.0010 \left( \frac{25}{f_{cc}} \right)^{0.1} \left( \frac{0.15}{x_{pu}} \right)^{1/3} \quad (3.4)$$

where  $f_{cc}$  is the compressive concrete strength and  $x_{pu}$  is the depth of the compression zone.

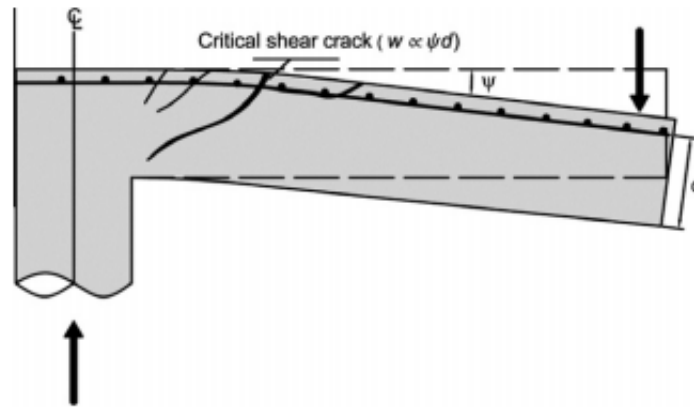
Broms (1990) also showed how the flexural reinforcement ratio affected the failure mode of a flat slab. Pure punching shear failure occurs in flat slabs with reinforcement ratios as high that the yield value of the steel is not reached in any point. This is characterized by a steep load-deformation curve that reaches high load capacities at low deformations before the brittle failure mode occurs.

Contrary to flat slabs with high reinforcement ratios, pure flexural failure occurs in flat slabs with ratios as low that all reinforcement yields before punching occurs. This is characterized by a flatter load-deformation curve with large deformations at lower loads.

### 3.1.4 Model by Muttoni

The proposed provisions are derived from the model by Muttoni. His theory for how punching shear occurs is based on the critical shear crack theory (CSCT) (Muttoni, 2008). The critical shear crack theory is based on the event where a diagonally flexural crack (critical shear crack), that is showed in figure 3.5, arises and disturbs the shear transfer action. This leads to reduction of the strength of the inclined concrete strut. Eventually the failure mode punching shear may occur. The amount of shear force that can be carried by the cracked concrete depends on

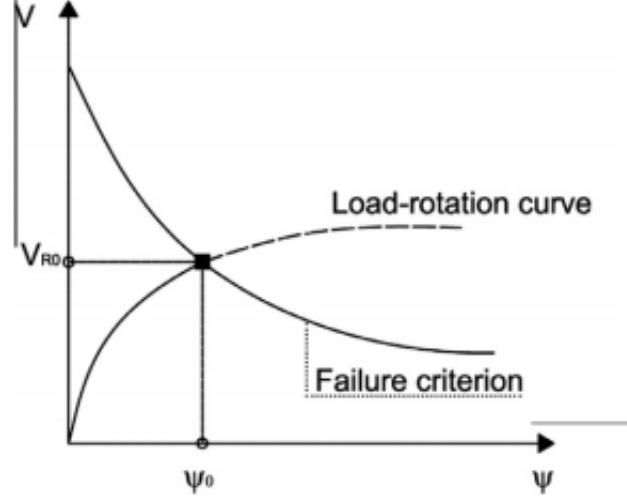
- the position of the crack
- the opening of the critical shear crack, and
- the roughness of the crack



**Figure 3.5** Critical shear crack from Fernández Ruiz et al. (2012)

The punching shear starts with small cracks that expand to a compressive conical failure (Fernandez et al., 2016). This failure consist of both sliding of the cone and opening of the shear crack. The opening and sliding process occur where the slab around the conical crack surface starts to rotate. The larger the angle of rotation is, the lower the punching shear strength becomes. This theory is used to calculate the capacity of the concrete to transfer shear forces through the crack.

To be able to calculate the punching shear resistance, the two expressions that needs to be considered are the failure criterion and the load-rotation relationship (Fernandez et al., 2016). In figure 3.6 the two expressions are represented by two curves. The intersection shows the punching shear resistance.



**Figure 3.6** Intersection showing the rotation capacity and the punching shear resistance (Fernández Ruiz et al., 2012)

The failure occurs in a narrow region which leads to a single failure criterion for the shear-transfer capacity:

$$\frac{V_{Rc}}{b_0 \cdot d \cdot \sqrt{f_c}} = \frac{\frac{3}{4}}{1 + 15 \cdot \frac{\psi \cdot d}{d_{g0} + d_g}} \quad (3.5)$$

where  $V_{Rc}$  is the punching shear resistance,  $b_0$  is the length of the control perimeter which is  $\frac{d}{2}$  from the edge of the column,  $d$  is the effective depth,  $f_c$  is the cylinder compressive strength for concrete,  $d_{g0}$  is a reference size equal to 16mm,  $d_g$  is the maximum aggregate size and  $\psi$  is the rotation of the slab (Muttoni, 2008). The equation for the load-rotation relationship can be simplified to

$$\psi = 1.5 \cdot \frac{r_s}{d} \cdot \frac{f_y}{E_s} \cdot \left( \frac{V}{V_{flex}} \right)^{\frac{3}{2}} \quad (3.6)$$

where the ratio between acting shear force and flexural shear capacity (Bentz, 2013) can be expressed as

$$f \left( \frac{V_E}{V_{flex}} \right) = k_m \cdot \left( \frac{m_s}{m_R} \right)^{\frac{3}{2}} \quad (3.7)$$

$r_s$  is the distance between the column axis to the line of contra flexure bending moment,  $f_y$  is the yielding strength of flexural reinforcement,  $E_s$  is the modulus of elasticity of flexural reinforcement,  $m_s$  is the moment for calculation of the bending reinforcement in the support strip,  $m_R$  is the bending strength,  $V_{flex}$  is the shear force associated to flexural capacity and  $V$  is the acting shear force (Muttoni, 2008).

The failure criterion is developed, based on some relationships (Muttoni et al., 1991). The critical shear crack opening ( $w$ ) is proportional to the effective depth ( $d$ ) multiplied with the rotation of the slab  $\psi$ . The width of the crack is also proportional to the punching shear force. The critical crack makes the failure load decrease.

The roughness of the crack is the factor that determines the amount of shear force that the cracked concrete can carry (Fernandez et al., 2016). The roughness of the critical shear crack depends on the aggregate properties and concrete type, for lightweight concrete  $d_{dg}$  is 16mm, normal weight concrete  $16 + D_{lower}$  and for high-strength concrete  $16 + D_{lower} \cdot (\frac{60}{f_{ck}})^2$ .

## 3.2 Finite Element Analysis

One way to find the distance from the column axis to the contra flexural location is to study the moment distribution over the column. In this thesis FEM was used for this purpose. It was also utilized to perform load combinations for a flat slab.

Finite element analysis, or as it is also called finite element method (FEM), is a numerical procedure for solving problems of engineering and mathematical behaviour (Cook et al., 2002)). The problems can for example involve stress analysis, structure analysis, heat transfer, electromagnetism and fluid flow. The mathematical model that is modelled in FEM is just an approximation of reality.

First step to be able to analyse the structure with FEM is to identify the problem. The problem is described by equations, either integrals or by differential equations. When the problem is identified, information needs to be gathered and on this basis create a model. To create the structure some inputs need to be known as the geometry of the structure, properties of different materials, loads acting on the structure and boundary conditions.

When the input is entered in the program the software prepares a mesh of finite elements. The elements are linked at points that are called nodes. Each element has matrices that describe its behaviour. The matrices are combined to one matrix equation that describes the whole structure. The system/structure that is analysed can either be linear or non-linear.

The matrix equation is solved and quantities are decided at the nodes. The solution is listed or graphically shown. In the end the results need to be analysed. Is the solution reasonable and is the solution free from errors?

Modelling errors are errors that occur when the real model is too simplified in FEM. To reduce this error the model can be modelled more exact.

When modelling the problem in FEM the structures are divided into finite elements. The details of these elements can be too poor which results in discretization error. To reduce this error more elements can be used.

In order to understand the application of finite element analysis to structure mechanics, a brief theory is given. Consider a slab in bending with its mid plane located

in the xy-plane. Rotations and displacements are small. The normal strain  $\varepsilon$  can be described as a deformation relative to the original length. The shear strain  $\gamma$  is the amount of change in a right angle. Assume that the deformations in x-, y- and z-directions are  $u$ ,  $v$  and  $w$ . Then the strains can be formulated as

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{bmatrix} = \begin{bmatrix} -z \frac{\partial^2 w}{\partial x^2} \\ -z \frac{\partial^2 w}{\partial y^2} \\ -2z \frac{\partial^2 w}{\partial x \partial y} \end{bmatrix} \quad (3.8)$$

where the relation to rotation  $\Psi$  is

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} -z \frac{\partial \Psi_x}{\partial x} \\ -z \frac{\partial \Psi_y}{\partial y} \\ -z \frac{\partial \Psi_x}{\partial y} + \frac{\partial \Psi_y}{\partial x} \end{bmatrix} \quad (3.9)$$

and the relation to curvature  $\kappa$  is

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} z \kappa_x \\ z \kappa_y \\ z \kappa_{xy} \end{bmatrix} \quad (3.10)$$

If the slab is linearly elastic, then the stresses can be formulated as function of the strains according to Hooke's law. This gives the following expression

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (3.11)$$

The moments per unit of length can be obtained by integration of the stresses over the thickness of the slab.

$$m_x = \int_{-t/2}^{t/2} \sigma_x z dz \quad (3.12)$$

$$m_y = \int_{-t/2}^{t/2} \sigma_y z dz \quad (3.13)$$

$$m_{xy} = \int_{-t/2}^{t/2} \tau_{xy} z dz \quad (3.14)$$

If the slab is thin in relation to its other dimensions and tranverse shear deformations are prohibited, then the moments according to Kirchhoff plate theory are expressed as

$$\begin{bmatrix} m_x \\ m_y \\ m_{xy} \end{bmatrix} = \begin{bmatrix} D & \nu D & 0 \\ \nu D & D & 0 \\ 0 & 0 & \frac{(1-\nu)D}{2} \end{bmatrix} \begin{bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ 2 \frac{\partial^2 w}{\partial x \partial y} \end{bmatrix} \quad (3.15)$$

where the flexural rigidity  $D = \frac{Et^3}{12(1-\nu^2)}$ . Looking at expressions 3.11 and 3.15 and considering the strain-deformation relationship it follows that vectors with quantities like for example stresses and moments can be expressed as functions of stiffness matrices and deformation vectors. Furthermore, for a structure with a large number of elements and nodes, the degree of freedom becomes large. Consequently, it is not unusual with large stiffness matrices when designing structures. This is where finite element analysis becomes useful to solve the equations.

### 3.3 Presentation of Eurocode 2 Section 6.4

This section presents the current Eurocode 2 for punching shear resistance. All equations and figures are obtained from section 6.4 in EN 1992-1-1(2004) and they are corrected with respect to EN 1992-1-1/AC2010 and EN 1992-1-1/A1.

A concentrated load that affect a slab of a small area, called loaded area, may cause the phenomenon punching shear.

When checking for punching shear controls are performed at the face of the column and at the basic control perimeter  $u_1$ . In cases where shear reinforcement is required a further perimeter  $u_{out,ef}$  should be found where shear reinforcement is no longer required.

#### CHECKS

Some checks need to be controlled when designing a slab structure.

For maximum punching shear stress the following condition must be satisfied

$$v_{Ed} \leq v_{Rd,max} \quad (3.16)$$

where  $v_{Ed}$  is the maximum shear stress and  $v_{Rd,max}$  is the maximum punching shear resistance.

Punching shear reinforcement is not necessary in the structure if the following condition is satisfied

$$v_{Ed} \leq v_{Rd,c} \quad (3.17)$$

where  $v_{Rd,c}$  is the punching shear resistance of a slab without shear reinforcement and  $v_{cs}$  is the punching shear resistance of a slab with shear reinforcement.

If this condition is not satisfied and shear reinforcements are needed, the flat slab should be designed with respect to section 6.4.5 in EN 1992-1-1 (2004).

#### EFFECTIVE DEPTH $d_{eff}$

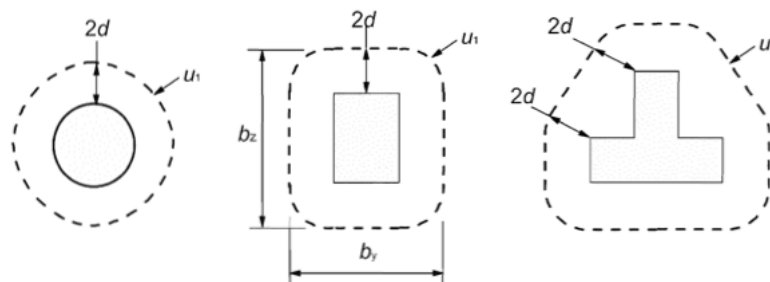
The effective depth of a slab is the average between  $d_y$  and  $d_z$  which is the effective depths of the reinforcement in y and z direction.

$$d_{eff} = \frac{(d_y + d_z)}{2} \quad (3.18)$$

#### LOAD DISTRIBUTION AND BASIC CONTROL PERIMETER

(1) When defining the basic control perimeter  $u_1$  it should be constructed in a way that minimises its length. The control perimeter is normally at a distance  $2d$  from the loaded area, see figure 3.7.

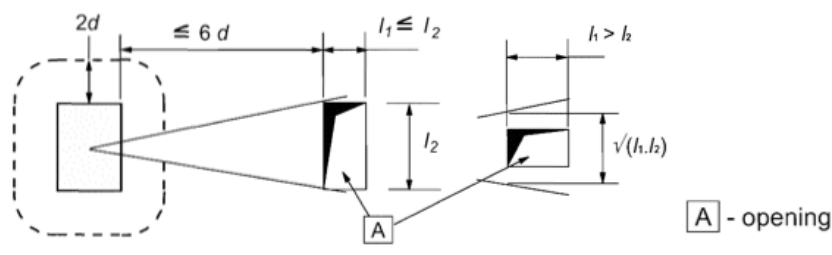




**Figure 3.7** Basic control perimeter

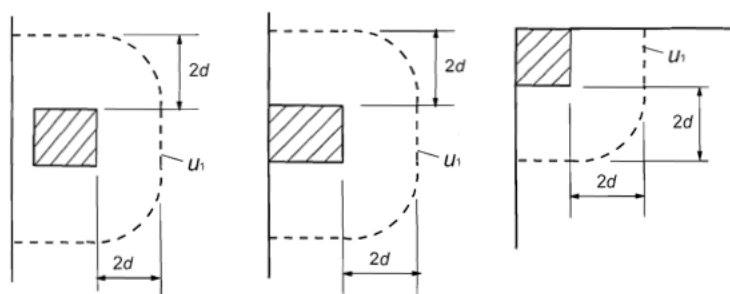
(2) If the concentrated force is opposed by a high pressure or by the effects of a load or reaction within a distance  $2d$  from the loaded area, the control perimeter should be considered at a shorter distance than  $2d$ .

If the loaded structure consist of openings, one check need to be done. If the minimum distance between the opening edge and the loaded areas perimeter is less than  $6d$  a part of the control perimeter is ineffective. This part of the control perimeter is the piece that occur when two lines are drawn from the center of the loaded area to the outline of the opening, see figure 3.8.



**Figure 3.8** Opening near the loaded area

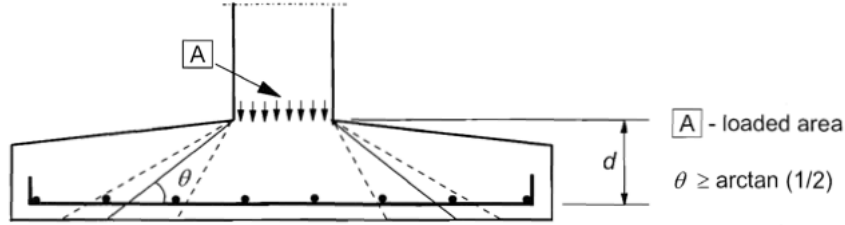
In the figure 3.9 below, basic control perimeters for loaded areas close to or at edge or corner is shown. This is only used if this control perimeter is less than the control perimeter that is derived from (1) or (2).



**Figure 3.9** Basic control perimeter for edge and corners

For loaded area that is situated less than  $d$  from a corner or edge, special reinforcement need to be used at the edge, see section 9.3.1.4 in EN 1992-1-1 (2004)

The control section is defined along the control perimeter and extends over the effective depth  $d$ . If the depth varies for slabs or footings other than step footings, the effective depth may be taken at the perimeter of the loaded area as shown in figure 3.10.



**Figure 3.10** Presentation a footing with different thickness

The basic control perimeter follows the shape of the loaded area but with rounded corners. This law applies to both perimeters inside and outside the basic control area.

Column heads with circular or rectangular shape where  $l_H < 2 \cdot h_H$ , control regarding the punching shear stresses need to be done, but only verification of stresses on the control section outside the column head. For a circular column head the length from the center of the column to the control section outside the column head  $r_{cont}$  is

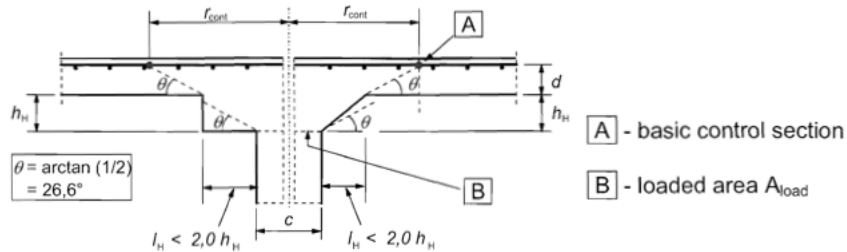
$$r_{cont} = 2 \cdot d + l_H + 0.5 \cdot c \quad (3.19)$$

and for a rectangular column head, the smallest of

$$r_{cont} = 2 \cdot d + 0.56 \cdot \sqrt{l_1 \cdot l_2} \quad (3.20)$$

$$r_{cont} = 2 \cdot d + 0.69 \cdot l_1 \quad (3.21)$$

are chosen where  $l_1 = c_1 + 2 \cdot l_{H1}$ ,  $l_2 = c_2 + 2 \cdot l_{H2}$ ,  $l_1 \leq l_2$ . In figure 3.11 slab where  $l_H < 2 \cdot h_H$  is satisfied is presented



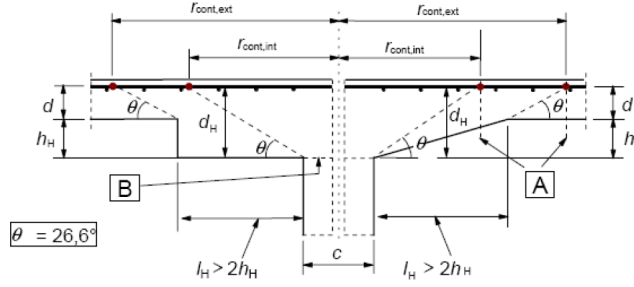
**Figure 3.11** Slab where  $l_H < 2 \cdot h_H$

When  $l_H > 2h_H$ , the control sections inside the column head and slab need to be controlled. Note that this is a correction from EN1992-1-1/AC2010. When checks are done inside the column head,  $d$  is represented by  $d_h$  instead.

A flat slab with circular column is seen below in figure 3.12 where  $l_H > 2h_H$ . The length between the center of the column to the control section is

$$r_{cont,ext} = l_H + 2 \cdot d + 0.5 \cdot c \quad (3.22)$$

$$r_{cont,int} = 2 \cdot (d + h_H) + 0.5 \cdot c \quad (3.23)$$



**Figure 3.12** Slab where  $l_H > 2h_H$

#### MAXIMUM SHEAR STRESS $v_{Ed}$

If the support reaction is eccentric regarding to the control perimeter, the maximum shear stress is

$$v_{Ed} = \beta \cdot \frac{V_{Ed}}{u_i \cdot d} \quad (3.24)$$

where  $V_{Ed}$  is the design shear force and  $d$  is the effective depth.

#### PARAMETER $\beta$

$$\beta = 1 + k \cdot \frac{M_{Ed}}{V_{Ed}} \cdot \frac{u_1}{W_1} \quad (3.25)$$

but  $\beta$  varies depending on different cases.

$\beta$  for internal circular column is

$$\beta = 1 + 0.6 \cdot \pi \cdot \frac{e}{D + 4 \cdot d} \quad (3.26)$$

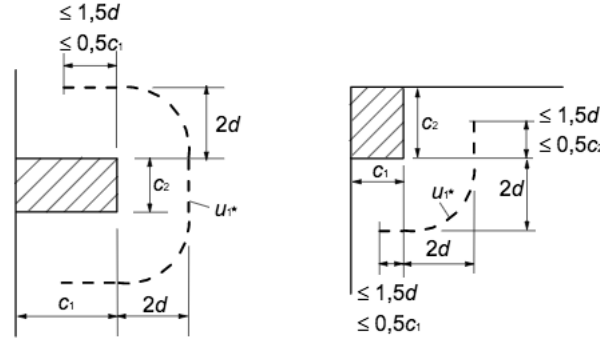
where  $e$  is  $\frac{M_{Ed}}{V_{Ed}}$  which is the eccentricity of the load and  $D$  is the diameter of the column.

For an inner rectangular column where the loading is eccentric to both axes, an approximate equation for  $\beta$  may be used

$$\beta = 1 + 1.8 \cdot \sqrt{\left(\frac{e_y}{b_z}\right)^2 + \left(\frac{e_z}{b_y}\right)^2} \quad (3.27)$$

where  $e_y$  is the eccentricity along y axis,  $b_z$  is the length of the control perimeter in z direction,  $e_z$  is the eccentricity along z axis and  $b_y$  is the length of the control perimeter in y direction.

For edge columns with eccentricity only towards the interior from a moment about an axis parallel to the slab edge, the punching force may be considered to be uniformly distributed along the control perimeter  $u_{1*}$  as shown in figure 3.13.



**Figure 3.13**  $u_{1*}$  for edge and corner column

$\beta$  for slabs with eccentricities in both directions is

$$\beta = \frac{u_1}{u_{1*}} + k \cdot \frac{u_1}{W_1} \cdot e_{par} \quad (3.28)$$

where  $u_{1*}$  is the control parameter but it is reduced according to figure 3.13 and  $e_{par}$  is the eccentricity parallel to the slab edge.

For corner columns with eccentric loading toward the interior of the slab, it is assumed that the punching force is uniformly distributed along the reduced control perimeter  $u_{1*}$ , as defined in Figure 3.13.  $\beta$  is calculated as

$$\beta = \frac{u_1}{u_{1*}} \quad (3.29)$$

For corner column connections, where the eccentricity is toward the exterior equation  $\beta = 1 + k \cdot \frac{M_{Ed}}{V_{Ed}} \cdot \frac{u_1}{W_1}$  is used.

Approximate values for  $\beta$  may be used if the lateral stability of the structure does not depend on frame action between the slabs and the columns, and where the lengths of the adjacent spans do not differ by more than 25%.

Approximate values for  $\beta$  are

$\beta = 1.15$  for inner column

$\beta = 1.4$  for edge column and

$\beta = 1.5$  for corner column

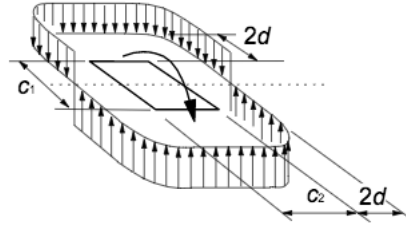
### VALUES OF k

The value for k for rectangular loaded area is given below in table 3.1

$\frac{c_1}{c_2}$	$\leq 0.5$	1	2	$\geq 3$
k	0.45	0.6	0.7	0.8

**Table 3.1** Presentation of the factor k

Where  $c_1$  and  $c_2$  are dimensions seen in figure 3.14



**Figure 3.14** Presentation of  $c_1$  and  $c_2$

### DISTRIBUTION OF SHEAR $W_i$

The distribution of shear is

$$W_i = \int_0^{u_1} |e| dl \quad (3.30)$$

that is illustrated in the figure 3.14.

The parameter  $dl$  is a length increment of the perimeter and  $e$  is the distance of  $dl$  from the axis about which the moment  $M_{Ed}$  acts.

$W_1$  varies depending on different cases.

For a column with rectangular shape  $W_1$  is

$$W_1 = \frac{c_1^2}{2} + c_1 \cdot c_2 + 4 \cdot c_2 \cdot d + 16 \cdot d^2 + 2 \cdot \pi \cdot d \cdot c_1 \quad (3.31)$$

For a edge column with rectangular shape,  $W_1$  is

$$W_1 = \frac{c_2^2}{4} + c_1 \cdot c_2 + 4 \cdot c_1 \cdot d + 8 \cdot d^2 + \pi \cdot d \cdot c_2 \quad (3.32)$$

In cases where the eccentricity perpendicular to the slab edge is toward the exterior, expression (6.39) applies. The eccentricity  $e$  should be measured with respect to the centroid axis of the control perimeter.

SOME GENERAL RULES ABOUT  $V_{Ed}$  and  $V_{pd}$

Reduction of the shear force according to 6.2.2 (6) and 6.2.3 (8) in EN 1992-1-1 (2004) is not valid if concentrated loads are applied close to a flat slab column support.

The vertical component  $V_{pd}$  resulting from inclined prestressing tendons may be regarded as a favourable action when crossing the control section.

#### PUNCHING SHEAR RESISTANCE OF SLABS WITHOUT SHEAR REINFORCEMENT

The punching shear resistance is calculated as

$$v_{Rd,c} = C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{\frac{1}{3}} + k_1 \cdot \sigma_{cp} \geq (v_{min} + k_1 \cdot \sigma_{cp}) \quad (3.33)$$

where  $k = 1 + \sqrt{\frac{200}{d}} \leq 2.0$  where  $d$  is in mm

$\rho_l = \sqrt{\rho_{ly} \cdot \rho_{lz}} \leq 0.02$  where  $\rho_{ly}$  and  $\rho_{lz}$  represent the bonded reinforcement ratio in y- and z- directions respectively. The values  $\rho_{ly}$  and  $\rho_{lz}$  should be calculated as mean values over a slab width equal to the column width plus  $3d$  on each side.

$\sigma_{cp} = \frac{(\sigma_{cy} + \sigma_{cz})}{2}$  where  $\sigma_{cy}$  and  $\sigma_{cz}$  are the normal concrete stresses acting on the critical section in y- and z- directions expressed in MPa and positive if compressive.

$$\sigma_{c,y} = \frac{N_{Ed,y}}{A_{cy}} \text{ and } \sigma_{c,z} = \frac{N_{Ed,z}}{A_{cz}}$$

where  $N_{Ed,y}$  and  $N_{Ed,z}$  are the longitudinal forces from loads or prestressing actions across a defined slab section which is the full bay for internal columns and the the control section for edge columns.  $A_c$  is the section area according to the definition of  $N_{Ed}$ .

$C_{Rd,c}$ ,  $v_{min}$  and  $k_1$  can be found in EKS10 (2016). Values that are recommended for  $C_{Rd,c} = \frac{0.18}{\gamma_c}$ ,  $\gamma_c = 1.5$ ,  $v_{min} = 0.035 \cdot k^{\frac{3}{2}} \cdot f_{ck}^{\frac{1}{2}}$  and  $k_1 = 0.1$ .

#### PUNCHING SHEAR RESISTANCE OF SLABS WITH SHEAR REINFORCEMENT

The design punching shear resistance for flats slabs where shear reinforcement is necessary, is calculated as

$$v_{Rd,cs} = 0.75 \cdot v_{Rd,c} + 1.5 \cdot \frac{d}{s_r} \cdot A_{sw} \cdot f_{ywd,ef} \cdot \left( \frac{1}{u_1 \cdot d} \right) \cdot \sin \alpha \leq k_{max} \cdot v_{Rd,c} \quad (3.34)$$

where  $A_{sw}$  is the section area of the shear reinforcement along one perimeter around the column.  $s_r$  is the radial spacing between perimeters of shear reinforcement and  $\alpha$  is the angle between the shear reinforcement and the plane of the slab.  $k_{max}$  is an amendment in EN 1992-1-1/A1 and set to 1.6 according to EKS10 (2016).  $f_{ywd,ef}$  is the effective design strength of the punching shear reinforcement formulated as  $f_{ywd,ef} = 250 + 0.25 \cdot d \leq f_{ywd}$

where  $f_{ywd}$  is the design tensile strength of the shear reinforcement obtained from a reduction of the characteristic strength with  $\gamma_s = 1.15$ .

The ratio  $\frac{d}{s_r}$  in the equation for the design punching shear resistance is given the value 0.67 if a line of bent-down bars exist.

In the section 9.4.3 is a detailed description about the punching shear reinforcement.

The punching shear resistance should not exceed the design value of the maximum punching shear resistance

$$v_{Ed} = \frac{\beta \cdot V_{Ed}}{u_0 \cdot d} \leq v_{Rd,max} \quad (3.35)$$

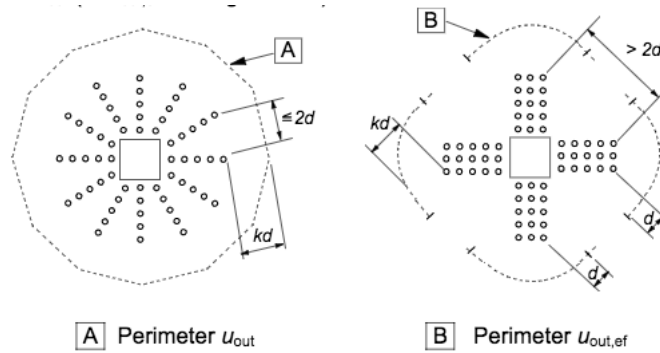
where  $u_0$  = enclosing minimum periphery. For edge column  $u_0 = c_2 + 3 \cdot d \leq c_2 + 2 \cdot c_1$  and corner column  $u_0 = 3 \cdot d \leq c_1 + c_2$

$v_{Rd,max}$  can be found in the National Annex and is in Sweden recommended to be  $0.5 \cdot \nu \cdot f_{cd}$  where  $\nu = 0.6 \cdot [1 - \frac{f_{ck}}{250}]$ .

When the flat slab do not consist of shear reinforcement, the control perimeter  $u_{out}$  is calculated as

$$u_{out,ef} = \beta \frac{V_{Ed}}{v_{rd,c \cdot d}} \quad (3.36)$$

The shear reinforcement should not be distributed more than a distance  $kd$  within an outer perimeter  $u_{out}$  or  $u_{out,ef}$ , see figure 3.15.



**Figure 3.15** Presentation of control perimeters at inner columns

The recommended value for  $k = 1.5$  in Sweden.

### 3.4 Presentation of PT1prEN 1992-1-1 Section 6.4

This section presents the proposed punching shear design provisions. All equations and figures are obtained from PT1prEN 1992-1-1 (2017).

#### CHECKS

For slabs without shear reinforcement the following condition should be satisfied

$$\tau_{Ed} \leq \tau_{Rd,c} \quad (3.37)$$

For slabs with shear reinforcement the following two conditions should be satisfied

$$\tau_{Ed} \leq \tau_{Rd,cs} \quad (3.38)$$

$$\tau_{Ed} \leq \tau_{Rd,max} \quad (3.39)$$

Since the design shear stress and the design punching shear stress resistance is calculated at a control section, the conditions above can be reformulated. Condition 3.37 can be expressed as

$$\beta V_{Ed} \leq V_{Rd,c} \quad (3.40)$$

where  $V_{Rd,c} = \tau_{Rd,c} \cdot d_v \cdot b_0$ .

Conditions 3.38 and 3.39 can similarly be expressed as

$$\beta V_{Ed} \leq V_{Rd,cs} \quad (3.41)$$

and

$$\beta V_{Ed} \leq V_{Rd,max} \quad (3.42)$$

where  $V_{Rd,cs} = \tau_{Rd,cs} \cdot d_v \cdot b_0$  and  $V_{Rd,max} = \tau_{Rd,max} \cdot d_v \cdot b_0$ .

#### SHEAR-RESISTING EFFECTIVE DEPTH $d_v$

The shear-resisting effective depth  $d_v$  is the average of the distance between the loaded area and the centroid of the flexural reinforcement bars in the x- and y-direction. If the column penetrates into the slab, this is only to account for if the penetration is larger than the tolerance according to EN 13670.

If the slab has variable depths, for example a slab with drop panels, control sections at a greater distance from the supported area may be governing.

The effective depth may be calculated as

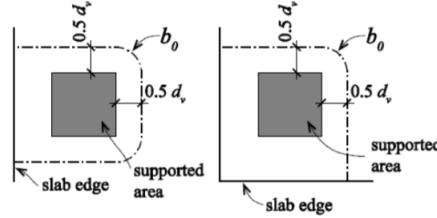
$$d_v = \frac{d_x + d_y}{2} \quad (3.43)$$

#### CONTROL PERIMETER $b_0$

The control perimeter should be constructed in a way that minimises its length  $b_0$ . It may normally be defined at a distance  $0.5d_v$  from the supported area.

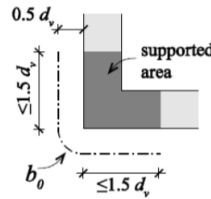


For a supported area near an edge or a corner of a slab, the control perimeter should be calculated according to figure 3.16.



**Figure 3.16** Length of the control section for an edge and a corner column.

The shear forces may be concentrated to the corners of large supported areas. This may be taken into account by reducing the lengths of the straight segments of the control perimeter assuming that they do not exceed  $3d_v$ . This could be the case for drop panels with large thickness because of the stiffness. The case with a corner of a wall is given in figure 3.17.



**Figure 3.17** Length of the control section for a corner of a wall.

Openings and inserts are dealt with in the same way as in Eurocode 2 in cases when the shortest distance between the control perimeter and the edge of the opening or the insert does not exceed  $5d_v$ .

#### DESIGN SHEAR STRESS $v_{Ed}$

The design shear stress  $v_{Ed}$  is calculated as

$$\tau_{Ed} = \beta \frac{V_{Ed}}{b_0 \cdot d_v} \quad (3.44)$$

Favourable effects of actions within the control perimeter may be accounted for by reducing the design shear force  $V_{Ed}$  acting at the control perimeter.

#### PARAMETER $\beta$

Approximated values for  $\beta$  may be used where the lateral stability does not depend on frame action of slabs and columns and where the lengths of the adjacent spans do not differ more than 25%. The approximated values are

$\beta = 1.15$  for inner columns

$\beta = 1.4$  for edge columns and for corners or ends of walls

$\beta = 1.5$  for corner columns

Provisions for calculating  $\beta$  for edge and corner columns are yet not available.

For inner columns not complying with the conditions stated above, parameter  $\beta$  shall be calculated as

$$\beta = 1 + \frac{e_b}{b_b} \quad (3.45)$$

$e_b$  is the eccentricity of the resultant of shear forces and calculated with respect to the centroid of the control perimeter. It may be calculated as

$$e_b = \sqrt{e_{b,x}^2 + e_{b,y}^2} \quad (3.46)$$

where

$$e_{b,x} = \frac{M_{Ed,y}}{V_{Ed}} \quad (3.47)$$

$$e_{b,y} = \frac{M_{Ed,x}}{V_{Ed}} \quad (3.48)$$

$b_b$  is the diameter of a circle which has the same area as the region enclosed by the control perimeter. Consequently it may be calculated as

$$b_b = 2\sqrt{\frac{A}{\pi}} \quad (3.49)$$

where  $A$  is the enclosed area.

### PUNCHING SHEAR RESISTANCE OF SLABS WITHOUT SHEAR-REINFORCEMENT

The design punching shear stress resistance is calculated as

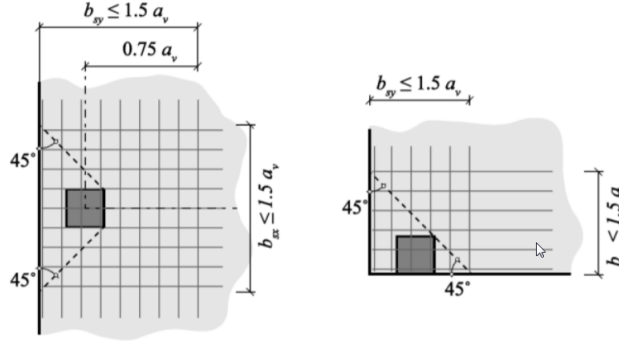
$$\tau_{Rd,c} = \frac{k_b}{\gamma_c} \left( 100\rho_l \cdot f_{ck} \cdot \frac{d_{dg}}{a_v} \right)^{\frac{1}{3}} \leq \frac{0.6}{\gamma_c} \sqrt{f_{ck}} \quad (3.50)$$

This expression assumes  $E_s = 200000$  MPa but it can be adapted for reinforcement types with a different modulus of elasticity.

#### PARAMETER $b_s$

The width of the support strip  $b_s$  is defined as  $1.5a_v$  but limited to the smallest concurring span length.

For columns near to slab edges,  $b_s$  is defined as in figure 3.18.



**Figure 3.18** Length of the support strip for a column near an edge

PARAMETER  $a_v$

$a_v$  refers to the location where the radial bending moment is equal to zero with respect to the support axis and should have the same units as  $d_{dg}$

$$a_v = \sqrt{a_{v,x} \cdot a_{v,y}} \geq 2.5d \quad (3.51)$$

The lengths  $a_{v,x}$  and  $a_{v,y}$  may be approximated when the lateral stability does not depend on frame action between the slabs and the columns and when fulfilling the condition  $0.5 \geq L_x/L_y \geq 2$ . The approximated values are  $0.22L_x$  and  $0.22L_y$  where  $L_x$  and  $L_y$  are the largest span lengths of the bays adjacent to the considered column.

In cases not complying with the conditions stated above, parameter  $a_v$  may be calculated using a linear elastic (uncracked) model.

For prestressed slabs or for slabs with compressive normal forces, parameters  $a_{v,x}$  and  $a_{v,y}$  to be used in  $v_{Rd,c}$  may be multiplied by the factor

$$\left( 1 + \mu \cdot \frac{\sigma_d}{\tau_{Ed}} \cdot \frac{d/6 + e_p}{b_0} \right)^3 \geq 0 \quad (3.52)$$

where

$\sigma_d$  is the average normal stress in the x- or y-directions over the width of the support strip  $b_s$ . Compression is negative.

$e_p$  is the eccentricity of the normal forces related to the centre of gravity of the section at control section in the x- or y-directions.

$$d = \frac{d_s^2 \cdot A_s + d_p^2 \cdot A_p}{d_s \cdot A_s + d_p \cdot A_p} \quad (3.53)$$

and  $d_s$ ,  $d_p$ ,  $A_s$  and  $A_p$  are the effective depths and the reinforcement areas for mild steel and bonded prestressed tendons, respectively.

In case of compressive membrane action around inner columns without openings,

parameter  $a_v$  to be used in  $v_{Rd,c}$  may be multiplied by the following factor

$$\frac{\sqrt{100\rho_l}}{50} \cdot \left( \frac{f_{yd}}{f_{ctd}} \right)^{3/4} \leq 1 \quad (3.54)$$

PARAMETER  $d_{dg}$

$d_{dg}$  is a coefficient taking account of concrete type and its aggregate properties. Its value is

- 32 mm for normal weight concrete with  $f_{ck} \leq 60$  MPa and  $D_{lower} \geq 16$  mm
- $16 + D_{lower} \leq 40$  mm for normal weight concrete with  $f_{ck} \leq 60$  MPa and  $D_{lower} < 16$  mm.
- $16 + D_{lower} (60/f_{ck})^2 \leq 40$  mm for normal weight concrete with  $f_{ck} > 60$  MPa
- 16 mm for lightweight concrete and for concrete with recycled aggregates

PARAMETER  $\rho_l$

The bonded flexural reinforcement ratio  $\rho_l$  may be calculated as

$$\rho_l = \sqrt{\rho_{l,x} \cdot \rho_{l,y}} \quad (3.55)$$

The ratios in the x- and y-directions shall be calculated as mean values over the width of the support strip  $b_s$ .

For prestressed slabs, the flexural reinforcement ratio to be used in  $\tau_{Rd,c}$  is defined as <sup>1</sup>

$$\rho_l = \frac{d_s \cdot A_s + d_p \cdot A_p}{b_s \cdot d^2} \quad (3.56)$$

where  $d_s$ ,  $d_p$ ,  $A_s$  and  $A_p$  are the effective depths and the reinforcement areas for mild steel and bonded prestressed tendons respectively.

PARAMETER  $\mu$

$\mu$  accounts for the shear force to bending moment ratio in the shear-critical region. It may be set to

- 8 for inner columns
- 5 for edge columns and for corner of walls
- 3 for corner columns"

PARAMETER  $k_b$

The shear gradient enhancement factor  $k_b$  is defined as

$$k_b = \sqrt{8 \cdot \mu \cdot \frac{d}{b_0}} \geq 1 \quad (3.57)$$

---

<sup>1</sup> $b_s$  is missing in the draft PT1prEN 1992-1-1 (2017) but has been added in order to obtain a unitless ratio

## PUNCHING SHEAR RESISTANCE OF SLABS WITH SHEAR-REINFORCEMENT

The resistance of shear reinforced flat slabs shall be calculated as

$$\tau_{Rd,cs} = \eta_c \cdot \tau_{Rd,c} + \eta_s \cdot \rho_w \cdot f_{ywd} \geq \rho_w \cdot f_{ywd} \quad (3.58)$$

### PARAMETER $\eta_c$

Parameter  $\eta_c$  accounts for how much of the concrete contribution that is developed at failure.

$$\eta_c = \tau_{Rd,c} / \tau_{Ed} \quad (3.59)$$

### PARAMETER $\eta_s$

Parameter  $\eta_s$  accounts for how much of the shear-reinforcement that is developed at failure. It is defined as

$$\eta_s = 0.10 + \left( \frac{a_v}{d} \right)^{1/2} \left( \frac{0.8}{\eta_c \cdot k_b} \right)^{3/2} \leq 0.8 \quad (3.60)$$

Where inclined distributed shear reinforcement is used,  $\eta_s$  may be multiplied by the factor

$$(\sin \alpha + \cos \alpha) \sin \alpha \quad (3.61)$$

### PARAMETER $\rho_w$

The transverse reinforcement ratio  $\rho_w$  is defined as

$$\rho_w = \frac{A_{sw}}{s_r \cdot s_t} \quad (3.62)$$

where

$A_{sw}$  is the area of one unit of shear reinforcement.

$s_r$  is the radial spacing of shear reinforcement between the first and second unit  $s_1$ . When the distance between the edge of the supported area and the first unit  $s_0$  is larger than  $0.5d_v$ ,  $s_r$  has to be replaced by  $s_0 + 0.5s_1$ .

$s_t$  is the average tangential spacing of perimeters of shear reinforcement measured at the control perimeter.

Where inclined distributed shear reinforcement is used,  $\rho_w$  may be multiplied by the factor

$$(\sin \alpha + \cos \alpha) \quad (3.63)$$

## MAXIMUM PUNCHING SHEAR RESISTANCE

The punching shear resistance is limited to

$$V_{Rd,max} = \eta_{sys} \cdot \tau_{Rd,c} \quad (3.64)$$

where

-  $\eta_{sys} = 1.5$  for stirrups

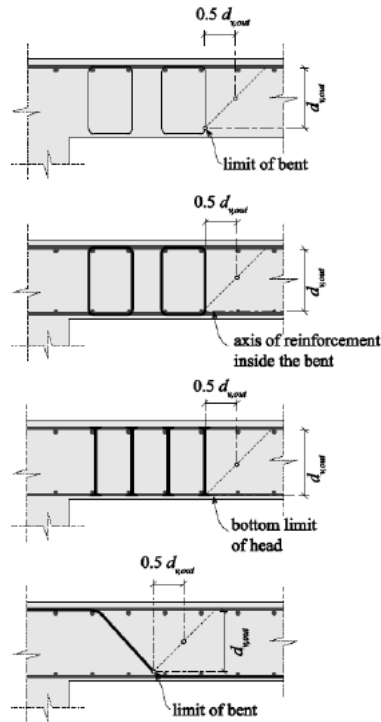
-  $\eta_{sys} = 1.8$  for studs

PARAMETER  $b_{0,out}$

The length of the outer perimeter  $b_{0,out}$  where shear reinforcement is not required is obtained from

$$b_{0,out} = \left( \frac{d_v}{d_{v,out}} \cdot \frac{1}{\eta_c} \right)^{3/2} \quad (3.65)$$

$d_{v,out}$  may be defined as in figure 3.19.



**Figure 3.19** Definition of  $d_{v,out}$  for different shear reinforced flat slabs.

# Chapter 4

## Case Study

### 4.1 Structure Presentation

Hästen 21, earlier known as Torsken 31, is a property situated adjacent to Nordiska kompaniet in the crossing Regeringsgatan and Mäster Samuelsgatan in Stockholm, see figure 4.1. The design of the current building Passagenhuset is the work of architect Bengt Lindroos on behalf of John Mattsson AB (Hästen 21, 2017), see figure 4.1. The load bearing properties of the building were decided by Sven Tyrén AB. The building was erected in 1973 and has nine floors with the lower three located below street level. The first two floors have car parks. The second floor also has loading bays, staging areas and a space for waste disposal. Shops are found on the third and fourth floor while the remaining five floors are used as office spaces. The central core of the building contains elevators and staircases.

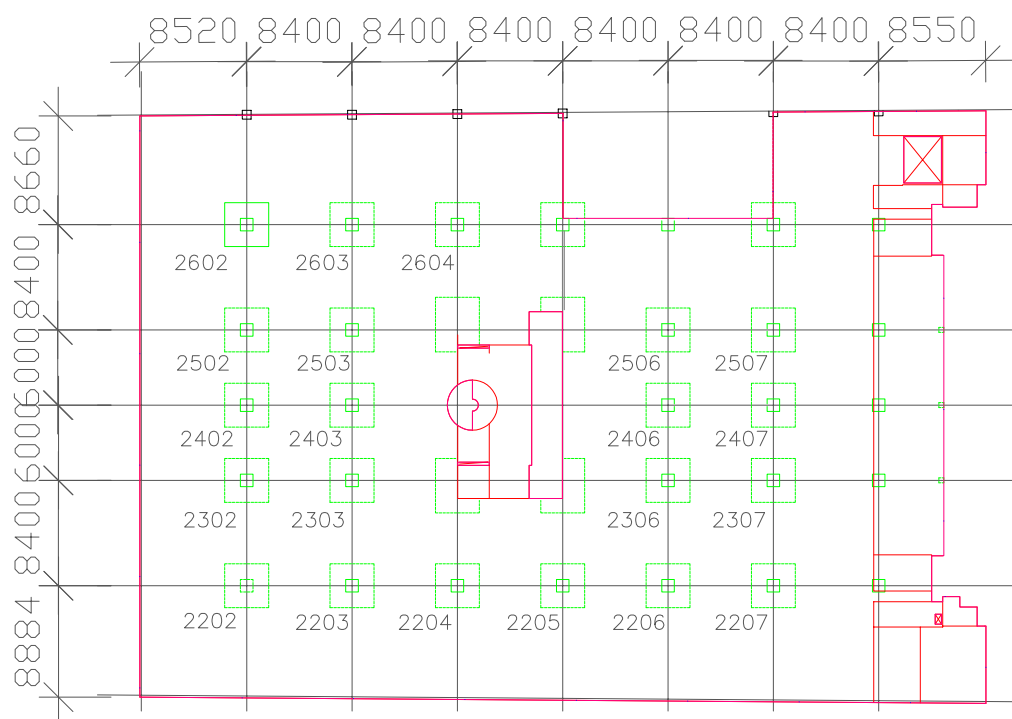
The second floor is an in situ normal weight concrete slab in varying thickness, see figure 4.5. The concrete is of quality K400 corresponding to C32/40. The slab rests on a set of concrete columns and load bearing concrete walls. 21 inner column locations are of interest concerning punching shear, see figure 4.2. The slab is provided with drop panels at all these locations. The total thickness of the slab and the drop panel at these locations is 400 mm. The depths from the bottom face of the drop panel to the centroids of the orthogonal flexural reinforcement bars are 362 mm and 374 mm in the x- and y-directions. The corresponding depths outside the drop panels are 262 mm and 274 mm for the slab parts with thickness 300 mm. The slab parts with thickness 270 mm have depths equal to 232 mm and 244 mm. The columns measure 1000x1000 mm and the drop panels measure 3500x3500 mm, see figure 4.3. What separates the column locations are the span lengths and the distribution of flexural reinforcement in the upper part of the slab.

The case study was aimed towards the second floor of Hästen 21. The aim of the study was to calculate the punching shear resistance in accordance with section 6.4 in Eurocode 2 (2004) and PT1prEN 1992-1-1 (2017). Furthermore dead loads

and variable loads were combined in order to estimate the load effects for some of the column positions. The variable loads comply with SS-EN 1991-1-1 and the load combinations were performed in accordance with SS-EN 1990. Ultimately, the resistances were compared with the load effects for both design provisions.

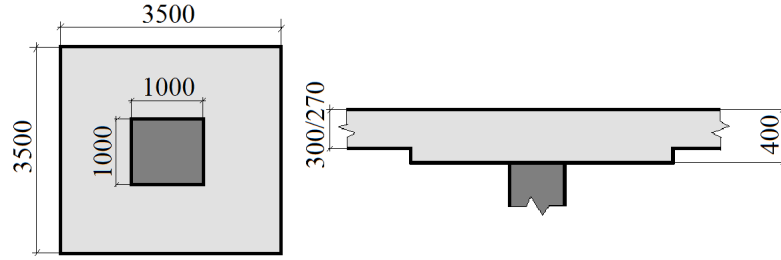


**Figure 4.1** Screenshot from Google Maps of Hästen 21 as seen from the crossing Regeringsgatan and Mäster Samuelsgatan



**Figure 4.2** Presentation of the dimensions of the flat slab





**Figure 4.3** Dimensions of the drop panels, columns and adjacent slab parts

## 4.2 Modelling Procedure

In order to obtain the load effect and the eccentricity factor  $\beta$ , which depends on the column reactions and the transferred moment, the flat slab was modelled in FEM Design 16 Plate. Another parameter that also was gathered through the analysis of the flat slab was  $a_v$ .

The first step in the modelling procedure was to draw the flat slab contours in AutoCAD. A digital AutoCAD drawing was provided by Tyréns. By studying the drawings, the border of the flat slab was decided and modified in the provided digital AutoCAD drawing. After this proceeded the AutoCAD file was transferred to FEM design. The slab was modelled with different thickness and with drop panels. The columns were then placed out. The connections between the columns and the slab were set to fixed in order to transfer the moment from the slab to the columns. All the load bearing walls were represented by hinged line supports.

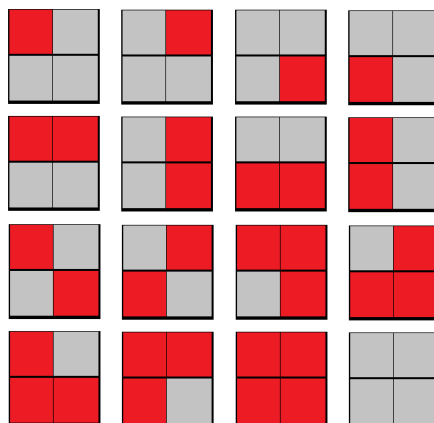
## 4.3 Load combination procedure

In order to obtain the design load effect and the transferred moment a load combination was required. The loads that affect the plate are the self weight and the live load. The self weight was applied on the whole slab and the live load was only applied to the four quadrants adjacent to the column considered, one column at a time. FEM Design was used to generate all possible combinations.

The first step in the procedure was to create the load cases. One represented the self weight, and the other four represented the live load on each quadrant adjacent to the column considered. The next step was to make the load groups where settings like permanent or temporary, favourable and unfavourable and  $\psi$  were set. The last step was to generate all the possible load combinations according to EKS10 (2016) equation 6.10 a and b, see figure 4.4.

After the generation was completed, the program calculated the column reactions. The results at the top node of the considered column were transferred to a list and

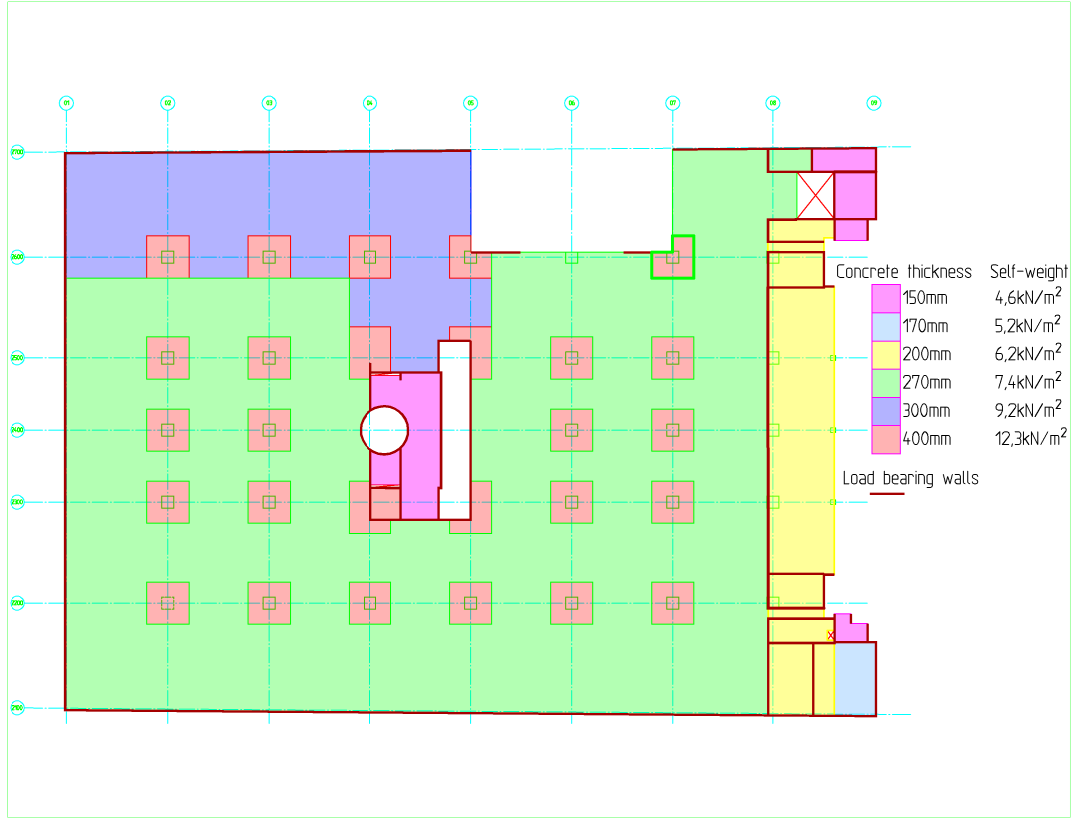
converted to a text-file. The text-file was imported to Excel and displayed as a number of columns. Since every single combination resulted in moments in both directions, it was concluded that they were all eccentric to both axis. Consequently equation 6.43 in Eurocode 2 (2004) was used to find the eccentricity factor  $\beta$ . The corresponding factor for the proposal was calculated from equation 6.56 in PT1prEN 1992-1-1 (2017). Each factor was then multiplied with the obtained column reaction force and the largest result was considered as the design shear force. The design shear force was checked against the resistance to see if shear reinforcement was required.



**Figure 4.4** Presentation of all considered load combinations with live load in red.

## 4.4 Loads considered in the load combination

The loads that were considered in the load combination process were the permanent load and the live load (imposed load). The permanent load is the total self-weight of structural and non-structural members, the self-weight of new coatings and/or distribution conduits that is applied on the structure after execution, the water level and the source and moisture content of bulk materials (Eurocode 1, 2002). The self weight for the second floor was taken from the calculation part provided by Tyréns, see figure 4.5. Imposed load consists of loads arising from occupancy. "Values given in this section, include: normal use by persons, furniture and moveable objects (e.g. moveable partitions, storage, the contents of containers), vehicles, anticipating rare events, such as concentrations of persons or of furniture, or the moving or stacking of objects which may occur during reorganization or redecoration" (Eurocode 1, 2002).



**Figure 4.5** Presentation of slab thickness and the self weight

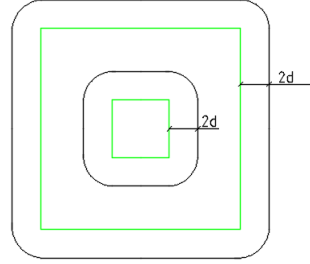
The second floor of Hästen 21 is a garage space. It was assumed that cars with a weight not more than 3 tonnes use this space. Thus, the floor belongs to category F for gross vehicle weight  $\leq 30$  kN (Eurocode 1, 2002). For this category the distributed load  $q_k$  is  $1.5 - 2.5 \text{ kN/m}^2$ . In the load combination the worst case was analysed, that is why the distributed load  $q_k = 2.5 \text{ kN/m}^2$  was used in the combination.

Since Hästen 21 is a multi-storey building that consists of offices, stories, and parking spaces it belongs to building type A according to part A paragraph 13 in EKS10. The safety class is taken into account by the partial factor  $\gamma_d$  and for a building of type A, safety class 2  $\gamma_d$  is 0.91 (EKS9, 2013). The  $\psi$ -factor is 0.7 for  $\psi_0$  and  $\psi_1$  and 0.6 for  $\psi_2$  according to table B-1, category F in EKS10 (2016).

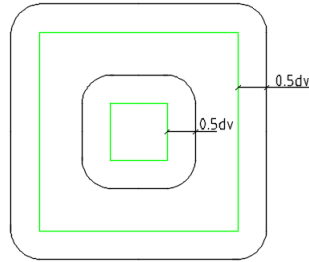
## 4.5 Calculation procedure of punching shear resistance

The calculation procedure of the punching shear resistance at the 21 inner columns according to section 6.4 in Eurocode 2 (2004) is given in appendix A. The corresponding procedure for section 6.4 in PT1prEN 1992-1-1 (2017) is given in appendix B.

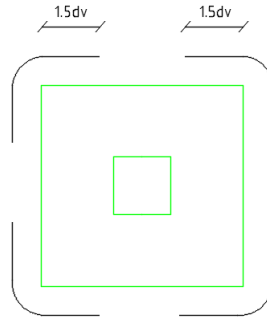
The resistance at one of the wall corners is calculated in appendix C. Parameters related to the concrete type were the characteristic compressive strength  $f_{ck} = 32MPa$  and the aggregate parameter  $d_{ag} = 32mm$ . For further indata, see section "structure presentation". The punching shear resistances were checked at an interior and an exterior control perimeter for both provisions, see figure 4.6-4.7. According to the proposal the exterior perimeter was reduced, see figure 4.8.



**Figure 4.6** Interior and exterior basic control perimeter according to Eurocode 2



**Figure 4.7** Interior and exterior control perimeter according to the proposal



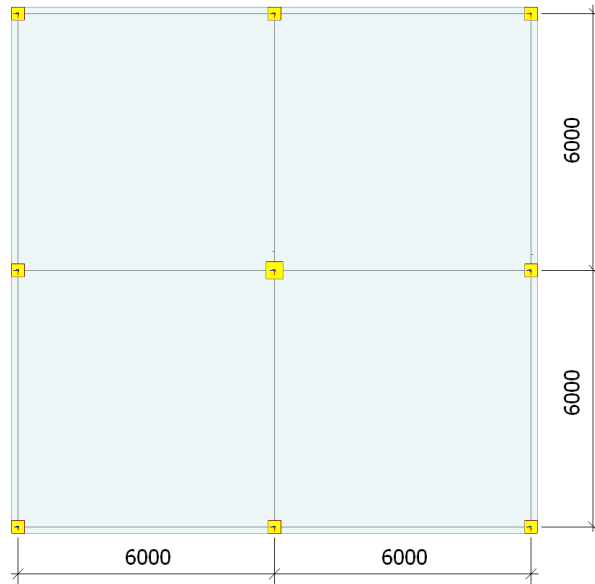
**Figure 4.8** Reduced exterior control perimeter according to the proposal

# Chapter 5

## Parametric Study

### 5.1 Description of the parametric study

For the parametric study a symmetric square flat slab  $12 \times 12\text{m}$  with one inner column, four edge columns and four corner columns was used, see figure 5.1. The flat slab was inspired by calculation example D in SCA (2010b). The thickness of the slab is 300mm. The width of the inner column is 400mm and the widths of the edge and corner columns are 300mm. The concrete strength class is C25/30 and it is assumed that the factor  $d_{dg}$  used in PT1prEN 1992-1-1 (2017) taking account of concrete type is 32mm.



**Figure 5.1** Presentation of the flat slab used in the parametric study

The parametric study was done for the following three cases:

- Flat slab without shear reinforcement
- Flat slab with shear reinforcement
- Prestressed flat slab

For each case, the punching shear resistance was calculated for an inner, edge and corner column. It was assumed that the lateral stability did not depend on frame action and thus the simplified values for  $a_v$  were used as instructed in 6.4.3(2) in PT1prEN 1992-1-1. In every case a set of parameters were varied, one at a time, while the others stayed constant. The calculations were performed by a written Matlab code considering the provisions in section 6.4 in Eurocode 2 and PT1prEN 1992-1-1. This allowed for plotting the graphs in a simple way. See presentations of the code in appendix D, E and F. The code was checked by hand calculations, see appendix G.

## 5.2 Without shear reinforcement

In the case without shear reinforcement, six parameters were varied. These were:

- Effective depth,  $d$
- Compressive strength,  $f_{ck}$
- Column width,  $B$
- Flexural reinforcement ratio,  $\rho$
- The distance to the contra flexural location  $a_v$
- Coefficient taking account of concrete type and its aggregate properties,  $d_{dg}$

The starting values of the parameters considered were taken from example D in SCA (2010b) mentioned above, see table 5.1. The Matlab code is presented in appendix D. The results are given in figures 6.1-6.24 in the Result section.

**Table 5.1** Input values for parametric study without shear reinforcement

	Inner column	Edge column	Corner column
$d$ [mm]	259	262	263
$f_{ck}$ [MPa]	25	25	25
$B$ [mm]	400	300	300
$\rho$ [-]	0.0044	0.004	0.0057
$a_v$ [mm]	1320	1320	1320
$d_{dg}$ [mm]	32	32	32

### 5.3 With shear reinforcement

In the case with shear reinforcement only studs were used as shear reinforcement. The five parameters that were varied were:

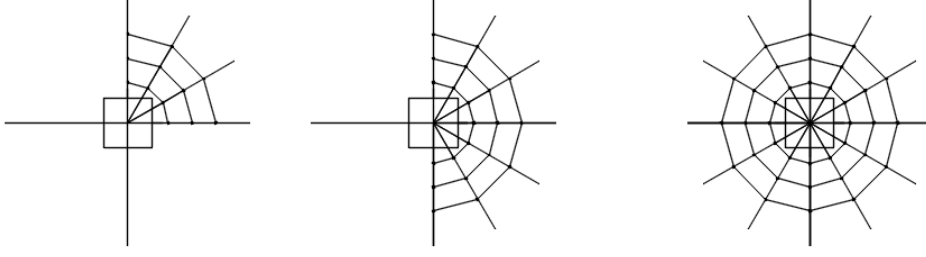
- The diameter of the reinforcement
- The distance to the contra flexural location  $a_v$
- Concrete contribution factor  $\eta_c$
- The radial spacing between the rows of studs  $s_r$
- Numbers of studs in a row

Regarding the starting values, it was assumed that the concrete contribution factor  $\eta_c$  was 0.75 as in equation 6.52 in Eurocode 2. Studs are available with diameters 10-25mm (Halfen, 2017). The starting diameter was thus chosen as 10mm.

The studs were distributed evenly in a circular pattern in compliance with sections 9.4.3 in Eurocode 2 and PT1prEN 1992-1-1 respectively, see figure 5.2. This meant that the radial distance  $s_r$  between the column face and the first row had to be  $0.3d - 0.5d$ . It was chosen as  $0.5d$  for all three columns. The radial distance between the rows had to be  $\leq 0.75d$  and the starting value was chosen as this. Furthermore, the tangential distance  $s_t$  at the control section had to be  $\leq 2d$  according to Eurocode 2. This led to a least amount of studs  $n/\text{row}$ . The starting values of the parameters considered are given in table 5.2. The Matlab code is presented in appendix E. The results are given in figures 6.25-6.62 in the Result section.

**Table 5.2** Input values for parametric study with shear reinforcement

	Inner column	Edge column	Corner column
$d$ [mm]	259	262	263
$f_{ck}$ [MPa]	25	25	25
$B$ [mm]	400	300	300
$\rho$ [-]	0.0044	0.004	0.0057
$a_v$ [mm]	1320	1320	1320
$d_{dg}$ [mm]	32	32	32
$\eta_c$ [-]	0.75	0.75	0.75
$s_r$ [mm]	194	196	197
$n$ [-]	12	7	4



**Figure 5.2** Principal sketch of the distribution of studs for the corner, edge and inner columns.

## 5.4 Prestressed

In the case with a prestressed flat slab, the slab was provided with post tensioned tendons evenly distributed over the distance  $2a_v$  in the x- and y-direction, see figure 5.3. Two parameters were varied and they were:

- Prestressing force  $N$
- Diameter of tendons  $d_p$

The slab was loaded in FEM Design 16 Plate with a distributed load of  $8kN/m^2$ . The reaction forces were determined at the inner, edge and corner columns. Each reaction was then multiplied with  $\beta$  as 1.15, 1.4 and 1.5 for the inner, edge and corner column respectively. The shear stresses  $\tau_{Ed}$  at each column location were determined by division of the control section area.

Flexural reinforcement bars were distributed evenly in both directions over the areas where the reinforcement ratio is calculated. Depths and spacings in the x- and y-direction were chosen in a way that the original effective depths and reinforcement ratios were obtained.

The tendons were given a parabolic shape. The shape was derived by assuming an eccentricity over the support axis and zero eccentricity at the contra flexure perimeter as measured in FEM Design 16. The general equation obtained for the eccentricity at a distance  $x$  from the support axis of the column is given as

$$e_p = \frac{-e_0}{l^2} \cdot x^2 + e_0 \quad (5.1)$$

where  $e_0$  is the eccentricity above the support axis and  $l$  is the measure distance to the location of contra flexure for each column. The derivative of the eccentricity is given as

$$e'_p = \frac{-2 \cdot e_0}{l^2} \cdot x \quad (5.2)$$



From this equation the horizontal tensioning force component can be obtained as

$$N_x = \sqrt{\frac{N^2}{1 + e'^2}} = \sqrt{\frac{N^2}{1 + \left(\frac{2 \cdot e_0}{l^2} \cdot x\right)}} \quad (5.3)$$

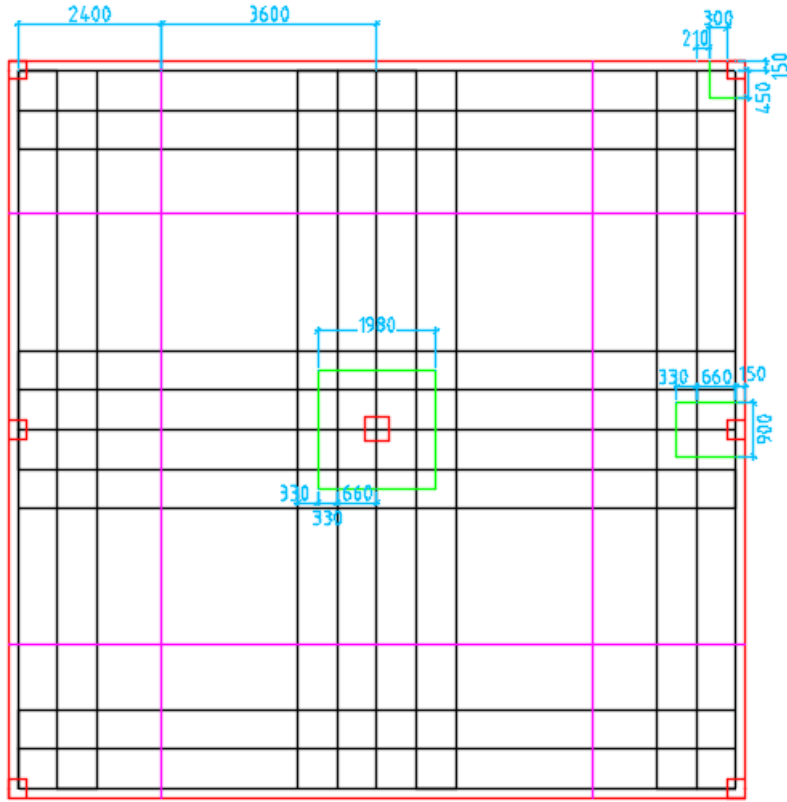
No friction losses are taken into account and it is assumed that the tensioning force  $N$  is constant along the tendon.

The starting values are given in table 5.3. The Matlab code is presented in appendix F. Figures 6.63-6.67 in the Result section presents the result when the tensioning force is varied and the tendon diameter is assumed to be close to zero. This is of course unrealistic, but it is only done to show the effect from the tensioning force on the resistance without any influence from the tendons on the reinforcement ratio.

Figures 6.68-6.70 in the result section presents the result when the tendon diameter is varied and the tensioning force is close to zero. This is done to show the effect on the resistance from the tendons without any influence from the tensioning force.

**Table 5.3** Input values for parametric study for a prestressed flat slab

	Inner column	Edge column	Corner column
$d_x$ [mm]	267	268	269
$d_y$ [mm]	251	256	257
$d$ [mm]	259	262	263
$\phi$ [mm]	16	12	12
$f_{ck}$ [MPa]	25	25	25
$B$ [mm]	400	300	300
$\rho$ [-]	0.0044	0.004	0.0057
$a_v$ [mm]	1320	1320	1320
$d_{dg}$ [mm]	32	32	32
$e_{0,x}$ [mm]	90	50	90
$e_{0,y}$ [mm]	50	10	50



**Figure 5.3** Drawing of the distribution of post tensioned tendons in the flat slab. The green borders indicate the widths of the support strips.

# Chapter 6

## Results

### 6.1 Case Study

#### 6.1.1 Current Eurocode 2

In this section, results from the calculation model for the current Eurocode 2 section 6.4. is presented. The calculation procedure is presented in appendix A. The column numbers are found in figure 4.2. The relevant indata is presented in chapter 4.

#### GEOMETRY

The calculated effective depths at the interior and exterior control perimeters are presented in table 6.1. The calculated lengths of the interior and exterior control perimeters are presented in table 6.2.

**Table 6.1** Effective depth at the interior and exterior basic control perimeter

Column	$d_H$ [mm]	$d$ [mm]
2202-2507	368	238
2602-2604	368	268

Control if  $l_H = 1250mm \geq 2 \cdot h_H$

$2 \cdot h_H = 260mm$

In this case  $l_H > 2 \cdot h_H$

**Table 6.2** Interior and exterior basic control perimeter

Column	$u_{1,int}$ [mm]	$u_{1,ext}$ [mm]
2202-2507	8624	16991
2602-2604	8624	17368

The column perimeter was

$u_0 = 4000mm$

## MAXIMUM PUNCHING SHEAR RESISTANCE

The strength reduction factor for concrete cracked in shear was

$$\nu = 0.523$$

The design value of the concrete compressive strength was

$$f_{cd} = 2.133 \cdot 10^7 \text{Pa}$$

The maximum punching shear resistance was

$$v_{Rd,max} = 5.581 \cdot 10^6 \text{Pa}$$

$$V_{Rd,max} = 8.215 \cdot 10^6 \text{N}$$

## CALCULATION OF PUNCHING SHEAR RESISTANCE AT THE INTERIOR BASIC CONTROL PERIMETER

The size effect factor was

$$k_{int} = 1.737 < 2.0$$

The reinforcement control width was

$$b_s = 3208 \text{mm}$$

The number of bars in x- and y-direction that fit within the control width is presented in table 6.3

**Table 6.3** Number of bars within the control width in the x-and y-direction

Column	$N_{x,i}$	$N_{y,i}$
2202-2207	33	33
2302-2303, 2306-2307	28	26
2402-2403, 2406-2407	23	17
2502	33	25
2503	28	25
2506	23	17
2507	27	25
2602, 2604	43	41
2603	41	41

The calculated reinforcement ratios in both directions are presented in table 6.4 and the mean ratio are presented in table 6.5.

**Table 6.4** Reinforcement ratio for longitudinal reinforcement in x- and y-direction

Column	$\rho_{l,i,x}$	$\rho_{l,i,y}$
2202-2207	$3.214 \cdot 10^{-3}$	$3.111 \cdot 10^{-3}$
2302-2303, 2306-2307	$2.727 \cdot 10^{-3}$	$2.451 \cdot 10^{-3}$
2402-2403, 2406-2407	$2.240 \cdot 10^{-3}$	$1.602 \cdot 10^{-3}$
2502	$3.214 \cdot 10^{-3}$	$2.357 \cdot 10^{-3}$
2503	$2.727 \cdot 10^{-3}$	$2.357 \cdot 10^{-3}$
2506	$2.24 \cdot 10^{-3}$	$1.602 \cdot 10^{-3}$
2507	$2.630 \cdot 10^{-3}$	$2.357 \cdot 10^{-3}$
2602, 2604	$4.188 \cdot 10^{-3}$	$3.865 \cdot 10^{-3}$
2603	$3.798 \cdot 10^{-3}$	$3.865 \cdot 10^{-3}$

**Table 6.5** Reinforcement ratio for longitudinal reinforcement

Column	$\rho_{l,int} \leq 2.0$
2202-2207	$3.162 \cdot 10^{-3}$
2302-2303, 2306-2307	$2.585 \cdot 10^{-3}$
2402-2403, 2406-2407	$1.895 \cdot 10^{-3}$
2502	$2.752 \cdot 10^{-3}$
2503	$2.535 \cdot 10^{-3}$
2506	$1.895 \cdot 10^{-3}$
2507	$2.489 \cdot 10^{-3}$
2602, 2604	$4.023 \cdot 10^{-3}$
2603	$3.831 \cdot 10^{-3}$

The minimum punching shear resistance was

$$v_{min,int} = 4.533 \cdot 10^5 \text{Pa}$$

Parameter  $C_{Rd,c}$  was

$$C_{Rd,c} = 0.12$$

No normal stresses acted on the slab, which meant that the normal concrete stress

$$\sigma_{cp} = 0$$

The calculated punching shear resistances at the interior control perimeter are presented in table 6.6.

**Table 6.6** Design value of punching shear resistance of a slab without punching shear reinforcement along the control section considered

Column	$v_{Rd,c,int}$ [Pa]	$v_{min,int} + k_1 \cdot \sigma_{cp}$ [Pa]	$V_{Rd,c,int}$ [N]
2202-2207	$4.51 \cdot 10^5$	$4.533 \cdot 10^5$	$1.439 \cdot 10^6$
2302-2303, 2306-2307	$4.22 \cdot 10^5$	$4.533 \cdot 10^5$	$1.439 \cdot 10^6$
2402-2403, 2406-2407	$3.80 \cdot 10^5$	$4.533 \cdot 10^5$	$1.439 \cdot 10^6$
2502	$4.30 \cdot 10^5$	$4.533 \cdot 10^5$	$1.439 \cdot 10^6$
2503	$4.19 \cdot 10^5$	$4.533 \cdot 10^5$	$1.439 \cdot 10^6$
2506	$3.80 \cdot 10^5$	$4.533 \cdot 10^5$	$1.439 \cdot 10^6$
2507	$4.16 \cdot 10^5$	$4.533 \cdot 10^5$	$1.439 \cdot 10^6$
2602, 2604	$4.89 \cdot 10^5$	$4.533 \cdot 10^5$	$1.551 \cdot 10^6$
2603	$4.81 \cdot 10^5$	$4.533 \cdot 10^5$	$1.526 \cdot 10^6$

### CALCULATION OF PUNCHING SHEAR RESISTANCE AT THE EXTERIOR BASIC CONTROL PERIMETER

The size effect factors are presented in table 6.7

**Table 6.7**  $k_{ext}$ 

Column	$k_{ext} \leq 2.0$
2202-2507	1.917
2602-2604	1.864

The reinforcement control widths are presented in table 6.8

**Table 6.8** Reinforcement control width

Column	$b_{s,e}$ [mm]
2202-2507	2428
2602-2604	2608

The number of bars in x- and y-direction that fit within the control width is presented in table 6.9

**Table 6.9** Numbers of bars within the control width in the x-and y-direction

Column	$N_{x,e}$	$N_{y,e}$
2202-2207	25	25
2302-2303, 2306-2307	22	20
2402, 2406	18	13
2403, 2407	17	13
2502	25	19
2503	22	19
2506	17	13
2507	21	19
2602, 2604	35	33
2603	33	33

The calculated reinforcement ratios in both directions are presented in table 6.10 and the mean ratio are presented in table 6.11

**Table 6.10** Reinforcement ratio for longitudinal reinforcement in x- and y-direction

Column	$\rho_{l,e,x}$	$\rho_{l,e,y}$
2202-2207	$5.019 \cdot 10^{-3}$	$4.773 \cdot 10^{-3}$
2302-2303, 2306-2307	$4.417 \cdot 10^{-3}$	$3.818 \cdot 10^{-3}$
2402, 2406	$3.614 \cdot 10^{-3}$	$2.482 \cdot 10^{-3}$
2403, 2407	$3.413 \cdot 10^{-3}$	$2.482 \cdot 10^{-3}$
2502	$5.019 \cdot 10^{-3}$	$3.627 \cdot 10^{-3}$
2503	$4.417 \cdot 10^{-3}$	$3.627 \cdot 10^{-3}$
2506	$3.413 \cdot 10^{-3}$	$2.482 \cdot 10^{-3}$
2507	$4.216 \cdot 10^{-3}$	$3.627 \cdot 10^{-3}$
2602,2604	$5.793 \cdot 10^{-3}$	$5.223 \cdot 10^{-3}$
2603	$5.462 \cdot 10^{-3}$	$5.223 \cdot 10^{-3}$

**Table 6.11** Reinforcement ratio for longitudinal reinforcement

Column	$\rho_{l,ext} \leq 0.02$
2202-2207	$4.894 \cdot 10^{-3}$
2302-2303, 2306-2307	$4.107 \cdot 10^{-3}$
2402, 2406	$2.995 \cdot 10^{-3}$
2403, 2407	$2.910 \cdot 10^{-3}$
2502	$4.267 \cdot 10^{-3}$
2503	$4.003 \cdot 10^{-3}$
2506	$2.910 \cdot 10^{-3}$
2507	$3.911 \cdot 10^{-3}$
2602,2604	$5.501 \cdot 10^{-3}$
2603	$5.314 \cdot 10^{-3}$

The minimum punching shear resistances are presented in table 6.12

**Table 6.12** Minimum punching shear resistance

Column	$v_{min,ext}$ [Pa]
2202-2507	$5.254 \cdot 10^5$
2602-2604	$5.038 \cdot 10^5$

The calculated punching shear resistances at the exterior control perimeter are presented in table 6.13.

**Table 6.13** Design value of punching shear resistance of a slab without punching shear reinforcement along the control section considered

Column	$v_{Rd,c,ext}$ [Pa]	$v_{min,ext} + k_1 \cdot \sigma_{cp}$ [Pa]	$V_{Rd,c,ext}$ [N]
2202-2207	$5.755 \cdot 10^5$	$5.254 \cdot 10^5$	$2.327 \cdot 10^6$
2302-2303, 2306-2307	$5.428 \cdot 10^5$	$5.254 \cdot 10^5$	$2.195 \cdot 10^6$
2402-2403, 2406-2407	$4.886 \cdot 10^5$	$5.254 \cdot 10^5$	$2.125 \cdot 10^6$
2502	$5.497 \cdot 10^5$	$5.254 \cdot 10^5$	$2.223 \cdot 10^6$
2503	$5.381 \cdot 10^5$	$5.254 \cdot 10^5$	$2.176 \cdot 10^6$
2506	$4.839 \cdot 10^5$	$5.254 \cdot 10^5$	$2.125 \cdot 10^6$
2507	$5.34 \cdot 10^5$	$5.254 \cdot 10^5$	$2.159 \cdot 10^6$
2602, 2604	$5.818 \cdot 10^5$	$5.038 \cdot 10^5$	$2.708 \cdot 10^6$
2603	$5.761 \cdot 10^5$	$5.038 \cdot 10^5$	$2.682 \cdot 10^6$

The resulting resistances are presented in table 6.14

**Table 6.14** The design value that is dimensioning

Column	$V_{Rd,max}$ [MN]	$V_{Rd,c,int}$ [MN]	$V_{Rd,c,ext}$ [MN]
2202-2207	8.215	1.439	2.327
2302-2303, 2306-2307	8.215	1.439	2.195
2402-2403, 2406-2407	8.215	1.439	2.125
2502	8.215	1.439	2.223
2503	8.215	1.439	2.176
2506	8.215	1.439	2.125
2507	8.215	1.439	2.159
2602, 2604	8.215	1.551	2.708
2303	8.215	1.526	2.682

$V_{Rd,c,int}$  was the design punching shear resistance in all cases.

## 6.1.2 Proposed Provisions

In this section, results from the calculation model for the proposed provision section 6.4. is presented. The calculation procedure is presented in appendix B. The column numbers are found in figure 4.2. The relevant indata is presented in chapter 4.

### GEOMETRY

The calculated effective depths at the interior and exterior control perimeters are presented in table 6.15. The calculated lengths of the interior and exterior control perimeters are presented in table 6.16. A reduced control perimeter was calculated due to concentration of shear forces to the corners of the drop panels and are presented in table 6.16.



**Table 6.15** Effective depth at the interior and exterior control perimeter

Column	$d_{v,int}$ [mm]	$d_{v,ext}$ [mm]
2202-2507	368	238
2602-2604	368	268

**Table 6.16** Interior and exterior control perimeter

Column	$b_{0,int}$ [mm]	$b_{0,ext}$ [mm]	$3 \cdot d_{v,ext}$ [mm]	$b_{0,ext,red}$ [mm]
2202-2507	5156	14748	714	3604
2602-2604	5156	14842	804	4058

### CALCULATION OF PUNCHING SHEAR RESISTANCE OF SLABS WITHOUT SHEAR REINFORCEMENT

The ratios between the longest span lengths adjacent to the column considered are presented in table 6.17

**Table 6.17** Ratio between  $L_{x,max}$  and  $L_{y,max}$ 

Column	$0.5 \leq \frac{L_{x,max}}{L_{y,max}} \leq 2$
2202	0.952
2203	0.932
2204	0.926
2205	0.92
2206	0.913
2207	0.907
2302	1
2303	1
2306	1
2307	1
2402	1.42
2403	1.4
2406	1.4
2407	1.4
2502	1
2503	1
2506	1.4
2507	1
2602	0.978
2603	0.959
2604	0.953

The calculated distances from the support axis to the contra flexural locations are presented in table 6.18.

**Table 6.18** Presentation of  $a_v$

Column	$a_{v,x}$ [mm]	$a_{v,y}$ [mm]	$a_v$ [mm]	$2.5 \cdot d_{v,int}$ [mm]
2202	1874	1968	1921	920
2203	1848	1982	1914	920
2204	1848	1996	1920	920
2205	1848	2009	1927	920
2206	1848	2023	1934	920
2207	1848	2037	1940	920
2302	1874	1848	1861	920
2303	1848	1848	1848	920
2306	1854	1848	1851	920
2307	1848	1848	1848	920
2402	1874	1320	1573	920
2403	1848	1320	1562	920
2406	1854	1320	1564	920
2407	1848	1320	1562	920
2502	1874	1848	1861	920
2503	1848	1848	1848	920
2506	1854	1848	1851	920
2507	1848	1848	1848	920
2602	1874	1916	1895	920
2603	1848	1928	1887	920
2604	1848	1939	1893	920

Table 6.19 shows the widths of the support strips.

**Table 6.19** Width of support strip

Column	$b_s$ [mm]
2202, 2204	2881
2203	2871
2205	2891
2206	2900
2207	2910
2302	2792
2303	2772
2306	2776
2307	2772
2402	2359
2403	2343
2406	2346
2407	2343
2502	2792
2503	2772
2506	2776
2507	2772
2602	2843
2603	2831
2604	2839

In table 6.20 the number of bars within the support strip is presented

**Table 6.20** Numbers of bars within the support strip in x-and y-direction

Column	$N_x$	$N_y$
2202-2205	29	29
2206-2207	30	30
2302-2303, 2306-2307	25	23
2402-2403, 2406-2407	17	13
2502	28	22
2503	25	22
2506	20	15
2507	24	22
2602, 2604	38	36
2603	36	36

## CALCULATION OF PUNCHING SHEAR RESISTANCE AT INTERIOR CONTROL PERIMETER

The calculated reinforcement ratios in both directions and the mean ratio are presented in table 6.21

**Table 6.21** Reinforcement ratio for longitudinal reinforcement

Column	$\rho_{l,i,x}$	$\rho_{l,i,y}$	$\rho_{l,int}$
2202, 2204	$3.145 \cdot 10^{-3}$	$3.044 \cdot 10^{-3}$	$3.094 \cdot 10^{-3}$
2203	$3.165 \cdot 10^{-3}$	$3.055 \cdot 10^{-3}$	$3.105 \cdot 10^{-3}$
2205	$3.134 \cdot 10^{-3}$	$3.034 \cdot 10^{-3}$	$3.084 \cdot 10^{-3}$
2206	$3.232 \cdot 10^{-3}$	$3.128 \cdot 10^{-3}$	$3.179 \cdot 10^{-3}$
2207	$3.221 \cdot 10^{-3}$	$3.117 \cdot 10^{-3}$	$3.169 \cdot 10^{-3}$
2302	$2.798 \cdot 10^{-3}$	$2.491 \cdot 10^{-3}$	$2.64 \cdot 10^{-3}$
2303, 2307	$2.818 \cdot 10^{-3}$	$2.509 \cdot 10^{-3}$	$2.659 \cdot 10^{-3}$
2306	$2.813 \cdot 10^{-3}$	$2.505 \cdot 10^{-3}$	$2.655 \cdot 10^{-3}$
2402	$2.251 \cdot 10^{-3}$	$1.666 \cdot 10^{-3}$	$1.937 \cdot 10^{-3}$
2403, 2407	$2.267 \cdot 10^{-3}$	$1.678 \cdot 10^{-3}$	$1.95 \cdot 10^{-3}$
2406	$2.264 \cdot 10^{-3}$	$1.676 \cdot 10^{-3}$	$1.948 \cdot 10^{-3}$
2502	$3.133 \cdot 10^{-3}$	$2.383 \cdot 10^{-3}$	$2.733 \cdot 10^{-3}$
2503	$2.818 \cdot 10^{-3}$	$2.400 \cdot 10^{-3}$	$2.600 \cdot 10^{-3}$
2506	$2.251 \cdot 10^{-3}$	$1.634 \cdot 10^{-3}$	$1.918 \cdot 10^{-3}$
2507	$2.705 \cdot 10^{-3}$	$2.400 \cdot 10^{-3}$	$2.548 \cdot 10^{-3}$
2602	$4.176 \cdot 10^{-3}$	$3.829 \cdot 10^{-3}$	$3.999 \cdot 10^{-3}$
2603	$3.973 \cdot 10^{-3}$	$3.845 \cdot 10^{-3}$	$3.908 \cdot 10^{-3}$
2604	$4.181 \cdot 10^{-3}$	$3.834 \cdot 10^{-3}$	$4.004 \cdot 10^{-3}$

The shear gradient enhancement factor was  
 $k_{b,int} = 2.137 > 1$

Table 6.22 shows the punching shear resistances at the interior control perimeter

**Table 6.22** Design punching shear resistance

Column	$\tau_{Rd,c,int} \leq \frac{0.6}{\gamma_c} \cdot \sqrt{f_{ck}} \text{ [Pa]}$	$\frac{0.6}{\gamma_c} \cdot \sqrt{f_{ck}} \text{ [MPa]}$	$V_{Rd,c,int} \text{ [MN]}$
2202	$7.814 \cdot 10^5$	2.263	1.483
2203	$7.833 \cdot 10^5$	2.263	1.486
2204	$7.815 \cdot 10^5$	2.263	1.483
2205	$7.797 \cdot 10^5$	2.263	1.479
2206	$7.868 \cdot 10^5$	2.263	1.493
2207	$7.85 \cdot 10^5$	2.263	1.49
2302	$7.49 \cdot 10^5$	2.263	1.421
2303	$7.525 \cdot 10^5$	2.263	1.428
2306	$7.518 \cdot 10^5$	2.263	1.426
2307	$7.525 \cdot 10^5$	2.263	1.428
2402	$7.145 \cdot 10^5$	2.263	1.356
2403, 2407	$7.178 \cdot 10^5$	2.263	1.362
2406	$7.171 \cdot 10^5$	2.263	1.361
2502	$7.576 \cdot 10^5$	2.263	1.438
2503	$7.47 \cdot 10^5$	2.263	1.417
2506	$6.745 \cdot 10^5$	2.263	1.28
2507	$7.419 \cdot 10^5$	2.263	1.408
2602	$8.55 \cdot 10^5$	2.263	1.622
2603	$8.497 \cdot 10^5$	2.263	1.612
2604	$8.557 \cdot 10^5$	2.263	1.624

#### CALCULATION OF PUNCHING SHEAR RESISTANCE AT THE EXTERIOR CONTROL PERIMETER

The calculated reinforcement ratios in both directions and the mean ratio are presented in table 6.23.

**Table 6.23** Reinforcement ratio for longitudinal reinforcement

Column	$\rho_{l,e,x}$	$\rho_{l,e,y}$	$\rho_{l,ext}$
2202, 2204	$4.907 \cdot 10^{-3}$	$4.665 \cdot 10^{-3}$	$4.785 \cdot 10^{-3}$
2203	$4.925 \cdot 10^{-3}$	$4.682 \cdot 10^{-3}$	$4.802 \cdot 10^{-3}$
2205	$4.891 \cdot 10^{-3}$	$4.650 \cdot 10^{-3}$	$4.769 \cdot 10^{-3}$
2206	$5.042 \cdot 10^{-3}$	$4.794 \cdot 10^{-3}$	$4.917 \cdot 10^{-3}$
2207	$5.025 \cdot 10^{-3}$	$4.778 \cdot 10^{-3}$	$4.900 \cdot 10^{-3}$
2302	$4.365 \cdot 10^{-3}$	$3.819 \cdot 10^{-3}$	$4.083 \cdot 10^{-3}$
2303, 2307	$4.397 \cdot 10^{-3}$	$3.846 \cdot 10^{-3}$	$4.112 \cdot 10^{-3}$
2306	$4.390 \cdot 10^{-3}$	$3.840 \cdot 10^{-3}$	$4.106 \cdot 10^{-3}$
2402	$3.512 \cdot 10^{-3}$	$2.554 \cdot 10^{-3}$	$2.995 \cdot 10^{-3}$
2403, 2407	$3.537 \cdot 10^{-3}$	$2.572 \cdot 10^{-3}$	$3.016 \cdot 10^{-3}$
2406	$3.532 \cdot 10^{-3}$	$2.568 \cdot 10^{-3}$	$3.012 \cdot 10^{-3}$
2502	$4.889 \cdot 10^{-3}$	$3.653 \cdot 10^{-3}$	$4.226 \cdot 10^{-3}$
2503	$4.397 \cdot 10^{-3}$	$3.679 \cdot 10^{-3}$	$4.022 \cdot 10^{-3}$
2506	$3.512 \cdot 10^{-3}$	$2.504 \cdot 10^{-3}$	$2.966 \cdot 10^{-3}$
2507	$4.221 \cdot 10^{-3}$	$3.679 \cdot 10^{-3}$	$3.940 \cdot 10^{-3}$
2602	$5.77 \cdot 10^{-3}$	$5.227 \cdot 10^{-3}$	$5.492 \cdot 10^{-3}$
2603	$5.489 \cdot 10^{-3}$	$5.249 \cdot 10^{-3}$	$5.368 \cdot 10^{-3}$
2604	$5.777 \cdot 10^{-3}$	$5.233 \cdot 10^{-3}$	$5.499 \cdot 10^{-3}$

The shear gradient enhancement factor is presented in table 6.24. The shear gradient enhance factor was effected by the reduced control perimeter.

**Table 6.24** Shear gradient enhancement factor

Column	$k_{b,ext} \geq 1$	$k_{b,ext,red} \geq 1$
2202-2507	1.016	2.056
2602-2604	1.075	2.056

Table 6.25 shows the punching shear resistances at the exterior control perimeter.

**Table 6.25** Design punching shear resistance

Column	$\tau_{Rd,c,ext}$ [Pa]	$\tau_{Rd,c,ext,red}$ [Pa]	$\frac{0.6}{\gamma_c} \cdot \sqrt{f_{ck}}$ [MPa]	$V_{Rd,c,ext}$ [MN]	$V_{Rd,c,ext,red}$ [MN]
2202	$4.297 \cdot 10^5$	$8.692 \cdot 10^5$	2.263	1.505	0.7455
2203	$4.307 \cdot 10^5$	$8.713 \cdot 10^5$	2.263	1.512	0.7473
2204	$4.297 \cdot 10^5$	$8.693 \cdot 10^5$	2.263	1.508	0.7456
2205	$4.287 \cdot 10^5$	$8.673 \cdot 10^5$	2.263	1.505	0.7439
2206	$4.326 \cdot 10^5$	$8.752 \cdot 10^5$	2.263	1.519	0.7507
2207	$4.317 \cdot 10^5$	$8.733 \cdot 10^5$	2.263	1.515	0.749
2302	$4.119 \cdot 10^5$	$8.332 \cdot 10^5$	2.263	1.446	0.7146
2303	$4.138 \cdot 10^5$	$8.371 \cdot 10^5$	2.263	1.452	0.718
2306	$4.134 \cdot 10^5$	$8.363 \cdot 10^5$	2.263	1.451	0.7173
2307	$4.138 \cdot 10^5$	$8.371 \cdot 10^5$	2.263	1.452	0.718
2402	$3.929 \cdot 10^5$	$7.948 \cdot 10^5$	2.263	1.379	0.6817
2403, 2407	$3.947 \cdot 10^5$	$7.985 \cdot 10^5$	2.263	1.386	0.6849
2406	$3.943 \cdot 10^5$	$7.977 \cdot 10^5$	2.263	1.384	0.6842
2502	$4.166 \cdot 10^5$	$8.428 \cdot 10^5$	2.263	1.462	0.7229
2503	$4.108 \cdot 10^5$	$8.31 \cdot 10^5$	2.263	1.442	0.7127
2506	$3.709 \cdot 10^5$	$7.504 \cdot 10^5$	2.263	1.302	0.6436
2507	$4.08 \cdot 10^5$	$8.253 \cdot 10^5$	2.263	1.432	0.7079
2602	$4.78 \cdot 10^5$	$9.142 \cdot 10^5$	2.263	1.901	0.9942
2603	$4.75 \cdot 10^5$	$9.085 \cdot 10^5$	2.263	1.89	0.988
2604	$4.784 \cdot 10^5$	$9.149 \cdot 10^5$	2.263	1.903	0.995

The resulting design punching shear resistance is presented in table 6.26

**Table 6.26** The design value that is dimensioning

Column	$V_{Rd,c,int}$ [MN]	$V_{Rd,c,ext}$ [MN]	$V_{Rd,c,rxt,red}$ [MN]
2202	1.483	1.508	0.7455
2203	1.486	1.512	0.7473
2204	1.483	1.508	0.7456
2205	1.479	1.505	0.7439
2206	1.493	1.519	0.7507
2207	1.49	1.515	0.749
2302	1.421	1.446	0.7146
2303, 2307	1.428	1.452	0.718
2306	1.426	1.451	0.7173
2402	1.356	1.379	0.6817
2403, 2407	1.362	1.386	0.6849
2406	1.361	1.384	0.6842
2502	1.438	1.462	0.7229
2503	1.417	1.442	0.7127
2506	1.28	1.302	0.6436
2507	1.408	1.432	0.7079
2602	1.622	1.901	0.9942
2603	1.612	1.89	0.988
2604	1.624	1.903	0.995

### 6.1.3 Differences in Punching Shear Resistance

The differences in punching shear resistance between current and proposed provisions are presented in table 6.27

**Table 6.27** Differences in punching shear resistance, for the inner perimeter, between the proposal and current Eurocode

Column	Differences in punching shear resistance [%]
2202-2205	-3
2206-2207	-4
2302-2307	1
2402	6
2403-2407	5
2502	0
2503, 2507	2
2606	11
2602, 2604	-5
2603	-6



### 6.1.4 Load combination

The design shear force and the average eccentricity factor are presented in table 6.28. Note that the eccentricity factors given are average values from each combination considered. When calculating the design shear force  $\beta$  for each combination was used, not the average.

**Table 6.28** Design shear force and mean eccentricity factor for columns 2202, 2302, 2406, 2506

Column	$V_{Ed}$ , EC2 [kN]	$\beta = 1 + \frac{e_b}{b_b}$ , EC2	$V_{Ed}$ , proposal [kN]	$\beta = 1 + \frac{e_b}{b_b}$ , proposal
2202	1366	1.138	1317	1.127
2302	1029	1.155	1015	1.139
2406	785	1.125	776	1.112
2506	1083	1.154	1068	1.138

The utilization factors with respect to the design shear force and resistance are presented in table 6.29

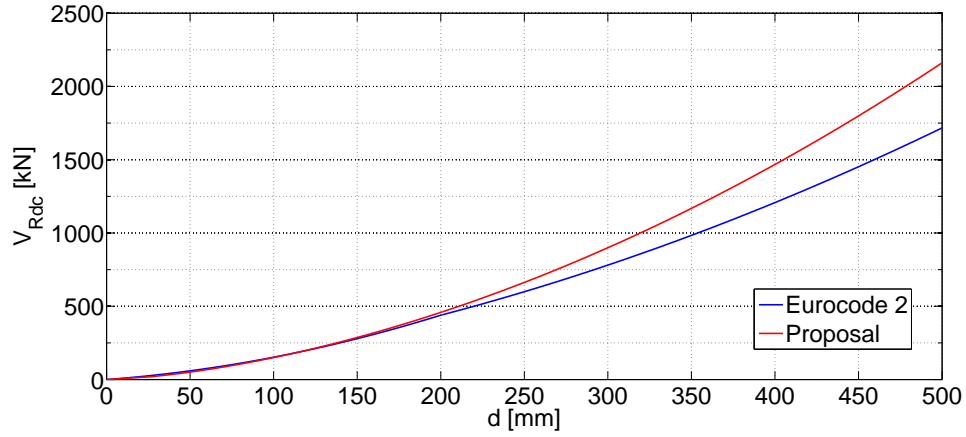
**Table 6.29** Utilization factors for columns 2202, 2302, 2406, 2506

Column	Utilization factor for EC2	Utilization factor for proposal
2202	0.95	0.89
2302	0.72	0.71
2406	0.55	0.57
2506	0.75	0.83

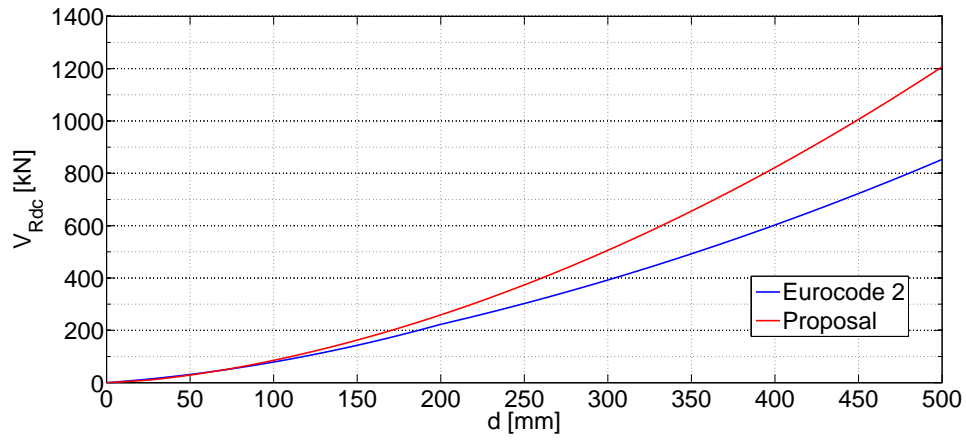
## 6.2 Parametric study

### 6.2.1 Without shear reinforcement

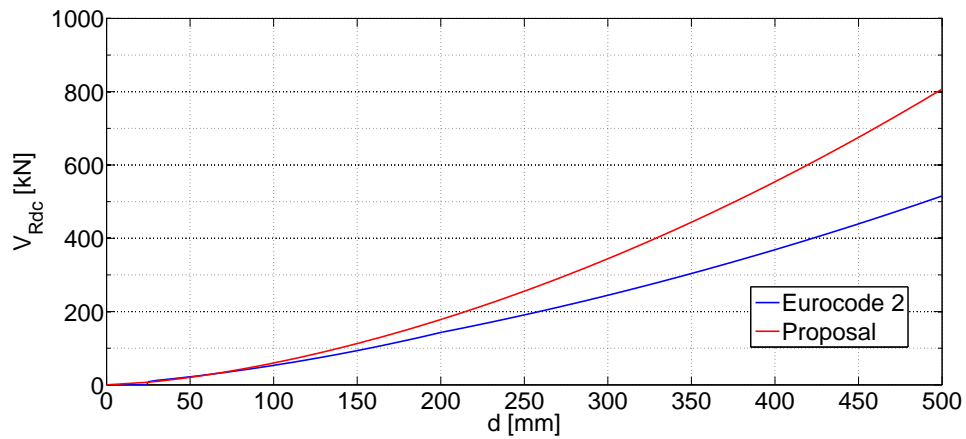
Figures 6.1-6.3 show the design punching shear resistances as function of the effective depth. The difference was within 25% when the depth was below 200 mm. At this point, the resistance according to Eurocode 2 changed inclination. The ratios between the resistances were largest for the corner columns and smallest for the inner column, see figure 6.4. Note that the resistance according to Eurocode 2 for the corner column was set to zero at very small depths. This is because the minimum resistance exceeded the maximum resistance. Consequently the resistance was invalid for this interval.



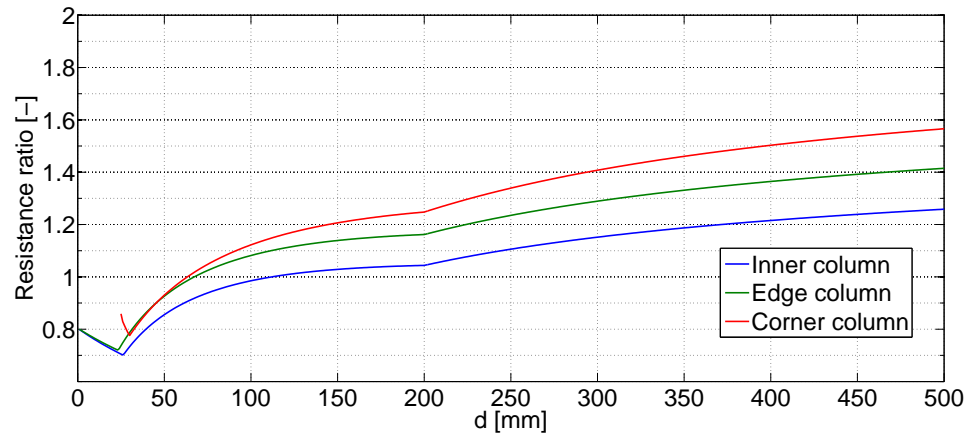
**Figure 6.1** Punching shear resistance for inner column as a function of the effective depth.



**Figure 6.2** Punching shear resistance for edge column as a function of the effective depth.

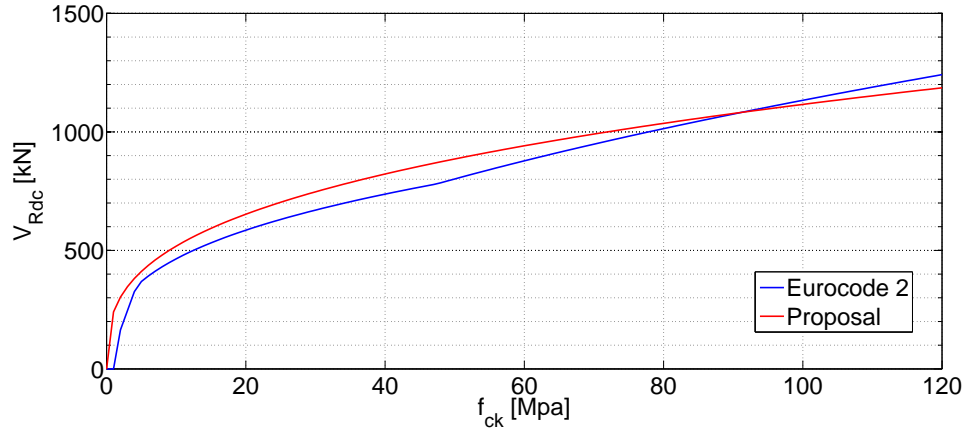


**Figure 6.3** Punching shear resistance for corner column as a function of the effective depth.

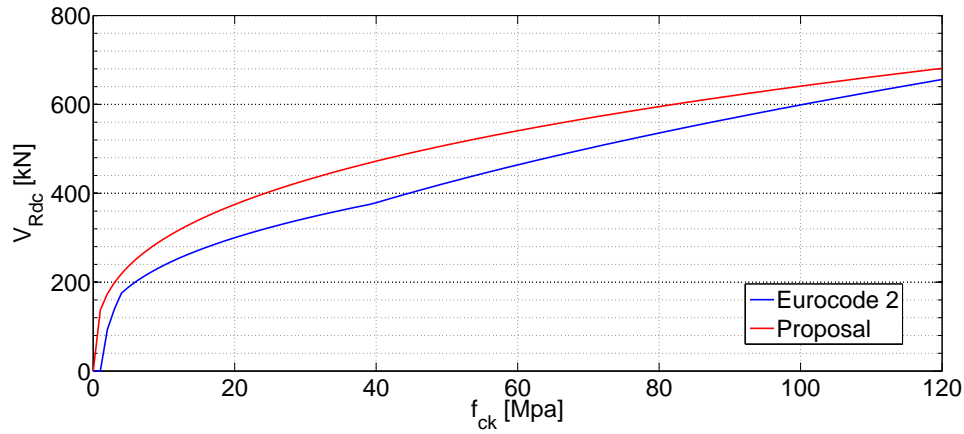


**Figure 6.4** Punching shear resistance ratio between the proposal and Eurocode 2 as a function of the effective depth.

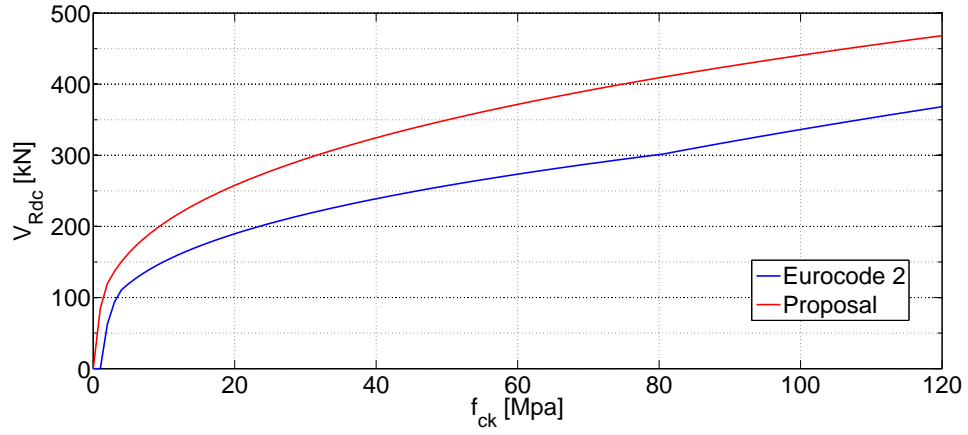
The design punching shear resistances as function of the characteristic compressive strength are presented in figures 6.5-6.7. The resistance according to Eurocode 2 changed inclination at one point for each column type. From this point the minimum punching shear resistance was governing. The ratios are shown in figure 6.8 and the difference was largest for the corner column and smallest for the inner column.



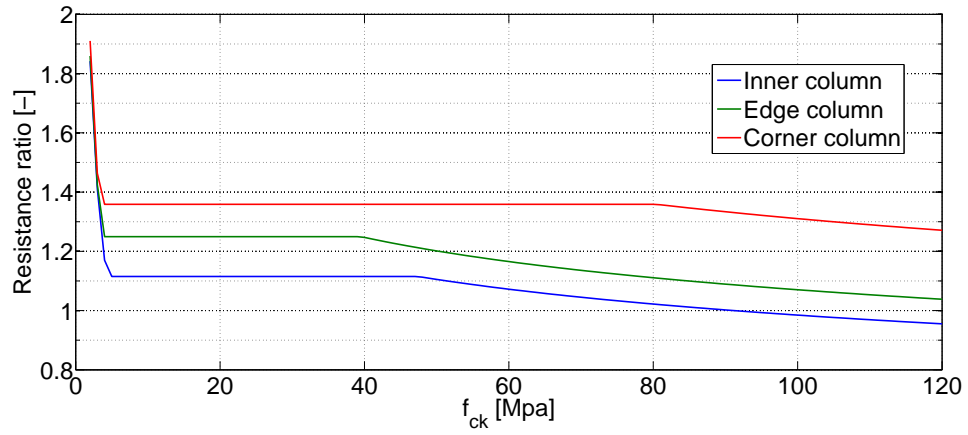
**Figure 6.5** Punching shear resistance for inner column as a function of the characteristic compressive cylinder strength.



**Figure 6.6** Punching shear resistance for edge column as a function of the characteristic compressive cylinder strength.

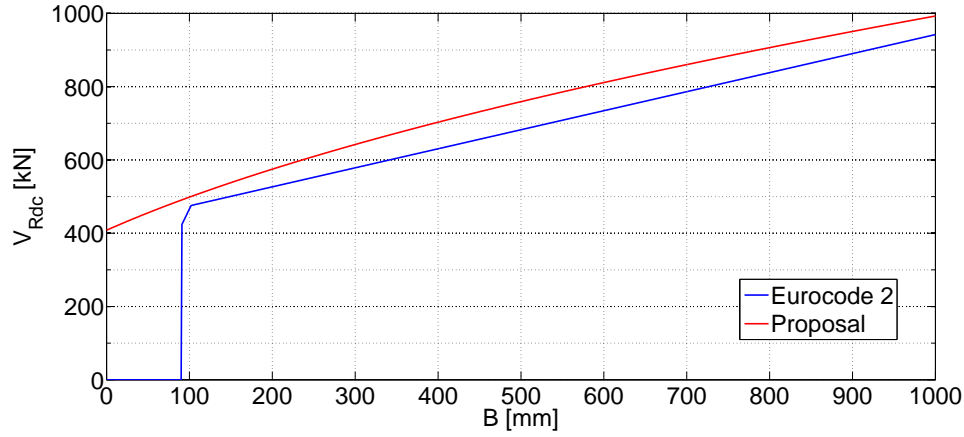


**Figure 6.7** Punching shear resistance for corner column as a function of the characteristic compressive cylinder strength.

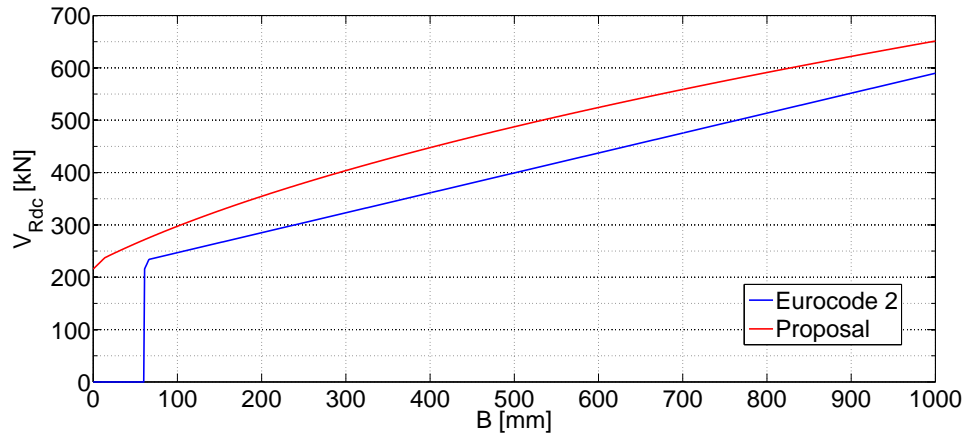


**Figure 6.8** Punching shear resistance ratio between the proposal and Eurocode 2 as a function of the characteristic compressive cylinder strength.

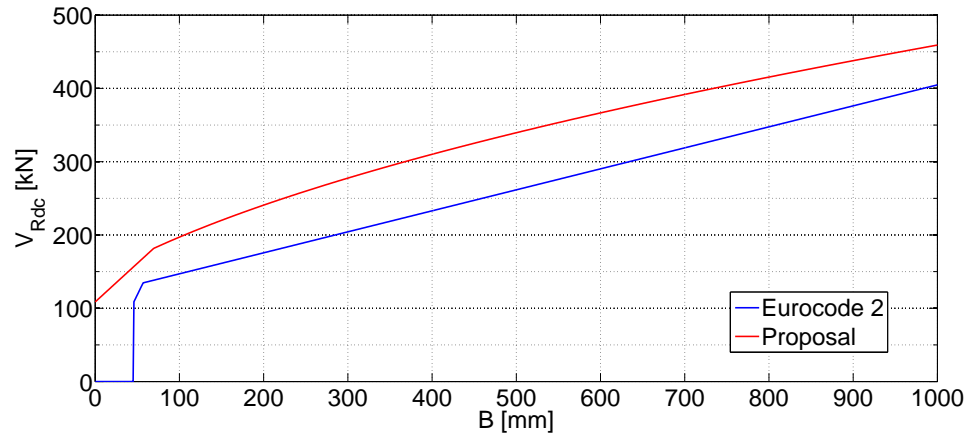
The punching shear resistances as function of the column width are shown in figures 6.9-6.11. At small widths, the resistance was invalid due to the minimum resistance exceeding the maximum resistance. This interval was set to zero. The ratios between the resistances are presented in figure 6.12. The inner column showed the smallest difference contrary to the corner column.



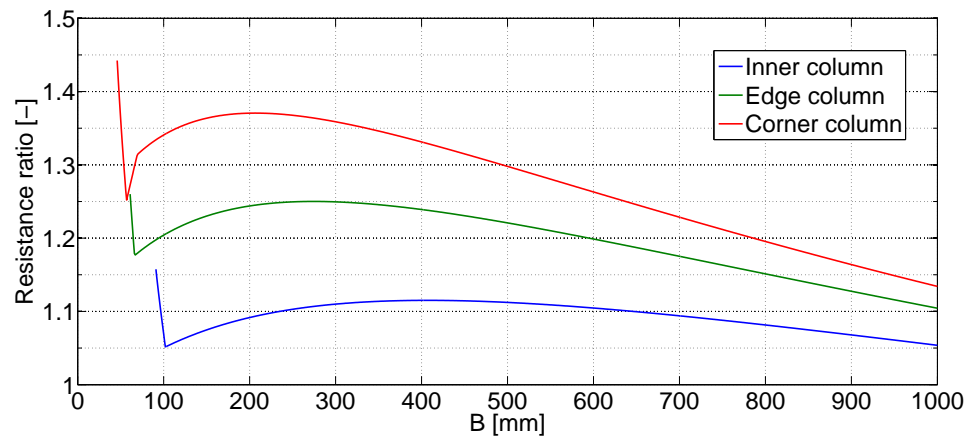
**Figure 6.9** Punching shear resistance for inner column as a function of the column width.



**Figure 6.10** Punching shear resistance for edge column as a function of the column width.

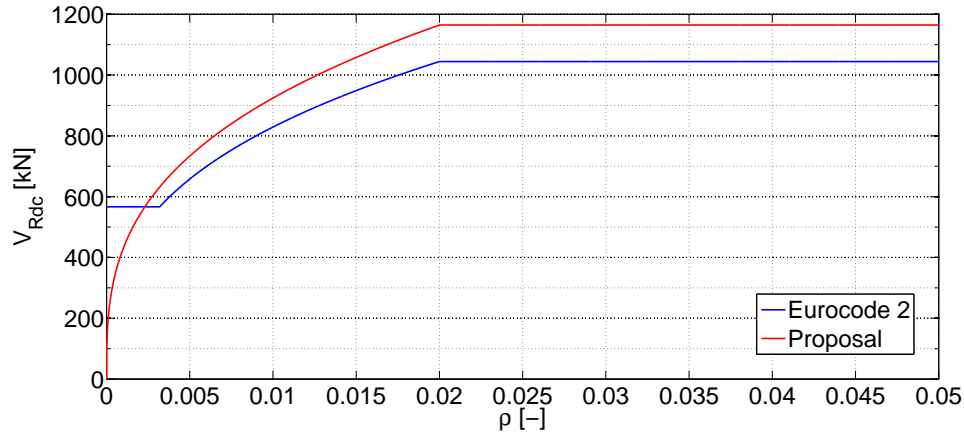


**Figure 6.11** Punching shear resistance for corner column as a function of the column width.

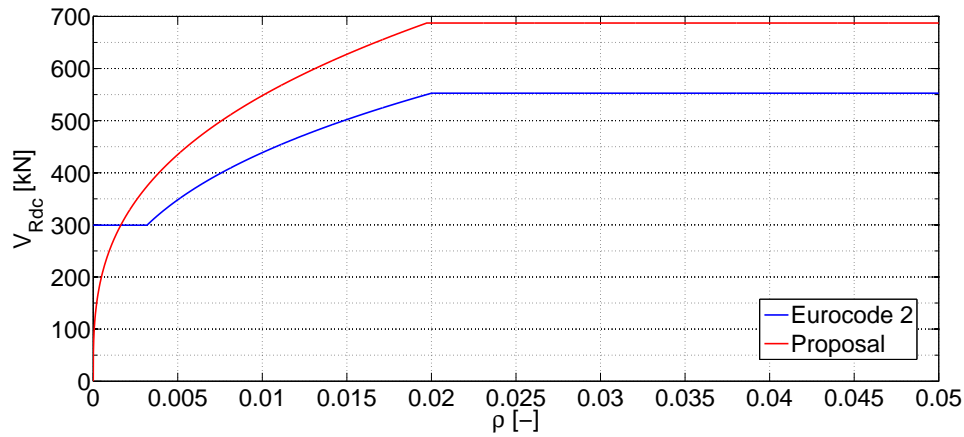


**Figure 6.12** Punching shear resistance ratio between the proposal and Eurocode 2 as a function of the column width.

Figures 6.13-6.15 present the punching shear resistance as function of the flexural reinforcement ratio. At low ratios, the resistance for Eurocode 2 was limited by the minimum resistance. Ratios larger than 2% do not contribute to larger resistance according to Eurocode 2. The resistance according to the proposal was limited by the maximum punching shear resistance which happened to be at 2%. The ratios between the resistances are shown in figure 6.16 where the inner column showed the smallest difference.

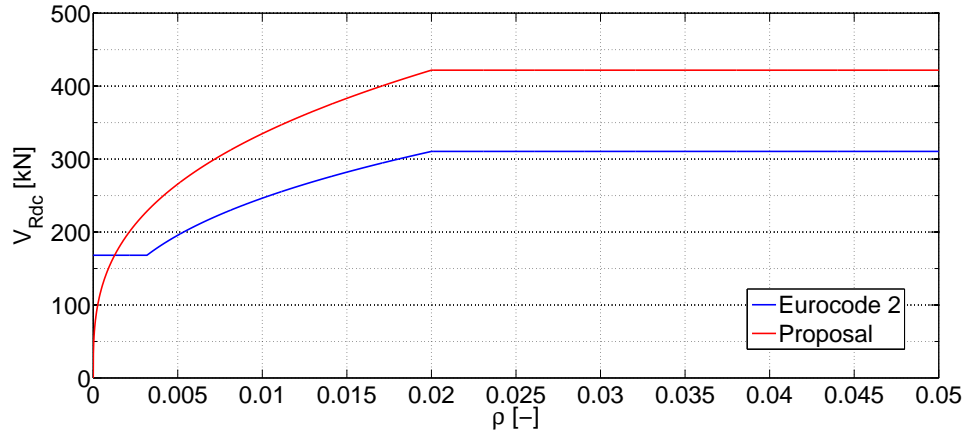


**Figure 6.13** Punching shear resistance for inner column as a function of the flexural reinforcement ratio.

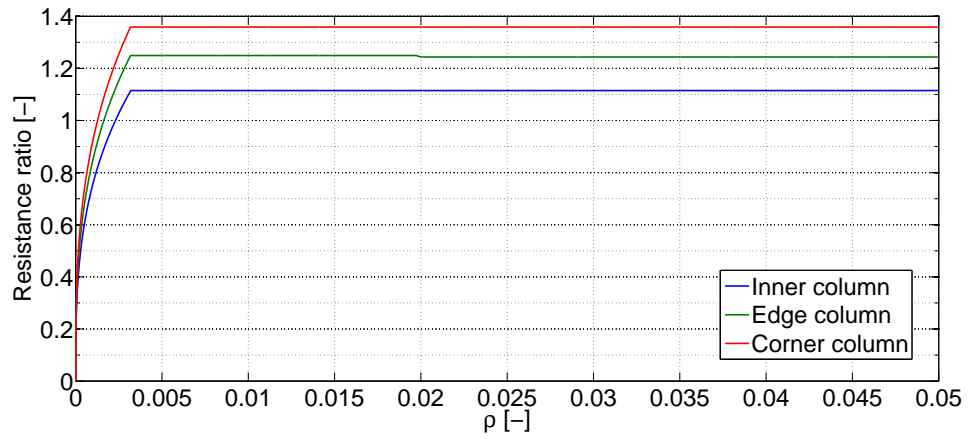


**Figure 6.14** Punching shear resistance for edge column as a function of the flexural reinforcement ratio.



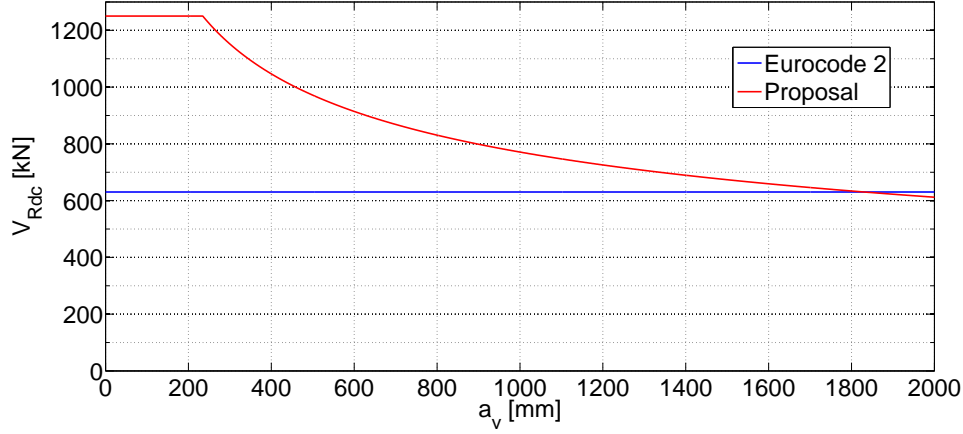


**Figure 6.15** Punching shear resistance for corner column as a function of the flexural reinforcement ratio.

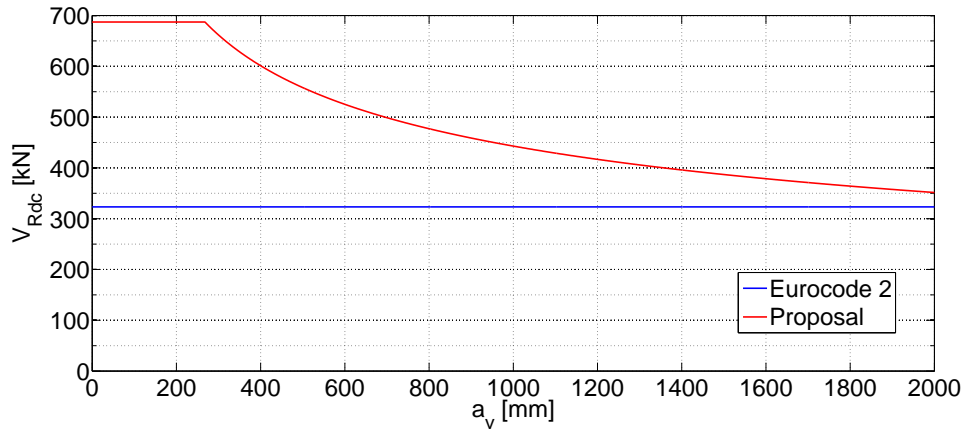


**Figure 6.16** Punching shear resistance ratio between the proposal and Eurocode 2 as a function of the flexural reinforcement ratio.

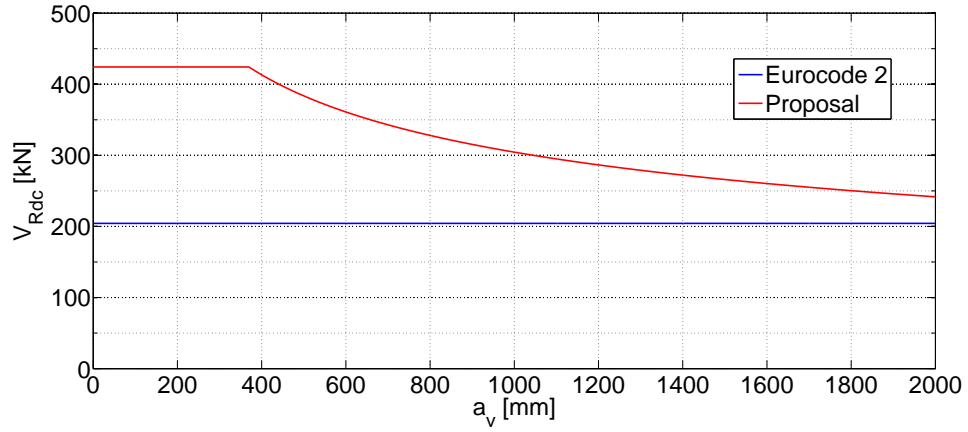
The resistance as function of the distance between the support axis and the contra flexural location are presented in figures 6.17-6.19. At small distances, the resistance according to the proposal was limited by the maximum resistance. Eurocode 2 was unaffected by the distance. Figure 6.20 shows the ratios between the resistances. The differences decreased at larger distances.



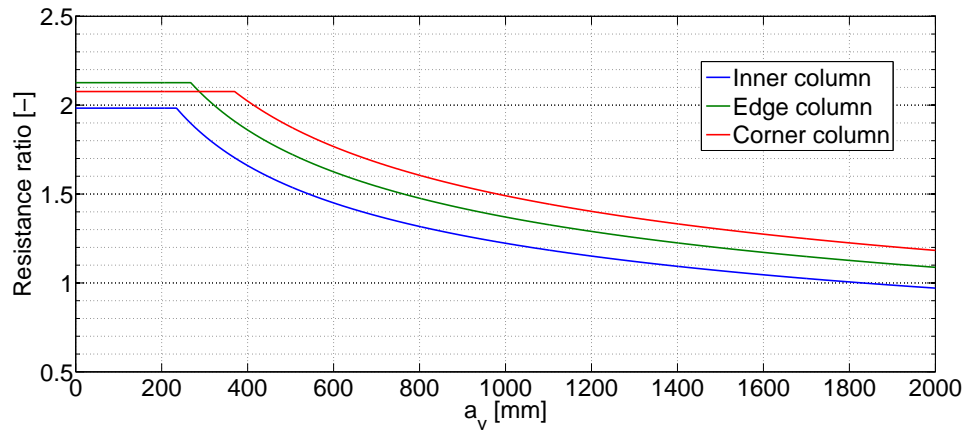
**Figure 6.17** Punching shear resistance for inner column as a function of the distance to the contra flexural location.



**Figure 6.18** Punching shear resistance for edge column as a function of the distance to the contra flexural location.

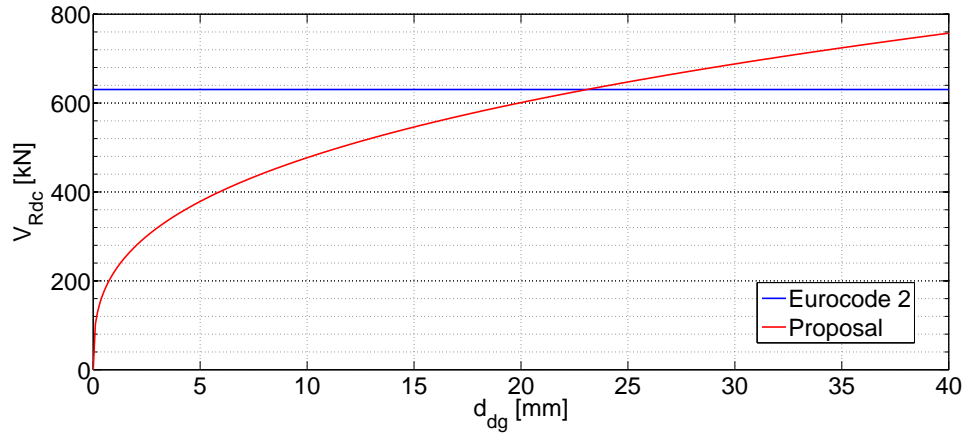


**Figure 6.19** Punching shear resistance for corner column as a function of the distance to the contra flexural location.

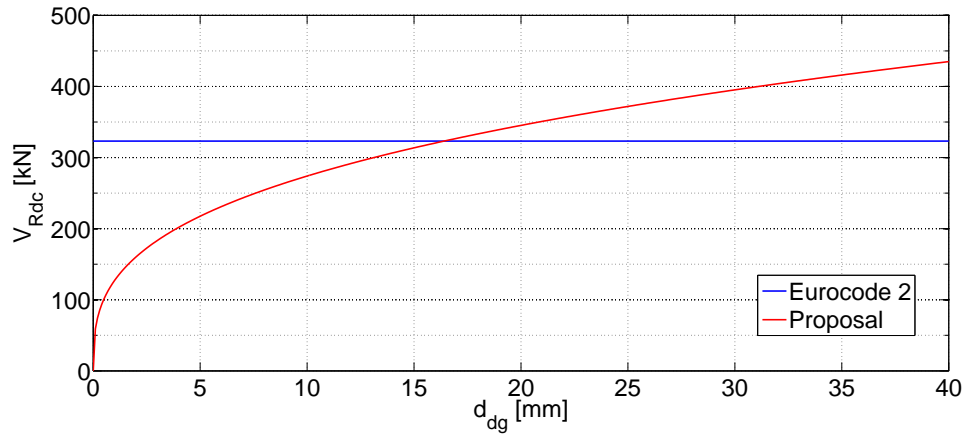


**Figure 6.20** Punching shear resistance ratio between the proposal and Eurocode 2 as a function of the distance to the contra flexural location.

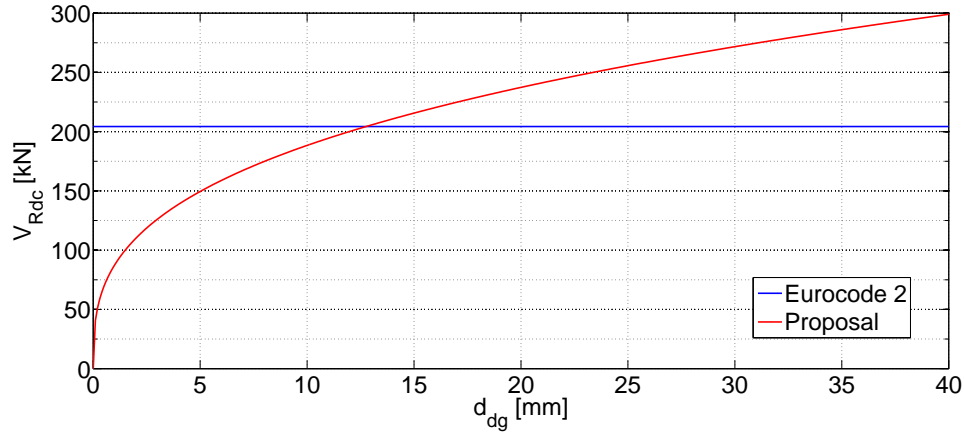
The resistance as function of coefficient  $d_{dg}$  are presented in figures 6.21-6.23. The resistance according to Eurocode 2 was unaffected, while the resistance according to the proposal increased with larger  $d_{dg}$ . Figure 6.24 shows the resistance ratios.



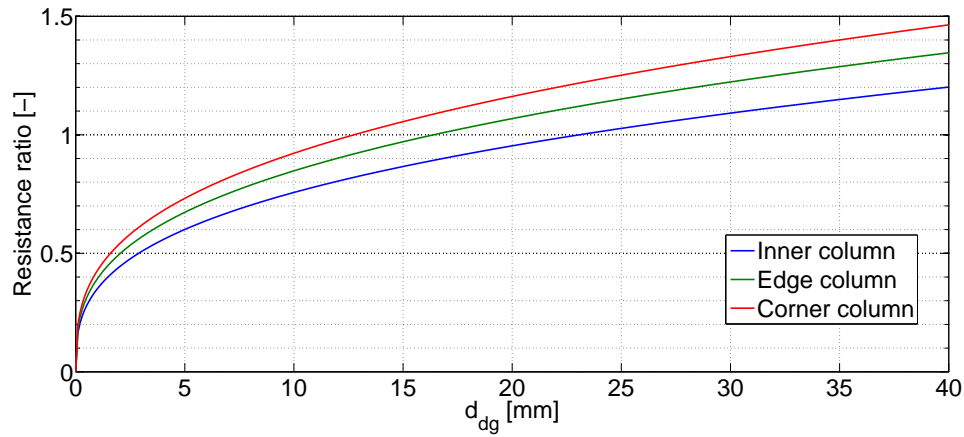
**Figure 6.21** Punching shear resistance for inner column as a function of the coefficient taking account of concrete type and its aggregate properties.



**Figure 6.22** Punching shear resistance for edge column as a function of the coefficient taking account of concrete type and its aggregate properties.



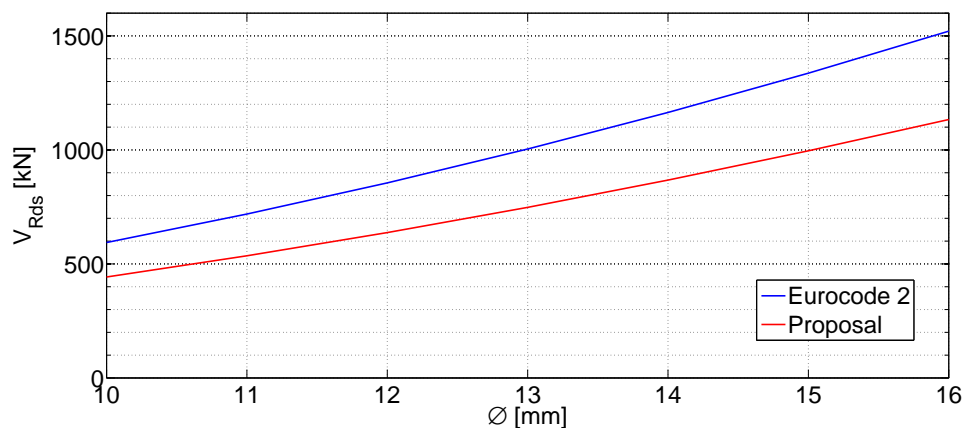
**Figure 6.23** Punching shear resistance for corner column as a function of the coefficient taking account of concrete type and its aggregate properties.



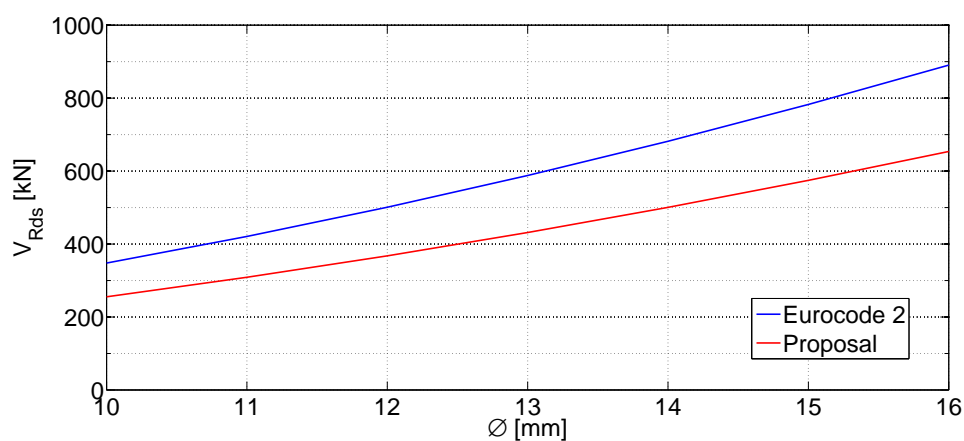
**Figure 6.24** Punching shear resistance ratio between the proposal and Eurocode 2 as a function of the coefficient taking account of concrete type and its aggregate properties.

### 6.2.2 With shear reinforcement

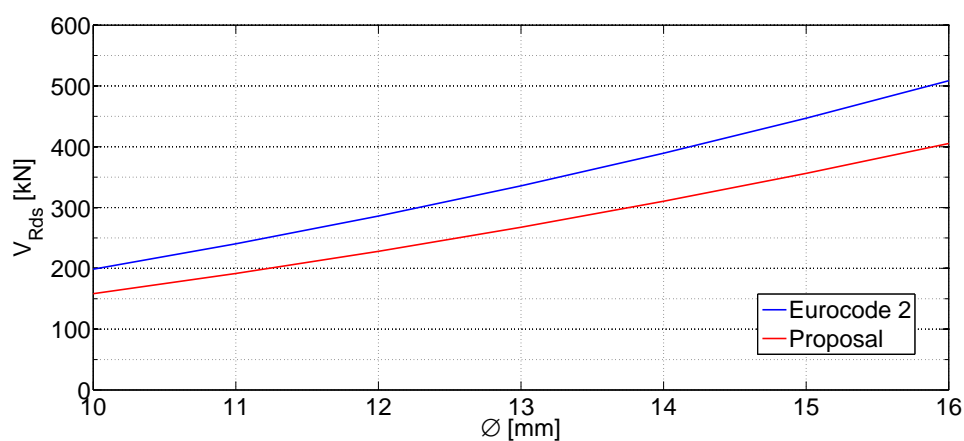
The shear reinforcement contribution as a function of the stud diameter are presented in figures 6.25-6.27. The contribution was larger according to Eurocode 2 and increased with the stud diameter. Figure 6.28 shows the ratios between the shear reinforcement contributions. The ratio was constant for all column types and the difference was lowest for the corner column.



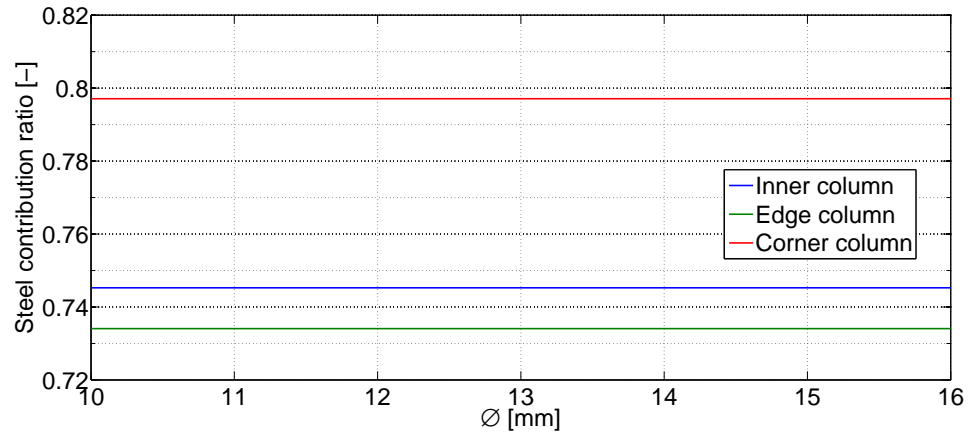
**Figure 6.25** Steel contribution for inner column as a function of the stud diameter.



**Figure 6.26** Steel contribution for edge column as a function of the stud diameter.

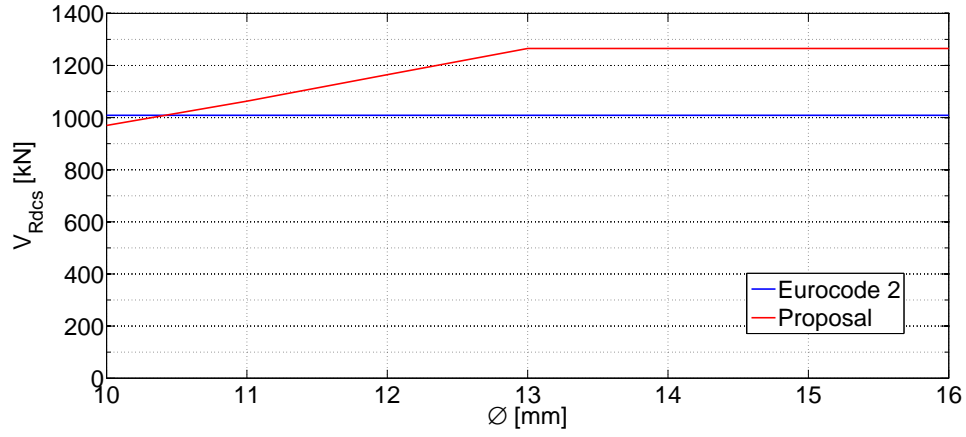


**Figure 6.27** Steel contribution for corner column as a function of the stud diameter.

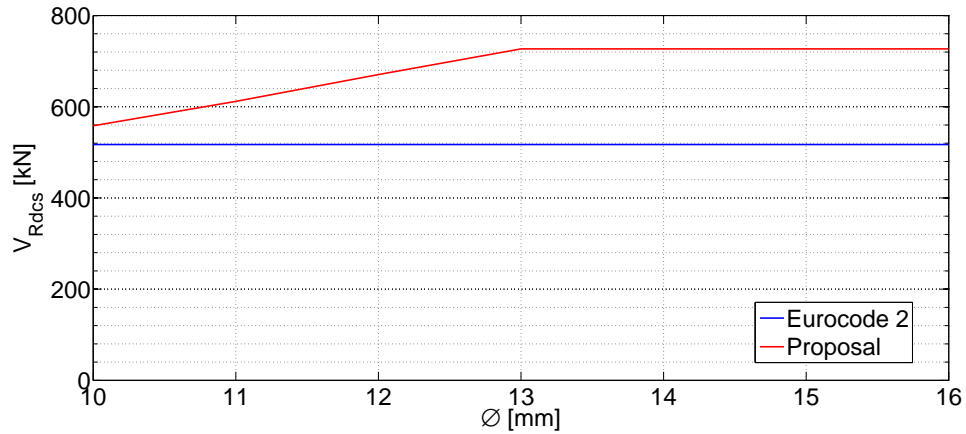


**Figure 6.28** Steel contribution ratio between the proposal and Eurocode 2 as a function of the stud diameter.

Figures 6.29-6.31 present the total punching shear resistance as function of the stud diameter. The resistance according to Eurocode 2 was limited by the maximum resistance over the whole interval. The resistance according to the proposal was generally larger than according to Eurocode 2. The difference between the resistances was smallest for the inner column, see figure 6.32.

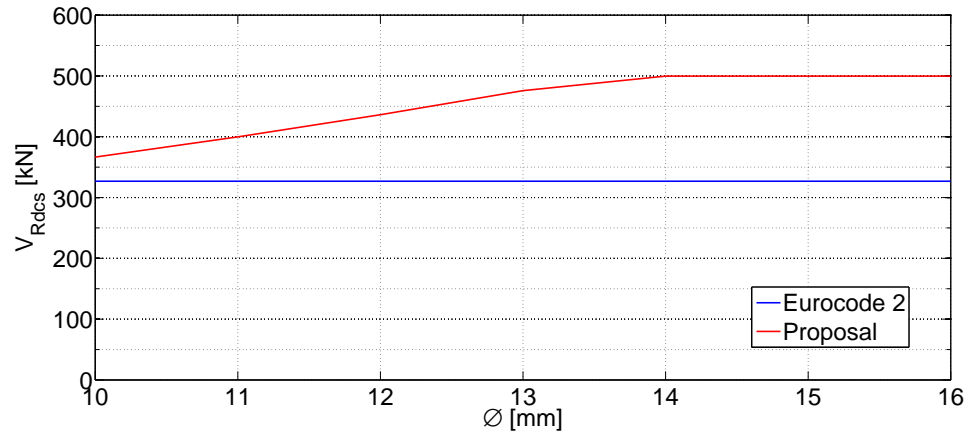


**Figure 6.29** Punching shear resistance for inner column as a function of the stud diameter.

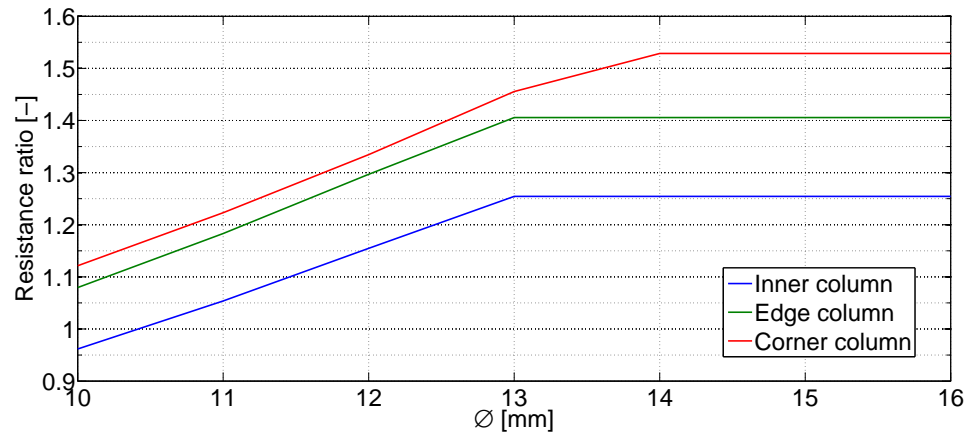


**Figure 6.30** Punching shear resistance for edge column as a function of the stud diameter.



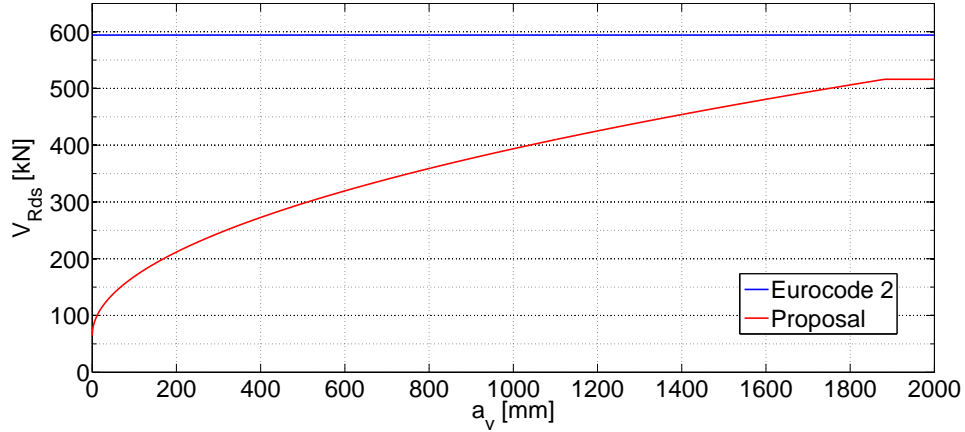


**Figure 6.31** Punching shear resistance for corner column as a function of the stud diameter.

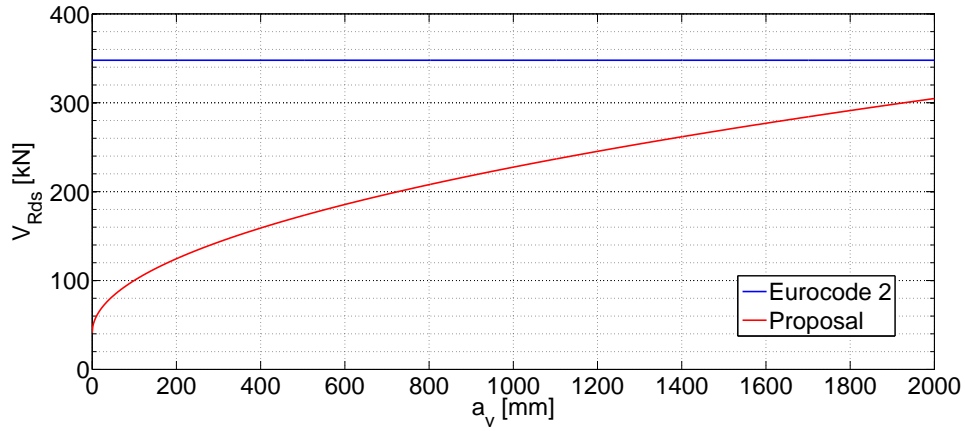


**Figure 6.32** Punching shear resistance ratio between the proposal and Eurocode 2 as a function of the stud diameter.

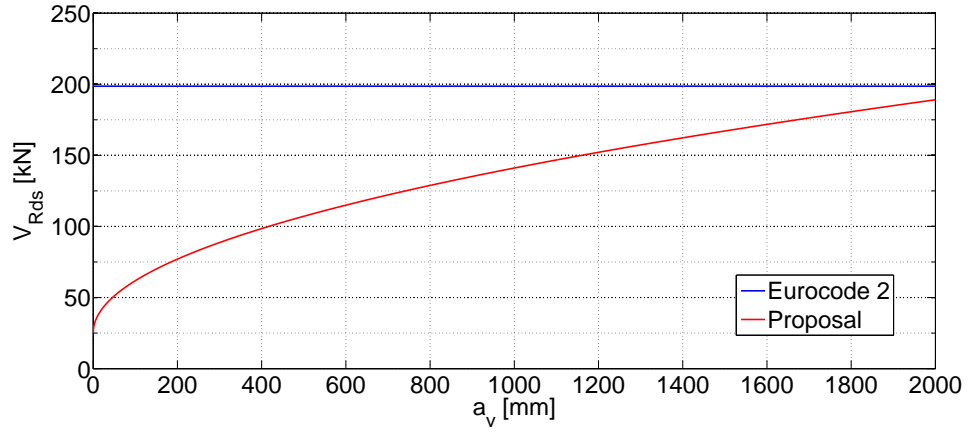
The shear reinforcement contribution as function of the distance between the support axis and the contra flexural location are shown in figures 6.33-6.35. The resistance according to Eurocode 2 was unaffected, while the resistance according to the proposal increased with larger distance. The ratios between the contributions are presented in figure 6.36. The difference was almost the same for the inner and edge columns.



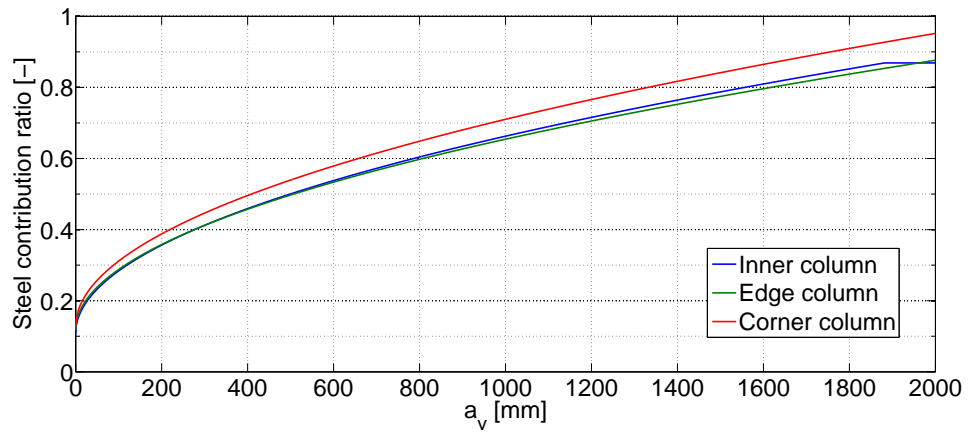
**Figure 6.33** Steel contribution for inner column as a function of the distance to the contra flexural location.



**Figure 6.34** Steel contribution for edge column as a function of the distance to the contra flexural location.

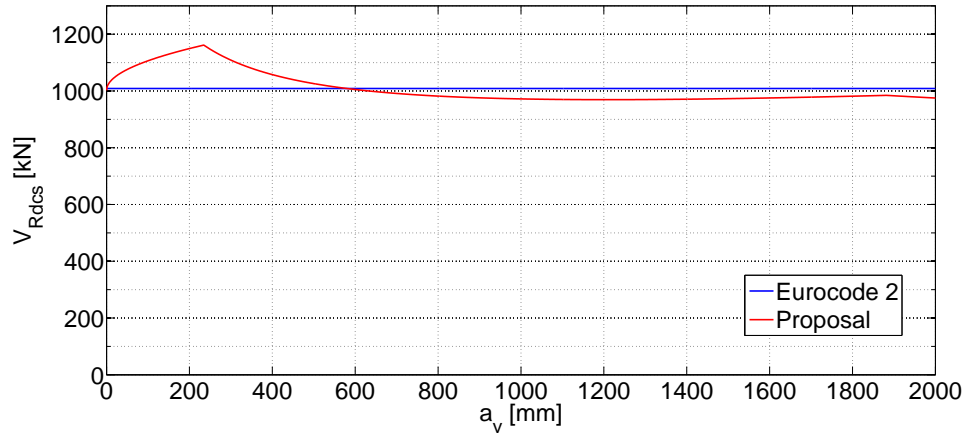


**Figure 6.35** Steel contribution for corner column as a function of the distance to the contra flexural location.

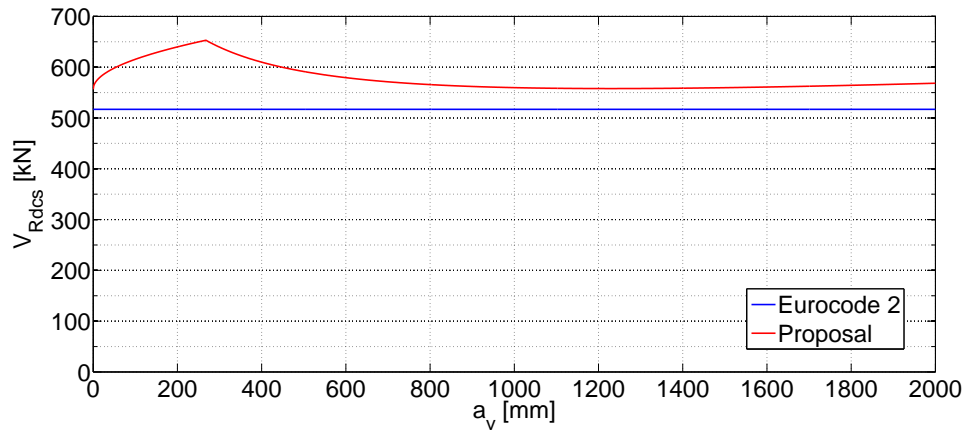


**Figure 6.36** Steel contribution ratio between the proposal and Eurocode 2 as a function of the distance to the contra flexural location.

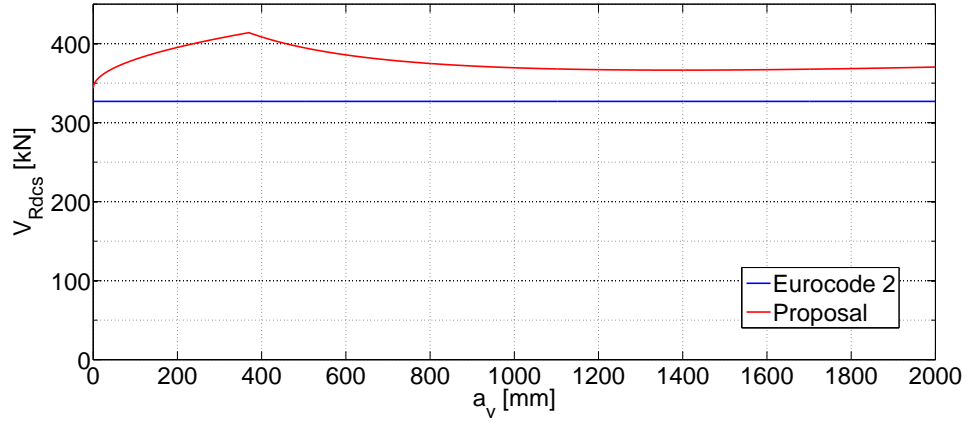
Figures 6.37-6.39 show the total punching shear resistances as function of the distance to the contra flexural location. The resistance according to the proposal peaked at a distance between 200-400 mm, then it decreased and flattened out. The resistance according to Eurocode 2 was unaffected by the distance and limited by the maximum resistance over the entire interval. The ratios between the resistances are presented in figure 6.40. The differences were almost constant at distances above 1000 mm.



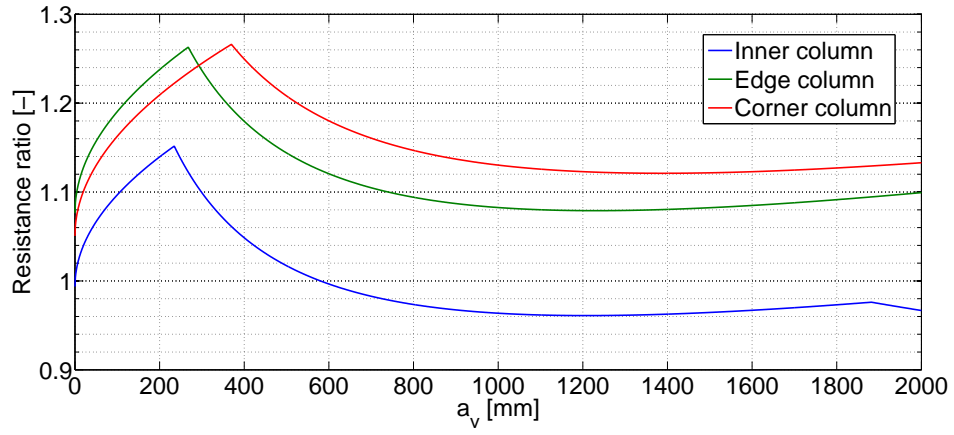
**Figure 6.37** Punching shear resistance for inner column as a function of the distance to the contra flexural location.



**Figure 6.38** Punching shear resistance for edge column as a function of the distance to the contra flexural location.

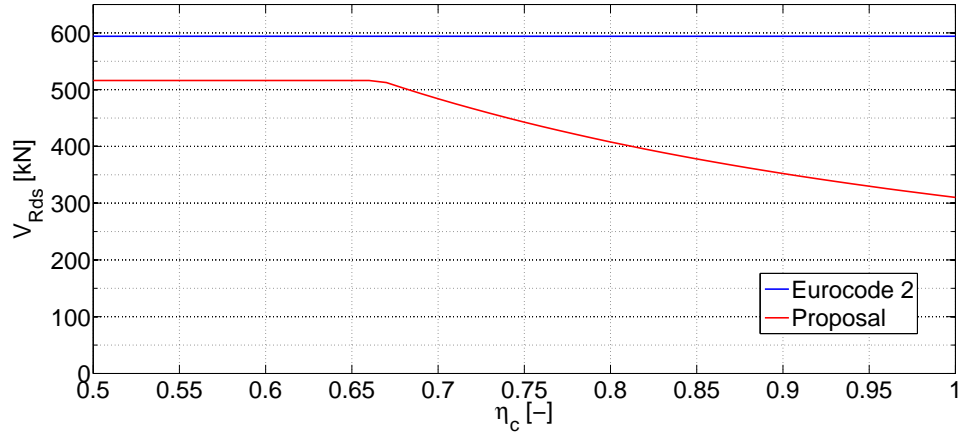


**Figure 6.39** Punching shear resistance for corner column as a function of the distance to the contra flexural location.

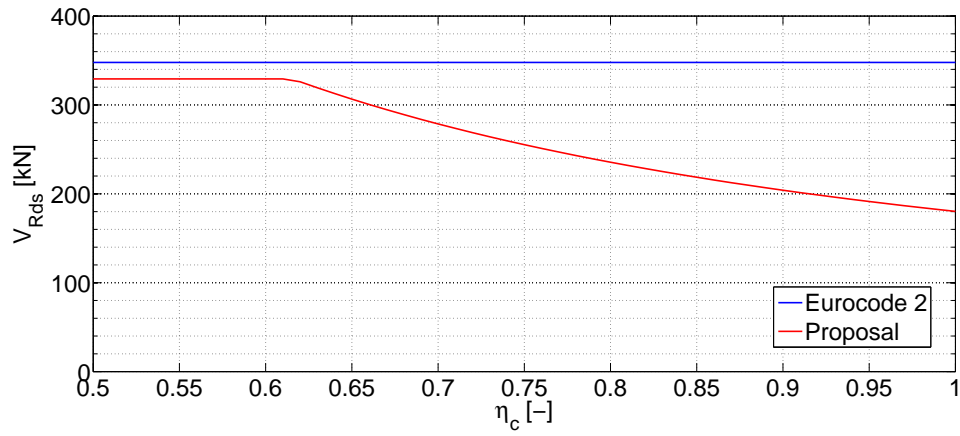


**Figure 6.40** Punching shear resistance ratio between the proposal and Eurocode 2 as a function of the distance to the contra flexural location.

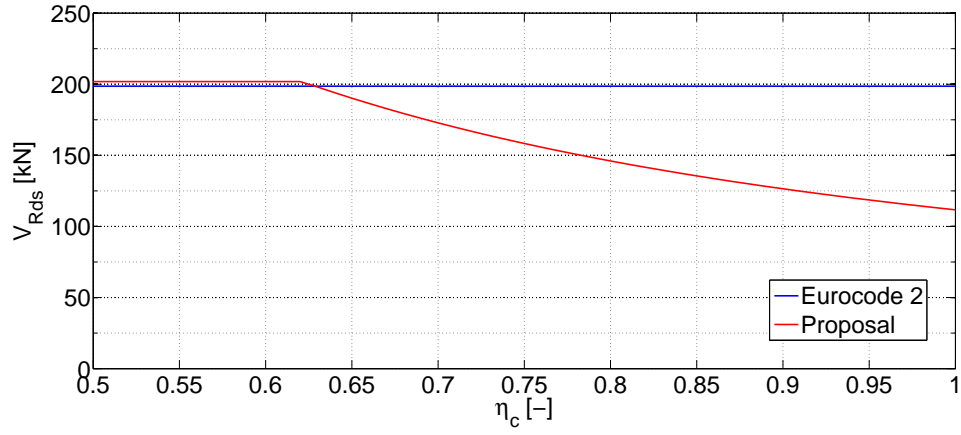
The shear reinforcement contributions as function of the concrete contribution factor are presented in figures 6.41-6.43. The contribution according to Eurocode 2 was unaffected. The contribution according to the proposal was limited at smaller factors. It decreased with larger concrete contribution factors. The ratios between the contributions are shown in figure 6.44 and the difference was constant for factors up until 63%.



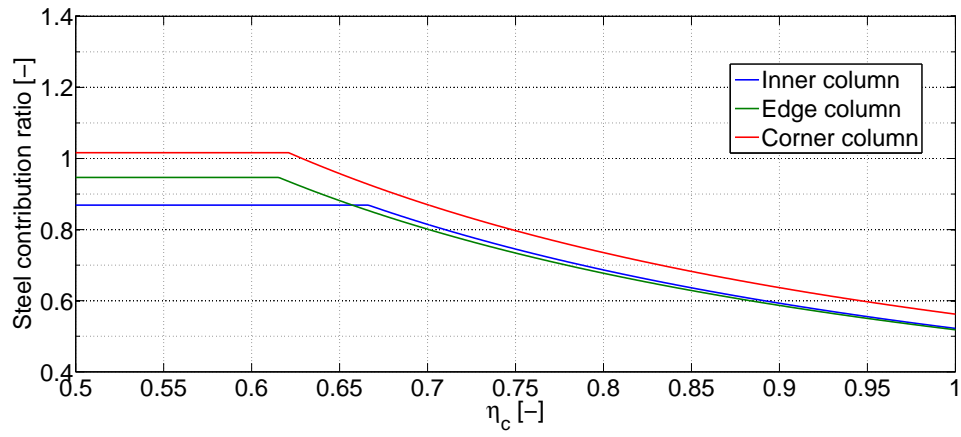
**Figure 6.41** Steel contribution for inner column as a function of the concrete contribution factor.



**Figure 6.42** Steel contribution for edge column as a function of the distance to the concrete contribution factor.

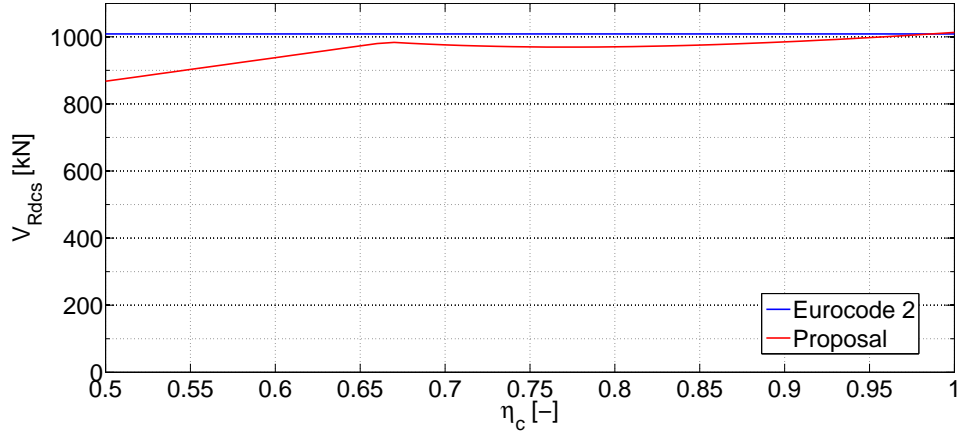


**Figure 6.43** Steel contribution for corner column as a function of the distance to the concrete contribution factor.

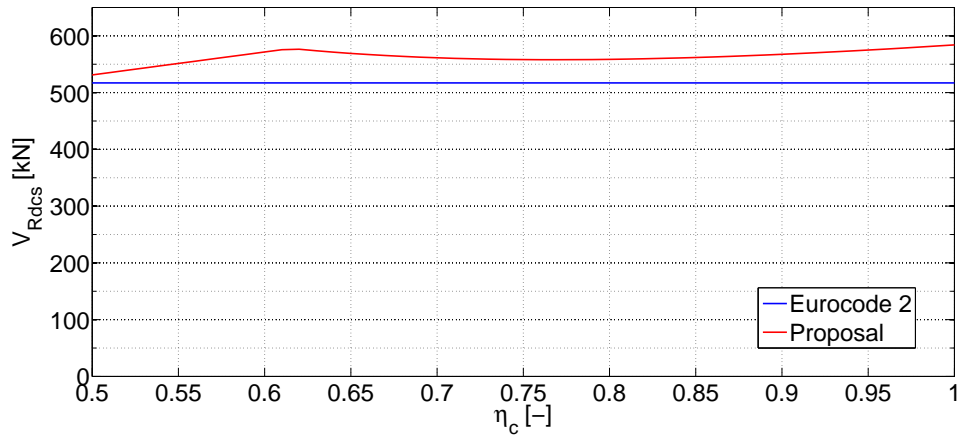


**Figure 6.44** Steel contribution ratio between the proposal and Eurocode 2 as a function of the concrete contribution factor.

Figures 6.45-6.47 show the total punching shear resistances as function of the concrete contribution factor. Eurocode 2 was unaffected by this factor and limited by the maximum resistance. The resistance according to the proposal increased at first, but flattened out. Figure 6.48 shows the ratios between the resistances. The difference was mostly smaller for the inner column than for the edge and corner columns.

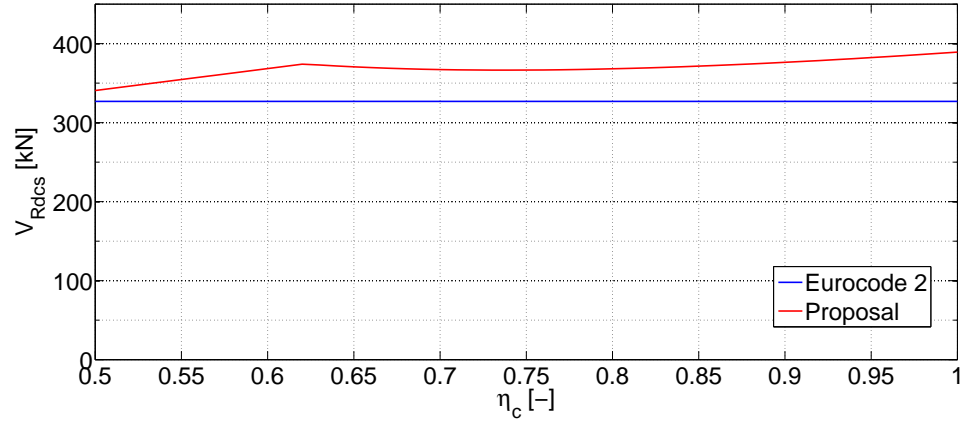


**Figure 6.45** Punching shear resistance for inner column as a function of the concrete contribution factor.

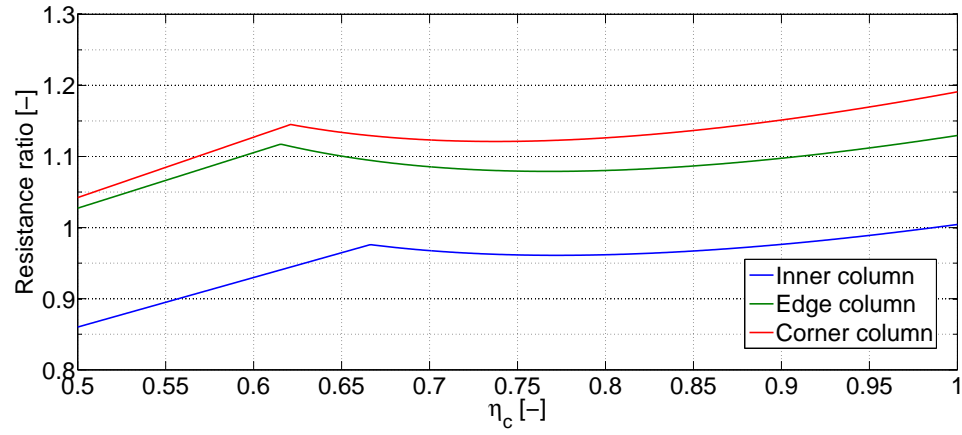


**Figure 6.46** Punching shear resistance for edge column as a function of the concrete contribution factor.



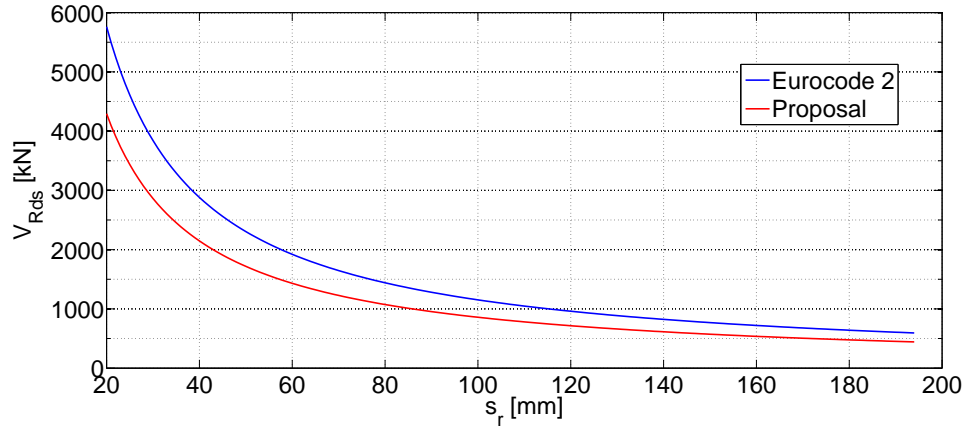


**Figure 6.47** Punching shear resistance for corner column as a function of the concrete contribution factor.

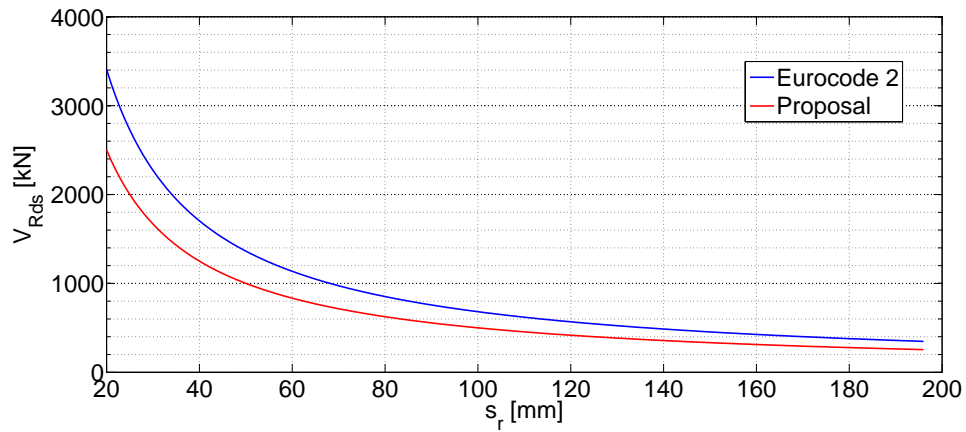


**Figure 6.48** Punching shear resistance ratio between the proposal and Eurocode 2 as a function of the concrete contribution factor.

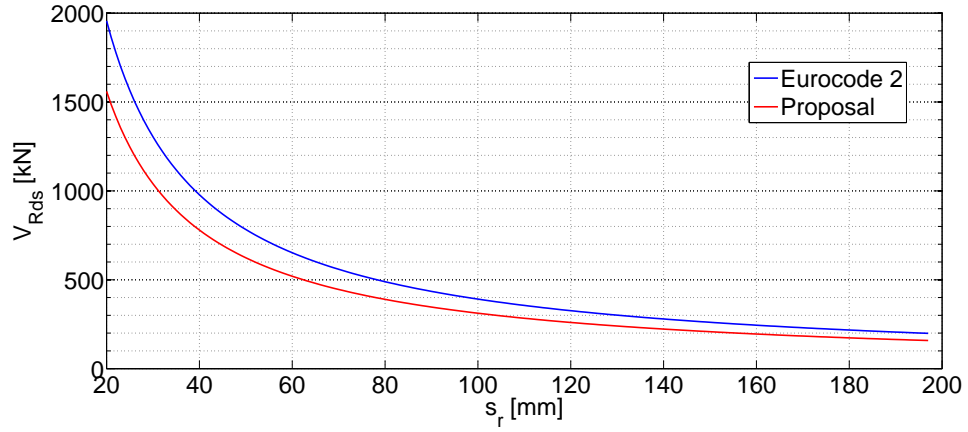
The shear reinforcement contributions as function of the radial distance are presented in figures 6.49-6.51. The contributions decreased rapidly at lower distances but flattened out. Figure 6.52 shows the ratios between the contributions. They were constant over the entire interval for all columns.



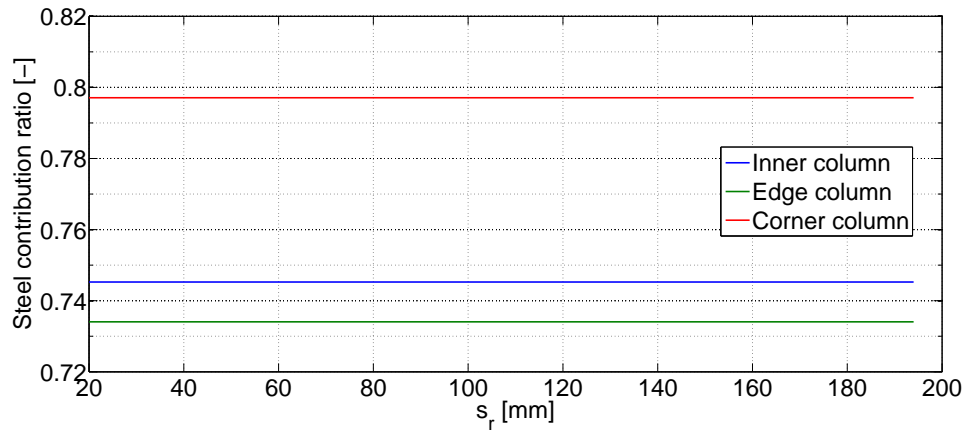
**Figure 6.49** Steel contribution for inner column as a function of the radial distance between the stud rows.



**Figure 6.50** Steel contribution for edge column as a function of the radial distance between the stud rows.

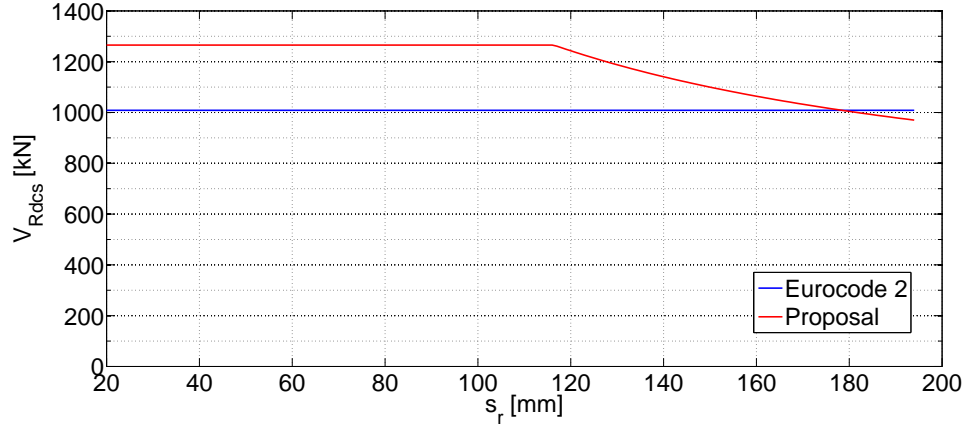


**Figure 6.51** Steel contribution for corner column as a function of the radial distance between the stud rows.

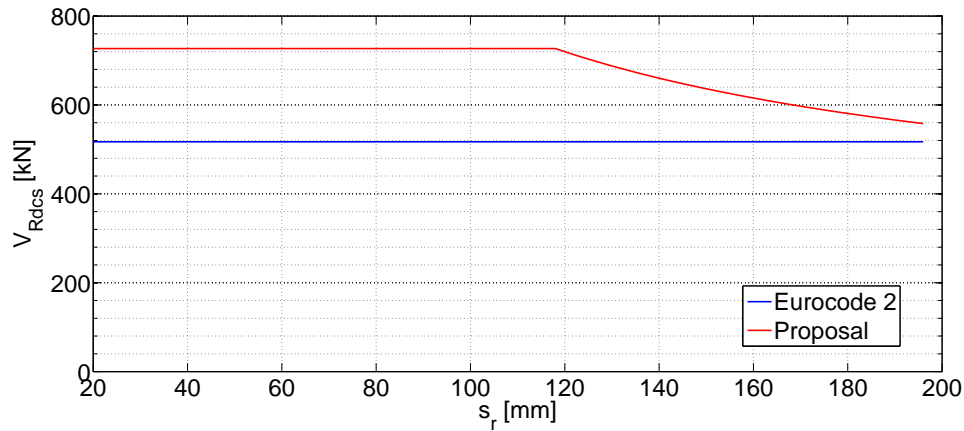


**Figure 6.52** Steel contribution ratio between the proposal and Eurocode 2 as a function of the radial distance between the stud rows.

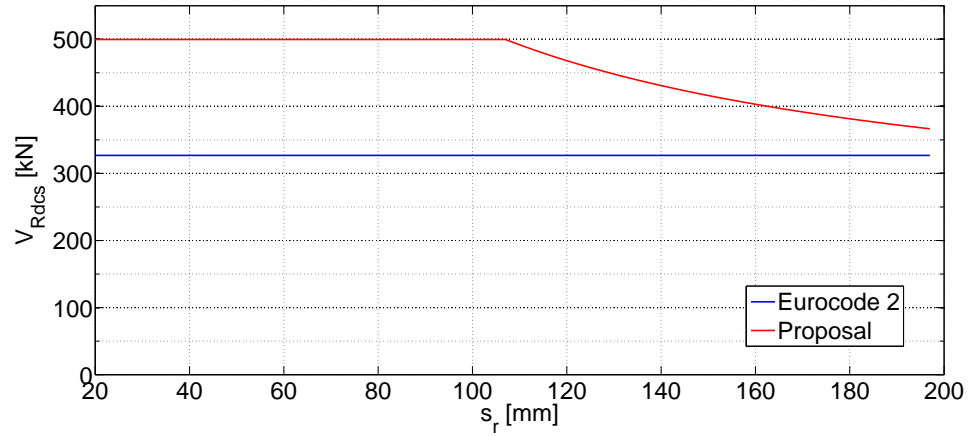
Figures 6.53-6.55 show the total punching shear resistances as function of the radial distance. The resistance according to Eurocode 2 was limited by the maximum resistance and so was the resistance according to the proposal at first. At larger distances, this resistance decreased. Figure 6.56 presents the ratios between the resistances. The differences were constant up until a distance of about 110 mm before decreasing.



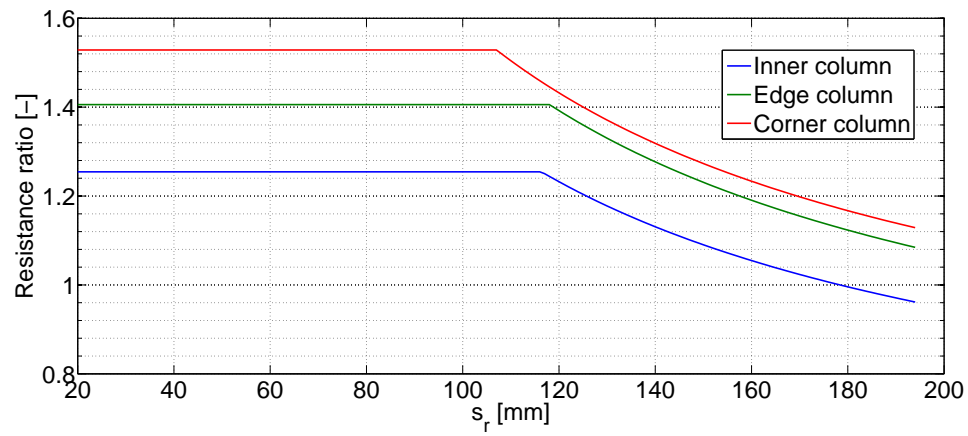
**Figure 6.53** Punching shear resistance for inner column as a function of the radial distance between the stud rows.



**Figure 6.54** Punching shear resistance for edge column as a function of the radial distance between the stud rows.

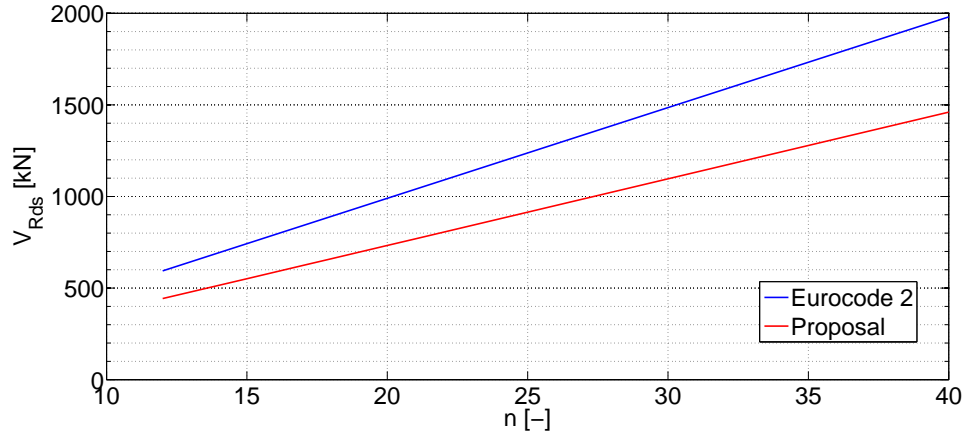


**Figure 6.55** Punching shear resistance for corner column as a function of the radial distance between the stud rows.

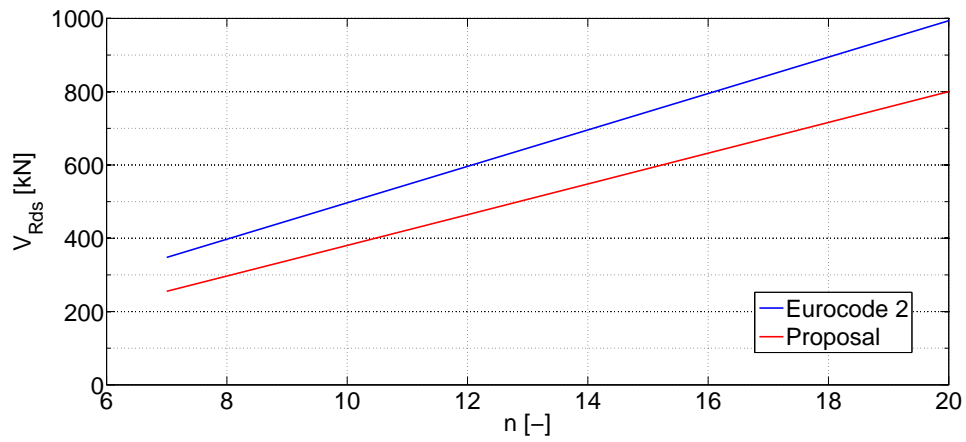


**Figure 6.56** Punching shear resistance ratio between the proposal and Eurocode 2 as a function of the radial distance between the stud rows.

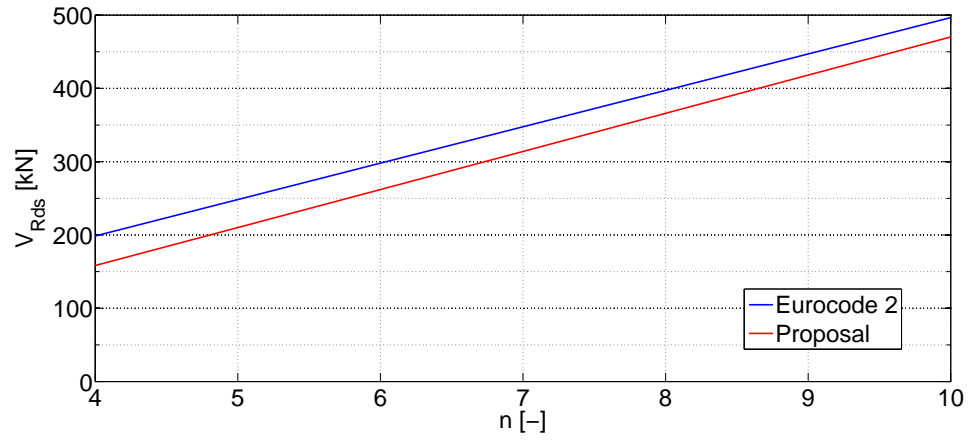
Figures 6.57-6.59 present the shear reinforcement contributions as function of the number of studs within each row. The resistance was linearly dependent according to both provisions.



**Figure 6.57** Steel contribution for inner column as a function of the number of studs in a row.

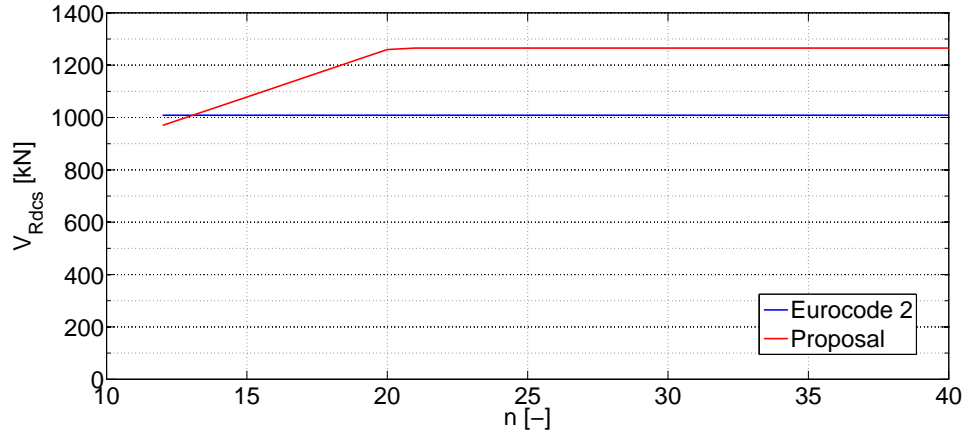


**Figure 6.58** Steel contribution for edge column as a function of the number of studs in a row.

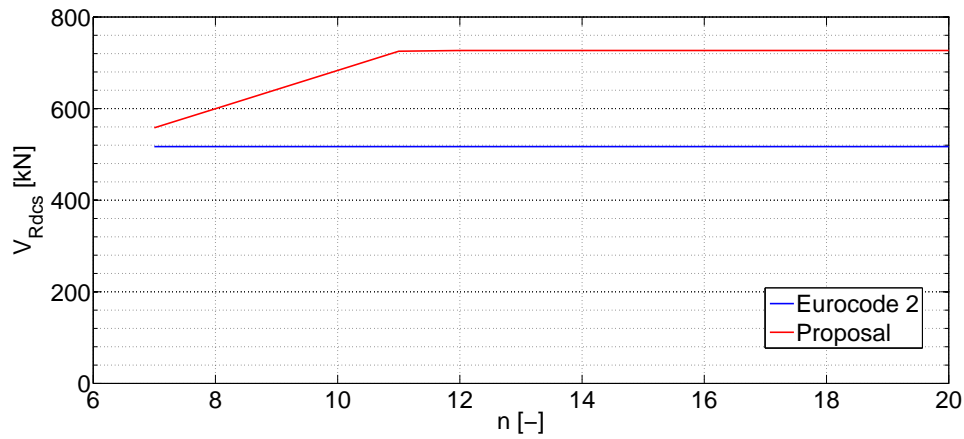


**Figure 6.59** Steel contribution for corner column as a function of the number of studs in a row.

The total punching shear resistances as function of the number of studs within each row are presented in figures 6.60-6.62. The resistance according to Eurocode 2 was limited by the maximum resistance over the entire interval. The resistance according to the proposal increased before being limited.

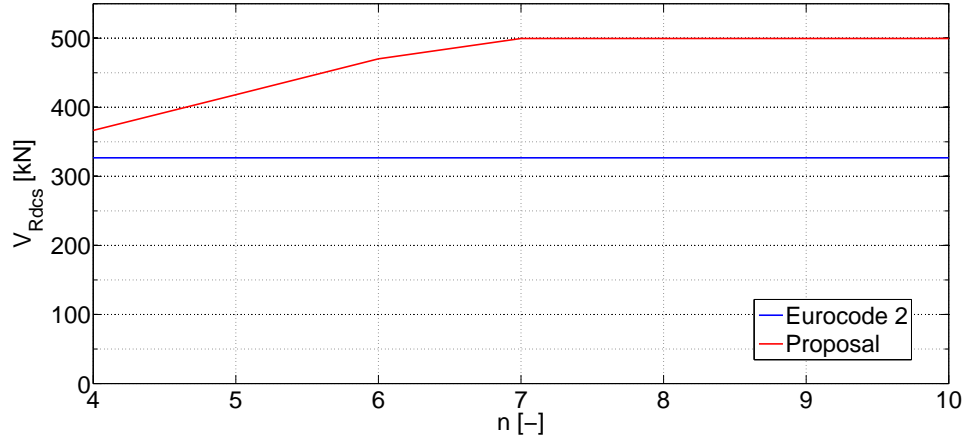


**Figure 6.60** Punching shear resistance for inner column as a function of the number of studs in a row.



**Figure 6.61** Punching shear resistance for edge column as a function of the number of studs in a row.

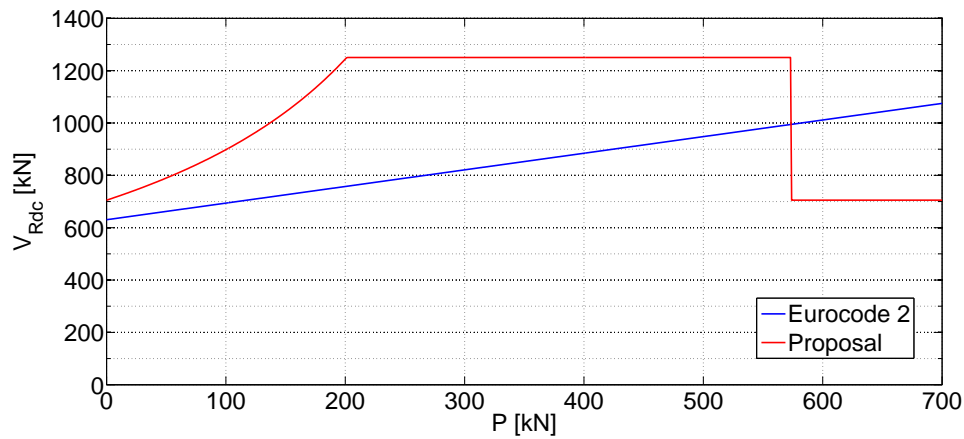




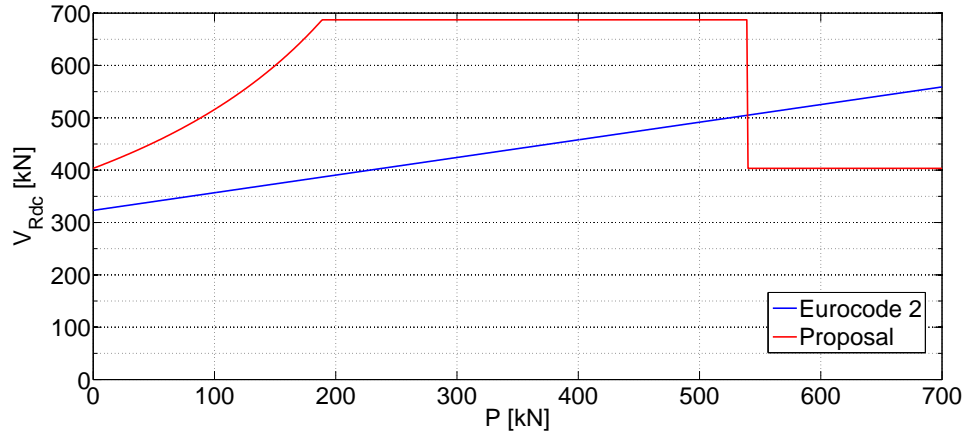
**Figure 6.62** Punching shear resistance for corner column as a function of the number of studs in a row.

### 6.2.3 Prestressed

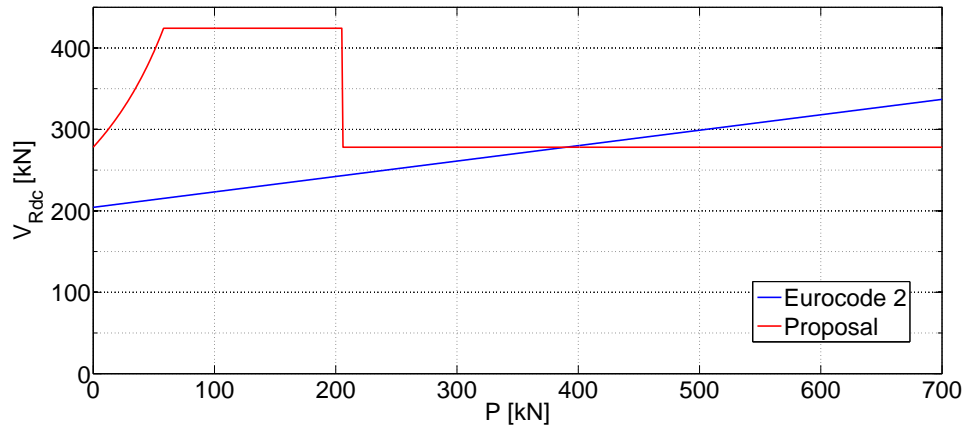
The punching shear resistances as function of the tensioning force are presented in figures 6.63-6.65. The tendon diameters were set to zero in order to neglect the reinforcement ratio contribution. The resistance according to Eurocode 2 increased linearly. For the proposal, the resistance increased rapidly before reaching the maximum resistance. Large tensioning forces did not contribute to any larger resistances. The ratios between the resistances are shown in figure 6.66.



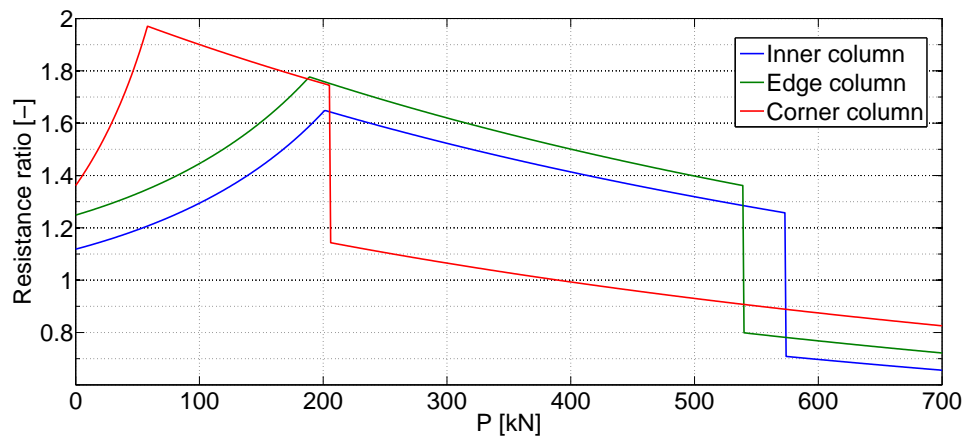
**Figure 6.63** Punching shear resistance for inner column as a function of the tensioning force. The tendon diameter is zero.



**Figure 6.64** Punching shear resistance for edge column as a function of the tensioning force. The tendon diameter is zero.

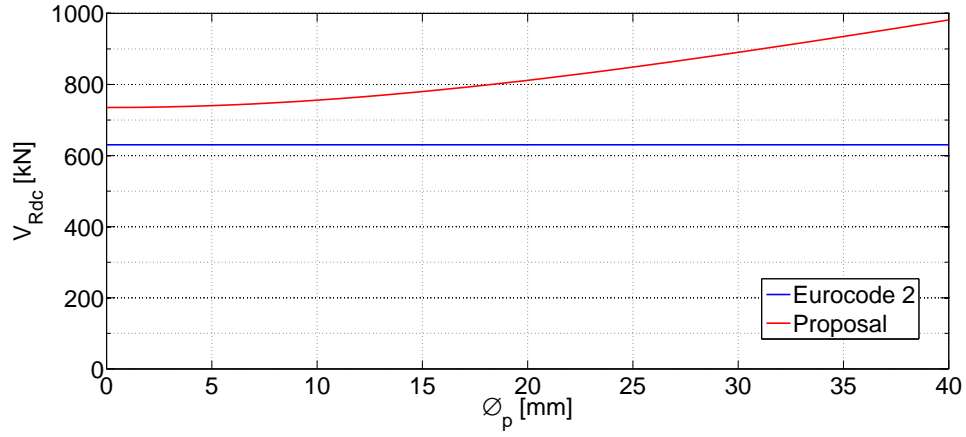


**Figure 6.65** Punching shear resistance for corner column as a function of the tensioning force. The tendon diameter is zero.

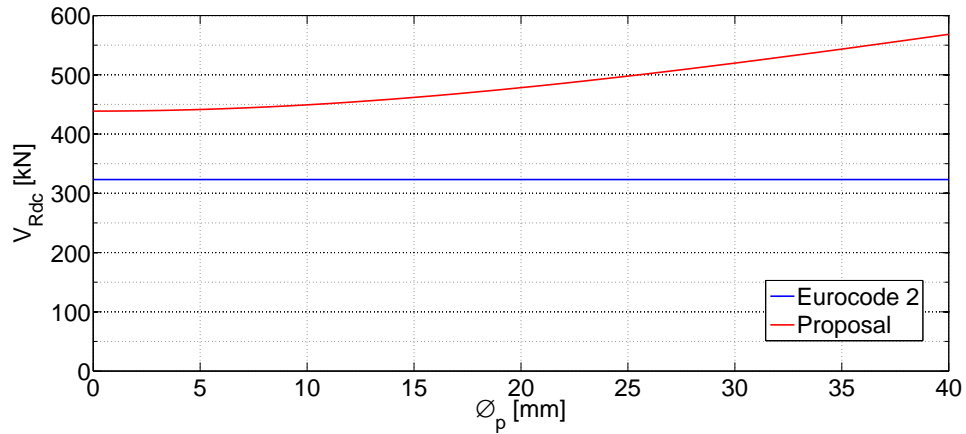


**Figure 6.66** Punching shear resistance ratio between the proposal and Eurocode 2 as a function of the tensioning force. The tendon diameter is zero.

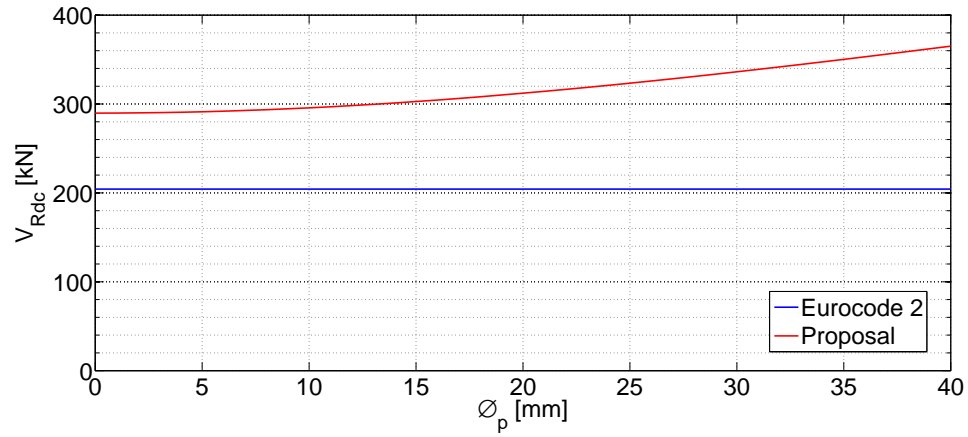
Figures 6.67-6.69 show the punching shear resistances as function of the tendon diameters. The tensioning forces were set to zero in order to neglect that resistance contribution. The resistance according to Eurocode 2 was unaffected. For the proposal, the resistance increased with larger diameters. The ratios between the resistances are shown in figure 6.70.



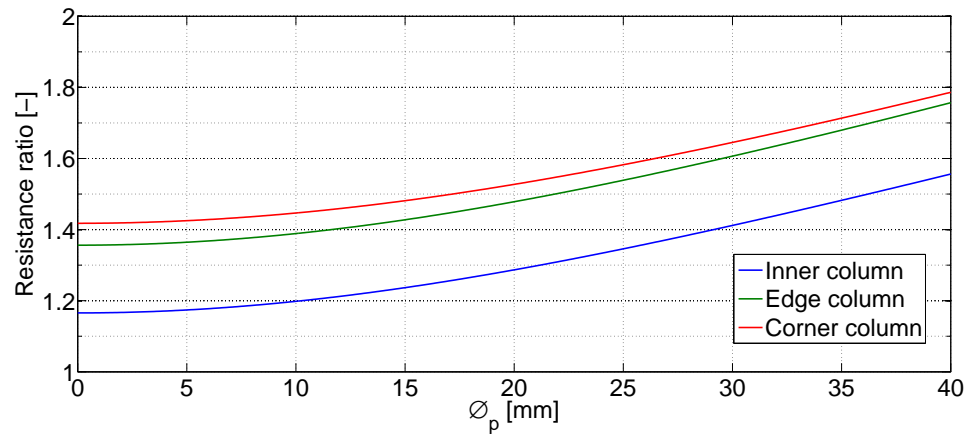
**Figure 6.67** Punching shear resistance for inner column as a function of the diameter of the tendons. The tensioning force is zero.



**Figure 6.68** Punching shear resistance for edge column as a function of the diameter of the tendons. The tensioning force is zero.



**Figure 6.69** Punching shear resistance for corner column as a function of the diameter of the tendons. The tensioning force is zero.



**Figure 6.70** Punching shear resistance ratio between the proposal and Eurocode 2 as a function of the diameter of the tendons. The tensioning force is zero.

# Chapter 7

## Discussion

### 7.1 Case study

Equations 6.47 in Eurocode 2 and 6.57 in PT1prEN 1992-1-1 (2017) have the factor  $100\rho_l f_{ck}$  in common. Since the reinforcement ratios did not differ that much between Eurocode 2 and the proposal, this value was similar in both equations. The factor  $100\rho_l f_{ck}$  is multiplied by the ratio  $\frac{d_{dg}}{a_v}$  in 6.57. This ratio decreased the factor by a magnitude 0.016-0.02 since the distance  $a_v$  was much larger than  $d_{dg}$ .

Looking at the interior control perimeter, the ratio between the perimeter lengths for Eurocode 2 and the proposal was 1.67 for most columns. Furthermore, the ratio between the factor  $C_{Rd,c}k$  in 6.47 and  $\frac{k_b}{\gamma_c}$  in 6.57 was between 0.14-0.17. Considering the exterior perimeter, the ratio between the perimeter lengths became 1.15-1.17. The ratio between  $C_{Rd,c}k$  and  $\frac{k_b}{\gamma_c}$  became 0.31-0.33.

The ratios show that the the factor  $\frac{d_{dg}}{a_v}$  had a large influence on the resistance according to the proposal. This factor decreased the punching shear resistance. However the differences between the resistances for Eurocode 2 and the proposal did not differ more than 11%.

Appendix H shows the case where the distances  $a_{v,x}$  and  $a_{v,y}$  were measured instead. The result showed that  $a_v$  would have become even larger and that would have decreased the resistance.

What is worth to mention is that the exterior control perimeter could be reduced in accordance with 6.4.2(4) in PT1prEN 1992-1-1 (2017). The results in table 6.25 show that this would have had a considerably more decreasing effect on the resistance.

The punching shear resistance according to Eurocode 2 was governed by the minimum resistance for almost every column. A separate calculation without accounting for this limit showed that the difference was negligible.

Although the outer perimeter was not governing, a comparison showed that the resistance was 42-63 % larger for Eurocode 2. The big difference between the exterior resistances mainly had to do with the shear gradient enhancement factor  $k_b$ . It decreases with increasing perimeter length  $b_0$ .

When performing the load combination, only the four quadrants adjacent to the column were loaded. An alternative would have been to combine loads over the whole slab or a larger area than the one considered in this study.

## 7.2 Parametric study

The impact on the punching shear resistance from the varied parameters is discussed for the cases prestressed, with and without shear reinforcement.

### 7.2.1 Without shear reinforcement

When varying the shear resisting effective depth  $d$  in figures 6.1-6.4, it was shown that the resistance according to Eurocode 2 changed inclination compared to the proposal from depths over 200 mm. This was because of the size effect factor  $k$  which was constant up to 200 mm and then decreased, see figure J.1 in appendix J. The size effects for both provisions were more easily detected when plotting the stress resistance, see figure J.2 in appendix J. The shapes of the punching shear resistance curves in figures 6.1-6.4 were largely influenced by the control section areas, see figure J.3 in appendix J.

The resistance was linearly proportional to the column width  $B$  for Eurocode 2, see figures 6.9-6.12. The proposal had a different shape due to the impact from the shear gradient enhancement factor  $k_b$  which decreases with increasing width. If  $a_v$  had not stayed constant in accordance with the provision 6.4.3(2), it would most likely have increased with larger column width, thus causing an even lower resistance growth rate. As the column width approaches zero, the control perimeter takes the shape of a circle and consequently the perimeter length never becomes zero. This is why a resistance could be detected even for an infinite small column width.

It was shown in figures 6.13-6.15 that the minimum resistance governed the design resistance for Eurocode 2 in the case of very low reinforcement ratios. The proposal approached zero, which may be questionable, especially for thick flat slabs with very low reinforcement ratios.

It was shown in figures 6.17-6.19 that the resistance according to the proposal decreased with larger distance  $a_v$  from the support axis to the contraflexural location. Appendix I presents the measured distances in the x- and y-directions, considering symmetry. The weighted distances would have been 1400 mm, 721 mm and 700 mm for the inner, edge and corner columns respectively in accordance with equation 6.59

in PT1prEN 1992-1-1 (2017). This is considerably different from the approximated distance 1320 mm used in the study. Consequently, this simplified method may have underestimated the resistance for the edge and corner columns.

### 7.2.2 With shear reinforcement

When varying the parameters in the case with shear reinforcement it was shown that the total punching shear resistance reached the maximum limit for Eurocode 2 over the entire intervals. Perhaps this was due to the generally large shear reinforcement contribution.

$a_v$  increases the reinforcement contribution in contrast to the concrete contribution. It was shown in figures 6.37-6.39 that this had a large influence on the total resistance for lower distances  $a_v$ . Distances larger than 1000 mm did not have any influence on the resistance.

Increasing the concrete contribution factor  $\eta_c$  decreases the steel contribution but increases the concrete contribution for the proposal. Figures 6.45-6.47 show that this did not affect the total resistance much for the proposal compared to Eurocode 2 which has a constant concrete contribution factor of 0.75.

Looking at equation 6.52 in Eurocode 2 shows that only the shear reinforcement rows within a radial distance  $1.5d$  contributes to the punching shear resistance. In the proposal, only the distribution of shear reinforcement between the first and second row closest to the column, affects the punching shear resistance. Consequently, additional rows outside these limits do not contribute to the resistance.

### 7.2.3 Prestressed

Figures 6.63-6.66 show that the impact from the tensioning force on the punching shear resistance was larger for the proposal than for Eurocode 2 up until a certain limit. The reason why tensioning forces larger than this limit did not affect the resistance, had to do with the factor in 6.4.3(4) PT1prEN 1992-1-1 (2017) which then became negative and thus set equal to one.

Figures 6.67-6.70 show that the tendon diameter has an increasing influence on the resistance according to the proposal. This is because the reinforcement ratio is affected and increased. Since the tensioning force was set to zero, only the reinforcement ratio contributed to the increase in resistance which varied similarly to the cases in figures 6.13-6.15.

### 7.3 Comparison of the provisions

Equation 6.57 in PT1prEN 1992-1-1 (2017) accounts for the concrete type and the aggregate properties. The properties of the concrete obviously affect the resistance. Eurocode 2 does not account for that.

In Eurocode 2 the reinforcement ratio  $\rho_l$  is calculated as a mean value over a length equal to the column width plus  $3d$  on each side of the column. That may be a problem if the top reinforcement is distributed over a larger area. In the case where a drop panel is used, it may be regarded as the loaded area and consequently the "column" in this case. That increases the length which then might exceed the area where flexural top reinforcement is distributed.

In the proposed provision  $\rho_l$  is calculated as the mean value over the width of the support strip  $b_s$  as  $1.5a_v$ . This makes sure that the strip stays within the area with tension in the upper slab edge.  $1.5a_v$  seems to have a stronger connection to the tension area than  $3d$ . Furthermore, the reinforcement ratio is limited to 2 % in Eurocode 2 compared to the proposal which does not include any limit.

The distance to the contra flexural location  $a_v$  accounts for the distance from the column axis where punching shear theoretically may occur. Since it has a big influence on the resistance, it is important that it is accurate. As seen in the parametric study  $a_v$  was estimated to a distance up to 85 % larger than the actual distance for the corner column. This was seen by comparing the approximated distance 1320 mm with the measured 700 mm. For the inner column, the actual distance was 1400 mm and larger than the estimated thus overestimating the resistance.

The parameter  $\mu$  accounts for the ratio between the force and the bending moment. It is different for each type of column, thus distinguishing each individual case. The resistance according to Eurocode 2 does not account for if it is an inner, edge or corner column apart from the length of the control perimeter  $u_1$ .

The proposed provisions instruct how to calculate the punching shear resistance for ends and corners of walls. In the calculation given in appendix C it was assumed that the support strip was  $1.5a_v$ . This is unclear, since no definition of the support strip is included for that case.

In the case with prestressing forces, the proposal accounts for several factors while Eurocode 2 only accounts for the normal stresses. As seen in the parametric study, the resistance growth rate is higher as a consequence. The fact that the bonded tendons affect the reinforcement ratio seems realistic. The factor in 6.4.3(4) becomes negative and set to one if the tensioning forces are large. Thus, above this limit no favourable effects will come from the prestressing forces. Furthermore, the factor should not be allowed to be zero.

6.4.3(4) refers to equations 6.14 and 6.15 for calculating a new effective depth and



reinforcement ratio respectively. In the parametric study, these were calculated separately in both principal directions. That is how the procedure was assumed to be since the depths and reinforcement areas may differ in both directions. Furthermore, in order for  $\rho_l$  to be unitless,  $b_s$  should probably be included in the denominator of 6.15.

In the case with studs as shear reinforcement, only the radial distance between the first and second row affects the resistance. That way it has no influence if the other rows are more or less dense. Eurocode 2 accounts for all rows within a distance of  $1.5d$ .

Considering the usability of the proposal compared to Eurocode 2, one of the most different procedures is the way the eccentricity factor  $\beta$  is calculated. The proposal currently has one formula for inner columns. Edge and corner columns are still to be announced. Eurocode 2 has several cases depending on the column position, shape and eccentricity direction.

More parameters are included in the proposal compared to Eurocode 2. Many of them are however simple to declare. For example,  $k_b$  depends on  $\mu$  which is taken as a standard value,  $d$  and  $b_0$  which are also included in Eurocode 2.  $d_{dg}$  has to be specified and is known. The most demanding parameter to decide is probably  $a_v$  which may be found by finite element analysis or estimated from the adjacent span lengths.

The case with prestressed flat slabs requires more parameters than just the normal stress, as in Eurocode 2. The eccentricity and the tendon section areas are however known from the prestress design. What may be somewhat demanding is to calculate the effective depth  $d_p$  of the tendons.

Also when it comes to shear reinforced flat slabs, more parameters are included in the proposed provisions than in Eurocode 2. Once again, they take almost no effort to declare.



# Chapter 8

## Conclusions

In the case study the punching shear resistances were very similar and did not differ more than 11 %. The punching shear resistance in the parametric study was larger for the proposal than for Eurocode 2. This was due to a smaller  $a_v$ .

The conclusion is that the ratio  $\frac{d_{dg}}{a_v}$  has a big influence on the punching shear resistance. Flat slabs with large span widths may obtain lower resistance in the proposal than in Eurocode 2.

Furthermore, larger control perimeters  $b_0$  contribute more to the resistance for Eurocode 2 than for the proposal, since the shear gradient enhancement factor  $k_b$  decreases the growth rate.

The reinforcement ratio  $\rho_l$  affect the resistance in the same manner for the proposal and Eurocode 2, but it has no upper limit in the proposal. Consequently, reinforcement ratios exceeding 2% still contributes to larger resistance in the proposal.

The parametric study with post-tensioning tendons showed that the tensioning force has a bigger positive influence on the resistance in the proposal. Besides, the tendons contribute with larger reinforcement ratio and has an increasing affect on the resistance in the proposal.

For shear reinforced slabs with studs, it is unnecessary to locate the rows outside the second row on a radial distance lower than  $0.75d$  since it has no effect on the resistance.

The area where the flexural reinforcement ratio is checked seems to have a stronger connection to the upper-edge-tensile area in the proposal.  $3d$  in Eurocode 2 seems to have no relation to the upper-edge-tensile area.

Setting  $a_v$  as  $0.22L$  may be a rough estimation in some cases. This was shown in the parametric study. Perhaps a more refined estimation is required.

It is unclear how the support strip is defined for ends and corners of walls. More

instructions for this case could be included. Furthermore, it may be questionable if the reduction of the perimeters for large supported areas, in accordance with 6.4.2(4) in PT1prEN 1992-1-1, results in too small resistances.

It is also unclear if the equations 6.14 and 6.15 in PT1prEN 1992-1-1 (2017), considering the modified effective depth and reinforcement ratio, should be calculated in two directions.

Considering the work load required for the proposed provisions, the calculation procedure involves more parameters than in the current Eurocode 2. However, it was concluded that they do not require much work to decide. The most demanding parameters to obtain might be the reinforcement ratio and the effective depth in relation to prestressed structures, and  $a_v$ .

# Chapter 9

## Proposed Further Research

When investigating the punching shear resistance, all parameters need to be in realistic intervals. This is why more case studies are suggested. Their advantage is the strong connection to reality.

Additional case studies should preferably be on pure flat slabs without capitals and with edge and corner columns. Moreover, it would be interesting to investigate the resistance for a shear reinforced flat slab, or perhaps a prestressed flat slab. One topic left out of this study worth to investigate was punching shear of footings.

Interesting properties to investigate further are the distances  $a_v$  for flat slabs. They should preferably be measured and not estimated. The factor  $d_{dg}$  may be analyzed by case studies on flat slabs with different concrete types and concrete properties.



# References

Bentz E., Fernandez Ruiz M., Foster S., Muttoni A., Sigrist V., (2013), "Background to fib Model Code 2010 shear provisions – part II: punching shear", *Structural Concrete* 14, No 3, pp.207, Berlin, May.

Broms C.E.,(1990), "Punching of flat plates: a question of concrete properties in biaxial compression and size effect", *ACI Structural Journal*, vol.87, nr 3, pp.296, American Concrete Institute, Detroit

Broms C.E.,(2005), "Concrete flat slabs and footings Design method for punching and detailing for ductility", Doctoral thesis, Bulletin 80, Department of Civil and Architectural Engineering, Royal Institute of Technology, Stockholm, Sweden.

Cook R.D, Malkus D.S, Plesha M.E, Witt R.J (2002), "Concepts and applications of finite element analysis", 4th edition, John Wiley and Sons, pp. 1, 11, 300-301, United States

EN 1992-1-1:2005/AC:2010 (2010), "Eurocode 2: Design of concrete structures – Part 1-1: General rules and rules for buildings", European Committee for Standardisation(CEN), Brussels.

EN 1992-1-1:2005/A1:2014 (2014), "Eurocode 2: Design of concrete structures – Part 1-1: General rules and rules for buildings", European Committee for Standardisation(CEN), Brussels.

Eurocode 0 (2002), "Eurocode:Basis of structural design, European Standard EN 1990", European Committee for Standardisation(CEN), Brussels.

Eurocode 1 (2002), "Eurocode 1:Actions on structures-Part 1-1: General actions-Densities, self-weight, imposed loads for buildings", European Standard EN 1991-1-1, European Committee for Standardisation(CEN), Brussels.

Eurocode 2 (2004), "Eurocode 2:Design of concrete structures, European Standard EN 1992-1-1", European Committee for Standardisation(CEN), Brussels.

Fernández Ruiz M., Muttoni A., (2016), "The Critical Shear Crack Theory for punching design: from a mechanical model to closed-form design expressions", pp.1-8 ,submitted for publication.

Fernández Ruiz M., Foster S.J., Maya L.F., Muttoni A., (2012), "Punching shear strength of steel fibre reinforced concrete slabs", Engineering Structures 40, pp.83-84, Sydney, Australia, March.

Halfen (2017), "HDB-Shear reinforcement", <http://www.halfen.com/se/1410/produkter/armeringssystem/skjuvankarsystem-hdb/inledning/>, accessed 2017-04-28

Hallgren M.,(1996), "Punching Shear Capacity of Reinforced High Strength Concrete Slabs", Doctoral thesis, Bulletin 23, Department of structural engineering, Royal Institute of Technology, Stockholm, Sweden

Hästen 21 (2017), <http://hasten21.se/vart-erbjudande/arkitekturen/>, accessed 2017-03-15

Kinnunen S., Nylander H., (1960), "Punching of Concrete Slabs Without Shear Reinforcement", Transactions of the Royal Institute of Technology, N.158, Stockholm, Sweden.

Muttoni A., and Schwartz J., (1991), "Behaviour of Beams and Punching in Slabs without Shear Reinforcement," IABSE Colloquium, V. 62, pp. 708, Zurich, Switzerland.

Muttoni A., (2008), "Punching shear strength of reinforced concrete slabs without transverse reinforcement", ACI Structural Journal, V. 105, N 4, pp. 440-450, USA, July-August.

PT1prEN1991-1-1:2017-04, "Eurocode 2: Design of concrete structures –Part 1-1: General rules, rules for buildings, bridges and civil engineering structures",European Standard EN 2017", European Committee for Standardisation(CEN), Brussels.

Swedish concrete association (2010a),"Svenska betongföreningens handbok till Eurokod 2", N 1, pp. 73-77

Swedish concrete association (2010b),"Svenska betongföreningens handbok till Eurokod 2", N 2

Swedish NA (2016), "Boverkets konstruktionsregler", BFS 2011:10 t.o.m. BFS 2015:6, EKS10, Boverket, Karlskrona

Wood, J.G.M. "Piper's Row Car Park, Wolverhampton: Quantitative Study of the Causes of the Partial Collapse on 20th March 1997", SSD Contract Report to HSE,<http://www.hse.gov.uk/research/misc/pipersrow.htm>, accessed 2017.



# Appendix A

## Calculation Model According to Eurocode 2

This is the calculation model for the punching shear resistance in the case study according to Eurocode 2 section 6.4. In this section the equations are referred to the Eurocode 2 (2004).

### GEOMETRY

Effective depth at the interior basic control perimeter according to equation 6.32 (Eurocode 2, 2004)

$$d_H = \frac{d_{x,i} + d_{y,i}}{2} \quad (\text{A.1})$$

Effective depth at the exterior basic control perimeter according to equation 6.32 (Eurocode 2, 2004)

$$d = \frac{d_{x,e} + d_{y,e}}{2} \quad (\text{A.2})$$

Control if  $l_H \geq 2 \cdot (h_H)$  according to 6.4.2(9) (Eurocode 2, 2004). If this was true, checks of the control section both within the drop panel and in the slab had to be performed. This was always true in the case study.

Interior basic control perimeter according to 6.4.2(1) (Eurocode 2, 2004)

$$u_{1,int} = 4 \cdot B + 2 \cdot \pi \cdot 2 \cdot d_H \quad (\text{A.3})$$

Exterior basic control perimeter according to 6.4.2(1) (Eurocode 2, 2004)

$$u_{1,ext} = 4 \cdot p + 2 \cdot \pi \cdot 2 \cdot d \quad (\text{A.4})$$

The column perimeter was

$$u_0 = 4 \cdot B \quad (\text{A.5})$$

## CALCULATION OF MAXIMUM PUNCHING SHEAR RESISTANCE

Strength reduction factor for concrete cracked in shear according to equation 6.6N (Eurocode 2, 2004)

$$\nu = 0.6 \cdot \left( 1 - \frac{f_{ck}}{250 MPa} \right) \quad (A.6)$$

Design value of concrete compressive strength, equation 3.15 (Eurocode 2, 2004)

$$f_{cd} = \frac{f_{ck}}{\gamma_c} \quad (A.7)$$

Maximum punching shear resistance according to EKS10

$$v_{Rd,max} = 0.5 \cdot f_{cd} \cdot \nu \quad (A.8)$$

$$V_{Rd,max} = v_{Rd,max} \cdot d_H \cdot u_0 \quad (A.9)$$

## CALCULATION OF PUNCHING SHEAR RESISTANCE AT THE INTERIOR BASIC CONTROL PERIMETER ACCORDING TO 6.4.4(1) (Eurocode 2, 2004)

The size-effect factor was

$$k_{int} = 1 + \sqrt{\frac{200}{d_H}} \leq 2 \quad (A.10)$$

Reinforcement control width was

$$b_{s,i} = B + 6 \cdot d_H \quad (A.11)$$

Number of bars within the control width in the x- and y-direction

$$N_{x,i} = trunc \left( \frac{b_{s,i}}{cc_x} \right) \quad (A.12)$$

$$N_{y,i} = trunc \left( \frac{b_{s,i}}{cc_y} \right) \quad (A.13)$$

Reinforcement ratio for longitudinal reinforcement in x- and y-direction

$$\rho_{l,i,x} = \frac{N_{x,i} \cdot (\pi \cdot \phi^2 \cdot \frac{1}{4})}{b_{s,i} \cdot d_{v,i,x}} \quad (A.14)$$

$$\rho_{l,i,y} = \frac{N_{y,i} \cdot (\pi \cdot \phi^2 \cdot \frac{1}{4})}{b_{s,i} \cdot d_{v,i,y}} \quad (A.15)$$

This led to the reinforcement ratio for longitudinal reinforcement

$$\rho_{l,int} = \sqrt{\rho_{l,i,x} \cdot \rho_{l,i,y}} \quad (A.16)$$

---

that should be  $\leq 0.02$

Minimum punching shear strength according to equation 6.3N (Eurocode 2, 2004)

$$v_{min,int} = 0.035 \cdot k_{int}^{\frac{3}{2}} \cdot f_{ck}^{\frac{1}{2}} \quad (A.17)$$

$$C_{Rd,c} = \frac{0.18}{\gamma_c} \quad (A.18)$$

Normal concrete stresses

$$\sigma_{cp} = 0 \quad (A.19)$$

Design value of the punching shear resistance of a slab without punching shear reinforcement along the control section considered, equation 6.47 (Eurocode 2, 2004)

$$v_{Rd,c,int} = C_{Rd,c} \cdot k_{int} \cdot (100 \cdot \rho_{l,int} \cdot f_{ck})^{\frac{1}{3}} + k_1 \cdot \sigma_{cp} \geq v_{min} + k_1 \cdot \sigma_{cp} \quad (A.20)$$

If this was true

$$V_{Rd,c,int} = v_{Rd,c,int} \cdot u_{1,int} \cdot d_H \quad (A.21)$$

If not

$$V_{Rd,c,int} = v_{min,int} \cdot u_{1,int} \cdot d_H \quad (A.22)$$

CALCULATION OF PUNCHING SHEAR RESISTANCE AT THE EXTERIOR BASIC CONTROL PERIMETER ACCORDING TO 6.4.4(1) (Eurocode 2, 2004)

The size-effect factor was

$$k_{ext} = 1 + \sqrt{\frac{200}{d}} \leq 2 \quad (A.23)$$

Reinforcement control width was

$$b_{s,e} = B + 6 \cdot d \quad (A.24)$$

Number of bars within the control width in the x- and y-direction

$$N_{x,e} = trunc \left( \frac{b_{s,e}}{cc_x} \right) \quad (A.25)$$

$$N_{y,e} = trunc \left( \frac{b_{s,e}}{cc_y} \right) \quad (A.26)$$

Reinforcement ratio for longitudinal reinforcement in x- and y-direction

$$\rho_{l,e,x} = \frac{N_{x,e} \cdot (\pi \cdot \phi^2 \cdot \frac{1}{4})}{b_{s,e} \cdot d_{v,e,x}} \quad (A.27)$$

$$\rho_{l,e,y} = \frac{N_{y,e} \cdot (\pi \cdot \phi^2 \cdot \frac{1}{4})}{b_{s,e} \cdot d_{v,e,y}} \quad (A.28)$$

This led to the reinforcement ratio for longitudinal reinforcement

$$\rho_{l,ext} = \sqrt{\rho_{l,e,x} \cdot \rho_{l,e,y}} \quad (A.29)$$

that should be  $\leq 0.02$

Minimum punching shear strength according to equation 6.3N (Eurocode 2, 2004)

$$v_{min,ext} = 0.035 \cdot k_{ext}^{\frac{3}{2}} \cdot f_{ck}^{\frac{1}{2}} \quad (A.30)$$

$$C_{Rd,c} = \frac{0.18}{\gamma_c} \quad (A.31)$$

Normal concrete stresses

$$\sigma_{cp} = 0 \quad (A.32)$$

Design value of the punching shear resistance of a slab without punching shear reinforcement along the control section considered, equation 6.47 (Eurocode 2, 2004)

$$v_{Rd,c,ext} = C_{Rd,c} \cdot k_{ext} \cdot (100 \cdot \rho_{l,ext} \cdot f_{ck})^{\frac{1}{3}} + k_1 \cdot \sigma_{cp} \geq v_{min} + k_1 \cdot \sigma_{cp} \quad (A.33)$$

If this was true

$$V_{Rd,c,ext} = v_{Rd,c,ext} \cdot u_{1,ext} \cdot d \quad (A.34)$$

If not

$$V_{Rd,c,ext} = v_{min,ext} \cdot u_{1,ext} \cdot d \quad (A.35)$$

# Appendix B

## Calculation model according to the Proposal

This is the calculation model for the punching shear resistance in the case study according to PT1prEN 1992-1-1 section 6.4. In this section the equations are referred to the PT1prEN 1992-1-1 (2017).

### GEOMETRY

Effective depth at the interior control perimeter according to 6.4.2(1) (PT1prEN 1992-1-1, 2017)

$$d_{v,int} = \frac{d_{v,i,x} + d_{v,i,y}}{2} \quad (B.1)$$

Effective depth at the exterior control perimeter according to (PT1prEN 1992-1-1, 2017)

$$d_{v,ext} = \frac{d_{v,e,x} + d_{v,e,y}}{2} \quad (B.2)$$

Interior control perimeter according to 6.4.2(3)(PT1prEN 1992-1-1, 2017)

$$b_{0,int} = 4 \cdot B + 2 \cdot \pi \cdot 0.5 \cdot d_{v,int} \quad (B.3)$$

Exterior control perimeter according to 6.4.2(3) (PT1prEN 1992-1-1, 2017)

$$b_{0,ext} = 4 \cdot p + 2 \cdot \pi \cdot 0.5 \cdot d_{v,ext} \quad (B.4)$$

The control perimeter may be reduced according to 6.4.2(4) (PT1prEN 1992-1-1, 2017) if

$$3 \cdot d_{v,ext} \leq p \quad (B.5)$$

According to 6.4.2(4) (PT1prEN 1992-1-1, 2017) the reduction of the control perimeter was

$$b_{0,ext,red} = 4 \cdot 3 \cdot d_{v,ext} + 2 \cdot \pi \cdot 0.5 \cdot d_{v,ext} \quad (B.6)$$

## CALCULATION OF PUNCHING SHEAR RESISTANCE OF SLABS WITHOUT SHEAR REINFORCEMENT

Assuming that lateral stability did not depend on frame action between slab and columns and that  $0.5 \leq \frac{L_{x,max}}{L_{y,max}} \leq 2$  according to 6.4.3(2) (PT1prEN 1992-1-1, 2017),  $a_v$  could be approximated as  $0.22 \cdot L_x$  and  $0.22 \cdot L_y$

The flat slab was modelled in FEM Design 16 Plate and the distance  $a_v$  was found according to 6.4.3(1) (PT1prEN 1992-1-1, 2017). In appendix H, two figures of the distance  $a_v$  are shown. The differences between the simplified and the measured distances were however negligible.

The simplified distances  $a_v$  in each direction according to 6.4.2(2) (PT1prEN 1992-1-1, 2017) were

$$a_{v,x} = 0.22 \cdot L_{x,max} \quad (B.7)$$

$$a_{v,y} = 0.22 \cdot L_{y,max} \quad (B.8)$$

This led to, equation 6.59 (PT1prEN 1992-1-1, 2017)

$$a_v = \sqrt{a_{v,x} \cdot a_{v,y}} \quad (B.9)$$

which should be  $\geq 2.5 \cdot d_{v,int}$

Width of the support strip

$$b_s = 1.5 \cdot a_v \quad (B.10)$$

Number of bars within the support strip in the x- and y-direction

$$N_x = trunc\left(\frac{b_s}{cc_x}\right) + 1 \quad (B.11)$$

$$N_y = trunc\left(\frac{b_s}{cc_y}\right) + 1 \quad (B.12)$$

## CALCULATION OF PUNCHING SHEAR RESISTANCE AT INTERIOR CONTROL PERIMETER

Reinforcement ratio for longitudinal reinforcement in x- and y-direction according to 6.4.3(1) (PT1prEN 1992-1-1, 2017)

$$\rho_{l,i,x} = \frac{N_x \cdot (\pi \cdot \phi^2 \cdot \frac{1}{4})}{b_s \cdot d_{v,i,x}} \quad (B.13)$$

$$\rho_{l,i,y} = \frac{N_y \cdot (\pi \cdot \phi^2 \cdot \frac{1}{4})}{b_s \cdot d_{v,i,y}} \quad (B.14)$$

This leads to the reinforcement ratio for longitudinal reinforcement, equation 6.58 (PT1prEN 1992-1-1, 2017)

$$\rho_{l,int} = \sqrt{\rho_{l,i,x} \cdot \rho_{l,i,y}} \quad (B.15)$$

---

Shear gradient enhancement factor, equation 6.60 (PT1prEN 1992-1-1, 2017)

$$k_{b,int} = \sqrt{8 \cdot \mu \cdot \frac{d_{v,int}}{b_{0,int}}} \quad (\text{B.16})$$

that should be  $\geq 1$

Design punching shear stress resistance, equation 6.57 (PT1prEN 1992-1-1, 2017)

$$\tau_{Rd,c,int} = \left( \frac{k_{b,int}}{\gamma_c} \right) \cdot \left( 100 \cdot \rho_{l,int} \cdot f_{ck} \cdot \frac{d_{dg}}{a_v} \right)^{\frac{1}{3}} \quad (\text{B.17})$$

should be  $\leq \left( \frac{0.6}{\gamma_c} \right) \cdot \sqrt{f_{ck}}$

Design punching shear resistance

$$V_{Rd,c,int} = \tau_{Rd,c,int} \cdot d_{v,int} \cdot b_{0,int} \quad (\text{B.18})$$

#### CALCULATION OF PUNCHING SHEAR RESISTANCE AT EXTERIOR CONTROL PERIMETER

Reinforcement ratio for longitudinal reinforcement in x- and y-direction according to 6.4.3(1) (PT1prEN 1992-1-1, 2017)

$$\rho_{l,e,x} = \frac{N_x \cdot (\pi \cdot \phi^2 \cdot \frac{1}{4})}{b_s \cdot d_{v,e,x}} \quad (\text{B.19})$$

$$\rho_{l,e,y} = \frac{N_y \cdot (\pi \cdot \phi^2 \cdot \frac{1}{4})}{b_s \cdot d_{v,e,y}} \quad (\text{B.20})$$

This led to the reinforcement ratio for longitudinal reinforcement, equation 6.58 (PT1prEN 1992-1-1, 2017)

$$\rho_{l,ext} = \sqrt{\rho_{l,e,x} \cdot \rho_{l,e,y}} \quad (\text{B.21})$$

Shear gradient enhancement factor, equation 6.60 (PT1prEN 1992-1-1, 2017)

$$k_{b,ext} = \sqrt{8 \cdot \mu \cdot \frac{d_{v,ext}}{b_{0,ext}}} \quad (\text{B.22})$$

and the reduced resistance was

$$k_{b,ext,red} = \sqrt{8 \cdot \mu \cdot \frac{d_{v,ext}}{b_{0,ext,red}}} \quad (\text{B.23})$$

that should be  $\geq 1$

Design punching shear stress resistance, equation 6.57 (PT1prEN 1992-1-1, 2017)

$$\tau_{Rd,c,ext} = \left( \frac{k_{b,ext}}{\gamma_c} \right) \cdot \left( 100 \cdot \rho_{l,ext} \cdot f_{ck} \cdot \frac{d_{dg}}{a_v} \right)^{\frac{1}{3}} \quad (\text{B.24})$$

and the reduced resistance was

$$\tau_{Rd,c,ext,red} = \left( \frac{k_{b,ext,red}}{\gamma_c} \right) \cdot \left( 100 \cdot \rho_{l,ext} \cdot f_{ck} \cdot \frac{d_{dg}}{a_v} \right)^{\frac{1}{3}} \quad (\text{B.25})$$

that should be  $\leq \left( \frac{0.6}{\gamma_c} \right) \cdot \sqrt{f_{ck}}$

Design punching shear resistance without reduction

$$V_{Rd,c,ext} = \tau_{Rd,c,ext} \cdot d_{v,ext} \cdot b_{0,ext} \quad (\text{B.26})$$

Design punching shear resistance with reduction

$$V_{Rd,c,ext,red} = \tau_{Rd,c,ext,red} \cdot d_{v,ext} \cdot b_{0,ext,red} \quad (\text{B.27})$$



# Appendix C

## Calculation of Resistance at the Corner of a Wall

The proposed provisions explain how to estimate the punching shear resistance for corners of walls. An attempt was made to calculate the resistance of the upper right wall corner belonging to the central core of the building. The procedure follows the provisions in section 6.4 of PT1prEN 1992-1-1

The effective depths by the corner were 362mm and 374mm in the x- and y-directions. The shear resisting effective depth according to 6.4.2(1) (PT1prEN 1992-1-1, 2017) became

$$d_v = \frac{362 + 374}{2} = 368mm \quad (C.1)$$

The wall was regarded as a large supported area, which was why the straight segments were reduced in accordance with 6.4.2(4) (PT1prEN 1992-1-1, 2017). The length of the control perimeter was

$$b_0 = 1.5d_v + 1.5d_v + \frac{1}{4}\pi d_v = 1393mm \quad (C.2)$$

A small dwell was located 800 mm from the corner. It was assumed to be no larger than 100mm in diameter. Since the vertical length of the control perimeter was 552mm, the tangents according to 6.4.2(5)(PT1prEN 1992-1-1, 2017) had no reducing effect on the length  $b_0$ .

It was assumed that the lateral stability did not depend on frame action between the slab and the columns. The span lengths from the centre of the loaded area to the centre of columns 2506 and 2605 were 8425mm and 6950mm. Since the difference was smaller than 25%, 6.4.3(2) (PT1prEN 1992-1-1, 2017) was used.

$$a_{v,x} = 0.22 \cdot 8425 = 1854mm \quad (C.3)$$

$$a_{v,y} = 0.22 \cdot 6950 = 1529mm \quad (C.4)$$

Equation 6.59 (PT1prEN 1992-1-1, 2017) gave the distance

$$a_v = \sqrt{a_{v,x} \cdot a_{v,y}} = 1683mm \quad (C.5)$$

which was larger than  $2.5d_v$ . The reinforcement ratio was calculated as instructed in 6.4.3(1) (PT1prEN 1992-1-1, 2017). The length of the support strip became

$$b_s = 1.5a_v = 2525mm \quad (C.6)$$

which was smaller than the smallest concurring span length.

The top reinforcement in the x-direction consisted of  $24\phi12c75 - 5600$ . All these bars fit inside the support strip so the reinforcement ratio became

$$\rho_{l,x} = \frac{24 \cdot \pi \cdot 12^2 / 4}{b_s \cdot 362} = 0.00297 \quad (C.7)$$

The corresponding reinforcement in the y-direction  $24\phi12c80 - 5600$  also fit inside the support strip. The corresponding ratio became

$$\rho_{l,y} = \frac{24 \cdot \pi \cdot 12^2 / 4}{b_s \cdot 374} = 0.00287 \quad (C.8)$$

The resulting bonded flexural reinforcement ratio was obtained from equation 6.58 (PT1prEN 1992-1-1, 2017) as

$$\rho_l = \sqrt{\rho_{l,x} \cdot \rho_{l,y}} = 0.0029 \quad (C.9)$$

6.4.3(1) (PT1prEN 1992-1-1, 2017) showed that  $\mu = 5$  for corners of walls. The shear gradient enhancement factor from equation 6.60 (PT1prEN 1992-1-1, 2017) became

$$k_b = \sqrt{8\mu \frac{d_v}{b_0}} = 3.25 \quad (C.10)$$

As mentioned, the characteristic compressive cylinder strength  $f_{ck} = 32MPa$  and the parameter  $d_{dg} = 16mm$ . The design punching shear stress resistance according to equation 6.57 (PT1prEN 1992-1-1, 2017) became

$$\tau_{Rd,c} = \frac{k_b}{1.5} \left( 100 \cdot \rho_l \cdot f_{ck} \cdot \frac{d_{dg}}{a_v} \right)^{1/3} = 967kPa \quad (C.11)$$

which was lower than the upper limit

$$\frac{0.6}{1.5} \sqrt{f_{ck}} = 2263kPa \quad (C.12)$$

Ultimately, the design punching shear resistance was obtained as

$$V_{Rd,c} = \tau_{Rd,c} \cdot d_v \cdot b_0 = 496kN \quad (C.13)$$

## Appendix D

### Matlab Code - Without Shear Reinforcement

On the next page the Matlab code for calculating the resistance in the parametric study is given. The code is for a flat slab without shear reinforcement.

```
clear all;
clc;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%   FLAT SLABS WITHOUT SHEAR REINFORCEMENT
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%   Asking for indata
%disp('The parameter you choose to vary should be inserted as
[start_value:step:end_value]')
d = input('Enter effective depth [mm]:');
f_ck = input('Enter compressive concrete strength [MPa]:');
B = input('Enter column width [mm]:');
ra = input('Enter a reinforcement ratio less than 0.02 [-]:');
a_v = input(['Enter the distance a_v to the contraflexural perimeter [mm]:']);
d_dg = input('Enter d_dg between 16 mm and 40 mm to account for aggregate
properties [mm]:');
type = input('Is it an inner (1), edge (2) or corner column (3)?');

%   Which variable is varied?
if length(d) > 1
    variable = d;
elseif length(f_ck) > 1
    variable = f_ck;
elseif length(B) > 1
    variable = B;
elseif length(ra) > 1
    variable = ra;
elseif length(a_v) > 1
    variable = a_v;
elseif length(d_dg) > 1
    variable = d_dg;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%   EUROCODE 2
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%   Column perimeter
if type == 1
    u_0 = 4*B;
elseif type == 2
    u_0 = B+3*d;
    max_u = 3*B;
    if length(u_0) ~= length(max_u)
        for i = 1:length(u_0)
            if u_0(i) > max_u
                u_0(i) = max_u;
            end
        end
    else
        for i = 1:length(u_0)
            if u_0(i) > max_u(i)
                u_0(i) = max_u(i);
            end
        end
    end
end
```

---

```

        end
    else
        u = 3*d;
        max_u = 2*B;
        u_0 = [];
        if length(u) == 1
            for i = 1:length(variable)
                u_0(i) = u;
            end
        else
            u_0 = u;
        end
        if length(u_0) ~= length(max_u)
            for i = 1:length(u_0)
                if u_0(i) > max_u
                    u_0(i) = max_u;
                end
            end
        else
            for i = 1:length(u_0)
                if u_0(i) > max_u(i)
                    u_0(i) = max_u(i);
                end
            end
        end
    end
end

% Interior basic control perimeter
if type == 1
    u_1 = 4*B+2*pi*2*d;
elseif type == 2
    u_1 = 3*B+pi*2*d;
else
    u_1 = 2*B+0.5*pi*2*d;
end

% Size-effect factor
k = 1+sqrt(200./d);
for i = 1:length(k)
    if k(i) > 2
        k(i) = 2;
    end
end

% Punching shear resistance excluding min and max
C_Rdc = 0.18/1.5;
v_Rdc = C_Rdc*k.*(100*ra.*f_ck).^(1/3);
V_Rdc = v_Rdc.*u_1.*d;

% Minimum punching shear resistance
v_min = 0.035*k.^(3/2)*f_ck.^(1/2);
v_Rdmin = [];
if length(v_min) == 1;
    for i = 1:length(variable)
        v_Rdmin(i) = v_min;
    end
end

```

```
else
    v_Rdmin = v_min;
end
V_Rdmin = v_Rdmin.*u_1.*d;

% Punching shear resistance accounting for min
for i = 1:length(v_Rdc)
    if v_Rdc(i) < v_Rdmin(i)
        v_Rdc(i) = v_Rdmin(i);
    end
end
V_Rdc = v_Rdc.*u_1.*d;

% Maximum punching shear resistance
v = 0.6*(1-f_ck/250);
f_cd = f_ck/1.5;
V_Rdmax1 = 0.5*f_cd.*v.*d.*u_0;
V_Rdmax2 = 1.6*v_Rdc.*u_1.*d;
V_Rdmax = [];

if length(V_Rdmax1) == 1
    for i = 1:length(variable)
        V_Rdmax(i) = V_Rdmax1;
    end
else
    V_Rdmax = V_Rdmax1;
end
if length(V_Rdmax) ~= length(V_Rdmax2)
    for i = 1:length(V_Rdmax)
        if V_Rdmax(i) > V_Rdmax2
            V_Rdmax(i) = V_Rdmax2;
        end
    end
else
    for i = 1:length(V_Rdmax)
        if V_Rdmax(i) > V_Rdmax2(i)
            V_Rdmax(i) = V_Rdmax2(i);
        end
    end
end

% Punching shear resistance accounting for min and max
for i = 1:length(V_Rdc)
    if V_Rdc(i) > V_Rdmax(i)
        V_Rdc(i) = V_Rdmax(i);
    end
end
for i = 1:length(V_Rdc)
    if V_Rdc(i) < V_Rdmin(i)
        V_Rdc(i) = 0;
    end
end

% In the case where the resistance is constant
if length(V_Rdc) == 1
    V = [];
```

---

```

        for i = 1:length(variable)
            V(i) = V_Rdc;
        end
        V_Rdc = V;
    end

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    %   THE PROPOSAL
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    %   Interior basic control perimeter
    if type == 1
        b_0 = 4*B+2*pi*0.5*d;
    elseif type == 2
        b_0 = 3*B+pi*0.5*d;
    else
        b_0 = 2*B+0.5*pi*0.5*d;
    end

    %   Parameter accounting for shear force and bending moments
    if type == 1
        mu = 8;
    elseif type == 2
        mu = 5;
    else
        mu = 3;
    end

    %   Shear gradient enhancement factor
    k_b = sqrt(8*mu.*d./b_0);
    for i = 1:length(k_b)
        if k_b(i) < 1
            k_b(i) = 1;
        end
    end

    %   Maximum punching shear resistance
    max_val = (0.6/1.5)*sqrt(f_ck);
    max_value = [];
    if length(max_val) == 1
        for i = 1:length(variable)
            max_value(i) = max_val;
        end
    else
        max_value = max_val;
    end
    Max_value = max_value.*b_0.*d;

    %   Punching shear resistance
    v_Rdc_new = (k_b/1.5).*(100*ra.*f_ck.*d_dg./a_v).^(1/3);
    V_Rdc_new = v_Rdc_new.*b_0.*d;
    for i = 1:length(V_Rdc_new)
        if V_Rdc_new(i) > Max_value(i)
            V_Rdc_new(i) = Max_value(i);
        end
    end
end

```

```
% In the case where the resistance is constant
if length(V_Rdc_new) == 1
    V = [];
    for i = 1:length(variable)
        V(i) = V_Rdc_new;
    end
    V_Rdc_new = V;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% RESULTS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Ratio
ratio = V_Rdc_new./V_Rdc;

% Show the numeric results
fprintf('\nPunching shear resistance - Eurocode 2 [kN]')
V_Rdc/1000
disp('Punching shear resistance - Proposal [kN]')
V_Rdc_new/1000
disp('Ratio proposal/Eurocode 2 [-]')
ratio
```



## Appendix E

### Matlab code - With Shear Reinforcement

On the next page the Matlab code for calculating the resistance in the parametric study is given. The code is for a flat slab with shear reinforcement. Note that the code from appendix D should be included in this code.

```
clear all;
clc;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%   FLAT SLABS WITH SHEAR REINFORCEMENT
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%   Asking for indata
disp('The parameter you choose to vary should be inserted as
[start_value:step:end_value]')
d = input('Enter effective depth [mm]:');
f_ck = input('Enter compressive concrete strength [MPa]:');
B = input('Enter column width [mm]:');
ra = input('Enter a reinforcement ratio less than 0.02 [-]:');
a_v = input(['Enter the distance a_v to the contraflexural perimeter [mm]:']);
d_dg = input('Enter d_dg between 16 mm and 40 mm to account for aggregate
properties [mm]:');

D = input('Enter the diameter of the shear studs [mm]:');
n_c = input('Enter the concrete contribution n_c < 1 [-]:');
s_r = input('Enter the radial stud distance < 195 mm [mm]:');
n = input('Enter the sum of studs in a row [-]:');

type = input('Is it an inner (1), edge (2) or corner column (3)?');

%   Which variable is varied?
if length(d) > 1
    variable = d;
elseif length(f_ck) > 1
    variable = f_ck;
elseif length(B) > 1
    variable = B;
elseif length(ra) > 1
    variable = ra;
elseif length(a_v) > 1
    variable = a_v;
elseif length(d_dg) > 1
    variable = d_dg;

elseif length(D) > 1
    variable = D;
elseif length(n_c) > 1
    variable = n_c;
elseif length(s_r) > 1
    variable = s_r;
elseif length(n) > 1
    variable = n;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%   HERE GOES THE CODE FOR WITHOUT SHEAR REINFORCEMENT
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%   FLAT SLABS WITH SHEAR REINFORCEMENT
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

---

```

f_yk = 500;          % Characteristic steel tensile strength [MPa]
f_yd = f_yk/1.15;    % Design steel tensile strength [MPa]
f_yeff = 250+0.25*d; % Effective steel tensile strength [Mpa]
if f_yeff > f_yd
    disp('The effective tensile strength is too high')
    f_yeff = f_yd;
end
%   s_r = 195;    % radial spacing [mm]

if type == 1
    alpha = 2*pi./n;
    s_t = 2*(200+0.5*d)*sin(alpha/2);
    s_t_max = 2*(200+2*d)*sin(alpha/2);
elseif type == 2
    alpha = pi./(n-1);
    s_t = 2*(150+0.5*d)*sin(alpha/2);
    s_t_max = 2*(150+2*d)*sin(alpha/2);
elseif type == 3
    alpha = pi./(2*(n-1));
    s_t = 2*(150+0.5*d)*sin(alpha/2);
    s_t_max = 2*(150+2*d)*sin(alpha/2);
end
A = (pi*D.^2)/4; % Area of a shear stud [mm^2]
A_min = 0.08*sqrt(f_ck).*(s_r.*s_t_max)./(1.5*f_yk); % Minimum area of a
shear stud [mm^2]
for i = 1:length(A)
    if A(i) < A_min
        disp('The reinforcement area is too small')
    end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%   EUROCODE 2 - With shear reinforcement
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%   Contribution from concrete
V_c = 0.75*V_Rdc;

%   Contribution from steel
V_s = (1.5*d./s_r).*n.*A.*f_yeff;
v_s = [];
if length(V_s) == 1
    for i = 1:length(variable)
        v_s(i) = V_s;
    end
    V_s = v_s;
end

%   Resulting punching shear resistance
V_cs = V_c + V_s;

%   Check with the maximum resistance
for i = 1:length(variable)
    if V_cs(i) > V_Rdmax(i)
        disp('The maximum resistance is reached for EC2')
    end
end

```

```
        V_cs(i) = V_Rdmax(i);
    end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%   THE PROPOSAL - With shear reinforcement
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%   Contribution from concrete
V_c_new = n_c.*V_Rdc_new;

%   Contribution from steel
n_s = 0.1+sqrt(a_v./d).*(0.8./(n_c.*k_b)).^(3/2);
if length(n_s) == 1
    if n_s > 0.8
        n_s = 0.8;
    end
elseif length(n_s) > 1
    for i = 1:length(variable)
        if n_s(i) > 0.8
            n_s(i) = 0.8;
        end
    end
end
V_s_new = n_s.*A.*f_yd.*b_0.*d./(s_r.*s_t);

%   Resulting punching shear resistance
V_cs_new = V_c_new + V_s_new;

%   Check with the maximum resistance
V_cs_max = 1.8*V_Rdc_new;
for i = 1:length(variable)
    if V_cs_new(i) > V_cs_max(i)
        disp('The maximum resistance is reached for the proposal')
        V_cs_new(i) = V_cs_max(i);
    end
end
end
```

# Appendix F

## Matlab Code - Prestressed

On the next page the Matlab code for calculating the resistance in the parametric study is given. The code is for a prestressed flat slab.

```
clear all;
clc;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%   PRESTRESSED FLAT SLABS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%   Asking for indata
disp('The parameter you choose to vary should be inserted as
[start_value:step:end_value]')
d_v = input('Enter effective depth [mm]:');
f_ck = 25; %input('Enter compressive concrete strength [MPa]:');
B = input('Enter column width [mm]:');
ra = input('Enter a reinforcement ratio less than 0.02 [-]:');
P = input('Enter a prestressing load [kN]:');
D_p = input('Enter the diameter of a tendon [mm]:');
e_0x = input('Enter the eccentricity over the support [mm]:');
d_dg = input('Enter d_dg between 16 mm and 40 mm to account for aggregate
properties [mm]:');
type = input('Is it an inner (1), edge (2) or corner column (3)?');
a_vx = 1320;
a_vy = 1320;
a_v = 1320;

%   Which variable is varied?
if length(d_v) > 1
    variable = d_v;
elseif length(f_ck) > 1
    variable = f_ck;
elseif length(B) > 1
    variable = B;
elseif length(ra) > 1
    variable = ra;
elseif length(a_v) > 1
    variable = a_v;
elseif length(d_dg) > 1
    variable = d_dg;
elseif length(P) > 1
    variable = P;
elseif length(D_p) > 1
    variable = D_p;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%   EUROCODE 2
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%   Column perimeter
if type == 1
    u_0 = 4*B;
elseif type == 2
    u_0 = B+3*d_v;
    max_u = 3*B;
    if length(u_0) ~= length(max_u)
        for i = 1:length(u_0)
            if u_0(i) > max_u
```

---

```

        u_0(i) = max_u;
    end
end
else
    for i = 1:length(u_0)
        if u_0(i) > max_u(i)
            u_0(i) = max_u(i);
        end
    end
end
else
    u = 3*d_v;
    max_u = 2*B;
    u_0=[];
    if length(u) == 1
        for i = 1:length(variable)
            u_0(i) = u;
        end
    else
        u_0 = u;
    end
    if length(u_0) ~= length(max_u)
        for i = 1:length(u_0)
            if u_0(i) > max_u
                u_0(i) = max_u;
            end
        end
    else
        for i = 1:length(u_0)
            if u_0(i) > max_u(i)
                u_0(i) = max_u(i);
            end
        end
    end
end
end

% Interior basic control perimeter
if type == 1
    u_1 = 4*B+2*pi*2*d_v;
elseif type == 2
    u_1 = 3*B+pi*2*d_v;
else
    u_1 = 2*B+0.5*pi*2*d_v;
end

% Size-effect factor
k = 1+sqrt(200./d_v);
for i = 1:length(k)
    if k(i) > 2
        k(i) = 2;
    end
end

% Prestressing
e_0x = 90;
e_0y = e_0x-40;

```

```
x = B/2+2*d_v;  
if type == 1  
    e_px = (-e_0x/1960000)*x^2+e_0x % 1400mm  
    e_py = (-e_0y/1960000)*x^2+e_0y % 1400mm  
    P_xcomp_x = sqrt(((P/1000).^2)/(1+((-2/1960000)*e_0x*x)^2))  
    P_xcomp_y = sqrt(((P/1000).^2)/(1+((-2/1960000)*e_0y*x)^2))  
elseif type == 2  
    e_0x = 50;  
    e_0y = e_0x-40;  
    e_px = (-e_0x/160000)*x^2+e_0x % 400mm  
    e_py = (-e_0y/1690000)*x^2+e_0y % 1300mm  
    P_xcomp_x = sqrt(((P/1000).^2)/(1+((-2/160000)*e_0x*x)^2))  
    P_xcomp_y = sqrt(((P/1000).^2)/(1+((-2/1690000)*e_0y*x)^2))  
elseif type == 3  
    e_px = (-e_0x/490000)*x^2+e_0x % 700mm  
    e_py = (-e_0y/490000)*x^2+e_0y % 700mm  
    P_xcomp_x = sqrt(((P/1000).^2)/(1+((-2/490000)*e_0x*x)^2))  
    P_xcomp_y = sqrt(((P/1000).^2)/(1+((-2/490000)*e_0y*x)^2))  
end  
  
t = 300;  
cc_p = 660;  
A = t*cc_p;  
O_cx = 1000*P/A;  
O_cy = 1000*P/A;  
O_cp = (O_cx+O_cy)/2  
  
% Punching shear resistance excluding min and max  
C_Rdc = 0.18/1.5;  
v_Rdc = C_Rdc*k.*(100*ra.*f_ck).^(1/3);  
v = [];  
for i = 1:length(variable)  
    v(i) = v_Rdc;  
end  
v_Rdc = v+0.1*O_cp;  
V_Rdc = v_Rdc.*u_1.*d_v;  
  
% Minimum punching shear resistance  
v_min = 0.035*k.^(3/2)*f_ck.^(1/2);  
v_Rdmin = [];  
if length(v_min) == 1;  
    for i = 1:length(variable)  
        v_Rdmin(i) = v_min;  
    end  
else  
    v_Rdmin = v_min;  
end  
V_Rdmin = v_Rdmin.*u_1.*d_v;  
  
% Punching shear resistance accounting for min  
for i = 1:length(v_Rdc)  
    if v_Rdc(i) < v_Rdmin(i)  
        v_Rdc(i) = v_Rdmin(i);  
    end  
end  
V_Rdc = v_Rdc.*u_1.*d_v;
```



---

```

% Maximum punching shear resistance
v = 0.6*(1-f_ck/250);
f_cd = f_ck/1.5;
V_Rdmax1 = 0.5*f_cd.*v.*d_v.*u_0;
V_Rdmax2 = 1.6*v_Rdc.*u_1.*d_v;
V_Rdmax = [];

if length(V_Rdmax1) == 1
    for i = 1:length(variable)
        V_Rdmax(i) = V_Rdmax1;
    end
else
    V_Rdmax = V_Rdmax1;
end
if length(V_Rdmax) ~= length(V_Rdmax2)
    for i = 1:length(V_Rdmax)
        if V_Rdmax(i) > V_Rdmax2
            V_Rdmax(i) = V_Rdmax2;
        end
    end
else
    for i = 1:length(V_Rdmax)
        if V_Rdmax(i) > V_Rdmax2(i)
            V_Rdmax(i) = V_Rdmax2(i);
        end
    end
end

% Punching shear resistance accounting for min and max
for i = 1:length(V_Rdc)
    if V_Rdc(i) > V_Rdmax(i)
        V_Rdc(i) = V_Rdmax(i);
    end
end
for i = 1:length(V_Rdc)
    if V_Rdc(i) < V_Rdmin(i)
        V_Rdc(i) = 0;
    end
end

% In the case where the resistance is constant
if length(V_Rdc) == 1
    V = [];
    for i = 1:length(variable)
        V(i) = V_Rdc;
    end
    V_Rdc = V;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% THE PROPOSAL
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Interior basic control perimeter
if type == 1

```

```
        b_0 = 4*B+2*pi*0.5*d_v;
elseif type == 2
        b_0 = 3*B+pi*0.5*d_v;
else
        b_0 = 2*B+0.5*pi*0.5*d_v;
end

% Parameter accounting for shear force and bending moments
if type == 1
    mu = 8;
elseif type == 2
    mu = 5;
else
    mu = 3;
end

% Prestressing
if type == 1
    beta = 1.15;
    V_Ed = 445000;
    v_Ed = beta*V_Ed/(d_v*b_0) %MPa
    b_sx = 1.5*a_vx;
    b_sy = 1.5*a_vy;
    d_sx = 267;
    d_sy = 251;
    cc_sx = 180;
    cc_sy = 170;
    cc_px = 660;
    cc_py = 660;
    D_s = 16;
elseif type == 2
    beta = 1.4;
    V_Ed = 131000;
    v_Ed = beta*V_Ed/(d_v*b_0) %MPa
    b_sx = 900;
    b_sy = 1140;
    d_sx = 268;
    d_sy = 256;
    cc_sx = 130;
    cc_sy = 90;
    cc_px = 660;
    cc_py = 660;
    D_s = 12;
elseif type == 3
    beta = 1.5;
    V_Ed = 46000;
    v_Ed = beta*V_Ed/(d_v*b_0) %MPa
    b_sx = 600;
    b_sy = 700;
    d_sx = 269;
    d_sy = 257;
    cc_sx = 75;
    cc_sy = 75;
    cc_px = 660;
    cc_py = 660;
    D_s = 12;
end
```

---

```

% Eccentricity and x-component
e_0x = 90;
e_0y = e_0x-40;
x = B/2+0.5*d_v;
if type == 1
    e_px = (-e_0x/1960000)*x^2+e_0x           % 1400mm
    e_py = (-e_0y/1960000)*x^2+e_0y           % 1400mm
    P_xcomp = sqrt(((P/1000).^2)/(1+((-2/196000)*e_0x*x)^2))
    P_xcompy = sqrt(((P/1000).^2)/(1+((-2/196000)*e_0y*x)^2))
elseif type == 2
    e_0x = 50;
    e_0y = e_0x-40;
    e_px = (-e_0x/160000)*x^2+e_0x           % 400mm
    e_py = (-e_0y/1690000)*x^2+e_0y           % 1300mm
    P_xcomp = sqrt(((P/1000).^2)/(1+((-2/160000)*e_0x*x)^2))
    P_xcompy = sqrt(((P/1000).^2)/(1+((-2/1690000)*e_0y*x)^2))
elseif type == 3
    e_px = (-e_0x/490000)*x^2+e_0x           % 700mm
    e_py = (-e_0y/490000)*x^2+e_0y           % 700mm
    P_xcomp = sqrt(((P/1000).^2)/(1+((-2/490000)*e_0x*x)^2))
    P_xcompy = sqrt(((P/1000).^2)/(1+((-2/490000)*e_0y*x)^2))
end

% Average stress
t = 300;
A = t*cc_px/(10^6)
O_dx = -P_xcomp/A
O_dy = -P_xcompy/A

% Tendon depths and reinforcement areas
d_px = t/2+e_px;
d_py = t/2+e_py;
A_s = (pi*D_s.^2)/4;
A_p = (pi*D_p.^2)/4;

% Number of bars and tendons
n_sx = b_sx/cc_sx;
n_sy = b_sy/cc_sy;
n_px = b_sx/cc_px
n_py = b_sy/cc_py

% Effective depths
d_x = ((d_sx^2*n_sx*A_s)+(d_px^2*n_px*A_p))/((d_sx*n_sx*A_s)+(d_px*n_px*A_p))
d_y = ((d_sy^2*n_sy*A_s)+(d_py^2*n_py*A_p))/((d_sy*n_sy*A_s)+(d_py*n_py*A_p))

% Reinforcement ratios
ra_x = (d_sx*n_sx*A_s+d_px*n_px*A_p)/(b_sx*d_x^2)
ra_y = (d_sy*n_sy*A_s+d_py*n_py*A_p)/(b_sy*d_y^2)
ra = sqrt(ra_x.*ra_y)

% Reduction factors
f_x = (1+(mu*O_dx/v_Ed)*(d_x/6+e_px)/b_0).^3
f_y = (1+(mu*O_dx/v_Ed)*(d_y/6+e_py)/b_0).^3
if length(f_x) == 1
    if f_x < 0

```

```
        disp('negative f_x')
        f_x = 1;
    end
else
    for i = 1:length(f_x)
        if f_x(i) < 0
            disp('negative f_x')
            f_x(i) = 1;
        end
    end
end
if length(f_y) == 1
    if f_y < 0
        disp('negative f_y')
        f_y = 1;
    end
else
    for i = 1:length(f_y)
        if f_y(i) < 0
            disp('negative f_y')
            f_y(i) = 1;
        end
    end
end

% a_v
a_vx = a_vx*f_x
a_vy = a_vy*f_y
a_v = sqrt(a_vx.*a_vy)

% Shear gradient enhancement factor
k_b = sqrt(8*mu.*d_v./b_0);
for i = 1:length(k_b)
    if k_b(i) < 1
        k_b(i) = 1;
    end
end

% Maximum punching shear resistance
max_val = (0.6/1.5)*sqrt(f_ck);
max_value = [];
if length(max_val) == 1
    for i = 1:length(variable)
        max_value(i) = max_val;
    end
else
    max_value = max_val;
end
Max_value = max_value.*b_0.*d_v;

% Punching shear resistance
v_Rdc_new = (k_b/1.5).*(100*ra.*f_ck.*d_dg./a_v).^(1/3);
V_Rdc_new = v_Rdc_new.*b_0.*d_v;
for i = 1:length(V_Rdc_new)
    if V_Rdc_new(i) > Max_value(i)
        V_Rdc_new(i) = Max_value(i);
    end
end
```

---

```
        end
    end

    % In the case where the resistance is constant
    if length(V_Rdc_new) == 1
        V = [];
        for i = 1:length(variable)
            V(i) = V_Rdc_new;
        end
        V_Rdc_new = V;
    end
```



# Appendix G

## Matlab Code - Hand Calculations

Hand calculations were performed in order to check the matlab code for the parametric study given in appendices D, E and F. Some of the results are given here. The results in the case without shear reinforcement are presented in table G.1, with shear reinforcement are presented in table G.2 and from the case with post tensioned tendons in table G.3.

Table G.1: Some of the results from the hand calculation for the case without shear reinforcement

d [mm]	<b>100</b>	262	263	259	262	263
$f_{ck}$ [MPa]	25	<b>90</b>	25	25	25	25
B [mm]	400	300	<b>500</b>	400	300	300
$\rho_l$ [-]	0.0044	0.004	0.0057	<b>0.015</b>	0.004	0.0057
$a_v$ [mm]	1320	1320	1320	1320	<b>800</b>	1320
$d_{dg}$ [mm]	32	32	32	32	32	<b>35</b>
Type [-]	Inner	Edge	Corner	Inner	Edge	Corner
<b>Eurocode 2</b>						
$u_0$ [mm]	1600	900	789	1600	900	600
$u_0 \cdot d$ [mm <sup>2</sup> ]	160000	235800	207507	414400	235800	157800
$u_1$ [mm]	2857	2546	1826	4855	2546	1426
$u_1 \cdot d$ [mm <sup>2</sup> ]	285700	667100	480300	1257400	667100	375100
k [-]	2	1.8737	1.8720	1.8787	1.8737	1.8720
$v_{min}$ [MPa]	0.495	0.8516	0.4482	0.4507	0.4488	0.4482
$v_{Rd,c}$ [MPa]	0.5538	0.8516	0.5446	0.7546	0.4844	0.5446
$v_{Rd,max}$ [MPa]	0.854	1.3626	0.8714	1.2074	0.7751	0.8714
$V_{min}$ [kN]	141	568	215	567	299	168
$V_{Rd,c}$ [kN]	152	568	262	949	323	204
$V_{Rd,max}$ [kN]	244	909	419	1518	517	327
<b>Proposal</b>						
$b_0$ [mm]	1914	1312	1207	2414	1312	807
$b_0 \cdot d$ [mm <sup>2</sup> ]	191400	343630	317330	625226	343744	212130
$\mu$ [-]	8	5	3	8	5	3
$k_b$ [-]	1.8285	2.8268	2.2872	2.6206	2.8268	2.7975
$\tau_{Rd,c}$ [MPa]	0.7846	1.8009	1.0699	1.6924	1.3885	1.3086
$\tau_{Rd,c,max}$ [MPa]	2	3.7947	2	2	2	2
$V_{Rd,c}$ [kN]	150	618	340	1058	477	278
$V_{Rd,c,max}$ [kN]	383	1304	635	1250	687	424



Table G.2: Some of the results from the hand calculation for the case with shear reinforcement

Diameter [mm]	<b>12</b>	10	10	10	10
$a_v$ [mm]	1320	<b>400</b>	1320	1320	1320
$\eta_c$ [-]	0.75	0.75	<b>0.6</b>	0.75	0.75
$s_r$ [mm]	195	197	197	<b>140</b>	197
$\eta$ [-]	12	7	4	12	<b>10</b>
Type [-]	Inner	Edge	Corner	Inner	Edge
<b>Eurocode 2</b>					
$u_0$ [mm]	1600	900	600	1600	900
$u_0 \cdot d$ [mm <sup>2</sup> ]	414400	235800	157800	414400	235800
$u_1$ [mm]	4855	2546	1426	4855	2546
$u_1 \cdot d$ [mm <sup>2</sup> ]	1257	667100	375100	1257400	667100
k [-]	1.8787	1.8737	1.8720	1.8787	1.8737
$V_{min}$ [kN]	567	299	168	567	299
$V_s$ [kN]	851	346	198	823	494
$V_c$ [kN]	473	242	153	473	242
$V_{Rd,cs}$ [kN]	1324	588	352	1296	737
$V_{Rd,max}$ [kN]	1009	517	327	1009	517
<b>Proposal</b>					
$b_0$ [mm]	2414	1312	807	2414	1311
$b_0 \cdot d$ [mm <sup>2</sup> ]	625140	343744	212130	625140	343630
$\mu$ [-]	8	5	3	8	5
$k_b$ [-]	2.6206	2.8268	2.7975	2.6206	2.8268
$V_s$ [kN]	634	158	202	613	379
$V_c$ [kN]	527	451	167	527	303
$V_{Rd,cs}$ [kN]	1162	609	368	1141	681
$V_{Rd,max}$ [kN]	1265	1082	500	1265	727

Table G.3: Some of the results from the hand calculation for the case with post-tensioned tendons

P [kN]	<b>100</b>	0
Diameter [mm]	0	<b>20</b>
Type [-]	Inner	Inner
<b>Eurocode 2</b>		
$u_0$ [mm]	1600	1600
$u_0 \cdot d$ [mm <sup>2</sup> ]	414400	414400
$u_1$ [mm]	4855	4855
$u_1 \cdot d$ [mm <sup>2</sup> ]	1257400	1257400
k [-]	1.8787	1.8787
$\sigma$ [Mpa]	0.5051	0
$V_{min}$ [kN]	567	567
$V_{Rd,c}$ [kN]	694	630
$V_{Rd,max}$ [kN]	1009	1009
<b>Proposal</b>		
$b_0$ [mm]	2414	2414
$b_0 \cdot d$ [mm <sup>2</sup> ]	625140	625140
$\mu$ [-]	8	8
$k_b$ [-]	2.6206	2.6206
$e_{px}$ [mm]	85.0	85.0
$e_{py}$ [mm]	47.2	47.2
$a_{v,mod}$ [mm]	639	1320
$\rho_{mod}$ [-]	0.0044	0.0065
$V_{Rd,c}$ [kN]	898	802

# Appendix H

## Distances to the Contra Flexural Locations - Case Study

The following figures presents the measured distances  $a_{v,x}$  and  $a_{v,y}$  to the contra flexure locations in the case study.

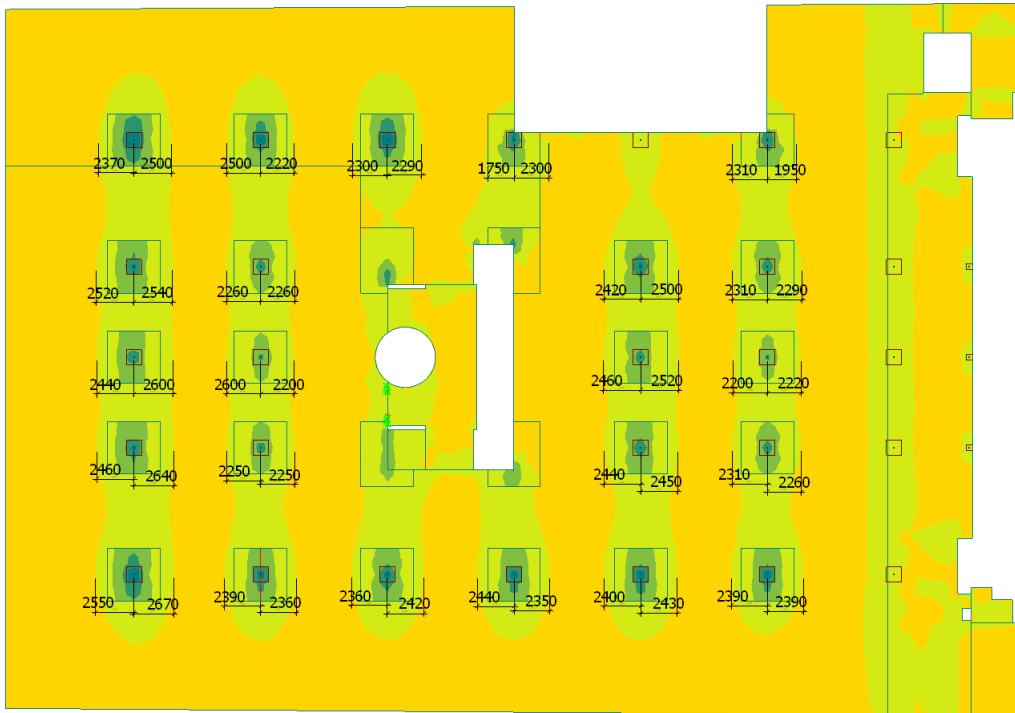


Figure H.1: Distances to the contra flexure locations in x-direction

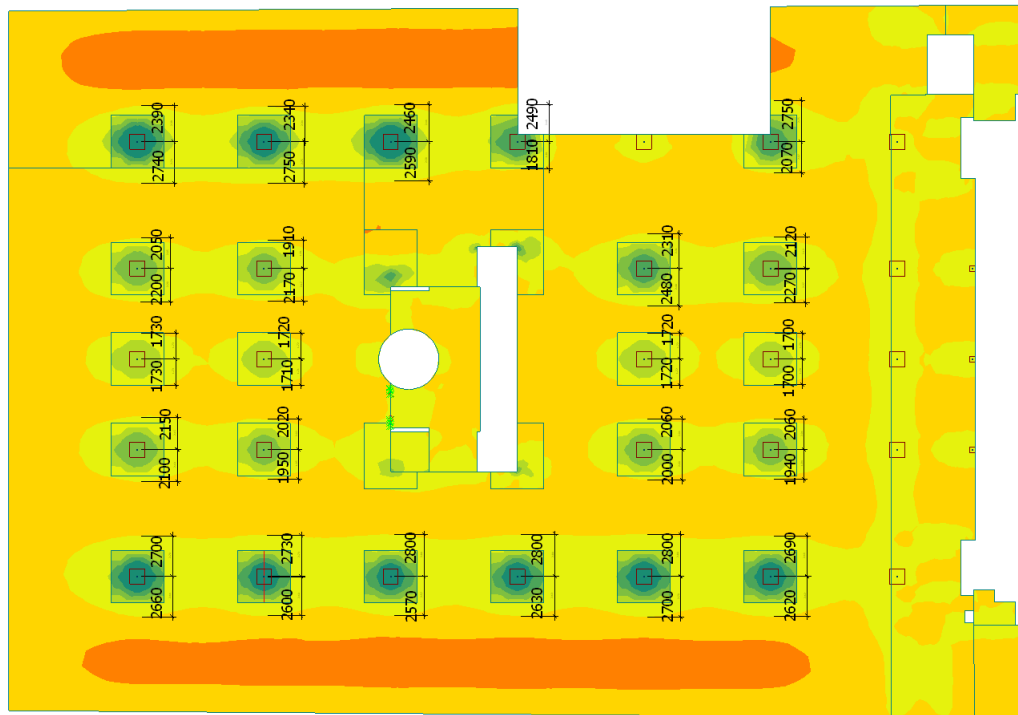


Figure H.2: Distances to the contra flexure locations in y-direction

# Appendix I

## Distances to the Contra Flexural Locations - Parametric Study

The distances  $a_{v,x}$  for the slab in the parametric study are presented in figure I.1. Note that the slab is symmetric.

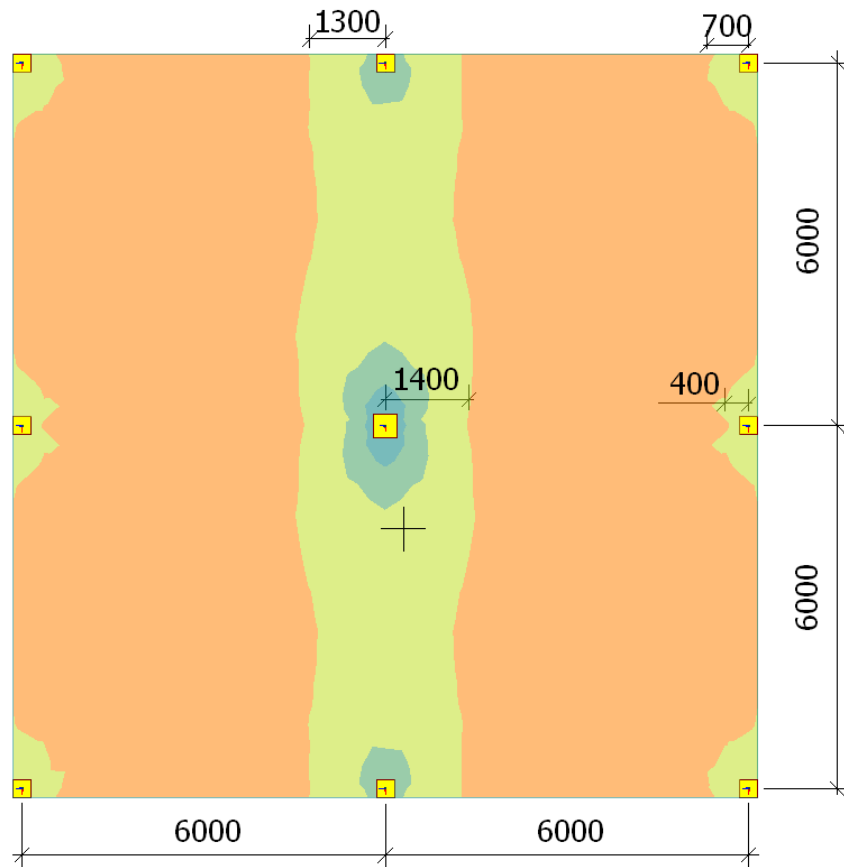


Figure I.1: Distances to the contra flexural locations for the parametric study



# Appendix J

## Explanation of Figures 6.1-6.4

Figures J.1-J.3 work as additional aid for understanding figures 6.1-6.4 in the parametric study section.

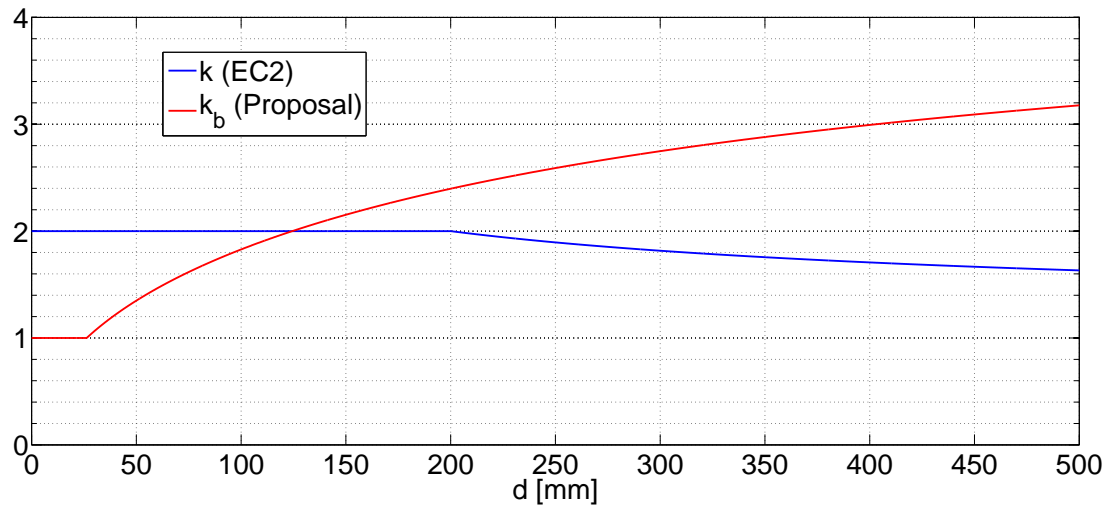


Figure J.1: Size effect factor  $k$  and shear gradient enhancement factor  $k_b$  as a function of the effective depth  $d$ .

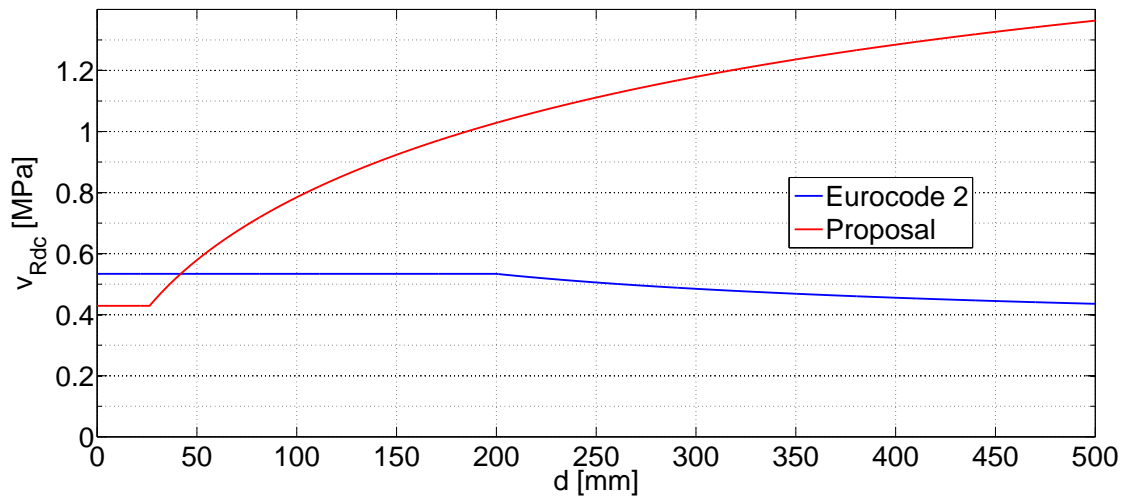
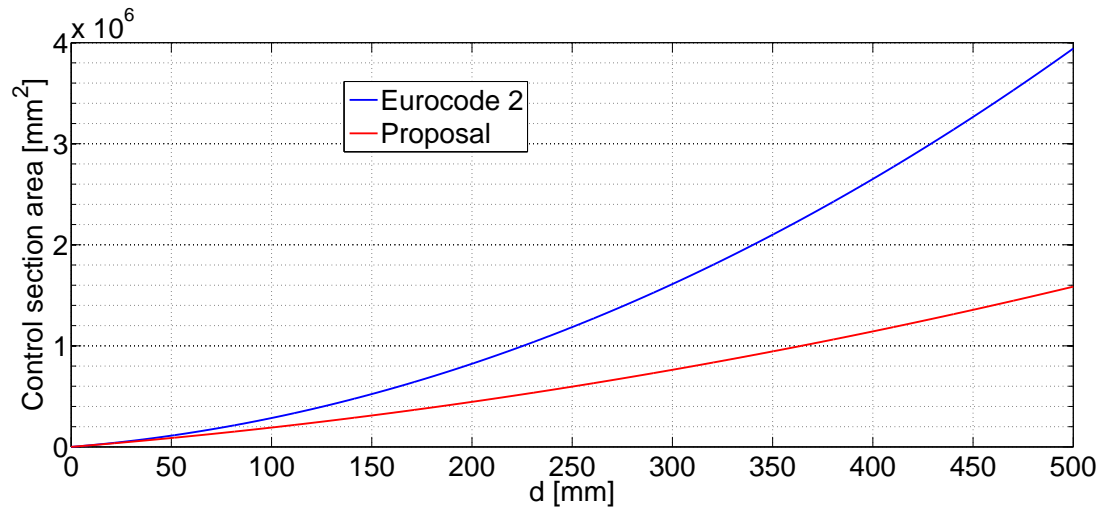


Figure J.2: Stress resistance as a function of the effective depth  $d$ Figure J.3: Control section area as a function of the effective depth  $d$ .