Determination of aerodynamic damping at high reduced frequencies

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high reduced frequencies**

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Abstract

Forced response which is blade vibration due to an external excitation can lead to blade failure. The estimation of the level of vibration is dependent on the determination of aerodynamic damping. This thesis investigates the determination of aerodynamic damping at high reduced frequencies in turbomachines. The aerodynamic damping was calculated by a linearized Navier-Stokes flow solver with exact 3D non-reflecting boundary conditions.

The method was validated using the two-dimensional test cases (Standard Configuration 5 and 8). Thereafter, two 3D profiles were also investigated: an aeroelastic turbine rig (AETR) which is a subsonic turbine case, and a virtual integrated compressor (VINK) which is a transonic compressor case. In AETR case, the first bending mode with reduced frequency 2.0 was studied. The 3D acoustic modes were calculated and the rate of decay was plotted as a function of nodal diameter and radial order. This plot identified six acoustic resonant points which included two points corresponding to the first radial order. The six resonance points correspond to six peaks in the damping curve. In VINK case, the fifth mode (1854 Hz, reduced frequency 3.1) was investigated. Acoustic resonance was predicted to occur for the first and second radial orders at the inlet. It was concluded that the higher order resonance points are influencing the damping curve. There were some inconsistencies in the results and grid convergence was not achieved. These inconsistencies were due to the difficulty in calculating the acoustic modes at the transonic inlet with an impinging shock.

Key words: Aerodynamic damping, reduced frequency, forced response, acoustic mode, acoustic resonance

Sammanfattning

Aerodynamiskt påtvingade vibrationer, som är bladvibrationer på grund av en extern excitation kan leda till haveri. Prediktering av vibrationen är beroende av bestämning av aerodynamisk dämpning. I detta arbete undersöks bestämningen av aerodynamisk dämpning vid höga reducerade frekvenser i turbomaskiner. Den aerodynamiska dämpningen beräknades genom en linjäriserad Navier-Stokes-lösare med exakta 3D icke-reflekterande gränsvillkor.

Metoden validerades med hjälp av de tvådimensionella testfallen (Standardkonfiguration 5 och 8). Därefter undersöktes två 3D-profiler: en aeroelastisk turbinrigg (AETR), som är en subsonisk turbinenhet och en virtuell integrerad kompressor (VINK) som är ett transoniskt kompressorfall. I AETR-fallet undersöktes det första böjningsformen med reducerad frekvens 2.0. 3D akustiska lägen beräknades och graden av förfall visades som en funktion av noddiameter och radiell grad. Denna metod identifierade sex akustiska resonanspunkter som innehöll två punkter som motsvarade den första radiella graden. De sex resonanspunkterna motsvarar sex toppar i dämpningskurvan. I VINK-fallet undersöktes den femte svängningsformen (1854 Hz, reducerad frekvens 3.1). Akustisk resonans förutspåddes inträffa för första och andra radiella graden vid inloppet. Slutsatsen drogs att de högre ordningens resonanspunkter påverkar dämpningskurvan. Det fanns vissa inkonsekvenser i resultaten och gridkonvergens uppnåddes inte. Dessa inkonsekvenser berodde på svårigheten att beräkna de akustiska svängningsformerna vid det transoniska inloppet med en stötvåg.

Nyckelord: Aerodynamisk dämpning, reducerad frekvens, aerodynamiskt påtvingade vibrationer, akustiskt svängningsform, akustisk resonans

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Nomenclature

Latin Symbols

b	blade length [m]
c	chord length [m]
$\bar{c}$	speed of sound [m/s]
C_p	unsteady pressure coefficient
f	frequency [Hz]
f_y	cut-off frequency [Hz]
h_{max}	maximum of blade mode shape amplitude [m]
I	identity matrix
j	-1
k	wavenumber in x-direction [m ⁻¹]
l	wavenumber in y-direction [m ⁻¹]
m	circumferential mode index
n	harmonic index
N	number of blades
p_0	total pressure [Pa]
p_1	static pressure [Pa]
p_{ref}	reference pressure [Pa]
$\tilde{p}$	unsteady pressure perturbation [Pa]
P	pitch [m]
r_m	mean radius [m]
R	time derivative of flow solution
t	time [s]
u	flow velocity
$\bar{u}$	flow speed in x-direction [m/s]
U	unsteady flow solution
$\bar{v}$	flow speed in y-direction [m/s]
w	local work coefficient
W_{aero}	aerodynamic work over one cycle [J]
x	grid point position [m]

Greek Symbols

α	non-dimensional amplitude of blade motion
μ	radial mode index
ρ	density [m ³ /s]
σ	interblade phase angle
ω	circular frequency
ω^*	reduced frequency based on full chord
Ξ	aerodynamic damping

Abbreviations

1F	first bending
1T	first torsion
2D	two dimensional
3D	three dimensional
AETR	aeroelastic turbine rig
CFD	computational fluid dynamics
EO	engine order
HCF	high cycle fatigue
IBPA	interblade phase angle
LUFT	linearized unsteady flow solver for turbomachinery
ND	nodal diameter
PS	pressure side
SS	suction side
STCF	standard configuration
VINK	virtual integrated compressor

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1. Introduction

1.1 Forced Response

Components in turbomachinery are always forced to vibrate under excitation, which is called forced response. Depending on the excitation level, the components might not withstand for surviving a required number of cycles and lead to a failure during operation, which is called high cycle fatigue (HCF).

This was the case that left engine damage in a SAAB Aircraft Co 340B aircraft. The damage was the result of the fatigue-related failure and separation of four of the stage 1 compressor blisk blade, as shown in Figure 1-1. The reason of the failures is reverse-bending fatigue, under the influence of high frequency aerodynamic vibrations (Australian Transport Safety Bureau, 2010).

Figure 1-1 Failed compressor blisk in a SAAB Aircraft Co 340B due to HCF (Australian Transport Safety Bureau, 2010)

In order to ensure safe and robust operation of engine with respect to HCF, forced response analysis is necessary.

Alternating stress

Vibrations of components and consequent alternating deformation lead to alternative stress, which in turn can lead to fatigue failure. The maximum tolerable alternating stress is a material property and dependent on the mean stress level as well as ambient conditions. According to Haigh diagram, for a larger level of mean stress the maximum tolerable alternating stress is smaller. If the alternating stress of a component is lower than the maximum, it means that the component is safe and can withstand a specific number of cycles, which is typically in order of 10^7 in turbomachinery.

Excitation forces

Flow in turbomachinery is spatial non-uniform due to wake, potential field, vortex, shock wave and so on. The spatial non-uniform flow in one frame of reference can lead to temporal unsteady flow in the other frame of reference. This kind of interaction exists between blade rows in turbomachinery, for example between stator and rotor. Excitation forces may come from interaction between blade rows, or any inhomogeneous distribution of flow parameter. The frequency of the excitation force is dependent on rotational speed and engine order (EO).

Structural dynamics

Natural frequency and mode shape of a structural component in turbomachinery can be determined through a free response analysis. Blade-alone mode shapes are usually referred to as 1st bending (1F), 1st torsion (1T) and so on. Disk-alone mode shapes are always referred to as nodal diameters (ND), which are the inflexion lines across disk. And bladed disk modes are combination of blade-alone modes and disk-alone modes.

If all blades are identical which referred to as tuned, they vibrate at the same frequency and mode so that a traveling wave mode is obtained. In the traveling wave mode, the phase lag between two adjacent blades is referred to as interblade phase angle (IBPA), which can be related to nodal diameter (ND).

$$\sigma = \frac{2\pi \cdot ND}{N}$$

where σ is the interblade phase angle, N is the number of the blades.

Resonance crossings

Figure 1-2 Campbell diagram illustration

When the frequencies of excitation forces coincide with the natural frequencies of the structure, resonance conditions are established, which can be identified as resonance crossings in Campbell diagram, as shown in Figure 1-2. The pick vibration amplitudes lead to peak alternating stresses, which in turn can lead to fatigue failure. It is close to impossible to avoid all resonance crossings while it is common to avoid crossings for low order modes. However, the occurrence of resonance at high frequencies is still possible.

1.2 Aerodynamic damping

The estimation of resonant vibration amplitude is done based on the level of the excitation forces and the overall damping level in the system, including aerodynamic damping and mechanical damping, as shown in Figure 1-3.

Figure 1-3 Effect of damping on oscillation amplitude during forced oscillation

Aerodynamic damping arises from interaction between blade vibration and flow response. The motion of the blade leads to unsteady pressure response. The resulting force on the blade surface, combined with the blade motion, can produce aerodynamic work. There might be a situation in which the phase of the force is leading the blade motion and destabilizing the system. It is referred to as flutter, when the aerodynamic damping is negative. If the aerodynamic damping is positive, the damping can regulate the vibration amplitude and the system is stable. One important parameter to measure flutter stability is so called reduced frequency ω^* , which is defined as

$$\omega^* = \frac{\omega c}{u}$$

where ω is the circular frequency, $\omega = 2\pi f$, c is the chord length and u is the flow velocity.

By assuming small harmonic unsteady response due to blade motion, traveling wave modes can be obtained. The aerodynamic damping can be calculated for every interblade phase angle, and plotted as damping curve versus interblade phase angle, or nodal diameter.

One of state-of-art methods to calculate aerodynamic damping is so called energy method. The aerodynamic work done by unsteady pressure on the blade surface over one cycle of blade oscillation is calculated. Positive aerodynamic work means energy added to the blade, indicating instability. And the aerodynamic damping is determined by normalizing the aerodynamic work.

$$\Xi = \frac{-W_{aero}}{\pi b h_{max}^2 p_{ref}}$$

where b is the blade length, h_{max} is the maximum of blade mode shape amplitude, and $p_{ref} = p_0 - p_1$, where p_0 is the average relative total pressure, and p_1 is the average static pressure at inlet.

2. Previous work

2.1 Standard Configurations

Since the “Fourth Aeroelasticity Symposium” in 1987, standard configurations have been used extensively by several researchers. Based on an updated report on standard configurations (Fransson and Verdon, 1991), the brief information of standard configurations has been summarized in Table 2-1, which shows basic flow parameters such as Mach number and reduced frequency for each standard configuration. Configurations with low reduced frequencies have been excluded, including fourth and sixth standard configurations. And it should be noted that only authors of experimental data have been listed in the table, due to much more theoretical results compared with experimental data.

Blade Profile	Theoretical	Experimental	Author	Year	Mach Number	ω^*
STCF1	√				0.17	0.144-1.206
		√	F.O. Carta	1982	..	0.144, 0.244, 0.302
STCF2	√				2.4 m/s	0.8
		√	Y. Hanamura H. Tanaka	1967	..	0.8
STCF3		√	H. Kobayashi	1990	0.68-1.39	0.056-0.914
STCF5	√	√	E. Széchényi	1980	0.5-1.0	0.28-2.04
STCF7	√	√	D.E. Riffel M.D. Rothrock	1980	1.315	0.86-0.9
STCF8	√				0.5-1.5	1 or 2
STCF9	√				0-1.5	1 or 2
		√	D.H. Buffum S. Fleeter	1989	0.65-0.8	0.37-1.114
STCF10	√				0.7 or 0.8	0.5, 1, 1.5, 2

Table 2-1 Summary of previous work on Standard Configurations

The first standard configuration is compiled from two dimensional compressor cascade experiments in low subsonic flow. The experiments were performed by F.O. Carta (Carta, 1982), in the linear low subsonic oscillating cascade at the United Technologies Research Center. All eleven blades oscillate in pitch mode and the reduced frequency varies from 0.144 to 0.302. Several prediction models have analysed this configuration for reduced frequency up to 1.206, and the theoretical results agree well with the experimental data.

The second standard configuration is a two dimensional compressor cascade in low subsonic flow, which has been measured in a water cascade tunnel at the University of Tokyo, by Y. Hanamura and H. Tanaka (Hanamura and Tanaka, 1967). The cascade consists of six stationary blades and eleven oscillating blades, which oscillate in pitch mode with reduced frequency at 0.8. However, no unsteady blade surface pressure has been measured, while damping and moment coefficients have been computed. Compared with theoretical results calculated by flat plate method (Smith, 1971), discrepancy exists especially for damping coefficient.

The third standard configuration consists of sixteen turbine blades in transonic flow, and has been tested in a Freon gas annular cascade test facility at the Tokyo National Aerospace Laboratory, by H. Kobayashi (Kobayashi, 1990). In the experiment, the reduced frequency was changed from 0.056 to 0.914, and the results show a good agreement with theoretical results calculated by D.S. Whitehead's finite element method (Whitehead and Newton, 1985)(Whitehead, 1990).

The fifth standard configuration is compiled from two dimensional compressor cascade experiments in transonic flow. The experiments were performed by E. Széchényi (Széchényi and Finas, 1981)(Széchényi and Girault, 1981), at the "Office Nationale d'Etudes et de Recherche Aérospatiale" (ONERA). In the experiment, only one blade vibrates with reduced frequencies from 0.28 to 2.04. The Mach number is between 0.5 and 1.0, and the incidence angle is between attached and fully separated flow. Several prediction models have analysed this configuration, and the difference between theoretical results and experimental data are larger at higher reduced frequency.

The seventh standard configuration is a compressor cascade in supersonic flow, and has been tested in linear air test facility at Detroit Diesel Allison, by R.E. Riffel and M.D. Rothrock (Riffel and Rothrock, 1980). In the experiment, the Mach number is 1.315 and the reduced frequency is from 0.86 to 0.90. However, it was difficult to keep experimental blade vibration amplitude and interblade phase angle constant between the 5 blades in the cascade, due to high frequency. So the agreement between experimental data and predictions is not good.

The eighth standard configuration deals with two dimensional cascade of flat plate, which is most common and important for validation of prediction models. Many

theoretical analyses have been performed in this configuration, with large variation of Mach number, 0.5 to 1.5. For high subsonic and supersonic flow, discrepancy has been found. The reduced frequency is either 1.0 or 2.0. High reduced frequency shows difference between different prediction models.

The ninth standard configuration is a continuation of the eighth, but includes thickness effect. The thickness/chord ratio varies from 0.01 to 0.10 for different Double Circular Arc profiles, and a few prediction models have analysed this configuration with Mach number ranging from 0 to 1.5, and reduced frequency fixed to 2.0. New cases with lower reduced frequency, 1.0, for the same configuration have been proposed by T.H. Fransson and J.M. Verdon (Fransson and Verdon, 1991). Experimental data has been provided by D.H Buffum and S. Fleeter, covering reduced frequencies from 0.37 to 1.114 at high subsonic flow (Buffum and Fleeter, 1989).

The tenth standard configuration is a two dimensional compressor cascade of modified NACA 0006 profile at subsonic and transonic flow. Two different steady state inlet operating condition have been chosen. For subsonic flow, the Mach number is 0.7 and flow angle is -55° , and for transonic flow, the Mach number is 0.8 and flow angle is -58° . For each of the two steady state inlet operating condition, reduced frequency is chosen among 0.5, 1.0, 1.5 and 2.0. Many theoretical analyses have been performed for this configuration, while no experimental data is available.

2.2 Recent work

S. Moffatt and L. He (Moffatt and He, 2003) have presented the methodology of an efficient frequency domain approach for prediction of blade forced response in industrial gas turbines. The interaction between fluid and structure is dealt with a loosely coupled manner, and the mode shape is interpolated from finite element analysis to the CFD mesh based on two-dimensional linear interpolation. The blade aerodynamic damping is calculated by solving linearized unsteady Navier-Stokes equations in frequency domain using a single passage approach. The unsteady blade modal force due to blade row interaction also can be determined. The NASA Rotor 67 transonic fan rotor has been investigated to demonstrate the forced response prediction methodology.

W. Ning and S. Moffatt (Ning *et al.*, 2003) have verified the forced response prediction methodology presented by S. Moffatt and L. He, through comparison of predicted blade surface pressure distributions with data measured on a VKI transonic turbine stage. Based on the verification, the last stage rotor blade of an ALSTOM three-stage transonic test compressor has been analysed. In the Campbell diagram shown in Figure 2-1, three resonant crossings of the rotor blade have been identified. The predicted mode shaped are shown in Figure 2-2. The aerodynamic damping, mechanical damping and modal force have been calculated. The resulting strains have been compared with strain gauge

measurements, and the errors of the prediction from measurements are 8%, 65% and 4% for mode 8, mode 9 and mode 10.

Figure 2-1 Campbell diagram for the last stage rotor in ALSTOM three-stage compressor (Ning et al., 2003)

Figure 2-2 Mode shape of the last stage rotor in ALSTOM three-stage compressor (Ning et al., 2003)

F. Figaschewsky and etc. (Beirow *et al.*, 2016)(Figaschewsky *et al.*, 2016) have introduced a simplified approach to estimate aerodynamic damping for high reduced frequency resonances at high inlet Mach number. For higher modes the difference between the minimum and maximum damping is often low and the mean value is a good approximation. So it calculates the mean value of aerodynamic damping based on steady state CFD solution, regardless of interblade phase angle due to low variation with interblade phase angle for high modes. Figure 1-4 shows comparison between CFD simulation and the simplified approach for an oscillating plane airfoil for inlet Mach number 0.3. And Figure 1-5 shows difference of calculated aerodynamic damping. The modified reduced frequency is the classic reduced frequency normalized by $(1-\text{Mach number}^2)$. It shows better agreement at high reduced frequency and for high inlet Mach number.

Figure 2-3 Unsteady pressure contours (top) compared with the approximation (bottom) (Figaschewsky et al., 2016)

Figure 2-4 Ratio of simplified estimated damping to damping derived from unsteady pressure (top) and resulting error (bottom) (Figaschewsky et al., 2016)

J. Li (Li, 2016) has analysed high order modes forced response problem for an embedded rotor in the Purdue three-stage axial compressor research facility. The Campbell diagram for the second stage rotor is shown in Figure 1-6. The first chordwise bending vibratory response to the second harmonic excitation EO88 has been investigated numerically and experimentally. A highly-efficient nonlinear frequency domain harmonic balance CFD code is used to predict the aerodynamic damping. Three operating points of the compressor have been investigated: Low loading, peak efficiency and high loading. The aerodynamic damping, modal force and response amplitude have been calculated and measured. Figure 2-6 shows predicted aerodynamic damping versus nodal diameter, while it is unreasonable that the curve is a smooth curve without any acoustic resonance

peaks at this high frequency. The comparison shows overall good agreement, while the percentages of difference between numerical and experimental results are 34.8%, 4.1% and 7.1% for three operating points, respectively.

Figure 2-5 Campbell diagram for the second stage rotor of Purdue three-stage compressor (Li, 2016)

Figure 2-6 Damping curve for the second stage rotor of Purdue three-stage compressor (Li, 2016)

3. Objectives and Approach

Components in turbomachinery are always forced to vibrate, and it is essential to ensure their safe and robust operation with respect to High Cycle Fatigue (HCF). The estimation of the resonant vibration amplitude is dependent on the determination of aerodynamic damping. Reliable prediction of aerodynamic damping is required, especially for potential resonance at high reduced frequencies. Until now, previous work on this problem is very limited and no validation is available.

It is challenging to predict aerodynamic damping accurately at high reduced frequencies, due to reflections from far-field and requirement of mesh. The acoustic modes are more likely to be cut-on at high reduced frequencies, and the reflections from far-field are more likely to pollute unsteady flow simulation if the acoustic modes are cut-on. And the requirement of mesh is dependent on the acoustic wavelength. Therefore, it is necessary to identify cut-on and cut-off acoustic modes at far-field, and determine the requirement of number of cells per wavelength.

A linearized unsteady flow solver for turbomachinery (LUFT) with exact 3D non-reflecting boundary conditions, has been employed. It has been validated first, by predicting local unsteady pressure distribution for two-dimensional test cases, standard configuration 5 and 8, and comparing with reference data. Further analysis has been done for standard configuration 8 with reduced frequency up to 16.0, including mesh convergence analysis, far-field location analysis and acoustic modes analysis.

Based on the validation, two 3D profiles have been selected to investigate in order to predict aerodynamic damping at high reduced frequencies. The first one is the aeroelastic turbine rig (AETR), which is a subsonic turbine case, and the other one is the virtual integrated compressor (VINK), which is a transonic compressor case. 3D acoustic modes have been calculated with 3D non-reflecting boundary conditions on a low resolution far-field mesh, and the rate of decay was plotted as a function of nodal diameter and radial order. The acoustic resonance points have been identified and compared with peaks in the damping plots. It is a good way to demonstrate that different peaks correspond to different resonance points due to different radial order acoustic modes.

4. Acoustic modes analysis

4.1 Introduction of Acoustic modes

The harmonic vibration of blades can generate acoustic waves propagating up and down in the turbomachinery. The acoustic waves either decay as they propagate, which is called cut-off, or maintain amplitudes as they propagate, which is called cut-on.

Circumferential modes

The circumferential mode index m is dependent on the nodal diameter of the traveling wave mode, as well as harmonic index, n , of scatter modes which means higher circumferential modes other than the fundamental one. Therefore, the circumferential mode index m can be calculated in a similar manner as Tyler-Sofrin formula (Tyler and Sofrin, 1962).

$$m = ND + n \cdot N$$

where m is the circumferential mode index, ND is the nodal diameter, n is the harmonic index, $n = -\infty, \dots, -1, 0, +1, \dots, +\infty$, and N is the number of the blades.

One important parameter to identify cut-on or cut-off modes is S , which is calculated as follows.

$$S = \sqrt{(\omega + \bar{v}l)^2 - l^2(\bar{c}^2 - \bar{u}^2)}$$

where ω is the circular frequency, $\bar{v}$ is the flow velocity in circumferential direction, $\bar{u}$ is the flow velocity in axial direction, $\bar{c}$ is speed of sound and l is the wavenumber in circumferential direction, which can be related to circumferential mode index, by mean radius r_m .

$$l = \frac{m}{r_m}$$

The sign of S^2 indicates whether the acoustic mode is cut-on or cut-off. If $S^2 > 0$, the mode is cut-on, else it is cut-off. The physical meaning can be easily understood by simplifying the equation for still flow, where $\bar{u} = 0$, $\bar{v} = 0$. Therefore, the equation can be rewritten

$$S = \sqrt{\omega^2 - l^2\bar{c}^2} = 2\pi\sqrt{f^2 - f_y^2}$$

where f_y is the frequency of the circumferential mode traveling with speed of sound, which is so called cut-off frequency.

When the driven frequency f is higher than cut-off frequency f_y , the acoustic mode is cut-on and can maintain amplitude as propagating. And when the driven frequency f is less than cut-off frequency f_y , the acoustic mode is cut-off.

Besides, the infinite scatter modes in circumferential direction are always cut-off for subsonic flow, while they are always cut-on for supersonic flow, which can be determined as follows.

$$\lim_{l \rightarrow \infty} S = \sqrt{l^2 [\bar{u}^2 + \bar{v}^2 - \bar{c}^2]}$$

Therefore, normalized wave amplitude at a certain distance from the blades can be determined, and plotted versus different circumferential mode index, showing different patterns for subsonic flow and supersonic flow, as shown in Figure 4-1. The value of normalized wave amplitude indicates whether the acoustic mode is cut-on or cut-off. The value 1 means the mode does not decay and is cut-on, and lower value indicates the mode would decay more rapidly. For subsonic flow, the acoustic modes are more likely to be cut-on with increased frequency, and reflections of these cut-on modes are more likely to affect the accuracy of unsteady simulation. For supersonic flow, the modes are more likely to be cut-off at high frequencies.

Figure 4-1 Normalized wave amplitude at different frequencies for subsonic (left) and supersonic (right) flow

Radial modes

All the above discussions are based on an assumption that radial mode index $\mu = 0$. The radial mode index μ gives the number of modes in radial direction. Higher radial modes have higher cut-off frequencies, so that the modes are more likely to be cut-off. The normalized wave amplitude is plotted for different radial modes, as shown in Figure 4-2. For subsonic flow, $\mu = 0$ modes decay at high circumferential modes, and higher radial modes decay more rapidly so that become insignificant. The radial modes might be cut-on, especially for low radial modes, if the driven frequency is high enough and higher than the cut-off frequency. For supersonic flow, the higher radial modes are more likely to be cut-off, while they always become cut-on at high circumferential modes. Therefore, there are theoretically infinite transition points between cut-on and cut-off in supersonic flow, while the number of transition points is limited in subsonic flow.

Figure 4-2 Normalized wave amplitude at different radial modes for subsonic (left) and supersonic (right) flow

Acoustic resonance

There might be a situation in which the acoustic wave travels only in circumferential direction, and there is no axial component in acoustic speed, which is referred to as acoustic resonance. In this kind of situation, the acoustic energy can not escape and it might cause peaks in aerodynamic damping curve.

The acoustic resonance occurs at the transition between cut-on and cut-off acoustic modes, which means the driven frequency is equal to cut-off frequency. The transition points can be identified in figures where the normalized wave amplitude plotted versus circumferential mode index, such as Figure 4-1, Figure 4-2.

At high frequencies, the acoustic modes are more likely to be cut-on in subsonic flow and there might be more than 2 acoustic resonance points, which in turn affect aerodynamic damping. For supersonic flow, there are theoretically infinite acoustic resonance points which makes it challenging to predict aerodynamic damping accurately.

4.2 Calculation of 2D acoustic modes

The acoustic modes for two dimensional flow can be solved theoretically (Giles, 1990), based on the linearized two dimensional Euler equations.

$$\frac{\partial \tilde{V}}{\partial t} + A \frac{\partial \tilde{V}}{\partial x} + B \frac{\partial \tilde{V}}{\partial y} = 0$$

where

$$\tilde{V} = \begin{bmatrix} \tilde{\rho} \\ \tilde{u} \\ \tilde{v} \\ \tilde{p} \end{bmatrix}, A = \begin{bmatrix} \bar{u} & \bar{\rho} & 0 & 0 \\ 0 & \bar{u} & 0 & \frac{1}{\bar{\rho}} \\ 0 & 0 & \bar{u} & 0 \\ 0 & \bar{\rho}\bar{c}^2 & 0 & \bar{u} \end{bmatrix}, B = \begin{bmatrix} \bar{v} & 0 & \bar{\rho} & 0 \\ 0 & \bar{v} & 0 & 0 \\ 0 & 0 & \bar{v} & \frac{1}{\bar{\rho}} \\ 0 & 0 & \bar{\rho}\bar{c}^2 & \bar{v} \end{bmatrix}$$

The coefficient matrices A and B are dependent on the steady state flow solution, which is assumed to be uniform steady flow, where ρ is density, $\bar{u}$ is flow speed

in x direction, $\bar{v}$ is flow speed in y direction, and $\bar{c}$ is speed of sound. $\tilde{V}$ is the unsteady flow perturbation, and a wave-like solution can be assumed for $\tilde{V}$.

$$\tilde{V} = V_m e^{j(kx + ly + \omega t)}$$

where k is wavenumber in x direction, which is unknown, $\omega = 2\pi f$, where f is the blade frequency, and l is wavenumber in y direction, determined by interblade phase angle and harmonic index.

$$l = \frac{\sigma + 2\pi n}{P}$$

where σ is interblade phase angle, n is the harmonic index in pitchwise direction and P is the pitch.

Therefore, the Euler equations can be rewritten and transformed to an eigenvalue problem.

$$(\omega I + kA + lB)V_m = 0$$

By prescribing the blade frequency and interblade phase angle, ω and l are known and wavenumber in x direction k can be solved.

$$\begin{aligned} k_1 &= k_2 = -(\omega + \bar{v}l)\bar{u} \\ k_3 &= \frac{(\omega + \bar{v}l)\bar{u} - \bar{c}S}{\bar{c}^2 - \bar{u}^2} \\ k_4 &= \frac{(\omega + \bar{v}l)\bar{u} + \bar{c}S}{\bar{c}^2 - \bar{u}^2} \end{aligned}$$

where

$$S = \sqrt{(\omega + \bar{v}l)^2 - l^2(\bar{c}^2 - \bar{u}^2)}$$

The wavenumber k_1 represents entropy wave which can only perturb entropy and k_2 represents vorticity wave which can perturb velocity, and k_3, k_4 represent acoustic waves propagating downstream and upstream respectively. If S is real and k_3, k_4 are also real, the acoustic waves are cut-on, which means the waves do not decay and maintain a constant amplitude. If S is complex and k_3, k_4 have imaginary parts, it indicates that the acoustic waves are cut-off, and their amplitude decay.

5. Numerical Prediction of Aerodynamic Damping

5.1 Description of Numerical Method

The numerical method used is a linearized unsteady flow solver for turbomachinery, so called LUFT, developed by Paul Petrie-Repar (Petrie-Repar and Carstens, 2004)(Petrie-Repar, 2006). Conventional time-marching method is computational expensive, so this efficient time linearized method has been chosen.

The flow equations can be transformed to frequency domain, by assuming small harmonic unsteady flow perturbations.

$$x = \bar{x} + \Re\{\tilde{x}e^{j\omega t}\}$$

$$U = \bar{U} + \Re\{\tilde{U}e^{j\omega t}\}$$

where x is the grid point position, and U is the unsteady flow solution.

The flow equations can be discretized using a cell-centered finite volume scheme, and the discretized time derivative of flow solution can be written as follows.

$$\frac{dU}{dt} = R(U, x, \dot{x})$$

And it can be linearized by performing a first-order Taylor expansion about the steady state flow values

$$R \approx \bar{R} + \frac{\partial R}{\partial U} \Delta U + \frac{\partial R}{\partial x} \Delta x + \frac{\partial R}{\partial \dot{x}} \Delta \dot{x}$$

where the time derivative of flow solution at steady state $\bar{R}$ is zero.

The resulting linearized flow equation can be written

$$\left[j\omega I - \frac{\partial R}{\partial U} \right] \tilde{U} \approx R(\bar{U}, \bar{x} + \tilde{x}, 0) + j\omega R(\bar{U}, \bar{x}, \tilde{x})$$

which can be solved by a linear solver to determine the unsteady flow perturbation.

Based on the unsteady flow perturbation, the aerodynamic work is defined as work done by unsteady pressure on blade over one cycle of oscillation, and can be calculated by integrating the pressure and blade displacement. And the aerodynamic damping is defined by normalizing the aerodynamic work.

$$\Xi = \frac{-W_{aero}}{\pi b h_{max}^2 p_{ref}}$$

where b is the blade length, h_{max} is the maximum of blade mode shape amplitude, and $p_{ref} = p_0 - p_1$, where p_0 is the average relative total pressure, and p_1 is the average static pressure at inlet.

5.2 Validation of Numerical Method

5.2.1 Comparison with reference data

Standard Configuration 8 test case

Figure 5-1 Eighth Standard Configuration: Cascade geometry (Bölcs and Fransson, 1986)

The cascade being investigated as the eighth standard configuration is shown in Figure 3-1. The test case parameters are shown in Table 5-1. D. Whitehead has presented results of this test case, by his linear unsteady cascade finite element potential flow model, called "FINSUP" (Whitehead and Newton, 1985)(Whitehead, 1990). J. Verdon has also applied his linearized unsteady aerodynamic analysis, called "LINFLO" (Verdon and Usab, 1986), to this test case. The results have been compared with LUFT. In this case, linearized Euler equations have been solved and two-dimensional non-reflecting boundary conditions (Giles, 1990) were applied at inlet and outlet.

Inlet Mach number	0.8
Pitch Ratio	0.75
Stagger angle	60°
Inlet angle	60°
Mode shape	Torsion
Torsion axis (x, y)	(0.5, 0.0)
Reduced Frequency ω^*	2.0
IBPA	90°

Table 5-1 Eighth Standard Configuration: Test case parameters

Figure 5-2 Eighth Standard Configuration: Imaginary unsteady pressure coefficient distribution ($\omega^*=2.0$, $IBPA=90^\circ$)

Figure 3-2 shows imaginary part of unsteady blade surface pressure distribution when reduced frequency ω^* is 2.0 and interblade phase angle is 90° , comparing results from LUFT with D. Whitehead and J. Verdon. The non-dimensional unsteady pressure coefficient C_p is defined as

$$C_p = \frac{\tilde{p}}{\alpha p_{ref}}$$

where $\tilde{p}$ is unsteady pressure, $p_{ref} = p_0 - p_1$, where p_0 is the average relative total pressure, and p_1 is the average static pressure at inlet, and α is non-dimensional amplitude of blade motion, which is defined as

$$\alpha = \frac{h_{max}}{c}$$

where h_{max} is the maximum of blade mode shape amplitude, and c is the chord length.

It is needed to mention that mesh 60×30 was used, which is fine enough for this case, and details about convergence and mesh independence will be given. The plot shows overall good agreement among different methods, while there are small differences, which is most likely due to the fact that the other two methods may not be mesh converged at such high reduced frequency. The overall agreement indicates that LUFT solves inviscid Euler equations correctly and the non-reflecting boundary conditions have been applied correctly.

Standard Configuration 5 test case

Figure 5-3 Fifth Standard Configuration: Overview of mesh

The test case investigated here is Standard Configuration 5 which is a two dimensional compressor cascade. The mesh is shown in Figure 3-3, which has been generated based on Double-J structure. The y^+ near wall is kept as around 1 in order to resolve the boundary layer, and turbulence model Spalart and Allmaras has been used. In this case, linearized Reynold's averaged Navier-Stokes equations have been solved and two-dimensional non-reflecting boundary conditions of Giles (Giles, 1990) were applied at inlet and outlet. The test case parameters have been shown in Table 5-2.

Inlet Mach number	0.5
Pitch Ratio	0.95
Stagger angle	59.3°
Inlet angle	63.3°
Mode shape	Torsion
Torsion axis (x, y)	(0.5, 0.0)
Reduced Frequency ω^*	2.04
IBPA	180°

Table 5-2 Fifth Standard Configuration: Test case parameters

The original experiments were performed by E. Széchenyi in 1980s (Széchenyi and Finas, 1981)(Széchenyi and Girault, 1981), at the "Office Nationale d'Etudes

et de Recherche Aérospatiale" (ONERA). He found that theoretical results do not agree well with experimental data, especially at high reduced frequencies. The difference between theoretical and experimental results become bigger with increased frequencies.

The steady pressure coefficient distribution is shown in Figure 5-4, and Mach number contour is plotted in Figure 5-5. The steady pressure coefficient is defined as steady pressure normalized by reference pressure, p_{ref} . The steady state results show good agreement between LUFT and experimental data, both on pressure side and suction side. The reason for small difference is most likely the accuracy of turbulence model. The Mach number contour shows clearly the boundary layer around the blade and the wake, and also indicates the incidence angle which is 4° in this case. And there is no separation and the flow is attached.

Figure 5-4 Fifth Standard Configuration: Steady pressure coefficient distribution

Figure 5-5 Fifth Standard Configuration: Mach number contour

Figure 5-6 shows distribution of imaginary unsteady blade surface pressure coefficient. D. Whitehead (Whitehead and Newton, 1985)(Whitehead, 1990) and J. Verdon (Verdon and Usab, 1986) have provided their own theoretical results, which are also included in the figure, showing bad agreement between theoretical results and experimental data. It should be noticed that at low reduced frequencies, such as 0.28, the methods from D. Whitehead and J. Verdon can predict accurate unsteady results as experiments did, especially when the incidence angle is low (Bölcs and Fransson, 1986). In this case presented here, the reduced frequency is 2.04, which is the highest reduced frequency in the experiments, and the difference between theoretical and experimental results is the biggest. The aerodynamic damping predicted by D. Whitehead and J. Verdon are 1.35 and 1.54, respectively, while in experiments the value is 1.23. One of the reasons may be the implementation of non-reflecting boundary conditions, and the other may be the mesh convergence, because it requires finer mesh at higher frequencies due to the resulting shorter wavelength.

Compared results from LUFT with others, it shows better agreement between LUFT and experimental data. Although the difference exists, which may be due to discrepancy of steady state results. The implemented non-reflecting boundary conditions are two dimensional Giles, the same as in Standard Configuration 8, which has been validated. The overall agreement demonstrates the linearized Navier-Stokes equations have been solved correctly.

Figure 5-6 Fifth Standard Configuration: Imaginary unsteady pressure coefficient distribution ($\omega^*=2.04$, $IBPA=180^\circ$)

5.2.2 Mesh convergence analysis

In order to study convergence and mesh independence, eight different mesh cases have been simulated for Standard Configuration 8, which is a two dimensional flat plate, as shown Table 5-3. The steady flow is the same as shown before, which means the inlet Mach number is 0.8 and flow angle is 60° . The mode shape is a rigid body pitching mode and the interblade phase angle is 90° . For four different reduced frequencies ω^* , 2.0, 4.0, 8.0 and 16.0, the calculated aerodynamic damping for each mesh case have been plotted in Figure 5-7.

Case	Mesh
1	30×15
2	60×30
3	120×60
4	180×90
5	240×120
6	300×150
7	360×180
8	420×210

Table 5-3 Different mesh cases for Eighth Standard Configuration

Figure 5-7 Mesh independence analysis for Eighth Standard Configuration

Figure 5-7 shows the value of aerodynamic damping becomes stable when mesh becomes finer, which indicates good convergence for all reduced frequencies. For lower reduced frequencies, such as 2.0 and 4.0, the aerodynamic damping is converged as long as mesh is finer than 60×30 , while for higher reduced frequencies, 8.0 and 16.0, mesh should be finer than 120×60 in order to get converged results. Combined with acoustic wavelength analysis, the requirement for number of cells per wavelength can be concluded at high reduced frequency.

Figure 5-8, Figure 5-9, Figure 5-10 and Figure 5-11 show local work coefficient distribution for several chosen mesh cases at different reduced frequencies, 2.0, 4.0, 8.0 and 16.0, respectively. Local work coefficient can be determined based on local unsteady pressure and local blade motion, and the integration of local work coefficient can be related to aerodynamic work as follows.

$$\int w dA = \frac{W_{aero}}{\pi \alpha^2 c p_{ref}}$$

where w is the local work coefficient, A is the blade surface, W_{aero} is the aerodynamic work, α is non-dimensional amplitude of blade motion, c is the chord length, and p_{ref} is the reference pressure.

Figure 5-8 shows mesh 30×15 can not resolve unsteady pressure accurately, especially near the leading edge and trailing edge, while mesh 60×30 can almost get same local work coefficient distribution as finer mesh. Figure 5-9 also shows mesh 60×30 is required to resolve unsteady pressure field accurately for reduced frequency 4.0.

Figure 5-8 Eighth Standard Configuration: Local work coefficient distribution for different meshes ($\omega^*=2.0$, $IBPA=90^\circ$)

Figure 5-9 Eighth Standard Configuration: Local work coefficient distribution for different meshes ($\omega^*=4.0$, $IBPA=90^\circ$)

Figure 5-10 and Figure 5-11 indicate that mesh 60×30 can not resolve unsteady pressure field accurately, especially near the leading edge and trailing edge. Therefore, it is needed to apply mesh 120×60 to predict aerodynamic damping accurately.

Figure 5-10 Eighth Standard Configuration: Local work coefficient distribution for different meshes ($\omega^*=8.0$, $IBPA=90^\circ$)

Figure 5-11 Eighth Standard Configuration. Local work coefficient distribution for different meshes ($\omega^*=16.0$, $IBPA=90^\circ$)

In Figure 5-12, aerodynamic damping at highest reduced frequency $\omega^* = 16.0$ has been plotted versus interblade phase angle. All mesh cases can capture overall shape of the curve and predict the acoustic resonance point accurately. However, mesh 30×15 and 60×30 can not determine aerodynamic damping accurately, while mesh 120×60 and 180×90 lead to exact same results for all interblade phase angle. It demonstrates that mesh 120×60 is fine enough for reduced frequency 16.0.

Figure 5-12 Eighth Standard Configuration: Mesh convergence analysis ($\omega^*=16.0$)

5.2.3 Far-field location analysis

Based on Standard Configuration 8, analysis on far-field location has been done at highest reduced frequency $\omega^* = 16.0$. The highest reduced frequency has been chosen because the acoustic waves are more likely to be cut-on at high frequency and the reflections of cut-on modes from far-field are more likely to pollute the unsteady solution.

In the original case, the far-field is one axial chord length away from blade both at inlet and outlet. A new case with half chord length away for far-field location has been simulated and compared with the original one, as shown in Figure 5-13. The agreement between different far-field location demonstrates that the non-reflecting boundary conditions have been implemented correctly both at inlet and outlet.

Figure 5-13 Eighth Standard Configuration: Far-field location analysis ($\omega^*=16.0$)

5.2.4 Acoustic analysis

The value $e^{-\text{imag}(k) \cdot 0.4c}$ has been chosen to show normalized wave amplitude at 40% chord length away from blade. If the value equals to 1, it means that the acoustic mode does not decay and it is cut-on, and if it is lower than 1, it means cut-off. Figure 5-14 shows normalized wave amplitude at 40% chord length away versus interblade phase angle, for different reduced frequencies. The left one is the result when harmonic index n is -1, which means scatter mode or higher circumferential mode. And the right one is for harmonic index n equals to 0. Compared with four different reduced frequencies, it shows that the acoustic modes are more likely to be cut-on at high reduced frequencies.

Figure 5-14 Eighth Standard Configuration: 2D acoustic modes analysis

The transition points between cut-on and cut-off are acoustic resonance points. The acoustic resonance has influence on aerodynamic damping, as shown in Figure 5-15. Each damping curve has one acoustic resonance point, and it can correspond to the transition points in Figure 5-14. For example, acoustic resonance can be identified at around -85° interblade phase angle for reduced frequency 4.0, due to the peak in the damping curve, and the transition point for reduced frequency 4.0 from two dimensional acoustic analysis is also around -85° . For the highest reduced frequency 16.0, all fundamental circumferential acoustic modes are cut-on, while there is a transition point when harmonic index n equals -1. And the transition point locates at around 20° interblade phase angle, which can correspond to the acoustic resonance point in the damping curve. It means the acoustic resonance resulting from scatter mode can still have influence on aerodynamic damping, especially at high frequencies.

Figure 5-15 Eighth Standard Configuration: Aerodynamic damping for all reduced frequencies

Figure 5-16 Eight Standard Configuration: Acoustic wavelength versus reduced frequency

Based on calculated wavenumber, the wavelength can be determined for different interblade phase angles and different reduced frequencies. In Figure 5-16 the calculated wavelength is plotted versus reduced frequency, when the interblade phase angle is 90° , both for waves going upstream and downstream. It indicates the acoustic waves going upstream always have shorter wavelength, which would be critical at high frequency. In Figure 5-17 the acoustic wavelength is plotted versus interblade phase angle, for the highest reduced frequency 16.0. It shows that the upstream acoustic wavelength at interblade phase angle 180° is shortest,

which is 0.2454 m. The shortest wavelengths for other reduced frequencies have also been calculated, which is 0.4328 m, 0.7374 m and 1.2710 m for reduced frequency 8.0, 4.0 and 2.0, respectively. Combined with mesh convergence analysis, it can be concluded that the recommended mesh requirement is 20 cells per wavelength.

Figure 5-17 Eighth standard configuration: Acoustic wavelength versus interblade phase angle

6. Aeroelastic Turbine Rig test case

6.1 Introduction

Aeroelastic Turbine Rig (AETR) is a test facility at KTH, for experimentally investigating aerodynamic damping in turbomachines. The AETR has been designed by Damian Vogt during his PhD studies (Vogt, 2005). It is world-unique as it allows studying the aeroelastic phenomena at relevant operating parameters while providing a great experimental freedom. Figure 6-1 depicts the blade and profile sections at various height.

Figure 6-1 AETR blade profile (Vogt, 2005)

In this numerical study, it is assumed that there are 80 blades in the row and the row is not rotating. The inlet total pressure is around 160 kPa which varies with radius, and the outlet static pressure is 109.1 kPa so that the outlet Mach number is kept as 0.75. Inlet flow angle is fixed at -26° and inlet total temperature is 303 K, and first bending mode has been investigated at different reduced frequencies.

6.2 Description of method

6.2.1 CFX Transient Blade Row Modeling

The simulations have been carried out on ANSYS CFX, using transient blade row modeling, with and without Fourier transformation method. Conventional method without Fourier transformation applies standard periodic boundary conditions, and simulating different IBPA requires different number of passages. It makes this method very costly. For modeling with Fourier transformation method, it applies phase shifted boundary conditions, so that two instances of passage are needed in the simulation. The data is collected at the interface between two passages and reconstructs flow solution at the periodic boundaries using the accumulated Fourier coefficients (He, 1989).

The non-reflective boundary condition in ANSYS CFX is developed based on characteristic analysis by Poinso and Lele in 1992, and Thompson in 1990 (Poinso and Lele, 1992)(Thompson, 1990). They converted the standard conservation equations for compressible flow to characteristic form, and dropped the transverse and viscous terms. The resulting system of equations describes the amplitude variation of the characteristic waves, which is referred to as the Locally One Dimensional Inviscid (LODI) system. And CFX solves this system of equations for all non-reflecting values on the boundary, but uses only the non-reflecting pressure values as a physical condition.

6.2.2 LUFT 3D non-reflecting boundary condition

The test case has also been carried out using LUFT code, by solving linearized Euler equations. A three dimensional non-reflecting boundary condition has been implemented at inlet and outlet (Petrie-Repar, 2010). The aerodynamic modes at the inlet and outlet can be determined by solving semi-discretized flow equations on a two dimensional far-field mesh with a lower resolution in order to save computational resource. One of the unique features of this methodology is that the steady flow can be non-uniform. Based on the calculated modes, the unsteady flow can be decomposed and the direction of each mode can be determined. The non-reflecting boundary condition is applied by prescribing the incoming modes and extrapolating the outgoing modes.

Figure 6-2 Aeroelastic turbine rig: Acoustic mode $(+20, 1)$ calculated with 3D non-reflecting boundary condition ($\omega^*=2.0$)

One example of far-field acoustic mode at outlet for aeroelastic turbine rig test case is shown in Figure 6-2. The real part of the pressure perturbation for first bending at reduced frequency $\omega^* = 2.0$ is plotted. The far-field mesh is uniform with a resolution of 15×15 cells. The incoming circumferential wave number is prescribed as +20. It can be identified the mode is $(m, \mu) = (+20, 1)$, where the +20 is for circumferential direction and 1 is for radial direction. The imaginary part of axial wave number is around 0 so that the mode is cut-on.

By prescribing incoming circumferential wavenumber and identifying acoustic modes, different radial modes for each nodal diameter can be determined. Figure 6-3 shows normalized wave amplitude at 40% chord length away versus nodal diameter, comparing between analytical solution from two dimensional calculation and numerical solution from the three dimensional calculation, where μ represents radial mode. The 2D analytical solution agrees well with the fundamental mode from 3D calculation, which can validate the accuracy of the 3D calculation, while there is no 2D solution for higher radial modes. In this case, the first radial modes are always cut-off for all nodal diameters, and other higher modes decay more rapidly and become insignificant. With increased frequency, the modes are more likely to be cut-on, so it is interesting and useful to plot this type of figure to show cut-on/cut-off features for high radial modes and the transition points between cut-on and cut-off modes, which may cause peaks in aerodynamic damping curve.

Figure 6-3 Aeroelastic turbine rig: Acoustic modes at outlet ($\omega^*=0.3$)

6.3 Results

6.3.1 Steady state

Comparison between CFX and LUFT

Figure 6-4 Comparison of steady loading in CFX and LUFT at midspan of AETR

Steady loading obtained from CFX and LUFT agree well with each other, as shown in Figure 6-4. Both pressure side and suction side show good agreement between results from CFX and LUFT. It should be noticed that turbulent model $k-\epsilon$ has been used in CFX, which applies wall function, while in LUFT the simulation is inviscid. The difference loading near trailing edge is due to wake. And Figure 6-5 shows a contour plot of Mach number at midspan, indicating subsonic flow in the turbine rig, and the outlet Mach number is around 0.75.

Figure 6-5 Mach number contour at midspan of AETR

Mesh convergence analysis

In order to use numerical method confidently, it is necessary to show the convergence with increasing mesh density. In this case, a set of two different meshes has been generated in TurboGrid. The two meshes are referred to as “coarse” and “fine”. Both meshes used O-H multiblock structure with an O-domain around the blade and H-domains in the blade-to-blade passage and inlet and outlet section respectively. An overview of mesh parameters is included in Table 6-1. And midspan sections for two meshes are shown in Figure 6-6.

Mesh name	Number of cells	Cells along span	First cell y^+
Coarse	110040	30	100
Fine	508340	70	160

Table 6-1 AETR mesh parameters

Figure 6-6 AETR coarse (left) and fine (fine) meshes at midspan

Figure 6-7 shows steady loading near hub, at midspan and near shroud for “coarse” and “fine” meshes. The steady loading curves agree well between two meshes, while there are minor differences, especially around suction peak near shroud. The comparison suggests converged results with increased mesh resolution.

Figure 6-7 Steady loading of AETR blade for different mesh (10%, 50%, 90% span)

6.3.2 Unsteady state ($\omega^* = 1.0$)

Reduced frequency $\omega^* = 1.0$ has been simulated both in ANSYS CFX and LUFT. In CFX, two different methods have been applied, which are time domain method and Fourier transformation method. The convergence of simulation becomes worse when reduced frequency increases, especially for CFX with Fourier transformation. Therefore, for higher reduced frequency $\omega^* = 2.0$, only results from LUFT are presented.

Comparison between CFX and LUFT

Figure 6-8 shows aerodynamic damping curve, comparing results from CFX and LUFT. For CFX with Fourier transformation, average value of damping calculated from two blades has been plotted. It shows good agreement between Fourier transformation method and time domain method in CFX. However, difference exists between results from CFX and LUFT, especially for negative nodal diameter. One of the reasons is the bad convergence in CFX, where the aerodynamic damping differs for blades.

Figure 6-8 Aeroelastic turbine rig: Aerodynamic damping versus Nodal Diameter ($\omega^*=1.0$)

The biggest difference between LUFT and CFX Fourier transformation located around nodal diameter -16. The local work coefficient is plotted in Figure 6-9. It shows difference exists between two blades in CFX Fourier transformation method, which indicates the solution is not well converged. The negative local work is mostly done on the concave side, which is the pressure side, while the positive work is mostly done on the convex side, which is the suction side. Compared with results from LUFT, it shows similar overall distribution of local work while CFX predicts more negative local work coefficient on the pressure side so that in CFX it is more stable and the aerodynamic damping is higher.

Figure 6-9 Aeroelastic turbine rig: Local work coefficient at 90% span ($\omega^*=1.0$, $ND=-16$)

For CFX time domain method, the comparison of local work coefficient at 90% span for nodal diameter -20 is plotted in Figure 6-10. It should be noticed that there are four passages and blades for nodal diameter -20 in CFX time domain method and the local work coefficient distributions for four blades are so similar that only one is plotted here. It shows good agreement on the pressure side while LUFT predicts more positive local work on the suction side, which makes damping lower compared with CFX.

Figure 6-10 Aeroelastic turbine rig: Local work coefficient at 90% span ($\omega^*=1.0$, $ND=-20$)

Mesh convergence analysis

Coarse and fine meshes have been simulated for nodal diameter -16 and the local work coefficient distributions of two meshes are plotted in Figure 6-11. Good agreement indicates that it is mesh converged.

Figure 6-11 Aeroelastic turbine rig: Local work coefficient at 90% span ($\omega^*=1.0$, $ND=-16$)

3D acoustic mode analysis

Acoustic modes have been calculated with 3D non-reflecting boundary conditions, based on a low resolution far-field mesh 15×15 . Different radial modes have been identified and the normalized wave amplitude at 40% chord length away is plotted versus nodal diameter in Figure 6-12. Only two radial modes $\mu = 0, 1$ are plotted at inlet and outlet, since the first radial modes are all cut-off for all nodal diameters and all the other higher radial modes must be cut-off and decay more rapidly. The fundamental modes at inlet could be cut-on for nodal diameter from -6 to +5, and at outlet the fundamental modes could be cut-on for nodal diameter from -3 to +21. So nodal diameter -6, -3, +5 and +21 are four transition points between cut-on and cut-off modes, which can correspond to four acoustic resonance points in damping curve, as shown in Figure 6-8. It indicates that the calculation of aerodynamic damping can predict the acoustic resonance correctly.

Figure 6-12 Aeroelastic turbine rig: Acoustic modes at inlet and outlet ($\omega^*=1.0$)

6.3.3 Unsteady state ($\omega^* = 2.0$)

The unsteady simulation has been carried out using LUFT code, with two different meshes. The calculated aerodynamic damping for first bending mode shape is plotted versus nodal diameter in Figure 6-13. The reduced frequency ω^* is 2.0, and the blade frequency is 1440.4 Hz. There is no result from ANSYS CFX due to bad convergence. The simulation with coarse mesh has been done for 90 different nodal diameters, by an interval of 4° interblade phase angle, so that acoustic resonance points can be identified. Compared with damping curve at low frequencies, the aerodynamic damping becomes larger which means it is more stable, and more acoustic resonance points can be identified as peaks which indicates the acoustic modes are more likely to be cut-on, especially for higher radial modes. In this case, there are six acoustic resonance points shown in the damping curve. Besides, the simulation with fine mesh has also been done for 8 nodal diameters. The results agree well with the coarse mesh.

Figure 6-13 Aeroelastic turbine rig: Aerodynamic damping versus Nodal Diameter ($\omega^*=2.0$)

The local work coefficient at 90% span is plotted for nodal diameter 0, as shown in Figure 6-14. Most aerodynamic work is done near shroud, and the result shows good agreement. It can be concluded that aeroelastic turbine rig case is mesh converged at reduced frequency $\omega^* = 2.0$.

Figure 6-14 Aeroelastic turbine rig: Local work coefficient at 90% span ($\omega^*=2.0$, $ND=0$)

Figure 6-15 shows the acoustic modes calculated with 3D non-reflecting boundary condition. Three radial modes, $\mu = 0, 1, 2$ are plotted at outlet and two radial modes, $\mu = 0, 1$ are plotted at inlet. The normalized wave amplitude at 40%

chord length away indicates whether the modes are cut-on or cut-off and how much the modes decay. It shows the fundamental modes at inlet and outlet, and the first radial modes at outlet would be cut-on for a certain range of nodal diameter, while higher radial modes, such as $\mu = 2$ at outlet and $\mu = 1$ at inlet, are always cut-off. The other higher radial modes are not included in this figure, because it is obvious that they are all cut-off for all nodal diameters, and decay more rapidly. The transition points between cut-on and cut-off modes represent acoustic resonance, and all six transition points can correspond to six acoustic resonance points in the damping curve, Figure 4-10. It should be noticed that the nodal diameter is expanded from +40 to +50, in order to capture one transition point at around nodal diameter +44. It is one of the scatter modes, higher circumferential modes, by prescribing incoming nodal diameter -36. Therefore, there is a peak at nodal diameter -36 in the damping curve, which means the scatter modes can affect the aerodynamic damping.

Figure 6-15 Aeroelastic turbine rig: Acoustic modes at inlet and outlet ($\omega^*=2.0$)

6.4 Discussion and Conclusion

Aeroelastic turbine rig (AETR) has been investigated for first bending mode at two different reduced frequencies 1.0 and 2.0. The outlet Mach number is around 0.75 and the flow is subsonic.

The simulation has been carried out using ANSYS CFX and LUFT. In CFX, two methods have been applied, which are time domain method and Fourier transformation method. The non-reflecting boundary condition and turbulence model k- ϵ were applied. In LUFT, linearized Euler equations have been solved and 3D non-reflecting boundary conditions were applied at inlet and outlet. Two

different meshes, coarse mesh and fine mesh, have been generated and simulated, and good mesh convergence has been achieved for this case.

The steady pressure distributions agree well between results from CFX and LUFT, while there are small differences near trailing edge due to wake effects. For reduced frequency 1.0, it shows bad convergence in CFX Fourier transformation method. The convergence in CFX time domain method is better, but it is very computational costly. The comparison with LUFT shows overall good agreement while there are differences especially for negative nodal diameter. For reduced frequency 2.0, the simulation has only been carried out using LUFT and it shows good mesh convergence.

The acoustic modes for two reduced frequencies have been calculated with 3D non-reflecting boundary conditions on a low resolution far-field mesh 15×15 . The normalized wave amplitude at 40% chord length away has been plotted versus nodal diameter, and it identified the acoustic resonance points between cut-on and cut-off modes. The resonance points from acoustic modes analysis can correspond to peaks in the damping curve. There are four acoustic resonance points for reduced frequency 1.0, due to the fundamental modes ($\mu = 0$) at inlet and outlet. All the other higher radial modes are cut-off. For reduced frequency 2.0, the first radial modes ($\mu = 1$) at outlet can be cut-on. Therefore, two more resonance points exist compared with reduced frequency 1.0. So there are six acoustic resonance points for reduced frequency 2.0, which indicates the acoustic modes are more likely to be cut-on with increased reduced frequency in subsonic flow. As far as the authors are aware, this is the first time that resonance points due to first order radial acoustic modes have been presented in the literature. Besides, one of the six resonance points locates in the scatter modes, which are higher circumferential modes. It indicates that the scatter modes can also have influence on aerodynamic damping.

7. Virtual Integrated Compressor

7.1 Introduction

The VINK compressor is a publicly available compressor geometry designed as part of a collaboration project with partners from Swedish universities and industry (Chalmers University of Technology, KTH Royal Institute of Technology, Lund University, GKN Aerospace Sweden AB, and Swerea).

It is a three stage, high speed, high efficiency booster with design pressure ratio of 2.8. Only first stage of VINK compressor with the plenum has been considered in this case. There are 51 blades in the first rotor row and 88 blades in the stator row. The overview of the first stage is illustrated in Figure 5-1. At design point, the rotational speed is 6242 rpm, and the inlet total temperature is 282.4 K. The inlet total pressure is around 47750 Pa, which varies with radius.

Figure 7-1 Overview for the first stage of VINK

The modal analysis with blade only modes has been done in previous work (Lejon *et al.*, 2017), and investigation on flutter stability of the rotor blades also has done for the two lowest mode shapes, 1F and 1T. The blade frequencies are 370 Hz and 766 Hz, respectively, and aerodynamic damping calculations were carried out using the LUFT code. The results show that the aerodynamic damping is positive for all nodal diameters of the traveling wave mode, which means the rotor blades of the first stage are aerodynamically stable.

However, from a forced response point of view, there are potential resonance crossings over the operating range, as shown in the Campbell diagram in Figure 7-2. It indicates there are no crossings for the lower order modes, while dangerous resonance crossings can be identified for the higher mode shapes. For example, the EO88 line, corresponding to 88 blades of the stator located in the downstream, crosses the mode 26 line at near the design rotational speed. Determination of how critical the level of these resonance crossings is vital during the compressor

aeromechanical design, and it is dependent on calculation of aerodynamic damping.

Figure 7-2 Campbell diagram for VINK compressor first stage rotor

Therefore, VINK compressor is a valuable and challenging case for determination of aerodynamic damping at high frequencies. Based on modal analysis, the fifth mode has been chosen due to high deflection at the tip region and near the leading edge of the first rotor blade, as shown in Figure 7-3. The frequency is 1853.9 Hz, which corresponds to reduced frequency $\omega^* = 3.1$, where the reference velocity is 371.7 m/s and reference length is 0.0987, which is also the chord length.

Figure 7-3 Total displacement of VINK first stage rotor (mode 5)

7.2 Description of method

The simulation has been carried out using the LUFT code. Full three dimensional steady state and linearized Reynold's averaged Navier-Stokes equation have been solved. The turbulence model used is Spalart and Allmaras without transition model. A periodic boundary condition has been applied to account for the number of blade passages. And the interfaces between the plenum, rotor and stator are mixing planes, The exact steady 3D non-reflecting boundary conditions(Petrie-Repar, 2010) have been applied only at the mixing planes, which are the interfaces between the plenum, rotor and stator.

For the first rotor, two meshes have been generated and simulated, listed in Figure 7-1 and shown in Figure 7-4. The meshes for plenum and stator are kept same for two cases. The number of cells are 29700 and 445968, respectively.

Mesh name	Number of cells	Cells along span	First cell y^+
Coarse	400785	55	0.7
Fine	853000	100	0.8

Table 7-1 VINK first rotor mesh parameters

Figure 7-4 VINK first rotor coarse (left) and fine (right) meshes

7.3 Steady state results

Blade loading for VINK first stage rotor at 50% span is shown in Figure 7-5 and the Mach number contour at 50% span is shown in Figure 7-6. The flow is supersonic at the inlet and an unattached shock wave occurs near the leading edge, propagating to the suction side of the neighbouring blade and propagating upstream to the mixing plane between the plenum and rotor. The flow becomes subsonic at the outlet. The results from two meshes show different shock wave position on the suction side, as well as slight differences near the leading edge on the pressure side.

Figure 7-5 VINK first stage rotor: Blade loading at 50% span

Figure 7-6 VINK first stage rotor: Mach number contour at 50% span for coarse (left) and fine (right) mesh

The shock wave propagating upstream to the mixing plane between the plenum and rotor, can be identified in the pitchwise distribution of steady pressure at rotor inlet, and rotor inlet pressure contour, as shown in Figure 7-7 and Figure 7-8. There are slight differences between the results from coarse mesh and fine mesh, while overall agreements are good. The large scale abrupt increase of pressure due to shock, makes it challenging to calculate 3D acoustic modes at inlet, which in turn leads to difficulties of implementation of 3D non-reflecting boundary conditions.

Figure 7-7 VINK first stage rotor: Inlet pressure at 50% span

Figure 7-8 VINK first stage rotor: Inlet pressure contour for coarse (left) and fine (right) mesh

7.4 Unsteady state results

Aerodynamic damping

Unsteady simulation with the coarse mesh has been done and the aerodynamic damping is plotted versus nodal diameter in Figure 7-9. Different far-field meshes have been created in order to investigate unsteady aerodynamic modes at the inlet and outlet. The shock propagating upstream and the resulting abrupt increase of pressure at the inlet make it challenging to calculate the modes correctly so that pitchwise averaging is applied, which means the acoustic modes are calculated based on pitchwise averaged flow.

In Figure 7-9, increasing resolution of far-field mesh can lead to different results, especially near nodal diameter 0 and +10. The different results from far-field mesh 15×15 and 30×15 indicate there are high radial modes can not be determined properly by a low resolution far-field mesh. And the difference between 30×15 and 30×30 also indicates that high circumferential modes have influence on damping while they might not be solved properly on a low resolution far-field mesh.

Figure 7-9 VINK first stage rotor: Aerodynamic damping versus Nodal Diameter (mode 5)

ND +12 has been chosen to analyse due to high discrepancy among different resolutions of far-field meshes. The local work coefficient at 90% span on the pressure side (PS) and suction side (SS) is plotted in Figure 7-10. The distributions of local work agree well on the suction side for all cases, while differ on the pressure side. The difference between far-field mesh 15×15 and 30×15 indicates that 15 cells in radial direction can not solve acoustic modes properly. And the

agreement between far-field mesh 30×15 and 30×30 indicates that 15 cells in circumferential direction can already solve modes properly.

Figure 7-10 VINK first stage rotor: Local work coefficient at 90% span (mode 5, ND=+12)

Acoustic modes analysis

The 3D acoustic modes have been determined on a far-field mesh 25×15 , as shown in Figure 7-11 and Figure 7-14. The inlet flow is supersonic while the outlet flow is subsonic. Therefore, two different patterns of acoustic modes can be seen at the inlet and outlet.

Figure 7-11 VINK first stage rotor: Acoustic modes at inlet (mode 5)

At the inlet, there are four resonance points from ND -25 to +25, which locate at ND -13 ($\mu = 0$), ND -9 ($\mu = 1$), ND -1 ($\mu = 2$), and ND +11 ($\mu = 3$). The higher order radial modes are the reasons why there are discrepancies near ND 0 and ND +10 between different far-field meshes in the damping curve. Only 15 cells in radial direction can not solve the modes properly, especially for $\mu = 3$, which can be concluded from Figure 7-12. That is the reason why local work distributions differ among different far-field meshes at ND +12. Besides, Figure 7-13 shows the pressure magnitude of acoustic mode ($m = +12$, $\mu = 3$), which is still cut-on in this case, and it is important to consider this high order radial mode in order to calculate aerodynamic damping correctly.

Figure 7-12 VINK first stage rotor: Acoustic modes at the inlet for two far-field meshes ($\mu=3$)

Figure 7-13 VINK first stage rotor: Acoustic mode plot at inlet ($m=+12$, $\mu=3$)

Figure 7-14 VINK first stage rotor: Acoustic modes at outlet (mode 5)

At the outlet, there are three acoustic resonance points from ND -25 to ND +25, which locate at ND -13 ($\mu = 0$), ND -9 ($\mu = 1$), and ND +10 ($\mu = 2$). The $\mu = 3$ modes are always cut-off and all the other higher radial modes are cut-off. The resonance points ND +10 ($\mu = 2$) at the outlet, and ND +11 ($\mu = 3$) at the inlet are so closed that might affect aerodynamic damping.

Mesh convergence analysis

Figure 7-15 VINK first stage rotor: Mesh convergence analysis (ND=0)

Unsteady simulation with the fine mesh has also been done, and Figure 7-15, Figure 7-16 show comparisons of local work distributions at nodal diameter 0 and -10. It indicates good agreement between two meshes although they predict different shock position in steady simulation. However, the local work distributions differ with each other near the leading edge, on the pressure side (PS). The reason might be the difference between steady state results.

Figure 7-16 VINK first stage rotor: Mesh convergence analysis (ND=-10)

7.5 Discussion and Conclusion

The fifth mode for the VINK case, has been investigated, with reduced frequency 3.1. The case was challenging because of the transonic relative Mach number at the inlet and the fact that the bow shock at the leading edge propagated upstream and crossed the computational inlet plane. Acoustic resonance points were predicted to occur for the first and second radial orders at the inlet. There were more resonance points due to higher radial order modes while they were difficult to solve by low resolution far-field meshes. The plot of aerodynamic damping versus nodal diameter was calculated and it was concluded that the higher order resonance points are influencing the damping curve. There were some inconsistencies in the results among different far-field meshes and grid convergence was not achieved. These inconsistencies were due to the difficulty in calculating the acoustic modes at the transonic inlet with an impinging shock.

8. Conclusion and future work

The aerodynamic damping was calculated by a linearized Navier-Stokes flow solver (LUFT) with exact 3D non-reflecting boundary conditions. The method was first validated on the two-dimensional flat plate (Standard Configuration 8). Good agreement with the reference data at reduced frequency 2.0 was achieved and grid converged solutions with reduced frequency up to 16.0 were obtained. It was concluded that at least 20 cells per wavelength is required.

Based on the validation, two 3D profiles were also investigated: an aeroelastic turbine rig (AETR), which is a subsonic turbine case, and a virtual integrated compressor (VINK) which is a transonic compressor case.

For the AETR case, the first bending mode with reduced frequency 2.0 was studied. The 3D acoustic modes were calculated and the rate of decay was plotted as a function of nodal diameter and radial order. This plot identified six acoustic resonant points which included two points corresponding to the first radial order. The aerodynamic damping as a function of nodal diameter was also calculated and plotted. There were six distinct peaks which occurred in the damping plot and these peaks correspond to the six resonance points. As far as the authors are aware, this is the first time that resonance points due to first order radial acoustic modes have been presented in the literature. Besides, one of the six resonance points locates in the scatter modes, which are higher circumferential modes. It indicates that the scatter modes can also have influence on aerodynamic damping.

In VINK case, the fifth mode (1854 Hz, reduced frequency 3.1) was investigated. The case was challenging because of the transonic relative Mach number at the inlet (1.13) and the fact that the bow shock at the leading edge crossed the computational inlet plane. Acoustic resonance was predicted to occur for the first and second radial orders at the inlet. The plot of aerodynamic damping versus nodal diameter was calculated and it was concluded that the higher order resonance points are influencing the damping curve. There were some inconsistencies in the results and grid convergence was not achieved. These inconsistencies were due to the difficulty in calculating the acoustic modes at the transonic inlet with an impinging shock.

Therefore, acoustic modes analysis is a good way to interpret aerodynamic damping curve, especially the peaks due to acoustic resonance. And it is important to consider the acoustic resonance of higher order radial modes in order to correctly calculate aerodynamic damping at high reduced frequencies.

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