Optimal Power Control for D2D Communications under Rician Fading: a Risk Theoretical Approach

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***Abstract*—Device-to-device communication is a technology that allows users in close proximity to establish a direct communication link instead of passing through the base station. Because direct communications are likely to have a strong line-of-sight component in the received signal, it is reasonable to model the direct channel with Rician fading. In this paper, we propose a power-control scheme for device-to-device communications on a shared channel. Our allocation minimizes the total power consumption while limiting the link outage probability due to Rician fast fading. By leveraging the concept of conditional-value-at-risk from the field of finance, we obtain a linear programming formulation which can be efficiently solved. Through simulation results we show the benefit of the proposed power allocation compared to a deterministic power control that does not account for the random channel variations. Moreover, we provide insights into how the network topology and the parameter settings affect the performance and feasibility of the power allocation.**

***Index Terms*—Device-to-Device communication, Rician fading, Rayleigh fading, channel uncertainty, power control, conditional-value-at-risk, outage probability, optimization.**

I. Introduction

Device-to-device (D2D) communication is a technology for next-generation (5G) cellular systems that allows users in close proximity to communicate directly, bypassing the base station [1]. The introduction of D2D communication into the architecture of traditional cellular networks brings several advantages, such as larger coverage, higher throughput, lower transmission latency, and increased energy efficiency [2,3].

Because of the cost and scarcity of licensed spectrum, it is preferable to implement D2D communication with frequency channel reuse. There are two main approaches: *underlay D2D communication*, in which D2D and cellular users share the same channels; and *overlay D2D communication*, in which a fraction of the cellular spectrum is dedicated to the D2D users (spectrum sharing can be enforced only among the D2D links).

When D2D transmissions share the spectrum, they suffer from co-channel interference, which can significantly impact the communication quality. A proper power allocation is therefore needed to manage the interference and provide a satisfactory quality-of-service (QoS) to the users, while making efficient use of the limited battery capacity of mobile devices. Most of the power-control schemes in the literature, whether for traditional cellular networks or D2D-enabled networks, base their allocations on the assumption of perfectly known channel state information (CSI) of all links, and that the channel gains remain stationary between the power allocation updates [4]–[6]. However, wireless channels are characterized by fast fading, which means that the channel gains can randomly change quickly (within milliseconds). On the one hand, considering a fast power control scheme that adapts to all channel variations can be impractical and costly because of the increased signalling overhead (needed to collect all instantaneous CSI and reassign the powers), and the resulting use of valuable bandwidth and power resources. On the other hand, neglecting the channel variations due to fast fading can lead to situations in which the assigned powers become inadequate and give unsatisfactory communications.

The common way to deal with the random variations of the channels is to consider constraints on the outage probability of the users. Many works in the literature have focused on analyzing and computing the outage probability in Rayleigh fading environments [7]–[10]. In particular, the authors of [7] have shown that the problem of minimizing the total power consumption subject to outage probability constraints in the interference-limited scenario (i.e., neglecting the noise) can be posed as a geometric program.

Modeling the communication links with Rayleigh fading is reasonable when the line-of-sight (LoS) components of the signals are negligible. In D2D communication, however, the short distance between the transmitter and the receiver and the low antenna heights are likely to cause a prominent LoS component. For this reason, it is more appropriate to model the D2D links as Rician fading channels. Analysis of the performance of D2D communication in Rician fading environment can be found in [11]–[14]. Using stochastic geometry, the above literature derives analytical results for the outage probability and for the average achievable throughput. However, those performance metrics are analyzed for a given power allocation. To the best of our knowledge, there is a lack in the literature of power-control schemes that account for channel uncertainty under Rician fading. This is mainly because the Rician fading model leads to closed-form expressions of the outage probability that are difficult to include in optimization formulations (e.g., because of lack of convexity).

To overcome the complexity of the outage probability constraints under Rician fading, we propose an approach based on the connection between the risk-theoretical concept of uncertainty and the uncertainty that exists in wireless communication. In particular, we discuss how the notion of *coherent* *measure of risk* [15,16] can be employed to derive a convex robust power-control formulation. Our main contributions are:

- we propose a power-control scheme for D2D communications with bounds on the risk of outage in noisy Rician-Rayleigh fading environments;
- we optimally solve the proposed power-control problem;
- we show, in a simulation study, that accounting for the channel uncertainty in the power-control formulation can indeed reduce the outage probability of the D2D users.

Finally, we believe that the novel approach we propose in this work may open up for more applications of tools from risk theory in the field of finance to telecommunication problems.

II. System model and problem formulation

We consider a cellular network with overlay D2D communications in which communication modes (that is, *D2D mode* if two users communicate directly and *cellular mode* if they communicate via the base station) and frequency channels have been assigned to the transmitters. In this formulation, the D2D links have a dedicated spectrum and do not interfere with the traditional cellular users. However, the D2D links share the same frequency channels (to increase the overall spectral efficiency of the system) and thus may interfere.

Without loss of generality, we focus on the set of N D2D links assigned to a certain channel. We refer to the two users forming the i^{th} D2D link ($i = 1, \ldots, N$) as transmitter i and receiver i, respectively. We indicate with P_i the transmission power level of transmitter i. All transmitters have the upper bound $P^{\max}$ on the transmission power.

Because of the power attenuation, typical of wireless networks, the power received at any receiver j from transmitter i is given by $P_i G_{ij}$, where G_{ij} is the channel gain of the radio link between the two users. In addition, we assume that all receivers are affected by noise, which we model as Gaussian white noise with power σ^2. We indicate with $\mathbf{P}$ the $N \times 1$ vector of transmission powers, and with $\mathbf{G}$ the $N \times N$ gain matrix: $[\mathbf{P}]_i \triangleq P_i$, $[\mathbf{G}]_{i,j} \triangleq G_{ij}$. The signal-to-interference-plus-noise ratio (SINR) at receiver i is given by

$$\gamma_i(\mathbf{P}, \mathbf{G}) = \frac{P_i G_{ii}}{\sum_{j \neq i} P_j G_{ji} + \sigma^2}.$$

Finally, we assume that the QoS requirement of link i is met when the SINR exceeds a given threshold γ^{tgt}. In other words, we consider the link i to be in *outage* when $\gamma_i(\mathbf{P}, \mathbf{G}) < \gamma^{\text{tgt}}$. For the sake of readability, in the sequel we omit the dependence of γ_i on $(\mathbf{P}, \mathbf{G})$.

A. Channel model

The power received at receiver j from transmitter i can be expressed as

$$P_i G_{ij} = P_i \bar{G}_{ij} F_{ij},$$

where the term $\bar{G}_{ij}$ represents the distance-dependent power attenuation (slow fading), and F_{ij} models the fast fading of the radio channel. We consider the standard path-loss model $\bar{G}_{ij} = D_{ij}^{-\alpha}$, where D_{ij} is the distance between transmitter i and receiver j, and $\alpha > 0$ is the path-loss exponent. The term F_{ij} is a random variable whose distribution depends on whether or not there is a LoS component between the transmitter i and the receiver j.

Given that the users forming a D2D link are usually physically close to each other, we can assume that their communication link is dominated by the LoS component. Therefore, we model the direct D2D links as *Rician fading* channels, which means that F_{ii} is a random variable with non-central χ^2 distribution with expectation Ω. The probability density function of F_{ii} is given by

$$f_{F_{ii}}(x) = \frac{K+1}{\Omega} \exp\left(-K - \frac{(K+1)x}{\Omega}\right) I_0\left(\sqrt{\frac{4K(K+1)x}{\Omega}}\right), \quad (1)$$

where K is the Rician K-factor representing the ratio of the power of the LoS component to the power of the scattered components, while $I_0(\cdot)$ is the zeroth order modified Bessel function of the first kind. Note that the stronger the LOS path component, the larger the Rician K-factor. All scattered components are Gaussian random variables with variance 1, which reduces the term Ω to $2(K+1)$ [17].

On the other hand, we assume that the signals coming from the interfering links do not have a strong LoS component and, similarly to [12,13], we model the cross links as *Rayleigh fading* channels. Hence, the random variables F_{ji} for $j \neq i$ are exponentially distributed with mean λ:

$$f_{F_{ji}}(x) = \frac{1}{\lambda} \mathrm{e}^{-x/\lambda}.$$

The considered Rician-Rayleigh fading model might not be accurate when there is a LoS also between the interfering users. However, a proper frequency channel assignment should avoid such scenarios to reduce the mutual interference. Nevertheless, the solution approach developed in this paper would apply equally well to a Rician-Rician fading environment .

B. Problem formulation

We consider the problem of minimizing the total transmission power subject to SINR constraints and power bounds. This optimization problem can be formally expressed as

$$\begin{aligned} \underset{\mathbf{P}}{\text{minimize}} \quad & \sum_{i=1}^{N} P_i \\ \text{subject to} \quad & \gamma_i \geq \gamma^{\text{tgt}}, \quad i = 1, \ldots, N, \\ & 0 \leq P_i \leq P^{\max}, \quad i = 1, \ldots, N. \end{aligned} \quad (2)$$

By defining the matrix $\mathbf{H}$ with entries $h_{ij} = \gamma^{\text{tgt}} G_{ji}/G_{ii}$ if $j \neq i$, and zero otherwise, and the vector $\boldsymbol{\eta}$ with entries $\eta_i = \gamma^{\text{tgt}} \frac{\sigma^2}{G_{ii}}$, the SINR constraints in (2) can be rewritten as $(\mathbf{I} - \mathbf{H})\mathbf{P} \geq \boldsymbol{\eta}$, where $\mathbf{I}$ is the N-dimension identity matrix, and the inequalities are component-wise.

For a given matrix gain $\mathbf{G}$, the above well-known linear programming problem can be solved efficiently, both in centralized and in distributed manners, if the feasible set is non-empty (see, for example, [4]). A necessary and sufficient condition for the existence of a feasible solution to (2) is related to the largest real eigenvalue of $\mathbf{H}$ [18, Ch. 2] and, thus, to the gain matrix $\mathbf{G}$. Therefore, if the channel gains change, the power allocation might not only cease to be optimal, but it may also become infeasible. For this reason, the power-control formulation in (2) is useful under the assumption of stationary (or with very slow fading) communication channels. Unfortunately, this is not always reasonable in practice as the channel gains may change rapidly.

In LTE systems, this fast variation of the channel gains is compensated by fast scheduling rather than fast power control. When D2D communications are integrated into cellular networks, it is preferable to avoid frequent resource reassignments (be it power or frequency channel) even more than in the traditional cellular networks. This is mainly because D2D-enabled networks require a higher signalling overhead to obtain the CSI of both the direct and the interfering links. This is even worse in *network-assisted D2D communications* where the resource assignments are centralized at the base station. In this case, feedback information is also necessary. Therefore, for D2D communications, a slow power-control that is robust to fast fading variations can be beneficial to reduce both power and scheduling updates.

In the next section we show how to handle the channel uncertainty, without resorting to frequent resource allocation updates, using a probabilistic approach.

III. Power control under uncertainty

A. The traditional approach: outage probability constraints

A common approach to account for the uncertainty of channel gains is to replace the SINR constraints with QoS requirements expressed in terms of a maximum *outage probability*. Specifically, the outage probability constraint of link i is defined as the chance constraint

$$\mathrm{Prob}(\gamma_i \leq \gamma^{\mathrm{tgt}}) \leq 1-\rho, \tag{3}$$

where ρ is the minimum required probability of successful communication.

The power-control problem in (2), with the SINR constraint replaced by (3), has been studied for the Rayleigh fading environment and efficient solutions have been proposed [7,10]. The advantage of assuming only Rayleigh fading is that it leads to mathematically treatable closed-form expressions of the outage probability. When considering Rician fading, the computation of the outage probability becomes much harder because of the integration of (1). Even when a closed-form expression can be derived (see, for example, [12]), it does not lead to a convex formulation of the optimization problem.

Nevertheless, we notice that constraint (3) resembles the value-at-risk constraint used in the field of finance. This observation opens up for a study of the power-control problem from a risk theoretical perspective.

B. A risk theoretical approach: VaR and CVaR constraints

The objective in risk theory is to make a decision while minimizing the risk of loss. In our case, we aim at assigning the transmission powers while reducing the risk of having users in outage. We leverage the results in the literature of risk theory to obtain a robust and treatable approach.

1) Preliminaries on risk theory: To define the concept of *risk*, we consider a function $\Gamma(\mathbf{P},\mathbf{G})$. This function quantifies the loss we occur in when we assign power $\mathbf{P}$ to a scenario with gain matrix $\mathbf{G}$. Given that we consider $\mathbf{G}$ as a random variable, $\Gamma(\mathbf{P},\mathbf{G})$ is also a random variable for fixed $\mathbf{P}$. The objective of risk theory is to condense the variations of this loss, due to $\mathbf{G}$, into a number. This is done by means of a measure function $\mathcal{R}(\cdot)$, which must satisfy certain properties to give significant information about the risk of loss associated with the choice $\mathbf{P}$. In the literature on risk theory, these functions are called *coherent measures of risk* [19]:

Definition 1. *A function $\mathcal{R}(\cdot)$ is a* coherent measure of risk *if and only if*

(i) $\mathcal{R}(K) = K$, *for all constant* K,

(ii) for any two square integrable Γ_1 *and* Γ_2 *and any* $\lambda \in (0,1)$,

$$\mathcal{R}\big((1-\lambda)\Gamma_1 + \lambda\Gamma_2\big) \leq (1-\lambda)\mathcal{R}(\Gamma_1) + \lambda\mathcal{R}(\Gamma_2)\,,$$

(iii) $\mathcal{R}(\Gamma_1) \leq \mathcal{R}(\Gamma_2)$ *when* $\Gamma_1 \leq \Gamma_2$,

(iv) $\mathcal{R}(\Gamma) \leq 0$ *when* $\|\Gamma^k - \Gamma\|_2 \to 0$ *with* $\mathcal{R}(\Gamma^k) \leq 0$ *for all* k.

(v) $\mathcal{R}(\lambda\Gamma) = \lambda\mathcal{R}(\Gamma)$ *for* $\lambda > 0$.

Two common measures of risk are the *value-at-risk* (VaR) and the *conditional-value-at-risk* (CVaR). For a given $\rho \in (0,1)$, we define the ρ-VaR and the ρ-CVaR as

$$\rho\text{-}\mathrm{VaR}(\Gamma) \triangleq \min\left\{\gamma \in \mathbb{R}\colon\ \mathrm{Prob}\{\Gamma \leq \gamma\} \geq \rho\right\}\,, \tag{4}$$

$$\rho\text{-}\mathrm{CVaR}(\Gamma) \triangleq \frac{1}{1-\rho}\int_\rho^1 \beta\text{-}\mathrm{VaR}(\Gamma)\mathrm{d}\beta\,. \tag{5}$$

Intuitively speaking, ρ-$\mathrm{VaR}(\Gamma)$ is the ρ-quantile of the random variable Γ, while ρ-CVaR is the expected value of Γ conditioned on the $1-\rho$ tail of its distribution.

From (4) and (5), we note that ρ-CVaR accounts more properly for the lack of knowledge on Γ than ρ-VaR, since ρ-CVaR is affected by the shape of the distribution over the whole $1-\rho$ tail. Moreover, unlike ρ-VaR, ρ-CVaR is a coherent measure of risk [19] and, thus, it fulfills the convexity property (ii) in Definition 1. This is a key characteristic of ρ-CVaR for our purposes. In addition, from the definitions in (4) and (5), it is easy to verify that

$$\rho\text{-}\mathrm{VaR}(\Gamma) \leq \rho\text{-}\mathrm{CVaR}(\Gamma). \tag{6}$$

All these properties make the ρ-CVaR a natural candidate for defining a robust power-allocation scheme which accounts for the lack of knowledge on the channel gains.

2) Robust power control: To apply the concepts of VaR and CVaR to the power-control formulation in (2), we define for each link i the loss function

$$\Gamma_i(\mathbf{P},\mathbf{G}) = \sum_{j\neq i} P_j h_{ij} - P_i + \eta_i.$$

If $\Gamma_i < 0$, the SINR constraint for link i is fulfilled.

For a given power allocation $\mathbf{P}$, the function $\Gamma_i(\mathbf{P}, \mathbf{G})$ is a random variable with a distribution induced by the random vector $\mathbf{G}$. Following the idea of the chance constraint (3), we want to avoid positive losses for each D2D link i (that is, situations of outage) at least with probability ρ. In terms of ρ-VaR, we express this constraint as $\rho\text{-VaR}_i(\mathbf{P}) \leq 0$, that is

$$\min\{\gamma \in \mathbb{R} : F_{\Gamma_i}(\mathbf{P}, \gamma) \geq \rho\} \leq 0, \tag{7}$$

where $F_{\Gamma_i}(\mathbf{P}, \gamma)$ is the cumulative distribution function of $\Gamma_i(\mathbf{P}, \mathbf{G})$, for a given $\mathbf{P}$. Replacing the SINR constraints in (2) with (7) leads to the same intractable formulation as the equivalent chance constraint (3) (in fact, the two constraints are equivalent). This follows from the fact that the VaR is not a coherent measure of risk and does not preserve the convexity of the original linear constraints.

We therefore turn our attention to the CVaR, to leverage its superior mathematical properties. Considering CVaR constraints in place of VaR constraints gives a more conservative solution as shown in (6). This can be beneficial for applications such as vehicular-to-vehicular communication [20], for which the communication reliability might have much higher priority than the power consumption. However, the most significant advantage of using CVaR in our power-control problem is that it leads to a convex constraint on the risk of outage.

To compute the CVaR with its general definition (5), we need to integrate $F_{\Gamma_i}(\mathbf{F}, \gamma)$. This integration is difficult (if not infeasible) in the case at hand. However, we can replace the CVaR constraints $\rho\text{-CVaR}_i(\mathbf{P}) \leq 0$, for each link i, with a simpler expression using the following auxiliary function

$$\xi_\rho^{(i)}(\mathbf{P}, C) = C + (1-\rho)^{-1}\mathbf{E}\left[\max\{0, \Gamma_i(\mathbf{P}, \mathbf{G}) - C\}\right], \tag{8}$$

where C is a real number and $\mathbf{E}[\cdot]$ is the expected value. The useful relations between $\xi_\rho^{(i)}(\mathbf{P}, C)$ and $\rho\text{-CVaR}_i$ are studied in [15] and summarized in the following theorem[1]:

Theorem 1 ([15], pp. 24–26)**.** *The function $\xi_\rho^{(i)}(\mathbf{P}, C)$ is convex and continuously differentiable in C. Furthermore, provided that $\Gamma_i(\mathbf{P}, \mathbf{G})$ is convex in $\mathbf{P}$, $\xi_\rho^{(i)}(\mathbf{P}, C)$ is convex with respect to $(\mathbf{P}, C)$ and the $\rho\text{-CVaR}_i(\mathbf{P})$ is convex in $\mathbf{P}$. Moreover*

$$\rho\text{-CVaR}_i(\mathbf{P}) = \min_{C \in \mathbb{R}} \xi_\rho^{(i)}(\mathbf{P}, C).$$

Using the above results, the robust power-control problem for D2D communication under Rician-Rayleigh fading can be formulated as follows:

$$\begin{aligned} \underset{\{P_i\},\{C_i\}}{\text{minimize}} \quad & \sum_{i=1}^{N} P_i \\ \text{subject to} \quad & \xi_\rho^{(i)}(\mathbf{P}, C_i) \leq 0, \quad i = 1, \ldots, N, \\ & 0 \leq P_i \leq P^{\max}, \quad i = 1, \ldots, N. \end{aligned} \tag{9}$$

The importance of Theorem 1 is that function $\xi_\rho^{(i)}(\mathbf{P}, C_i)$ is not only convex in $(\mathbf{P}, C_i)$, and therefore preserves the convexity of the original problem formulation, but it is also easy to approximate numerically. In fact, drawing a large number N_s of samples from the distribution $f_{\mathbf{G}}(G)$, we can approximate the expected value in (8) as follows

$$\mathbf{E}[\max\{0, \Gamma_i(\mathbf{P}, \mathbf{G}) - C\}] \approx \frac{1}{N_s} \sum_{s=1}^{N_s} \max\{0, \Gamma_i(\mathbf{P}, \mathbf{G}^s) - C\}, \tag{10}$$

where $\mathbf{G}^s$ is the s-th sample of the random vector $\mathbf{G}$. The sum in (10) is an unbiased estimator of the expected value and its standard error is inversely proportional to $\sqrt{N_s}$ [21, Ch. 11].

Finally, using (10) and linearizing the max function, we can rewrite (9) as the following linear programming problem

$$\begin{aligned} \underset{\{P_i\},\{C_i\},\{\beta_i^s\}}{\text{minimize}} \quad & \sum_{i=1}^{N} P_i \\ \text{subject to} \quad & C_i + a\sum_{s=1}^{N_s} \beta_i^s \leq 0, \quad i = 1, \ldots, N, \\ & \beta_i^s \geq \Gamma_i(\mathbf{P}, \mathbf{G}^s) - C_i, \quad \begin{cases} s = 1, \ldots, N_s \\ i = 1, \ldots, N \end{cases}, \\ & \beta_i^s \geq 0, \quad s = 1, \ldots, N_s, \, i = 1, \ldots, N, \\ & 0 \leq P_i \leq P^{\max}, \quad i = 1, \ldots, N, \end{aligned} \tag{11}$$

where $a = \left[(1-\rho)N_s\right]^{-1}$.

We assume that the base station solves (11) and then feeds back the optimal power levels to the D2D transmitters. Although the base station must gather information about the channel condition of all links, it only needs the quasi-stationary channel gains $\bar{G}_{ij}$, which can be updated on a larger time scale than the fast fading time scale. This means that by solving (11) we are able to both take into account the fast variations of the channels and avoid frequent and costly updates of the instantaneous CSI. It is also worth mentioning that in contrast to the approaches in [7,11,12], our scheme allows to account for the effect of additive noise at the receiver without any additional computational cost.

Finally, note that while Problem (2) can be infeasible depending on the gain matrix $\mathbf{G}$, the feasibility of Problem (11) depends on both the average channel gains and on the selected parameter ρ. Intuitively, for scenarios with very low direct gains and large cross-gains (that is, poor average SINRs) it might be impossible to ensure a large success probability to all links. This issue can be tackled by reducing the confidence level ρ, or by using an admission control scheme that shuts off the links for which to the CVaR constraints cannot be fulfilled.

IV. Numerical results

To evaluate the performance of the proposed power control, we consider a set of D2D links randomly placed in a square area and assigned to the same frequency channel. The default parameters used in our simulations are listed in Table I.

First, we discuss how the advantages of the proposed robust power allocation depends on the network topology and system parameters. Our simulations show that in some favorable cases it might be unnecessary to account for the channel variation

[1] We have adapted the results in [15] to the notation used in this work.

due to fast fading because the solutions of (2) and (11) coincide. These cases mainly occur for low SINR target values (up to 0 dB) and short D2D-link distances (up to 50 m). Although this parameter setup has been considered in related works (e.g., [11,12]), our simulation study focuses on less advantageous scenarios in which the distances of D2D links can vary up to 150 m and the SINR target exceeds 0 dB.

Figure 2 compares the performance of the solutions of Problem (2) and Problem (11). Specifically, it shows the cumulative distribution function (CDF) of the SINR of 6 D2D links, assigned to the same channel and placed in the network as shown in Figure 1. The solid curves in Figure 2 represent the CDF of each user when using the optimal power allocation from (2), while the dashed curves are obtained by the robust power allocation. We can see that D2D link 6 is particularly affected by the fast fading. This is not only because of variations of its direct-link gain, but also due to the random variations of the (strong) cross links between transmitters 1 and 5 and receiver 6. Under the deterministic power control allocation, the outage probability of user 6 is above 20%, while the robust power allocation shifts her CDF towards higher SINRs and guarantees an outage probability of 7%.

Another interesting result shown in Figure 2 is that the robust power control allocation increases the fairness in the system by shaping the SINR distributions of the links. In particular, the robust power control shifts the CDFs of the links with higher outage probability towards higher SINRs, while it shifts the CDFs of the D2D links that are always experiencing high QoS towards smaller SINR values.

As discussed in III-B2, the feasibility of the robust power control optimization depends both on the selection of the success parameter ρ and on the network topology. In particular, if in a given network there are some *weak* D2D links (weak because they either perceive high interference or they have small direct-link gain), it might not be possible to ensure every desired probability of outage. In fact, those weak links might make the feasible set of Problem (9) empty because their ρ-CVaR is alway positive. In Figure 3 we show how the maximum ρ for which Problem (9) is feasible depends on the SINR target and the Rician K-factor. We consider a set of 10 D2D links sharing the same channel. As expected, the minimum success probability that the robust power allocation

TABLE I
SIMULATION PARAMETERS

Parameter	***Value***
Allowed distance range for D2D links	$[50, 150]$ m
No. of D2D links	6 and 10
SIR target	$[-6, 9]$ dB
Path loss coefficient	4
Rician K-factor	2 dB
Noise power level	1 pW
Max transmission power	250 mW
No. of Monte Carlo runs to compute Eq. (10)	10000
No. of Monte Carlo runs to evaluate the performance	50000

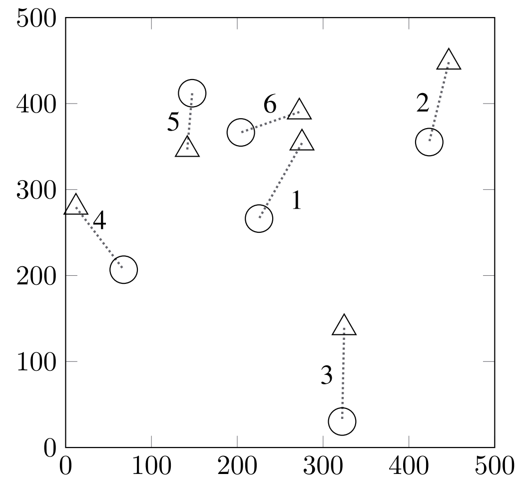

Fig. 1. Example of a 6 D2D links placed in 500 m^2 area and sharing the same frequency channel. Triangle and circle indicate transmitter and receiver of each D2D link, respectively.

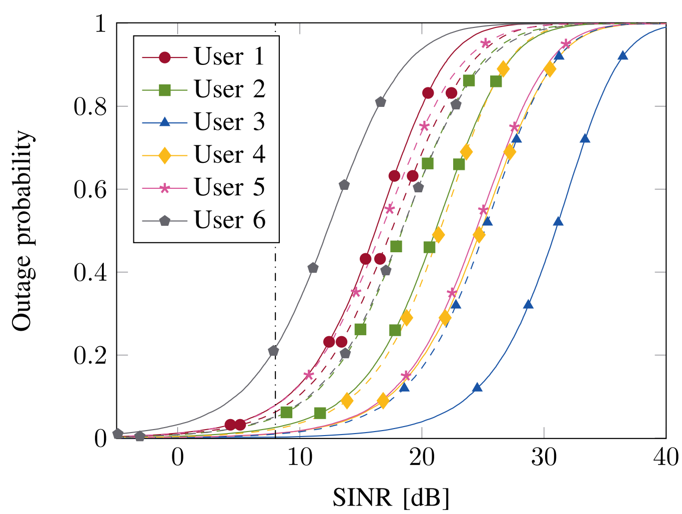

Fig. 2. Per-user CDF of the SINR, with the transmission power levels assigned with the deterministic power control (solid curves) and with the proposed robust power control (dashed curves). The vertical twodashed line indicates the SINR target of 8 dB.

problem can ensure decreases with the SINR target. Moreover, the maximum ρ increases with the Rician K-factor because larger values of K correspond to stronger LoS components. In Figure 3, $K = -\infty$ dB corresponds to the case where the Rician distribution degenerates to a Rayleigh distribution, that is, where there is no LoS between transmitters and receivers [17]. In the considered network topology, for $K = -\infty$ dB and SINR target larger than 5 dB, Problem (9) becomes infeasible (we consider infeasible all cases when the optimal ρ is smaller than the chosen tolerance value 0.01).

V. Conclusion and Future Work

In this paper we proposed a robust power allocation scheme for overlay D2D communication under Rician-Rayleigh fading channels. The optimal power allocation minimizes the

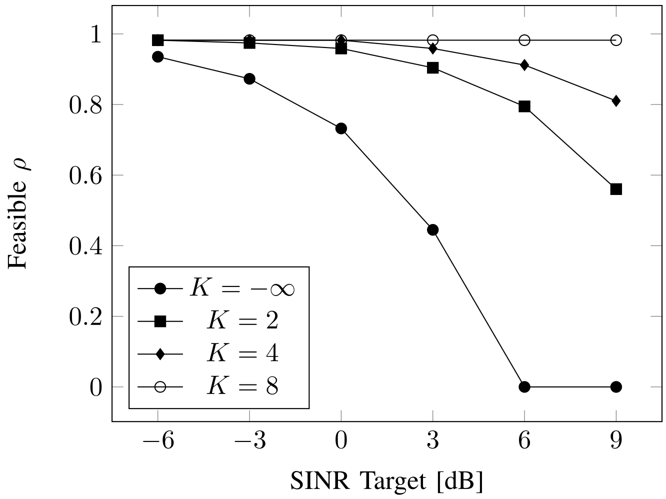

Fig. 3. Maximum feasible probability of success ρ for different values of the SINR target and different Rician K-factors. The results are related to 10 D2D links sharing the same channel.

total power consumption while ensuring a minimum success probability to each D2D link. To overcome the complexity of the outage probability constraints under Rician fading, we applied the concept of coherent-measure-of-risk from the field of finance, thus obtaining a convex and efficiently solvable optimization formulation. To the best of our knowledge, this is the first paper that provides optimal power allocation for D2D communications under Rician-Rayleigh fading. The outage probability achieved with the proposed scheme has been evaluated and compared to the deterministic power allocation that does not take into account the fast fading. Simulation results showed that the performance and the feasibility of the robust power allocation depend on the network topology and parameter set up. In particular, higher success probability can be achieved by D2D links with a high Rician K-factor, that is, a strong LoS component.

There are several extensions of this work that we hope to return to in the future. For example, considering the power-control problem for D2D underlay communication where traditional cellular communications are also involved; and simulating scenarios where different links are modeled with different K-factor values (thus obtaining different combinations of Rician and Rayleigh models). Furthermore, we would like to study the possibility of developing a distributed solution and deriving an analytical feasibility condition of the proposed problem. Finally, we wish to investigate the application of the risk-theoretical approach to other problems in the field of cellular networks; such as, for example, D2D-relaying networks.

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