Implementing SPC for non-normal processes with the I-MR chart:
A case study

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Acknowledgements

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Axel Elisson
Abstract

The application of statistical process control (SPC) requires normal distributed data that is in statistical control in order to determine valid process capability indices and to set control limits that reflects the process’ true variation. This study examines a case of several non-normal processes and evaluates methods to estimate the process capability and set control limits that is in relation to the processes’ distributions. Box-Cox transformation, Johnson transformation, Clements method and process performance indices were compared to estimate the process capability and the Anderson-Darling goodness-of-fit test was used to identify process distribution. Control limits were compared using Clements method, the sample standard deviation and from machine tool variation. Box-Cox transformation failed to find a transformation that resulted in normality for all processes. For some processes, Johnson transformation was successful. For most processes, the Anderson-Darling goodness-of-fit test failed to fit the data into specific distributions, making the capability estimations less reliable. However, compared to the general theory, the applied methods provided more accurate capability results. Control limits by either Clements method, the sample standard deviation or by machine tool variation provided good results when compared to historical data, thus improving the control chart’s ability to detect and alarm the user of special cause variation and to minimize the number of false alarms.
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<th>Abbreviations</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMM</td>
<td>Coordinate measuring machine</td>
</tr>
<tr>
<td>Cp</td>
<td>Process capability index (potential)</td>
</tr>
<tr>
<td>Cpk</td>
<td>Process capability index</td>
</tr>
<tr>
<td>H6</td>
<td>Heller milling machine 6</td>
</tr>
<tr>
<td>I-MR</td>
<td>Individuals and moving range</td>
</tr>
<tr>
<td>Ku</td>
<td>Kurtosis</td>
</tr>
<tr>
<td>LCL</td>
<td>Lower control limit</td>
</tr>
<tr>
<td>LSL</td>
<td>Lower specification limit</td>
</tr>
<tr>
<td>PCI’s</td>
<td>Process capability indices</td>
</tr>
<tr>
<td>Pp</td>
<td>Process performance index (potential)</td>
</tr>
<tr>
<td>Ppk</td>
<td>Process performance index</td>
</tr>
<tr>
<td>Sk</td>
<td>Skewness</td>
</tr>
<tr>
<td>SPC</td>
<td>Statistical process control</td>
</tr>
<tr>
<td>UCL</td>
<td>Upper control limit</td>
</tr>
<tr>
<td>USL</td>
<td>Upper specification limit</td>
</tr>
</tbody>
</table>
1 Introduction
Normality is critical in the application of statistical process control (SPC) as the setup of control limits and the calculation of process capability in the basic theory is under the assumption that the process is in statistical control and follows a normal distribution. In this case study, performed at the brake manufacturer Haldex, the brake caliper machining process was studied and a method to implement SPC for non-normal processes was presented.

In this chapter, the problem definition (section 1.1), objectives (section 1.2) and limitations (section 1.3) of the study is stated. The current state of SPC at Haldex is described in section 1.4 followed by a theoretical introduction to SPC in section 1.5.

1.1 Problem definition
At the site, control charts for the caliper are implemented and measurements are continuously executed. However, due to the setup of the current procedure to control the processes and evaluate process capability, several issues follow when compared to the SPC theory:

1. The control limits are set to 66,67% of the specification limits, thus, the control limits do not reflect the true part-to-part variation of the process. The consequence of this is that; (1) the control chart might give false alarm for common variation, and/or (2) the control chart will fail to alert special cause variation. Consequently, operators and management are reacting, or not reacting, without statistical reliance.
2. There are processes that are non-normally distributed and/or unstable, making the process capability indices invalid.
3. The sampling is not executed at a constant rate, making it difficult to identify process distribution and process behaviour, thus resulting in an un-predictable process.
4. There is no event log for the processes available, making it difficult to analyse historical data as special cause variation and major changes in the process is not documented.

1.2 Objectives
Based on the problem definition, the objective of this of study is to implement a valid procedure to run the processes with SPC. Hence, the following objectives are to be concerned:

1. Identify process behaviour, cycles and distribution.
2. Perform a process capability study of the current state.
3. Develop a method to set control limits corresponding to the process variation.
4. Develop a method to calculate valid process capability indices.

1.3 Limitations
This study has the following limitations:

- The I-MR chart is the only control chart that will be considered in this study.
- Out of the six Heller milling machine (H6-H11) with four fixtures each, the study is based on data only from the two fixtures machining left calipers in H6.

1.4 Background
At the site, a fully automated production line, consisting of six Heller milling machines and one CMM, is machining calipers. Each Heller milling machine (H6-H11) is machining two calipers at once (one right and one left). A rotatable fixture table is used to load parts simultaneously with the machining, resulting in four different fixture positions, as shown in Figure 1. Due to
the setup, fixture 1 and fixture 3 (left calipers) will have the same coordinates in the machine as well as for fixture 2 and fixture 4.

![Diagram of Heller milling machine with fixture positions labeled]

*Figure 1. Illustration of fixtures in the Heller milling machine.*

Since four different fixture positions must be considered, it results in four control charts from each machine and part feature, giving a total of 24 control charts for each part feature (six machines). To reduce the number of control charts to analyse, data from different fixtures can be combined and analysed together, e.g. both fixtures for the left caliper can be combined as well all four. It should be noted that errors and variation that is fixture or position specific is added when combining data from different fixtures.

There are currently nine features on the brake caliper that SPC has been applied to, tabulated in Table 1 with specifications and capability requirements. There are eight diameters and one distance feature. The selected features are chosen because they have a product characteristic or a process parameter that may affect user safety or compliance with regulations, fit, function, performance or subsequent processing. For long term capability requirements, the $C_{pk}/P_{pk}$ indices must be at least 1,33 as well as for the short-term requirements with the exception of Ø92 where the short-term requirement is 1,67.

*Table 1. Caliper features controlled by SPC with given tolerances and capability requirements.*

<table>
<thead>
<tr>
<th>Feature</th>
<th>Dimension [mm]</th>
<th>Tolerance [mm]</th>
<th>Short term</th>
<th>Long term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Ø39</td>
<td>39</td>
<td>+0,11, +0,08</td>
<td>1,33</td>
<td>1,33</td>
</tr>
<tr>
<td>3. Ø42 D side</td>
<td>42</td>
<td>+0,039, +0</td>
<td>1,33</td>
<td>1,33</td>
</tr>
<tr>
<td>10. Ø42 E side</td>
<td>42</td>
<td>+0,062, -0,15</td>
<td>1,33</td>
<td>1,33</td>
</tr>
<tr>
<td>12. Ø45,2</td>
<td>45,2</td>
<td>+0,039, +0</td>
<td>1,33</td>
<td>1,33</td>
</tr>
<tr>
<td>21. Ø30</td>
<td>30</td>
<td>+0,084, +0</td>
<td>1,33</td>
<td>1,33</td>
</tr>
<tr>
<td>26. Ø9</td>
<td>9</td>
<td>+0,015, +0</td>
<td>1,33</td>
<td>1,33</td>
</tr>
<tr>
<td>31. Distance 24</td>
<td>24</td>
<td>+0,1, -0,1</td>
<td>1,33</td>
<td>1,33</td>
</tr>
<tr>
<td>43. Ø25</td>
<td>25</td>
<td>+0,092, +0,04</td>
<td>1,33</td>
<td>1,33</td>
</tr>
<tr>
<td>46. Ø92</td>
<td>92</td>
<td>+0,1, -0,1</td>
<td>1,67</td>
<td>1,33</td>
</tr>
</tbody>
</table>

The machine tool has a significant influence on the process behaviour due to tool wear and high tool-to-tool variation. Tools have, over a long period, been evaluated to set a life-time expressed...
in number of parts it is capable of machining. In Table 2, the tool life-time for each part is tabulated, ranging from 700 parts to 10 000. Tools with lower life-time usually have faster tool wear that is easily identifiable on the control chart, whereas the tool wear is difficult to recognize for tools with higher life-time. Note that one tool machines parts from all four fixtures, therefore the number of parts machined with the same tool in a fixture is a fourth of the tabulated values.

**Table 2. Tool life-time of features run with SPC.**

<table>
<thead>
<tr>
<th>Feature</th>
<th>Tool life-time [parts]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Ø39</td>
<td>5 000</td>
</tr>
<tr>
<td>3. Ø42 D side</td>
<td>5 000</td>
</tr>
<tr>
<td>10. Ø42 E side</td>
<td>700</td>
</tr>
<tr>
<td>12. Ø45,2</td>
<td>700</td>
</tr>
<tr>
<td>21. Ø30</td>
<td>10 000</td>
</tr>
<tr>
<td>26. Ø9</td>
<td>10 000</td>
</tr>
<tr>
<td>31. Distance 24</td>
<td>4 000</td>
</tr>
<tr>
<td>43. Ø25</td>
<td>8 200</td>
</tr>
<tr>
<td>46. Ø92</td>
<td>1 200</td>
</tr>
</tbody>
</table>

1.4.1 SPC procedure
The I-MR chart is currently implemented and the moving range subgroup size is set to two. Due to the known complexity of process behaviour and distributions, the control limits are set to 66.67% of the specification limits. This setup is more a non-conformance control instead of process control as the control limits are based on the specification limits instead of the process variance (Down et al., 2005).

Measurements that is outside control limits but inside tolerance limits are marked as yellow and require attention from the operator. If any measurement is outside specification limits, immediate action is taken. Parts from the concerned machine and fixture is inspected and necessary adjustments are made to restore the process.

1.5 Theoretical introduction to SPC
Final inspection and testing of manufactured products is preventing defective products to reach the customer. This strategy does however not improve the quality, and to improve quality the process itself must be considered since it is there quality is defined. By changing strategy to a prevention approach, where concerned processes are observed to control the output, it is possible to improve quality (Down et al., 2005). A common application to control process output is the application of SPC. The concept of SPC is to ensure and improve quality through statistical tools, such as the control chart, introduced by Walter A. Shewhart in 1930. The control chart allows users to separate part-to-part variation (predictable variation) from special cause variation, i.e. variation that is not predictable. Shewhart defined the objective of SPC as: “eliminating causes of variability which need not be left to chance, making possible more uniform quality and thereby effecting certain economies” (Shewhart, 1930).

1.5.1 The control chart
There are several available control charts to apply for continuous data. However, in this study the individuals and moving-range (I-MR) chart is concerned. The I-MR chart is used for time dependent processes of continuous data and plots individual values on the I-chart and the MR-
chart plots the difference between two consecutive points (in the case of moving range subgroup of 2) according to:

\[ MR_i = |x_i - x_{i-1}|, \quad i = 2..n \]  

(1)

where \( x_i \) is an individual measurement and \( n \) is the number of individual measurements.

The essential elements of the control chart are seen in Figure 2, where LSL and USL are the lower and upper specification limit respectively. \( \bar{X} \) is the mean/center line and LCL and UCL are the lower and upper control limits respectively, calculated as three standard deviations from the mean as

\[ LCL = \bar{X} - 3\hat{\sigma}_c \]  

(2)

and

\[ UCL = \bar{X} + 3\hat{\sigma}_c \]  

(3)

respectively, where \( \hat{\sigma}_c \) is the estimate of the standard deviation of a stable process using the mean of the moving range, \( \bar{MR} \) (Down et al., 2005), given by:

\[ \hat{\sigma}_c = \frac{MR}{d_2} \]  

(4)

\( d_2 \) and the factors \( D_3 \) and \( D_4 \) are dependent by the moving range subgroup size and are tabulated in Table 3 below. In this study, the I-MR chart with a moving range subgroup size of two is used.

![I-MR Control Chart Example](image)

Figure 2: I-MR control chart example of a normal distributed process.

For the MR-chart, the moving range control limits are given by

\[ LCL_{MR} = D_3 \bar{MR} \]  

(5)

and

\[ UCL_{MR} = D_4 \bar{MR} \]  

(6)

The devisor \( d_2 \) and the factors \( D_3 \) and \( D_4 \) are dependent by the moving range subgroup size and are tabulated in Table 3 below. In this study, the I-MR chart with a moving range subgroup size of two is used.
Table 3. Constants for I-MR chart control limits dependent on the moving range subgroup size (Down et al., 2005).

<table>
<thead>
<tr>
<th>MR subgroup size</th>
<th>Devisor to estimate $\hat{\sigma}$</th>
<th>Factors for control limits</th>
<th>$D_3$</th>
<th>$D_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1,128</td>
<td></td>
<td>0</td>
<td>3,267</td>
</tr>
<tr>
<td>3</td>
<td>1,693</td>
<td></td>
<td>0</td>
<td>2,574</td>
</tr>
<tr>
<td>4</td>
<td>2,059</td>
<td></td>
<td>0</td>
<td>2,282</td>
</tr>
<tr>
<td>5</td>
<td>2,326</td>
<td></td>
<td>0</td>
<td>2,114</td>
</tr>
<tr>
<td>6</td>
<td>2,534</td>
<td></td>
<td>0</td>
<td>2,004</td>
</tr>
<tr>
<td>7</td>
<td>2,704</td>
<td></td>
<td>0,076</td>
<td>1,924</td>
</tr>
<tr>
<td>8</td>
<td>2,847</td>
<td></td>
<td>0,136</td>
<td>1,864</td>
</tr>
<tr>
<td>9</td>
<td>2,970</td>
<td></td>
<td>0,184</td>
<td>1,816</td>
</tr>
<tr>
<td>10</td>
<td>3,078</td>
<td></td>
<td>0,223</td>
<td>1,777</td>
</tr>
</tbody>
</table>

1.5.2 Nelson rules

The application of SPC and calculation of control limits is generally based on the conditions of a stable process along with normal distributed data, as in Figure 2. Stability is obtained when the variation and the mean is approximately constant, i.e. when the variation applied to every part is constant, called common variation. In this case the process is said to be in statistical control since it is possible to predict future measurements within a certain range. Processes that is out of statistical control has special cause variation applied to some parts, causing the measurements of these parts to differ remarkably from the common, part-to-part, variation (Montgomery, 2009).

The purpose of the control chart is to detect unexpected behaviour. Nelson (1984) presented eight rules applied for the control chart in order to find behaviour that is not expected, summarized in Table 4. The Nelson rules provides warnings for behaviour that is very unlikely for a random, normal distributed process. For processes with non-normal behaviour and/or distribution the rules should be chosen carefully to avoid false alarms.


<table>
<thead>
<tr>
<th>Rule #</th>
<th>Description</th>
<th>Control chart example</th>
<th>Problem indicated</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 point more than 3 standard deviations from the mean</td>
<td><img src="chart1.png" alt="chart" /></td>
<td>Out of statistical control, it is very likely that special cause variation occurred</td>
</tr>
<tr>
<td>2</td>
<td>9 points in a row on the same side of the mean</td>
<td><img src="chart2.png" alt="chart" /></td>
<td>There is a change in the trend</td>
</tr>
<tr>
<td>3</td>
<td>6 points in a row steadily increasing or decreasing</td>
<td><img src="chart3.png" alt="chart" /></td>
<td>There is a drifting trend</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>14 points in a row altering up and down</td>
<td>Process oscillation has increased</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2 out of 3 points in a row more than 2 standard deviations from the mean</td>
<td>There is a chance that special cause variation occurred</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>4 out of 5 points in a row more than 1 standard deviation from the mean</td>
<td>There is a chance that special cause variation occurred</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>15 points in a row within 1 standard deviation from the mean</td>
<td>The common, part-to-part variation has decreased</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>8 points in a row more than 1 standard deviation from the mean</td>
<td>The common, part-to-part variation has increased</td>
<td></td>
</tr>
</tbody>
</table>

### 1.5.3 Process capability indices

To determine how well a process produces parts in relation to given specifications (LSL and USL), process capability indices (PCI’s) are used. There are generally four PCI’s to determine process capability as follows:

- The process capability index: $C_p$
- The process capability index: $C_{pk}$
- The process performance index: $P_p$
- The process performance index: $P_{pk}$

These PCIs are ratios of how much of the specification range (USL-LSL) that is covered by the process’ common variation. The process capability, $C_p$ and $C_{pk}$, determines capability in relation to the estimated standard deviation $\hat{\sigma}_c$, that is dependent on the average moving range, according to equation (4), as follows:

$$C_p = \frac{USL - LSL}{6\hat{\sigma}_c}$$ \hspace{1cm} (7)

and

$$C_{pk} = \min \left( \frac{USL - \bar{X}}{3\hat{\sigma}_c}, \frac{\bar{X} - LSL}{3\hat{\sigma}_c} \right).$$ \hspace{1cm} (8)

The process performance, $P_p$ and $P_{pk}$, given by:

$$P_p = \frac{USL - LSL}{6\hat{\sigma}_p}$$ \hspace{1cm} (9)

and
\[ P_{pk} = \min \left( \frac{USL - \bar{X}}{3\hat{\sigma}_p}, \frac{\bar{X} - LSL}{3\hat{\sigma}_p} \right) \]  

(10)

determines capability in relation to the estimated sample standard deviation \( \hat{\sigma}_p \):

\[ \hat{\sigma}_p = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n - 1}} \]  

(11)

where \( n \) is the sample size.

\( C_{pk} \) and \( P_{pk} \) takes the decentralization of the process mean into calculation, providing the actual process capability/performance. \( C_p \) and \( P_p \) indicates the potential process capability for a perfectly centralized process, i.e. what is possible to achieve. Consequently, for a process with a centralized mean, \( C_p = C_{pk} \) and \( P_p = P_{pk} \). Generally, the process capability, \( C_p \) and \( C_{pk} \), is used for short term and real-time capability analysis whereas the process performance, \( P_p \) and \( P_{pk} \), is used for long-term analysis. Therefore, for a perfectly normal distributed process, \( C_p \approx P_p \) and \( P_{pk} \approx C_{pk} \) (Down et al., 2005).

The PCI’s \( C_{pk} \) and \( C_p \) requires the process to be normally distributed and in statistical control. On the contrary, the process performance indices \( P_{pk} \) and \( P_p \) do not assume that the process is normally distributed and in statistical control since they are calculated from the sample standard deviation (Keats & Montgomery, 1996). Therefore, calculations of \( C_{pk} \) and \( C_p \) for non-normal data provides misleading process capability results as process performance indices are usually significant lower than capability indices (Majstorovic & Sibalić, 2012).

For the normal distribution, the ± 3\( \sigma \) range covers 99.73% of all data (Montgomery, 2009). Usually, a process capability of \( C_{pk} \geq 1,33 \) is considered capable to produce part in relation to specifications. The probability of a part to exceed the specification limits in a process with \( C_{pk} = 1,33 \) is approximately 0.006%. In terms of part per million (ppm) out of specification, the 1.33 ratio corresponds to 63 ppm for a centralized process. Figure 3 illustrates the relationship between process capability, specification limits and ppm for a normal distributed process. Note that ppm is decreasing exponentially with increasing process capability.
<table>
<thead>
<tr>
<th>$C_p$</th>
<th>Spec. limits</th>
<th>Percent inside spec. limits</th>
<th>ppm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,33</td>
<td>±1σ</td>
<td>68,27</td>
<td>317 311</td>
</tr>
<tr>
<td>0,67</td>
<td>±2σ</td>
<td>95,45</td>
<td>45 500</td>
</tr>
<tr>
<td>1,00</td>
<td>±3σ</td>
<td>99,73</td>
<td>2 700</td>
</tr>
<tr>
<td>1,33</td>
<td>±4σ</td>
<td>99,9937</td>
<td>63</td>
</tr>
<tr>
<td>1,67</td>
<td>±5σ</td>
<td>99,999943</td>
<td>0,57</td>
</tr>
<tr>
<td>2,00</td>
<td>±6σ</td>
<td>99,9999998</td>
<td>0,002</td>
</tr>
</tbody>
</table>

Figure 3. Centralized normal distribution with relation between process capability, specification limits and ppm (Montgomery, 2009).

### 1.5.4 Event log

When running processes with SPC it is essential to gather information regarding the process behaviour in order to understand it. The event log should consist of any change, adjustment and event that may affect the process, such as; shift changes, tool changes, new material lots, maintenance, adjustments etc. When information is collected, it is possible to know under which circumstances a process is run (Down et al., 2005).
2 Method
In this chapter the methodology of the study is described. The first section (2.1) describes the processes’ behaviour and how distribution is determined as a foundation for the remaining sections. In 2.2 and 2.3, a method to evaluate short- and long-term capability is presented. In the final section (2.4) different approaches to calculate control limits are presented.

2.1 Process identification
In the current I-MR chart setup, all control limits are set to 66.67% of the specification limits, acting as warnings when the process is approaching specification limit. Therefore, alarms in the current state should not be confused with violation of the first Nelson rule. As seen in Table 5, the processes are generally out of statistical control and belongs to unknown distributions making it difficult to apply the theory in section 1.1.

Table 5. Control charts and descriptions of processes with control limits set as 66.67% of the specification limits.

<table>
<thead>
<tr>
<th>Control chart</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-MR Chart of D39</td>
<td>Variance and mean are not constant, process is out of statistical control.</td>
</tr>
<tr>
<td></td>
<td>Tool changes are difficult to locate.</td>
</tr>
<tr>
<td>I-MR Chart of D42D</td>
<td>Variance and mean seems to be constant within certain ranges.</td>
</tr>
<tr>
<td></td>
<td>The shifts in mean is assumed to be caused by tool variation.</td>
</tr>
<tr>
<td>I-MR Chart of D42E</td>
<td>Variance and mean seems to be close to constant within cycles.</td>
</tr>
<tr>
<td></td>
<td>Tool changes can be located.</td>
</tr>
<tr>
<td></td>
<td>The tool wear is making the process decrease over time.</td>
</tr>
<tr>
<td>I-MR Chart of D45.2</td>
<td>Variance and mean are not constant, process is out of statistical control.</td>
</tr>
<tr>
<td></td>
<td>The tool wear is making the process decrease over time.</td>
</tr>
<tr>
<td></td>
<td>Tool changes are difficult to locate.</td>
</tr>
<tr>
<td>Tool wear is making the process slowly decrease.</td>
<td></td>
</tr>
<tr>
<td>Process is significantly decentralized.</td>
<td></td>
</tr>
<tr>
<td>Variation is small in relation to specifications.</td>
<td></td>
</tr>
<tr>
<td>Variance and mean is not constant, process is out of statistical control.</td>
<td></td>
</tr>
<tr>
<td>Tool changes are difficult to locate.</td>
<td></td>
</tr>
<tr>
<td>Variance and mean are close to constant.</td>
<td></td>
</tr>
<tr>
<td>Tool changes are difficult to locate.</td>
<td></td>
</tr>
<tr>
<td>Tool wear is making the process slowly decrease.</td>
<td></td>
</tr>
<tr>
<td>Variation is small in relation to specifications.</td>
<td></td>
</tr>
<tr>
<td>Variance seems to be constant within certain ranges.</td>
<td></td>
</tr>
<tr>
<td>The shifts in the mean is assumed to be caused by tool variation.</td>
<td></td>
</tr>
</tbody>
</table>

The data was tested to follow a specific distribution with the Anderson-Darling goodness-of-fit test (AD-test). The AD-test, developed by Anderson and Darling (1954), is a method to statistically measure if a set of data follow a specified continuous distribution function. The purpose of the AD-test is either accept or reject the null hypothesis, $H_0$, stated as: *The data follow a specified distribution*. To determine whether to accept or reject the null hypothesis, the probability of that the null hypothesis is true, is calculated, denoted as the $p$-value. The $p$-value has a range from 0 to 1, where higher value indicates higher probability that the null hypothesis is true. Generally, if $p$-value $\geq 0.05$, at a 95% confidence interval, the null hypothesis is accepted and it is assumed that the data follow a certain distribution.
Since the data is continuous, it is tested if the processes follow any of the following continuous probability distributions that is of importance in SPC (Montgomery, 2009):

- The normal distribution
- The exponential distribution
- The lognormal distribution
- The gamma distribution
- The Weibull distribution

To determine PCI’s of other distributions than the normal, the mean and variance must be calculated. In Table 6, the mean and variance of other continuous distributions are presented.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Parameters</th>
<th>Mean, μ</th>
<th>Variance, σ²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>λ</td>
<td>(\mu = \frac{1}{\lambda})</td>
<td>(\sigma^2 = \frac{1}{\lambda^2})</td>
</tr>
<tr>
<td>Lognormal</td>
<td>(\theta, \omega)</td>
<td>(\mu = e^{\theta + \omega^2/2})</td>
<td>(\sigma^2 = e^{2\theta + \omega^2}(e^{\omega^2} - 1))</td>
</tr>
<tr>
<td>Gamma</td>
<td>(r, \lambda)</td>
<td>(\mu = \frac{r}{\lambda})</td>
<td>(\sigma^2 = \frac{r}{\lambda^2})</td>
</tr>
<tr>
<td>Weibull</td>
<td>(\theta, \beta)</td>
<td>(\mu = \theta \Gamma \left(1 + \frac{1}{\beta}\right))</td>
<td>(\sigma^2 = \theta^2 \left(\Gamma \left(1 + \frac{2}{\beta}\right) - \left(\Gamma \left(1 + \frac{1}{\beta}\right)\right)^2\right))</td>
</tr>
</tbody>
</table>

The processes were tested with the AD goodness-of-fit test for the mentioned continuous distributions, p-values tabulated in Table 7. The null hypothesis was accepted for distance 24 at the normal distribution and for Ø 42 E-side at the Weibull distribution.

<table>
<thead>
<tr>
<th>Feature</th>
<th>Normal</th>
<th>Weibull</th>
<th>Gamma</th>
<th>Lognormal</th>
<th>Exponential</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Ø39</td>
<td>&lt;0,005</td>
<td>&lt;0,010</td>
<td>-</td>
<td>&lt;0,005</td>
<td>&lt;0,003</td>
</tr>
<tr>
<td>3. Ø42 D side</td>
<td>&lt;0,005</td>
<td>&lt;0,005</td>
<td>&lt;0,005</td>
<td>&lt;0,005</td>
<td>&lt;0,003</td>
</tr>
<tr>
<td>10. Ø42 E side</td>
<td>&lt;0,005</td>
<td>0,392</td>
<td>&lt;0,005</td>
<td>&lt;0,005</td>
<td>&lt;0,003</td>
</tr>
<tr>
<td>12. Ø45,2</td>
<td>&lt;0,005</td>
<td>&lt;0,010</td>
<td>&lt;0,005</td>
<td>&lt;0,005</td>
<td>&lt;0,003</td>
</tr>
<tr>
<td>21. Ø30</td>
<td>&lt;0,005</td>
<td>&lt;0,010</td>
<td>-</td>
<td>&lt;0,005</td>
<td>&lt;0,003</td>
</tr>
<tr>
<td>26. Ø9</td>
<td>&lt;0,005</td>
<td>&lt;0,010</td>
<td>&lt;0,005</td>
<td>&lt;0,005</td>
<td>&lt;0,003</td>
</tr>
<tr>
<td>31. Distance 24</td>
<td>0,063</td>
<td>0,014</td>
<td>0,063</td>
<td>0,047</td>
<td>&lt;0,003</td>
</tr>
<tr>
<td>43. Ø25</td>
<td>&lt;0,005</td>
<td>&lt;0,010</td>
<td>-</td>
<td>&lt;0,005</td>
<td>&lt;0,003</td>
</tr>
<tr>
<td>46. Ø92</td>
<td>&lt;0,005</td>
<td>&lt;0,010</td>
<td>&lt;0,005</td>
<td>&lt;0,005</td>
<td>&lt;0,003</td>
</tr>
</tbody>
</table>

### 2.2 Short term capability analysis

As stated, the reliability of the PCI’s is dependent on the normality of the process data. Therefore, the p-value of the normal distribution is compared to the \(C_p/C_{pk}\) indices. The p-value, \(C_p\) and \(C_{pk}\) are continuously calculated for 30 data points, i.e. for \(x_1 \ldots x_{30}, x_2 \ldots x_{31}, \ldots, x_{n-30} \ldots x_n\) where \(n\) is the sample size.
2.3 Long term capability analysis

When analysing larger data sets, the complexity increases as more factors of variation is added which affects the validity of the PCI’s. This section describes the methods that were used to estimate the long-term PCI’s of the given processes. The methods in section 2.3.3 and 2.3.4 calculates the PCI’s by first transforming the data into normal.

2.3.1 Capability estimation by process performance indices

The capability was estimated by the process performance indices $P_p$ and $P_{pk}$ as stated in equation (9) and equation (10). The process performance indices are applicable for all kinds of data as it does not require normal distributed or stable data.

2.3.2 Capability estimation by machine tool intervals

For processes where the variation from different machine tools is significant greater than the part-to-part variation (as in Ø 42 D-side and Ø 92), the data was separated by each tool change, resulting in $k$ intervals (one for each machine tool). The mean of each interval is:

$$
\bar{x}_j = \frac{1}{n_j} \sum_{i=1}^{n_j} x_i
$$

(12)

where $j = 1, 2, \ldots, k$ and $n_j$ is the sample size of each interval.

For each interval, the process performance $P_{pk,i}$ was calculated, as stated in equation (10). The overall capability of the of the process is determined by the minimum of $P_{pk,i}$, as:

$$
P_{pk} = \min(P_{pk,i})
$$

(13)

To determine the potential process performance index, $P_p$, the data was first standardized by subtracting the mean from each data point in each interval. $P_p$ is then determined from the entire, standardized, data set.

From the given data, there is no information regarding location of tool changes which is crucial for this method. Therefore, the locations have been assumed in the two obvious cases Ø 42 D-side and Ø 92 where the tool changes can be seen clearly in Table 5.

2.3.3 Capability estimation by Box-Cox transformation

Box and Cox (1964) introduced a power transformation method to transform a positive response variable $X$, into normal, defined by:

$$
X^{(\lambda)} = \begin{cases} 
\frac{X^\lambda - 1}{\lambda}, & \lambda \neq 0 \\
\log(X_i), & \lambda = 0 
\end{cases}
$$

(14)

The transformation depends on a single parameter $\lambda$. For some unknown value of $\lambda$, it is assumed that the transformed observations $X^{(\lambda)}$ will be normally distributed. The probability density function (PDF) of the transformed data is obtained by multiplying the normal PDF with the Jacobian of the transformation as:
\[ f(X \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(X^\lambda - \frac{1}{\lambda} - \mu)^2}{2\sigma^2}\right) J(\lambda; X), \quad (15) \]

where

\[ J(\lambda; X) = \frac{\partial X_i^{(\lambda)}}{\partial X_i} = X_i^{\lambda - 1} \quad (16) \]

The parameter \( \lambda \) is estimated with maximum likelihood estimation (MLE) by assigning values to \( \lambda \) from a selected range, generally: \(-5 \leq \lambda \leq 5\). The likelihood function \( L \) is the reverse probability density function, defined as:

\[ L(\lambda \mid X_i) = \prod_{i=1}^{n} f(X \mid \mu, \sigma^2) \]

\[ = \frac{1}{(2\pi\sigma^2)^{n/2}} \exp\left(-\frac{1}{2\sigma^2} \sum_{i=1}^{n} \left(\frac{X_i^{\lambda} - \frac{1}{\lambda} - \mu}{\lambda} \right)^2\right) \prod_{i=1}^{n} X_i^{\lambda - 1} \quad (17) \]

By maximizing the likelihood, the best fit estimators \( \hat{\sigma} \) and \( \hat{\mu} \) for a normal distribution with chosen lambda is found. However, due to the complexity of maximizing the likelihood function the logarithm is used instead, called the log likelihood. The product is therefore eliminated as follows in equation (18):

\[ l = \log(L) \]

\[ = -\frac{n}{2} \ln(2\pi) - \frac{n}{2} \ln(\hat{\sigma}^2) - \frac{n}{2} + (\lambda - 1) \sum_{i=1}^{n} \ln(X_i) \quad (18) \]

Since the logarithm function is continuous increasing, the values maximizing the likelihood will also maximize the logarithm of the likelihood. The first and third term is constants and will therefore not have any impact for the estimated parameters and may be neglected, resulting in equation (19):

\[ l = -\frac{n}{2} \ln(\hat{\sigma}^2) + (\lambda - 1) \sum_{i=1}^{n} \ln(X_i) \quad (19) \]

The mean and variance are estimated by

\[ \hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} \frac{X_i^\lambda - 1}{\lambda} \quad (20) \]

and

\[ \hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{X_i^\lambda - 1}{\lambda} - \hat{\mu}\right)^2 \quad (21) \]
respectively.

The Box-Cox transformation aims to make a skewed normal distribution more symmetric by stretching either the lower or upper tail. If $\lambda$ is less than one, the transformation will pull in a stretched-out upper tail and stretch out the lower tail. For $\lambda$ greater than one, the transformation will stretch out the upper tail and pull in the lower tail. Therefore, the method will find a $\lambda$ less than one if the distribution is right-skewed (positive) and a $\lambda$ greater than one for left-skewed (negative).

Box-Cox transformation is only applicable on data that is non-zero and positive. If any variable $X$ from the data is less or equal to zero, a constant is added in order to make the data positive.

If the transformation is successful, meaning that the null hypothesis is accepted for the transformed data, the specification limits are transformed with chosen $\lambda$. The PCI’s are estimated by using the mean and standard deviation of the transformed data (Hosseinifard et al. 2009) as in equation (9) and (10).

Box-Cox transformation failed for all non-normal processes. The optimal $\lambda$ did not result in $p$-value $\geq 0.05$ (Table 8), therefore the null hypothesis was rejected and no PCI’s were determined.

Table 8. Box-Cox transformation of the non-normal processes.

<table>
<thead>
<tr>
<th>Feature</th>
<th>$\lambda$ (optimal)</th>
<th>$p$-value</th>
<th>$H_0$</th>
<th>$P_{pk}$</th>
<th>$P_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Ø39</td>
<td>98,6205</td>
<td>&lt;0,005</td>
<td>Rejected</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3. Ø42 D side</td>
<td>96,6341</td>
<td>&lt;0,005</td>
<td>Rejected</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>10. Ø42 E side</td>
<td>96,3545</td>
<td>&lt;0,005</td>
<td>Rejected</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>12. Ø45,2</td>
<td>-6,0808</td>
<td>&lt;0,005</td>
<td>Rejected</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>21. Ø30</td>
<td>-6,0955</td>
<td>&lt;0,005</td>
<td>Rejected</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>26. Ø9</td>
<td>164,1899</td>
<td>&lt;0,005</td>
<td>Rejected</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>43. Ø25</td>
<td>-7,0155</td>
<td>&lt;0,005</td>
<td>Rejected</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>46. Ø92</td>
<td>-5,2716</td>
<td>&lt;0,005</td>
<td>Rejected</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Wang et al. (2016) found that Box-Cox transformation provided good capability estimations for gamma and lognormal distributions whereas the capability estimation for Weibull distributions was significant lower than the actual value.

2.3.4 Capability estimation by Johnson transformation

Johnson (1949) developed a system of distributions used to fit an unknown distribution and transform it into normal, called Johnson transformation. The system contains of three transformation; lognormal, unbounded and bounded, as seen in Table 9.

Table 9. The Johnson systems with corresponding transformations.

<table>
<thead>
<tr>
<th>Johnson system</th>
<th>Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_L$ (Lognormal)</td>
<td>$Y(y^<em>,\delta,\varepsilon) = y^</em> + \delta \ln(X - \varepsilon)$</td>
</tr>
<tr>
<td>$S_U$ (Unbounded)</td>
<td>$Y(y,\delta,\varepsilon,\lambda) = y + \delta \sinh^{-1}\left(\frac{X - \varepsilon}{\lambda}\right)$</td>
</tr>
<tr>
<td>$S_B$ (Bounded)</td>
<td>$Y(y,\delta,\varepsilon,\lambda) = y + \delta \ln\left(\frac{X - \varepsilon}{\lambda + \varepsilon - X}\right)$</td>
</tr>
</tbody>
</table>
The Johnson system is chosen dependent on the distribution where:

- Lognormal systems cover the lognormal family.
- Unbounded systems cover distributions that goes from negative infinity to infinity from lower to upper tail, e.g., t and normal distribution.
- Bounded systems cover distributions that have a fixed boundary on either the upper or lower tail, or both, e.g., gamma and Weibull distributions.

The transformation is dependent on four parameters where γ and δ indicates shape, λ shape and ε location (George & Ramachandran, 2011). Slifker and Shapiro (1980) presented a method to select a suitable Johnson system and estimate the four Johnson parameters. The idea was to distinguish bounded from unbounded systems through evaluating the tails of the unknown distribution. The selection algorithm contains the following operations:

1. Chose a z-score (0 < z < 1) and create the four points ±z and ±3z. As a rule of thumb, the chosen value of z should take greater values the larger the number of observations.
2. Determine the probabilities \( P_\zeta \), where \( \zeta = \{-3z, -z, z, 3z\} \), from the standard normal table and let \( x_\zeta \) be the corresponding percentiles of the data values.
3. Define the discriminant \( d \), calculated as

\[
d = \frac{mn}{p^2}
\]

where \( m = x_{3z} - x_z \), \( n = x_z - x_{-z} \) and \( p = x_z - x_{-z} \). The Johnson system is chosen dependent on the value of the discriminant. If \( d \) is less than 0.999, the bounded system is chosen. If \( d \) is greater than 1.001, the unbounded is chosen and for any value in between, the lognormal system is chosen.

For selected Johnson system below (1-3), the four parameters are estimated as follows:

1. **Johnson S_L distribution**

\[
\hat{\delta} = \frac{2z}{\ln \left( \frac{m}{p} \right)}
\]

\[
\hat{p}^* = \hat{\delta} \ln \left( \frac{m - 1}{p \left( \frac{m}{p} \right)^{1/2}} \right)
\]

\[
\hat{\varepsilon} = \frac{x_z + x_{-z}}{2} - \frac{p m + 1}{2 \frac{m}{p} - 1}
\]

2. **Johnson S_U distribution**

\[
\hat{\delta} = \frac{2z}{\cosh^{-1} \left( \frac{1}{2} \frac{m}{p} + \frac{n}{p} \right)}
\]
\[ \hat{\gamma} = \hat{\delta} \sinh^{-1} \left( \frac{n - m}{2 \left( \frac{mn}{p} - 1 \right)^{1/2}} \right) \]  

(27)

\[ \hat{\lambda} = \frac{2p \left( \frac{mn}{p} - 1 \right)^{1/2}}{\left( \frac{m}{p} + \frac{n}{p} - 2 \right) \left( \frac{m}{p} + \frac{n}{p} + 2 \right)^{1/2}} \]  

(28)

\[ \hat{\varepsilon} = \frac{x_{z} + x_{-z}}{2} + \frac{p \left( \frac{n}{p} - m \right)}{2 \left( \frac{m}{p} + \frac{n}{p} - 2 \right)} \]  

(29)

3. Johnson SB distribution

\[ \delta = \frac{z}{\cosh^{-1} \left( \frac{1}{2} \left( 1 + \frac{p}{m} \right) \left( 1 + \frac{p}{n} \right) \right)^{1/2}} \]  

(30)

\[ \hat{\gamma} = \delta \sinh^{-1} \left( \frac{\left( \frac{p}{n} - \frac{p}{m} \right) \left( 1 + \frac{p}{m} \right) \left( 1 + \frac{p}{n} \right) - 4}{2 \left( \frac{pp}{mn} - 1 \right)} \right) \]  

(31)

\[ \hat{\lambda} = \frac{p \left( \left( 1 + \frac{p}{m} \right) \left( 1 + \frac{p}{n} \right) - 2 \right)^{2} - 4}{\frac{p}{mn} - 1} \]  

(32)

\[ \hat{\varepsilon} = \frac{x_{z} + x_{-z}}{2} - \frac{\hat{\lambda}}{2} + \frac{p \left( \frac{n}{p} - \frac{m}{p} \right)}{2 \left( \frac{pp}{mn} - 1 \right)} \]  

(33)

After estimating the Johnson parameters, the data set \( X \) can be transformed and for the transformed data \( Y \), the Anderson-Darling test is performed to evaluate normality. The procedure is repeated with a feasible step size and number of iterations with a new \( z \) value. The Johnson transformation function corresponding to the \( z \) value with highest p-value will be selected.

As opposed to the Box-Cox transformation, Johnson transformation allows transformation from the entire skewness-kurtosis plane (Down et al., 2005), but at the cost of increased complexity.

The PCI’s are estimated by using the mean and variance of the transformed, normal distributed data in equation (9) and (10).
For Ø 39, Ø 42 D-side, Ø 9 and Ø 92 no Johnson system were found that satisfied the condition \( p \)-value \( \geq 0.05 \), therefore, the null hypothesis was rejected and no PCI's were determined. For Ø42 E-side, Ø45.2, Ø30 and Ø25 a Johnson transformation was found that satisfied the condition \( p \)-value \( \geq 0.05 \) and the null hypothesis was accepted and PCIs was calculated accordingly, shown in Table 10.

**Table 10.** Johnson transformation of the non-normal processes.

<table>
<thead>
<tr>
<th>Feature</th>
<th>z value</th>
<th>Johnson system</th>
<th>( p )-value</th>
<th>( H_0 )</th>
<th>( P_{\mu} )</th>
<th>( P_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Ø39</td>
<td>0.42</td>
<td>SU</td>
<td>0.011</td>
<td>Rejected</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3. Ø42 D side</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Rejected</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>10. Ø42 E side</td>
<td>0.56</td>
<td>SB</td>
<td>0.469</td>
<td>Accepted</td>
<td>1.00</td>
<td>1.61</td>
</tr>
<tr>
<td>12. Ø45.2</td>
<td>0.66</td>
<td>SB</td>
<td>0.863</td>
<td>Accepted</td>
<td>1.36</td>
<td>1.40</td>
</tr>
<tr>
<td>21. Ø30</td>
<td>0.79</td>
<td>SU</td>
<td>0.546</td>
<td>Accepted</td>
<td>1.29</td>
<td>1.87</td>
</tr>
<tr>
<td>26. Ø9</td>
<td>0.51</td>
<td>SU</td>
<td>0.045</td>
<td>Rejected</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>43. Ø25</td>
<td>0.63</td>
<td>SU</td>
<td>0.317</td>
<td>Accepted</td>
<td>1.37</td>
<td>1.59</td>
</tr>
<tr>
<td>46. Ø92</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Rejected</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

### 2.3.5 Capability estimation by Clements method

For a perfectly normal distributed process, the skewness and kurtosis is both zero, the mean is equal to the median (0.5 percentile) and the percentiles 0.135 and 99.865, denoted by \( X_{0.135} \) and \( X_{99.865} \) respectively, will be equal to the values of \( \pm 3\sigma \) (Figure 4).

![Figure 4. For a perfectly normal distribution, -3σ and 3σ takes the same value as \( X_{0.135} \) and \( X_{99.865} \) respectively.](image)

For non-normal processes, Clements (1989) proposed a percentile estimation method to replace the six-sigma interval by estimating the mentioned percentiles with given skewness, \( Sk \), and kurtosis, \( Ku \). The estimated percentiles are given by

\[
X_{0.135} = \bar{X} - sL'_p, \quad (34) \\
X_{99.865} = \bar{X} + sU'_p \quad (35)
\]

and

\[
X_{0.5} = \begin{cases} 
\bar{X} + sM', & Sk < 0 \\
\bar{X} - sM', & Sk \geq 0 
\end{cases} \quad (36)
\]

where \( L'_p \) is the standardized 0.135-percentile, \( U'_p \) is the standardized 99.865-percentile and \( M' \) is the standardized median, tabulated in Appendix 1, Appendix 2 and Appendix 3, derived from Pearson’s family of continuous distributions. Note that for positive skewness, the sign is reversed when calculating the estimated median.
The estimated PCIs are thereafter estimated as
\[
\hat{C}_p = \frac{USL - LSL}{X_{99.865} - X_{0.135}}
\]
and
\[
\hat{C}_{pk} = \min\left(\frac{X_{0.5} - LSL}{X_{0.5} - X_{0.135}}, \frac{USL - X_{0.5}}{X_{99.865} - X_{0.5}}\right).
\]
For practical use, Clements recommends bilinear interpolation to find the standardized percentile \( P(Sk, Ku) \):
\[
P(Sk, Ku) = C_1 + C_2 Sk + C_3 Ku + C_4 SkKu
\]
where the four coefficients can be found by solving the system
\[
\begin{bmatrix}
1 & Sk_1 & Ku_1 & Sk_1Ku_1 \\
1 & Sk_1 & Ku_2 & Sk_1Ku_2 \\
1 & Sk_2 & Ku_1 & Sk_2Ku_1 \\
1 & Sk_2 & Ku_2 & Sk_2Ku_2
\end{bmatrix}
\begin{bmatrix}
C_1 \\
C_2 \\
C_3 \\
C_4
\end{bmatrix}
= \begin{bmatrix}
P(Q_{11}) \\
P(Q_{12}) \\
P(Q_{21}) \\
P(Q_{22})
\end{bmatrix}
\]
The advantage with Clements’ method is that the data doesn’t require any transformation. Therefore, it provides results that is more user-friendly. As opposed to Johnson and Box-Cox transformations, Clements’ method reflects the process’ true distribution. In a comparative study of Clements’ method by Czarski (2009), the percentiles \( X_{0.135}, X_{0.5} \) and \( X_{99.865} \) confirmed very good accuracy when compared to the theoretical percentiles of known continuous distributions (exponential, gamma, and Weibull) except for the lognormal distribution. Moreover, Senvar and Sennaroglu (2016) as well as Wang et al. (2016) found that Clements’ method provided more accurate capability estimations of Weibull distributions in comparison with Box-Cox and Johnson transformation.

2.3.6 Capability estimation from known distribution
Only Ø 42 E-side was assumed to follow a distribution other than the normal distribution. The process was described by the Weibull distribution, the mean and variance are calculated as stated in Table 6 and used to estimate the PCI’s stated in equation (9) and (10).

2.4 Control limit calculations
Control limits were calculated with four different approaches:

1. By the average moving range
2. By the estimated sample standard deviation
3. By machine tool intervals
4. By Clements method

Control limits calculated from the average moving range is the standard for the I-MR chart (Down et al., 2005), stated by equation (2) and equation (3). The second approach is to use the estimated sample standard deviation, stated in equation (1). The lower and upper control limit are
\[
LCL = \bar{X} - 3\hat{\sigma}_p
\]
and...
\[ UCL = \bar{X} + 3\hat{\sigma}_p \] 

respectively. In the third approach, the control limits are calculated with respect to machine tool variation. The lower and upper control limits are set to

\[ LCL = \min(\bar{X}_j) - 3\hat{\sigma}_p \] 

and

\[ UCL = \max(\bar{X}_j) + 3\hat{\sigma}_p \]

respectively, where \( \bar{X}_j \) is the mean of each machine tool interval and \( \hat{\sigma}_p \) is the sample standard deviation, calculated from the standardized data. Control limits by the fourth approach, Clements method, are set to

\[ LCL = X_{0.135} = \bar{X} - s L_p' \] 

and

\[ UCL = X_{99.865} = \bar{X} + s U_p' \]

respectively, as mentioned in section 2.3.7.
3 Results

In this chapter the results of the methods applied on the concerned processes are presented. In section 3.1 the short-term capability analysis is presented, followed by the long-term capability analysis (section 3.2) and control limit calculations (section 3.3).

3.1 Short term capability analysis

The mean and minimum values of the PCI's of the short-term capability analysis is tabulated in Table 11. The PCI's over time are visualized in Figure 5 - Figure 13. For the short-term capability analysis, Ø 45,2 (Figure 8) and distance 24 (Figure 11) failed the capability requirements twice and once respectively in the past months.

Ø 39 (Figure 5) reaches a minimum of $C_{pk}$ at 1,60, which might be critical as the normality cannot be assumed during that period. Likewise, Ø 9 (Figure 10) has a minimum of $C_{pk}$ at 1,45 under non-normal conditions. Remaining processes resulted in high PCI’s, with process capability of $3.51 \leq C_{pk} \leq 5.37$.

As stated in section 3.1, only distance 24 was assumed to follow a normal distribution. However, for smaller samples, normality can often be assumed for processes that is non-normal under long-term analysis.

Table 11. Mean and minimum values of PCI’s of the short-term capability analysis. Yellow: PCI less than 1,67 (5 sigma process). Red: PCI less than 1,33 (4 sigma process).

<table>
<thead>
<tr>
<th>Feature</th>
<th>Mean $C_p$</th>
<th>Mean $C_{pk}$</th>
<th>Min $C_p$</th>
<th>Min $C_{pk}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Ø39</td>
<td>5,52</td>
<td>5,23</td>
<td>1,69</td>
<td>1,60</td>
</tr>
<tr>
<td>3. Ø42 D side</td>
<td>12,31</td>
<td>9,37</td>
<td>5,11</td>
<td>4,41</td>
</tr>
<tr>
<td>10. Ø42 E side</td>
<td>8,05</td>
<td>7,12</td>
<td>4,72</td>
<td>3,51</td>
</tr>
<tr>
<td>12. Ø45,2</td>
<td>3,02</td>
<td>2,28</td>
<td>1,29</td>
<td>1,02</td>
</tr>
<tr>
<td>21. Ø30</td>
<td>12,80</td>
<td>7,41</td>
<td>8,67</td>
<td>4,52</td>
</tr>
<tr>
<td>26. Ø9</td>
<td>4,47</td>
<td>4,05</td>
<td>2,27</td>
<td>1,45</td>
</tr>
<tr>
<td>31. Distance 24</td>
<td>2,25</td>
<td>1,58</td>
<td>1,68</td>
<td>1,07</td>
</tr>
<tr>
<td>43. Ø25</td>
<td>13,60</td>
<td>10,38</td>
<td>6,07</td>
<td>5,37</td>
</tr>
<tr>
<td>46. Ø92</td>
<td>15,50</td>
<td>10,70</td>
<td>7,47</td>
<td>3,55</td>
</tr>
</tbody>
</table>
Figure 5. Short term capability of Ø 39. If $H_0=1$, normality may be assumed.

Figure 6. Short term capability of Ø 42 D-side. If $H_0=1$, normality may be assumed.

Figure 7. Short term capability of Ø 42 E-side. If $H_0=1$, normality may be assumed.

Figure 8. Short term capability of Ø 45.2. If $H_0=1$, normality may be assumed.

Figure 9. Short term capability of Ø 30. If $H_0=1$, normality may be assumed.

Figure 10. Short term capability of Ø 9. If $H_0=1$, normality may be assumed.
Figure 11. Short term capability of distance 24. If $H_0=1$, normality may be assumed.

Figure 12. Short term capability of Ø 25. If $H_0=1$, normality may be assumed.

Figure 13. Short term capability of Ø 92. If $H_0=1$, normality may be assumed.

3.2 Long term capability analysis

The estimated PCI’s of the selected methods are compared in Figure 14 ($C_p$) and Figure 15 ($C_{pk}$). As distance 24 is assumed to follow the normal distribution, only the capability indices $P_p$ and $P_{pk}$ calculated by the estimated sample standard deviation is considered.

The estimated potential capability $C_p$ was greater than 1.33 for all features and methods except for $P_p$ of Ø 45.2.
Ø 39, Ø 9 and Ø 25 all resulted in $C_{pk}$ greater than 1.33 for each method. For remaining features, at least one method provided $C_{pk}$ less than 1.33.

Table 12 shows the tabulated PCI estimations of each method as well as the $C_p$ and $C_{pk}$ indices, as stated in equation (7) and (8). Note that the $C_p$ and $C_{pk}$ is significantly higher than the estimated PCI’s.
Table 12. Tabulated PCI's for each method. PCI’s less than 1,33 are marked with red. CM=Clements method, JT=Johnson transformation, TV=Machine tool variation.

<table>
<thead>
<tr>
<th>Feature</th>
<th>Cp</th>
<th>Pp</th>
<th>Cp CM</th>
<th>Pp JT</th>
<th>Pp TV</th>
<th>Cpk</th>
<th>Ppk</th>
<th>Cpk CM</th>
<th>Ppk JT</th>
<th>Ppk WD</th>
<th>Ppk TV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Ø39</td>
<td>3.83</td>
<td>1.45</td>
<td>1.43</td>
<td>-</td>
<td>-</td>
<td>3.74</td>
<td>1.41</td>
<td>1.36</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3. Ø42 D side</td>
<td>10.71</td>
<td>1.43</td>
<td>2.60</td>
<td>-</td>
<td>-</td>
<td>2.41</td>
<td>9.57</td>
<td>1.28</td>
<td>2.00</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>10. Ø42 E side</td>
<td>7.17</td>
<td>1.78</td>
<td>2.36</td>
<td>1.61</td>
<td>-</td>
<td>6.34</td>
<td>1.58</td>
<td>1.56</td>
<td>1.00</td>
<td>0.98</td>
<td>-</td>
</tr>
<tr>
<td>12. Ø45,2</td>
<td>2.58</td>
<td>1.27</td>
<td>1.56</td>
<td>1.40</td>
<td>-</td>
<td>1.93</td>
<td>0.96</td>
<td>1.47</td>
<td>1.36</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>21. Ø30</td>
<td>12.04</td>
<td>5.65</td>
<td>6.46</td>
<td>1.87</td>
<td>-</td>
<td>6.92</td>
<td>3.25</td>
<td>2.67</td>
<td>1.29</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>26. Ø9</td>
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<td>1.63</td>
<td>1.50</td>
<td>-</td>
<td>-</td>
<td>3.44</td>
<td>1.50</td>
<td>1.40</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>31. Dist 24</td>
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<td>1.62</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.59</td>
<td>1.17</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>43. Ø25</td>
<td>11.43</td>
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<td>4.73</td>
<td>1.59</td>
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<td>8.94</td>
<td>3.57</td>
<td>4.00</td>
<td>1.37</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>46. Ø92</td>
<td>12.51</td>
<td>1.47</td>
<td>2.20</td>
<td>2.54</td>
<td>8.92</td>
<td>1.05</td>
<td>2.06</td>
<td>-</td>
<td>-</td>
<td>1.92</td>
<td>-</td>
</tr>
</tbody>
</table>

3.3 Control limit calculations
The calculated control limits for each method is presented in appendix 4-12. Control limits by average moving range, sample standard deviation and Clements method were calculated for all non-normal distributed processes. As machine tool changes were only safely assumed for Ø 42 D-side and Ø 92, control limits by machine tool intervals were calculated for those.
4 Conclusions
In this chapter, conclusions are made based on the presented results.

4.1 Short-term capability
The short-term capability analysis provided overall acceptable results as the mean of the PCI’s were very high in comparison with the requirements. When the processes are suddenly shifting mean, the short-term capability is greatly affected since normality cannot be assumed. Therefore, it is difficult to determine whether a process that is barely exceeding $C_{pk}=1.33$ under non-normal conditions is capable since these special cause variations might be expected.

4.2 Long-term capability
The results from the long-term capability analysis is difficult to interpret as the distribution is unknown and the Johnson transformation as well as Clements method have only proved validity for processes of known continuous distributions.

The following conclusions regarding long-term capability for each process is made:

- The three processes were PCI’s confirmed to be at least 1.33 for all methods ($\varnothing 39$, $\varnothing 9$ and $\varnothing 25$) is assumed to be capable as the contradictory could not be proven.
- The $P_{pk}$ of the sample standard deviation of $\varnothing 42$ D-side and $\varnothing 92$ were less than 1.33 which states that the processes are not capable. However, if the assumed machine tool changes are in fact true, the $P_{pk}$ determined from each tool interval is a strongly considerable PCI in terms of validity and the processes would therefore be considered capable.
- Distance 24 follows the normal distribution, therefore the $P_{pk}$ is accepted and the process is determined to incapable since $P_{pk} \leq 1.33$.
- Acceptable capability of $\varnothing 45.2$ is determined to be incapable as it is out of statistical control and $P_{pk}$ by the sample standard deviation is less than 1.33.
- $\varnothing 42$ E-side follows a Weibull distribution due to tool wear and the $P_{pk}$ by Weibull is less than 1.33. The process is however in statistical control and the part-to-part variation is very low in comparison with the specification range. Furthermore, Clements method and Johnson transformation estimated the process to be capable.
- $\varnothing 30$ is estimated to be capable by Clements method and incapable by Johnson transformation. Therefore, process capability cannot be safely determined.

4.3 Control limits
Clements method to set control limits provided good results for processes where the behaviour and distribution of the process is unknown and where the common variation is difficult to distinguish from special cause variation. Processes where Clements method is preferable to set control limits are:

- $\varnothing 39$
- $\varnothing 45.2$
- $\varnothing 30$
- $\varnothing 9$
- $\varnothing 25$
Clements method to set control limits provided poor results for remaining processes as the control limits was either too narrow, resulting in false alarms, or too wide, resulting in special cause variation not being detected.

The control limits of Ø 42 D-side and Ø 92 were set according to machine tool variation since it is the only valid method due to the shifting mean of the processes. Control limits of distance 24 were calculated by the average moving range since the process is assumed to follow the normal distribution. The control limits of Ø 42 E-side were calculated with the sample standard deviation as this was the only method not providing any false alarms.
5 Discussion
The main issue in the current state is the lack of information regarding the processes’ behaviours, resulting in difficulties of identifying process cycles and distributions. With information of machine tool changes, new material lots and other significant factors that relates to special cause variation in the event log, the capability analysis would be easier to interpret and the control limits would be set with higher accuracy. Therefore, the operators have a significant role in solving the “lack of information” problem as they have constant supervision of the machines. Another important factor that affects the process distribution is the sampling plan. Sampling should be executed at a constant rate so that cycles and distribution easier can be found.

The Nelson rules are difficult to apply as they are made for the normal distribution (except for rule #1 which is applied to all processes). However, processes with rapid and continuous tool wear (processes with low tool life-time) could use a modification of Nelson rule #4, *14 points in a row altering up and down*, where the number of points is instead drastically decreased since each point is expected to take a lower value than the previous. In this case the operator would obtain alarms when the process is no longer decreasing continuously. Processes with the opposite behaviour is usually those with high tool life-time could use Nelson rule #3, *6 points in a row steadily increasing or decreasing*. Since the tool slowly wears, continuously decreasing points is very unlikely. Thus, Nelson rule #3 would alert the user when there is a drift in the process.

It should be noted that the processes will change over time. Consequently, control limits will be out-dated as they no longer represent the variation of the process. Therefore, it is of important to continuously observe the processes and update the control limits whenever needed. The long-term goal should be to narrowing the control limits (and therefore increasing process capability) as this is an effect of decreasing variation.

In this study, the short-term capability analysis has been made unknowingly over major changes in the process, such as machine tool and material lot changes. In practise, these changes should not be considered when calculating the short-term capability. When the capability calculation is independent on these major changes it is more likely to achieve normality during short periods and thus making the PCI’s more valid.

Box-Cox and Johnson transformation is powerful methods to transform data into normal. However, not all processes will (or should) follow the normal probability distribution, therefore it is of importance to not force a process to normality as this will result in false alarms and making unnecessary adjustments of the process. The aim is to find the distribution and possible cycles of a process, only then the process will be predictable. From a predictable process, it is easier to determine control limits that separates common from special cause variation and to evaluate the process capability. Statistical process control is about improving processes, and the more that is known about a certain process the easier it is to improve.
References


## Appendix 1: Standardized tails of Pearson curves: Table 1a

<table>
<thead>
<tr>
<th>$K_1$</th>
<th>0.0</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
<th>4.5</th>
<th>5.0</th>
<th>5.5</th>
<th>6.0</th>
<th>6.5</th>
<th>7.0</th>
<th>7.5</th>
<th>8.0</th>
<th>8.5</th>
<th>9.0</th>
<th>9.5</th>
<th>10.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{p}$ (0.135 percentile) for $Sk &gt; 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{p}$ (99.865 percentile) for $Sk &lt; 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Notes
- $L_{p}$ represents the percentile of the distribution.
- $K_1$ is used as an index for different distributions.
- The table provides percentiles for skewness values above and below zero, as indicated by $Sk$.

### Calculations
- The table entries are calculated based on the standardized tails of Pearson curves, which are functions of $K_1$ and $Sk$.

### Usage
- The table can be used to determine percentile values for different skewness conditions.
- For example, to find the 0.135 percentile for a positive skewness ($Sk > 0$), one would look up the corresponding $K_1$ value in the table.

### Example
- To find the 0.135 percentile for a skewness of 2.0, one would look in the row for $K_1 = 2.0$ and find the corresponding value in the column for $Sk > 0$.
## Appendix 2: Standardized tails of Pearson curves: Table 1b

<table>
<thead>
<tr>
<th>$K_u$</th>
<th>0.0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
<th>1.6</th>
<th>1.7</th>
<th>1.8</th>
<th>1.9</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U^p$ (99.665 percentile for $Sk &gt; 0$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Notes
- $K_u$ is the standardized skewness.
- $U^p$ is the upper $p$-percentile of the Pearson distribution for the given skewness.
- The table provides values for $Sk > 0$ and $0 < Sk < 0$. 
- For $Sk > 0$, $U^p$ is the upper $p$-percentile of the distribution.
- For $0 < Sk < 0$, $U^p$ is the lower $p$-percentile of the distribution.
## Appendix 3: Standardized median of Pearson curves

### M’ (50 percentile), Change sign for Sk > 0

<table>
<thead>
<tr>
<th>Kurtosis (Ku)</th>
<th>Skewness (Sk)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>-1.4</td>
<td>0.000 0.293 0.111 0.184 0.282 0.424 0.627 0.764</td>
</tr>
<tr>
<td>-1.2</td>
<td>0.000 0.039 0.082 0.132 0.196 0.294 0.412 0.591 0.727</td>
</tr>
<tr>
<td>-1.0</td>
<td>0.000 0.032 0.065 0.103 0.151 0.212 0.297 0.419 0.588</td>
</tr>
<tr>
<td>-0.8</td>
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<tr>
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<td>0.000 0.020 0.041 0.064 0.091 0.122 0.161 0.212 0.290</td>
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<tr>
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<tr>
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<tr>
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</tr>
<tr>
<td>1.2</td>
<td>0.000 0.006 0.017 0.028 0.040 0.052 0.074 0.101 0.144</td>
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<tr>
<td>1.4</td>
<td>0.000 0.005 0.015 0.025 0.037 0.048 0.067 0.092 0.132</td>
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<td>1.6</td>
<td>0.000 0.004 0.013 0.021 0.033 0.043 0.059 0.083 0.121</td>
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<td>1.8</td>
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<tr>
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<td>3.0</td>
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Appendix 4: Control limits of Ø 39

Control limits dependent by the average moving range

Control limits by estimated sample standard deviation
Control limits by Clements method
Appendix 5: Ø 42 D-side

Control limits dependent by the average moving range

Control limits by the estimated sample standard deviation
Control limits by Clements method

Control limits by machine tool variation
Appendix 6: Ø 42 E-side

Control limits dependent by the average moving range

Control limits by the estimated sample standard deviation
Control limits by Clements method
Appendix 7: Ø 45,2

Control limits dependent by the average moving range

Control limits by estimated sample standard deviation
Control limits by Clements method
Appendix 8: Ø 30

Control limits dependent by the average moving range

Control limits by the estimated sample standard deviation
Control limits by Clements method

I-MR Chart of D30

- UCL = 30.056
- LCL = 30.056
- UCL = 30.049
- LCL = 30.063
- USL = 30.084
- LSL = 30.000

Date:
- 2017-02-24
- 2017-04-05
- 2017-04-19
- 2017-05-09
- 2017-05-24

Lower Control Limit (LCL) = 0
Upper Control Limit (UCL) = 0.00799
Average Range (AR) = 0.00244
Appendix 9: Ø 9

Control limits dependent by the average moving range

Control limits by the estimated sample standard deviation
Control limits by Clements method
Appendix 10: Distance 24

Control limits dependent by the average moving range
Appendix 11: Ø 25

Control limits dependent by the average moving range

Control limits by the estimated sample standard deviation
Control limits by Clements method
Appendix 12: Ø 92

Control limits dependent by the average moving range

Control limits by the estimated sample standard deviation
Control limits by Clements method

I-MR Chart of D92

Control limits by machine tool variation

I-MR Chart of D92