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Abstract—High integration of rooftop photovoltaic (PV) plants in distribution systems leads to new technical challenges: reverse-active power and voltage rise in low-voltage (LV) and medium-voltage (MV) grids. These challenges limit the maximum amount of power that can be produced by PVs in LV and MV grids, called the hosting capacity (HC). Battery storage systems (BSSs) have been used in many studies to decrease the reverse power and improve the HC by controlling the active power. However, the influence of a central BSS on the HC can be greatly improved by using a quadratic power control, simultaneous active and reactive power control, and by selecting the optimal battery size, the converter size, and the place of the central BSS. The effectiveness of the quadratic power control was not seen in previous works due to the fact that grids with one level of voltage without modeling of MV/LV transformers were simulated. This paper develops a method to select the optimal size of the battery and converter unit as well as the optimal place of an LV-central BSS having an optimal quadratic power control. The simulation results show considerable effects of the optimal selection of an LV-central BSS on the HC improvement.

Index Terms-- Battery storage system, distribution grids, hosting capacity, reactive power control, renewable energy sources.

I. INTRODUCTION

The worldwide trend of PV installations is increasing exponentially, while the majority of them, particularly in Europe, are connected to LV grids [1]. These LV connected PVs, which are usually ranged up to 10 kW, are called rooftop PVs. Several advantages of rooftop PVs, e.g. loss reduction, make power system policy leaders prefer them to large-scale PVs [2].

However, the high integration of rooftop PVs in LV grids creates some technical challenges such as reverse power flow and voltage rise [3]. This reverse power might lead currents to exceed the thermal limit of power lines, and could also make a considerable voltage rise in customer sides of the grid [4]. This voltage rise can also violate the standard voltage restriction (e.g. 90% to 110% by EN 50160 [5]) and limit the hosting capacity (HC) of PVs in LV and MV grids.

Many projects such as the SNOOPI project [6], which is researching to find a practical solution to install more PVs in LV grids, are working to handle these technical challenges. Among all different solutions for increasing the HC in distribution grids [7-10], battery storage systems (BSSs) have been attracting a great deal of attention in recent years. Although the cost of the BSS is steadily decreasing, the main concern is still the initial investment cost. Consequently, finding a strategy to optimize the size and place of BSSs in order to improve the HC with the minimum cost is of the prime importance.

There are several works on sizing, placement, or the performance analysis of BSSs in PV systems [11-24], which can be divided in two categories: stand-alone and grid-connected systems. Stand-alone systems, which rely on PVs as the only power source of systems, are beyond of the scope of this research. Grid-connected studies can be divided into two subcategories: solar farm and rooftop PVs.

In solar farms, it is important to keep the production in the level of the market commitment and maximize the expected revenue. In this regard, a capital budget allocation for PV panels and storage systems was presented by [11].

On the other hand, in the residential area, BSSs are typically used to handle technical challenges of rooftop PVs and improve the HC. There are two main topologies for using BSSs in rooftop PVs. The first topology is installing an individual BSS in each PV; and the second strategy is having one (or a few) central BSS(s) in the LV grid.

Regarding an individual storage in rooftop PVs, a simple local control was proposed in [2] to change the active output power of the BSS proportional to the PV production. Two similar distributed control methods based on the consensus technique were presented in [12, 13]. A central method was studied in [14] to increase the HC in LV grids by controlling individual storages based on a sensitivity analysis. A new control strategy for residential BES to solve over-voltage issues without adversely affecting the local objectives of the storage owner is presented in [15]. A method was presented in [16] to flatten the net active power profile of each customer and to mitigate the voltage rise using a central control. A discussion is also performed in [16] about advantages and disadvantages of local, distributed, and central control methods. Although having an individual BSS in PVs can cope with the overvoltage problem and improve the HC, it is complicated to coordinate them for voltage regulation and also to integrate them in the ancillary market.

Many studies have researched the contribution of a central BSS in a distribution grid. For example, a dynamic control of a...
central BSS was proposed in [17] to improve inertial response. A local control of a central storage system in distribution grid is presented in [18] to improve the voltage and frequency regulation. A central BSS connecting to all PVs by a DC link was developed in [19] to mitigate the neutral voltage rise.

Because of high investment cost of storage systems, some research focused on optimizing the size and place of a central BSS. A method for sizing a storage system was presented in [20] to regulate voltage and to reduce the number of on-line tap changer (OLTC) switching. A central BSS was applied in [21] to reduce the operating cost using a linearized backward/forward sweep method. The BSS size (both energy and power) is selected in [22] by a stochastic dynamic programming algorithm, while the time series of renewable generation is modeled as a Markov chain.

However, all these research [17-22] used only the active power control and did not use the reactive power control ability of BSS’s converter to improve voltage regulation. A quadratic power control can simultaneously control the active and reactive power of the converter to improve the voltage regulation. In this regard, an optimal placement and sizing method for a central BSS was proposed in [23] to regulate voltage and minimize the cost using a quadratic power control. This method used a Fourier series-based approach to simplify the optimization. Another optimization approach was proposed in [24] for placement of a central BSS in a simple system using a quadratic power control.

These aforementioned research [23, 24] simulated only the LV grids and assume that the voltage of MV/LV substations is kept fixed. In this case, because of the high R/X ratio of LV distribution lines, the reactive power does not have considerable effects on voltage regulation in compare to the active power control. While in practice distribution grid, only the voltage of HV/MV substations is kept fixed, usually by using an OLTC. Therefore, the cumulative impedance of MV lines, the MV/LV transformer, and LV lines has an effect on the voltage regulation. Due to high reactance of transformers, the cumulative R/X ratio is not so high, and as a result, the reactive power of the central BSS has a large effect on voltage regulation, which was not fully studied in previous work.

In order to fulfill these gaps, this paper develops an optimal sizing and placement method using a quadratic power control for a central BSS in LV grids to improve the HC of rooftop PVs. This research was done as a part of a project called SNOOPI, Smart Network Control with Coordinated PV Infeed [6], and the proposed method was simulated in three LV grids in Germany. This project selects two large MV feeders having about 80% integration of renewable energy sources, based on yearly energy production, with details of three LV grids. Then, the consumption and generation profiles were estimated and verified by using historical data and some measurements [25].

The proposed optimal BSS selection method is tested in these LV grids using the actual production and consumption profiles, but due to space limitation, only results of one LV grid are presented in this paper. The results show that the proposed method can considerably improve the HC of rooftop PVs in LV grids and also demonstrate the importance of selecting appropriate size of the converter in comparison to the battery size. To the authors’ best knowledge, the influence of the converter size on the HC improvement is not investigated in previous works. The main contributions of this paper can be summarized as follow:

- Formulate the sizing and placement of a LV-central BSS using a quadratic power control to maximize the HC of rooftop PVs in LV grids, as an optimization problem.
- Simulate the proposed method in several real networks considering the effect of MV lines, MV/LV transformer, and LV lines.
- Investigate the influence of different converter sizes in comparison to the storage sizes on the HC.
- Analyze economic benefit of a LV-central BSS.

The rest of this paper is organized as follows: Section II defines the maximum HC of rooftop PVs in LV grids. The BSS and its model are presented in section III. The proposed method for optimal sizing, placement, and control of a LV-central BSS are described in section IV. The simulation set-up and the results are respectively shown in section V and VI. Section VII concludes the paper.

II. HOSTING CAPACITY

A. Current constraints

The electrical production peak of PVs is usually occurred at midday when the residential consumption is typically low. Therefore, in distribution grids with a high penetration of rooftop PVs, the production can exceed the consumption and can make reverse current [26]. This reverse current can become two or three times higher than the maximum demand, and might exceed the current limit of the MV and LV lines.

B. Voltage constraints

The reverse active power can cause a voltage rise in distribution systems. Fig 1 shows a typical distribution feeder when active power is injected in the customer side of the grid. This injected power from the LV side to the upper network produces a voltage rise, which can be calculated from the following approximate equation:

$$\Delta V = V_{LV,k} - V_{HV} \approx R_{k}I_{P_{inj,k}} + X_{k}I_{Q_{inj,k}}.$$  \hspace{1cm} (1)

Here, $V_{LV,k}$ is the voltage magnitude (in pu) of $k$-th LV bus injecting active power ($P_{inj,k}$) and reactive power ($Q_{inj,k}$); $V_{HV}$ is the voltage magnitude (in pu) of the high voltage side of the
transformer; \( R_{c,k} \) and \( X_{c,k} \) are respectively the cumulative resistance and reactance from the HV bus to the \( k \)-th LV bus in pu, as follows:

\[
R_{c,k} = R_{HV/MV} + R_{MV,i} + \ldots + R_{MV,j} + R_{MV/LV} + R_{LV,j} + \ldots + R_{LV,k-1},
\]

\[
X_{c,k} = X_{HV/MV} + X_{MV,i} + \ldots + X_{MV,j} + X_{MV/LV} + X_{LV,j} + \ldots + X_{LV,k-1}.
\]

Here, the subscripts HV/MV and MV/LV refer respectively to the high voltage to medium voltage transformer and medium voltage to low voltage transformers; also, the subscripts MV,\( i \) and LV,\( i \) refer respectively to \( i \)-th MV lines and LV lines.

In a practical distribution network, HV/MV transformers are usually equipped with on-line tap changers and therefore, the voltage of the HV bus (or MV \(_1\) in Fig. 1) is almost fixed at 1 pu. Consequently, when a PV plant injects active power into the grid, according to (1), the voltage of all other busses increases. However, as stated by the voltage disturbances standard, the voltage of distribution grids should be kept in an acceptable range (e.g. 0.9 pu to 1.1 pu in [5]).

In contrast to the voltage rise caused by power injection, active and reactive power consumption can be used for voltage regulation in distribution grids. However, most research in this area has concluded that the reactive power has little influence in voltage regulation due to the high R/X ratio of distribution lines. This conclusion was made because those research only simulated a network with one level of voltage (LV or MV grids), without considering the transformer effects. While the resistance in LV and MV lines is higher than the reactance, transformers are highly inductive and the pu cumulative ratio of distribution grids \( (R_{c,k} / X_{c,k}) \) is around 1 (see Fig. 3). Consequently, both active and reactive power control can be used as effective solutions for improving voltage rise in distribution grids.

It is noteworthy to mention that since the cumulative resistance is not negligible, using (1) leads to a large error. Here, this equation is just used to explain the importance of the transformer modeling and the influence of the reactive power control in voltage regulation. The proposed method will use the power flow equations without any simplification.

C. Hosting Capacity Definition

Regarding the aforementioned current and voltage constraints in distribution systems, the amount of power could be produced by PVs installed in LV and MV grids without causing any technical problem, is limited. Therefore, the HC of LV grids can be defined as the maximum accumulated nominal power of all PVs installed in the LV grids, without power curtailment, while the current and voltage constraints are satisfied [27], as follows:

\[
HC = \text{Max}_{k} \left( \sum_{i} P_{PV,i} \right) \quad \forall t,
\]

s.t. 

\[
F_{i} (P_{i}(t) - k_{r} P_{PV,i}(t) - P_{ass,i}(t), Q_{i}(t) - k_{r} Q_{PV,i}(t) - Q_{ass,i}(t)) = 0 \quad \forall i,
\]

\[
I_{i}(t) < I_{max} \quad \forall k.
\]

\[
V_{min} < V_{i}(t) < V_{max} \quad \forall t.
\]

Here, \( F_{i} \) stands for the power flow equations, which are used to calculate the current of lines and voltage of nodes; \( P_{i}(t) \) and \( Q_{i}(t) \) refer respectively to active and reactive power of \( i \)-th node at time \( t \); the subscripts L, PV, and BSS refer respectively to loads, PVs, and storage systems; \( I_{i}(t) \) is the current of the \( k \)-th line at time \( t \) and should always be kept less than the maximum limit of the line current \( (I_{k,max}) \); \( V_{i}(t) \) is the voltage of the \( i \)-th LV node at time \( t \) and should be kept in the acceptable range \( (V_{min}, V_{max}) \) for all time; \( P_{k PV,i} \) is the nominal power of \( i \)-th PV and assumed that it is equal to maximum power of PV; and \( k_{r} \) is the increasing factor of the PV, which is installed in \( i \)-th LV node.

In order to calculate the HC, the power produced by PVs should be increased until one of the inequality constraints (voltage or current) at any time reaches its limit. However, as shown in (1), the connecting node of PVs has a major effect on the voltage rise. For example, a production near to MV/LV transformer \( (LV_{j} \) at Fig. 1) has negligible effect on LV voltage rise while a PV plant in the end of LV feeder \( (LV_{k+1} \) at Fig. 1) has a large effect. Therefore, the place where a new PV is installed will change the value of the HC. The influence of different PV extension scenarios on the HC has been studied in [25]. Here, since there is no data about the future extension of PV plants, it is assumed that all existing PVs are increased by the same factor \( (k_{1}=k_{2}=\ldots=k_{a}) \).

III. CENTRAL BATTERY STORAGE SYSTEM

A central BSS can reduce the reverse current and regulate the voltage by absorbing the extra power when the production is higher than the consumption, and also by a reactive power control. Generally, a central BSS includes a battery bank, which stores a limited amount of energy and a power converter unit.

A. Battery Bank

There are various types of batteries that can be used as electrical energy storage. Since the purpose of this study is not to discuss about details of different types; here, Li-Ion batteries due to high energy density, long life time, and relatively high efficiency are selected and a general model is reviewed.

The available energy of a battery at time \( t \), called the state of charge (SOC), can be calculated as:

\[
\text{SOC}(t+1) = \begin{cases} 
\text{SOC}(t) + \eta_{B} P_{BSS}(t) & \text{Charging} \\
\text{SOC}(t) - \frac{P_{BSS}(t)}{\eta_{B}} & \text{Discharging}
\end{cases}
\]

Here, \( P_{BSS}(t) \) is the active output power of the battery in time \( t \) and \( \eta_{B} \) is the battery’s charging/discharging efficiency. In order to optimize life time of a battery, its SOC should be always kept between a maximum \( (\text{SOC}_{m}) \) and minimum \( (\text{SOC}_{n}) \) value:

\[
\text{SOC}_{n} \leq \text{SOC}(t) \leq \text{SOC}_{m}.
\]

In addition, since the BSS should be ready for the next day
operation, the initial SOC of the next day (the SOC after the end of the current day) should be the same as the initial SOC:

\[ \text{SOC}(T) = \text{SOC}(0). \]  

Here, \( T \) is the number of time interval per day.

**B. Power Converter Unit**

Power converter units are mainly used to connect DC batteries to AC grids; they are also in charge of control the voltage, current, and other power characteristics of BSSs. The converter units have different structures based on the battery technologies. However, in a simple way, a switching process can be assumed ideal and the power converter unit can be modeled by some constraints and power losses. Regarding modeling of the loss, \( \eta_B \) in (4) can be replaced by \( \eta_C \eta_B \eta_C \), where \( \eta_C \) is the efficiency of the power converter. The constraints of the power converter unit are as follows:

\[
\begin{align*}
P_m & \leq P_{\text{BSS}}(t) \leq P_M, \\
Q_m & \leq Q_{\text{BSS}}(t) \leq Q_M, \\
\sqrt{P_{\text{BSS}}(t)^2 + Q_{\text{BSS}}(t)^2} & \leq S_M.
\end{align*}
\]

Here, \( Q_{\text{BSS}}(t) \) is the reactive output power of the converter in time \( t \); \( P_m \) and \( Q_m \) are respectively the maximum active and reactive power of the charging mode; \( P_M \) and \( Q_M \) are respectively the maximum active and reactive power of the discharging mode; and \( S_M \) is the maximum converter’s apparent power.

**IV. PROBLEM FORMULATION**

This section formulates the proposed method of the sizing, placement, and setting of the quadratic control of an LV-central BSS to maximize the HC of rooftop PVs in LV grids. The core of this method is an optimization problem, which finds the optimum settings of the BSS connected to bus \( j \) and had a given size of the battery bank \((\text{SOC}_M)\) and the converter size \((S_M)\), in different time, as follows:

\[
\begin{align*}
\text{Max.} \quad & \text{HC}\{P_{\text{BSS},j}(t), Q_{\text{BSS},j}(t), j, \text{SOC}_M, \text{SOC}(0), S_M \} \\
\text{s.t.} \quad & \\
\text{SOC}(t+1) = \begin{cases} 
\text{SOC}(t) + \eta P_{\text{BSS},j}(t) & \text{Charging} \\
\text{SOC}(t) - \frac{P_{\text{BSS},j}(t)}{\eta} & \text{Discharging} 
\end{cases} \\
\text{SOC}_0 \leq \text{SOC}(t) \leq \text{SOC}_M \quad \forall t, \\
\text{SOC}(T) = \text{SOC}(0), \\
P_m \leq P_{\text{BSS},j}(t) \leq P_M \quad \forall t, \\
Q_m \leq Q_{\text{BSS},j}(t) \leq Q_M \quad \forall t, \\
\sqrt{P_{\text{BSS},j}(t)^2 + Q_{\text{BSS},j}(t)^2} \leq S_M \quad \forall t, \\
F_i(P_{\text{L},i}(t) - k_i P_{\text{PV},i}(t) - P_{\text{BSS},j}(t), \\
Q_{\text{L},i}(t) - k_i Q_{\text{PV},i}(t) - Q_{\text{BSS},j}(t)) = 0 \quad \forall i, \forall t, \\
I_{i}\leq I_{i,max} \quad \forall k, \forall t, \\
V_{\text{min}} < V(t) < V_{\text{max}} \quad \forall i, \forall t.
\end{align*}
\]

Here, \( P_{\text{BSS},j}(t) \) and \( Q_{\text{BSS},j}(t) \) are settings of the BSS control and they are respectively the active and reactive output power of the central BSS installed in bus \( j \), in time \( t \). HC is calculated from (3) , i.e. a maximization of a maximization.

As it is shown in (8), HC is a function of several variables; and this optimization problem considers active and reactive output power of the BSS in each time interval as designing parameters while the other variables are assumed as given inputs in (8). In other words the optimum setting of the BSS controller, active and reactive output power, can be found by solving (8). Then the proposed method finds the optimum size and place of the central BSS by solving (8) several times for all possible combinations of places and sizes of the battery bank and converter unit. This strategy is selected to avoid an optimization problem with too many discrete design parameters.

The place of the central BSS is one of the major factors within designing an optimal storage system. Different installation place for a central BSS can make dissimilar improvement in voltage and current due to the grid specifications. Here, the best place is selected by solving (8) for all possible places. Other placement methods, such as sensitivity analysis, sometimes result in a wrong place. Sensitivity analysis can only consider one critical node and therefore if the grid has two or more critical nodes, the sensitivity analysis cannot handle the placement problem.

Furthermore, it is of prime importance to select the optimal size of converter unit and battery bank such that the LV grid has the maximum HC with the minimum possible cost. To the authors’ best knowledge, this is the first time that the influence of the converter size in comparison to the size of the battery bank is investigated for the HC improvement. The cost of the central BSS \((\text{Cost}_{\text{BSS}})\) can be calculated as follows:

\[
\text{Cost}_{\text{BSS}} = C_B (\text{SOC}_m - \text{SOC}_a) + C_C S_M.
\]

Here, \( C_B \) is the cost of the useful energy capacity of the battery bank per kWh (about 1000 Euro for Li-ion [28]) and \( C_C \) is the cost of the power converter unit per kVA (about 350 Euro [28]).

The cost can be considered in the optimization goal function to select the optimal place and size of the BSS. However, the proposed method solves the optimization problem (8) for different combinations of places, and sizes of the battery bank and converter unit. Then, a cost analysis is performed to find the optimum combination with minimum cost.

The objective function in (8) calculates the power flow equations for each time interval to find the HC. Increasing the power system scale leads to a bit slower calculation in the power flow equations and thereby slightly decreases the speed of the optimization solver. However, the number of time interval has a major effect on the optimization solver efficiency. In this study, time interval is selected equal to 1 hour and therefore, the optimization problem (8) has 48 design parameters for each given places and size of the central BSS, the average active and reactive power of the central BSS for
24 hours. Selecting a shorter time interval, directly, makes the problem much more complex. For example, 15-minute time interval in a day leads to an optimization problem with 192 design parameters.

It is noteworthy to mention that the LV-central BSS, especially its reactive power control, affects the energy loss. However, since this effect is negligible, as shown in the results, the proposed method does not consider the cost of the energy loss.

V. SIMULATION SET-UP

In SNOOPI project, two radial MV feeders in Germany, with details of three LV grids have been investigated. However, only the results of one LV grid are presented here. The corresponding MV feeder has several small PVs connected to LV grids and a 9.6 MW wind park connected to MV feeder [25]. The penetration level of renewable energy sources in this feeder totally becomes 88% of the yearly energy consumption. The reverse active power in the beginning of this MV feeder reaches 7.9 MW while its demand peak is 5.7 MW. The MV grid supplies about 40 MV/LV substations, which are simulated as an aggregated load and generation profiles except two. One of these substations, which are simulated with all LV details, is shown in Fig. 2. This LV grid has 283 nodes including 141 junctions and 142 consuming buses; it also has 18 PVs with maximum production of 151 kW.

In this study the consumption and generation profile of the critical day, which has the maximum reverse power, are estimated by using the standard profiles according to the method explained in [25]. Since the HC is limited by the maximum current and voltage bound (as shown in (3)), considering the critical day is the most conservative situation and is enough for calculating the HC improvement.

As discussed in section II.B, the effectiveness of the reactive power control in voltage regulation compared to the active power control depends largely on the ratio of the cumulative resistance over reactance ($R_c/X_c$) of the grid. In this MV feeder, the $R/X$ ratio of lines is in the range of 0.9 to 1.7, while this value for LV feeder is in the range of 2.6 to 8.2. In addition, the MV/LV transformer has large impedance with the $R/X$ ratio of 0.2. Therefore the cumulative ratio of the resistance to reactance ($R_c/X_c$) in this grid, calculated by (2), is about 1, as shown in Fig. 3. This ratio results that the reactive power control can be more effective than the active power control in many buses, particularly buses located near the MV/LV transformer.

It is worth mentioning that the reactive power control ability of the PV convertor is not considered in this research. Because there is no obligation in the existing regulations for distribution generators to control the reactive power and even if the regulation changed, it would be not practical to change all existing convertors.

VI. SIMULATION RESULTS

In order to investigate the effectiveness of a central BSS, the maximum HC of the LV grid without any BSS should first be calculated in the critical day. For this purpose, the $k$ factors in (3) are increased until the voltage of one of the LV buses or the current of one of the LV lines reaches its limit. Fig. 4 shows the voltages of the LV grid having the maximum HC without a central BSS for the critical day. As expected, the voltages in several buses are near the limit and the central BSS has to improve all these critical voltages.

In all simulations, the battery bank is assumed to have the half of its charge at the beginning of the day ($SOC(0) = 0.5\times SOCM$) and the $SOC_m$ is assumed to be zero.

In order to solve (8), the $fmincon$ function of MATLAB is used due to the fairly good speed. However, the optimization problem in (8) is a non-convex optimization and $fmincon$ function cannot guarantee to find the global optimum in this...
situation. To indicate the performance of \texttt{fmincon} function, its results in some cases are compared with results of Genetic algorithm (GA). The difference between the results is less than 0.5%, but the \texttt{fmincon} function is much faster. The simulations are conducted on a Laptop core i7 with 16 GB RAM and each time of solving (8) by \texttt{fmincon} function takes about 230 seconds, using parallel calculation, while the GA is 40 times slower.

A. Reactive power control effects

In order to demonstrate the effect of reactive power control in the HC improvement, a LV-central BSS with different size of the battery bank and the converter is installed in the best place. Table I shows the HC improvement (percentage) when the BSS have a quadratic control and supply both active and reactive power, while Table II lists the results for the same BSS having just the ability of active power control.

In table II, when there is no reactive power control, having the converter size larger than the battery size does not have any effect on the BSS output. The comparison of these results shows that using both active and reactive power control in a LV-central BSS improves the HC about two times.

B. Battery and Converter Size

The sensitivity analysis performed in Table I shows the importance of the converter size in the HC improvement, when the BSS can also provide the reactive power control. Increasing the battery size slightly improves the HC while the converter size has more effect on the HC improvement. In order to compare the effect of the converter size and battery bank size, four different cases with their approximate cost, calculated by (9), are defined as follows:

- Case 1: 80 kWh battery bank, 20 kVA converter (87 k €)
- Case 2: 20 kWh battery bank, 80 kVA converter (48 k €)
- Case 3: 80 kWh battery bank, 80 kVA converter (108 k €)
- Case 4: 160 kVA converter without battery bank (56 k €)

Table I shows that case 1, having cost of 87,000 euros, improves the HC 12.3%, while case 2, having cost of 48,000 euros (55 % of case 1), improves the HC 29.2 % (2.37 times more than case 1). This comparison demonstrates that despite of the lower cost of the power converter unit, its size is even more effective in the HC improvement compared to the battery size. In other words, the effectiveness of the battery bank is much less in compare to the converter unit for the HC capacity improvement. Furthermore, even if the cost of the battery bank decreased to about one third and the cost of the converter unit stayed the same, case 2, having the larger converter unit, would improve the HC more than two time of case 1, having the larger battery bank, with the same cost.

This better improvement results from the ability of the converter to provide the reactive power. The reactive power control in LV grids can improve voltage regulation almost as much as the active power control as mentioned in section II.B. However, providing the active power control has more restrictions because battery banks can provide a limited amount of energy while converter units can give a limited amount of power. For instance, an 80 kWh battery bank (case 1) can provide only 20 kW active power for 4 hours (with unit efficiency); however, an 80 kVA converter can provide 80 kVAR reactive power for all period.

Comparing case 2, improving the HC 29.2%, and case 3, improving the HC 29.5%, shows that increasing the size of the battery bank without increasing the converter size has little influence on the central BSS performance. This little improvement comes from the last term of (7), which states that if the converter provides active power, its ability for reactive power production is reduced. Therefore, a converter unit having a size in the range of the battery bank size limits the reactive power control ability and leads to little improvement effects. The same reason leads to have almost no HC improvement when the converter size kept fixed at \( S_M = 60 \) kVA and the battery bank is increased from \( \text{SOC}_M = 40 \) to 80
Therefore, using a large converter to provide the reactive power even without battery bank like case 4 could be very effective. A central BSS without any battery bank is indeed a distributed static synchronous compensator or D-STATCOM, one of the flexible alternating current transmission system (FACTS) devices used to enhance power system controllability. Case 4, improving the HC 47.6%, shows that a D-STATCOM can considerably improve the HC of this LV grid with lower cost in comparison with the BSS.

The optimal active and reactive power of a central BSS in these four cases are shown in Fig. 5. All cases show that the central BSS absorbs reactive power to control the voltage rise in the critical day (day with the largest reverse power). In other words, even instead of a D-STATCOM, which can absorb or inject reactive power, a thyristor controlled reactor (TCR) or any kind of variable reactor, which has only the ability of reactive power injection, might provide the similar HC improvement with lower cost. Although selecting the best device needs dynamic study, which will be the subject of the authors’ future work.

Reactive power consumption can improve the voltage regulation, but it increases the current in the grid. The following subsections analyze the influence of the optimal BSS on energy loss and on the current and voltage constraints of the grid.

C. Current and voltage constraints

Fig. 6 and 7 show the maximum and minimum of the LV grid voltages during the critical day with and without an optimal BSS. A central BSS by controlling both active and reactive power improves the voltage regulation and allows the LV grid has more PVs.

Table III shows the effect of the central BSS on the current of the MV/LV transformer. Increasing the battery size leads to slightly decrease in the maximum current of the transformer while increasing the converter size lead to lightly increase in this current. In other words, in the larger battery size, the active power control decreases the current peak while in the larger converter size, the reactive power control increases the current magnitude.

Simulations on other grids with in SNOOPI project show similar results. The increase in the converter size of a central BSS enhances the reactive power control and consequently improves the voltage regulation, while slightly magnifies the current limitation. On the other hand, increasing the battery bank, which is much more expensive, has little positive effect on both voltage and current.

D. Energy loss

A central BSS in LV grids affects energy loss in two ways. First, since it controls the active and reactive power of the coupling node, the current of the lines and subsequently their losses will be changed. An active power control decreases the maximum current and therefore reduces the energy loss while a reactive power control increases the maximum current and the energy loss. Secondly, as it is mentioned in section II, a converter unit and a battery bank are not ideal and thus the BSS has efficiency ($\eta$) less than one. Assuming $\eta$=90%, Table IV lists the percentage of energy loss in regard to the total energy produced by PVs using an optimal BSS with different battery bank and converter sizes.

This table shows that increasing the converter size increase the energy loss due to the fact that the reactive power consumption is increased as well. However, by increasing the size of the battery bank, the loss of lines is decreased due to the reduction of the maximum current, but the loss of the BSS is increased. The maximum energy loss in Table IV is 0.264 %, about two times more than the minimum energy loss (0.135 %), and can be neglected in compare to the energy production.

<table>
<thead>
<tr>
<th>$S_M$ (kVA)</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
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<tbody>
<tr>
<td>20</td>
<td>0.48</td>
<td>-0.49</td>
<td>-0.96</td>
<td>1.15</td>
<td>1.34</td>
</tr>
<tr>
<td>40</td>
<td>1.04</td>
<td>-0.29</td>
<td>-0.97</td>
<td>1.50</td>
<td>1.79</td>
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<tr>
<td>60</td>
<td>1.68</td>
<td>0.14</td>
<td>-0.61</td>
<td>0.61</td>
<td>0.61</td>
</tr>
<tr>
<td>80</td>
<td>2.39</td>
<td>1.64</td>
<td>1.35</td>
<td>1.34</td>
<td>1.34</td>
</tr>
<tr>
<td>100</td>
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<td>2.38</td>
<td>2.04</td>
<td>1.86</td>
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<tr>
<td>120</td>
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<td>3.47</td>
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</tr>
<tr>
<td>160</td>
<td>6.09</td>
<td>4.70</td>
<td>4.25</td>
<td>3.92</td>
<td>3.83</td>
</tr>
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</table>
in Fig. 8, due to a long distance to substation (large cumulative impedance). Therefore, installing a battery for the large PV plants could be a non-optimal solution. Fig. 8 shows the results of the case 1-4 when the central BSS is installed in a bus in different areas, a group of buses located near each other and have almost same effect on the HC. Different areas of this LV grid are shown in Fig. 2. A BSS installed in area 1 has the highest effect on the HC, as shown in Fig. 8, due to a long distance to substation (large cumulative impedance) and closeness to large PVs. Although, a large cumulative impedance or closeness to large PVs is important to select the optimum place, the best place cannot be easily selected without considering all possible places. For example, Fig. 8 shows the BSS installed in the largest PV, located in area 4, improves the HC less than the one installed in area 1. Therefore, installing a battery for the large PV plants could be a non-optimal solution.

Installing a central BSS in an area near to the MV/LV substation does little influence in the HC due to low cumulative impedance of those nodes and it is not shown in Fig. 8.

E. Central BSS Place

The place of the LV-central BSS has a major effect on the HC improvement. Therefore, in this study, All 283 buses are selected as a possible place for BSS and in each place 40 different combinations of the battery and converter size are considered.

Fig. 8 shows the results of the case 1-4 when the central BSS is installed in a bus in different areas, a group of buses located near each other and have almost same effect on the HC. Different areas of this LV grid are shown in Fig. 2. A BSS installed in area 1 has the highest effect on the HC, as shown in Fig. 8, due to a long distance to substation (large cumulative impedance) and closeness to large PVs. Although, a large cumulative impedance or closeness to large PVs is important to select the optimum place, the best place cannot be easily selected without considering all possible places. For example, Fig. 8 shows the BSS installed in the largest PV, located in area 4, improves the HC less than the one installed in area 1. Therefore, installing a battery for the large PV plants could be a non-optimal solution.

Installing a central BSS in an area near to the MV/LV substation does little influence in the HC due to low cumulative impedance of those nodes and it is not shown in Fig. 8.

F. Discussion

In summary, the proposed method investigates the influence of the converter size, the battery bank size, and the placement of an LV-central BSS using an optimal quadratic control in improving the HC of the LV grids. The simulation results can be summarized as follows:

(1) Since the beginning of MV feeders has a fixed voltage in practice, the large inductance of MV/LV transformers has a major effect on the voltage regulation. In these circumstances, the active and reactive power have almost the same effect in the voltage regulation.

(2) Although the battery bank has a higher price than the power converter unit, its size has less effect on the voltage regulation compared to the converter size. In addition, a large battery bank (active power control) can slightly reduce the maximum current, while a large power converter (reactive power control) increases slightly the maximum current.

(3) A central BSS by controlling the active power reduces the lines’ losses. However, the reactive power control and the internal loss of the BSS can slightly increase the total energy loss of the LV grid.

(4) An optimal BSS has only a negligible effect on current reduction and a converter without battery (such as case 4) can considerably improve the voltage regulation. Therefore, a device, which is only controlling the reactive power can improve the HC with much lower cost. Optimal reactive power in Fig. 5 shows that a D-STATCOM, TCR, or any kind of variable reactor might improve the HC in a similar ways to an expensive central BSS. However, selecting an appropriate device for this purpose needs more study especially in dynamic behavior, which will be the subject of authors’ future work.

(5) The reactive power consumed by a LV-central BSS is small in comparison to the total active power of the LV grid and it cannot significantly change the total power factor of the LV grid, e.g. in case 4 the power factor of the MV/LV substation change less than 1 percentage. Furthermore, the BSS regulates locally voltage over the LV grid and therefore, the voltage regulation by OLTC over distribution grids becomes less difficult. In other words, the LV-central BSS has no adverse effect on the upper grid.

VII. Conclusion

This paper proposes a method for optimal sizing and placement of a LV-central BSS using an optimal quadratic control in order to increase the HC of rooftop PVs in LV grids. For this purpose, the output active and reactive power, place, the battery size, and converter size are formulated as a multi-variable non-linear constrained optimization problem. The simulation results demonstrate that, although the reactive power control increases negligibly the maximum current and energy loss, it is as effective as the active power control to regulate the voltage profile due to the large inductance of MV/LV transformers. The sensitivity analysis of a central BSS installed in the optimum place also shows that although the power converter is much less expensive than the battery bank, its size has more influence on improving the HC compared to the size of the battery bank.

It can be concluded from this simulation that a device that
provides only reactive power control (such as a D-STATCOM or TCR) might improve the HC of distribution grids with much lower investment cost of a central BSS. However, such device slightly increases the maximum current and power loss. It is noteworthy to mention that this paper only simulates the steady-state conditions and the dynamic studies are necessary for designing a controller for the HC improvement as well.

REFERENCES


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