Characterization and Modeling of Temperature Induced Non-Reciprocal Effects in Astrix Fiber-Optic Gyroscopes

RÉMI DURIEUX
Acknowledgements

First, I want to express my sincere thanks to Armelle P, Head of Sensors & Actuators Engineering for her kindness, her help, and her confidence in accepting my internship application. She devoted lots of time to helping me with my future professional choices.

As an Airbus Defence & Space supervisor, I want to thank Philippe F, who warmly welcomed me since the very first day. He has always been receptive to my questions and has helped me resolve administrative issues before the thesis started. I sincerely thank Franck B, Head of Inertial Sensors & Actuators, for allowing me to conduct this Master thesis within Airbus Defence & Space.

As an everyday supervisor, Dr. Steve M supported me in this work and gave me encouragement and advice despite his busy agenda. I want to thank him for always being receptive to my ideas and for providing me valuable suggestions for this study.

I want to express my sincere thanks to Philippe L, for generously sharing his expertise and for his helpful suggestions whenever I had problems. He spent much time to discuss the physical origins of the phenomena, which was of a great help.

I wish to thank Nicolas D, who took lots of time explaining me valuable ideas and who greatly contributed to the tests I have performed (thank you also for giving me the opportunity to win a couple of badminton games). Many thanks go to Guillaume B for his contribution to the writing of the test plans despite his busy agenda. I would like to acknowledge the help of Anthony P who offered me relevant suggestions and who always took time to answer my questions about thermal issues. I want to say thank you to Ina T for her daily support and for always being in a good mood since the very first day. I want to extend my sincere thanks to all members of the department, and especially to Marion B, Benjamin I, Nathalie V, Guillaume L, Gilbert C, Cécile M, Daniel B. I want to thank Anaïs A, who wrote this internship subject and who has welcomed me during the first months.

As my KTH supervisor, I express my sincere gratitude to Professor Gunnar Björk for allowing me to conduct this Master thesis under his supervision. I also would like to acknowledge Professor Gunnar Tibert and Elin Wiljergard for helping me resolving administrative issues.

Finally, I want to thank my professors from Institut d’Optique Graduate School Thierry Lépine, Mathieu Hebert and Rafaël Clerc for this second year at the University.
Abstract

Swedish version


Under temperaturvariationer uppstår vissa fysiska utvidgningar i FOGs optiska fiberspol, vilket har en tendens att minska sensorens precision. Eftersom en signifikant 100-faldig förbättring av FOGs prestationer observerats under de senaste åren är karaktäriseringen och modelleringen av sådana temperaturinducerade effekter i FOG ett viktigt ämne som behöver studeras i djupet. Denna examensarbete analyserar det fysiska ursprunget för de termiska effekterna i Astrix FOGs. ADS använder högpresterande test i en vakuumkammare för att simulera värmen från solen. Testresultaten har använts för att bättre karaktärisera effekterna för att slutligen utveckla effektiva modeller.

English version

When a spacecraft is orbiting the Earth or cruising towards a planet, its attitude must be precisely monitored. Therefore, sensors and actuators are used to measure the spacecraft orientation and to apply torques to re-orient if necessary. Fiber-Optic Gyroscopes (FOGs) sensors have rapidly emerged in the 1990s as an efficient option for spacecraft attitude control. For more than 15 years, Airbus Defence & Space (ADS) has been supplying its highest-precision FOGs – called Astrix FOGs - to various customers, including the European Space Agency (ESA). Since the spacecraft is sometimes facing the Sun and other times eclipsed by the Earth or another planet, it experiences a wide range of temperatures; so does the FOG. Under temperature variations, some physical dilatations occur in the FOG optical fiber coil, which tends to decrease the sensor precision. As a significant 100-fold improvement of FOG’s performances has been observed in the last few years, the characterization and the modelling of such temperature-induced effects in FOGs is an essential topic that needs to be studied in more depth. This Master thesis analyses the physical origin of the thermal effects in Astrix FOGs. ADS uses high-performance test benches in a vacuum chamber in order to simulate the heat coming from the Sun: the test results have been used to better characterize the effects in order to finally develop efficient models.
Table of content

1. Introduction

2. Theory of Fiber-Optic Gyroscopes
   2.1. Introduction to spacecraft attitude dynamics
   2.2. Principle of the Fiber-Optic Gyroscope
      2.2.1. Sagnac effect: the basic principle of the FOG
      2.2.2. Presentation of Astrix 200 and 1090
      2.2.3. Noise, drift and scale factor
      2.2.4. Reciprocity of a FOG is the origin of bias
   2.3. Time transience-related (non-reciprocal) effects
      2.3.1. Mission specifications
      2.3.2. Thermally induced non-reciprocal effects

3. Analytical study of temperature-induced non-reciprocal effects in Astrix 200 and 1090 FOGs
   3.1. Thermal characterization of Astrix 200 and 1090
      3.1.1. Thermal sensors on Astrix
      3.1.2. Thermal architecture
   3.2. Thermally induced effects characterization
      3.2.1. Equipment test facilities and temperature profiles
      3.2.2. Thermal cycles on VTPIE
      3.2.3. “Shupe tests” on T3A
      3.2.4. VTPIE and T3A comparison
   3.3. Thermal test results (rate-wise)
      3.3.1. Thermal tests results: the difference of temperature propagation
      3.3.2. Thermal tests results: the impact of temperature variation on the rate

4. Modelling and characterization of Mohr and parasitic effects
   4.1. Modelling of thermal effects on Astrix FOGs
      4.1.1. Thermal sensitivity of bias
      4.1.2. Modeling method
      4.1.3. Modelling results on residual error
   4.2. Specific test design for the characterization of the parasitic effect on Astrix 1090
      4.2.1. Thermal test design for the study of the parasitic effect on Astrix 1090
      4.2.2. Parasitic effect test results and interpretation
   4.3. Specific test design for the characterization of the parasitic effect on Astrix 200
      4.3.1. Thermal test design for the study of the parasitic effect on Astrix 200
      4.3.2. Parasitic effect test results and interpretation

5. Conclusion

6. References
ACRONYMS

FEM  Finite Element Model
FOG  Fibre Optic Gyroscope
ICU  Inertial Core Unit
IMU  Inertial Measurement Unit
IOC  Integrated Optical Circuit
SIA  Sagnac Interferometer Assembly
GEU  Gyroscope Electronic Unit
GOH  Gyroscope Optical Harness
AOCS  Attitude and Orbit Control System
1. INTRODUCTION

The gyroscope, an instrument measuring angular rotations, is a key sensor in modern navigation systems. It has a wide spectrum of applications, in particular in aerospace engineering. Research groups and space agencies made many investigations to develop innovative concepts about gyroscopes. In the last 1960s, the development of the fiber optic gyroscope (FOG) was started at US Naval Research Laboratories, Washington (USA). The use of optical fibers to realize an optical angular rate-sensor was investigated with the hope of reducing cost and increasing accuracy. In a FOG, the angular rate is estimated by measuring the rotation-induced phase shift between two beams that counter-propagate in a fiber coil. Since this phase shift is detected by an interferometric technique, these FOGs are sometimes called interferometric FOGs (IFOGs).

FOG performance is limited by the presence of nonreciprocal effects in the fiber coil. Time-varying temperature gradients, vibrations, polarizations instabilities, Rayleigh backscattering and Kerr effect are noise sources limiting FOG resolution. Generally the FOG design is focused on the reduction of all noise sources below the photodetector shot noise, so that FOG can operate in the so called shot noise limited mode.

A FOG’s performance can be very high. Unfortunately the problem of a FOG’s relatively high sensitivity to temperature changes and vibrations has not been completely solved at the moment. Major discoveries concerning temperature-induced effects have been performed by Shupe in 1984 and more recently by Mohr in 2004. Time-dependent (spatial) thermal gradients are at the origin of the first effect. The second one is due to the temporal derivative of the temperature. Using specific coil windings, the Shupe effect can be eliminated.

This Master thesis focuses on the characterization and the modeling of such non-reciprocal temperature-induced effects in the FOGs produced by Airbus Defense & Space, called the Astrix FOGs. Two types of FOGs are produced within the department. The difference lies in the number of optical coil used in the instrument. One optical coil provides information about the rotation around its axis. Only three axes are necessary for determining any rotation on a spacecraft. For redundancy matters, either a second FOG is used on the spacecraft (Astrix 200), or a fourth axis is implemented within the FOG (Astrix 1090). Thermal tests are performed on Astrid FOG within the department.

The thermal propagation within the two types of FOGs has first been achieved using thermal Finite Element Models (FEM). Then, the test results have been used to estimate the Mohr effect. Some adapted post-processing tools have been developed for this activity. The correlation between the thermal sensor measurements and the thermal effects has then been studied. A compensation model has been developed according to the physics of the effect determined in the state of the art phase previously performed. A specific test procedure as well as an analysis algorithm has finally been written in order to validate the full method on cleanroom equipment.
2. **THEORY OF FIBER-OPTIC GYROSCOPES**

2.1. **INTRODUCTION TO SPACECRAFT ATTITUDE DYNAMICS**

Fiber-Optic Gyroscopes provide key data for Attitude and Orbit Control System (AOCS). Two kinds of sensors are used for AOCS: optical sensors (e.g. star trackers, sun sensors) and inertial sensors (e.g. FOGs). These sensors provide information on satellite dynamics. The values are compared with the desired values of attitude and then corrected with the help of actuators (e.g. reaction wheels, control moment gyroscopes, mass ejection systems).

Figure 1 describes the basic principle of AOCS.

![Figure 1 - AOCS block diagram](image)

2.2. **PRINCIPLE OF THE FIBER-OPTIC GYROSCOPE**

2.2.1. Sagnac effect: basic principle of the FOG

Detection of rotation with light was demonstrated by Sagnac in 1913 [1]. He showed that two waves acquired a phase difference by propagating in opposite directions around a loop interferometer, which was rotating about its axis as shown in Figure 2.

![Figure 2 - The original Sagnac setup of a ring interferometer (S stands for surface)](image)
For example, Michelson and Gale used an interferometer like the one showed in Figure 2, having a perimeter of over 1 mile. A ring laser provides a dramatic improvement in sensitivity, allowing the use of small loops, by internally converting the phase difference to a frequency difference and measuring this frequency difference precisely.

An alternative and convenient method for increasing the sensitivity of Sagnac systems uses an optical fiber which may be wrapped many times around a small cylinder to increase the phase difference produced by rotation, as shown on Figure 3. The first demonstration of such a system was made by Vali and Shorthill in 1976 [2]. Fiber-optic gyroscopes (FOGs) are now of substantial interest for potential practical applications [3].

In the FOG, the insertion of the fiber medium into the optical path of a Sagnac interferometer does not alter the Sagnac effect directly, but gives rise to misleading signals. Proper consideration of optical reciprocity allowed FOGs to measure far below earth rate. To further increase the sensitivity to present levels, a number of other physical effects had to be identified, such as the Faraday effect in the fiber material in the presence of the Earth magnetic field.

![Figure 3 - The optical coil has multiple turns (N turns and an area A)](image)

The Sagnac effect has a simple kinematic description [1],[4]. Let us consider a hypothetical interferometer with a circular optical path in vacuum. When the interferometer is at rest in an inertial frame of reference the path lengths of the counterpropagating waves are equal and, since light travels at the same speed $c$ in both directions around the loop, both waves return to the point they have been injected in phase. When the interferometer is rotating at a rate $\Omega$ and the observer is motionless in the original inertial frame, the injection point (where the light is splitted) moves at a rotation rate $\Omega$ during the propagation time $\tau$. Therefore, the difference between the propagation times of the counterpropagating waves is [6]:

$$\Delta \tau \sim \frac{4A \Omega}{c^2}$$

In the case of an ideal circular path with $N = 1$.

For continuous waves of frequency $\omega$, this corresponds to a phase shift

$$\Delta \phi = \omega \Delta \tau = \frac{4A \omega}{c^2} \Omega$$

This result remains unchanged when the interferometer is filled with a medium of refractive index $n$. There is no frequency difference between the counterpropagating waves in the rotating frame, the rest frame of the medium, and hence no dependence on dispersion.
As the Sagnac effect is proportional to the flux of the rotation rate vector $\Omega$, it can be enhanced with a multiturn path, just as the flux of a B field is enhanced in a multiturn inductance coil. In this case of a multiturn path (Figure 3), the Sagnac phase difference is still the same but now the actual area $A$ is $N$-fold the area of a single loop. It is often expressed as:

$$\Delta \phi = \frac{8\pi NA}{\lambda c} \Omega = \frac{2\pi LD}{\lambda c} \Omega$$

where $\lambda$ is the wavelength in vacuum, $D$ is the coil diameter and $L$ is the fiber coil length. Therefore, by simply scaling up or down the effective area of the fiber coil, the actual sensitivity may be shifted and tuned to the need of the application.

2.2.2. Presentation of Astrix 200 and 1090

Astrix FOGs perform rotation rate detection around one axis only. On a spacecraft, any rotation around one of the three axes has to be detected, as shown on FIG…

**FIGURE 4 - THREE ROTATIONS CORRESPONDING TO THE THREE SATELLITE AXES HAVE TO BE DETECTED**

This study only concerns rotation sensors. On a spacecraft, the rotation around the three axes needs to be monitored, as shown on Figure 4.
2.2.2.1. Optical architecture

The general principle of the optical architecture is summed up on Figure 5.

This part details the principle of the angular rate detection around one axis (i.e. one fiber coil).

Three parts can be distinguished: the light is emitted from the source and passes through a 50/50 coupler. Then the light goes to the SIA (Sagnac Interferometer Assembly), sometimes called Interferometer, composed of the optical coil and the IOC (Integrated Optical Circuit). The IOC separates, combines (Y-junction), polarizes and modulates the optical signal. It is composed of a polarizer and of electrodes. The SIA is an interferometer so the phase shifts is converted into an amplitude variation. An interference pattern is obtained after the IOC.

**Figure 5 - Schematic of FOG main structure**

The Gyro Optical Harness (GOH) is the wire making the link between the FOG Electronics Module (FEM) and the SIA, as shown on Figure 6. The IOC is situated in the bar in the middle of the optical coil.

**Figure 6 - Schematic of SIA, GOH and FEM organization**
Airbus Defence & Space has been building several kinds of FOGs: only Astrix 200 and Astrix 1090 will be presented in this present report. The cost and performances of a FOG differ from one model to the other. The Astrix name corresponds to the diameter of the fiber coil (about 200mm and 90mm, respectively). These two models contain design difference that will be detailed in the next part. The design has a direct impact on the thermal behavior.

In order to ensure a robust redundancy, two solutions are possible:

- Either two FOGs are used with three optical coil (with perpendicular axes) like in Astrix 1090
- Or only one FOG is used with four axes like in Astrix 200

### 2.2.2.2. Astrix 200

The ASTRIX 200 is the very high performance member of a Fibre Optics Gyroscopes (FOG) family based on a same electronics associated to different size of fibre optic coils, which determines the performances.

Figure 7 shows an Astrix 200.

![Astrix 200](https://www.google.fr/search?q=astrix+200+airbus&source=lnms&tbm=isch&sa=X&ved=0ahUKEwi36oLphbXZAhWEVQBKHZLWBWwQ_AUICigB&biw=1280&bih=855#imgrc=y0J3-WQuUlJHM_&spf=1519148676520)

The Astrix 200 is constituted of two units connected by a composite harness called GOH. The rotation sensitive part is called Inertial Core Unit (ICU) and its driving electronics is called Gyroscope Electronic Unit (GEU). The figure above presents the ASTRIX 200 Inertial Measurement Unit (IMU).
The GEU groups together the four FEM and the ICU is composed of the four SIA mounted on a tetrahedron structure fitted to the satellite pattern through bipods. For thermal constraints, the ICU is covered by a Multi-Layer Insulation (MLI).

The gyroscopic measurement is done in the SIA but the FEM participates to the inertial performance. The following equipment outputs are available through a digital link:

- Inertial raw and filtered angles,
- Several temperature data including SIA and FEM ones,
- Other set-up and management data.

The GOH allows separating thermally dissipating elements in order to place the SIA as close as possible to the satellite payload.

2.2.2.3. Astrix 1090

The Astrix 1090 high performance inertial measurement unit has been mainly developed to fit Telecommunication satellites needs but it also fits to a large range of space applications, just like the Astrix 200 unit.

Astrix 1090 features three sensing axes allowing computation of the spacecraft angular rates along the three orthogonal directions, as shown on Figure 8.

Figure 8 - Astrix 1090 SIAs, ICU and GEU

https://www.google.fr/search?biw=1280&bih=855&tbm=isch&sa=1&ei=GF-MWrswhaBSsP8qAo&q=astrix+1090+airbus&oq=astrix+1090+airbus&gs_l=psy-ab.3...1693.1693.0.2874.1.1.0.0.0.0.99.99.1.10...0.1c.1.64.psy-ab..0.0.0...0.92FuPNrJ7A#imgref=V0j9HreLhxdXM;spf=1519148827871
Astrix 1090 unit is based on a compact, single box concept. The electronics module (including the source module) is packaged in a hexagonal box (GEU) mounted horizontally on spacecraft support structure. The 3 SIA are mounted on a corner cube shaped structure (constituting the ICU), fixed on top of GEU through 3 elastomeric isolation mounts.

The ICU is linked to the GEU by three isolators. These isolators are composed by two frames and a layer of elastomer. This means that there is no harness (GOH) for Astrix 1090 unit: SIA is situated directly on the GEU. This will have a direct impact on the thermal propagation in the FOG.

2.2.2.4. Responsibilities

The manufacturing and validation of ASTRIX FOGs are made in four main steps:

► AIRBUS Defence & Space is in charge of the driving electronics (FEM) design and characterization.
► AIRBUS Defence & Space is in charge of FEM manufacturing and of the functional tests performed at board level.
► IXSPACE is in charge of sensor design and manufacturing (SIA and optoelectronic parts) and of the tests at channel level (inertial performance tests of the gyroscopic loop with the SIA coupled to the FEM).
► AIRBUS Defence & Space is in charge of ICU and GEU assembling and of all the inertial performance tests and environment tests performed at equipment level.

2.2.3. Noise, drift and scale factor: vocabulary and temperature dependency

The signal bias of a fiber gyro is the output signal at rest. It is a random function that is the sum of a white noise (with the theoretical limit of the photon shot noise) and a slowly varying function to take into account the long-term drift of the mean value.

Another very important characteristic of a gyro is the scale factor. Compared to other sensors, a gyroscope needs a much better accuracy over a much wider dynamic range: the important measurement is the integrated rotation angle, and any past error degrades future information. It is important to have low noise and low drift to measure a very low rate, but it is also important to have an accurate measurement of high rates (i.e., an accurate scale factor).

The scale factor and the bias are supposed to be sensitive to the environment:

- the scale factor depends on the time, the temperature, the input rate
- the bias depends on the time, the temperature, the magnetic field, the mechanical acceleration

Some of these sensitivities are modelled; some other ones are just measured.
2.3. **TIME TRANSIENCE-RELATED (NON-RECIPROCAL) EFFECTS**

2.3.1. Mission specifications

Astrix FOGs are used for different mission types, such as scientific missions, Earth observation, Telecommunication, or navigation missions. The orbit choice is mission-dependent. According to the mission type, the orbit will not be the same. Most common orbits around the Earth are Low Earth Orbit (LEO), Medium Earth Orbit (MEO), Geostationary Earth Orbit (GEO) and Highly Elliptical Earth Orbit (HEO). These orbits have different parameters (e.g. eccentricity, semimajor axis, inclination) which have an impact on the temperature variation experienced by the satellite and the FOG. For this type of missions, the temperature variation is low (a variation of 5 or 10°C during 24 hours, i.e. 0.001°C/min).

Future missions plan to fly Astrix FOGs for interplanetary missions, such as a scientific mission on Mars surface. In such missions, the temperature variations can become much greater, as it is the case in atmosphere re-entry (see Figure 9). Temperature variation rates can become as high as 20°C/min in the case of landing modules re-entry.

Therefore, this present Shupe and Mohr effects study on Astrix FOGs is a R&D study for future missions.

![Figure 9 - Example of Mars landing mission thermal constraints](https://atmospheres.research.ltu.se/pages/news/exomars_landing.php)

2.3.2. Thermally induced non-reciprocal effects

2.3.2.1. Effect of temperature transience

There exists two ways for phase delay to occur. Either the wave speed is changed because of a change of the fiber index (Shupe effect, discovered in 1980); or the optical path is changed because of a physical deformation of the optical coil (Mohr effect, discovered in 2004).

2.3.2.1.1. The Shupe effect

The two counterpropagating paths are only equalized in a FOG if the system is time-invariant. Parasitic phase shifts generated by the environment can occur, particularly with nonuniform temperature change as analyzed very early by Shupe [1]. The two interfering waves do not see the perturbation at exactly the same time, unless it is applied in the middle of the coil.
An elementary fiber segment of elementary curvilinear coordinate $\delta z$ accumulates a phase:

$$\delta \Phi = 2\pi \frac{n\delta z}{\lambda}$$

With a temperature change $dT$, this accumulated phase has a variation:

$$d(\delta \phi) = \frac{2\pi}{\lambda} \left( \frac{dn}{dT} + n\alpha_{SiO_2} \right) dT \cdot \delta z$$

where the temperature dependence of the index of silica $\frac{dn}{dT}$ is $8.5 \times 10^{-6} / ^\circ C$, and the thermal expansion coefficient $\alpha_{SiO_2} = 0.5 \times 10^{-6} / ^\circ C$ [2].

To simplify equations, we may define a coefficient $\alpha_T$ of thermal dependence of the accumulated phase

$$\alpha_T = \frac{dn}{dT} + n\alpha_{SiO_2}$$

which is about $9 \times 10^{-6} / ^\circ C$, and:

$$d(\delta \phi) = \frac{2\pi}{\lambda} \alpha_T dT \cdot \delta z$$

At the coordinate $z$, the time delay $\Delta t(z)$ between both interfering waves is (cf Figure 10):

$$\Delta t(z) = \frac{L - z}{c/n} - \frac{z}{c/n} = \frac{L - 2z}{c/n}$$

Where $L$ is the coil length. It is zero in the middle of the coil where $z = L/2$, and maximum at the end where $z = 0$.

![Figure 10 - Effect of an asymmetrical perturbation](image)

If the temperature varies at a rate $dT/dt$ it yields an elementary phase difference error $\delta \phi_e$ in the interferometer:

$$\delta \phi_e = \frac{2\pi}{\lambda} \alpha_T \frac{dT(z)}{dt} \Delta t(z) \delta z$$

And then

$$\delta \phi_e = \frac{2\pi}{\lambda} \alpha_T \hat{T}(z) \frac{L - 2z}{c/n} \delta z$$

This elementary phase error $\delta \phi_e$ is proportional to the temporal derivative of temperature $\hat{T}$. Taking, for example, a coil of 200m composed of 40 layers of 50 turns, and assuming that the first external layer is heated solely at a moderate rate of $0.01^\circ C/s$, a phase error as high as $5 \times 10^{-6}$ rad results.
These elementary phase errors have to be summed over the fiber coil length $L$, and the good way to do it is to consider pairs of symmetrical elementary segments at the coordinates $z$ and $(L - z)$ (as shown on Figure 10). For such a pair, the phase error generated in the interferometer is:

$$\delta\phi_{ep}(z) = \delta\phi(z) + \delta\phi(L - z)$$

which yields:

$$\delta\phi_{ep}(z) = \frac{2\pi}{\lambda} \alpha_T \frac{L - 2z}{c} \left(\dot{T}(z) - \dot{T}(L - z)\right) \delta z$$

since

$$L - 2(L - z) = -(L - 2z)$$

This result is very important as it shows that, actually, the Shupe effect does not depend on $\dot{T}$ but on the difference between the temporal derivatives of the temperature at $z$ and at $(L - z)$: it is a delta T-dot effect.

This has a first implication because the difference of the derivatives of two functions is the derivative of the difference between these two functions.

The elementary rotation rate error $\delta\Omega_{ep}$ induced by the pair of symmetrical elementary segments $\delta z$ at $z$ and $L - z$ is proportional to

$$d(T(z) - T(L - z))/dt$$

In inertial navigation the rate is integrated to yield the accumulated angle of rotation. Because in a temperature change, the temperature difference between $z$ and $(L - z)$ is zero before the change and back to zero at the end, the Shupe effect does not yield an error on the accumulated angle that is calculated by integration, even if it induces a transient error on the rate measurement; the average rate error is nulled out.

![Figure 11 - Effect of a symmetrical pair of asymmetrical temperature changes in (z) and (L-z)](image)

### 2.3.2.1.2. Symmetrical windings

The equations involving $\delta\phi_{ep}$ show that if the same temperature change $\dot{T}$ is experienced by symmetrical segments at equal distance from the middle of the coil, the effect is canceled out, as $\delta\phi(z)$ becomes opposite to $\delta\phi(L - z)$. This compensation is obtained with a symmetrical winding as analyzed by Shupe [1]: the fiber is wound from the middle, and by alternating layers coming from each half-coil length, this places symmetrical segments in proximity, as shown on Figure 12.
Compared to ordinary winding, this dipolar winding reduces the Shupe effect by a factor approximately equal to the number of layers [3]. An even better compensation, approximately equal to the square of the number of layers, is obtained with a quadrupolar winding proposed by Frigo [2] as shown on FIG... (b). With a pair of symmetrical layers in a dipolar winding, a radial heat propagation always first reaches the layer of the same half-coil length, which yields a residual transient sensitivity. With the quadrupolar winding, the layer order is reversed from pair to pair, which further improves the compensation. To obtain the best results, the number of layers must be a multiple of four and all the layers must have the same number of turns. Some additional benefit is also obtained by thermally insulating the coil to slow down its rate of temperature change.

2.3.2.2. Non uniform temperature-induced stresses: the Mohr effect (or T-dot effect)

In practice, the Shupe effect is almost not experienced in a fiber gyro with a quadrupolar winding [2] as the variation of the difference of temperature, the delta T dot, is very small between the two adjacent layers of the symmetrical points and also because of its thermal inertia the packaging slows down the temperature change, which yields a good temperature uniformity in the coil. However, the expression of \( \delta \phi_{ep} \) which describes the Shupe effect, is incomplete. It assumes that the coefficient \( \alpha_T \) of thermal dependence of the accumulated phase is the same for the two elementary segments of a pair of symmetrical points, which is not at all the case because of non-uniform temperature-induced stresses in the coil that change the actual index dependence \( dn/dT \) as well as the actual thermal expansion \( \alpha_{SiO2} \).

This was described quite early by Cordova and Swabian [4] and analyzed in more details recently by Mohr and Schadt [5]. For a pair of symmetrical elementary elements \( \delta z \) at positions \( z \) and \( L - z \), instead of the previous expression of \( \delta \phi_{ep} \) of a pure Shupe effect, the elementary phase error \( \delta \phi_{ep}(z) \) induced in the interferometer is actually:

\[
\delta \phi_{ep}(z) = \int_{z}^{L-z} \alpha_T \frac{dn}{dT} dt
\]
\[
\delta \phi_{ep}(z) = \frac{2\pi}{\lambda} \left[ \alpha_T(z) \hat{T}(z) - \left[ \alpha_T(L - z) \hat{T}(L - z) \right] \right] \frac{L - 2z}{c/n} \delta z
\]

which can be approximated by:

\[
\delta \phi_{ep}(z) = \frac{2\pi}{\lambda} \left[ \langle \alpha_T \rangle \cdot \Delta \hat{T} + \Delta \alpha_T \cdot \langle \hat{T} \rangle \right] \frac{L - 2z}{c/n} \delta z
\]

The first term \( \langle \alpha T \rangle \cdot \Delta \hat{T} \), where \( \langle \alpha_T \rangle \) = \((\alpha_T(z) + \alpha_T(L - z))/2\) is the mean thermal dependence of the phase at \( z \) and \( L - z \), is the pure Shupe effect, while the second term \( \Delta \alpha_T \cdot \langle \hat{T} \rangle \), where \( \langle \hat{T} \rangle = (\hat{T}(z) + \hat{T}(L - z))/2 \) is the mean temperature derivative, is the effect of temperature-dependent stresses which induces a difference \( \Delta \alpha_T(z) = \alpha_T(z) - \alpha_T(L - z) \) of the temperature dependence of the phase between the position at \( z \) and the symmetrical one at \((L - z)\). To summarize, the elementary phase error created by a symmetrical pair of elementary segments \( \delta z \) is the sum:

\[
\delta \phi_{ep}(z) = \delta \phi_{Sh}(z) + \delta \phi_{Tdot}(z)
\]

with the pure Shupe effect:

\[
\delta \phi_{Sh}(z) = \frac{2\pi}{\lambda} \langle \alpha_T \rangle \cdot \Delta \hat{T} \frac{L - 2z}{c/n} \delta z
\]

which depends on the delta \( T \) dot, \((\Delta \hat{T})\), and the effect of the temperature-induced non-uniformity of the stress in the coil, which depends on the mean \( T \) dot, \((\langle \hat{T} \rangle)\):

\[
\delta \phi_{Tdot}(z) = \frac{2\pi}{\lambda} \Delta \alpha_T \cdot \langle \hat{T} \rangle \frac{L - 2z}{c/n} \delta z
\]

In practice, this \( T \)-dot effect \( \delta \phi_{Tdot} \) is much larger than the pure Shupe effect \( \delta \phi_{Sh} \), which depends on the delta \( T \) dot. This \( T \)-dot dependence is actually good news as it can be modeled much more easily than the \( \Delta \hat{T} \) dot. Knowing \( \alpha_T(z) \) along the whole fiber coil requires a good understanding of the complex composite structure of the coil composed of the silica fiber, its polymer coating and the supporting frame of the coil with usually a potting material [5]. Symmetrical windings like the quadrupolar winding are also very efficient because the elementary segments of a symmetrical pair are close to each other, which drastically reduces \( \Delta \alpha_T(z) = \alpha_T(z) - \alpha_T(L - z) \). Finally, the two effects, Shupe and \( T \) dot, have a difference in temporal signature as outlined by Mohr and Schadt [5]. To understand it, we are going to recall the basics of heat propagation, which is actually a process of diffusion.
3. **ANALYTICAL STUDY OF TEMPERATURE-INDUCED NON-RECURSIVE EFFECTS IN ASTRIX 120/200 AND 1090 FOGS**

3.1. **THERMAL CHARACTERIZATION OF ASTRIX 120/200 AND 1090**

3.1.1. Thermal sensors on Astrix

3.1.1.1. Temperature measurement

During the tests this difference is not so important because both GEU and ICU are in the equipment test facility (close to each other, almost touching). However on the satellite the distance between ICU and GEU is much more important which leads to bigger temperature differences. For more precise test measurements, it would be interesting to separate physically ICU and GEU.

![Figure 13 - Thermistor Equivalent Electrical Circuit](image)

The temperatures are evaluated by measuring the voltage across a thermistor supplied by a voltage reference $V_{ref}$ dropped by a fixed resistor $R_{bias}$. This voltage is acquired with an Analog-to-digital converter (ADC). The digital value representative of the SIA temperature $V_{RSIA}$ is called “SIA temperature”. The value representative of the electronics temperature $V_{Rorg}$ is called “Organiser temperature”. Figure 13 shows the thermal sensor electrical equivalent circuit.

From these values on 12 bits considered as unsigned integers, the thermistor values can be inferred:

$R_{SIA}$ is the thermistor value (Ohms) placed in the IOC:

$$R_{SIA} = R_{bias} \frac{V_{RSIA}}{V_{ref} - V_{RSIA}}$$

$R_{org}$ is the measured thermistor value (Ohms) placed on the organiser:

$$R_{org} = R_{bias} \frac{V_{Rorg}}{V_{ref} - V_{Rorg}}$$

Finally, the temperature measurements are:

$$T_{SIA} = \frac{1}{A + B \ln(R_{SIA}) + C \ln(R_{SIA})^3} - 273.15$$

with $T_{SIA}$ the SIA temperature and

$$T_{org} = \frac{1}{A + B \ln(R_{org}) + C \ln(R_{org})^3} - 273.15$$

with $T_{ORG}$ the organiser temperature.

3.1.1.2. Thermal sensor location

Different thermal sensor locations are used when the FOG is tested on ground:
- in the baseplate where the FOG is placed (called “Baseplate temperature”)
- in the board where the electronic board is (called “Board temperature”)
- in the organizer where the optical source is (called “Organizer temperature”)
- in the IOC close to the optical coil (called “SIA temperature”).

These telemetries have an accuracy of 0.05°C.

3.1.2. Thermal architecture

In space, only conduction and radiation have to be taken into account since there is no convection in vacuum. Different components are used in order to reduce these two heat transfers.

3.1.2.1. Astrix 200
3.1.2.1.1. Material analysis

The choice of the component is essential for the conduction and radiation properties of the FOG. The goal is to keep down heat exchanges within the optical coil. Both conduction and radiation heat transfers must be considered.

The bipods need to not be conductive, so that satellite temperature variations are smoothed out. A Multi-Layer Insulation (MLI) is used as well to reduce radiative exchanges.

3.1.2.1.2. FEM results

Finite Element Models (FEM) calculations have been performed in order to evaluate the effectiveness of the materials. The main gradients are obtained at bipod level (low conductivity) which makes it an efficient material since it filters temperature propagation. The Figure… shows the thermal propagation in a steady state case with all nodes (point from the FOG where the temperature is controlled) set at 20°C. The temperature is homogeneous after and before the bipods on the thermal path.

SIA 4 temperature is slightly lower than the other SIAs due to the radiative exchange of the external face of the external face of the SIA 4 with the harness support.

![Figure 14 - Thermal FEM result: homogeneous temperature in the SIAs](image-url)
As seen in the Figure 14 and Figure 15, the temperature in the SIA is very homogeneous (temperature gradients below CONFIDENTIAL °C. Because of IOC high power density, the maximum gradients are obtained in the lower part of the SIA. This is an important detail as the SIA temperature sensor is placed in the IOC (this will be detailed in the next section).

**Figure 15 - Thermal FEM result: homogeneous temperature in a SIA: the heat comes from the IOC**

Thermal gradients in the blades (making the connexion between the structure and the SIA) are below CONFIDENTIAL °C. As detailed in Figure 16, the gradients are identical in the two symmetrical blades (those close from IOC) but it is slightly lower in the upper blade, which may have an impact in thermo-elastic distortions.

**Figure 16 - Thermal FEM result: blades thermal behavior**

3.1.2.2. Astrix 1090: thermal test using TRP sensors

Very few FEM calculations have been performed on Astrix 1090 FOG. These FEM calculations only concern GEU thermal behavior. The temperature that needs to be known is the coil temperature for Mohr effect modeling.

Since no thermal analysis had been realized for Astrix 1090, an extra thermal test has been performed to make sure that the SIA temperature was close to the optical coil temperature. Indeed, previous FEM calculations had shown that a difference of a couple of degrees can occur between the SIA sensor location and the optical coil support (see Figure 17) due to the IOC heat dissipation.
Extra sensors have been placed as close from the optical coil as possible, on the thermal path (as detailed in Figure 18).

In the case of Astrix 1090, the GEU and the SIA are not separated like for Astrix 200 FOGs. This has a major consequence on temperature propagation. The temperature filtering is not as efficient in Astrix 1090 as it is in Astrix 200.

The Figure 19 shows the evolution of the temperatures of the two extra sensors as well as the SIA temperature.
The evolution of the difference between the temporal derivative of the extra sensor and that of the SIA sensor are plotted in Figure 20.

These differences stay positive most of the time. Even if IOC dissipates energy, TRP sensors come first on the thermal path (heat comes from the baseplate, where the FOG is placed). The biggest differences occur during transient phases. This difference always stays smaller than confidential °C though. When the temperature is stabilized (steady-state case), the difference decreases.
For the study of the Mohr effect, temporal derivatives of the temperatures need to be studied, as shown in Figure 21. The corresponding differences are plotted in Figure 22.
The biggest differences occur during transient phases. This difference always stays smaller than $\text{CONFIDENTIAL } ^\circ C/s$ though. When the temperature is stabilized (steady-state case), the difference decreases.

Since the difference stays smaller than $\text{CONFIDENTIAL } ^\circ C/s$ this sensor can be used for Mohr effect modelling.

### 3.2. **THERMAL TEST ANALYSIS ON ASTRIX 200 AND 1090**

The response of the Astrix to the temperature control command is different. The Figure 23 shows the evolution of the control temperature, the organizer temperature, the SIA temperature and the baseplate temperature.

![Figure 23 - Response of thermal sensors to the temperature control command](image)

#### 3.2.1. **Equipment test facilities and temperature profiles**

Different tests are performed for the study of time transience-related thermally induced effects:

- Thermal cycles in vacuum (from $\text{CONFIDENTIAL } ^\circ C$ to $\text{CONFIDENTIAL } ^\circ C$) are performed in the test mean called “VTPIE” (these tests are called “VTPIE test” or “thermal cycle test”). Figure 24 shows the VTPIE (CONFIDENTIAL picture).

- A specific test for the study of Mohr and Shupe effects is performed in vacuum in the test mean called “the 3-axis table (T3A)”. This test is only performed on Astrix 200. They are called “T3A tests” or “Shupe tests”. The Figure 27 shows the T3A (CONFIDENTIAL picture).

Both tests are performed in a vacuum (like in space).
3.2.2. Thermal cycles in VTPIE

3.2.2.1. Thermal cycle description

The VTPIE tests are performed in order to make sure that the FOG can resist to a wide range of temperatures. Thermal cycles are applied to the FOG in the vacuum, the minimum and the maximum temperature depending on the mission specification. The FOG is at rest.

![The VTPIE test bench](image)

**Figure 24 - The VTPIE test bench**

The advantage of such a test for the study of the Mohr effect is the broad range of temperature. The temperature varies usually between -20°C and +80°C. One cycle usually lasts for 11 hours, as shown in Figure 25.

![Evolution of the organizer temperature (°C) during a VTPIE test on an Astrix 200 FOG](image)

**Figure 25 - Evolution of the organizer temperature (°C) during a VTPIE test on an Astrix 200 FOG**

In the VTPIE the FOG is at rest (i.e. not rotating in any direction).
3.2.3. “Shupe tests" on T3A: a more precise analysis of the thermal effect (for Astrix 200 only)

3.2.3.1. Technical description of T3A

The T3A is a test facility to perform thermal tests in vacuum on a table that can move in the three directions with a high stability. In the case of the thermal tests for the study of the Shupe and the Mohr effects, the FOG has to be fixed so the T3A does not move during the test.

![Three-axis Table Test Bench](image)

**Figure 26 – Three-axis Table Test Bench**

The Figure 27 shows a simplified description of the T3A.

The GEU and the ICU are mounted on the same side of the T3A internal base plate as shown on Figure 27 so that both structures see the same temperature variation. To avoid any deformation of the assembly, a mechanical mass is fitted on the other side of the base plate when a FOG is tested alone.

The temperature rate can be chosen in the range \([0.1 ; 2]\) °C/min with steps of 0.1 °C/min.

During thermal tests for the study of the Shupe and Mohr effects the T3A is at rest (i.e. not rotating in any direction) in order to only measure the temperature-induced output rate.
3.2.3.2. “Shupe test” description

In order to measure the bias sensitivity to thermal variation, some tests are performed in the T3A. These tests are called “Shupe tests” even if they should be called “Mohr tests” to be more precise. Shupe effect usually means thermal effect in general. The Mohr effect is sometimes called the “constraint Shupe”, hence the ambiguity. The tested parameter is called the Shupe coefficient and is measured in (°/h)/(K/min). Figure 28 shows the evolution of the temperature during a Shupe test. As described on the Figure, the Astrix unit has a filtering behavior (regarding the temperature). The Astrix unit has been designed so that a high temperature variation in the GEU gives rise to a small temperature variation in the SIA (where the fiber coil and the precise optical interferences occur).
Unless one axis has a different behavior (either rate or temperature-wise), only the results for one axis are shown in the report.

### 3.2.4. VTPIE and T3A test comparison

Note that the amplitude variation and temporal duration is different between the two test benches, as shown on Figure 29.

![Figure 29 - Evolution of the organizer temperature (blue) and of the SIA temperature (green) (°C) in the VTPIE (high amplitude cycles) and in the T3A (°C) (small amplitude variations) for Astrix 200](image)

### 3.3. Thermal test results

#### 3.3.1. Thermal tests results: the difference of temperature propagation between Astrix 200 and Astrix 1090

##### 3.3.1.1. VTPIE

Figure 30 shows the response of the SIA temperature to the command temperature in the Astrix 1090 during a VTPIE test. The SIA temperature almost reaches the same values as the organizer and board temperatures.
3.3.1.2. T3A

Figure 31 shows the evolution of SIA, organizer and board temperatures in Astrix 1090 during a Shupe test. As the thermal transitions are faster, the SIA temperature does not have time to reach the same values as the organizer temperature.

Figure 30 - Evolution of SIA, Organizer and Board Temperatures (°C) in Astrix 1090 during a VTPE test

Figure 31 - Evolution of SIA, Organizer and Board Temperatures (°C) in Astrix 1090 during a Shupe test
3.3.2. Thermal tests results: the impact of temperature variation on the rate

3.3.2.1. Astrix 1090 results: Mohr effect is highlighted

As described in 2.3.2.2., the Mohr effect pops up during transient thermal periods. When the temperature is constant, no rate variation occurs, as shown on Figure 32.

![Figure 32](image_url)

**Figure 32 - Evolution of the rate (°/h) (green) and the SIA temperature (°C) (blue) for an Astrix 1090**

As described in Figure 33 and Figure 34, the Mohr effect is proportional to the temporal derivative of the temperature.
The proportionality factor can be obtained by plotting the rate as a function of $\dot{T}_{SIA}$. The slope of this line is the Shupe coefficient. In order to respect the mission specification, this value must stay smaller than requirement.

3.3.2.2. Astrix 200 results: Mohr effect and another parasitic effect are highlighted

3.3.2.2.1. VTPIE tests results

25 thermal cycle tests and 20 Shupe tests have been analyzed for the study of the Astrix 200. Each test contains data for each of the four axes (180 axes data in total). The temporal derivative of the temperature has been compared to the rate in order to identify the Mohr effect. Thermal cycle data show that the rate follows $\dot{T}_{SIA}$ just like with Astrix 1090. The Mohr effect is highlighted on Astrix 200 FOGs as predicted by the theory.
Most of the time the result is different from that described on Figure 34 for an Astrix 1090 though.

The Mohr effect is always visible for Astrix 200 thermal tests but some parasitic peaks, sometimes greater than those of the Mohr effect, appear in the different tests. The peaks always pop up both in Shupe tests and in thermal cycles tests but they are not always visible on the four axes. The amplitude of the peak is also axis-dependent. No conjecture has been made on the axis on which this parasitic effect pops up preferably. Figure 35 and Figure 36 show the parasitic effect with different amplitudes.

The parasitic peak pops up in each transition of thermal cycle tests but not in each transition of Shupe effect tests.

In Astrix 200 thermal tests, a parasitic effect pops up, as shown on Figure 35. The rate is supposed to follow $\dot{T}_{SIA}$ (Mohr effect) but some periodic peaks are observed in the beginning and in the end of each cycle.

![Image](image_url)

**Figure 35 - Evolution of the rate (°/h) (green) and $T_{SIA}$ (°/s) (blue) for an Astrix 200 during a thermal cycle: the parasitic effect stays smaller than the Mohr effect**

In some cases, this parasitic effect becomes greater than the Mohr effect, as shown on Figure 36.
Figure 36 - Evolution of the rate ($^\circ$/h) (green) and $T_{S1A}$ ($^\circ$C/s) (blue): The parasitic effect becomes greater than the Mohr effect for an Astrix 200

In these two last figures, the Mohr effect is still visible (even in the last one) because the rate varies with $\dot{T}_{S1A}$ out of the parasitic peaks.

This parasitic effect can be characterized with the study of the evolution of the rate as a function of $\dot{T}_{S1A}$ as shown on Figure 37. Therefore, this effect can be distinguished from the Mohr effect.

Figure 37 - Evolution of the rate ($^\circ$/h) as a function of $T_{S1A}$ ($^\circ$C/s): A parasitic effect can be observed as the proportionality is not respected for an Astrix 200

In Figure 37, the points situated close to the fitting curve function correspond to the Mohr effect. The points that do not fit the curve function correspond to the parasitic effect.

The analysis of all the tests allowed noticing that these peaks occur as $\ddot{T}_{ORG}$ (i.e. the second temporal derivative of the organizer temperature) reaches its maximum values, as showed on Figure 38. Since the effect is only visible in the
beginning of each transition, this could mean that the parasitic peaks are due thermo-elastical effects in the Astrix 200 bipods. The heat would not have time to propagate to the SIAs so this cannot be due to a fiber coil dilatation. According to the peaks value in °/h and their temporal duration, assuming that the Astrix 200 has a length of 20 cm, we can derive an order of magnitude of the move: it corresponds to a 10 µm displacement within 30 minutes.

**FIGURE 38 - EVOLUTION OF T$_{ORG}$ (°C/s$^2$) (blue) AND THE RATE (°/h) (green) DURING A VTPIE TEST FOR AN ASTRIX 200:**
THE PARASITIC PEAKS DEPEND ON T$_{ORG}$(°C/s$^2$)

3.3.2.2.2. T3A tests results

In Shupe tests analysis, the parasitic effect is visible in the beginning of the tests with great amplitude. The amplitude of the peaks decreases as the test goes though, as shown on Figure 39.

**FIGURE 39 - EVOLUTION OF T$_{SIA}$ (°C/s) (blue) AND RATE (°/h) (green): PARASITIC PEAKS APPEAR IN THE BEGINNING OF Shupe TESTS. THE RATE IS NOT FOLLOWING T$_{SIA}$ PERFECTLY FOR AN ASTRIX 200**
\( \dot{\theta}_{ORG} \) values decrease as the test goes, as shown on Figure 40. This shows that the parasitic effect only pops up for specific (high enough) \( \dot{\theta}_{ORG} \) values.

![Figure 40 - Evolution of \( \dot{\theta}_{ORG} \) (°/s²) (blue) and rate (°/h) (green) during a Shupe test: parasitic peaks appear in the beginning of Shupe tests. The rate is not following \( T_{SHA} \) perfectly for an Astrix 200](image)

The Mohr effect has been modeled and the parasitic effect has been characterized through a specific test. This will be detailed in the next section.
4. **Modeling and Characterization of Mohr and parasitic effects**

4.1. **Modeling of thermal effects on Astrix FOGs**

4.1.1. Thermal sensitivity of bias

The operating point of FOG components (diode, electronics, resistances, etc.) vary with respect to the temperature. This variation is modeled in the on-board computer of the Astrix. The thermal sensitivity of bias is characterized in a specific test during 51 hours. This test cannot be used for the study of the Mohr effect because the FOG is set to rotate. This multi-temperatures calibration is done with the generic thermal profile defined to reach the required temperatures at GEU and ICU levels. The thermal model of the equipment is extracted from this multi-temperatures calibration performed before environmental tests.

As the Mohr effect is a change of the bias only (and not the scale factor), the thermal sensitivity of the bias model has been analyzed. This model is a fourth order polynomial equation:

\[
\text{Bias}(T) = \alpha_0 + \alpha_1 T_{\text{ORG}} + \alpha_2 T_{\text{ORG}}^2 + \alpha_3 T_{\text{ORG}}^3 + \alpha_4 T_{\text{ORG}}^4 \\
+ \beta_0 + \beta_1 T_{\text{SIA}} + \beta_2 T_{\text{SIA}}^2 + \beta_3 T_{\text{SIA}}^3 + \beta_4 T_{\text{SIA}}^4
\]

4.1.2. Modeling method

4.1.2.1. Van der Monde matrices and modelling principle

In order to model the rate dependence to thermal variations, an interpolation using the Vandermonde matrix has been used.

In order to determine the coefficients \(a_0, a_1, \ldots, a_n\) of a polynomial

\[
P_n(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n
\]

such that it interpolates the \(n+1\) points \((x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)\)

let us write the following linear system of equations:

\[
\begin{align*}
P_n(x_0) &= y_0 \\
P_n(x_1) &= y_1 \\
& \quad \quad \vdots \\
P_n(x_n) &= y_n
\end{align*}
\]

Which can also be written

\[
\begin{align*}
a_0 + a_1 x_0 + a_2 x_0^2 + \cdots + a_n x_0^n &= y_0 \\
a_0 + a_1 x_1 + a_2 x_1^2 + \cdots + a_n x_1^n &= y_1 \\
& \quad \quad \vdots \\
a_0 + a_1 x_n + a_2 x_n^2 + \cdots + a_n x_n^n &= y_n
\end{align*}
\]
The rate has been fitted with the SIA temperature, the organizer temperature and the temporal derivative of the SIA.

Most of the tests that have been used contain about 250 000 points (thermal cycling) or 70 000 points (Shupe tests). The acquisition frequency is 1 Hz so it corresponds to about 70 hours or 18 hours tests respectively.

The rate has been fitted with the SIA temperature, the organizer temperature and the temporal derivative of the SIA temperature. All these measurements correspond to the $x_i$. Second order polynoms have been used for $\dot{T}_{SIA}$ and fourth order polynoms for temperatures. Using higher orders does not improve the residual error.

There is currently no temporal derivative of temperature sensor in the market. $\dot{T}_{SIA}$ values have been derived from $T_{SIA}$ ones:
\[
\dot{T}_{\text{SIA}_i} = \frac{T_{\text{SIA}_{i+1}} - T_{\text{SIA}_i}}{\Delta t}
\]

Since the acquisition frequency is 1 Hz :
\[
\Delta t = 1s
\]

4.1.2.2. Filtering duration choices

Let the condition for a proper Shupe residual observation be:

\[
\text{Filtered ARW} < \frac{1}{3} \text{Shupe Residual Error}
\]

i.e. for a Shupe residual error of \(10^{-2}^\circ/\text{h}\) (which would be a good outcome)

\[
\text{Filtered ARW} < 3 \times 10^{-3}^\circ/\text{h}
\]

The filtering duration is chosen such that ARW is not hiding the residual error, so the residual error has to be greater than the filtered ARW 3-sigma.

Let \(\tau\) be the filtering duration

\[
\text{Filtered ARW} = 3 \times ARW_{200} \times \frac{1}{\sqrt{\tau}}
\]

Since the performance of an Astrix 200 FOG is

\[
ARW_{200} = \text{CONFIDENTIAL} (^\circ/\sqrt{\text{h}})
\]

It yields to the following condition

\[
\tau > \text{CONFIDENTIAL}s
\]

A filtering duration of \(\text{CONFIDENTIAL} s\) can be chosen.

4.1.3. Modeling results on residual error

4.1.3.1. Astrix 1090 results

4.1.3.1.1. Conclusion and comparison between the two models

Ten Astrix 1090 thermal cycle tests have been analyzed (no Shupe test has been performed on Astrix 1090). Each test contains data for each of the three axes (30 axes data in total). For each axis, the output rate has been analyzed with the FOG at rest and during constant temperature phases (i.e. no Mohr effect stimulation) in order to evaluate the noise of the FOG for the chosen filtering duration. Then the rest of the test has been analyzed: the temporal derivative of the temperature has been compared to the rate in order to identify the Mohr effect. The two models have been implemented (both separately and together) to the rate data. The 3 sets of residual errors (one for the Mohr effect model, one for the thermal sensitivity of bias model and one for both models together) have been analyzed and compared in order to figure out which model is the more efficient.
Beforehand, the output rate had been analyzed during constant temperature phases (i.e. no Mohr effect stimulation) in order to evaluate the noise of the FOG for the chosen filtering duration. Tests with constant temperature and no rotation have been used in order to evaluate the noise of the FOG in the test facility. The noise amplitude (peak to peak) is equal to confidential °/h for a 1090 Astrix in average (for a filtering duration of confidential).

The conclusion is that the residual error after any of the two compensations is of the same order of magnitude as the noise, as shown in Figure 41. This means that both models compensate the Mohr effect the same way. Adding the two models together does not improve the result.

In the test below, the peaks due to the Mohr effect are those appearing for high values of T SIA (four positive peaks and four negative peaks in this test). These peaks are (slightly) better compensated with the Mohr effect model than with the other model but there is no major difference (only a 1.5-fold decrease in average).

![Figure 41 - Comparison between the evolutions of T SIA (top left), the rate (top right), the residual error of the bias thermal model (bottom left) and the residual error of the Mohr model (bottom right) for Astrix 1090 thermal cycle](image)

The rest of the peaks correspond to the filtered noise. The bias sensitivity thermal model compensates the Mohr effect even if it does not contain the terms in T SIA. Bias sensitivity model or Mohr effect model reduce the Mohr effect by a factor of 12 for Astrix 1090 which is a positive result.
4.1.3.1.2. Interpretation

For each of the 30 axes, $\dot{T}_{\text{SIA}}$ and $T_{\text{ORG}} - T_{\text{SIA}}$ have been analyzed. A similarity between the two measurements has been identified as shown on Figure 42. Even if the unit of the two parameters is not the same, the two data sets appear to be the same in the Vandermonde matrix. Therefore, since the temporal behavior is similar, no improvement is obtained while using $\dot{T}_{\text{SIA}}$ data (Mohr effect model) in addition to $T_{\text{ORG}}$ and $T_{\text{SIA}}$ data (thermal sensitivity of bias model).

The Astrix FOG has a filtering behavior temperature-wise in order to reduce the thermal inertia.

The difference $T_{\text{SIA}} - T_{\text{ORG}}$ shall not be interpreted as a (spatial) thermal gradient in the coil since the organizer is separated from the SIA.

The difference between the two normalized curves of Figure 42 has been plotted in Figure 43. This difference can be written (in %):

$$\Delta = \left| \frac{\dot{T}_{\text{SIA}}}{\|\dot{T}_{\text{SIA}}\|} - \frac{T_{\text{ORG}} - T_{\text{SIA}}}{\|T_{\text{ORG}} - T_{\text{SIA}}\|} \right| \times 100$$

The rate was plotted as well (left axis) in order to see when the maximum values of $\Delta$ appear. The maximum values of $\Delta$ corresponds to naught values of rate (no Mohr effect). When the Mohr effect becomes important (positive or negative rate peaks), then $\Delta$ tends to 0.
Therefore, the two measurements ($\dot{T}_{\text{SIA}}$ and $T_{\text{ORG}} - T_{\text{SIA}}$) have a similar temporal behavior when the Mohr effect pops up. This explains the similarity between the residual errors of the two models.

To conclude, the residual errors are the same for both models and a 12-fold decrease of the Mohr effect is obtained with them.

4.1.3.2. Astrix 200 modeling results

4.1.3.2.1. Conclusion and comparison between the two models

The two models have been implemented (both separately and together) to the rate data. The 3 sets of residual errors (one for the Mohr effect model, one for the thermal sensitivity of bias model and one for both models together) have been analyzed and compared in order to figure out which of the models is the more efficient.

4.1.3.2.1.1. Thermal cycles tests modeling analysis

In thermal cycle tests, the parasitic effect pops up in each transition. The amplitude of the peaks is the same during all the tests. This amplitude varies from one Astrix 200 to another though.

After modeling the Mohr effect, only the parasitic effect is still visible in the residual error. The parasitic effect prevents from assessing the efficiency of the two models, as its value stays greater than the Mohr effect residual error, as shown in Figure 44.
FIGURE 44 - COMPARISON BETWEEN THE EVOLUTIONS OF $T_{SLA}$ (TOP LEFT), THE RATE (TOP RIGHT), THE RESIDUAL ERROR OF THE BI-THermal MODEL (BOTTOM LEFT) AND THE RESIDUAL ERROR OF THE Mohr MODEL (BOTTOM RIGHT) FOR ASTRIX 200 THERMAL CYCLE

The Mohr effect has been compensated since the residual error is close to 0 outside the parasitic peaks. The compensation of the Mohr effect (i.e. outside the parasitic peaks) is slightly better with the Mohr effect model than with the other model (by a factor 2 in average).

4.1.3.2.1.2. Shupe test modeling analysis

In the beginning of Astrix 200 Shupe tests, unexpected high rate values appear, as shown on Figure 45. This is correlated to high values and high frequencies of $\ddot{T}_{ORG}$. When $\ddot{T}_{ORG} >$ CONFIDENTIAL °C/s$^{-2}$ the parasitic effect can be observed and becomes higher than the Mohr effect. The parasitic effect is also visible for smaller values of but do not exceed Mohr peaks.

In the case of the Shupe tests, no gain is obtained with either of the two models, as shown on Figure 45.
4.1.3.2.2. Interpretation

4.1.3.2.2.1. Astrix 200 filters the temperature just like Astrix 1090 does

The two measurements ($\dot{T}_{\text{SIA}}$ and $T_{\text{ORG}} - T_{\text{SIA}}$) have a similar temporal behavior just like in the case of Astrix 1090, as shown on Figure 46. This explains the similarity between the residual errors of the two models.
4.1.3.2.2. Parasitic effect appears on all thermal cycles transitions and not all Shupe tests transitions because the $\dot{T}_{\text{ORG}}$ values are not the same for the two tests.

The parasitic effect seems to appear in phase with $\dot{T}_{\text{ORG}}$ peaks which amplitude is high enough and temporal width short enough.

In thermal cycle tests, this maximum value of $\dot{T}_{\text{ORG}}$ is always the same, as shown on Figure 47 and two local maximum are visible for CONFIDENTIAL $^\circ$C/s$^{-2}$ and CONFIDENTIAL $^\circ$C/s$^{-2}$ . The width of one peak is also constant and equal to 5000 s.

In Shupe tests, this maximum value of $\dot{T}_{\text{ORG}}$ changes as the test goes, as shown on Figure 48. Seven local maximum are visible ranging from CONFIDENTIAL $^\circ$C/s$^{-2}$ to CONFIDENTIAL $^\circ$C/s$^{-2}$ . The width of the peaks also changes as the test goes, ranging from 1000 s to 11000 s.
The two conditions for the parasitic effect to pop up are always respected in thermal cycle tests and are only respected in the three first peaks of Shupe tests, hence the result.

**Figure 47 - The amplitude and the temporal width of $T_{\text{ORG}} \left( ^\circ\text{C} / \text{s}^2 \right)$ is a criteria for the parasitic effect to pop up (Thermal Test Explanation)**

**Figure 48 - The amplitude and the temporal width of $T_{\text{ORG}} \left( ^\circ\text{C} / \text{s}^2 \right)$ is a criteria for the parasitic effect to pop up (Shupe Test Explanation)**
4.2. **Specific test design for the characterization of the parasitic effect on Astrix 1090**

The parasitic effect has been studied on an Astrix 1090 first. Even if no parasitic peaks were detected, different values of $\ddot{T}_{\text{DRG}}$ have been stimulated.

4.2.1. Thermal test design for the study of the parasitic effect on Astrix 1090

In order to define criteria on temporal width and amplitude for which the parasitic effect pops up, a specific test has been designed in T3A. In this test, thermal cycles are performed in the T3A. This had never been done before with an Astrix 1090 in a (fixed) T3A.

In order to design a thermal test, three parameters must be determined: the temperature of each step, the rate to reach the desired temperature, and the duration of the total step, as described on Figure 49.

![Figure 49 - Thermal test step design (parameters to be set)](image)

Here is the test plan for this specific test.
4.2.2. Parasitic effect test results and interpretation

Figure 50 shows the evolution of the organizer and the SIA temperatures.

Different temperature rates have been applied in order to stimulate a wide range of $T_{\text{SIA}}$ values and show the Mohr effect (cf Figure 51). In a regular thermal cycle, the Mohr effect is visible two times per cycle because one cycle contains two transitions. In this specific test, intermediary steps have been added in order to double the number of transition per cycle.

Moreover, the succession of the rates values has been chosen so that higher values of $T_{\text{ORG}}$ are obtained, as shown on Figure 53. The slope of the thermal transitions differ from one transition to the other in order to stimulate different values of $T_{\text{ORG}}$. High $T_{\text{ORG}}$ values have been stimulated.

The maximum $T_{\text{ORG}}$ experienced by an Astrix 1090 in the T3A so far was CONFIDENTIAL °C/s$^{-2}$. The goal of this specific test was to apply different values of $T_{\text{ORG}}$ values to an Astrix 1090 ranging from CONFIDENTIAL °C/s$^{-2}$ to CONFIDENTIAL °C/s$^{-2}$. 

<table>
<thead>
<tr>
<th>Identifier</th>
<th>SetPoint(°)</th>
<th>Rate(°)</th>
<th>Stab_Hour</th>
<th>Stab_Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>THERM_STEP</td>
<td>33.0</td>
<td>2.0</td>
<td>1.0</td>
<td>4.0</td>
</tr>
<tr>
<td>THERM_STEP</td>
<td>31.0</td>
<td>0.7</td>
<td>0.0</td>
<td>3.0</td>
</tr>
<tr>
<td>THERM_STEP</td>
<td>15.0</td>
<td>2.0</td>
<td>3.0</td>
<td>8.0</td>
</tr>
<tr>
<td>THERM_STEP</td>
<td>-10.0</td>
<td>2.0</td>
<td>7.0</td>
<td>0.0</td>
</tr>
<tr>
<td>THERM_STEP</td>
<td>-17.0</td>
<td>1.0</td>
<td>4.0</td>
<td>0.0</td>
</tr>
<tr>
<td>THERM_STEP</td>
<td>10.0</td>
<td>2.0</td>
<td>1.0</td>
<td>15.0</td>
</tr>
<tr>
<td>THERM_STEP</td>
<td>14.0</td>
<td>1.2</td>
<td>0.0</td>
<td>5.0</td>
</tr>
<tr>
<td>THERM_STEP</td>
<td>30.0</td>
<td>2.0</td>
<td>7.0</td>
<td>0.0</td>
</tr>
<tr>
<td>THERM_STEP</td>
<td>15.0</td>
<td>2.0</td>
<td>1.0</td>
<td>10.0</td>
</tr>
<tr>
<td>THERM_STEP</td>
<td>11.0</td>
<td>1.4</td>
<td>0.0</td>
<td>10.0</td>
</tr>
<tr>
<td>THERM_STEP</td>
<td>0.0</td>
<td>2.0</td>
<td>7.0</td>
<td>0.0</td>
</tr>
<tr>
<td>THERM_STEP</td>
<td>20.0</td>
<td>1.0</td>
<td>6.0</td>
<td>0.0</td>
</tr>
<tr>
<td>THERM_STEP</td>
<td>15.0</td>
<td>2.0</td>
<td>1.0</td>
<td>10.0</td>
</tr>
<tr>
<td>THERM_STEP</td>
<td>11.0</td>
<td>1.7</td>
<td>0.0</td>
<td>10.0</td>
</tr>
<tr>
<td>THERM_STEP</td>
<td>10.0</td>
<td>1.0</td>
<td>7.0</td>
<td>0.0</td>
</tr>
<tr>
<td>THERM_STEP</td>
<td>30.0</td>
<td>2.0</td>
<td>1.0</td>
<td>10.0</td>
</tr>
<tr>
<td>THERM_STEP</td>
<td>34.0</td>
<td>1.9</td>
<td>0.0</td>
<td>10.0</td>
</tr>
<tr>
<td>THERM_STEP</td>
<td>51.0</td>
<td>2.0</td>
<td>7.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
Different values of $\dot{T}_{SIA}$ are stimulated which is realized by setting different temperature rates $\alpha$.

**Figure 50 - Evolution of the organizer (blue) and the SIA (green) temperatures (°C) of an Astrix 1090 during the specific test**

**Figure 51 - Evolution of the organizer temperature (°C) (blue) and $\dot{T}_{SIA}$ (°C/s) (green) temperatures of an Astrix 1090 during the specific test**
Even for high $\tilde{T}_{\text{ORG}}$ values, no parasitic effect was observed (cf Figure 53). The 3 SIA are mounted on top of GEU through 3 elastomeric isolation mounts. This tends to stabilize the whole structure of the unit. Therefore, when the test bench baseplate moves, no rotation is detected whatever the value of $\tilde{T}_{\text{ORG}}$ is.

Figure 52 - Evolution of Organizer temperature ($^\circ$C) (blue) and $T_{\text{ORG}}$($^\circ$C/s$^2$) (green): higher $T_{\text{ORG}}$ values as usual are obtained as desired

Figure 53 - Evolution of $T_{\text{SIA}}$ ($^\circ$C/s) (blue) and rate ($^\circ$/h) (green) during the specific test on Astrix 1090: no parasitic effect was observed even for high values of $T_{\text{DRG}}$
5. CONCLUSION

It has been shown that thermally induced non-reciprocity places a practical limit on the Fiber-Optic Gyroscope (FOG) sensitivity that is well above the shot-noise limit. Astrix FOGs supplied by Airbus Defense & Space are not limited by these effects today, but this could be the case in the future in a particularly harsh environment like it is the case during a planetary atmosphere re-entry.

A distinction between two physical effects – the Shupe and the Mohr effects – has been highlighted. Because of these effects, a non-zero rotation rate is indicated by a FOG at rest. The Shupe effect depends on the temporal derivative of thermal gradients between symmetrical points in the optical coil. The Mohr effect depends on the temporal derivative of the optical coil mean temperature. Shupe induced errors can be nulled out using a symmetrical winding. In practice, only the Mohr effect is experienced by a FOG.

Thermal tests are performed in vacuum in order to better understand the response of the FOGs to temperature transience. Different models have been used on the thermal test results. The thermal effects can be easily modeled in the case of Astrix 1090 FOGs (where transient temperature induced errors are important) but not in the case of Astrix 200 FOGs. The non-efficiency of the Mohr effect model with Astrix 200 has two causes. First, the observed transient errors are low thanks to the thermal design of the FOG. Moreover, a parasitic effect has been discovered during temperature transience. It could be due to mechanical displacements of the bipods induced by thermo-elastic effects.

A specific thermal test has been designed in order to better understand this parasitic effect and the temperature induced errors in general.

In the future, a temporal derivative of temperature sensor could be used in order to provide more accurate values. The Mohr effect modeling would be more precise and a higher order could be used in the model. Placing the thermal sensors closer to the optical coil would increase the measurement accuracy.

The subject shows that a multidisciplinary approach is required, involving physics, optics, signal processing, thermal and mechanical engineering; this makes it an exciting challenge for the future.
6. REFERENCES