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A quality process for assessing mathematics in a study programme

Magnus Andersson and Anna Delin

Abstract—**We present two methodologies to assess the use of mathematics in a study programme. Firstly, we use a relatively simple methodology to assess how students show their ability to use mathematics in their degree project reports. Secondly, we present a methodology to assess how mathematics is used during a study programme. We have applied the first methodology on the mathematics content in 114 randomly chosen bachelor degree reports from 6 different study programmes within the fields of electrical engineering and computer engineering at KTH. For the 3-year bachelor degree programmes in computer engineering, we find clear deficits in the way students use mathematics in their bachelor degree reports as compared to the other programmes in our study. Through the second methodology, we were able to relate the deficits in the bachelor degree reports to a programme structure where skills in mathematics have not been sufficiently demanded in the engineering courses of the programme.**

Index Terms—**Bachelor degree reports, mathematics, quality processes, computer engineering, electrical engineering**

I. Introduction

KNOWING how to use mathematics is a central skill for an engineer. This fact is also reflected in the Swedish Higher Education Ordinance ("Högskoleförordningen") [1], which states that students should *be able to demonstrate relevant knowledge of mathematics* in order to gain a bachelor degree in engineering. However, in recent years, there has been a number of alarming reports about the decrease in knowledge of mathematics among upper high school students [2,3] – an issue that has also been addressed at the highest policy level [4]. When a general decline in mathematics knowledge from upper high school is combined with an increasing fraction of a yearly cohort entering university, it is not surprising that many first-year students require additional support to pass the first university courses in mathematics [5]. These effects adds to the subject-specific gap between upper secondary school and university mathematics [6]. Although there has been suggestions for how to bridge this gap from either an instrumental perspective [7] or through a special support programme [8], such approaches may not be sufficient to deal with the low examination rates in introductory university courses in mathematics.

However, in this work we will focus on another and potentially more serious consequence of these problems and ask if students are able to demonstrate relevant knowledge of mathematics at the end of their education, i.e. if they fulfill the requirements set by the Swedish Higher Education Ordinance [1]. If so, their educational programme has been able to compensate for possible deficits in mathematics from upper high school. If not, students do not fulfill the requirements for an engineering exam and are not employable as *engineers* in industry (although they can still be employable at other positions) and there is an obvious need for programme development. Our main focus is to develop methods that can be used to assess the use of mathematics in a study programme and our aim is to gather information that is practically useful for teachers when working with programme development.

Firstly, to assess how students within a programme show their ability to use mathematics, we developed a methodology to analyze the mathematics content in their bachelor thesis reports. We found that a substantial number of students had clear deficits in their ability to show the connection between their thesis work and the underlying mathematics. The deficits were particularly clear in two 3-year bachelor degree programmes in computer engineering. In addition, we developed a methodology to assess the use of mathematics in courses during a study programme. From this analysis, we see that the low mathematics content in the reports from one of the programmes could be related to a low demand on students to use mathematics in other courses than the mathematics courses themselves. The principles behind the methodologies developed here should probably also be useful for others who want to assess the learning of core content in a study programme.

II. Methodology

We have analyzed the mathematics content in a number of bachelor theses (all written after 3 years of university studies) from 6 different study programmes at KTH. Four of them were 3-year engineering programmes (Högskoleingenjör) and two were 5-year engineering programmes (Civilingenjör). For two of the bachelor programmes, we also analyzed how mathematics was used within programme courses. The two methodologies used in our investigation will now be described in some detail.

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M. Andersson is with the Department of Applied Physics, School of Engineering Sciences, KTH, Stockholm, Sweden (corresponding author; e-mail: magnusan@ kth.se).

A. Delin is with the Department of Applied Physics, School of Engineering Sciences, KTH, Stockholm, Sweden (e-mail: annadel@kth.se).

A. *Analyzing mathematics content in theses reports*

After three years of study, students write a bachelor thesis report as part of a mandatory bachelor thesis course. Data from the administrative systems (Ladok on Web) were used to find the names of the students from each programme that had passed the appropriate bachelor thesis courses during the years 2015-2016. All thesis reports available in digital form were downloaded from the DiVA database [9] and assigned an integer number from one up to the total number of downloaded reports for each study programme. Among the found reports, we randomly chose and read 20 reports (for programmes with less than 20 found reports, we read all the reports). The randomization was executed through a computer script using the built-in pseudo-randomizing function "random.randint(min,max)" in the Python programming language, which is based on a Mersenne twister random number operator [10-11]. For each of the chosen reports, we gathered the following information:

Quantitative information
• Was a company involved? Name of company.
• Mathematical concepts found in the reports.
• Number of displayed equations.
• Number of figures with mathematical content.
• Number of tables with mathematical content.
• Have students performed simulations?
• Have students written computer code?
Qualitative information
• Missing mathematical concepts that should have been included in the theses according to us.
• Judgement if an ability to use relevant mathematics is shown convincingly in the reports?
Meta information
• Time for analyzing the reports

B. *Analyzing mathematics content within courses*

To be able to judge the importance of mathematics in a study programme, we asked the examiners in mandatory courses to rate how important each of the mathematical concepts taught in the mathematics courses were for the student's ability to understand the course content in their courses. The grading was according to the following scale:

0: The concept is not at all used in the course.
1: The concept is less important for the course (students can cope without it)
2: The concept is important for the course (student have difficulties to cope without it)
3: The concept is central for the course (students will not cope without it)

These estimates were later gathered in a matrix, with programme progession on the abscissa and progression of mathematical concepts on the ordinate. This gives a schematic description of the use of mathematics in different courses and allows for the calculation of a "math usage index" which can be compared between different programmes.

TABLE I
STATISTICS FROM THE SEARCH IN THE DIVA DATABASE.

Program	Students	Reports in digital form
CDATE	326	179
CELTE	95	51
TIDAA	58	30
TIDAB	61	29
TIEDB	26	14
TIELA	43	20

III. RESULTS

The methodology was applied to four 3-year bachelor degree programmes and to bachelor level theses from two 5-year master degree programmes. All programmes were in the fields of computer engineering (TIDAA, TIDAB and CDATE) or electrical engineering (TIEDB, TIELA and CELTE). Digital versions of thesis reports were found in the DiVA database [9] and Table I show the statistics from this search. In total, 609 students had passed the bachelor thesis courses during years 2015-2016. Among them, we found 593 students in DiVA and 581 students had a digital version of their report registered in DiVA. This means that 2,6% of the thesis work had not been properly registered (the registration of name and title is mandatory in DiVA) and 2% of the students had decided to not publish a full-text digital version of their thesis report in DiVA. However, the total number of reports (323) is significantly lower, since about 80% of the reports were written jointly by two students. It is worth mentioning that we tried different types of search criteria in DiVA to get all relevant reports from a study programme into one single list, but such attempts failed due to several reasons (one being incorrect registration of entries in DiVA). Instead, we started from a search list that was as close as possible to the desired one and downloaded all the reports found in that list. Missing reports were later found by searching for student name. Finally, we randomly chose and read 20 of these reports from each of the programmes.

Quantitative and qualitative data as described above was gathered for each report. However, in this study we look for quality measures relevant for an educational programme and not for a single thesis report. The most striking result came from analyzing the portion of thesis reports with displayed equations as seen in Fig. 1.

Fig. 1 shows a simple measure of mathematics usage in a thesis report – the existence of at least *one* displayed mathematical equation in the report. For all programmes, we found reports without any displayed equations at all, including both the 3-year programmes (TIEDB, TIELA, TIDAA and TIDAB) and the 5-year programmes (CDATE and CELTE). However, when making this comparison, we have to remark that the examination method of the CELTE programme differs from the one in the remaining programmes. In the CELTE programme, the bachelor thesis report is written in the form of a 10-page research article following the IEEE format, while the remaining programmes require a traditional thesis report. Furthermore, we did not have access to some of the appendices in the CELTE reports. This implies that we may have underestimated the number of thesis with at least one displayed equation for the CELTE programme when comparing with other programmes in Fig. 1. This will, however, not affect the main conclusions of our paper.

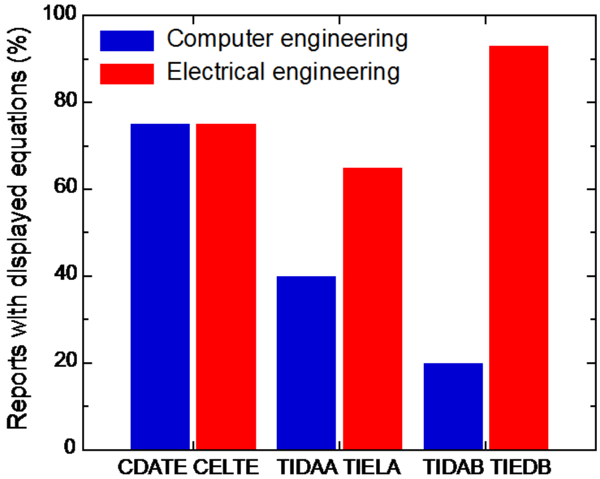

Fig. 1. The percentage of the bachelor thesis reports that had at least *one* displayed equation. The CELTE programme uses a different form of thesis presentation, which may cause a systematic underestimation of the number of theses with displayed equation (see text for details).

In Fig. 1, there is a clear difference between progammes within electrical engineering and those within computer engineering. The programmes in electrical engineering and the 5-year computer engineering programme CDATE have a higher degree of reports with at least one displayed equation, while the two 3-year bachelor programmes in computer engineering (TIDAA and TIDAB) have a significantly lower number of reports with displayed equations. Based on this finding, we also made a search in DiVA for bachelor thesis reports from 3-year bachelor programmes in computer engineering at other universities in Sweden during years 2015-16. Among the 74 reports found in DiVA, only 16% had at least one displayed equation, which is comparable to the results from the TIDAB programme.

Among the reports that we have studied, there was a wide distribution in the number of displayed equations (between 0 and 94), the number of figures (between 0 and 30) and the number of tables (between 0 and 52) with mathematical content. We didn't further investigate into some of these numbers, since we were not able to define sufficiently precise criteria for how to evaluate the math content in these cases. Let us give a few practical examples of some problems that we encountered. In one report, there was a figure that had more than 30 panels, spreading over 5 pages and with text in between the panels. These students had obviously selected to show all their results using one figure number, while other students had distributed similar information oven tens of figure numbers. There was no general practice to highlight computer code snippets that may contain mathematical content in the reports – sometimes they were shown as figures, sometimes as part of the running text and sometimes as program listings in an appendix. It was also difficult to draw a borderline if a figure or a table contained sufficient mathematical content to be counted in our analysis. Therefore, counting figures or tables did not provide sufficiently useful information.

Counting equations was a more useful measure. However, the number of equations did not scale with more mathematical content or mathematical sophistication in the reports. For example, in some theses, derivations from textbooks were written out in full detail. Hence, the simple criteria used above for having at least one displayed equation in the thesis report was found to be the most reliable one. However, to confirm the observed differences between the CDATE and the TIDAB programmes, we performed a hypothesis test using the Mann-Whitney double-side test with a null hypothesis of no difference between the two groups. When applying this test to either the number of equations in the reports or to our judgements, we ended up with $z > 3.07$, which means that the differences are robust within an error probability of $p < 0.003$.

Another useful way to analyze the mathematics content was to look at the level of sophistication of the mathematical concepts used in the theses, combined with a qualitative assessment of mathematical concepts that (in our opinion) should have been better described in the reports. From a programme level perspective, such an analysis qualitatively seems to agree with the trends in Fig. 1. While the CDATE students in many cases had selected highly mathematical subjects for their theses, the TIDAA and TIDAB bachelor theses were often lacking advanced mathematical content. As an example, although several theses from TIDAA and TIDAB use concepts such as graphs and trees from discrete mathematics, they appear primarily in figures and as background illustrations without any deeper discussions. In contrast, problems based on discrete mathematics form the central theme in a substantial part of the examined CDATE theses. Hence, it seems as the CDATE students have better consolidated their knowledge of discrete mathematics.

Similar trends were also found regarding the use of calculus and linear algebra in the reports. Concepts from those courses could be used to better explain algorithms in more detail, but that was actually seldom done. On the contrary, compression and machine learning algorithms were often treated as black boxes and used without explaining much about how they work internally. A feeling we got from reading the reports was also that some students had the attitude to not bother about what's inside an algorithm. Statistics was used in a large number of theses, but mostly on a very basic level actually taught in high school. Averages were often calculated, but less commonly standard deviations, variances, medians or max/min values, even when those measures would have been relevant for the theses. In a few exceptional cases, we found more advanced uses of statistics, such as graphs of probability distributions and Bayesian analysis. Finally, in 40% of the reports from the

5-year programmes, we found no reasons for remarking on missing mathematics, while only 13% of the reports from the 3-year programmes passed our analysis without remarks.

To get an idea of how time consuming our analyses were, we clocked the total time for evaluating each thesis. The precision in our time measurements was set to whole minutes, since it was not meaningful to determine the time more precisely than that. In Fig. 2, we show the statistical distribution of the evaluation times (we have excluded three cases in this analysis, when the clock didn't start properly or when we were interrupted by phone calls). The median time for making our analysis was 10 minutes, and the distribution of times follows quite well a Gaussian distribution. From our data, we obtained a mean value of 10.0 minutes and a standard deviation of 2.8 minutes.

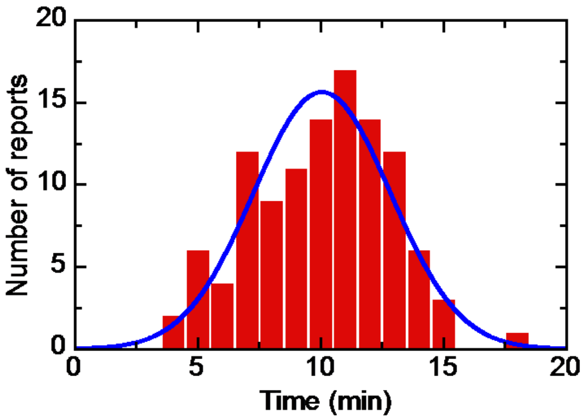

Fig. 2. Statistical distribution of the time it took to evaluate a thesis report. The median is 10 minutes. The blue curve shows a Gaussian distribution with the same mean value and standard deviation as the original data.

For the TIDAB and TIEDB programmes, we also asked the teachers in each of the mandatory courses about the importance of different mathematical concepts for student learning in their courses. These data were gathered in matrix form in an Excel sheet as shown for the TIDAB programme in Fig. 3. Starting from the upper left corner, the curriculum development of the programme is shown towards the right, where each of the mandatory courses during the 3 years (Y1-Y3) is shown for each of the four periods (P1-P4) during the academic year. In addition to these courses, students also take elective courses not shown here. The mathematics concepts (like e.g. complex numbers, matrices, minimal trees or normal distribution) are listed below each of the four mathematics courses in the programme (for a full list of the concepts, see Appendix A). The couloring has the following meaning:

Green (0): The concept is not at all used in the course.
Yellow (1): The concept is less important for the course
Orange (2): The concept is important for the course
Red (3): The concept is central for the course
Brown: The concept is learnt in this course
White: Data is gathered from other source than teacher

Math content	Y1	P1	P1	P2	P2	P3	P3	P4	P4	Y2	P1	P1	P2	P2	P3	P3	P4	Y3	P2	P2	P3		Sum
Calculus																							
Concept 1		2		3	3	2	1	0	0		0	3	3	1	1	3	0		0	3	0		25
Concept 2		0		3	3	2	0	0	0		0	3	3	0	0	3	0		0	3	0		20
Concept 3		0		2	2	0	0	0	0		0	1	0	0	0	0	0		0	3	0		8
Concept 4		0		0	2	1	0	0	0		0	2	0	0	0	0	1		0	3	0		9
Concept 5		0		0	2	0	0	0	0		0	0	0	0	0	0	0		0	0	0		2
Concept 6		0		0	0	0	0	0	0		0	3	0	0	0	0	0		0	3	0		6
Concept 7		0		0	0	0	0	0	0		0	2	0	0	0	0	0		0	3	0		5
Concept 8		0		0	0	0	0	0	0		0	0	0	0	0	0	0		0	1	0		1
Concept 9		0		0	0	0	0	0	0		0	0	0	0	0	0	0		0	0	0		0
Concept 10		0		0	0	0	0	0	0		0	0	0	0	0	0	1		0	3	0		4
Concept 11		0		0	0	0	0	0	0		0	0	0	0	0	0	0		0	1	0		1
Concept 12		0		0	0	0	0	0	0		0	2	0	0	0	0	0		0	0	0		2
Concept 13		0		0	0	3	0	0	0		0	3	0	0	0	0	0		0	2	0		8
Concept 14		0		0	0	0	0	0	0		0	1	0	0	0	0	0		0	3	0		4
Linear algebra																							
Concept 1					2		2	3			3	3	3	1	2	3	0		3	3	3		31
Concept 2					2		1	0			0	3	3	1	0	3	0		0	3	0		16
Concept 3								0			0	3	0	0	0	2	0		0	3	0		8
Concept 4								3			3	3	0	0	0	2	0		3	2	3		19
Concept 5								0			0	1	0	0	0	2	0		0	1	0		4
Concept 6								0			0	0	0	0	0	2	0		0	0	0		2
Concept 7								0			0	2	0	0	0	2	0		0	3	0		7
Concept 8								0			0	0	0	0	0	0	0		0	3	0		3
Concept 9								0			0	0	0	0	0	0	0		0	0	0		0
Discrete mathematics																							
Concept 1				1							0		3	0	0	3	0		0	2	0		9
Concept 2				1	2		1				0		3	0	0	3	0		0	2	0		12
Concept 3					2						0		0	0	0	0	0		0	2	0		4
Concept 4				1							0		3	0	0	3	0		0	2	0		9
Concept 5											0		1	0	0	0	0		0	0	0		1
Concept 6											0		1	0	0	0	0		0	0	0		1
Concept 7											0		1	0	0	0	0		0	0	0		1
Concept 8											0		0	0	0	0	0		0	0	0		0
Concept 9											0		3	0	1	3	0		0	0	0		7
Concept 10											0		3	0	1	3	0		0	0	0		7
Concept 11											0		3	0	1	3	0		0	0	0		7
Statistics																							
Concept 1																			0		0		0
Concept 2																			0		0		0
Concept 3																			0		0		0
Concept 4																			0		0		0
Concept 5																			0		0		0
Concept 6																			0		0		0
Concept 7																			0		0		0
Concept 8																			0		0		0
Sum		2	0	11	20	8	5	6	0		6	35	33	3	6	40	2		6	54	6		243

Fig. 3. The use of mathematical concepts in mandatory courses in the TIDAB programme. Starting from the upper left corner, curriculum development is shown to the right and currciculum development in mathematics is shown downwards in the way the courses appears in the programme. Numbers show how important teachers evaluate the concepts to be for student learning in their courses. Columns with brown marking are mathematics courses. See text for more details.

It is easy to identify mathematical concepts that is needed in earlier engineering courses in this matrix, since such concepts falls below the diagonal in the figure. This means that teachers in those courses have to spend some time to explain concepts that are later on explained in more detail in the mathematics courses. Hence, this matrix is useful for discussing programme development. However, in this paper we are going to use it in another way. The numbers in each column and row are summarized to give a course mathematics usage number (lower sum) and a concept usage number for all courses (right hand sum). Finally, those numbers are summed up to a total mathematics usage number (243 in the case of the TIDAB programme) that is a measure of the total importance of mathematics within the mandatory courses in the programme.

Although this number can be critized for being imprecise, it gives us a possibility to make comparisons between different study programmes (in our case between the TIDAB and the TIEDB programmes). An interesting factor in such an analysis is how much mathematics is actually used outside of the pure mathematics courses in the programmes. To get a reasonable estimate of that, we start by removing the mathematics concept connections between different mathematics courses and the mathematics concepts trained in earlier courses (below the diagonal). For the TIDAB programme in Fig. 3, we get the number 139 from the total mathematics usage number (243) minus the mathematics usage number coming from other courses in mathematics (54+35) minus the mathematics usage number from earlier courses (15). Finally, we normalize this value to the maximum possible mathematics usage number in non-mathematics courses. This gives a mathematics usage index of 139/381=0.36 for the TIDAB programme. This index falls always in the range between 0 (no mathematics concepts are used in any course) to 3 (all mathematics concepts are central to all courses). Doing exactly the same analysis for the TIEDB programme (not shown here) give a mathematics usage index of 0.78, i.e. more than twice as large as for the TIDAB programme.

IV. Discussion

We start our discussion with the low number of displayed equations in bachelor thesis reports written by students at 3-year programmes in computer engineering. Although our analyses are made within a KTH context, we observed the same low (or even slightly lower) numbers in bachelor theses from 3-year computer engineering programmes at other universities in Sweden. Hence, we have no reasons to believe that KTH is an exceptional case. When comparing bachelor thesis reports from programmes within electrical engineering, we only see modest differences between 5-year and 3-year programmes. The 5-year programme in computer engineering at KTH has a similar percentage of reports with displayed equations as programmes in electrical engineering, showing that it is fully possible for students to use mathematics in their reports even within the field of computer engineering. Hence, this measure should not be considered as research field dependent (which the average number of equations in thesis reports could be). Furthermore, students at the 3-year TIEDB programme within the field of electrical engineering and the 3-year TIDAB programme within the field of computer engineering have exactly the same courses in mathematics and take them jointly. Hence, there is no difference in the mathematics education between these two programmes. Despite this, students from the TIEDB programme show a clearly higher ability to use mathematics in their thesis reports than the students from the TIDAB programme do.

Another important aspect is that one of the hallmarks of being an engineer is to be able to use mathematics in a proper and relevant way. This is important for the development of our society and is also explicitly mentioned in the Swedish Higher Education Ordinance [1] by phrasing that an engineer should *be able to demonstrate relevant knowledge of mathematics* in order to gain a bachelor degree in engineering. The same phrasing is used as a learning outcome in the bachelor theses courses for the four 3-year programmes in our study (TIDAA, TIDAB, TIEDB and TIELA). Hence, we can demand students to demonstrate relevant knowledge of mathematics in their bachelor thesis reports – the problem is that this is not visible in the thesis reports from the TIDAA and TIDAB programmes. Our observations point towards a quality problem, since the complete lack of equations in many reports may indicate an attitude that mathematics is not a central engineering skill. For the TIDAB programme, we also found that the mathematics being taught, seems to be used (and thereby consolidated) only marginally in subsequent engineering courses, which could suggest that this attitude is also spread among teachers. If that is the case, it is not surprising if students get the impression that mathematics in unrelevant and have little practical use within the field.

In our view, it should be possible to put higher requirements on the thesis reports in computer engineering. It is not unrealistic to demand that computer engineering students should be able to read a description of an algorithm (relevant for their thesis work), understand the way the algorithm works and briefly describe it in their report. This would inform the reader and make it easier to understand the context. At the same time, students can show that they have the necessary engineering skill to be able to understand what is going on "under the hood". In addition, we think that a larger fraction of the thesis reports should have specified some sort of variance measure of statistical data.

One may also ask the question why the 5-year CDATE programme comes out better in this evaluation. One explanation is found in the programme syllabus, since there were more mandatory mathematics courses in the CDATE programme (42 credits) than in the TIDAB (30 credits) or the TIDAA (24 credits) programmes during the years relevant for this study [12]. However, this is probably not the only explanation, since the TIDAA programme (with less mathematics) comes out slightly better than the TIDAB programme in our analysis. One reason could also be the local culture, since the programmes in the field of computer engineering (CDATE, TIDAA and TIDAB) are taught at three different campuses at KTH.

Although it was possible to evaluate a thesis report in 10 minutes on average, it also took time to find, download and organize the downloaded theses. A reasonable estimate is that this on average took about 2 additional minutes per thesis after having learnt how to effectively search the DiVA database. This means that the analysis presented in this paper could have been made at a rate of 5 theses per hour. Although 12 minutes may appear to be a rather short time per report, it adds up to a workload that is hardly realistic for a typical university teacher. In our case (with 114 evaluated reports and 323 downloaded reports), it means almost 30 hours of total work. However, in reality it took more time, since we were simultaneously developing the methodology. Although we tried to develop a time efficient evaluation method to keep down the time for evaluation, the gathering of a sufficient amount of data is probably out of reach for a single teacher. There are three principally different solutions to this

i) Distribute the workload within a teacher team.
ii) Pay others (like e.g. students) to do the evaluation work.
iii) Develop a computer algorithm that automatically extracts the mathematics content from a pdf file.

All these solutions have their advantages and disadvantages. An advantage by creating a teacher team working together (e.g. through a workshop or as part of a pedagogic course) is that it increases the awareness of the problems among the teachers. It also gives them the opportunity to see both bad and good examples in the reports and inspire them to suggest creative ideas about how to improve the mathematics content in bachelor thesis reports. However, the total workload for the teachers as a group will not change. Paying others to do the evaluation work, frees the teacher's time but may result in less accurate evaluations. A computer algorithm that scans the reports for mathematics content has the advantage of being able to handle a large number of reports in a short period of time. It can also avoid some problems related to human judgement. These advantages can possibly compensate for the additional costs due to software development time. Hence, there needs to be a large demand for developing such a software solution in order to motivate the development costs.

The methodology developed and presented here, creates a possibility to calculate quality indicators relevant for a whole study programme. Such quality indicators are important since they give a solid background for programme development, and identifies issues that need to be improved. It should also be possible to develop similar quality indicators for other central concepts in an engineering education. Hence, with a computer algorithm for extracting relevant data from thesis reports, it would be possible to support both programme directors and external reviewers with proper data for their decisions.

V. Implications for policy

It is clear from our discussion that the way mathematics is used in these thesis reports needs to be improved. This implies a change in both student culture and teacher culture. For students, it is important that they start to self-reflect about how and why they are using mathematics as engineers. In particular, they should be able to connect their thesis work to relevant mathematical concepts that they have learnt in their mathematics courses. This could be done through a self-reflection part in their thesis report (in a similar manner as they are asked to self-reflect on the connection between their thesis work and sustainable development). One could also require them to map their thesis work to relevant university mathematics. Another way could be to show examples of good practice, where mathematics is properly used in thesis reports.

Another way to get students to focus more on mathematics is to better include it in other engineering courses. This requires, however, a change in teacher's attitude and how other courses are taught, which can be difficult to achieve. One may suggest curriculum discussions on programme level, workshops on how to include mathematics in courses and a database of good examples.

From a quality perspective, it would be useful to monitor the use of mathematics in thesis reports from different engineering educations on a national (or an international) level. This would clearly be possible using some of the methods developed here together with an algorithm that could automatically gather and display the relevant statistics from thesis reports. With such a tool, it would also be possible to make continuous evaluations on a large scale and follow trends. The indicators from such an analysis could be helpful for identifying potential quality problems already at an early stage.

VI. Conclusions

We have developed a methodology to investigate mathematics content in thesis reports and to investigate mathematics content within a study programme. From our analysis of 6 engineering programmes at KTH, we find that there is an obvious deficit in the use of mathematics in thesis report written by students from bachelor programmes in computer engineering. We also give som suggestions about how to improve the situation.

Appendix A

The mathematics concepts used in Fig. 3. are listed below in the same order as they occur in the figure.

Calculus
1. Elementary functions, polynomials
2. Elementary functions, logarithms, exponentials
3. Elementary functions, trigonometric
4. Derivatives of elementary functions
5. Complex numbers
6. Sketching functions, tangentials
7. Find extreme values
8. Taylor formula
9. Integral approximation, Riemann sums
10. Integrals of primitive functions
11. Solve linear differential equations
12. Calculate limiting values
13. Inverse of functions
14. Convergence or divergence of sums

Linear algebra
1. Basic logic
2. Basic theory of sets
3. Solve linear systems of equations
4. Handle matrices and determinants
5. Vector algebra in two and three dimensions
6. Coordinate systems and change of base
7. Scalar and vector product
8. Solve over- or under-determined systems
9. Eigenvalue problems

Discret mathematics
1. Calculate permutations and combinations
2. Use set notation
3. Use Venn diagram
4. Induction principle
5. Recognize groups, rings and bodies
6. Determine subgroups and ideals
7. Elemental order in a group
8. Use the Chinese remainder theorem
9. Determine minimal trees
10. Determine shortest path in a graph
11. Use graph theoretical models

Mathematical statistics

1. Use elementary stochaistic models
2. Use normal approximation
3. Statistical models for experiments
4. Describe data sets and measures
5. Present data graphically
6. Calculate point and interval estimates
7. Calculate error probability for hypothesis tests
8. Regression and correlation coefficients

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