Valuation Practices of IFRS 17

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Abstract

This research assesses the IFRS 17 Insurance Contracts standard from a mathematical and actuarial point of view. Specifically, a valuation model that complies with the standard is developed in order to investigate implications of the standard on financial statements of insurance companies. This includes a deep insight into the standard, construction a valuation model of a fictive traditional life insurance product and an investigation of the outcomes of the model.

The findings show firstly that an investment strategy favorable for valuing insurance contracts according to the standard may conflict with the Asset & Liability Management of the firm. Secondly, that a low risk adjustment increases the contractual service margin (CSM) and hence the possibility of smoothing profits over time. Thirdly, that the policy for releasing the CSM should take both risk-neutral and real assumptions into account.

Keywords: IFRS 17, Valuation, Life insurance, Insurance contracts, Contractual service margin (CSM), Risk adjustment, Traditional life, Unit-linked, Insurance state model, Economic scenario generator (ESG).
Sammanfattning

I denna rapport ansätts redovisningsstandarden *IFRS 17 Insurance Contracts* utifrån ett matematiskt och aktuariellt perspektiv. En värderingsmodell som överensstämmer med standarden konstrueras för att undersöka standardens implikationer på ett försäkringsbolags resultaträkning. Detta inkluderar en fördjupning i standarden, konstruktion och modellering av en fiktiv traditionell livförsäkringsprodukt samt undersökning av resultaten från modellen.


*Nyckelord:* IFRS 17, Värdering, Livförsäkring, Försäkringskontrakt, Avtalsmässig servicemarginal (CSM), Risk justering, Traditionell försäkring, Stadiemodel, Ekonomisk scenario generator (ESG).
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Jimmy & Björn
Limitations

The conclusions in this report are our own and not WTW’s or KTH’s. The model and calculations presented are only to illustrate the main new features of the new accounting standard. Hence, there are limitations and simplifications. The reader should be aware that this is a developing subject where a consensus has not yet been developed. Therefore market practice will emerge and evolve over time.
# Contents

1 Introduction .......................................................... 1
   1.1 Background ......................................................... 1
   1.2 Financial statement under IFRS 17 .............................. 1
       1.2.1 Valuation ..................................................... 2
       1.2.2 Recognition of profit and loss ............................ 2
   1.3 Objective and scope ............................................... 2
   1.4 Outline of the thesis ............................................ 3

2 IFRS 17 Insurance Contracts .......................................... 4
   2.1 Scope of the standard ............................................ 4
   2.2 Separation of investment component ............................ 5
   2.3 Level of aggregation ............................................. 6
   2.4 Measurement approaches ......................................... 7
   2.5 Fulfillment cash flows ........................................... 7
       2.5.1 Estimated future cash flows ............................... 7
       2.5.2 Discount rate .............................................. 8
       2.5.3 Risk adjustment ........................................... 10
   2.6 Contractual Service Margin ...................................... 11
   2.7 Market Variables ................................................ 12

3 Product and measurements .............................................. 14
   3.1 Pension product .................................................. 14
       3.1.1 Application under IFRS 17 ................................ 15
   3.2 Present value of future cash flows ............................. 16
   3.3 Discount Rate ...................................................... 18
       3.3.1 Estimating the liquidity premium ......................... 19
   3.4 Risk adjustment .................................................. 20
       3.4.1 Conceptual approach ....................................... 20
       3.4.2 Measurement .............................................. 21
   3.5 CSM and Profit & Loss .......................................... 22
       3.5.1 Measurement at initial recognition ....................... 22
       3.5.2 Subsequent measurement .................................. 22
List of Figures

2.1 Liquidity premium. ................................................. 9
3.1 Extrapolation of interest rates. ................................. 18
4.1 State model. ...................................................... 26
4.2 Aggregated decremental model. ............................... 27
4.3 Assumptions and results of the state model. ................ 29
4.4 Rates with the CIR-model. .................................. 35
4.5 Results from the ESG. ........................................... 37
5.1 Estimated future benefits and cash flows. .................. 45
5.2 State model. ...................................................... 45
5.3 Initial measurements. ........................................... 46
5.4 Allocation in a bank account: Estimated future benefits and cash flows. 47
5.5 Allocation in a bank account: Initial measurements. .......... 47
5.6 Onerous contracts: Estimated future benefits and cash flows. . 48
5.7 Onerous contracts: State model. .............................. 48
5.8 Onerous contracts: Initial measurements. .................... 48
5.9 Series of subsequent measurements. .......................... 49
5.10 Onerous contracts: Series of subsequent measurements ......... 50
5.11 Market conditions up to subsequent measurement. ......... 51
5.12 Base: Estimated future benefits and cash flows. .......... 51
5.13 Base: State model. .............................................. 52
5.14 Base: Subsequent measurement. .............................. 52
5.15 Onerous: Estimated future benefits and cash flows. ....... 53
5.16 Onerous: Subsequent measurement. .......................... 53
# List of Tables

2.1 Risk categorization. .................................................. 11

3.1 Premium on swap rates. ............................................. 19

4.1 Contract parameters ................................................. 25
4.2 Parameters of the financial model. ................................. 35
4.3 Fund management model. ............................................ 36
4.4 Parameters of the insurance product. ............................. 43
Chapter 1

Introduction

1.1 Background

In the second quarter of 2017, the International Accounting Standards Board (IASB) published a new International Financial Reporting Standard: IFRS 17 Insurance Contracts [27], here referred to as IFRS 17 or the standard\(^1\).

The standard presents principles for recognition, measure, presentation and disclosure of insurance contracts. The objective is to provide accurate information to stakeholders and investors [§IN1]. Furthermore, the standard will harmonize how risks are recognized and measured. Thus, the standard is believed to increase the comparability of insurance companies.

The standard will be mandatory from the year of 2021 and will apply to all insurance contracts including reinsurance contracts held and issued by entities [§3]. It is not yet fully clear how the standard will be implemented in Sweden, although it is expected to be applied to group reporting for listed entities.

1.2 Financial statement under IFRS 17

Important impacts of the standard are how to value insurance contracts and when to recognize profits and losses. With the principles of the standard, the financial statements of insurance companies will be largely affected.

\(^1\)Reference with the prescript § refers to paragraphs of the standard
1.2.1 Valuation

The standard proposes a hybrid of market consistent valuation and book value accounting. Specifically, insurance contracts are valued by its fulfillment cash flows which are derived from §32:

- The best estimate of future cash flows.
- An adjustment for time value of money and financial risks.
- An adjustment for non-financial risk.

The best estimate of future cash flows refers to an unbiased estimation of future cash flows using the probability weighted mean. The adjustment for time value of money and financial risks is carried out with a discount rate that should be consistent with the liquidity characteristics of the future cash flows. Thus, the return from bearing financial risks is captured by the discounted estimated future cash flows. Similarly, the return from bearing non-financial risks is captured in the non-financial risk adjustment.

1.2.2 Recognition of profit and loss

Under IFRS 17, profits are not recognized at the initial recognition of a contract. Instead, any surplus in fulfillment cash flows §38a is captured in an item of the balance sheet named contractual service margin (CSM). The CSM is subsequently adjusted for changes in fulfillment cash flows, changes of the fair value of the underlying item and changes in time value and financial risks. The CSM is also gradually released as the service is being fulfilled. Similarly, the risk adjustment is released as the contracts are released from non-financial risks. Profits are in turn recognized as the release of the CSM and the risk adjustment.

The fact that surplus is captured in the CSM gives the issuer a possibility of smoothing profits over time. Contrary, when there is a deficit in fulfillment cash flows a loss should be recognized instantly.

1.3 Objective and scope

With the recent release of the standard, IFRS 17 is unexplored to many within the insurance industry. Thus, the effects of the standard on the financial statements of insurance companies and strategies for reaching objectives such as profit smoothing are yet unknown. Furthermore, the standard is principle based and does hence not specify a practice. Therefore, determine a practice that complies with the standard and enables achievements of desired objectives is vital.
The aim of this research is therefore to provide an insight in the principles of IFRS 17 and possible practices. Furthermore, the implications of the standard will be analyzed and evaluated with respect to a traditional life and pension products. This requires a deeper insight into the standard, a construction of fictive insurance product and a determination of measurements techniques that compiles with the standard.

The research will be limited to techniques for valuing a traditional life product. Valuation techniques for non-life products and disclosures of a financial statement are hence left outside the scope.

1.4 Outline of the thesis

In Chapter 2, a deeper insight into the standard is provided. In Chapter 3, the product is introduced and a general model that complies with IFRS 17 is proposed. In Chapter 4, an insurance model is conducted. This includes; constructing an insurance state model, a financial model and a product model. In Chapter 5, the results from the model are presented along with discussions and further investigations. Lastly, in Chapter 6, the main conclusions from the research are summarized.
Chapter 2

IFRS 17 Insurance Contracts

This chapter covers the most central parts of the standard: Recognition of insurance contracts, separation of investment components, aggregation of insurance contracts, the measurement approaches, estimates of future cash flows and the new measures of IFRS 17; the risk adjustment and the contractual service margin (CSM). The purpose is to provide an insight into the standard necessary in order to value a traditional life insurance contract.

2.1 Scope of the standard

Standard insight 1: Scope of the standard.

The standard applies to insurance contracts, reinsurance contracts and investment contracts with discretionary participation features, provided that the entity also issues insurance contracts [§3].

Under IFRS 17, an insurance contracts is defined as a contract under which one party (the issuer) accepts significant insurance risk from another party (the policy holder) by agreeing to compensate the policy holder if a specified uncertain future event adversely affects the policyholder [§B17].

Insurance risk is in turn defined as risks other than financial risks [§B7]. Furthermore, if the event associated with insurance risk mean that significant additional amounts would be payable by the issuer; then the insurance risk is also significant [§B18].

IFRS 17 will apply to issuers and holders of insurance contracts, i.e. contracts where a significant insurance risk is transferred. Contracts sold by insurance companies are
commonly divided into non-life insurance contracts and life insurance contracts. Non-life insurance contracts are usually associated with a casualty such as medical expense or property damage, thus associated with insurance risks.

Contrary, life insurance contracts usually promise a death or longevity benefit to the policyholder. However, a life insurance contract may also include or be limited to an investment contract. Thus, a life insurance contract may transfer insurance risks and/or financial risks to the issuer. E.g. a contract with a guaranteed minimum rate of return transfers financial risks to the issuer since the issuer is obligated to pay the amount of which the guarantee may exceed the policyholder’s account balance. If the contract furthermore promises a death benefit that may exceed the account balance, then there is also an insurance risk in form of mortality risk associated with the contract [§B9].

2.2 Separation of investment component

Standard insight 2: Separating investment components from an insurance contract.

The entity shall separate an investment component and apply IFRS 9 for the separated investment component if and only if the investment component is distinct, thus;

- if the investment component and the insurance component are not highly interrelated; and
- if there is a contract with equivalent terms which is sold, or could be sold, separately on the same market [§11, §B31].

Any cash flows from a separated component of the host insurance contract shall also be separated, thereafter IFRS 17 shall apply to all the remaining components of the host insurance contract [§12, §13].

As stated in Section 2.1, insurance contracts may consist of an investment component and an insurance component. The standard sets out principles on how to separate components of an insurance contract that if removed, is seen as independent and covered by other accounting standards.

Depending on the core business of the applying company, practices of when to separate a component will have significant impacts. The desired outcome of the practice is likely to vary within different insurance companies. Larger corporations, e.g. banks with business in the insurance sector, might prefer to account for the investment component under IFRS 9 Financial Instruments, since they might already have a business plan for how to handle and account for investment contracts.
2.3 Level of aggregation

Standard insight 3: Level of aggregation of insurance contracts.

The standard requires that portfolios comprising contracts subject to similar risks and managed together are identified [§14].

A portfolio of contracts should in turn be divided into a minimum of:
- a group of contracts that at initial recognition are onerous;
- a group of contracts that at initial recognition have no significant possibility of becoming onerous subsequently; and
- a group of the remaining contracts [§16].

It’s not permitted to include contracts issued more than one year apart in the same group [§22].

The standard requires that issuers and holder of insurance contract assess the contracts and divide them into portfolios and groups. A portfolio should comprise of contracts exposed to similar risks, i.e. comprise of contracts that may be affected similarly if the event of risk occurs. This entails that contracts within the same product line are expected to be within the same portfolio.

Furthermore, a portfolio of contracts should be divided into a minimum of three groups; contracts that are onerous at initial recognition, contracts that would become onerous if a likelihood changes in assumptions would occur subsequently and other contracts. This will force entities into assess the level of onerousness of a contract at initial recognition. It’s permitted to assess the level of onerousness for a set of contracts if the entity can conclude that the contracts in the set will be within the same group [§17]. Thus, a set of contracts can be assessed from one model point.

Since a key feature of the standard is that surpluses are captured in the CSM as future profit, whereas losses are recognized instantly; this separation is vital to accurately release profits and losses. It’s permitted to further subdivide the contracts with respect to profitability or to which extent different contracts are to be onerous under different scenarios [§21]. However, it’s not permitted to reassess the composition of a group subsequently [§24]. Hence, an accurate assessment at initial recognition is important.

Lastly, contracts issued more than one year apart should not be included in the same group. Thus, contracts must to a minimum be separated with regards to:
- Type of risk exposure.
- Level of onerousness.
- Year of issue.
2.4 Measurement approaches

Under IFRS 17, there are three measurement approaches:

– The *premium allocation approach* (PAA).
– The *building block approach* (BBA).
– The *variable fee approach* (VFA).

The PAA should only be used in order to measure contracts with a boundary period of one year or less. Thus, the PAA is likely to apply to insurance contracts related to casualty and property. The BBA should be used in order to measure contracts with a boundary of one year or more. Lastly, the VFA is exclusively used to measure contracts that have direct participation features (see Standard insight 4). Thus, the traditional life product, which will later be considered, should be measured with the VFA.

**Standard insight 4: Insurance contracts with direct participating features.**

Insurance contracts with direct participation features are contracts under which the entity’s obligation to the policyholder is the net of:

(a) the obligation to pay the policyholder an amount equal to the fair value of the underlying items; and

(b) a variable fee that the entity will deduct from (a) in exchange for the future service provided by the insurance contract [§B101].

2.5 Fulfillment cash flows

The fulfillment cash flows of insurance contracts comprise of estimates of future cash flows, a discount rate which adjusts for time value of money and financial risks and a risk adjustment which adjusts for non-financial risks [§32-37].

2.5.1 Estimated future cash flows

According to the standard, future cash flows should be determined from a hybrid of markets observations and the unbiased probability weighted mean of possible outcomes. Specifically:

– To the extent that market observations are available and relevant, the estimate should be consistent with the observations.

– Where market observations are not available and relevant, realistic assumptions that reflect the perspective of the entity should be used.

The estimates of future cash flows should [§33]:

– In an unbiased way incorporate all reasonable and supportable information.
– Reflect the perspective of the entity, provided that any market variable is consistent with market prices.
– Reflect conditions existing at the measurement date.
– Be estimated separate from the adjustment for time value of money and financial risk and the adjustment for non-financial risks.

The fact that market observations should be used when available and relevant implies that the market risk-premium will be included in the estimates of future cash flows. Contrary, where realistic assumptions are used, no risk-premium is included. Since the purpose of the discount rate and the risk adjustment is to capture time value of money, financial and non-financial risks; it’s important that these do not capture any risk-premium included in the estimates of future cash flows.

The estimates of future cash flows should be current. Thus, the measure is forward looking only. However, for non-onerous contracts, changes in the estimates are not immediately recognized as profit or loss. Instead, changes are passed to the CSM (see Section 2.6).

2.5.2 Discount rate

Standard insight 6: Discount rate.

Estimates of future cash flows should be adjusted with a discount rate that reflects time value of money and financial risks not included in the estimates of future cash flows. The discount rate should be determined such that [§36]:

– it reflects the time value of money, the characteristics of the cash flows and the liquidity characteristics of the insurance contracts;
– it’s consistent with observable current market prices; and
– exclude the effect of factors influencing observable market prices but not future cash flows.

According to the standard, the discount rate should be estimated from a maximized use of observable inputs and reflect current market conditions from the perspective of a market participant [§B78]. Furthermore, the discount rate should be adjusted with a liquidity premium for the dissimilarities between the cash flows being measured and the observed market prices [§B83]. Specifically, factors such as premium for credit risk and liquidity on observed market prices should not be reflected in the discount rate, whereas the liquidity characteristics of the insurance contracts and the cash flows should.
**Liquidity premium**

The adjustment for liquidity premium is based on the assumption that insurance companies use bonds to match their long term obligations and are therefore likely hold bonds to maturity. Thus, the exposure to liquidity risk of bond holders that would sell their bonds on an illiquid market before maturity should not be reflected in the discount rate \([§B79]\). This implies that an entity would be able to obtain an extra risk-free return on the risk-free yield if it holds bonds to maturity.

The adjustment can be carried out with one of the following approaches:

- The bottom-up approach where a liquidity premium is added to the risk-free yield curve \([§B80]\).
- The top-down approach where the fair value of a reference portfolio, e.g. a portfolio of corporate bond, with similar characteristics as the cash flows of the insurance contracts is used to reflect the market rates of return \([§B81]\).

**Figure 2.1: Liquidity premium.**

The bottom-up approach uses the risk-free yield to which a liquidity premium is added. The underlying assumption is that the risk-free yield is liquid whereas insurance contracts are not \([28]\). The approach is only allowed for cash flows that do not vary based on the return on underlying items.

The top-down approach uses the yield of a reference portfolio reduced for credit default risk. Specifically, it captures the fact that the spread between government lending rates and corporate lending rates generally are higher than the credit default cost on corporate bonds. However, the difference between the spread and expected credit default cost consist not only of a liquidity premium but also of a premium on credit default cost \([28]\). Therefore, determine an accurate liquidity premium has difficulties.

The use of liquidity premiums within the insurance industry has raised criticisms since it’s argued that it would contradict with market consistency \([28]\). Furthermore, since the concept of business of insurance companies is built on uncertainty, an entity cannot be
certain that it can hold bonds to maturity [37]. However, since the discount rate should only adjust for financial risks, an interpretation is that liquidity arisen from non-financial risks should not be reflected in discount rate.

**Discounting different types of cash flows**

According to the standard, the discount rate should adjust cash flows for time value of money and financial risks. This is, however, dependent of the type of cash flow. E.g. the time value of money and financial risk of an option is different from the time value of money and financial risk of a fixed cash flow. Cash flows that do not vary based on returns on underlying items should be discounted with a single discount rate with previously prescribed adjustment [§B84].

However, cash flows that vary based on returns on underlying items should either;

- be discounted with a rate that adjusts for the variability [§B75]; or
- be adjusted for the variability and discounted with the same rate as other cash flows [§B77].

The last option requires risk-neutral or stochastic modeling techniques. One of the key features of a risk-neutral technique is that market consistent valuation of varying cash flows and fixed cash flows are obtained with a single discount rate. It’s believed that this option will likely be preferred for valuation of products with direct participating features, since discounting with a single discount rate simplifies the model and only requires that the assumptions of drift of market variables are changed.

### 2.5.3 Risk adjustment

#### Standard insight 7: Risk adjustment.

The risk adjustment measures the compensation an entity would require to become indifferent between [§§B87];

- fulfilling a liability that has a range of possible outcomes arising from non-financial risks; and
- fulfilling a liability that will generate fixed cash flow.

The risk adjustment is a forward looking component which represents the compensation for bearing the uncertainty about the amount and timing of the cash flows arising from non-financial risks [§37], where non-financial risks comprise of e.g. insurance risks, lapse risk, persistency risk and expense risk [§B86]. Consequently, the risk adjustment captures the expected future profit from bearing non-financial risks.
Foroughi et al. [18] proposes three possible techniques for the determination of the risk adjustment;  
- confidence level;  
- conditional tail expectation; and  
- cost of capital.

Regardless of the techniques chosen; it is required that the confidence level to which the risk adjustment corresponds is disclosed [§B92]. Thus, determine a risk adjustment from a confidence level is likely to be tempting. However, due to the high skewness in the distributions of non-financial risks [8], using a confidence level approach might be insufficient.

Although the risk adjustment might seem similar to the risk margin under the directive Solvency II there are major differences. Firstly, the risk margin is determined from a prescribed technique whereas the risk adjustment is determined from proscribed principles. Secondly, the risk margin is to ensure that the value of technical provisions is equal to a current exit value. Thus, the approach of the risk margin is that another entity should be willing to hold the insurance contracts. Contrary, the risk adjustment focuses solely on the entity holding them. Thirdly, the risk margin applies to non-hedgeable risks whereas the risk adjustment applies to non-financial risks. As shown in Table 2.1, non-hedgeable and non-financial risks does not necessary coincide.

Table 2.1: Risk categorization.

<table>
<thead>
<tr>
<th>Financial risks</th>
<th>Non-financial risks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedgeable</td>
<td>Non-hedgeable</td>
</tr>
<tr>
<td>e.g. equity</td>
<td>e.g. interest rates with long maturities</td>
</tr>
<tr>
<td></td>
<td>e.g. traded reinsurance contracts</td>
</tr>
</tbody>
</table>

2.6 Contractual Service Margin

A main difference between IFRS 17 and other standards, e.g. Solvency II and IFRS 4, is when profits and losses are recognized. The standard proposes that profits should be recognized as the service is delivered and not at the initial recognition. This gave birth to a new measure called contractual service margin (CSM). The CSM is a liability component that stores future profits from insurance contracts. At initial recognition, the CSM is thus equal to the net fulfillment cash flows. During the boundary period of the contract, the CSM is subsequently adjusted and gradually released. The released amount is recognized as insurance revenue and thus contributes to profit.
The subsequent adjustment of the CSM depends on the measurement approach (see Section 2.4). With the PAA and the BBA, the CSM is adjusted for interest on the carrying amount [§44b]. Contrary, with the VFA, the CSM is instead adjusted for the entity’s share of change in fair value of the underlying item [§45b] (see Standard insight 8). This is natural since change in fair value captures change in time value.

**Standard insight 8: Adjustment of the contractual service margin with the variable fee approach.**

The contractual service margin is at subsequent measurement adjusted for:
- New contracts added to the group.
- The entity’s share of the change in fair value of the underlying item.
- Changes in fulfillment cash flows relating to future services.
- Currency exchange differences arising on the contractual service margin.
- The amount recognized as insurance revenue.

The CSM does only capture future profits and can thus not be negative. Hence a loss will be recognized instantly if the net of fulfillment cash flows is negative at initial recognition or if the subsequent adjustment amount of the CSM exceeds the carrying amount of the CSM [§48]. Any recognized loss is stored in a loss component [§49]. If there is a loss component, it needs to be reverted before a new CSM can be established. The reversion of the loss component is not recognized as insurance revenue but as reversal of losses. However, the reversion contributes to profit.

Two things are worth to be noticed. Firstly, unlike the loss component, the CSM buffers changes in assumptions. Thus, the CSM has a smoothing effect on profits. Secondly, the carrying amount of the CSM is affected by risk adjustment. Thus, a high risk adjustment will increase the likeliness that a contract becomes onerous.

### 2.7 Market Variables

The standard requires that estimations of future cash flows are consistent with observable market prices [§33b]. An entity shall maximize the use of observable inputs and not use their own estimate of market data as substitute; except, as described in paragraph 79 of *IFRS 13 Fair Value Measurement* [§B44]. Thus, when variables need to be derived, it shall be consistent with IFRS 13 *Fair Value Measurement*.

The standard states that the market prices reflect a blended range of possible future outcomes thus reflect the market price of risk. Hence, if the actual outcome differs from the previous market price, it does not mean that the market price was ‘wrong’ [§B45].

The fair value is, according to IFRS 13, the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at
the measurement date. When measuring the fair value of an asset or a liability, the entity
shall use the same assumptions as other market participants would use if they are acting
in their own economic best interest.

If there exists a replicating portfolio, i.e. a portfolio which exactly matches the replicated
cash flows in all scenarios with respect to amount, timing and uncertainty; the entity
can choose to use the fair value of the replicating portfolio to measure the present value
of future cash flows, instead of explicitly estimating the cash flows and the discount rate
[§B46].

The standard does not require the use of a replicating portfolio. However, if there exists
a replicating portfolio and the entity uses another technique; the entity must ensure itself
that the technique used is unlikely to produce a result that significantly differs from the
outcome of the replicating portfolio [§B47]. Furthermore, the standard recognize that
other portfolio techniques such as stochastic modeling techniques can be more robust
when there are strong correlations between cash flows that vary with returns on assets
and other cash flows [§B48].
Chapter 3

Product and measurements

In the following chapter, one pension product is firstly selected for further research. Secondly, a measurement approach of the product that is believed to comply with IFRS 17 is selected. This includes measurement of present value of future cash flows, discount rate, risk adjustment and CSM.

3.1 Pension product

The two most common life insurance products in Sweden are the unit-linked insurance\(^1\) and traditional life insurance\(^2\). These are retirement products where a policyholder deposits premiums during a specific period in order to accumulate a capital that will later paid out to the policyholder, e.g. during the policyholder’s retirement. The policyholder’s account balance does hence consist of deposited premiums. Furthermore, the policyholder’s account is linked to a set of underlying assets, e.g. a fund. However, the policyholder’s benefit amount is not necessarily directly linked to the account balance.

If the policyholder suffers from death before the contract matures, the policyholder’s benefit amount will usually be paid out to the policyholder’s inheritors. Similarly, the policyholder may have the possibility to surrender the contract, e.g. by transfer its benefit amount to another insurance company. Furthermore, the policyholder may choose to cease with its premium payments earlier than scheduled.

For the unit-linked insurance, the policyholder’s benefit amount is equal to the policyholder’s account balance. Hence, the payouts to the policyholder do purely depend on the return of the underlying assets. Thus, even if the value of the assets develops with negative return, the issuer has no direct obligations to the policyholder.

\(^1\)Fondförsäkring

\(^2\)Traditionell försäkring
The traditional life insurance is usually also linked to underlying items, but does also promise a minimum guaranteed amount to the policyholder.

Thus, the policyholder’s benefit amount is floored. However, if the return of the underlying assets is high a bonus will add to the benefit amount. This entails a non-linear relation between the benefit amount and the account balance which has similarities with financial option contracts. Furthermore, since the issuer is obligated to cover the guarantee if the account balance does not cover the guarantee, the non-linearity is passed on to the issuer’s financial risk.

### 3.1.1 Application under IFRS 17

**Scope of the standard**

From the discussion regarding the scope of the standard carried out in Section 2.1, IFRS 17 does only apply to contracts that transfer a significant insurance risk. Thus, IFRS 17 will not apply to contracts where the issuer is not in any case obligated to pay an amount that exceeds the policyholder’s account balance. Furthermore, for contracts where the issuer is obligated to pay an amount that may exceed the policyholder’s account balance, IFRS 17 will only apply if the payment is triggered by an insured event, e.g. death.

For unit-linked contracts, the issuer is usually not obligated to pay an amount that exceeds the policyholder’s account balance. However, unit-linked contracts commonly have an extra death benefit attached. Hence, the scope of the standard with regards to unit-linked products is still not clear.

Due to the guarantee of traditional life products, the issuer is obligated to pay an amount that may exceed the policyholder’s account balance. Furthermore, the payment can usually be triggered by insured events e.g. death. The cover for the guarantee may also be significant, which thus makes the insurance risk significant. Hence, it’s believed that a traditional life insurance product will be within the scope of IFRS 17. From this conclusion, the traditional life insurance product is selected for this research.

**Separation of an investment component**

The traditional life insurance does have similarities with investment products. The standard requires that investment components should be separated from an insurance contract, unless the investment components are interrelated with the remaining components of the contract (see Section 2.2). Thus, a possible separation would be to separate cash flows of regular pension payouts from cash flows related to occurrence of insured events. However, this is problematic since regular pensions payouts depend on whether an insured event occurs. E.g. if a policyholder suffers from death, the policyholder will not be subject to regular pension payouts. Thus, it’s concluded that the investment component
is interrelated with the insurance component. Hence, the interpretation of the standard is that an investment component cannot be separated from a traditional life insurance and a conventional unit-linked insurance, provided that the products fall within the scope of IFRS 17.

3.2 Present value of future cash flows

Cash flows from a group of contracts of the traditional life product described in previous section derive from financial and non-financial risks. Thus, these risks are needed to take into consideration when estimating the present value of future cash flows.

In a general approach, let \( t_0, t_1, \ldots, t_m \) be time steps within the boundary of the contracts, where \( t_0 \) is the time of issue and \( t_m \) is the time of maturity. Furthermore, let the vector \( \mathbf{X}_i \) represent the financial states at time \( t_i \), e.g. interest rates and prices of traded assets, and the vector \( \mathbf{Y}_i \) represent the non-financial states at time \( t_i \), e.g. number of premium paying customers.

Let furthermore all randomness of the contracts be captured by the vector set

\[
\mathbf{Z}_{0:m} = \{ \mathbf{Z}_0, \mathbf{Z}_1, \ldots, \mathbf{Z}_m \},
\]

where

\[
\mathbf{Z}_i = \left( \mathbf{X}_i^\top, \mathbf{Y}_i^\top \right)^\top, \quad i = \{0, 1, \ldots, m\}.
\]

For every point in time, outcomes of the contracts, such as cash flows and policyholders’ accounting balance, can be derived from the present and all previous outcomes.

If the randomness of financial and non-financial outcomes are measurable on the physical probability measures \( P \) and \( P' \) respectively, then the randomness of \( \mathbf{Z}_{0:m} \) can be evaluated under the physical probability measure

\[
P = (P \times P').
\]

When \( \mathbf{Z}_{0:m} \) is evaluated under \( P \), real world assumptions are used for both financial and non-financial states. This is common within Asset & Liability Management (ALM), e.g. to obtain an investment strategy that minimizes the risk that the benefit amount exceeds the account balance.

However, if risk-neutral techniques are used on financial risks, IFRS 17 allows all cash flows to be discounted with one single discount rate (see Section 2.5.2). Therefore, \( \mathbf{Z}_{0:m} \) will instead be evaluated under the probability measure

\[
Q = (Q \times P'),
\]

where \( Q \) is the risk-neutral probability measure under which the discounted values of traded assets are martingales [24]. The interpretation of using the probability measure \( Q \) is that financial risks are considered with assumptions of the risk-neutral world whereas non-financial risks are considered with assumptions of the real world.
With the risk-neutral technique, the discounted estimates of future cash flows $PV_i$ at
time $t_i$, is the estimation of:

$$PV_i = \sum_{j=i+1}^{m} E^Q \left[ b(i, j) CF_j \mid F_i \right],$$  \hspace{1cm} (3.4)

where

$$b(i, j) = e^{-\sum_{k=i}^{j-1} r_k (t_{k+1} - t_k)}$$

is the discount factor with $r_k$ as the discount rate between time $t_k$ and $t_{k+1}$ and $CF_j$ is the net cash flow at time $t_j$.

If the discount rate is assumed to be stochastic, the discount factor in (3.4) cannot be
moved outside the expectation. Thus, to disclose the estimates of future cash flows and
the discount rate separate, as required under IFRS 17 (see Section 2.5.1), (3.4) needs to
be adjusted to:

$$PV_i = \sum_{j=i+1}^{m} P(t_i, t_j) E^Q \left[ \frac{b(i, j)}{P(t_i, t_j)} CF_j \mid F_i \right],$$  \hspace{1cm} (3.5)

where

$$P(t_i, t_j) = e^{-\sum_{k=i}^{j-1} f_{i,k} (t_{k+1} - t_k)}$$

is the price at time $t_i$ of a zero-coupon bond that matures at time $t_j$ and $f_{i,k}$ is the
forward rate between time $t_k$ and $t_{k+1}$ at time $t_i$.

This is, in fact, a change in numeraire of the martingale measure such that for cash flows
at time $t_j$

$$E^Q \left[ \frac{b(i, j)}{P(t_i, t_j)} CF_j \mid F_i \right] = E^{Q_{t_j}} \left[ CF_j \mid F_i \right],$$  \hspace{1cm} (3.6)

where $Q_{t_j}$ is the corresponding $t_j$-forward measure of $Q$ and

$$\frac{b(i, j)}{P(t_i, t_j)} = \frac{d Q_{t_j}}{d Q}$$  \hspace{1cm} (3.7)

is the Radon–Nikodym derivative [4].

Thus, the disclosed discount rates are the zero rates of the bonds and the disclosed
estimates of future cash flows are the estimates of the expected cash flows under $Q$
adjusted with the Radon–Nikodym derivative.
3.3 Discount Rate

The standard requires a maximal use of relevant observable market data when estimating the discount rate. However, estimating forward rates for long durations is problematic since markets for bonds and interest rate swaps with long durations are not deep and liquid. Since lack of deep and liquid markets entails subjectivity \[4\] and since the standard does not provide any further guidelines except for consistency with IFRS 13, the estimation of interest rates for long durations in accordance with the standards will likely be a subject for future research.

Under Solvency II, the forward structure is extrapolated towards an ultimate forward rate (UFR), which for 2017 is 4.2% in Sweden \[6\]. Such extrapolation would underestimate the present value of future liabilities since market rates currently are lower than the UFR. Furthermore, available market data would be ignored. Thus, it’s doubtful that the term structure of Solvency II would compile with the standard.

With present data from Bloomberg on Swedish interest rate swaps and the Smith-Wilson smoothing method \[6, 13\], the forward rate structure shows a sharp rise in nearest time and a decline in thirty years. A natural choice could be extrapolated the forward structure from the last observed forward rate of 2.15%. However, it might seem unlikely that the market actually forecasts declining rates. The decline could rather be derived from illiquidity on the thirty years interest rate swaps. Thus, another option could be to extrapolate the forward structure from its peak of 2.73%.

Interest rates smoothed with the Smith Wilson-method

\[(a)\] Zero rates

\[(b)\] Forward rates

Figure 3.1: Extrapolation of interest rates.
3.3.1 Estimating the liquidity premium

The standard requires that the observed market rates should be adjusted for liquidity differences between the market of the observed rates and the insurance contracts. As previously stated, the allowed methods for adjustment are the bottom-up approach, where a liquidity premium is added to the estimated risk-free rate, and the top-down approach, where premiums for credit default costs are subtracted from the observed yield of an asset portfolio. Methods for carrying out the top-down approach include the use of observed prices on credit default swaps, covered bonds and options [5].

The method used in this research is a simple proxy method where a top-down approach is used to estimate the spread of corporate bonds adjusted for default risk. Thereafter, the adjusted spread is added to the risk-free rate. The method is carried out as follows:

– Firstly, the average historic spreads between the risk-free rate and the yield on corporate bonds with rating Aaa and Baa is conducted [19].

– Secondly, average historic default rate of similar corporate bond is conducted [7]. Assuming a recovery rate of 30%, the credit default cost is assumed to be 70% of the default rate.

– Thirdly, the risk premium for credit default costs is assumed to be 50% of the spread subtracted for default costs [5].

– Lastly, in order to estimate the risk-free rate. The rates obtained from swaps are adjusted downwards with 0.35% for counter-party risk [17].

The result is presented in Table 3.1. It should be noted that this method is indeed proxy since it’s assumed that the liquidity premium is constant for every maturity and since the bonds used for estimating the spread does not perfectly correspond with the bonds used for estimating the credit default cost.

<table>
<thead>
<tr>
<th>Table 3.1: Premium on swap rates.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>Average spread</td>
</tr>
<tr>
<td>Average default cost</td>
</tr>
<tr>
<td><strong>Spread after default</strong></td>
</tr>
<tr>
<td>Liquidity premium (50%)</td>
</tr>
<tr>
<td>Counter-party risk on swaps</td>
</tr>
<tr>
<td><strong>Premium on swap rates</strong></td>
</tr>
</tbody>
</table>

19
3.4 Risk adjustment

According to the standard, the risk adjustment should be the amount which would make
the entity indifferent between having a fixed cash flow and a cash flow that varies depend-
ing on non-financial risks. The adjustment could be seen as similar to the adjustment
for financial risks, which was carried out with the risk-neutral measure. However, mea-
suring non-financial risks under a risk-neutral measure has difficulties since markets for
non-financial risks are less effective. The standard is therefore flexible with respect to
measurement techniques. However, the technique should be such that [§B91]:

- Risk with low frequency and high severity will result in higher risk adjustment than
  risks with high frequency and low severity.
- For similar risk, contracts with a longer duration will result in higher risk adjust-
  ment than contracts with a shorter duration.
- Risks with a wider probability distribution will result in higher risk adjustment
  than with narrower probability distribution.
- As experience reduces uncertainty about the amount and timing of the cash flow,
  the risk adjustment will decrease and vice versa.

3.4.1 Conceptual approach

For the traditional life insurance product considered, non-financial risks are to some
extent risk with respect to timing. E.g. if a policyholder surrenders, the policyholder’s
benefit amount will be paid out instantly instead of during the policyholder’s retirement.
Recall that the present value of future cash flows captures financial risks only, thus is
estimated as if non-financial outcomes were known. This is indeed captured by the
liquidity premium, where insurance contracts are assumed to be illiquid. Therefore,
a suitable measurement technique for the risk adjustment would be a technique that
captures potential costs for the uncertainty with respect to amount and timing.

The proposed approach is therefore based on the entity’s cost for any deviations between
the expected cash flows and the actual cash flows. Since the entity is not financially
affected when amounts are withdrawn from the policyholder’s account balance, it’s suit-
able to consider cash flows that have a financial impact on the entity. E.g. charges and
guarantee covers rather than the total cash flows of the contract. Furthermore, only
future costs should be considered since previous outcomes have already been recognized
as profit or loss.

From time $t_i$, the cost for deviation between the actual and expected cash flows can be
assumed to accumulate as follows:

$$\begin{align*}
\rho_{j+1} &= (\rho_j + E^Q[c_{f_j} | F_{t_i}] - c_{f_j}) e^{\gamma_j (t_{j+1} - t_j)} \\
\rho_i &= 0
\end{align*}$$

(3.8)
where $\rho_j$ is the accumulated cost at time $t_j$, $cf_j$ is the net cash flow to the entity at time $t_j$ and $\gamma_j$ is a forward rate associated with the risk in the time interval $[t_j, t_{j+1})$.

The rate associated with the risk should in some sense capture the entity’s financing cost, e.g. the entity’s weighted average cost of capital (WACC). However, after the time of maturity $t_m$ there is no risk. Thus, after maturity $\gamma_j$ equals the risk-free rate. Hence, at time $t_i$ the present value of the total cost is:

$$P(t_i, t_m)\rho_m,$$

(3.9)

where $P(t_i, t_m)$ is the price at time $t_i$ of a zero-coupon bond that matures at time $t_m$.

The risk adjustment $RA_i$ at time $t_i$ can thus be measured by an estimation of (3.9), i.e:

$$RA_i = P(t_i, t_m)\hat{\rho}_m,$$  

(3.10)

where $\hat{\rho}_m$ is an estimation of $\rho_m$.

3.4.2 Measurement

One possible way of estimating $\rho_m$ is to produce one or multiple scenarios. However, since the risk adjustment must be positive $\rho_m$ must also be positive. This could be obtained by ensuring that the scenarios produced are unfavorable or by assuming that an entity’s rate of return on a surplus is different from the cost of capital for covering a deficit.

The measurement conducted in this research is produced from one unfavorable ‘stressed’ scenario on which the cost in (3.9) is estimated. As for the expected cash flow, the stressed scenario should be based on what is known at the time of measurement. Thus, the estimated cost $\hat{\rho}_m$ is accumulated from

$$\begin{cases} \rho_{j+1} = (\rho_j + E^Q[cf_j|F_i] - \hat{cf}_j^i) e^{\gamma_j(t_{j+1} - t_j)} \\
\rho_i = 0 \end{cases},$$

where $\hat{cf}_1^i, \hat{cf}_2^i, \ldots, \hat{cf}_m^i$ are cash flows of a stressed scenario based on the known outcomes up to time $t_i$.

This model has benefits since it captures the relation between the risk-free rate and the cost of capital. Furthermore, the model captures the timing uncertainty, i.e. even if variations are only limited to timing there will still be a positive risk adjustment. Additionally, contracts with long duration will likely be measured with a higher risk adjustment than contracts with short duration since longer duration implies a higher deviation with respect to timing. A drawback is, however, that that there is no guarantee that the estimator $\hat{\rho}_m$ is unbiased. Furthermore, a more accurate estimation of future costs for deviations may be obtained by using real world assumptions.
3.5 CSM and Profit & Loss

In this section, the measurement of the contractual service margin is stated with regards to the traditional life insurance product. As previously stated in Section 2.4, insurance contracts with direct participating features should be measured with the variable fee approach (VFA). In this simplified model, the measures of interest are therefore the present value of future cash flows, the risk adjustment and the entity’s share of change in fair value of the underlying item.

3.5.1 Measurement at initial recognition

At initial recognition, any surplus in fulfillment cash flows is captured in the CSM. However, if there is a deficit in fulfillment cash flows a loss component is established and a loss is recognized instantly. Thus, at time $t_0$:

$$\begin{align*}
CSM_0 &= \max(PV_0 - RA_0, 0) \quad LC_0 = - \min(PV_0 - RA_0, 0) \quad P&L_0 = -LC_0,
\end{align*}$$

(3.11)

where $LC_0$ is the loss component, $P&L_0$ is the profit recognized, $PV_0$ is the present value of future cash flows and $RA_0$ is the risk adjustment.

3.5.2 Subsequent measurement

Since a financial component is not separated from the insurance contract, the entity’s share of change in fair value of the underlying item is derived from the fund in which the policyholders’ premiums are invested. However, covers for guarantees and the entity’s charges on the fund should also be included. Hence, the entity’s share of change in fair value of the underlying item between time $t_{i-1}$ and $t_i$; $\Delta FV_i$, is stated as follows:

$$\Delta FV_i = F_i - F_{i-1} + cf_i,$$

(3.12)

where $F_i$ is the fund balance after charges and $cf_i$ is the cash flow to the entity at time $t_i$.

Furthermore, let

$$\begin{align*}
\Delta PV_i &= PV_i - PV_{i-1} \\
\Delta RA_i &= RA_i - RA_{i-1},
\end{align*}$$

(3.13)

denote the change in present value of future cash flows and the change in risk adjustment between time $t_{i-1}$ and $t_i$ respectively.

With simplified calculations, the adjustment of the CSM and the profit is computed as follows:

$$\begin{align*}
CSM_i &= CSM_{i-1} + \Delta PV_i + \Delta FV_i - R_i \quad P&L_i = R_i - \Delta RA_i,
\end{align*}$$

22
where $R_i$ is the amount released from the CSM. However, this calculation is only accurate for non-onerous contracts. Recall that in a general case, if there is a loss component present; the loss component must be reverted before a CSM is established. Furthermore, changes in the loss component conduces to profit. Thus, in a general case, the calculations are as follows:

\[
LC_i = \max \left( LC_{i-1} - \Delta PV_i - \Delta FV_i + \Delta RA_i - cf_i, 0 \right)
\]

\[
CSM_i = \max \left( CSM_{i-1} + \Delta PV_i + \Delta FV_i - LC_{i-1}, 0 \right) - R_i \tag{3.14}
\]

\[
P\&L_i = R_i - \Delta RA_i + \min \left( CSM_{i-1} + \Delta PV_i + \Delta FV_i - cf_i, LC_{i-1} \right).
\]

The amount released from the CSM should reflect the service carried out between time $t_{i-1}$ and $t_i$. Furthermore, the amount released should not be such that the CSM becomes negative. In this research, this is implemented such that the amount released is a partition $\phi_i$ of the CSM in the previous time step. In order to release the CSM as the service is delivered, the partition is chosen as the number of active contracts $AC_i$, i.e. contracts that has not matured and where the policyholder has not surrendered or suffered from death, relative to the sum of expected future active contracts. Hence,

\[
R_i = \min \left( \phi_i CSM_{i-1}, \max(CSM_{i-1} + \Delta PV_i + \Delta FV_i - LC_{i-1}, 0) \right), \tag{3.15}
\]

where

\[
\phi_i = \frac{AC_i}{\sum_{j=1}^{m} E^Q[AC_j | F_i]}.
\]
Chapter 4

Insurance model

In the following chapter, the model necessary to determine the measures of the traditional life insurance product is presented. This includes constructing a non-financial insurance state model, a financial model and a product model.

As concluded in Section 2.3, the lowest level of aggregation under which a group of insurance contracts can be considered is a level where the contracts share the same level of onerousness, risk exposure and duration. This interprets that if a group of newly issued insurance contracts are of same type regarding benefits and share similar assumptions, the contracts can be modeled together on an aggregated level.

As stated in Section 3.2, all randomness of the product is assumed to be derived from financial and non-financial outcomes. Thus, the outcomes purely depend on the assumptions of the product, the outcomes of the insurance state model and the outcomes of financial model.

The insurance states are modeled as deterministic, i.e. modeled only with respect to expected values. However, the financial outcomes are modeled as stochastic since there is a non-linear relationship between the return of the underlying item and the cash flows of the contracts. From the stochastic model an economic scenario generator (ESG) is constructed in order to produce a large number of economic scenarios. With occasionally high volatility on capital markets and new regulations such as Solvency II, this method has become increasingly common [31] both for market consistent valuation and real world models [32, 34].

The expected value of future cash flows are estimated with a regular Monte Carlo method, i.e. estimated from the mean of all outcomes. The regular Monte Carlo method has benefits since it’s easy to implement, however a drawback is the slow rate of convergence [21].
4.1 Base assumptions

It’s assumed that cash flows of the policyholder and the issuer are monthly. Therefore, it’s natural to choose a discrete model with monthly time steps and annual time units. Thus, if the contract is issued at time $t_0$ and matures at time $t_m$ the time series can be denoted as follows:

$$t_{i+1} = t_i + \Delta t, \quad i = \{0, 1, \ldots, m - 1\}, \quad (4.1)$$

where $\Delta t = 1/12$.

The contracts are assumed to be a part of an employment benefit. At time $t_0$ the policyholder starts as a premium paying customer. The monthly premium amount at time $t_0$ is $P_0$, thereafter the premium is coupled to the policyholders’ wage. According to the fictive terms of this product, there is a predefined claim age $x_c$ where the policyholder reaches the claim state. Claim payouts will thus start the next turn in month after the policyholder has reached the claim state. Furthermore, there is a predefined maturity age $x_m$ when the contract matures. At the next turn in month after the policyholder has reached its maturity age; any remaining fund balance will be paid out to the policyholder.

Thus, if the policyholder is of age $x$ at time $t_0$ and $t_c$ represents the time for when the first claim is paid out, then;

$$c = \lceil (x_c - x)/\Delta t \rceil .$$

Similarly, if $t_m$ is the time of maturity, then;

$$m = \lceil (x_m - x)/\Delta t \rceil .$$

However, there is also randomness: At any time, premium payments may cease, e.g. if the policyholder leaves its working position, this is here from referred to as the paid-up state. In such case the benefit amount and the capital from previous premiums are kept for claim payouts. Contrary, if the policyholder suffers from death prior to reaching the age $x_m$, the policyholder’s benefit amount will be withdrawn from the fund and accrue to the policyholder’s inheritors. Similarly, the policyholder has the possibility to surrender the contract at any time, e.g. by transfer its benefit amount to another insurance company.

The model is run on a group of $N$ contracts where $N/2$ contracts are held by men and $N/2$ contracts are held by women. The contract parameters are shown in Table 4.1.

<table>
<thead>
<tr>
<th>Table 4.1: Contract parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>$x = 35$</td>
</tr>
<tr>
<td>$x_c = 65$</td>
</tr>
<tr>
<td>$x_m = 75$</td>
</tr>
<tr>
<td>$\frac{N}{2} = 1000$</td>
</tr>
<tr>
<td>$P_0 = \text{CU} 1000$</td>
</tr>
</tbody>
</table>
4.2 Insurance state model

As previously stated, the policyholder may leave the contract for several reasons. Since the policyholder’s benefits differ depending on the cause of leaving the contract, the various ways of leaving the contract need to be captured in the state model \[10\]. However, the model does not need to capture what happens after the policyholder has left the contract. E.g. the model does not need to recognize if policyholder that already has surrendered suffers from death. Thus, the exit states of the contract are absorbing states.

The state space \( \mathbf{E} \) is defined as follows \[14\]:

\[
\mathbf{E} = \{ \text{Premium, Paid-up, Claim, Mature, Surrender, Death} \},
\]

with short hand notation:

\[
\mathbf{E} = \{ p, pup, c, m, s, d \}. \tag{4.2}
\]
The vector of non-financial states at time $t_i$ is thus:

$$Y_i = \left( Y_i^p, Y_i^{pup}, Y_i^c, Y_i^m, Y_i^s, Y_i^d \right)^\top,$$

where the elements represent the number of policyholders in each state respectively.

Furthermore, let

$$A_i = \begin{pmatrix} p_i^{p,p} & \cdots & p_i^{p,d} \\ \vdots & \ddots & \vdots \\ p_i^{d,p} & \cdots & p_i^{d,d} \end{pmatrix}$$

be the transition matrix at time $t_i$, where $p_i^{k,l}$ is the probability that a policyholder transfers from state $k \in E$ to state $l \in E$ in the time interval $[t_i, t_{i+1})$. One should notice that transitions are not possible between every state, thus the probability of transfer from one state to another can be zero. An overview of the possible transitions is shown in Figure 4.1.

![Figure 4.2: Aggregated decremental model.](image)

To construct the state model, multiple two-state models are constructed and aggregated together into a multi-stage model [11, 25]. This has benefits if it’s easier to model one transition conditional on another transition. E.g. if the reasoning is that for a policyholder to choose to surrender the policyholder must be alive, it’s easier to estimate surrender rates on the condition that the policyholder has not suffered from death. In this case, to the extent that the transition is possible it’s assumed that the unconditional
transition probabilities $\tilde{p}^{k,l}_i$ do not depend on which state the policyholder is transforming from. Using the order of aggregation in Figure 4.2 and Bayes theorem [29], the probabilities of the transitions are stated as follows:

$$p_{i}^{p,d} = \tilde{p}^{d}_i$$
$$p_{i}^{p,s} = (1 - p_{i}^{p,d})\tilde{p}^{s}_i$$
$$p_{i}^{p,c} = (1 - p_{i}^{p,d} - p_{i}^{p,s})\tilde{p}^{c}_i$$
$$p_{i}^{p,pup} = (1 - p_{i}^{p,d} - p_{i}^{p,s} - p_{i}^{p,c})\tilde{p}^{pup}_i$$

(4.5)

$$p_{i}^{pup,d} = \tilde{p}^{d}_i$$
$$p_{i}^{pup,s} = (1 - p_{i}^{pup,d})\tilde{p}^{s}_i$$
$$p_{i}^{pup,c} = (1 - p_{i}^{pup,d} - p_{i}^{pup,s})\tilde{p}^{c}_i$$

(4.6)

$$p_{i}^{c,d} = \tilde{p}^{d}_i$$
$$p_{i}^{c,s} = (1 - p_{i}^{c,d})\tilde{p}^{s}_i$$
$$p_{i}^{c,m} = (1 - p_{i}^{c,d} - p_{i}^{c,s})\tilde{p}^{m}_i$$

(4.7)

### 4.2.1 Assumptions and results

This research considers contract issued to men and women of the age of 35. The transfers to the claim and maturity states occur at a predefined time, namely at time $t_c$ and $t_m$ respectively. Thus,

$$\tilde{p}_i^c = \begin{cases} 1 & \text{if } i = c \\ 0 & \text{else} \end{cases} \quad \tilde{p}_i^m = \begin{cases} 1 & \text{if } i = m \\ 0 & \text{else} \end{cases}. \quad (4.8)$$

The probabilities of deaths are obtained from DUS 14 [12]. The assumptions and the results are shown in Figure 4.3. As seen, surrender is the highest cause for leaving the contract whereas a very little partition reaches the death state. The high rates of surrender and paid-up are mainly caused by the assumption that the contracts are tied to the policyholders’ employment. The low rate of death at the beginning of the contract boundary is derived from the fact that the policyholders are young at the time of issue.

\footnote{Obligatoriskt försäkrade}
4.3 Financial Model

To construct an ESG, a stochastic financial model is needed. Since the purpose of the ESG is to obtain a market consistent valuation, the dynamics under the risk-neutral measure $Q$ is vital. Furthermore, to obtain a subsequent scenario the dynamics under the physical probability measure $P$ is also of interest. The model is highly inspired by Gerstner et al. [20] and the master thesis of Gip Orreborn [22].

4.3.1 Interest rate model

Similarly to the market model presented in [20], the interest rate model chosen is Cox-Ingersoll-Ross (CIR). Under the CIR-model the interest rate is assumed to have the following dynamic [9]:

\[
\begin{align*}
    dr_t &= \kappa(\theta - r_t)dt + \sigma_r \sqrt{r_t}dW_t, \\
    r_{t0} &= r_0.
\end{align*}
\]

The CIR-model is a mean reverting stochastic process where $\theta$ is the level of convergence, $\kappa$ is the speed of convergence and $\sigma_r$ is a scale factor of the diffusion. Unlike e.g. Hull White, the CIR-model cannot be perfectly fitted to a term structure [4]. However, a benefit of the CIR-model is that it accurately captures the higher volatility that is common when interest rates are high. Furthermore, the model has an Affine Term Structure where a zero-coupon bond at time $t$ that matures at time $T$ can be priced as follows [4]:

\[
P(t, T) = A(T - t; \kappa, \theta, \sigma_r)e^{-B(T-t; \kappa, \theta, \sigma_r)r_t},
\]

where

\[
A(T - t; \kappa, \theta, \sigma_r) = \left( \frac{2he^{(\kappa + h)(T-t)/2}}{2h + (\kappa + h)(e^{h(T-t)} - 1)} \right)^{2\kappa\theta/\sigma_r^2},
\]

Figure 4.3: Assumptions and results of the state model.
\[ B(T - t; \kappa, \theta, \sigma_r) = \frac{2(e^{h(T-t)} - 1)}{2h + (\kappa + h)(e^{h(T-t)} - 1)}, \]

and \( h = \sqrt{\kappa^2 + 2\sigma_r^2} \).

A drawback of the CIR-model in the current low interest rate environment is that the CIR-model does not allow for negative interest rates. A remedy to this issue is presented in [22] where a CIR-model with a displacement parameter \( \delta \) is used. The interest rate is thus assumed to have the following representation:

\[ r_t = r'_t - \delta, \tag{4.11} \]

where the displaced rate \( r'_t \) is given the dynamic of the CIR-model, i.e.

\[ dr'_t = \kappa(\theta' - r'_t)dt + \sigma_r\sqrt{r'_t}dW_t. \tag{4.12} \]

From (4.11) it can easily be concluded that:

\[ dr_t = dr'_t. \tag{4.13} \]

Thus, if \( \theta' = \theta + \delta \),

\[ \tag{4.14} \]

the undisplaced rate \( r_t \) will have the following dynamic:

\[
\begin{cases}
    dr_t = \kappa(\theta - r_t)dt + \sigma_r\sqrt{r_t}dW_t + \delta dW_t \\
    r_{t_0} = r_0
\end{cases}
\]  

\tag{4.15}

Using the fact that the price of a zero-coupon bond is the discount of the interest rates from present time to maturity, this model entails that a bond is priced as follows:

\[
P(t, T) = e^{-\int_t^T r_s ds} = e^{-\int_t^T (r'_s - \delta) ds} = e^{-\int_t^T r'_s ds + \delta(T - t)} = P'(t, T)e^{\delta(T - t)}, \tag{4.16}
\]

where

\[ P'(t, T) = A(T - t; \kappa', \theta', \sigma_r)e^{-B(T - t; \kappa', \theta', \sigma_r)r'_t} \]

is the price of a bond in a world with displaced interest rates. Similarly, the zero interest rate at time \( t \) to time \( T \) can be represented as follows:

\[
R(t, T) = -\frac{\log P(t, T)}{T - t} = -\frac{\log P'(t, T) + \delta(T - t)}{T - t} = R'(t, T) - \delta, \tag{4.17}
\]

where \( R'(t, T) \) is the displaced zero rate. Thus, it can be concluded that \( \delta \) also is a displacement of the entire interest rate term structure.
Dynamic under $P$ and $Q$

When considering a CIR-model under $P$ and $Q$, a possible model for the price of risk is $\rho_t = \rho \sqrt{r_t}$. This model has the benefit that the interest rate model remains a CIR-model after being transferred from $P$ to $Q$ and vice versa [20].

To investigate whether the CIR-model with a displacement remains a CIR-model with a displacement after being transferred from $P$ to $Q$, let a zero-coupon bond be represented by the stochastic process $F^K$, i.e.

$$p(t, T) = F^K(t, r_t; T) \quad (4.18)$$

where $F^K$ is the solution to the stochastic differential equation

$$\begin{cases} dF^K_t = \alpha_K F^K_t dt + \sigma_K F^K_t dW_t \\ F(T, r_T; T) = 1 \end{cases} \quad (4.19)$$

Given that the interest rate has the $P$-dynamic

$$dr_t = \mu_P(t, r_t)dt + \sigma(t, r_t) dW^P_t, \quad (4.20)$$

the drift and diffusion in (4.19) are as follows [4]:

$$\alpha_K = \frac{\partial F^K}{\partial t} + \mu_P \frac{\partial F^K}{\partial r} + \frac{1}{2} \sigma^2 \frac{\partial^2 F^K}{\partial r^2}, \quad \sigma_K = \frac{\sigma F^K}{F}. \quad (4.21)$$

Under $Q$ the drift is however equal to the risk-free rate [4]. Thus, given that the interest rate has the $Q$-dynamic

$$dr_t = \mu_Q(t, r_t)dt + \sigma(t, r_t) dW^Q_t, \quad (4.22)$$

the following relationship must hold:

$$r_t = \frac{\partial F^K}{\partial t} + \mu_Q \frac{\partial F^K}{\partial r} + \frac{1}{2} \sigma^2 \frac{\partial^2 F^K}{\partial r^2}. \quad (4.23)$$

Furthermore, $\alpha_K$ and $r_t$ are related as follows [4]:

$$\alpha_K - r_t = \rho_t \sigma_K. \quad (4.24)$$

Entering (4.21) and (4.23) into (4.24) and simplifying, the relation between $\mu_P$ and $\mu_Q$ is thus:

$$\mu_P(t, r_t) - \mu_Q(t, r_t) = \rho_t \sigma(t, r_t). \quad (4.25)$$

Under the CIR-model with a displacement in (4.15), the parameters $\mu_P$, $\mu_Q$ and $\sigma$ are given by:

$$\mu_P(t, r_t) = \kappa^P (\theta^P - r_t) \quad \mu_Q(t, r_t) = \kappa^Q (\theta^Q - r_t) \quad \sigma(t, r_t) = \sqrt{r_t + \delta \sigma_r}. \quad (4.26)$$
Thus, a natural choice for modeling the price of risk would in this case be to set:

\[ \rho_t = \rho \sqrt{r_t + \delta}. \]  \hspace{1cm} (4.27)

Entering (4.26) (4.27) into (4.25) yields:

\[ \kappa^P(\theta^P - r_t) - \kappa^Q(\theta^Q - r_t) = \rho \sigma_r (r_t - \delta). \]  \hspace{1cm} (4.28)

Thus,

\[ \kappa^Q(\theta^Q - r_t) = \kappa^P(\theta^P - r_t) - \rho \sigma_r (r_t + \delta) \]
\[ = \kappa^P \theta^P - \rho \sigma_r \delta - r_t (\kappa^P + \rho \sigma_r). \]  \hspace{1cm} (4.29)

Hence, if \( \rho_t \) is modeled as in (4.27) and \( r_t \) is modeled under \( P \) as in (4.15); \( r_t \) will remain a CIR-model with a displacement under \( Q \) but with the parameters

\[ \kappa^Q = \kappa^P + \rho \sigma_r, \quad \theta^Q = \frac{\kappa^P \theta^P - \rho \sigma_r \delta}{\kappa^Q}. \]  \hspace{1cm} (4.30)

**Discretization and simulation**

As in the thesis of Gip Orreborn [22], the interest rate model is discretized using the Milstein Scheme. The Milstein scheme is an extension of the Euler-Maruyama scheme where the second term of the stochastic Taylor series is added [26]. The progression of a stochastic process \( X \) is hence estimated as follows:

\[ X_{i+1} = X_i + \mu(t_i, X_i) \Delta t + \sigma(t_i, X_i) \Delta W_i + \frac{1}{2} \sigma(t_i, X_i) \frac{\partial \sigma}{\partial x}(t_i, X_i)(\Delta W_i^2 - \Delta t). \]  \hspace{1cm} (4.31)

Inserting the parameters of the interest rate model in (4.15) and using that

\[ \frac{\partial \sigma}{\partial r}(t_i, r_i) = \frac{1}{2 \sqrt{r_i + \delta}} \sigma_r \]

and

\[ \Delta W_i = \sqrt{\Delta t} \xi_{r,i} \]

where \( \xi_{r,i} \) has a standard normal distribution, the interest rate can thus be simulated as follows:

\[ r_{i+1} = r_i + \kappa(\theta - r_i) \Delta t + \sigma_r \sqrt{r_i + \delta} \sqrt{\Delta t} \xi_{r,i} + \frac{1}{4} \sigma_r^2 \Delta t(\xi_{r,i}^2 - 1). \]  \hspace{1cm} (4.32)

**4.3.2 Equity model**

A very common way for modeling the price of equity is to use a *Geometric Brownian Motion* (GBM). A GBM is the solution to the stochastic differential equation

\[ dS_t = \mu(t, S_t)dt + \sigma(t, S_t)dW_t, \]
\[ S_{t_0} = s_0, \]  \hspace{1cm} (4.33)
which is \( [4] \):

\[
S_t = S_0 e^{\int_0^t (\mu(u,S_u) - \frac{1}{2}\sigma^2(u,S_u))du + \int_0^t \sigma(u,S_u)dW_u}.
\]  

(4.34)

In the general model, the drift and diffusion are functions of time and the price of the equity. To simplify the model, constant volatility is assumed, i.e. \( \sigma_s(t,S_t) = \sigma_s \). The drift parameter will however depend on the current interest rate, therefore the drift parameter can only be simplified to \( \mu(t,S_t) = \mu_t \).

**Dynamic under Q and P**

Under the risk-neutral measure \( Q \), the drift term equals the risk-free interest rate \( r_t \). Thus,

\[
dS_t = r_t S_t dt + \sigma_s S_t dW^Q_t,
\]

(4.35)

where \( r_t \) is obtained from the interest model.

Using the price of risk \( \rho_t \) from (4.27) and the relationship in (4.24), the dynamic of the equity price under \( P \) can be stated as follows:

\[
dS_t = \alpha_t S_t dt + \sigma_s S_t dW^P_t,
\]

(4.36)

where \( \alpha_t = \rho_t \sigma_s + r_t \).

**Correlation with the interest rate**

As often assumed there is a correlation \( c \) between the interest rate and the equity price such that \( dW^s_{s,t} dW^r_{r,t} = cd t \). One way of model this correlation is to reformulate the progression of the equity price in (4.33) as follows [20]:

\[
dS_t = \mu_s t S_t dt + \sigma_s S_t (cdW^s_{s,t} + \sqrt{1-c^2} dW^r_{r,t}),
\]

where \( W^r_{r,t} \) is the Wiener process driving the interest rate and \( W^s_{s,t} \) is an exclusive random source of the price of equity.

**Discretization and simulation**

Since there exists an explicit solution of the GBM, the differential of the process does not need to be discretized. Instead the discretization can be obtained from the explicit solution. Thus, the progression of the equity price can be simulated as follows [20]:

\[
S_{t+1} = S_t e^{(\mu_s - \frac{\sigma^2_s}{2})\Delta t + \sigma_s \sqrt{\Delta t} \xi_{r,i} + \sqrt{1-c^2} \xi_{s,i})},
\]

(4.37)

where \( \xi_{r,i} \) also is the random parameter of the interest rate (4.32) and where \( \xi_{r,i} \) and \( \xi_{s,i} \) has a standard normal distribution.
4.3.3 Inflation model

Models for inflation can either be explicit, where e.g. AR-processes are used [36] or implicit through Fisher parity [33]. Since 1993, the central bank of Sweden\(^2\) has operated under the objective of achieving price stability with an annual inflation of 2%. Therefore, an assumption that future inflation rates on prices coincides with the objective might be accurate. Furthermore, since cash flows are close to linear with inflation, it can be assumed that the price index follows a deterministic model. The price index \(\Pi^c_t\) at time \(t_i\) based at time \(t_0\) can thus be stated as follows:

\[
\begin{align*}
\Pi^c_{t+1} &= \Pi^c_t (1 + \pi^c) \Delta t \\
\Pi^c_0 &= 1,
\end{align*}
\]

where \(\pi^c\) is the constant annual rate of inflation on prices. Since wages generally rises faster than prices, it is suitable to let wages follow a wage inflation index:

\[
\begin{align*}
\Pi^w_{t+1} &= \Pi^w_t (1 + \pi^w) \Delta t \\
\Pi^w_0 &= 1,
\end{align*}
\]

where \(\pi^w\) is the constant annual rate of inflation on wages.

Lastly, since there are no diffusions; the processes \(\Pi^c_t\) and \(\Pi^w_t\) will behave the same under \(Q\) as under \(P\).

4.3.4 Assumptions

As previously stated, the CIR-model cannot be fitted perfectly to an observed term structure [4]. Instead, to accurately calibrate the CIR-model, techniques such as the Kalman Filter are required [35, 1]. Since the main purpose of this thesis is only to illustrate valuation under IFRS 17; the parameters used will mainly be the same as the parameters used in [20]. Furthermore, since the ESG only models the characteristics of deep and liquid markets, the liquidity premium needs to be included in the interest rate model [34]. Thus, \(\theta^Q\) will be set to the convergence level of the forward structure of 2.73% plus the liquidity premium on swaps of 0.44% (see Section 3.3). Lastly, \(\kappa^Q\) and \(\sigma_r\) are adjusted to fit the observed term structure including the liquidity premium.

Similarly, the parameters of the equity model are those used in [20]. The price of risk \(\rho\) is set to 1.5 which would make the expected equity return 8% when the interest rate is 2%. The parameters of the models under \(P\) are provided implicitly from (4.27), (4.30) and (4.36). Lastly, the parameters of inflation follows on the discussion carried out in Section 4.3.3.

\(^2\)Riksbanken
Results from the CIR-model compared with Smith Wilson-smoothing

(a) Zero rates

(b) Forward rates

Figure 4.4: Rates with the CIR-model.

<table>
<thead>
<tr>
<th>Interest rate model</th>
<th>Equity model</th>
<th>Inflation model</th>
<th>Interacting parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa^Q = 0.2084$</td>
<td>$\sigma_s = 20%$</td>
<td>$\pi_c = 2%$</td>
<td>$\rho = 1.5$</td>
</tr>
<tr>
<td>$\theta^Q = 3.17%$</td>
<td>$\sigma_r = 2%$</td>
<td>$\pi_w = 3%$</td>
<td>$c = -0.1$</td>
</tr>
<tr>
<td>$\delta = 2%$</td>
<td>$r_0 = -0.08%$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.3.5 Model of the underlying fund

The underlying item of a traditional life insurance product is generally an underlying fund or other underlying assets of the entity. Commonly, there is also an investment strategy which depends on the current financial state. Thus, in order to obtain an accurate valuation, simulations may need to include a dynamic investment strategy. This would however require a nested stochastic model [16]. Therefore, this research will be carried out with a static investment strategy similar to [22].

In this research, it’s assumed that the underlying item is a fund which consists of a portfolio of equities and a portfolio of bonds. The partitions of the portfolios in the fund are $\omega_S$ and $\omega_B$ respectively. Furthermore, the portfolio of bonds consists of sub-portfolios with partitions $\omega_{B,1}, \omega_{B,2}, \ldots, \omega_{B,q}$ of bonds with durations $\tau_1, \tau_2, \ldots, \tau_q$ respectively.
Let the parameters be captured in vectors as follows:

\[ \omega = (\omega_B, \omega_S) \]  \hspace{1cm} (4.40)

\[ \omega_B = (\omega_{B,1}, \omega_{B,2}, \ldots, \omega_{B,q}) \]  \hspace{1cm} (4.41)

\[ \tau = (\tau_1, \tau_2, \ldots, \tau_q) . \]  \hspace{1cm} (4.42)

Furthermore, let

\[ R_{B,i} = \frac{P(t_{i+1}, t_i + \tau_1)}{P(t_i, t_i + \tau_1)} \cdot \frac{P(t_{i+1}, t_i + \tau_2)}{P(t_i, t_i + \tau_2)} \cdots \frac{P(t_{i+1}, t_i + \tau_q)}{P(t_i, t_i + \tau_q)} - 1 \]  \hspace{1cm} (4.43)

be a vector of returns of the sub-portfolios of bonds in the time period \([t_i, t_{i+1})\), where

\[ P(t, T) = A(T - t; \kappa^Q, \rho^Q + \delta, \sigma_r) e^{-B(T - t; \kappa^Q, \rho^Q + \delta, \sigma_r)(r_t + \delta) + \delta(T - t)} \]

is the price of a bond at time \(t\) that matures at time \(T\). Thus, the return of the portfolio of bonds is:

\[ R_{B,i} = \omega_B \omega_B^\top . \]  \hspace{1cm} (4.44)

Similarly, let

\[ R_{S,i} = \frac{S_{i+1}}{S_i} - 1 \]  \hspace{1cm} (4.45)

be the return of the portfolio of equities in the time period \([t_i, t_{i+1})\), where \(S_i\) is the price of equity at time \(t_i\). The return of the fund in the time interval \([t_i, t_{i+1})\) is thus:

\[ R_i = \omega(R_{B,i}, R_{S,i})^\top . \]  \hspace{1cm} (4.46)

<table>
<thead>
<tr>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_B = (0.25, 0.25, 0.25, 0.25) )</td>
</tr>
<tr>
<td>( \tau = (5, 10, 15, 20) )</td>
</tr>
<tr>
<td>( \omega = (0.5, 0.5) )</td>
</tr>
</tbody>
</table>

### 4.3.6 Results of the ESG

The results of the ESG are shown in Figure 4.5. The outcomes of the fund model may be realistic given its defensive allocation strategy. However, the interest rate model does not capture any extreme scenarios. Thus, although the interest model may be sufficient with respect to estimating an expected value, the model would likely be insufficient for model extreme scenarios.
The model is verified under $Q$ by a martingale test. The test is performed to ensure that the expected discounted unit fund balance, estimated by the average, does not drift.

(a) Interest rates  
(b) Fund return  
(c) Martingale test

Figure 4.5: Results from the ESG.


4.4 Product model

4.4.1 Introduction to the traditional life insurance product

The model presented in this section is a model with the key features of a simplified general Swedish traditional life insurance product. The model is constructed with the purpose to illustrate the implications of IFRS 17.

Associated to the product there is a fund balance, a guaranteed amount and a total benefit amount. The total benefit amount is the sum of the guaranteed amount and the bonus. With the subscript $i$ referring to the state at time $t_i$, the following notations are introduced:

- The fund balance $F_i$.
- The benefit amount $V_i$.
- The guaranteed amount $G_i$.
- The insurance capital $I_i$.
- The bonus $B_i$.
- The guaranteed rate of return $r^G$.
- The guaranteed partition of the deposited premiums $\lambda$.
- The rate of return of the insurance capital $z_i$.

The fund balance $F_i$ is accumulated from the return of the underlying assets, deposited premiums and withdraws for benefits and charges. The guaranteed amount $G_i$ is derived from an obligation of the issuer that a partition $\lambda$ of the deposited premiums is guaranteed. The guaranteed amount is not linked to the return of the underlying fund; instead the guaranteed amount grows with a predefined guaranteed rate $r^G$. Contrary, the insurance capital $I_i$ grows with a rate $z_i$ that is linked to the return of the underlying fund through a smoothed process. Lastly, the bonus $B_i$ is the potential exceeding amount of the insurance capital to the guaranteed amount, i.e.

$$B_i = \max(I_i - G_i, 0).$$  (4.47)

The benefit amount $V_i$ is usually communicated as the sum of a guaranteed amount and a bonus, i.e.

$$V_i = G_i + B_i,$$  (4.48)

which rewrites as

$$V_i = \max(I_i, G_i).$$  (4.49)

In order to determine the rate of return of the insurance capital $z_i$, the collective consolidation level (CCR) is commonly used [22]. The (CCR) is a ratio of assets to liabilities.
associated with insurance contracts. For this model, the CCT at time $t_i$ is stated as follows:

$$ CCR_i = \frac{F_i}{V_i}. $$  

(4.50)

The rate $z_i$ is usually declared with an aim reaching a target ratio $CCR^T$ [3, 23]. For this model, $z_i$ is determined as follows:

$$ z_i = \max \left( \left( \frac{CCR_i}{CCR^T} \right)^{\frac{1}{\nu}} (1 + r^F) - 1, 0 \right), $$  

(4.51)

where $\nu$ is a smoothing parameter which can be seen as the number of years for which the issuer aims reach the target quote. The factor $(1 + r^F)$ compensates for the forecasted growth $r^F$ in the time period $[t_i, t_i+1]$ for which $z_i$ applies. Lastly, the maximum operator ensures that $z_i$ only takes positive values.

### 4.4.2 Progression of the amounts

As previously stated, the progression of the fund balance consists of returns of the underlying assets, premiums, benefits paid out and charges. The amounts paid out to the policyholders are to some extent equal to the transactions of the fund. However, potential guarantee covers is also included. The progression of the guaranteed amount and the insurance capital is similar to the progression of the fund balance; however, these transactions are only technical.

With the subscript $i$ referring to transactions at time $t_i$, let:

- $P_i$ denote the aggregated premium amount.

- $C_i$ denote the aggregated claim amount to the policyholder; and $F^C_i, G^C_i$ and $I^C_i$ the claim amounts associated with the fund, the guarantee and the insurance capital respectively.

- $M_i$ denote the aggregated maturity benefit to the policyholder; and $F^M_i, G^M_i$ and $I^M_i$ the maturity benefits associated with the fund, the guarantee and the insurance capital respectively.

- $D_i$ denote the aggregated death benefit to the policyholder; and $F^D_i, G^D_i$ and $I^D_i$ the death benefits associated with the fund, the guarantee and the insurance capital respectively.

- $S_i$ denote the aggregated surrender benefit to the policyholder; and $F^S_i, G^S_i$ and $I^S_i$ the surrender benefits associated with the fund, the guarantee and the insurance capital respectively.

- $\nu^{\text{tax}}_i$ denote the aggregated real tax charge; and $\nu^F_i, \nu^G_i$ and $\nu^I_i$ the tax charges associated with the fund, the guarantee and the insurance capital respectively.
– Λᵢ denotes the issuer’s aggregated real charge; and Λᵢ^F, Λᵢ^G, and Λᵢ^I denote the charges associated with the fund, the guarantee and the insurance capital respectively.

With the introduced notations, the progression of the fund balance, the guaranteed amount and the insurance capital are stated as follows:

\[
\begin{align*}
F_{i+1} & = \left( F_i - \nu_i^{F,\text{tax}} - P_i - F_i^C - F_i^M - F_i^D - F_i^S - \Lambda_i^F \right) (1 + R_i) \\
F_0 & = 0 \tag{4.52}
\end{align*}
\]

\[
\begin{align*}
G_{i+1} & = \left( G_i - \nu_i^{G,\text{tax}} + \lambda P_i - G_i^C - G_i^M - G_i^D - G_i^S - \Lambda_i^G \right) (1 + r_G)^\Delta t \\
G_0 & = 0 \tag{4.53}
\end{align*}
\]

\[
\begin{align*}
I_{i+1} & = \left( I_i - \nu_i^{I,\text{tax}} + P_i - I_i^C - I_i^M - I_i^D - I_i^S - \Lambda_i^I \right) (1 + z_i)^\Delta t \\
I_0 & = 0 \tag{4.54}
\end{align*}
\]

where \( R_i \) is the return of the fund in the time interval \([ti, t_{i+1})\).

Tax

The annual tax is assumed to be charged as a partition \( \beta_i^{\text{tax}} \) of the underlying amount. Recalling the relation \( V_i = \max(I_i, G_i) \), the tax charge is assumed to be as follows:

\[
\begin{align*}
\nu_i^{G,\text{tax}} & = I_a \beta_i^{\text{tax}} G_i \\
\nu_i^{I,\text{tax}} & = I_a \beta_i^{\text{tax}} I_i \\
\nu_i^{\text{tax}} & = \max(\nu_i^{G,\text{tax}}, \nu_i^{I,\text{tax}}) \\
\nu_i^{F,\text{tax}} & = \nu_i^{\text{tax}} \tag{4.55}
\end{align*}
\]

where \( I_a \) is an indicator function ensuring that taxes are only charged annually.

Premiums

The premium amount is \( P_0 \) at time \( t_0 \). Thereafter, given that the premiums are paid as an employment benefit, the premium amount is assumed to be coupled to wage inflation. Thus:

\[
P_i = \Pi_i^w Y_i^p P_0 \tag{4.56}
\]

where \( Y_i^p \) is the number of policyholder in the premium state at time \( t_i \) and \( \Pi_i^w \) is the index of wage inflation based at time \( t_0 \).
Claim amount

The claim is a partition of the underlying amount after tax. Thus, the transactions associated with claims are as follows:

\[
G_i^C = (G_i - \nu_i^{G,\text{tax}}) \chi_i \quad I_i^C = (I_i - \nu_i^{I,\text{tax}}) \chi_i
\]

\[
C_i = \max(G_i^C, I_i^C) \quad F_i^C = \min\left(C_i, (F_i - \nu_i^{F,\text{tax}}) \chi_i\right)
\]

where

\[
\chi_i = \mathcal{I}_{\{t_i \in [c, m]\}} \frac{1}{\sum_{j=i}^{m-1} (1 + rF) \Delta t(j-i)}
\]

is the payout rate for claims at time \(t_i\). The rate is set using the concept of commutations functions\(^3\) [2, 30, 15] with \(r^F\) as the forecasted rate of return. \(\mathcal{I}_{\{t_i \in [c, m]\}}\) is an indicator function ensuring that claims are only paid out in the time interval \([t_c, t_m]\). This choice of payout rate entails that if the return of the underlying amount is the forecasted rate, then the policyholder’s claim amounts will be constant. It should be noted that the claim amount withdrawn from the fund is floored. Thus, there is a possibility that the entity is forced to cover the remain of the claim amount \(C_i\).

Maturity benefit

The maturity benefit is the remain of fund balance at maturity. Thus,

\[
G_i^M = \mathcal{I}_{i=m} G_i \quad I_i^M = \mathcal{I}_{i=m} I_i
\]

\[
F_i^M = \mathcal{I}_{i=m} F_i \quad M_i = F_i^M
\]

where \(\mathcal{I}_{i=m}\) is an indicator function ensuring that the maturity benefits are only paid out at the time of maturity.

Death benefit

The death benefit is the share of the underlying amount after tax of those who has suffered from death since previous time step. Furthermore, the benefit paid out to the policyholder’s inheritors is the maximum of the policyholder’s share in the fund and the guaranteed amount. Thus,

\[
G_i^D = (G_i - \nu_i^{G,\text{tax}})(Y_i^d - Y_i^{d-1}) \quad I_i^D = (I_i - \nu_i^{I,\text{tax}})(Y_i^d - Y_i^{d-1})
\]

\[
F_i^D = (F_i - \nu_i^{F,\text{tax}})(Y_i^d - Y_i^{d-1}) \quad D_i = \max(G_i^D, F_i^D)
\]

\(^3\)The commutation functions used are the Discounted life (often denoted \(D_i\)) and the Sum of discounted life (often denoted \(N_i\)). However, since the share of those who suffer from death is paid out; the life function \(l_i\) in \(D_i\) and \(N_i\) is omitted.
where $Y^d_i - Y^d_{i-1}$ is the number of deaths in the time period $[t_{i-1}, t_i)$. As for claims, the entity may be forced to cover the difference of the amount withdrawn from the fund and the amount paid out.

**Surrender benefit**

Contrary to the death benefit, the surrender benefit is the policyholder’s fund balance. Thus,

\[
G^S_i = (G_i - \nu^{G,\text{tax}}_i)(Y^s_i - Y^s_{i-1}) \quad I^S_i = (I_i - \nu^{I,\text{tax}}_i)(Y^s_i - Y^s_{i-1})
\]

\[
F^S_i = (I_i - \nu^{F,\text{tax}}_i)(Y^s_i - Y^s_{i-1}) \quad S_i = F^S_i,
\]

where $Y^s_i - Y^s_{i-1}$ is the number of surrenders in the time period $[t_{i-1}, t_i)$.

**Charges**

The issuer’s charge is the sum of a quarterly variable charge based on a constant rate of the underlying amount and an annual fixed charge coupled to cost inflation. The actual charge is based on the maximum of the amounts charged to the guaranteed amount and the benefit amount. However, the real charge is not allowed to exceed the fund balance after other transactions. Thus,

\[
\Lambda^G_i = I_q \Lambda^G G_i + I_a \Lambda^{fix} \Pi^c_i
\]

\[
\Lambda^I_i = I_q \Lambda^G I_i + I_a \Lambda^{fix} \Pi^c_i
\]

\[
\Lambda^F_i = \min \left( F_i - \nu^{tax}_i + P_i - F^C_i - F^M_i - F^D_i - F^S_i, \max(\Lambda^G_i, \Lambda^I_i) \right)
\]

\[
\Lambda_i = \Lambda^F_i,
\]

where $I_q$ is an indicator function ensuring that the variable charge is only charged quarterly, $I_a$ is an indicator function ensuring that the fixed charge is only charged annually, $\Lambda^G$ is the rate of the variable charge, $\Lambda^{fix}$ is the fixed charge at time $t_0$ and $\Pi_i$ is the index of cost inflation based at time $t_0$.

**4.4.3 Cash flows**

Since an investment component is not separated, the net cash flow of the insurance contract consists of all transactions to and from the entity including transactions to and from the fund. Thus, the net cash flow at time $t_i$ can be stated as follows:

\[
CF_i = P_i - C_i - M_i - D_i - S_i - \nu^{tax}_i.
\]
The proposed model for the risk adjustment in Section 3.4 is based on cash flows that affect the entity financially. These cash flows include the entity’s charge and the amounts the entity pays in order to cover for the guarantee. The amount paid out to cover for guarantees is the difference between the amount paid out to policyholders and the amount charged to the fund. Thus, since the cover only applies to claim payouts and death benefits, the net cash flow that affects the entity financially is stated as follows:

\[
cf_i = \Lambda_i - (C_i - F_i^C) - (D_i - F_i^D)
\]  

(4.63)

### 4.4.4 Parameters

The parameters are presented in Table 4.4. According to the Swedish tax legislation. The tax rate at time \( t_i \) is based on the current government lending rate \( r^G_i \), floored to 0.5%. The government lending rate is furthermore assumed to be the risk-free rate without the liquidity premium.

<table>
<thead>
<tr>
<th>Product parameters</th>
<th>Cash flow parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r^G ) = 1.25%</td>
<td>( \beta^{tax} = 0.15 \max(0.5%, r^G_i) )</td>
</tr>
<tr>
<td>( r^F ) = 0.30%</td>
<td>( \Lambda^{fix} = 0.25% \ (1.0% \ \text{annually}) )</td>
</tr>
<tr>
<td>( \lambda ) = 95%</td>
<td>( \Lambda^{fix} = \text{CU 300} )</td>
</tr>
<tr>
<td>( CCR^T ) = 1.15</td>
<td></td>
</tr>
<tr>
<td>( \nu ) = 1</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 5

Result & Discussion

In this chapter, the results of the model presented in Chapter 3 and Chapter 4 are shown with the purpose of exploring possible implications of the standard and to illustrate how the measures of IFRS 17 behave. Firstly, the results of a valuation at initial recognition are presented. Thereafter, some base assumptions are modified in order to illustrate how the measures may be affected. Secondly, the future behavior of the measures and their impact on profits is evaluated. Lastly, from a realistic ten year scenario, a subsequent valuation of the contracts is obtained.

5.1 Initial Recognition

5.1.1 Base assumptions

The first results presented are obtained with the base assumptions of the model. As described in Section 3.4, the risk adjustment is measured by the present value of future costs of capital for deviations in net cash flow to the entity arising from a stressed scenario. The stressed scenario is for this case obtained with;

- death rate stressed upward 25%;
- surrender rate stressed upward 25%; and
- a cost of capital of 8%.

Figure 5.1a shows that bonuses are expected to be declared throughout the progression of the contract. However, as the partition of active insurance contracts decreases over time, the growth of the benefit amount decreases.

Figure 5.1b shows that there is a net inflow during the premium period whereas there is a net out flow during the claim period. It is also seen that cash inflows decreases with
the partition of active contracts. However, with the premium amount coupled to wage inflation, the decrease is mitigated.

Figure 5.1c shows the difference in cash flows to the entity between the base assumptions and the stressed assumptions. As seen, the stressed scenario produces a scenario that is unfavorable with respect to timing and amount. The net decrease in cash inflows is derived from outstanding charges. The change in cash outflows is derived from the entity’s obligation to cover for guarantees. Since the benefit at death is floored to the guarantee, the entity may fill remaining balance. The cash outflows are higher with the stress scenario at the beginning. However, since the benefit of surrender is not floored to the guarantee, the higher surrender rates do not affect cash out flows at the beginning. Nonetheless, the lower rate of active contracts in the stressed scenario results in a lower cash outflows during the claim period since the cover for guarantee is less.

Figure 5.2: State model.
According to the standard, at initial recognition the CSM equals the present value of cash inflows less outflows and risk adjustment. As seen in Figure 5.3, there is a positive net and a CSM is therefore established.

![Initial recognition graph]

Figure 5.3: Initial measurements.

5.1.2 Discussion: Allocation in a bank account

As previously stated, the traditional life insurance product has similarities with a financial option where the fund is the underlying item. A main difference is however that the issuer has the ability to change the underlying items of the fund in order to adjust expected return and volatility. In the risk-neutral world of valuation, the expected return is however always the risk-free rate. Therefore, minimizing the volatility and thus the risk that the guaranteed amount exceeds the fund balance would create a more favorable valuation.

One hypothetic way of minimizing the volatility is to only invest in a bank account. The outcome of such strategy is illustrated in the following scenario. The results in Figure 5.4 shows that bonuses are expected to be lower which hence have a decreasing effect on the expected value of future cash outflows. Contrary, the present value of cash inflows is not affected.

The outcome of this strategy with regards to the CSM (Figure 5.5) shows that the CSM is affected positively; thus, the strategy is favorable with risk-neutral assumptions. However, the strategy is unlikely to be favorable with real world assumptions. One notable remark in Figure 5.5 is the higher risk adjustment, even if the financial risk is reduced the greater net present value of future cash flows increases non-financial risks as a surrendered contract impacts future profits more than in the base case.
5.1.3 Discussion: Onerous contracts

With the previous assumptions, the contracts turned out to be non-onerous. In order to illustrate a set of onerous contracts a hypothetic product is created where the variable charge is omitted. As seen in Figure 5.6, by omitting the float charge; bonuses and cash outflows increases. The unfavorable condition of this hypothetic product results in that an entity would prefer a high level of surrender. Thus, since the stressed scenario needs to be produced such that it results in a less favorable scenario, the stress on surrender rates is switched from a 25% upward stress to a 25% downward stress.

As seen in Figure 5.6c, since the float charge is omitted the new surrender stress does not result in a significant gain of cash inflows. However, as fewer policyholders surrenders the expected cover for guarantee increases.

The modified product results in a loss component (see Figure 5.8), which would force the issuer to take on an instant loss.
Figure 5.6: Onerous contracts: Estimated future benefits and cash flows.

(a) Benefit amount
(b) Expected Cash Flows
(c) Stressed Cash Flows

Figure 5.7: Onerous contracts: State model.

(a) Base
(b) Stressed

Figure 5.8: Onerous contracts: Initial measurements.
5.2 Future measurements

In order to produce a series of subsequent measurements, it’s assumed that future outcomes are the expected outcomes under $Q$ and that the released partition of the risk adjustment equals the released partition of the CSM. It should be noted that this scenario is unrealistic and only presented to illustrate the relations of the measures of IFRS 17.

5.2.1 Base assumptions

As seen in Figure 5.9a, under $Q$ the fair value of the underlying item is to some extent the inverse of the in present value of future cash flows. This is logic since the value of the underlying item accumulates from deposited premiums whereas deposited premiums are not included in future cash flows. There are variations in change of fair value of the underlying item. These variations are likely derived from the stochastic results of the ESG.

Figure 5.9b shows the future amounts of the CSM and the risk adjustment. Due to variations in change of fair value of the underlying item, the decrease of the CSM is not as smooth as the decrease in the risk adjustment. Furthermore, as seen in Figure 5.9c, the smoothing effect of the CSM on profits declines with time. Thus, if the time to maturity is short, the profits recognized will be more affected by the changes in fair value of the underlying item than if time to maturity is long.
5.2.2 Onerous contracts

The future scenario of the onerous contracts without variable charge is produced with the same method. As before, the changes in fair value of the underlying item cancels the change in present value of future cash flows (see Figure 5.10a). As previously stated, a loss is recognized at initial recognition. Thereafter, profits equal the net change in the loss component. Thus, there is no smoothing on profits (see Figure 5.10c).

5.3 Realistic scenario and a subsequent measurement after 10 years

The future scenario in previous section was obtained from assumptions under $\mathbb{Q}$. Such scenario is however unlikely. Therefore, it’s interesting to instead consider a realistic scenario with assumptions under $\mathbb{P}$. In order to obtain outcomes for every time step, a two stage model would be required; one that evolves with assumptions under $\mathbb{P}$; and one that for every time step obtains a valuation under $\mathbb{Q}$.

A simpler way is therefore to assume that financial outcomes takes their expected value under $\mathbb{P}$ for ten years. Given the outcomes, a subsequent valuation 10 years from issue is obtained.

The progression of the unit fund and the interest rates are shown in Figure 5.11a and Figure 5.11b respectively. As seen, under $\mathbb{P}$ the unit fund has progressed with higher return than under $\mathbb{Q}$ whereas the interest rate has progressed in a lower path under $\mathbb{P}$ than under $\mathbb{Q}$. Thus, a new term structure is recognized at the subsequent measurement (see Figure 5.11c).
5.3.1 Base assumptions

The subsequent valuation of the insurance contracts is shown in Figure 5.12: The estimates have similar characteristics as at initial recognition. However, expected bonuses are slightly lower since the interest rates are lower than assumed at initial recognition.

Subsequent measurement after 10 years

As seen in Figure 5.14, the effect of 10 years of returns under $P$ results in a change in fair value of the underlying item that increases the CSM. Thus, the closing balance of the CSM is higher than the opening balance even after the release to profit.

The non-financial states of the policyholders have not changed since the state model has progressed with its initial assumptions. However, the new stressed scenario has a base at the subsequent measure and has thus started to converge against the base assumptions (see Figure 5.13). The closing balance of profit and loss equals the amounts released from the CSM and the risk adjustment. Only a fraction of the risk adjustment is released due to the characteristics of the uncertainty and a shift in amount and timing as a result of the new assumptions.
Discussion

A question raised at the subsequent measurement is the increase of the CSM. In Section 5.2.1 it was shown that the proposed policy of CSM release results in a rather constant profit. However, with the CSM being adjusted upwards in a real-world scenario, it is more likely that the proposed policy of release would lead to accelerating profits as the entity is expected to release 29.9M over 20 years instead of 25.8M over 30 years. Consequently, a proposed remedy to achieve a constant profit over time could be to adjust the partition of the CSM released for expected deviations between returns under the risk-neutral measure and the real world measure.

5.3.2 Onerous contracts

The subsequent valuation of the onerous contracts results in increased bonus. Consequently, the expected benefit amount diverges from the guarantee and the net effect of
the stressed cash flow is therefore reduced.

The most remarkable change compared with the result in Section 5.2.2 is that the change in fair value now acts as a reversal of the entire loss component and establishes a CSM. The change in the loss component is recognized as profit or loss together with the released risk adjustment.

Subsequent measurement after 10 years:
Loss Scenario

![Figure 5.15: Onerous: Estimated future benefits and cash flows.](image)

Subsequent measurement after 10 years:
Loss Scenario

![Figure 5.16: Onerous: Subsequent measurement.](image)
Chapter 6

Conclusions

In this research, the standard IFRS 17 has been assessed with respect to a traditional life product.

When assessing the product of choice, it’s obvious that the traditional life product transfers financial risk. Furthermore, from the application guidance of the standard [§B9], insurance risk in form of mortality risk was recognized since the death benefit may exceed the policyholder’s account balance. Hence, the traditional life was chosen for further research as it is within the scope of IFRS 17.

Furthermore, it was concluded that the investment component of a guaranteed unit-linked product with death benefit has interrelations with the insurance component. Since the standard prevents separation of an investment component if the investment component is highly interrelated with the insurance component, it was concluded that an investment component cannot be separated from a traditional life product with death benefit.

A central part of the standard is the new measures contractual service margin (CSM), loss component and the non-financial risk adjustment. At initial recognition the net present value of future cash flows less risk adjustment become CSM if positive, or loss component if negative. Therefore, the CSM and the loss component are at initial recognition dependent on the non-financial risk adjustment. Consequently, a poorly assessed risk adjustment may result in an unrepresentative CSM or an instant loss.

The risk adjustment was assessed through an attempt to capture future costs for deviations between actual and expected cash flows. The method used was simple but could be extended to a multiple scenario analysis. However, since the risk adjustment is the amount that would make an entity indifferent to non-financial risks, the adjustment will be under the subjective risk preference of the entity. From the results it could be seen that this modeled resulted in a risk adjustment which amount was proportional to the expected profit of the contracts.

On estimating the discount rate, the standard requires that a maximum of market vari-
ables are used. Thus, since the term structure of Solvency II ignores market data after the last liquid point, it was concluded that the term structure of Solvency II does not compile with the standard. Nonetheless there the question of to which level the term structure should be extrapolated and when market observations should be seen as irrelevant due to lack of depth and liquidity on financial markets remains.

The liquidity premium was estimated with a proxy method by comparing historic spreads with historic default rates. A more accurate estimation may include assessing the liquidity premium for different durations and benchmarking with results from other methods.

On estimating the present value of future cash flows, the standard allows every cash flow to be discounted with the same discount rate if risk-neutral techniques are used. It was however shown that by considering risks in the risk-neutral world, contradictions with the real world arose. E.g. it was shown that a favorable investment strategy in the risk-neutral world contradicts with a favorable investment strategy in the real world.

The use of a risk-neutral technique raised further issues. Firstly, if an entity aims to release profits linearly with the number of active contracts: The policy of release should include the deviation between the expected return in the risk-neutral world and the expected return in the real-world. Secondly, in order to more accurately measure the risk adjustment as the cost of deviations in cash flows, real world assumptions may instead be used.
Bibliography


