Sensor fusion for positioning of an autonomous vehicle

Design and implementation of an unscented Kalman filter

FREDRIK MATSSON
Sensor fusion for positioning of an autonomous vehicle

Design and implementation of an unscented Kalman filter

FREDRIK MATSSON
Abstract

The automotive industry has currently a high focus on automating road vehicles. Positive environmental impact can be achieved if car-sharing becomes more common, aiding fewer cars on the roads. When the human factor in driving decreases, positive effects may be seen in traffic safety. But many challenges remain, for example the questions of liability. The vehicles must be able to detect their surroundings and the sensors need redundancy. Sensor fusion techniques increase the reliability of measurement results by combining measurement results from multiple different sensors. This thesis uses inertial sensors to calculate position and heading. An unscented Kalman filter has been designed and implemented on a demonstrator. The demonstrator consists of an r/c car with autonomous functions. It has a forward-facing camera and it can follow road sidelines. The Kalman filter incorporates measurements from two incremental encoders, a gyroscope and a steering angle sensor. The result shows that the combination of sensor measurements provides a better estimation of position and direction of travel.
Sammanfattning

Acknowledgements

The work that was leading up to this report was performed at ALTEN Sweden AB in Kista, Stockholm. I would like to thank Hani Abou-Dabous for making this thesis possible. For giving me the opportunity to do this thesis i would like to thank Detlef Scholle. Last but not least i would like to thank Anisse Taleb for support throught this thesis work.
### Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADC</td>
<td>Analog to Digital Converter</td>
</tr>
<tr>
<td>ARM</td>
<td>Advanced RISC Machine</td>
</tr>
<tr>
<td>C-ITS</td>
<td>Cooperative Intelligent Transport Systems</td>
</tr>
<tr>
<td>CPS</td>
<td>Cyber-Physical System</td>
</tr>
<tr>
<td>ECEF</td>
<td>Earth Centered Earth Fixed</td>
</tr>
<tr>
<td>ECI</td>
<td>Earth Centered Inertial</td>
</tr>
<tr>
<td>ENU</td>
<td>East North Up</td>
</tr>
<tr>
<td>FPGA</td>
<td>Field-Programmable Gate Array</td>
</tr>
<tr>
<td>GLONASS</td>
<td>Globalnaya Navigatsionnaya Sputnikovaya Sistema</td>
</tr>
<tr>
<td>GNSS</td>
<td>Global Navigation Satellite System</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>I2C</td>
<td>Inter-Integrated Circuit</td>
</tr>
<tr>
<td>IMU</td>
<td>Inertial Measurement Unit</td>
</tr>
<tr>
<td>ITS</td>
<td>Intelligent Transport Systems</td>
</tr>
<tr>
<td>MEMS</td>
<td>Micro-Electro-Mechanical Systems</td>
</tr>
<tr>
<td>MEO</td>
<td>Medium Earth Orbit</td>
</tr>
<tr>
<td>R/C</td>
<td>Radio Controlled</td>
</tr>
<tr>
<td>SoC</td>
<td>System on Chip</td>
</tr>
<tr>
<td>UKF</td>
<td>Unscented Kalman Filter</td>
</tr>
<tr>
<td>USD</td>
<td>United States Dollar</td>
</tr>
<tr>
<td>WGS</td>
<td>World Geodetic System</td>
</tr>
</tbody>
</table>
Contents

1 Introduction .......................................................... 12
  1.1 Background ...................................................... 12
  1.2 Motivation ....................................................... 14
  1.3 Goals ............................................................ 14
  1.4 Methodology ..................................................... 14
  1.5 Delimitations .................................................... 15

2 Theory .................................................................. 16
  2.1 General concepts ................................................. 16
    2.1.1 Cyber-Physical Systems .................................... 16
    2.1.2 Intelligent Transport Systems ......................... 16
  2.2 Earth Geodesy ..................................................... 16
    2.2.1 Earth Models .................................................. 16
    2.2.2 Meridians and Parallels of latitudes ..................... 17
  2.3 Navigation ........................................................ 17
    2.3.1 Dead reckoning techniques ............................... 18
    2.3.2 Position fixing techniques ................................ 19
  2.4 Coordinate systems .............................................. 20
    2.4.1 Vector notation ............................................. 20
    2.4.2 Coordinate transformations ............................... 20
    2.4.3 Earth centered inertial, ECI .............................. 21
    2.4.4 Earth Centered Earth Fixed ............................... 22
    2.4.5 Geodetic coordinate system ............................... 22
    2.4.6 Local East-North-Up ....................................... 23
    2.4.7 Body-fixed Coordinate System ............................ 23
  2.5 Sensors .......................................................... 24
    2.5.1 Inertial Sensors .............................................. 24
    2.5.2 Absolute measurements .................................... 26
  2.6 Sensor fusion .................................................... 27
2.6.1 Kalman filtering ........................................ 28
2.6.2 Non-linear Kalman filtering ......................... 29
2.7 Modeling .................................................... 33
  2.7.1 Vehicle Model ........................................ 33
  2.7.2 Sensor models ....................................... 34

3 Methodology and Implementation .................... 36
  3.1 Experimental setup .................................. 36
  3.2 Design .................................................. 38
    3.2.1 Problems solved .................................. 40

4 Results .................................................... 44
  4.1 Discussion ............................................. 49

5 Conclusion and Future Work ......................... 51

6 References ................................................ 53
1 Introduction

1.1 Background

Vehicles are becoming more autonomous[1][2][3]. But vehicles are far from becoming driverless. As the development of the technology proceeds, questions about morality and liability emerge[4].

Future visions include autonomous cars that can be requested for pick-up and drop-off at desired locations[5]. This future would facilitate the everyday life of many people. Owning a car or even knowing how to drive would be less common.

Today’s usage of vehicles for personal purposes is not sustainable, where people are driving their own cars to and from their work without passengers. As cities are growing the problems of traffic contamination are becoming more evident[6][7]. A more effective use of cars will be necessary, where autonomous vehicles can aid carsharing and reduce the amount of cars on the roads.

A system for autonomous driving can be divided into three major parts; Driving policy, mapping and sensing[8].

Driving policy deals with the decisions that an autonomous vehicle must take on its detected environment. Traffic is a multi-agent setting where each agent needs negotiation skills to cooperate and to avoid accidents. The majority of the agents today are human but the future will probably include a larger proportion of robot agents. The robot agents will be obliged to interpret human behavior and negotiate in human-like manners to achieve their individual goals safely. The driving policy also includes the difficult challenges of ethical dilemmas that emerges when human lives are put in danger[9]. For example in a situation where an accident is inevitable and the robot agent is designed to minimize the total damage, then it might sacrifice its own human passengers to save the lives of multiple other humans. Will humans be equally valued in this problem or might babies be valued more?
These are just some of the questions that arise within the field of driving policy.

Mapping of the traffic environment provides redundancy of what is detected by the vehicles sensors. This is critical for autonomous driving[11]. For example for object detection where an object that is detected by a camera with a classifying algorithm can get redundancy from other sensors such as a Lidar scanner. The drivable surface, the surface area where the vehicle safely can be driven, can be detected by a camera with a line detection algorithm. The redundancy of the drivable surface can be obtained from detailed maps. The majority of maps today are used for navigation and they might be connected to an application that will provide the best route and show points of interests. The resolution of these maps is not sufficient for autonomous driving. There are also maps available that has centimeter precision that could be used for autonomous driving. But if a map is to be used for autonomous driving, then the map is part of a safety-critical system and it must always be correct. To achieve this it needs to be updated continuously. This can be solved with the help of crowd-sourcing, where each agent contribute to correcting the map when differences are detected. The maps are 3-d images created with the help of lidars and cameras and they can include landmarks of stationary objects, for example road signs and light poles[10].

The sensing segment of autonomous driving concerns the sensing of the vehicles environment and the creation of a model of the sensed environment. The vehicle must detect objects in its surroundings and classify them as for example stationary or moving or if it is a bus or a pedestrian. Is also needs to detect the drivable surface, even when no sidelines are available and it needs to detect and classify obstacles on this drivable surface. The sensing segment also includes estimation of position och heasing. To achieve all this sensor fusion techniques can be applied[12].
1.2 Motivation

Many safety systems are requiring precise vehicle state information. For driver assistance systems like lane keeping or vehicle safety systems like crash avoidance by active steering, the position is an important state[13].

Global Navigation Satellite Systems is currently the most widely used technology for vehicle positioning[17]. However, GNSS suffers from outages in some environments. Urban areas cause multi-signal propagation which is decreasing performance. Road infrastructure such as roads under highways and road tunnels blocks the satellite’s line-of-sight. The accuracy of GPS is around 5m and is not sufficient for autonomous driving.

Complementary positioning solutions is necessary to achieve redundancy. One solution can be to fuse own inertial measurements with map-information including landmarks[14]. This thesis will cover the design and implementation of a positioning system that uses inertial, wheel speed and steering angle measurements. This positioning system can be integrated with absolute measurements or map-information.

1.3 Goals

The goal of this thesis is the design of a positioning system that will be implemented on a demonstrator consisting of an r/c car that is able to autonomously drive on a track.

1.4 Methodology

This thesis will take a quantitative research approach and applied research will be conducted to attempt to improve the accuracy of the positioning system which will be using inertial, wheel speed and steering angle measurements. The work will be divided into three parts, a Literature study, Design and Implementation.
Literature study
A literature study will be conducted. Sources of information will be Internet and KTH’s library database of book and articles. Literature related to vehicle positioning, inertial navigation and sensor fusion will be studied.

Design
The design of the vehicles positioning system will be based on knowledge retrieved from the literature study.

Implementation
The design will be implemented on a demonstrator consisting of an R/C car equipped with the following sensors:
GPS, accelerometers, gyroscopes, magnetometers, wheel encoders, lidar and camera. It also has autonomous functions for lateral control by using a line detection algorithm.

1.5 Delimitations
The project has a deadline that is set to 13th of December 2017.
The design shall be implemented on an existing demonstrator platform.
The problem will be simplified to a 2-d problem.
2 Theory

2.1 General concepts

2.1.1 Cyber-Physical Systems

Cyber-Physical Systems is the integration of physical processes together with computation and communication. These systems are embedded systems that sense and influence the physical world. The potential to improve human daily life is vast and the application areas are wide. There are also considerable economic potentials and major investments are being made worldwide.[15]

2.1.2 Intelligent Transport Systems

These systems are Cyber-Physical Systems that provide innovative services by using sensing and communication technologies. The aim is to increase safety and to decrease the growing emission and congestion problems. The extension of ITS is the Cooperative addition, C-ITS, where road users and infrastructure communicate and cooperate. Information will be shared between both road users and infrastructure to improve transport efficiency.[16]

2.2 Earth Geodesy

2.2.1 Earth Models

The earth is oblate due to the rotation around the poles. The Earth radius varies with about 22 km between the poles and the equator. This corresponds to about 0.3% difference in radius.

Physical Surface: This is the Earth’s actual surface. It is the land or sea that is in contact with the atmosphere.

Geoid: The Geoid is a model of Earth that corresponds the equipotential
gravity field surface that is best fitted to the global mean sea level. It is an imaginary surface beneath the land surface. It corresponds to the shape that the oceans would take if there were no solid land but only the Earth’s gravity field. The Geoid is too complex for the use in navigational computations.

**Reference Ellipsoid:** The Geoid can be approximated by an ellipsoid with its minor axis coinciding with the Earth’s rotational axis and its center coinciding with Earth’s mass center. The ellipsoid model is suitable for navigational purposes due to its analytical expressions.

### 2.2.2 Meridians and Parallels of latitudes

A Meridian is an arc that stretches between the north pole and the south pole and is perpendicular to the equator. The Prime Meridian passes through Greenwich, England and is set to be $0^\circ$ longitude.

A Parallel of latitude is an imaginary circle that is parallel to the equator. The equator is set to be $0^\circ$ latitude which makes the poles $90^\circ$ latitude.

### 2.3 Navigation

The problem of navigation is everywhere on the Earth. For example, the migrating animals such as birds like the Arctic tern that yearly navigates from the Arctics to the Antarctic or the sea vessels crossing the oceans. Finding the way has always been crucial to life.

Navigation can be self-contained or by using absolute measurements of external objects. Fusion of the two navigation techniques can give accurate estimation of position and velocity. A common technique for fusing the two navigation techniques is by using Kalman filtering. Kalman filtering was first used during the Apollo program when navigating the lunar transfer orbit. The Apollo guidance computer fused measurements from an IMU and a sextant. It merged inertial measurements with absolute measurements using celestial navigation.
Two methods of navigation are dead reckoning and position fixing.

**Dead reckoning**
By starting at a known position and measuring heading and velocity at known time intervals the position can be estimated.

**Position fixing**
By measuring the distances or angles to multiple stationary object, the position can be estimated using trilateration or triangulation.

### 2.3.1 Dead reckoning techniques
Dead-reckoning is self-contained and does not use any external measurements. It uses measurements such as heading and velocity to estimate the displacement between the measurements. Any errors will propagate and accumulate during the calculations, causing the error to grow unbounded.

**Odometry**
The distance of a displacement of a wheeled vehicle can be estimated by counting the wheel rotations. The rotation of the wheel axes can be counted by using rotary encoders. The measured rotation is translated to linear displacement by integration. One problem is wheel slippage. As mentioned earlier, any errors will grow unbounded. Advantages are low cost, high sampling times and short range accuracy.

**Inertial navigation**
Inertial navigation uses inertial sensor such as accelerometers and gyroscopes. These sensors provide measurements of specific forces and rotation rates respectively. They have a wide range of cost, from very accurate and very expensive to affordable low accuracy sensors.

Gyroscopes commonly used in aeroplanes are Ring Laser gyroscopes.
They are highly accurate, the drift is around 1°/hour but the price is around 20'000 USD. The MEMS gyroscopes are much more affordable, around 10 USD, but less accurate, around 70°/hour. Measurements from gyroscopes are integrated to provide Attitude.

2.3.2 Position fixing techniques

Position fixing techniques uses absolute measurements to estimate position.

Electronic compass
Electronic compasses uses magnetometers to sense the local magnetic field. They can provide heading measurements relative to Earth’s magnetic north. One disadvantage is that the local magnetic field is easily distorted by nearby power lines or metal structures. This makes electronic compasses errors unreliable for heading measurements for land vehicles.

Global Navigation Satellite Systems
GNSS satellites are broadcasting coded radio frequency signals that can be divided into three main components. The carrier wave, a ranging code and navigation data. The carrier wave is a sinusoidal radio frequency signal in the L-band. The ranging code allows the receiver to determine the travel time of the signal to estimate distance. The navigation data includes the satellites position and velocity. The receiver can then use the information in the GNSS signal to estimate its position by using trilateration. Accuracy for low-cost commercial receiver is a few meters.

Celestial navigation
Celestial navigation is using angular measurements between the horizon and a celestial body to determine position.
2.4 Coordinate systems

The position of a point in space can be described with a vector originating from origo of a reference frame. The position of the point will then be relative to the used reference frame.

An inertial reference frame has zero net force acting on it, resulting in zero velocity or a constant velocity motion. Inertial sensors provide measurements relative to an inertial frame. A non-inertial reference frame can be exposed to forces causing accelerations relative to an inertial frame.

The following reference frames will be described:
- ECI - Earth Centered Inertial
- ECEF - Earth Centered Earth Fixed
- WGS - World Geodetic System
- Local ENU - East North Up
- Body fixed

2.4.1 Vector notation

Vectors are expressed with bold lowercase letters and vector components are expressed with non-bold lowercase letters. Both are expressed with a superscript that represents the reference frame that the vector belongs to. For example, a three-dimensional vector \( r \) originating from origo in an arbitrary reference frame \( k \) is expressed as:

\[
 r^k = [x^k \ y^k \ z^k]^T
\]

2.4.2 Coordinate transformations

Vector transformations between different reference frames is necessary in navigational computations. A transformation can be described by a transformation matrix. Transformation matrices will be expressed with bold upper-
case letters. A subscript will represent the reference frame that the vector belonged to before the transformation and a superscript will represent the reference frame that the vector will be transformed into. For example a transformation matrix transforming vectors in reference frame k into an arbitrary reference frame m is expressed as:

\[ R^m_k \]

Transforming a vector belonging to the reference frame k into the reference frame m is given by:

\[ r^m = R^m_k \cdot r^k \]

If both reference frames are mutually orthogonal, the transformation matrix will also be orthogonal. The inverse of an orthogonal matrix is also its transpose:

\[ (R^m_k)^{-1} = (R^m_k)^T \]

The inverse transformation, transforming from the reference frame m back into the reference frame k is given by

\[ r^k = R^k_m \cdot r^m = (R^m_k)^T \cdot r^m \]

### 2.4.3 Earth centered inertial, ECI

This inertial reference frame is not fixed with the rotation of the earth, it can therefore be used to position objects in the near-earth environment, such as satellites. Object positions can be described with both Cartesian and spherical coordinates. Using spherical coordinates, a position can be described in terms of right ascension and declination. The right ascension is the angular distance from the vernal equinox going eastward along the equatorial plane. The declination is the angle between the equatorial plane and earths rotational axis, positive on the north hemisphere.
**Table 1: ECI**

<table>
<thead>
<tr>
<th>Origo</th>
<th>Fixed at Earth mass center.</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-axis</td>
<td>Points through the equatorial plane towards the vernal equinox.</td>
</tr>
<tr>
<td>Y-axis</td>
<td>Points through the equatorial plane completing the right-hand rule.</td>
</tr>
<tr>
<td>Z-axis</td>
<td>Points along earths rotational axis.</td>
</tr>
</tbody>
</table>

**2.4.4 Earth Centered Earth Fixed**

This reference frame is also centered at the earth mass center but it rotates together with the earth. It is a right-handed orthogonal system that uses Cartesian coordinates.

<table>
<thead>
<tr>
<th>Origo</th>
<th>Fixed at Earth mass center.</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-axis</td>
<td>Points through the equatorial plane at the prime meridian.</td>
</tr>
<tr>
<td>Y-axis</td>
<td>Points through the equatorial plane completing the right-hand rule.</td>
</tr>
<tr>
<td>Z-axis</td>
<td>Points along earths rotational axis.</td>
</tr>
</tbody>
</table>

**Table 2: ECEF**

**2.4.5 Geodetic coordinate system**

A Geodetic coordinate system has the same axes as ECEF. A point near the Earth’s surface is described in terms of the angles latitude and longitude and the elevation. The longitude of a point is the angle between the prime meridian and an other meridian passing through that point. The latitude of a point is the angle between the equatorial plane and a latitude parallel that passes through that point. The geodetic coordinate system can be used to locate objects near the earth surface.

WGS84 is a standard widely used in GPS navigation. It is an ellipsoidal approximation of a Geoid defined by the following parameters:
<table>
<thead>
<tr>
<th>a</th>
<th>Semi-major Axis</th>
<th>6378137.0 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/f</td>
<td>Flattening Factor of the Earth</td>
<td>298.257223563</td>
</tr>
<tr>
<td>ω</td>
<td>Nominal Mean Angular Velocity</td>
<td>7292115 10-11 rad/s</td>
</tr>
<tr>
<td>GM</td>
<td>Geocentric Gravitational Constant</td>
<td>3986004.418 108 m3/s2</td>
</tr>
</tbody>
</table>

Table 3: WGS84

2.4.6 Local East-North-Up

This is a local earth reference frame with the origin fixed to a point on the surface. The East North directions spans a plane tangent to the Earth’s surface and the Up direction points away from Earth perpendicular to the East-North plane.

<table>
<thead>
<tr>
<th>Origo</th>
<th>Arbitrary point on earth surface.</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-axis</td>
<td>Points east</td>
</tr>
<tr>
<td>Y-axis</td>
<td>Points north.</td>
</tr>
<tr>
<td>Z-axis</td>
<td>Points up</td>
</tr>
</tbody>
</table>

Table 4: Local ENU

2.4.7 Body-fixed Coordinate System

The body-fixed coordinate is fixed with the vehicles body. The x-axis points in the forward direction, the y-axis points in the transversal direction, the z-axis points up. The orientation of this frame can be described relative to the local ENU frame by using Euler angles or quaternions.

<table>
<thead>
<tr>
<th>Origo</th>
<th>Fixed at a vehicles center of gravity</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-axis</td>
<td>Points in the longitudinal direction</td>
</tr>
<tr>
<td>Y-axis</td>
<td>Points in the lateral direction</td>
</tr>
<tr>
<td>Z-axis</td>
<td>Points up</td>
</tr>
</tbody>
</table>

Table 5: Body fixed ENU
2.5 Sensors

A sensor can detect the physical environment and forward the information for processing. Sensors are commonly consisting of two components. A sensitive element and a transducer. The sensitive element interacts with the input and the transducer translates the input into an output signal that can be read by a data acquisition system.

Sensor error
The absolute sensor error is the difference between the sensor output and the true value. The relative sensor error is the difference divided by the true value.

Noise
Sensor noise is unwanted fluctuation in the sensor output signal when the true value is kept constant. The variance of the noise is an important parameter in sensor characteristics. White noise is a random signal where all frequencies contain equal intensity.

Drift
Sensor drift is an unwanted change in sensor output while the true value is kept constant.

Resolution
The resolution is the minimal change that the sensor can detect.

2.5.1 Inertial Sensors

Accelerometers
An accelerometer measures its own acceleration relative to an inertial reference frame. The function is comparable to a damped mass on a spring.
When the sensor is exposed to an acceleration, the mass will be displaced. The displacement can be measured by using the capacitive or piezoresistive effects. A capacitive accelerometer is utilizing the moving mass as a capacitor, changing the capacitance as it moves. A piezoresistive accelerometer is utilizing the change in a materials electrical resistivity when it is deformed.

**Gyrosopes**

A Gyroscope is a device that measures angular rate relative to an inertial reference frame. Early gyroscopes used a spinning mass supported by gimbals. The conservation of angular momentum keeps the spinning mass leveled when the support is tilted and the angular difference can be measured.

**Optical gyroscopes**

Optical gyroscopes uses the Sagnac effect. If two pulses of light are sent in opposite directions around a stationary circular loop, they will travel the same inertial distance and they will arrive at the end simultaneously. But if the loop is rotating and two light pulses are again sent in opposite directions, the light pulse traveling in the same direction as the rotation will travel a longer inertial distance and will arrive at the end later. Using interferometry the differential phase shift can be measured and translated into angular velocity. This type of gyroscopes are usually used in aeroplanes.

**Vibrating gyroscopes**

MEMS gyroscopes are commonly vibrating gyroscopes. This type of gyroscopes consists of a vibrating mass mounted on a spring. If the mass is oscillating in the x-axis and a rotation about the z-axis is applied, then an acceleration in the y-axis is induced. This acceleration is called Coriolis acceleration and is given by

\[ a_{cor} = 2v \cdot \Omega \]

where \( v \) is the velocity of the mass and \( \Omega \) is the angular rate of rotation. The angular rate is thus given by the velocity of the oscillating mass and by
measuring the force which induces the Coriolis acceleration

**Rotary encoders**
Rotary encoders can be divided into absolute and incremental encoders. Absolute encoders can indicate the angular position of its shaft. The position is given by an encoded disc that is rotating together with the shaft. Different techniques are used to read the encoded disc, for example mechanical or optical techniques. The incremental encoder can not indicate the angular position but it will indicate incremental changes in angular rotation. Each increment of angular rotation produces an impulse in the sensor output.

### 2.5.2 Absolute measurements

**Global Navigation Satellite Systems**
Operational systems with global coverage today are United State’s GPS and Russia’s GLONASS. Several other system is scheduled to be operational by year 2020, for example, Europe’s Galileo and China’s BeiDou-2. Other countries such as India, Japan and France are also developing their own GNSS.

GPS
The GPS is divided into three segments, a space segment, a control segment and a user segment.

The space segment
The space segment was originally consisting of 24 satellites divided into 6 circular orbits, with 4 satellites in each orbits. Today there are a total of 31 operational satellites in the GPS constellation. The satellites are orbiting in the Medium Earth Orbit at an altitude of approximately 20’200 km. The orbits have a 55° inclination from the equator and the orbital period is 12h. The constellation ensures that at least 4 satellites are visible at any place on earth at any given time.
The control segment

The control segment is a global network of ground facilities. Its purpose is to control and maintain the system. The control segment consists of Monitoring stations, Ground antennas and a Master control station. There are 16 Monitoring stations and 11 Ground antennas spread around the world.

The monitoring stations track the satellites and collect GPS signals and forwards the information to the Master control station.

The Ground antennas communicates with the satellites via the S-band. They to send commands and uploads navigation data and program code.

The Master control station is located in Schriever Air Force Base in Colorado, USA. It provides commands and control of the satellites. It also collects data from the monitoring stations and computes the location of each satellite. The system is monitored to ensure system health and accuracy. Reposition of satellites can be commanded to maintain optimal constellation.

The user segment

The user segment consists of the receivers of the GPS signals. The receivers receive the coded signals and estimates position, velocity and time.

Magnetometers

Magnetometers can measure the local magnetic field by using the hall effect. They consist of a thin sheet of semi-conducting material. In a magnetic-free environment, the electrons in the thin sheet is evenly distributed and the potential difference is zero. When a magnetic field is present the electrons will distribute unevenly, inducing a potential difference. The potential difference is measured and translated into magnetic flux density.

2.6 Sensor fusion

Sensor fusion is the merging of numerical data from multiple sensors to achieve an information gain relative to using each sensor individually. This can be done with a probabilistic approach using statistical inference from multiple observations. The most common technique is the Kalman filter that was
introduced by Rudolf Kalman in 1960. The Kalman filter was first implemented in the Apollo guidance computer that brought the humans to the moon.

2.6.1 Kalman filtering

Kalman filtering can be used to estimate a variable that cannot be measured directly. It can also be used to estimate a variable by combining measurements from multiple sensors.

Kalman filtering is a Bayesian recursive estimation algorithm. It estimates the state $X$ of a discrete-time controlled linear dynamical process. The process can be described by a linear stochastic difference equation in state space form:

$$X_k = F_{k-1}X_{k-1} + G_{k-1}U_{k-1} + v_{k-1}$$

where, $F$ is the dynamical transition matrix, $G$ is the input vector and $U$ is the control input vector.

The sensors of this process is providing measurements $Z$:

$$Z_k = H_kX_k + e_k$$

where $H$ is the observation model.

The stochastic variables $v_k$ and $e_k$ represent process noise and measurement noise respectively.

$$v \sim \mathcal{N}(0, Q), \quad e \sim \mathcal{N}(0, R)$$

They are assumed to be white noise, independent of each other and having gaussian probability distributions. Their covariances are given by $Q$ and $R$ respectively.

The filter estimates a posteriori estimate using a priori estimate and a weighted difference between a new measurement and the predicted measurement.
\[ \dot{x}_k = \hat{x}_k + K(y_k - H\hat{x}_k) \]

The weight or gain K is chosen such that the state covariance is minimized.

The algorithm is divided in two parts. One part is a prediction phase where a priori estimation is calculated by evolution of state dynamics. The other part is an observation phase where data from new measurements is used to correct the priori estimate. The algorithm is initialized by providing initial values for the state vector, X and the state error covariance matrix, P.

Algorithm 1 Kalman filter

Initialize:
\[ \hat{X}_{1|0} = \text{E}(x_0), \ P_{1|0} = \text{Cov}(x_0) \]

Loop:

Predict:
\[ \hat{X}_{k|k-1} = A_{k-1}\hat{X}_{k-1|k-1} + B_{k-1}U_{k-1} \]
\[ P_{k|k-1} = A_{k-1}P_{k-1|k-1}A_{k-1}^T + Q_{k-1} \]

Observe and correct:
\[ K_k = P_{k|k-1}H_k^T (H_kP_{k|k-1}H_k^T + R_k)^{-1} \]
\[ \hat{X}_{k|k} = \hat{X}_{k|k-1} + K_k(Y_k - H_k\hat{X}_{k|k-1}) \]
\[ P_{k|k} = P_{k|k-1} - K_kH_kP_{k|k-1} \]

2.6.2 Non-linear Kalman filtering

A non-linear system can be described in state-space form with additive noise.

\[ X_k = f(X_{k-1}, u_{k-1}) + v_{k-1} \]
\[ y_k = h(X_k) + e_k \]
When a gaussian distributed stochastic variable propagates through a non-linear process the outcome will no longer be gaussian as in the linear case. One method that has been a standard for many years is to construct a first-order linear approximation of the non-linear function, about the current mean and covariance estimation. This technique is called an Extended Kalman Filter.

One other type of Kalman filter is the Unscented Kalman Filter. Instead of propagating a gaussian variable through a linearized function, a set of weighted points representing the gaussian variable is propagated through the actual non-linear function. A new gaussian can now be computed from the propagated points. The set of points is called sigma points. There are different methods for computing the sigma points and their respective weights. This thesis uses a method proposed by R. Merwe in 2004[18]. This method uses three parameters, $\alpha, \beta, \kappa$ to control the distribution and weighting of the sigma points. A matrix $\mathcal{X}$ is generated, containing $2L+1$ sigma vectors $\mathcal{X}_i$. The first vector of sigma points is the mean of the input gaussian

$$\mathcal{X}_0 = \bar{x}$$

and the remaining $2L$ sigma vectors is given by

$$\mathcal{X}_i = \bar{x} + \left(\sqrt{(L+\lambda)P_x}\right)_i \quad i = 1, \ldots, L$$

$$\mathcal{X}_i = \bar{x} - \left(\sqrt{(L+\lambda)P_x}\right)_{i=L} \quad i = L + 1, \ldots, 2L$$

where $L$ is the dimension of the state and $\lambda = \alpha^2(L + \kappa) - L$ is a scaling parameter. The parameter $\alpha$ determines the spread of sigma points around the mean and is usually set to a small positive value of order 1e-3. The parameter $\kappa$ is a secondary scaling parameter usually set to 3-L. The matrix square root can be calculated by using the Cholesky decomposition. The corresponding weights have a superscript notation, $m$ or $c$, for weights belonging to state mean or state covariance respectively

$$W_0^{(m)} = \lambda/(L + \lambda)$$
\[ W_0^{(c)} = \lambda/(L + \lambda) + (1 - \alpha^2 + \beta) \]
\[ W_i^{(m)} = W_i^{(c)} = 1/\{2(L + \lambda)\} \]

where the parameter \( \beta \) determines the prior knowledge of the distribution of the state. For Gaussian distributions, \( \beta = 2 \) is optimal.

The sigma points are propagated through the non-linear function
\[ \mathcal{Y}_i = g(\mathcal{X}_i) \quad i = 0, ..., 2L \]

and an approximation of the state mean and covariance for \( y \) is given by a weighted sample mean and covariance of the posterior sigma points
\[ \bar{y} \approx \sum_{i=0}^{2L} W_i^{(m)} \mathcal{Y}_i \]
\[ P_y \approx \sum_{i=0}^{2L} W_i^{(c)} \{\mathcal{Y}_i - \bar{y}\}\{\mathcal{Y}_i - \bar{y}\}^T \]
Algorithm 2 Unscented Kalman filter

Initialize:
\[ \hat{X}_0 = \text{E}(x_0), \ P_0 = \text{Cov}(x_0) \]

Calculate sigma points:
\[ \mathcal{X}_{k-1} = [\hat{X}_{k-1} \ \hat{X}_{k-1} + \sqrt{(L + \lambda)P_{k-1}} \ \hat{X}_{k-1} - \sqrt{(L + \lambda)P_{k-1}}] \]

Loop:

Predict:
\[ \mathcal{X}_{k|k-1} = F[\mathcal{X}_{k-1}, u_{k-1}] \]
\[ \hat{X}_k = \sum_{i=0}^{2L} W_i^{(m)} \mathcal{X}_{i,k|k-1} \]
\[ P_k^- = \sum_{i=0}^{2L} W_i^{(c)}[\mathcal{X}_{i,k|k-1} - \hat{x}_k^-][\mathcal{X}_{i,k|k-1} - \hat{x}_k^-]^T + R^u \]
\[ Y_{k|k-1} = H[\mathcal{X}_{k|k-1}] \]
\[ \hat{y}_k = \sum_{i=0}^{2L} W_i^{(m)} Y_{i,k|k-1} \]

Observe and correct:
\[ P_{\hat{y}_k \hat{y}_k} = \sum_{i=0}^{2L} W_i^{(c)}[\hat{y}_{i,k|k-1} - \hat{y}_k^-][\hat{y}_{i,k|k-1} - \hat{y}_k^-]^T + R^n \]
\[ P_{x_k \hat{y}_k} = \sum_{i=0}^{2L} W_i^{(c)}[\mathcal{X}_{i,k|k-1} - \hat{x}_k^-][\hat{y}_{i,k|k-1} - \hat{y}_k^-]^T \]
\[ K_k = P_{x_k \hat{y}_k} P_{\hat{y}_k \hat{y}_k}^{-1} \]
\[ \hat{X}_k = \hat{X}_k + K_k(Y_k - \hat{Y}_k^-) \]
\[ P_k = P_k^- - K_k P_{\hat{y}_k \hat{y}_k} K_k^T \]
2.7 Modeling

Modeling is mathematically describing a system so that the system can be analyzed and optimized. A precise and complex model might be closer to reflecting reality but will require more computational power to process. In the following sections rather simple but powerful models are introduced.

2.7.1 Vehicle Model

The bicycle model is simplifying a 4-wheel vehicle by modeling the front and rear wheel pairs as single wheels. It is a three degree of freedom model including x and y positions and the yaw angle, $\theta$.

![Figure 1: Bicycle model](image)
$f_f$ and $f_r$ are the lateral tire forces. The lateral tire forces can be assumed linear functions of slip angle:

$$f_f = -C_f \alpha_f$$
$$f_r = -C_r \alpha_r$$

where $C_f$ and $C_r$ are the cornering stiffnesses and the slip angles are given by:

$$\alpha_f = \tan^{-1}\left(\frac{\dot{y}^b + \dot{\delta}_f}{\ddot{x}^b}\right) - \delta$$
$$\alpha_r = \tan^{-1}\left(\frac{\dot{y}^b + \dot{\delta}_r}{\ddot{x}^b}\right)$$

where $\delta$ is the steering angle, $l_f$ is the distance between the center of gravity and the front wheel axle and $l_r$ is the distance between the center of gravity and the rear wheel axle.

### 2.7.2 Sensor models

**Odometry model**

Wheel velocity can be modeled as

$$V_o = \omega r_{eff} + n_o$$

Where $\omega$ is the angular rate measured by the incremental encoder, $n_o$ is the measurement noise and $r_{eff}$ is the effective rolling radius. Systematic errors include unequal wheel diameters, wheelbase uncertainty, misalignment of wheels and finite encoder resolution. Non-systematic errors include traveling on an inclined plane, wheel-slippage, external forces. The wheels should ideally be knife-edge thin and not compressable to minimize errors.

**Gyroscope**

The angular rate can be modeled as

$$\dot{\theta}_g = \dot{\theta} + b_g + n_g$$
where $\dot{\theta}$ is the angular rate measured by the gyroscope, $b_g$ is the sensor bias and $n_g$ is the measurement noise.

**Steering angle**
The steering angle is measured by a potentiometer inside the steering servo and the angle can be modeled as

$$\delta_s = \delta + n_s$$

where $\delta$ is the steering angle measured by the potentiometer and $n_s$ is the measurement noise.
3 Methodology and Implementation

3.1 Experimental setup

The demonstrator consists of an r/c car equipped with a Zedboard. The Zedboard is using a Xilinx Zynq-7000 All programmable System on Chip. The system on chip includes an FPGA and a dual ARM Cortex-A9 processor. One core of the ARM processor is used to run safety-critical processes and the other core is used to run a petalinux for entertainment applications.

The demonstrator has the following sensors: Four incremental encoders are measuring the angular rate of each wheel axis. The encoder resolution is 500 pulses per rotation. Two low-cost 9-axis inertial measurement units consisting of accelerometers, gyroscopes and magnetometers are mounted over each wheel pair axle. A lidar is mounted on a servo and attached to the front of the vehicle. A camera is also mounted on the front of the vehicle. The steering angle is measured using the potentiometer inside the steering servo.
Figure 2: The demonstrator
3.2 Design

The position of the vehicle will be estimated by using dead reckoning techniques. The orientation of the vehicle is estimated by integration of measurements of the angular rate. The sensor used are the two rear wheel encoders, the gyroscope in the yaw-axis and the potentiometer measuring the steering angle. The wheel encoders are connected to the FPGA, where the signal is decoded into angular rate. The gyroscope is using the I2C communication protocol and the potentiometer measuring the steering angle is connected to an ADC. The raw signal from the gyroscope is preprocessed removing known biases.

The following assumptions are made: The aerodynamic forces are assumed zero. Longitudinal velocity is assumed constant. The vehicle moves in the xy-plane with the center of gravity at zero height. The lateral slip angles of the wheels are assumed zero. This means that the lateral tire forces are zero and that the velocity vectors of each wheel is parallel to the wheel direction.

The kinematic model is given by:

\[ \dot{x} = v_x \cos \theta \]
\[ \dot{y} = v_x \sin \theta \]
\[ \dot{\theta} = \frac{v_x \tan \delta}{L} \]

where \( \delta \) is the steering angle, \( v_x \) is the longitudinal velocity and \( L \) is the distance between the front and read wheel axles.

The Unscented Kalman Filter is given by

\[ X_{k+1} = f(X_k, u_k) + v_k \]
\[ Z_k = h(X_k) + e_k \]

The state-vector is

\[ X = [x, y, \theta, \dot{\theta}] \]
The input is the wheel speeds and the steering angle

\[ u_k = [v_{l,k}, v_{r,k}, \delta_k] \]

and the dynamical transition of the process is

\[ x_{k+1} = x_k + v_{x,k} \cdot \cos \theta_k \cdot \Delta k \]
\[ y_{k+1} = y_k + v_{x,k} \cdot \sin \theta_k \cdot \Delta k \]
\[ \theta_{k+1} = \theta_k + \dot{\theta}_k \cdot \Delta k \]
\[ \dot{\theta}_{k+1} = \frac{v_{x,k} \tan \delta_k}{L} \cdot \Delta k \]

where

\[ v_{x,k} = \frac{v_{l,k} + v_{r,k}}{2} \]

The wheel encoders and the gyroscope provide angular rate measurements

\[ Z_k = [g_{z,k}, o_{z,k}] \]

The measurements from the gyroscope are

\[ g_{z} = g\dot{\theta} + g\epsilon \]

The measurements from the wheel encoders are

\[ o_{z} = o\dot{\theta} + o\epsilon \]

where

\[ o\dot{\theta} = \frac{v_{l,k} - v_{r,k}}{W} \]

and

\[ [v_l, v_r] = [\omega_l \cdot r_{eff,l}, \omega_r \cdot r_{eff,r}] \]

and \( W \) is the distance between the rear wheels.

The process noise and the measurement noise are set in the process noise covariance matrix and the measurement noise covariance matrix. Both matrices are diagonal indicating that the noises are uncorrelated.

The initial states and the initial covariance matrix are set to zero.
3.2.1 Problems solved

Wheel encoder measurement
The resulting trajectories given by the wheel encoder measurements are shown in figure 3.

![Figure 3: Encoder measurements](image)

The figure shows multiple runs in both directions plotted on the same graph. Figure 4 shows the signal from one wheel encoder.

![Figure 4: Encoder signal before](image)

The signal is noisy and is including large spikes, increasing the variance.
The wheel encoders were found to be very sensitive to lateral forces on their axes. Initially the wheel encoders was mounted as shown in figure 5.

Figure 5: Encoder attachment before

One gear is attached to the wheel axis and one gear is attached to the wheel encoder. The hole of the gears was not correctly centered and the contact force between the gears was large. To eliminate mechanical error sources, the wheel encoders was mounted as shown in figure 6.
Figure 6: Encoder attachment after

A flexible PVC hose was used to directly connect the wheel axis to the wheel encoder, minimizing lateral forces on the wheel encoder’s axles. The large spikes in the signal from the wheel encoders disappeared as shown in figure 7.

Figure 7: Encoder signal after
Driving direction differences
The resulting trajectories from the gyroscope and the steering angle measurements showed different results depending on driving direction. Therefore the driving of the vehicle is divided into three modes, turning left, turning right and driving straight. The Unscented Kalman Filter will use different covariance matrices for each mode. In the straight driving mode, the state covariance for theta is larger, giving less thrust in changes of theta. In the turning mode, the state covariance for theta is lower, giving more influence in changes of theta. Also the covariance matrices are set different for turning left or turning right. The covariances was set in a purely heuristic manner.
4 Results

The demonstrator was driven on a rectangular track and it was controlled by a line following algorithm. The rectangular track had a size of 4.7 x 8.0 m. The 90 degrees curves had a radius of 1 m. Data from eight runs was collected, four in each direction and the data was processed offline. The resulting trajectories from driving in both directions is plotted on the same graph where one driving direction is mirrored both horizontally and vertically.

The resulting trajectories from the wheel encoder data are shown in figure 8.

Figure 8: Encoder measurements
The resulting trajectories based on the gyroscope is shown in figure 9. The heading is given by the gyroscope and the vehicle velocity is given by the average of the wheel encoders.

Figure 9: Gyroscope measurements
The resulting trajectories based on the steering angle is shown in figure 10. The heading is given by the steering angle, using the kinematic bicycle model and the vehicle velocity is given by the average of the wheel encoders.

Figure 10: Styr measurements
The resulting trajectories from the Unscented Kalman Filter is shown in figure 11.

Figure 11: UKF results
The resulting trajectory from the Unscented Kalman Filter when driving three laps is shown in figure 12.

Figure 12: UKF, one result, three laps
4.1 Discussion

The rectangular track was placed on the floor inside the office of Alten. The size of the rectangle was constrained by the free space in between working desks and shelves. A tape measure was used to measure the sides of the rectangle and the radius of the corner arcs.

The method used for placing the track allows for some deviation from a perfect rectangular shape. The office environment inhibited the cross measurement of the corners.

The line following algorithm did not center the vehicle between the outer lines. The lateral control is only based on the angle of the observed lines and it does not compensate for lateral position errors. This allowed the vehicle to cut corners.

The encoder measurements was improved significantly after the gears was removed. But still the variance of the measurements was not satisfactory. Another error source was the soft tires allowing variable wheel radius. The resolution of the encoders are 500 pulses per revolution giving one pulse per centimeter. A higher encoder resolution is preferred.

The measurements based on the gyroscope showed best repeatability compared to the encoder and the steering angle measurements. The shape of the trajectories is different when driving in each direction. Since the vehicle is driving in the same track the shapes should be each others mirror. A probable cause is different sensor scaling when rotating in different directions.

The measurements based on the steering angle has lower variance in one direction compared to the other direction. This is probably due to some slack in the linkage between the wheels and the steering angle sensor.

The resulting trajectories from the Unscented Kalman Filter showed significantly better trajectories then each of the measurements individually. The filter was heuristically tuned to fit the track. Still there are some small differences between the driving directions which is induced by the same type of direction differences as in the individual measurements.

The last plot shows the trajectory of the vehicle driving three laps around
the track. As expected the accuracy of the estimation diverges. The trajec-
tory leaves the track after about one and a half lap. Using a road side sensor
to reset the estimation each lap would allow for continuous positioning if the
vehicle is driving in constant speed. This could be used for demonstration
purposes on the given track.
5 Conclusion and Future Work

A positioning system has been designed and implemented on a demonstrator. An Unscented Kalman Filter has been used to fuse three types of angular rate measurements of the heading to improve the estimation. The three measurements are obtained from a gyroscope placed over the rear wheel axis, from the differential wheel speeds of the rear wheels and from the steering angle by using a kinematic model. Data was acquired by the demonstrator when it was autonomously driving on a rectangular track in both directions and the vehicle was controlled with a line following algorithm. The data showed different results depending on the driving direction. Therefore the design included three driving modes: driving straight, turning left and turning right. The driving modes has different covariance matrices in the Unscented Kalman Filter calculations and which driving mode used is decided by the steering angle. The Uncented Kalman Filter fusion of three separate measurements of angular rate showed significantly better results then the results of each individual measurement.

Future work

Due to the availability of both ARM processors and FPGA on the Zynq SoC, future work should include an implementation on both. The computations can be divided such that heavy matrix calculations such as the Cholesky decomposition can be computed on the FPGA and the backbone of the algorithm can be run on the ARM.

A drawback in the current setup was the accuracy of the sensors. The encoder measurements could give lower variance with higher resolution. A more accurate gyroscope can give more likewise results when rotating in both directions.

The next step is to integrate this estimation with absolute measurements and map-information. Absolute measurements can be achieved by combining camera and lidar where the camera can detect unique stationary objects.
and the lidar can measure the distance to these objects. Map-information containing the position of unique stationary objects will enable robust positioning of the vehicle.
6 References


[15] Cyber-physical systems

http://cyberphysicalsystems.org

[16] Intelligent Transportation systems

https://ec.europa.eu/transport/themes/its_en

