Footbridge Dynamics

Human-Structure Interaction

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Abstract

For aesthetic reasons and due to an increased demand for cost-effective and environmentally friendly civil engineering structures, there is a trend in designing light and slender structures. Consequently, many modern footbridges are susceptible to excessive vibrations caused by human-induced loads. To counteract this, today’s design guidelines for footbridges generally require verification of the comfort criteria for footbridges with natural frequencies in the range of pedestrian step frequencies. To ensure that a certain acceleration limit is not exceeded, the guidelines provide simplified methodologies for vibration serviceability assessment.

However, shortcomings of these methodologies have been identified. First, for certain footbridges, human-structure interaction (HSI) effects might have a significant impact on the dynamic response. One such effect is that the modal properties of the bridge change in the presence of a crowd; most importantly, the damping of the bridge is increased. If this effect is neglected, predicted acceleration levels might be overestimated. Second, as a running person induces a force of greater amplitude than a walking person, a single runner might cause a footbridge to vibrate excessively. Hence, the running load case is highly relevant. These two aspects have in common that they are disregarded in existing design guidelines.

For the stated reasons, the demand for improvements of the guidelines is currently high and, prospectively, it might be necessary to require the consideration of both the HSI effect and running loads. Therefore, this licentiate thesis aims at deepening the understanding of these subjects, with the main focus being placed on the HSI effect and, more precisely, on how it can be accounted for in an efficient way.

A numerical investigation of the HSI effect and its impact on the vertical acceleration response of a footbridge was performed. The results show that the HSI effect reduces the peak acceleration and that the greatest reduction is obtained for a crowd to bridge frequency ratio close to unity and a high crowd to bridge mass ratio. Furthermore, the performance of two simplified modelling approaches for consideration of the HSI effect was evaluated. Both simplified models can be easily implemented and proved the ability to predict the change in modal properties as
well as the structural response of the bridge. Besides that, the computational cost was reduced, compared to more advanced models.

Moreover, a case study comprising field tests and simulations was performed to investigate the effect of runners on footbridges. The acceleration limit given in the design guideline was exceeded for one single person running across the bridge while a group of seven people walking across the bridge did not cause exceedance of the limit. Hence, it was concluded that running loads require consideration in the design of a footbridge.

**Keywords:** Dynamics, vibration, footbridge, pedestrian bridge, human-structure interaction, runner, running loads, simplified models.
Sammanfattning


Brister i dessa beräkningsmetoder har emellertid identifierats. För det första kan olika typer av människa-bro-interaktion (HSI)¹ ha en betydande inverkan på responsen hos vissa broar. Exempel på en HSI-effekt är att brons modala egenskaper förändras när människor befinner sig på bron; i huvudsak sker en ökning av brons dämpning. Om denna effekt inte tas i beaktande föreligger stor risk att överskatta förväntade accelerationsnivåer. För det andra är kraften från en löpare större än kraften från en gående person vilket gör att en ensam löpare på en gångbro kan ge upphov till accelerationsnivåer som överskriver gränsvärdena för komfort. Löpande personer är därför ett mycket relevant lastfall. Befintliga normer uttrycker inte explicit att någon av dessa aspekter bör tas i beaktande.

Behovet av förbättrade riktlinjer för hur normerna bör tillämpas är därför mycket stort och i framtiden kan det bli nödvändigt att kräva att både HSI-effekter och löparlastar tas i beaktande. Därför syftar denna licentiatavhandling till att bidra till en fördjupad förståelse inom dessa två ämnen, med huvudfokus på ovan nämnda HSI-effekt i allmänhet och hur den kan beaktas på ett enkelt, noggrant och tidseffektivt sätt i synnerhet.

En numerisk undersökning av HSI-effekten och dess inverkan på den vertikala responsen hos en gångbro genomfördes. Resultaten visar att HSI-effekten reducerar

¹Av engelskans Human-Structure Interaction.

Effekten av löpare på gångbroar studerades genom en fallstudie med fältmätningar. Utifrån resultaten från dessa fältmätningar kunde det konstateras att accelerationsgränsen som anges i normerna överskreds när en ensam löpare sprang över bron men inte när en grupp på sju personer gick i takt över samma bro. Därför drogs slutsatsen att löparlaster bör tas i beaktande vid dimensionering av en gångbro.

Nyckelord: Dynamik, vibration, gångbro, människa-bro-interaktion, löpare, löparlaster, förenklade modeller.
Preface

The research work presented in this thesis was carried out at the Department of Civil and Architectural Engineering, KTH Royal Institute of Technology. The research was conducted under the supervision of Professor Raid Karouni, Dr. Mahir Ülker-Kaustell and Dr. Andreas Andersson to whom I would like to express my sincerest gratitude for their support and professional guidance which has truly been invaluable for me.

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Stockholm, April 2018

Emma Zäll
List of Relevant Publications


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<td>Coupled crowd-bridge model</td>
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<td>Deterministic crowd</td>
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<td>Dynamic load factor</td>
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<td>Degree of freedom</td>
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Chapter 1

Introduction

1.1 Background

Due to their slenderness, pedestrian bridges are often prone to excessive vibrations caused by human-induced loads; two well-known examples are the London Millennium Bridge (Dallard et al., 2001a,b) and the Solférino Bridge in Paris (Sétra, 2006). The liveliness of such bridges is strongly connected to various kinds of interaction effects. One such effect is that in the coupled crowd-bridge system, the modal properties of the crowd and the bridge vary while the pedestrians walk across the bridge. This human-structure interaction (HSI) effect is mainly related to high pedestrian densities (Van Nimmen et al., 2017). Moreover, if this HSI effect is not accounted for in the dynamic analysis of a footbridge subjected to human-induced loads, the predicted acceleration response might be significantly under- or overestimated (Shahabpoor and Pavic, 2012; Živanović, 2012).

Today’s design guidelines for footbridges generally require verification of the vibration serviceability limit state for bridges with natural frequencies in the range for the pedestrian step frequency, commonly below 5 Hz. To ensure the comfort for the pedestrians on the bridge, the expected acceleration level is required to be below a certain limit. In the vertical direction, a common value for this limit is 0.5 m/s². Simplified methodologies for verification of the vibration serviceability limit state are provided in the design guidelines. These methodologies do, however, despite the need mentioned above, not recommend to account for any HSI effects.

Furthermore, due to an increased interest in recreational running, jogging people have become a more frequent load case on footbridges lately. Several studies have indicated that runners on pedestrian bridges might cause very high acceleration levels to occur. In spite of this, runners is most often not a required load case in the design phase of a footbridge (EN, 1991; Sétra, 2006; Feldmann et al., 2008).
1.2 Aims and scope

The long term aim of this research project is to develop an improved design methodology that can be incorporated in the design guidelines for pedestrian bridges. The prospected improvements concern the consideration of HSI effects and running loads, which are both neglected in existing design guidelines. The aim of this licentiate thesis is to deepen the understanding of the two subjects respectively to take a step in the direction of a future improvement of the design guidelines. More precisely, taking into consideration the amount of previous knowledge and performed research within the two fields, the objectives of this licentiate project are to:

1. Develop time efficient models for prediction of the dynamic response of a footbridge under consideration of the HSI effect and evaluate their performance.
2. Perform measurements on an in-service footbridge to investigate if there is a need for further research about running loads on pedestrian bridges.

The present study is subject to a number of limitations and simplifying assumptions. First, it is limited to consider vertical vibrations; no lateral, torsional or longitudinal modes are studied. Second, the numerical study of the HSI effect considers exclusively simply supported bridges, which is, however, a reasonable limitation since many pedestrian bridges are of such kind. Third, the crowd is assumed to be uniformly distributed. This assumption is due to the main focus being on the HSI effect, which is mainly related to high pedestrian densities and, hence, uniformly distributed crowds. Finally, the experimental work is limited to the case study related to running loads. Further verification of remaining numerical results is not within the scope of this thesis, but belongs to the continuation of the research work.

1.3 Scientific contribution

The performed research work has resulted in the following scientific contribution:

- A thorough analysis of the HSI effect and its impact on the structural response of a simply supported footbridge subjected to the load from a uniformly distributed crowd has been performed.
- A simple and time efficient, but yet accurate model for consideration of the HSI effect has been developed and its performance has been evaluated in terms of accuracy of the results as well as computational time.
- The performance of a model that accounts for the HSI effect in a very simple and time efficient manner has been evaluated in terms of accuracy and computational time.
- The need to consider running loads in the vibration serviceability assessment of footbridges has been experimentally and numerically verified.
1.4 Outline of thesis

The present chapter gives a brief introduction to the subject which aims at motivating the performed research work as well as presenting its aims and limitations.

Chapter 2 highlights the different aspects related to footbridge dynamics and gives the reader an overview of the research field. For the aspects being directly related to the research work performed in the present licentiate project, a fairly thorough review will be given while more peripheral aspects will only be briefly reviewed. Furthermore, in the presentation of the research field, it will be specified how each topic is related to the performed research work and, if relevant, how it has been incorporated.

The main purpose of Chapter 3 is to prepare for the evaluation of the performance of the simplified modelling approaches why a deep understanding of the HSI effect as such is required. To this end, the theoretical background to two commonly used modelling approaches for dynamic analysis of footbridges subjected to human-induced loads is first presented. One of the modelling approaches accounts for the HSI effect while the other one does not. Second, these two models are used to perform a parametric study and a Monte Carlo simulation with the objective to gather a profound knowledge about the HSI effect and its impact on the structural response. Furthermore, the impact of pedestrian inter-subject variabilities on the structural response is also studied.

The model mentioned above that accounts for the HSI effect is then used as a reference model in Chapter 4, which concerns two simplified modelling approaches for prediction of the dynamic response of a footbridge under consideration of the HSI effect. First, the theoretical background to the two simplified modelling approaches is presented. Second, the models are compared to the reference model. The comparison comprises a parametric study with a deterministic crowd as well as a Monte Carlo simulation with stochastic crowds. The performance of the simplified models is evaluated based on the accuracy of the predicted HSI effect and acceleration response. In addition to that, the computational time is also compared.

Chapter 5 presents the performed research work related to running loads. With the aim of motivating further research about running loads on pedestrian bridges, the objectives of the chapter are to investigate if the effect of runners on a footbridge is negligible with respect to vibration amplitudes and resonance and to evaluate three modelling approaches for dynamic analysis of footbridges subjected to loads induced by running people. Besides that, a review of how running is considered in today’s design guidelines is presented.

Finally, in Chapter 6, general conclusions from the performed research work are summarised and suggestions for further research are given.
Chapter 2

Dynamic analysis of footbridges

This chapter aims at highlighting the different aspects related to footbridge dynamics and giving the reader an overview of the field. Simply put, the different subproblems related to footbridge dynamics are; first, the footbridge; second, the crowd; and third, different kinds of interaction effects that appear both between different pedestrians in the crowd and between the crowd and the footbridge. There exist several excellent review papers covering parts of as well as the whole research field (Živanović et al., 2005b; Ingólfsson et al., 2012; Venuti and Bruno, 2009; Racic et al., 2013, 2009; Sachse et al., 2003). Therefore, this literature review focuses mainly on the subjects related to the performed research work and aims at presenting the information needed for the reader to more easily follow the rest of the thesis.

2.1 Footbridge dynamics

Generally, footbridges are light and slender structures. As a consequence, they are prone to excessive vibrations caused by human-induced loads. Therefore, in the design of a footbridge, it is often required to perform a dynamic analysis to verify the vibration serviceability of the bridge. The dynamic behaviour of a footbridge is governed by its modal properties, i.e. mass, frequency and damping. A concise and structured overview of the modal properties of more than 160 footbridges was provided by Van Nimmen (2015). Based on the large set of data, it could be concluded that the first set of natural frequency is often found below 5 Hz and that footbridges are lightly damped structures. A majority of the studied bridges had damping ratios below 1% and the highest value was no higher than 3%. The susceptibility to dynamic pedestrian loading which is something that many footbridges have in common is a result of these low damping ratios in combination with their natural frequencies being in the range for the pedestrian step frequency.
In the dynamic analysis of footbridges, modal analysis is commonly used, meaning that the bridge is described by its mode shapes. Generally, to obtain an accurate prediction of the structural response, it suffice to include a limited number of mode shapes, having frequencies in the range of the frequency content of the dynamic load. More precisely, for dynamic analysis of footbridges, only mode shapes having a frequency in the range for pedestrian step frequencies have to be considered. In the research work summarised in this thesis, modal analysis was used, with no exceptions.

2.2 Crowd dynamics

The load from a walking crowd changes simultaneously in time and space. The variation in space is caused by the collective evolution of the crowd along the walkway, e.g. along a footbridge. Due to the pedestrians’ intelligent behaviour, such motion features their ability to interact with each other and the surroundings, their ability to adjust their velocity depending on the situation and also the inhomogeneity in their interaction with the surroundings (Venuti and Bruno, 2009; Cristiani et al., 2010). These abilities give rise to interaction effects; first, there are human-human interactions, meaning that the pedestrians adjust their walking trajectory in order to avoid colliding, and that nearby pedestrians tend to adapt to each others step frequencies (Zivanovic, 2015). Second, the pedestrians tend to synchronise their step frequency with the movements in the bridge, resulting in a resonant amplification of the vibrations which may lead to discomfort for the pedestrians. This interaction effect is commonly known as the lock-in phenomenon. However, lock-in is self-limiting because people tend to lose their balance and stop walking when large vibrations occur (Živanović et al., 2005b).

Crowd related phenomena can be observed with reference to three different scales; macroscopic, mesoscopic and microscopic; where macroscopic refers to describing the crowd by means of average quantities, mesoscopic refers to describing the state of the system by means of probability distributions, and microscopic refers to describing each individual pedestrian separately. For application with footbridges, microscopic and macroscopic models are commonly used (Bruno et al., 2016; Venuti and Bruno, 2007; Bruno et al., 2011). Modelling of crowd dynamics is, however, out of the scope of this research project.

2.3 Human-induced loads

The variation in space, caused by the evolution of the crowd along the walkway, obviously leads to a variation in time. Besides this time dependency, each person produces a dynamic, time varying force, usually referred to as the ground reaction force (GRF). It goes without saying that the GRF differs for different activities;
walking and running will be considered in the present work. Other activities such as bobbing, jumping and vandal loading are out of the scope of this thesis.

2.3.1 Walking loads

The research work about the walking GRF aims at, 1) develop measurement techniques and methodologies that are reliable not only for determination of the force from a single pedestrian but from a whole crowd, and 2) mathematically describe the force in an as simple as possible manner. In two extensive literature reviews, Racic et al. (2009, 2013) present advances within both experimental and analytical approaches.

The GRF is characterised by inter-subject variability, which refers to its variation from one pedestrian to another, and intra-subject variability, which refers to an individual’s inability to produce two identical steps. If these variabilities are not accounted for in the vibration serviceability assessment of a footbridge, the dynamic response might be overestimated (Van Nimmen et al., 2014b). Moreover, due to human-structure interaction, the GRF on a perceptibly vibrating surface is different compared to the one on a rigid floor. Similarly as for the inter- and intra-subject variabilities, neglecting this effect might lead to overestimation of the dynamic response (Caprani et al., 2015; Živanović et al., 2005a).

Thanks to the (almost) periodic nature of the pedestrian footfalls, a sum of Fourier harmonic components is commonly used to mathematically describe the walking load in the time domain according to

\[ F_{pl}(t) = G + \sum_{i=1}^{\infty} Ga_i \sin(2\pi f_{pl}t + \theta_i), \] (2.1)

where \( G \) is the pedestrian’s weight, \( f_{pl} \) is the pedestrian’s step frequency, \( i \) is the harmonic considered and \( a_i \) and \( \theta_i \) are respectively the dynamic load factor (DLF) and the phase shift of the \( i \):th harmonic (Živanović et al., 2005a). In this research project, the pedestrian step frequencies and the phase shifts were generated from, respectively, the normal and uniform distributions used by Caprani and Ahmadi (2016) according to

\[ f_{pl} \in \mathcal{N}(1.96, 0.209) \text{ Hz}, \quad \theta_i \in \mathcal{U}(0, 2\pi). \] (2.2)

The simplest possible modelling alternative is to use only the first harmonic term in (2.1). The dynamic response of a footbridge might, however, be significantly overestimated if such a method is applied (Pimentel, 1997; Dey et al., 2016). Therefore, a common choice is to account for the first four harmonics. Furthermore, a great amount of experimental studies has been performed, aiming at determining the values of the DLFs. Comprehensive summaries can be found in the literature (Živanović et al., 2005b; Racic et al., 2009). Throughout the work with this thesis,
the GRF caused by a walking pedestrian was described by the first four harmonics in (2.1) and the DLFs proposed by Young according to

\[
\begin{align*}
    a_1 &= 0.41(f_{pl} - 0.95), \\
    a_2 &= 0.069 + 0.0056f_{pl}, \\
    a_3 &= 0.033 + 0.0064f_{pl}, \\
    a_4 &= 0.013 + 0.0065f_{pl}.
\end{align*}
\]  

These DLFs are valid for step frequencies in the range 1 - 2.8 Hz. Moreover, under the assumption that \( f_{pl} \) and \( \theta_i \) are generated from appropriate distributions, the mathematical model (2.1) accounts for inter-subject variabilities. However, intra-subject variabilities as well as human-structure interaction effects are neglected.

A sum of Fourier harmonic components (2.1) gives a periodic function while the GRF is only near periodic due to the intra-person variability. Van Nimmen et al. (2014b) developed an imperfect, near-periodic function which proved to give a more accurate representation of the GRF. Other examples of more advanced force models that account for inter- and intra-subject variabilities as well as variations in the GRF caused by liveliness of the structure, have been proposed by Živanović et al. (2007) and Racic and Brownjohn (2011).

2.3.2 Running loads

Despite the increased interest in recreational running and the fact that footbridge dynamics has been the center of attention for many researchers during the latest couple of decades, a majority of existing publications concerns walking loads only; regarding running loads, there is not very much to read. One reason for this could be that current design guidelines have no or little information about running loads and it is most often not a required load case in the design phase (EN, 1991; Sétra, 2006; Feldmann et al., 2008). Current design guidelines neglect running loads either without any motivation or with the motivation that the time it takes for a runner to cross a footbridge is not long enough for the steady state response to be reached.

Besides footbridge dynamics, running loads are related to several other fields of study and, hence, it is a well-investigated topic. For example, researchers within the fields of biomechanics have provided the characterization of the running GRF and also how it relates to other relevant parameters such as speed and foot placement (Keller et al., 1996; McClay and Cavanagh, 1994). Nevertheless, the number of studies related to running loads in applications with footbridges is limited.

In early works concerning running loads, the GRF is mathematically described in a similar way as is usually done for walking, see (2.1). Though, an important difference between walking and running is the discontinuity in the running GRF, corresponding to the “flying” phase in between two consecutive steps (Racic and Morin, 2014). Occhiuzzi et al. (2008) proposed a load model which takes this discontinuity into account and used it to predict vertical bridge deck accelerations.
of a footbridge excited by running pedestrians, though without reaching very good agreement with measurements.

To the best of the author’s knowledge, the most comprehensive study published so far, addressed this issue by presenting a database including no less than 458 individual running GRF signals recorded on a rigid floor, together with two attempts to mathematically describe the measurements (Racic and Morin, 2014). The first modelling strategy was a traditional, Fourier-based, perfectly periodic, deterministic load model while the second one was based on the database and featured a more realistic near-periodic, stochastic load model. The stochastic model proved to be able to replicate the measured forces.

Furthermore, there are studies mainly related to walking loads, indicating that running loads might be crucial for the dynamic performance of footbridges. For example, in an experimental study, four times higher acceleration levels were reached for a group of people running, compared to the same group of people walking (Lai et al., 2017). Similar results were shown in a study considering the assessment of vibration comfort criteria for steel footbridges (Pańtak et al., 2012). In neither of these cases, the results related to running were further analysed.

### 2.4 Coupled crowd-bridge system

In the coupled crowd-bridge system, the dynamic properties of the two subsystems change while the crowd walks along the footbridge. This human-structure interaction (HSI) effect has been the main focus throughout the performed research work, and in the remainder of this thesis, HSI refers to this effect. The effect has been quite intensively studied lately, both experimentally and numerically. Sachse et al. (2003) summarise advances in a literature review, which also covers how the GRF is affected by liveliness of the supporting structure.

In an experimental study, Ellis and Ji (1997) performed in-situ measurements on a grandstand, subjected to crowd loading, and laboratory measurements on a simply supported beam, loaded by one person adopting different postures and activity rates. A change in frequency and damping for the grandstand was observed. For the beam, the natural frequency changed due to the presence of a passive person, but not an active person. Therefore, it was concluded that in case of active persons, HSI can be neglected. However, later works have indicated that also active people affect the modal properties. Živanović et al. (2009) studied the effect by comparing frequency response functions (FRFs) obtained from measurements performed on an empty structure and the same structure loaded by a standing and a walking crowd. It was concluded that both a standing and a walking crowd increased the effective damping of the structure, though more significantly for a standing crowd. In another study, the same author studied modelled FRFs, obtained by using a
sophisticated force model (Živanović et al., 2007) in combination with an SDOF model of the bridge, adjusted to have its measured frequency. Great discrepancies between modelled and measured FRFs were obtained. This was concluded to be caused by the HSI effect, which could also be quantified in terms of an increase in damping from 0.26% to 1.02% (Živanović, 2012).

Moreover, Georgakis and Jørgensen (2013) performed a series of forced vibration tests with walking pedestrians in laboratory conditions and concluded that the damping induced by each pedestrian was positive for all amplitudes and that the added mass of a single pedestrian was invariant to the amplitude of vibration and the pedestrian flow rate. Furthermore, in contradiction with the conclusion in (Živanović et al., 2009), Kasperski (2014) performed experiments and concluded that an active pedestrian affects the effective damping more than a passive pedestrian and that the effect increases with the number of people on the bridge. Shahabpoor et al. (2017b) verified these statements in a study based on measured as well as analytically generated FRFs. They also reported that the frequency of the joint moving human-structure system was higher than that of the empty structure while it was lower when the same people were stationary.

A straightforward approach to modelling footbridge dynamics is to describe the crowd as a time dependent, distributed load. This method is adopted in current design guidelines (Sétra, 2006; Feldmann et al., 2008). Such models neglect the HSI effect and it has been shown that it might result in an overestimation of the predicted acceleration response (Shahabpoor and Pavic, 2012). Another approach is to describe the pedestrians as time dependent moving loads, which again means that the HSI effect is neglected and the predicted acceleration response might be overestimated (Živanović, 2012).

Several methods to account for the HSI effect have been proposed, for example, to describe the walking pedestrian as an inverted pendulum (Qin et al., 2013). In the approach most commonly used in today’s research work (Živanović, 2015; Venuti et al., 2016; Shahabpoor et al., 2017a; Jiménez-Alonso et al., 2016), including this thesis, a pedestrian is modelled as a spring-mass-damper (SMD) system. The SMD enables to account for the fact that the human is a dynamic system having its own dynamic properties. This idea was initially proposed by Archbold (2004, 2008).

Many studies have focused on determination of the dynamic properties of an SMD representing a pedestrian in different postures (Shahabpoor et al., 2016; Van Nimmen, 2015). An extensive literature review on the subject was performed by Jones et al. (2011); they concluded that reported values of the natural frequency are in the range 3.3-10.4 Hz and damping ratios are in the range 33-69%. In this research project, a normal distribution for the pedestrian mass, previously used by Venuti et al. (2016), and normal distributions for frequency and damping ratio for an active
body posture, as reported by Van Nimmen (2015), were adopted according to

\[
m_p \in \mathcal{N}(75, 15) \text{ kg}, \quad f_p \in \mathcal{N}(3.25, 0.32) \text{ Hz}, \quad \xi_p \in \mathcal{N}(0.3, 0.05).
\]  

Furthermore, Agu and Kasperski (2011) investigated the influence of the inter-subject variabilities in the dynamic properties of the pedestrians on the HSI effect. They concluded that if the inter-subject variabilities are neglected, too high damping ratios might be obtained, which in turn might result in estimations of the structural response that are on the unsafe side. Moreover, several authors have studied how the frequency and damping of the bridge vary in time while a single pedestrian or a crowd walk across a footbridge and concluded that the presence of the crowd causes the frequency of the bridge to decrease and the damping to increase (Qin et al., 2013; Venuti et al., 2016; Caprani and Ahmadi, 2016). The maximum acceleration response does, however, not always coincide with the pair of minimum frequency and maximum damping, which was shown by Ahmadi and Caprani (2015). To the best of the author’s knowledge, the most comprehensive analysis of the HSI effect and its impact on the dynamic response of footbridges to pedestrian excitation was performed by Van Nimmen et al. (2017). The results showed that for footbridges with frequencies below 6 Hz, the HSI effect leads to an effective damping ratio of the bridge which is higher than the inherent damping ratio of the empty structure, and that, in many cases, accounting for the HSI effect leads to a significant reduction of the structural response.

2.5 Design guidelines

In general, international building and bridge codes of today claim that for footbridges with natural frequencies in the range for the pedestrian step frequency, verification of the vibration serviceability limit state is required. For footbridges, the dynamic performance and comfort level are evaluated based on the expected maximum acceleration. Hence, an acceleration limit is given. The proper value of this limit is related to the experienced comfort for a pedestrian, which is of course very subjective since it is characterised by a person’s perceptiveness to vibrations (Živanović et al., 2005b). This issue can be considered one of the slightly more peripheral aspects related to footbridge dynamics and it is, hence, not within the scope of this thesis.

The frequency range in which comfort assessment is required and also the acceleration limit differ from one design guideline to another. Some examples are, first, the European Standard Eurocode, in which it is stated that for bridges with a fundamental frequency below 3 Hz, verification of the comfort criteria shall be performed, for frequencies in the range 3 - 5 Hz, it may be specified for the particular project, and for frequencies above 5 Hz, it can be assumed that no verification is needed. Furthermore, regarding an appropriate limit for the acceleration levels, it should be defined for the individual project, but a recommended maximum value for vertical
acceleration for any part of the bridge deck is given as \( \min(0.5\sqrt{f_0}, 0.7) \) m/s\(^2\) where \( f_0 \) is the fundamental frequency of the bridge (EN, 1991).

Second, according to the French Sétra Guideline, assessment of the comfort criteria is required for bridges with a natural frequency below 5 Hz. The acceleration limit varies depending on the footbridge class, related to the level of traffic, and the comfort level, determined by the owner of the bridge. As an example, for vertical vibrations, the highest level of comfort requires a maximum acceleration no larger than 0.5 m/s\(^2\) (Sétra, 2006).

Third, according to the European HiVoSS (Human-induced Vibration of Steel Structures) guideline, it is stated that for natural frequencies in the range 1.25 - 2.3 Hz, dynamic analyses are required. It is mentioned that also the second harmonic in the walking load can cause resonance and to avoid this, the range should instead be 1.25 - 4.6 Hz. However, it is indicated that this does not need to be considered since there are no reported cases in the literature. Furthermore, similarly to the Sétra guideline, both the level of traffic and the required comfort level affect the acceleration limit in the comfort assessment, and also in this guideline, the highest level of comfort requires a maximum vertical acceleration of 0.5 m/s\(^2\) (Feldmann et al., 2008).

Moreover, some of the guidelines provide simplified methodologies for verification of the vibration serviceability limit state. Regarding walking, most guidelines have agreed to cover this issue by modelling the crowd as an equivalent, deterministic load, evenly distributed on the bridge deck. The load is scaled from single person force measurements and adjusted to different amplitudes, depending on the density of the crowd. The movement of the crowd is most often not considered in the guidelines and interaction effects are commonly neglected.

Several evaluations of existing guidelines point at the need for improvements, e.g. by consideration of the uncertainty in the predicted modal properties of the footbridge (Van Nimmen et al., 2014a) and by consideration of the HSI effect (Shahabpoor and Pavic, 2012; Pimentel et al., 2001). Another shortcoming which many guidelines have in common is that not much attention is paid to characterisation and modelling of running loads. A more thorough review of how running loads are treated in existing design guidelines is presented in section 5.1.
Chapter 3

Modelling of the crowd-bridge system

The objectives of this chapter are, first, to present two models for dynamic analysis of pedestrian bridges, and second, to investigate how the modal properties of a footbridge change in the presence of a crowd and what effect the changes have on the structural response. First, section 3.1 presents a simple moving loads model which does not account for the human-structure interaction (HSI) effect mentioned above, i.e. the change in modal properties of the footbridge. Thereafter, section 3.2 describes a coupled crowd-bridge model which is based on a model initially proposed by Caprani and Ahmadi (2016). As opposed to the moving loads model, the coupled crowd-bridge model accounts for the HSI effect. Finally, in section 3.3, the two models are used to perform an analysis of the HSI effect and its influence on the dynamic response of a pedestrian bridge.

In this thesis, unless otherwise stated, a 30 m long, simply supported footbridge with a density of 500 kg/m$^3$ and a rectangular cross section of width 3 m and height 1 m is considered. The bridge is described by its first bending mode according to $\phi_b(x) = \sin(\pi x / L)$, where $L$ is the length of the bridge. The natural frequency and damping ratio of the first bending mode are, respectively, 2 Hz and 1%. Accordingly, the bridge has a mass per unit length of 1500 kg/m and an approximate bending stiffness of 2 GNm$^2$. This bridge is henceforth referred to as the reference bridge; its properties were chosen to be representative for existing footbridges, see section 2.1, and for the bridge to be susceptible to the HSI effect and to pedestrian excitation. In the rest of this thesis, the studied bridges are introduced by stating only the properties that differ from the reference bridge.

Furthermore, with no exceptions, a uniformly distributed crowd consisting of pedestrians walking on a line with an equal inter-pedestrian distance of 1 m and a speed
of 1.25 m/s is considered. The influence of the inter-pedestrian distances and speeds is outside the scope of this thesis why this walking pattern was kept throughout the whole study. Nevertheless, a minor study with other pedestrian speeds was performed to verify that the pedestrian speed does not have a significant impact on the results. The results are, however, not presented herein.

3.1 Moving loads model

All modelling approaches used in the research work are based on the assumption that modal analysis with a reduced number of mode shapes suffices to describe the bridge. The moving loads model, which is henceforth referred to as ML, does however differ from the other models in that it also relies on an assumption that the HSI effect can be neglected, which is not true for any of the other models. Consequently, ML comprises a modal domain representation of the bridge, combined with time dependent moving loads to describe the pedestrians. The equation of motion for one mode of the bridge is written as

\[ m_b \ddot{q}_b + c_b \dot{q}_b + k_b q_b = \phi_b \sum_{r=1}^{N_p} F_{p,r}(t), \]

where \( m_b, c_b \) and \( k_b \) are the modal mass, damping and stiffness of the considered bridge mode, having a unity normalised mode shape \( \phi_b \). The modal properties of the \( n \):th mode of the bridge are calculated as

\[ m_{b,n} = \int_0^L m(x) [\phi_{b,n}(x)]^2 dx, \quad c_{b,n} = 2\xi_{b,n} m_{b,n} \omega_{b,n}, \quad k_{b,n} = \omega_{b,n}^2 m_{b,n}, \]

where \( L \) and \( m(x) \) are the length and mass per unit length of the bridge while \( \phi_{b,n}(x), \omega_{b,n} \) and \( \xi_{b,n} \) are, respectively, the unity normalised mode shape, angular natural frequency and damping ratio for the \( n \):th mode of the bridge. Furthermore, \( q_b, \dot{q}_b \) and \( \ddot{q}_b \) in (3.1) are the generalised displacement, velocity and acceleration of the considered bridge mode while \( F_{p,r}(t) \) is the force applied by the \( r \):th pedestrian at time \( t \). Any available model for the pedestrian loading \( F_{p,r}(t) \) may be used. Throughout this research project, the model defined by (2.1) with four harmonics in the Fourier series and the dynamic load factors proposed by Young (Živanović et al., 2005b) was used, exclusively.

The dynamic response of the considered bridge mode, represented in the modal domain, is found by solving (3.1). For that purpose, numerous different methods can be used. In this work, however, numerical time integration using Newmark’s average acceleration method was applied to solve the equation of motion, with no exceptions. To include several mode shapes in the analysis of the dynamic
response, this procedure is done for each mode shape separately and the total dynamic response of the bridge is found according to

\[ u(x,t) = \sum_{n=1}^{N_m} \phi_{b,n}(x)q_{b,n}(t), \]

(3.3)

where \( q_{b,n}(t) \) is the generalised displacement of the \( n \):th mode of the bridge at time \( t \), i.e. the solution to (3.1).

### 3.2 Coupled crowd-bridge model

The coupled crowd-bridge model, which is henceforth referred to as CCBM, relies on the assumption that a pedestrian can be well approximated by a spring-mass-damper (SMD) system (Brownjohn, 2001) combined with a time dependent force corresponding to the load from the footstep. Such description of a pedestrian enables to account for the HSI effect. Consequently, CCBM comprises a modal domain representation of the bridge which is coupled to the SMD systems representing the pedestrians. Considering a case with \( N_m \) modes for the bridge and \( N_p \) pedestrians, this results in a system of order \( N = N_m + N_p \).

The equation of motion for a pedestrian (idealised as a SMD system) is written as

\[
m_p \ddot{q}_p + c_p \left( \dot{q}_p - \sum_{n=1}^{N_m} \phi_{b,n} \dot{q}_{b,n} \right) + k_p \left( q_p - \sum_{n=1}^{N_m} \phi_{b,n} q_{b,n} \right) = 0, \tag{3.4}
\]

where \( m_p, c_p \) and \( k_p \) are respectively the mass, damping and stiffness of the pedestrian. The vertical displacement, velocity and acceleration of the pedestrian are designated \( q_p, \dot{q}_p \) and \( \ddot{q}_p \). Furthermore, \( q_{b,n} \) is the modal coordinate describing the generalised displacement of the \( n \):th displacement normalised mode shape of the bridge, denoted \( \phi_{b,n} \). Accordingly, \( \dot{q}_{b,n} \) is the generalised velocity of the \( n \):th mode.

Similarly, the equation of motion for one mode of the bridge is written as

\[
m_b \ddot{q}_b + c_b \dot{q}_b + k_b q_b = \phi_b \sum_{r=1}^{N_p} \left( F_{p,r}(t) - m_{p,r} \ddot{q}_{p,r} \right), \tag{3.5}
\]

where \( m_b, c_b \) and \( k_b \) are the modal mass, damping and stiffness of the considered bridge mode, having the mode shape \( \phi_b \). Moreover, \( q_b, \dot{q}_b \) and \( \ddot{q}_b \) in (3.1) are the generalised displacement, velocity and acceleration for the considered bridge mode, while \( m_{p,r} \) and \( \ddot{q}_{p,r} \) are the mass and vertical acceleration of the \( r \):th pedestrian, which applies the force \( F_{p,r}(t) \) at time \( t \). By combining (3.4) and (3.5), the equation of motion for the coupled crowd-bridge system is obtained as

\[
M \ddot{q} + C \dot{q} + K q = F, \tag{3.6}
\]
The mass, damping and stiffness matrices \( M, C \) and \( K \) as well as the force vector \( F \) are time dependent, which is caused by the time dependency of the pedestrians’ positions. Though, for simplicity, the time dependency is suppressed in the following equations. Consequently, the mass, damping and stiffness matrices in (3.6) are given by

\[
M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, \quad C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}, \quad K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix},
\]

where \( M_{11} = \text{diag}(m_{b,n}), \quad M_{12} = m_{p,r}\phi_{b,n}(x_{p,r}), \quad M_{21} = 0_{N_p \times N_m}, \quad M_{22} = \text{diag}(m_{p,r}), \)

\[
C_{11} = \text{diag}(c_{b,n}), \quad C_{12} = 0_{N_m \times N_p}, \quad C_{21} = -c_{p,r}\phi_{b,n}(x_{p,r}), \quad C_{22} = \text{diag}(c_{p,r}),
\]

\[
K_{11} = \text{diag}(k_{b,n}), \quad K_{12} = 0_{N_m \times N_p}, \quad K_{21} = -k_{p,r}\phi_{b,n}(x_{p,r}), \quad K_{22} = \text{diag}(k_{p,r}),
\]

where \( n = 1, \ldots, N_m, \quad r = 1, \ldots, N_p \) and \( m_{p,r}, \quad c_{p,r}, \quad k_{p,r} \) and \( x_{p,r} \) are the mass, damping, stiffness and position of the \( r \):th pedestrian. Furthermore, \( m_{b,n}, \quad c_{b,n} \) and \( k_{b,n} \) are the modal mass, damping and stiffness for the \( n \):th mode of the bridge, calculated according to (3.2), while \( q \) and \( F \) in (3.6) are the displacement and load vectors according to

\[
q = \begin{bmatrix} q_{b,n} \\ q_{p,r} \end{bmatrix}, \quad F = \begin{bmatrix} \sum_{r=1}^{N_p} F_{p,r}(t)\phi_{b,n}(x_{p,r}) \\ 0_{N_p \times 1} \end{bmatrix}.
\]

The dynamic response of the coupled crowd-bridge system, represented in the modal domain, is found by solving (3.6), with \( F_{p,r}(t) \) being, in this work, modelled as (2.1). Finally, (3.3) is used to obtain the total dynamic response of the bridge.

### 3.2.1 Eigenvalue analysis of the coupled system

In the coupled crowd-bridge system, the modal properties of the different subsystems vary while the pedestrians walk across the bridge. The instantaneous modal properties can be found by using the state-space method (Qin et al., 2013). To express the coupled crowd-bridge system in the state-space form, a new variable \( u = [q \quad \dot{q}]^T \) is introduced. This enables to rewrite (3.6) as

\[
\dot{u} = Au + B,
\]

where \( A \) and \( B \) are the system matrix and input matrix according to

\[
A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -M^{-1}F \end{bmatrix}.
\]

The instantaneous modal properties of the system are obtained by performing an eigenvalue analysis of the system. That is done by solving the homogeneous form of (3.12), i.e. solving for \( B = 0 \). A general solution is assumed according to

\[
q = q_0 e^{\lambda t} \implies \dot{q} = \lambda q_0 e^{\lambda t},
\]
where $\lambda$ is the eigenvalue and $q_0$ is a $N \times 1$ vector with the amplitude of vibration for each degree of freedom (DOF). This gives

$$u = \begin{bmatrix} q_0 \\ \lambda q_0 \end{bmatrix} e^{\lambda t} = \bar{u} e^{\lambda t}, \quad \dot{u} = \begin{bmatrix} \lambda q_0 \\ \lambda^2 q_0 \end{bmatrix} e^{\lambda t} = \lambda \bar{u} e^{\lambda t},$$

(3.15)

where a vector $\bar{u} = [q_0 \quad \lambda q_0]^T$ has been introduced. Substitution into the homogeneous form of (3.12) gives a general eigenvalue problem according to

$$\lambda \bar{u} = A \bar{u},$$

(3.16)

where $\lambda$ and $\bar{u}$ are the complex eigenvalue and corresponding eigenvector of the system matrix $A$. For the under-damped case, the $2N$ eigenvalues of $A$ appear in complex conjugate pairs and the $n$:th natural frequency $\omega_n$ and damping ratio $\xi_n$ are found according to

$$\omega_n = |\lambda_n|, \quad \xi_n = \frac{|\text{Re}(\lambda_n)|}{|\lambda_n|}.$$  

(3.17)

By solving (3.12) - (3.17) for each instant in time, the time variations of the modal properties of the bridge are obtained.

### 3.2.2 Tracking of eigenvalues

To know which eigenvalue corresponds to the bridge mode, a tracking in time is needed. For that purpose, the method proposed by Caprani and Ahmadi (2016) was used. For simplicity, the method is herein explained using the same nomenclature as was used by them. In their method, the minimum euclidean norm of the bridge frequency and damping ratio vector between two consecutive times, $t$ and $\delta t$ is used to track the evolution of the pairs. Denoting $f(t)$ and $\xi(t)$ as a frequency and damping pair at time $t$, the algorithm begins with the known properties of the empty bridge for $t_0 = 0$. The subsequent tracking in two-dimensional space is then defined by

$$h_p(t_i) = \min_j \|h_j(t_i) - h_k(t_{i-1})\|,$$

(3.18)

in which

$$h_p(t_i) = [f(t_i), \xi(t_i)]^T,$$

(3.19)

is the vector of bridge frequency and damping found at the current time step to be at system DOF $p$; and

$$h_k(t_{i-1}) = [f(t_{i-1}), \xi(t_{i-1})]^T,$$

(3.20)

is the vector of bridge frequency and damping known from the previous time step to be at system DOF $k$; and

$$h_j(t_i) = [f_j(t_i), \xi_j(t_i)]^T,$$

(3.21)
is the vector of frequency-damping pair \( j \in \{1, \ldots, N\} \) from the \( N \) system DOFs, found from (3.17) at the current time step \( t_i \).

The robustness of the tracking method was evaluated by letting a crowd consisting of 30 pedestrians walk across a footbridge while the frequency and damping of the bridge and the pedestrians were tracked during the whole event. The pedestrians walked in phase with each other, with a step frequency of 2 Hz and their mass, natural frequency and damping ratio were generated from the distributions given in (2.5). The natural frequency of the considered bridge was equal to the mean value of the pedestrians’ natural frequencies. The robustness of the tracking method is verified by Figure 3.1 where it is shown how the frequency-damping pairs change during the passage of the crowd. Due to the great difference in the damping ratio of the bridge compared to the pedestrians, the risk of mixing up the frequency-damping pairs is insignificant.

![Figure 3.1: Tracking of the frequency-damping pairs of the bridge and the pedestrians for the case of a crowd consisting of 30 pedestrians crossing a footbridge.](image)

3.3 The human-structure interaction effect

In this section, CCBM and ML are used to investigate the HSI effect. Measures of the effect, in terms of the change in modal properties; \( \Pi_f \) and \( \Pi_\xi \) and its influence on the structural response \( \Pi_{\ddot{u}} \), are herein defined as

\[
\Pi_f = \frac{f_{\text{eff}}}{f_b}, \quad \Pi_\xi = \frac{\xi_{\text{eff}}}{\xi_b}, \quad \Pi_{\ddot{u}} = \frac{\ddot{u}_{\text{HSI}}}{\ddot{u}},
\]

(3.22)

where \( f_b \) and \( \xi_b \) are the natural frequency and modal damping ratio of the empty bridge, while \( f_{\text{eff}} \) and \( \xi_{\text{eff}} \) are the effective natural frequency and damping ratio of the
3.3. THE HUMAN-STRUCTURE INTERACTION EFFECT

bridge mode in the coupled system. Furthermore, \( \ddot{u}_{\text{HSI}} \) and \( \ddot{u} \) are the acceleration response obtained with and without consideration of the HSI effect, i.e. for CCBM and ML, respectively. Among the measures of the effect, the change in modal properties, \( \Pi_f \) and \( \Pi_\xi \), are governed by the properties of the bridge and the crowd while the effect on the structural response \( \Pi_\ddot{u} \) is also related to the applied load.

To facilitate for the investigation of the HSI effect, three dimensionless parameters are introduced. First, \( \Gamma \) is the ratio of the mass of the crowd and the modal mass of the bridge. Second, \( \Omega \) is the ratio of the average natural frequency of the pedestrians in the crowd and the natural frequency of the bridge. Third, \( \Omega_l \) is the ratio of the average step frequency of the pedestrians in the crowd and the natural frequency of the bridge. The three dimensionless parameters are calculated as

\[
\Gamma = \sum_{r=1}^{N_p} \gamma_{p,r}, \quad \Omega = f_b^{-1} N_p^{-1} \sum_{r=1}^{N_p} f_{p,r}, \quad \Omega_l = f_b^{-1} N_p^{-1} \sum_{r=1}^{N_p} f_{pl,r},
\]

(3.23)

where \( N_p \) is the number of pedestrians and \( \gamma_{p,r} \) is the ratio of the mass of the \( r \):th pedestrian and the modal mass of the bridge, calculated according to \( \gamma_{p,r} = m_{p,r}/m_b \). Furthermore, the natural frequency and step frequency of the \( r \):th pedestrian are denoted \( f_{p,r} \) and \( f_{pl,r} \), respectively.

3.3.1 Parametric study

First, a parametric study was performed to investigate the HSI effect on the reference bridge when subjected to the load from a deterministic crowd consisting of 30 equal pedestrians having a constant damping ratio of 30\%. To be able to separate the effect of changing \( \Omega \) and \( \Omega_l \), the properties of the bridge were held constant throughout the parametric study. Hence, the values of the dimensionless parameters were controlled by changing the properties of the crowd. More precisely, mass ratios in the range \( 0.02 < \Gamma < 0.16 \) were obtained by changing the value of \( m_p \), frequency ratios in the range \( 0.5 < \Omega < 1.75 \) were obtained by changing the value of \( k_p \) and frequency ratios in the range \( 0.5 < \Omega_l < 1.4 \) were obtained by changing the value of \( f_{pl} \). If an average pedestrian mass of 75 kg is assumed, the chosen mass ratios \( \Gamma \) correspond to pedestrian densities in the range 0.07 - 0.53 pedestrians/m\(^2\). Furthermore, the range for \( \Omega_l \) is a consequence of the range of validity for the chosen dynamic load factors, i.e. 1 - 2.8 Hz, see section 2.3. Hence, for a bridge with another frequency, the range for \( \Omega_l \) would be different. To summarise; the values of all parameters except stiffness, mass and step frequency of the pedestrians were held constant, and all pedestrians were equal.

For each combination of \( \Gamma \), \( \Omega \) and \( \Omega_l \), the change in modal properties of the bridge as well as the steady state acceleration response with and without taking the HSI effect into consideration were calculated.
CHAPTER 3. MODELLING OF THE CROWD-BRIDGE SYSTEM

The effect of HSI on the modal properties of the bridge

Figure 3.2 shows how the modal properties of the bridge vary with the frequency ratio $\Omega$ for the considered values of the mass ratio $\Gamma$. The HSI effect increases with increasing $\Gamma$ and is most prominent for $\Omega$ close to unity. The maximum change in damping is obtained for $\Omega$ slightly lower than unity while, conversely, the maximum change in frequency is obtained for $\Omega$ slightly higher than unity. These results are invariant to changing the bridge parameters, except for the damping ratio $\xi_b$. Or, with other words, for a simply supported bridge with 1% damping, a uniformly distributed crowd consisting of equal pedestrians and these combinations of $\Gamma$ and $\Omega$, the HSI effect on the modal properties is described by Figure 3.2.

Illustrative example

To facilitate for the interpretation of the results and to verify the generality of the system before continuing with the parametric study, an example is first presented and analysed. A mass ratio $\Gamma = 0.1$, a frequency ratio $\Omega = 1.25$ and a varying frequency ratio $\Omega_l$ were considered. CCBM and ML were used to calculate the steady state acceleration response with and without consideration of the HSI effect as well as the impact of the HSI effect on the structural response $\Pi_u$. The reference bridge and the deterministic crowd were considered and the generality of the system was verified by changing the bridge parameters one at the time.

The system proved to be invariant to changes in the mass of the bridge, i.e. changes in density or length of the bridge and changes in width or height of the cross section of the bridge deck. However, a change in the damping ratio affects the results, see Figure 3.3, where results obtained for the reference bridge and a bridge with twice the damping are shown. The difference between these two bridges was expected.

![Figure 3.2: The effect of HSI on the frequency (left) and damping ratio (right) of the bridge for different mass ratios $\Gamma$.](image)
since the damping ratio is not included in any of the dimensionless parameters. The response obtained with ML has a resonance peak at $\Omega_l = 1$ and its amplitude is halved for the bridge with twice the damping. For the response obtained with CCBM, the resonance peak is shifted to the left since the effective frequency is slightly lower than the frequency of the empty structure, as was shown in Figure 3.2.

Furthermore, the amplitude of the resonance peak is decreased and so is also the difference between the amplitudes of the two peaks. Because of the shift of the resonance peak, $\Pi_\ddot{u}$ has a maximum value for $\Omega_l$ slightly lower than unity and a minimum value for $\Omega_l = 1$. The interpretation of this is that for step frequencies slightly lower than the frequency of the empty structure, CCBM gives higher acceleration than ML while for step frequencies equal to the frequency of the empty structure, it is the other way around. It is worth pointing out that CCBM gives higher acceleration than ML in a range of $\Omega_l$ where ML gives low amplitudes of vibration, which means that even though CCBM gives, for example three times higher acceleration than ML, it does not necessarily have to be a very high acceleration level. Hence, the value of $\Pi_\ddot{u}$ does not indicate whether or not the HSI effect reduces the maximum acceleration. It rather shows the effect of HSI for each frequency in the considered range of step frequencies. However, from a designer’s perspective, it is also of great interest to see how the maximum obtained acceleration for any step frequency changes due to the HSI effect, which means comparing the maximum value obtained with each model. For that purpose, a reduction factor was introduced according to

$$R = \frac{\max_{\Omega_l}(\ddot{u}_{\text{HSI}})}{\max_{\Omega_l}(\ddot{u})}. \quad (3.24)$$

For the two bridges shown in Figure 3.3, the reduction factors are 0.33 and 0.48 for the reference bridge and the bridge with twice the damping, respectively.

Finally, changing the frequency of the bridge to 2.5 Hz gives a change in the amplitude of the resonance peak for both models. Recalling that the dynamic load factors increase with increasing step frequency, see (2.4) in section 2.3, and that the resonance peak appears at a higher step frequency for a bridge with a higher frequency, an increased amplitude is expected. The HSI effect is, however, almost invariant to a change in the bridge frequency, see Figure 3.4, where results obtained for the reference bridge and a bridge with a frequency of 2.5 Hz are shown.

The decision to perform the parametric study with the reference bridge and varying crowd parameters was based on this example. It was concluded that, in a qualitative sense, the results will be valid for any other combination of a simply supported bridge and a uniformly distributed crowd consisting of equal pedestrians. Though, in a quantitative sense, the results will differ for bridges with other damping ratios, and the same applies to the acceleration response for bridges with other frequencies.
The effect of HSI on the structural response

In Figure 3.5, it is shown how the effect of HSI on the structural response varies as a function of $\Omega_I$, for different combinations of $\Omega$ and $\Gamma$. For high values of the frequency ratio ($\Omega > 1$), the HSI effect increases significantly with increasing mass ratio, whereas for low values of the frequency ratio ($\Omega < 1$), the mass ratio does not have as great influence on the results. Furthermore, the HSI effect is most significant during resonance with any of the harmonics in the pedestrian loading. As was previously discussed, this is due to the shift in the resonance peak for the case when HSI is considered. This means that for high frequency ratios $\Omega$ and a step frequency slightly lower than the frequency of the empty structure, CCBM gives higher acceleration level than ML and as a result, $\Pi_u > 1$. 

Figure 3.3: Steady state acceleration response when HSI is neglected (left) and considered (middle) and the HSI effect on the structural response (right), for $\Gamma = 0.1$ and $\Omega = 1.25$, for bridges with damping ratio $\xi_b = 1\%$ (blue) and $\xi_b = 2\%$ (red).

Figure 3.4: Steady state acceleration response when HSI is neglected (left) and considered (middle) and the HSI effect on the structural response (right), for $\Gamma = 0.1$ and $\Omega = 1.25$, for bridges with frequency $f_b = 2\text{ Hz}$ (blue) and $f_b = 2.5\text{ Hz}$ (red).
3.3. THE HUMAN-STRUCTURE INTERACTION EFFECT

Figure 3.5: The effect of HSI on the structural response, for varying frequency ratio $\Omega_l$, for different combinations of the frequency ratio $\Omega$ and the mass ratio $\Gamma$.

Figure 3.6: The reduction factor on the structural response, for different combinations of the frequency ratio $\Omega$ and the mass ratio $\Gamma$. 
Moreover, the reduction in the peak acceleration for CCBM compared to ML was calculated for the same combinations of $\Omega$ and $\Gamma$, see Figure 3.6. The largest reduction is obtained for frequency ratios close to unity and a large mass ratio. For the considered values of $\Omega$, $\Gamma$, and $\Omega_l$, the HSI effect on the structural response is always beneficial from a vibration serviceability perspective, i.e. the peak acceleration is always decreased due to the HSI effect, which corroborates the results and conclusions presented by Van Nimmen et al. (2017).

Figure 3.7: The effect of HSI on the structural response, for varying frequency ratio $\Omega$, for different combinations of the frequency ratio $\Omega_l$ and the mass ratio $\Gamma$. 
3.3. THE HUMAN-STRUCTURE INTERACTION EFFECT

Figure 3.7 shows the effect of HSI on the structural response as a function of the frequency ratio $\Omega$ for different combinations of $\Omega_l$ and $\Gamma$. These results corroborate the conclusions from Figure 3.5. Again, the HSI effect increases with increasing mass ratio, especially for resonant loading and for high values of $\Omega$. The shift of the resonance peak which gives higher acceleration levels for CCBM than ML for $\Omega_l$ slightly lower than unity is here seen in the subplot showing results for $\Omega_l = 0.875$, where $\Pi_\delta > 1$. By comparing Figure 3.7 and Figure 3.5, it can be concluded that the effect of HSI on the structural response $\Pi_\delta$ is more affected by a change in $\Omega_l$ than a change in $\Omega$, even though the effect itself is only directly related to $\Omega$.

3.3.2 Monte Carlo simulation

In some sense, the results from the parametric study represent extreme cases since the deterministic crowd consists of equal pedestrians walking with the same step frequency and in phase with each other. In reality, crowd loading is characterised by inter- and intra-subject variabilities, see section 2.3. This section aims at investigating how the results presented in section 3.3.1 are changed if a more realistic crowd loading is considered. For this purpose, three types of stochastic crowds are introduced and compared to the previously introduced deterministic crowd. The stochastic crowds are characterised by the degree to which the inter-subject variabilities are accounted for. However, the intra-subject variabilities are still neglected. The four types of crowds that will be considered in the subsequent analysis are defined as:

- **Deterministic crowd (DC)**
  Having deterministic properties according to
  \[
  m_p = 75 \text{ kg}, \quad f_p = 3.25 \text{ Hz}, \quad \xi_p = 0.3, \quad f_{pl} = 1.96 \text{ Hz}, \quad \theta_i = 0. \quad (3.25)
  \]

- **Semi-stochastic crowd 1 (SSC1)**
  Having stochastic dynamic properties and a deterministic load according to
  \[
  m_p \in \mathcal{N}(75,15) \text{ kg}, \quad f_p \in \mathcal{N}(3.25,0.32) \text{ Hz}, \quad \xi_p \in \mathcal{N}(0.3,0.05), \quad f_{pl} = 1.96 \text{ Hz}, \quad \theta_i = 0. \quad (3.26)
  \]

- **Semi-stochastic crowd 2 (SSC2)**
  Having deterministic dynamic properties and a stochastic load according to
  \[
  m_p = 75 \text{ kg}, \quad f_p = 3.25 \text{ Hz}, \quad \xi_p = 0.3, \quad f_{pl} \in \mathcal{N}(1.96,0.209) \text{ Hz}, \quad \theta_i \in \mathcal{U}(0,2\pi). \quad (3.29)
  \]

- **Stochastic crowd (SC)**
  Having stochastic properties according to
  \[
  m_p \in \mathcal{N}(75,15) \text{ kg}, \quad f_p \in \mathcal{N}(3.25,0.32) \text{ Hz}, \quad \xi_p \in \mathcal{N}(0.3,0.05), \quad f_{pl} \in \mathcal{N}(1.96,0.209) \text{ Hz}, \quad \theta_i \in \mathcal{U}(0,2\pi). \quad (3.31)
  \]
A Monte Carlo simulation with 450 stochastic crowds, their semi-stochastic counterparts and the deterministic crowd was performed. A set of 351 bridges with frequencies in the range 1.5-5 Hz were considered. For each combination of bridge and crowd, the change in modal properties of the bridge as well as the structural response with and without considering the HSI effect were calculated. In the Monte Carlo simulation, the structural response was defined as the maximum acceleration response obtained during one passage of the crowd, instead of the steady state acceleration response, which was the case for the parametric study with the deterministic crowd. Furthermore, a constant mass ratio $\Gamma = 0.1$ was considered, since it was clear from the analysis presented in section 3.3.1 that the HSI effect increases monotonically with an increasing mass ratio.

The effect of HSI on the modal properties of the bridge

Figure 3.8 illustrates the HSI effect on the modal properties of the bridge for the 450 stochastic crowds and the deterministic crowd. The data is lognormally distributed and, besides the raw data, mean value as well as upper and lower 95% confidence intervals are also shown. It can be concluded that the spread caused by the inter-subject variability in the dynamic properties of the pedestrians is smaller for $\Pi_f$ than for $\Pi_\xi$. Within a 95% confidence interval, the maximum effect of HSI on the effective frequency of the bridge differs around 0.3% for the different crowds. Corresponding value for the effective damping is approximately 12%. Furthermore, the average effect from the stochastic crowds is slightly decreased compared to the effect caused by the deterministic crowd. This corroborates the conclusion by Agu and Kasperski (2011), saying that if the inter-subject variabilities are neglected, too high damping ratios might be obtained.

Figure 3.8: The effect of HSI on the frequency (left) and damping ratio (right) of the bridge for a mass ratio $\Gamma = 0.1$, for DC (red dash-dotted) and SC, shown in terms of the mean (black solid), upper and lower 95% confidence intervals (black dashed) and each of the 450 crowds (grey dotted).
The effect of HSI on the structural response

The variation in maximum acceleration response obtained with ML and CCBM for the deterministic crowd, the two semi-stochastic crowds and the stochastic crowd is shown in Figure 3.9. Again, the data is lognormally distributed and the raw data as well as mean value and upper and lower 95% confidence intervals are shown. First, by comparing the left and right columns, it can be concluded that, for all types of crowds, the HSI effect is beneficial from a vibration serviceability perspective, i.e. the maximum acceleration response is always lower for CCBM than for ML. This corroborates the conclusion from the parametric study. Moreover, from the second and third row, it can be concluded that the spread for SSC1 is insignificant compared to the spread for SSC2. With other words, difference in acceleration response caused by inter-subject variabilities in the dynamic properties of the pedestrians in the crowd is insignificant in comparison with the difference in acceleration response caused by inter-subject variabilities in the pedestrian step frequencies. Consequently, for SSC2 and SC, the results vary a lot for the 450 crowds while for SSC1, the 450 crowds give almost the same result.

Furthermore, also the reduction of the resonance peak is mainly related to the variation in the pedestrian step frequencies rather than the variation in their dynamic properties or the HSI effect. This effect is further illustrated in Figure 3.10, where the average acceleration response obtained with ML and CCBM for the deterministic crowd, the 450 stochastic crowds and their semi-stochastic counterparts are shown, i.e. the black solid lines in Figure 3.9 are shown in Figure 3.10. The effect mentioned above can be seen by comparing the acceleration levels obtained for SSC2 and SC to those obtained for DC and SSC1. Furthermore, looking at the difference between CCBM and ML at the resonance peaks, it is greater for DC and SSC1 than for SSC2 and SC, which implies that the HSI effect at resonance is greater for a crowd with synchronised pedestrians. Moreover, the reduction of the resonance peak is greater for $\Omega_l = 0.5$ than for $\Omega_l = 1$, which indicates that the HSI effect has a greater impact on the second than the first harmonic in the pedestrian loading (2.1). This can be understood by recalling that the HSI effect on the damping is most prominent for $\Omega$ slightly lower than unity. For the properties of the stochastic crowds considered in this Monte Carlo simulation, $\Omega_l = 0.5$ corresponds to $\Omega \approx 0.83$ while $\Omega_l = 1$ corresponds to $\Omega \approx 1.66$. Hence, greater effect of HSI can be expected for the second harmonic.

Finally, Figure 3.11 shows the reduction factors, see (3.24), for the 450 stochastic crowds, their semi-stochastic counterparts and the deterministic crowd. The latter was previously shown in Figure 3.6. In accordance with the results shown in Figure 3.9, the reduction factor experiences a greater spread for SSC2 and SC than for SSC1. The reduction for SSC1 is almost equal to the one obtained for DC, i.e. different dynamic properties of the pedestrians in the crowd does not have a big impact on the reduction factor. On the contrary, for SSC2 and SC, the reduction is
most often smaller than for DC, which happens since the pedestrians are not synchronised. For the stochastic and semi-stochastic crowds, the dashed lines show the upper 98% confidence intervals, which could be of great interest from a designer’s perspective since they indicate what would be a conservative value of the reduction factor in the design of a footbridge.

Figure 3.9: The maximum acceleration obtained with ML (left) and CCBM (right) for a mass ratio $\Gamma = 0.1$, for DC (first row), SSC1 (second row), SSC2 (third row) and SC (fourth row), shown in terms of the mean (black solid), upper and lower 95% confidence intervals (black dashed) and each of the 450 crowds (grey dotted).
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Figure 3.10: Mean values of the maximum obtained acceleration response with ML (solid) and CCBM (solid with markers), for DC (blue) and the 450 different SSC1 (red), SSC2 (black) and SC (green), for a mass ratio $\Gamma = 0.1$.

Figure 3.11: Reduction factor for a mass ratio $\Gamma = 0.1$, for DC (blue), SSC1 (red), SSC2 (black) and SC (green), shown in terms of upper 98% confidence interval (dashed) and each of the 450 crowds (markers).
3.3.3 Concluding remarks

In this section, the effect of HSI on the modal properties and structural response of a simply supported footbridge subjected to the load from a uniformly distributed crowd was investigated. Two different models, ML which neglects the HSI effect and CCBM which accounts for the effect, were used to perform a parametric study with a deterministic crowd as well as a Monte Carlo simulation with four types of crowds. The crowds were characterised by the degree to which the inter-subject variabilities were accounted for. Hence, the influence of inter-subject variabilities on the modal properties and structural response as well as its relation to the HSI effect could also be studied. In this investigation, the characteristics of the coupled crowd-bridge system were defined by three dimensionless parameters. First, the ratio of the mass of the crowd and the modal mass of the bridge $\Gamma$, second, the ratio of the average natural frequency of the pedestrians in the crowd and the natural frequency of the bridge $\Omega$, and third, the ratio of the average step frequency of the pedestrians in the crowd and the natural frequency of the bridge $\Omega_l$.

Recalling that this investigation was limited to consider the combination of a simply supported footbridge and a uniformly distributed crowd, the generality of the conclusions is limited to such situation. The main conclusions are summarised as:

1. The HSI effect reduces the peak acceleration.
2. The effect of HSI on the modal properties of the bridge as well as the reduction in peak acceleration are largest for $\Omega \approx 1$ and increase monotonically with increasing $\Gamma$.
3. The effect of HSI on the structural response is most significant during resonance, i.e. for $\Omega_l = 1$.
4. The HSI effect is reduced for a crowd of pedestrians walking with different step frequencies compared to a crowd walking in phase, i.e. if the inter-pedestrian variability in the step frequency is accounted for.

These conclusions indicate that, in a design situation, it is important to account for the HSI effect and inter-subject variabilities in the pedestrians’ step frequencies. On the contrary, inter-subject variabilities in the pedestrians’ dynamic properties can possibly be neglected. For the purpose of accounting for these effects, an improved version of Figure 3.11 could be valuable in the future design guidelines for footbridges.
Chapter 4

Simplified modelling approaches

With the aim of reducing the complexity as well as the computational cost of vibration serviceability assessment of footbridges under consideration of the human-structure interaction (HSI) effect, the performed research work has focused on development of simplified approaches for modelling of the coupled crowd-bridge system. The objectives of this chapter are, first, to introduce two simplified modelling approaches, and second, to evaluate their performance in terms of accuracy and time efficiency, as compared to the coupled crowd-bridge model (CCBM), presented in section 3.2. The two simplified models have in common that the introduced approximations and simplifications concern how the HSI effect is accounted for. The first model, described in section 4.1, features an approximation of the HSI effect which is easily implemented and time efficient but still accurate. In the second model, presented in section 4.2, a larger degree of approximation is introduced. Hence it is even easier to implement and more time efficient, but with a lower level of accuracy as a consequence. Finally, in section 4.3, the performance of the simplified models is evaluated, both concerning accuracy of the results and computational time.

4.1 Moving Loads and a Sum of 2DOF systems

In this simplified modelling approach, a new methodology for considering the HSI effect is proposed. The main idea is to account for the HSI effect by updating the modal properties of the bridge during the passage of the crowd. This enables to describe the pedestrians as moving loads, which is very time efficient. The time variations of the modal properties of the bridge are obtained from several 2DOF systems, each of them consisting of one pedestrian (described as a spring-mass-damper system) and the generalised coordinate of one mode of the bridge. This is done by determining the maximum interaction effect from each pedestrian, scaling the effect depending on the pedestrian’s position and adding the contributions from all pedestrians to determine the interaction effect caused by the whole crowd.
Consequently, besides the assumptions stated for CCBM, it is here assumed that:

1. The HSI effect can be accounted for by using time dependent modal properties of the bridge.

2. The change in modal properties of the bridge varies linearly with the size of the crowd.

A consequence of the first assumption is that the pedestrians can be described as time dependent moving loads, which is very time efficient. The first assumption also implies that the variations in the modal properties of the pedestrians are neglected, i.e. the modal properties of the pedestrians are time independent, which is of course an approximation. Furthermore, the second assumption means that the effective modal properties of the bridge can be calculated for each pedestrian separately and that the time dependent modal properties of the bridge under the influence of the whole crowd are then found by adding the contribution from each pedestrian in each instant in time.

This model is henceforth referred to as MLS2 (short for Moving Loads combined with a Sum of 2DOF systems to account for the HSI effect). The methodology is presented with two examples, first, a single pedestrian crossing a bridge, and second, a crowd of pedestrians crossing a bridge.

### 4.1.1 Single pedestrian

To find the response in one bridge mode during the passage of a single pedestrian, the change in modal properties of the bridge mode is needed. For a 2DOF system, the matrices given in (3.7) are

$$
M_{2DOF} = \begin{bmatrix} m_b & m_p \phi_b \\ 0 & m_p \end{bmatrix}, \quad C_{2DOF} = \begin{bmatrix} c_b & 0 \\ -c_p \phi_b & c_p \end{bmatrix}, \quad K_{2DOF} = \begin{bmatrix} k_b & 0 \\ -k_p \phi_b & k_p \end{bmatrix}, \quad (4.1)
$$

where, for brevity, the pedestrian’s position $x_p$ was suppressed, i.e. $\phi_b = \phi_b(x_p)$. Accordingly, the 2DOF version of the system matrix $A$, given in (3.13) is written

$$
A_{2DOF} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -\left( \frac{k_b}{m_b} + \frac{k_p \phi_b^2}{m_p} \right) & \frac{k_p \phi_b}{m_b} & -\left( \frac{c_b}{m_b} + \frac{c_p \phi_b^2}{m_p} \right) & \frac{c_p \phi_b}{m_p} \\ -\frac{k_p \phi_b}{m_p} & -\frac{k_p}{m_p} & -\frac{c_p \phi_b}{m_p} & -\frac{c_p}{m_p} \end{bmatrix}. \quad (4.2)
$$

To facilitate the subsequent analysis, a number of dimensionless parameters are introduced according to

$$
\omega_b = \sqrt{\frac{k_b}{m_b}}, \quad \omega_p = \sqrt{\frac{k_p}{m_p}}, \quad \xi_b = \frac{c_b}{2m_b \omega_b}, \quad \xi_p = \frac{c_p}{2m_p \omega_p}, \quad \gamma_p = \frac{m_p}{m_b}. \quad (4.3)
$$
4.1. MOVING LOADS AND A SUM OF 2DOF SYSTEMS

Substituting (4.3) into (4.2) gives

\[
A_{2DOF} = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-\omega_p^2 & \gamma_p \omega_p \phi_b \\
\omega_p^2 \phi_b & -\omega_p^2 & -2 \left( \omega_b \xi_b + \gamma_p \omega_p \xi_p \phi_b^2 \right) & 2 \gamma_p \omega_p \xi_p \phi_b \\
\end{bmatrix}.
\] (4.4)

The instantaneous modal properties of the system are obtained from the eigenvalues of the system matrix \(A_{2DOF}\). The eigenvalues are found by solving \(\det(A_{2DOF}) = 0\), which means finding the roots of the 4th order polynomial

\[
a \lambda^4 + b \lambda^3 + c \lambda^2 + d \lambda + e = 0,
\] (4.5)

where the coefficients are

\[
a = 1, \quad b = 2 \xi_p \omega_p \eta + 2 \xi_b \omega_b, \quad c = \omega_p^2 \eta + \omega_b^2 + 4 \xi_b \xi_p \omega_b \omega_p,
\] (4.6)

\[
d = 2 \omega_b \omega_p (\xi_p \omega_b + \xi_b \omega_p), \quad e = \omega_b^2 \omega_p^2,
\] (4.7)

where a new variable \(\eta = \gamma_p \phi_b^2 + 1\) was introduced for the sake of conciseness. By introducing a number of constants according to

\[
p_1 = 2c^3 - 9bde + 27ad^2 + 27b^2e - 72ace,
\] (4.8)

\[
p_2 = p_1 + \sqrt{-4 \left( c^2 - 3bd + 12ae \right)^3 + p_1^2},
\] (4.9)

\[
p_3 = \frac{c^2 - 3bd + 12ae}{3a} + \frac{3\sqrt{p_2}}{3a},
\] (4.10)

\[
p_4 = \sqrt{\frac{b^2}{4a^2} - \frac{2c}{3a} + p_3},
\] (4.11)

\[
p_5 = \frac{b^2}{2a^2} - \frac{4c}{3a} - p_3,
\] (4.12)

\[
p_6 = \frac{-b^3}{2a^3} + \frac{4bc}{a^2} - \frac{8d}{3a},
\] (4.13)

the roots of (4.5) can be calculated as

\[
\lambda_{1,2} = -\frac{b}{4a} \pm \frac{p_4}{2} \pm \frac{\sqrt{p_5 - p_6}}{2}, \quad \lambda_{3,4} = -\frac{b}{4a} \pm \frac{p_4}{2} \pm \frac{\sqrt{p_5 + p_6}}{2}.
\] (4.14)

The eigenvalue corresponding to the bridge mode is found as

\[
\lambda_b = \min_j \left| \frac{\text{Re}(\lambda_j)}{|\lambda_j|} \right|, \quad j \in \{1, \ldots, 4\},
\] (4.15)

where it is assumed that for the 2DOF system, the damping ratio of the bridge mode is lower than the damping ratio of the spring-mass-damper system describing
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the pedestrian, see section 2.1 and 2.4. By using (3.17), the modal properties of
the bridge mode are calculated according to

\[ \omega_p^b = | \lambda_b |, \quad \xi_p^b = \frac{| \text{Re}(\lambda_b) |}{| \lambda_b |}, \quad (4.16) \]

where the superscript \( p \) indicates that a pedestrian is present. Recalling that the
eigenvalues are time dependent due to the time dependency of the pedestrian’s
position, i.e. \( \lambda_b = \lambda_b(\phi_b(x_p(t))) \), then so are also the resulting frequency and
damping of the bridge according to

\[ \omega_p^b(\lambda_b(\phi_b(x_p(t)))) \Rightarrow \omega_p^b(t), \quad \xi_p^b(\lambda_b(\phi_b(x_p(t)))) \Rightarrow \xi_p^b(t). \quad (4.17) \]

To compute the time dependent modal properties, \( \omega_p^b(t) \) and \( \xi_p^b(t) \), the procedure
described by (4.1) - (4.16) has to be repeated for each instant in time. However, if
a simply supported bridge is considered, an alternative, more efficient, approach is
obtained by introducing a third assumption according to:

3. In a coupled system consisting of a simply supported bridge and a pedestrian,
the effective modal properties of the bridge vary with the mode shape squared.

Under this assumption, it is required to perform (4.1) - (4.16) only once, for the
case when the pedestrian stands at an antinode of the displacement normalised
mode shape. It means solving (4.1) - (4.16) for \( \phi_b(x_p(t)) = 1 \), which results in

\[ \tilde{\omega}_b^p = | \lambda_b(\phi_b(x_p(t)) = 1) |, \quad \tilde{\xi}_b^p = \frac{| \text{Re}(\lambda_b(\phi_b(x_p(t)) = 1)) |}{| \lambda_b(\phi_b(x_p(t)) = 1) |}, \quad (4.18) \]

where the \( \tilde{\bullet} \) indicates that it is the maximum change compared to the modal
properties of the empty structure. Thereafter, the dependency of the pedestrian’s
position, and hence the time, is added simply by scaling the maximum values with
the mode shape squared according to

\[ \omega_b^p(t) = \tilde{\omega}_b^p \phi_b^2(x_p(t)), \quad \xi_b^p(t) = \tilde{\xi}_b^p \phi_b^2(x_p(t)). \quad (4.19) \]

By using the time dependent modal properties of the bridge and describing the
pedestrian as a time dependent moving load, the dynamic response in the considered
bridge mode is calculated as

\[ \ddot{q}_b + 2\xi_b^p(t)\omega_b^p(t)\dot{q}_b + \omega_b^p(t)^2 q_b = \phi_b(x_p(t))F_p(t)/m_p, \quad (4.20) \]

where \( F_p(t) \) is in this work described by (2.1) and \( \omega_b^p(t) \) and \( \xi_b^p(t) \) are calculated
according to (4.17) in the general case and according to either (4.17) or (4.19) when
a simply supported bridge is considered.

The correspondence between the two approaches for finding the time dependent
modal properties of the bridge, i.e. using (4.17) or (4.19), is illustrated in Figure 4.1, where the time dependent modal properties of the bridge are shown for the
case of a single pedestrian crossing the reference bridge introduced in Chapter 3. The pedestrian speed, step frequency, mass, frequency and damping ratio were respectively 1.25 m/s, 2 Hz, 75 kg, 2 Hz and 30%. Due to the equality in the results, see Figure 4.1, the time efficiency and the fact that the research work is limited to consider simply supported bridges, the use of (4.19) was adopted for MLS2 in this research project.

![Graph showing frequency and damping ratio comparison.]

Figure 4.1: Comparison between (4.17) (blue curves) and (4.19) (red dots), shown in terms of frequency (top) and damping ratio (bottom) of the bridge mode during the passage of a single pedestrian.

### 4.1.2 Crowd of pedestrians

To find the dynamic response of a bridge mode during the passage of a crowd consisting of \( N_p \) pedestrians, the procedure is extended to include several pedestrians. In the list below, it is described how this is done for an arbitrary mode of a simply supported bridge, having the displacement normalised mode shape \( \phi_b(x) \), the natural frequency \( f_b \) and the modal damping ratio \( \xi_b \).

1. For each pedestrian \( r \),
   a) Solve the eigenvalue problem given by (3.16) for the system matrix given in (4.4), for the pedestrian standing at the antinode of the mode shape according to
   \[
   \lambda \ddot{\mathbf{u}} = \mathbf{A}_{2DOF} \ddot{\mathbf{u}}.
   \]
b) Find the eigenvalue corresponding to the bridge mode by using (4.15) according to

$$\lambda_b = \min_j \frac{|\text{Re}(\lambda_j)|}{|\lambda_j|}, \quad j \in \{1, \ldots, 4\}.$$

c) Calculate the modal properties of the bridge mode by using (4.18) according to

$$\tilde{\omega}^r_b = | \lambda_b |, \quad \tilde{\xi}^r_b = \frac{|\text{Re}(\lambda_b)|}{|\lambda_b|}, \quad (4.22)$$

where the superscript \(r\) indicates that the \(r\):th pedestrian is present.

d) Add the time dependency by using (4.19) according to

$$\omega^r_b(t) = \tilde{\omega}^r_b \phi^2_b(x_{p,r}(t)), \quad \xi^r_b(t) = \tilde{\xi}^r_b \phi^2_b(x_{p,r}(t)), \quad (4.23)$$

where \(x_{p,r}(t)\) is the position of the \(r\):th pedestrian at time \(t\).

The output from step 1, i.e. (4.23), describes how the modal properties vary throughout the passage of the \(r\):th pedestrian. Hence, there will be as many \(\omega^r_b(t)\) and \(\xi^r_b(t)\) as there are pedestrians.

2. At time \(t\), sum over all pedestrians \(r\) to get the instantaneous modal properties according to

$$\omega^c_b(t) = \omega_b + \sum_{r=1}^{N_p} (\omega^r_b(t) - \omega_b), \quad \xi^c_b(t) = \xi_b + \sum_{r=1}^{N_p} (\xi^r_b(t) - \xi_b), \quad (4.24)$$

where the superscript \(c\) denotes that the whole crowd was taken into account. Given that the position of each pedestrian in each time step is known, the output from step 2, i.e. (4.24), describes how the modal properties vary throughout the passage of the whole crowd.

3. By using the time dependent modal properties of the bridge and describing the pedestrians as time dependent moving loads, the dynamic response in this bridge mode is calculated according to

$$\ddot{q}_b + 2\xi^c_b(t)\omega^c_b(t)\dot{q}_b + \omega^2_b(t)q_b = F_p(t), \quad (4.25)$$

where the external force \(F_p(t)\) caused by the crowd is calculated as

$$F_p(t) = \sum_{r=1}^{N_p} F_{p,r}(t)\phi_b(x_{p,r})/m_{p,r}, \quad (4.26)$$

where \(F_{p,r}(t)\) is the load caused by the \(r\):th pedestrian, being, in this work, modelled according to (2.1). The output from step 3, i.e. the solution to (4.25), is the dynamic response in the considered mode shape, caused by the whole crowd.
Finally, to include several mode shapes in the analysis of the dynamic response, this procedure is done for each mode shape separately and then (3.3) is used to obtain the total dynamic response of the bridge.

4.2 Moving Loads and an equivalent 2DOF system

The second simplified modelling approach is thought of as a possible design approach. Therefore, the HSI effect is accounted for in the simplest possible and most time efficient way. The idea is to account for the HSI effect by describing the crowd as an equivalent single degree of freedom (SDOF) system and calculate the effective modal properties of the bridge when the crowd is present. These (time independent) effective modal properties are then adopted for the bridge while the pedestrians are described as moving loads. A similar approach was adopted by Van Nimmen et al. (2017) though they accounted for the pedestrians’ positions on the bridge deck which is not the case here. Accordingly, this modelling approach relies on the following assumptions:

1. The HSI effect can be accounted for by using time independent effective modal properties of the bridge.
2. The crowd is uniformly distributed on the bridge deck.
3. For consideration of the HSI effect, the crowd can be well approximated by an equivalent SDOF system.

From the first assumption it follows that any variations in time are neglected and that the maximum interaction effect is assumed to be valid for the whole passage of the crowd. Besides that, any variations in the modal properties of the pedestrians are neglected, as was the case for MLS2. From the second and third assumptions it follows that the dependency of the pedestrians’ positions is neglected.

This model is henceforth referred to as ML2 (short for Moving Loads combined with an equivalent 2DOF system to account for the HSI effect). An example, which illustrates how to use the methodology is presented in section 4.2.1.

4.2.1 Crowd of pedestrians

To find the response in one bridge mode during the passage of a crowd consisting of $N_p$ pedestrians, the effective modal properties of the bridge are needed. To find these, it is assumed that the crowd is uniformly distributed on the bridge deck and the crowd is then described as an SDOF system. The equivalent mass of the crowd is calculated as the modal mass of the crowd while the equivalent frequency and damping are calculated as mean values of, respectively, frequency and damping for
the pedestrians in the crowd. Hence, the properties for an equivalent SDOF system describing the crowd are calculated according to

\[
m_{c} = \sum_{r=1}^{N_p} m_{p,r} \phi_b^2(x_{p,r}), \quad \omega_{c} = N_p^{-1} \sum_{r=1}^{N_p} \omega_{p,r}, \quad \xi_{c} = N_p^{-1} \sum_{r=1}^{N_p} \xi_{p,r}, \quad (4.27)
\]

\[
c_{c} = 2\xi_{c}m_{c}\omega_{c}, \quad k_{c} = \omega_{c}^{2}m_{c}, \quad (4.28)
\]

where the mode shape of the considered bridge mode is denoted \( \phi_b \) and \( m_{p,r}, \omega_{p,r}, \xi_{p,r} \) and \( x_{p,r} \) are, respectively, the mass, frequency, damping ratio and position of the \( r \):th pedestrian. According to the assumption about a uniformly distributed crowd, the pedestrians’ positions are equally distributed on the bridge deck. Furthermore, in the case when \( \phi_b \) is the first vertical bending mode of a simply supported bridge, the equivalent mass of the SDOF system describing the crowd simplifies to \( m_{c} = 0.5 \sum_{r=1}^{N_p} m_{p,r} \).

A 2DOF system consisting of the equivalent SDOF system describing the crowd and one mode of the bridge is created; in this setting, the system matrix given in (4.2) becomes

\[
A_2 = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-\left( \frac{k_b}{m_b} + \frac{k_c}{m_c} \right) & \frac{k_b}{m_b} & -\left( \frac{c_b}{m_b} + \frac{c_c}{m_c} \right) & \frac{c_b}{m_b} \\
\frac{k_c}{m_c} & -\frac{k_b}{m_b} & -\frac{c_b}{m_c} & -\frac{c_c}{m_c}
\end{bmatrix}.
\]

(4.29)

By following the procedure described by (4.3) - (4.16), the effective modal properties of the bridge are found as

\[
\omega_{b}^{c1} = |\lambda_b|, \quad \xi_{b}^{c1} = \frac{|\text{Re}(\lambda_b)|}{|\lambda_b|},
\]

(4.30)

where the superscript \( c1 \) indicates that a crowd is present and that it was described as an equivalent SDOF system. By using the effective modal properties of the bridge and describing the pedestrians as time dependent moving loads, the dynamic response in the considered bridge mode is calculated according to

\[
\ddot{q}_b + 2\xi_{b}^{c1}\omega_{b}^{c1}\dot{q}_b + \omega_{b}^{c12}q_b = F_p(t),
\]

(4.31)

where the external force \( F_p(t) \), caused by the presence of the crowd, is calculated according to (4.26).

Again, to include several mode shapes in the analysis of the dynamic response, this procedure is done for each mode shape separately and then (3.3) is used to obtain the total dynamic response of the bridge.
4.3 Performance of the simplified models

The aim of this section is to evaluate the performance of the two simplified modelling approaches, MLS2 and ML2. For that purpose, the models are compared to CCBM concerning accuracy of the predicted changes in modal properties and acceleration levels as well as the computational time. The procedure for comparing the accuracy of the modelling approaches starts with a comparison in the time domain. Thereafter, the remainder of the comparison is similar to the procedure explained in section 3.3, i.e. it comprises a parametric study with a deterministic crowd and a Monte Carlo simulation with more realistic, stochastic crowds.

4.3.1 Comparison in time domain

With the aim of comparing the performance of the models in the time domain, they were used to calculate the change in modal properties as well as the structural response of the reference bridge under the load from one of the stochastic crowds, see (3.30)-(3.31). The results clearly illustrate the difference between the two simplified models, being that ML2 uses time independent effective modal properties of the bridge while MLS2 tracks how they change in time, see Figure 4.2. Regarding the
acceleration, MLS2 gives accurate predictions compared to CCBM, whereas ML2 gives qualitatively similar results, but a small quantitative error. The change in frequency is well predicted by MLS2 during the whole event and so is also the maximum change in frequency predicted by ML2. Regarding the damping ratio, MLS2 overestimates the change compared to the damping of the empty bridge while, conversely, ML2 underestimates the maximum change in damping ratio.

4.3.2 Parametric study

In the parametric study, the reference bridge and the deterministic crowd introduced in section 3.3.1 were considered. Again, the values of the dimensionless parameters (3.23) were controlled by changing the properties of the crowd. Mass ratios in the range $0.04 < \Gamma < 0.16$ and frequency ratios in the range $0.5 < \Omega < 1.75$ were considered. Regarding the frequency ratio $\Omega_l$, two load cases were considered. In the first load case, the pedestrian step frequency was equal to the frequency of the empty structure. Hence, near-resonant loading was obtained since the effective frequency of the bridge is somewhat different in the presence of the crowd. In the second load case, resonant loading was considered, meaning that the pedestrian step frequency was equal to the effective frequency of the bridge obtained with the considered model.

For each combination of $\Gamma$ and $\Omega$ and for both load cases, the change in modal properties as well as the steady state acceleration response were calculated. Figure 4.3 shows the obtained HSI effect on frequency and damping for varying frequency ratio $\Omega$ and different values of the mass ratio $\Gamma$. Compared to CCBM, MLS2 gives an error that increases with increasing $\Gamma$, which shows that the variation in the modal properties of the bridge does not increase linearly with the size of the crowd. Hence, MLS2 is more suitable for small crowds than for large crowds. On the contrary, ML2 gives exactly the same result as CCBM, which shows that, for calculation of the HSI effect on the modal properties of a simply supported bridge, a deterministic, uniformly distributed crowd can be exactly represented as an equivalent SDOF system. These results were expected since ML2 is based on an assumption that the crowd is uniformly distributed. The results would, however, differ for a randomly distributed crowd.

Furthermore, Figure 4.4 shows how the models perform considering prediction of the steady state acceleration response. For both load cases, MLS2 gives results that are similar to CCBM for the considered mass ratios $\Gamma$, while ML2 gives an error which increases with increasing $\Gamma$, especially for $\Omega$ slightly lower than unity. Moreover, it is interesting to notice that very high acceleration levels are reached for resonant loading, large values of $\Gamma$ and $\Omega > 1$. This happens since the effective damping decreases for increasing values of $\Omega$. However, in reality the pedestrians would stop walking before such high acceleration levels are reached.
Figure 4.3: The effect of HSI on the frequency (left) and damping ratio (right) for mass ratios $\Gamma = 0.04$ (top), $\Gamma = 0.1$ (middle) and $\Gamma = 0.16$ (bottom), for CCBM (blue solid), MLS2 (red dash-dotted) and ML2 (yellow dashed).
Figure 4.4: Steady state acceleration response during near-resonant (solid) and resonant (dashed) loading for mass ratios $\Gamma = 0.04$ (top), $\Gamma = 0.1$ (middle) and $\Gamma = 0.16$ (bottom), for CCBM (blue), MLS2 (red) and ML2 (yellow).
4.3. PERFORMANCE OF THE SIMPLIFIED MODELS

4.3.3 Monte Carlo simulation

In the Monte Carlo simulation, the deterministic crowd (3.25) and the set of 450 stochastic crowds (3.30)-(3.31), generated in section 3.3.2, were considered. Similarly, the set of 351 bridges with frequencies in the range 1.5-5 Hz was considered. For each combination of bridge and crowd, the change in modal properties of the bridge as well as the structural response were calculated. Again, a mass ratio $\Gamma = 0.1$ was considered and the structural response was defined as the maximum acceleration during one passage of the crowd.

Figure 4.5: The effect of HSI on the frequency (left) and damping ratio (right) of the bridge for a mass ratio $\Gamma = 0.1$, for 450 stochastic crowds, shown in terms of the mean (black solid), upper and lower 95% confidence intervals (black dashed) and each crowd (grey dotted), for CCBM (top), MLS2 (middle) and ML2 (bottom).
Figure 4.5 illustrates the HSI effect on the modal properties of the bridge for the 450 stochastic crowds. For all modelling approaches, the obtained data follows a lognormal distribution and, besides the raw data, mean value as well as upper and lower 95% confidence intervals are shown. First, it can be concluded that, in a qualitative sense, the three models give similar results and that the results are characterised by a low degree of variation. The simplified models, MLS2 and ML2, give a slightly lower spread in the results than CCBM. Within a 95% confidence interval, the maximum effect of HSI on the effective frequency of the bridge for the different crowds differs around 0.3% for MLS2 and 0.2% for ML2, compared to approximately 0.3% for CCBM. Corresponding values for the effective damping ratio are approximately 10% and 7% for MLS2 and ML2, respectively, compared to 12% for CCBM. Furthermore, to enable a quantitative comparison, Figure 4.6 shows the average HSI effect on the modal properties obtained with the different models. Compared to CCBM, MLS2 slightly underestimates the effect while ML2 slightly overestimates the effect.

Figure 4.7 shows the maximum acceleration response of the bridge for the 450 stochastic crowds; raw data as well as mean values and upper and lower 95% confidence intervals for lognormal distributions are shown. From a qualitative perspective, the models give similar results, though the results spread slightly more for ML2, compared to MLS2 and CCBM. From a quantitative perspective, the two simplified models slightly overestimate the response. At the resonance peak, MLS2 gives an error of around 4%, compared to approximately 7% for ML2. This is further illustrated in Figure 4.8 where the average acceleration response obtained with the different models is shown. From a designer’s point of view, overestimation of the response is, however, to prefer, compared to underestimation of the response.

![Figure 4.6: The effect of HSI on the frequency (left) and damping ratio (right) of the bridge for a mass ratio $\Gamma = 0.1$, shown in terms of the mean value for 450 stochastic crowds, for CCBM (blue solid), MLS2 (red dash-dotted) and ML2 (yellow dashed).](image-url)
4.3. PERFORMANCE OF THE SIMPLIFIED MODELS

Figure 4.7: The maximum acceleration of the bridge for a mass ratio $\Gamma = 0.1$, for 450 stochastic crowds, shown in terms of the mean (black solid), upper and lower 95% confidence intervals (black dashed) and each crowd (grey dotted), for CCBM (top), MLS2 (middle) and ML2 (bottom).
Figure 4.8: The maximum acceleration of the bridge for a mass ratio $\Gamma = 0.1$, shown in terms of the mean value for 450 stochastic crowds, for CCBM (blue solid), MLS2 (red dash-dotted) and ML2 (yellow dashed).

### 4.3.4 Computational time

The computational time changes with the size of the system and the length of the simulation why there was no need to consider a stochastic crowd in this comparison since it does not affect the results. Therefore, with the objective to compare the computational time, two scenarios including a 50 m long footbridge subjected to the load from the deterministic crowd (3.25) were studied.

First, the effect of changing the length of the simulation was investigated. For that purpose, a crowd consisting of 50 pedestrians, giving a mass ratio $\Gamma = 0.1$, was considered. The time required to calculate the time dependent modal properties as well as the structural response was measured for increasing length of the simulation. Second, the effect of changing the size of the crowd was investigated. For that purpose, the simulation time was held constant at 100 s and for an increasing number of pedestrians in the crowd, up to a mass ratio $\Gamma = 0.1$, the time required to calculate the time dependent modal properties as well as the structural response of the bridge was measured.

The computational time for calculation of the modal properties for the two scenarios mentioned above are shown in Figure 4.9. Due to the great difference between the models, the results are shown in logarithmic scale. The computational time for ML2 is invariant to changes in the size of the crowd and the length of the simulation since it calculates only one value for the effective modal properties of the bridge,
no matter the size of the system and the length of the simulation. For MLS2, the computational time increases slightly with increasing crowd size and simulation time. However, compared to CCBM, the computational time for both simplified models is significantly shorter. For CCBM the computational time increases rapidly with increasing number of pedestrian and length of the simulation. For a crowd consisting of 50 pedestrians and a simulation time of 100 s, ML2 and MLS2 are around 500000 and 6000 times faster than CCBM, respectively.

The computational time for calculation of the acceleration response for the two scenarios are shown in Figure 4.10. Here, the computational time is approximately equal for the simplified models while CCBM requires slightly more time. The difference between the models is, however, clearly smaller than for calculation of the modal properties. The computational time for the simplified models increases linearly with the simulation time while it is almost invariant to an increasing number of pedestrians. The latter is explained by the size of the system being invariant to a change in the size of the crowd for these two models. For CCBM, the computational time increases linearly for both scenarios, however, faster with increasing simulation time than increasing size of the crowd. In this case, there is only a modest time gain from using any of the simplified models. More precisely, for a crowd consisting of 50 pedestrians and a simulation time of 100 s, the simplified models are around 30% faster than CCBM. However, recalling the stochastic nature of crowd loading and, hence, the need for Monte Carlo simulations with a huge number of runs, this time gain might still be valuable.

![Computational time for calculation of the time dependent modal properties of the bridge, shown as a function of the number of pedestrians (left) and the total time of the simulation (right), for CCBM (blue solid), MLS2 (red dash-dotted) and ML2 (yellow dashed).](image)
4.3.5 Concluding remarks

In this section, the performance of two simplified modelling approaches for prediction of the dynamic response of a footbridge subjected to human-induced loads was evaluated concerning accuracy of the results as well as computational time. For comparison, CCBM was used as a reference model. The models differ in how they account for the HSI effect; in CCBM, each pedestrian is described as a spring-mass-damper system having its own dynamic properties which enables to obtain time-variant system matrices and, hence, to account for the HSI effect. Both simplified models, MLS2 and ML2, describe the pedestrians as moving loads and account for the HSI effect by adjusting the modal properties of the bridge. For this purpose, ML2 uses time independent modal properties corresponding to the maximum possible change caused by the considered crowd while MLS2 uses time dependent modal properties of the bridge, calculated by adding the contribution from each pedestrian to get the total impact from the whole crowd.

First and foremost, it should be mentioned that the main advantage with the simplified modelling approaches in general and ML2 in particular is that they are easily implemented why they can preferably be used in a design situation. However, for academic purposes and if as accurate results as possible are desired, it is recommended to use CCBM since it is the most detailed of the models.

In the comparison of the models, the combination of a simply supported footbridge and a uniformly distributed crowd was considered. Hence, the generality of the conclusions is limited to such situation. The main conclusions are summarised as:

Figure 4.10: Computational time for calculation of the acceleration response, shown as a function of the number of pedestrians (left) and the total time of the simulation (right), for CCBM (blue solid), MLS2 (red dash-dotted) and ML2 (yellow dashed). Note the different scales on the y-axis.
1. In the time domain, the acceleration response as well as the modal properties of the bridge can be well predicted by MLS2.

2. For calculation of the effect of HSI on the modal properties of the bridge, a deterministic crowd can be exactly represented as an equivalent SDOF system. Hence, ML2 is recommended for such calculations.

3. When stochastic crowd loading is considered, both simplified models fairly accurately predict the acceleration response as well as the change in modal properties of the bridge. However, for the acceleration response, MLS2 is slightly more accurate than ML2, as compared to CCBM.

4. Compared to CCBM, both simplified models give a significant reduction in computational time for calculation of the effective modal properties of the bridge and a modest reduction in computational time for calculation of the acceleration response.

In short, these conclusions indicate that, in a design situation, ML2 is probably the most proper choice of model while MLS2 is most useful for calculation of the time dependent modal properties, especially for a Monte Carlo simulation. Finally, if there is no need for a simple model and there are no limitations in time, CCBM is the most proper choice of model. However, further studies with other types of bridges and crowds are required before such general statements can be verified.
Chapter 5

Running loads on footbridges

The objectives of the present chapter are to investigate if the effect of runners on a footbridge is negligible with respect to vibration amplitudes and resonance and to evaluate three modelling approaches for vibration serviceability assessment related to running loads. To this end, a review of how running is considered in today’s design guidelines is first presented in section 5.1. Thereafter, section 5.2 presents a case study comprising field measurements and simulations of the vertical dynamic response of an in-service footbridge subjected to human-induced loads.

5.1 Running loads in design guidelines

Generally, the design guidelines claim that for footbridges with natural frequencies in the range for the pedestrian step frequency, verification of the vibration serviceability limit state is required and an acceleration limit which must not be exceeded is given. Design guidelines for pedestrian bridges was previously reviewed in section 2.5. Herein, the focus is on how they treat the running load case.

5.1.1 The European Standard Eurocode

The European Standard Eurocode states that forces exerted by pedestrians with a frequency identical to one of the natural frequencies of the bridge should be taken into account for limit state verification in relation with vibrations and that appropriate dynamic models of pedestrian loads and comfort criteria should be defined. The guideline does not include any information about running loads in particular, neither that they should nor should not be considered. Nevertheless, it suggests that the design situation should be selected depending on the expected level of pedestrian traffic and that besides walking, “other traffic categories” should be considered, which, of course, involves running or jogging people. However, no modelling approaches or design methodologies are given (EN, 1991).
5.1.2 The French Sétra guideline

The French Sétra guideline claims that the running load case should not be systematically retained, indicating that it should not be considered. The reason given is the short crossing time, implying short disturbance time for other people and low probability of reaching the steady state response. Furthermore, it is indicated that exceptional events such as marathons should be separately considered.

Without offering a design methodology, two possible mathematical descriptions of the vertical component of the time varying force are given. First, a semi-sinusoidal approach is suggested. It features a discontinuous function, described as half a sinus in the period of foot-ground contact and zero elsewhere according to

\[ F(t) = k_p G_0 \sin(\pi t / t_p) \quad \text{for} \quad (j - 1)T_m \leq t \leq (j - 1)T_m + t_p, \quad (5.1) \]
\[ F(t) = 0 \quad \text{for} \quad (j - 1)T_m + t_p \leq t \leq (j - 1)T_m, \quad (5.2) \]

where \( k_p, j \) and \( G_0 \) are respectively the impact factor, step number and pedestrian weight while \( T_m \) and \( t_p \) are the period and period of contact for one step. Furthermore, \( k_p = F_{\text{max}} / G_0 \) and \( T_m = 1 / f_m \), where \( F_{\text{max}} \) is the maximum load and \( f_m \) the step frequency. It is suggested to approximate the period of contact according to \( t_p = T_m / 2 \). However, it is declared that such a simplification is an overestimation of the period of contact compared to experimentally measured values why the semi-sinusoidal approach is conservative. The procedure for determination of the impact factor \( k_p \) is claimed to be delicate and is just briefly described with reference to a graphical representation of the relation between the impact factor and the pacing frequency, based on measured values. Second, it is suggested using a Fourier series without its negative parts according to

\[ F(t) = G_0 + \sum_{i=1}^{n} G_i \sin(2\pi f_m t) \quad \text{for} \quad (j - 1)T_m \leq t \leq (j - 0.5)T_m, \quad (5.3) \]
\[ F(t) = 0 \quad \text{for} \quad (j - 0.5)T_m \leq t \leq jT_m, \quad (5.4) \]

where average values for the amplitudes of the first three harmonics can be found according to (Sétra, 2006)

\[ G_1 = 1.6 G_0, \quad G_2 = 0.7 G_0, \quad G_3 \approx 0.2 G_0. \quad (5.5) \]

5.1.3 The European HiVoSS guideline

The European HiVoSS (Human-induced Vibrations of Steel Structures) guideline highlights running loads as an important load case in the assessment of the comfort criteria. The running load features are briefly described and a time domain description is suggested in terms of a Fourier series with coefficients adapted to running, similar to those given in (5.5). However, a difference is that the force is continuous in time.
Unlike other guides, HiVoSS also suggests an approach with a moving load. A group of \( n \) runners, perfectly synchronised in frequency and phase with the footbridge’s natural frequency \( f \) is then modelled as

\[
P(t, v) = P \cos(2\pi ft)n'\Psi,
\]

where \( P \) is the constant force for a runner with step frequency equal to the natural frequency of the bridge; for the vertical direction, a value is given according to \( P = 1250 \) N. Furthermore, \( n' \) is the equivalent number of pedestrians (\( n' = n \) in case of perfect synchronization) on the loaded surface \( S \), and \( \Psi \) is the reduction coefficient, accounting for the probability that the step frequency approaches the natural frequency under consideration. The reduction coefficient \( \Psi \) increases linearly from 0 to 1 in the range 1.9 - 2.2 Hz, equals 1 in the range 2.2 - 2.7 Hz, decreases linearly from 1 to 0 in the range 2.7 - 3.5 Hz and equals 0 elsewhere. Furthermore, the recommended speed of the joggers is 3 m/s, but it is also declared that many cases allow placing the load at the antinode of the considered mode shape. However, no further specifications are given regarding when such simplification is valid (Feldmann et al., 2008).

5.2 Case study

The bridge subjected to the case study is an in-service footbridge crossing the railway tracks close to the commuter train station in Barkarby, Stockholm, Sweden. It is a simply supported steel truss with a single span of length 37 m and a footway of width 3.5 m, see Figure 5.1.

![Pedestrian bridge in Barkarby, Stockholm, Sweden.](image)
5.2.1 Field measurements

Field tests were performed on the bridge, using two triaxial accelerometers (Sensr CX1), referred to as sensor \(a_1\) and \(a_2\), placed in three different sensor setups. Sensor \(a_1\), serving as a reference sensor, was kept in the same position at the south edge beam and 3.12 m to the east of the midspan location. This point was chosen to record vibrations in as many modes as possible. For example, the second bending mode has an antinode at midspan. Sensor \(a_2\) was kept at the north edge beam and concerning its longitudinal position, it was placed at midspan, the east quarter point and the west quarter point for respectively Setup A, B and C, see Figure 5.2.

First, aiming at identifying the modal properties of the bridge, it was subjected to impulse loading, consisting of a person performing a single jump at the center line of the deck in the longitudinal position of sensor \(a_1\), for the same reason as was given above. To capture the behaviour of higher modes, all three sensor setups were used for this load case. For each setup, three jumps were performed. In between each jump, a pause was introduced in order for the free decay to be completely attenuated. The acceleration signals were analysed and the response was concluded to be clearly dominated by the first bending mode, having a frequency of 2.7 Hz and a damping ratio of 0.36%, according to the half-power bandwidth method.

Second, with the aim of evaluating the dynamic performance of the bridge, walking and running tests with 1, 4 and 7 people were performed. According to normal pacing frequency, a step frequency of 2 Hz was adopted for the walking tests. For running, 2.7 Hz was chosen since it is within the frequency range for running and coincides with the first natural frequency of the bridge and hence it corresponds to an important load case. Perfect synchronization within the group was aimed for and the correct pacing frequencies were obtained using a metronome. Furthermore, the lateral positions of the participants were consistently chosen to be symmetrical with respect to the center line, i.e. on the center line for 1 person and in two lines on each side of the center line for 4 and 7 people. For these tests, Setup A was used and for each load case, the bridge was crossed three times, again with a pause in between each passage for the energy in the free vibrations to entirely dissipate.

![Figure 5.2: Sensor positioning.](image-url)
5.2. CASE STUDY

5.2.2 Numerical analyses

The numerical part of this study aimed at evaluating three modelling approaches for vibration serviceability assessment related to running pedestrians why exclusively the running tests were simulated. The considered load cases were 1 and 4 people running across the bridge.

A modal domain description of the bridge, limited to include only the first bending mode, was used for the simulations. This choice was motivated by the acceleration signals from the field tests being clearly dominated by the fundamental frequency of the bridge. The mode shape was approximated as \( \phi_b(x) = \sin(\pi x/L) \), where \( L \) is the span length. The natural frequency and damping ratio were respectively 2.7 Hz and 0.36 \%, chosen according to the measured acceleration signals, see section 5.2.1. Furthermore, the modal mass was found to be approximately 77 tonnes, calculated according to (3.2).

The running people were described as moving loads. Three different models, henceforth referred to as model A, B and C, were used for the time dependency of the load from a running person. The time dependency was described according to (5.1)-(5.2) for model A, (5.3)-(5.5) for model B and (5.6) for model C, see Figure 5.3 for a graphical representation.

Apart from the difference in the time dependency of the running load, the three modelling approaches are identical. The spatial dependency of the load was described by a constant velocity of 3.6 m/s and a longitudinal inter-pedestrian distance of 1.3 m. These values as well as remaining input parameters, being the pedestrian step frequencies and masses, were chosen in accordance with the field tests. However, the pedestrians’ lateral positions were disregarded; the pedestrians were assumed to run on the center line of the bridge.

![Figure 5.3: Models for time dependency of running load.](image-url)
Furthermore, in model A and B, a contact period of 0.23 s was used (Bachmann and Ammann, 1987); in model A, an approximate value of the impact factor \( k_p = \frac{\pi T_m}{2t_p} \approx 2.5 \) was used (Occhiuzzi et al., 2008); and in model C, a reduction factor \( \psi = 1 \) was used and perfect synchronization was assumed, i.e. \( n' = n \).

### 5.2.3 Results and discussion

Regarding the field tests, only the signals from sensor \( a_2 \) were considered since the maximum acceleration is the decisive parameter and the response was dominated by the first bending mode, having its maximum amplitude at midspan, i.e. at the position of sensor \( a_2 \). The signals were low-pass filtered with an upper limit of 10 Hz. Acceleration signals for one passage of 4 people walking and 1, 4 and 7 people running are shown in Figure 5.4, where from it is clear that for this particular bridge, the running load case is worse than the walking load case. The acceleration level was around 11 times larger for 4 people running, compared to (the same group of) 4 people walking. Moreover, it is indicated that also for the running load case interaction effects that affect the dynamic response may appear. The vibration amplitude did not increase linearly with the size of the group, which was of course partly caused by the participants’ different masses and the slightly varying running speeds and hence different duration for application of the load. However, it was thought to be mainly caused by interaction effects. For the case of 7 people running across the bridge, there were two peaks instead of one; the reason for this was most likely that for large acceleration levels, the participants failed to keep the correct step frequency. Therefore, when the vibration amplitude increased, the synchronization level decreased and vice versa.

![Figure 5.4: Measured vertical bridge deck acceleration at midspan for 4 people walking and 1, 4 and 7 people running across the bridge.](image-url)
5.2. CASE STUDY

Table 5.1: Measured bridge deck acceleration at midspan for different load cases.

<table>
<thead>
<tr>
<th>No. people</th>
<th>Walking, 2 Hz</th>
<th>Running, 2.7 Hz</th>
<th>Max acc. (m/s²)</th>
<th>Avg. peak acc. (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>✓</td>
<td></td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>4</td>
<td>✓</td>
<td></td>
<td>0.16</td>
<td>0.06</td>
</tr>
<tr>
<td>7</td>
<td>✓</td>
<td></td>
<td>0.10</td>
<td>0.08</td>
</tr>
<tr>
<td>1</td>
<td>✓</td>
<td></td>
<td>0.59</td>
<td>0.47</td>
</tr>
<tr>
<td>4</td>
<td>✓</td>
<td></td>
<td>1.79</td>
<td>1.37</td>
</tr>
<tr>
<td>7</td>
<td>✓</td>
<td></td>
<td>2.52</td>
<td>1.71</td>
</tr>
</tbody>
</table>

The horizontal (black dashed) lines in Figure 5.4 show the acceleration limit for maximum comfort according to the Sétra and HiVoSS guidelines. The limit was exceeded for all running tests, but for none of the walking tests. Furthermore, due to the low level of damping, the acceleration level was above the limit for almost half a minute when a group of people was running across the bridge. A low level of damping is something that many footbridges have in common. Moreover, the vertical bridge deck accelerations at midspan for all passages are summarised in Table 5.1; the maximum reached acceleration and average peak acceleration from all passages are given.

The envelope of the vertical bridge deck acceleration for one passage of 1 and 4 people running, obtained from the field measurements and model A, B and C are shown in Figure 5.5. It can be concluded that, for this particular bridge, model A and B give fairly accurate results while model C overestimates the response and that all three models give a greater error for a group of people running than for a single person running. This could possibly be caused by lacking synchronization between the runners in the field tests while in the simulations, the runners were assumed to be perfectly synchronised. Furthermore, for this particular bridge, model A and B give better correspondence between simulations and measurements than model C.

![Figure 5.5: Envelope of vertical bridge deck acceleration at midspan due to 1 (left) and 4 (right) people running across the bridge, obtained from field measurements and simulations. Note the scaling on the y-axis.](image-url)
Table 5.2: Steady state and maximum acceleration for different load cases.

<table>
<thead>
<tr>
<th></th>
<th>1 person running</th>
<th></th>
<th>4 people running</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max acc. (m/s²)</td>
<td>Steady state (m/s²)</td>
<td>Max acc. (m/s²)</td>
<td>Steady state (m/s²)</td>
</tr>
<tr>
<td>Model A</td>
<td>0.60</td>
<td>1.43</td>
<td>1.95</td>
<td>4.60</td>
</tr>
<tr>
<td>Model B</td>
<td>0.63</td>
<td>1.42</td>
<td>2.02</td>
<td>4.59</td>
</tr>
<tr>
<td>Model C</td>
<td>0.80</td>
<td>2.26</td>
<td>3.19</td>
<td>9.03</td>
</tr>
<tr>
<td>Field test</td>
<td>0.59</td>
<td>-</td>
<td>1.79</td>
<td>-</td>
</tr>
</tbody>
</table>

To see if the short crossing time had any impact on the vibration amplitude, the steady state response for each of the numerical models was calculated by applying the time dependent loads at midspan, as compared to moving loads for previous simulations. The results are shown in Table 5.2 and it could be concluded that, due to the short crossing time, the steady state was not reached in any of the cases.

5.2.4 Concluding remarks

With the objective to investigate if the effect of runners on footbridges is negligible, an in-service footbridge was subjected to a case study comprising field measurements and numerical simulations. Consulting today’s codes of practice, verification of comfort during running is not required for the bridge in question. More precisely, the European HiVoSS guideline states that the fundamental frequency of this bridge is high enough for dynamic analyses not to be required. According to the French Sétra guideline, running loads do not need to be considered in any case, motivated by the short crossing time. Conversely, the European standard Eurocode claims that, based on the natural frequencies of the bridge, verification of the comfort criteria shall be performed. However, neither is it clearly stated that running loads in particular should be considered, nor how it could be done.

In spite of this, the field measurements showed that the acceleration limit stated in the guidelines was exceeded for one single person running across the bridge and that a group of people walking synchronised did not cause exceedance of the limit. Therefore, the main conclusion from this study was that running loads cannot in all cases be regarded as a negligible load case in the design phase of a pedestrian bridge. This motivates that, for pedestrian bridges having natural frequencies in the range for running step frequencies, the design guidelines should require verification of the comfort criteria during running. Furthermore, it was concluded that the crossing time was too short for steady state to be reached. However, the acceleration limit was still exceeded and due to the low level of damping, the vibration amplitude was above the limit for a considerable amount of time. Therefore, the short crossing time referred to in the Sétra guideline can be considered a poor motivation for neglection of the running load case. Besides this, two of the applied modelling approaches proved the ability to fairly accurately predict the dynamic response of a footbridge induced by running loads from 1 and 4 people whereas the third modelling approach vastly overestimated the acceleration levels.
Chapter 6
Concluding remarks

In this final chapter, general conclusions from the performed research work are summarised and some suggestions for further research are given.

6.1 Conclusions

The performed research work considers two subjects: the human-structure interaction (HSI) effect and running loads, with the main focus being placed on the former and, more precisely, on how it can be accounted for in an efficient way in the prediction of the dynamic response of a footbridge. First, the HSI effect and its impact on the structural response were analysed, and second, the performance of two simplified modelling approaches was evaluated. Moreover, a case study was performed to investigate the effect of runners on footbridges. Herein, the conclusions are very briefly summarised since they have been elaborated in further detail in previous chapters. The main conclusions from the present work are:

- For the combination of a simply supported footbridge and a uniformly distributed crowd, the HSI effect is beneficial from a vibration serviceability perspective in the sense that it reduces the peak acceleration. The reduction is strongly related to the load characteristics as well as the dynamic properties of the crowd and the bridge. The effect is most beneficial during resonant loading and for a crowd to bridge frequency ratio close to unity and a high crowd to bridge mass ratio. Furthermore, the peak acceleration as well as the impact of the HSI effect on the structural response of the bridge decrease if the inter-subject variability in the pedestrian step frequency is accounted for.

- Both simplified modelling approaches for consideration of the HSI effect fairly accurately predict the change in modal properties as well as the structural response of a simply supported footbridge subjected to the load from a uniformly distributed crowd. The models can be easily implemented why they
can preferably be used in a design situation. Finally, the computational time for calculation of the modal properties is significantly reduced while for the calculation of the acceleration response, a modest reduction in computational time is obtained.

- Running loads cannot in all cases be regarded as a negligible load case in the design phase of a pedestrian bridge. Furthermore, in spite of the crossing time being too short for the steady state response to be reached, the acceleration limit can still be exceeded. In such situation, the vibration amplitude might stay above the limit for a considerable amount of time as a result of the low level of damping which is something that many footbridges have in common. Finally, it was indicated that HSI effects might occur also for the running load case.

### 6.2 Further research

The proposed recommendations for further research are based on the conclusions from the present study in combination with the amount of previous knowledge and performed research within the two studied fields respectively.

#### 6.2.1 Human-structure interaction

Considering the HSI effect, a great amount of publications exist, see section 2.4. There is a consensus that the effect is of great importance and should be accounted for in the design of a footbridge. Nevertheless, field measurements as well as numerical studies aiming at deepening the understanding of the effect are still valuable. Furthermore, advanced models have been developed and have proven to accurately predict the dynamic response of a footbridge subjected to the load from walking people. Considering this, the main focus during the performed researched work was placed on the development of simplified modelling approaches. Consequently, the main contribution from this work is simplified, time efficient models. However, further verification of the performance of these models is still needed and so is also the development of the models to cover more aspects, and preferably with avoidance of an increased level of complexity. Accordingly, further research questions are identified as:

- **Further verification of simplified models**

  Until now, the simplified HSI models have shown promising results for prediction of effective modal properties and structural response of a footbridge. However, this study was limited to consider the first vertical bending mode of a simply supported footbridge subjected to the load from a uniformly distributed crowd. The next step is to include more modes in the analysis and to study other types of bridges and crowd distributions. In the end, the proposed design methodology should be presented together with information about its
6.2. FURTHER RESEARCH

limitations, an expected size of the error and for what kind of bridges and load scenarios it is applicable.

- **Further development of simplified models to cover more aspects**
  The simplified models account for one type of HSI effect and pedestrian loading. The models could be adjusted to account for several HSI effects or to consider other types of human-induced loads. For example, to consider running loads, the load from a footstep is different, but so is also the HSI effect due to the discontinuity in the foot-ground contact. Hence, the techniques for finding the effective modal properties require changes, for the models to consider the running load case under the consideration of HSI effects. However, it might also be the case that the change in modal properties is not of significant importance for the running load case while other kinds of HSI effects are.

6.2.2 Running loads

The research about running loads on pedestrian bridges is certainly in an earlier stage of its way to consideration in the design guidelines. Therefore, the contribution from this work could be seen as a preliminary study, performed with the aim of motivating further research within the field, by verifying the need of consideration of running loads in the design of a footbridge. However, to further deepen the understanding of the subject, field measurements and laboratory tests are still of highest priority. Hence, recommended further research subjects are identified as:

- **Further investigation of running loads**
  Full-scale measurements on in-service footbridges as well as tests performed in laboratory conditions are required to enlarge the amount of measured data that could later be used for verification of numerical models. The ground reaction force during running as well as its effect on the structural response of a footbridge are of great interest to measure. Additional aims with the field measurements are to further verify the need of consideration of running loads in the design of a pedestrian bridge and to further investigate the interaction phenomenon appearing for high acceleration levels during the tests presented in section 5.2.1. Could one possibly find out at which level of acceleration the runner cannot stick to its pace?

- **Further development of numerical models**
  Existing models for running loads do not account for the interaction effect mentioned above. Given that the field measurements indicate that there is a certain level of acceleration when the runners change their step frequency, this effect should be accounted for in the models. For that purpose, an active modelling technique, where some threshold is incorporated to make the runner stop in case of high acceleration levels, is required.


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