Spatial Variability of shotcrete thickness

MAXIMILIAN KLAUBE
Abstract

An important task during the construction process is to validate the dimensions and properties of a given structure. The dimensions like for instance the thickness of a construction element should be measured after finishing building it. The aim is to compare the measured value with the design value to avoid that elements do not correspond to the input requirements. Moreover, the measurements are helpful to analyse the statistical distribution of the investigated geometrical property by computing e.g. a histogram, which visualises the dispersion and enable the calculation of the probability of failure for a specific structure or element.

In this work, a shotcrete layer has been analysed in order to provide information about the homogeneity of the shotcrete thickness in a pre-determined tunnel section. The calculation method is based on two laser scans, before and after applying the shotcrete. Due to the construction process, the shotcrete layer will not be totally equal, which might be a safety problem. Especially, when the shotcrete layer is thinner than required and hence, the actual variation of the shotcrete must be considered and verified. To determine the statistical distribution, correlograms and histograms have been computed for a wall area in a tunnel in Southern Sweden. The correlogram shows the distance where the values have a correlation to each other and usually this distance is called scale of fluctuation. For the wall section, this scale of fluctuation has been calculated for the length (0.8m) as well as the height (0.8m). Compared to the original sample distance, e.g. distance of the rock bolts, the variance for the calculation of the probability of failure might be reduced.

Keywords: Spatial variability, Scale of fluctuation, Correlogram, Tunnel engineering, Reliability-based design, Shotcrete
Table of content

1  Introduction .................................................................................................................. 1
  1.1  Background .............................................................................................................. 1
  1.2  Aim ............................................................................................................................ 3
  1.3  Disposition .............................................................................................................. 3
  1.4  Limitations ............................................................................................................... 3
2  Design and support of underground excavations in rock .............................................. 4
  2.1  Rock support .......................................................................................................... 4
  2.2  Shotcrete ................................................................................................................. 4
  2.3  Uncertainties in rock engineering ........................................................................... 5
  2.4  Design principles of the rock support ..................................................................... 7
        2.4.1  Prescriptive method ....................................................................................... 8
        2.4.2  Observational method ................................................................................... 8
        2.4.3  Calculation methods ...................................................................................... 9
  2.5  Design of shotcrete layer for block stability ............................................................. 17
3  Spatial variability ........................................................................................................ 20
  3.1  Variance reduction ................................................................................................. 20
  3.2  Estimation of the scale of fluctuation ..................................................................... 21
4  Method ........................................................................................................................ 27
  4.1  Preparation of the data .......................................................................................... 27
        4.1.1  Identifying suitable data points ...................................................................... 27
        4.1.2  Calculating the shotcrete thickness ............................................................... 31
  4.2  Correlogram .......................................................................................................... 31
5  Results ........................................................................................................................ 34
6  Discussion ..................................................................................................................... 39
7  Conclusions and further research .............................................................................. 41
8  List of figures ................................................................................................................. 42
<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>List of tables</td>
<td>............................................................................</td>
<td>43</td>
</tr>
<tr>
<td>10</td>
<td>Publication bibliography</td>
<td>............................................................................</td>
<td>44</td>
</tr>
<tr>
<td>11</td>
<td>Appendix MATLAB files</td>
<td>............................................................................</td>
<td>46</td>
</tr>
</tbody>
</table>
1 Introduction

1.1 Background

In the long history of humanity, the demand for mobility has always been an important factor for development. Especially, the wish to travel safer, faster, but also to be independent of weather conditions, increase the need for civil engineering structures such as bridges or tunnels. Tunnels for mobility can be classified more general as underground excavations. These can be built as well for other purposes which can be nuclear storage facilities or sewage systems. These are all structures with long traditions within the field of civil engineering and often it is a challenge for engineers to design and construct such a complex structure. Also, here the designers and the entrepreneurs always search for the best solution to create safe, durable, and cost-efficient excavations. Especially, the design requires reliable methods to avoid any kind of damage during the construction as well under the utilisation phase. Therefore, engineering standards and guidelines have been established to cope with all different issues which might occur during the lifetime of the structure.

Even though intensive research in the field of rock mechanics and the design of underground excavations have been done to enhance the comprehension of the actual behaviour of the rock mass, uncertainties are still present in the constructions. Usually, different rock support systems like concrete or rock bolts are used to improve the stability of the tunnel. A frequent design is to use shotcrete and rock bolts as initial support to avoid failures like block falls during the construction phase which is especially, a matter of labour safety. Finally, a second concrete lining (e.g. pre-casted elements) is added for the utilisation phase. However, in some cases the initial support is sufficient, and a second concrete lining is not necessary. That mainly depends on the quality and the strength of the rock mass. However, it is crucial that shotcrete always fulfil the requirements such as required thickness, strength, or adhesion of the shotcrete to the rock. Especially, the layer should have a thickness which is equal or larger than the design value. However, due to several practical reasons such as the unevenness of the rock surface, the knowledge and ability of the operator, the thickness of shotcrete will not be constant. For instance, the unevenness of the rock
surface after blasting is shown in Figure 1, which is part of the analysed tunnel section. Thus, it might be possible that the thickness is thinner than demanded and will thus not fulfil the safety requirements. This difficulty should be included in a design method by considering the statistical distribution of the shotcrete thickness. One possibility is the calculation of the scale of fluctuation since it provides information about the homogeneity. It describes the distance between the values which are correlated to each other and can be used as input parameter for enhancing the calculation of the probability of failure since it can help to reduce the variation.

Figure 1 Tunnel section at the ÄSPÖ hard rock laboratory
1.2 Aim
As described before, the following research work deals with the possibility to consider the actual shotcrete thickness in a probabilistic-based design method. The main aim will be to develop a concept to calculate the scale of fluctuation of a shotcrete layer in a blasted tunnel by computing correlograms which visualise the correlation. This can be used to check if the variation can be reduced, which influences the calculation of the probability of failure.

1.3 Disposition
The work can be divided into three main parts. In the first part, an introduction to underground excavations in rock and design of tunnel support will be given. The focus will be laid on the illustrations of deterministic design methods as well as on reliability-based methods. The second part deals with the calculations of the actual shotcrete thickness distribution and the scale of fluctuation. Important results during this part are the visualisation of the correlation with correlograms and the statistical distribution with histograms. They are crucial for the calculation of the scale of fluctuation. In the third and final part, the results are discussed and reviewed to point out important findings and to figure out weak points of the developed concept. Furthermore, some ideas for a further research will be given as well, e.g. how exactly the scale of fluctuation can be used to calculate the reliability of failure.

1.4 Limitations
The outcome of this work will contribute information to establish a method considering the homogeneity of a shotcrete layer in the design, but the results might be restricted in their significance. One limitation of the outcome might be to transfer the findings of this specific wall section with some singularities to a more general real-world situation. Especially, the situation for the roof might be different and block fall-related problems occur mostly within the roof section. However, the aim is to provide a first a concept and the basis for further investigations since no similar research work deals with this topic.
2 Design and support of underground excavations in rock

2.1 Rock support
Underground excavations in hard rock mass are frequently constructed using drilling and blasting but also in some cases by using a Tunnel Boring Machine (Hemphill, 2012). The decision which method might be preferable depends on different factors, such as tunnel length or required flexibility regarding the rock quality and the alignment (Palmström and Stille, 2010). Afterwards, the rock support can be installed by applying a combination of rock bolts and shotcrete and usually, a second concrete lining, e.g. pre-casted concrete elements, is added (Nord, 2017). The cases when the second concrete lining can be omitted, depending on if the rock strength and quality are high enough. In these cases, the combination of the shotcrete and rock bolts do not aim to carry the whole rock weight but rather stabilise the rock mass and to avoid failure (Ellison, 2017). Common failures which might be prevented by using rock support are gravity-driven failures such as block falls or due to too high stresses such as slabbing (Palmström and Stille, 2010). Since this research work concerns the thickness of a shotcrete layer, a short overview is given to illustrate the function.

2.2 Shotcrete
Shotcrete is a mixture of cement, ballast, water, and additives and should meet high requirements to fulfil the demanded duties (Ellison, 2017). The mixture should be stable so that a tendency for segregation of water and material should be reduced. Especially, this stability should be given during the whole construction, mainly for transporting, pumping, and spraying. After spraying the mixture with high velocity, the adhesion forces to the rock surface but also to other shotcrete layers should be high enough to reduce rebound effects. Therefore, the shotcrete mixture will be compacted under high pressure and should develop fast good strength properties to provide a safe labour environment. But also, in general, shotcrete should feature good strength properties, especially compressive strength. However, by adding fibres, bars, or a grid also the tensile strength can be increased significantly.
2.3 Uncertainties in rock engineering

The main idea of a design process is to find a way to construct a structure like tunnels durable and safe. Basically, the durability refers to the fact that a structure fulfils the design requirements for a pre-determined lifespan, e.g. 100 years. The safety of a structure, however, concerns the possibility to minimise the likelihood of a failure which is given through the society's demands on an acceptable level of safety. However, the design should be economically reasonable and affordable. In fact, this failure probability versus code trade-off issue is very difficult, since human life cannot be determined easily in costs (Fenton and Griffiths, 2008).

A design concept takes these uncertainties into account by establishing a specific level of safety, and usually different approaches are given in a national building code. The main idea of these safety concepts is shown in the following chapter. But it is important to remember that a failure does not only mean directly a complete destruction with possible loss of life rather also it could also include unacceptable deformations (Fenton and Griffiths, 2008).

First of all, a general overview about uncertainties which can occur during the building process is given since they are very common in the field of civil engineering, and especially, in rock engineering. Thus, it is very helpful to identify and to classify different uncertainties and to be aware of them. However, at some point, it can be quite challenging to consider all uncertainties since they are manifold, and structures have unique conditions and properties. In practical usage, uncertainties can be divided into eight different types (Melchers, 1999).

The phenomenological uncertainty is related to the problem that some actions or events are unknown during the design and construction phase. That can happen for instance under extreme weather conditions or due to a lack of knowledge like unexpected ground conditions. Related to this problem is the decision uncertainty which means that some actions are considered as irrelevant and have been neglected. However, these actions are important and the taken decision is wrong. Another problem which can occur during the design stage is modelling uncertainty which can also be related to the two first ones. Nowadays, national codes like the Eurocode,
require some assumptions and simplifications to enable a better handling of mechanical behaviour, e.g. enable the usage of an analytical approach. It is obvious that assumptions can be wrong since the simplifications always neglect some effects. The following uncertainty seems to be quite similar since it deals with a problematic prediction of a specific behaviour. Prediction uncertainties include that some predictions about future conditions are not correct. However, not only the design process itself can be problematic but also the evaluation of material parameters and data in general. Physical uncertainties are related to the natural variation of material parameters or actions especially natural loads. Moreover, statistical uncertainties can be related to the topic of inappropriate and not representative data. Thus, such a problem is also often described as a bias within the field of statistics. Results seem to be accurate since the variation is rather small but is not very close to the real situation. The last category deals with problems and uncertainties due to human error which is related to inappropriate human behaviours like carelessness or insufficient knowledge.

A strategy to enhance the handling of these different uncertainties would be to classify them. Mainly, two groups of uncertainties can be detected. On one hand, there are epistemic uncertainties and on the other hand, aleatory uncertainties. The main difference is that epistemic uncertainties can be reduced by increasing the knowledge about it, for instance with more studies or experiments. Whereas, aleatoric uncertainties cannot be reduced since the results are distributed randomly (Spross, 2017b). Even though it might be difficult to determine the properties of natural materials correctly, it can be concluded that uncertainties present in rock tunnel design are mainly epistemic (Johansson et al., 2017). Mostly, uncertainties which are related to natural behaviour can be defined as aleatory. Whereas human behaviour might be determined as a more special case since it does not belong to one of both categories. In fact, the knowledge of failures due to human behaviour is limited and thus, normally uncertainties due to human behaviour cannot be accounted in the design. Therefore, the risk for human errors should be reduced e.g. by internal and external reviews (Johansson et al., 2016). However, many uncertainties shown above can be reduced with better guidelines or more research, e.g. through a better understanding of shotcrete distribution.
The given problem which is investigated in this research can also be seen as uncertainty within rock mechanics. The thickness of the shotcrete layer is not constant and hence, the distribution is uncertain and a risk if the layer is too thin. This difficulty is visualised in Figure 2.

![Figure 2 Variation of the shotcrete thickness due to uneven rock mass surface (Sjölander et al., 2017)](image)

**2.4 Design principles of the rock support**

In countries within the European Union and some affiliated countries, the Eurocodes (SS-EN 1990) is the base for the design of structures. The main reason is to reduce trade and building code restrictions within the European market. According to the Swedish Standards Institute (SIS, 2018), there are in total 10 different Eurocodes, the Eurocode 2 for instance deals with concrete structures and Eurocode 7 is used for geotechnical structures. Thus, Eurocode 7 can also be used for underground excavation purposes and has been adapted for instance by the Swedish Transportation Administration for the Swedish market, which is responsible for the design of many tunnels in Sweden. The Eurocode 7 (SS-EN 1997) indicates which design method that can be used for geotechnical structures and an overview is given in Figure 3.

![Figure 3 Available design methods according to Eurocode 7 (SS-EN 1997)](image)
2.4.1 Prescriptive method

Based on experiences but also on research, the need for rock support can be estimated by using traditional methods which have been appropriate for many different underground excavation situations. A very common possibility is to use a classification system like the RMR or Q rating system. Both systems have different categories which assess different properties of the rock mass, i.e. the rock mass quality or the influence of joints (Palmström and Stille, 2010). The required rock support can be estimated by using these rating systems. The Norwegian Geotechnical Institute introduced the Q system in 1974 by providing among others a chart which shows the connection between a certain rock class and the rock support (Barton et al., 1974).

2.4.2 Observational method

In some cases, it can be difficult to apply such generalised rating systems, especially if the prediction of the rock mass behaviour might be difficult. In areas with a high degree of varying and heterogeneous geotechnical ground conditions, this method can be very reasonable and useful (Spross, 2017a), since it adapts the design according to the in-situ conditions. Therefore, in such situations, the Eurocode allows using the Observational method (SS EN-1997). The observational method is an acceptable verification method to verify limit states in Eurocode 7 (Spross and Johansson, 2017). Ralph Peck introduced this observational method in 1969 (Spross, 2017a) and roughly spoken, the observational method consists of three important parts. Within the first part, the guidelines but also experiences help to predict the behaviour and to establish limit states. Whereas, in the second part, these limits are monitored during the construction process and therefore, the performance of those parameters must be measurable. The third part concerns pre-defined countermeasures which should be applied if the limits of acceptable behaviour are exceeded (Spross, 2017a).
2.4.3 Calculation methods

As shown in Figure 5, the verification of limit states can be based on three different mathematical concepts which are the analytical, semi-empirical and the numerical solution. In general, the design controls that the resistance of a structure is larger than the acting loads by including a safety margin.

In this research work, the focus will be laid on the deterministic and reliability-based methods.

In general, both methods try to implement a structural safety even though the concepts are quite different. The deterministic method usually uses pre-determined safety factors to yield a single solution which fulfils the safety requirements. However, the probabilistic design method assess the probability of failure considering the actual level of safety for each design situation (Johansson et al., 2016). Thus, for geotechnical issues, the reliability-based method might be more suitable because deterministic safety factors are not able to capture the uncertainty related to the loads and resistance in the individual case (Johansson et al., 2016). Both methods are presented more detailed in the following chapters.

2.4.3.1 Deterministic design method

Safety factor method

The safety factor method describes the method to establish a certain level of safety by applying a deterministic factor \( F \). Basically, \( F \) describes how much the average resistance \( R \) should be larger than the average actions \( S \) (Eq. 1) (Johansson et al., 2016).

\[
F = \frac{R}{S} \quad (1)
\]
Usually, these values can be found in engineering standards and they are based on long-term experiences and expert judgment (Johansson et al., 2016).

### 2.4.3.2 Reliability-based design methods

In general, the random variables representing the resistance and the loads are collected in a vector \( \mathbf{X} = [X_1, \ldots, X_n] \). The reliability-based design methods can be defined as the evaluation of the probability of failure. This can be solved by the multi-dimensional integral (Eq. 2) of the unsafe region which is defined by \( G(\mathbf{X}) \leq 0 \) since the limit state is defined as \( G(\mathbf{X}) = 0 \) (Melchers, 1999).

\[
p_f = P [G(\mathbf{X}) \leq 0] = \int \ldots \int_{G(\mathbf{X}) \leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \quad (2)
\]

where \( f_{\mathbf{X}}(\mathbf{x}) \) is the joint probability density function for all random variables. For many cases, this equation cannot be solved analytically so that some approximation methods have been developed and to some extent, they can consider for instances non-linear limit states or non-normal variables. These methods can be categorised into four different levels, based on their complexity (Johansson et al., 2016).

- **Level I methods** consider uncertainties by adding safety margins to the resistance as well to the actions, e.g. partial factor method.
- **Level II methods** use mean value, standard deviation, and correlation coefficients to model uncertainties even though it assumes a normal distribution of the variables, e.g. simplified reliability index.
- **Level III methods** are based on the joint distribution function of all parameters to model uncertainties, e.g. Monte Carlo Simulation and first-order reliability method (FORM).
- **Level IV methods** include the consequences of failure into the analysis and enable also cost-benefit analyses.
In this research work, the Monte Carlo simulation and FORM are presented, since they are commonly used (Johansson et al., 2016). Moreover, the partial factor is explained since it is the basis for the Eurocode (SS-EN 1990).

However, it must be noted that in rock and geotechnical engineering design, S and R often depend on the deformation and therefore there are not separable (Johansson et al., 2016) and thus, it might be difficult to reduce every problem in civil engineering to simple R versus S reliability equations (Melchers, 1999). For instance, the active earth pressure acting as a load on a structure depends at the same time on its strength, which determines the resistance (Katzenbach, 2016).

**Reliability-based partial factor method**

Basically, the resistance of a structure is compared with a specific load case. It is obvious that the resistance ($R$), commonly also described as bearing capacity, should be larger than the affecting load ($S$). As an equation, this limit-state concept can be expressed as:

$$R - S \geq 0 \quad (3)$$

The distributions of the $S$ and the $R$ have usually a distribution which can be described by a normal distribution. An exemplified distribution of $R$ and $S$ is visualised in Figure 6. However, the bell curves are not always the same because the accuracy of estimating the load or the resistance can be different as well. In the given Figure 6, the Gaussian bell curve for the load is more accurate than for the resistance. However, this relationship between $S$ and $R$ is valid for many other design situations. This is related to the topic of uncertainties in the structure and the ability to predict the future behaviour. For instance, the Gauss bell curve for steel will usually be much more accurate than for concrete since the distribution of material properties of steel is much more homogeneous. Hence, the partial factor of steel would be smaller than for concrete. Thus, these partial factors ($\gamma$) are used to compute the design values from the characteristic values, which is given in equation 4.
\[ S \times \gamma_S \leq \frac{R}{\gamma_R} \] (4)

Figure 4 also tries to emphasize the “design effect”, which means that the actions are increased, and the resistance is decreased. In the sketch, the design values are given as \( S_d \) and \( R_d \). These both values are the results of applying these partial coefficients on the mean values. The design value of the resistance should be thereby larger than the design value of the action. However, the risk is not zero so that at some point the actions can be nevertheless larger than the resistance. Therefore, the level of safety does not say that a structure would never collapse, it is more about to reduce the likelihood for a failure and to comply the demands on safety.

\[ S_{d} \leq R_{d} \] (5)

Figure 4 Sketch of the distribution of the resistance (R) and the load (S) (Vagnon et al., 2017)
Moreover, if more actions are impacting on the structure, then some combination factors can be used since the actions can occur at the same time. However, it is unlikely that all the actions occur in full strength at the same time so that some actions can be reduced by using these combination factors.

The magnitude of these partial factors is based on the reliability index, which considers the reference period and different probability targets (Vagnon et al., 2017). These targets are determined for instance by three different reliability classes (RC1 to RC3) through the Eurocode, which indicate the hazardousness of consequences due to a failure. Hence, structures which can be classified as RC3 have the highest requirements (SS-EN 1990).

The requirements from the Eurocode regarding the reliability index are given in the following table:

<table>
<thead>
<tr>
<th>Reliability Class</th>
<th>Minimum values for $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 year reference period</td>
</tr>
<tr>
<td>RC3</td>
<td>5.2</td>
</tr>
<tr>
<td>RC2</td>
<td>4.7</td>
</tr>
<tr>
<td>RC1</td>
<td>4.2</td>
</tr>
</tbody>
</table>

The reliability index provides information about the failure probability ($p_f$) since the reliability index is defined as a relationship between the mean value ($\mu_m$) of a specific parameter and the standard deviation ($\sigma_m$), which is visualised in Figure 5. The distance between the mean value of the resistance distribution and the failure point is equal to a multitude of the standard deviation. Thus, the reliability index can be expressed by the following equation:

$$\mu_m = \beta \ast \sigma_m \iff \beta = \frac{\mu_m}{\sigma_m}$$ (6)
FORM

The FORM method calculates the second-order moments, which are usually determined as mean value and as standard deviation (Melchers, 1999). This approach is based on the idea that the limit state function has more than two basic random variables. For instance, the linear limit state is shown (Eq. 7):

$$ G(x) = Z(x) = a_0 + \sum_{i=1}^{n} a_i * x_i $$

The limit state function is set equal to zero in order to calculate these second-order moments. However, the calculation might be very challenging since $G(x)=0$ is mostly not linear. Thus, an approximation of the limit state function can be linearized by using the first-order Taylor series expansion ($f$) and to transform everything into the standard normal space. Afterwards, the shortest distance ($\beta$), between the origin which is the point of the maximum likelihood and the transformed function can be calculated and gives the safety index (Johansson et al., 2016). This concept is shown in Figure 6.
The equations are shown to calculate the transformed normalised variables (Eq. 8 - 9) as well as the safety index (Eq. 10) (Spross, 2016).

For the resistance:

\[ Z_R = \frac{R - \mu_R}{\sigma_R} \]  

(8)

For the actions:

\[ Z_S = \frac{S - \mu_S}{\sigma_S} \]  

(9)

And finally, the shortest distance can be calculated by using, e.g. for a linear limit state, following expression which provides the safety index:

\[ \beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \]  

(10)
Monte-Carlo Simulation

The Monte Carlo simulations can be described as a computational approach and aim to check the probability of failure by applying the limit state function. This limit state function, $G(\hat{x})$, with random values ($\hat{x}$) is set equal to zero (Eq. 11) and if the limit state is exceeded then the structure has failed. The simulation counts the number of failure ($n$) for a determined number of repetition ($N$) (Johansson et al., 2016). The probability of failure can be expressed (Eq. 12) as a relationship of counted number of failure and total number of repetition.

$$G(\hat{x}) = 0$$ (11)

$$p_f = \frac{n \times (G(\hat{x}_i) \leq 0)}{N}$$ (12)

It seems to be obvious that the number of repetition must be high enough to get representative results. Moreover, the accuracy will be better if the failure is unlikely (Eq. 13). Thus, the error probability ($\varepsilon$) for an observed value is quite important for this concept (Melchers, 1999) and can be expressed as:

$$\varepsilon = \sqrt{\frac{k (1 - p_f)}{N}}$$ (13)

In this equation represents the number of performed repetitions ($N$) and the likelihood of a failure ($p_f$). The error possibility also includes the accuracy of the estimator by showing the error in the simulation. That is also known as the confidence interval which defines how true a value is (Fische and Hebuterne, 2010) and e.g. a confidence interval of 90% gives a $k$ equal to 1.64 by using a standard normal table. Moreover, it might be important to know how many repetitions are needed with a given confidence level ($C$) and failure likelihood ($p_f$) (Eq. 14) (Melchers, 1999)

$$N > \frac{-\ln(1 - C)}{p_f}$$ (14)
2.5 Design of shotcrete layer for block stability

As described before, the design relates loads to the bearing capacity by including a safety margin. One major issue which governs the thickness of a shotcrete layer, is the fall of rock blocks. Through the design, a suitable thickness of the shotcrete layer can be calculated so that it can cope with this issue. However, there can be different reasons for block fall when a combination of shotcrete and rock bolts are used. In Figure 7, these main failure modes are illustrated, and they are mainly related to a bad adhesion of shotcrete to the rock surface or to inappropriate rock strength properties. Some failure modes are related to each other, for instance, the flexural failure can only occur after an adhesion failure. These failure modes explain also the need for reinforcement since tensile stresses occur e.g. through bending moments (Barrett and McCreath, 1995).

Figure 7 Principal failure modes of a shotcrete layer (Barrett and McCreath, 1995)
These failure modes can also be described as design models for block fall. During the last years, some limit states have been developed in order to include these failure modes and to determine a suitable thickness for the shotcrete layer (Barrett and McCreath, 1995). An overview of some limit state equations is given to estimate the capacity of shotcrete.

Direct shear failure

\[ C_{ds} = 4 \times \tau_{ds} \times s \times t \]  

(15)

The direct shear failure includes parameter about the shear stresses, \( \tau_{ds} \), as well about the rock bolt spacing, \( s \), and shotcrete thickness, \( t \). This limit state describes the situation when the loads exceed the shear strength of the shotcrete (Barrett and McCreath, 1995).

Punching shear failure

\[ C_{pf} = \sigma_{ps} \times 4(c + d)t \]  

(16)

This design model is used to describe a debonded shotcrete layer where the shear stress at the rock bolts are higher than the shear capacity of the shotcrete layer. This limit state is based on the diagonal shear stress \( \sigma_{ps} \), the perimeter of a potential failure surface, \( c \) and \( d \), as well as the shotcrete thickness \( t \) (Barrett and McCreath, 1995).

Flexural failure

\[ C_{Flex} = 0.9 \times \frac{R_{10/5} \times R_{30/10}}{200} \times \frac{\sigma_{flex} \times t^2 \times s}{12} \]  

(17)

The design model is used to determine the capacity of a shotcrete layer against a flexural failure due to a lack of adhesion. The \( R \) values stand for flexural toughness factors according to ASTM C1018 (Barrett and McCreath, 1995). Also, here \( t \) stands for the shotcrete layer thickness and \( s \) for the spacing of rock bolts. The flexural strength of shotcrete has also to be included into this limit state.
Furthermore, it is important as well to estimate the loads, e.g. here the weight of a potential rock block. For this purpose, a load model has been developed, which is visualised in Figure 8.

![Figure 8 Load model rock block (Barrett and McCreath, 1995)](image)

According to this design model, the weight of the rock block can be expressed as given in Equation 18, which is based on the specific weight $\gamma$ and the rock bolt distance $s$.

$$w = \frac{\gamma \times \sqrt{6} \times s^3}{6}$$

(18)
3 Spatial variability

3.1 Variance reduction

Previous design methods are mainly based on the mean value and the standard deviation as an indicator for the variance of a chosen parameter. Usually, the shotcrete layer thickness is assumed to be constant. Also, for instance, in soil mechanics, soil layers are modelled as a homogeneous layer with constant soil properties (Vanmarcke, 1977). In fact, the in-situ soil characteristics are rarely homogeneous (Nie et al., 2015) so that a parameter, the scale of fluctuation, has been introduced to measure the persistence of soil properties from point to point, but it can also be used for other purposes. That means, with the distance of this parameter, the properties are very similar to each other (Vanmarcke, 1977). Furthermore, the scale of fluctuation can be used to analyse the variation of the mean value.

However, the scale of fluctuation is based on the thickness of the investigated layer, so that the sample size influences the outcome (Shuwang and Linping, 2015). Normally, the sample size determines the number of sample intervals and a larger size obviously allows a higher number of sample intervals. This issue is very important for the calculation because a high number of sample intervals gives more points available for the calculation so that the analysis will be more accurate (Nie et al., 2015).

The relation of the scale of fluctuation and the sample size can be used to describe the influence of the variance of the mean value. For that reason, a variance reduction factor has been introduced. That is shown in Equations 19 - 20, which includes \( \delta \) as the scale of fluctuation, \( s \) as the sample distance, e.g. the distance of rock bolts, and the variance reduction factor, \( \Gamma^2 \) (Vanmarcke, 1977). These equations are valid for a one-dimensional case.

\[
\Gamma^2(s) = \begin{cases} 
1 & \text{for } s \leq \delta \\
\frac{\delta}{s} & \text{for } s \geq \delta 
\end{cases}
\]

(19)  

(20)
Thus, it can be concluded that the variance reduction factor can be decreased by increasing the sample size, which would also decrease the probability of failure. However, it is important to mention that this concept is based on the assumptions that the mean value is governing the failure and not the smallest value (*weakest link*).

### 3.2 Estimation of the scale of fluctuation

The statistical analysis requires usually different methods to describe and to characterise a given data set, which can be obtained e.g. during measurements and is usually the base for the calculation of the probability of failure. Since the raw material might be difficult to understand, some tools are available to summarise important information and belong mostly to the field of descriptive mathematics (Fische and Hebuterne, 2010). Some common possibilities to describe a data set in a quantitative way are mean, median, or mode value but also the variance which describes the dispersion (Fische and Hebuterne, 2010). However, these values might only be sufficient by studying one random parameter so that a method is required to describe the relationship between two or more random variables or how it changes in time and space.

Therefore, a spatial variability analysis has been performed, which is a concept within the field of geostatistics. The main idea is to combine statistical and geometrical methods which help to understand how variables are distributed in space or in time and space (Chilès and Delfiner, 2012). Thus, the aim of geostatistics can be mainly described as analysis how different points for instance in a defined space or plain are correlated to each other. This can be used for different proposals such as e.g. the investigation of the different geological layer. Hence, geostatistics provides a methodology to investigate the spatial correlation of natural variables. It mainly tries to solve questions as what an observation at one point tells us about values at neighbouring points and if we can expect continuity (Chilès and Delfiner, 2012). One main assumption can be determined that quite often for instance high values are surrounded by other high values so that an analyse of clustering effects can be performed.

Moreover, one main aspect is then also to estimate and interpolate values in the surrounding which is usually called *Kriging*. The interpolation function
is the probabilistic approach to predict data which is based on a covariance or variogram model derived from measured/calculated data (Chilès and Delfiner, 2012). However, interpolation of data is not necessary for this work so that methods as Kriging are only mentioned and will not be further evaluated. The focus will be laid more on a method to investigate the spatial variability and correlation of variables in space.

The correlation plays in the following analyse a key role since it helps to understand how a variable, e.g. the thickness of shotcrete, is distributed in a determined space. Since this project work aims to analyse the correlation of the thickness with itself, this phenomenon is often described as *Autocorrelation* and can be visualised in a correlogram. The chosen approach is based on a more practical comprehension of calculating the scale of fluctuation (Isaaks and Srivastava, 1989). This value gives the distance where the investigated property reveals a strong correlation or dependency to each other (Larsson et al., 2012).

Geostatistics is based on the assumptions that in fact, one point contains only the information about their location in space but also the investigated value (e.g. the thickness of shotcrete). In the first step, a mesh grid must be established with $i$ rows and $j$ columns. The number of rows and columns usually depend on the size of the analysed area and the required accuracy. Hence, the area contains a defined number of points which can be defined by their position within the rows and columns. In this research work, the points stand for the shotcrete thickness and can be displayed in a more general matter as $t_{ij}$. These values can be collected for one row or column in a vector. Then, these values should be paired with a surrounding value which has a pre-defined distance $h$ to the initial point. Since the mesh grid distances are determined by certain constant length, the $h$-distance is based on this mesh grid. Thus, by using, for instance, a $1\times1\,\text{m}$ mesh grid, the $h$-distance will be a multiple of $1\,\text{m}$. These points can be collected again in a vector, e.g. for one row, and these values can be written as $t_{ij+h}$. The corresponding values can be assembled as a pair such as $P(t_{ij} | t_{ij+h})$ and all these pairs can be visualised in a Cartesian coordinate system, which is called then $h$-scatterplot (Isaaks and Srivastava, 1989). The $h$-distance is also called *separation distance* or *lag distance* (Chilès and Delfiner, 2012) and usually, the distance is increased during the analysis. The first lag distance corresponds to the distance between two neighbouring points whereas, for instances, the second lag distance skips one point and the third
lag distance skips two points. An h-scatterplot can also be computed for these lag distances which are the base to calculate the linear correlation coefficient. This concept has been visualised (Figure 9) for four different h values [0.1,0.2,0.3,0.4]. In this exemplified case, the first lag distance corresponds to an h distance of h = 0.1m. The followings lag distances correspond to a multiple of the initial lag distance.

In Figure 9, the point clouds for four different lag distances has been illustrated in an h-scatterplot. Along the x-axis, the vector, $V(t)$, has been displayed which contains the initial values, $t_{ij}$. However, the values of the vector, $V(t+h)$, which contains the shifted values, $t_{ij+h}$, has been attributed to the y-axis. Moreover, a 45-degree line (dash line) has been added which can be used as first rough regression line as well to show that the point cloud is not symmetric and depends on the direction of construction (Isaaks and Srivastava, 1989).
By studying these different diagrams, it is obvious that the linear correlation is getting lower with higher h distances since the point cloud is getting more widespread. Hence, this method is already a simple way to analyse the autocorrelation along one direction. However, at some point, it might more useful to choose a more quantitative approach. Mostly, the h-scatterplot can be used as a base to compute a correlogram or a variogram, especially to enable a comparison of the linear correlation coefficients for different lag distances.

A correlogram is used to visualise the relationship between the correlation coefficient for a certain distance and the distance itself. Thereby, it is possible to identify the scale of fluctuation. By using the diagrams in figure 10, the key values as the correlation coefficient, standard deviation and moment of inertia can be calculated and are used to plot the diagrams. Especially, the calculation of the linear correlation factor is crucial for the developed concept.

The correlation factor \( r_{xy} \) is a real value (Fische and Hebuterne, 2010) which is based on a linear approximation of two different observed properties, \( x \) and \( y \). The calculation itself is based on the mean value (\( \bar{x} \) and \( \bar{y} \)), the number of point pairs (\( n \)), variance (\( s_x^2 \) and \( s_y^2 \)) and covariance (\( s_{xy} \)) (Eq. 21 – 23) (Finckenstein et al., 2006).

\[
\begin{align*}
    r_{xy} &= \frac{s_{xy}}{s_x \cdot s_y} \quad (21) \\
    s_{xy} &= \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x}) \cdot (y_i - \bar{y}) \quad (22) \\
    s_x^2 &= \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \quad and \quad s_y^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2 \quad (23)
\end{align*}
\]

\( x_i \) represent all values of one property, whereas \( y_i \) stands for the second property and the aim is to calculate the linear correlation between both properties. Moreover, the variance and the covariance require the mean value of each variable. Finally, the correlation factor can be calculated and will be between: \(-1 \leq r_{xy} \leq 1\).

The point pairs can be approximated and visualised with a regression line. Since a linear relationship (Eq. 24 - 26) has been assumed, the equation for the regression line can be expressed as:

\[
f(x) = ax + b \quad (24)
\]
with:
\[
\begin{align*}
a &= \frac{s_{xy}}{s_x^2} \quad \text{(25)} \\
b &= \bar{y} - a \bar{x} \quad \text{(26)}
\end{align*}
\]

The regression line and the correlation factor can help to visualise the correlation. For instance, a correlation of 1 or -1 shows a perfect correlation between two variables, whereas, a correlation of zero means that there is no correlation between two properties. Thus, either all points are on the regression line or the point cloud cannot be approximated by a regression line. If the sign is positive, e.g. 1, there will be a perfect positive correlation. However, a negative correlation factor stands for a negative correlation and thus, the sign indicates the inclination of the regression line.

Since the scale of fluctuation is based on an autocorrelation with different lag distances, the Equations 18 – 20 have to be adapted. The correlation factor is described then as correlation function \( p(h) \), which is based on the values separated by the distance \( h \). In fact, this function can be defined as the covariance function \( C(h) \), standardised by the appropriate standard deviations (Isaaks and Srivastava, 1989). The correlation and covariance function are shown in Equations 21 – 23.

**Correlation function:**
\[
p(h) = \frac{C(h)}{\sigma^{-h} \cdot \sigma^{+h}} \quad \text{(27)}
\]

with:
- \( \sigma^{-h} \) standard deviation of values \( v(t) \)
- \( \sigma^{+h} \) standard deviation of values \( v(t+h) \)

**Covariance function:**
\[
C(h) = \frac{1}{N(h)} \sum_{(i,j)|h_i,j=h} v_i \cdot v_j - m_{-h} \cdot m_{+h} \quad \text{(28)}
\]

with:
- \( N(h) \) Number of pairs
- \( v_i \) values of \( v(t) \)
- \( v_j \) values of \( v(t+h) \)
- \( m_{-h} \) mean value of \( v(t) \)
- \( m_{+h} \) mean value of \( v(t+h) \)
Both equations are based on the equations to calculate the linear correlation factor and they are used to compute the correlogram. For the cases with a high number of points, the mean value and standard deviation of \( v(t) \) and \( v(t+h) \) will be similar.

The correlation function, \( p(h) \), covariance function, \( C(h) \), and the moments of inertia, \( \gamma(h) \), are plotted all in Figure 10 to visualise how these diagrams can, in general, look like.

The values in the diagram for the moment of inertia (c) are increasing, which is the opposite development compared to the diagrams (a) and (b). But this difference is logical since a low correlation factor means a large point cloud and a large point cloud has a higher moment of inertia. Usually, the moment of inertia is the basis for the semivariogram.

![Figure 10 Diagrams of different values (a: correlation coefficient; b: covariance function; c: moment of inertia) (Isaaks, Srivastava 1989)](image-url)
4 Method

4.1 Preparation of the data

4.1.1 Identifying suitable data points

The analysis of the data aims to provide information about the homogeneity of a shotcrete layer of one part of a tunnel wall. In the given case, a tunnel section of around 25m in the ÄSPÖ laboratory (Oskarshamn), has been measured with laser scanning before and after applying shotcrete. The difference between both scans gives the actual thickness of the shotcrete layer. In this case, such a measurement contains a very large point cloud with X, Y, and Z coordinates. However, the aim is to calculate the scale of fluctuation for one wall so that only the points are needed which refer to the wall. Therefore, a filter has been developed to gather all required points.

In the first step, points containing information about the roof, the floor and the second wall were removed, so that only points are kept for a representative wall section. That required that the selected wall section should not have larger singularities which may disturb the results. A 3-D sketch of the tunnel opening with the different directions is given in Figure 11 even though the calculations have been performed mostly in a 2-D plane.

![Sketch of the tunnel before using shotcrete](image)

Figure 11 Sketch of the tunnel opening before using shotcrete
Basically, the view from above (X-Y view) is used in order to determine one wall so that the second wall can be removed. The points in the X-Y plane are used to calculate a linear regression line (Chapter 3) which cut the point cloud into halves. This is shown in Figure 12, where the whole point cloud is shown with the regression line. Moreover, this figure shows also the chosen wall where the second wall is removed. Thus, the regression line is used as cutting line through the number of points in the X-Y plane.

![X-Y with trendline](image)

**Figure 12 Ground plan showing the removed second wall**

Furthermore, the side elevation is used (X-Z view) to identify the height of the wall. In this case, the Z coordinate is verified and low as well as high values have been excluded since they are part of the roof and floor. By studying the general sketch and the data points, the limits for the roof and floor can be set up and the new dataset contains only one wall without roof and floor. This is shown in the first sketch in Figure 13, which is again given as view from above (X-Y view). Then, it is also important, to control the length of this wall. Thus, it should be possible to increase or to decrease the length by establishing a boundary condition.
In this case, the cutting lines have been assumed as sufficient to be perpendicular to the regression line. Hence, a basic approach from the field of linear algebra has been applied to solve this problem. These cutting lines can be calculated by using the rule that the dot product of two lines must be zero to be perpendicular to each other. The definition of the dot product is given to (Finck von Finckenstein, 2006):

$$\vec{u} \cdot \vec{v} = |u| \cdot |v| \cdot \cos \alpha$$

(29)

The angle $\alpha$ describes the angle between the lines who cross among themselves. In a perpendicular case, the angle $\alpha$ will be 90 and thus, the equation will be zero (Finck von Finckenstein, 2006). Usually, it might be very helpful to work in these cases with parametrised equations. In the second sketch in Figure 13, the two cutting lines as well with the maintained part of the wall are visualised. A 3-D of the final wall which is used for the calculation of the scale of fluctuation is shown in Figure 14.

These calculations have been developed with MATLAB and that can be found in the Appendix MATLAB files, in part 1.

Figure 13 Ground plan of the filtered wall
Figure 14 3D Sketch of the final wall section
4.1.2 Calculating the shotcrete thickness

Before the actual thickness can be calculated, it is important to flip the wall into the X-Y plain since the wall is standing in the X-Y plain (Figure 14) and then a calculation mesh grid function could not be used. Thus, first, the wall is rotated so that it is standing approximately on the X-axis. This has been achieved by subtracting the Y-values of the wall by the Y-values of the regression line (Figure 13). Second, the Y and Z-values have been exchanged so that the wall is laying in the X-Y plane and the Z coordinates provide information about the height of the wall.

During the second step, the actual shotcrete thickness has been calculated by using a mesh grid function. The points of the wall section create a plain in the three-dimensional space where the Z coordinate provides information about the height. Furthermore, this function establishes a mesh grid, which is identical both walls, hence for the wall with and without the shotcrete. Thus, a simple subtraction of the Z coordinates of an identical point leads to the required shotcrete thickness. Hence, this step consists of applying this function to calculate the thickness, which can be used afterwards for the spatial variability analysis.

This analysis has been done with MATLAB and can be found in the Appendix MATLAB files, in the second part.

4.2 Correlogram

After calculating the thickness of the whole area of the chosen wall, the analysis provides the correlation within the shotcrete distribution. This can be visualised in a correlogram where the scale of fluctuation can be read. This has been done in the x-direction (length) as well as in the y-direction (height) to analyse the variation in both directions. As described in chapter 3, the scale of fluctuation aims to compare the different correlation coefficients for different lag distances. However, the situation is a little bit more complicated compared with the given example since the mesh grid provides much more parallel lines, which must be combined to get one correlogram. Thus, the h-scatterplot has been drawn including every row (x-direction) or column (y-direction). An exemplified mesh grid is given in Figure 15.
As described in Chapter 3, a point in this mesh grid provides the information about its position in one direction (either x or y) and the corresponding value, which is, in this case, the thickness of shotcrete. The first task is to create the h-scatterplot for one specific lag distance. Thus, for a certain lag, e.g. lag 1, a vector $v(t)$ as well as a vector $v(t+h)$ can be computed which contain values form all over the rows. That means, both vectors can be assembled for instances for row 1 and then the values of the second row, third and so forth have been added to these two vectors. Finally, one h-scatterplot can be obtained for one lag distances containing the values of every row. Afterwards, the lag distances can be increased and for every lag distance, the correlation coefficient can be calculated.

The results can also be visualised in a correlogram, which helps to detect the scale of fluctuation. The approach calculating only one correlation coefficient for every lag distance is based on the idea to present a more overall result. Thus, it is possible to make a statement about the homogeneity of the shotcrete thickness along the chosen area since the scale of fluctuation provide information about the distribution. Basically, a high value for the scale of fluctuation means that the distance with very similar points is large. By comparing the scale of fluctuation with the original sample distance, e.g. rock bolt distance, the variation can be reduced with respect to the average thickness within this distance.
By studying the variation of the shotcrete thickness in the histogram, it can be detected which distribution fits best to the analysed data. Furthermore, the mean value and the standard deviation can be calculated.

These investigations are given as MATLAB script in the third part of the Appendix MATLAB files.
5 Results

First of all, a suitable area must be identified to avoid disturbances. The border area has been excluded due to some irregularities and the transmission area to the roof and floor. Moreover, larger rock blocks have been withdrawn from the second wall which makes it inappropriate. The chosen limits for the wall section are given in Table 2.

Table 2 Limitations of the chosen wall section

<table>
<thead>
<tr>
<th></th>
<th>Max. Value</th>
<th>Min. Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Length</strong></td>
<td>1931m</td>
<td>1905m</td>
</tr>
<tr>
<td><strong>Height</strong></td>
<td>446m</td>
<td>444m</td>
</tr>
</tbody>
</table>

These values are based on more global reference system so that they only provide indirect information about the tunnel geometry. However, the differences between the maximum and minimum value still give the size of the area. Thus, in this case, the wall area is 2m high and 26m long.

By using the different methods which have been introduced in the previous chapter, the thickness distribution is calculated and shown in a 3D (Figure 16) as well in a 2D figure (Figure 17). These figures are adapted to the actual distribution so that the values around the mean value are green whereas, the extreme values are red (larger values) or blue (smaller values) (Table 3).
Figure 16 3D presentation of the shotcrete layer thickness
Figure 17 2D presentation of the shotcrete layer thickness
The scale of fluctuation can also be visualised in a correlogram (in both directions), which are shown in Figures 18 and 19.

Figure 18 Correlogram in x-direction

Figure 19 Correlogram in y-direction
The correlogram in the y-direction has been visualised for all lag distances since the total width is two meters. Whereas, the correlogram in the x-direction only shows the first 3.25m because a diagram containing every lag distance is very large and confusing as well as only the beginning is necessary to identify the scale of fluctuation.

Moreover, the histogram of the shotcrete thickness with normal distribution has been plotted, Figure 20, to enable the calculation of the probability of failure. The mean value and the standard deviation is given for the normal distribution in Table 3 even though the extreme values have been excluded. These values can be explained with practical reasons as e.g. scaling of rock blocks (for small and negative values).

Table 3 Key values of the statistical distribution

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Mean Value [m]</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>0.11</td>
<td>0.019</td>
</tr>
</tbody>
</table>

Figure 20 Histogram of the shotcrete thickness distribution
6 Discussion

The analyses provide some values for the scale of fluctuation so that there is now a possibility to include the actual shotcrete distribution into a reliability-based design method. However, some weak points of this method should be considered and discussed in order to reduce and even to avoid any kind of disturbance within the calculation process.

Firstly, the analyses have to cope with the scale dependency of the results and with irregularities of the wall. The term scale dependency refers to the problem that the scale of fluctuation depends on the degree of variation in the chosen area. Thus, the outcome can be different, by choosing, for instance, a very small section (e.g. 1m by 1m wall section) with quite homogeneous conditions. Within this area, the measured values are much closer to each other compared with a much bigger area. This issue can be explained by using quantiles of a given distribution. The mean value, or the 50% quantile, of a certain distribution, will be correlated to the 60% quantile but not the 95% quantile. However, this approach does not take into consideration, how close these values are in more physical manner. For instance, the second wall holds some large irregularities so that the results will be not suitable. By choosing a smaller part the results will be different and might be wrong. Moreover, those irregularities must be identified by the engineer who is performing the analyses. However, that depends on his experiences how to deal with such problems.

Secondly, it might be difficult to compare the scale of fluctuation in x- and y-direction since these values are based on different scales. Hence, this issue is related to the first-mentioned problem dealing with the influence of different scales. The value of x refers to a length of 26m whereas the scale of fluctuation in the y-direction refers to a width of 2m. Thus, it might be quite challenging to combine these values, but which is necessary to calculate the probability of failure.

Thirdly, another problem arises by identifying a suitable limit for the scale of fluctuation. This value has been defined as distance where the correlation coefficient will be zero. However, the correlation coefficient is never getting zero and the correlation function behaves asymptotically to the x-axis. Thus, a limit has been established where the correlation is more or less zero, especially from a practical point of view. In this analysen, the limit has been
set to 0.2. The scale of fluctuation has been calculated for the wall in the x-direction (length) as well as in the y-direction (height). The results are given in Table 4:

Table 4 Scale of fluctuation for the right wall

<table>
<thead>
<tr>
<th>h [m]</th>
<th>Lag in x</th>
<th>Lag in y</th>
<th>Sof in x [m]</th>
<th>Sof in y [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>16</td>
<td>16</td>
<td>0.8</td>
<td>0.8</td>
</tr>
</tbody>
</table>

However, it might be more reasonable that a limit of 0.01 or 0.1 is more suitable. In these cases, the scale of fluctuation can vary a lot. By using 0.1 as a limit, the scale of fluctuation is 1.05m and for 0.01 the scale of fluctuation is 3m. Compared to the calculated scale of fluctuation of 0.8m, the variation is significantly large. Whereas for the scale of fluctuation in the y-direction, the variation is much more stable and different limits reveal the same scale of fluctuation even though the smallest value is around a limit of 0.02. The reason might be because the analysed width in the y-direction is much shorter as the length in the x-direction. Thus, the questions are then, which limit is suitable and could the limits be different for the x and y-direction.

It also important to discuss the significance of the scale of fluctuation for the calculation of the probability of failure. The scale of fluctuation itself has the problem that it does not provide any information about the thickness so that the layer could be, e.g., very homogenous but too thin. That would be a big problem since the thickness of a larger area would not correspond to the requested design value. Thus, the scale of fluctuation has to be analysed in conjunction with other parameters like for instance, mean value.

As described in chapter 3, the scale of fluctuation has to be compared with the sample size. That could be the distance of rock bolts if the block stability is analysed between the rock bolts. In the case, the sample size is higher, the variation of the mean value of the thickness can be reduced since the number of points which can be analysed makes the calculation more reliable.
7 Conclusions and further research

By interpreting the results, some conclusions can be drawn but also some tasks for the future arise. The main assignment was to develop a method to calculate the scale of fluctuation for a specific wall section of a part of a tunnel. These values are a crucial step to calculate the probability of failure by considering the actual statistical distribution. For this wall area, the scales of fluctuation have been determined to 0.8m (x-direction) and 0.8m (y-direction). That implies that the variance of the average shotcrete thickness can be reduced within a systematic bolt pattern, if block stability is analysed.

Another important point which must be investigated more detailed is the mechanical system of the shotcrete, which describes the failure. That can be an averaging system, where the mean value is the governing value or is it, e.g., the smallest and hence, the weakest value which governs the failure. The reduction of the variance is based on the assumption that the mean value is the reference value for the bearing capacity. Probably, the truth lies somewhere in between.

Moreover, a similar method should be developed as well for the roof part since the risk for failures is there much higher than compared to a wall.
8 List of figures

Figure 1 Tunnel section at the ÅSPÖ hard rock laboratory ........................................ 2
Figure 2 Variation of the shotcrete thickness due to uneven rock mass surface (Sjölander et al., 2017) .................................................................................. 7
Figure 3 Available design methods according to Eurocode 7 (SS-EN 1997) ........ 7
Figure 4 Sketch of the distribution of the resistance (R) and the load (S) .... 12
Figure 5 Visualisation of the reliability index ............................................................... 14
Figure 6 Visualisation of FORM .................................................................................. 15
Figure 7 Principal failure modes of a shotcrete layer (Barrett and McCreath, 1995) .......................................................................................................................... 17
Figure 8 Load model rock block (Barrett and McCreath, 1995) ......................... 19
Figure 9 Four h-scatterplots with four ...................................................................... 23
Figure 10 Diagrams of different values (a: correlation coefficient; b: covariance function; c: moment of inertia) (Isaaks, Srivastava 1989) ............. 26
Figure 11 Sketch of the tunnel opening before using shotcrete ......................... 27
Figure 12 Ground plan showing the removed second wall ................................ 28
Figure 13 Ground plan of the filtered wall ............................................................... 29
Figure 14 3D Sketch of the final wall section ......................................................... 30
Figure 15 mesh grid with rows and columns .............................................................. 32
Figure 16 3D presentation of the shotcrete layer thickness .................................. 35
Figure 17 2D presentation of the shotcrete layer thickness .................................. 36
Figure 18 Correlogram in x-direction ..................................................................... 37
Figure 19 Correlogram in y-direction ..................................................................... 37
Figure 20 Histogram of the shotcrete thickness distribution ............................ 38
9 List of tables

Table 1 Reliability index according to $RC$ and reference period (SS-EN 1990) ......................................................................................................................... 13
Table 2 Limitations of the chosen wall section .............................................. 34
Table 3 Key values of the statistical distribution ........................................... 38
Table 4 Scale of fluctuation for the right wall................................................. 40
10 Publication bibliography


11 Appendix MATLAB files

Part 1 - Preparation of the data

Note: In fact, there are two MATLAB scripts since one is dealing with the situation without shotcrete and the second one, with shotcrete. They are practical identical, even though they have different input values. Due to clarity, only one is shown here.

```matlab
%before shotcrete
%saving values from .txt in vector
before_scan=fopen('Skanning_Fore_Betong.txt');  %importing the file
A1=textscan(before_scan,'%f%f%f');  %scanning the file with
Floating-point number double
X1=A1{1};  %vector for x coordinates (length)
Y1=A1{2};  %vector for y coordinates (height)
Z1=(-1)*A1{3};  %vector for z coordinates (width) -->
shall be positive
n=length(X1)/8;  %number of points
XYZ1=[X1 Y1 Z1];  %matrix containing all values
[xyz1, ix] = sort(XYZ1(:,1));  %sorting according to the 1st row
(length)
XYZ1n = XYZ1(ix,:);  %figure(1)
scatter3(X1(1:15000,1),Y1(1:15000,1),Z1(1:15000,1))
scatter3(XYZ1n(1:n,1),XYZ1n(1:n,2),XYZ1n(1:n,3))
title('Sketch of the tunnel before using shotcrete (bs)')
xlabel('X (length)');
ylabel('Y (width)');
zlabel('Z (height)');

%start calculation of linear regression in 2D plains
%x-y
X1n=XYZ1n(:,1);  %extracting X values
Y1n=XYZ1n(:,2);  %extracting y values
[p1, s1] = polyfit(X1n, Y1n, 1);  %calculating regression values
Output = polyval(p1, X1n);
Correlation1 = corrcoef(Y1n, Output);
reg1=zeros(length(X1n),1);
for i=1:length(X1n)
reg1(i,1)=i;
reg1(i,2)=p1(1)*XYZ1n(i,1)+p1(2);  %calculating regression line
```
Before Shotcrete is used:

The trendline for X-Y is y = \text{num2str}(p1(1)) \times x + \text{num2str}(p1(2))

Before Shotcrete is used:

The trendline for X-Z is z = \text{num2str}(p2(1)) \times x + \text{num2str}(p2(2))

% excluding unneeded values: under the regression line
% x-y plain

Y1t=zeros(length(X1n),1);
Y1f1=zeros(length(X1n),1);
X1f1=zeros(length(X1n),1);
Z1f1=zeros(length(X1n),1);

for i=1:length(X1n)
    Y1t(i,1)=p1(1)*X1n(i)+p1(2);
    if Y1t(i,1) >= Y1n(i);
        Y1f1(i,1) = Y1n(i);
        X1f1(i,1) = X1n(i);
        Z1f1(i,1) = Z1n(i);
    else
        Y1f1(i,1)=0;
        X1f1(i,1)=0;
        Z1f1(i,1)=0;
    end
end

X1f1(X1f1==0) = [];
Y1f1(Y1f1==0) = [];

X1n=XYZ1n(:,1);
Z1n=XYZ1n(:,3);
[p2, s2] = polyfit(X1n, Z1n, 1);
Output = polyval(p2, X1n);
Correlation2 = corrcoef(Z1n, Output);
reg2=zeros(length(X1n),1);
for i=1:length(X1n)
    reg2(i,1)=i;
    reg2(i,2)=p2(1)*XYZ1n(i,1)+p2(2);
end

The trendline for X-Z is z = \text{num2str}(p2(1)) \times x + \text{num2str}(p2(2))

X1n=XYZ1n(:,1);
Z1n=XYZ1n(:,3);
[p1, s1] = polyfit(X1n, Z1n, 1);
disp(f3)
disp(f1)

end

The trendline for X-Y is y = \text{num2str}(p1(1)) \times x + \text{num2str}(p1(2))

disp(f1)
Z1f1(Z1f1==0) = [];  %merge the three vectors to one matrix  
XYZf1 = [X1f1 Y1f1 Z1f1];
figure(2)
hold on
grid on
subplot(2,1,1)
plot(X1n, Y1n, X1n, reg1(:,2))
title('X-Y with trendline (bs)')
subplot(2,1,2)
plot(X1f1, Y1f1, X1n, reg1(:,2))
title('X-Y with lower part only')

%{  
%x-z plain  
Z1t=zeros(length(X1n),1);  
Y1f2=zeros(length(X1n),1);  
X1f2=zeros(length(X1n),1);  
Z1f2=zeros(length(X1n),1);  
for i=1:length(X1n)  
    Z1t(i,1)=p2(1)*X1n(i)+p2(2);  %formula of the trendline in x-z case  
    if Z1t(i,1) <= Z1n(i);  %check if z is above or under the regression line  
        Z1f2(i,1) = Z1n(i);  
        Y1f2(i,1) = Y1n(i);  
        X1f2(i,1) = X1n(i);  
    else  
        Z1f2(i,1)=0;  
        X1f2(i,1)=0;  
        Y1f2(i,1)=0;  
    end  
end  
X1f2(X1f2==0) = [];  %remove 0 from the vector  
Z1f2(Z1f2==0) = [];  
Y1f2(Y1f2==0) = [];  
XYZf2 = [X1f2 Y1f2 Z1f2];  %merge the three vectors to one matrix  
figure(3)
hold on
grid on
subplot(2,1,1)
plot(X1n, Z1n, X1n, reg2(:,2))
title('X-Z with trendline (bs)')
subplot(2,1,2)
plot(X1f2, Z1f2, X1n, reg2(:,2))
title('X-Z with upper part only')

% check both matrices which rows are equal
[Lia]=ismember(XYZf1,XYZf2,'rows');

% reduce the XYZf1 matrix
XYZr=zeros(length(XYZf1),3);
for i=1:length(XYZf1)
    if Lia(i) == 1
        XYZr(i,:) = XYZf1(i,:);
    else
        XYZr(i,:) = 0;
    end
end

% remove 0 from the matrix XYZr
Xr = XYZr(:,1);
Xr(Xr==0) = [];
Yr = XYZr(:,2);
Yr(Yr==0) = [];
Zr = XYZr(:,3);
Zr(Zr==0) = [];
XYZrn = [Xr Yr Zr];
%

Xr=XYZf1(:,1);
Yr=XYZf1(:,2);
Zr=XYZf1(:,3);
XYZrn = [Xr Yr Zr];

% cut height (z-direction) --> from 442.8190 to 448.6350
% going 2m from the regression line
XYZrH=zeros(length(Xr),3);
for i=1:length(Xr)
    if and(Zr(i)>= 443.5, Zr(i)<=447)
        XYZrH(i,:) = XYZrn(i,:);
    else
        XYZrH(i,:) = 0;
    end
end

% remove 0 from the matrix XYZrH
XrH = XYZrH(:,1);
XrH(XrH==0) = [];
YrH = XYZrH(:,2);
YrH(YrH==0) = [];
ZrH = XYZrH(:,3);
ZrH(ZrH==0) = [];
XYZrHn = [XrH YrH ZrH];

%define the length of wall strip: 30m (1900-1930)
%defining the length of the regression line
%lower line
XrL1=1903;       %chosen x start point on the regression line
YrL1=p1(1)*XrL1+p1(2);       %calculating the start y point on the regression
bL=XrL1/(YrL1-p1(2));       %grade of perpendicular vector (same for both lines)
iL1=(bL*XrL1)+YrL1;        %intercept of perpendicular vector
pLy1=zeros(1903,1);
pLx1=zeros(1903,1);
for i=1898:1910
    pLy1(i)=-bL*i+iL1;
pLx1(i)=i;
end

%upper line
XrL2=1930;       %chosen x start point on the regression line
YrL2=p1(1)*XrL2+p1(2);       %calculating the start y point on the regression
iL2=(bL*XrL2)+YrL2;
pLy2=zeros(1930,1);
pLx2=zeros(1930,1);
for i=1920:1935
    pLy2(i)=-bL*i+iL2;
pLx2(i)=i;
end

%delete the lower and upper part
%lower part
XYZd=zeros(length(XrH),3);
Y1d=zeros(length(XrH),1);
for i=1:length(XrH)
    Y1d(i,1)=-bL*XYZh(i,1)+iL1;
if Y1d(i,1) <= XYZrHn(i,2)
    XYZd(i,:) = XYZrHn(i,:);
else
    XYZd(i,:) = 0;
end
end

% remove 0 from the matrix
X1d = XYZd(:,1);
X1d(X1d==0) = [];
Y1d = XYZd(:,2);
Y1d(Y1d==0) = [];
Z1d = XYZd(:,3);
Z1d(Z1d==0) = [];
XYZdn = [X1d Y1d Z1d];

% upper part
XYZu = zeros(length(X1d),3);
Y2u = zeros(length(X1d),1);
for i=1:length(X1d)
    Y2u(i,1) = -bL*XYZdn(i,1)+iL2;
    if Y2u(i,1) >= XYZdn(i,2)
        XYZu(i,:) = XYZdn(i,:);
    else
        XYZu(i,:) = 0;
    end
end

% remove 0 from the matrix
X2u = XYZu(:,1);
X2u(X2u==0) = [];
Y2u = XYZu(:,2);
Y2u(Y2u==0) = [];
Z2u = XYZu(:,3);
Z2u(Z2u==0) = [];
XYZun = [X2u Y2u Z2u];

figure(4)
hold on
grid on
subplot(2,1,1)
plot(XYZrHn(:,1), XYZrHn(:,2), X1n, reg1(:,2), pLx1(1898:1910),
title('X-Y with lower part only with limits for chosen wall section (bs)')
subplot(2,1,2)
plot(XYZun(:,1), XYZun(:,2), X1n, reg1(:,2), pLx1(1898:1910),
title('X-Y with lower part only with cutted areas')

{%
% rotation of the wall from the x-y plain into the x-z plain
% x-y plain
f=round(length(XYZun(:,1))/2);                              % half point of the total point number
XYnorm=zeros(2,length(XYZun(:,2)));                           % x-y plain
alpha=zeros(length(XYZun(:,2)),1);
alpha=-atan(XYZun(f,2)/XYZun(f,1));                          % angle of the x-y regression line compared to the x-axis in radians
rm=[cos(alpha) -sin(alpha); sin(alpha) cos(alpha)];           % 2D rotation matrix
for i=1:length(XYZun(:,1))
    Pc(1,1)=XYZun(i,1);  % extracting the points to one new point
    Pc(2,1)=XYZun(i,2);
    XYnorm(:,i)=rm*Pc;    % calculating the rotated points
end
% transpose vectors
Xnorm1=transpose(XYnorm(1,:));
Ynorm1=transpose(XYnorm(2,:));
%
% create trendline through the wall points
% x-y
Xn=XYZun(:,1);
Yn=XYZun(:,2);
Yt=zeros(length(Yn),1);
Yline=zeros(length(Xn),1);
for i=1:length(XYZun(:,1))
    Yt(i)=Yn(i)-(p1(1)*Xn(i)+p1(2));  % calculating the points on the regression line
    Yline(i)=p1(1)*Xn(i)+p1(2);
end
calculating the distance between regression line and points

v1 = zeros(length(XYZun(:,1)),1);
x1 = zeros(length(XYZun(:,1)),1);
y1 = zeros(length(XYZun(:,1)),1);
d1 = zeros(length(XYZun(:,1)),1);
XrL1 = XYZun(1,1);
YrL1 = XYZun(1,2);
XPy = YrL1 - p3(2);
for i = 1:length(XYZun(:,1))
    v1(i) = ((-XYZun(i,1))*XrL1 + (p3(2) - XYZun(i,2))*XPy) / (- (XrL1^2 + XPy^2));
    x1(i) = v1(i)*XrL1 - XYZun(i,1);
    y1(i) = (p3(2) + v1(i)*XPy) - XYZun(i,2);
    d1(i) = sqrt(x1(i)^2 + y1(i)^2);
end

m1 = p1(1);
n1 = p1(2);
m2 = -1/m1;
n2 = zeros(length(Yn));
xo = zeros(length(Yn));
yo = zeros(length(Yn));
d1 = zeros(length(Yn));
for i = 1:length(Xn)
    n2(i) = Yn(i) - m2*Xn(i);
    xo(i) = (n2(i) - n1) / (m1 - m2);
    yo(i) = m1*xo(i) + n1;
    d1(i) = sqrt((xo(i) - Xn(i))^2 + (yo(i) - Yn(i))^2);
end

checking if the point is lower or higher than the regression line

d = zeros(length(d1),1);
for i = 1:length(Xn)
    if Yn(i) >= Yline(i)
        d(i) = d1(i);
    else
        d(i) = -d1(i);
    end
end

figure(5)
subplot(2,1,1)  
plot(Xn, Yn)  
title('original point cloud')  
subplot(2,1,2)  
plot(Xn, Yt)  
title('new points by subtracting')  
ylim([-1 5])

%flip wall (laying in the x-y plain)  
%change Z and Y (Y=d and Z=0)  
Zfinal=Yt;  
Yfinal=XYZun(:,3);  
figure(6)  
scatter3(Xn,Yfinal,Zfinal)  
title('wall is laying in the X-Y plain')  
zlim([-1 5])  
figure(13)  
plot(Xn, Yfinal,'.')  
title('2D sktech of wall in the x-y plain (bs)')

%final matrix with all the data  
result_bs=[Xn, Yfinal, Zfinal];  
figure(3)  
scatter3(XYZun(:,1),XYZun(:,2),XYZun(:,3))  
title('chosen wall section')

Part 2 – Calculation of the shotcrete thickness

%Analys av scanned data för tunnel section

%Skapar x,y,z vektorer med mätdata  
data=result_bs;  
Xu=data(:,1);  
Yu=data(:,2);  
Zu=data(:,3);

data=result_as;  
Xo=data(:,1);  
Yo=data(:,2);  
Zo=data(:,3);
%Skapar rutnät för punkter (section of the tunnel)
%has to be insert manually

\texttt{ti=0.1;}
\texttt{txu=1905:ti:1931;}
\texttt{tyu=444:ti:446;}

\texttt{txo=1905:ti:1931;}
\texttt{tyo=444:ti:446;}

%Genererar x och y vektorer i rutnätet
\texttt{[XIu,YIu]=meshgrid(txu,tyu);} \\
\texttt{[XIo,YIo]=meshgrid(txo,tyo);} \\

%Genererar höjdata efter angivet rutnät
\texttt{ZIu=griddata(Xu,Yu,Zu,XIu,YIu);} \\
\texttt{[Nxu,Nyu,Nzu]=surfnorm(XIu,YIu,ZIu);} \\
\texttt{ZIo=griddata(Xo,Yo,Zo,XIo,YIo);} \\
\texttt{[Nxo,Nyo,Nzo]=surfnorm(XIo,YIo,ZIo);} \\

%Beräknar matedness
\texttt{XD=XIo;}
\texttt{YD=YIo;}
\texttt{ZD=ZIu-ZIo;}

%create colormap
\texttt{mymap=[}
\texttt{0 0 1 } \quad \%dark blue - final step \\
\texttt{0 0.6 1 } \quad \%pale blue \\
\texttt{0 1 1 } \quad \%turquoise \\
\texttt{0 0.5 0 } \quad \%green - mean \\
\texttt{1 0.84 0 } \quad \%gold \\
\texttt{1 0.55 0 } \quad \%darkorange \\
\texttt{1 0 0}; } \quad \%red \\
\texttt{hold on}

%Plottar sprickytan
\texttt{figure(15)} \\
\texttt{surf(XD,YD,ZD);} \\
\texttt{title(['shotcrete thickness 3D with h = ' num2str(ti)]))}
Appendix MATLAB files

```matlab
colorbar
colormap(mymap)
caxis([-.05 .25])
axis equal
figure(16)
contourf(XD,YD,ZD);
title(['shotcrete thickness 2D with h = ' num2str(ti)])
colorbar
colormap(mymap)
caxis([-.05 .25])
axis equal
%view(2)
%light
%shading interp
%figure(16)
LL=length(txu)*length(tyu);
LLL=(length(txu)-1)*(length(tyu)-1);
ZDD=reshape(ZD,LL,1);
ZDDD=sort(ZDD);
G=ones(length(ZDDD),1)/LLL;
GG=cumsum(G);
%plot(GG,ZDDD)

Part 3 – Calculation of the Scale of fluctuation

%define the limits/max
%ZD(isnan(ZD(:,1)),:)=[];
out = ZD(:, all(~isnan(ZD), 1));
[zc1, zc2]=size(ZD);

%correlogram in x
zend=zc1;
lg=zc2-1;
CY=zeros(lg+1,1);
Lag=zeros(lg+1,1);
ZDnew=ZD(1:end,:);
CY(1)=1;
for z=1:lg
    hold on
    maxlg=floor(lg/z);
```

```
tail=zeros((maxlg*zend),1);
head=zeros((maxlg*zend),1);
j=1;

% 2nd loop: creating the h-scatterplot for every lag distance
for a=1:zend
    for b=1:z:((maxlg)*z)
        tail(j)=ZDnew(a,b);
        head(j)=ZDnew(a,b+z);
        j=j+1;
    end
end

[p6, s6] = polyfit(tail, head, 1);
Output = polyval(p6, tail);
CorrelationCX = corrcoef(head, Output);
CY(z+1)=CorrelationCX(1,2);  % calculating the correlation coefficient
Lag(z+1)=z;
clearvars tail head
end

CYlength=length(CY);
figure(17)
plot(Lag(1:CYlength), CY)
title(['Correlogram in x-direction with h = ' num2str(ti)])
ylim([-0.1 1.5])
xlim([0 (lg/8)])
xlabel('lag distances h ');
ylabel('correlation coefficient');

CYlim1=find(CY < 0.05);
f3=['The first lag distance with a correlation below 0.05 (correlogram in x),
lag = ' num2str(CYlim1(1))];
disp(f3)

% correlogram in y (17 points)
%
for z=1:20
    hold on
    ZDnew=ZD(2:18,z);  % extracting Z values
    lg=15;  % number of total lag distances
    CY2=zeros(lg+1,1);
    CY2(1)=1;

Lag=zeros(lg+1,1);
devtail=zeros(lg+1,1);
devhead=zeros(lg+1,1);
for i=1:lg
    maxlg2=floor(17-i);
tail2=zeros(length(maxlg2)-1,1);
    head2=zeros(length(maxlg2)-1,1);
    for j=1:maxlg2
        tail2(j)=ZDnew(j);
        head2(j)=ZDnew(j+i);
    end
    [p7, s7] = polyfit(tail2, head2, 1);
    Output = polyval(p7, tail2);
    CorrelationCY = corrcoef(head2, Output);
    CY2(i+1)=CorrelationCY(1,2);
    Lag(i+1)=i;
    clearvars tail2 head2
end
Lag(1)=0;
CYlength2=length(CY2);
figure(19)
plot(Lag(1:CYlength2), CY2, 'DisplayName', 
    ['column = ' num2str(z)])
title('Correlogram in y-direction')
ylim([-0.1 1.5])
xlabel('lag distances h (h=0.1m)');
ylabel('correlation coefficient');
end
legend('show')%}

zend2=zc2; %zc2 columns in the y direction
lg=zc1-1; %number of total lag distances (max 15)
CY2=zeros(lg+1,1); %final matrix with the correlation coefficients
Lag=zeros(lg+1,1); %vector describing the number of lags
CY2(1)=1; %first correlation coefficient is one (correlation with itself)
for z=1:lg %1st loop: increasing from lag 1 to final lag
    hold on
    maxlg=floor(lg/z);
tail2=zeros((maxlg*zend2),1);
    head2=zeros((maxlg*zend2),1);
j=1; %2nd loop: creating the h-scatterplot for every lag distance
for b=1:zend2
    for a=1:z:((maxlg)*z)
        tail2(j)=ZD(a,b);
        head2(j)=ZD(a+z,b);
        j=j+1;
    end
end

[p7, s7] = polyfit(tail2, head2, 1);
Output = polyval(p7, tail2);
CorrelationCX = corrcoeff(head2, Output);
CY2(z+1)=CorrelationCX(1,2); %calculating the correlation coefficient
Lag(z+1)=z;
clearvars tail2 head2
derm

CYlength=length(CY2);
figure(18)
plot(Lag(1:CYlength), CY2)
title(['Correlogram in y-direction with h = ' num2str(ti)])
ylim([-0.1 1.5])
xlim([0 lg])
xlabel('lag distances h');
ylabel('correlation coefficient');

CYlim2=find(CY2 < 0.05);
f4=['The first lag distance with a correlation below 0.03 (correlogram in y),
lag = ' num2str(CYlim2(1))];
disp(f4)

%histogram and lognormal Distribution
figure(19)
x=(ZD(1:end,:));
x(x<=0) = [];
xn=reshape(x,[],1);
%relative frequencies
numOfBins = 100;
[histFreq, histXout] = hist(xn, numOfBins);
pd = fitdist(xn, 'lognormal');
xvalues=0:0.001:0.3;
yvalues=pdf(pd,xvalues); %histogram(xn)
subplot(2,1,1)
histfit(xn,100,'lognormal')
title('histogram with lognormal distribution')
subplot(2,1,2)
histfit(xn,100,'normal')
title('histogram with Gauss distribution')