HYPOID GEARS

Developing an FE model to study the effect of press-fit on a hypoid gear set

MAADHAV PADMANABHAN
Hypoid växelsats

Utveckla en FE-modell för att studera Effekten av presspassning på en hypoid växelsats

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HYPOID GEARS

Developing an FE model to study the effect of press-fit on a hypoid gear set

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Abstract

Gears are an integral part of devices ranging from simple wrist watches to complex systems like automotive and wave energy converters. They play a very important role in the transfer of torque. There are several types of gears to be chosen from, depending on the application. This thesis work deals exclusively with hypoid gears. A hypoid gear is a type of bevel gear, that transmits torque between two non-parallel shafts. It is similar to a spiral bevel gear, except that the pinion axis can be offset. It is this ability to offset, that renders hypoid gears as highly sought-after gears for automotive applications. It is crucial that these hypoid gears are designed efficiently and that their life before failure is predictable, to a reasonable degree of accuracy. To enable such predictions, this thesis makes an effort to build a finite element (FE) model. With the developed FE model, a study of the effect of press-fit on the ring gear’s root bending stresses, that is present in a hypoid gear set of a car’s Rear-Drive Unit (RDU), was carried out.

The FE model was built on three different software - Transmission 3D (T3D), Hypoid Face Milled (HFM) and MSC Marc (Marc). The effect of press-fit on root bending stresses of the ring gear was first analyzed on T3D. To determine if the press-fit was correctly induced, the model was rebuilt and analyzed on Marc. HFM was used to determine the effect of inclusion of different components on the root bending stresses. Additionally, the HFM model was also replicated on Marc and a press-fit of 100\(\mu\)m was induced. This was done to see if modelling the press-fit on HFM gave similar results to that of T3D and if using HFM in conjunction with Marc lead to a better modelling procedure.

It was found that the maximum root bending stresses increased linearly with increasing press-fit dimension. It was also found that the inclusion of different parts does not cause a significant increase in root bending stresses, except for the inclusion of differential cage. The effect of press-fit could not be quantified despite knowing that it affects the root bending stresses. When the same analysis done on HFM or T3D was done on Marc, there was a 10% increase in stresses at highly stressed zone in Marc model of T3D; there was 10 – 15% increase in stresses at highly stressed zone in the HFM model of Marc. Hence, quantification remained an impediment. It was not possible to quantify the error that occurred during the migration of analysis from HFM or T3D to Marc. However, potential causes of these errors could be the error in computation of bearing forces and difference in the definition of contacts in the software.

Owing to large computational time and limited working period, all the analysis on Marc was carried out for one position and for the first time step of the gear mesh. If the errors in migration of analysis from one software to another could be quantified, then this modelling can be used to estimate the contribution of press-fit to root bending stresses on the ring-gear. Although the contact pattern comparison between the virtual models and the physical test suggests that HFM is a more trustworthy software, it is recommended to conduct some strain gauge measurements on different ring gear teeth and then compare the results with those of the virtual model.

Keywords: Hypoid gears, HFM, transmission 3D, MSC Marc, FE analysis, root bending stresses
Sammanfattning


FE-modellen byggdes på tre olika programvär: Transmission 3D (T3D), Hypoid Face Milled (HFM) och MSC Marc (Marc). Effekten av presspassning på ringbøjspänningarna hos ringväxeln analyserades först på T3D. För att bestämma om presspassningen var korrekt inducerad, byggdes modellen och analyserades på Marc. HFM användes för att bestämma effekten av inkludering av olika komponenter på rotbøjspänningarna. Dessutom replikerades HFM-modellen också på Marc och en presspassning på 100 microns inducerades. Detta gjordes för att se om modellering av presspass på HFM gav liknande resultat som T3D och om man använde HFM i kombination med Marc leder till ett bättre modelleringförfarande.

Det visade sig att de maximala rotbøjspänningarna ökade linjärt med ökande presspassningsdimension. Det konstaterades också att införandet av olika delar inte orsakar en signifikant ökning av rotbøjspänningar, utom införandet av differentialbur. Effekten av presspassning kunde inte kvantifieras trots att man vet det påverkar rotbøjspänningarna. När samma analys gjord på HFM eller T3D gjordes på Marc, fanns det en 10% ökning av spänningar vid starkt stressed zon i Marc-modellen av T3D; det var 10-15% ökning av spänningar vid starkt stressed zon i HFM-modellen av Marc. Därför förblev kvantifiering ett hinder. Det var inte möjligt att kvantifiera felet som inträffade under analysanalysen från HFM eller T3D till Marc. Dock kan potentiella orsaker till dessa fel vara felet vid beräkning av bärkräfter och skillnad i definitionen av kontakter i programvaran.

På grund av stor beräkningstid och begränsad arbetstid utfördes all analys på Marc för bara en position och för det första steget av växels nät. Om felet i migration av analys från en programvara till en annan kunde kvantifieras, då kan denna modellering användas för att beräkna bidragen från presspassning till rotbøjspänningar på ringväxeln. Dessutom om kontaktmönstersjämförelsen mellan de virtuella modellerna och det fysiska testet föreslår att HFM är en mer pålitlig programvara, rekommenderas det att utföra en viss belastning Mätningarna på olika ringväxeltänder och sedan jämföra sedan resultaten med dem från virtuella modellen.

Nyckelord: Hypoid växlar, HFM, transmission 3D, MSC Marc, FE analys, rotbøjspänningar
Acknowledgement

My work at GKN Driveline Köping AB over the past five months has been a rewarding experience. I began with rudimentary knowledge on gears and ended up developing a proclivity towards the design and manufacturing of hypoid gears. This accumulation of knowledge would not have been possible without the help of several individuals who played a key role in motivating me and helping with problems when I was stuck.

I thank Amelin Begovic, my manager at GKN Driveline for granting me the opportunity to work on this thesis; Karthik Pingle - my supervisor at GKN Driveline, for helping me define my tasks and patiently answering all my queries, however silly they seemed. I would also like to extend my gratitude to my colleagues - Niklas Svensson and Yi Ma, without whose assistance, setting up of the model on the software would not have been possible. Stefan Björklund’s (my supervisor at KTH) visit to GKN Driveline buoyed my spirits and motivated me to carry out my work diligently. Last but not the least, my sincere thanks to Ulf Sellgren for granting the approval to carry out my Master Thesis at GKN Driveline.

This section will seem expurgated without the mention of my parents. Without them none of this would have been possible. I thank them for their love and support.

Tack så mycket

Maadhav Padmanabhan
Stockholm, Sweden
### Nomenclature

#### NOTATIONS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_m$</td>
<td>Mean spiral angle (deg)</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>Spiral angle (deg)</td>
<td></td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Pitch angle (deg)</td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>Pressure angle (deg)</td>
<td></td>
</tr>
<tr>
<td>$\psi$</td>
<td>Mean spiral angle at pitch surface (deg)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{FP}$</td>
<td>Permissible tooth root stress (MPa)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{F_{1,2}}$</td>
<td>Tooth root stresses (MPa)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_F$</td>
<td>Encountered tooth root stress (MPa)</td>
<td></td>
</tr>
<tr>
<td>$\Sigma, \alpha$</td>
<td>Shaft angle (deg)</td>
<td></td>
</tr>
<tr>
<td>$D_m$</td>
<td>Mean pitch diameter of the ring gear (mm)</td>
<td></td>
</tr>
<tr>
<td>$d_m$</td>
<td>Mean pitch diameter of the pinion (mm)</td>
<td></td>
</tr>
<tr>
<td>$E$</td>
<td>Pinion offset (mm)</td>
<td></td>
</tr>
<tr>
<td>$F$</td>
<td>Face width (mm)</td>
<td></td>
</tr>
<tr>
<td>$G$</td>
<td>Gear mounting distance (mm)</td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td>Stiffness (N/m)</td>
<td></td>
</tr>
<tr>
<td>$K_A, K_V, K_{F_{\beta}}, K_{F_\alpha}$</td>
<td>Force factors</td>
<td></td>
</tr>
<tr>
<td>$P$</td>
<td>Pinion mounting distance (mm)</td>
<td></td>
</tr>
<tr>
<td>$S_F$</td>
<td>Safety factor for tooth root bending stress</td>
<td></td>
</tr>
<tr>
<td>$S_{F_{\text{min}}}$</td>
<td>Minimum safety factor</td>
<td></td>
</tr>
<tr>
<td>$T_G$</td>
<td>Torque on the ring gear (Nm)</td>
<td></td>
</tr>
<tr>
<td>$T_P$</td>
<td>Torque on the pinion (Nm)</td>
<td></td>
</tr>
<tr>
<td>$W_r$</td>
<td>Radial force (N)</td>
<td></td>
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<tr>
<td>$W_z$</td>
<td>Axial force (N)</td>
<td></td>
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<tr>
<td>$W_{t_G}$</td>
<td>Tangential force on ring gear (N)</td>
<td></td>
</tr>
<tr>
<td>$W_{t_P}$</td>
<td>Tangential force on pinion (N)</td>
<td></td>
</tr>
<tr>
<td>$Y_\epsilon$</td>
<td>Contact-ratio factor</td>
<td></td>
</tr>
<tr>
<td>$Y_{Fa_{1,2}}, Y_{Sa_{1,2}}$</td>
<td>Factors of tooth form</td>
<td></td>
</tr>
<tr>
<td>$F_{vnt}$</td>
<td>Tangential force (N)</td>
<td></td>
</tr>
</tbody>
</table>

#### ABBREVIATIONS

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AWD</td>
<td>All Wheel Drive</td>
</tr>
<tr>
<td>BDF</td>
<td>Bulk Data File</td>
</tr>
</tbody>
</table>
$CVJ$ Continuous Velocity Joints

$e - drive$ Electric Drive

$FE$ Finite Element

$FEA$ Finite Element Analysis

$FZG$ Forschungsstelle für Zahnräder und Getriebebau

$HFM$ Hypoid Face Milled

$IST$ Instrument

$LTCA$ Loaded Tooth Contact Analysis

$PH$ Pin Hole

$PTU$ Power Take-off Unit

$RBE$ Rigid Body Element

$RDU$ Rear Drive Unit

$SPA$ Special Analysis

$T3D$ Transmission 3D

$WC$ Window Center

$WE$ Window Edge
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1. INTRODUCTION

This chapter describes the background, purpose, limitations and the method(s) used in the presented project.

1.1 Background

This master thesis was carried out at GKN Driveline Köping AB. GKN plc is a British multinational company active in automotive, aerospace, metallurgy and land systems. GKN Driveline specializes in the development and manufacturing of:

- Electrified Drive (e-Drive)
- All-Wheel Drive Systems (AWD)
- Continuous Velocity Joints (CVJ)
- Motorsport driveline solutions

GKN Driveline Köping AB specializes in AWD systems. An AWD system can be seen in Fig. 1.1. This AWD system is one of the many variants that GKN produces. In this thesis, a driveline similar to the one shown in Fig. 1.1 is the one under scrutiny. The main function of AWD is to distribute the power from the engine to all four wheels of the car. However, a functionality wherein the car can intelligently toggle between all-wheel drive and two-wheel drive, is enabled by the disconnect unit. Initially, the power from engine is transferred to the power take-off unit (PTU) via transmission. The PTU is located on the front axle and directly distributes power to the front wheels and the prop-shaft. To transfer power to the prop-shaft which is perpendicular to the direction of front axle, hypoid gears are used. Power from the prop-shaft is in-turn transmitted to the rear axle, by the means of a hypoid pinion-ring gear pair, which along with a differential constitute the rear drive unit (RDU). The differential enables the wheels on the same axle to move at different angular speeds while negotiating a corner. The differential unit on the front axle fulfills a similar purpose. When the front-axle’s disconnect unit is active, power is transferred only to its wheels. In such a case, it is important that the RDU does not rotate the prop-shaft by any means. Hence a side-mounted clutch is provided so that during the two-wheel drive, the wheels on the rear-axle rotate independent of each other, thus enabling them to rotate at different angular speeds when negotiating a corner.

Figure 1.1: One of the all wheel drive solutions by GKN
In both the PTU and the RDU, hypoid gear pairs are used to transmit power (Fig. 1.2). It is these hypoid gears that are a major area of interest in this project. When subjected to high loads, these gears are highly stressed. It becomes important to estimate the life of these gears with reasonable accuracy, for they are the parts that enable transmission of power from engine to all the wheels.

![Figure 1.2: Power take-off Unit (PTU) [left]; Rear Drive Unit (RDU) [right]](image)

In this project, a virtual model of the RDU was built taking into account the micro-geometry of the gears. Additionally, the variation of the root bending stresses on the ring-gear were studied.

### 1.2 Purpose

When a customer demands weight reduction, certain design changes have to be made in the RDU (Fig. 1.3) and PTU. It is important to predict with reasonable accuracy, the effect of these changes on bending strength and fatigue strength of the gears. So, the broad goal of this project was to build a virtual model of the RDU that would enable such predictions. The RDU was chosen to be modelled owing to its complexity. The ring gear in the RDU unit is press-fit onto a differential cage, whose stiffness in the radial direction is not uniform. Also, the rear of the rim is welded onto the disc-shaped face of the differential cage.

![Figure 1.3: RDU](image)
The objectives of this project were:-

- Build an FE model of the RDU gear-set with the inclusion of gear-micro geometry, press-fit at the rim-differential cage interface to
  1. study the variation of root bending stresses at different teeth on the gear.
  2. find if the press-fit makes a significant contribution to the root bending stresses.

1.3 De-limitations

The RDU is a complex unit consisting of several parts. Since the stresses on gear teeth are of major interest, some components were not modelled into the system with the assumption that their contribution to the stresses on gears may not be significant.

The contribution of housing stiffness to the stresses on gears was not modelled into the system. The pinion drives inside the differential cage were modelled as perfectly rigid bodies, while in reality, they undergo minor displacement. But it was assumed that this would not significantly contribute to varying stresses on the gear teeth.

The back face of the rim is welded onto a disc-shaped face of the differential cage. This weld may influence the stresses on gear teeth. But this is not modelled in the system as it was considered to be an independent investigation by itself. So, the analysis is done based on the assumption that this weld does not significantly affect the stresses on the gear.

1.4 Method

This project involves the use of a variety of software in order to build the virtual model. The method(s) used are as follows:-

1. The RDU unit was modelled using Transmission 3D. This model includes the micro-geometry of the gears and press-fit at the rim- differential cage interface.

2. The model setup was validated by the comparison of contact pattern obtained from T3D with that of the contact pattern from physical tests.

3. The distribution of root bending stresses on the gear teeth in the T3D model was analyzed.

4. A similar model inclusive of the micro-geometry of the gear was built on MSC MARC and stress values between the software were correlated.

5. The model was also built on Hypoid Face Milled without the press fit and the variation of root bending stresses are studied. This software is also used to study the inclusion of various other parts in the FE analysis.

6. The model from HFM was setup on Marc, and a press-fit of 100\(\mu\)m was induced. The results of this were compared with those of the T3D model to observe, which among the two, yielded a more conservative estimate.

Each of the aforementioned software serves a specific purpose. Extensive training was imparted in order to use these software to model the RDU.

- T3D is a software package developed by Advanced Numerical Solutions LLC. It enables the modelling of complex transmission systems that include hypoid gears.

- HFM is a software designed by Advanced Numerical Solutions LLC., that is used to carry out gear contact analysis on Hypoid Gears that are made using face milling process.

- Marc is a finite element software developed by MSC Software.

Extensive literature survey was done to gain fundamental knowledge needed to build the virtual model and analyze the results. Regular meetings were held with the supervisor at GKN DRIVELINE and KTH to discuss the progress of the project.
2. FRAME OF REFERENCE

This chapter elucidates the basic knowledge needed to understand the problem. The basics of hypoid gear terminology, how hypoid gears are designed, micro-geometry and their relevance to the industry is explained.

"Gears are toothed, cylindrical wheels used for transmitting motion and power from one rotating shaft to another. The teeth of the driving gear mesh in spaces between the teeth of the driven gear." [1]. Gears in general are used to transfer power between machine elements. Depending on the relation between the axes of the gear pair in contact, shape of the blank on which the teeth are cut, the curvature of teeth and many other special features, gears are classified as follows [2]:-

- Spur gears
- Helical gears
- Worm gears
- Rack and Pinion
- Bevel Gears

Figure 2.1: Different types of gears [1]

Different types of gears are seen in Fig.2.1. This thesis deals exclusively with hypoid gears. In order to understand what hypoid gears are, it is essential to delve into bevel gears. There is essentially just one feature that delineates hypoid gears from bevel gears, which shall be discussed in the successive sections.

2.1 Bevel Gears

Bevel gears are used to transfer power between two non-parallel shafts, usually at 90° to one another [1]. Bevel gears can also be used to transfer power between shafts that are at angles other than 90° (seen in Fig.2.3). A bevel gear pair is seen in Fig.2.2. Bevel gears can be classified based on several criteria, but the broadest of them is as follows:

1. Straight Bevel Gears
2. Spiral Bevel Gears
3. Zerol Bevel Gears
4. Hypoid Gears
2.1.1 Straight Bevel Gears

They are the simplest form of bevel gears. They have straight or tapered teeth, which if extended inward will meet at a common point on the gear axis [4]. This can be seen in Fig. 2.4.

2.1.2 Spiral Bevel Gears

These bevel gears have teeth that are curved/oblique, as seen in Fig. 2.5. These gears ensure smoother transmission of motion, owing to the additional overlapping tooth action [4]. Hence, there is less noise at high speeds.

2.1.3 Zerol Bevel Gears

These are nothing but spiral bevel gears with zero spiral angle. So, they have curved teeth in the same general direction as straight bevel teeth (seen in Fig. 2.6) [4]. Gears with spiral angle less than 10 degrees are sometimes referred to as zerol bevel gears [4].
2.1.4 Hypoid Gears

Hypoid gears are similar to spiral bevel gears, except for one difference. The pinion axis in hypoid gears is offset either above or below the gear axis. The direction of offset is always determined by looking at the gear set in such a way that the pinion is to the right [4], this can be seen in Fig. 2.7.

Hypoid gears have the following major advantages over other gears:

- Since the pinion shaft can be lowered with respect to the gear shaft, hypoid gears are used in automotive. If the drive shaft can be lowered with respect to the driven gear shaft (rear axle), then the body of the car can be lowered which in-turn ensures lower center of gravity and thus more stability [2].

- In industrial applications, if there is sufficient offset between the two shafts, then the shafts may pass one another with sufficient clearance thus enabling the use of straddle mounting on the gear pair [2]. It also enables the use of Gang Drive, where in several gears can be mounted on the same shaft, and each of these gear pair can serve a different purpose [2].

It is also interesting to note that, the hypoid pinion is designed to have a larger spiral angle than the gear [2]. This makes the pinion diameter large, thus resulting in a stronger pinion than the corresponding spiral bevel pinion [2].
2.2 Bevel Gear Terminology

A multitude of parameters have to be known and computed in order to build a bevel gear to suit a specific purpose. It is important to be familiar with a few parameters that define the geometry of the bevel gear. Some important terms have been explained briefly in this section. These terms are common to the hypoid gears also.

- **Pitch angle** ($\Gamma$) - It is the angle between face of the pitch surface and axis of the gear/pinion. Usually, the mid-face is taken as the pitch surface. This can be seen in Fig. 2.9.

- **Face width** ($F$) - It is the length of the teeth measured along a pitch cone element [4]. This can be seen in Fig. 2.9.

- **Crossing point** - In case of the bevel gears it is the point at which the axes of the pinion and gear cross each other. However, in case of hypoid gears it can be defined as the point at which the axes of the gear and the pinion apparently cross. The point $Q$ in Fig. 2.9 is the crossing point.

- **Mounting distance** - It can be defined as the distance between the rear face of the gear element (gear or pinion) and the crossing point. The pinion mounting distance is shown in Fig. 2.9, marked with the letter M.

- **Shaft angle** ($\Sigma$) - The angle at which the axes of the pinion and ring gear intersect, in case of bevel gears, is termed as shaft angle. In case of hypoid gears, it is the angle at which the axes of the pinion and the gear apparently intersect.

- **Backlash** - If a gear tooth is made as thick as the space between two adjacent gear teeth, then the gears would mesh leaving no gap thus hindering the ability to lubricate the gears. This would lead to different kinds of failure. In order to prevent this, the space between two adjacent gear teeth is made larger, thus creating some gap. This gap is termed as backlash [1]. This can be seen in Fig. 2.9.

- **Pressure angle** ($\phi$)

  The angle between the tangent to the pitch circle and the line drawn normal to the surface of gear tooth is termed as pressure angle [1]. This normal line, termed as line of action, is nothing but the path along which the point of contact between the pinion and gear teeth moves as the gears mesh and transmit power. This path is a straight line in case of cylindrical gears and not a straight line in case of bevel gears [5].

![Figure 2.8: Pressure angle ($\phi$) [1]](image)

- **Gear teeth features**

  *Top land* is the top surface of the gear tooth. The bottom surface of the tooth is termed as *bottom land* and the part along fillet, over the bottom land is termed as the *gear root*. *Toe* is the part of the gear tooth that is closest to the crossing point while the *heel* is the one that is farther away from the crossing point [6]. The *face* and *profile* directions shown in Fig. 2.10 are the coordinate system along the face width of the gear teeth and come in handy when the stress distribution on the loaded side of the teeth is plotted.
Figure 2.9: Bevel gear nomenclature [4]
• **Hand of spiral** - In order to define the hand of spiral, one has to look at the teeth in 12 o’clock position, from the apex of the pitch cone. If the teeth, seen from front to back, curves to the right then it is a right hand of spiral and if it curves to the left then it is left hand of spiral [3]. This can be seen in Fig.2.11.

![Figure 2.11: Hand of Spiral : (left) Left hand Pinion; (right) Right hand Ring gear ) [3]](image)

• **Offset**

The ability to move the pinion shaft above or below the ring gear shaft is termed as offset. There are two types of offset:-

- **Positive offset**

In this case the pinion axis is displaced along the direction of the spiral angle of the gear, seen in Fig.2.12 [3]. In such cases the mean spiral angle of the pinion is larger than that of the gear, thus resulting in a larger pinion diameter than its corresponding spiral bevel pinion [5].

![Figure 2.12: Positive offset [3]](image)

- **Negative Offset**

In this case the pinion axis is displaced in a direction opposite to that of the spiral angle of the gear, seen in Fig.2.13 [3]. In such cases the mean spiral angle of the pinion is smaller than that of the gear, thus resulting in a smaller pinion diameter than its corresponding spiral bevel pinion [5].
• **Spiral angle** ($\beta$) - It is the angle between the tangent drawn to any point along the tooth flank and the line of the tangent point to the apex of the pitch angle [5]. In general the spiral angle is specified at the center of the tooth width [5], this is termed as *Mean Spiral angle* ($\beta_m$). This can be seen in Fig.2.14.

![Spiral Angle](image)

These are some of the terms that are to be known in order to understand the design and assembly of a bevel gear set. Besides these there are a few other jargon that have to be known in order to carry out the necessary analysis on a pair of hypoid/bevel gears.

## 2.3 Contact pattern

Contact pattern on a gear tooth is that part of the gear tooth flank that participates in the transfer of force and motion [3]. In simple words, it indicates the part of the gear tooth flank that is in contact when the pinion and ring-gear are in mesh. Physical testing is carried out by applying a marking compound or a paint on the gear teeth and operating them under loaded or unloaded conditions in a test rig. The paint or the marking compound is rubbed off in the places where the contact occurs, thus indicative of the contact area. This can be seen in Fig.2.15.

The contact pattern helps discern the load carrying capacity of the gear. Ideally, the contact pattern should be spread over a large area of the flank of the gear, in order to have superior load carrying capacity. However, in order to achieve this contact pattern optimization must be carried out. Optimization of tooth contact pattern is an open problem in the design of hypoid gears [7]. In the actual operating conditions there is a three dimensional pinion vs gear displacement [8]. In order to compensate for deflections/misalignment during actual working conditions, several counter measures were developed. One such method is the *flank modification* or *ease-off identification*, which shall be discussed in the successive sections.
2.4 Drive side and Coast side

Drive side and the coast side (seen in Fig. 2.16) are indicative of the flank on the pinion/ring gear that transmits load in a given working scenario of an engine. There are two working scenarios of the engine:

- Drive - When the engine moves the vehicle forward [3]
- Coast - When the engine brakes the moving vehicle [3]

For spiral bevel gears with positive hypoid offset, concave tooth flank of the pinion driving the convex tooth flank of the ring gear are considered favourable load conditions [3]. Thus, in car axles the concave tooth flank of the pinion is chosen to be the drive side and the convex tooth flank is chosen to be the coast side.

2.5 Forces in Hypoid gears

There are three forces that act on the gear teeth in contact - tangential, axial and radial force (seen in Fig. 2.18). The axial and the radial forces are dependant on the curvature of the loaded tooth face [4]. The configuration of loaded face used in the RDU unit that is modelled in this thesis is underlined in Tab. 2.1.

- Tangential force

  The tangential force on the gear (larger element or the one with more number of teeth) is given by Eq. 2.1,

\[
W_{tg} = \frac{2T_G}{D_m}
\]  

(2.1)
Table 2.1: Load face on the gear [4]

<table>
<thead>
<tr>
<th>Pinion hand of spiral</th>
<th>Rotation of driver</th>
<th>Loaded face</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Driver</td>
<td>Driven</td>
</tr>
<tr>
<td>Right</td>
<td>clockwise</td>
<td>convex</td>
</tr>
<tr>
<td></td>
<td></td>
<td>concave</td>
</tr>
<tr>
<td></td>
<td>counterclockwise</td>
<td>convex</td>
</tr>
<tr>
<td>Left</td>
<td>clockwise</td>
<td>concave</td>
</tr>
<tr>
<td></td>
<td>counterclockwise</td>
<td>convex</td>
</tr>
</tbody>
</table>

where $T_G$ is the torque transmitted by the gear (Nm) and $D_m$ is the mean pitch diameter (m) of the gear. The tangential force on the pinion is given by Eq. 2.2,

$$W_{tP} = \frac{2T_P}{d_m}$$  \(2.2\)

where $T_P$ is the torque transmitted by the pinion (Nm) and $d_m$ is the mean pitch diameter (m) of the pinion.

- **Radial force**

The radial forces on the ring gear/pinion depend on which face is the loaded face. If the loaded face is the concave face, then the radial force can be computed by Eq. 2.3 [4]. If, the loaded face is the convex face, then the radial force can be computed using Eq. 2.4 [4],

$$W_r = \frac{W_t}{cos(\psi)} [tan(\phi)cos(\gamma) - sin(\psi)sin(\gamma)]$$  \(2.3\)

$$W_r = \frac{W_t}{cos(\psi)} [tan(\phi)cos(\gamma) + sin(\psi)sin(\gamma)]$$  \(2.4\)

- $W_r$ is the radial force on the corresponding gear element
- $W_t$ is the tangential force on the corresponding gear element
- $\psi$ is the mean spiral angle at pitch surface of the corresponding gear element
- $\gamma$ is the pitch angle of the corresponding gear element

In Eq. 2.3 & Eq. 2.4 the parameters of the corresponding gear element - ring gear or pinion can be plugged in, in order to obtain the radial force. If the radial force has a positive sign (+), then it indicates that the force is acting away from the mating member and this force is termed as separating force [4]. Alternatively, a negative sign (−) indicates that the force is acting towards the pitch apex [4].

- **Axial force**

The axial forces on the ring gear/pinion also depend on which face is the loaded face. If the loaded face is the concave face, then the axial force can be computed by Eq. 2.5 [4]. If, the loaded face is the convex face, then the axial force can be computed using Eq. 2.6 [4],

$$W_x = \frac{W_t}{cos(\psi)} [tan(\phi)sin(\gamma) + sin(\psi)cos(\gamma)]$$  \(2.5\)

$$W_x = \frac{W_t}{cos(\psi)} [tan(\phi)sin(\gamma) - sin(\psi)cos(\gamma)]$$  \(2.6\)

- $W_x$ is the axial force on the corresponding gear element
- $W_t$ is the tangential force on the corresponding gear element
- $\psi$ is the mean spiral angle at pitch surface of the corresponding gear element
- $\gamma$ is the pitch angle of the corresponding gear element

In Eq. 2.5 & Eq. 2.6 the parameters of the corresponding gear element - ring gear or pinion can be plugged in, in order to obtain the axial force. A positive sign (+) is indicative of the fact that the thrust is acting away from the pitch apex [4]. Alternatively, a negative sign (−) indicates that the thrust is acting towards the pitch apex [4].
The direction of the aforementioned forces is determined by the hand of spiral and direction of rotation. The direction of rotation of the gear element is determined by viewing it in such a way that the apex of the gear/pinion lay ahead [4]. This is shown in Fig.2.17.

Figure 2.17: Determining the direction of rotation [4]

Figure 2.18: Forces in hypoid gears [4]
2.6 Axle Deflection

Axle deflections are one of the most critical issues in the design and analysis of hypoid and spiral bevel gears [9]. In vehicles, the driveline has to operate under heavily loaded conditions. These loads can cause deflections on the shafts and hence the gears which are mounted on these shafts. Besides the load induced deflections, the manufacturing tolerances on the housing also result in relative displacement of the gear set [3]. The possible deflections on the gear set are represented in Fig.2.19.

- E - This is indicative of the pinion offset
- P - This is indicative of the pinion mounting distance
- G - This is indicative of the gear mounting distance
- $\alpha$ - This is indicative of the shaft angle

If the E, P, G, $\alpha$ are positive, it can be discerned that the deflections are causing an increase in pinion offset, gear/pinion mounting distance and the shaft angle [9]. Alternatively, a negative sign indicates a decrease in the aforementioned parameters.

These deflections will alter the gear contact pattern in terms of contact area, contact path and transmission error [9]. Change in the contact pattern can alter the maximum stress level in the gear. In a study conducted at FZG (Gear Research Laboratory, Munich) on a set of spiral bevel gears, it was found that an alteration in the axial displacement of pinion (P) could cause the maximum stress level to vary in a range of about 16% on the gear and about 20% on the pinion [10]. Such variations are undesirable, as they can have a significant impact on the gear root strength [9]. Multitude of commercial gear design & analysis software are available - KIMoS, HFM (Hypoid Face Milled), into which pre-determined E, P, G, $\alpha$ can be fed as input. These software, help in predicting the contact pattern under loaded conditions for a given set of E, P, G, $\alpha$ values, with reasonable accuracy. With the predicted contact pattern, it can be determined as to how much the stresses vary due to the deflections. However, the question remains, “How can these deflections be compensated for?” One of the methods used for this very purpose is the introduction of flank modifications.

2.7 Flank Modifications

There are five different modifications that can introduced on the flank of a hypoid gear [3]. They are as follows:

- Length wise crowning
- Profile crowning
- Angle of twist
- Alteration of the Pressure angle
- Alteration of the Spiral angle
Alteration of these parameters changes the contact pattern. For example, in order to elongate the contact pattern in the face width direction, the lengthwise crowning is reduced [3]. When the flanks mesh with each other (after the introduction of flank modifications on them), at each instance of mesh the minimum gap between the pinion and gear flank are measured, and finally a plot of all these minimum distances is made over the entire mesh, thus yielding the ease-off. In simpler terms, ease-off is nothing but the deviation in the tooth geometry from its otherwise flat surface, after the introduction of flank modifications. This can be seen in Fig. 2.20. The profile crowning is achieved by changing the blade profile curvature [9]; angular twist/bias can be achieved by changing cutter tilt or using higher order machine motions [9]; lengthwise crowning can be achieved by varying the cutter head tilt along with blade angle [9].

2.8 Design of hypoid/bevel gears

The detailed procedure of design/dimensioning of hypoids/bevels is deemed esoteric to be included as a part of this thesis. Hence, the design procedure is described in its broadest sense as follows [5]:

- **Dimensioning**

  In this phase the gear macro geometry is computed. This involves the computation of various diameters, angles, module, number of teeth etc that determine shape of the blank and teeth. All these computations are made based on the type of gear system chosen. The type of gear system is chosen based on the application.

- **Standard Calculation**

  In order to design the gear against different kinds of failure, various safety factors have to be computed. These safety factors vary depending on the type of failure the gear is being designed against and also the macro geometry. If the safety factors that have to be considered are too high, then first step is followed again, thus altering the macro geometry in order to accommodate the design against different kinds of failure.

![Figure 2.20: Flank modifications that describe ease off [3]](image-url)
• **Ease-off Synthesis**

In order to obtain the desired contact pattern, accommodating for axle deflections, manufacturing inaccuracies and the like, the flank modifications are introduced. Then the effect of the introduced flank modifications is studied using the ease-off synthesis and discerning how the contact pattern looks and where it occurs. If the contact pattern obtained is undesirable, then the process must be repeated.

• **Loaded Tooth Contact Analysis (LTCA)**

The final step, is to study the effect of load on the designed gear. Analysis is done so as to see if the occurring stresses are within acceptable limits for the decided micro (flank modifications) and macro geometries. If acceptable then the design and dimensioning is done, else the micro or macro geometries have to be re-computed in order to minimize stresses.

In the aforementioned procedure, most of the times, toggling between steps occur. It is not always possible to swiftly decide the dimensions - macro and micro geometries in one attempt.

### 2.9 Types of failure in Bevel gears

Some of the common types of failure found in bevel gears (seen in Fig. 2.21) are:

- Pitting
- Tooth root breakage
- Scuffing
- Tooth internal fatigue

![Figure 2.21: Different types of failures in bevel gears [5]](image)

This project largely deals with tooth root breakage. So, a glimpse of the safety factor computation for designing the gear set against tooth root breakage shall follow.

The safety factor for tooth root bending $S_F$ is calculated as seen in Eq. 2.7 [5]:

$$S_F = \frac{\sigma_{FP}}{\sigma_F} \geq S_{F_{\text{min}}}(2.7)$$

where, $\sigma_{FP}$ is the permissible tooth root stress, $\sigma_F$ is the encountered tooth root stress and $S_{F_{\text{min}}}$ is the minimum safety factor. $S_{F_{\text{min}}}$ is usually a value agreed upon between the customer and the designer [5]. The tooth root stresses are computed using Eq. 2.8 [5],

$$\sigma_{F1,2} = \frac{F_{vmt} \times Y_{Fa1,2} \times Y_{Sa1,2} \times Y_e \times Y_{BS} \times Y_{LS} \times K_A \times K_V \times K_{F\beta} \times K_{F\alpha}}{b_v \times m_{mn}}(2.8)$$
where, \( F_{vmt} \) is the tangential force; \( Y_{Fa1,2} \) \& \( Y_{Sa1,2} \) are factors of tooth form, stress concentration, stress distribution; \( Y_c \) is Contact-ratio factor; \( Y_{BS} \) is Spiral angle factor; \( Y_{LS} \) is Load sharing factor and \( K_A, K_V, K_{F\beta}, K_{Fa} \) are Force factors. All these factors are estimated from standard tables, charts or using definite formulae.

Similar calculation procedures exist for the design of gears against different kinds of failure. However, those are beyond the scope of the current work.

### 2.10 Finite Element Analysis

This thesis involves the use of finite element method to setup the model and to carry out analysis. Finite element method is a numerical method that is used to find an approximate solution to problems that are governed by partial differential equations & boundary conditions, in various fields like- structural engineering, solid mechanics, fluid mechanics etc. In general, finite element analysis can be said to comprise of three phases [11]:

- **Pre-processing**

  In this stage the user basically builds the model, creates a mesh for the model and applies the boundary conditions on the model.

- **Solver**

  Here the equations are solved by the FE solver in question by using a set of governing equations. Their governing equations are different in different applications.

- **Post-Processing**

  Here the results from the solver are visually represented. Based on observations/interpretations of these graphical results, conclusions are drawn or the model is subjected to further investigation.

It is not within the scope of this thesis to expound the process of FEA but rather delineate certain terms that might be useful for the reader.

In FEA the model/body in question is discretized i.e. divided into smaller parts. Then each of these parts are analyzed using the governing equations. Consequently the effect on all these smaller parts is representative of the effect on the entire body. In order to better understand this, the concept of nodes and elements are outlined below.

#### 2.10.1 Nodes and Elements

Any given body can be discretized, i.e. divided into smaller pieces of finite dimension. These smaller pieces of finite dimension are known as *finite elements* [12]. The entire body can be considered as an assembly of these small elements. These elements are connected together through number of joints called *nodes*. These are better seen in Fig. 2.22. During discretization it is assumed that elements are attached to other elements only at nodal points [12]. The density of nodes at a particular region on the domain/body depends on stress variation in the region, if large then the density of nodes are higher, else the density of nodes are lesser [12]. This discretization into smaller elements, is what constitutes a *mesh*.

Each element in the discretized body, has the same properties as the body [12]. There are several types of elements available in FEA, however they can broadly be classified as [12]

- One dimensional(1D) elements
- Two dimensional(2D) elements
- Three dimensional(3D) elements

The choice of the elements depends on the kind of problem being solved.

Loads and other boundary conditions are applied on the *nodes*. The displacement of these nodes are described using a polynomial function. The choice of the polynomial function depends on the type of element in use [12]. For example, in case of a 1D element having single degrees of freedom (dof) with two nodes (seen in Fig. 2.23), the displacement function can be \( a_0 + a_1x \) [12]. The displacement of the nodes causes displacement on the elements, which in-turn is representative of the strain occurring in the body.
2.10.2 Stiffness Matrix

In all the finite element solvers, an attempt is made to compute the nodal displacements. In highly simplified terms this process is governed by Eq. 2.9, where \( K, x, F \) are the stiffness (N/m), displacement (m) and the load (N) matrices respectively.

\[
[K][x] = [F]; \implies [x] = [F][K]^{-1}
\] (2.9)

Consider an element as shown in Fig. 2.24. It is assumed that the element connecting the two nodes has some stiffness. So, when a load is applied on node 1 causing it to displace, then the displacement can be computed if the force and the stiffness of the element are known using \( F = k.x \implies k = \frac{F}{x} \). There will be several such elements over the discretized domain which are assumed to connect nodes with some finite stiffness. All these stiffnesses together combine to form the stiffness matrix of a component.

These stiffness matrices have some unique properties [12]:

- They are symmetric and square
- All diagonal elements are positive

These stiffness matrices are largely dependant on the type of element used in the analysis. In the context of this thesis work, the bearing stiffness matrices are obtained readily from a software called Shaft, gear and bearing concepting (SABR).

This chapter does not present in-depth knowledge of everything that was assimilated in order to carry out the analysis. It only outlines the concepts that were deemed crucial for the understanding of the method and results that are annotated in the successive chapters. The reader is urged to note that there will arise two definitions of the word \textit{mesh} in this thesis. Depending on context it can refer to:

- the process where the pinion tooth begins and ends its contact on the corresponding gear tooth
- the discretization of the body chosen for analysis
3. METHOD

This chapter describes how the model was built in different software - T3D, HFM & Marc.

A broad description on the method followed, is listed as follows:

- The model was built on T3D and validated with physical contact pattern test.
- The analysis was carried out on T3D for varying levels of press-fit and the results were summarized.
- The teeth at the highly stressed zone in T3D was imported to Marc, and a similar stress analysis was done on it. This was done to determine if the press-fit established was correct. The results were compared.
- To know if inclusion of several parts in the analysis caused an increase in stress values, the model was rebuilt in 3 stages on HFM.
  - Stage 1 - Just the teeth
  - Stage 2 - Teeth + Rim
  - Stage 3 - Complete model like in T3D, but without press-fit
- The HFM model was also validated using contact pattern comparison.
- The teeth, from the same position as in T3D, was imported from HFM into Marc.
  - A comparison was made between HFM and the corresponding Marc model.
  - Then, press-fit was induced and a comparison was made between the T3D model and HFM’s Marc model.

3.1 Modelling on HFM

The modelling procedure in HFM and T3D is quite similar except for a few minor differences. The case of HFM is described first as it is easier to understand.

3.1.1 Modelling the pinion and the gear teeth

The GUIDE application of HFM is used to build the model. The EDIT menu in the guide has four sub-menus:

- SPAFILE
- SYSTEM
- PINION
- GEAR

SPAFILE sub-menu enables the user to select the units of working and also import the Special Analysis file (SPA file). This SPA file comprises of all the basic macro-geometry parameters of the gear and pinion teeth - no. of teeth, spiral angle, root angle, etc. HFM reads all these values from the SPA file. The working units chosen in this case are Nmm.

SYSTEM sub-menu enables the user to specify the driver, shaft angle, offset, hand of spiral of the pinion, input torque and rpm.

PINION sub-menu in-turn has four other sub-menus: COMMON, CONCAVE, CONVEX and RIM. COMMON menu has all the data read from the SPA file. The CONCAVE and CONVEX both lead to three sub-menus- MACHINE, CUTTER and MODIFICATIONS. The machine and cutter parameters were pre-fed and not altered, for both concave and convex side. Flank modifications were done by enabling the DOTOPMODFN check-box under the MODIFICATIONS menu, which was present in both CONCAVE and CONVEX menu. This allowed flank modifications on the CONVEX and CONCAVE side to be uploaded as a text(.txt) file which contained all ease-off modifications from an instrument (IST) file. RIM menu shall be discussed in the subsequent sections.
GEAR sub-menu, in-turn, had four other sub-menus: COMMON, CONCAVE, CONVEX and RIM. These worked just like the PINION menu, except for the fact that all the data fed and read here corresponded to the ring gear.

Following the above steps lead to the setting up of just the pinion and ring gear teeth (seen in Fig. 3.1). It is interesting to note that no constraints need to be explicitly specified to run an analysis with just the teeth. In this case, the PINION is the driver. The pinion has 12 teeth and the ring gear, 31 teeth. The input torque is 750 Nm. The pinion is left handed while the ring gear is right handed.

![Figure 3.1: Model with just the teeth from HFM](image)

### 3.1.2 Modelling of the pinion and gear rim

**Pinion Rim**

Under the PINION menu, there was a sub-menu named RIM that allowed the user to build the rim when DORIM check box was enabled. The rim geometry as specified in the software is shown in Fig. 3.2. The rim had to be modelled in segments. All the necessary dimensions were obtained from the CAD drawing of the pinion blank. The number of segments is left to user’s convenience. In this model, the pinion rim was divided into three segments, and the dimensions for each segment were estimated from the CAD drawing.

**Gear Rim**

Under the GEAR menu, there was a sub-menu named RIM that allowed the user to build/import the rim when DORIM check box was enabled. For ring gear, the rim was readily available as a .bdf file. This was imported into HFM and positioned appropriately.

Once both the pinion and the ring gear rim were modelled, analysis could be run just with the teeth and rim. The teeth with rim model is shown in Fig. 3.3.

### 3.1.3 Modelling the pinion shaft

Under the PINION menu, ENABLESHAFT check-box was turned on. This, in-turn, enabled a new button called SHAFT. The SHAFT button allows user to build the shaft based on certain guidelines as shown in Fig. 3.4. The shaft is divided into several segments and begins only from the rear face of the pinion rim (this is specified by the SHAFT OFFSET as seen in Fig. 3.4). To model the shaft, it was divided into 12 segments. The dimensions for each of these segments were obtained from the CAD model based on the guidelines specified in Fig. 3.4. The shaft menu also allowed the specification of material properties of the shaft. A Young’s Modulus of 210 GPa was used, in conjunction with - Poisson’s ratio of 0.3 and a density of $7.84 \times 10^{-9} \text{kgm}^{-3}$. The model with the pinion shaft is shown in Fig. 3.5. Additionally, the shaft was constrained linearly along its axis of rotation.
3.1.4 Modelling the differential carrier

Under the GEAR menu, ENABLEDIFFCARRIER checkbox was enabled to model the differential cage. Under DIFFCARRIER menu that appeared, the user was allowed to import an FE model of the differential cage. Additionally, there were three sub-menus- PINIONHOLES, CONICALRACES, CYLINDRICALRACES to position the carrier appropriately. The distance of each of these entities from the crossing point was measured with the help of a CAD model. Each of these entities are marked appropriately in Fig.3.6. The circular disc shaped surface was modelled as a CONICALRACE with a cone angle of 90°. The model with the differential cage is shown in Fig.3.7.
3.1.5 Modelling of the bearings

There are two bearings present on the pinion shaft and two on the differential cage. They are shown in Fig. 3.8.

Head and Tail Bearing (associated with pinion shaft)

The head and tail bearings are present on the third and sixth segment of the pinion shaft respectively. While modelling the pinion shafts, at segments three and six the BEARINGRIGID option under OUTERCONNECTIONS was selected under the SHAFT menu. Additionally, the distance from one end of the segment to its center were to be specified under BRGOFFSET that was enabled after selecting BEARINGRIGID, in each case. Once this was done, the stiffness matrices of each of these bearings were uploaded as a .brg file.

Button and Gear Bearing (associated with differential cage/housing)

The DOBRGCYLINDRICARACE check box was turned on under DIFFCARRIER option for both the cylindrical races here. The bearing position was specified at ZPOSNBRG in each case, by measuring the distance between the crossing point and the center of the cylindrical race. Finally, the bearing stiffness matrices were uploaded as a .brg file. Additionally, the entire model was constrained by the button and gear bearing in HFM, i.e. the bearings were fixed by applying a fixed displacement condition (fixing all 6 degrees of freedom) on the outer ring of the bearings.
This completes the model on HFM. HFM does not allow modelling of the press-fit. This shall be done using T3D. After the model was setup, the analysis was run. Consequently the results were obtained using POSTPROC menu.

The analysis in case of both HFM and T3D were chosen to be run on the teeth at four different positions (seen in Fig. 3.9) - window edge 1, window center, window edge 2 and pin hole. The teeth close to these positions were chosen because there was some change in geometry of the differential cage at these positions, making the study of those teeth interesting. It was computationally intensive to run the analysis for every single tooth. When a pinion tooth begins to come in contact with the corresponding ring gear tooth, the analysis begins. The start-end of this contact was divided into eleven time steps in both HFM and T3D. When importing the teeth with forces to Marc, only one time step was chosen as a matter of convenience.
3.2 Modelling on Transmission3D

Modelling on T3D was similar to HFM, with minor differences. Only these differences in modelling are enunciated.

- T3D does not allow the analysis of just the teeth or teeth with rim. Only complete model analysis can be done.
- T3D introduces the concept of ROTORS. In T3D ROTOR refers to an entity that rotates about a fixed axis [13]. Shafts, carriers and gears are associated with a ROTOR. The ROTOR as a whole is then fixed to the global coordinate system. So in this model, 2 ROTORS were used.
- ROTOR1 was associated with the ring gear and ROTOR2 with the pinion.
- ENABLEHYPOIDS option had to be enabled in both the ROTORS, to build ring gear and pinion teeth [13]. A HYPOID button popped up. Within the menu, the same sub-menus as under PINION/GEAR in HFM appeared - COMMON, CONVEX, CONCAVE and RIM. So, under ROTOR 1, there was HYPOID1 under which all the gear parameters were defined. As in HFM there was a provision to upload the SPA file and txt format of the IST file. The IST file contained all the flank modification data.
- The rim in case of ring gear was imported, similar to HFM. In order to import rim of the gear, CARRIER option under ROTOR1 was used. The CARRIER option was enabled when the ENABLECARRIERS check box was tick marked under ROTOR1.
- The rim on the pinion was modelled as a shaft. To do this, the ENABLESHAFTS check box had to be enabled under ROTOR2.
- The pinion shaft was modelled in the same way as in HFM. The guidelines to building the shaft are also the same in T3D as they are in HFM.
- The carrier option under ROTOR1 was used again to import FE file of the differential carrier. Then, the setting up of the differential cage was similar to that of HFM.
- To model the bearings, CONNECTORS option under each ROTOR menu was to be enabled [13]. Once that was done, RACES were to be created at appropriate positions depending on whether it was on the pinion shaft or differential cage. Once that was done, stiffness matrices were directly keyed in.
- In T3D, to fix the differential cage and components connected to it, to the rotor, a few nodes on the pin hole of the differential cage were found and constrained in all six degrees of freedom. The entire system was attached to global coordinate system by constraining the bearings.
- Press-fit

The press-fit was modelled in ROTOR1 by the use of shafts. Four sectors of concentric shafts were created as seen in Fig.3.10. Inner surface of the inner shaft sectors were attached to the differential cage, while outer surface of the outer shaft sectors were attached to the rim. Then, an interference was introduced at the interface of the two shaft sectors. This interference was introduced in three steps - 10µm, 55µm and 100µm. Analysis was done for each of these press-fit levels.
In both, T3D and HFM, flank modification data from the IST file is fit on to the gear teeth by means of a \textit{RMS polynomial fit} function. Once this analysis was done, the model was setup on Marc.

\subsection*{3.3 Modelling on Marc}

Marc was used to replicate the models in both HFM and T3D. HFM and T3D models were built on Marc in the same way with minor difference in boundary conditions.

The components used to build the model on Marc were:

- Ring gear teeth with forces on the convex surface of the teeth at \textit{window edge 2} position
- Differential cage
- Inner rings of the button and the gear bearing
- Gear rim

The pinion and its components were not included in the model. This is because, forces due to driving motion of the pinion are already exported as an FE mesh from both T3D and HFM.

\subsubsection*{3.3.1 Ring gear teeth with forces on the convex face}

The teeth at the zone of interest (both in HFM & T3D) - \textit{window edge 2} were found. Then, the HFM or T3D model was solved for that particular position. EXPORTFEMODEL option under the POSTPROC menu was used to export the necessary tooth as a \textit{Nastan bulk data file} (.bdf). The GOTOPOSN option enables the user to select the time step at which the forces on the tooth are of interest. For convenience the first time step was chosen, and each of the 31 teeth on the ring gear at this instance of time, were imported as bdf files. In these files, the forces were present on the teeth only at the position at which the analysis was run on HFM or T3D, \textit{window edge 2}. Each of these teeth were imported into HYPERMESH and built into a single ring gear set. It is to be noted that the .bdf file of each tooth, yields the tooth with the mesh. This mesh is generated within HFM and T3D. In both the cases, the mesh is finer at the gear root than the rest of the tooth. Compared to T3D, HFM yields a tooth with larger number of nodes and elements (mesh statistics for this can be found in APPENDIX A). The final models are seen in Fig. 3.11.
3.3.2 Differential Cage

The differential cage was meshed on SIMLAB. Fine mesh was retained at all the contact interfaces of the differential cage - cylindrical races, weld zone and press-fit zone (shown in Fig. 3.12). At the areas of finer mesh, preserve entities option was used so that the geometry of these zones remain unaltered. Initially a surface mesh was generated i.e. a shell was generated (seen in Fig. 3.13). Then the quality of the mesh was checked. Once the required quality was achieved, this shell was converted to a solid mesh. Final model of the differential cage is seen in Fig. 3.14.

3.3.3 Inner rings for button and ring gear bearings

The button bearing and the gear bearing utilize the same type of bearing. So, it was sufficient to mesh one of them. For the purpose of a simplified model, the bearing as a whole is not used. Only the inner rings of the bearings were meshed and used in the analysis. In order to build a mesh of the inner ring of bearing, initially a 2D mesh of its cross-section was built using the automesh feature in HYPERMESH and this 2D mesh is spun over 360°. Also, the number of elements over the entire spin step can be specified. In order to obtain a highly refined mesh, this value must be higher. However, these bearing seats are not an area of interest, so 100 elements were chosen in this case. The final mesh can be seen in Fig. 3.15.

3.3.4 Gear rim

The gear rim was built in a way similar to the bearings. The CAD model of the rim was exported into HYPERMESH, a 2D mesh of the cross section was created and finally this 2D mesh was spun over 360°. If the fineness of the rim is not sufficient then, the same procedure can be followed except that in the spin step, the number of elements are increased. It must be noted that once the spin is complete, there will be two copies of the mesh, comprising of the same set of nodes, elements at the location from where the spin began. Due to the 360° spin, the start and end are at the same location. So, the common set of nodes at this point must be paired. This is done using the equivalence option under faces tab on HYPERMESH. The final mesh can be seen in Fig. 3.16.
Figure 3.12: Zones at which the geometry is preserved and fine mesh is used

Figure 3.13: Surface mesh of the differential cage

Figure 3.14: Final model of the differential cage
Figure 3.15: Inner ring of button bearing and gear bearing

Figure 3.16: Meshed model of the rim
3.3.5 Contact conditions

The definition of contact between different components was one of the most important tasks in setting up the model on Marc. Once the contact was setup, a contact status check was done so as to see if the contact has been established correctly. The contacts that are modelled in Marc are as follows:

- **Glued contact**
  - This contact exists between the gear rim and the teeth, thus ensuring they are glued together as a single body.
  - The weld zone was simplified to a glued contact. So, the back face of the rim was just stuck to the flat circular disc on the differential cage.

- **Touching Contact**
  - The press-fit between the ring gear and the differential cage was modelled using this contact condition. An interference of 100µm was established.
  - The bearings are press-fit onto the differential cage. So, this contact condition was used to model the radial press-fit between the bearings and the differential cage.
  - The bearings also made an axial touch to the differential cage. This was also modelled using this contact condition.

These contacts were established in two steps. In the first step, the nodes and elements on the corresponding contact surfaces were found and in the next step the radial interference of 100µm was introduced at the bearing-differential cage interface and the gear rim-differential cage interface. The contact status check was done after the interference had been established.

The contact status check is seen in Fig.3.17 (clearer picture in APPENDIX C). The yellow color is indicative of a good contact.
3.3.6 Boundary Conditions

The boundary conditions are inclusive of the loads and fixed displacements used in the analysis. The teeth with forces are exported from both T3D and HFM into their corresponding Marc models. There exists a minor difference in how the HFM and the T3D models are setup and this difference is in the constraining of the model.

Transmission 3D

In T3D the gear and pinion are initially fit onto an imaginary rotor, which in turn is connected to the global coordinate system. So, to constrain ring gear to the local coordinate system, the nodes at the face of the pin hole are constrained in six degrees of freedom. The entire system is held in position with respect to the global coordinate system by constraining at the bearings.

T3D has two coordinate systems - local and global. The local coordinate system is associates with each of the ROTOR and its components while the global coordinate system is the one in which both the rotors are present.

The aforementioned conditions were modelled in MSC MARC as follows (seen in Fig.3.18):-

- The nodes at the pin hole position on either sides were constrained in all six degrees of freedom. This is done using a RBE2 (Rigid Body Element).
- The bearing forces are applied on the bearings using a RBE2 element. The RBE2 element was placed 9mm inward from either end of the differential cage and tied to nodes on the bearing ring, where the rolling elements would be present.
- The gear loads are imported from T3D as a bdf file.

In the T3D model built on Marc, the bearing forces were used to balance the gear forces in the loading step. The bearing forces are obtained from SABR, a software developed by Ricardo plc.

Hypoid Face Milled

In HFM there was no necessity to lock the differential cage at the pin hole position like in T3D. It was enough to constrain the system at the bearings in order to run the analysis. When the HFM model was built on Marc, the nodes at the pin hole could not be constrained as it was decided that it would be a good practice to replicate the conditions as in HFM. Owing to this, the bearing forces could not be used. Instead of applying the bearing forces on the RBE2 element (similar to T3D, it is placed 9mm inward from either ends of the differential cage), a fixed displacement constraint (constraining all six degrees of freedom) was applied (seen in Fig.3.19). The forces on the gear teeth are imported from HFM into Marc.
Once the models are setup on Marc, the analysis was run on a calculation cluster as it required high computing power. The results from the cluster were then post processed using HYPERVIEW. The maximum principal stresses are checked for in the result of Marc.

### 3.4 Materials

As the model being setup was an already produced model, the materials used in case of each of the components were readily available. For all the materials, linear elastic properties were assumed. The materials assigned to the components in the analysis are summarized in Table 3.1.

Table 3.1: Materials used on the components in the analysis

<table>
<thead>
<tr>
<th>Material</th>
<th>Component(s)</th>
<th>Young’s Modulus (GPa)</th>
<th>Poisson’s Ratio</th>
<th>Yield Strength (MPa)</th>
<th>Ultimate Tensile Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel GGG-70</td>
<td>Differential cage</td>
<td>170</td>
<td>0.275</td>
<td>370</td>
<td>700</td>
</tr>
<tr>
<td>Case Hardened steel</td>
<td>Pinion, ring gear, pinion shaft, bearing race</td>
<td>210</td>
<td>0.3</td>
<td>1510</td>
<td>2500</td>
</tr>
</tbody>
</table>
4. RESULTS & DISCUSSION

This chapter summarizes the results obtained from the analyses - effect of press fit on root bending stresses and also makes an effort to explain some patterns in the result.

Firstly, the results from T3D and its corresponding Marc model are summarized and discussed. Later the results from HFM and its corresponding Marc model are summarized and discussed.

4.1 Transmission 3D

As explained in the previous chapter, the T3D model was analyzed at four different positions (seen in Fig. 4.1) - Window Edge 2 (WE2); Window Center (WC); Window Edge 1 (WE1) and Pin Hole (PH) and at different levels of press-fit -0µm, 10µm, 55µm, 100µm.

![Figure 4.1: Transmission 3D model](image)

4.1.1 Model Validation

In order to validate the virtual model, the contact pattern from the analysis at 500Nm drive torque was compared with the contact pattern available from a physical test at the same torque. In Fig. 4.2 it can be observed that, the contact pattern from the virtual model nearly matches the one from the physical test. The minor differences between the virtual model contact pattern and the physical test contact pattern are that:

- In the virtual model there seems to be some contact closer to the toe while such a contact is absent in the physical test
- The contact at the heel seems nearly complete in the physical test, while in the virtual model it is not

These differences may be due to the inherent inability of the RMS polynomial fit to function effectively within Transmission 3D. This function is responsible for the fitting of data points from the flank modifications file, onto the curvature of the teeth using an appropriate polynomial function. This fit may not be perfect every time and hence susceptible to error. In the current analysis, the error encountered by the function, while trying to fit the data points from the flank modification file of the gear, was quite large, about 20%. This could be contributing to the observed difference in contact pattern between the virtual model and the physical test.

This validation is crucial, as it is indicative of the correctness of the setup. A large deviation in contact pattern can significantly alter stress on the teeth, ergo leading to poor design. Post validation, analyses were carried out at 750Nm drive torque in order to discern the influence of press-fit on the root-bending stresses at various positions.
4.1.2 Influence of press fit

The model was first subjected to a torque of 0.1Nm (low-load condition) at different levels of press fit interference. The results from this analysis, are seen in Fig.4.3 (clearer picture in APPENDIX C). It can be observed from Fig.4.3 that:

- The stresses increase with increase in press-fit dimension
- The distribution of stresses on either sides of the cross-pin axis is similar, or the stress distribution in diametrically opposite directions are similar
- The press-fit causes expansion of ring-gear at the pin-hole position and compression at the window center position

The variation of maximum root bending stresses with varying press fit dimension at low load (0.1Nm) was plotted, in order to examine the nature of variation of root bending stresses with variation in press fit dimension. It can be seen from Fig.4.4 that variation in root bending stresses with increasing press fit dimension, is linear.

The load on the gear, acts on the convex face thus pushing the teeth. This causes a displacement on the tooth towards the window at window edge 2, while it causes a displacement towards the pin hole or away from the window at window edge 1. In that case, it can be observed from Fig.4.1 that tooth at window edge 1 would have more support than the tooth at window edge 2, thus enabling lesser strain at window edge 1 than at window edge 2. Lesser strain, would imply lesser stresses ergo tooth at window edge 1 is stressed lesser than the one at window edge 2 when loaded. The strain plot can be seen in Fig.4.6.
Figure 4.4: Variation in root bending stresses at low load with varying press fit

Table 4.1: Maximum root bending stress values at different positions on the ring gear

<table>
<thead>
<tr>
<th>PRESS-FIT</th>
<th>MAX. GEAR ROOT BENDING STRESSES (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Position</td>
</tr>
<tr>
<td>Tooth No.</td>
<td></td>
</tr>
<tr>
<td>0µm</td>
<td></td>
</tr>
<tr>
<td>10µm</td>
<td></td>
</tr>
<tr>
<td>55µm</td>
<td></td>
</tr>
<tr>
<td>100µm</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.5: Variation in root bending stresses with varying press fit at 750Nm drive torque
The no-press fit data points are omitted owing to the fact that some amount of interference always exists in reality. Additionally, from the output files spewed by transmission3D, bending stress maps were plotted at different locations in order to visualize the spread of stress over the gear root. These maps indicate - the position at which the maximum stresses occur on the teeth, the magnitude of stresses at different positions.

**Bending Stress Maps**

The stresses from the output files of the tooth in complete contact at each of the four positions, for a drive torque of 750 Nm was utilized in order to build the bending stress map. The observations made from these bending stress maps are summarized as follows:

- **At 100 µm of press fit** (seen in Fig. 4.7)
  - The maximum stress levels at different positions viz. pin hole, window center, window edge 1 and window edge 2 are different.
  - The maximum stress for the tooth in full contact, at all positions, occurs closer to the heel. However, the exact point at which it occurs is different.
  - The tooth at pin hole and window edge 2 positions are subjected to higher levels of stresses than the tooth at other two positions. Additionally, it can be observed that at Window edge 2 the highly stressed zone is spread over a larger area than at the pin hole position.
  - Similarly, the highly stressed zone is spread out over a larger area at the window center position than at window edge 1.

- **At 55 µm of press fit** (seen in Fig. 4.8)
  - While the magnitude of maximum stress has reduced at all positions, the tooth at window edge 2 still has a small concentration of high stresses (about 1200 MPa).
  - The highly stressed zone has reduced by a small amount at the window center position when compared to the corresponding plot at 100 µm of press fit. The stress map at window edge 1 position has not undergone any major change.

- **At 10 µm of press fit** (seen in Fig. 4.9)
  - The maximum stress level at window edge 2 position has dropped. However, the highly stressed zone still remains large in pin hole and window edge 2 positions.
Figure 4.7: Bending Stress Map at 100µm

Figure 4.8: Bending Stress Map at 55µm
The above made observations can be summarized graphically. In order to obtain the area over which the highly stressed zone is spread, a stress limit was defined. Any point on the root stressed beyond 1000 MPa was considered to be highly stressed. The area over which these points are spread, is expressed as percentage(%) of total root area (which is nothing but the total area of the bending stress map). A sample plot is shown in Fig. 4.10. The area of spread of the highly stressed zone, in each position and at different levels of press-fit was computed and summarized as seen in Tab. 4.2. This data is graphically represented for better understanding. From Fig. 4.11 it can be observed that the highly stressed zone decreases in size with increasing press-fit dimension, at all positions. Additionally, from Fig. 4.12 it can be observed that the teeth closer to window edge 2 and pin hole positions have a larger concentration of high stresses, while teeth closer to window edge 1 have the least concentration of highly stressed zone. These shall remain mere findings, as the reason for such a trend continues to be inexplicable.

Finally the maximum root bending stresses, when each of the 31 teeth on the ring gear is in full contact, was found and a plot of the same was made. Owing to long computational time, this analysis was carried out for all 31 teeth only at 100 μm of press fit. Fig. 4.13 shows that the teeth are subjected to cyclic loading at 100 μm of press fit and 750 Nm of drive torque. It was predicted that at other levels of press-fit, under the same load of 750 Nm, the teeth will be subjected to cyclic loading. In order to verify this prediction, the maximum root bending stresses at different press fit levels must be carried out.

<table>
<thead>
<tr>
<th>PRESS-FIT</th>
<th>STRESS CONCENTRATION AS A % OF TOTAL ROOT AREA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Position Window edge 2 Window center Window edge 1 Pin Hole</td>
</tr>
<tr>
<td>Tooth No.</td>
<td>T19</td>
</tr>
<tr>
<td>10 μm</td>
<td>13.34</td>
</tr>
<tr>
<td>50 μm</td>
<td>10.34</td>
</tr>
<tr>
<td>100 μm</td>
<td>8.46</td>
</tr>
</tbody>
</table>

Table 4.2: Stress concentration of highly stressed zone at different position and press-fit
Figure 4.10: Finding the zone enclosed by stresses $> 1000MPa$

Figure 4.11: Variation of stress concentration with varying press fit
The reason for this cyclic distribution of stress can be attributed to the variation in stiffness of the differential cage in the radial direction. This variation in stiffness causes different levels of strain in different positions and hence different stresses. It is interesting to observe that both the peaks in this cyclic distribution occur on the teeth located close to window edge 2.

In order to double check the analysis, verify if the press fit induced in the model was correct, the weakest position was chosen and an equivalent analysis was carried out in Marc. The model was setup in Marc as explained in the previous chapter, and results of the model were compared with the T3D model.
4.1.3 Transmission 3D versus MSC MARC

Fig. 4.14 shows the stress distribution in both the T3D and the corresponding Marc model (loaded conditions i.e. when the gear loads are present). It can be observed that the distribution of stresses are quite similar and the contact patterns match well on the tooth in complete contact. However there is a difference in the stress values, over the highly stressed zone, between transmission 3D and Marc. The values over the highly stressed zone are 5 – 10% higher in the Marc model than in the actual T3D model. This discrepancy may be attributed to two potential causes:

- **The error in the computation of the mesh position**

  The bearing forces, are obtained from SABR - a software developed by Ricardro plc. SABR calculates the bearing forces as a function of meshing position. Ideally, meshing is the process where the pinion tooth begins to make contact with the gear tooth and finally extinguishes the contact to move on to the adjacent gear tooth. This process usually traces a definitive path - a line in case of cylindrical gears. SABR, however, reduces the meshing position to an equivalent loading point on the tooth that is in full contact, and this point is shifted so as to achieve equilibrium. In the current analysis, the meshing position on SABR and the actual meshing point are offset - by a tooth. Hence, the bearing forces computed are not entirely accurate. These aforementioned reasons may have manifested themselves in Marc model, thus altering the stress values.

- **The inherent difference in the definition of contact, in Marc and T3D.**

![Figure 4.14: Comparison between T3D and Marc models](image)

4.2 Hypoid Face Milled (HFM)

Transmission 3D was used to analyze the entire system with and without press-fit. However, the effect of inclusion of each additional component - rim, pinion shaft, differential cage and bearings could not be determined.

HFM was used to determine how the addition of various components changes the root bending stress value. As described in the previous chapter, HFM does not allow the modeling of press-fit. Once the effect of inclusion of various components was established, the teeth from HFM were imported to Marc and press fit was induced. The results of this were compared with the T3D model in order to determine which among these would be a better method to quantify the effect of press-fit on root bending stresses.

4.2.1 Validation of the model

As done in T3D, a contact pattern comparison was done between the virtual model and the physical test on HFM also. The analysis was run at 500 Nm drive torque.
It can be observed from Fig. 4.16, that the contact pattern from the virtual model in HFM resembles the contact pattern from the physical test. Also, this contact pattern differs from that of the transmission3D contact pattern. The reason for this being, the RMS polynomial fit functions effectively in HFM, keeping the error in the lower limits (3 – 5%). Owing to this, most of the data points from the flank modification files are fit correctly on the curvature of the tooth.

4.2.2 Effect of including different components

This analysis was carried out to check the effect on inclusion of rim, differential cage, pinion shaft and bearings. The analysis was carried out in 3 stages:

- Only teeth - This model just takes into account the teeth on the pinion and the gear. There is no rim, shaft, differential cage or bearings.
- Teeth and rim - This is a small addition to the previous step. Only the pinion and gear rim are included along with the teeth
- Complete setup - This contains all the aforementioned parts.

In all the cases the analyses were carried out at 750 Nm drive torque. The results are summarized in Tab. 4.3. It can be seen that between the only teeth and teeth+rim model there is no significant difference. The inclusion of rim causes a 0.2% increase in stress. Also, it can be observed that in the first two models, the maximum root
Table 4.3: Effect of inclusion of other parts

<table>
<thead>
<tr>
<th>MODEL</th>
<th>MAX. ROOT BENDING STRESS (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Position</td>
</tr>
<tr>
<td>Tooth No.</td>
<td>T21</td>
</tr>
<tr>
<td>Only teeth</td>
<td>1079</td>
</tr>
<tr>
<td>Teeth + Rim</td>
<td>1081</td>
</tr>
<tr>
<td>Complete model</td>
<td>1068</td>
</tr>
</tbody>
</table>

bending stresses are constant at all positions. This is because there is a near constant stiffness in the radial direction and the design is symmetric.

With the addition of the differential cage, pinion shaft and the bearing stiffness matrices, the stresses at different positions change. When compared against the only teeth model:-

- At window edge 2 position there is a 1% decrease in stress.
- At window center position there is a 2% decrease in stress.
- At window edge 1 position there is nearly 2% decrease in stress.
- At pin hole position there is a 0.5% decrease in stress.

The difference in stresses between different positions can be attributed to the inclusion of the differential cage, that has varying stiffness in radial direction. It can be seen from Fig. 4.17 when there is no press-fit and the gear is under load, the stresses at different positions behave the same way. However at no-press fit conditions, stresses in HFM are lower than in T3D. From Fig. 4.17 it can also be observed that, in T3D window edge 2 position has a higher stress than pin hole position, while in HFM it is the vice-versa. The difference in stresses between the pin-hole position and the window edge 2 position, in HFM, is minor (0.5%). The bending stress maps in Fig. 4.18 look similar to that of those from T3D with press-fit. They are similar in the sense that the high levels of stress are concentrated nearly over the same area, at all positions. The level of stresses are however lower than in the case of no press-fit. In Fig. 4.18, the −1 on the x-axis label is indicative of the toe and 1, the heel while the y-axis label 0 − 16 is indicative of position along the root upto the root fillet.

It can be generalized that, although the stresses are at lower levels on HFM than in T3D, the nature of behaviour of these stresses are similar in both the software. This can also be seen from Fig. 4.19, where there is a cyclic variation of maximum root bending stresses when no press-fit is induced (max. root bending stresses only on alternate teeth from tooth 13 to tooth 31 are shown, as computation at each tooth was tedious). The levels of stress are lower than those seen in T3D model with 100µm of press-fit but the nature of behaviour of the stresses are the same. So, with the inclusion of press-fit the stresses must behave in a similar way. Unfortunately owing to large computational time and limited working period, the forces at the teeth in just one position was chosen.
It was decided that if, good correlation was achieved, then a procedure could be adopted to estimate the effect of press fit on the root bending stresses.

To make the comparison easier, the teeth with its forces at window edge 2 position was chosen to be imported from HFM into Marc.

Figure 4.19: Cyclic distribution of stress in HFM
4.2.3 HFM versus MSC MARC

It must be noted that, similar to the T3D analysis, only the teeth with forces at the first time step is imported to Marc.

HFM to corresponding MSC MARC model comparison

It can be observed from Fig. 4.20, the contact pattern and the stress distribution look similar. However, there is a significant error in the magnitude of stresses in the highly stressed zone. The stress levels in Marc model of HFM varies from actual HFM model by 10 − 15%.

One of the main reasons for this is the contact defined in Marc, between the rim and the teeth. The teeth imported from HFM have a highly refined mesh, while the rim itself has a very coarse mesh. This difference in mesh, makes it difficult for Marc to establish a perfect glued contact between the interfaces. This will in-turn affect the stresses on the gear. This can be seen in Fig. 4.21. With a coarse mesh on the rim, the stresses in the highly stressed zone were off by about 20 − 30%. But with a finer mesh, the error is reduced to 10 − 15%, we can observe from Fig. 4.21 that the stress distribution in the model with the refined mesh on the rim, does not have the grey zone at the root (stresses greater than 1200 MPa) anymore. Further refining the mesh in order to close the gap at the glued contact (between rim and teeth), will further lower the stresses. This hypothesis was not tested owing to large computational time and limited working period.

However, the refinement of mesh itself might not give an exact correlation of data. This can be due to the inherent difference in the definition of the contact between the different software.

![Figure 4.20: HFM vs HFM model on MSC MARC: 0µm press fit and 750 Nm drive torque](image)

It has been established that there lies some error when the analysis is moved to Marc, from HFM or T3D. If the rim is re-meshed and the contact gap is extinguished in the HFM model of Marc, then the error in stresses will be minimized. It can also be seen that this error is mostly due to the inherent difference in the definition of contact conditions between Marc and HFM/T3D.

In order to be able to compare press-fit induced HFM model on Marc with the the T3D model, the error in migration of analysis from HFM to Marc must be quantified. In order to do this, the teeth at different positions, at all the 11 times steps at each positions must be imported to Marc. Once imported, the complete model must be setup for all the cases and with identical rim mesh. Then the analysis must be run to check if the error in stresses at the highly stressed zone is constant among all these models for a given mesh of the rim. If it proves to be constant then the error can be accounted for and the press-fit can be induced.

It is also important to carry out a similar analysis on T3D in order to see if the error remains constant over different positions and at different time steps. Only then a comparison between T3D model and the press-fit induced HFM model on Marc would be appropriate.
A press-fit of 100µm was induced in the HFM Marc model. As mentioned before, the comparison between HFM Marc model and the T3D model is not appropriate to be made at this moment. Additionally, it can be observed from Fig. 4.22 that the stress distribution is different from that of T3D, thus indicative of difference in contact pattern between T3D and HFM.

Despite being unable to make the comparison, it is safe to say that the HFM model is closer to reality than the T3D model. This is because of the contact pattern match between the physical tests and the virtual model on HFM. Once the aforementioned analysis is done and the error is estimated then the exact contribution of press-fit can be determined.

Unfortunately, the aforementioned suggestions could not be implemented owing to large computational time and limited working period of the project. If the suggestions are implemented, the model would be perfectly capable of quantifying the effect of press-fit on the root bending stresses of the ring gear.

One cannot help but wonder “What about the pinion?” Although, the press-fit should not have any influence on the root bending stresses of the pinion, the varying stiffness of the differential cage may have some effect.
The results for the pinion were also obtained and can be seen in APPENDIX B of this report. However, it was more relevant for this project work to analyze and discuss the effect of press-fit on the ring-gear.
5. CONCLUSIONS & FUTURE WORK

This chapter draws definitive conclusions from the results that were summarized. It also states what must be done in the future to improve the model.

5.1 Conclusions

This thesis was successful in developing a model in order to determine the effect of press-fit on root bending stresses. However, this model needs to be refined to quantify the contribution of press fit to the root bending stresses. The conclusions are summarized as follows:-

- The root bending stresses increase with increase in press fit dimension.
- The variation of maximum root bending stresses on the ring gear with increasing press fit dimension is linear.
- The maximum root bending stresses show a cyclic variation over 31 teeth of the ring gear. However, the maximum root bending stress values occur at different position on different teeth.
- Zone between window edge 2 and pin hole is the highly stressed zone on the ring gear
- The addition of the rim to the teeth does not largely influence the root bending stress in the FE analysis. However, the addition of the differential cage causes the stresses at different positions to change.
- The behaviour of stresses in the no press fit condition in HFM is similar to that in transmission 3D. However, HFM can be deemed trust worthy owing to the contact pattern match of the virtual model with the physical test.
- Although, the magnitude of stresses are different between HFM and T3D for the same conditions, the behaviour of stress with changing positions are comparable.
- There exists some error when migrating the analysis to MARC from HFM or T3D. Unless this error is quantified correctly, it would not be possible to estimate the exact contribution of press-fit to the root bending stresses.

5.2 Future Work

There is ample scope for refinement of the model in order to quantify the contribution of press fit to the root bending stresses.

- Error check

There exists about 9% increase in stresses in the T3D Marc model when compared to the actual T3D model and a 15% increase in stresses in the HFM Marc model when compared to actual HFM model.

In order to quantify the contribution of press-fit to the root bending stresses, the error in migration of analysis from HFM to Marc and T3D to Marc must be quantified. In order to quantify the error, as discussed in the previous chapter, the teeth with forces from both T3D and HFM at different positions and at all time steps at each positions must be obtained and used to setup the corresponding model for each position, time step on Marc. Then the model must be analyzed so as to see if the error is constant in each time-step and position for HFM/T3D. Once the error in migration of analysis from HFM to Marc and T3D to Marc are quantified, then a meaningful comparison can be made.

- Weld analysis

The weld between the differential cage and the rim has been ignored in the analysis. This weld may contribute to the root bending stresses. In order to confirm this hypothesis, a weld analysis must be carried out.
• Physical testing

Owing to limited time, and all the testing slots being taken, no physical strain gauge measurements could be made in order to validate the virtual analysis. If physical tests are made, then the correctness of the analysis can be easily judged.

• Ease-off Analysis

Efforts can be made to minimize the root bending stresses by finding the optimum ease-off.

• Housing stiffness

The stiffness of the housing has been ignored in this thesis work. In order to refine this model, the housing stiffness must be accounted for. Both HFM and T3D allow the inclusion of housing stiffness in the form of stiffness matrix.

With the aforementioned points taken into consideration, the model can be further refined and the actual contribution of the press-fit to the root bending stresses can be found. The current model built on Marc can be used to carry out fatigue life analysis on FEMFAT. However, it would be more appropriate to use the time-step at which the maximum root bending stresses occur at each position in order to compute the fatigue life.

This thesis was successful in building an FE model to study the influence of press-fit on the root bending stresses of the hypoid gear set in the rear drive unit of a car. Although, the model could not quantify the press-fit’s contribution to the increase in root bending stresses some suggestions were made as to how this can be done.
APPENDIX A

Bearing Forces Computed from SABR

<table>
<thead>
<tr>
<th>Components of Button Bearing Forces</th>
<th>Value (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_x$</td>
<td>-11999</td>
</tr>
<tr>
<td>$F_y$</td>
<td>9980</td>
</tr>
<tr>
<td>$F_z$</td>
<td>7945</td>
</tr>
<tr>
<td>$M_x$</td>
<td>122738</td>
</tr>
<tr>
<td>$M_y$</td>
<td>144439</td>
</tr>
</tbody>
</table>

Table 5.1: Button Bearing forces

<table>
<thead>
<tr>
<th>Components of Gear Bearing Forces</th>
<th>Value (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_x$</td>
<td>-11976</td>
</tr>
<tr>
<td>$F_y$</td>
<td>13233</td>
</tr>
<tr>
<td>$F_z$</td>
<td>-10374</td>
</tr>
<tr>
<td>$M_x$</td>
<td>-178570</td>
</tr>
<tr>
<td>$M_y$</td>
<td>-165431</td>
</tr>
</tbody>
</table>

Table 5.2: Gear Bearing forces

System Parameters

Table 5.3: System Parameters Defined in the Analysis

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Torque</td>
<td>750Nm</td>
</tr>
<tr>
<td>rpm</td>
<td>2000</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.3</td>
</tr>
<tr>
<td>Pinion hand of spiral</td>
<td>left</td>
</tr>
<tr>
<td>Gear hand of spiral</td>
<td>right</td>
</tr>
<tr>
<td>Driver</td>
<td>Pinion</td>
</tr>
<tr>
<td>No. of teeth on Pinion</td>
<td>12</td>
</tr>
<tr>
<td>No. of teeth on ring gear</td>
<td>31</td>
</tr>
</tbody>
</table>
\( \mu \) - The coefficient of friction at the pinion-ring gear teeth interface.

Mesh Statistics

Table 5.4: Ring gear teeth Mesh obtained from T3D and HFM

<table>
<thead>
<tr>
<th>Teeth Model</th>
<th>Elements</th>
<th>Nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>T3D</td>
<td>35712</td>
<td>175584</td>
</tr>
<tr>
<td>HFM</td>
<td>349184</td>
<td>1549008</td>
</tr>
</tbody>
</table>

Table 5.5: Rim mesh statistics before and after refining in order to solve contact issue

<table>
<thead>
<tr>
<th>Rim model</th>
<th>Elements</th>
<th>Nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before refining (default)</td>
<td>3050</td>
<td>3850</td>
</tr>
<tr>
<td>After refining (to solve contact issues)</td>
<td>68600</td>
<td>77600</td>
</tr>
</tbody>
</table>
APPENDIX B

Gear Analysis

Max. root bending stress variation with increasing press-fit interference at low load (0.1Nm)

Max. root bending stress variation with increasing press-fit interference at 750 Nm drive torque
Strain variation with varying press fit interference at 750 Nm drive torque

Max. Root bending stress variation on the ring gear with 100µm press fit at 750Nm drive torque
Variation of stress concentration at different positions

Variation of stress concentration with varying press fit interference at 750Nm drive torque
Pinion Analysis

Pinion root bending stress variation with change in position at varying levels of press fit and 750Nm drive torque

Pinion root bending stress variation with change in press fit levels at different positions and 750Nm drive torque
APPENDIX C

T3D Marc Model vs. Actual T3D model - No load

T3D – 100μm press fit, No Load

Marc model of T3D – 100μm press fit, No Load
T3D model with varying levels of press-fit

Without press-fit

With press-fit 10µm

With press-fit 55µm

With press-fit 100µm

T3D model with 0.1Nm load and varying levels of press-fit

Deformation factor 200X

<table>
<thead>
<tr>
<th>Deformation factor (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAXPPL NORMAL</td>
</tr>
<tr>
<td>9.0000e+002</td>
</tr>
<tr>
<td>5.4000e+002</td>
</tr>
<tr>
<td>3.6000e+002</td>
</tr>
<tr>
<td>2.2500e+002</td>
</tr>
<tr>
<td>1.3500e+002</td>
</tr>
<tr>
<td>9.0000e+001</td>
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<tr>
<td>2.2500e+001</td>
</tr>
<tr>
<td>1.3500e+001</td>
</tr>
<tr>
<td>7.2000e+000</td>
</tr>
<tr>
<td>0.0000e+000</td>
</tr>
</tbody>
</table>
Contact status check

- Glued contact between rim and teeth
- Press-fit zone
- Weld zone
- Axial touch (Button bearing)
- Axial touch (Gear bearing)
- Radial press-fit (bearings)
Bibliography


