Numerical simulations of Carbon Fiber Reinforced Polymers under dynamic loading

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Abstract

The ability to withstand dynamic loading represents an important design criteria for crucial applications such as those adopted in the automotive and aerospace industries. Numerical simulations can lead to a reduction of time and costs for designing composite structures by replacing testing campaigns that are performed in order to assess whether the design requirements of the structure are met.

The present thesis deals with the development of a robust simulation methodology within the FE explicit commercial code PAM-CRASH in order to predict the damage behaviour of Carbon Fiber Reinforced Polymers when loaded dynamically. The strain rate dependence of the carbon/epoxy composite under study is identified and a material-characteristic strain rate model is developed starting from experimental data. A delay damage model based on a Continuum Damage Mechanics approach is used to predict the response of composite laminates under dynamic loading. The simulation methodology is validated against experimental data from a patch to a coupon level by using solid elements to model the plies of the laminate. A dynamic three-point bending simulation is performed at the sub-component level by modelling the composite structure through the use of solid elements for the plies and cohesive elements for the interfaces between them.

Rather good agreement is found in terms of stiffness and strength between the results from the numerical simulations and those obtained from the experimental tests. Limitations are identified in the sensitivity of the strain rate model to the damage limits set to stop the scaling of the lamina elastic moduli and in severe dynamic effects, e.g. stress waves, which affected the simulations at high strain rates.
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## 5 Results and Discussion

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Chapter 1

Introduction

1.1 Background

The growing demand for lighter structures has resulted in the increasing replacement of metals by composite materials in the past years. This trend has mainly involved the aerospace and automotive industry. In particular, the total global consumption of composite materials used in transportation equipments has increased by 5.7% between 2006 and 2011 [1]. Reduced structural weight results in improved structural efficiency and lower environmental impact. However, other applications such as civil and maritime engineering as well as high-performance sport goods have also been subjected to the above trend.

Composites have many potential advantages over traditional materials (e.g. metals) due to their structural constitution: a reinforcing material (typically continuous in case of structural applications) is combined with a relatively compliant matrix material (a resin) which transfers the load between the reinforcements and protects them from damage [2]. Composites are usually built up of separate thin layers of fibers and matrix, called ply or lamina. A lamina with only one orientation of the fibers is called unidirectionally (UD) reinforced. Often laminae of different fiber orientations are stacked together to form a laminate. The stacking sequence of plies has to be optimised thus giving the laminate the desired stiffness and strength for a given application. Composite materials present high strength and stiffness to weight ratio, good fatigue performance and excellent corrosion resistance [3]. In addition, composites also offer advantages in terms of noise and vibration reduction, impact resistance and energy absorption capabilities. Examples of successful substitutions of metal components by composites structures, especially Carbon Fiber Reinforced Polymers (CFRP), in the civil aeronautic industry are the newly developed Airbus A350 XWB and Boeing 787 Dreamliner, for which more than 50% of the structural weight consists of composites [4], as shown in Figure 1.1. Composite materials have been introduced in the automotive industry, e.g. in the BMW i3, which is featuring a CFRP passenger cell, roof and interiors. Figure 1.2 shows the CFRP passenger cell of an early version of the BMW i3 undergoing internal crash test evaluations.
The reduction of weight offered by the use of composite structures must not compromise safety and crashworthiness. In particular, the ability to withstand dynamic loading is an important design criteria in case of crucial applications such as those introduced above. Classic dynamic loading scenarios are impacts and crashes. Moreover, other events such as dropped tools, handling accidents and hail impacts might also result in relevant dynamic solicitations of the structures.

Thus, correctly understanding and modelling the mechanical behaviour of composites in case of dynamic loading results to be of fundamental importance in order to ensure the development of structures compliant with the safety regulations. In particular, composite materials present complex damage and failure mechanism which have not been completely understood when compared to those of metals. Moreover, composites are known for their strain rate dependent material behaviour, which depends on the material constituents. A characterisation of the strain rate dependence of their mechanical properties is therefore crucial in order to model...
Numerical tools, such as Finite Element (FE) analyses, are still being developed in order to reduce the development costs and time needed for the introduction of new composite structures. The use of numerical simulations, in fact, could decrease the penalties coming from extensive physical test campaigns and enhance the performance and safety of the finished products. However, the numerical methodologies implemented in the past years are often insufficient to realistically predict the mechanical response and damage resistance of composite materials subjected to dynamic loading.

1.2 Objective of the thesis

The objective of the following thesis work is to implement and validate within the explicit FE commercial code PAM-CRASH a robust simulation methodology in order to predict the damage behaviour of CFRP under dynamic loading scenarios. In particular, the material under study is a high-performance unidirectionally reinforced carbon fiber/epoxy composite. It is worth noting that the industrial name of the material is omitted from the text due to confidentiality reasons through the use of the following acronym: MATCFE (MATerial Carbon Fiber Epoxy).

Reaching the above objective requires the following tasks to be addressed within the thesis:

- identification of the material strain rate dependent behaviour from experimental tests and development of a strain rate model
- implementation of the above strain rate model into the explicit FE commercial code PAM-CRASH
- investigation of the delay damage effect which predict the propagation of damage inside the material under dynamic loading and identification of the parameters that govern the model associated to the above effect
- validation of the strain rate and delay damage model with respect to experimental tests

1.3 Layout of the thesis

The present thesis is organised as follows. Chapter 2 provides a literature review of the published work regarding the strain rate dependence of CFRP and numerical simulations of composite laminates under dynamic loading. The theoretical and mathematical background on which this thesis work is based is presented in Chapter 3. Focus is placed on the modified Ladevèze damage model for the elementary ply of composite laminates under quasi-static loading scenarios, the formulation of the strain rate model for MATCFE and the delay damage model for composites under dynamic loading. Chapter 4 provides a chronological description of the
procedure used to reach the objective of the thesis presented in Section 1.2. The procedure performed for the development of the strain rate model and its implementation in the FE commercial code PAM-CRASH are described. Moreover the effect of the delay on the damage evolution inside the plies of the laminate when loaded dynamically is investigated and the results validated against experimental data. A sub-component level simulation is carried out to evaluate the effect of the strain rate and delay damage models on the response of the laminate. The results of the work carried out are presented and discussed in Chapter 5. Finally, Chapter 6 provides the main conclusions and suggestions for further research.
Chapter 2

Literature review

The following chapter covers a review of the current theoretical and methodological contributions to numerical simulations of composites focusing on their dynamic strain rate dependent behaviour. The review is carried out in the form of a synthesis and analysis of relevant published works. The work of several authors on the above topic is compared and discussed in order to present an overview of the state of the art of dynamic simulations of composite materials, with focus on CFRP.

2.1 Strain rate dependence of CFRP: experimental assessment

In their entire life cycle, composite structures might be subjected to impact and crash events which cause high speed deformations of the structures. These phenomena are of particular importance mainly for the automotive and aerospace industry. A deep understanding of the dynamic behaviour of the materials selected for such applications is therefore fundamental to ensure a reliable design of the structures that should not fail prematurely at high loading rates, fulfilling the strict requirements on crashworthiness and safety. The above argument underlines the need for a dynamic characterisation of CFRP to understand the strain rate effects on their mechanical properties. Partly due to experimental difficulties, only a few studies on the strain rate dependence of the mechanical properties of carbon fiber/thermoset polymer composites are present in the literature. High-speed mechanical testing of composite laminates arises specific difficulties related to inertial effects, non-uniform stress-strain distributions within the specimen, strain measurement technique and clamping mechanism.

Most of the work that has been carried out has employed servo-hydraulic machines and split Hopkinson pressure bars for testing at intermediate and high strain rates. Figure 2.1 shows the load train of a servo-hydraulic machine. It consists of a load washer, shoulder grips and a slack adaptor, which includes a hollow tube and a sliding bar. A tensile specimen is placed into the shoulder grips. The dynamic load is introduced to the lower grip through the slack adaptor. When the machine is
actuated, the hollow tube travels freely with the hydraulic head over a distance to reach a predefined speed before coming into contact with the cone-shaped surface of the sliding bar. A layer of damping tape is placed at the cone-shaped surface of the bar to create a damper. The slack adaptor eliminates the inertial effect of the lower portion of the load train in its acceleration stage. However, the sudden engagement with the upper portion of the load train can generate a high amplitude stress wave, causing oscillations at the natural frequency of the system, i.e. system ringing. The inertial responses of the grips and load cells increase with the test speed and may obscure the correspondent outputs, causing their analysis to be difficult and inaccurate due to non-uniformity of stress and strain within the specimen [7]. To reduce inertial effects, lightweight grips are recommended in case of dynamic tensile tests [8]. A tensile split Hopkinson bar, instead, consists of a compressed air device, an input (or incident) bar, an output (or transmission bar) and a data acquisition system. The specimen is placed between the ends of the two bars, as shown in Figure 2.2. The compressed air device impels a striker against one end of the incident bar generating a tensile stress pulse that travels along it towards the specimen. When the pulse reaches the specimen, it splits into two smaller waves, one of which is reflected back into the incident bar, while the other is transmitted through the specimen and the transmission bar. The wave travelling through the system triggers data collection from the strain gauges located on both the incident and transmission bars. Stress, strain, and strain rate are then calculated from the data collected by the strain gauges. However, during the initial stages of loading, the stress in the specimen is non-uniform (axially), which may lead to inaccuracies.

Figure 2.1. Load train of a servo-hydraulic machine [8]
in the early part of the stress-strain curve. This is due to the propagations and reflections of the stress weaves within the specimen and errors in the time shifting of the strain gauges signals [9]. In order to achieve a dynamic stress equilibrium within the specimen, which is an essential condition for validating the acquired data from dynamic tests, the stress wave should travel back and forth inside it a minimum number of times before failure occurs. However, it should be noted that there is no quantitative criterion yet to define the exact number of travels needed to assure a dynamic stress equilibrium. Gray [10] suggested a minimum number of three when performing high-strain rates testing on metals. Xiao [8] verified that the above criterion is also applicable to dynamic tensile tests on plastic materials by using servo-hydraulic machines. Moreover, bar-specimen interface friction might also cause critical inaccuracies in the collected data due to unexpected transverse deflections of the Hopkinson bars.

Zhang et al. [11] experimentally investigated the unidirectional tensile properties of carbon fiber/epoxy laminates from quasi-static to intermediate strain rates. Quasi-static and low-speed tensile tests were conducted at strain rates varying from $10^{-5}$ to $0.07 \, s^{-1}$ using an Instron hydraulic machine. Intermediate-speed tests were instead performed on an Instron high-speed servo-hydraulic machine at strain rates from 10 to $240 \, s^{-1}$. It was concluded that the tensile strength and stiffness in the fiber direction were insensitive to the loading speed when the strain rate was less than $50 \, s^{-1}$. However, when the strain rate was over $50 \, s^{-1}$, both the tensile strength and stiffness in the fiber direction increased with the strain rate.

The tensile mechanical properties of UD carbon fiber/epoxy prepreg laminates were also tested by Taniguchi et al. [12] at quasi-static and intermediate strain rates up to $100 \, s^{-1}$ by means of a tensile split Hopkinson pressure bar. The experimental results demonstrated that the tensile modulus and strength in the longitudinal direction were independent of the strain rate. In contrast, the above properties increased with the strain rate in the transverse and shear directions. Moreover, it was observed that the strain rate dependence of the shear strength was more pronounced than that of the transverse strength, with values up to 19% for the former and 78% for the latter over the respective static ones.

Al-Zubaidy et al. [13] experimentally determined the tensile mechanical properties of CFRP sheets at quasi-static and intermediate strain rates. Quasi-static tensile tests were performed on carbon fiber/epoxy specimens at the strain rate of...
about $10^{-4} \text{ s}^{-1}$ using an Instron hydraulic machine. Impact tensile experiments at strain rates up to $87.4 \text{ s}^{-1}$ were instead carried out utilising a drop-mass rig. It was found that the tensile properties of CFRP in the fiber direction were strain rate dependent. In particular, increased strain rates led to increased longitudinal tensile strength, tensile stiffness, strain to failure and energy absorption, even though their general trends and percentages of enhancement differed.

The dynamic response of carbon fiber/epoxy composites at higher strain rates was evaluated by Daniel et al. [14] by using two different test methods. In the first test method, thin carbon/epoxy laminates were dynamically tested in tension under longitudinal, transverse, and in-plane shear loading at strain rates up to $500 \text{ s}^{-1}$. In the longitudinal direction the modulus increased moderately with the strain rate (up to 20% over the static value), but the strength and ultimate strain did not vary significantly. The modulus and strength increased sharply over the static values in the transverse direction but the ultimate strain only increased slightly. There was a 30% increase in the in-plane shear modulus and strength. A second test method was used for dynamic testing of thin laminates in compression. Longitudinal properties were obtained up to a strain rate of $90 \text{ s}^{-1}$. Within that range, the longitudinal modulus increased with strain rate (up to 30% over the static value), however the strength and ultimate strain were equal to or a little lower than static values. The dynamic modulus and strength at $210 \text{ s}^{-1}$ increased sharply over static values in the transverse direction while the ultimate strain was lower than the static one.

Melin and Asp [15] investigated the dependence on strain rate of the mechanical properties of a high performance carbon fiber/epoxy composite loaded in transverse direction at strain rates between 100 and $800 \text{ s}^{-1}$. The dynamic tests were performed in a split Hopkinson bar. Moreover, a moiré technique combined with high-speed photography, at framing rates of $0.25 – 1 \text{ MHz}$, was used for extraction of the local strain fields. The transverse mechanical properties were found to have weak or no dependence on the strain rate. Moreover, the average transverse modulus did not depend on strain rate, whereas the strain to and stress at failure were found to increase slightly with increased strain rate.

High strain rates ($750 – 900 \text{ s}^{-1}$) were tested on UD and quasi-isotropic carbon/epoxy laminates by Gómez-del Río et al. [16]. Tensile properties at room ($20 ^\circ \text{C}$) and low temperature ($-60 ^\circ \text{C}$) were evaluated by using a modified split Hopkinson bar. The results of the dynamic tests showed little influence of strain rate and temperature on the tensile strength of unidirectional laminates loaded in the fiber direction. In contrast, the strength increased up to 110% over the static value in the transverse direction at high strain rate. Moreover, a slight increase with the strain rate was observed in the tensile strength of the quasi-isotropic laminates. However, the temperature resulted to have little effect on their in-plane dynamic properties.

Gilat et al. [17] studied the strain rate dependent behaviour of IM7/977-2 carbon/epoxy matrix composite in tension from quasi-static to high strain rates. Tensile tests were conducted with a hydraulic machine at quasi-static strain rates of approximatively $10^{-5} \text{ s}^{-1}$ and intermediate strain rates of $1 \text{ s}^{-1}$. High strain rate tests were instead performed by using the tensile split Hopkinson bar technique at
2.1. STRAIN RATE DEPENDENCE OF CFRP: EXPERIMENTAL ASSESSMENT

strain rates ranging from approximatively 400 to 600 s\(^{-1}\). Resin and carbon/epoxy composite specimens with layups of 90°, 10°, 45° and [±45°] were used for the tests. Overall, the experimental results showed that the strain rate significantly affected the response of the carbon/epoxy system selected. In all of the configurations tested, higher stiffness was observed with increasing strain rate. Only a small increase in the maximum stress with increasing strain rate was registered in the tests with the resin, the 90°, and the 10° specimens. The tests conducted on the 45° and [±45°] specimens showed instead a more significant effect of the strain rate on the strength. Moreover, the maximum strain at all strain rates in the tests with the latter specimens was much larger than in all the other types of specimens.

The results of the works found in literature and summarised in Table 2.1 are ambiguous. Comparisons between those are difficult since variations in fiber volume fractions and materials are prone to influence the assessed strain rate dependencies, especially in the matrix-dominated properties, i.e. transverse and shear ones. Moreover, no standard methodology is currently available for dynamic tensile testing of composite materials, resulting in the use of different techniques by different authors, which makes complicated a generalised evaluation of the results obtained experimentally. In particular, high strain rate testing arises severe difficulties in the execution and data interpretations. Important factors such as specimen size and geometry, wave propagations, inertial effects, loading devices, load measurement, system ringing, clamping mechanisms and strain measurement techniques may strongly affect the data collected from the tests, leading to inaccurate results.

A rather common pattern can however be found in most of the works previously described: CFRP sensitivity to strain rate seems to be driven by the viscous behaviour of the matrix. In particular, transverse and shear stiffness and strength generally increase with increasing strain rate. Thus, although those trends might be recursive for CFRP, a complete characterisation of the mechanical properties of each specific composite is necessary in order to correctly identify their strain rate dependence.
Table 2.1. Summary of published data about the effect of strain rate on the mechanical properties of CFRP laminates under tension loading

<table>
<thead>
<tr>
<th>Reference</th>
<th>Range of strain rates tested $[s^{-1}]$</th>
<th>Fiber direction</th>
<th>Transverse direction</th>
<th>Shear direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Stiffness</td>
<td>Strength</td>
<td>Stiffness</td>
</tr>
<tr>
<td>Zhang et al. [11]</td>
<td>$10^{-5} - 240$ (from $50 \ s^{-1}$)</td>
<td>$\uparrow$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Taniguchi et al. [12]</td>
<td>$0 - 100$</td>
<td>=</td>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
</tr>
<tr>
<td>Al-Zubaidy et al. [13]</td>
<td>$10^{-4} - 87.4$</td>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
<td>-</td>
</tr>
<tr>
<td>Daniel et al. [14]</td>
<td>$10^{-4} - 500$</td>
<td>$\uparrow$</td>
<td>=</td>
<td>$\uparrow \uparrow$</td>
</tr>
<tr>
<td>Melin and Asp [15]</td>
<td>$100 - 800$</td>
<td>-</td>
<td>-</td>
<td>$\uparrow \uparrow$</td>
</tr>
<tr>
<td>Gómez-del Río et al. [16]</td>
<td>$750 - 900$</td>
<td>-</td>
<td>=</td>
<td>-</td>
</tr>
<tr>
<td>Gilat et al. [17]</td>
<td>$10^{-5} - 600$</td>
<td>-</td>
<td>$\uparrow$</td>
<td>$\uparrow \uparrow$</td>
</tr>
</tbody>
</table>

(=) = no variation with respect to static value  
$\uparrow$ = increase with respect to static value  
$\uparrow \uparrow$ = high increase with respect to static value  
- = data not available
2.2 Numerical simulations of composite laminates under dynamic loading

In literature, numerical simulation of composite materials under dynamic loading is a well-known topic comprising various difficulties, e.g. correct modelling of intralaminar ply failure, such as fiber rupture, matrix/fiber debonding, matrix microcracking and inter-laminar delamination with respect to damage initiation and evolution. Interface delamination is probably the most critical and insidious failure mechanism, since it may severely degrade the strength and integrity of the structure. Several modelling methodologies are applied in literature in order to correctly simulate dynamic scenarios and the resulting damage mechanisms that occur within the material. Explicit FE-solvers are generally used to simulate those scenarios, being able to successfully handle large deformations and non-linear behaviours due to damage evolution.

Schueler et al. [18] simulated high velocity blunt impact on flat toughened carbon/epoxy prepreg plates by using Abaqus/Explicit solver. The modelling of the material consisted in stacked shell elements for the plies to capture intra- and inter-laminar damage as well as their interrelations and cohesive interface layers for the interface between them to simulate delamination. In particular, the Ladevèze continuum damage mesoscale model was implemented for the plies. However, no material strain rates effects were included in the numerical method. The model was validated with experimental data from high velocity tests. As a result, it was concluded that the evolution of intra- and inter-laminar damage with increasing impact energy was well captured, while the size of the damage zone was not.

A similar approach was adopted by Waimer et al. [19] to simulate CFRP components made of woven fabrics subjected to crash loads. Stacked shell elements were used to represent the plies and cohesive elements for representation of delamination failure between the plies. A damage evolution law in the fiber direction based on fracture energies as well as damage and plasticity laws in the shear direction were implemented for the plies. A cohesive zone modelling approach was instead used for the cohesive elements. Similarly to Schueler et al. [18], the strain rate dependence of the material was not implemented in the model. Validation of the numerical approach was performed on the basis of an extensive test program of CFRP crush absorbers on the structural level. In conclusion, the validation of the simulation method identified an acceptable agreement of the simulation with the test results over a wide range of design and loading parameters, yet without capturing all failure effects in detail.

Johnson [20] used stacked shell elements for the plies and TIED slide lines for the interface to simulate high velocity impact on UD and fabric carbon/toughened epoxy laminates in the dynamic explicit solver PAM-CRASH. Damage and plasticity models were implemented for the plies while interface damage models were
implemented using fracture mechanics principles. The model focused also on the strain rate dependence of the material. Rate dependent failure modes were included based on rate dependent semi-empirical scale functions applied to several mechanical properties. The FE modelling and simulations were validated against structural impact tests performed on composite plates at different loading speeds resulting in good agreement of impact failure modes in both low and high velocity impacts.

A different approach was used by Feng et al. [21] to predict the structural response and failure mechanisms of graphite/epoxy composite laminates subjected to low-velocity impacts: solid and cohesive elements with zero thickness were selected for the plies and the interface between them respectively. Intra-laminar damage models based on energy-based continuum mechanics and inter-laminar damage laws were implemented in Abaqus/Explicit in order to capture the planar extent and the detailed through the thickness distribution of damage induced by impacts. The model did not make use of any material constitutive law with the strain rate dependence of the mechanical properties implemented in it. Validation of the numerical methodology was conducted by comparison between FE simulations and experimental data obtained by drop-weight tests and stereoscopic X-radiography. The developed FE model provided a correct prediction of the structural impact response of laminated samples over the range of impact energies examined. Rather good agreement was also achieved in terms of shapes, orientations and sizes of delamination induced by impact at the different interfaces.

Duodu et al. [22] developed a three-dimensional rate dependent damage constitutive model for simulating the response of cross-ply Kevlar/epoxy composite laminates subjected to high-velocity impacts. The model was implemented in Abaqus/Explicit and consisted of cohesive interface elements inserted between adjacent plies with different orientations, which were modelled with eight-node linear solid elements. The simulation methodology was validated against existing experimental data found in literature. It was concluded that the model was able to effectively simulate the fiber failure and delamination behaviour under high strain rates. On the other hand, the methodology implemented was not able to exactly predict the size of the damage zone.

Thus, it appears evident that all the methodologies described above and summarised in Table 2.2 present one aspect in common: the modelling of the laminate is performed by using different elements for the plies and for the interfaces between them in order to correctly capture the different types of damage that occur within the plies and at their interfaces, i.e. intra- and inter-laminar damage respectively. Moreover, while cohesive elements result to be the most common approach adopted to model the interfaces, solid and shell elements are almost equally used for representation of the plies. Solid elements allow to predict through the thickness stresses and strains and may be more suitable than stacked shell elements for simulating the damage evolution in the plies. On the other hand, they require a higher computational cost which can make the simulations extremely expensive in terms of time and CPU demand.

All the methodologies previously described are claimed to be able to effectively
2.2. NUMERICAL SIMULATIONS OF COMPOSITE LAMINATES UNDER DYNAMIC LOADING

represent, with different degrees of accuracy, the structural impact response of the laminate and the evolution of the damage within it. However, none of those can correctly predict the size and shape of the damage zone due to impact. Moreover, most of the modelling approaches do not take into account any rate-dependent effect on the mechanical properties of the material as well as on the evolution of the damage, which seems to be a fundamental point in case of a highly dynamic process.
<table>
<thead>
<tr>
<th>Reference</th>
<th>Modelling strategy</th>
<th>Strain rate dependence implemented</th>
<th>Results</th>
</tr>
</thead>
</table>
| Schueler et al. [18]| Stacked shell elements | Cohesive elements | No | Evolution of damage well captured  
|                    |                    |                                    | Imprecise size of damage zone                   |
| Waimer et al. [19] | Stacked shell elements | Cohesive elements | No | Acceptable agreement of simulations with test results  
|                    |                    |                                    | Damage not captured in detail                    |
| Johnson [20]       | Stacked shell elements | TIED slide lines | Yes | Good agreement of failure modes at low and high velocity  
|                    |                    |                                    | Correct prediction of the structural impact response |
| Feng et al. [21]   | Solid elements     | Cohesive elements | No | Acceptable agreement in terms of shapes, orientations and sizes of delamination  
|                    |                    |                                    | Good agreement in terms of fiber failure and delamination behaviour |
| Duodu et al. [22]  | Solid elements     | Cohesive elements | Yes | Imprecise size of damage zone |
Chapter 3

Theory

The following chapter covers the theoretical and mathematical background on which this thesis work is based. The different scales adopted to model composite laminates are firstly introduced. The mechanics of damage for composites under quasi-static loading is then described by focusing on the two main types of damage that might occur in a laminate, i.e. intra- and inter-laminar damage. Finally, a delay damage model for composite laminates is presented in order to model their behaviour in dynamic scenarios.

3.1 Scale modelling for composite laminates

When dealing with composite laminates, a fundamental aspect is represented by the scale adopted to analyse them, since that would impose the level at which the calculations should be performed. The analysis of a laminate made of UD plies can be carried out on three different levels, as shown in Figure 3.1. On the macrolevel, the composite is considered as a single homogeneous material whose properties are obtained with through the thickness homogenisation, e.g. by using Classical Lamination Theory. The laminate is instead analysed in the greatest detail on the microlevel, where distinction is made between fibers and matrix. Between those

![Figure 3.1. Three scales of analysis for composite laminates made of UD plies](image-url)
levels, there is an intermediate modelling scale called the mesolevel. This level is associated with the thickness of the layer and the thickness of the different interlaminar interfaces.

A macrolevel modelling approach results to be limited in case of dynamic simulations which involve the development of damage mechanisms within the laminate, such as matrix microcracking, fiber/matrix debonding, fiber rupture and delamination. In fact, the concept of through the thickness homogenisation only holds when cracks propagate through the thickness, which is not the case of delamination. Microlevel analyses instead allow to precisely capture the mechanical behaviour of composites during damage. However, difficulties in the determination of the material micromechanical properties and severe limitations with respect to computational costs arise in this case.

In the following thesis work, laminate analyses are performed at the mesolevel, where the main damage mechanisms listed above appear nearly uniform throughout the thickness of each constituent. In particular, the mesomodel is defined by means of two mesoconstituents, as shown in Figure 3.2: a single layer, which is assumed to be homogeneous and orthotropic and an interface, which is a mechanical surface connecting two adjacent layers and which depends on the relative orientation of the fibers.

3.2 Damage mechanics of composite laminates

Damage in fiber-reinforced composites can be caused by many different sources that include static and fatigue loading, low and high energy impact, crash as well as environmental factors such as moisture absorption and corrosion.

In order to successfully design a composite structure it is fundamental to correctly identify the different types of damage that occur within it and how they develop. Damage in fiber-reinforced composites can be divided into two main types: intra- and inter-laminar damage depending on whether the damage occurs within a ply or at the interface between two adjacent plies. Intra-laminar damage involves several damage mechanisms depending on different loading situations. When loaded in tension along the fiber direction, a unidirectional lamina can exhibit fibers fragile rupture due to their brittle behaviour (Figure 3.3(a)) and matrix microcracking which generates cracks perpendicular to the fibers. The latter mechanism can occur also in tension transverse to the fibers but in this case with cracks that propagate
3.2. DAMAGE MECHANICS OF COMPOSITE LAMINATES

Figure 3.3. Intra-laminar damage mechanisms in tension and compression [25]: (a) fiber fragile rupture, (b) matrix microcracking, (c) fiber/matrix debonding, (d) fiber buckling along the fiber direction (Figure 3.3(b)). Fiber/matrix debonding (Figure 3.3(c)) represents another dominant damage mechanism in this loading scenario and it consists in a decohesion between fiber and matrix at the interface. Thus, for a unidirectional lamina tensile failure in the fiber direction is mainly governed by the strength of the fibers, while in transverse direction by the matrix properties. In compression, the damage mechanisms are quite different than in tension. Individual fibers of fiber bundles tend to buckle as the load increases leading to fiber microbuckling (Figure 3.3(d)) or eventually to buckling failure.

Inter-laminar damage, or delamination, instead consists in the propagation of a crack along the bonding interface between two adjacent plies due to the development of inter-laminar stresses under in-plane compression, through the thickness tension or shear. Inter-laminar stresses may also be caused by the so-called "free edge effect", which mainly occurs in cross ply (e.g. [0/90]_s) and angle ply laminates, i.e. [±θ]. In the former, a Poisson mismatch between the layers causes the development of a inter-laminar shear stress and a transverse stress in order to ensure equilibrium, with the transverse one approaching infinity at the lamina free edges. On the other hand, the latter laminates are affected by a mismatch in the in-plane shear which causes the development of a inter-laminar normal stress whose magnitude tends to infinity at the free-edges. Figure 3.4 shows the delamination damage at the interface between +45° and 90° layers in a CFRP composite subjected to impact.

Intra- and inter-laminar damage mechanisms may develop simultaneously and may be coupled within a composite laminate under static or dynamic loading making it difficult to correctly predict not only their onset but also their evolution.
and magnitude. An accurate modelling of these two types of damage is therefore necessary in order to obtain reliable results from numerical simulations.

3.2.1 Intra-laminar damage modelling

A Continuum Damage Mechanics (CDM) approach is used in this thesis work to model the intra-laminar damage in composite laminates. Namely, CDM is a branch of continuum mechanics used to describe the damage and fracture process ranging from the initiation of microcracks to the final fracture in the material caused by the development of macrocracks [27]. The procedure on which Continuum Damage Mechanics is based consists in the representation of the damage state of the material in terms of properly defined damage variables. A characterisation of the mechanical behaviour of the damaged material and the further development of the damage is then performed by the use of the above variables and thermodynamic forces. In particular, a modification of the Ladevèze damage model [28] of the elementary ply for laminated composites under quasi-static loading is adopted. CDM theory is used to describe the onset and evolution of three main damage mechanisms: fiber fragile rupture, matrix microcracking and fiber/matrix debonding. Moreover, three scalar

![Figure 3.4. Delamination between $+45^\circ$ (top) and $90^\circ$ (bottom) layers caused by impact [26]](image1)

![Figure 3.5. Stiffness degradation of the material due to damage (adapted from [23])]
3.2. DAMAGE MECHANICS OF COMPOSITE LAMINATES

damage variables, assumed to be constant throughout the thickness of the ply, are introduced: \( d_1 \), associated with cracks orthogonal to the fiber direction representing their fragile failure; \( d_2 \) and \( d_{12} \) associated with cracks parallel to the fibers, representing the last two damage mechanisms taken into account by the model, i.e. matrix microcracking and fiber/matrix debonding respectively. The above damage variables are linked to the material stiffness reduction as shown in Figure 3.5: the development of the damage within the ply leads to a stiffness decrease which occurs until complete failure. Moreover, the modified Ladevèze model takes the differences between the tension and compression in the fiber direction into account.

The strain energy density \( E_{3D}^{d} \) of the 3D damaged ply is expressed in the following form:

\[
E_{3D}^{d} = \frac{1}{2} \left[ \frac{\sigma_{11}^2}{E_1^0(1 - d_1)} - 2\frac{\nu_{12}^0}{E_1^0} \sigma_{11} \sigma_{22} - 2\frac{\nu_{12}^0}{E_1^0} \sigma_{11} \sigma_{33} + \frac{\langle \sigma_{22} \rangle_+^2}{E_2^0(1 - d_2)} + \frac{\langle \sigma_{22} \rangle_-^2}{E_2^0} \\
- 2\frac{\nu_{23}^0}{E_2^0} \sigma_{22} \sigma_{33} + \frac{\langle \sigma_{33} \rangle_+^2}{E_3^0(1 - \lambda d_2)} + \frac{\langle \sigma_{33} \rangle_-^2}{2E_3^0} \right] + \frac{\sigma_{12}^2}{G_{12}^0(1 - d_{12})} + \frac{\sigma_{13}^2}{2G_{13}^0(1 - \lambda d_{12})} + \frac{\sigma_{23}^2}{2G_{23}^0(1 - \lambda d_{12})} \right]
\]

(3.1)

where \( \langle \cdot \rangle_+ \) and \( \langle \cdot \rangle_- \) represent a tensile and compressive stress respectively. The superscript "0" above each lamina elastic \( E_i \) and shear \( G_{ij} \) modulus represents their respective value when the damage in the lamina is equal to zero. Moreover, the parameter \( \lambda \), which is equal to 1 or 0, allows either taking into account the damage associated to through the thickness stresses either disregarding it. By assuming a plane stress state, a 2D expression of the strain energy density is determined:

\[
E_{2D}^{d} = \frac{1}{2} \left[ \frac{\sigma_{11}^2}{E_1^0(1 - d_1)} - 2\frac{\nu_{12}^0}{E_1^0} \sigma_{11} \sigma_{22} + \frac{\langle \sigma_{22} \rangle_+^2}{E_2^0(1 - d_2)} \\
+ \frac{\langle \sigma_{22} \rangle_-^2}{E_2^0} + \frac{\sigma_{12}^2}{G_{12}^0(1 - d_{12})} \right]
\]

(3.2)

Thus for 2D problems, the damage energy release rates associated with \( d_1 \), \( d_2 \) and \( d_{12} \) are expressed as follows:

\[
\begin{align*}
Y_1 &= \frac{\partial E_d}{\partial d_1} |_{\sigma} = \frac{\sigma_{11}^2}{2E_1^0(1 - d_1)^2} \\
Y_2 &= \frac{\partial E_d}{\partial d_2} |_{\sigma} = \frac{\langle \sigma_{22} \rangle_+^2}{2E_2^0(1 - d_2)^2} \\
Y_{12} &= \frac{\partial E_d}{\partial d_{12}} |_{\sigma} = \frac{\sigma_{12}^2}{2G_{12}^0(1 - d_{12})^2}
\end{align*}
\]
The associated damage energy release rates $Y_1$, $Y_2$ and $Y_{12}$ are analogous to thermodynamic forces and they govern the damage development just as energy release rates govern cracks propagation. The evolution of damage with respect to the thermodynamic forces under static loading is given by:

\[
\begin{align*}
    d_1 &= \begin{cases} 
    0 & \text{if } Y_1 \leq Y_{t1} \text{ and } Y_1 \leq Y_{c1} \\
    1 & \text{if } Y_1 > Y_{t1}, \sigma_{11} > 0 \text{ or } Y_1 > Y_{c1}, \sigma_{11} < 0 
    \end{cases} \\
    d_2 &= \begin{cases} 
    b_3 d_{12} & \text{if } d_{12} < 1, d_2 < 1, Y_2 < Y_s^2 \\
    1 & \text{otherwise}
    \end{cases} \\
    d_{12} &= \begin{cases} 
    f(\sqrt{Y}) & \text{if } d_{12} < 1, d_2 < 1, Y_{12} < Y_{12}^s \\
    1 & \text{otherwise}
    \end{cases}
\end{align*}
\]  

(3.4)

where $Y = b_2 Y_2 + Y_{12}$ is an equivalent thermodynamic force defined as a combination of the effects in shear and in the transverse direction. The static identification of the evolution law (i.e. the identification of $f$) and the material parameters ($Y_{t1}$, $Y_{c1}$, $Y_s^2$, $Y_s^{12}$, $b_2$ and $b_3$) is carried out by tests performed on the macroscale in tension-compression on different stacking sequence of the laminate [28]. Then, Classical Lamination Theory is used to obtain information on the elementary ply scale. As from Eq. (3.4), the damage variables vary from 0 (undamaged state) to 1 (fully damaged state). Moreover, the behaviour in the fiber direction is assumed to be independent of the transverse and shear one. On the contrary, the model introduces a coupling between the evolution of $d_2$ and $d_{12}$ through the parameters $b_2$ and $b_3$, being on the average both associated with the same type of cracks. Figure 3.6 shows the evolution of the three damage variables and the coupling between the damage in transverse and shear direction. In particular, a sudden failure occurs along the fibers direction due to their brittle behaviour at a damage energy release rate value that equals the critical one in tension or compression, i.e. $Y_{t1}$ or $Y_{c1}$ (Figure 3.6(a)). On the contrary, the damage associated with cracks along the fiber direction, such as matrix microcracking and fiber/matrix debonding exhibits an initial non-linear development followed by sudden failure when the thermodynamic forces in the above directions reach their critical value, i.e. $Y_s^2$ and $Y_s^{12}$ respectively (Figure 3.6(b) and (c)).

In order to model the inelastic strains induced by the damage, a plasticity model is introduced. Permanent deformations are associated to loading in the transverse and shear directions and assumed to be isotropic. For 2D problems, the elasticity domain is defined by the function $f$ such that:

\[
f = \sqrt{\sigma_{12}^2 + a^2 \sigma_{22}^2} - R_0 - R(p) \leq 0 \tag{3.5}
\]

where $R(p) = R_0 + K p^\gamma$ is a material-characteristic function depending on the accumulated plastic strain $p$. $R_0$ is the initial plasticity threshold, $a$ is a coupling coefficient between shear and transverse effect, $K$ and $\gamma$ are material parameters.
Moreover, the plasticity criterion in Eq. (3.5) is based on the effective stress $\tilde{\sigma}$:

$$\tilde{\sigma} = \begin{bmatrix} \frac{\sigma_{11}}{1 - d_1} \\ \frac{\langle \sigma_{22} \rangle_+ + \langle \sigma_{22} \rangle_-}{1 - d_2} \\ \frac{\sigma_{12}}{1 - d_{12}} \end{bmatrix}$$

Finally, the non-linear elastic behaviour in the fiber direction is modelled through the introduction of two material parameters, $\xi^+$ and $\xi^-$ for tension and compression respectively. In particular, the model takes into account an increase of stiffness in tension and a decrease in compression due to the realignment of the crystalline molecules within the fiber along the direction of loading, as shown in Figure 3.6(d). The fragile behaviour of the fibers is thus expressed by:

$$E_1 = E_1^0 \left(1 + \xi^+ \langle \varepsilon_{11} \rangle_+ + \xi^- \langle \varepsilon_{11} \rangle_- \right)$$

where the undamaged modulus $E_1^0$ is assumed to have the same value in tension and compression.
3.2.2 Inter-laminar damage modelling

As for intra-laminar damage, a Continuum Damage Mechanics approach is used to model the damage at the interface between two plies. In particular, a modification of the model proposed by Allix and Ladevège [30] is used in this thesis work. The interface is modelled as a two-dimensional surface which ensures stress and displacement transfers from one ply to another. Moreover, its mechanical behaviour depends on the relative orientations of the upper and lower plies. The interface is assumed to be orthotropic, with the axes $N_1$ and $N_2$ bisectors of the angle between the directions of the fibers of the adjacent layers and $N_3$ perpendicular to the surface plane, as shown in Figure 3.7. The deterioration of the interface is assumed to occur according to the three modes of crack separation known in fracture mechanics, namely modes I (opening), II (shearing) and III (tearing). Thus, three damage variables are introduced to describe the damage within the interface: $d_3$ associated with fracture mode I; $d_{13}$ and $d_{23}$ associated with mode of fracture II and III respectively. The model takes into account the difference in the damage behaviour in tension and compression by means of tension- or compression-energy. Thus, the energy per unit area at the interface is expressed by:

$$E_d = \frac{1}{2} \left[ \frac{\langle \sigma_{33} \rangle^2}{k_3^0 (1 - d_3)} + \frac{\langle \sigma_{33} \rangle^2}{k_3^0 (1 - d_3)} + \frac{\sigma_{13}^2}{k_1^0 (1 - d_{13})} + \frac{\sigma_{23}^2}{k_2^0 (1 - d_{23})} \right]$$  \hspace{1cm} (3.8)

where $k_1^0$, $k_2^0$ and $k_3^0$ are the undamaged stiffness. The thermodynamic forces associated with the dissipation are:

$$\begin{align*}
Y_3 &= \frac{\partial E_d}{\partial d_3} \bigg|_\sigma = \frac{\langle \sigma_{33} \rangle^2}{2k_3^0 (1 - d_3)^2} \\
Y_{13} &= \frac{\partial E_d}{\partial d_{13}} \bigg|_\sigma = \frac{\sigma_{13}^2}{2k_1^0 (1 - d_{13})^2} \\
Y_{23} &= \frac{\partial E_d}{\partial d_{23}} \bigg|_\sigma = \frac{\sigma_{23}^2}{2k_2^0 (1 - d_{23})^2}
\end{align*}$$  \hspace{1cm} (3.9)

In case of mixed loading, the evolution of the damage is related to the critical energy release rates relative to the three fracture modes, i.e. $G_{IC}$, $G_{IIC}$ and $G_{IIIC}$. Thus, a simple modelling approach consists in considering that the damage evolution is
3.2. DAMAGE MECHANICS OF COMPOSITE LAMINATES

Figure 3.8. Cohesive laws for damage evolution (adapted from [29]): (a) polynomial, (b) bilinear, (c) exponential

governed by an equivalent thermodynamic force $Y$:

$$Y = \sup_{\tau \leq t} G_{IC} \left\{ \left( \frac{Y_3}{G_{IC}} \right)^\alpha + \left( \frac{Y_{13}}{G_{IIC}} \right)^\alpha + \left( \frac{Y_{23}}{G_{IIIC}} \right)^\alpha \right\}^{1/\alpha}$$  \hspace{1cm} (3.10)

where $\alpha$ is equal to 1 in the present work since it is assumed that there is no coupling between the damage associated with the three modes of fracture. Moreover, it is also assumed that all the damage variables have the same evolution over the loading; a single damage variable $d$ is therefore used to describe delamination. Thus, the damage evolution law is defined by:

$$\begin{cases} d_3 = d_{13} = d_{23} = d = g(\sqrt{Y}) & \text{if } d < 1 \\ d_3 = d_{13} = d_{23} = d = 1 & \text{otherwise} \end{cases}$$  \hspace{1cm} (3.11)

where $g(\sqrt{Y})$ can be a polynomial, bilinear or exponential cohesive law. Figure 3.8 shows the stress-relative displacement (i.e. the displacement between two homologous points in the upper and lower layer) curves associated to the three laws. For the first two cohesive models (Figure 3.8(a) and (b)), the damage occurs after a linear elastic behaviour of the interface. On the contrary, the exponential model (Figure 3.8(c)) does not present any elastic threshold so that the damage occurs
CHAPTER 3. THEORY

immediately when the interface is loaded. Initially equal to zero, the damage variable $d$ equals one when the load-carrying capacity of the interface is completely deteriorated.

3.3 Damage model for composite laminates under dynamic loading

In order to correctly model the material response under dynamic loading scenarios, e.g. impacts and crashes, an enhancement of the mesomodel previously described for static loading is adopted in this thesis work. Due to the small scale at which the intra-laminar damage modelling is performed (i.e. mesoscale) and to the small dimensions of the cracks that develop within the plies of the laminate, a static description of the damage is in fact assumed to remain valid even for high loading rates during which high strain rates occur in the material. Since the damage evolution due to high-rate force solicitation is not instantaneous, a delay damage model developed by Allix et al. [24] is introduced. In addition, a material-characteristic strain rate model is developed to capture the variation of stiffness of the laminate with the strain rate. As introduced in Section 2.1 in fact, CFRP generally show a strain rate dependence of the mechanical properties along the in-plane transverse and shear directions.

3.3.1 Strain rate model for lamina elastic moduli

It is shown from tensile experimental tests that the material under study, namely MATCFE, exhibits a strain rate dependence of the lamina in-plane elastic properties in the transverse (i.e. perpendicular to the fiber) and shear directions (22- and 12-direction respectively), where the matrix viscous behaviour is predominant over the fibers brittle one [31]. Figure 3.9(a) and (b) show the stress-strain curves obtained from tensile tests performed at the Department of Materials, Textiles and Chemical Engineering of Ghent University at different strain rates in the shear and transverse direction respectively, where a clear increase of the lamina elastic moduli with the strain rate is identified. On the contrary, the material behaviour in the fiber direction is assumed to be strain rate independent. Along the above direction, in fact, the response of the material is driven by the fibers brittle behaviour which is predominant over the matrix viscous one.

A strain rate model is developed in order to predict the variation of the lamina elastic moduli in the transverse and shear directions, i.e. $E_2$ and $G_{12}$, with the strain rate $\dot{\varepsilon}_i$. The influence of the strain rate on the above material properties is represented by a scaling function $f_i$ which is function of the strain rate:

$$E_i (\dot{\varepsilon}_i) = f_i (\dot{\varepsilon}_i) E_i^{(0,\text{ref})} \quad \text{for } i = 2, 12$$

(3.12)

where $E_i^{(0,\text{ref})}$ represents the undamaged elastic modulus at a reference strain rate value, i.e. in quasi-static loading conditions.
Figure 3.9. Stress-strain curves of MATCFE under tensile loading at different strain rates along the lamina in-plane: (a) shear and (b) transverse direction.
It is possible to express Eq. (3.12) along the in-plane transverse and shear directions in the local coordinate system as follows:

\[ E_2 (\dot{\varepsilon}_2) = f_2 (\dot{\varepsilon}_2) E_2^{(0,\text{ref})} \]  

(3.13a)

\[ G_{12} (\dot{\gamma}_{12}) = f_{12} (\dot{\gamma}_{12}) G_{12}^{(0,\text{ref})} \]  

(3.13b)

The above equations only represent the variation of stiffness with the strain rate and they do not include the time dependent material response as a consequence of the matrix viscous behaviour.

Finally, due to the absence of a characterisation of the mechanical response in compression, it is assumed that MATCFE presents the same strain rate dependence as in tension. The amount of published data in the literature about the strain rate characterisation of CFRP under compressive loading is insufficient to validate the above assumption. This is due to the extreme difficulties encountered in the execution of the experimental tests and successive validation of the results. Daniel et al. [14] performed dynamic tests on CFRP thick laminates in compression up to \( 80 \, \text{s}^{-1} \). It was found that the transverse modulus increased up to 18% over the static value. Thus, the assumption made within the present thesis work seems to agree with the limited results found in literature. As a consequence, the strain rate model is used also in case of compressive loading.

### 3.3.2 Delay damage model

Classical damage models, i.e. local and time-independent such as that presented in Section 3.2.1, suffer from mesh dependence and usually lead to localisation of the deformation into a single element when used in FE codes [32]. In particular, they allow the damage rate to increase indefinitely. This characteristic has no physical sense. In order to obtain a physically consistent computational damage approach in case of dynamic loading, a damage model with delay effect is introduced. The main assumptions of the delay damage model are the following [24]:

- the evolution of damage due to variations of forces is not instantaneous
- a maximum damage rate exists, just as a maximum crack velocity exists

The introduction of the delay effect leads to a modification of the quasi-static damage evolution laws for the in-plane transverse and shear directions presented in Eq. (3.4). In particular, two delay parameters are introduced: \( \tau^c_i \) and \( a_i \). The modified equations of damage evolution can be written as:

\[
\begin{align*}
\dot{d}_2 &= \frac{1}{\tau^c_2} \left\{ 1 - \exp \left[ -a_2 (b_3 d_{12} - d_2) \right] \right\} \quad \text{if} \quad d_2 < 1 \\
\dot{d}_2 &= 1 \quad \text{otherwise} \\
\dot{d}_{12} &= \frac{1}{\tau^c_{12}} \left\{ 1 - \exp \left[ -a_{12} \left( f(\sqrt{Y}) - d_{12} \right) \right] \right\} \quad \text{if} \quad d_{12} < 1 \\
\dot{d}_{12} &= 1 \quad \text{otherwise}
\end{align*}
\]  

(3.14)
The physics of this type of model is such that the damage evolution is not instantaneous, but is governed by the internal characteristic time \( \tau_i^c \). The damage rate \( \dot{d}_i \) is calculated from the difference between the damage without delay and that with delay effect. In this way, a fast variation of the damage energy release rate will not lead to an immediate evolution of the damage. The damage will evolve with a certain delay fixed by the characteristic time, which is independent of the mesh. A maximum damage rate \( \dot{d}_{i}^{\text{max}} \) exists such that:

\[
\dot{d}_{i}^{\text{max}} = \frac{1}{\tau_i^c}
\] (3.15)

The limitation on the damage rate allows the stress to increase more than in the classical model previously presented, so that the nearest element will damage when several elements are used, avoiding a localisation of the deformation in one single element. Moreover, the more or less brittle character of the damage evolution laws is governed by \( a_i \). The above delay damage model is consistent with the static analysis since for a quasi-static evolutions of damage (\( \dot{d}_i \simeq 0 \)) the static evolution laws are verified.

It is worth noting that within the present thesis work the delay effect is applied only to the evolution of the damage inside the plies along the in-plane transverse and shear directions. Thus, no damage delay model is used along the fiber direction and for the interface between two plies.
Chapter 4

Method

The following chapter provides a chronological description of how the work was carried out in the framework of this thesis. Firstly, the procedure used for the determination of the scaling functions for the lamina elastic moduli is described. The implementation of the strain rate model in the FE code PAM-CRASH is then presented and verified on a patch level. The delay damage parameters are determined and the results from the numerical simulations performed on a full-size specimen are validated against the experimental data. Lastly, a dynamic three-point bending test is simulated to analyse the effect of the strain rate and delay damage models on the response of the laminate with respect to quasi-static results.

4.1 Determination of scaling functions for elastic moduli

The determination of the scaling functions for the material-characteristic strain rate model introduced in Eq. (3.12) was performed on a phenomenological level starting from the experimental curves obtained from tensile tests performed on MATCFE specimens at various strain rates and presented in Figure 3.9. The determination was carried out for the in-plane shear and transverse directions in the local coordinate system separately, i.e. 12- and 22- direction respectively.

4.1.1 Shear direction

In order to determine the scaling function for the lamina elastic modulus in the in-plane shear direction $G_{12}$, experimental data were made available from dynamic tensile tests performed until failure on $[\pm 45]_2$ specimens at four strain rates ranging from $0.00107$ to $2.94 \text{ s}^{-1}$. The tests were carried out by the Department of Materials, Textiles and Chemical Engineering of Ghent University using an Instron High Strain Rates servo-hydraulic machine. The strain fields were acquired through Digital Image Correlation (DIC) and at least five repetitions were performed for each strain rate tested. The experimental data provided were the result of an averaging of the data from each repetition whose results were considered reliable. They consisted of
the longitudinal stress $\sigma_x$ and strain $\varepsilon_x$ in the laminate global coordinate system. No averaged data on the transverse strain $\varepsilon_y$ were made available due to difficulties regarding the acquisition of the above strain field through DIC, especially at high strain rates.

In order to determine $G_{12}$ at each strain rate, the engineering shear stress $\tau_{12}$ and strain $\gamma_{12}$ in the lamina local system were calculated. From Classical Lamination Theory, the following equalities hold for a $\left[\pm 45\right]_2s$ laminate:

$$\tau_{12} = \frac{\sigma_x}{2}$$  \hspace{1cm} (4.1)

and

$$\gamma_{12} = \varepsilon_x - \varepsilon_y$$  \hspace{1cm} (4.2)

Due to the lack of data regarding the transverse strain, by using the definition of Poisson’s ratio $\nu_{xy}$ of the laminate:

$$\nu_{xy} = -\frac{\varepsilon_y}{\varepsilon_x}$$  \hspace{1cm} (4.3)

it was possible to rewrite Eq. (4.2) as:

$$\gamma_{12} = \varepsilon_x (1 + \nu_{xy})$$  \hspace{1cm} (4.4)

In particular, it was determined from static tests performed by third parties on MATCFE $\left[\pm 45\right]_2s$ specimens that $\nu_{xy} = 0.75$. Additionally, it was assumed that

**Figure 4.1.** Stress-strain curves of MATCFE with $\left[\pm 45\right]_2s$ layup loaded in tension at various strain rates and strain ranges according to ASTM D3518/D3518M
the Poisson’s ratio exhibits no strain rate dependence, i.e. it remains constant as the strain rate varies. The shear modulus $G_{12}$ was determined at each strain rate by using two different methods:

- calculation of the chord modulus of elasticity in the in-plane shear direction according to ASTM D3518/D3518M [33]
- determination of a polynomial regression for each stress-strain curve and calculation of their slope at the origin of the axes

According to ASTM D3518/D3518M, the chord shear modulus was calculated as follows:

$$G_{12}^{\text{chord}} = \frac{\Delta \gamma_{12}}{\Delta \tau_{12}}$$ (4.5)

where $\Delta \gamma_{12}$ is the difference between two engineering shear strain points and $\Delta \tau_{12}$ is the difference in applied engineering shear stress between the above two shear strain points. Based on the standard, the calculation of the chord shear modulus must be performed over a strain range of 0.004±0.0002, starting with the lower strain point in the range of 0.0015 to 0.0025. Thus, two engineering strain ranges were selected for the analysis as shown in Figure 4.1: 0.0015 to 0.0055 and 0.0020 to 0.0060. The second method instead consisted in determining a polynomial regression for each stress-strain curve and calculating the value of the first-order derivative at the origin of the axes. By applying the two methods described above, a recursive trend was found in the variation of the lamina shear modulus with the strain rate, as shown in Figure 4.2. However, only the values determined through polynomial regressions were selected for further analysis. In fact, the value of the undamaged shear modulus $G_{12}^{(0, \text{ref})}$ at the lowest strain rate, which was assumed to be the reference value (i.e. quasi-static, $\dot{\gamma}_{12}^{\text{ref}} = 0.00107$ s$^{-1}$) for the strain rate in the shear direction, was the closest to the one reported after static tests conducted by third parties on the same material [29]. In particular, the value determined from static tests was obtained by using the same polynomial regression-based methodology as that used in this thesis work. Moreover, the above methodology allowed to determine the value of the undamaged shear modulus to be inserted into Eq. (3.1)-(3.3), i.e. when the damage within the material was equal to zero.

Hence, three different expressions for the scaling function $f_{12}$ to be inserted into Eq. (3.13b) were used to fit the selected values of the shear modulus obtained from the experimental tests:

$$f_{12}^{(1)} = A_1 \left[ 1 + A_2 \left( \ln \frac{\dot{\gamma}_{12}}{\dot{\gamma}_{12}^{\text{ref}}} \right)^{A_3} \right]$$ (4.6a)

$$f_{12}^{(2)} = 1 + B_1 \left( \ln \frac{\dot{\gamma}_{12}}{\dot{\gamma}_{12}^{\text{ref}}} \right)^{B_2}$$ (4.6b)

$$f_{12}^{(3)} = C_1 + C_2 \left( \log \frac{\dot{\gamma}_{12}}{\dot{\gamma}_{12}^{\text{ref}}} \right) + \log C_3$$ (4.6c)
Eq. (4.6a) was introduced by Kwon et al. [34] to predict the variation of the tensile properties of CFRP under strain rates ranging from 0.001 to 100 s$^{-1}$. It consists of three coefficients, which are determined by fitting them to the experimental data. The second expression for the scaling function, which is presented in Eq. (4.6b), is a simplification of Eq. (4.6a) being characterised by two coefficients instead of three. The elimination of the first coefficient from Eq. (4.6a) allows the scaling function to be exactly equal to one when the strain rate equals its reference value. Moreover, the lower number of coefficients makes the implementation of the strain rate model in any FE code faster. Finally, Eq. (4.6c) was used to model the strain rate dependence of the elastic properties of metals. It again presents three coefficients to be determined.

A non-linear least-squares data-fitting routine implemented in MATLAB was then used to find the values of the coefficients of $f_{12}^{(1)}$, $f_{12}^{(2)}$ and $f_{12}^{(3)}$ that best fit the shear modulus values at different strain rates (Appendix A). Figure 4.3 shows the minimum residual fit of the three expressions for the scaling function in the shear direction to the selected experimental values of the shear modulus $G_{12}$ normalised with respect to the undamaged one at the reference strain rate, i.e. $G_{12}^{(0,\text{ref})}$, according to Eq. (3.13b).

### 4.1.2 Transverse direction

The procedure used to determine the scaling function for the lamina elastic modulus $E_2$ in the transverse direction followed the same path as that used for the determination of $G_{12}$. Dynamic end-loaded tensile tests on [90]$_8$ specimens loaded along the
4.1. DETERMINATION OF SCALING FUNCTIONS FOR ELASTIC MODULI

Figure 4.3. Fit to $G_{12}/G_{12}^{(0,\text{ref})}$ experimental data as function of the strain rate (log scale)

$0^\circ$-direction were performed at Ghent University by using an Instron High Strain Rates servo-hydraulic machine at six strain rates varying from 0.00108 to 86.9 s$^{-1}$. The strain fields were again recorded using DIC. Experimental data regarding the laminate longitudinal stress $\sigma_x$ and strain $\varepsilon_x$ were provided following the same type of averaging performed on the data along the shear direction.

In order to determine the value of $E_2$ at each strain rate, the transverse stress $\sigma_{22}$ and strain $\varepsilon_{22}$ in the local coordinate system were calculated according to Classical Lamination Theory:

$$\sigma_{22} = \sigma_x$$  \hspace{1cm} (4.7)

and

$$\varepsilon_{22} = \varepsilon_x$$  \hspace{1cm} (4.8)

Two methods were again used to determine the values of the elastic modulus in the transverse direction at different strain rates:

- calculation of the tensile chord modulus of elasticity according to ASTM D3039/D3039M [35]
- determination of a polynomial regression for each stress-strain curve and calculation of their slope at the origin of the axes

ASTM D3039/D3039M prescribes the determination of the tensile chord modulus of elasticity $E_{2\text{chord}}$ for balanced and symmetric laminates with respect to the test direction as follows:

$$E_{2\text{chord}} = \frac{\Delta \sigma_{22}}{\Delta \varepsilon_{22}}$$  \hspace{1cm} (4.9)
where $\Delta \varepsilon_{22}$ is the difference between two strain points and $\Delta \sigma_{22}$ is the difference in the applied tensile stress between those strain points. In particular, a strain range of 0.002 is prescribed, with the lower and upper points at 0.001 and 0.003 respectively, as shown in Figure 4.4. Polynomial regressions were performed for the averaged stress-strain curves at the two highest strain rates (44.2 and 86.9 $s^{-1}$) due to their oscillating behaviour. Thus, the stress values to be used in Eq. (4.9) for the above strain rates were determined from those new curves. The second method instead followed exactly the same path as the one previously used for the determination of the shear modulus $G_{12}$.

A recursive trend was again found in the variation of the transverse elastic modulus $E_2$ with the strain rate by applying the two methods described above, as shown in Figure 4.5. However, as for the lamina shear modulus, only the values of $E_2$ determined through polynomial regressions were considered for the determination of the scaling function. This was because the value of the undamaged elastic modulus $E_{2}^{(0,\text{ref})}$ at the reference strain rate ($\dot{\varepsilon}_{22}^{\text{ref}} = 0.00108$ $s^{-1}$) determined through this method resulted to be the closest one to the value reported from static tests [29], which was again calculated by using the same interpolation-based methodology. As for the shear direction, the use of polynomial regressions allowed to determine the value of the elastic modulus at each strain rate when the material was not damaged yet. Hence, the same formulation of the expressions used for the determination of the scaling function in the shear direction were adapted along the transverse
4.2 Strain rate model implementation

Once the determination of the elastic moduli scaling functions was completed, the strain rate model described in Section 3.3.1 was implemented in the user-defined material subroutine MAT80 of the explicit FE commercial code PAM-CRASH. The above subroutine is built for an eight-node solid element, used as constitutive element for the layers of the composite laminates and modelled according to the

\[
\begin{align*}
\hat{f}_2^{(1)} &= A_1 \left[ 1 + A_2 \left( \ln \frac{\dot{\varepsilon}_{22}}{\dot{\varepsilon}_{22}^{\text{ref}}} \right)^{A_3} \right] \\
\hat{f}_2^{(2)} &= 1 + B_1 \left( \ln \frac{\dot{\varepsilon}_{22}}{\dot{\varepsilon}_{22}^{\text{ref}}} \right)^{B_2} \\
\hat{f}_2^{(3)} &= C_1 + C_2 \left( \log \frac{\dot{\varepsilon}_{22}}{\dot{\varepsilon}_{22}^{\text{ref}}} \right) + \log C_3
\end{align*}
\]

Figure 4.6 shows the minimum residual fit of \( \hat{f}_2^{(1)} \), \( \hat{f}_2^{(2)} \) and \( \hat{f}_2^{(3)} \) to the selected values of the elastic modulus \( E_2 \) normalised with respect to the undamaged one at the reference strain rate \( E_2^{(0,\text{ref})} \). The values of the coefficients for the three expressions reported in Eq. (4.10) were again determined by using a similar least-squares data-fitting procedure as that used for the shear data (Appendix B).

Figure 4.5. Variation of the lamina elastic modulus in the transverse (to the fiber) direction with the strain rate (log scale)
modified Ladevèze intra-laminar damage model presented in Section 3.2.1. The implementation of the strain rate model was performed for both the in-plane shear and transverse directions. The scaling of the elastic moduli in each direction was executed only when the strain rate along them was equal or higher than the corresponding reference values ($\dot{\gamma}_{12}^{ref}$ and $\dot{\varepsilon}_{22}^{ref}$). The value of the two scaling functions was then computed based on the value of the strain rate calculated at each time step $n$. Finally, the undamaged elastic moduli at the reference strain rates ($G_{12}^{(0,ref)}$ and $E_{2}^{(0,ref)}$) in the shear and transverse directions were scaled at each time step by the scaling functions values. Moreover, the same scaling value used for the elastic modulus in transverse direction was applied to the one through the thickness ($E_{3}$) as well, which was assumed to be equal to $E_{2}$ due to out-of-plane isotropy of the ply. This was done in order to assure a correct scaling of the elastic moduli in case of bending of the elements.

Due to instability problems in the solutions of the analyses run with the strain rate model implemented as above, it was decided to scale the elastic moduli only until a certain value of damage. In particular, simulations performed on coupons loaded in tension at strain rates higher than the reference one resulted in unstable responses of the material which were characterised by vibrations that led to premature failure of the laminates, as shown in Figure 4.7. Thus, further simulations were carried out firstly by disabling the plasticity effects (but keeping the damage enabled) and lastly by applying the opposite scenario in order to identify the cause of the vibrating response. The deactivation of the damage and plasticity was achieved by setting their threshold values several order of magnitudes higher than

Figure 4.6. Fit to $E_{2}/E_{2}^{(0,ref)}$ experimental data as function of the strain rate (log scale)
those used by default. It was found that disabling the plasticity had no influence on the response of the material except for an expected more linear behaviour and a lower value of the strain to failure. On the contrary, the vibrating behaviour was almost completely eliminated when the damage was disabled leading to a more stable response of the material. Thus, it was concluded that the simultaneous action of the strain rate model and damage was the source of the instability. In particular, the strain rate model induced an increase of the stiffness of the elements at each time step when loaded in tension. However, at the same time, the damage led to a progressive decrease of stiffness according to the damage model presented in Section 3.2.1, which caused an increase in strain rate. The above increase led to higher stiffness of the elements and therefore to an increase in damage resulting in an unstable loop. Thus, the action of those two effects resulted to be in conflict with each other, leading to continuous oscillations of the stiffness of the elements and therefore of the stresses within them.

In order to avoid instabilities in the response of the material, the scaling of the lamina elastic moduli was executed only when the damage within the material was lower than a determined value. The determination of the above damage limit was carried out through simulations performed on a coupon level with $[\pm 45]_2$ and $[90]_8$ layups at various strain rates. Thus, a damage limit of 0.00003 and 0.00001 was set for the shear and transverse direction respectively. Beyond the above values of damage, the scaling of the elastic moduli was stopped. The last elastic moduli values scaled were taken as the reference ones for the next time steps. Figure 4.8 shows the flow chart of the strain rate model implemented in the user-defined ma-
Figure 4.8. Flow chart of the strain rate model implemented in user-defined subroutine MAT80

4.3 Strain rate model verification

The strain rate model implemented in the user-defined subroutine MAT80 was verified on a patch level along the in-plane shear and transverse directions in the lamina...
4.3. STRAIN RATE MODEL VERIFICATION

local system. A 1 mm × 1 mm × 0.225 mm single element with fibers oriented along the 90°-direction was used to perform the verification. In particular, the element thickness was set equal to the thickness of a single layer of MATCFE. The element was end-loaded in tension by imposing the strain rates experimentally tested for both the shear and transverse directions. Moreover, the load was applied as a constant velocity which was given as input to the solver in the form of a linear time-displacement curve. The velocity \( v \) necessary to obtain the targeted strain rate was determined by using the following relation [2]:

\[
v = \dot{\varepsilon}L
\]

where \( \dot{\varepsilon} \) is the desired strain rate in the selected direction and \( L \) is the length of the specimen under study. The output of the analyses performed by the solver at each strain rate consisted of the force-displacement \((F - \delta)\) curve recorded in the direction of the load at the section where it was applied.

4.3.1 Shear direction

The shear condition was obtained by clamping one side of the element parallel to the fibers direction and pulling the opposite one in parallel direction as shown in Figure 4.9(a). Poisson’s contraction through the thickness of the element was allowed.

The force-displacement curves were transformed to engineering stress-strain ones according to:

\[
\tau_{12} = \frac{F}{2A_0}
\]

where \( F \) is the force at the section where the velocity is applied and \( A_0 \) is the initial cross-section area of the specimen. Moreover, due to the small scale of the problem under study, the engineering shear strain \( \gamma_{12} \) was determined as follows:

\[
\gamma_{12} = \frac{\delta_1}{L}
\]
where $\delta_1$ is the displacement along 1-direction recorded at the section where the velocity is applied, as shown in Figure 4.10. Hence, the strain rate model along the shear direction was verified by determining the value of the lamina shear modulus $G_{12}$ at each strain rate simulated and comparing it with the correspondent value calculated analytically by plugging Eq. (4.6b) into Eq. (3.13b). The values of $G_{12}$ were determined through the use of polynomial regressions of the stress-strain curves and calculation of their slope at the origin of the axes.

### 4.3.2 Transverse direction

In order to load the element in the transverse direction, the velocity profile was applied perpendicularly to the fibers (Figure 4.9(b)). The element was allowed to contract along the width and through the thickness so that no reaction forces were generated along these directions.

The stress-strain curves were obtained from the force-displacement ones by using the following relations:

\[ \sigma_{22} = \frac{F}{A_0} \]  

(4.14)

and

\[ \varepsilon_{22} = \frac{\delta_2}{L} \]  

(4.15)

where $\delta_2$ is the displacement along the 2-direction recorded at the section where the velocity is applied. The verification of the strain rate model along the transverse direction followed the same path as that performed for the shear one. The values of the lamina elastic modulus in the transverse direction obtained from simulations and determined through polynomial regressions were compared to those calculated analytically by using Eq. (3.13a) and (4.10b).
4.4 Identification of delay damage parameters

The identification of the delay damage parameters $\tau_i^c$ and $a_i$ that govern the damage evolution in case of dynamic loading according to Eq. (3.14) was carried out proceeding from a patch level to a coupon one. In particular, dynamic tensile simulations were performed until failure on samples with different dimensions, as showed in Table 4.1 and Figure 4.11. The samples had the same layups as those used for the dynamic experimental tests described in Sections 4.1.1 and 4.1.2, i.e. $[\pm 45]_2$, and $[90]_8$. The plies of the laminates were modelled by using MAT80 eight-node solid elements with the strain rate model previously implemented. Moreover, the element size utilised for the mesh was the same as the one used in Section 4.3 for the verification of the above model. No interface was included between the plies. One short side of the samples was constrained against translation along the longitudinal direction, i.e. the $x$-direction in the global coordinate system of the laminates. Poisson’s contraction was allowed along the width and through the thickness of the samples. Moreover, a constant velocity profile was imposed at the unconstrained short side for each strain rate along the longitudinal direction, by again using Eq. (4.11).

The two sets of delay parameters were determined separately for the in-plane shear and transverse direction respectively following the procedure described in Figure 4.12. First, simulations at different strain rates were carried out without delay effect and compared to the experimental data presented in Figure 3.9 in order to perform a validation of the results and/or identify discrepancies. The delay damage model was kept disabled (as default) in PAM-CRASH. In particular, $\tau_i^c$ and $a_i$ were left unchanged from the default setting and equal to zero and one respectively. Then, simulations were performed iteratively with different values of the delay parameters in order to replicate the same response of the laminates as that obtained from the experimental tests, especially in terms of strength and strain to failure. In particular, the combination of $\tau_i^c$ and $a_i$ providing the best fit of the simulated curves to the experimental ones was searched.

It is worth noting that the identification of the above parameters was performed at the highest strain rate used in the experimental tests for both the shear and transverse direction in the lamina local system, i.e. 2.94 and 86.9 $s^{-1}$ respectively. This was done in order to reduce the computation time needed for each iteration. In fact, the higher the strain rate, the lower the time needed by the solver to

<table>
<thead>
<tr>
<th>Sample</th>
<th>$L$ [mm]</th>
<th>$W$ [mm]</th>
<th>$T$ [mm]</th>
<th>No. of elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>1</td>
<td>1.8</td>
<td>8</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>1</td>
<td>1.8</td>
<td>40</td>
</tr>
<tr>
<td>C</td>
<td>54</td>
<td>20</td>
<td>1.8</td>
<td>8640</td>
</tr>
</tbody>
</table>

Table 4.1. Dimensions and number of elements of the samples used for the identification of the delay damage parameters ($L = $ length, $W = $ width, $T = $ thickness)
compute the solution of the analyses. The effects of the above procedure on the parameters identification will be discussed in Chapter 5. For the same reason, the identification process was carried out starting from a patch level. At this level, in fact, each iteration required a consistently lower computational time than if performed at the coupon one (in the order of minutes instead of hours). A rough value of both the delay parameters was determined from simulations carried out at the patch level and used as input for those performed at the coupon one, where more accurate values were expected to be obtained along with longer time needed for computations. Finally, it is worth to be noticed that the dimensions of the coupon reported in Table 4.1 (Sample C) are those of the section of the specimen used in the experimental tests included between the grips of the servo-hydraulic machine, i.e. the gauge length. Thus, the grips were not taken into account and were not modelled for the simulations at the coupon level.

As previously described, the procedure used for the identification of the delay parameters was the same for both the shear and transverse direction. The only
difference was represented by the post-processing of the output data from the simulations, which consisted of the force-displacement curves recorded in the direction of the load at the section where was applied.

### 4.4.1 Shear direction

In order to compare to the experimental data the results from the simulations performed at different strain rates on the three samples previously described, the engineering shear stress-strain curves were determined. In particular, the in-plane shear stress in the lamina local system was again calculated by using Eq. (4.1), where the longitudinal stress $\sigma_x$ in the laminate coordinate system was calculated as:

$$\sigma_x = \frac{F}{A_0} \quad (4.16)$$

The shear strain was instead determined through Eq. (4.4). It is worth noting that the laminate Poisson’s ratio $\nu_{xy}$ was assumed to be strain rate independent as already done for the calculation of the engineering shear strain of the specimen used.
for the experimental tests in Section 4.1.1. A value of 0.75 was again used for the above ratio. The longitudinal strain $\varepsilon_x$ to be inserted into Eq. (4.4) was calculated as follows:

$$\varepsilon_x = \frac{\delta_x}{L}$$

(4.17)

where $\delta_x$ is the displacement in the longitudinal direction recorded at the section where the velocity is applied.

As previously introduced, the identification of the delay damage parameters was performed by using an iterative procedure which involved the combination of different values of $\tau_{c12}$ and $a_{12}$. The above procedure was carried out by taking into account the effects of the delay parameters on the response of the laminates when loaded dynamically.

**Effect of $\tau_{c12}$ on the laminate response**

The effect of the characteristic time on the response of the laminate was investigated on Sample A with layup $[\pm45]_{2s}$ at the highest strain rate used in the experimental test for the shear direction. This was done by varying the value of $\tau_{c12}$ while keeping the $a_{12}$ parameter constant and equal to 3.0. Figure 4.13 shows the influence of the above delay parameter on the stress-strain response of the laminate. The variation of $\tau_{c12}$ had no effect on the initial stiffness of the laminate. However, the non-linear behaviour of the curves was mitigated as the characteristic time increased leading to a clear increase in strength. The value of strain at the maximum stress also increased.
4.4. IDENTIFICATION OF DELAY DAMAGE PARAMETERS

Figure 4.14. Influence of $a_{12}$ on the response of Sample A with $[\pm45]_{2s}$ layup when loaded in tension at $\dot{\gamma}_{12} = 2.94 \text{ s}^{-1}$ ($\tau_{12}^c = 3.0 \text{ ms constant}$)

as the characteristic time increased. Overall, a smoother behaviour characterised the response of the laminate with increasing $\tau_{12}^c$ before complete failure.

Effect of $a_i$ on the laminate response

The effect of the $a_{12}$ parameter on the response of the composite laminate was investigated on the same sample, layup and at the same strain rate as those used for the corresponding study on $\tau_{12}^c$. The procedure used consisted again in varying the delay parameter under study while keeping the other one equal to a constant value, i.e. 3.0 ms. As for the characteristic time, the variation of $a_{12}$ did not affect the initial stiffness of the laminate, as shown in Figure 4.14. The non-linear behaviour of the stress-strain curves was instead mitigated for decreasing values of $a_{12}$. Thus, the strength increased as the above delay parameter decreased. On the other hand, the value of strain at the maximum stress remained constant. Moreover, the response of the laminate was not characterised any more by a sudden failure. In particular the load-carrying capacity of the laminate decreased gradually with increasing $a_{12}$ until complete failure for strain values higher than the one at the maximum stress.

Thus, the same effects were obtained on the response of the laminate for values of $\tau_{12}^c$ and $a_{12}$ increasing and decreasing respectively and with different magnitudes. In particular, the effect of $\tau_{12}^c$ on the response of the laminate was more pronounced than the one of $a_{12}$. This led to a rather complex response of the sample when the effects of the two delay parameters were combined.
4.4.2 Transverse direction

As for the shear direction, the stress-strain curves were obtained from the force-displacement ones by using Classical Lamination Theory. In particular, the in-plane transverse stress in the lamina local system was determined by plugging Eq. (4.16) into Eq. (4.7). Finally, the transverse strain was calculated through Eq. (4.8) and (4.17). The effect of $\tau_c$ and $a_2$ on the laminate response resulted to be the same as that described in the previous Section for the corresponding delay parameters along the in-plane lamina shear direction. Thus, an analogous description of the influence of the above parameters on the response of the sample when loaded dynamically in tension as the one performed for the shear direction will not be performed.

4.5 Dynamic three-point bending simulation

Once the strain rate and delay damage models were validated at the coupon level against experimental data, a dynamic simulation was performed at the subcomponent level to evaluate the effect of the above models on the response of the composite laminate. In particular, a dynamic three-point bending simulation was performed and the results compared to those obtained from a corresponding dynamic experimental test and quasi-static simulation.

4.5.1 Simulation set-up

The dynamic-three point bending test was simulated by reproducing the same set-up as that used for the actual experimental test. A 600 mm $\times$ 100 mm $\times$ 100 mm bar with rectangular cross-section and rounded edges was used, as shown in Figure 4.15. The laminate was made of six plies of MATCFE with $[+60/-60/0]$ layup. The two supports and the pusher were reproduced by using hollow semi-cylinders with different diameters in order to avoid excessive indentation or failure of the laminate due to stress concentrations directly under the pusher. The pusher had a total weight of 500 kg and it was free-falling due to its own weight. The simulation was set to reach a maximum vertical displacement of 60 mm at the midspan of the bar. The output of the analysis performed by the solver consisted of the force-displacement curve recorded along the vertical direction at the point of contact between the laminate and the pusher.

4.5.2 Modelling strategy

The laminate was modelled by using MAT80 eight-node solid elements for the plies and COS3D eight-node cohesive elements for the interface between them, as shown in Figure 4.16. As explained in Section 2.2 in fact, the use of cohesive elements at the interface between two plies is fundamental in order to correctly simulate the initiation and development of delamination, which is one of the most severe damage mechanisms in case of out-of-plane bending of the laminate. In particular,
the thickness of MAT80 elements was set equal to 0.22 mm while that of COS3D ones was equal to $xxxxx$ mm. It is worth noting that MAT80 elements presented the strain rate model previously implemented and the delay damage model enabled with the delay parameters $\tau_i$ and $a_i$ set equal to the values determined according to the procedure described in Section 4.4. On the other hand, COS3D elements did not have any strain rate dependence of their elastic properties implemented. The delay effect was not applied to those elements. Moreover, the element size utilised for the mesh of the laminate was set equal to $1.25 \text{ mm} \times 1.25 \text{ mm}$. Contact with friction was implemented between the pusher/supports and the laminate by using a coefficient of friction $\nu$ which was set equal to 0.3. Self-friction was instead implemented between the plies and the interfaces by using the same value of the friction coefficient, i.e. $\nu = 0.3$. An elimination rule was set for the MAT80 elements: the elimination of a single element occurred when all the three damage variables ($d_1, d_2$ and $d_{12}$) were greater or equal to $0.999 - 0.00001$, being 0.999 the maximum value of damage at which failure occurs.
Chapter 5

Results and Discussion

The results of the work carried out in this thesis are presented and discussed in the following chapter by using the same logical structure as that adopted in Chapter 4. Relationship between the results and the theoretical models introduced in Chapter 3 are discussed in order to identify agreements and differences.

5.1 Strain rate model for lamina elastic moduli

As introduced in Section 3.3.1, MATCFE exhibits a clear dependence of the lamina elastic moduli along the in-plane shear and transverse directions with the strain rate. In particular, dynamic tensile tests show an increase of the moduli with increasing strain rate in both the directions (Figure 3.9(a) and (b)) which agrees with the overall trend deduced from the data found in literature and presented in Section 2.1 about the strain rate sensitivity of the mechanical properties of CFRP.

5.1.1 Scaling functions for lamina elastic moduli

Shear direction

The variation of the lamina shear modulus $G_{12}$ with the strain rate determined through the three methods described in Section 4.1.1 is presented in Table 5.1. An increase of the shear modulus with the strain rate is clearly identified by all the

<table>
<thead>
<tr>
<th>$\dot{\gamma}_{12}$ [s$^{-1}$]</th>
<th>$G_{12}^{ASTM,1}$ [GPa]</th>
<th>$G_{12}^{ASTM,2}$ [GPa]</th>
<th>$G_{12}^{Pol}$ [GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00107</td>
<td>3.86</td>
<td>3.53</td>
<td>4.92</td>
</tr>
<tr>
<td>0.0467</td>
<td>3.99</td>
<td>3.91</td>
<td>4.69</td>
</tr>
<tr>
<td>0.445</td>
<td>4.46</td>
<td>4.08</td>
<td>5.29</td>
</tr>
<tr>
<td>2.94</td>
<td>4.81</td>
<td>4.48</td>
<td>5.44</td>
</tr>
</tbody>
</table>

Table 5.1. Variation of $G_{12}$ with the strain rate for MATCFE
methods. However, relevant discrepancies are found between the magnitude of the values obtained through each methodology. The value of the shear modulus at the reference strain rate determined by using polynomial regressions results to be the closest to the one obtained from static tests which is equal to 5.31 GPa [29] making sense since it was calculated with the same methodology. However, it presents an underestimation of 7% with respect to its static counterpart. The reason for this discrepancy is not clear. The values calculated by applying ASTM D3518/D3518M methodology instead present an higher underestimation. Moreover, the strain range selected for the calculations of $G_{12}$ is found to have an important influence on its final value. This is due to the oscillating behaviour that characterises the stress-strain curves even at the lowest strain rates (Figure 4.1). The above behaviour is attributed to difficulties in the execution of dynamic tensile tests, especially at high strain rates, such as those introduced in Section 2.1. Thus, it is concluded that the methodology prescribed by ASTM D3518/D3518M is not suitable for the calculation of the lamina shear modulus from dynamic tests, being developed for quasi-static loading conditions only. Moreover, the strain range prescribed for the determination of $G_{12}$ in case of static loading does not ensure that the damage inside the material is equal to zero when subjected to dynamic loading. On the other hand, the use of a methodology based on the determination of polynomial regressions for each stress-strain curve and the calculation of their first-order derivative at the origin of the axes results to be more robust in case of oscillating behaviour of the curves. Moreover, it ensures that the calculation of the shear modulus is performed when the material is not damaged yet. From the selected values of $G_{12}$, it is determined

![Figure 5.1. Fit of the scaling function for the lamina shear modulus to the experimental data](image)
that the shear modulus increases up to 11% with respect to the quasi-static value. Uncertainties might affect the values of $G_{12}$ due to the assumption on the strain rate insensitivity of the laminate Poisson’s ratio $\nu_{xy}$. The amount of published data present in the literature regarding the above topic is insufficient to draw reliable conclusions. The only reliable work found is the one carried out by Okoli and Smith [36], who performed tensile tests on glass/epoxy laminates at different strain rates. It was concluded that the laminate Poisson’s ratio was strain rate insensitive. The presence of the fibers in the composite was suggested as cause of the above insensitivity.

By interpolating the selected values of the normalised lamina shear modulus at different strain rates, it is determined that $f_{12}^{(2)}$ gives the best fit, especially at low strain rate values, as shown in Figure 4.3. Thus, $f_{12}^{(2)}$ is selected as scaling function for the lamina shear modulus, i.e. $f_{12} = f_{12}^{(2)}$. The minimum residual values of the coefficients of the above function are listed in Table 5.2. In particular, the expression of $f_{12}^{(2)}$ is particularly suitable for the application under study since it equals one when the strain rate $\dot{\gamma}_{12}$ equals the reference one $\dot{\gamma}_{12}^{ref}$, so that no scaling of the elastic modulus is executed. Figure 5.1 shows the fit of $f_{12}$ to the experimental data as function of the strain rate, which is here limited to 100 s$^{-1}$. A steep behaviour characterises the part of the curve between 0.00107 and 10 s$^{-1}$ approximately.

Transverse direction

The values of the lamina elastic modulus in the transverse direction determined at different strain rates by applying the two methods presented in Section 4.1.2 are listed in Table 5.3. As for the shear modulus, a clear increase in $E_2$ is identified by

<table>
<thead>
<tr>
<th>$\dot{\varepsilon}_{22}$ [s$^{-1}$]</th>
<th>$E_2^{ASTM}$ [GPa]</th>
<th>$E_2^{Pol}$ [GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00108</td>
<td>8.08</td>
<td>9.75</td>
</tr>
<tr>
<td>0.0419</td>
<td>8.70</td>
<td>10.55</td>
</tr>
<tr>
<td>0.287</td>
<td>9.12</td>
<td>11.05</td>
</tr>
<tr>
<td>4.23</td>
<td>9.67</td>
<td>12.67</td>
</tr>
<tr>
<td>44.2</td>
<td>12.58</td>
<td>13.36</td>
</tr>
<tr>
<td>86.9</td>
<td>14.24</td>
<td>16.24</td>
</tr>
</tbody>
</table>

Table 5.2. Minimum residual values of the coefficients of $f_{12}^{(2)}$

<table>
<thead>
<tr>
<th>$B_1$ [-]</th>
<th>$B_2$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000104</td>
<td>3.3836</td>
</tr>
</tbody>
</table>

Table 5.3. Variation of $E_2$ with the strain rate for MATCFE
Table 5.4. Minimum residual values of the coefficients of $f_2^{(2)}$

<table>
<thead>
<tr>
<th>$B_1$ [-]</th>
<th>$B_2$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0029</td>
<td>2.1797</td>
</tr>
</tbody>
</table>

Figure 5.2. Comparison between scaling functions for the elastic modulus in transverse and shear direction

both the methods. Even more relevant discrepancies are found between the elastic modulus values determined by applying ASTM D3039/D3039M methodology and the one based on polynomial regressions. The value of $E_2$ at the reference strain rate is found to be closer to the one determined from static tests, which is equal to 9.66 GPa [29], when calculated by using the latter methodology. This is again attributed to fact that the value of $E_2$ determined from static tests was calculated by using an interpolation-based methodology in order to ensure the condition of no damage within the material. In particular, the determined elastic modulus results to be overestimated by 1% with respect to the static value. Moreover, the unreliable results obtained by using ASTM D3039/D3039M are to be attributed again to the oscillating behaviour of the stress-strain curves, especially at the two highest strain rates and to the presence of damage inside the material at strain values within the range prescribed by the standard, as shown in Figure 4.4. At these strain rates, in fact, stress waves propagate within the specimen causing non-uniform stress and strain distributions, leading to inaccurate results as introduced in Section 2.1. Hence, the strain rate is not spatially constant into the tested specimen. As for the calculation of the shear modulus, the use of an interpolation-based methodology is
found to be more robust when an extremely oscillating behaviour characterises the stress-strain curves under study. The selected values of $E_2$ determined by applying the above methodology show a maximum increase of 67% with respect to the value at the reference strain rate.

Among the three expressions selected to interpolate the values of the normalised elastic modulus in the transverse direction, $f_{2}^{(2)}$ gives again the best fit especially at the lowest strain rates, as shown in Figure 4.6. Thus, $f_{2}^{(2)}$ is selected as scaling function for the lamina elastic modulus in transverse direction. The correspondent values of the coefficients to be inserted in Eq. (4.10b) are listed in Table 5.4.

The variation of $f_2$ with the strain rate is presented and compared to that of $f_{12}$ in Figure 5.2. The scaling function presents a steeper behaviour in the transverse direction than in the shear one within the range of $0.00108$ to $10 \text{ s}^{-1}$. Moreover, $f_2$ has higher magnitude than $f_{12}$ over the entire strain rate range considered. Thus, MATCFE exhibits a more pronounced strain rate dependence of the lamina elastic modulus along the transverse (to the fiber) direction than in the shear one, where the material response is purely dominated by the matrix viscous behaviour.

5.1.2 Model verification

The correct functioning of the strain rate model implemented in the user-defined material subroutine MAT80 is verified. The verification is carried out by comparing the values of the lamina elastic moduli obtained from the numerical simulations with those analytically calculated for each strain rate tested.

Shear direction

The stress-strain curves obtained from numerical simulations of a single element loaded in shear at different strain rates as explained in Section 4.3.1 are shown in Figure 5.3. The initial shear modulus increases with increasing strain rates. Good agreement is found between the values calculated analytically for each strain rate and those determined from the numerical simulations, as shown in Table 5.5. In particular, the simulated values at the highest strain rates are overestimated by less than 1% with respect to the analytical ones. Moreover, the shear stress-strain curves show high non-linearity with a slightly pronounced plateau region at a stress
level that increases as the strain rate increases. The above non-linear behaviour is attributed to the progressive damage and plasticity developments within the element. The material is degraded up to a maximum value of damage which is set to 0.999 for the three damage variables ($d_1$, $d_2$ and $d_{12}$) to avoid numerical instabilities. A sudden drop in the load-carrying capabilities is obtained afterwards. Finally, the strength increases with the rate of loading, while the strain to failure decreases. In fact, the stress within the material increases with the strain rate for the same value of strain and so does the damage. Thus, the energy release rate $Y_{12}$ increases as the strain rate increases. Hence, the strain at which the critical value of the energy release rate $Y_{12}$ is reached decreases with the strain rate, i.e. the maximum value of damage at which failure occurs is reached at lower strains for the high strain rates than for the low ones. However, it is worth noting that the above results do not include the delay damage effect introduced in Section 3.3.2 and therefore are not yet completely representative of the real behaviour of MATCFE under dynamic loading.

**Transverse direction**

The response of the single element loaded in tension at different strain rates along the transverse direction as described in Section 4.3.2 is shown in Figure 5.4. The stress-strain curves show a more pronounced increase with the strain rate in the initial lamina elastic modulus with respect to the shear direction. This is due to the higher strain rate dependence of the modulus along the transverse direction.
than along the shear one. Good agreement is again found between the values of the transverse modulus analytically calculated and those determined from numerical simulations at different strain rates, as shown Table 5.6. As for the shear direction, the simulated values of the elastic modulus are found to be overestimated by less than 1% with respect to the analytical ones. Moreover, the maximum overestimation is observed at the highest strain rate tested. The transverse stress-strain curves show a slightly non-linear behaviour of the material which might be attributed to limited plasticity effects. The above behaviour reflects the typical weak response of composite laminates when loaded in tension perpendicularly to the fibers. The transverse strength shows an increase up to 25% with respect to the quasi-static

<table>
<thead>
<tr>
<th>$\varepsilon_{22} [s^{-1}]$</th>
<th>$E_{2}^{\text{Analyt}}$ [GPa]</th>
<th>$E_{2}^{\text{Simul}}$ [GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00108</td>
<td>9.75</td>
<td>9.75</td>
</tr>
<tr>
<td>0.0419</td>
<td>10.22</td>
<td>10.23</td>
</tr>
<tr>
<td>0.287</td>
<td>10.94</td>
<td>10.95</td>
</tr>
<tr>
<td>4.23</td>
<td>12.56</td>
<td>12.57</td>
</tr>
<tr>
<td>44.2</td>
<td>14.59</td>
<td>14.62</td>
</tr>
<tr>
<td>86.9</td>
<td>15.28</td>
<td>15.33</td>
</tr>
</tbody>
</table>
value. Finally, the strain to failure decreases with the strain rate in a more pronounced way than along the shear direction. This behaviour might be caused by a higher increase in the energy release rate with the strain rate along the transverse direction, especially at strain values close to failure. The more pronounced increase of stress (and therefore damage) with the strain rate for the same value of strain when compared to the shear direction is at the basis of the above phenomenon. As for the shear direction, the results discussed above do not include the effect of the delay damage model on the response of the material, but they only take into account the variation of the lamina stiffness due to the material strain rate dependence when the above is loaded at strain rates higher than the quasi-static one.

5.2 Delay damage model

As described in Section 3.3.2, a delay damage model is introduced (in addition to the strain rate model previously presented) in order to model the damage evolution within the ply of the composite laminate along the in-plane shear and transverse direction in case of dynamic loading. First, the results of the numerical simulations carried out without delay effect on the three samples described in Section 4.4 are presented and discussed for both the shear and transverse direction. Then, the delay damage effect is introduced and the results are compared and validated with respect to the experimental data.

5.2.1 Numerical simulations without delay effect

Shear direction

The engineering stress-strain curves obtained from numerical simulations of the samples with \([\pm 45]_{2s}\) layup performed without delay effect at various strain rates are presented in Figure 5.5. The initial stiffness results to be correctly scaled with the strain rate along the lamina in-plane shear direction for all the three samples. In particular, the undamaged lamina shear modulus of the three samples results to be overestimated with respect to the analytical value predicted by using Eq. (3.13b) up to a maximum of 5\% at a strain rate equal to 0.445 s\(^{-1}\) when the simulations are performed on Sample C. Moreover, the strength increases while the strain to failure decreases with the strain rate as discussed in Section 5.1.2. However, the samples show in the entire range of strain rates tested lower strength and strain to failure than those obtained from the experimental tests. A maximum underestimation of strength (up to 15\%) is observed at the highest strain rate tested. Moreover, the underestimation decreases with decreasing strain rate. The strain to failure decreases as the length of the samples increases. This might be related to the procedure used to calculate the shear strain, i.e. by the use of Eq. (4.4) which depends on the longitudinal strain and laminate Poisson’s ratio only. However, a similar path as that observed for the strength obtained from numerical simulations might be identified for the strain to failure. In fact, the lowest underestimation of
5.2. DELAY DAMAGE MODEL

![Shear stress-strain curves](image)

**Figure 5.5.** Comparison between the shear stress-strain curves obtained from numerical simulations performed on the samples under study with \([±45]_2s\) layup at various strain rates without delay effect and the experimental data.

The above strain is again found at the lowest strain rate, i.e., 0.0467 s\(^{-1}\). Thus, the underestimation of strength and strain to failure with respect to the experimental data can be attributed to the high damage rate that develops in the laminates when loaded at strain rates higher than the quasi-static one. The damage propagates faster than expected within the plies along the shear direction according to Eq. (3.4), leading to a premature failure of the samples. This effect increases with increasing strain rate. Thus, the introduction of the delay damage effect is expected to be beneficial for the overall simulated response of the laminates since it will result in an increase in strength and strain to failure, as explained in Section 4.4.1.

**Transverse direction**

The stress-strain curves resulting from the numerical simulations performed on the three samples with \([90]_3s\) layup at different strain rates without the introduction of
any delay effect are shown in Figure 5.6. Rather relevant discrepancies are observed between the results of the samples, especially in terms of strain to failure and initial stiffness. Overall, the strain to failure and strength are overestimated with respect to the experimental data for each strain rate tested. The above overestimations result to be constant with the strain rate. Moreover, by considering the stress-
strain curves obtained from the simulations performed on Sample C, the strain to failure and strength are found to be overestimated up to a maximum of 35 and 61% respectively over the corresponding experimental values.

The overestimation regarding the initial stiffness that affects the response of Sample C even at the lower strain rates tested can be attributed to the propagation of stress waves inside the sample along the longitudinal direction. The loading of the laminate with a constant velocity is found to be the cause of the above waves. In particular, the transition from zero velocity (i.e. sample not loaded, time step zero) to a constant velocity in one time step induces the generation of stress waves that travel back and forth along the sample several times until failure. The propagation of the stress waves leads to non-uniform stress and strain distributions inside the sample, providing unreliable results characterised by oscillations in the stress-strain curves especially at the higher strain rates tested (4.23, 44.2 and 86.9 s\(^{-1}\)) where the effect of the above waves are clearly more pronounced. Figure 5.7 shows the propagation of a stress wave along the laminate longitudinal direction (x-direction). After a first propagation from the side where the velocity is applied to the opposite one (Figure 5.7(a) and (b)), the stress wave travels back in the opposite direction (Figure 5.7(c) and (d)). The same phenomenon is repeated several times before failure of the sample. Thus, numerical simulations performed at the highest strain rates present the same difficulties encountered during the experimental tests performed on MATCFE specimens with [90]_8 layup described in Section 5.1.1. The generation of stress waves could be avoided by applying an increasing velocity in the form of an exponential time-displacement function. By doing this, the sample would not be subjected any more to a shock going from zero velocity to a high one in only one time step. However, the above solution is not applicable within the present thesis work due to the strain rate model implemented for the lamina elastic moduli, which is enabled only when the strain rate in the shear and transverse direction is greater or equal than the corresponding reference values. Moreover, an exponential loading function would provide a non-constant strain rate which would make rather complex the validation against experimental data explicitly obtained at precise strain rate levels.

Due to the overestimation of strain to failure and strength reported from numerical simulations with respect to the experimental data, it is decided not to apply the delay effect along the lamina in-plane transverse direction. The application of the above effect, in fact, would result in a delay of damage leading to higher strength and strain to failure, as described from the effect of \(\tau_i^c\) and \(a_i\) in Section 4.4.

5.2.2 Numerical simulations with delay effect

As discussed in Section 5.2.1, it is decided to introduce the delay effect only in the lamina shear direction where simulations performed without the above effect provided an overall underestimation of strength and strain to failure when compared to the experimental data. The set of delay parameters which resulted in the best fit of the stress-strain curves obtained from simulations performed on Sample C with
Figure 5.7. Variation of the laminate normal stress $\sigma_x$ with the time inside Sample C with [90]$_s$ layup when loaded in tension at 86.9 s$^{-1}$. The propagation of a stress wave is visible at different times: (a) 0.0028 ms, (b) 0.0136 ms, (c) 0.0264 ms and (d) 0.0368 ms
Table 5.7. Value of the delay parameters selected for the lamina shear direction

<table>
<thead>
<tr>
<th>$\tau_{12}$ [ms]</th>
<th>$a_{12}$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.8</td>
<td>11.1</td>
</tr>
</tbody>
</table>

Figure 5.8. Comparison between the shear stress-strain curves obtained from numerical simulations performed on Sample C with $[\pm 45]_{2s}$ layup at various strain rates and delay effect enabled and the experimental data. The delay parameters used for the simulations are: $\tau_{12} = 5.8$ ms and $a_{12} = 11.1$

delay effect enabled to those resulting from the experimental tests is presented in Table 5.7. Figure 5.8 shows the comparison between the stress-strain curves determined from simulations carried out with the above values of $\tau_{12}$ and $a_{12}$, those resulting from simulations without delay effect and the experimental data at different strain rates. The effect of the delay on the response of the laminate increases with increasing strain rate. This can be attributed to the fact that the delay effect acts on the damage rate by limiting it below a maximum value as described in Sec-
The damage rate is higher at high strain rates than at low ones, resulting in a more pronounced limitation and consequently in a higher delay of damage evolution in the former loading scenario. The introduction of the delay effect leads to an increase of strength and to a smoother failure response of the laminate, validating the study carried out in Section 4.4.1 about the effect of $\tau_{12}$ and $a_{12}$ on the response of the laminate when the delay of damage is enabled. In particular, an increase of strength up to 18% with respect to the results from the simulations without delay effect is found at the highest strain rate, leading to an overestimation of less than 1% against the experimental data. The lower strain rates (0.0467 and 0.445 s$^{-1}$) present instead a rather low increase in strength and strain to failure which are still found to be underestimated with respect to the experimental data. It is worth noticing that the identification of the delay parameters was performed at the highest strain rate tested as explained in Section 4.4.1. A parameter identification carried out at a lower strain rate, in fact, would have resulted in an overestimated response of the composite laminate at the higher ones.

The response of Sample C is found to be sensitive to the damage limit $d_{12}^{lim}$ implemented in the strain rate model to stop the scaling of the lamina elastic modulus along the shear direction, as described in Section 4.2. In particular, the undamaged stiffness of the laminate increases as the damage limit increases, as shown in Figure 5.9 where only the initial part of the stress-strain curves is reported. The oscillations that characterise the part of the curves under study (which are not clearly visible from Figure 5.8) are to be attributed to stress waves propagating inside the laminate. However, the propagation of the above waves gradually decreases and
finally ends at strain values approximatively equal to 30\% of the strain to failure leading to a dynamic stress equilibrium within the sample and consequently to reliable results in terms of stress and strain distributions. It is worth to be noticed that the above sensitivity affects the entire range of strain rates tested as well as the simulations performed to study the response of the laminate along the lamina transverse direction by using [90]_{8} samples. Thus, the values of the damage limit along the in-plane shear and transverse directions in the lamina local coordinate system implemented in the strain rate model (i.e 0.00003 and 0.00001 respectively) are calibrated on Sample C (i.e. full-size specimen) to best fit the experimental data, as already introduced in Section 4.2.

### 5.3 Dynamic three-point bending simulation

The dynamic three-point bending simulation described in Section 4.5 stopped when the vertical displacement reached 37 mm (after 3.9 ms from the start of the simulation) due to an error regarding negative volume of one element. The evolution in time of the damage variable $d_{2}$ along the lamina in-plane transverse direction until that point is shown in Figure 5.10. First, the damage mainly propagates under the pusher and longitudinally along the bar. This can be attributed to an initial stress wave that travels in the longitudinal direction inside the laminate (Figure 5.10(a) and (b)). Then, the damage is found to be concentrated in the zone of contact between the pusher and the bar (especially at the edges) and in the one between the supports and the bar. An increase in the magnitude of the damage is observed as the midspan deflection of the laminate increases.

The results of the dynamic simulation are compared to those obtained from the corresponding experimental test and the quasi-static simulation in terms of force-displacement curves, as shown in Figure 5.11. In particular, the curve resulting from the experimental test was filtered to eliminate excessive noise and vibrations that could obscure the results of the test. Moreover, the quasi-static simulation was performed without any strain rate model implemented and with the delay damage model disabled for the MAT80 eight-node solid elements used to model the plies of the laminate. An initial peak in force is observed in the response of the bar from the dynamic simulation for displacement values lower than 1 mm. The above peak can be attributed to the propagation of a stress wave inside the bar as previously introduced. The response of the laminate then stabilises for higher values of the vertical midspan displacement. A clear increase in the initial stiffness and strength is observed with respect to the force-displacement curve obtained from the quasi-static simulation. The increase in initial stiffness is to be attributed to the effect of the strain rate model which was implemented for the MAT80 solid elements. Similarly, the increase in strength is mainly due to the effect of the delay on the evolution of damage within the plies of the laminate. Finally, rather good agreement is found between the results of the dynamic simulation and those obtained from the dynamic experimental test. In particular, good fit is observed for displacement values be-
Figure 5.10. Variation with the time of the damage variable $d_2$ along the lamina in-plane transverse direction: (a) 0.9 ms, (b) 1.9 ms, (c) 2.9 ms and (d) 3.9 ms
5.3. DYNAMIC THREE-POINT BENDING SIMULATION

Figure 5.11. Comparison between three-point bending force-displacement (vertical) curves obtained from: dynamic simulation, dynamic experimental test and quasi-static simulation.

between 5 and 37 mm. The overall response of the laminate results to be rather well captured by the simulation performed under dynamic loading conditions. However, an overestimation of initial stiffness is observed in the initial part of the curve obtained from the simulation performed dynamically which was clearly affected by the propagation of a stress wave. An improved handling of the above dynamic effect within the FE solver is therefore necessary to correctly capture the initial response of the laminate.
Chapter 6

Conclusions

Within the present thesis, a literature review on the strain rate dependent behaviour of CFRP under tension loading was performed. Although most of the published data reported a clear strain dependence of CFRP, no general agreement was found between the results of the works carried out. That necessarily led to a characterisation of the rate dependent behaviour of the material under study in the present work. Moreover, a review regarding the modelling strategies used in the literature to simulate the mechanical response of composite laminates in case of dynamic loading, e.g. impact and crashes, was carried out in order to identify a suitable strategy.

The strain rate dependent behaviour of the high-performance carbon/epoxy composite MATCFE when loaded in tension was investigated showing an increase of the lamina elastic modulus along the in-plane shear and transverse directions, i.e. 12 and 22-direction respectively, as the strain rate increased. A material-characteristic strain rate model was developed to predict the variation of the lamina stiffness with the strain rate along the above directions. The strain rate model was implemented in the FE explicit commercial code PAM-CRASH for eight-node solid elements. Those elements were first modelled by using a modification of the Ladevèze damage model for the intra-laminar damage evolution in quasi-static loading conditions. A delay damage model was then used to model the mechanical behaviour of composite laminates in case of dynamic loading. The effect of delay on the damage evolution within the plies of the laminate was investigated in tension. An overall increase in strength and strain to failure was observed from the numerical simulations when the above effect was applied. Thus, the delay parameters were identified at the coupon level for the lamina shear and transverse directions separately.

The strain rate and delay damage models were validated against experimental data from a patch to a coupon level. Rather good agreement was found in terms of stiffness and strength between the results from the numerical simulations and those from the experimental tests. The strain rate model was proved to be sensitive to the damage limits set to stop the scaling of the lamina stiffness. Moreover, severe dynamic effects, such as stress waves, affected the response of the laminate along the lamina transverse direction at the coupon level when loaded at high strain rate,
leading to unreliable results.

A dynamic three-point bending simulation was performed to evaluate the effect of the strain rate and delay damage models on the response of the laminate at the sub-component level. Encouraging results were obtained with respect to the corresponding quasi-static simulation due to a clear increase in stiffness and strength. Moreover, rather good agreement was found with the results from the dynamic experimental test performed on the same set-up as that used for the dynamic simulation. In particular, the overall response of the laminate was well captured.

Dynamic effects were again found to affect the initial response of the material.

The strain rate model developed was demonstrated to correctly predict the variation of the lamina elastic moduli with the strain rate. A modification of the above model is necessary in order to scale the lamina stiffness until complete failure of the laminate. Moreover, a more extensive characterisation of the strain rate dependent behaviour of MATCFE at high strain rates is needed to improve the strain rate model implemented within the present thesis work. Finally, an identification of the strain rate dependence of the mechanical properties of the interface between the plies should also be performed and a material-characteristic strain rate model developed in order to improve the prediction of the mechanical behaviour of the composite laminates under dynamic loading.
References


REFERENCES


Appendix A: determination of $f_{12}$

```matlab
clc
clear
close all
set(0,'defaulttextinterpreter','latex')

%% DETERMINATION OF SCALING FUNCTION FOR ELASTIC MODULI: SHEAR DIRECTION

% Author: Edoardo Taddei
% Date: March 2017
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% 3001 Leuven, Belgium

% The following script performs the determination of
% the lamina elastic modulus along the in-plane shear
% direction (12-direction) at different strain rates for
% xxxx from stress-strain curves experimentally
% determined on [+45/-45]_2s specimens loaded along the
% 0°-direction.

% Two methods are used:
% - calculation of the chord modulus of elasticity
% in the in-plane shear direction according to
% ASTM D3518/D3518M
% - determination of a polynomial regression for each
% stress-strain curve and calculation of their slope
% at the origin of the axes

% Finally, the script calculates the parameters for
% three different scaling functions by using a
% non-linear least-squares data fitting algorithm

%% Calculation of G12
```
% Loading the stress-strain values from % the .xls file

CP=xlsread('170310_CE45_StressStrainOverviewData.xlsx','
   2:200');

ncolumn=ones(1,size(CP,2));
CP=mat2cell(CP,length(CP),ncolumn);

% Removing the NaN terms from each column of the .xls file

for i=1:size(CP,2)
    CP{i}(isnan(CP{i}))=[];
end

% Selecting the longitudinal strains from the .xls file and calculating the shear strains [-]

gamma=cell(1,size(CP,2)/2);
for i=1:size(CP,2)/2
    j=2*i-1;
    gamma{i}=CP{j}+(-CP{j}*0.75);
end

% Selecting the longitudinal stresses from the .xls file and calculating the shear stresses [MPa]

tau=cell(1,size(CP,2)/2);
for i=1:size(CP,2)/2
    j=2*i;
    tau{i}=CP{j}/2;
end

% Determining the shear strain range of 0.004+-0.0002 % starting from 0.0015

gammae_1500=cell(1,length(gamma));
for i=1:length(gamma)
gammae_1500{i}=gamma{i}(gamma{i}>0.00147 & gamma{i}<0.0057);
end
% Determining the corresponding stresses

\[ \text{taue}_{1500} = \text{cell}(1, \text{length}(...)) \]

\[ \text{for } i = 1 : \text{length}(...), \]

\[ \text{taue}_{1500}(i) = \text{tau}(i)(\text{gamma}(i) > 0.00147 \& \text{gamma}(i) < 0.0057); \]

\[ \text{end} \]

% Calculating G12 [MPa] within the strain interval
% 0.0015 - 0.0055 according to ASTM D3518/D3518M

\[ \text{G12}_{1500} = \text{zeros}(1, \text{length}(...)); \]

\[ \text{for } i = 1 : \text{length}(...), \]

\[ \text{G12}_{1500}(i) = (\max(...(i)) - \min(...(i))) / (\max(...(i)) - \min(...(i))); \]

\[ \text{end} \]

% Determining the shear strain range of 0.004+/-0.0002
% starting from 0.002

\[ \text{gammae}_{2000} = \text{cell}(1, \text{length}(...)); \]

\[ \text{for } i = 1 : \text{length}(...), \]

\[ \text{gammae}_{2000}(i) = \text{gamma}(i)(\text{gamma}(i) > 0.00197 \& \text{gamma}(i) < 0.00627); \]

\[ \text{end} \]

% Determining the corresponding stresses

\[ \text{taue}_{2000} = \text{cell}(1, \text{length}(...)); \]

\[ \text{for } i = 1 : \text{length}(...), \]

\[ \text{taue}_{2000}(i) = \text{tau}(i)(\text{gamma}(i) > 0.00197 \& \text{gamma}(i) < 0.00627); \]

% calibrate these two values

\[ \text{end} \]

% Calculating G12 [MPa] within the strain interval
% 0.002 - 0.006 according to ASTM D3518/D3518M

\[ \text{G12}_{2000} = \text{zeros}(1, \text{length}(...)); \]

\[ \text{for } i = 1 : \text{length}(...), \]

\[ \text{G12}_{2000}(i) = (\max(...(i)) - \min(...(i))) / (\max(...(i)) - \min(...(i))); \]

\[ \text{end} \]

% Calculating G12 [MPa] by using polynomial refressions
% of the stress-strain curves
par = cell(1, length(gamma));
G12_polyfit = zeros(1, length(gamma));
for i = 1:length(gamma)
    par{i} = polyfit(gamma{i}, tau{i}, 3);
    G12_polyfit(i) = par{i}(3)/1000;
end

% Loading the values of the strain rate tested [1/s]
edot = [0.00107, 0.0467, 0.445, 2.94];

% Setting the reference value of G12
% as the lowest one (quasi-static)
G12_1500_ref = G12_1500(1);
G12_2000_ref = G12_2000(1);
G12_polyfit_ref = G12_polyfit(1);

% Determination of f12
% Automating the procedure for the three different
% set of values of G12
G12_tot = [G12_1500, G12_2000, G12_polyfit];
G12_ref_tot = [G12_1500_ref, G12_2000_ref, G12_polyfit_ref];

x0 = {[0.00001, 0.1], [0.00001, 0.1], [0.00001, 0.1]};
y0 = {[1, 1, 1], [1, 1, 1], [1, 1, 1]};
z0 = {[0.8, 0.04, 0.1], [0.8, 0.04, 0.1], [0.8, 0.04, 0.1]};

x = cell(1, size(G12_tot, 2));
y = cell(1, size(G12_tot, 2));
z = cell(1, size(G12_tot, 2));
SF1 = cell(1, size(G12_tot, 2));
SF2 = cell(1, size(G12_tot, 2));
SF3 = cell(1, size(G12_tot, 2));
for i = 1:size(G12_tot, 2)
    x10 = x0{i};
y20 = y0{i};
z30 = z0{i};
end

% Calling for non-linear least-squares data fitting
159  % algorithm: SF1
160
161  x{i} = lsqcurvefit(@scalingf1,x10,edot,G12_tot{i}/
162         G12_ref_tot(i));
163  SF1{i}=scalingf1(x{i},linspace(edot(1),edot(end),200000))
164          ;
165
166  % Calling for non-linear least-squares data fitting
167  % algorithm: SF2
168
169  y{i} = lsqcurvefit(@scalingf2,y20,edot,G12_tot{i}/
170         G12_ref_tot(i));
171  SF2{i}=scalingf2(y{i},linspace(edot(1),edot(end),200000))
172          ;
173
174  % Calling for non-linear least-squares data fitting
175  % algorithm: SF3
176
177  z{i} = lsqcurvefit(@scalingf3,z30,edot,G12_tot{i}/
178         G12_ref_tot(i));
179  SF3{i}=scalingf3(z{i},linspace(edot(1),edot(end),200000))
180          ;
181  end
182
183  % where:
184
1     % scalingf1=f12^2
2     function f = scalingf1(x,edot)
3     edot_ref=edot(1);
4     C=x(1);
5     m=x(2);
6     f=(1+C*log(edot/edot_ref).^m);
7
8     % scalingf2=f12^1
9     function g = scalingf2(y,edot)
10    edot_ref=edot(1);
11    D=y(1);
12    D
n=y(2);
p=y(3);

g=p.*(1+D*(log(edot/edot_ref).^n));

% scalingf3=f12^(3)

function h = scalingf3(z,edot)
edot_ref=edot(1);
a=z(1);
D=z(2);
n=z(3);

h=a+D*log10(edot/edot_ref)+log10(n);
Appendix B: determination of $f_2$

```matlab
clc
clear
close all
set(0,'defaulttextinterpreter','latex')

%% DETERMINATION OF SCALING FUNCTION FOR ELASTIC MODULI: TRANSVERSE DIRECTION

% Author: Edoardo Taddei
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% The following script performs the determination of the lamina elastic modulus along the in-plane transverse direction (22-direction) at different strain rates for xxxx from stress-strain curves experimentally determined on [90]_8 specimens loaded along the 0°-direction.

% Two methods are used:
% - determination of a polynomial regression for each stress-strain curve and calculation of their slope at the origin of the axes
% - calculation of the tensile chord modulus of elasticity according to ASTM D3039/D3039M

% Finally, the script calculates the parameters for three different scaling functions by using a non-linear least-squares data fitting algorithm

%% Calculation of E2
```

79
% Loading the stress-strain values from the .xls file
UD = xlsread('170310_CE90_StressStrainOverviewData.xlsx', '2:200');
ncolumn = ones(1, size(UD, 2));
UD = mat2cell(UD, length(UD), ncolumn);

% Removing the NaN terms from each column of the .xls file
for i = 1:size(UD, 2)
    UD{i}(isnan(UD{i})) = [];
end

% Selecting the longitudinal strains from the .xls file [-]
eps = cell(1, size(UD, 2)/2);
for i = 1:size(UD, 2)/2
    j = 2*i - 1;
    eps{i} = UD{j};
end

% Selecting the longitudinal stresses from the .xls file [MPa]
sig = cell(1, size(UD, 2)/2);
for i = 1:size(UD, 2)/2
    j = 2*i;
    sig{i} = UD{j};
end

% Calculating a second-order regression for the first four curves
p = cell(1, 4);
E2 = zeros(1, 4);
for i = 1:4
    p{i} = polyfit(eps{i}, sig{i}, 2);
end

% Calculating the lamina transverse elastic modulus as the value of the derivative at the origin
E2(i) = p(i)(2)/1000;

% Calculating a second-order regression for the 5th and 6th stress-strain curve
eps5 = eps{5};
fun5 = @(x5, eps5) x5(1) * eps5^2 + x5(2) * eps5 - 1;
x0 = [1 1];
x5 = lsqcurvefit(fun5, x0, eps{5}, sig{5});

eps6 = eps{6};
fun6 = @(x6, eps6) x6(1) * eps6^2 + x6(2) * eps6 - 1.3;
x0 = [1 1];
x6 = lsqcurvefit(fun6, x0, eps{6}, sig{6});

% Adding the E2 values for the 5th and 6th curve to those already determined
E2 = [E2 x5(2)/1000 x6(2)/1000];

% Setting the reference value of the transverse elastic modulus as the lowest one (quasi-static)
E2_ref = E2(1);

% Determining the strain values included within 0.001 and 0.003 (only for the first four curves)
epse_1000 = cell(1, 4);
for i = 1:6
    epse_1000{i} = eps{i}(eps{i} > 0.0009 & eps{i} < 0.0031);
end

% Determining the corresponding stresses
sige_1000 = cell(1, 4);
for i = 1:4
    sige_1000{i} = sig{i}(eps{i} > 0.0009 & eps{i} < 0.0031);
end

% Determining the stresses for the last two curves
fun5cell = fun5(x5, linspace(0, eps{5}(end)));  
fun6cell = fun6(x6, linspace(0, eps{6}(end)));  
sige_1000{5}= fun5cell(34:end);  
sige_1000{6}= fun6cell(31:end);  

% Calculating E2 [MPa] within the strain interval  
% 0.001-0.003 according to ASTM D3039/3039M  
E2_1000 = zeros(1,6);  
for i=1:6  
    E2_1000(i) = (max(sige_1000{i}) - min(sige_1000{i}))/1000;  
end  

% Loading the values of the strain rate tested [1/s]  
edot90 = [0.00108 0.0419 0.287 4.23 44.2 86.9];  

% Setting the reference value of the strain rate as the lowest one (quasi-static)  
edot90_ref = edot90(1);  

%% Determination of f2  
% Calling for non-linear least-squares data fitting  
% algorithm: SF1 (sfun1=f2^2)  
sfun1 = @(a, edot90) (1+a(1)*log(edot90/edot90_ref).^a(2));  
a0 = [0.00001, 0.1];  
a = lsqcurvefit(sfun1, a0, edot90, E2/E2_ref);  
SF1 = sfun1(a, linspace(edot90(1), edot90(end), 60000));  

% Calling for non-linear least-squares data fitting  
% algorithm: SF2 (sfun2=f2^1)  
sfun2 = @(b, edot90) (b(3).*((1+b(1))*(log(edot90/edot90_ref).^b(2))));  
b0 = [1, 1, 1];  
b = lsqcurvefit(sfun2, b0, edot90, E2/E2_ref);  
SF2 = sfun2(b, linspace(edot90(1), edot90(end), 60000));  

% Calling for non-linear least-squares data fitting  
% algorithm: SF3 (sfun3=f2^3)
sfun3=@(c, edot90)(c(1)+c(2)*log10(edot90/edot90_ref)+
    log10(c(3)));
c0=[0.4, 0.04, 0.1];
c=lsqcurvefit(sfun3,c0,edot90,E2/E2_ref);
SF3=sfun3(c,linspace(edot90(1),edot90(end),60000));