Of Pipes and Bends

by

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May 2018
Technical Reports
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Cover: Chaos running after order: turbulent slug coexisting with a nonlinear travelling wave. The flow is in a toroidal pipe with curvature 0.022 and Reynolds number $Re = 5050$. The travelling wave is the result of a Hopf bifurcation at $Re = 5032$; it is stable and has a finite basin of attraction. The slug is a symptom of subcritical transition, it expands and suppresses the wave, but it eventually dissipates and the wave is restored. Isocontours of opposite values of streamwise velocity are depicted in red and blue, while the white isocontours are of negative $\lambda_2$. The fluid is flowing from right to left.

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“Well, we ain’t got any,” George exploded.
“Whatever we ain’t got, that’s what you want. God a’mighty.”

— John Steinbeck
Of Mice and Men
Of Pipes and Bends

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Abstract
This work is concerned with the transition to turbulence of the flow in bent pipes, but it also includes an analysis of large-scale turbulent structures and their use for flow control.

The flow in a toroidal pipe is selected as it represents the common asymptotic limit between spatially developing and helical pipes. The study starts with a characterisation of the laminar flow as a function of curvature and the Reynolds number $Re$, since the so-called Dean number is found to be of little use except for infinitesimally low curvatures. It is found that the flow is modally unstable and undergoes a Hopf bifurcation for any curvature greater than zero. The bifurcation is studied in detail, and an effort to connect this modal instability with the linearly stable straight pipe is also presented.

This flow is not only modally unstable, but undergoes subcritical transition at low curvatures. This scenario is found to bear similarities to straight pipes, but also fundamental differences such as weaker turbulent structures and the apparent absence of puff splitting. Toroidal pipe flow is peculiar, in that it is one of the few fluid flows presenting both sub- and supercritical transition to turbulence; the critical point where the two scenarios meet is therefore of utmost interest. It is found that a bifurcation cascade and featureless turbulence actually coexist for a range of curvature and $Re$, and the attractors of the respective structures have a small but finite basin of attraction.

In $90^°$ bent pipes at higher $Re$ large-scale flow structures cause an oscillatory motion known as swirl-switching. Three-dimensional proper orthogonal decomposition is used to determine the cause of this phenomenon: a wave-like structure which is generated in the bent section, and is possibly a remnant of a low-$Re$ instability.

The final part of the thesis has a different objective: to reduce the turbulent frictional drag on the walls of a channel by employing a control strategy independent of $Re$-dependent turbulent scales, initially proposed by Schoppa & Hussain [Phys. Fluids 10:1049–1051 (1998)]. Results show that the original method only gives rise to transient drag reduction while a revised version is capable of sustained drag reduction of up to 18%. However, the effectiveness of this control decreases rapidly as the Reynolds number is increased, and the only possibility for high-$Re$ applications is to use impractically small actuators.

Key words: nonlinear instability, bifurcation, flow control
Rör och krökningar

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Sammanfattning
Detta arbete behandlar omslaget till turbulens hos strömmingen i krökta rör, men det inkluderar även en analys av storskaliga turbulenta strukturer samt dess användning inom strömningskontroll.


Strömmningen är inte bara modalt instabil, utan genomgår även ett subkritiskt omslag för låga kurvaturer. Detta scenario visar sig ha likheter med raka rör, men även fundamentalta skillnader såsom svagare turbulenta strukturer samt en till synes frånvarande delning av turbulenta puffar. Strömmningen i torusformade rör är besynnerlig i det avseendet att det är en av de få strömmningar som uppvisar så väl sub- som superkritiskt omslag till turbulens; den kritiska punkten där de två scenarierna möts är därför av ytterst intresse. Det konstateras att en bifurkationskaskad faktiskt samexisterar med turbulens utan särdrag för ett spann av $Re$-tal, och att attraktorerna av de respektive strukturerna har en liten men ändlig attraktionsbassäng.


Nyckelord: icke-linjär instabilitet, bifurkation, strömningskontroll
Preface

This thesis deals with transition to turbulence and skin-friction-drag reduction. A brief introduction and summary of the results is presented in the first part. The second part contains the journal articles written during this work. The papers are adjusted to comply with the present thesis format for consistency, but their contents have not been altered as compared with their original counterparts.


May 2018, Stockholm

*Jacopo Canton*
Division of work between authors
The main advisor for the project is Dr. Philipp Schlatter (PS). Dr. Ramis Örlü (RÖ) acts as co-advisor.

Paper 1. The code was developed by Jacopo Canton (JC) who also performed the computations. The paper was written by JC with feedback from PS and RÖ.

Paper 2. The stability code was developed by JC, the nonlinear simulation code by Azad Noorani and JC. All computations were performed by JC. The paper was written by JC with feedback from PS and RÖ.

Paper 3. The second stability code was developed by JC with help from Enrico Rinaldi (ER) and PS. The computations were done by JC, who wrote the paper with feedback from PS.

Paper 4. The codes were developed by JC and ER, the code for the nonlinear adjoints was based on a code written by Oana Marin (OM) and Michel Schanen. ER and JC also performed the computations and wrote the paper, with feedback from PS.

Paper 5. The codes were written by JC who also did the computations. The paper was written by JC with feedback from ER, PS and RÖ.

Paper 6. This work was started during the Master’s Thesis of Lorenz Hufnagel (LH) who was supervised by JC and PS. The code was developed by LH and JC starting from a code written by OM; LH and JC performed the simulations. The paper was written by JC with feedback from LH, OM, EM, PS and RÖ.

Paper 7. The code was developed by PS and JC, JC performed the simulations. The paper was written by JC with feedback from PS, RÖ and the external co-authors.

Paper 8. JC performed the simulations using the code developed by PS and JC. The paper was written by JC with feedback from Cheng Chin, PS and RÖ.
Other publications
The following papers, although related, are not included in this thesis.


Conferences
Part of the work in this thesis has been presented at the following international conferences. The presenting author is underlined.


J. Canton, R. Örlü, C. Chin & P. Schlatter. Reynolds number dependence of large-scale friction control in turbulent channel flow. ERCOFTAC


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Part I

Overview and summary
Chapter 1

Introduction

Fluid mechanics is a vast and fascinating subject and ultimately a field of classical physics. Differently from other branches of classical mechanics, though, it still presents unanswered questions and open problems. Some are of fundamental nature such as why and how does a flow transition from a laminar to a turbulent state, while others echo in everyday life: how can an aeroplane fly or fluid be transported while consuming less energy?

Owing to the inherent difficulty of describing the complex motion of a continuum, fluid mechanics is approached from different routes depending on the nature of the problem at hand. Analytical, i.e. exact, solutions can, in fact, be computed only for particularly simple cases while most problems require a numerical solution or experimental approach. Fluid mechanics itself is subdivided into branches: fluids can be regarded as Newtonian or non-Newtonian, viscous or inviscid, compressible or incompressible; flows can also be categorised as wall-bounded or unbounded, laminar or turbulent; and these are but a few examples. The present work deals with Newtonian, viscous, incompressible fluids in wall-bounded flows and sheds some light on the laminar, transitional and the turbulent regimes.

A flow is deemed laminar when it is either steady, i.e. time-invariant, or its unsteady motion appears to be ordered, easily described; conversely, turbulent flows are chaotic and comprise a wide range of spatial and temporal scales which interact with each other and render the description of the motion much more complicated. Traditionally, two different methodologies are employed to approach each regime: laminar flows are considered ‘simple enough’ to be described from a deterministic point of view and dynamical system theory is often used to analyse this regime. Turbulent flows, on the other hand, are too complex for a deterministic description, hence they are treated as a random process and are usually studied with tools developed by probability theory.

Figure 1.1 illustrates the difference between laminar and turbulent regimes: the plume of smoke rising from a pipe is initially laminar and rises in a quasi-rectilinear direction, it then becomes unstable and forms a couple of vortex rings before transitioning towards a turbulent state.
Figure 1.1: Plume of smoke rising from a pipe. The picture illustrates different flow regimes: laminar (close to the pipe), transitional (at the height of the vortex rings), and turbulent (towards the top of the frame). Own photograph.
The equations describing the motion of a Newtonian, viscous, incompressible fluid, be it in the laminar or turbulent regime, are the incompressible Navier–Stokes equations, written here in non-dimensional form:

\[
\frac{\partial u}{\partial t} + (u \cdot \nabla) u - \frac{1}{Re} \nabla^2 u + \nabla p = f, \quad (1.1a)
\]

\[
\nabla \cdot u = 0. \quad (1.1b)
\]

The unknowns are the velocity and pressure fields \((u, p)\), \(f\) represents a force field and \(Re\) is a non-dimensional group named Reynolds number. Clearly, the equations also need to be provided with appropriate initial and boundary conditions, which depend on the geometry and the flow being examined.

The Reynolds number can be interpreted as the ratio between the orders of magnitude of the nonlinear (inertial) and viscous terms; this explains how this system of equations can describe both kinds of flows: the laminar regime, dominated by viscosity, and the turbulent regime, where nonlinear effects are prevalent (see, for example, Batchelor 2000).

The Navier–Stokes equations represent a nonlinear dynamical system describing the evolution of \((u, p)\). As it often happens with nonlinear dynamical systems, the qualitative nature of the solution can be significantly altered by the parameters that describe the system. Moreover, although the equations describe a deterministic phenomenon, \(i.e.\) one where any subsequent state can be exactly determined provided the knowledge of an initial state, the evolution of the solution can be highly sensitive to the initial condition. This is another qualitative difference between laminar and turbulent regimes: while the former is unaffected by this problem, the smallest uncertainty on the initial datum for a turbulent flow entails the impossibility of the exact knowledge of its evolution (see, for example, Strogatz 1994; Kuznetsov 2004).

The chapters that follow present a summary of the thesis, where these concepts are applied to the flow in bent pipes (chapters 2-5) and the turbulent flow in a channel (chapter 6). Chapter 7 presents a summary of the journal articles included in Part II. Finally, this overview concludes in chapter 8 which includes a summary and an outlook on future work. Part II of this manuscript forms a collection of the journal articles written on these subjects.

1.1. Supercritical transition and hydrodynamic stability

Hydrodynamic stability is concerned with the discovery and analysis of the mechanisms that lead a flow from a laminar state through the first stages of transition towards turbulence. This section provides a brief overview on some key concepts used in chapter 2. Exhaustive discussions on the subject and on dynamical systems in general can be found, for instance, in the books by Strogatz (1994) and Schmid & Henningson (2001).

In a very schematic way, and considering the Reynolds number as the sole parameter for the Navier–Stokes equations, the initial evolution of some fluid mechanical systems can be described as follows:
For a small enough $Re$ there exists only one steady, stable solution. Here *stable* means that any perturbation to this solution will, eventually, disappear, either due to convection or to diffusion. These steady states comprise most of the solutions to the Navier–Stokes equations that can be computed analytically. When the solution cannot be obtained analytically, as is the case for the flow in a toroidal pipe presented in this thesis, the equations have to be solved numerically. Two common approaches in this case are employing Newton’s method or integrating the equations in time until convergence to the steady state is reached. In some simple cases, for very low Reynolds numbers, the nonlinear term in the equations has such a small influence that it can be completely neglected; the flows belonging to this category are called Stokes flows.

For larger values of $Re$ the initial steady state becomes unstable and another solution (or more than one) appears.

If this new solution is a steady state, then a steady state bifurcation has occurred. This scenario, with the Reynolds number replaced by the curvature, occurs in bent pipes: for low $Re$ the flow in a straight pipe is described by the axisymmetric Poiseuille solution (Batchelor 2000), when the pipe is bent into a torus this solution becomes unstable and is substituted by a different steady state, described analytically by Dean (1927), and provided with only a mirror symmetry.

Instead of a steady state a time-periodic solution can appear, in this case a Hopf bifurcation has occurred. When the bifurcation is supercritical the flow settles onto a stable limit cycle where it oscillates at one determined
frequency. Toroidal pipes present this scenario as well: when the Reynolds number is increased the steady state becomes unstable and the flow undergoes a Hopf bifurcation. The nature of this bifurcation and the exact values of the parameters involved are discussed in chapter 2.

Typically these systems then undergo a period-doubling cascade (see, e.g. Strogatz 1994; Kuznetsov 2004) or follow a Ruelle–Takens–Newhouse route to chaos (Ruelle & Takens 1971; Newhouse et al. 1978). A period-doubling cascade is a succession of bifurcations where the system moves to new periodic attractors with twice the period of the previous limit cycle. An example of period-doubling is illustrated in figure 1.2 which presents the bifurcation diagram for the Rössler system (Rössler 1976). This is a system of three differential equations, with just one nonlinear term, designed by Otto Rössler to exhibit the simplest possible strange attractor. The flow inside of a torus, instead, undergoes a Ruelle–Takens–Newhouse route to turbulence (Ruelle & Takens 1971; Newhouse et al. 1978), where a succession of Hopf bifurcations moves the system to quasi-periodic attractors with one additional period per each bifurcation. This is the main topic in chapter 2 and papers 2, 3 and 5.

1.2. Subcritical transition and intermittency

The occurrence of a bifurcation can be predicted by a linear stability analysis, which allows the determination of the bifurcation point and a description of the flow following the instability. In some cases, though, a different scenario can occur: the stable solution may not undergo any bifurcation, and the steady state, which was the only solution for low Re, can remain stable for any Reynolds number. In these cases other solutions can appear in the form of more complicated, at times chaotic, attractors. Separating the steady solution from this new attractor there can be a saddle boundary, so-called edge of chaos (Skufca et al. 2006). This is the scenario observed in many wall-bounded shear flows such as straight pipes (see, among others, Wygnanski & Champagne 1973; Wygnanski et al. 1975; Hof et al. 2004; Avila et al. 2011; Barkley et al. 2015; Barkley 2016), channel flow, plane Couette flow (for a review see Manneville 2016).

Straight pipe flow, for example, has a linearly stable laminar velocity profile (Meseguer & Trefethen 2003), i.e. all small perturbations decay and no critical Reynolds number can be defined using linear theory. However, experiments and simulations show that subcritical transition to turbulence can occur for $Re \gtrsim 1700$ if perturbations are sufficiently large. Several experimental studies have detailed the transition scenario and found a distinction between turbulent patches that do not grow in size, so-called puffs, and patches that expand in the surrounding laminar flow, slugs (see, e.g. Lindgren 1969; Wygnanski & Champagne 1973; Wygnanski et al. 1975; and the review by Mullin 2011). Only recently a statistical description of the intermittent flow in pipes has provided an accurate estimate of a critical Reynolds number, $Re \approx 2040$ (Avila et al. 2011). Below this threshold, the probability of a puff decaying outweighs
the probability of a new puff being generated through a splitting mechanism, depicted in figure 1.3. On the other hand, if the Reynolds number is higher than 2040, the probability of splitting rapidly increases and puffs proliferate. If \( Re \gtrsim 2300 \) puffs turn into slugs, which rapidly fill the pipe thereby marking the onset of sustained turbulence.

A similar transition scenario takes place in toroidal pipes for low curvatures, this is the main topic of chapter 3 and papers 4 and 5.

1.3. A description of the flow

As a first step in the investigation of the flow inside bent pipes, the focus is on an idealised toroidal setup, depicted in figure 1.4. This shape, albeit rarely encountered in industrial applications, is representative of a canonical flow. This makes it relevant for the research on the onset of turbulence since it deviates from a straight pipe by the addition of one parameter only: the curvature. Moreover, the torus constitutes the common asymptotic limit of two flow cases: the curved (spatially developing) pipe and the helical pipe. Analysing a toroidal pipe allows us to identify the effect that the curvature has on the flow, separating it from that of the torsion (as in helical coils) and the developing length (as in spatially developing curved pipes).

Following the first experimental investigations by Eustice (1910, 1911), Dean (1927, 1928) analysed this flow analytically. In both of his papers the curvature of the pipe, defined as the ratio between pipe and torus radii (\( \delta = R_p/R_t \), see figure 1.4), was assumed to be very small. By means of this and successive approximations Dean was able to derive a solution to the incompressible Navier–Stokes equations. Dean’s approximate solution depends on a single parameter, called Dean number and defined as \( De = Re\sqrt{\delta} \). Dean was also the first to demonstrate the presence of secondary motion and found it to be in the form of two counter-rotating vortices that were then given his name (see Kalpakli Vester et al. 2016, for an extensive review with an historical perspective).
1.3. A description of the flow

Figure 1.4: **Left:** Sketch of the toroidal pipe with curvature $\delta = R_p/R_t = 0.3$. The ‘equatorial’ plane of the torus corresponds to the $x - y$ plane. **Right:** Corresponding base flow for $Re = 3379$, streamwise (top) and in-plane (bottom) velocity magnitude. Reprinted from Canton *et al.* (2017b) with permission from Elsevier.

Later, Adler (1934), Keulegan & Beij (1937) and other experimentalists proceeded to measure the frictional resistance offered by the fluid when flowing through a curved pipe. The most notable of these works were included in a seminal paper by Ito (1959). One of the major findings presented in this paper is that the Fanning friction factor for the laminar flow scales with the Dean number up to $De = 2 \times 10^3$. As it will be shown in paper 1, this is actually not entirely correct.

Di Piazza & Ciofalo (2011) were the first to present an analysis on instability encountered by this flow. These authors investigated two values of curvature (0.1 and 0.3) by direct numerical simulation and observed, in both cases, a transition from stationary to periodic, quasi-periodic and then chaotic flow. However, as indicated by Kühnen *et al.* (2015) and as will be shown in chapter 2 and paper 2, their results were inaccurate, with the exception of the symmetry characteristics observed in the flow. The only experiments employing toroidal pipes in the context of the present work are those by Kühnen *et al.* (2014, 2015). The experimental difficulty to impose a bulk flow in the torus, led these authors to sacrifice the $2\pi$ (streamwise) periodicity and the mirror symmetry of the system by introducing a steel sphere in the tube to drive the fluid. Their results for $0.028 < \delta < 0.1$ confirmed the findings of Webster & Humphrey (1993), *i.e.* that the first instability leads the flow to a periodic regime, while for $\delta < 0.028$ subcritical transition was observed, as in Sreenivasan & Strykowski (1983).

One of the most relevant quantities for the flow through curved pipes is the friction encountered by the fluid. Ito (1959) and Cieślicki & Piechna (2012) report friction factors ($f$) measured in experiments (the former) and numerical computations (the latter), along with theoretical and empirical regression lines. Their conclusion is that the data collapse onto one line, confirming Dean’s finding of a single non-dimensional number governing the flow. The actual
1. Introduction

Figure 1.5: Fanning friction factor as a function of Dean number, with same scaling and axes as in figure 6 in Ito (1959). Individual lines are coloured by the corresponding curvature. The ‘band’ of lines becomes thinner for high De because the maximum Re was limited to 7000 for all values of δ, resulting in a different maximum De dependent on the curvature. Reprinted from Canton et al. (2017b) with permission from Elsevier.

picture is, however, different: as can be seen in figure 1.5 the lines do indeed show a common trend, but do not scale with De. The data represent a wide band, where the value of f changes with curvature. Even at De = 10 the friction range is still quite wide, with 0.004 ≤ f ≤ 1.74 and a relative difference with respect to Cieślicki & Piechna’s formula (f ≈ 0.50) of -100% and 290% respectively. From this and other results, presented in paper 1, it can be concluded that the Dean number is not suitable as a scaling parameter for this flow: Reynolds number and curvature need to be considered as separate parameters.

As was mentioned before, the curvature is the only parameter that differentiates a toroidal pipe from a straight one. It therefore appears natural to
ask the question of “when can $\delta$ be considered low enough that it does not influence the flow?” It can be demonstrated analytically that the secondary motion characterising this flow is always present, for any Reynolds number and curvature larger than zero (see paper 1, §3.3). Hence, the mathematical answer to the question is that the two flows can never be considered similar, but how would a physicist or engineer answer the same question?

Figure 1.6 shows the friction Reynolds number ($Re_\tau$) of the laminar flow as a function of $\delta$ and $Re$. The white lines represent isocontours of $Re_\tau$ corresponding to the friction Reynolds number of a straight pipe with fluid flowing at the bulk Reynolds number indicated by the $y$-labels, i.e., 1000, 2000, ..., 7000. The markers and dashed white line indicate a departure of more than 1% from the straight pipe $Re_\tau$. Reprinted from Canton et al. (2017b) with permission from Elsevier.
Chapter 2

Hydrodynamic stability

While hydrodynamic stability and transition to turbulence in straight pipes — being one of the classical problems in fluid mechanics — has been studied extensively, the stability of curved pipe flow has received less attention. The technical relevance of this flow case is apparent from its prevalence in industrial applications: bent pipes are found, for example, in power production facilities, air conditioning systems, and chemical and food processing plants. Vashisth et al. (2008) presents a comprehensive review on the applications of curved pipes in industry. A second fundamental area of research where bent pipes are relevant is the medical field. Curved pipes are, in fact, an integral part of vascular and respiratory systems. Understanding the behaviour of the flow in this case can aid the prevention of several cardiovascular problems (Berger et al. 1983; Bulusu et al. 2014).

2.1. Investigation methods

The flow is driven at a constant volume rate by a force field directed along the streamwise direction. The steady solution, which is stable for low Reynolds numbers, inherits from the geometry the invariance with respect to $s$ and the symmetry with respect to the equatorial plane of the torus (see figure 1.4). This allows the solution to be computed on a two dimensional section (retaining three velocity components) sensibly reducing the computational cost.

The steady solutions to the Navier–Stokes equations (1.1) are computed via Newton’s method. Introducing $\mathcal{N}(x)$ as a shorthand for the terms without time derivative, with $x = (u, p)$, and separating the linear and quadratic parts, the equations can be written as:

$$\mathcal{N}(x) = Q(x, x) + L(x) - f = 0,$$

where

$$Q(x, y) = \begin{pmatrix} (u \cdot \nabla)v \\ 0 \end{pmatrix}, \quad L(x) = \begin{pmatrix} -\frac{1}{Re} \nabla^2 u + \nabla p \\ \nabla \cdot u \end{pmatrix},$$

(2.1)
2.1. Investigation methods

and \( y = (v, q) \). The non-incremental formulation of Newton’s method reads:

\[
J|_{x_n}(x_{n+1}) = Q(x_n, x_n) + f, \tag{2.3}
\]

where \( J|_{x_n} \) is the Fréchet derivative of \( \mathcal{N}(x) \) evaluated at \( x_n \) corresponding, after spatial discretisation, to the Jacobian matrix \( J \). The convergence criterion is based on the infinity norm of the residual, i.e. \( \|\mathcal{N}(x_n)\|_{L^\infty} \), and the tolerance is set to \( 10^{-15} \) for all computations.

Once a steady state has been computed, its stability properties are investigated by modal stability analysis. Employing a normal modes ansatz, the perturbation fields are defined as:

\[
u'\left(r, \theta, z, t\right) = \hat{u}(r, z) \exp\left\{i(k\theta - \omega t)\right\}, \tag{2.4a}
\]

\[
p'\left(r, \theta, z, t\right) = \hat{p}(r, z) \exp\left\{i(k\theta - \omega t)\right\}, \tag{2.4b}
\]

where \( k \in \mathbb{Z} \) is the streamwise wave number, \( \omega = \omega_r + i\omega_i \in \mathbb{C} \) is the eigenvalue and \( \hat{x} = (\hat{u}, \hat{p}) \in \mathbb{C} \) is the corresponding eigenvector. Although \( k \) is in principle a real number, the \( 2\pi \)-periodicity of the torus restricts it to integer values.

The (spatially discretised) linearised Navier–Stokes equations are then reduced to a generalised eigenvalue problem of the form

\[
i\omega M\hat{x} = L_k\hat{x}, \tag{2.5}
\]

where \( \hat{x} \) is the discretised eigenvector, \( M \in \mathbb{R}^{dof \times dof} \) is the (singular) generalised mass matrix, and \( L_k \in \mathbb{C}^{dof \times dof} \) represents the discretisation of the linearised Navier–Stokes operator, parametrised by \( k \). Finally, ‘dof’ is a shorthand for the total number of degrees of freedom, accounting for both velocity and pressure nodes. The eigensolutions are computed to machine precision through an interface to the ARPACK library (Lehoucq et al. 1998).

Once a bifurcation is identified, its neutral curve can be traced in parameter space, separating the stable and unstable regions. In order to track a bifurcation, an augmented set of equations describing the system at the bifurcation point needs to be solved. Equation (2.6) presents this system in the case of a Hopf bifurcation, which is the type of bifurcation encountered in toroidal pipe flows:

\[
Nx = 0, \tag{2.6a}
\]

\[
i\omega_r M\hat{x} = L_k\hat{x}, \tag{2.6b}
\]

\[
\phi \cdot \hat{x} = 1, \tag{2.6c}
\]

with \( N = N(Re, \delta, x) \), \( M = M(\delta) \), and \( L_k = L_k(Re, \delta, x) \). The unknowns for this system are: the base flow \( x = (u, p) \in \mathbb{R}^{dof} \), the frequency of the critical eigenmode \( \omega_c \in \mathbb{R} \), and the corresponding eigenvector \( \hat{x} = (\hat{u}, \hat{p}) \in \mathbb{C}^{dof} \). The roles of \( Re \) and \( \delta \) can be interchanged: one is given while the other is obtained as part of the solution. Equation (2.6a) is a real equation determining that \( x \) is a steady state. Equation (2.6b) is a complex equation which represents the eigenvalue problem (2.5) when a pair of complex conjugate eigenvalues has zero growth rate. This equation forces the solution of the system to be on the
2. Hydrodynamic stability

Neutral curve. Equation (2.6c) is a complex equation as well, it fixes the phase and amplitude of the eigenvector. The constant vector $\phi$ is chosen as the real part of the initial guess for $\hat{x}$, such that (2.6c) mimics an $L^2$ norm.

Newton’s method is employed for this system as well, without assembling the Jacobian matrix. Instead, a block Gauss factorisation and linear algebra are used to split the resulting system into five dof $\times$ dof linear systems, two real and three complex, presented in (2.7a–e). Beside reducing memory requirements, these systems have matrices with the same sparsity pattern as those already used for the solution of the steady state and the eigenvalue problem. Vectors $\alpha$ through $\epsilon$ are then used to compute the updates for the unknowns (2.7f–i). The tolerance for the solution of this system is chosen as to have an uncertainty on $\text{Re}$ on the neutral curve of $\pm 10^{-4}$%.

$$\alpha = -J^{-1}N,$$  \hspace{1cm} (2.7a)

$$\beta = -J^{-1}\frac{\partial N}{\partial \delta},$$  \hspace{1cm} (2.7b)

$$\gamma = -[L_k + i\omega_r M]^{-1}iM\hat{x},$$ \hspace{1cm} (2.7c)

$$\delta = -[L_k + i\omega_r M]^{-1}\left(\frac{\partial L_k \hat{x}}{\partial x} + i\omega_r \frac{\partial M \hat{x}}{\partial x}\right)\alpha,$$ \hspace{1cm} (2.7d)

$$\epsilon = -[L_k + i\omega_r M]^{-1}\left(\frac{\partial L_k \hat{x}}{\partial x} + i\omega_r \frac{\partial M \hat{x}}{\partial x} + \left(\frac{\partial L_k \hat{x}}{\partial \delta} + i\omega_r \frac{\partial M \hat{x}}{\partial \delta}\right)\beta\right);$$ \hspace{1cm} (2.7e)

$$\Delta \delta = \frac{-\text{Re}(\phi \cdot \gamma) \Im(\phi \cdot \delta) - \Im(\phi \cdot \gamma)(1 - \text{Re}(\phi \cdot \delta))}{\text{Re}(\phi \cdot \gamma) \Im(\phi \cdot \epsilon) - \Im(\phi \cdot \gamma) \Re(\phi \cdot \epsilon)},$$ \hspace{1cm} (2.7f)

$$\Delta \omega_r = \frac{\Im(\phi \cdot \delta) \Re(\phi \cdot \epsilon) - \Im(\phi \cdot \epsilon)(1 - \Re(\phi \cdot \delta))}{\Re(\phi \cdot \gamma) \Im(\phi \cdot \epsilon) - \Im(\phi \cdot \gamma) \Re(\phi \cdot \epsilon)},$$ \hspace{1cm} (2.7g)

$$\Delta \hat{x} = -\hat{x} + \gamma + \Delta \omega_r \delta + \Delta \delta \epsilon,$$ \hspace{1cm} (2.7h)

$$\Delta x = \alpha + \Delta \delta \beta.$$ \hspace{1cm} (2.7i)

2.2. The Hopf bifurcation and the neutral curve

Preliminary eigenvalue computations reveal the presence of unstable pairs of complex-conjugate eigenmodes, indicating the presence of a Hopf bifurcation. Employing the method described in §2.1, the neutral curve of each critical mode has been traced in the parameter space as a function of curvature and Reynolds number. The result is a complex picture, depicted in figure 2.1. It presents five families and three isolated modes, and their envelope constitutes the global neutral curve for the flow. All eigenmodes represent a travelling wave, but their properties are modified along the neutral curve. A family comprises eigenmodes that share common characteristics, while the eigenvalues designated as isolated are those which contribute to the global neutral curve while being dissimilar to the modes belonging to the neighbouring neutral curves. In more detail, all modes belonging to a given family display the same
spatial structure and symmetry properties, they have approximately equal phase speed and comparable wavelength. In addition, while the wavenumber does not vary monotonically along the global neutral curve, it does so inside families. Furthermore, eigenvalues in the same family lie on the same branch and, possibly even more characteristic, the eigenmodes forming a family have neutral curves with a very similar trend ($\delta, Re(\delta)$). This can readily be observed in figure 2.1 where each neutral curve is purposely plotted beyond the envelope line to illustrate this feature. Nonlinear direct numerical simulations were also performed and show excellent agreement with these results: all of the characteristics of the bifurcation are observed in the nonlinear flow and the accuracy on the critical Reynolds number is confirmed as well.

Given that straight pipe flow is linearly stable at least up to $Re = 10^7$ (Meseguer & Trefethen 2003), it appears natural to ask how the neutral curve behaves when the curvature tends to zero. It is hard to see with the $\delta$-axis in linear scale, but the leftmost line in figure 2.1 ends at curvature 0.002. As was mentioned in chapter 1, there actually is a limit for $\delta$, depending on Re, below which the flow in a toroidal pipe can be well approximated by that in a straight pipe (see figure 1.6). Paper 3 provides a description of this problem, some of the solutions that were adopted, and preliminary results.

Studying the behaviour of the flow in a torus becomes more complicated after the first Hopf bifurcation: linear stability tools can still be used, but the flow is now periodic. Instead of eigenvalues, Floquet multipliers of the periodic orbit need to be computed, and these, in general, can only be found by numerical
integration (Strogatz 1994). This method only allows to find a possible second Hopf bifurcation, beyond which the trajectories of the system are on a toroidal orbit, i.e. there are now two incommensurable frequencies, resp. periods. To study the stability of this torus and find successive bifurcations, Lyapunov exponents have to be computed, to know if neighbouring orbits remain close together or separate exponentially fast. This is a very expensive analysis to carry out on a large system such as a fluid flow, where the number of degrees of freedom is typically very large.

An alternative is to simply “observe” the flow, either experimentally or via numerical simulations, and measure quantities that are good indicators of the state of the system which can be used for a reduced-order analysis of the flow. Examples of these quantities include the kinetic energy, dissipation, or even simply velocity components in a selection of points of the domain. This is the procedure that was followed to analyse the flow inside a torus. After the first bifurcation, this system undergoes what appears to be a Ruelle–Takens–Newhouse route to chaos (Ruelle & Takens 1971; Newhouse et al. 1978),
with new, incommensurable frequencies appearing as the Reynolds number is increased. Figure 2.2 provides an overview of the results, which are further detailed in paper 5. The figure depicts power spectral densities (PSD), and the corresponding phase space trajectories, for the flow at $\delta = 0.05$ and $Re = 4000, 4500$ and 6000. Figure 2.2(a) highlights the importance of the neutral curve for the nonlinear flow: after the first bifurcation there is a single, stable limit cycle which attracts all initial conditions. The 2-periodic, toroidal attractor in figure 2.2(b), instead, is indicative of a second Hopf bifurcation following the first one identified by the neutral curve. The corresponding PSD also highlights two incommensurable frequencies measured in the flow. This particular value of curvature is selected in paper 2 to verify the accuracy of the neutral curve and provide more details on the relevance of the linear analysis for the nonlinear flow.
In Chapter 2 it was shown that the flow inside a toroidal pipe is linearly unstable for all curvatures greater than zero. However, this is not the only transition mechanism in bent pipes.

White (1929) was the first to observe that the flow in a bent pipe at low curvatures can be maintained in a laminar state for higher Reynolds numbers than in a straight pipe. Later, Sreenivasan & Strykowski (1983) showed that even a turbulent flow coming from a straight pipe can be fully relaminarised after entering a coiled pipe section. Besides reporting this observation, made possible by the injection of a dye streak, Sreenivasan & Strykowski (1983) also provided estimates for the critical Reynolds number for transition to turbulence for curvatures up to $\delta = 0.12$. Since the transition mechanism was unknown at the time, these authors reported critical Reynolds numbers based on three different criteria. They defined a “conservative lower” critical $Re$ for the appearance of the first ‘burst’ of turbulence near the outer wall of the pipe, a “liberal lower” transitional $Re$ corresponding to the first appearance of turbulence on the whole cross-section of the pipe under investigation, and finally an “upper” limit as the lowest $Re$ at which the flow becomes fully turbulent. The reason for these three distinct limits is that the transitional flow at low curvatures displays a degree of intermittency, depending both on the Reynolds number and the curvature, which made it difficult to define a precise limit for transition to turbulence.

Straight pipes present a similar problem. The Hagen-Poiseuille velocity profile is linearly stable at least up to $Re = 10^7$ (Meseguer & Trefethen 2003), i.e. all small perturbations decay and no critical Reynolds number can be defined using linear theory. However, experiments and simulations show that subcritical transition to turbulence can occur for $Re \gtrsim 1700$ if perturbations are sufficiently large. Different structures have been observed at transitional Reynolds numbers: turbulent patches that do not grow in size and are now known as puffs, and structures that expand in the surrounding laminar flow, so-called slugs (see, e.g. Lindgren 1969; Wygnanski & Champagne 1973; Wygnanski et al. 1975; and the review by Mullin 2011). Only in recent years a statistical description of the flow in straight pipes has provided an accurate estimate of a critical Reynolds number, $Re \approx 2040$ (Avila et al. 2011). Below this threshold, the probability of a puff decaying outweighs the probability of a new puff being generated through a
splitting mechanism. On the other hand, if the Reynolds number is higher than the threshold, the probability of splitting increases superexponentially and puffs proliferate. Theoretical models have been proposed and quantitatively capture this subcritical transition scenario, which falls into the directed percolation universality class (Barkley 2011; Barkley et al. 2015; Shih et al. 2015; Barkley 2016).

Building on top of this knowledge, Kühnen et al. (2015) repeated the analysis by Sreenivasan & Strykowski (1983) and reported that subcritical transition dominates in bent pipes for \( \delta \lesssim 0.028 \), while above this value transition to turbulence is supercritical. Subcritical transition in bent pipes was described as being “very similar as in straight pipes, where laminar and turbulent flows can coexist” (Kühnen et al. 2015). However, an in-depth analysis of this regime is still missing.

### 3.1. Investigation methods

Subcritical transition is more expensive to study than supercritical transition: linearisation cannot be used and the flow has to be simulated by solving the full time-dependent Navier–Stokes equations (1.1). For this reason only one value of curvature was chosen to investigate this regime. In order to ensure that the nature of transition investigated is subcritical, the curvature was set to \( \delta = 0.01 \). This value is sufficiently smaller than the threshold for the onset of supercritical transition, \( \delta \approx 0.028 \) according to Kühnen et al. (2015). At the same time, it introduces a significant deviation of the laminar flow from the one of a straight pipe (see chapter 1 and paper 1), which for this curvature becomes linearly unstable for \( Re = 4257 \) (see chapter 2 and paper 2).

The study is again fully numerical, and direct numerical simulations (DNS) are performed using the spectral element solver Nek5000 (Fischer et al. 2008), which was previously validated on turbulent straight and bent pipes (El Khoury et al. 2013; Noorani et al. 2013) and in transitional regimes (chapter 2 and paper 2). In order to observe the large scale evolution of puffs and slugs, the length of the computational domain is \( L_s = 100D \) and \( L_s = \pi d/3 \approx 105D \) for straight and bent pipes, respectively (the subscript \( s \) indicates the streamwise direction). The spatial resolution satisfies typical DNS requirements for fully turbulent flows at Reynolds numbers slightly higher than the ones considered here, for details see paper 4.

The scalar quantity \( q \) is used as an indicator of the level of turbulence in accordance with the literature on transitional straight pipes (see, e.g. Barkley 2011), and its definition is adapted to the case of bent pipes as:

\[
q(s, t) = \sqrt{\int_0^{2\pi} \int_0^R \left( (u_r - U_r)^2 + (u_\theta - U_\theta)^2 \right) r \, dr \, d\theta}.
\] (3.1)

Here \( s, r \) and \( \theta \) indicate the streamwise, radial and azimuthal directions in toroidal coordinates. The instantaneous velocity components are \( u_s = u_s(s, r, \theta, t) \), \( u_r = u_r(s, r, \theta, t) \) and \( u_\theta = u_\theta(s, r, \theta, t) \); capital letters denote the
3. Subcritical transition

Figure 3.1: Space-time evolution of the cross-flow velocity fluctuations $q$, defined by equation (3.1), for exemplary puffs (a) and slugs (b) in straight and curved pipes. Colours represent $\log_{10} q$, white corresponds to laminar flow, dark colours to high fluctuations. Panels (c,d) report the spatial distribution of $q$ sampled at four time instants. The (horizontal) scale used to indicate the magnitude of $q$ is the same for the straight and curved pipes.

3.2. The collapse of strong fronts

Figure 3.1 illustrates one of the most relevant findings of this analysis. It depicts the evolution of the cross-flow velocity fluctuations, $q$, for puffs (figure 3.1(a)) and slugs (figure 3.1(b)) in both straight and bent pipes. The turbulence intensity $q = q(s - u_f t, t)$ is computed in a frame of reference that moves with a constant streamwise velocity $u_f$, and the same range of colour levels is used for straight and bent pipes to allow for a direct visual comparison. One-dimensional profiles of $q$, sampled at several subsequent times, are reported in figure 3.1(c) and (d), and help the comparison between the two flows.

Localised turbulent structures in bent pipes bear qualitative similarities to those in straight pipes in that they appear in the form of puffs and slugs that are sustained by an instability at their upstream front. However, a clear and distinctive feature differentiates puffs and slugs between the two pipes: the absence of a strong upstream front if the pipe is bent. The space–time diagrams show the well-known concentration of turbulent fluctuations, indicated by the dark tone of the colour, at the upstream front in straight pipes (Barkley 2011,
3.2. The collapse of strong fronts

Figure 3.2: Turbulent kinetic energy production (left halves) and dissipation (right halves) at the front of slugs in straight and bent pipes. All quantities are averaged over time by tracking the slug, and over the cross-section taking into account the mirror symmetry of the torus, the axial invariance of the pipe has not been used in order not to alter the comparison. The text labels on the bottom report the maxima of $P_k$ and $\varepsilon_k$ on each section.

2016; Song et al. 2017). Conversely, the flow in bent pipes shows no evidence of this strong front and is characterised by a somewhat uniform distribution of $q$.

As reported by Sreenivasan & Strykowski (1983), turbulent fluctuations in bent pipes appear first in the outer portion of the bend, while they pervade the whole cross-section only for higher Reynolds numbers. To investigate the connection between this observation and localised structures, figure 3.2 presents the time averaged distributions of production and dissipation over a cross-section of the pipe. The panels in figure 3.2 are computed at the upstream front of slugs for different Reynolds numbers. In a straight pipe $P_k$ and $\varepsilon_k$ are mainly concentrated in the near-wall region and in a ring around the centre of the pipe. Conversely, the budget in a bent pipe shows a high localisation towards the outside of the bend and lower peak values. The spatial localisation and lower local values of $P_k$ and $\varepsilon_k$ also suggest that an additional mechanism must come into play in sustaining localised turbulent structures, and this is likely to simply be the secondary motion created by the Dean vortices. As the curvature is increased from a straight to a bent pipe, the linear and nonlinear optimal perturbations (see paper 4) become increasingly localised in the same region where the peaks of $P_k$ and $\varepsilon_k$ are located. It therefore appears that this region, where the recirculating flow impinges on the wall of the pipe, is highly receptive to flow perturbations and is responsible for their amplification. The fluctuations are then transported around the walls of the pipe and lifted up towards the inner section.

Paper 4 provides more details of this study, but there is a fundamental difference between straight and bent pipes worth mentioning here: the apparent
absence of puff splitting when the pipe is bent, at least for the choice of parameters investigated. The absence of puff splitting appears to be connected to the weak and localised upstream fronts. Turbulent structures that leave a mother puff have a low probability of entering the small region of high amplification located near the outer wall, which would trigger the instability that sustains a puff. Moreover, due to the secondary motion, the few vortical structures that visit this region do not linger for long enough to generate a new puff.
Chapter 4

The critical point

The flow inside of a toroidal pipe presents both sub- and supercritical transition to turbulence, as discussed in chapters 2 and 3, and detailed in papers 2–5. The number of fluid flows with this characteristics is quite small.

There are some flows which are modally stable for all Reynolds numbers, meaning that according to a linear analysis they should never become turbulent. This is the case, for example, of straight pipe flow, which is modally stable at least up to $Re = 10^7$ (Meseguer & Trefethen 2003) but undergoes subcritical transition for $Re \gtrsim 1700$ (see, e.g., Barkley 2016, and references therein). For these flows a linearised analysis fails entirely.

Other flows, instead, are actually linearly unstable but still undergo subcritical transition for lower Reynolds numbers. One of the most famous examples is plane Poiseuille flow, which has a critical Reynolds number of 5772 but actually becomes naturally turbulent for $Re \approx 3300$ (Kim et al. 1987), and can even be partially turbulent for Reynolds numbers as low as 1200 (Kleiser & Zang 1991; Tsukahara et al. 2005). For these flows a linearised analysis fails in the sense that it provides results which are “irrelevant” in a nonlinear simulation or experiment. This is what happens to the flow in the torus for low curvatures: there is a linear instability, as for all other curvatures, but in experiments and DNS the flow undergoes transiton to turbulence at lower Reynolds numbers. Even when above the neutral curve, nonlinear simulations do not show any trace of the modes that should be unstable according to linear analysis.

What makes the flow inside of a torus peculiar is that for curvature above approximately 0.025 a linear analysis does actually provide the correct results, as was discussed in chapter 2. For curvatures above this value the nonlinear flow undergoes a Hopf bifurcation, exactly as predicted by a modal analysis.

To the best of our knowledge, there are not many other flows that undergo both transition scenarios by changing only one parameter, examples are Taylor-Cuette flow (Coles 1965; Andereck et al. 1986) and rotating Couette flow (Tsukahara et al. 2010; Tsukahara 2011). There still is one important difference between Taylor-Couette, rotating Couette, and the flow in a torus: the first two do indeed present both subcritical and supercritical transition, but the regimes are well separated. Subcritical transition is present if the outer cylinder is rotated in anti-clockwise direction, while supercritical transition appears when
Figure 4.1: Portion of the $\delta - Re$ parameter space of the flow in a toroidal pipe. Experimental and numerical data from the literature are reported as well as the location of the present computations. The data by Cioncolini & Santini (2006) refers to the first discontinuity in their friction measurements, while the data by Sreenivasan & Strykowski (1983) is the curve they refer to as the “conservative lower critical limit”. Point A is supercritical, and illustrated in figure 2.2(b), while point B is critical and is detailed in figure 4.2.

4.1. Investigation methods

In this region of parameter space the flow has to be studied in its full nonlinear regime, since linear and nonlinear mechanisms coexist. The neutral curve was computed with PaStA, as detailed in chapter 2, while the nonlinear simulations are all performed with Nek5000, as in chapter 3. The same meshes and degree of spatial accuracy, validated in the previous chapters, are also employed.

4.2. Bifurcation cascades and intermittency

We now turn our attention to the region of parameter space where the neutral curve meets the lines indicating subcritical transition, i.e. $\delta \approx 0.025$ and
4.2. Bifurcation cascades and intermittency

Figure 4.2: Left: phase space for $\delta = 0.022$ and $Re = 5050$ (point B in figure 4.1). All trajectories start in the neighbourhood of the travelling wave, the closest being the black line (panel (a)). The yellow trajectory is turbulent for $t \approx 100D/U$ and then slowly returns to the stable limit cycle (panel (b)), while the blue trajectory remains turbulent for $t > 1000D/U$ (panel (c)). Right: snapshots of the flow field along the three phase space trajectories. Red and blue colours are isocontours of streamwise velocity for two opposite values, $i.e.$ $u_s = \pm 0.005$; white isocontours are of negative $\lambda_2$ (Jeong & Hussain 1995).

$4000 < Re < 5000$. As a first step it is necessary to verify that both sub- and supercritical behaviours can still be isolated. We therefore perform nonlinear simulations for $\delta = 0.022$ and $Re = 4500$, with two domains of different length, about $10D$ and $20D$, respectively. The simulations are initialised with puffs computed by Rinaldi et al. (2018) which, for both domains, grow in length in the form of slugs and turn the whole domain to turbulent flow, confirming once more the subcritical transition lines by Sreenivasan & Strykowski (1983); Kühnen et al. (2015) and the findings by Rinaldi et al. (2018). The second verification is at $\delta = 0.028$ and $Re = 4600$, just above the neutral curve, which for this curvature marks the linear instability at $Re = 4570$, and below the subcritical transition thresholds. Here the flow is initialised with a paraboloidal profile perturbed with random noise, and converges to the nonlinear travelling wave created by the Hopf bifurcation, as previously explained for $\delta = 0.05$.

We therefore proceed by lowering the curvature, while remaining above the neutral curve, and investigate three more pairs of $(\delta, Re)$, $i.e.$ $(0.026, 4750)$, $(0.024, 4900)$, and $(0.022, 5050)$. The last pair of values corresponds to point B in figure 4.1, and is illustrated in figure 4.2. In this region of parameter space the two transition scenarios coexist. The beginning of the Ruelle–Takens–Newhouse route to chaos can be observed in the form of stable, nonlinear travelling waves, while subcritical transition is found in the form of expanding slugs. In point B, for example, a simulation initialised with a randomly perturbed parabolic velocity profile slowly converges to a nonlinear travelling wave, as predicted by the modal analysis. This process is illustrated by the black trajectory in the phase space of figure 4.2 and with a snapshot of the flow field in figure 4.2(a). On the other hand, if the simulation is initialised with a localised disturbance,
the disturbance grows and invades the whole pipe, turning the flow into a fully turbulent state; in agreement with the location of the lines indicating subcritical transition.

The intermediate cases are what makes this flow unique. When the travelling wave is perturbed with a highly energetic puff the subcritical transition scenario dominates: the puff expands and creates a fully turbulent flow. For this particular combination of $\delta$ and $Re$, highly energetic means that the kinetic energy of the puff, $E_{k,puff}$, has to be more than approximately 6.3 times that of the travelling wave, $E_{k,wave}$. This is the case represented by the blue trajectory in figure 4.2 and the corresponding flow field in panel (c), where the white $\lambda_2$ isocontours (Jeong & Hussain 1995) illustrate the spatially chaotic nature of turbulence, while the temporal disorder is visualised by the trajectory. Conversely, if the puff has lower energy, i.e. $E_{k,puff} < 6.3E_{k,wave}$, it will grow in size but only transiently, before diffusing and disappearing completely. The yellow trajectory in figure 4.2 illustrates this process: it starts from an initial condition close to that represented in panel (b), then travels through the turbulent region of phase space (main panel and upper insert), and finally returns to the attracting limit cycle of the travelling wave (lower insert). It can therefore be concluded that both transition mechanisms, sub- and supercritical, can be observed in this critical region of parameter space. The structures corresponding to these processes are all stable, and their basins of attraction are complementary and finite.
Chapter 5

Large-scale structures in turbulent flow

After investigating transition to turbulence, the attention is now moved to a different problem: the presence of large-scale, low-frequency structures in fully turbulent curved pipe flow, known as *swirl-switching*. The focus is now in understanding the nature of these structures, similarly to what was done with puffs and slugs in low-curvature pipes. A study on the origin of these oscillations is still underway and is outlined towards the end of this chapter.

Swirl-switching is a low-frequency oscillatory phenomenon which affects the Dean vortices in bent pipes and may cause fatigue in piping systems (Kalpakli Vester *et al.* 2016). Despite thirty years worth of research, the mechanism that causes these oscillations and the frequencies that characterise them remain unclear. The Dean vortex alternation was initially, and unexpectedly, observed by Tunstall & Harvey (1968), who experimentally studied the turbulent flow through a sharp, L-shaped bend ($\delta = 1$). These authors measured “low random-frequency” switches between two distinct states, and were able to identify an either clockwise or anti-clockwise predominance of the swirling flow following the bent section. Tunstall & Harvey attributed the origin of the switching to the presence of a separation bubble in the bend and to the “occasional existence of turbulent circulation entering the bend”. It was Brücker (1998) who coined the term “swirl-switching”, and identified the oscillations as a continuous transition between two mirror-symmetric states with one Dean cell larger than the other.

Rütten *et al.* (2001, 2005) were the first to numerically study the phenomenon and showed that the switching takes place even without flow separation. Moreover, Rütten and co-workers found that the structure of the switching is more complex than just the alternation between two distinct symmetric states, since the outer stagnation point “can be found at any angular position within $\pm 40^\circ$”. Rütten *et al.* attributed the switching to a shear-layer instability. However, their simulations were performed by using a “recycling” method, where the results from a straight pipe simulation were used as inflow condition for the bent pipe. These periodic straight pipes likely introduced a forcing with frequency comparable to that of the switching. Sakakibara *et al.* (2010) were the first to analyse the flow by means of two-dimensional proper orthogonal decomposition (2D POD). In a subsequent work (Sakakibara & Machida 2012) they conjectured that the swirl-switching is caused by very large-scale motions (VLSM) formed
in the straight pipe preceding the bend. Hellström et al. (2013) and Kalpakli & Örlü (2013); Kalpakli Vester et al. (2015) also presented results based on 2D POD. The former found non-symmetric modes resembling a tilted variant of the Dean vortices corresponding to the shear-layer instabilities found by Rütten et al. (2005). Kalpakli & Örlü (2013); Kalpakli Vester et al. (2015) also found antisymmetric modes as most dominant structures. Carlsson et al. (2015) performed LES in a geometry similar to that of Kalpakli & Örlü (2013), namely, with a short straight section following the bend, for four different curvatures. The inflow boundary condition was generated by means of a recycling method, as in Rütten et al. (2001, 2005). The three lower curvatures were therefore dominated by the spurious frequencies artificially created in the straight pipe by the recycling method, while the frequencies measured for $\delta = 1$ were in the same range identified by Hellström et al. (2013) but were found to be mesh dependent. Noorani & Schlatter (2016) were the first to investigate the swirl-switching by means of direct numerical simulations (DNS). By using a toroidal pipe they showed that swirl-switching is not caused by structures coming from the straight pipe preceding the bend, but is a phenomenon inherent to the curved section.

### 5.1. Investigation methods

The present analysis is performed by means of direct numerical simulations, where the equations are discretised with the spectral-element code NEK5000 (Fischer et al. 2008). Figure 5.1 shows the computational domain, which is a 90° bent pipe with curvature $\delta = 0.3$. A straight pipe of length $L_i = 7D$ precedes the bent section and a second straight segment of length $L_o = 15D$ follows it. The Reynolds number is fixed to $Re = 11,700$, corresponding to a friction Reynolds number $Re_\tau \approx 360$ (referred to the straight pipe sections). In order to avoid the excitation of unphysical phenomena or a modification of the frequencies inherent to the swirl-switching, the velocity field at the inlet boundary of the straight pipe preceding the bend is prescribed via a divergence-free synthetic eddy method (DFSEM). This method, introduced by Poletto et al. (2011) and based on the original work by Jarrin et al. (2006), works by prescribing a mean flow modulated in time by fluctuations in the vorticity field. Details are provided in paper 6.

Snapshot POD (Lumley 1967; Sirovich 1987) is used to extract coherent structures from the DNS flow fields and identify the mechanism responsible for swirl-switching. POD decomposes the flow into a set of orthogonal spatial modes and corresponding time coefficients ranked by kinetic energy content in decreasing order. The most energetic structure extracted by POD corresponds to the mean flow and will be herein named “zeroth mode”, while the term “first mode” will be reserved for the first time-dependent structure. The symmetry of the pipe about the $I − O$ plane, which results into a statistical symmetry for the flow, is used to improve the convergence of the decomposition by storing an additional mirror image for each snapshot (Berkooz et al. 1993).
5.2. Swirl-switching is a wave-like structure

The four most energetic modes are depicted in figure 5.2 by means of pseudo-colors of normal and streamwise velocity components, as well as streamlines of the in-plane velocity. It can be observed that the modes come in pairs: 1-2 and 3-4, as is usual for POD modes and their time coefficients in a convective flow. The first coherent structure extracted by the POD is formed by modes 1 and 2 and constitutes a damped wave-like structure that is convected by the mean flow. The spatial structure of these modes is qualitatively sinusoidal along the streamwise direction $s_0$, with a wavelength of about 7 pipe diameters. This wave-like structure is formed by two counter-rotating swirls, visible in the 2D cross-sections in figure 5.2, which are advected in the streamwise direction while decaying in intensity and, at the same time, move from the inside of the bend towards the outside, as can be seen in the longitudinal cuts in figure 5.2.

As can be observed from figure 5.2, the modes do not present any connection to the straight pipe section preceding the bend. This is in direct contrast with
Figure 5.2: Four most energetic three-dimensional POD modes. The four longitudinal cuts show pseudocolours of the normal velocity component \( u_n \), while the eight cross-sections display the in-plane streamlines and are coloured by streamwise velocity \( u_s \). Hufnagel et al., The three-dimensional structure of swirl-switching in bent pipe flow, *J. Fluid Mech.* 835:86–101 (2017), reproduced with permission.

The present findings are also in contrast with previous conclusions drawn from flow reconstructions based on 2D POD modes that invoke Taylor’s frozen turbulence hypothesis (see, e.g., Hellström et al. 2013). This is due to the fact that the 2D analysis mixes convection and true temporal variation, and thus cannot reveal the full three-dimensional structure of travelling modes. This does

the findings of Carlsson et al. (2015), whose results were likely altered by the use of a recycling inflow.
not only apply to the present flow case, but to any streamwise inhomogeneous flow in which 2D POD is utilised in the cross-flow direction.

A final comment is that the wave-like structure found in the present study is different from those observed in transitional flows (see, e.g. Hof et al. 2004). It is simply a coherent structure extracted by POD from a turbulent background flow, as opposed to an exact coherent state or an eigenmode. Nevertheless, this wavy structure may be a surviving remnant of a global instability caused by the bend, as discussed in chapter 2.
Chapter 6

Friction control in turbulent channel flows

Turbulent flows are massively present in engineering applications which require fast and effective solutions. As an example the energy and transport industries have seen a steady development over the years. The naval shipping and aeronautical industries alone consume a combined 572 billion liters of fuel per year (Kim & Bewley 2007). The shear size of this figure indicates that even marginal improvements aimed at fuel saving have an enormous impact on the economy and the environment.

The turbulent flow in a channel, similarly to that inside of a straight pipe, is one of the most studied internal flows in fluid mechanics (Moser et al. 1999). This is because, despite its simplicity, it presents most of the features found in wall-bounded turbulent flows, it can therefore be used as a model for more complex geometries and, among other things, as a test case for control and drag reduction strategies. In fact, a large number of numerical and experimental studies have been performed in channel flows to find effective control mechanisms for wall-shear-stress modifications (see, for example, Gad-el Hak 2007). Most of these control strategies, though, have only been tested for relatively low Reynolds numbers.

6.1. Prelude

Turbulent flows are studied with a different approach: these flows are chaotic and it is nearly impossible to accurately describe the kinematics and dynamics of each of the numerous structures that constitute them.

When analysing turbulent flows it is customary to decompose the velocity field into its mean and fluctuating components (see, for example, Pope 2000). Each of these satisfies a ‘modified version’ of the Navier–Stokes equations, where different terms can be identified as contributing to the production, transport and dissipation of turbulent kinetic energy. It is by analysing the time averaged effect of these terms, and the action of the fluid on solid surfaces, that turbulent flows are commonly studied. A different but complementary approach is to analyse what is known as coherent structures. These are structures, first identified by flow visualisation (Kline et al. 1967) or other eduction techniques, which can interact between each other in a self-sustaining process (Waleffe 1997) and regulate the turbulence intensity close to the wall.
6.2. A description of the flow

In the present chapter the goal is not only to analyse the flow, but also to find a way to modify it and reduce the viscous drag that affects the surfaces in contact with the fluid. Different approaches to this problem have been proposed in the past decades. A first major distinction can be made between active and passive strategies: the former class comprises solutions that require an input of energy, while methods pertaining to the latter category are based on modifications of the geometry of the flow (Gad-el Hak 2007).

A few examples of effective active techniques comprise blowing and/or suction of fluid on the wetted surface (Kametani et al. 2015; Stroh et al. 2015), applying volume forces to modify the velocity field (Choi et al. 1994), and introducing spanwise oscillations (Jung et al. 1992) or travelling waves (Quadrio et al. 2009) at the wall. Passive techniques, instead, usually involve the addition of surface elements such as riblets (Choi et al. 1993; García-Mayoral & Jiménez 2011) or a non-uniform distribution of roughness (Vanderwel & Ganapathisubramani 2015).

The drag on a moving object can be divided into two major contributions: one caused by pressure differences between the front and backward facing parts of the body, and the other due to friction generated by the relative movement between the object and the surrounding fluid. The second part of the present thesis, and all of the flow control techniques mentioned above, deal with the second cause, also known as skin-friction drag. It has been long known, though, that turbulent flows for relatively low Reynolds numbers are affected by phenomena which disappear when Re is increased (Moser et al. 1999), so called low-Re effects. It is then very important to verify the effectiveness of the proposed control strategies at higher Re and/or as a function of Re (Iwamoto et al. 2002; Gatti & Quadrio 2013).

6.2. A description of the flow

This chapter is concerned with the ideal flow through a channel of height 2h, and ‘infinite’ length and width. This infinitely large channel is numerically simulated with a box periodic in the streamwise and spanwise directions (see figure 6.1). In the uncontrolled case the mean flow has only one component (U) in the streamwise (x) direction and most of the turbulent production takes place close to the channel walls, in an area known as viscous wall region.

Following the initial idea by Schoppa & Hussain (1998) (sometimes referred to as SH in the following), a series of streamwise-invariant, counter-rotating vortices are superposed on the flow. In the original implementation by these authors, the vortices were introduced by performing numerical simulations with the shape of the streamwise mean flow fixed in time. This strategy, referred to as “frozen vortices” in the following, can only provide transient drag reduction and required re-thinking, as will be shown in paper 7. In the present implementation, the large-scale vortices are introduced via a volume forcing provided with variable intensity and spanwise wavelength. Applying a volume
The force field that generates the large-scale vortices is defined as:

\[
\begin{align*}
  f_x &= 0, \\
  f_y(y, z) &= A\beta \cos(\beta z)(1 + \cos(\pi y/h)), \\
  f_z(y, z) &= A\pi/h \sin(\beta z) \sin(\pi y/h),
\end{align*}
\]  

(6.1)

where \( A \) is the forcing amplitude and \( \beta \) the wavenumber along \( z \). A sketch of the control vortices is depicted in figure 6.1. The wavenumber is chosen such as to have vortex periods \( \Lambda = 2\pi/\beta \) between 1.1h and 9.9h, corresponding to inner-scaled wavelengths \( \Lambda^+ \) between 120 and 3630. Since \( A \) does not correspond to a measurable flow quantity, the rescaled maximum wall-normal mean velocity is used to characterise the strength of the vortices, i.e. \( \max \left| \langle v \rangle_{x,t} \right| / U_b \), where \( \langle \cdot \rangle_{x,t} \) denotes the average in the streamwise direction and time.

The objective of this study is to achieve drag reduction \( DR \). This is measured as the relative reduction in the streamwise pressure gradient \( p_x \) necessary to achieve a certain bulk flow. This index can be related to the ratio of the wall-integrated strain rate \( \Omega_w \) between the controlled and uncontrolled cases, employed by Schoppa & Hussain (1998), with the following expression:

\[
\begin{align*}
  DR &= \frac{p_{x,\text{unc}} - p_{x,\text{con}}}{p_{x,\text{unc}}} = 1 - \frac{p_{x,\text{con}}}{p_{x,\text{unc}}} = 1 - \frac{\Omega_{w,\text{con}}}{\Omega_{w,\text{unc}}}; \\
  \Omega_w &= \frac{1}{L_x(z_2 - z_1)} \int_0^{L_x} \int_{z_1}^{z_2} \left. \frac{\partial u}{\partial y} \right|_{y=0} \, dx \, dz.
\end{align*}
\]

(6.2)  

(6.3)

Clearly, positive values of \( DR \) correspond to a favourable effect while negative ones indicates drag increase.
6.4. Large-scale friction control

The uncertainty on DR is quantified as (Tropea et al. 2007):

$$\sigma^2_{\langle DR \rangle} = \frac{1}{T} \int_{-T}^{T} \left( 1 - \frac{|\tau|}{T} \right) C_{\text{DR DR}}(\tau) d\tau$$

where $C_{\text{DR DR}}(\tau)$ is the autocovariance of DR, $\tau$ the signal time, and $T$ is the length of the integration time (see table 6.1).

6.3. Investigation methods

All simulations are performed using the fully spectral code SIMSON (Chevalier et al. 2007), where periodicity in the wall-parallel directions $x$ (streamwise) and $z$ (spanwise) is imposed, and the no-slip condition is applied on the two channel walls. The wall-parallel directions are discretized using Fourier series, where aliasing errors are removed by using 1.5 times the number of modes prescribed, while a Chebyshev series is employed in the wall-normal direction. The temporal discretisation is carried out by a combination of a third order, four stage Runge–Kutta scheme for the nonlinear term, and a second order Crank–Nicolson scheme for the linear terms. The time step is chosen such that the Courant number is always below 0.8.

Four values of the bulk Reynolds number, based on bulk velocity $U_b$, channel half-height $h$ and fluid viscosity $\nu$, are employed: $Re = 1518, 2800, 6240$ and $10000$, such as to result in a friction Reynolds number, based on friction velocity $u_\tau$, $h$ and $\nu$, corresponding to $Re_\tau \approx 104, 180, 360$ and 550. The lowest Reynolds number, corresponding to a barely turbulent flow, is the value employed in the original study by Schoppa & Hussain (1998). Details of the domain sizes and spatial resolution employed for the four sets of simulations are reported in table 6.1.

6.4. Large-scale friction control

As a first step, the forcing method is applied to a channel flow for the same Reynolds number as employed by Schoppa & Hussain (1998), i.e. $Re_\tau = 104$, to compare the two methods. Six different spanwise wavelengths were chosen, including the wavelength identified by SH as the best performing for the frozen vortices, i.e. $\Lambda^+ = 400$. For each vortex size the amplitude was varied in order to determine an optimum value. Figure 6.2(b) shows that the friction on the channel walls is unchanged for very low amplitudes of the imposed vortices (approximately below 0.5%). Negative results are also obtained for strong amplitudes: above 8% of the bulk velocity all cases show significant negative DR, i.e. drag increase. An inspection of the flow fields, presented in paper 7, clearly shows that negative DR is caused by the strong shear created by the vortices in the near-wall region, rather than increased turbulence. In fact, for the strongest amplitude of the forcing, turbulence disappears altogether, but the frictional drag is sensibly increased. Relevant for the present study is the intermediate region of amplitudes, for which all vortex sizes (except $\Lambda = 1.1h$) show at least a 10% reduction of the drag. The maximum efficiency for this Reynolds number is reached for the case with $\Lambda = 2.2h$, which yields DR $\approx 16\%$. 
6. Friction control in turbulent channel flows

<table>
<thead>
<tr>
<th>$Re_\tau$</th>
<th>Integration time</th>
<th>Domain size</th>
<th>Grid points</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T [h/U_b]$</td>
<td>$L_x/h$, $L_z/h$</td>
<td>$N_x$, $N_y$, $N_z$</td>
</tr>
<tr>
<td>104</td>
<td>10500</td>
<td>8, 3.832</td>
<td>48, 65, 48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8, 6.6</td>
<td>48, 65, 60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8, 9.9</td>
<td>48, 65, 96</td>
</tr>
<tr>
<td>180</td>
<td>1500</td>
<td>12, 6.6</td>
<td>128, 97, 96</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12, 9.9</td>
<td>128, 97, 144</td>
</tr>
<tr>
<td>360</td>
<td>1000</td>
<td>12, 6.6</td>
<td>300, 151, 200</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12, 9.9</td>
<td>300, 151, 300</td>
</tr>
<tr>
<td>550</td>
<td>400</td>
<td>12, 6.6</td>
<td>432, 193, 300</td>
</tr>
</tbody>
</table>

Table 6.1: Details of the numerical discretisation employed for the simulations. $T$ corresponds to the duration of the controlled simulations; $N_x$ and $N_z$ represent the number of Fourier modes employed in the wall-parallel directions (values before dealiasing), while $N_y$ is the order of the Chebyshev series used for the wall-normal direction.

6.5. Reynolds number dependence of the control method

The same analysis is repeated for $Re_\tau = 180, 360$ and 550. The results for $Re_\tau = 180$, presented in figure 6.2(c), are similar to those for $Re_\tau = 104$, presenting approximately the same range of effective wavelengths and forcing amplitudes. An even greater reduction in drag, corresponding to $\text{DR} \approx 18\%$, is achieved for $\Lambda = 6.6h$ and a forcing amplitude of just $0.04U_b$.

Although the large-scale vortices can provide a good level of drag reduction for $Re_\tau = 104$ and 180, their performance degrades rapidly with Reynolds number. For $Re_\tau = 360$ the maximum DR is only 8%, and for $Re_\tau = 550$ no more than 0.4% can be obtained for any of the wavelengths investigated. This is illustrated in figure 6.2(a) which shows, in semi-log scale, the maximum achievable value of DR as a function of $Re_\tau$.

Valuable insight on the effect that the Reynolds number has on the large-scale vortices can be obtained by analysing the wall-shear stress $\tau_w$ as a function of $Re_\tau$. Figure 6.3 presents the profiles of $\tau_w$ measured at the lower channel wall ($y = -1$) for the four values of $Re_\tau$ investigated. The analysis is conducted on vortices with $\Lambda = 6.6h$ which better highlight the Reynolds-number dependence, but the same trend is observed for all other wavelengths. Two phenomena are clearly observable as the Reynolds number is increased: a reduction in the variation in $\tau_w$ and a modification of the profiles as a function of $z$.

The first phenomenon is most probably a direct consequence of the increase in $Re$: as the Reynolds number increases the height of the viscous sublayer, measured in outer units, decreases as $1/Re_\tau$. The maximum variation in $\tau_w$ (for a positive DR) at the centre of two vortices, where the fluid is pushed towards the wall, does also decrease monotonically with $Re_\tau$. This appears
6.5. Reynolds number dependence of the control method

Figure 6.2: Panel (a): maximum achievable drag reduction as a function of friction Reynolds number. The uncertainty on DR is quantified by equation (6.4); the x-axis is in logarithmic scale. Panels (b) to (e) depict DR as a function of control strength, $\max |\langle v \rangle_{x,t}|/U_b$, and wavelength of the vortices, $\Lambda$. $Re_\tau = 104, 180, 360$ and 550 are reported in (b), (c), (d) and (e), respectively. In (b–e) red and blue colors correspond to positive, resp. negative, values of DR, values are reported on the isocontours; black points correspond to simulations. Reprinted with permission from Canton et al., Phys. Rev. Fluids, 1, 081501, 2016. Copyright by the American Physical Society.

to indicate that the large-scale vortices cannot penetrate the viscous sublayer as its height decreases, and thus cannot affect the near-wall region as $Re_\tau$ is increased beyond a limit value. Such a result is unfortunate as the idea behind this method was to use large, Reynolds number-independent, scales to control the flow.

The second phenomenon is well illustrated by comparing figures 6.3a,b with 6.3c,d. For controlled flows at low Reynolds numbers the minimum in $\tau_w$ is achieved close to the vertical vortex axes (located at $1/4$ and $3/4$ of $\Lambda$) at $z/\Lambda \approx 0.16$ and 0.84; in this area the vortices are pushing the fluid in the spanwise direction only. As the Reynolds number is increased this minimum is moved to the extrema of the period, where the vortices are lifting the fluid
6. Friction control in turbulent channel flows

Figure 6.3: Wall shear stress at the lower channel wall \((y = -1)\) normalised by the value assumed in the uncontrolled case \((\tau_w = 1)\). The four panels (a–d) correspond to increasing Reynolds numbers, \(Re_\tau = 104, 180, 360\) and 550, and show the measurements for control vortices with \(\Lambda = 6.6h\), each line corresponding to a different forcing amplitude. A similar picture is observed for the other wavelengths investigated. The curves are colour-coded with their value of DR, reported in the legends. The dark red line in (d) corresponds to a drag reduction of only 0.02\%, the highest achievable value for \(Re_\tau = 550\) for vortices with \(\Lambda = 6.6h\). Reprinted with permission from Canton et al., Phys. Rev. Fluids, 1, 081501, 2016. Copyright by the American Physical Society.

away from the wall. Concurrently, the area where the wall-shear stress is increased becomes wider than \(\Lambda/2\), extending over the vertical vortex axes. This modification is clear at the intermediate \(Re_\tau = 360\) (figure 6.3c) where both low- and high-Reynolds profiles of \(\tau_w\) can be observed.

If the first of these phenomena can be easily attributed to an increase of \(Re_\tau\) alone, and simply results in a decrease in amplitude, the modification of the \(\tau_w\) profiles is an indication of a more fundamental change in the flow mechanics affected by the large-scale vortices. Clearly, low Reynolds number effects are still present for \(Re_\tau = 104\) and 180 (see also Moser et al. 1999) and the present large-scale control is able to affect the low-\(Re\) wall cycle and thereby reduce the turbulence intensity. For higher Reynolds numbers these low \(Re\) effects disappear and the present control strategy cannot provide any drag reduction.

6.6. Large- vs Small-scale control

At the end of paper 8 we suggested that “a technique capable of generating fluid lift up directly at the wall, differently from the present one based on motion
Besides the amplitude $A$ and the spanwise wavelength $\Lambda_z$, the vortices are now controlled by one more parameter: the distance of their centre of rotation from the wall, $y_c$. Panel (b) presents the drag reduction provided by seven size combinations for different amplitudes. The best performing vortices for this $Re$ have the same viscous scales of the best vortices for $Re = 180$.

created outside of the viscous layer, would probably be more successful for high Reynolds numbers”.

We tested our idea by performing simulations at $Re = 550$ and controlling the flow with small vortices close to the wall. Figure 6.4(a) depicts a sketch of the small-scale vortices, while preliminary results are presented in figure 6.4(b). As we expected, the small-scale vortices perform quite well for $Re = 550$, with a maximum DR of about 15%, while the large-scale version of the method could not provide any drag reduction. In particular, figure 6.4(b) reports results for three values of the streamwise wavelength $\Lambda_z/h$, and for the best of the three, $\Lambda_z/h = 2.2$, different vortex heights. Even without doing a thorough
optimisation of the control scheme, it can be observed that the best performance is obtained for $\Lambda^+_z \approx 1200$ and $100 \lesssim y_c^+ \lesssim 180$. These are the same values that provide the best performance for $Re_\tau = 180$. We can therefore conclude that this control scheme does not scale in outer units, as observed in section 6.5, but in viscous units. Therefore, this is clearly a small-scale control, and not – as labelled by others (Yao et al. 2017) – a large-scale control.

To make a numerical example, the experimental data and empirical relation by Dean (1978) can be used to estimate the friction coefficient as a function of Reynolds number: $Re_\tau \approx 0.09 Re^{0.88}$. By using this approximation, we can estimate the size that the actuators would need to have for practical applications knowing that

$$\Lambda_z = \Lambda_z^+ \delta_\nu = \frac{\Lambda_z^+ \delta}{Re_\tau}, \quad \text{and} \quad y_c = y_c^+ \delta_\nu = \frac{y_c^+ \delta}{Re_\tau},$$

(6.5)

where $\delta_\nu$ denotes the viscous length scale. If, for example, one wanted to employ this control scheme on the wings of a plane (typical cruising $Re$ of about $10^{7}$), substituting numerical values for optimal performance in equation (6.5), results in a spanwise wavelength $\Lambda_z = \mathcal{O}(10^{-4})$ m and distance from the wall $y_c = \mathcal{O}(10^{-5})$ m. It is clear that actuators of this size are utterly impractical.
Summary of the papers

Paper 1

*Characterisation of the steady, laminar incompressible flow in toroidal pipes covering the entire curvature range*

This paper presents an in-depth analysis of the steady solution in curved pipe flows. A large database of numerical solutions, computed without any approximation, is presented spanning, for the first time, the entire curvature range and for Reynolds numbers ranging from 10 to 7000, where the flow is known to be unsteady. The analysis shows that by increasing the curvature the flow is fundamentally changed. Moderate to high curvature solutions are not only quantitatively, but also qualitatively different from low curvature solutions. It is therefore concluded that the Dean number and the Dean similarity are only relevant for infinitesimally low curvature. A complete description of some of the most relevant flow quantities is provided. Most notably Ito’s [J. Basic Eng. 81:123–134 (1959)] plot of the friction factor for laminar flow in curved pipes is reproduced, the influence of the curvature on the friction factor is quantified and the scaling is discussed at length.

Paper 2

*Modal instability of the flow in a toroidal pipe*

This paper presents a study of the first instability encountered by the incompressible flow in a toroidal pipe. It is found that, differently from the flow inside a straight pipe, this flow is modally unstable for all curvatures, the lowest investigated being 0.002, and that the steady solution undergoes a Hopf bifurcation leading to a limit cycle for a bulk Reynolds number of about 4000. The instability is therefore studied by means of linear stability analysis with classical eigenvalue computations and a novel bifurcation tracking algorithm. The analysis reveals that several families and isolated modes contribute to the neutral curve corresponding to the Hopf bifurcation. Comparison to nonlinear DNS shows excellent agreement, confirming the linear analysis and proving its significance for the nonlinear flow. Experimental data from the literature are also shown to be in considerable agreement with the present results.
7. Summary of the papers

Paper 3

Approaching zero curvature: modal instability in a bent pipe

The objective of the present work is to investigate the behaviour of the linear instability as the curvature of the pipe tends to zero. In order to do so, a stability code solving the equations in toroidal coordinates is presented. Results indicate that the toroidal pipe remains linearly unstable for curvatures as low as $10^{-7}$. While the critical Reynolds number $Re$ necessary for the instability grows with an approximately algebraic trend below $\delta = 0.002$, the neutral curve also closes in onto the limit of negligible curvature. It therefore appears that there could be values of $\delta$ and $Re$ where a linearly unstable toroidal flow could be connected to the linearly stable straight pipe flow.

Paper 4

The collapse of strong turbulent fronts in bent pipes

Isolated patches of turbulence in transitional straight pipes are sustained by a strong instability at their upstream front, where the production of turbulent kinetic energy (TKE) is up to five times higher than in the core. Direct numerical simulations presented in this paper show no evidence of such strong fronts if the pipe is bent. This paper details the temporal and spatial evolution of puffs and slugs in a toroidal pipe with pipe-to-torus diameter ratio $\delta = D/d = 0.01$ at several subcritical Reynolds numbers. Results show that the upstream overshoot of TKE production is at most one-and-a-half times the value in the core and that the average cross-flow fluctuations at the front are up to three times lower if compared to a straight pipe, while attaining similar values in the core. Localised turbulence can be sustained at smaller energies through a redistribution of turbulent fluctuations and vortical structures by the in-plane Dean motion of the mean flow. This asymmetry determines a strong localisation of TKE production near the outer bend, where linear and nonlinear mechanisms optimally amplify perturbations. A substantial reduction of the range of Reynolds numbers for long-lived intermittent turbulence is also observed, in agreement with experimental data from the literature. Moreover, no occurrence of nucleation of spots through splitting are detected in the range of parameters considered. Based on the present results, we argue that this mechanism gradually becomes marginal as the curvature of the pipe increases and the transition scenario approaches a dynamical switch from subcritical to supercritical.
Paper 5

A critical point for bifurcation cascades and intermittency

Transition to turbulence in straight pipe flow is a subcritical process, dominated by the proliferation of isolated patches of turbulence. However, experiments and computations suggest that when the pipe is bent above a pipe-to-coiling diameter ratio $\delta \gtrsim 0.028$ a supercritical Hopf bifurcation initiates a cascade, leading through increasing flow complexity and, ultimately, to a fully turbulent state. This paper proves this speculation correct by presenting the reconstruction, from nonlinear simulations at $\delta = 0.05$, of a stable two-periodic attractor and measurements of incommensurable frequencies in the flow. We also report evidence, in a limited region of the $\delta - Re$ parameter space, of the coexistence of fully turbulent flow and of a stable supercritical limit cycle; both being attracting states with finite and complementary basins. This is the first time that a fluid flow is shown to be stable in both turbulent and laminar states (other than the steady solution) for a fixed combination of flow parameters.

Paper 6

The three-dimensional structure of swirl-switching in bent pipe flow

Swirl-switching is a low-frequency oscillatory phenomenon which affects the Dean vortices in bent pipes and may cause fatigue in piping systems. Despite thirty years worth of research, the mechanism that causes these oscillations and the frequencies that characterise them remain unclear. This paper shows that a three-dimensional wave-like structure is responsible for the low-frequency switching of the dominant Dean vortex. The present study, performed via direct numerical simulation, focuses on the turbulent flow through a 90° pipe bend preceded and followed by straight pipe segments. A pipe with curvature 0.3 (defined as ratio between pipe radius and bend radius) is studied for a bulk Reynolds number $Re = 11700$, corresponding to a friction Reynolds number $Re_T \approx 360$. Synthetic turbulence is generated at the inflow section and used instead of the classical recycling method in order to avoid the interference between recycling and swirl-switching frequencies. The flow field is analysed by three-dimensional proper orthogonal decomposition (POD) which for the first time allows the identification of the source of swirl-switching: a wave-like structure that originates in the pipe bend. Contrary to some previous studies, the flow in the upstream pipe does not show any direct influence on the swirl-switching modes. It is also shown that a three-dimensional characterisation of the modes is crucial to understand the mechanism, and that reconstructions based on 2D POD modes are incomplete.
Paper 7

On large-scale friction control in turbulent wall flow in low Reynolds number channels

This is the first of two papers on turbulent skin-friction control via large-scale, counter-rotating vortices. The idea of a control strategy not involving viscous, and thus Reynolds number-dependent, flow scales was advanced in a paper by Schoppa & Hussain [Phys Fluids 10:1049–1051 (1998)]. The original implementation of the vortices consisted in the numerical imposition of the mean flow in the simulations. This approach is revisited in the present paper where the vortices are re-implemented as a volume force since the original implementation was found to be unphysical and lead only to transient drag-reduction at most. The control is tested in turbulent channel flows for friction Reynolds numbers $Re_\tau$ of 104 (as in the original study) and 180. A parameter study involving forcing amplitude and wavelength are performed for both Reynolds numbers, in search for optimal values. The vortices prove to be effective, providing a drag reduction of up to 18% for a viscous-scaled wavelength of 1200 and a forcing strength of only about 4% of the bulk velocity. An analysis of the flow quantities altered by the control is presented in an effort to elucidate the mechanics of operation of the vortices.

Paper 8

Reynolds number dependence of large-scale friction control in turbulent channel flow

This paper is concerned with the Reynolds-number dependence of the large scale vortex control scheme proposed by Schoppa & Hussain [Phys Fluids 10:1049–1051 (1998)] and is a natural extension of Paper 3. Two new sets of Direct Numerical Simulations have been performed for friction Reynolds numbers $Re_\tau$ of 360 and 550. These simulations, along with the ones carried out for lower $Re_\tau$, constitute an extensive database that allows an in-depth analysis of the method as a function of the control parameters (amplitude and wavelength) and the Reynolds number. Results show that the effectiveness of the method is reduced as the Reynolds number increases above $Re_\tau = 180$ and no drag reduction can be achieved for $Re_\tau = 550$ for any combination of the parameters controlling the vortices. An analysis of the effects of $Re_\tau$ on the mechanics of the control is presented as a function of both outer and inner (viscous) scaling.
Conclusions and outlook

Stability and transition to turbulence in bent pipes have been studied with linear and nonlinear methods. Regarding lower Reynolds numbers, the focus is on the flow in a toroidal pipe, as it represents the common asymptotic limit between spatially developing and helical pipes. It is shown that a Dean number ($De$)-based analysis is inadequate even for the laminar flow where quantities that were believed to scale with $De$, e.g. the friction factor, are actually a function of curvature and Reynolds number separately.

The flow inside a toroidal pipe is analysed close to its first instability for the complete range of curvatures. The results of the stability analysis show that an infinitesimal curvature is sufficient to make this flow linearly unstable. It is found that a Hopf bifurcation leads the flow to a periodic regime for all curvatures. A novel bifurcation tracking algorithm is introduced and employed to trace the neutral curve associated to this bifurcation. Several different modes are found, with differing properties and eigenfunction shapes. Some eigenmodes are observed to belong to groups with common characteristics, deemed families, while others appear as isolated. These findings show excellent agreement with nonlinear simulations and previous experimental results. As the curvature tends to zero, towards the straight pipe limit, the flow remains linearly unstable, but the critical Reynolds number grows nearly exponentially.

A different kind of analysis is performed in the low curvature range, where subcritical transition precedes the modal instability. Here the flow becomes turbulent while being linearly stable, with laminar and turbulent flows coexisting in an intermittent fashion. Localised turbulent structures in bent pipes bear qualitative similarities to those in straight pipes in that they appear in the form of puffs and slugs that are sustained by an instability at their upstream front. However, the most striking difference is a substantial weakening of the front if the pipe is bent. Furthermore, there appears to be no puff splitting, at least for the choice of parameters investigated. The absence of puff splitting is likely to be connected to the weak and localised upstream fronts: turbulent structures that leave a mother puff have a low probability of entering the small region of high amplification located near the outer wall, which would trigger the instability that sustains a puff.
The phenomenon of swirl-switching in turbulent bent pipe flow is also studied in the present work. The method of choice is three-dimensional proper orthogonal decomposition, performed on snapshots extracted from nonlinear simulations, where synthetic turbulence is prescribed at the inflow. Results show that the switching is caused by the presence of a wave-like structure with a wave length of about 7 pipe diameters. Differently from results reported in the literature, this mode does not present any connection to the straight pipe preceding the bend. Moreover, it is concluded that it is due to the three-dimensional, travelling structure of this single mode that this phenomenon was confused with multiple two-dimensional modes in previous studies.

Finally, a study on drag reduction via large-scale vortices in turbulent channel flow is also presented. It is shown that, for low Reynolds numbers, substantial skin-friction drag reduction can be achieved by imposing counter-rotating, streamwise-invariant vortices as a volume force. Drag reduction is achieved via a delicate balance between the region where the vortices lift turbulence away from the wall, reducing the wall-shear stress, and the region where, despite causing relaminarisation, the vortices induce a higher skin-friction drag. It is also found that the performance of this control strategy rapidly decreases with Reynolds number, since the large-scale vortices are only effective in flows where low-Re effects are still present, and cannot induce any drag reduction for \( Re_\tau \geq 550 \). For high values of \( Re_\tau \) the control scheme can still work, but only if the size of the vortices and their distance from the wall are scaled in viscous units. This renders the method Reynolds number-dependent, and necessitates small and impractical actuators for typical industry applications.

**Outlook on future work**

As it is often the case, some rather interesting questions remain unanswered.

It is still not entirely clear how the neutral curve of the flow in a toroidal pipe connects to the \( Re \)-independent stability of the flow in a straight pipe. Present results indicate that zero curvature is an asymptotic limit for the neutral curve, which approaches \( \delta = 0 \) with \( Re \to \infty \) algebraically. However, the number of data points currently available is not sufficient to draw firm conclusions. An additional analysis could also be performed by separating the effects that the curvature has on the base flow and on the perturbation fields. Moreover, the code written in toroidal coordinates can be readily adapted to solve the equations in helical coordinates, allowing the study of helically coiled pipes and of the effect of torsion on the transition scenario.

No puff splitting was observed in our simulations of the subcritical regime at low curvatures. We conjectured that this phenomenon is actually absent, and provided a possible explanation. Our claim should be verified by repeating our simulations for longer times or, even better, experimentally, as done in straight pipes. An experimental apparatus with a helically coiled pipe could also be used, since low torsion does not affect the transition scenario; this could
be built using a very long pipe, therefore allowing the observation of puffs for long times and the possibility of numerous repetitions with different initial perturbations. It is also important to understand whether puff splitting is absent for any curvature greater than zero, confining it to a purely straight pipe phenomenon, or if instead, and more likely, it is gradually affected by the introduction of curvature.

The critical point where sub- and supercritical transition intersect also requires additional investigations. We documented the coexistence of a bifurcation cascade and featureless turbulence, but an in-depth explanation of the dynamics of the system in the neighbourhood of this point could shed new light on both transition processes. Here as well the code written in toroidal coordinates could help, by reducing the computational cost of the many simulations required.

Regarding swirl-switching, we could determine the nature of these oscillations, but the origin of this wave-like structure remains, at the moment of writing, unknown. We conjectured that this structure, until now observed in relatively turbulent flows, could actually appear at lower Reynolds numbers and be caused by a modal instability of the flow; this is currently under active investigation.
Acknowledgements

I would like to thank my advisors, Philipp Schlatter and Ramis Örlü for choosing me as a PhD student. During the past four years they have taught me a lot, not only about fluid mechanics; they have also provided me with constant support and helped me whenever I needed it, in and out of the office. Lastly, they have also sent me around the globe to conferences and workshops which greatly enriched me as a researcher and as a person. Special thanks to Shervin Bagheri who reviewed both my these, providing helpful suggestions, and took me twice to Montestigliano workshops. Lastly, Henrik Alfredsson who, with the help of Ramis and Antonio Segalini, did his best to bring me back to the real world, and asked to be chairman at my defence.

I am grateful to my many friends and colleagues at the department, past and present, to all the people who lent me their couch in times of need, to the Royal Technicians for the distraction, and to the graphic designers for helping with the cover page. And, in no particular order, to Enrico, with whom it was nice to work and exchange opinions in the past year, Mattias, who translated all the Swedish things that needed translation and accepted only gelato as form of payment, Prabal, Giandomenico, Elektra, Nicolas, Pierluigi, Luca, Marco, Walter, Nicolò, Mehdi, Ali, Kristina, Ninge, Francesco, Marco, Pedro, Ricardo, Evelyn, Lorenz, Velibor, Matteo, Emanuele, Ekaterina, Uğis, Erik, Azad, Daniel, Ellinor, Iman, Jean Cristophe, Luis.

I would like to thank my family, for the support you always give me and for teaching me how to become who I am. Last, but not least, thank you Agnese, always with me even when two thousand kilometers away, for the love and the infinite patience.

This study has been supported by the Swedish Research Council (VR; Diarienummer: 621-2013-5788). Parts of the thesis have also been funded by the Knut and Alice Wallenberg Foundation via the Wallenberg Academy Fellow Programme. Computer time was provided by the Swedish National Infrastructure for Computing (SNIC) and PRACE. The Petersohns Minne Foundation is acknowledged for travel stipends. GKN Aerospace is acknowledged for awaring the price in mechanics at the 25th Svenska Mekanikdagarna.
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