# The Effect of Instrumental Precision on Optimisation of Epoch-Wise Displacement Networks

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## Abstract

In order to detect the geo-hazards, different deformation monitoring networks are usually established. It is of importance to design an optimal monitoring network to fulfil the requested precision and reliability of the network. Generally, the same observation plan is considered during different time intervals (epochs of observation). Here, we investigate the case that instrumental improvements in sense of precision are used in two successive epochs. As a case study, we perform the optimisation procedure on a GPS monitoring network around the Lilla Edet village in the southwest of Sweden. The network was designed for studying possible displacements caused by landslides. The numerical results show that the optimisation procedure yields an observation plan with significantly fewer baselines in the latter epoch, which leads to saving time and cost in the project. The precision improvement in the second epoch is tested in several steps for the Lilla Edet network. For instance, assuming two times better observation precision in the second epoch decreases the number of baselines from 215 in the first epoch to 143 in the second one.

## Methodology

In this study, we are about to investigate the case that more precise measurements can be performed in the next epoch of two-epoch observations. Suppose that the observations in the first epoch have the weight matrix $\mathbf{P}_1$, and the Variance-Covariance (VC) matrix $\mathbf{C}_{\mathbf{x}_1}$. If we increase the weight of observations in the second epoch $\mathbf{P}_2$, we will acquire a more precise network by the second observation plan, so we can write:

$$\mathbf{P}_2 = \frac{1}{k}\mathbf{P}_1 \quad \rightarrow \quad \mathbf{C}_{\mathbf{x}_2} = k\,\mathbf{C}_{\mathbf{x}_1} \quad \text{with} \quad k < 1$$

where $\mathbf{C}_{\mathbf{x}_2}$ represents the VC matrix of the network in the second epoch, and $k$ is a coefficient that scales down the variances of the net points in the first epoch.

The displacement vector is written as: $\mathbf{d} = \mathbf{x}_2 - \mathbf{x}_1 \rightarrow \mathbf{C}_\mathbf{d} = \mathbf{C}_{\mathbf{x}_1} + \mathbf{C}_{\mathbf{x}_2}$.

The above equation can be linearised by expanding to a Taylor series as:

$$\mathbf{C}_\mathbf{d} - k\,\mathbf{C}_{\mathbf{x}_1} = \mathbf{C}_{\mathbf{x}_1} = \mathbf{C}^0_{\mathbf{x}_1} + \sum_{i=1}^{n} \frac{\partial \mathbf{C}_{\mathbf{x}_1}}{\partial \mathbf{P}_i} \Delta \mathbf{P}_i$$

We try to minimise the differences between the VC matrix of the first epoch and a defined ideal criterion matrix ($\mathbf{C}_s$):

$$\left\| \mathbf{C}_s - \mathbf{C}^0_{\mathbf{x}_1} \right\|_2 = \min$$

subjected to precision, reliability and physical constraints.

## Study Area

In order to test the idea of this study in reality, the GPS monitoring network of Lilla Edet region in the southwest of Sweden is chosen. This area is well-known for its landslides, which have been happening during the years, and since the year 2000, the risky area has been settled under monitoring controls in different time intervals (epochs). The existing GPS monitoring network of the area consists of 35 observation stations, where 6 of them are fixed points. Totally, 245 independent baselines are observed in this network by neglecting the correlation influence amongst the GPS receivers. It has been assumed that each observation session includes two GPS receivers and the instruments are moved to perform the next session.

## Results and Discussions

To start the optimisation procedure, first we need to numerically introduce our criterion matrix. The displacement VC matrix, $\mathbf{C}_\mathbf{d}$, was inserted into the criterion matrix by using the statistical method, where the minimum displacement of 5 millimetres in all directions was assumed to be detected for each point in this network. We also assumed different values for $k$ to investigate its effect on the optimised observation plan.

The single-objective optimisation model of precision, which was constrained to reliability, was used to optimise this monitoring network in order to redesign observation plans for two subsequent epochs considering better precision for instruments in the second one.

As can be seen in Table 1, in the first step, we assume the equal weight matrix for the both epochs i.e. no instrumental precision improvement is considered. In order to have a network to be able to detect 5 millimetre displacements, it is enough to observe 215 baselines in each epoch of measurements.

Table 1 Number of required observations in the first and second epochs after optimisation procedure according to precision improvements. The number of baselines before optimisation is 245.

| $k < 1$ | $P_2 = \frac{1}{k} P_1$ | No. of observations in | |
|---|---|---|---|
| | | first epoch | second epoch |
| 1 | $P_2 = P_1$ | 215 | 215 |
| 0.9 | $P_2 = 1.1\,P_1$ | 215 | 204 |
| 0.8 | $P_2 = 1.2\,P_1$ | 215 | 193 |
| 0.7 | $P_2 = 1.4\,P_1$ | 215 | 175 |
| 0.6 | $P_2 = 1.7\,P_1$ | 215 | 154 |
| 0.5 | $P_2 = 2\,P_1$ | 215 | 143 |

In the next steps, we try to indicate the effect of precision improvements on the second epoch of the observation plan. The larger we made the weight matrix in the second epoch, the less number of baselines we required for measuring. Figures 1 and 2 are the illustration of the above table, when $k$ is equal to 0.5 and 0.7, respectively.

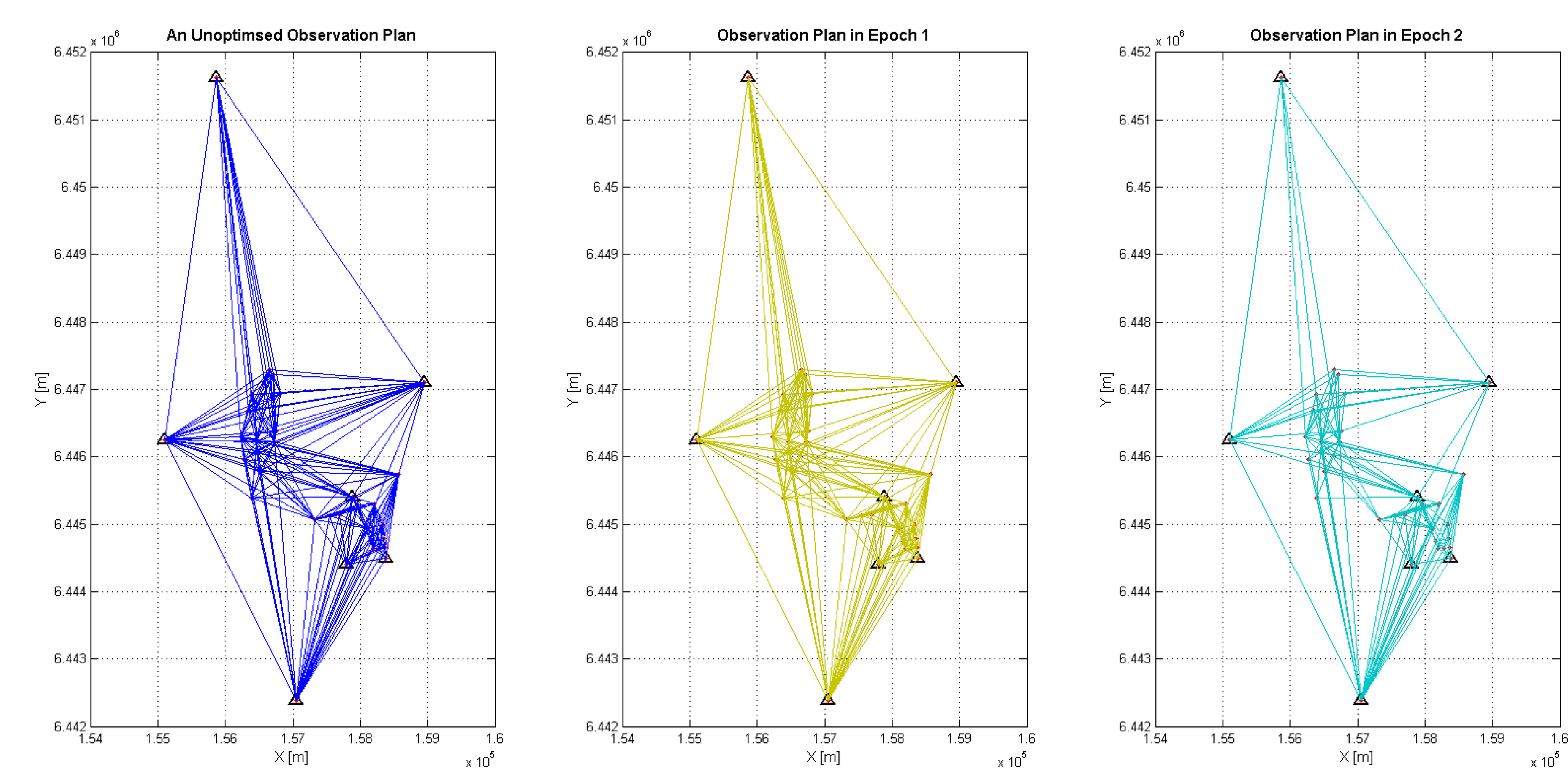


Figure 1 The panels in the figure display the unoptimised and optimised networks in first epoch, and observation plan for second epoch considering $k = 0.5$, respectively, from left to right.

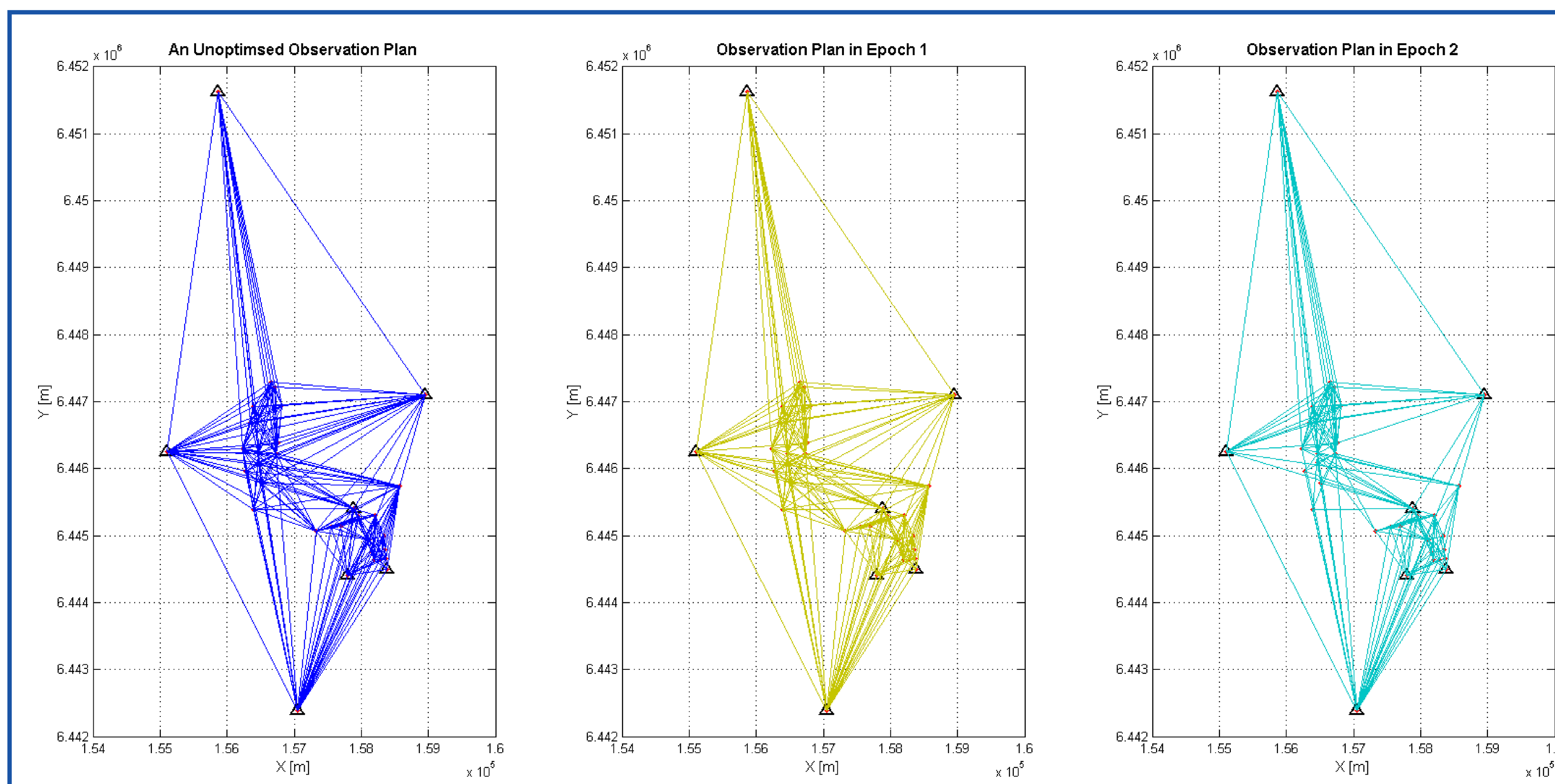


Figure 2 The panels in the figure display the unoptimised and optimised networks in first epoch and observation plan for second epoch considering $k = 0.7$, respectively, from left to right.

It should be clarified here that it is improbable these days to double the weight matrix for the next epoch. In other words, with available precise measuring devices in the market, it is very rare to be able to increase the precision of the instruments very much within a short time interval. However, it has been investigated theoretically in this study to express the idea and bring up the thoughts around this issue.

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