Buckling of End-Bearing Retaining Walls in Clay

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Master Thesis, 2018
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Abstract
The design of back-anchored retaining walls in Sweden has traditionally not included global elastic instability of the retaining wall as a possible failure mode. Eurocode 3 part 5 (SS-EN 1993-5) requires design of steel structural members for retaining walls to assess the risk of buckling if the normal force exceeds 4% of the critical buckling load of the retaining wall.

The geological conditions in Eastern Sweden are characterized by the intersection of very hard Precambrian rock and very soft Holocene clays. Thus often ground anchors anchored in rock at a 30-50 degree angle to the vertical plane are used to support retaining walls, resulting in a very high utilization of the ground anchor and a significant normal force in the retaining wall. The threshold value for buckling risk is consequently frequently exceeded and the specific failure mode, of global buckling, is often limiting the use of the structural members in practical design.

The buckling load can either be calculated using Euler’s second or third buckling mode, or by modelling the soil-structure interaction by a suitable model. Since no such model is specified in the code, the aim of this thesis was to develop a model which takes into account the stabilizing effect of the soil for the calculation of the buckling force and to model the soil-structure interaction with a beam-spring model connected to Winkler springs.

The model simulations show that the soil has a significant influence on the critical load, especially when the retaining wall base is driven to depths greater than 2 meters below excavation depth. The model simulations suggest that higher utilization, with up to 4 times greater critical load, of the steel members is possible for some specific cases and an idealized design factor is also elaborated.

Keywords
Berliner walls, Critical loads, buckling, finite element model, Eurocode 3.
Sammanfattning

Dimensioneringen av bakåtförsänkade spontväggar har traditionellt sett i Sverige inte tagit hänsyn till risken för global knäckning. I och med införandet av Eurokod 3 kapitel 5 (SS-EN 1993-5) som styrande dokument vid dimensionering av sponter måste risken för knäckning nu mera beaktas när normalkraften överstiger 4 % av den kritiska knäckningslasten.

De geologiska förhållandena i de östra delarna av Sverige, med lösa leror som täcker hårt berg, leder till att bakåtförsänkade sponter med brant lutande stag ofta används. Detta leder till en hög utnyttjandegrad av ankaret och också stora normalkrafter i sponten, vilket leder till att knäckning ofta blir dimensionerande brottmod för sponten.

Metoden för att beräkna knäckningslasten kan enligt SS-EN 1993-5 göras med Eulers andra eller tredje knäckningsfall eller med en modell som tar hänsyn till jordens stabiliserande effekt. Idag finns ingen sådan numerisk modell att hitta i litteraturen, varför målet med detta arbete har varit att finna en lämplig modell för att ta hänsyn till jordens inverkan vid bestämning av knäckningslasten. För att modellera samverkan mellan jorden och sponten användes en balkmodell med Winkler fjädrar.

Simuleringarna visar att jorden har en signifikant inverkan på den kritiska knäckningslasten, särskilt när nedslagsdjupet är större än 2 meter. Flera simulerade geometrier har gett drygt fyra gånger högre knäcklast jämfört med den knäcklast som erhålls om SS-EN 1993-5 följs. Om jorden tas hänsyn till i dimensioneringen av en spont skulle således slankare konstruktioner kunna användas.

Nyckelord

Berliner spont, Rörspont, kritisk knäckningslast, knäckning, finit element modell, Eurocode 3
Preface

This work has been done as the final part on the Master of Science in Civil Engineering program at the Royal Institute of Technology, KTH, in Stockholm. The thesis has been written for the division of Soil and Rock Mechanics at the school of Architecture and the Built Environment in collaboration with ELU Konsult AB. The problem under investigation was first raised by Jimmie Andersson during the work with the project which is described in chapter 2.4.

We would like to express our greatest gratitude towards Anders Beijer-Lundberg and Jimmie Andersson at ELU Konsult AB for their exceptional co-operation during the whole process and the fast responses and never tiring work to help us in our writing and understanding of the problem. We would also like to thank Christoffer Svedholm at ELU Konsult AB for the kind assistance during the modelling phase.

At the division of Soil and Rock Mechanics we would like to thank our supervisor Stefan Larsson for the help and good comments that was given when needed and for being one of the main reasons why we chose geotechnology as our specialization.

Some KTH-classmates also deserves some gratitude, not only for the immensely fun five years at KTH but also for inspiration during the work with the thesis, especially Sofie Johansson and Tobias Nilsson for the good company in the KTH computer room during the work on the ABAQUS models and Louise Larsson for keeping the motivation high during the days at ELU Konsult AB.

Finally we would like to thank Fredrik Resare at ELU Konsult AB who introduced us at ELU Konsult AB and without whom we might not have been given the chance to write this thesis.

Stockholm, May 2018

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Nomenclature

Abbreviations
BW Berliner Wall
FEM Finite Element Method
RW Retaining Wall
SPW Sheet Pile Wall

Latin Symbols
A Cross section area \([m^2]\)
c\text{\textsubscript{u}} Undrained shear strength \([Pa]\)
d Pile diameter \([m]\)
D RW driven depth below excavation bottom \([m]\)
E Young’s modulus \([Pa]\)
e \text{2\textsuperscript{nd} order deflection} \([m]\)
e\text{\textsubscript{0}} Initial maximum deflection \([m]\)
f\text{\textsubscript{y}} Yield strength \([Pa]\)
f\text{\textsubscript{y,red}} Reduced yield strength \([Pa]\)
f\text{\textsubscript{yd}} Design value for yield strength \([Pa]\)
F\text{\textsubscript{Anchor}} Force in anchor \([N]\)
F\text{\textsubscript{Q,Ed}} Additional horizontal force regarding non-sway buckling mode of a SPW \([N]\)
G Shear modulus \([Pa]\)
H Excavation depth \([m]\)
h Overall height of a sheet pile cross section \([m]\)
l Second area moment \([m^4]\)
K Buckling length factor \([-]\)
k\text{\textsubscript{k}} Bedding modulus \([N/m^3]\)
k\text{\textsubscript{yy}} Interaction factor \([-]\)
k\text{\textsubscript{yz}} Interaction factor \([-]\)
l Distance between lowest horizontal support (wale beam, anchor) and toe of a SPW \([m]\)
l\text{\textsubscript{cr}} Buckling length of equivalent centrally compressed RW \([m]\)
M\text{\textsubscript{c,Rd}} Design moment resistance of the sheet pile cross section \([Nm]\)
M\text{\textsubscript{Ed}} Design value of the maximum moment \([Nm]\)
M\text{\textsubscript{N,Rd}} Reduced design value of the resistance to bending moments \([Nm]\)
M\text{\textsubscript{Rd}} Design moment resistance \([Nm]\)
M\text{\textsubscript{y,Ed}} Design value of the maximum moment around y-axis \([Nm]\)
$M_y,\text{Rk}$ Characteristic moment resistance around y-axis [Nm]

$M_z,\text{Ed}$ Design value of the maximum moment around z-axis [Nm]

$M_z,\text{Rk}$ Characteristic moment resistance around z-axis [Nm]

$N$ Normal force [N]

$N_{\text{cb}}$ Stability factor [-]

$N_{\text{cr}}$ Critical normal force [N]

$N_{\text{Ed}}$ Design value of normal force [N]

$N_{\text{pl,Rd}}$ Design plastic resistance to normal force [N]

$N_{\text{Rk}}$ Characteristic normal force resistance [N]

$q_d$ External load [Pa]

$q_k$ Maximum soil stress [Pa]

$T$ Tensile force work [J]

$t_f$ The flange thickness of a sheet pile cross section [m]

$t_w$ The web thickness of a sheet pile cross section [m]

$U$ Strain energy [J]

$u$ Deflection in horizontal direction [m]

$V_{\text{Ed}}$ Design value of shear force resistance [N]

$V_{\text{pl,Rd}}$ Design value of plastic shear resistance [N]

$W_{\text{el}}$ Elastic section modulus [m$^3$]

$W_{\text{pl}}$ Plastic section modulus [m$^3$]

$W_y$ Section modulus around y-axis [m$^3$]

$x$ Distance from top of BW to maximum displacement [m]

**Greek Symbols**

$\alpha$ Imperfection factor [-]

$\beta_B$ Reduction factor of section modulus [-]

$\beta_{cr}$ Correlation factor between $l_{cr}$ from FEM and EN 1993-5 [-]

$\beta_D$ Reduction factor of bending stiffness [-]

$\gamma$ Unit weight of soil [N/m$^3$]

$\gamma_{M0}$ Partial factor for resistance of cross section whatever the class is [-]

$\gamma_{M1}$ Partial factor for resistance of members to instability assessed by member checks [-]

$\gamma_{\text{Sd,\text{Ncb}}}$ Partial factor for stability [-]

$\varepsilon$ Strain [-]

$\eta$ Generalized factor for initial imperfection [-]
\( \eta_b \) Shape factor for bending [-]
\( \eta_c \) Shear strength correction factor [-]
\( \bar{\lambda} \) Slenderness factor [-]
\( \rho \) Reduction factor [-]
\( \sigma \) Stress [Pa]
\( \tau \) Shear stress [Pa]
\( \tau_{fu} \) Shear strength [Pa]
\( \varphi_d \) Friction angle [°]
\( \chi \) Reduction factor for the relevant buckling mode [-]
\( \chi_{LT} \) Reduction factor for lateral torsional buckling [-]
\( \chi_y \) Reduction factor for flexural buckling around y-axis [-]
1. Introduction

Different types of retaining structures are usually used in deep excavations to avoid steep slopes and still ensure a stable surrounding soil. There are different types of retaining walls, RW, where in this report one type is specifically managed and analyzed, namely the Berliner Wall, BW. Another commonly used RW is the sheet pile wall, SPW. Both can work as a cantilever or as a continuously supported beam, with anchors or struts on one or several levels. Ground anchors can either be inclined or horizontal, since the bedrock in the Stockholm area is mostly shallow (Johansson, 1984) one common type are inclined anchors drilled into bedrock (Bredenberg, 1984). Typical Berliner Walls with steel pipes and plates as main elements and tied back anchors are shown in Figure 1.

Figure 1 - A berliner wall with three anchor levels (left) and a detail of whale beam and anchors (right), pictures provided by ELU Konsult AB.
To assure that the RW can withstand all imposed loads from retained soil and surcharge loads, the importance of having a safe design is crucial for safety of the workers and surrounding structures. However, being too conservative can lead to an overly expensive structure with a very high material use. Failures that can occur are both associated with the structural capacity of the RW and all of its components; anchors, wale beams and dowels, and also the total stability of the excavation (Ryner, Fredriksson, & Stille, 1996).

In Sweden, the suggested design method of RWs has gone through significant changes during the past decades. Until 1996 “Förankrade Sponter” (Sahlström & Stille, 1979) was the most commonly used guidelines, followed by the handbook “Sponthandboken” adapting the methods in “Förankrade Sponter” to the partial coefficient method introduced in BKR 94 (Ryner, Fredriksson, & Stille, 1996). In 2011 the old Swedish standards were fully substituted by Eurocode as the methods that should be followed when designing new constructions in Sweden (Paulsson & Engström, 2016). For the design of geotechnical structures this meant that Eurocode 7 was introduced as the governing document to follow during the design process. Eurocode 7 specifies that all structural parts in an RW made of steel should be designed according to Eurocode 3 part 5, EN-1993-5. The section introduces, in Sweden, a previously unconsidered failure mode in the design - global buckling (European Comittee for Standardization, 2007). The method in EN 1993-5 is presented fully in chapter 2.1.3.

Situations where buckling can become a potential failure mode arise when the normal force, transferred by the inclined anchors, is significant (European Comittee for Standardization, 2007). This can for instance occur when the anchors, due to surrounding structures, must be heavily inclined, for example as in the project presented in chapter 2.4, or when fewer anchors with higher loads are used. In the suggested Eurocode model no consideration is taken to the surrounding soil, consequently the critical buckling force often becomes low. One of the effects of the new rules is that the amount of steel needed in the structures increases significantly compared to when the previous Swedish standards are used. Since no
records can be found of an RW that has failed due to buckling in the scientific literature, a justified question is how well the model represents reality. The surrounding soil should have an impact on the critical buckling force and therefore the normal force bearing capacity.

This study has focused on end-bearing BWs in homogenous clay and by using several Finite Element Models the behavior under axial loading has been analyzed. The outcome is a presentation of how the buckling load varies with various soil conditions.

1.1. Objectives

Since the introduction of the Eurocode brought about big changes to the design of an RW, with the addition of a novel failure mode to be considered, several questions arise regarding the old and new model. The aim of the thesis is to study whether the new recommended model might be too conservative and not taking the soil-structure interaction well enough into account during the design of the Berliner walls. The aim can be narrowed down to the following main research question:

How can the soil-structure interaction be accounted for in the determination of the critical buckling load for an end bearing berliner wall in homogenous clay?

1.2. Methodology

The testing was carried out using a Winkler spring foundation in the Finite Element program ABAQUS 6.14-2. The springs are used to model the surrounding soil both as linearly elastic in a linear buckling step and in a post-buckling analysis as non-linear elastic to model the plastic properties of the soil. The results are compared to the model described in EN 1993-5.
1.3. Structure

The first chapter, introduction, gives some initial information on the problem and why it is of interest to study.

The second chapter, background, gives the relevant information regarding classical linear instability theory and how it is implemented in the Eurocode. The background also contains soil mechanics that is relevant for the work, as well as a general description of typical Swedish soil conditions and a case study description related to the thesis.

The third chapter, methodology, describes the method used in detail, both how the various parameters have been determined for nine different cases as well as a thorough description of the modelling procedure and a convergence test. It is followed by a fourth chapter, results, firstly presenting summarized results to put the work in context followed by all results from the nine different cases.

The final chapter, discussion and conclusions, contains a discussion of the results, conclusions and suggestions for further work in the area. The discussion also contains a comparison between various methods to determine the critical load and how well they reflect the results achieved in the simulations.
2. Background

The background chapter gives the necessary background information to make interpretations of the results and selection of method. The chapter contains several subchapters all describing various aspects of the problem. The theory starts with a basic description of the theory of elastic instability followed by a description of methods by which the problem is currently considered in the design. It is followed by a part describing typical soil conditions of interest for the study, followed by a section describing the relevant theories in soil mechanics. The background is concluded with a brief description of a project carried out by ELU Konsult AB where global elastic instability caused problems in the design.

2.1. Elastic Stability

2.1.1. Critical Loads

The theory of bending of a beam, or other structures, describes the stresses in the structure as directly proportional to the applied loads. The requirement for this to be true is that the deformations, due to the applied loads, do not change the load deformation response. If a beam only has a lateral load this can be said to be true, since, as long as small deformations are assumed, the stresses in the beam will only depend on the bending moment. If the beam however is also subjected to axial forces, the deflection due to the lateral forces will cause an extra bending moment to arise in the beam. This will increase the deflection even more and therefore a bending moment and resulting stress will arise due to the axial force’s eccentricity. The forces applied to the beam will hence change the way the beam reacts to the forces. If the deflection is increased only the slightest, the stresses will increase due to a larger bending moment from the axial
force. This effect on the moment due to the axial force is called second order effects (Timoshenko & Gere, 2012).

The subject of the behavior of beams under these conditions was comprehensively studied and described by Timoshenko (Timoshenko & Gere, 2012). Due to the use of high strength steel and especially the urge to use less steel in the construction, the problem of elastic stability is of great importance. The reason for which is that slender structures are affected by buckling to a higher extent, while structures with a low slenderness usually have a plastic failure (Luís, Rui, & Helena, 2010), see Figure 3 where this relationship is visualized. The theories concerning effects of stability have been known for a long time and are still studied today. It is the theory of elastic stability which constitutes the basis for the models used to calculate the utilization of a compressed and laterally loaded structure both in EN 1993-5, for RWs, and EN 1993-1-1, for regular beams.

The basis of the effect when having both a lateral and axial load is the effect on the stresses in the beam due to an increase in the first order deflection caused by the eccentricity of the axial force. The increase in deflection can be expressed using an amplification factor. This amplification of the initial deflection, \( \xi \), has been shown to be as presented in Equation 1, where \( N_{cr} \) is the critical Euler buckling load. As can be seen the amplification increases rapidly when the applied axial force approaches the critical load.

\[
\xi = \frac{1}{1 - N/N_{cr}} \tag{1}
\]

This amplification is accurate if used on the initial deflection, from which it is possible to determine the total maximum bending moment and with the use of the Navier equation determine the total maximum stress in the cross section. Since the moment is the second derivative of the deflection the solution loses some of its accuracy if applied directly on the moment (Timoshenko & Gere, 2012).

The critical load of a beam is defined as the load at which a non-eccentrically compressed perfectly straight beam in equilibrium deflects
into an instable state. Euler (1759) based his model on a beam clamped in one end and free at the other and found that the load at which this occurs is as presented in Equation 2, where the factor $K$ is equal to 2. The subject was further studied by Lagrange (1770) who developed a solution to determine the same type of load for a perfectly straight beam which is simply supported, where the factor $K$ in Equation 2 is equal to 1. The solution was then combined for several different combinations of clamped and pinned supports resulting in different values of $K$ depending on the support conditions. The factor $K$ describes how big a part of the total beam length, $L$, is subjected to the buckling phenomenon, where half a sinusoid is formed along the length $KL$ (Mariano & Galano, 2015). It depends only on the elastic modulus, $E$, and the second area of moment, $I$.

$$N_{cr} = \frac{\pi^2 EI}{(KL)^2}$$ (2)

For beams which do not fulfill any of the support types for which a value of $K$ can be directly found the energy method can be used to determine the critical load. This is carried out by estimating the shape of the deflection curve of the buckled bar and setting the strain energy of bending equal to the work done by the axial force due to vertical displacement, $\lambda$. Knowing that the bending moment in any point along the beam is equal to the deflection at that point times the axial force and that the work done in the axial direction is equal to the axial load times the axial displacement, the value of the critical load can be found. The solution is exact for a known shape of the deflection curve, but as long as the approximation of the deflection curve is quite accurate the solution will only be marginally wrong (Timoshenko & Gere, 2012). The expression for the total strain energy along the bar due to bending moment, $\Delta U_1$, most suitable when only an approximate shape of the deflection curve is used, is shown in Equation 3, where $M$ is the moment and $EI$ is the stiffness. The energy resulting from axial deformation, $\Delta T$, can be expressed as in Equation 5, $N_{cr}$ is then found by solving Equation 6. In Figure 2 the axis directions and principal working of the method is displayed, along with explanation of the variables.
Equation 4 is the work done due to deflection of the elastic springs, where $k$ is the spring coefficient.

$$\Delta U_1 = \int_0^l \frac{M^2}{2EI} \, dx = \int_0^l \frac{N^2(d - y)^2}{2EI} \, dx$$

$$\Delta U_2 = \frac{k}{2} \int_0^l y^2 \, dx$$

$$\Delta T = \frac{N}{2} \int_0^l \left(\frac{dy}{dx}\right)^2 \, dx$$

$$\Delta U_1 + \Delta U_2 = \Delta T$$

Figure 2 - The principle of the energy method for determination of critical load.
By appropriate modifications to Equation 4, the same analysis can be done for a beam which is only partially restrained by springs or with different spring coefficients for the various springs. Also, if there are several loads at different positions along the beam the critical loads can be determined using the energy method (Timoshenko & Gere, 2012). Using the energy method to determine the critical load for a system with springs along the whole, or part of the beam, requires that the springs are elastic and therefore have a linear force-deflection relationship. The expression for the critical load of a bar with continuous elastic restraints can be expressed as in Equation 7, where \( m \) represents the number of half sinusoids that forms along the beam when it buckles and \( \beta \) the modulus of foundation, for a beam with several springs with a given distance between them it is the spring coefficient divided by the distance between the springs (Timoshenko & Gere, 2012). The value of \( m \) can be determined by the use of Equation 8.

\[
N_{cr} = \frac{\pi^2 E_s I}{L^2} \left( m^2 + \frac{\beta \cdot L^4}{m^2 \cdot \pi^4 \cdot E_s I} \right) \tag{7}
\]

\[
\frac{\beta L^4}{\pi^4 EI} = m^2 (m + 1)^2 \tag{8}
\]

2.1.1.1. Buckling of Partially Embedded Piles

An implementation of the methods with an elastic foundation has been performed for piles in the handbook “Pålgrundläggning”, both for fully embedded piles where Equation 7 is used, and for partially embedded piles for which Equation 9, 10 and 11 are used (Olsson & Holm, 1993). For calculations, the soil is seen as an elastic medium represented by a subgrade modulus. The modulus character depends on the type of soil, linearly increasing in depth for friction soils and constant for clays, and is defined as a material dependent constant, \( k_{bm} \). For partially embedded piles, when the pile has been driven down at the depth of \( h > 2.5l_s \), the pile is seen as clamped at the depth \( l_s \), calculated using Equation 9 for clays and Equation 10 for friction soils where \( EI \) is the pile stiffness and \( d_p \) the pile diameter (Olsson & Holm, 1993).
\[ l_s = 1.4 \sqrt[4]{\frac{EI}{k_{bm} \cdot d_p}} \]  

\[ l_s = 1.8 \sqrt[5]{\frac{EI \cdot h}{k_{bm} \cdot d_p}} \]

For determination of the buckling load for the partially embedded pile, the buckling length, \( l_{cr} \), depends on the depth where the pile is seen as clamped, \( l_s \), and the height above the ground surface, \( l_0 \) as in Equation 11. The buckling load is determined by using the regular Euler buckling, with the calculated buckling length \( l_{cr} \) as \( KL \) in Equation 2 (Olsson & Holm, 1993).

\[ l_{cr} = \frac{l_0 + l_s}{2} \]

2.1.2. Effect of Imperfections

Since beams, and RWs, are not in reality perfectly straight the methods used to determine the bearing capacity of centrally compressed real members also include a factor that accommodates for imperfections (Luís, Rui, & Helena, 2010). The reason is that the model formulated by Euler implies that no transversal deflection due to an axial force occurs until the value of \( N_{cr} \) is reached, followed by a great deflection when \( N_{cr} \) is reached. If a real beam with longitudinal imperfections is loaded with an axial force, a lateral deflection will occur before the load has reached the value of \( N_{cr} \) since a moment due to the resulting eccentricity of the force will develop. The force-displacement relationship is therefore nonlinear even though the critical load not yet has been reached. Through extensive experiments the effect of these imperfections on the bearing capacity have been studied. In Eurocode, different types of cross sections are classed with different values of an imperfection factor, \( \alpha \), which combined with the slenderness of the beam makes it possible to calculate the actual axial bearing capacity (Luís, Rui, & Helena, 2010). In Figure 3 this is demonstrated, where the...
theoretical bearing capacity is represented by the thick lines depending on either Euler buckling or plasticity, and the dots below these lines represent the result from experiments on real beams.

In Eurocode the relation between actual stress at failure, $\sigma$, and yield stress, $f_y$, is represented by a factor, $\chi$, determined by the use of five different curves (European Comittee for Standardization, 2005). The slenderness, $\bar{\lambda}$, used to determine this factor, represented on the x-axis in Figure 3, is calculated using Equation 12.

![Diagram](image)

**Figure 3** - The relationship between slenderness and bearing capacity, the dots represent experimental results (Luís, Rui, & Helena, 2010).
As shown in Figure 3 a beam with low slenderness will not have problems with buckling rather \( f_y \) is the limiting factor. However, the greater \( f_y \), the greater is the risk for instability being the main issue since higher yield strengths results in having more slender structures (Luís, Rui, & Helena, 2010). The risk for buckling can be considered to have increased historically due to the development of high steel strengths and more regular use of them in structures (Tsai, Lin, Lin, Lin, & Hsiao, 2008). As can also be seen some experimental results are above both theoretical lines, because of strain hardening, an effect that is not accounted for in the design, instead the maximum allowable stress, \( \sigma \), is limited to \( f_y \) (European Committee for Standardization, 2005).

\[
\lambda = \sqrt{\frac{f_y A}{N_{cr}}} \tag{12}
\]

To determine \( \chi \) both the effect of an initial deflection, \( e_o \), and of initial stresses caused by the forming of the steel member must be considered. The initial stresses are caused by processes during the steel forming of the cross-section. They are in internal equilibrium but can cause an uneven stress distribution when a load is applied, meaning that some fibers have a higher compressive stress than \( N_{Ed}/A \) even if no geometric imperfections exists. Effects of the geometric imperfections are considered using the Ayrton-Perry equation (1886) where the initial deformation is considered to be a sinusoid with maximum deflection \( e_o \), as shown in Figure 4, and the deflection achieved when the axial force, \( N_{Ed} \), is applied is denoted \( e \) (Luís, Rui, & Helena, 2010). Using Equation 1 the total deflection can be expressed as given in Equation 13. Assuming that the beam is only subjected to an axial force and not affected by any initial stresses the yield criterion can be set using the Navier equation as in Equation 14.
Figure 4 - Definition of $e_0$ and a comparison between a perfectly straight beam and a real beam with initial imperfections.

Equation 14 can be rewritten as a non-dimensional relationship as given in Equation 16, which is done using Equation 4 in combination with the previous definition of $\chi$ as the relation between actual stress at failure and theoretical yielding stress and the slenderness $\bar{\lambda}$ as defined in (12), firstly by dividing each side with the yield strength, leading to Equation 15.

$$e = e_0 * \xi = \frac{e_0}{1 - \frac{N}{N_{cr}}} \tag{13}$$

$$\frac{N}{A} + \frac{N * e}{W_{el}} = f_y \tag{14}$$

Equation 14 can be rewritten as a non-dimensional relationship as given in Equation 16, which is done using Equation 4 in combination with the previous definition of $\chi$ as the relation between actual stress at failure and theoretical yielding stress and the slenderness $\bar{\lambda}$ as defined in (12), firstly by dividing each side with the yield strength, leading to Equation 15.

$$\frac{N}{N_{pl}} + \frac{N * e_0 * A}{W_{el} * (1 - \frac{N}{N_{pl} N_{cr}}) N_{pl}} = 1 \tag{15}$$

$$\chi + \frac{\chi * e_0 * A}{(1 - \chi * \bar{\lambda}^2) * W_{el}} = 1 \tag{16}$$
By rearranging (16) the expression given in 17 is achieved, in which \( \eta \) is a representation of the initial imperfections accommodating for both the geometrical deflections and residual stresses. The term \( \eta \) has been chosen to be expressed as in Equation 18, in which \( \alpha \) is an imperfection factor which in Eurocode 3 depends on cross-section type, size, buckling plane and yield strength.

\[
(1 - \chi)(1 - \chi \cdot \bar{\lambda}^2) = \frac{e_0 \cdot A}{W_{el}} \cdot \chi = \eta \chi
\]  
(17)

\[
\eta = \alpha(\bar{\lambda} - 0.2)
\]  
(18)

The 0.2 represents the length of the plateau seen in Figure 3 where the value of \( \chi \) is constant. Combining Equations 17 and 18 results in the minimum solution to the quadratic Equation 17 as presented in Equation (19).

\[
\chi = \frac{1}{0.5(1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2) + \sqrt{0.5(1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2) - \bar{\lambda}^2}}
\]  
(19)

The factor \( \chi \) can then be used to reduce the axial force bearing capacity with regards to the risk for buckling. The value of \( \alpha \) has been determined by experimental studies during later part of 1970’s where numerous trials of real beams were conducted. The end result was five different curves, each with its own value of \( \alpha \) (Maquoi & Rondal, 1978), (Luís, Rui, & Helena, 2010) and (European Comittee for Standardization, 2005).
2.1.3. Calculation of Bearing Capacity According to SS-EN 1993-5

According to the old Swedish design guide “Spønshåndboken” seven different failure mechanisms for the structural members of the RW are to be checked (Ryner, Fredriksson, & Stille, 1996). Concerning the structural bearing capacity, the RW should, according to Spønshåndboken, be checked so that the external moment due to earth pressure is not greater than the RW bearing capacity. This can be expressed using the standard notations from Eurocode as in Equation 20. The principal working of this type of failure is presented in Figure 5.

\[ M_{Rd} > M_{Ed} \]  \hspace{1cm} (20)

Figure 5 - Principal of structural failure of the RW according to Spønshåndboken, parameters in parenthesis indicates that they are not accounted for in the design.
For a given external moment, the required cross section for the sheet pile could then be determined since the value of $M_{Rd}$ is defined as:

$$M_{Rd} = \eta \ast W_y \ast f_{yd}$$  \hspace{1cm} (21)

According to Eurocode 3 part 5, if $N_{Ed}$ is sufficiently large, the risk for a failure in form of flexural buckling should however be considered. When taking into account the combination of bending moment, $M_{Ed}$, and $N_{Ed}$ the requirement is that the total utilization of the sheet piles bearing capacity, $N_{pl,Rd}$ and $M_{c,Rd}$, is less than 100 %. The suggested way of determining the utilization is described in Equation 22 (European Committee for Standardization, 2007). As can be seen the utilization in Eurocode 3 also includes the normal force utilization with respect to buckling as well as the utilization with respect to external moment with an additional interaction factor. The interaction factor, which in the case of sheet piles, is set to a fixed value of 1.15 is supposed to take second order effects into account.

$$\frac{N_{Ed}}{\chi \ast N_{pl,Rd}} + 1.15 \frac{M_{Ed}}{M_{c,Rd}} \leq 1$$  \hspace{1cm} (22)

In Figure 6 the failure mechanism according to EN 1993-5 is presented. As can be seen it is similar to that presented in Figure 5 with the difference that the effect of $N_{Ed}$ is included. As shown in Figure 6 the term $M_{Rd}$ has been exchanged to the term $M_{Ed}$. The reason is that the model defined in EN 1993-5 prohibits $M_{Ed}$ from reaching $M_{Rd}$ while $N_{Ed}$ is applied as well.
The suggested simplified model can be used if the supports can be regarded as rigid, which is possible if the additional deformation of a horizontal support is not more than $1/500^{\text{th}}$ of the critical buckling length, $l_{cr}$, as well as being driven to bedrock or if the footing is able to withstand a horizontal additional force $F_{Q,Ed}$, calculated according to Equation 23, due to passive earth pressure or friction. If this is not obtained, either the sheet pile can be driven longer so that the demand of being able to withstand an additional force is achieved, or the methods described in Eurocode 3 chapter 1 part 1, EN 1993-1-1, should be used.

$$F_{Q,Ed} = \pi * N_{Ed} * \left(\frac{d}{l} + 0.01\right)$$  \hspace{1cm} (23)

The needed parameters to determine the utilization of the RW is described in Equation 24 and 25 below.

$\chi$ is the reduction due to buckling determined from curve “d” in EN 1993-1-1, 6.3.1.2, in which the slenderness, $\lambda$, is determined using Equation
12. $N_{pl,Rd}$ is the plastic normal force bearing capacity determined by Equation 24, and $M_{c,Rd}$ is the cross-sectional bearing capacity with respect to external moment as described by Equation 25. $N_{Ed}$ and $M_{Ed}$ are the external normal force and maximum external bending moment respectively. In Equation 25 $\beta_B$ is a reduction factor which accommodates for possible lack of shear force transmission in the interlocks and are given in the national annex. According to the Swedish national annex they should be decided by testing (Trafikverket, 2011).

$$N_{pl,Rd} = \frac{f_y \cdot A}{\gamma M_0} \quad (24)$$

$$M_{c,Rd} = \frac{\beta_B \cdot W_{el/pl} \cdot f_y}{\gamma M_0} \quad (25)$$
Figure 7 - Determination of buckling length according to EN 1993-5 for simply supported (above) and clamped end (below).

In the determination of $\bar{\lambda}$, the critical buckling load $N_{cr}$ must also be determined. This can, according to EN 1993-5, be made either through an appropriate model of the surrounding soil or by using a simplified modelled as described in Equation 26. This is based on Euler-Bernoulli buckling load where the critical length, $l_{cr}$, is either set to the distance
between the last anchor and bottom of the sheet pile or, if the RW is fixed in bedrock or soil, 70 % of the length from last anchor and bottom of the RW, the two simplified models are displayed in Figure 7.

In the determination of $N_{cr}$ a reduction factor, $\beta_D$, depending on the interlocks ability to transfer axial forces is also applied depending on the type of interlock. As can be seen any effect of the adjacent soil on the RWs critical buckling length is not accounted for in the EN 1993-5 suggested model resulting in a lower critical buckling load than what might be the real case.

$$N_{cr} = \frac{\beta_D \cdot E_s I \cdot \pi^2}{l_{cr}^2} \tag{26}$$

If the external axial force, $N_{Ed}$, is less than 4 % of the critical buckling force, determined as in Equation 26, the risk for buckling of the RW need not to be checked, but instead the bearing capacity is only controlled with regards to moment, similar to the old Swedish guideline. However, if $N_{Ed}$ is greater than 10 % of $N_{pl,Rd}$, as described in Equation 24, for cross-sections of Z-type in class 1 and 2, 25 % for cross sections of U-type in class 1 or 2 or 10 % for cross sections in class 3 the contribution of $N_{Ed}$ must also be accommodated for by reducing $M_{c,Rd}$ of the RW. The determination of this reduced moment capacity, $M_{N,Rd}$, is presented with Equation 27, 28 and 29 for Z cross-sections of class 1 or 2, U cross-sections of class 1 or 2 and all cross-sections of class 3 respectively.

$$M_{N,Rd} = 1.11 \cdot M_{c,Rd} \cdot \left(1 - \frac{N_{Ed}}{N_{pl,Rd}}\right), \text{ but } M_{N,Rd} \leq M_{c,Rd} \tag{27}$$

$$M_{N,Rd} = 1.33 \cdot M_{c,Rd} \cdot \left(1 - \frac{N_{Ed}}{N_{pl,Rd}}\right), \text{ but } M_{N,Rd} \leq M_{c,Rd} \tag{28}$$

$$M_{N,Rd} = M_{c,Rd} \cdot \left(1 - \frac{N_{Ed}}{N_{pl,Rd}}\right) \tag{29}$$
Furthermore, if the externally applied transversal force, $V_{Ed}$, is greater than 50 % of the RWs transversal force capacity, $V_{Rd}$, the yield strength of the RW should be reduced as described in Equation 30 for all of the above uses of the steel strength.

$$f_{y,red} = (1 - \rho) * f_y$$  \hspace{1cm} (30)

Where the value of the reduction factor $\rho$ is determined as in Equation 31, in which the plastic transversal force capacity, $V_{pl,Rd}$, is determined by the use of Equation 32.

$$\rho = \left( \frac{2 * V_{Ed}}{V_{pl,Rd}} - 1 \right)^2$$  \hspace{1cm} (31)

$$V_{pl,Rd} = \frac{t_w * (h - t_f) * f_y}{\sqrt{3} * \gamma_{M0}}$$  \hspace{1cm} (32)

2.1.4. Comparison to EN 1993-1-1

If the supports in form of anchors and footing cannot be regarded as rigid it is recommended that the buckling analysis is made in accordance to the methods described in EN 1993-1-1. The method in EN 1993-1-1 is similar to that in EN 1993-5 since it is also an interaction formula that is used to calculate the total utilization of the bearing capacity, as displayed in Equation 33 (European Comittee for Standardization, 2005).

$$\frac{N_{Ed}}{\chi_y(N_{Rk})/\gamma_{M1}} + k_{yy} \frac{M_{y,Ed}}{\chi_{LT} M_{y,Rk}/\gamma_{M1}} + k_{yz} \frac{M_{z,Ed}}{M_{z,Rk}/\gamma_{M1}} \leq 1$$  \hspace{1cm} (33)

The last term in Equation 33 can in the case of RW be disregarded since only bending around one axis is viable, also the term $\chi_{LT}$ can be set to 1 since no problem of lateral torsional buckling should be considered. The term $k_{yy}$ is an interaction factor taking account the second order effects,
due to the combination of external bending moment and axial force. This term can be compared to, in Equation 22, the factor 1.15 in the simplification used in EN 1993-5. This interaction factor is determined by the combination of several factors accounting for the structures behavior, either elastic or plastic depending on cross sectional class, as well as load situation. The model is however only elaborated for simply supported double-symmetric cross sections (Luís, Rui, & Helena, 2010), which is why the adaption on a RW might yield results that are erroneous, since the RW might not have a doubly symmetrical cross section and neither have a moment distribution similar to those described in EN 1993-1-1. Furthermore, if the conditions for using Equation 22 are not fulfilled, the RW might not be simply supported, since the supports might rather work as an elastic foundation, thus making the adaption even more difficult.

2.1.5. Buckling Analysis Using FEM

To determine $N_{cr}$ for structures with complex geometry Finite Element Methods, FEM, can be used (Novoselac, Ergić, & Baličević, 2012). For an elastic behavior of an ideal structure an eigenvalue buckling analysis can be performed, where the program solves for which eigenvalue, load factor, the stiffness matrix become singular. Each degree of freedom will have a load factor and from this several buckling modes can be obtained; usually the first mode is the one of interest since this has the lowest eigenvalue (Ellobody, Feng, & Young, 2014).

According to Novoselac et al. (2012) the eigenvalue buckling analysis is suitable for a fast determination of the structures buckling force. If, however, one wants to include nonlinear material behavior, plasticity or imperfections a post buckling analysis must be made. This can be done using the Riks method (Crisfield, 1981), which for example is included in the finite element program ABAQUS (Dassault Systèmes, 2014). Furthermore, the eigenvalue linear buckling analysis only gives deformations in a normalized scale, since it predicts the buckled shape of the structure and puts the largest deformation to unity (Ellobody, Feng, & Young, 2014). This can be compared to the energy method described
above, where an approximate value of the buckling load can be calculated if the deformed buckled shape can be predicted.

Riks (1972) developed a method to solve equilibrium equations beyond stability limit points, which the traditional Newton method cannot. Riks (1972) proposed a modified Newton method to solve the equilibrium and uses both the load and displacement as unknowns in the step analysis. The method is based on moving along an equilibrium path defined by the nodal and loading parameters.

A modified Riks method is often used in Finite Element programs, example given ABAQUS (Dassault Systèmes, 2014), where several methods have been developed. One method was developed by Crisfield (1981) where a modified Newton-Raphsons method is used to solve the non-linear differential equations to solve the equilibrium. It uses a user estimated arc-length to estimate the first load proportionality factor and then automatically calculates the parameters needed for the continuation of the process.

2.1.5.1. Mode Jumping

In the post buckling behavior of a structure, the displacement profile can suddenly move from one buckle mode to another. This phenomenon occurs when several states of buckling modes exists in equilibrium but only a few are stable for the applied load. When the load further increases, the structure will move away from the previous stable path to the new path leading to a momentary motion to another, now stable, path. This behavior can continue to arise during the continuous loading of the structure as more instability points are reached and several mode jumps can thus occur during the loading of the structure (Riks, Rankin, & Brogan, 1996).

2.2. Soil Conditions

Due to the Weichselian ice age covering most part of northern Europe, the soil layer in Scandinavia is mainly glacially and post-glacially originated soil layers with a layer of moraine above the bed rock. In areas where the retreating ice sheet was creating lakes layers of fine-grained glacial or post-
Glacial sediments of clay and silt covers the moraine (Sveriges Geologiska Undersökning, 2018). In the eastern part of Sweden these valleys of clay-layered soils are mostly shallow with bedrock close to the ground surface, often resulting in inclined anchors and RWs being driven down to the bedrock in the construction of an RW. Furthermore, the two types of soil, clay and friction soil, will be expected in different extents when constructing RWs in these areas.

In Figure 8 a typical soil sequence in the Mälardal-region in eastern Sweden is shown as described in Handboken Bygg Geoteknik, chapter G02 (Johansson, 1984). It should be noted that the measurements of length and meter above sea level are only approximate to give an estimation of layer thicknesses. It should also be noted that there is no big height variations and that there can be areas where clay interacts directly with hard rock.

![Figure 8 - Typical soil layer sequence in the Mälardal region as described in Handboken Bygg Geoteknik.](image-url)
2.2.1. Clay

Clays are the least permeable of soils due to its high content of fine grained cohesive minerals. The undrained shear strength, obtained by cohesion between the grains, $c_u$, is the primary strength contribution for the clay soil mass, with values from $<10$ kPa, extremely low, to $300$ kPa, extremely high, where the latter is the limit for the soil mass to be defined as weak rock (Larsson, 2008).

In undrained conditions, the strength, as well as the stress-strain state, are highly dependent on the stress history of the clay, a result of how the soil was deposited, erosion and past engineering activities in that particular area. The behavior of the soil is thereby associated with the maximum stress it has been subjected to in the past, the preconsolidation pressure, $\sigma'_c$, in combination with the present stress state, $\sigma'_v$. The ratio of the two is denoted as the overconsolidation ratio, $OCR$, which have been shown is directly related to the mobilized shear stress when there have been no past engineering activities (Kullingsjö, 2007).

As previously stated, the deformation character of the clay differs depending on the $OCR$. Generally, the soil has a deformation of an elastic nature when the present stress state is smaller than the preconsolidation pressure, $\sigma'_c$, i.e. overconsolidated. The elastic modulus, $M_0$, of this stage is often idealized and calculated using Equation 34 (Larsson, 2008).

$$M_0 = 250c_u$$ (34)

However, the clay starts to creep when the $OCR$ is close to 1, i.e. normally consolidated, followed by a more plastic deformation character when the ratio is larger than 1 (Larsson, 2008). This phenomenon is shown in a semi-logarithmic scale in Figure 9 of both the real soil behavior to the left and an idealized behavior to the right, where $e$ is the deformation and $\sigma'_0$ is the initial vertical pressure. The figure also shows the elastic character of a soil unloading swelling response (Helwany, 2007).
2.2.2. Bedrock

In eastern Svealand, the middle-eastern part of Sweden, the bedrock most commonly consists of hard granite or gneiss and is generally shallow (Johansson, 1984). The compressive strength of granite has a wide spectrum of variation concerning experimental results from different places around the world. However, the majority of granites have a compressive strength of the intact rock ranging from 150 – 250 MPa (Bäckblom, Franzén, & Stille, 1984). The Young’s modulus of metamorphic and igneous intact rock is often in the range of 30-100 GPa and can therefore be considered adequate for all engineering requirements (Bell, 1992).

2.3. Soil Mechanics

2.3.1. Elasticity and Plasticity

Although soils are anisotropic and have material properties of a non-linear behavior, the behavior can sometimes be idealized and described with
elastic strains using Hooke’s law. This is a rough estimation of the real soil behavior although this assumption of linear elastic relation between stress and strain has been used historically when solving the problem with finite element methods (Kullingsjö, 2007).

The strain of the elastic material, $\varepsilon$, is completely described in three dimensions with the generalized Hook’s law presented in Equation 35, depending on Young’s modulus, $E$, Poisson’s ratio, $\nu$, and the shear modulus, $G$, subjected to stresses, $\sigma$, and shear stresses, $\tau$, with the stress direction conventions shown in Figure 10.

Figure 10 - The stress directions in a 3D-space (Helwany, 2007).
For structures with a uniform cross section along one axis, as for many soil structures, the strains could be simplified and reduced into two dimensions i.e. plane strain conditions. The reduced stress-strain state is presented in Equation 36 (Helwany, 2007).

\[
\begin{pmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\tau_{12} \\
\tau_{13} \\
\tau_{23}
\end{pmatrix} = \begin{bmatrix}
1/E & -v/E & -v/E & 0 & 0 & 0 \\
-v/E & 1/E & -v/E & 0 & 0 & 0 \\
-v/E & -v/E & 1/E & 0 & 0 & 0 \\
0 & 0 & 0 & 1/2G & 0 & 0 \\
0 & 0 & 0 & 0 & 1/2G & 0 \\
0 & 0 & 0 & 0 & 0 & 1/2G
\end{bmatrix} \begin{pmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\tau_{12} \\
\tau_{13} \\
\tau_{23}
\end{pmatrix}
\]

(35)

For linearly elastic strains, the strains can be fully recovered, while for perfectly plastic strains, it cannot. A plasticity model, however, consists of both elastic and plastic behavior.

### 2.3.2. Mohr Coulomb Failure Envelope

The shear strength of soil is defined as the maximum shear stress, \( \tau_{fu} \), it resists until it fails. This can be determined by the Mohr Coulomb failure envelope, see Equation 37. Here the failure envelope is linearly defined with the cohesion, \( c' \), the effective stress at failure, \( \sigma' \), and the friction angle, \( \varphi' \) (Helwany, 2007).

\[
\tau_{fu} = c' + \sigma' \tan(\varphi')
\]

(37)

The soil is drained when the pore water is allowed to dissipate so that the pore water pressure changes with time under loading. Friction soils such as moraine and sand are, in the state of relatively slow loading and time, under drained conditions. However, for an undrained state, for a small time the water is too tightly bound so that the water is contained
within the soil. Because of the water pressure affecting the soil strength, this phenomenon divides the strength of the soil into drained and undrained strength. Concerning Equation 37, this, for soil under undrained conditions, e.g. normally consolidated clays, result in no effective stress. However, for friction soils this is not the case where the drained shear strength can be accounted for (Larsson, 2008).

2.3.3. P-u Method

To obtain the non-linear behavior of the soil, one can use the p-u method representing the soil with non-linear springs. The relation between the soil pressure, \( p \), and the deflection, \( u \), along a pile is shown in Equation 38, where \( EI \) is the pile stiffness and \( E_s \) the soil modulus.

\[
p = EI \frac{d^4 u(x)}{dx^4} = -E_s u
\]  

(38)

Matlock (1970) derived curves from empirical data and known boundary conditions for soft normally consolidated marine clay. Furthermore, Reese et al. (1974) and (1975) used the same method for sand and stiff clay solving the soil modulus in Equation 38 assuming it as a one or two parameter function presented in Equation 39 and 40. The aim of the two different soil modulus functions was to represent the behavior of sand, Equation 39, and stiff clay, Equation 40, respectively collecting the spring constant, \( k \), along the pile, here denoted as the x-axis, and of the \( n \)th order for in the case of the stiff clay.

\[
E_s = kx
\]  

(39)

\[
E_s = kx^n
\]  

(40)

In a designing guide from the Swedish Transport Administration (2009), TR Bro, an implementation of the p-u method is described to calculate the lateral resistance of soil. The method is, for cohesive soil, divided into long term and short term action and is based on calculating a
resistance against the piles surface area, $k_k$, shown in Equation 41 for short term and 42 for long term, and a maximum stress, $q_k$, that the soil can take, shown in Equation 43 for short term and 44 for long term. Since the resistance is calculated as resistance acting along the surface area the value is given in N/m³ and the maximum stress in Pa. The value of $d$ represents, for a circular pile, the diameter. For the subgrade moduli and maximum stresses the subscripts $ltl$ and $stl$ are used for long term and short term load respectively.

$$k_{k, stl} = 200 \frac{c_u}{d} \quad (41)$$

$$k_{k, ltl} = 50 \frac{c_u}{d} \quad (42)$$

$$q_{k, stl} = 9 c_u \quad (43)$$

$$q_{k, ltl} = 6 c_u \quad (44)$$

2.4. Case Study

A BW was constructed by ELU in the southern part of Stockholm, displayed in Figure 11. The majority of the soil is friction material down to bedrock. Section F is the BW with the highest utilization why that has been studied. The total length of the BW is there approximately 15 m and the distance between the lowest wale beam and bedrock is 10.2 m. Since the primary elements are drilled down in the bedrock they were, in the calculation of the critical load, considered to be rigidly supported.

The initial results in section F resulted in a needed cross section of type KCKR 220x12.5 as depicted in Figure 12 of steel with a yield strength of 460 MPa. The reason for which the utilization became a problem in this case was the presence of several pipes behind the BW, resulting in a needed use of heavily inclined anchors and thus high normal forces in the king piles.
Figure 11 - The project site and berliner wall in the project, picture provided by ELU Konsult AB.

Figure 12 - Circular pipe of type KCKR 220x12.5, as used as king piles in the BW. The measurements are given in mm.
3. Methodology

The method is here described starting with a general explanation of how the numerical simulations were divided into nine different cases followed by a description of the various clay shear strengths used for each case. The last section describes the modelling phase and how the results were extracted and processed, as well as a convergence test and a methodological assessment.

3.1. Parameter Study

The testing of the BW has been divided into nine main cases, where each case has seven different values of $c_u$; resulting in a total of 63 different models. The parameters that were changed in the nine main cases were number of anchor levels, 1-3, and the driven depth, $D$. For each of the nine cases the cross-section type was kept constant to the same type as used in the project described in section 2.4, KCKR 220x12.5, and was normalized in the results, the KCKR 220x12.5 was used due to it being a standard cross-section often used in BWs. Also the distance between the lowest anchor and excavation level and the offset between the individual anchors were kept constant.

The bedrock was assumed to be flat in the models, and the anchor length followed from the vertical length between the end of the sheet pile and the anchor level. In reality the bedrock would most likely not have been flat. However, the anchor stiffnesses, which was the only parameter affected by the length, was always high enough to almost be considered as a pinned support regardless of the total length.
3.1.1. Model Parameter Variation

Figure 13 shows the parameter variation of models equipped with one, two and three anchor levels. Table 1 shows the values of each variable defined in Figure 13.

![Figure 13 - BW with either one, two or three anchor levels as used in the modelling.](image)

Table 1 - Values for each of the parameters specified in Figure 12 for all nine cases.

<table>
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<tr>
<th></th>
<th>H</th>
<th>D</th>
<th>A</th>
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<th>hanchor1</th>
<th>lanchor2</th>
<th>hanchor2</th>
<th>lanchor3</th>
<th>hanchor3</th>
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<td>5.2</td>
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<td>-</td>
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</tr>
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<td>8.1</td>
<td>2.5</td>
<td>5.2</td>
<td>2.5</td>
<td>-</td>
<td>-</td>
<td>30°</td>
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<tr>
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<td>1.5</td>
<td>12.7</td>
<td>2.5</td>
<td>9.8</td>
<td>2.5</td>
<td>-</td>
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<td>14.4</td>
<td>2.5</td>
<td>30°</td>
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</table>
## 3.2. Soil Parameter Variation

A Winkler spring foundation was used to model the soil with a 10 cm distance between the springs. To determine the spring stiffness variation to be used for each of the various shear strengths the behavior was seen as 100 % short term action. The spring stiffnesses was calculated using Equation 41 and 43 from section 2.3.3 resulting in the load-deformation curves displayed in Figure 14. The surface area of the KCKR 220x12.5 pipe was subsequently divided into 10 cm long pieces over which each soil spring acted, since only one direction of moment was considered the surface area was calculated as the diameter multiplied with the length between springs.

To model the soil above the excavation bottom only half the curve was used, either in positive or negative direction. The load-deformation behavior is displayed in positive direction in Figure 15. In the following section, 3.3.2, the method used to determine whether the springs above excavation bottom was to act in negative or positive direction is described.

![Figure 14 - Load-Displacement for soil springs with various shear strength.](image-url)
3.3. Finite Element Modelling

All finite element modelling has been carried out using the software ABAQUS 6.14-2 (Dassault Systèmes, 2014). The processing of data has been done using MS EXCEL and MATHCAD PRIME 4.0 (PTC, 2017).

3.3.1. Definition of Structural Part

The structural part that has been defined in the program was the pile of type KCKR 220x12.5 as described in chapter 2.4, the cross-section is of cross-sectional class 1, and $f_y$ has been used for the definition of steel properties. A 2D deformable wire type part was created with the total length as needed for the specific case, furthermore points were defined every 10 cm to be able, in the later stages, to attach springs and connectors representing anchors and soil.

A profile was then defined as a thin walled circular pipe with the radius and thickness as shown for KCKR 220x12.5 in Figure 12. The material used for the section varied depending on which step that was run, for the linear
buckling and post buckling without plasticity the steel was only modelled using \( E = 210 \text{ GPa} \) and Poisson’s ratio of 0.3, for the post buckling step with plasticity included also \( f_y \) of 460 MPa was defined.

The part was meshed using two-noded linear beam elements, B21, with a mesh size of 1cm.

3.3.2. Definition of Interaction Parameters

The anchors were modeled through springs active only in the horizontal direction, the reason for which was that the anchors are not supposed to resist any of the applied normal force, while still retaining the structure from lateral movement. The anchor cross-sectional area that has been used is that from a steel anchor of type MAI T76N, with an effective area of 1835mm\(^2\) and Young’s modulus of 210 GPa.

The surrounding soil was modeled through axial connectors in the horizontal direction with a 10 cm c/c distance. For the linear buckling step the connectors were defined as linear elastic springs with a spring stiffness as calculated for the specific shear strength defined in part 3.2. For the majority of load cases, only the soil below excavation level was used, however for the cases with one anchor, springs was also used above the anchor and for case 3.3 soil springs were used between the lowest and middle anchor when the shear strength was 31.4-78.6 kPa. Whether springs could be used above excavation level during the linear buckling step was controlled by running the linear buckling simulation step initially with springs placed below the excavation level and then analyzing the shape of the buckled structure.

Above the excavation level the direction towards which the largest deformation occurred was defined as the direction without soil restraint, the excavated side, if any part of the structure above that buckled in the opposite direction, soil springs were put in at those points. The simulation step was then performed again and if none of the points with new springs buckled towards the excavation the model was accepted, if it at any of those points buckled towards the excavation the springs were removed at those points and the same procedure was repeated until only nodes that buckled
towards the soil above the excavation level were equipped with soil springs. The procedure is described in Figure 16.

During the subsequent two post buckling steps the connectors were defined using the non-linear elastic properties described above. Soil springs could then be defined along the whole length of the pile. Above the excavation level the piles were defined to only resist loads towards the soil side as shown in Figure 16. The different models defined in ABAQUS are shown in Figure 17, Figure 18 and Figure 19.

![Figure 16 - Method to determine whether soil springs were needed above the excavation level.](image)

![Figure 17 - Principle of one anchor level linear buckling (left) and post buckling (right) model.](image)
3.3.3. Load and Boundary Conditions

The loads applied in the various anchors varied depending on the stiffness of the specific anchor and the stiffness of the remaining anchors. In the linear buckling step the forces were defined so that the sum of all forces
was 1 and the specific anchors’ force was proportional to the stiffness relation. In the two post buckling steps the buckling force resulting from the linear buckling step was multiplied with the same proportionality factor for each anchor. The values of the normalized anchor forces and spring stiffnesses used in all anchors are displayed in Table 2.

In all steps the boundary condition was defined for the end of the pile as a displacement/rotation type, constrained from movement in the x- and y-direction.

Table 2 – Normalized anchor forces and horizontal spring stiffnesses for each of the nine cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>$k_{anchor1}$ [10^7 N/m]</th>
<th>$N_{anchor1}$ $N_{tot}$ [-]</th>
<th>$k_{anchor2}$ [10^7 N/m]</th>
<th>$N_{anchor2}$ $N_{tot}$ [-]</th>
<th>$k_{anchor3}$ [10^7 N/m]</th>
<th>$N_{anchor3}$ $N_{tot}$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1.1</td>
<td>3.71</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Case 1.2</td>
<td>1.96</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Case 1.3</td>
<td>1.34</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Case 2.1</td>
<td>2.38</td>
<td>0.391</td>
<td>3.71</td>
<td>0.609</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Case 2.2</td>
<td>1.52</td>
<td>0.436</td>
<td>1.96</td>
<td>0.564</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Case 2.3</td>
<td>1.11</td>
<td>0.454</td>
<td>1.34</td>
<td>0.546</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Case 3.1</td>
<td>1.76</td>
<td>0.224</td>
<td>2.38</td>
<td>0.304</td>
<td>3.71</td>
<td>0.472</td>
</tr>
<tr>
<td>Case 3.2</td>
<td>1.24</td>
<td>0.262</td>
<td>1.52</td>
<td>0.322</td>
<td>1.96</td>
<td>0.416</td>
</tr>
<tr>
<td>Case 3.3</td>
<td>0.954</td>
<td>0.280</td>
<td>1.11</td>
<td>0.327</td>
<td>1.34</td>
<td>0.393</td>
</tr>
</tbody>
</table>
3.3.4. Simulation Steps

The linear buckling simulation step was defined as a linear perturbation – buckle in ABAQUS. Five eigenvalues were requested and the subspace eigensolver was used with 10 vectors per iteration and a maximum of 300 iterations. The default model field output for the step type was requested, however a node file was also saved for the displacement to be used in the subsequent steps as an imperfection file. The node file was created using the keyword editor in which a new line was added under the output request requesting a node file for the variable U.

The subsequent simulation step was the first of the two post buckling steps. The post buckling with linear elastic steel and plastic soil was defined as a static RIKS step using the arc length method. The estimated total arc length was set to the default of 1 and the arc length increment was set to a maximum of 0.1 and minimum of $1 \times 10^{-5}$ and the total number of increments was limited to 250.

The default model output request was used, generating values of the displacements, stresses, forces and contact stresses and displacements. The nodal file containing the displacement from the linear buckling step was used as an initial imperfection of the pile. This was executed through the use of the keyword editor in which, above the definition of the post buckling simulation step, a line was added calling for the imperfection:

```
*IMPERFECTION, FILE=Linear_Buckling, Step=1
1, (H+D)/100
```

The second line in the expression defined that the first eigenvalue from the linear buckling simulation was to be used and that the imperfection shape was to be multiplied by the total length of the pile divided by 100, since that was the equivalent initial imperfection as defined in EN 1993-1-1 for imperfection curve d.

The second post buckling step was defined similarly as the first, with the difference that the yield stress was added for the steel. For both cases no stop value was defined, the first post buckling simulation step would consequently complete all 250 increments and the load proportionality file
was checked to assure that the bifurcation point had been reached. For the post buckling including steel plasticity the simulation was often aborted due to several nodes reaching plasticity, the load proportionality factor was then controlled to make sure that the maximum load had been reached.

### 3.3.5. Simulations and Results

The lowest eigenvalue was recorded when the linear buckling job had been completed and inserted in the post buckling simulation steps, described in section 3.3.3. The post buckling simulations could then be run and the load proportionality factor was retrieved from the history output and multiplied with the linear buckling load. In order to create a load-deflection curve, the nodal displacement was taken from the node with largest horizontal displacement at the increment, with either the maximum load, for model with plastic steel or when the buckling behavior was unstable. When a stable buckling behavior was achieved the increment where the load-deflection curve had a dramatic change in inclination was used. The bending moment was also recorded from the model with steel plasticity using a path created along the whole pile at the increment where the maximum load was reached.

An equivalent ultimate normal force from the suggested Eurocode 3 model, as described in section 2.1.3 using Equation 26 was calculated for each of the nine cases. The aim was to compare these to the ultimate load achieved from the FEM.

A value of the imperfection factor $\chi$ was then calculated using the ultimate load obtained from the ABAQUS simulation and the plastic normal force capacity of the pile. Using the calculated factor $\chi$ and the relationship between $\chi$ and the slenderness, $\lambda$, as described in Equation 19, an equivalent Euler critical buckling length, $l_{cr}$, was calculated with the use of Equation 2. In Appendix A the calculation procedure is presented in detail. The critical buckling length calculated from the simulation results was then compared to the suggested buckling length in Eurocode. A correlation factor, $\beta_{cr}$, was then plotted against the subgrade modulus.
3.3.6. Simulation Convergence Control

A convergence control was carried out on case 1.2 to find a suitable mesh size and distance between the soil springs. Distances between soil springs was set to 50 cm, 10 cm and 5 cm in the linear buckling analysis, resulting in mode shapes and linear buckling loads. For the mesh size convergence test mesh sizes of 10 cm, 1 cm and 0.5 cm was used for the two noded beam elements, B21, also a 1 cm mesh with quadratic three noded beam elements, B22, was tested for the linear buckling analysis, giving the linear buckling load.

For both controls approximate relative errors between the tested geometrical size and the subsequent tested geometrical size, $\epsilon_A$, were calculated related to the chosen mesh size and type and the distance between the soil springs. In Table 3 the relative errors are displayed and in Figure 20 the deformed shapes are displayed. Note that the mode shape of the mesh type and size convergence test is not displayed, since the difference in deformation was close to zero between the different mesh types and sizes.

Table 3 - Convergence control results.

<table>
<thead>
<tr>
<th>Soil Spring Distance [cm]</th>
<th>Mesh Size [cm]; and Element Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>10 ; B21</td>
</tr>
<tr>
<td>10</td>
<td>1 ; B21</td>
</tr>
<tr>
<td>5</td>
<td>0.5 ; B21</td>
</tr>
<tr>
<td></td>
<td>1 ; B22</td>
</tr>
<tr>
<td>$N_{cr}$ [kN]</td>
<td>7486</td>
</tr>
<tr>
<td></td>
<td>7130</td>
</tr>
<tr>
<td></td>
<td>7080</td>
</tr>
<tr>
<td>$\epsilon_A$ [%]</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>
3.4. Methodological Assessment

Since there was a need for a control of both the background variables, such as the possibility to increase the axial load without changing the geometry, and the resulting outcomes, a FEM-model was concluded to be an appropriate method to conduct the tests. From the idealized FEM results all forces that contribute to the failure can be modelled. The background variables can also be controlled, such as soil stiffness. This implied that the model was flexible and consequently a generalization around a parameter variation was easy to carry out from the results.

One of the major problems that the use of FEM had was the challenge to achieve a good representation of the real behavior of soil and steel, and
mainly the interaction between the two. It was therefore important to calibrate the models in order to minimize the errors. The model contained idealizations of the real behavior of the materials and one of the main objectives was therefore to minimize these.

Alternative methods which could have been used also result in several limitations, such as the alternative of conducting an experiment with a scale model of a BW. This way of conducting the study would also have made it possible to have some control over the relevant background variables. However, if a scaled experiment had been carried out there would instead be problems concerning how well the model represented the reality, especially since the soil behavior is dependent on the gravity and thus the scale, and how much that could be generalized based on the results.

Another problem with the use of an experiment would have been the difficulties of controlling what kind of failure that occurs, since the aim of the study was to improve the model used for dimensioning against buckling the experiment required that the sheet pile actually failed by buckling. This was much more straight forward to control using a FEM-program.

The alternative of using a field study would guarantee that the soil-structure interaction would have been correct. However, there would not have been possible to properly control the background variables, and the interpretation of the results would have been much more cumbersome. If the scale-experiment would have difficulties with ensuring a buckling type failure the field experiment would have the same problem to an even larger extent. Even if several full-scale field tests would have been carried out there would be no guarantee that any of them would have failed by buckling and then no conclusion other than that buckling is unlikely could have been drawn. A full-scale test would have been more appropriate to validate the results from a FEM-analysis, since the FEM-analysis gave an idea of what can induce a buckling failure.

Given all the above reasons the FEM-model was thought of as the best way of doing the study since it gave both good control of all the background parameters and an easier interpretation of the results. The failure type was
also easier to control and therefore a buckling failure was more likely to occur.
4. Results

The results from the FEM analysis and the subsequent calculations are displayed in the current chapter. The two first parts contain the main results with normalized factors describing the load bearing capacity and how it differs from the load bearing capacity according to Eurocode 3, as well as how the critical buckling force differs, expressed by a comparison between critical lengths. The following nine parts contain individual results from all the various cases, mainly to demonstrate the behavior of the BW during axial loading.

The results are concluded with calculations carried out on the project presented in part 2.4.

4.1. Ultimate load

In Figure 21 the ultimate loads, $N_{pl,Rd,ab}$, collected from the Abaqus models for all cases, normalized by the related Eurocode ultimate load, $N_{pl,Rd,EN}$, are presented in comparison with the various subgrade modulus of the clay. Note that for case 1.1, 2.1 and 3.1, the cases with the shortest driven depth, the ultimate load quota has values below 1 for small values of $c_u$. Values of the ultimate loads are specified, for $f_y = 460$MPa, in tables in the following sub chapters for each case separately.
4.2. Critical Buckling Length

The factor $\beta_{cr}, l_{cr,ab}/l_{cr,EN}$, for each case with varying shear strength is plotted in Figure 22, with increasing shear strength $\beta_{cr}$ decreases just as the ultimate load shown above increases. The curves are divided in the nine subcases just as the ultimate load graph above is. As can be seen case 1.1, 2.1 and 3.1 have some values on $\beta_{cr}$ that are higher than 1 for low shear strengths, for all other cases $\beta_{cr}$ is always below 1.
Figure 22 - $\beta_{cr}$ as a function of $k_k$ for all nine cases.

### 4.3. Case 1.1

Table 4 displays the results for case 1.1 of the critical load from the linear buckling model, $N_{cr}$, the maximum moment, $M_{pl,Ed}$, the distance from the pile top to where the maximum moment is reached, $x$, the ultimate load, $N_{pl,Rd}$, and the maximum deflection when the ultimate load is reached, $d$, for the different input values of the clay shear strength. The table also includes calculated critical load and ultimate load using EN 1993-5.
Table 4 – Compiled results for case 1.1.

<table>
<thead>
<tr>
<th>$c_u$ [kPa]</th>
<th>$N_{cr}$ [kN]</th>
<th>$x$ [m]</th>
<th>$d$ [mm]</th>
<th>$N_\text{pl,Rd}$ [kN]</th>
<th>$M_\text{pl,Ed}$ [kNm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9</td>
<td>5440</td>
<td>3.75</td>
<td>34.53</td>
<td>1563</td>
<td>146.8</td>
</tr>
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<td>15.7</td>
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<td>3.55</td>
<td>29.23</td>
<td>1802</td>
<td>129.9</td>
</tr>
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<td>3.46</td>
<td>28.51</td>
<td>1927</td>
<td>125.8</td>
</tr>
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<td>31.4</td>
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<td>23.24</td>
<td>2038</td>
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</tr>
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<td>18.76</td>
<td>2261</td>
<td>101.1</td>
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<td>3.02</td>
<td>18.59</td>
<td>2527</td>
<td>97.51</td>
</tr>
<tr>
<td>Eurocode</td>
<td>4401</td>
<td></td>
<td></td>
<td>1900</td>
<td></td>
</tr>
</tbody>
</table>
In Figure 23 the results from the different post buckling models of case 1.1 are illustrated where the cases without plastic properties of the steel, i.e. elastic steel, show a stable buckling behavior.
4.4. Case 1.2

For case 1.2 the results obtained from the Abaqus model and the Eurocode calculations are presented in Table 5. The load-displacement curves from the post buckling results of case 1.2 are presented in Figure 24 showing an unstable buckling behavior where only elastic material properties of the steel are considered.

Table 5 – The compiled results for case 1.2.

<table>
<thead>
<tr>
<th>$c_u$ [kPa]</th>
<th>$N_{cr}$ [kN]</th>
<th>$x$ [m]</th>
<th>$d$ [mm]</th>
<th>$N_{pLRd}$ [kN]</th>
<th>$M_{pLED}$ [kNm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9</td>
<td>5846</td>
<td>3.78</td>
<td>64.88</td>
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</tr>
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<td>1530</td>
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</tr>
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<td>23.6</td>
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<td>52.61</td>
<td>1634</td>
<td>169.7</td>
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<td>10985</td>
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<td>824</td>
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</table>
Figure 24 – The load-displacement curves from the post buckling results of case 1.2 with different steel properties and various clay shear strengths.
4.5. Case 1.3

Compiled results from the numerical model and the suggested Eurocode model for case 1.3 are presented in Table 6 with varying clay shear strength. The different load-displacement curves for the case are presented in Figure 25 where one curve represents a specific input value of $c_u$ and one type of material definition of the steel. For all load-displacement curves, where the steel is considered linearly elastic, an unstable buckling behavior is shown.

Table 6 - Results for case 1.3.

<table>
<thead>
<tr>
<th>$c_u$ [kPa]</th>
<th>$N_{cr}$ [kN]</th>
<th>$x$ [m]</th>
<th>$d$ [mm]</th>
<th>$N_{pl,Rd}$ [kN]</th>
<th>$M_{pl,Ed}$ [kNm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9</td>
<td>5548</td>
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<td>1091</td>
<td>179.1</td>
</tr>
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<td>59.91</td>
<td>1301</td>
<td>180.2</td>
</tr>
<tr>
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<td>60.83</td>
<td>1388</td>
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</tr>
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<td>Eurocode</td>
<td>570</td>
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<td></td>
<td></td>
<td>435</td>
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</table>
Figure 25 – Load-displacement curves for case 1.3 with elastic and plastic material properties of the steel and various clay shear strength.
4.6. Case 2.1

Furthermore, the results from the numerical model and the Eurocode model calculations for case 2.1 are compiled and stated in Table 7. From the numerical model the results of \( N_{cr}, x, d, N_{pl,Rd} \) and \( M_{pl,Ed} \) are presented with varying \( c_u \). Figure 26 is displaying the load-displacement curves for each value of \( c_u \) and steel material type, where for the curves with elastic steel a stable buckling behavior is shown.

Table 7 - Results for case 2.1.

<table>
<thead>
<tr>
<th>( c_u ) [kPa]</th>
<th>( N_{cr} ) [kN]</th>
<th>( x ) [m]</th>
<th>( d ) [mm]</th>
<th>( N_{pl,Rd} ) [kN]</th>
<th>( M_{pl,Ed} ) [kNm]</th>
</tr>
</thead>
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<tr>
<td>7.9</td>
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<td>156.1</td>
</tr>
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</tr>
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</tr>
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<td>2107</td>
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<td></td>
<td></td>
<td></td>
<td>1900</td>
</tr>
</tbody>
</table>
Figure 26 – Load-displacement curves for case 2.1.
4.7. Case 2.2

The results for case 2.2 are presented in Table 8 and the different load-displacement curves are shown in Figure 27 with varying clay shear strength. Note that for values of $c_u$ from 10 to 50 kPa where the steel material is considered linearly elastic an unstable buckling behavior is shown, whereas the load-displacement curve with a $c_u$ equal to 5 kPa a stable behavior is shown. For the elastic steel curves with $c_u$ equal to 40 and 50 kPa a mode jumping behavior is shown, where the node for which the displacement is shown changes displacement direction after the limit load.

Table 8 – Results for case 2.2.

<table>
<thead>
<tr>
<th>$c_u$ [kPa]</th>
<th>$N_{cr}$ [kN]</th>
<th>$x$ [m]</th>
<th>$d$ [mm]</th>
<th>$N_{pL,Rd}$ [kN]</th>
<th>$M_{pL,Ed}$ [kN/m]</th>
</tr>
</thead>
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<td>1392</td>
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</tr>
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<td>54.93</td>
<td>1714</td>
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<td>47.73</td>
<td>1734</td>
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<td>1876</td>
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</tr>
<tr>
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<td>40.82</td>
<td>2067</td>
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<td>5.70</td>
<td>38.53</td>
<td>2088</td>
<td>138.7</td>
</tr>
<tr>
<td>Eurocode</td>
<td>1234</td>
<td></td>
<td></td>
<td></td>
<td>824</td>
</tr>
</tbody>
</table>
Figure 27 – Load-displacement curves from the Abacus results for case 2.2.
**4.8. Case 2.3**

For case 2.3 the results are presented in Table 9. Load-displacement curves for the numerical models with elasto-plastic and elastic steel behavior respectively are displayed in Figure 28. For the results with elastic steel behavior the curves are showing an unstable buckling behavior and for the cases with greater values of $c_u$ several mode jumps takes place also in this case. The BW went from having several buckles, to only having one larger in several variations.

Table 9 – Results for case 2.3.

<table>
<thead>
<tr>
<th>$c_u$ [kPa]</th>
<th>$N_{cr}$ [kN]</th>
<th>$x$ [m]</th>
<th>$d$ [mm]</th>
<th>$N_{pl,Rd}$ [kN]</th>
<th>$M_{pl,Ed}$ [kNm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9</td>
<td>5592</td>
<td>13.61</td>
<td>100.58</td>
<td>1222</td>
<td>191.2</td>
</tr>
<tr>
<td>15.7</td>
<td>7924</td>
<td>14.34</td>
<td>53.17</td>
<td>1350</td>
<td>192.6</td>
</tr>
<tr>
<td>23.6</td>
<td>8989</td>
<td>6.26</td>
<td>56.56</td>
<td>1543</td>
<td>178.5</td>
</tr>
<tr>
<td>31.4</td>
<td>9939</td>
<td>3.98</td>
<td>57.58</td>
<td>1643</td>
<td>180.4</td>
</tr>
<tr>
<td>47.1</td>
<td>11410</td>
<td>5.90</td>
<td>58.42</td>
<td>1744</td>
<td>163.5</td>
</tr>
<tr>
<td>62.8</td>
<td>12369</td>
<td>5.77</td>
<td>53.67</td>
<td>1796</td>
<td>160.0</td>
</tr>
<tr>
<td>78.6</td>
<td>13068</td>
<td>3.74</td>
<td>51.68</td>
<td>1828</td>
<td>160.3</td>
</tr>
<tr>
<td>Eurocode</td>
<td>570</td>
<td></td>
<td></td>
<td>435</td>
<td></td>
</tr>
</tbody>
</table>
Figure 28 – Load-displacement curves from Abaqus results for case 2.3.
4.9. Case 3.1

In Table 10 results for case 3.1 are summed up from both the numerical model with varying shear strength and the suggested Eurocode model. In Figure 29 the load-displacements curves are shown, where one curve represents one input value of the clay shear strength and type of steel behavior. Here, all the different cases are showing a stable buckling behavior.

Table 10 - Results for case 3.1.

<table>
<thead>
<tr>
<th>$c_u$ [kPa]</th>
<th>$N_{cr}$ [kN]</th>
<th>$x$ [m]</th>
<th>$d$ [mm]</th>
<th>$N_{pl,Ed}$ [kN]</th>
<th>$M_{pl,Ed}$ [kNm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9</td>
<td>7199</td>
<td>9.14</td>
<td>50.73</td>
<td>1610</td>
<td>173.1</td>
</tr>
<tr>
<td>15.7</td>
<td>7944</td>
<td>9.05</td>
<td>48.79</td>
<td>1699</td>
<td>165.2</td>
</tr>
<tr>
<td>23.6</td>
<td>8670</td>
<td>8.93</td>
<td>46.26</td>
<td>1786</td>
<td>158.1</td>
</tr>
<tr>
<td>31.4</td>
<td>9375</td>
<td>8.84</td>
<td>43.11</td>
<td>1870</td>
<td>151.1</td>
</tr>
<tr>
<td>47.1</td>
<td>10712</td>
<td>8.66</td>
<td>39.21</td>
<td>2024</td>
<td>139.7</td>
</tr>
<tr>
<td>62.8</td>
<td>11934</td>
<td>8.48</td>
<td>32.52</td>
<td>2163</td>
<td>128.4</td>
</tr>
<tr>
<td>78.6</td>
<td>13019</td>
<td>8.34</td>
<td>32.65</td>
<td>2267</td>
<td>122.3</td>
</tr>
<tr>
<td>Eurocode</td>
<td>4401</td>
<td></td>
<td></td>
<td>1900</td>
<td></td>
</tr>
</tbody>
</table>
Figure 29 – Load-displacement curves from the numerical results for case 3.1.
4.10. Case 3.2

For case 3.2 all cases displayed an unstable buckling behavior and as can be seen in Figure 30 the two highest values of $c_u$ generated some mode jumps. In Table 11 the numerical values achieved during the FEM simulation are summed up along with the related value from EN 1993-5.

Table 11 – Results for case 3.2.

<table>
<thead>
<tr>
<th>$c_u$ [kPa]</th>
<th>$N_{cr}$ [kN]</th>
<th>$x$ [m]</th>
<th>$d$ [mm]</th>
<th>$N_{pl,Ed}$ [kN]</th>
<th>$M_{pl,Ed}$ [kNm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9</td>
<td>6503</td>
<td>12.59</td>
<td>72.18</td>
<td>1409</td>
<td>172.3</td>
</tr>
<tr>
<td>15.7</td>
<td>7890</td>
<td>12.94</td>
<td>64.55</td>
<td>1637</td>
<td>194.2</td>
</tr>
<tr>
<td>23.6</td>
<td>8875</td>
<td>8.80</td>
<td>32.58</td>
<td>1584</td>
<td>174.5</td>
</tr>
<tr>
<td>31.4</td>
<td>9769</td>
<td>8.66</td>
<td>52.50</td>
<td>1713</td>
<td>166.3</td>
</tr>
<tr>
<td>47.1</td>
<td>11279</td>
<td>8.43</td>
<td>51.21</td>
<td>1845</td>
<td>156.7</td>
</tr>
<tr>
<td>62.8</td>
<td>12337</td>
<td>6.14</td>
<td>49.95</td>
<td>1895</td>
<td>156.5</td>
</tr>
<tr>
<td>78.6</td>
<td>13052</td>
<td>6.09</td>
<td>48.19</td>
<td>1919</td>
<td>158.1</td>
</tr>
<tr>
<td>Eurocode</td>
<td>1234</td>
<td></td>
<td></td>
<td>824</td>
<td></td>
</tr>
</tbody>
</table>
Figure 30 – Load-displacement curves for case 3.2 with elastic and plastic material properties of the steel.
4.11. Case 3.3

In case 3.3 all values on $c_u$ generated an unstable buckling behavior and several of the various values on the shear strength exhibits some mode jumps, as can be seen in Figure 31. The numerical values from the simulation is summed up in Table 12 along with the related values according to EN 1993-5.

Table 12 - Results for case 3.3.

<table>
<thead>
<tr>
<th>$c_u$ [kPa]</th>
<th>$N_{cr}$ [kN]</th>
<th>$x$ [m]</th>
<th>$d$ [mm]</th>
<th>$N_{pl,Ed}$ [kN]</th>
<th>$M_{pl,Ed}$ [kNm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9</td>
<td>5582</td>
<td>16.10</td>
<td>91.29</td>
<td>1174</td>
<td>188.8</td>
</tr>
<tr>
<td>15.7</td>
<td>7885</td>
<td>12.71</td>
<td>71.34</td>
<td>1212</td>
<td>208.5</td>
</tr>
<tr>
<td>23.6</td>
<td>8909</td>
<td>8.74</td>
<td>67.08</td>
<td>1434</td>
<td>186.5</td>
</tr>
<tr>
<td>31.4</td>
<td>10398</td>
<td>8.61</td>
<td>68.86</td>
<td>1521</td>
<td>180.1</td>
</tr>
<tr>
<td>47.1</td>
<td>12161</td>
<td>6.25</td>
<td>67.07</td>
<td>1632</td>
<td>174.2</td>
</tr>
<tr>
<td>62.8</td>
<td>13385</td>
<td>6.18</td>
<td>62.66</td>
<td>1685</td>
<td>173.4</td>
</tr>
<tr>
<td>78.6</td>
<td>14322</td>
<td>6.14</td>
<td>58.24</td>
<td>1718</td>
<td>170.5</td>
</tr>
<tr>
<td>Eurocode 570</td>
<td>435</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 31 – Load-displacement curves from the Abaqus results for case 3.3.
4.12. Case Study

A complementary calculation carried out during the scope of this work shows that the pipe described in the case study would have a utilization of 1.02 if every aspect in SS-EN 1993-5 is followed. The reason for this is that in the original design the buckling curve used to determine $\chi$ was not curve $d$ but rather curve $a$, since the pipes are drilled and not vibrated. A comparison has also been made where the methods in Sponthandboken were used, the cross-section needed then would have a third of the steel weight compared to the pipe that was required according to Eurocode for this specific yield mode.

The method used in SS-EN 1993-5 was also compared with the interaction method from SS-EN 1993-1-1 where the interaction factor $k_{\text{yy}}$ was calculated by approximating the sheet pile to be a simply supported beam with a moment distribution as a parabola, which according to the data achieved from the original PLAXIS model used is a good approximation. This calculation yielded an interaction factor much higher than the 1.15 used in Equation 22 as can be seen in Table 13. A third analysis was made using Equation 7 with an approximate modulus of foundation to determine a critical load, the critical load was then, compared to the method described in Eurocode, approximately six times higher which meant that the risk of buckling still is accommodated for, but the utilization is approximately 50% instead of 102%, implying that a smaller cross-section could likely be used. The primary elements are however not fully embedded in soil, and the results from that calculation should not be seen as an actual critical load but rather as a method to approximately accommodate for the adjacent soil in the model.

This result was also compared to a calculation of the critical load if the pile would have been only partially embedded in soil and completely free in the other part, also giving a critical load approximately nine times higher than that from Eurocode. This was done using the method described in
part 2.1.1.1, with a length above ground level of 2.3 m and below ground 7.9 m.

Values from each calculation is displayed in Table 13, the cross-sections in the right column are of standard type KCKR. For all these calculations the soil, consisting mainly of sand, was considered having a sub-grade modulus of 12 MN/m$^3$ as suggested in TR BRO 2009 (Trafikverket, 2009).

All calculations can be found in Appendix B.

<table>
<thead>
<tr>
<th>Modell used</th>
<th>$N_{cr}$ [kN]</th>
<th>$N_{Ed}/N_{cr}$</th>
<th>Interaction factor [-]</th>
<th>Utilization [-]</th>
<th>Cross-section $(dxt)$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS-EN 1993-5</td>
<td>1204</td>
<td>0.51</td>
<td>1.15</td>
<td>1.02</td>
<td>220x12.5</td>
</tr>
<tr>
<td>SS-EN 1993-1-1</td>
<td>1204</td>
<td>0.51</td>
<td>1.57</td>
<td>1.12</td>
<td>220x12.5</td>
</tr>
<tr>
<td>Sponthandboken</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.23*/0.89**</td>
<td>168x6</td>
</tr>
<tr>
<td>Timoshenko elastic foundation</td>
<td>10500</td>
<td>0.058</td>
<td>1.15</td>
<td>0.50</td>
<td>220x12.5</td>
</tr>
<tr>
<td>Partially embedded</td>
<td>10700</td>
<td>0.057</td>
<td>1.15</td>
<td>0.50</td>
<td>220x12.5</td>
</tr>
</tbody>
</table>

* With the actual cross-section used
** With the suggested cross-section
5. Discussion and Conclusions

Here the results presented are discussed and compared with other methods. Some recommendations on how the results can be used during the design process are also presented. Conclusions are also presented. Finally, suggestions for future work to further increase the understanding of the problem are presented.

5.1. Load Bearing Capacity

As is clearly seen in Figure 21 the strength and geometrical extension of the surrounding soil have a significant impact on the axial load capacity of the Berliner wall. Having reached peak values around four times higher bearing capacity compared to when the soil is not taken into account, it is obvious that, with soil support, more slender structures could be used. It should however also be noted that a small driven depth in combination with soft soil, 7.9-23.6 kPa shear strength, leads to lower bearing capacities than the model in Eurocode suggests, one reason for this might be the difference between having a roller support, as in EN 1993-5, or a spring, as in the ABAQUS model, as a representation of the anchor. When a spring is used, for even a very stiff spring, displacement in the horizontal direction is possible implying that the moment and thus the stress increases. The difference in capacity between the ABAQUS model and EN 1993-5 is in the cases with lower bearing capacities quite small, the largest being 5 % and using a spring instead of a roller support increases the similarity to the real case since an actual anchor can move both vertically and horizontally.

From the individual cases the linear buckling load achieved from ABAQUS never results in the same load bearing capacity as the post buckling analyses. This is due to the fact that in the linear buckling simulation the clay never becomes plastic, whereas in the post buckling
simulations it does. One should therefore not use only a linear buckling model to calculate the buckling load for a Berliner wall, since the plasticity of the soil has a severe impact.

The individual cases also display, in many cases, “strange” load-displacement curves when the steel was kept linearly elastic, where the selected node changes direction. It always occurred when there was an unstable buckling behavior and two nodes with almost the same magnitude of displacement but in opposite directions. At the instances where the direction of motion changes a sudden mode shape change occurs, often going from two sinusoids to one. This could be because of the plasticity in the soil, that the soil on one side becomes plastic and therefore requires less energy to buckle in that direction. As Riks et.al. (1996) stated in their analysis of Steins (1959) experiment on the post buckling behavior of shells there can be several stable post buckling modes for one structure. When a limit point, for imperfect structures, are reached a jump often occurs to a, at the time, stable mode.

All three cases with 2 meter driven depth display a stable buckling behavior, whereas all other cases display an unstable buckling. This indicates that when the driven depth is small the soil does not affect the behavior in a significant way, rather the steel structure itself dictates the behavior. When, however, the driven depth is large the plastic behavior of the soil most likely provokes an unstable behavior, when a sudden drop in stiffness occurs due to the plasticity of the soil.

5.2. Comparison with “Pålgrundläggning”

One model used today which takes the soil into account for the determination of critical load is the end bearing pile model described in chapter 2.1.1.1. The comparison of this method on a Berliner wall is however not entirely satisfactory. One reason is that the method is based on the top of the pile being the load insertion point, whereas a Berliner wall can have several loads with different points of application. Furthermore, the top of the pile where the load is applied should be regarded as a pinned
support and the pile should be free to deform along the rest of the length, for a Berliner wall the anchors all work as intermediate supports.

These difficulties can lead to several ways of applying the method, one would be similar to the model in EN 1993-5 to regard the lowest anchor as the force application point. This will however lead to no change in buckling length between Berliner walls with different whale beam levels. This gives some variation in the buckling length. Another application would be to regard the total length above excavation bottom as the length \( l_0 \) in Equation 11, but then the effect of the intermediate supports will be disregarded. Given all these arguments and having tried to apply the method on the studied Berliner walls with no success in getting a suitable correlation we, the authors, would not recommend using the method from “Pålgrundläggning” with partially embedded piles to calculate the buckling load for a Berliner wall since it often yields a higher buckling load than the one obtained from the ABAQUS simulations.

Since there is great variation of the critical load depending on the surrounding soil, with almost all of the cases with 10 meter driven depth having half the critical length compared to EN 1993-5, the suggestion is that more research should be focused on the subject of finding a good method to calculate the critical load with account of the soil. Some suggestions for further work is given in part 5.4.

### 5.3. Conclusions

From the simulations made it can be concluded that the soil surrounding a Berliner wall has a significant effect on the critical buckling load. With small driven depth, i.e. \( \leq 2 \)m, the effect is however almost negligible and the method described in EN 1993-5 can be used without affecting the efficiency of the design. When the driven depth increases the soil plays an important part, especially in soils with high shear strengths. If the soil is taken into account when the driven depth is between 6 and 10 meter the critical load can be four times higher compared to what the method described in EN 1993-5 would yield. Finding a method to properly calculate the critical load depending on the soil properties would require more future
research and a recommendation today would be to calculate the bearing capacity using FEM.

The greatest contributing factor for the variation of critical load has been the driven depth and in Table 14 the mean value of the introduced parameter $\beta_{cr}$, the correlation factor between the critical length achieved in ABAQUS and that given in EN-1993-5, for the various driven depths is presented, regardless of the amount of whale beams. For the cases with 2 meter driven depth the variation is rather great but both 6 and 10 meter driven depth induces a less sensitive $\beta_{cr}$ that does not vary that much depending on the shear strength.

Table 14 - Mean values and standard deviations of $\beta_{cr}$ for the various driven depths.

<table>
<thead>
<tr>
<th>D [m]</th>
<th>$\beta_{cr}$ [-]</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td>0.93</td>
<td>0.14</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>0.57</td>
<td>0.06</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>0.45</td>
<td>0.05</td>
</tr>
</tbody>
</table>
5.4. Suggestions for Further Research

Although there are cases where there is a thin layer of soft clay above the hard rock, there are also ground conditions in the Stockholm area where only moraine cover the bedrock or both. Therefore, an additional work of interest to this study would be to analyze the buckling behavior of an RW modelling the soil springs as a representation of friction soil instead of clay. This would result in soil springs with varying stiffness proportional with the depth.

In the study 3D effects are not considered. Anchors with a large center to center distance relative the distance between the piles create a situation where there are more than two piles between the anchors. For a deeper understanding of the effect of the whole structure and the force distribution on the piles in between anchors in particular, a suggestion of further work is to set up a 3D-model to analyze the problem. Such a model, equipped with solid elements in order to model the soil, should also include the surface friction between the structural elements and the soil since not the whole normal force necessarily is taken by the pile tip. This would generate a more realistic distribution of the normal force also internally.

Moreover, analyses on other types of RWs are of interest for further work, e.g. buckling analyses on SPWs. Although the second moment of area only is an input value in the numerical calculations, this would strengthen the validity of the model.
References

PTC. (2017). PTC Mathcad Prime 4.0 [Software].
Sveriges Geologiska Undersökning. (den 16 01 2018). *Om geologi*. Hämtat från SGU: sgu.se
Appendix A

Below is the calculation of $l_{cr}$ presented for case 1.1 with $c_u = 7.9$ kPa. The same has been done for all other cases as well, with the ultimate load achieved in the ABAQUS simulation for that specific case.

**Calculation of $l_{cr}$**

Case 1.1 with $c_u = 7.9$ kPa

$E := 210 \text{ GPa}$

$I_y := 4.3 \cdot 10^7 \text{ mm}^4$

$f_y := 460 \text{ MPa}$

$A := 0.008113 \text{ m}^2$

$N_{\text{ultimate, ABAQUS}} := 1563 \text{ kN}$

$\chi_{\text{calculated}} := \frac{N_{\text{ultimate, ABAQUS}}}{f_y \cdot A} = 0.419$

$\Phi(\lambda) := 0.5 \cdot (1 + \alpha \cdot (\lambda - 0.2) + \lambda^2)$

$\chi(\lambda) := \frac{1}{\Phi(\lambda) + \sqrt{(\Phi(\lambda))^2 - \lambda^2}}$

$\lambda_{\text{new}} := \text{root} \left( \chi(\lambda) - \chi_{\text{calculated}}, \lambda, 0, 2 \right) = 1.1$

$\chi(\lambda_{\text{new}}) = 0.419$

$N_{\text{cr, new}} := \frac{f_y \cdot A}{\lambda_{\text{new}}^2} = 3083 \text{ kN}$

$N_{\text{cr, lcr}}(l_{cr}) := \frac{E \cdot I_y \cdot \pi^2}{l_{cr}^2}$

$L_{cr} := \text{root} \left( N_{\text{cr, lcr}}(l_{cr}) - N_{\text{cr, new}}, l_{cr}, 0.1 \text{ m}, 50 \text{ m} \right) = 5.377 \text{ m}$
Appendix B

Below are all parts of the case study calculations presented.

\[ d_{outer} = 219.1 \text{ mm} \quad \text{Outer diameter of pipe} \]
\[ t = 12.5 \text{ mm} \quad \text{Pipe thickness} \]
\[ L = 10.7 \text{ m} \quad \text{Approximated length between lowest wale beam and bedrock} \]
\[ E_s = 210 \text{ GPa} \quad \text{Young's modulus for steel} \]
\[ f_y = 460 \text{ MPa} \quad \text{Yield strength of the steel} \]
\[ \alpha = 0.76 \quad \text{Reduction factor taking into account initial imperfections} \]
\[ \frac{d_{outer}}{t} = 17.528 \quad \text{Limit for cross-sectional class 1} \]

\[ \Rightarrow \text{Cross-section Class 1} \]

**Buckling**

\[ d_{inner} = d_{outer} - 2 \cdot t = 194.1 \text{ mm} \quad \text{Inner diameter of the pipe} \]
\[ I_y = \frac{\pi}{64} \left( d_{outer}^4 - d_{inner}^4 \right) = (4.3 \cdot 10^7) \text{ mm}^4 \quad \text{Second area of moment for the pipe} \]
\[ W_{el} = \frac{\pi}{32} \cdot \frac{d_{outer}^4 - d_{inner}^4}{d_{outer}} = (3.97 \cdot 10^5) \text{ mm}^3 \quad \text{Elastic section modulus} \]
\[ A = \left( \frac{d_{outer}}{2} \right)^2 \cdot \pi - \left( \frac{d_{inner}}{2} \right)^2 \cdot \pi = 8113 \text{ mm}^2 \quad \text{Cross-sectional area} \]

\[ N_{pl,Rel} = f_y \cdot A = 3732 \text{ kN} \quad \text{Plastic axial force bearing capacity} \]
\[ W_{pl} = d_{outer}^2 \cdot t = (6 \cdot 10^5) \text{ mm}^3 \quad \text{Plastic section modulus} \]
\[ M_{pl,ld} = W_{pl} \cdot f_y = 276 \text{ kN} \cdot \text{m} \quad \text{Plastic moment capacity} \]
\[ N_{cr} = 0.75 \cdot E_s \cdot I_y \cdot \frac{\pi^2}{(0.7 \cdot L)^2} = 1204 \text{ kN} \quad \text{Critical buckling load if the primary elements are seen as rigid in the bottom, due to being drilled down in the bedrock} \]
\[ \lambda = 2 \sqrt{\frac{N_{pl,Rd}}{N_{cr}}} = 1.761 \]  
Sienderness

\[ \phi := 0.5 \left( 1 + \alpha \cdot (\lambda - 0.2) + \lambda^2 \right) \]

\[ \chi := \frac{1}{\phi + \sqrt{\phi^2 - \lambda^2}} = 0.217 \]  
Reduction factor of the axial force bearing capacity due to risk of buckling

\[ M_{c,Rd} := M_{pl,Rd} \]  
No reduction needed

\[ N_{Ed} := 610 \text{ kN} \]  
Data collected from the original documents

\[ M_{Ed} := 64.4 \text{ kN} \cdot \text{m} \]

\[ \frac{N_{Ed}}{N_{cr}} = 0.507 \]

\[ \frac{N_{Ed}}{X \cdot N_{pl,Rd}} + 1.15 \cdot \frac{M_{Ed}}{M_{c,Rd}} = 1.02 \]  
Utilization according to SS-EN 1993-5

**Using the old Swedish guidelines to determine needed cross-section**

\[ W_{pl,spont} := 2 \text{ mm}^2 \]  
Start guess for the MathCad root function to work

\[ M_{Rd} (W_{pl,spont}) := W_{pl,spont} \cdot J_g \]

Determination of needed cross-section according to the old Swedish guideline Sporthandboken

\[ W_{pl,spont} := \text{root} \left( \frac{M_{Ed}}{M_{Rd} (W_{pl,spont})} - 1, W_{pl,spont} \cdot 5000 \text{ mm}^3, 917 \cdot 10^3 \text{ mm}^4 \right) = 140 \cdot 10^3 \text{ mm}^3 \]

\[ \frac{71.5 \text{ kg}}{\text{m}} = 3 \]

\[ \frac{24 \text{ kg}}{\text{m}} \]  
Comparisson between cross-sectional weight for the used profile (71.5 kg/m) and the one needed if Sporthandboken would have been used (24 kg/m). Weights taken from Tibnor construction tables
If the old Swedish guidelines from Spøhandboken had been used the profile could have been a KCKR 188x6 instead of a 220x12.5 meaning that the new rules give approximately three times the needed steel weight compared to Spøhandboken

\[
\frac{M_{Ed}}{f_y \cdot 158 \cdot 10^3 \text{mm}^3} = 0.89 \quad \frac{M_{Ed}}{M_{pl,Rd}} = 0.233
\]

**Using EN 1993-1-1 for the interaction factors**

\[N_{pl,Rd} := f_y \cdot A = 3732 \text{ kN}\]

\[W_{pl} := d_{outer}^2 \cdot t = (6 \cdot 10^5) \text{mm}^3\]

\[M_{pl,Rd} := W_{pl} \cdot f_y = 276 \text{ kN} \cdot \text{m}\]

\[w_y := \frac{W_{pl}}{W_{el}} = 1.513\]

\[w_y := 1.5\]

\[n_{pl} := \frac{N_{Ed}}{N_{pl,Rd}} = 0.163\]

\[\lambda = 1.761\]

\[C_{mu,0} := 1 + 0.03 \frac{N_{Ed}}{N_{cr}} = 1.015\]

\[C_{mu} := C_{mu,0}\]

\[C_{yy} := 1 + (w_y - 1) \cdot \left( 2 - \frac{1.6}{w_y} \cdot C_{my}^2 \cdot \lambda - \frac{1.6}{w_y} \cdot C_{my}^2 \cdot \lambda^2 \right) \cdot n_{pl} = 0.727\]

\[\mu_y := \frac{1 - \frac{N_{Ed}}{N_{cr}}}{1 - \lambda \cdot \frac{N_{Ed}}{N_{cr}}} = 0.554\]

\[k_{yy} := C_{my} \cdot \frac{\mu_y}{N_{Ed}} \cdot \frac{1}{C_{yy}} = 1.57\]
\[ \frac{N_{Ed}}{X \cdot N_{pl,Rd}} + k_{yy} \cdot \frac{M_{Ed}}{M_{pl,Rd}} = 1.12 \]

Utilization according to SS-EN 1993-1-1

Approximating a new critical buckling load using methods for a continuously elastic supported beam

\[ \beta := 12 \frac{MN}{m^3} \cdot d_{outer} = 2.629 \text{ MPa} \]

12 \( \frac{MN}{m^3} \) is an approximate subgrade modulus for medium packed friction soil along the length of the pile

\[ m_{sin} := 3 \]

Number of half sine waves formed during the buckling that minimizes the buckling load

Using modulus of foundation and the solution by Timoshenko (2012)

\[ P_{cr,approximate} := \frac{\pi^2 \cdot E_s \cdot I_y}{L^2} \cdot \left( m_{sin}^2 + \frac{\beta \cdot L^4}{m_{sin}^2 \cdot \pi^4 \cdot E_s \cdot I_y} \right) = 10.5 \text{ MN} \]

\[ \frac{N_{Ed}}{P_{cr,approximate}} = 0.058 > 0.04 \rightarrow \text{Risk for buckling} \]

Determining risk of buckling between first and second anchor level

\[ N_{cr,anchors} := \frac{\pi^2 \cdot E_s \cdot I_y}{(3 \text{ m})^2} = 10005.166 \text{ kN} \]

Critical buckling load between the two anchor levels after the final stage of excavation

\[ N_{Ed,level1} = 243 \text{ kN} \]

\[ \frac{N_{Ed,level1}}{N_{cr,anchors}} = 0.024 \rightarrow \text{No risk of buckling} \]
Critical load if the guideline "Handbok för Pågrundläggning" from Swedgeo is used, for a partially embedded pile

\[ l_0 := 2.3 \, \text{m} \]

\[ l_{\text{embedded}} := 7.9 \, \text{m} \]

\[ k := 12 \frac{MN}{m^3} \]

\[ n_h := \frac{k \cdot d_{\text{outer}}}{l_{\text{embedded}}} = 332.81 \frac{kN}{m^3} \]

\[ l_s := 1.8 \cdot 5 \sqrt{\frac{E_s \cdot I_y}{n_h}} = 3.49 \, \text{m} \]

\[ l_k := \frac{l_s + l_0}{2} = 2.895 \, \text{m} \]

\[ P_{\text{cr, partially}} := \frac{\pi^2 \cdot E_s \cdot I_y}{l_k^2} = 10.7 \, MN \]

\[ \frac{N_{Ed}}{P_{\text{cr, partially}}} = 0.057 \]

\[ \lambda_{\text{new}} = \sqrt{\frac{f_y \cdot A}{P_{\text{cr,approximate}}}} = 0.597 \quad \Rightarrow \quad \chi_{\text{new}} := 0.7 \]

\[ \frac{N_{Ed}}{\chi_{\text{new}} \cdot N_{pl,Rd}} + 1.15 \cdot \frac{M_{Ed}}{M_{pl,Rd}} = 0.502 \]

\[ \lambda_{\text{new,2}} := \sqrt{\frac{f_y \cdot A}{P_{\text{cr, partially}}}} = 0.589 \quad \Rightarrow \quad \chi_{\text{new,2}} := 0.72 \]

\[ \frac{N_{Ed}}{\chi_{\text{new,2}} \cdot N_{pl,Rd}} + 1.15 \cdot \frac{M_{Ed}}{M_{pl,Rd}} = 0.495 \]