Optimizing the net interest margin of a bank

An extension of the Black-Litterman model with financial regulations

JOSEFIN HANSSON

ANNIE ZHANG
Optimizing the net interest margin of a bank
An extension of the Black-Litterman model with financial regulations

JOSEFIN HANSSON
ANNIE ZHANG
Abstract

A bank’s business model is based on borrowing and lending, and by borrowing funds at a lower rate and lending these funds at a higher rate, the bank makes a profit. Thus, a key task in each bank’s operations is to maximize its net interest margin. In addition, in optimizing the net interest margin, banks must take into account financial regulations, among others the Net Stable Funding Ratio (NSFR).

In cooperation with Handelsbanken, this bachelor thesis in applied mathematics and industrial engineering aims to examine how the bank’s net interest margin can be optimized. The Black-Litterman model, an asset allocation model, is used and modified in order to model the bank’s balance sheet. Historical data of Swedish government bonds serves to estimate the variances and covariances of the bond returns. From there, two approaches are used to model the returns, the first assuming that the bonds are held to maturity and the second assuming that the bonds are held for trading.

The results show that bond returns cannot be approximated with a normal distribution when held to maturity. Thus, a fundamental assumption of the Black-Litterman model is violated. Instead, the thesis concludes that the bank’s net interest margin can be optimized with the Black-Litterman model only if it is assumed that the bank’s balance sheet is a portfolio of bonds where the bonds are held for trading. Further, in order to ensure that the requirements of the NSFR are met, the NSFR can be incorporated into the model as a constraint.
Sammanfattning


Acknowledgements

Foremost, we would like to express our gratitude to Handelsbanken, to Magnus Hanson who gave us the opportunity to write this thesis at his department, and to Christian Alexandersson who has been a great support throughout the work, providing us with all the necessary facilities for the research and taking the time to answer all our questions.

We would also like to thank our thesis advisor Johan Karlsson for the valuable guidance during the process of writing this thesis.
# Table of contents

## Contents

1. **Introduction** ......................................................... 6
   1.1. Background ...................................................... 6
   1.2. Aim .......................................................... 7
   1.3. Problem statement ........................................... 7
   1.4. Scope and Limitations ....................................... 7

2. **Economic Theory** .................................................... 8
   2.1. Bonds ........................................................ 8
   2.2. A bank’s borrowing and lending .................................. 10
   2.3. Financial regulations ......................................... 11
      2.3.1. The Basel Committee on Banking Supervision ............. 11
      2.3.2. The Net Stable Funding Ratio ............................. 12

3. **Mathematical Theory** ............................................... 14
   3.1. Markowitz Portfolio Theory .................................... 14
   3.2. The Black-Litterman Model .................................... 18

4. **Methodology** .......................................................... 24
   4.1. Data processing ................................................ 24
      4.1.1. Approach 1 – assuming that the bonds are held to maturity .......... 24
      4.1.2. Approach 2 – assuming that the bonds are held for trading .......... 26
   4.2. Optimization with the Markowitz model ...................... 27
   4.3. Optimization with the Black-Litterman model ................. 28
      4.3.1. Short selling not allowed .................................. 28
      4.3.2. Short selling allowed ..................................... 30
   4.4. Optimization with regards to NSFR ........................... 31

5. **Results** ............................................................... 34
   5.1. Data processing ................................................ 34
      5.1.1. Approach 1 – assuming that the bonds are held to maturity ........ 34
1 Introduction

1.1 Background

Handelsbanken is one of the largest banks in Sweden and was founded in Stockholm in 1871. Today the bank operates across more than 20 countries, where Sweden, the UK, Denmark, Finland and the Netherlands are considered as the bank’s primary markets. Handelsbanken’s business model, similar to other banks, is based on borrowing and lending which is represented on the balance sheet by interest-bearing liabilities and interest-bearing assets, respectively. The borrowing rate is the interest rate paid by the bank on customer deposits and other borrowings, and the lending rate is the interest rate paid by the customers to the bank on their loans. By borrowing at a lower rate and lending the funds at a higher rate, the bank makes a profit. The difference between the lending rate and the borrowing rate is the net interest margin. As such, maximizing the net interest margin and thereby profits, while assuming as low a risk as possible, is a key task in each bank’s operation.

Since 1974, the banking systems around the world have been supervised by the Basel Committee on Banking Supervision. The objective of the committee is to develop standards for banking regulations, such as minimum capital requirements and other requirements associated with different types of risks. After the 2008 financial crisis, the capital requirements were tightened and in addition, new regulations on leverage and liquidity measures were added. One of the main measures on liquidity introduced by the committee was the Net Stable Funding Ratio (NSFR). The NSFR aims to create a sustainable funding profile in relation to the bank’s assets over a one-year horizon \[4\]. Thus, banks must take these regulations into account when optimizing their net interest margins.

An important task for Handelsbanken is to optimize its net interest margin with regards to financial regulations. In order to do that, an appropriate allocation model must be used. One of the most well-known portfolio allocation models is the Markowitz mean-variance portfolio model developed by Harry Markowitz in 1952. In the model, it is assumed that investors want as high an expected return as possible at as low a risk as possible \[13\]. Investors can either minimize the risk given a specified expected return or maximize the expected return given a specified level of risk. However, practical use of the model has proven to result in issues such as input sensitivity, error maximization and highly-concentrated portfolios \[12\]. The use in practice is therefore limited, even though the model is very intuitive from a theoretical perspective. Since the idea of mean-variance optimization is still very appealing, attempts to improve the model have continued. In 1992, Fischer Black and Robert Litterman developed the Black-Litterman model, which is an asset allocation model based on the Markowitz’ model. The idea was to develop a better-behaved asset allocation model and to overcome the practical problems associated
with the Markowitz model. The main difference between the Markowitz model and the Black-Litterman model is that the Black-Litterman model enables the investor to have his or her own views regarding the performance of the assets, views that may differ from the market’s expected returns. The Black-Litterman model has proven to be successful in several aspects, and this study aims to examine how the Black-Litterman model can be used to optimize the net interest margin of Handelsbanken, taking into account the requirements of financial regulations.

1.2 Aim

The aim of this study is to examine how the Black-Litterman model can be used for optimizing Handelsbanken’s net interest margin. Since the Black-Litterman model is an asset allocation model primarily used for optimizing portfolios of stocks, the aim is to investigate which assumptions and modifications are needed to make the model applicable to optimizing the bank’s balance sheet. The purpose is not to provide Handelsbanken with a complete model, but instead to give the bank an indication of how the Black-Litterman model can be used for the intended purpose.

1.3 Problem statement

The study seeks to answer the following research questions:

- Which assumptions and modifications of the Black-Litterman model are needed in order to make the model applicable to optimizing Handelsbanken’s net interest margin?
- How can the liquidity requirement, the Net Stable Funding Ratio, be incorporated in the model?

1.4 Scope and Limitations

The objective of the Basel Committee on Banking Supervision is to develop standards for banking regulations. These standards are referred to as Basel Accords. There are several new requirements in the latest Basel III accords that banks must follow. One of them is the Net Stable Funding Ratio. The Net Stable Funding Ratio is a liquidity measurement and the plan was to implement the NSFR by 1 January 2018, but the date has since been postponed and the timing of the implementation is currently not clear. One limitation in this thesis is that the study only examines how NSFR can be incorporated in the model, and does not examine any other requirements.
2 Economic Theory

In order to understand the structure of the bank’s balance sheet, this section consists of basic theory of bonds and the bank’s borrowing and lending. Further content in this section consist of the financial regulations that banks must follow, in particular the Net Stable Funding Ratio.

2.1 Bonds

Bond definition

A bond is a debt obligation between two parties, a borrower and a lender. The owner of the bond is the debtholder and the issuer is the party raising money. The issuers are typically companies, municipalities, states, and sovereign governments. Bond purchases are usually made for investment purposes since the issuer provides the owner interest payments, referred to as coupons, in return for acquired financing.

The price of a bond depends on the bond’s risk, which consists of risk in interest rates and the credit risk of the issuer. Changes in interest rates affect the bond’s price, where increasing interest rates requires a higher discount rate, resulting in a lower price, and vice versa. As such, the term to maturity of a bond affects the price, where a longer maturity implies a greater risk for interest rates fluctuations and thus, warrants a lower price. The credit risk affects the bond’s price based on the issuer’s creditworthiness. For example, a bond issued by the U.S. government is considered (almost) default-free, i.e. the issuer has a high credit rating, and therefore trades at a higher price (the yield is lower). A corporate bond on the other hand, has a certain risk of default, i.e. a lower credit rating, and therefore trades at a lower price (the yield is higher).

The terms of a bond are described in the bond certificate and typically includes:

- **Face value**, the amount paid at the end of the bonds life, also referred to as the principal value or the par value
- **Term to maturity**, the remaining life of the bond at any given time after issuance
- **Maturity date**, the date when the issuer redeems the bond and pays the holder of the bond the face value
- **Coupon rate**, the rate of interest the issuer pays on the face value of the bond
- **Coupon dates**, the fixed dates on which the issuer will make coupon payments
- **Issue price**, the price of the bond at the emission date
Zero-coupon bonds

A zero-coupon bond is the most basic interest rate derivative. A zero-coupon doesn’t pay interest, i.e. no coupons. Instead the bond is issued at a discount, meaning that the purchase price of the bond is below the par value. The difference between the issue price and the face value is the investor’s return. A zero-coupon bond contains only two cashflows, the first at the time of the purchase and the second at the maturity date when receiving the face value (if held to maturity).

Fixed coupon bonds

A fixed-coupon bond is the most basic coupon bond. The difference between a zero-coupon bond and a coupon bond is that, in addition to the face value payed at the maturity date, a coupon bond pays fixed coupons during the bond’s term to maturity. Thus, a coupon bond can be viewed as a portfolio of zero-coupon bonds.

Bond pricing with the Yield to maturity formula

**Definition 2.1** The yield to maturity of a bond is the discount rate that sets the present value of the promised bond payments equal to the current market price of the bond.

The yield to maturity (YTM) can be viewed as the internal rate of return (IRR) of an investment in a bond. If the YTM is known, the price of an \(n\)-year-fixed-coupon bond at time \(j\), is calculated according to the formula:

\[
D_j = \sum_{i=1}^{n} \left( \frac{c_i}{(1 + r_j)^i} \right) + \frac{K}{(1 + r_j)^n}
\]  

(1)

Thus, the price of an \(n\)-year-zero-coupon bond at time \(j\) is:

\[
D_j = \frac{K}{(1 + r_j)^n}
\]  

(2)

where

- \(c_i\) is the \(i\):th coupon payment, \(i = 1, \ldots, n\)
- \(r_j\) is the YTM of the bond at time \(j\)
- \(K\) is the face value
- \(n\) is the number of years until the maturity date
2.2 A bank’s borrowing and lending

A bank’s business model is based on borrowing and lending, represented by the interest-bearing liabilities and the interest-bearing assets respectively on the balance sheet. The borrowing rate is the interest rate paid by the bank on customer deposits and other borrowings and the lending rate is the interest rate paid by the customers to the bank on their loans. By borrowing at a lower rate and lending the funds at a higher rate, the bank makes a profit. The difference between the lending rate and the borrowing rate is the net interest margin.

Handelsbanken issues mortgage loans, private loans, car and leisure loans and different types of credit. Housing mortgage loans constitute the largest part of the loans issued and often have a long term to maturity, for example 30 years. Furthermore, the loans can either have a fixed or a variable interest rate. If the interest rate is fixed, the rate will remain the same for a part of the term of the loan or for the entire term. If the interest rate is variable, the rate adjusts on a periodic basis based on an underlying benchmark index or interest rate, typically the prime rate in the country.

A bank usually issues loans with a total amount which is 25 times as large as the amount it has. Therefore, the loans need funding. This is achieved mainly through deposit accounts, bonds and covered bonds. A deposit account is an account that allows the account holder to deposit and withdraw money. Examples of deposit accounts are current accounts, savings accounts, money market accounts and call deposit accounts. The balance on a deposit account represents the amount which the bank owes to the customer and is recorded as a liability for the bank. The customers receive an interest on the deposits. A bond is a fixed income investment in which an entity borrows money from an investor at a fixed or variable interest rate for a defined period of time. A covered bond is a debt security issued by a mortgage institution or a bank and collateralised against a pool of assets that cover claims at any point of time in case of failure of the issuer.

Basing its business on borrowing and lending, Handelsbanken assumes interest rate risk, credit risk and liquidity risk. Interest rate risk is the risk that the spread between the interest rate received on loans and the interest rate paid on borrowings will change over time. Credit risk is the probability that the bank will lose the principal amount that was lent and any interest earned because the borrower defaults on the debt. Liquidity risk is the risk that the bank does not have enough cash to carry out daily operations, such as meeting payment obligations from depositors and issuing loans. The Net Stable Funding Ratio, indicates whether the bank owns enough assets that can be converted into cash easily within a year. The bank’s objective is to maximize the profit, by maximizing the net interest margin, while assuming as low a risk as possible.
2.3 Financial regulations

2.3.1 The Basel Committee on Banking Supervision

The Basel Committee on Banking Supervision (BCBS) is an international committee formed in 1974. The committee was formed by the G10 countries, i.e. the ten wealthiest countries in the world at that time. Today, members of the BCBS include 27 countries and the European Union. The objective of the committee is to develop standards for banking regulations. These standards are referred to as Basel Accords.

The first Basel Accords, *Basel I*, was introduced in 1988 and implemented in the first member countries in the following years. The focus of Basel I was to set a minimum capital requirement standard for banks, in order to minimize the credit risk and keep banks solvent during times of financial stress. This was done by creating an asset classification system, dividing various assets into different risk categories.

The second Basel Accords, *Basel II*, was formed in 2004 and was in the process of being implemented when the 2008 financial crisis occurred. The Basel II was an extension of the Basel I reform and included additional regulation for the minimum capital requirements, but also additional standards for regulatory supervision and market discipline. The addition of regulatory supervision provided standards for dealing with different types of risks, including systematic risks, liquidity risks and legal risks, while the addition of market discipline included different requirements for banks risk assessment processes and risk exposures.

The third Basel Accords, Basel *Basel III*, was formed in 2009, agreed upon in 2011 and was planned to be implemented during 2013-2015. However, due to certain disagreements the implementation of the reform has been delayed and negotiations are still ongoing. In 2017, a new update of the reform was introduced and implementation will take place gradually until 2027 [3]. The focus of Basel III is to improve the regulation, supervision and risk management of the previous reforms. Since the 2008 financial crisis occurred despite the existing Basel-regulations, BCBS had to review and strengthen the framework. The capital requirements for the banks are now even tighter and in addition to the improvements of previous reforms, new regulations on leverage and liquidity measures will be added. The purpose of the new requirements is to ensure that the banks do not borrow excessively and to ensure that banks have sufficient liquidity during financial stress. The two main measures on liquidity introduced in Basel III are the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR). The LCR must be implemented by 2019 and the plan was to implement the NSFR by 1 January 2018, but the date has since been postponed and the timing of the implementation is currently not clear. Figure [1] summarizes the main features of Basel III, and the focus of this study is the liquidity requirement NSFR, listed in the fourth column in the figure.
2.3.2 The Net Stable Funding Ratio

Although the bank’s complied with the existing capital requirements during 2007, they experienced severe liquidity problems in the 2008 financial crisis. As a result, a new standard of liquidity was set following the crisis. The Net Stable Funding Ratio (NSFR) aims to create a sustainable funding profile in relation to the bank’s assets, over one-year horizon [4]. One of the reasons for the massive liquidity crisis during 2008 was that the banks had placed their assets on long-term maturities and covered it with short-term financing [15]. The purpose of the NSFR is to reduce the likelihood of future funding stress by making long-term exposures correspond to long-term funding. The plan was to implement the NSFR by 1 January 2018, but the date has since been postponed and the timing of the implementation is currently not clear.
The definition of NSFR is,
\[
\frac{\text{Available amount of stable funding}}{\text{Required amount of stable funding}} \geq 100\% \quad (3)
\]

The Available amount of stable funding (ASF) is calculated by dividing the liabilities into five different categories associated with an ASF factor, where a greater factor implies a more stable form of funding. The liabilities are weighted by their corresponding ASF factor and the amounts are summed, resulting in the overall ASF. See Figure 2 for categorization.

<table>
<thead>
<tr>
<th>ASF factor</th>
<th>Components of ASF category</th>
</tr>
</thead>
<tbody>
<tr>
<td>100%</td>
<td>Total regulatory capital, other capital instruments and liabilities with effective residual maturity of one year or more</td>
</tr>
<tr>
<td>95%</td>
<td>Stable non-maturity (demand) deposits and term deposits with residual maturity of less than one year provided by retail and SME customers</td>
</tr>
<tr>
<td>90%</td>
<td>Less stable non-maturity deposits and term deposits with residual maturity of less than one year provided by retail and SME customers</td>
</tr>
<tr>
<td>50%</td>
<td>Funding with residual maturity of less than one year provided by non-financial corporate customers, operational deposits, funding with residual maturity of less than one year from sovereigns, public sector entities (PSEs), and multilateral and national development banks, other funding with residual maturity of not less than six months and less than one year not included in the above categories, including funding provided by central banks and financial institutions</td>
</tr>
<tr>
<td>0%</td>
<td>All other liabilities and equity not included in above categories, including liabilities without a stated maturity, derivatives payable net of derivatives receivable if payables are greater than receivables</td>
</tr>
</tbody>
</table>

Figure 2: Summary of liability categories and associated ASF factors [4]

The Required Stable funding (RSF) is based on the liquidity risk of the banks’ assets. The RSF is calculated in the same way as the ASF, by first dividing the assets into seven different categories associated with an RSF factor, where a smaller factor implies less liquidity risk. The assets are weighted by their corresponding RSF factor and the amounts are summed, resulting in the overall RSF. See Figure 3 for categorization.
### Summary of asset categories and associated RSF factors

<table>
<thead>
<tr>
<th>RSF factor</th>
<th>Components of RSF category</th>
</tr>
</thead>
</table>
| 0%         | - Coins and banknotes  
|            | - All central bank reserves  
|            | - Unencumbered loans to banks subject to prudential supervision with residual maturities of less than six months |
| 5%         | - Unencumbered Level 1 assets, excluding coins, banknotes and central bank reserves |
| 15%        | - Unencumbered Level 2A assets |
| 50%        | - Unencumbered Level 2B assets  
|            | - HQLA encumbered for a period of six months or more and less than one year  
|            | - Loans to banks subject to prudential supervision with residual maturities six months or more and less than one year  
|            | - Deposits held at other financial institutions for operational purposes  
|            | - All other assets not included in the above categories with residual maturity of less than one year, including loans to non-bank financial institutions, loans to non-financial corporate clients, loans to retail and small business customers, and loans to sovereigns, central banks and PSFs |
| 65%        | - Unencumbered residential mortgages with a residual maturity of one year or more and with a risk weight of less than or equal to 35%  
|            | - Other unencumbered loans not included in the above categories, excluding loans to financial institutions, with a residual maturity of one year or more and with a risk weight of less than or equal to 35% under the Standardised Approach |
| 85%        | - Other unencumbered performing loans with risk weights greater than 35% under the Standardised Approach and residual maturities of one year or more, excluding loans to financial institutions  
|            | - Unencumbered securities that are not in default and do not qualify as HQLA including exchange-traded equities  
|            | - Physical traded commodities, including gold |
| 100%       | - All assets that are encumbered for a period of one year or more  
|            | - Derivatives receivable net of derivatives payable if receivables are greater than payables  
|            | - All other assets not included in the above categories, including non-performing loans, loans to financial institutions with a residual maturity of one year or more, non-exchange-traded equities, fixed assets, pension assets, intangibles, deferred tax assets, retained interest, insurance assets, subsidiary interests, and defaulted securities |

Figure 3: Summary of asset categories and associated RSF factors

### 3 Mathematical Theory

#### 3.1 Markowitz Portfolio Theory

Markowitz’s mean-variance portfolio model is the base of a lot of research within portfolio theory, and it is also the model from which the Black-Litterman model was developed. To
understand the Black-Litterman model, it is therefore important to first give a background to Markowitz’s model.

The Markowitz Model

In 1952, Harry Markowitz published the article “Portfolio Selection”, and from then on, portfolio theory became an academic field. Before Markowitz, there had been little research on the mathematical relations between the assets in a portfolio. Markowitz claims that the co-movements of the assets, represented by the covariances, should be taken into consideration when selecting a portfolio, not solely the characteristics of individual assets. Instead of a selection of individual securities, Markowitz considers the portfolio as a whole when finding an optimal portfolio. When covariances are taken into consideration, Markowitz argues that it is possible to form portfolios with a lower risk with the same expected return or portfolios with a higher expected return at the same risk level [13]. Including the covariances increases the possibility of obtaining a well-diversified portfolio. In Markowitz’s model, the risk is the variance of the portfolio, which depends on the variances of and the covariances between the assets’ returns.

In the model, it is assumed that investors want as high an expected return as possible at as low a risk as possible [13]. Investors can minimize the risk given a specified expected return or maximize the expected return given a specified level of risk. The inputs needed in the model are the expected return of each asset, the variance of each asset and the covariances between each pair of assets. One way to estimate these parameters is to use historical data. However, this approach assumes that past performance is a good approximation of future performance [13].

Minimizing the risk, or the variance of the portfolio, given a certain expected return involves solving the problem:

\[
\begin{align*}
\min_{x} & \quad x^T \Sigma x \\
\text{subject to} & \quad x^T \mu = r
\end{align*}
\]  

while maximizing the expected return of the portfolio given a certain level of risk or variance involves solving the problem:

\[
\begin{align*}
\max_{x} & \quad x^T \mu \\
\text{subject to} & \quad x^T \Sigma x = \sigma^2
\end{align*}
\]  

where

\[x\] is the column vector of portfolio weights
µ is the column vector of expected returns
Σ is the covariance matrix
σ² is the variance of the portfolio
r is the expected return of the portfolio

Figure 4: All attainable combinations of expected return and standard deviation

Figure 4 shows all attainable combinations of expected return and standard deviation (which is the square root of the variance), the attainable set. The combinations on the curve are the minimum variance set since these points represent the minimum attainable variance for each level of expected return. The upper part of the curve is the efficient frontier. All the portfolios on the efficient frontier are regarded as efficient since they generate the maximum expected return for a given risk level. All portfolios that are not on the efficient frontier are called inefficient since it is possible to have a lower risk given the expected return or a higher expected return given the same risk.

Instead of (4) or (5), the portfolio optimization problem can also be formulated as:

$$\max_{x} x^T \mu - \frac{\delta}{2} x^T \Sigma x$$

where δ is the risk aversion parameter measuring the risk-return trade-off. He and Litterman assume the parameter to be 2.5.
Problems with The Markowitz Model

Even though the Markowitz mean-variance model is a one of the most well-known models in modern portfolio theory, practical use of the model has proven to result in serious issues. The use in practice is therefore limited, even though the model is very intuitive from a theoretical perspective.

One of the most common problems with the model is that it often generates highly-concentrated portfolios. When optimizing the problem and allowing short selling, i.e. the constraint $x \geq 0$ is removed, the resulting allocation vector almost always have several negative weights. Since taking short positions is often not permitted, adding the constraint often results in an allocation vector with the majority of zero weights, which leads to a few assets with excessively large weights. In the most extreme case, the optimization leads to a vector with allocation in only one asset, an unreasonable portfolio that practitioners would not prefer.

Since the expected returns, variances and covariances are all estimates, they all involve a certain error, and another common problem with the model is estimation error maximization. The model has shown to overweight assets that contribute to the largest estimation errors, which is assets with high expected return and negative correlation, and underweight those with low expected return and positive correlation. However, this should be in accordance with investor’s preferences, as they should want to overweight the assets associated with high expected returns. From this point of view, the argument may seem contradictory, but either way, no practitioners of the model seek to maximizing the error.

It is also common to use the mean of historical data to estimate the market’s expected return, which also has shown to contribute to the error maximization.

Neither does the model take the assets market capitalization weights into account. Since the model has shown to overweight assets with high expected returns and negative correlation, an asset that also has a low level of market capitalization, often leads to a too high allocation weight, with regard to the asset’s level of capitalization.

Lastly, a common problem with the model is input-sensitivity. Small changes in the input, especially in the expected return, may generate a completely different portfolio, thus, the model is often very unstable. This is usually explained by an ill-conditioned covariance matrix, estimated with insufficient historical data.

Even though the model has severe problems, the idea of mean-variance optimization is still very appealing and attempts to improve the model have continued.
3.2 The Black-Litterman Model

The Black-Litterman model, hereafter referred to as the B-L model, is an asset allocation model developed by Fischer Black and Robert Litterman in 1992. The idea behind the model was to develop a better-behaved asset allocation model and to overcome the practical problems associated with the Markowitz’ model, such as input sensitivity, error maximization and highly-concentrated portfolios [12]. The main difference between the Markowitz’ model and the B-L model is that the B-L model models a new expected return vector, hereafter referred to as the B-L expected return vector. The B-L expected return vector is optimized in the same mean-variance way as in the Markowitz’ model. The B-L expected return vector combines the markets expected returns, referred to as the market equilibrium, and the investors unique views regarding the performance of the assets, as illustrated in Figure 5.

Below is the theory and notations needed to understand the B-L model and the so-called B-L formula, which calculates the B-L expected returns.

Assumptions

The main assumptions in the B-L model are presented below. Several of the assumptions made in the B-L model are the same as for other financial and quantitative portfolio models, the assumptions in italics are specific to the B-L model.

- Assets returns are normally distributed
- The market is free of arbitrage
- Only risk and expected return are used in investment decisions
- A portfolio’s risk can be measured by the variance and the covariance between the assets’ rate of return
- Investors have views about the assets they believe can lead to a better portfolio and risk ought to be taken in these assets
- The market is not totally efficient [12]
- Investors are never completely sure on any view and a level of confidence can be estimated to each view
The B-L formula

The B-L formula presented below is the formula for calculating the B-L expected return vector. Hereafter N represents the number of assets invested, and K the number of views.

$$E[R] = \left[ (\tau \Sigma)^{-1} + P'\Omega^{-1}P \right]^{-1} \left[ (\tau \Sigma)^{-1} \Pi + P'\Omega^{-1}Q \right]$$  \quad (7)$$

where

$$E[R]$$ is the new combined return vector, also referred to as the B-L expected return vector (Nx1 column vector)
\( \tau \) is a scalar and referred to as the weight-on-views  
\( \Sigma \) is the covariance matrix of the market returns (N x N matrix)  
\( P \) is a matrix representing the views, where each row contains the weights of assets of one view (K x N matrix)  
\( \Omega \) is a diagonal covariance matrix representing the uncertainty in each view (K x K matrix)  
\( \Pi \) The market equilibrium return vector, also referred to as the markets expected returns (N x 1 column vector)  
\( Q \) is the view vector (K x 1 column vector)  

The market equilibrium return  

The B-L model uses “equilibrium” returns as a starting point and according to Litterman, equilibrium returns is the state where supply equals demand [12]. There are several ways to estimate the equilibrium returns, and the most common approach is to use the Capital Asset Pricing Model (CAPM). This is the expected returns provided by the market if all investors act in a mean-variance way.  

The investor views  

The view vector \( Q \)  

The B-L model enables the investor to have views on the expected return of one or several assets in the portfolio. A view can either be expressed in absolute or relative terms. An example of an absolute view is, "Asset A will have an excess return of 2%", and an example of a relative view is, "Asset A will outperform asset B with 5%".  

Each view is associated with a level of uncertainty, \( \varepsilon \), where the collection of error terms \( \varepsilon \) are independent and normally distributed random variables, with mean zero and covariance matrix \( \Omega \).  

Thus, the views have the form:  
\[
Q + \varepsilon = \begin{bmatrix} Q_1 \\ \vdots \\ Q_k \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_k \end{bmatrix}
\]  

According to the example above, \( Q \) has the form:
\[
Q + \varepsilon = \begin{bmatrix} 2 \\ 5 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}
\]

The view matrix \( P \)

Each row in the matrix \( P \) corresponds to a view and each column represents the assets in the portfolio. Thus, \( P \) is a \( K \times N \) matrix. Each view corresponds to one row in the matrix, where assets with no view are set to zero. For an absolute view in asset \( A \), “1” is set to the corresponding column \( A \) in that row. Thus, the sum of the row in an absolute view should equal one. For a relative view in asset \( A \) and \( B \), “1” is set to the corresponding outperforming asset in column \( A \), and “-1” in set to the corresponding underperforming asset in column \( B \). Thus, the sum of the row in a relative view should equal zero.

This weighting is done according to the same principle when dealing with multiple assets in the same view. There are different methods for weighting and one approach is to weight the assets equally, regardless of the assets market capitalization. This means that a relative view containing four assets should be equally weighted with a factor of \( \pm 0.5 \). Another approach is to weight the assets in proportion to the assets market capitalization, in percent. However, the main principle is still that the sum of the row should equal one or zero.

Thus, the matrix \( P \) has the form:

\[
P = \begin{bmatrix}
p_{1,1} & p_{1,2} & \cdots & p_{1,N} \\
p_{2,1} & p_{2,2} & \cdots & p_{2,N} \\
\vdots & \vdots & \ddots & \vdots \\
p_{K,1} & p_{K,2} & \cdots & p_{K,N}
\end{bmatrix} = \begin{bmatrix}
\cdots & p_1 & \cdots \\
\cdots & p_2 & \cdots \\
\vdots & \vdots & \vdots \\
\cdots & p_K & \cdots
\end{bmatrix}
\]

Considering the two views mentioned earlier, if there are four assets in the portfolio, \( A \), \( B \), \( C \) and \( D \), the \( P \) matrix has the form:

\[
P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}
\]

The covariance matrix \( \Omega \)

To form the covariance matrix of the views, the model first assumes that the views are independent. The variance of the error terms \( \varepsilon \) describe the uncertainty in the views and are denoted with the variable \( \omega \).

Thus, the covariance matrix \( \Omega \) is a diagonal matrix with zeros on the off-diagonal elements:
Calculating $\omega$ is one of the most difficult and complex parts of the B-L model, and there are several approaches presented in different papers. One approach is to calculate the variance of an individual view portfolio, $p_k \Sigma p_k^T$, where $p_k$ is the k:th row of the $P$ matrix and $\Sigma$ is the covariance matrix of the markets returns, and then weight the variance with the weight-on-views scalar $\tau$ \cite{8}. This method is used by He and Litterman, and is also the most common method presented in different literature \cite{16}.

According to this method, the covariance matrix $\Omega$ has the form:

$$\Omega = \begin{bmatrix} \omega_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \omega_k \end{bmatrix}$$

The greater the uncertainty, the closer the B-L expected return vector will be to the market equilibrium return vector. The lower the uncertainty, the closer the B-L expected return vector will be to the views \cite{8}.

The weight-on-views scalar $\tau$

The scalar $\tau$ represents the weight-on-views and the easiest way to deal with the B-L model is to make an assumption regarding the value of $\tau$ \cite{8}. However, there is no clear way in existing literature on how to assume the constant, and ideas on how to set the variable differs. Black and Litterman suggests that $\tau$ should be set close to zero, Stachell and Scowcraft suggest that $\tau$ should be set to one, Lee on the other hand, sets $\tau$ between $0.01$ and $0.05$ \cite{8}.

When the covariance matrix $\Omega$ is calculated using the method presented in this paper, the value of $\tau$ becomes irrelevant, since only $\tau^{-1}\omega$ enters the B-L formula. Thus, changing $\tau$ will only affect the covariance matrix $\Omega$, but the B-L expected return is unaffected.

The B-L optimal portfolio

When all the necessary inputs are determined, the B-L expected returns is calculated according to the B-L formula \cite{7} on p. \cite{19}. The B-L expected returns are then optimized according to formula \cite{6}, presented in the Markowitz section on p. \cite{16}:

$$\max x^T \mu - \frac{\delta}{2} x^T \Sigma x$$
Where $\mu$ represents the B-L expected returns. In most papers, $\Sigma$ is also used to describe the covariance of the B-L expected returns. He and Litterman, on the other hand, argue that $\Sigma$ should be replaced with $\Sigma_p$, where $\Sigma_p = \Sigma + (\tau \Sigma)^{-1} + P^T \Omega^{-1} P)^{-1}$, where the later term is the posterior variance, which is the uncertainty in the posterior mean estimate.
4 Methodology

In order to model the bank’s balance sheet, the bank’s lending and borrowing is seen as a portfolio of bonds with different terms to maturity, where a short position in a bond represents borrowing and a long position represents lending. To simplify the study, it is assumed that the bonds have a fixed coupon rate and a fixed maturity date. This assumption is a simplification since the bank, in reality, both issues and takes out variable interest rate loans and loans with no fixed maturity date. Furthermore, it is assumed that the terms to maturity are limited to 1 month, 3 months, 6 months, 1 year, 2 years, 5 years, 7 years and 10 years and that there is only yield to maturity for each term to maturity, which is the same for a short- and long position. The latter implies that a loan issued by the bank with a certain term to maturity has the same interest rate regardless of the creditworthiness of the borrower and that the borrowing rate and the lending rate are the same. In reality, however, the lending rate is higher for a borrower with a low creditworthiness and the lending rate is always higher than the borrowing rate, which is how the bank makes a profit.

The balance sheet of the bank is thus modeled as a portfolio consisting of short- and long positions in the 8 different bonds, and it is the allocation between these 8 bonds which will be optimized using the Black-Litterman model.

4.1 Data processing

Let $B_1$, $B_2$, ..., and $B_8$ be the bonds with 1 month-, 3 months-, 6 months-, 1 year-, 2 years-, 5 years-, 7 years- and 10 years to maturity respectively. To estimate the variances and covariances of the returns of the bonds, historical data of Swedish government bonds from Thomson Reuters is used, provided by Handelsbanken. The data consists of the yields to maturity each month from January 1988 to January 2018 for Swedish government bonds with 1 month-, 3 months-, 6 months-, 1 year-, 2 years-, 5 years-, 7 years- and 10 years to maturity. Each row in the data represents a certain month and consists of the yields to maturity for the 8 different bonds that month. The data thus consists of 361 rows.

4.1.1 Approach 1 – assuming that the bonds are held to maturity

The first approach involves assuming that the bonds are held to maturity. This is a reasonable assumption since the bank is tied to and cannot sell a loan which it has issued. In order to be able to compare the return on bonds with different times to maturity, the data has to be modified so that the returns are given with the same time horizon. The time horizon 1 year is chosen. This implies that the 1-month-bond has to be bought 12 times, the 3-month-bond 4 times and the 6-month-bond 2 times to constitute an investment with
Figure 6: Historical yields to maturity of Swedish government bonds in percent

<table>
<thead>
<tr>
<th>Time</th>
<th>1 month</th>
<th>3 months</th>
<th>6 months</th>
<th>1 year</th>
<th>2 years</th>
<th>5 years</th>
<th>7 years</th>
<th>10 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>1988 April</td>
<td>10.50</td>
<td>10.90</td>
<td>10.66</td>
<td>10.80</td>
<td>11.12</td>
<td>11.40</td>
<td>11.54</td>
<td>11.60</td>
</tr>
<tr>
<td>1988 May</td>
<td>10.00</td>
<td>10.28</td>
<td>10.28</td>
<td>10.40</td>
<td>10.80</td>
<td>11.21</td>
<td>11.38</td>
<td>11.47</td>
</tr>
<tr>
<td>1988 June</td>
<td>10.25</td>
<td>10.35</td>
<td>10.33</td>
<td>10.53</td>
<td>10.78</td>
<td>11.06</td>
<td>11.16</td>
<td>11.27</td>
</tr>
<tr>
<td>1988 September</td>
<td>10.40</td>
<td>10.48</td>
<td>10.64</td>
<td>10.97</td>
<td>11.18</td>
<td>11.39</td>
<td>11.49</td>
<td>11.56</td>
</tr>
<tr>
<td>1988 December</td>
<td>10.40</td>
<td>10.35</td>
<td>10.65</td>
<td>10.95</td>
<td>11.06</td>
<td>10.87</td>
<td>10.82</td>
<td>10.78</td>
</tr>
<tr>
<td>1989 January</td>
<td>10.30</td>
<td>10.38</td>
<td>10.49</td>
<td>10.57</td>
<td>10.51</td>
<td>10.45</td>
<td>10.43</td>
<td>10.37</td>
</tr>
<tr>
<td>1989 February</td>
<td>11.25</td>
<td>11.25</td>
<td>11.30</td>
<td>11.34</td>
<td>11.17</td>
<td>10.94</td>
<td>10.91</td>
<td>10.82</td>
</tr>
</tbody>
</table>

... (Data continues for the other months)

For example, the yields to maturity for the 3-month-bond in January 1988 is replaced with the average of the yields to maturity for the 3-month-bond in January 1988, April 1988, July 1988 and October 1988. The equivalent procedure is

a 1-year-time horizon. However, the 1-year-bond, 2-year-bond, 5-year-bond, 7-year-bond and 10-year-bond only need to be bought once, at the beginning of the year, since their term to maturities are longer than or equal to 1 year. The returns on the 1-month-bond, 3-month-bond and 6-month-bond are calculated as the average of the yields to maturity of the bonds bought during the year. Thus, in each row of the data, the yields to maturity for the bonds with 1 month, 3 months and 6 months to maturity are modified, whereas the yields to maturity for the bonds with 1 year, 2 years, 5 years, 7 years and 10 years to maturity are left unchanged. For example, the yields to maturity for the 3-month-bond in January 1988 is replaced with the average of the yields to maturity for the 3-month-bond in January 1988, April 1988, July 1988 and October 1988. The equivalent procedure is
used for the 1-month-bond and the 6-month-bond and is repeated in each row of the data. The calculations are summarized below.

Let $r_{i,j}$ be the YTM of bond $B_i$ at time $j$, where $j = 1$ represents January 1988, $j = 2$ represents February 1988 and so on. Let $p_{ij}$ be the calculated annual return of bond $B_i$ at time $j$.

For $i = 1$, $p_{1j} = \frac{r_{1,j} + r_{1,j+1} + \ldots + r_{1,j+11}}{12}$ where $j = 1, \ldots, 350$.

For $i = 2$, $p_{2j} = \frac{r_{2,j} + r_{2,j+3} + r_{2,j+6} + r_{2,j+9}}{4}$ where $j = 1, \ldots, 352$.

For $i = 3$, $p_{3j} = \frac{r_{3,j} + r_{3,j+6}}{2}$ where $j = 1, \ldots, 355$.

For $i = 4, 5, \ldots, 8$, $p_{ij} = r_{i,j}$ where $j = 1, \ldots, 361$.

However, plotting the resulting returns $p_{ij}$ in one histogram for each bond $B_i$ in Figure 7 on p. 34 shows that the returns cannot be approximated with a normal distribution. Hence, a fundamental assumption in the Black-Litterman model is violated. Another problem with assuming that the bonds are held to maturity is that the returns on the bonds with a term to maturity longer than or equal to the time horizon are known and are given by the yields to maturity ($p_{ij} = r_{i,j}$ for $i = 4, 5, \ldots, 8$ above). Since the returns on these bonds are not stochastic ($r_{i,j}$ is known at time $j$), it is not possible to speak of variances and covariances of the returns. This is not a problem for bonds with a term to maturity shorter than the time horizon. Even though the bond bought at the beginning of the time period has a known YTM, if the time horizon is longer than the term to maturity, it is necessary to buy at least one more bond in the future with a YTM which is unknown today (for example, for $i = 3$, $r_{3,j+6}$ is unknown at time $j$). Therefore, the average of the yields to maturity is still stochastic.

### 4.1.2 Approach 2 – assuming that the bonds are held for trading

Since the returns on the bonds are not approximately normally distributed, another approach is used where the bonds are assumed to be held for trading. In this approach, the portfolio of bonds is seen as that of an ordinary investor, meaning that the bonds can be bought and sold at any time and need not be held to maturity. To calculate the returns, it is assumed that the bonds are zero-coupon bonds and that they are bought one month and sold the next. The price of each bond each month is calculated using the bonds’ yields to maturity:

$$\text{Price} = \frac{\text{Face value}}{(1+\text{YTM})^n}$$
where \( n \) is the number of years until the maturity date. Assuming that every bond has a face value of 100, the price of bond \( B_i \) at time \( j \), \( D_{i,j} \), is calculated as

\[
D_{i,j} = \frac{100}{(1+r_{i,j})^n_i}
\]

where \( j = 1, \ldots, 361 \) and \( n_1 = \frac{1}{12}, n_2 = \frac{3}{12}, \ldots, n_8 = 10 \).

Then, the monthly return on bond \( B_i \) at time \( j \), \( q_{ij} \), is calculated as the sum of the percentage change in price and a twelfth of the YTM:

\[
q_{ij} = \frac{D_{i,j+1} - D_{i,j}}{D_{i,j}} + \frac{r_{i,j}}{12}
\]

where \( j = 1, \ldots, 360 \).

A twelfth of the yield is added because when calculating the price change, the original price, \( D_{i,j} \), is compared to the price, \( D_{i,j+1} \), of a bond next month with the same term to maturity, whereas the correct method would be to compare with the price of a bond with one month less to maturity. This is because when \( B_i \) is sold at time \( j + 1 \), it has a term to maturity which is one month shorter compared to when it was bought. The price change calculated will therefore be smaller than it should be and adding a twelfth of the yield compensates for this. Plotting the resulting returns \( q_{ij} \) in one histogram for each bond \( B_i \) in Figure 8 on p. 35 shows that the returns are approximately normally distributed, which is why approach 2 of calculating the returns will be used henceforth.

The variances and covariances of the returns on the 8 bonds are now estimated as the sample variances and covariances of the historical returns. Let \( \sigma_{ii} \) the variance of the return on bond \( B_i \) and \( \sigma_{ik} \) be the covariance of bond \( B_i \) and bond \( B_j \).

\[
\sigma_{ii} = \frac{\sum_{j=1}^{360} (q_{ij} - \overline{q_i})^2}{360-1} \quad \text{for } i = 1, \ldots, 8
\]

\[
\sigma_{ik} = \frac{\sum_{j=1}^{360} (q_{ij} - \overline{q_i})(q_{kj} - \overline{q_k})}{360-1} \quad \text{for } i = 1, \ldots, 8, k = 1, \ldots, 8 \text{ and } i \neq k
\]

\[
\overline{q_i} = \frac{\sum_{j=1}^{360} q_{ij}}{360}.
\]

An estimate of the covariance matrix of the returns is then calculated as

\[
\Sigma = \begin{bmatrix}
\sigma_{11} & \sigma_{12} & \cdots & \sigma_{18} \\
\sigma_{21} & \sigma_{22} & \cdots & \sigma_{28} \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_{81} & \sigma_{82} & \cdots & \sigma_{88}
\end{bmatrix}
\]

### 4.2 Optimization with the Markowitz model

To see how the B-L model affects the allocation between the 8 bonds, the Markowitz model is used as a comparison. At first, a simple portfolio where short positions are not allowed is considered. The optimization problem is formulated as:
minimize \( x \) \(- (x^T \mu - \frac{\delta}{2} x^T \Sigma x) \)
subject to \( e^T x = 1 \)
\( x \geq 0 \)

(8)

with \( x = [x_1, x_2, \ldots, x_8]^T \), where \( x_1, x_2, \ldots, x_8 \) are the weights (or proportions) of \( B_1, B_2, \ldots, B_8 \) respectively, \( \mu = [\mu_1, \mu_2, \ldots, \mu_8]^T \), where \( \mu_1, \mu_2, \ldots, \mu_8 \) are the expected returns on \( B_1, B_2, \ldots, B_8 \) respectively, \( \delta \) is the risk aversion parameter set to 2.5 \([7]\), \( \Sigma \) is the covariance matrix calculated above using the historical data and \( e = [1, 1, \ldots, 1]^T \). The constraint \( e^T x = 1 \) means that the sum of all weights equal 1 and the constraint \( x \geq 0 \) means that there can be no negative weights, that is, short selling is not allowed.

Using CVX, a package for specifying and solving convex programs, the optimization problem is solved for the following two vectors of expected returns:

\[
\mu_A = \frac{1}{100} \begin{bmatrix} 0.0270 & 0.0270 & 0.0270 & 0.0270 & 0.0270 & 0.0270 & 0.0443 & 0.0765 \end{bmatrix}^T, \quad (9)
\]

and

\[
\mu_B = \frac{1}{100} \begin{bmatrix} -0.0535 & -0.0550 & -0.0550 & 0 & -0.0253 & 0.0270 & 0.0443 & 0.0765 \end{bmatrix}^T. \quad (10)
\]

The elements in the vectors are given in percent and therefore divided by 100. Since the covariance matrix is estimated using the historical monthly returns on the bonds, the expected returns are given per month as well. For the results, please refer to the Result section.

4.3 Optimization with the Black-Litterman model

4.3.1 Short selling not allowed

Proceeding with the B-L model, the optimization problem is still:

minimize \( x \) \(- (x^T \mu - \frac{\delta}{2} x^T \Sigma x) \)
subject to \( e^T x = 1 \)
\( x \geq 0 \)

28
Table 1: The market’s expected monthly returns

<table>
<thead>
<tr>
<th>Bond</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>B5</th>
<th>B6</th>
<th>B7</th>
<th>B8</th>
</tr>
</thead>
<tbody>
<tr>
<td>YTM</td>
<td>1 month</td>
<td>3 months</td>
<td>6 months</td>
<td>1 year</td>
<td>2 years</td>
<td>5 years</td>
<td>7 years</td>
<td>10 years</td>
</tr>
<tr>
<td>Return (%)</td>
<td>0.0270</td>
<td>0.0270</td>
<td>0.0270</td>
<td>0.0270</td>
<td>0.0270</td>
<td>0.0270</td>
<td>0.0443</td>
<td>0.0765</td>
</tr>
</tbody>
</table>

Table 2: Investor views 1

<table>
<thead>
<tr>
<th>Bond</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>B5</th>
</tr>
</thead>
<tbody>
<tr>
<td>YTM</td>
<td>1 month</td>
<td>3 months</td>
<td>6 months</td>
<td>1 year</td>
<td>2 years</td>
</tr>
<tr>
<td>Return (%)</td>
<td>-0.0535</td>
<td>-0.0550</td>
<td>-0.0550</td>
<td>0</td>
<td>-0.0253</td>
</tr>
</tbody>
</table>

where \( x, \mu, \delta, \Sigma \) and \( e \) are defined as above. The only difference is that \( \mu \) is now the B-L expected return vector, calculated according to formula (7):

\[
\mu = E[R] = [(\tau \Sigma)^{-1} + P' \Omega^{-1} P]^{-1}[(\tau \Sigma)^{-1} \Pi + P' \Omega^{-1} Q]
\]

where \( \Pi \) is the market’s expected returns. Since the expected returns estimated by the market are given by the bonds’ current yields to maturity, \( \Pi \) is given and need not be calculated as in the case when the assets in the portfolio to be optimized are stocks. \( \tau \) is the weight-on-views, set to 0.01.

The market’s expected returns are given by table 1. This yields the \( \Pi \) - vector:

\[
\Pi = \frac{1}{100} [0.0270 \ 0.0270 \ 0.0270 \ 0.0270 \ 0.0270 \ 0.0443 \ 0.0765]^T.
\]

Notice that \( \Pi = \mu_A \).

First, the problem is solved for the absolute investor views on the monthly returns given by table 2.

The investor thus has the same view as the market on the expected returns on the bonds with 5-, 7- and 10 years to maturity. Notice that investor views 1 corresponds to \( \mu_B \) above.

This yields the following P-matrix and Q-vector:

\[
P_1 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\]

and

29
Table 3: Investor views 2

<table>
<thead>
<tr>
<th>Bond</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>B5</th>
</tr>
</thead>
<tbody>
<tr>
<td>YTM</td>
<td>1 month</td>
<td>3 months</td>
<td>6 months</td>
<td>1 year</td>
<td>2 years</td>
</tr>
<tr>
<td>Return (%)</td>
<td>-0.0120</td>
<td>-0.0121</td>
<td>-0.0087</td>
<td>-0.0078</td>
<td>0.0149</td>
</tr>
</tbody>
</table>

\[
Q_1 = \frac{1}{100} \begin{bmatrix} -0.0535 & -0.0550 & -0.0550 & 0 & -0.0253 \end{bmatrix}^T
\]  

(12)

Notice that in each row, the P-matrix has a 1 in the column that corresponds to the bond on which the investor has an absolute view, and the entries in the Q-vector correspond to the expected returns estimated by the investor.

The matrix \( \Omega_1 \) is then calculated using the formula:

\[
\Omega = \begin{bmatrix}
(p_1 \Sigma p_1^T)\tau & 0 & 0 \\
0 & \ddots & 0 \\
0 & 0 & (p_k \Sigma p_k^T)\tau
\end{bmatrix}
\]

where \( \tau = 0.01 \). The B-L expected return vector, \( \mu_{v_1} \), is then calculated using formula (7), and the optimization problem is solved as before. For the results, please refer to section 5.

In the same way, the problem is also solved for a second set of absolute investor views on the monthly returns given by table 3. This yields \( P_2 = P_1, \Omega_2 = \Omega_1 \) and

\[
Q_2 = \frac{1}{100} \begin{bmatrix} -0.0120 & -0.0121 & -0.0087 & -0.0078 & 0.0149 \end{bmatrix}^T
\]  

(13)

The B-L expected return vector, \( \mu_{v_2} \), is then calculated using formula (7), and the optimization problem is solved as before. For the results, please refer to section 5.

4.3.2 Short selling allowed

Now, assume that short selling is allowed, which means that there can be negative components in the x-vector. Let \( x = [x_1 \ x_2 \ \ldots \ x_8]^T \) where \( x_i \) is the difference between the weight of the asset and the weight of the liability of bond \( B_i \). A positive x-component means that the weight of the asset is larger than the weight of the liability. Allowing short selling is more realistic since in this way, both the bank’s assets and liabilities can be modelled. When short selling is not allowed, only the bank’s assets are considered.
Table 4: The given weights of the assets

<table>
<thead>
<tr>
<th>Bond</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>B5</th>
<th>B6</th>
<th>B7</th>
<th>B8</th>
</tr>
</thead>
<tbody>
<tr>
<td>YTM</td>
<td>1 month</td>
<td>3 months</td>
<td>6 months</td>
<td>1 year</td>
<td>2 years</td>
<td>5 years</td>
<td>7 years</td>
<td>10 years</td>
</tr>
<tr>
<td>Weight</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The bank’s assets, consisting of loans issued by the bank, are often predetermined, and only
the liabilities can be optimized. Therefore, it is now assumed that the weights of the assets
are given by

\[ a = [a_1 \ a_2 \ \ldots \ a_8]^T \]

where \( a_i \) is the weight of the asset of bond \( B_i \). It is also assumed that \( x_i, \ i = 1, \ldots, 8 \), cannot be greater than \( a_i \). The x-vector is optimized
and the optimal weights of the liabilities are then calculated as \( a - x \). The weights of the
assets are assumed to be given by table 4.

That is,

\[ a = [0.1 \ 0.3 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1]^T. \]

The problem is defined as:

\[
\begin{align*}
\text{minimize} \quad & \quad - (x^T \mu - \frac{\delta}{2} x^T \Sigma x) \\
\text{subject to} \quad & \quad e^T x = 0 \\
& \quad x \leq a
\end{align*}
\]

(14)

The B-L expected return vector, \( \mu \), is calculated using formula (7). The constraint \( e^T x = 0 \)
means that the sum of the differences between the weights of the assets and the weights of
the liabilities equal zero, that is, total assets equal total liabilities. The market’s expected
returns are assumed to be the same as before, given by table 1. The resulting expected
return of the portfolio, \( x^T \mu \), is the net interest margin of the bank.

The problem is solved for Investor views 1 and Investor views 2 defined in table 2 and 3 respectively. For the results, please refer to section 5.

4.4 Optimization with regards to NSFR

As a last step, the NSFR is incorporated in the optimization problem. As available stable
funding, the 1-month-bond, 3-month-bond and 6-month-bond \( (B_1, B_2 \) and \( B_3) \) belong to
the category “Funding with residual maturity of less than one year from sovereigns, public
sector entities (PSEs), an multilateral and national development banks” and as required
stable funding they belong to the category “All other assets not included in the above
categories with residual maturity of less than one year, including loans to non-bank financial
institutions, loans to non-financial corporate clients, loans to retail and small business
customers, and loans to sovereigns, central banks and PSEs”. Therefore, \( B_1, B_2 \) and \( B_3 \)
receive an ASF factor of 0.5 and a RSF factor of 0.5. The 1-year-bond, 2-year-bond, 5-year-
bond, 7-year-bond and 10-year-bond \( (B_4, B_5, B_6, B_7 \) and \( B_8) \) belong to the category “Other
Table 5: The ASF factors and RSF factors

<table>
<thead>
<tr>
<th>Bond</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>B5</th>
<th>B6</th>
<th>B7</th>
<th>B8</th>
</tr>
</thead>
<tbody>
<tr>
<td>YTM</td>
<td>1 month</td>
<td>3 months</td>
<td>6 months</td>
<td>1 year</td>
<td>2 years</td>
<td>5 years</td>
<td>7 years</td>
<td>10 years</td>
</tr>
<tr>
<td>ASF factor</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>RSF factor</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
</tr>
</tbody>
</table>

capital instruments and liabilities with effective residual maturity of one year or more” as available stable funding and to “Other unencumbered performing loans with risk weights greater than 35% under the Standardised Approach and residual maturities of one year or more, excluding loans to financial institutions” as required stable funding. Therefore, $B_4, B_5, B_6, B_7$ and $B_8$ receive an ASF factor of 1 and a RSF factor of 0.85. The factors are summarized in the table.

The constraint

$$NSFR = \frac{\text{Available amount of stable funding}}{\text{Required amount of stable funding}} \geq 100\%$$

is equivalent to \(\text{Available amount of stable funding} \geq \text{Required amount of stable funding}\).

The available stable funding consists of the liabilities and the required stable funding consists of the assets. The NSFR constraint is equivalent to

$$0.5(a_1 - x_1 + a_2 - x_2 + a_3 - x_3) + (a_4 - x_4 + a_5 - x_5 + a_6 - x_6 + a_7 - x_7 + a_8 - x_8) \geq 0.5(a_1 + a_2 + a_3) + 0.85(a_4 + a_5 + a_6 + a_7 + a_8)$$

which reduces to

$$-0.5(x_1 + x_2 + x_3) - (x_4 + x_5 + x_6 + x_7 + x_8) + 0.15(a_4 + a_5 + a_6 + a_7 + a_8) \geq 0.$$

Assuming that the given weights of the assets are $a = [0.1 \ 0.3 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1 \ 0.1]^T$ as above, the constraint is

$$-0.5(x_1 + x_2 + x_3) - (x_4 + x_5 + x_6 + x_7 + x_8) + 0.15(0.1 + 0.1 + 0.1 + 0.1 + 0.1) \geq 0$$

or simply

$$0.5(x_1 + x_2 + x_3) + x_4 + x_5 + x_6 + x_7 + x_8 \leq 0.075$$

The problem is defined as:
minimize \[ x^T \mu - \frac{\delta}{2} x^T \Sigma x \]
subject to \[ e^T x = 0 \]
\[ x \leq a \]
\[ 0.5(x_1 + x_2 + x_3) + x_4 + x_5 + x_6 + x_7 + x_8 \leq 0.075 \]

where \( x = [x_1 \ x_2 \ \ldots \ x_8]^T \), where \( x_i \) is the difference between the weight of the asset and the weight of the liability of bond \( B_i \).

The B-L expected return vector, \( \mu \), is calculated from (7). The market’s expected returns are assumed to be the same as before, given by table 1. The problem is solved for Investor views 1 and Investor views 2 defined in table 2 and 3 respectively. For the results, please refer to section 5.
5 Results

5.1 Data processing

5.1.1 Approach 1 – assuming that the bonds are held to maturity

Using approach 1 described in the methodology with the time horizon 1 year, the histograms obtained are shown in Figure 7. Apparently, when the bonds are held to maturity, the resulting returns cannot be approximated with a normal distribution.

![Histograms of the returns when using approach 1.](image)

Figure 7: Histograms of the returns when using approach 1.

5.1.2 Approach 2 – assuming that the bonds are held for trading

Using approach 2, where the bonds are held for trading and the time horizon is 1 month, the histograms obtained are shown in Figure 8. When the bonds can be bought and sold at any time, the resulting returns can be approximated with a normal distribution.

![Histograms of the returns when using approach 2.](image)
The covariance matrix calculated from the returns is given by (16).

\[
\Sigma = 10^{-3} \begin{bmatrix}
0.0287 & 0.0155 & 0.0166 & 0.0190 & 0.0683 & 0.0305 & 0.0485 & 0.0571 \\
0.0155 & 0.0134 & 0.0134 & 0.0140 & 0.0202 & 0.0155 & 0.0177 & 0.0189 \\
0.0166 & 0.0134 & 0.0137 & 0.0146 & 0.0268 & 0.0175 & 0.0220 & 0.0242 \\
0.0190 & 0.0140 & 0.0146 & 0.0162 & 0.0393 & 0.0216 & 0.0300 & 0.0341 \\
0.0683 & 0.0202 & 0.0268 & 0.0393 & 0.7593 & 0.1419 & 0.3877 & 0.5347 \\
0.0305 & 0.0155 & 0.0175 & 0.0216 & 0.1419 & 0.0519 & 0.0926 & 0.1135 \\
0.0485 & 0.0177 & 0.0220 & 0.0300 & 0.3877 & 0.0926 & 0.2274 & 0.2946 \\
0.0571 & 0.0189 & 0.0242 & 0.0341 & 0.5347 & 0.1135 & 0.2946 & 0.4026 \\
\end{bmatrix}
\] (16)

5.2 Optimization with the Markowitz model

With \( \mu_A \) given by (9), \( \delta = 2.5 \), \( \Sigma \) given by (16) and \( e = [1 \ 1 \ \ldots \ 1]^T \), the solution to problem (8) is \( x = [0.7386 \ 0.0001 \ 0.0000 \ 0.0000 \ 0.0000 \ 0.0000 \ 0.0000 \ 0.2612]^T \).
See Figure 9.

Figure 9: The weights in the optimal portfolio using the Markowitz model with expected returns given by $\mu_A$.

If $\mu$ is changed to $\mu_B$, given by (10), the solution to (8) is

$$x = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.5910 & 0.0000 & 0.0000 & 0.0000 & 0.4090 \end{bmatrix}^T.$$  See Figure 10.

### 5.3 Optimization with the Black-Litterman model

#### 5.3.1 Short selling not allowed

With the first set of investor views, given by table 2, $P_1$ and $Q_1$ are given by (11) and (12) respectively, and $\Omega_1$ is given by

$$\Omega_1 = 10^{-6} \begin{bmatrix} 0.1345 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1374 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1621 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.2870 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.5191 \end{bmatrix}$$  (17)

Furthermore, the results are
Figure 10: The weights in the optimal portfolio using the Markowitz model with expected returns given by $\mu_B$.

\[
\mu_{v_1} = 10^{-3} \begin{bmatrix} -0.2793 & -0.2813 & -0.3035 & -0.3010 & -0.3145 & -0.3372 & -0.1767 & 0.1441 \end{bmatrix}^T
\]
\[
x = \begin{bmatrix} 0.7777 & 0.0002 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.2221 \end{bmatrix}^T
\]

The total expected return of the portfolio = $-1.8530 \times 10^{-4} = -0.018530\%$.

The volatility of the portfolio = $\sqrt{x^T \Sigma x} = 0.0073 = 0.73\%$.

See Figure 11 for the graph of the portfolio weights.

With the second set of investor views, given by table 3 $\Omega_2 = \Omega_1$ since $P_2 = P_1$ $Q_2$ is given by (13).

Furthermore, the results are

\[
\mu_{v_2} = 10^{-3} \begin{bmatrix} -0.0013 & -0.0022 & -0.0137 & -0.0422 & -0.0020 & -0.0023 & 0.1728 & 0.5076 \end{bmatrix}^T
\]
\[
x = \begin{bmatrix} 0.7311 & 0.0001 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.2687 \end{bmatrix}^T
\]

The total expected return of the portfolio = $1.3545 \times 10^{-4} = 0.013545\%$.

The volatility of the portfolio = $\sqrt{x^T \Sigma x} = 0.0084 = 0.84\%$.

See Figure 12 for the graph of the portfolio weights.
5.3.2 Short selling allowed

With investor views 1, given by table 2, \( \Omega_1 \) is the same as above, given by (17). The result is:

\[
\mu_{v1} \text{ is the same as above.}
\]

\[
x = \begin{bmatrix} 0.1000 & 0.3000 & -0.3859 & 0.1000 & 0.1000 & -0.4140 & 0.1000 & 0.1000 \end{bmatrix}^T
\]

The given assets = \[
\begin{bmatrix} 0.1000 & 0.3000 & 0.1000 & 0.1000 & 0.1000 & 0.1000 & 0.1000 & 0.1000 \end{bmatrix}^T
\]

The liabilities = \[
\begin{bmatrix} 0.0000 & 0.0000 & 0.4859 & 0.0000 & 0.0000 & 0.5140 & 0.0000 & 0.0000 \end{bmatrix}^T
\]

The total expected return of the portfolio = the net interest margin = \( 7.9624 \times 10^{-5} = 0.0079624\% \).

The volatility of the portfolio = \( \sqrt{x^T \Sigma x} = 0.0016 = 0.16\% \).

Figure 13 illustrates the portfolio weights.

With investor views 2, given by table 3, \( \Omega_2 \) is the same as above, given by \( \Omega_2 = \Omega_1 \) in (17).
The result is:

\( \mu_{v_2} \) is the same as above.

\[ x = \begin{bmatrix} 0.1000 & 0.3000 & 0.1000 & -0.5922 & 0.1000 & -0.2077 & 0.1000 & 0.1000 \end{bmatrix}^T \]

The given assets = \[ \begin{bmatrix} 0.1000 & 0.3000 & 0.1000 & 0.1000 & 0.1000 & 0.1000 & 0.1000 \end{bmatrix}^T \]

The liabilities = \[ \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.6922 & 0.0000 & 0.3077 & 0.0000 & 0.0000 \end{bmatrix}^T \]

The total expected return of the portfolio = the net interest margin = \( 9.1172 \times 10^{-5} = 0.0091172\% \).

The volatility of the portfolio = \( \sqrt{x^T \Sigma x} = 0.0022 = 0.22\% \).

Figure 14 illustrates the portfolio weights.

### 5.4 Optimization with regards to NSFR

With investor views 1, given by table 2 \( \Omega_1 \) is the same as above, given by \[ 17 \].

The result is:
Figure 13: The blue bars represent the weight of the assets and the yellow bars represent the weight of the liabilities in the optimal portfolio using investor views 1.

\( \mu_{v1} \) is the same as above.

\[
x = \begin{bmatrix} 0.1000 & 0.3000 & -0.3859 & 0.1000 & 0.0999 & -0.4140 & 0.1000 & 0.1000 \end{bmatrix}^T
\]

The given assets = \[ \begin{bmatrix} 0.1000 & 0.3000 & 0.1000 & 0.1000 & 0.1000 & 0.1000 & 0.1000 \end{bmatrix}^T \]

The liabilities = \[ \begin{bmatrix} 0.0000 & 0.0000 & 0.4859 & 0.0000 & 0.0001 & 0.5140 & 0.0000 & 0.0000 \end{bmatrix}^T \]

NSFR = 1.1216

The total expected return of the portfolio = the net interest margin = \( 7.9624 \times 10^{-5} = 0.0079624\% \).

The volatility of the portfolio = \( \sqrt{x^T \Sigma x} = 0.0016 = 0.16\% \).

Figure 15 illustrates the portfolio weights.

With investor views 2, given by table 3, \( \Omega_2 \) is the same as above, given by \( \Omega_2 = \Omega_1 \) in (17).

The result is:

\( \mu_{v2} \) is the same as above.
Figure 14: The blue bars represent the weight of the assets and the yellow bars represent the weight of the liabilities in the optimal portfolio using investor views 2.

\[ x = \begin{bmatrix} 0.1000 & 0.3000 & 0.1000 & -0.5922 & 0.1000 & -0.2077 & 0.1000 & 0.1000 \end{bmatrix}^T \]

The given assets = \( \begin{bmatrix} 0.1000 & 0.3000 & 0.1000 & 0.1000 & 0.1000 & 0.1000 & 0.1000 \end{bmatrix}^T \)

The liabilities = \( \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.6922 & 0.0000 & 0.3077 & 0.0000 & 0.0000 \end{bmatrix}^T \)

NSFR = 1.4815

The total expected return of the portfolio = the net interest margin = \( 9.1172 \times 10^{-5} = 0.0091172\% \).

The volatility of the portfolio = \( \sqrt{x^T \Sigma x} = 0.0022 = 0.22\% \).

Figure 16 illustrates the portfolio weights.
Figure 15: The blue bars represent the weight of the assets and the yellow bars represent the weight of the liabilities in the optimal portfolio using investor views 1 and optimizing with regards to NSFR.
Figure 16: The blue bars represent the weight of the assets and the yellow bars represent the weight of the liabilities in the optimal portfolio using investor views 2 and optimizing with regards to NSFR.
6 Discussion

6.1 Interpretation of the results

Analyzing the results, it is apparent that when short selling is not allowed, the investor views do not affect the optimal portfolio construction essentially. This can be seen by noting that almost the same portfolio construction is obtained when optimizing using the Markowitz model with the expected returns $\mu_A$ and when optimizing using the B-L model with investor views 1 and investor views 2 and with the market’s expected returns, $\Pi$, equal to $\mu_A$. When optimizing with the Markowitz model with $\mu_A$ as the expected returns, the optimal portfolio has approximately 74% of the capital invested in $B_1$ and 26% invested in $B_8$. When optimizing with the B-L model with investor views 1, approximately 78% of the capital is invested in $B_1$ and 22% in $B_8$. With investor views 2, approximately 73% of the capital is invested in $B_1$ and 27% in $B_8$. The B-L optimal portfolio is a weighted combination of the investor views and the market portfolio [12]. The weight of the investor views on the B-L optimal portfolio depends on the weight-on-views, $\tau$, and the level-of-unconfidence, represented by $\Omega$. The stronger the confidence assigned to the views and to the weight-on-views, the larger the deviance between the B-L optimal portfolio and the market portfolio [12]. One possible explanation of the result is thus that $\tau$ and $\Omega$ are such that the confidence assigned is too small, making the B-L optimal portfolio almost equal to the market portfolio.

Proceeding to the results when optimizing using the B-L model with investor views 1 and investor views 2 when short selling is allowed, it is observed that the views do make a difference in the B-L optimal portfolio. This is seen by noting that, in contrast with the case when short selling was not allowed, the optimal portfolio when using investor views 1 and investor views 2 are different. When investor views 1 is used, short positions are taken in $B_3$ and $B_6$, and when using investor views 2, short positions are taken in $B_4$ and $B_6$.

Analysing the results when optimizing with regards to NSFR, it is noted that when NSFR is added to the constraints, the optimal portfolio is unchanged for both investor views 1 and investor views 2. This means that the optimal portfolios before adding the NSFR-constraint already satisfy

$$\frac{\text{Available amount of stable funding}}{\text{Required amount of stable funding}} \geq 100\%$$

and therefore the constraint does not affect the optimal portfolio construction. This is probably due to the fact that in the optimal portfolios for both investor views 1 and 2, short positions are taken in bonds with a relatively long term to maturity: 6 months and 5
years and 1 year and 5 years respectively. This makes the NSFR large, since bonds with a long term to maturity have large ASF-factors. It is likely that the NSFR-constraint would have made an impact on the portfolio constructions if the optimal portfolios before adding the constraint had short positions in bonds with short terms to maturity.

6.2 The impact of the input data

The covariance matrix $\Sigma$

As mentioned in previous parts of the report, volatility and correlation are inputs to the model and need to be estimated. There are different methods for estimating volatility and correlation (i.e. the covariance matrix) and the choice of method gives rise to different covariance matrices. Thus, the choice of estimation method is critical and a well-estimated covariance matrix can lead to more efficient portfolio allocations [10]. For example, in the study, the inputs are estimated based on historical observations on bond returns, and another approach might have given rise to a different covariance matrix and in turn a different allocation. Even after the choice of method is made, additional important considerations must be taken, one such being how long a period of historical data to use. In general, there appears to be a consensus that both too long and too short data periods can increase the risk of affecting the covariance matrix negatively. Makarov [11] argues that it is possible to have too much historical data if the market’s volatility varies significantly over time. Data from the past may not reflect the current market conditions, thus reducing accuracy of the covariance matrix. To this point, Alexander [1] argues that since the interest rates have decreased radically over the last decades, from over 15% in the 1980’s to below 3% in the 20th century, it is not desirable to include the entire range when making estimation. However, when making estimates for a portfolio with a relatively long future holding period, it may be appropriate to include old data that contains “economic shocks” in the market, since there is a chance for “economic shocks” to occur again in the future. Excessively limiting the range of historical data may not produce reliable results because of the short period of observation.

Litterman and Winkelmann argues for another approach, which retains a relatively large range of data but, where more weight is given to observations that have occurred recently and less weight to observations that have occurred in the past, i.e. weighting by a declining function of time [10]. According to Litterman and Winkelmann volatilities and correlations tend to vary over time and the older the returns are, the less relevant they are for revealing what the covariance is today, and therefore this weighting approach is favoured.

To summarize, how long a period of historical data to use is a critical question. This study is based on the returns of bonds from 1988 onwards, and it is evident that opinions of whether this is an optimal choice of time period may vary. As outlined by Litterman and
Winkelmann, there is no general optimal approach and no optimally estimated covariance matrix, but the selected approach should make as much sense as possible for the task at hand. There is a possibility that a different data period or a more advanced method for calculating the covariance matrix, for example by weighting with the declining function of time, would lead to a more accurate estimation of the covariance matrix. Thus, the estimated covariance matrix may have affected the results to a certain extent, but further research on the subject must be made before a conclusion can be reached.

The covariance matrix $\Omega$

Similar to the covariance matrix $\Sigma$, the covariance matrix $\Omega$, which again represents the uncertainty in the investor’s views, must be estimated. There is no single approach to estimate $\Omega$, and in the basic Black-Litterman model the choice of method is left to the investor. Over the years different methods for estimating $\Omega$ have been established, which are described in various research concerning the B-L model. Walters [16] has summarized the four most well-known methods; Proportional to the variance of the market equilibrium portfolio, Use a confidence interval, Use the variance of residuals in a factor model and Idzorek’s method to specify the confidence along the weight dimension. This study is based on the first of these four methods, i.e. proportional to the variance of the market equilibrium portfolio. The main reasons for the choice of this method for the study is that there has been no room for examining several methods, and that this method is said to be the most common choice, used by both He and Litterman and Meucci [16]. It should be noted that a different method for estimating the covariance matrix $\Omega$ may have given rise to another matrix, and thus may have impacted the result. As in the previous discussion about $\Sigma$, it is difficult to determine whether the said choice of method was the one generating the most accurate $\Omega$ without any further examination.

6.3 Method improvements and further research

As mentioned earlier in the result, optimizing with regards to NSFR does not affect the optimal portfolio. The NSFR is added as a constraint to the optimization setup and as long as the constraint is satisfied, in this case NSFR $\geq 1$, the actual size of the NSFR is of no significance. This means that NSFR could be both equal to, or very far from, the minimum requirement - none of which is desirable for the bank. Handelsbanken would likely want some margin on NSFR, however an excessively high margin would result in higher funding costs. A suggestion for method improvement is to incorporate the NSFR into the objective function so that the size of NSFR would also have an impact on the problem.

Several assumptions regarding the bonds have been made in the study. First, it is assumed
that the bonds have a fixed coupon rate and a fixed maturity date. This assumption is a simplification since the bank both lends and borrows through variable interest rate loans and loans with no fixed maturity date. Furthermore, it is assumed that there are only eight different terms to maturity, and that there is only one yield to maturity for each term to maturity, which is the same for a short and long position. The latter implies that a loan issued by the bank with a certain term to maturity has the same interest rate regardless of the creditworthiness of the borrower, and that the borrowing rate and the lending rate are equal. In reality, the lending rate is higher for a borrower with a low creditworthiness and the lending rate is always higher than the borrowing rate, which is how the bank makes a profit. A suggestion for method improvement is thus to include more than eight different terms to maturity, where the bonds can have either fixed or floating interest rates, and interest rate periods which may differ from the bond’s terms to maturity. Additional lengths of the terms to maturity can lead to the NSFR constraint having a greater impact on the problem. Furthermore, a way to improve the model would be to incorporate different credit ratings and to have a higher lending rate than borrowing rate. By expanding these simplifications on which the study is based, the model would reflect the reality more accurately.

Lastly, further research in the field of the Black-Litterman model in non-normal markets would be useful for Handelsbanken, as this reflects the bank’s reality in a more realistic way. This study is based on the basic Black-Litterman model, where one of the fundamental assumptions is that the returns can be approximated with a normal distribution. In order to meet this requirement, the assumption that the bonds are held for trading has been made. A more realistic approach would have been to assume that the bonds are held to maturity, but since this approach violates the fundamental assumption of the Black-Litterman model, the study has been simplified assuming that bonds are held for trading. There are extensions on the Black-Litterman model in non-normal markets [14], but these extensions are not within the scope of this study.
7 Conclusion

The aim of this thesis is to examine how the Black-Litterman model can be used for optimizing Handelsbanken’s net interest margin. The study has shown that bond returns cannot be approximated with a normal distribution when held to maturity. Since the assumption of normalized returns are a fundamental assumption of the Black-Litterman model, another approach to model the bank’s balance sheet has been necessary.

The thesis concludes that the bank’s net interest margin can be optimized with the Black-Litterman model if it is assumed that the bank’s balance sheet is a portfolio of bonds, and instead of being held to maturity, the bonds are held for trading. By allowing short selling and letting the optimized variable represent the difference between assets and liabilities, both assets and liabilities, i.e. the bank’s balance sheet, is modeled. Since the bank’s assets consist of loans issued by the bank, they are often predetermined and were therefore given. By optimizing the difference between the given assets and the unknown liabilities, the liabilities are indirectly optimized. In other words, given the bank’s assets, it is possible to use the Black-Litterman model to examine how Handelsbanken should finance its debt in order to optimize its net interest margin.

In addition, the aim is to examine how the model can be used to optimize the bank’s net interest margin taking into account the requirements of the Net stable funding ratio (NSFR). The net stable funding ratio is a liquidity measurement in the Basel III accords, initiated by The Basel Committee on Banking Supervision. The plan was to implement the NSFR by 1 January 2018, but the date has since been postponed and the timing of the implementation is currently not clear. The conclusion is that the NSFR requirement can be incorporated into the model as a constraint, where each liability and each asset is weighted according to the weights in Figure 2 and 3 where the weights are based on the different risk profiles of assets and liabilities.
8 Bibliography
