

Effective Neutrino Mixing and Oscillations in Dense Matter

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Abstract

We investigate the effective case of two-flavor neutrino oscillations in infinitely dense matter by using a perturbative approach. We begin by briefly summarizing the conditions for the three-flavor neutrino oscillation probabilities to take on the same form as the corresponding two-flavor probabilities. Then, we proceed with the infinitely dense matter calculations. Finally, we study the validity of the approximation of infinitely dense matter when the effective matter potential is large, but not infinite, this is done by using both analytic and numeric methods.

Key words:

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1 Introduction

The large experimental collaborations [1–10] in neutrino oscillation physics have successfully used effective two-flavor neutrino oscillation formulas in their analyzes in order to determine the fundamental neutrino oscillation parameters. However, we now know that there exists (at least) three active neutrino flavors in Nature, which, in principle, means that the analyzes have to be carried out using three-flavor neutrino oscillation formulas. Thus, it is important to carefully examine the effective two-flavor formulas in order to determine their validity in different situations, especially since neutrino oscillation physics has now entered the era of precision measurements. For example, the

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Super-Kamiokande [1–3], SNO [4–6], K2K [7], and KamLAND [8–10] collaborations have significantly pinned down the errors on the fundamental parameters and future results of the above mentioned experiments as well as other long-baseline experiments will continue to decrease the uncertainties of these parameters.

Furthermore, it is known that the presence of matter can result in large alterations in the behavior of neutrino oscillations [11]. An example of this is the Mikheyev–Smirnov–Wolfenstein (MSW) effect [11, 12], which is the most plausible description of the oscillations of solar electron neutrinos into neutrinos of different flavors. The matter effects are proportional to the energy of the neutrinos as well as the matter density. Thus, for large neutrino energy and dense matter, the matter effects become important. Note that, in Ref. [13], approximate mappings between the three-flavor neutrino parameters in vacuum and matter have been obtained. In addition, in Refs. [14–21], exact treatments of three-flavor neutrino oscillations in constant matter density have been performed, whereas, in Refs. [13, 22–34], different analytic approximations of three-flavor neutrino oscillation probability formulas have been investigated for both constant and varying matter density.

In this Letter, we study the matter effects on neutrino mixing and oscillations in the limit where the matter density becomes infinite and the resulting neutrino oscillation probabilities are actual two-flavor formulas. We also investigate how well these two-flavor formulas reproduce the actual neutrino oscillation probabilities when neutrinos propagate through matter with large, but not infinite, matter density.

2 Effective Two-Flavor Formulas

With three neutrino flavors, there are six fundamental parameters which influence neutrino oscillations, two mass squared differences (Δm_{21}^2 and Δm_{31}^2), three mixing angles (θ_{12} , θ_{23} , and θ_{13}), and one CP -violating phase (δ). For the leptonic mixing matrix U , we adopt the standard parameterization given in Ref. [35]. In addition, we introduce the mass hierarchy parameter $\alpha \equiv \Delta m_{21}^2/\Delta m_{31}^2$ as well as the parameter $\Delta \equiv \Delta m_{31}^2/(2E)$, where E is the neutrino energy. We also use the abbreviations $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$, $s_\delta \equiv \sin \delta$, and $c_\delta \equiv \cos \delta$.

In some special cases, it is possible to write one or more of the neutrino oscillation probabilities as effective two-flavor formulas, *i.e.*, the probabilities

can be written as

$$P_{\alpha\beta} = \delta_{\alpha\beta} + (1 - 2\delta_{\alpha\beta}) \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2}{4E}L\right), \quad (1)$$

where $P_{\alpha\beta}$ is the probability of the transition $\nu_\alpha \rightarrow \nu_\beta$, $\sin^2(2\theta)$ defines the oscillation amplitude, Δm^2 defines the oscillation frequency, and L is the length of the path travelled by the neutrinos. In the case of three-flavor neutrino oscillations in vacuum, the survival probability $P_{\alpha\alpha}$ is given by

$$P_{\alpha\alpha} = 1 - 4 \sum_{1 \leq i < j \leq 3} |U_{\alpha i}|^2 |U_{\alpha j}|^2 \sin^2\left(\frac{\Delta m_{ij}^2}{4E}L\right). \quad (2)$$

This probability takes the form of Eq. (1) if any of the elements $U_{\alpha i}$ of the leptonic mixing matrix or any of the mass squared differences equals zero. The same argument holds for the neutrino oscillation probability $P_{\alpha\beta}$. If one of the conditions is satisfied, then the leptonic mixing matrix can be made real and there is no CP -violation in neutrino oscillations (Majorana phases, which do not influence neutrino oscillations, could still be present if neutrinos are Majorana particles). In that case, the neutrino oscillation probability is given by

$$P_{\alpha\beta} = -4 \sum_{1 \leq i < j \leq 3} U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \sin^2\left(\frac{\Delta m_{ij}^2}{4E}L\right). \quad (3)$$

Note that if $U_{\alpha i} = 0$, then only the oscillations involving the neutrino flavor ν_α will become effective two-flavor cases.

The two-flavor cases mentioned above are realized if we make either of the approximations $\theta_{13} \rightarrow 0$ or $\alpha \rightarrow 0$, see Table 1 for amplitudes and frequencies. These approximations can be used when analyzing neutrino oscillation experiments neglecting matter effects, for example, the oscillations of electron anti-neutrinos of reactor experiments such as KamLAND [8–10] ($\theta_{13} \rightarrow 0$) and oscillations of atmospheric neutrinos in experiments such as Super-Kamiokande [1–3] ($\alpha \rightarrow 0$).

The above argument also holds for neutrinos propagating through matter of constant density if exchanging the fundamental neutrino parameters for their effective counterparts in matter. This gives us another important two-flavor case, namely the limit of an effective matter potential approaching infinity, where all neutrino oscillation probabilities reduce to the two-flavor form of Eq. (1). The remainder of this Letter is devoted to study this limit in detail.

$P_{\alpha\beta}$	$\sin^2(2\theta)$	Δm^2
$\theta_{13} = 0$		
P_{ee}	$\sin^2(2\theta_{12})$	Δm_{21}^2
$P_{e\mu} = P_{\mu e}$	$c_{23}^2 \sin^2(2\theta_{12})$	Δm_{21}^2
$P_{e\tau} = P_{\tau e}$	$s_{23}^2 \sin^2(2\theta_{12})$	Δm_{21}^2
$\alpha = 0$		
$P_{e\mu}$	$s_{23}^2 \sin^2(2\theta_{13})$	$\Delta m_{31}^2 = \Delta m_{32}^2$
$P_{\mu\tau}$	$c_{13}^4 \sin^2(2\theta_{23})$	$\Delta m_{31}^2 = \Delta m_{32}^2$

Table 1

The neutrino oscillation amplitudes and frequencies for different two-flavor formulas in vacuum. In the $\alpha = 0$ case, all channels are two-flavor channels and the amplitudes and frequencies can be deduced from the channels in this table (see Ref. [34]).

3 Mixing in Dense Matter

In matter, the Hamiltonian of three flavor neutrino evolution consists of two parts, the kinematic term $H_k = \Delta U \text{diag}(0, \alpha, 1) U^\dagger$ and the interaction term $H_m = \text{diag}(V, 0, 0)$, where $V = \sqrt{2} G_F n_e$, G_F is the Fermi coupling constant, and n_e is the electron number density, resulting from coherent forward-scattering of neutrinos. When $2EV \gg \Delta m_{31}^2$, the term H_k can be thought of as a perturbation to the term H_m . Since H_m has two degenerate eigenvectors, we can use degenerate perturbation theory to find approximate eigenvectors and eigenvalues of the total Hamiltonian. The degenerate sector of H_m is spanned by $\nu_\mu = (0, 1, 0)^T$ and $\nu_\tau = (0, 0, 1)^T$. In this sector, the Hamiltonian is of the form

$$H_d = \begin{pmatrix} a & b^* \\ b & d \end{pmatrix}, \quad (4)$$

where a , b , and d depend on the fundamental neutrino parameters as well as the effective matter potential V and the neutrino energy E .

In the case of two-flavor neutrino oscillations in vacuum, the Hamiltonian takes the form

$$H = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{pmatrix}, \quad (5)$$

where Δm^2 and θ are the mass squared difference and mixing angle in the two-flavor scenario, respectively. Defining θ_∞ as the effective mixing angle in the degenerate sector of H_m and Δm_∞^2 as the effective mass squared difference,

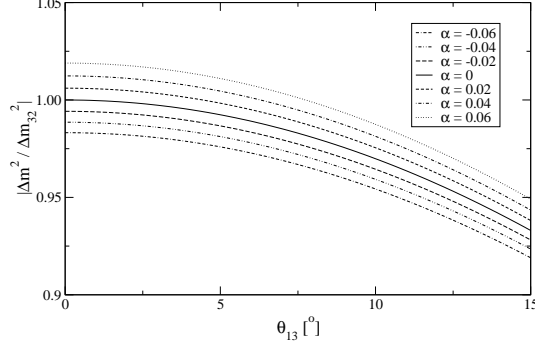


Fig. 1. The quotient $|\Delta m_\infty^2/\Delta m_{32}^2|$ as a function of the leptonic mixing angle θ_{13} for different values of the mass hierarchy parameter α . In this plot, the mixing angle θ_{12} has been set to 33° .

by comparing Eqs. (4) and (5), we obtain

$$\begin{aligned} \sin^2(2\theta_\infty) &= \frac{4|b|^2}{(a-d)^2 + 4|b|^2} \\ &= 4 \frac{\{s_{23}c_{23}[c_{13}^2 - \alpha(c_{12}^2 - s_{12}^2s_{13}^2)] - \alpha s_{12}c_{12}s_{13}c_\delta \cos(2\theta_{23})\}^2}{[c_{13}^2 - \alpha(c_{12}^2 - s_{13}^2s_{12}^2)]^2 + 4\alpha c_{12}^2s_{12}^2s_{13}^2} \\ &\quad + 4 \frac{\alpha^2 s_\delta^2 s_{12}^2 c_{12}^2 s_{13}^2}{[c_{13}^2 - \alpha(c_{12}^2 - s_{13}^2s_{12}^2)]^2 + 4\alpha c_{12}^2s_{12}^2s_{13}^2}, \end{aligned} \quad (6)$$

$$\begin{aligned} \Delta m_\infty^2 &= 2E\sqrt{(a-d)^2 + 4|b|^2} \\ &= |\Delta m_{31}^2| \sqrt{c_{13}^4 - 2\alpha c_{13}^2(c_{12}^2 - s_{13}^2s_{12}^2) + \alpha^2(1 - c_{13}^2s_{12}^2)^2} \\ &= |\Delta m_{31}^2| \sqrt{[c_{13}^2 - \alpha(c_{12}^2 - s_{13}^2s_{12}^2)]^2 + 4\alpha c_{12}^2s_{12}^2s_{13}^2}. \end{aligned} \quad (7)$$

We note that even for $\theta_{13} = 0$ or $\alpha = 0$, the effective two flavor parameters Δm_∞^2 and θ_∞ are not equal to Δm_{32}^2 and θ_{23} , which might have been expected. However, for reasonable values of the fundamental neutrino parameters, θ_∞ does not deviate significantly from θ_{23} . In Fig. 1, the quotient $|\Delta m_\infty^2/\Delta m_{32}^2|$ is plotted as a function of θ_{13} for different α . From this figure, we conclude that the sign of the correction to the approximation $\Delta m_\infty^2 = |\Delta m_{32}^2|$ depends on the values of the parameters θ_{13} and α . It should be noted that if the absolute value of the fundamental parameter Δm_{32}^2 and the effective parameter Δm_∞^2 were to be measured (*i.e.*, Δm_∞^2 determined in an experiment where the matter is extremely dense and $|\Delta m_{32}^2|$ is determined by vacuum or close to vacuum experiments), then the quotient $|\Delta m_\infty^2/\Delta m_{32}^2|$ could provide valuable information on the mass hierarchy (*i.e.*, the sign of α) and the leptonic mixing angle θ_{13} . It is also interesting to observe that for $\alpha \geq 0$ (normal mass hierarchy), there exists a value of θ_{13} such that $\Delta m_\infty^2 = |\Delta m_{32}^2|$.

In Fig. 2, we plot the effective neutrino parameters in matter as a function of the product EV . For neutrinos at large values of EV , the effective mixing

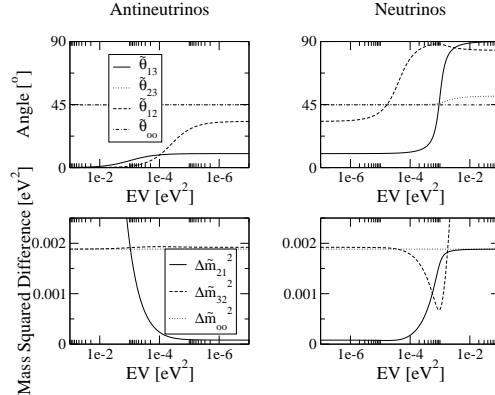


Fig. 2. Numerical results for the effective neutrino parameters in matter as a function of EV for a normal mass hierarchy with the vacuum parameters $\Delta m_{21}^2 = 8 \cdot 10^{-5} \text{ eV}^2$, $\Delta m_{31}^2 = 2 \cdot 10^{-3} \text{ eV}^2$, $\theta_{12} = 33^\circ$, $\theta_{23} = 45^\circ$, $\theta_{13} = 10^\circ$, and $\delta = 0$.

angles $\tilde{\theta}_{12}$ and $\tilde{\theta}_{23}$, where the tilde denotes that these are the effective matter parameters, do not play a very important role, since $\tilde{\theta}_{13} \rightarrow 90^\circ$ when $EV \rightarrow \infty$. Instead, in this case, the important mixing angle is the mixing between the first and second eigenstates, which for $\tilde{\theta}_{13} = 90^\circ$ becomes dependent on the effective mixing angles $\tilde{\theta}_{12}$ and $\tilde{\theta}_{23}$, as well as the effective phase $\tilde{\delta}$, this angle is clearly the angle θ_∞ .

As can be seen from Fig. 2, we may consider essentially three different regions for the parameter EV . The first region is $2EV \ll \Delta m_{21}^2$, where the fundamental neutrino parameters are essentially equal to the vacuum parameters, the second region is $2EV \gg \Delta m_{31}^2$, where ν_e is essentially an eigenstate to the Hamiltonian and there is two-flavor neutrino mixing between ν_μ and ν_τ . The last region is the region inbetween the other two, where resonance phenomena occur. In general, the results for an inverted mass hierarchy are similar to those of a normal mass hierarchy.

Observe that for atmospheric neutrinos passing through the Earth with a characteristic energy of $E \simeq 1 \text{ GeV}$, the product EV is typically in the upper part of the resonance region in which none of the effective mass squared differences, nor any of the effective mixing angles, are small. This effect, which is studied in, for example, Ref. [36] (for a recent study, see Ref. [37]), should be taken into account when analyzing data from atmospheric neutrino experiments.

4 Accuracy of the Two-Flavor Approximation

The accuracy of the two-flavor approximation in the limit $EV \rightarrow \infty$ is dependent on how close the leptonic matter mixing angle $\tilde{\theta}_{13}$ is to 90° , or in other

words, the value of \tilde{c}_{13} . The correction to $\nu_3 = \nu_e$ is given by

$$\nu_3 - \nu_e \simeq \frac{H_{e\mu}\nu_\mu + H_{e\tau}\nu_\tau}{V} \quad (8)$$

in first order non-degenerate perturbation theory (which can be used, since ν_e is a non-degenerate eigenstate of H_m), where $H_{e\alpha}$ is the e - α element of H_k . From this calculation follows that to first order in $\Delta m_{31}^2/(EV)$, the quantity \tilde{c}_{13} is given by

$$\tilde{c}_{13} \simeq \frac{\Delta m_{31}^2}{2EV} \sqrt{\frac{1}{4} \sin^2(2\theta_{13}) - 2\alpha s_{13}^2 c_{13}^2 s_{12}^2 + \alpha^2 c_{13}^2 s_{12}^2 (c_{12}^2 + c_{13}^2 s_{12}^2)}. \quad (9)$$

In Figs. 3 and 4, we plot the relative and absolute accuracies of the large matter density approximation for the probability P of a ν_μ to oscillate into some other flavor in the limit $EV \rightarrow \infty$ for two different values of L/E as a function of EV and θ_{13} . Since this oscillation probability is given by $P = 1 - P_{\mu\mu}$, the relative accuracy is given by $A_{\text{rel}} = A_{\text{abs}}/P = A_{\text{abs}}/(1 - P_{\mu\mu})$ where A_{abs} is the absolute accuracy. As for all relative accuracies, this definition clearly has a problem for $P = 0$. Since $P_{\mu\mu}$ is close to one in the regime studied in the plots, the relative accuracy for $P_{\mu\mu}$ is approximately the same as the absolute accuracy. The values used for L/E correspond roughly to values that might be used for future long-baseline experiments and neutrino factories. Note that these values are smaller than the oscillation length determined by Δm_{31}^2 .

The reason why the approximation becomes worse at large values of θ_{13} is the fact that if θ_{13} is large, then $\tilde{\theta}_{13}$ will approach 90° slowly when $EV \rightarrow \infty$ compared to when θ_{13} is small, see Eq. (9). Thus, larger values of EV will be required before the approximation $EV \rightarrow \infty$ becomes valid. For $EV \rightarrow \infty$, the region in which the two-flavor neutrino approximation $\Delta m^2 = \Delta m_{32}^2$ and $\theta = \theta_{23}$ is accurate is the region in which θ_{13} obtains the value for which $\Delta m_\infty \simeq |\Delta m_{32}^2|$. This is to be expected, since $\theta_\infty \simeq \theta_{23}$. When studying Figs. 3 and 4, one should keep in mind that the ν_μ survival probability $P_{\mu\mu}$ is larger for $L/E = 7000 \text{ km} / 50 \text{ GeV}$ than for $L/E = 3000 \text{ km} / 50 \text{ GeV}$ and that we have plotted the relative accuracy of the approximation.

The reason why the two-flavor neutrino approximation $EV \rightarrow \infty$ seems to be a good approximation even for small values of EV when θ_{13} is small has to do with the fact that for small values of L/E and $\theta_{13} = 0$, this approximation gives an error which is of the order α^2 to the $(L/E)^2$ dependence of the probability $P_{\mu\mu}$ when making a series expansion in the quantity L/E .

Presently, the number of experiments in which the approximation of $EV \rightarrow \infty$ can be justified is somewhat limited. One such example could be a neutrino factory setup with a ‘‘magic’’ baseline ($L \simeq 7250 \text{ km}$) [38] with a neutrino energy of about 50 GeV. For this energy, the value of $\log(EV/\text{eV}^2)$ typically

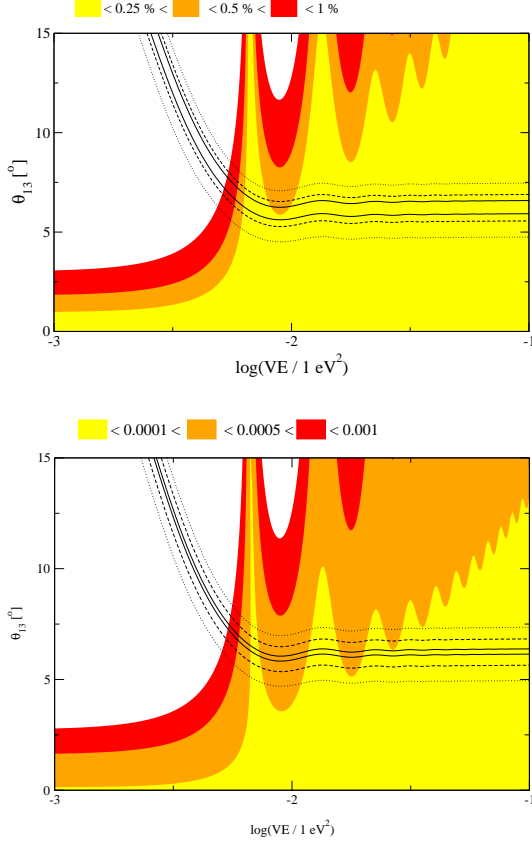


Fig. 3. The relative (upper panel) and absolute (lower panel) accuracy of using the approximation $EV \rightarrow \infty$ in the neutrino oscillation channel $1 - P_{\mu\mu}$ for $L/E = 7000 \text{ km} / 50 \text{ GeV}$. The fundamental neutrino parameters have been set to $\Delta m_{21}^2 = 8 \cdot 10^{-5} \text{ eV}^2$, $\Delta m_{31}^2 = 2 \cdot 10^{-3} \text{ eV}^2$, $\theta_{12} = 33^\circ$, $\theta_{23} = 45^\circ$, and $\delta = 0$. The colored regions correspond to different values of the relative accuracy $A_{\text{rel}} = |P_{\mu\mu,\text{num}} - P_{\mu\mu,\text{app}}| / (1 - P_{\mu\mu,\text{num}})$ and absolute accuracy $A_{\text{abs}} = |P_{\mu\mu,\text{num}} - P_{\mu\mu,\text{app}}|$ as indicated by the legends above the panels. The curves corresponds to the isocontours of the accuracy of the two flavor approximation $\theta = \theta_{23}$ and $\Delta m^2 = \Delta m_{32}^2$. The solid curves are the $A_{\text{rel}} = 0.25 \%$ isocontours, the dashed curves are the $A_{\text{rel}} = 0.5 \%$ isocontours, and the dotted curves are the $A_{\text{rel}} = 1 \%$ isocontours in the relative accuracy plot. In the absolute accuracy plot, the solid curves are the $A_{\text{abs}} = 0.0001$ isocontours, the dashed curves are the $A_{\text{abs}} = 0.0005$ isocontours, and the dotted curves are the $A_{\text{abs}} = 0.001$ isocontours.

varies between -2.5 and -1.8 along the baseline, which is basically the middle region of Figs. 3 and 4. Even though the real Earth matter density profile is not constant, the $EV \rightarrow \infty$ approximation should be at least as good as for the case of a constant matter density corresponding to the lowest density of the Earth matter density profile. The approximation has no direct application to solar or supernova neutrinos for which there is a larger V than that given by the Earth. This is due to the fact that these situations are well-described by adiabatic evolution and that the neutrino energies are lower than in the case of a neutrino factory. The approximation might also be applicable to de-

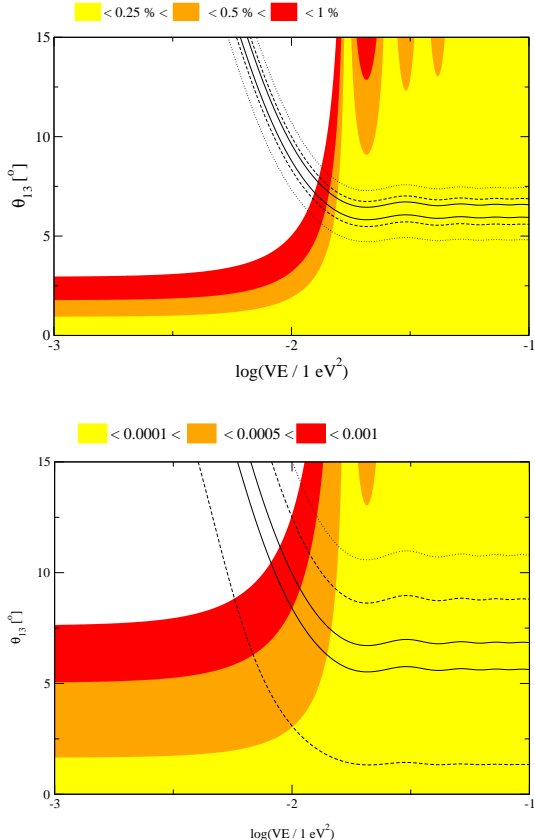


Fig. 4. The relative (upper panel) and absolute (lower panel) accuracy of using the approximation $EV \rightarrow \infty$ in the neutrino oscillation channel $1 - P_{\mu\mu}$ for $L/E = 3000$ km / 50 GeV. The areas and curves are the same as described in the figure caption of Fig. 3.

terminate the second order oscillation effects in high-energy neutrino absorption tomography of the Earth.

5 Summary and Conclusions

We have studied effective two-flavor neutrino mixing and oscillations in the limit $EV \rightarrow \infty$. In this limit, the neutrino oscillations take the form of two-flavor $\nu_\mu \leftrightarrow \nu_\tau$ oscillations with the frequency and amplitude given by Eqs. (6) and (7), respectively. It has been noted that, if measured, the quotient $|\Delta m_\infty^2 / \Delta m_{32}^2|$ could provide valuable information on the mass hierarchy in the neutrino sector and the leptonic mixing angle θ_{13} . We have also examined the validity of using the mentioned two-flavor formulas for large, but not infinite, matter density. The deviation from the two-flavor scenario is determined by the deviation of \tilde{c}_{13} from zero, which is given by Eq. (9) to first order in perturbation theory. The validity of the approximation has also been studied numerically with the results presented in Figs. 3 and 4.

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