Finite Element Analysis of the Dynamic Effect of Soil-Structure Interaction of Portal Frame Bridges

- A Parametric Study

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SHAHO RUHANI
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KTH Royal Institute of Technology
School of Architecture and the Built Environment
Department of Civil and Architectural Engineering
Division of Structural Engineering and Bridges
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SE-100 44, Stockholm, Sweden

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Abstract

In Sweden, the railway sector currently faces the challenge of developing its first high-speed railway line, in response to the need to provide faster domestic and international transport alternatives. High-speed train passages on railway bridges can cause resonance in the bridge superstructure, which induce high accelerations that should not exceed the limits stipulated in the current design code. The most common bridge type adopted in Sweden is the portal frame bridge, an integral abutment bridge confined by surrounding soil. The soil possesses inherent material damping and radiation damping that allows energy dissipation of train-induced vibrations. Both the damping and the natural frequency of the soil-structure system influence the acceleration response of the bridge superstructure. Therefore, it is necessary to investigate the effect of soil-structure interaction on portal frame bridges.

Within this thesis, a numerical parametric study was performed to gain knowledge of the dynamic effect of the relative deck-abutment stiffness on the soil-structure interaction of portal frame bridges. For four span lengths, three different boundary conditions were analyzed in the form of i) no soil, ii) backfill, and iii) half-space. The analysis was performed on two- and three-dimensional finite element models. The backfill and subsoil were modeled with both direct finite element approach, and with a simplified approach using Kelvin-Voigt models and frequency-dependent impedance functions. Furthermore, time was devoted to investigating the nonlinear compression-only behavior of the interaction between the backfill and the abutments to allow separation.

The results presented in the thesis illuminate the essence of including soil-structure interaction in the dynamic analysis as both the modal damping ratio and the natural frequency increased drastically. The effect of backfill on short span bridges has shown to be more prominent on the reduction of the train-induced vibrations. For longer spans, the subsoil proved to be more significant. For the simplified models the modal damping ratios of the different span lengths have been quantified as a logarithmic trend of the 1st vertical bending mode. Two-dimensional models have been problematic when using plane stress elements due to the sensitivity of the element thickness on the response. Thus, such models are only recommended if validation with corresponding three-dimensional models and/or field measurements are possible. By allowing separation of the soil-structure interface, the effect of contact nonlinearity on the acceleration response has been more suitable with direct finite element approach - in which static effects of the soil are accounted for - contrary to the simplified nonlinear models with compression springs.

Keywords: Soil-Structure Interaction; Railway Bridge; Portal Frame Bridge; Dynamic Analysis; Parametric Study; Nonlinear Analysis


Resultaten belyser vikten av att inkludera jord-struktur interaktionen i dynamiska analyser p.g.a. okningen den medför för den modala dämpningen och egenfrekvensen. För korta spännvidder, påvisades det att effekten av motfyllningen var mer framstående för reduktion av vibrationerna orsakade av tåg. För längre spännvidder framgick det däremot att underjorden hade en större påverkan. Effekten av jord-struktur interaktionen på spännvidderna kvantifierades som ett logaritmiskt samband för den modala dämpningen av första vertikala böjmoden. Tvådimensionella modeller har varit problematiska när plana spänningselement användes p.g.a. känsligheten i responsen orsakad av variationer i elementtjockleken. Därem rekommeras tvådimensionella modeller endast om validering mot tredimensionella eller fältmätningar är möjliga. När separation tillåts i gränsytan av jord-struktur interaktionen, visade det sig att direkten tillvägagångssätt med finita element var mer lämplig med hänsyn till det icke-linjära kontaktbeteendet. Detta eftersom de statiska effekterna av jorden påverkade accelerationsresponsen markant. De statiska effekterna har inte varit möjliga att simulera i dem förenklade icke-linjära modeller med tryckfjädrar.

Nyckelord: Jord-struktur interaktion; Järnvägsbro; Plattrambro; Dynamisk analys; Parametrisk studie; Icke-linjär analys
Preface

The work presented in this master thesis has been initiated by the engineering consultancy ELU Konsult AB and the Division of Structural Engineering and Bridges at KTH Royal Institute of Technology.

The work has been supervised by Licentiate of Engineering Abbas Zangeneh, to whom we would like to express our sincere gratitude for His guidance and invaluable support throughout this work. The same holds for Adjunct Professor Costin Pacoste, who have given time for reflections and fruitful discussions throughout this work. Furthermore, we would like to thank Him for introducing us to the subject of finite elements.

Moreover, we would also like to thank Head of Division of Structural Engineering and Bridges at KTH Professor Raid Karoumi, for acquainting and inspiring us to the subject of bridge design and structural dynamics which encouraged us to delve deeper into these fields of engineering science.

We are grateful to ELU Konsult AB, especially to Head of Division in Stockholm Dan Svensson, for giving the opportunity to write the thesis at their office, where the colleagues showed great hospitality, curiosity and helpfulness in this master thesis which formed an enjoyable environment. Also, we are thankful to ELU Konsult AB for providing licenses of the finite element software BRIGADE/Plus.

Last but not least, we would like to thank our families and friends for their support and patience during the work of this master thesis and during the entire study at KTH.

Stockholm, June 2018

Dağdelen, Turgay

Ruhani, Shaho
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Nomenclature

Roman Symbols

\(a\)  Tapered deck ratio; Dimensionless constant

\(A_{\text{trib}}\)  Tributary area

\(b\)  Footing width; Dimensionless constant

\(C\)  Damping matrix

\(c\)  Damping coefficient

\(\hat{C}\)  Modal damping matrix

\(c_{cr}\)  Critical damping

\(c_{K-V}\)  Damping coefficient of Kelvin-Voigt model

\(C_{La}\)  Lysmer analog wave velocity

\(C_{p}\)  Dilatational wave velocity

\(c_r\)  Radiation damping coefficient

\(C_s\)  Shear wave velocity

\(\tilde{c}\)  Dynamic dashpot coefficient

\(c(\omega)\)  Frequency-dependent damping impedance

\(D\)  Global vector of DOF

\(d\)  Soil depth

\(D_n\)  Normal damping coefficient

\(D_t\)  Tangential damping coefficient

\(E\)  Modulus of elasticity

\(\tilde{f}\)  Complex cyclic frequency

\(f\)  Cyclic frequency
NOMENCLATURE

\( f(t) \) External time-varying load
\( f_n \) Natural cyclic frequency
\( f_s \) Sampling frequency
\( F(\omega) \) Load spectrum
\( G \) Spring coefficient; Shear modulus
\( g \) Gap distance
\( G' \) Dashpot coefficient
\( H \) Abutment height
\( H(\omega) \) Complex frequency response function
\( I \) Second area moment of inertia
\( K \) Stiffness matrix
\( k \) Stiffness or spring coefficient
\( k_g \) Nonlinear spring coefficient
\( \hat{K} \) Modal stiffness matrix
\( k_{K-V} \) Spring coefficient of Kelvin-Voigt model
\( k_{st} \) Static spring coefficient
\( \bar{k}(\omega) \) Frequency-dependent stiffness impedance
\( k(\omega) \) Dynamic stiffness coefficient
\( L \) Theoretical span length
\( l \) Backfill length
\( M \) Mass matrix
\( m \) Mass
\( \hat{M} \) Modal mass matrix
\( P \) External load
\( p(t) \) External time-varying load
\( \mathbf{P}(t) \) External time-varying load vector
\( \hat{\mathbf{P}}(t) \) Modal time-varying load vector
\( \mathbf{P}(t_{k+1}) \) External load vector at time instance \( k + 1 \)
NOMENCLATURE

\( P(\omega) \) Load spectrum

\( q_n(t) \) nth time-varying modal coordinate

\( q_i(t) \) ith time-varying modal coordinate

\( \dot{q}_i(t) \) first time derivative of ith time-varying modal coordinate

\( \ddot{q}_i(t) \) second time derivative of ith time-varying modal coordinate

\( R \) External nodal load vector

\( r(u_{k+1}) \) Internal restoring force matrix

\( R \) Half-space radius

\( t \) Time; Bridge thickness

\( T_n \) Natural period (undamped)

\( t_k \) Time instance at \( k \)

\( t_{k+1} \) Time instance at \( k + 1 \)

\( \bar{u} \) Complex displacement

\( u \) Displacement

\( u \) Time-varying displacement

\( u \) Time-varying displacement vector

\( u_0 \) Initial displacement vector

\( u(t) \) Time-varying displacement vector

\( u_k \) Displacement vector at time instance \( k \)

\( u_{k+1} \) Displacement vector at time instance \( k + 1 \)

\( u_{k+1}^{(j)} \) Displacement vector at time instance \( k + 1 \) and iterate \( j \)

\( u_{k+1}^{(j+1)} \) Displacement vector at time instance \( k + 1 \) and iterate \( j + 1 \)

\( \bar{u}_{k+1} \) Corrector to displacement vector at time instance \( k + 1 \)

\( \dot{u} \) Time-varying velocity

\( \dot{u} \) Time-varying velocity vector

\( \dot{u}_0 \) Initial velocity vector

\( \dot{u}(t) \) Time-varying velocity vector

\( \dot{u}_k \) Velocity vector at time instance \( k \)
\( \dot{u}_{k+1} \) Velocity vector at time instance \( k + 1 \)

\( \dot{u}_{(j)}^{(k+1)} \) Velocity vector at time instance \( k + 1 \) and iterate \( j \)

\( \dot{u}_{(j+1)}^{(k+1)} \) Velocity vector at time instance \( k + 1 \) and iterate \( j + 1 \)

\( \hat{u}_{k+1} \) Corrector to velocity vector at time instance \( k + 1 \)

\( \ddot{u} \) Time-varying acceleration

\( \dot{w} \) Time-varying velocity

\( \ddot{u} \) Time-varying acceleration vector

\( \ddot{u}(t) \) Time-varying acceleration vector

\( \ddot{u}_{k} \) Acceleration vector at time instance \( k \)

\( \ddot{u}_{k+1} \) Acceleration vector at time instance \( k + 1 \)

\( \ddot{u}_{(j)}^{(k+1)} \) Acceleration vector at time instance \( k \) and iterate \( j \)

\( \ddot{u}_{(j+1)}^{(k+1)} \) Acceleration vector at time instance \( k + 1 \) and iterate \( j + 1 \)

\( \ddot{U}(\omega) \) Acceleration spectrum

\( v \) Train speed

\( v_{\text{cr}} \) Critical train speed

\( Z(\omega) \) Impedance function

**Greek Symbols**

\( \alpha \) Hilber-Hughes-Taylor parameter; Mass-proportional Rayleigh parameter

\( \alpha_n \) Dimensionless stiffness coefficient

\( \beta \) Newmark parameter; Stiffness proportional Rayleigh parameter; Frequency parameter

\( \beta_n \) Dimensionless damping coefficient

\( \delta \) Natural logarithm; Kronecker delta

\( \Delta \ddot{u}_{(j)}^{(k+1)} \) Acceleration increment at time instance \( k + 1 \) and iterate \( j \)

\( \Delta_{\text{el}} \) Element size

\( \Delta t \) Time step

\( \epsilon \) Small strain tensor

\( \epsilon_{(j)}^{(k+1)} \) Residual vector at time instance \( k + 1 \) and iterate \( j \)

\( \epsilon_{k+1} \) Residual vector at time instance \( k + 1 \)
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<thead>
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<th>Symbol</th>
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<tr>
<td>$\eta$</td>
<td>Loss factor</td>
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<tr>
<td>$\gamma$</td>
<td>Shear strain; Newmark parameter</td>
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<tr>
<td>$\lambda$</td>
<td>Boogie distance; Wavelength; Lagrange multiplier; Lamé constant</td>
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<tr>
<td>$\mu$</td>
<td>Elastic modulus; Lamé constant</td>
</tr>
<tr>
<td>$\mu'$</td>
<td>Loss modulus</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Circular frequency</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>Natural circular frequency</td>
</tr>
<tr>
<td>$\phi_n$</td>
<td>$n$th natural vibration mode</td>
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<tr>
<td>$\Phi$</td>
<td>Modal Matrix</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Phase lag</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Cauchy stress tensor</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Shear stress</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Incident angle</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Damping ratio</td>
</tr>
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</table>

**Abbreviations**

- 2D Two-Dimensional Space
- 3D Three-Dimensional Space
- API Application Programming Interface
- CPU Central Processing Unit
- DFT Discrete Fourier Transform
- DOF Degrees Of Freedom
- MDOF Multiple Degrees Of Freedom
- SDOF Single Degree Of Freedom
- DSS Direct Steady State
- DTI Direct Time Integration
- EOM Equation Of Motion
<table>
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<th>Acronym</th>
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<tr>
<td>FEM</td>
<td>Finite Element Method</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
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<tr>
<td>FRF</td>
<td>Frequency-Response Function</td>
</tr>
<tr>
<td>GUI</td>
<td>Graphical User Interface</td>
</tr>
<tr>
<td>HSLM</td>
<td>High Speed Load Model</td>
</tr>
<tr>
<td>IFFT</td>
<td>Inverse Fast Fourier Transform</td>
</tr>
<tr>
<td>IVP</td>
<td>Initial Value Problem</td>
</tr>
<tr>
<td>PP</td>
<td>Post-Processing</td>
</tr>
<tr>
<td>SLS</td>
<td>Serviceability Limit State</td>
</tr>
<tr>
<td>SSI</td>
<td>Soil Structure Interaction</td>
</tr>
<tr>
<td>ULS</td>
<td>Ultimate Limit State</td>
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Chapter 1

Introduction

1.1 Background

The railway sector in Sweden faces the development of its first high-speed railway, the European Corridor. The railway line, planned for inauguration in 2035, aims to connect the three major cities of Sweden - Stockholm, Gothenburg, and Malmö, with traveling time of 2-3 hours, see Figure 1.1. The new high-speed railway line will enable the development of intermediate regions and provide energy efficient transportation alternatives. Such traveling times requires a design speed of 320 km/h (SOU 2017:107). In 2017 the first part of the European Corridor was initiated, the so-called East Link project which connects Södertälje and Linköping.

Currently, one of the existing railway lines, the Bothnia Line, allows for speeds up to 250 km/h. In addition to new high-speed railway lines, train speeds on existing railway lines are planned to be upgraded above 200 km/h. Although high-speed trains are attractive, it entails some problems with the railway bridges. Railway bridge superstructures subjected to train passages above 200 km/h may enter resonance regime. This phenomenon occurs when the frequency of excitation from the passing train coincides with one of the natural frequencies of the bridge. At resonance, the bridge superstructure vibrates severely due to the periodic nature of the axle loads of the train. The consequences of a railway bridge experiencing resonance may lead to track degradation, ballast instability, increased maintenance cost and increased safety risk of passengers (Johansson et al., 2014). Thus, the structural integrity of the bridge system is violated.

EN1991-2, which is the current design code for new bridges in Sweden, stipulate design values that should not be exceeded. These values regard the maximum vertical bridge deck acceleration, deck twist, deflection and angular rotation. Generally, the vertical bridge deck acceleration is the most important parameter to consider. The specified limit is set to $3.5 \text{ m/s}^2$ and $5.0 \text{ m/s}^2$ for ballasted and ballastless tracks, respectively (CEN, 2003). The limits are based on field measurements and laboratory experiments. Ballast instability was observed on a French high-speed railway bridge with acceleration levels at approximately $7-8 \text{ m/s}^2$, to which a safety factor of 2 was applied (Johansson et al., 2013). To comply with these requirements, evaluation of railway bridges through dynamic analysis is thus necessary. The most commonly encountered
CHAPTER 1. INTRODUCTION

type of bridge along the Swedish railway network are short-span portal frame bridges, which is somewhat of a Swedish specialty (Sundquist, 2007). This type of bridge is an integral abutment bridge and is designed as a reinforced concrete rigid frame with wing walls, which is confined by an embankment, see Figure 1.2. Johansson et al. (2013) carried out an inventory, where a dynamic analysis was performed on bridges along the Bothnia Line. The simplified dynamic evaluation conducted in the report, where the effect of SSI was neglected, showed that 75% of the portal frame bridges exceeded the limiting design values of the design code. The modal parameters, i.e. the natural frequency and the damping ratio, governs the frequency content and amplitude of the vibrations in a railway bridge.

Fig. 1.1 The European Corridor connecting Stockholm, Gothenburg and Malmö. From © 2016 Europakorridoren AB.

Fig. 1.2 Portal frame bridge at Orrvik located on the Bothnia Line. From Andersson and Karoumi (2015).

Numerical investigations indicate that the modal damping ratio of short-span railway bridges is higher than the recommended design values due to energy dissipation at boundaries between the structure and surrounding soil. The dynamic response of portal frame bridges is thus highly dependent on the soil-structure interaction (SSI) (Zangeneh et al., 2017). However, it does not exist detailed guidelines nor reliable simple models to account for the effect of SSI, why classical boundary conditions often are assumed in dynamic analysis of railway bridges (Zangeneh, 2018). The dynamic response of the structural system is highly dependent on the damping ratios, particularly close to resonance regime. Currently, the damping ratios used in the design are based on the type of bridge and span length, and only considers the inherent material damping. The damping ratio chosen for the analysis of train-induced vibrations of railway bridges is limited to approximately 2.7% in CEN (2003) for reinforced concrete bridges. Train passages are often analyzed by a series of moving concentrated loads. The train-bridge interaction has a favorable

---

1Europakorridoren AB is a non-profit organization jointly run by municipalities, regions and industry representatives. The purpose of the association is to ensure that the European Corridor is expanded to provide Sweden with a modern, efficient, environmentally-friendly and long-term sustainable traffic system.
1.2 Previous Studies

Although there are extensive number of publications covering the concepts of SSI on the seismic response of bridges, few sources deal with the application of methods to account for the effect of SSI on train-induced vibration of railway bridges (Zangeneh, 2018). Galvín and Domínguez (2007) presented a general numerical model for the analysis of soil motion due to high-speed train passages. The soil, ballast and structure were represented by a three-dimensional time domain boundary element approach, and considerations of SSI were taken into account. The numerical results were to a large extent validated by experimental results. Ülker Kaustell (2009) developed a simplified 2D model to evaluate the effect of SSI of a portal frame bridge subjected to high-speed trains. The effect of the backfill were neglected, and frequency-dependent impedance functions were used to model the subsoil. The author concluded that the boundary conditions of the structure and the stiffness of the subsoil contributes significantly to the modal damping ratios of the system. Zangeneh (2018) investigated the dynamic SSI of portal frame bridges using coupled finite element-boundary element methods in three dimensions, as well as a full three-dimensional finite element model. Furthermore, controlled vibration tests were performed on a case study bridge. The results from the dynamic test were implemented as reference data which formed the ground for an automated finite element model updating procedure which calibrated the material properties. It was shown that the calibrated finite element model, simulated frequency response functions, which were in good agreement with measurements, albeit some discrepancies were observed for higher frequency content. The numerical analysis for short-span portal frame bridges showed that SSI has a fundamental influence on the dynamic response, where the influence of the backfill on the dynamic properties of the structure was emphasized. Moreover, this contributed to a significant reduction of the resonant response. Even though the effect of SSI on shorter bridges was illuminated, the effect of the relative deck and abutment thickness, governed mainly by the
span length, was yet to be demonstrated. Furthermore, a proposition of a simplified model to consider the effect of surrounding soil, which allows for a practically convenient alternative, was presented. The simplified model tended to overestimate the dissipative capacity of the backfill soil. In the study by Zangeneh et al. (2018) fully tied interaction was assumed between the backfill soil and abutments. This may, according to the authors, be a factor which overestimates the dissipation capacity of the backfill soil for vibration modes governed by the motion of the abutments and could be a reason for the discrepancy between measured and calculated results found at high frequency range. A possibility of including a gap formulation in the model by means of nonlinear contact between abutment and bridge was discussed as an attempt to reduce the observed discrepancy. Ülker Kaustell (2009) was of the same perception that one cause of mismatch between experimental and theoretical analysis might lie within nonlinearities emanating from soil-structure interaction.

1.3 Aims and Scope

In order to gain a deeper understanding of some of the issues as previously mentioned, the main objective of this thesis is to investigate the effect of SSI on portal frame bridges. This is investigated for short to long span lengths with different boundary conditions and modeling alternatives for the surrounding soil. Furthermore, this is done by evaluating the modal parameters and investigating the response of the systems in the mid- and quarter point due to high-speed train passages. The parameters of interest are the modal damping ratio $\zeta$, the natural frequency $f_n$ and the acceleration envelope generated from train passages of the high-speed load model (HSLM) of CEN at different speeds. Mainly, the study is performed in a two-dimensional space (2D). A parametric study is conducted, where the direct approach of finite element modeling is used for analysis of four different span lengths with three different assumptions of the surrounding soil. In addition to the full modeling of the soil, simplified models for including the surrounding soil is proposed for use in two dimensions. In particular, this is investigated for models assuming fully tied abutment-soil interaction and nonlinear contact interface. The work is extended to three-dimensional space (3D) of the different lengths with two assumptions regarding the surrounding soil. In 3D, simplified models are also investigated, however, no train passages are analyzed.

1.4 Model Description

The different modeling cases which will be referred to throughout the thesis are distinguished by the case number and denotation as depicted in Figure 1.4. The span lengths considered are 5-, 10-, 15- and 20 meters and the three different ways to consider the surrounding soil, are i) assuming no soil interaction, ii) only backfill, or iii) subsoil and backfill together creating a half-space. The characteristics of the models are briefly presented below. A more thorough definition of the geometrical- and material properties, as well as the boundary conditions applied for the different cases, is presented in Section 3.2, Section 3.3, and Section 3.5 and Section 3.7, respectively.

I. Frame Only: A 2D model of a portal frame bridge with clamped boundary conditions on the footings. It consists of deck, abutments and footings and no consideration of the surrounding soil is taken into account.
II. Backfill: A 2D model as a further development of Case I. Backfill is included in the model with clamped boundary conditions underneath, and a viscous boundary at the distant boundary of the model is applied.

III. Halfspace: A 2D model with a portal frame bridge, backfill and subsoil modeled as a half-space with a viscous boundary.

IV. Simplified Backfill: A 2D simplified model with clamped boundary conditions as Case I, with the inclusion of linear Kelvin-Voigt type springs and dashpots in parallel on the abutments to consider the backfill.

V. Simplified Halfspace: A 2D simplified model with linear Kelvin-Voigt type springs and dashpots in parallel on the abutments to consider the backfill. Frequency-dependent impedance functions are implemented on the footings to consider the subsoil.

VI. Simplified Backfill Nonlinear: A 2D simplified model of the portal frame bridge with clamped boundary conditions. The backfill soil is accounted for by nonlinear spring in series with linear Kelvin-Voigt type spring and dashpot connected in parallel.

VII. Backfill Contact Nonlinear: A 2D model as a development of case II with nonlinear contact formulation between the abutments and backfill soil.

Case I - VII conclude the models in 2D for all of which the modal parameters will be evaluated as well as the response due to high-speed train passage. The remaining cases, Case VIII - XI, are analyzed in 3D for which only the modal parameters were evaluated, i.e. no train analysis was performed.

VIII. 3D Frame Only: A 3D model of a portal frame bridge. The extension of Case I into three dimensions with clamped footings and no SSI.

IX. 3D Backfill: A 3D model of a clamped portal frame bridge and backfill with a viscous boundary. The 3D analogy of Case II.

X. 3D Simplified Backfill: A 3D simplified model with linear Kelvin-Voigt type springs and dashpots in parallel applied in three directions, analogous case IV.

XI. 3D Simplified Halfspace: A 3D model corresponding to its 2D counterpart in Case V with linear Kelvin-Voigt type springs and dashpots in parallel on the abutments to consider the backfill. The subsoil is considered through frequency-dependent impedance functions applied to footings.
Fig. 1.4 Models with corresponding case number and denotation throughout the thesis.
Chapter 2

Theoretical Background

In this chapter, some theoretical concepts and definitions are presented to form the foundation on which this thesis has been built upon. The chapter is initiated by a recapitulation of the most fundamentals of finite element theory and structural dynamics. The damping phenomena is elaborated, and some constitutive material models are presented in the following. The chapter continues with the basics of signal analysis. Subsequently, the reader is guided through some analysis procedures in the frequency- and time domain. Within the time-domain, some aspects of nonlinearities are presented followed by wave propagation theory in elastic media. Furthermore, the concepts of impedance functions and absorbing boundaries are discussed and finally, finishing the chapter by some theoretical aspects of simplified soil modeling.

2.1 Finite Element Method

In the following section, the theoretical background to describe the applications and concepts of the finite element method (FEM) is made in accordance with Cook et al. (2007).

FEM is a numerical method which provides approximate solutions to field problems. Mathematically speaking, field problems are expressed by partial differential equations for which the solution also satisfies the boundary condition, i.e. boundary value problems (BVP). Furthermore, a field problem requires that one must decide the spatial distribution of the dependent variables. However, finite element analysis (FEA) aims at only approximating the field quantity with a piecewise interpolation. In structural engineering applications, the field quantity of interest would for instance be to determine the distribution of displacements on a structure during load.

The general procedure, for structural analysis purposes, to solve a finite element model is to divide a domain into simpler parts referred to as finite elements. Each of the elements are then formulated in similar manner where the field variables are discretized at nodes and interpolated within the element. An individual element is identified by nodes, degrees of freedom (DOF) and a characteristic matrix, i.e. local stiffness matrix. Thereafter, all elements are assembled to a global domain, connected at nodes and arranged in a mesh. Finally, mechanical loads are applied, and nodal unknowns are solved by imposing boundary conditions. The set of static equilibrium equations in Equation (2.1) includes $K$, $D$ and $R$. They represent the global stiffness
matrix, global vector containing DOFs and the external nodal load vector respectively. Eventually, strains can be evaluated by computing the gradients of resulting field quantity. Finally, this renders mechanical stresses by multiplying the constitutive matrix containing elastic constants with strains.

\[
KD = R
\]  

The nodal field variables, governed from the solution, in combination with the approximated piecewise interpolation completely determines the spatial variation of the DOF within all elements in an average sense. It is crucial to understand that FEM only provides approximation of the field quantity contrary to more classical solution procedure which provides exact results to BVPs over the domain. The most significant benefit of using FEM is that it can replicate all kinds of geometries, thus making it suitable for structural engineering purposes.

The nature of finite element solutions is that equilibrium of nodal forces and moments are satisfied at the nodes. However, this is not always the case across the element boundaries or within the elements. Furthermore, compatibility is achieved at nodes but not necessarily across the interelement boundaries when different elements are combined in a finite element model. Thus, results computed by computational model contains errors compared to the exact solution of mathematical models. These errors are classified in different groups, e.g. modeling error, numerical error, ill-conditioning and discretization error which are important to consider when dealing with commercial finite element software. Discretization error mainly consists of dealing with convergence rates, i.e. for a sufficiently refined mesh, error in the solutions can be bounded. Numerical errors are associated with truncation of residuals during iteration schemes. Error considering ill-conditioning is present when the solution is sensitive to small adjustments in stiffness, i.e. small change in input leading to large change in output.

Throughout this thesis, dealing with issues concerning the aforementioned problems with finite element modeling (FE-modeling), the authors have been encouraged to ensure accuracy, reliability and robustness of the model. This indeed, is important to provide validity of the results obtained from FEA within research.

### 2.2 Structural Dynamics

In this section, the topics of dynamic equilibrium conditions, natural frequencies, damping, the procedures to evaluate the aforementioned and the concept of resonance will be briefly discussed. The following derivations are based on Chopra (2013).

**The Equation of Motion**

In structural dynamic analysis, structures exert oscillatory response caused by loads which disrupts the static equilibrium of the structure. As vibrations occur, the dynamic response is governed by three components, namely the mass, damping and stiffness. To understand the nature of dynamic behavior, it is fundamental to understand these key concepts. The structural response
of a system is time-dependent if the loading is time-dependent. For cyclic loading with frequency
less than approximately a quarter of the structure’s lowest natural frequency of vibration, the
dynamic response is seldom greater than the static response. Thus, a structure subjected to
dynamic loading can be encountered as quasi-static for which dynamic analysis is not a necessity.
However, for greater frequencies a dynamic analysis is required. In dynamic problems, the mass
and damping of the system are present in addition to the stiffness as mentioned in Section 2.1.
The main aim of dynamic analysis is to retrieve the natural frequencies of vibrations and their
mode shapes. These are obtained by solving an eigenvalue problem after which post processing of
results are enabled. Furthermore, the steady-state response to harmonic loading and transient
response to non-periodic or impact loads may be of interest in dynamic analysis.

Considering a system with mass, the number of degrees of freedom (DOF) are the number of
independent displacement required to define the displaced position of a mass relative to their
original position. Figure 2.1 illustrates a single degree of freedom (SDOF) system with a single
mass m, a massless single linear spring of with stiffness k, and a massless single linear viscous
damper with the viscous damping coefficient c. The viscous damper generates a resisting force
proportional to the rate of deformation. The linear spring exerts an elastic resisting force. With
an equilibrium equation for the forces and the use of D’Alembert’s principle to include the inertia
force, the equation of motion (EOM) is formulated according to Equation (2.2). Motion, i.e. the
displacement, is described by \( u = u(t) \) where velocity and acceleration are defined according to
Newton’s notation to indicate the first- and second time derivatives of the displacement u. A
dynamic load \( p(t) \) is applied to the system which also is known as the excitation or external load.
By the nature of initial value problems (IVP) - as the EOM is, the initial displacement \( u(0) \) and
initial velocity \( \dot{u}(0) \) at time zero must be specified to completely define the problem. Generally,
the structure is at rest prior the dynamic excitation setting these initial conditions equal zero.

\[
m\ddot{u} + c\dot{u} + ku = p(t) \tag{2.2}
\]

As the complexity of a system increase, the number of degrees of freedom increase naturally. The
second order differential equation stated in Equation (2.2) now becomes a set of equilibrium
equations that have to be solved. Thus, matrix approach is feasible. The matrix equation
represents the ordinary differential equations governing the displacements \( u_n \) where \( n \) indicates
the number of degrees of freedom. As for a SDOF-system, the matrix equation shown in
Equation (2.3) for a multiple degree of freedom (MDOF) system contains the inertial force,
damping force and internal force which need to be in equilibrium with the external force.
\[ \mathbf{M} \ddot{\mathbf{u}} + \mathbf{C} \dot{\mathbf{u}} + \mathbf{K} \mathbf{u} = \mathbf{P}(t) \] (2.3)

The general procedure of formulating the system of EOMs in Equation (2.3) is to initially discretize the structure and define the DOFs. The displacements of the nodes of the elements constituting the structural idealization, are the degrees of freedom which governs the solution in dynamic analysis. The three types of forces, i.e. the inertial, damping and internal forces, are formulated at each DOF and assembled into their respective matrices. The mass-, damping- and stiffness matrices are denoted as \( \mathbf{M} \), \( \mathbf{C} \) and \( \mathbf{K} \), respectively. The external forces acting on the DOFs are represented in the load matrix, \( \mathbf{P} \).

**Natural Frequencies and Mode Shapes**

The concept of natural frequencies and mode shapes are most easily explained by considering an undamped SDOF-system undergoing free vibration. A structure which has been disturbed from its static equilibrium position and is allowed to vibrate without external dynamic excitation is vibrating freely. Thus, the right hand side of Equation (2.2) is set to zero. For undamped systems, the damping force is neglected, setting the differential equation governing free vibration of the SDOF system as shown in Equation (2.4).

\[ m \ddot{u} + ku = 0 \] (2.4)

The solution to the linear, homogeneous second-order differential equation with constants coefficients is obtained by standard procedures when subjected to initial conditions \( u = u(0) \) and \( \dot{u} = \dot{u}(0) \):

\[ u(t) = u(0) \cos \omega_n t + \frac{\dot{u}(0)}{\omega_n} \sin \omega_n t \] (2.5)

where the natural circular frequency \( \omega_n \) is defined as:

\[ \omega_n = \sqrt{\frac{k}{m}} \] (2.6)

Equation (2.5) is visualized in Figure 2.2. The system exerts oscillatory motion about its static equilibrium, it is seen that the motion is repeated every \( 2\pi/\omega_n \) seconds. One cycle of free vibration is completed during the natural period of vibration of the system, \( T_n \). The natural period is related to the natural circular frequency accordingly:

\[ T_n = \frac{2\pi}{\omega_n} \] (2.7)

Evidently, a system thus executes \( 1/T_n \) cycles in 1 second, rendering the expression for the natural cyclic frequency of vibration:

\[ f_n = \frac{1}{T_n} \] (2.8)

An undamped system oscillates between its maximum and minimum displacements, where the deflected shape at the local extrema is the natural mode of vibration. Note that the term natural
emphasize that the vibration properties are properties of the structure in free vibration, i.e. they only depend on the mass and stiffness.

In the expansion to MDOF-systems, the natural frequencies and modes are obtained by the solution of an eigenvalue problem. The free vibration of an undamped system is mathematically represented by Equation (2.10) where the mode shape \( \phi_n \) is time independent and the time variation of the displacements are defined by simple harmonic function, see Equation (2.9).

\[
q_n(t) = A_n \cos(\omega_n t) + B_n \sin(\omega_n t) \tag{2.9}
\]

\[
u(t) = q_n(t) \phi_n \tag{2.10}
\]

By combining Equation (2.3) and Equation (2.10) for a undamped system, the real eigenvalue problem can be formulated according to Equation (2.11). The non-trivial solution provides the eigenvalues and eigenvectors, i.e. the natural frequencies and natural modes of vibration. Thus, each characteristic deflected shape is the natural mode, related to a unique natural frequency. However, for systems with non-proportional damping, the eigenvalue problem should be distinguished by the usage of the term complex eigenvalue problem.

\[
|K - \omega_n^2 M| \phi_n = 0 \tag{2.11}
\]

### Damping Ratio

The differential equation governing the free vibration of SDOF systems with damping is obtained by setting \( p(t) = 0 \) in Equation (2.2), dividing with \( m \) and with \( \omega_n \) defined as in Equation (2.6), the EOM becomes

\[
\ddot{u} + 2\zeta \omega_n \dot{u} + \omega_n^2 u = 0 \tag{2.12}
\]

where \( \zeta \) is the damping ratio which is a fraction of the critical damping \( c_{cr} \).

\[
\zeta = \frac{c}{c_{cr}} = \frac{c}{2m \omega_n} \tag{2.13}
\]
\[ c_{cr} = 2\sqrt{km} \]  

(2.14)

The energy dissipated in a cycle of free vibration or in a cycle of forced harmonic vibration is represented in the damping constant \( c \). Evidently, the damping ratio which is a dimensionless measure of damping, is dependent of the stiffness and the mass of the system. The magnitude of the damping ratio \( \zeta \) governs three different types of damped systems, namely underdamped, critically damped and overdamped systems. For \( \zeta < 1 \) the system oscillates about its position of equilibrium with progressively decreasing amplitude and is said to be underdamped. This type of damping is of interest, as damping within structural engineering is within the limits of underdamped systems. For \( \zeta > 1 \), the system is overdamped and does not exhibit oscillatory motion. Critically damped systems are defined by \( \zeta = 1 \), and constitutes the transition from oscillatory to non-oscillatory motion.

**Resonance Regime**

In structural dynamics, one might seek the amplitude of response to a cyclic load of known magnitude \( p_0 \) and forcing frequency \( \omega \). Thus, the general EOM from Equation (2.2) is rewritten in the form of Equation (2.15) and is referred to as EOM for forced vibration.

\[ m\ddot{u} + c\dot{u} + ku = p_0 \sin(\omega t) \]  

(2.15)

As the forcing frequency approaches the natural frequencies of a structure, the amplitude of vibrations increases. These, frequencies are the resonant frequencies, i.e. the forcing frequency at which the largest response amplitude occurs. For systems without damping at resonant frequency, the amplitude of vibration will theoretically grow infinitely with time. Damping is all that prevents the growth of vibration amplitude. However, at large amplitudes the response may be limited by nonlinearities that occur. With increased amplitude the system would fail at any time instance if the system is brittle. For ductile systems, yielding would occur and the stiffness would decrease. Consequently, the natural frequency would decrease and no longer coincide with the forcing frequency and the resonant state would be exited. In damped systems, the amplitude of vibrations increase with time according to an envelope function which asymptotically reaches the steady-state amplitude, which is shown in Figure 2.3.

![Resonance Response](image)

*Fig. 2.3 Resonance Response for \( \zeta = 0.05 \). From Chopra (2013).*
In train-induced vibration on the bridge superstructure, the critical speed \( v_{cr} \) by a series of moving loads, which causes resonance of a simply supported railway bridge, can be estimated with Equation (2.16) where \( f \) is the natural frequency of the bridge, \( \lambda \) is the boogie axle distance and \( i \) is an integer multiple.

\[
v_{cr} = \frac{f\lambda}{i}
\]  

(2.16)

**Evaluation of the Damping Ratio**

One of the fundamental findings attained from dynamic analysis is the damping ratio. The damping ratio can be evaluated from the time history of a response of systems subjected to dynamic excitation, or by regarding the response curve with respect to the forcing frequency. In the time domain, the decay of motion enables one to find a relation between the ratio of two peaks of damped free vibration. Considering the time response, tentatively the displacement at any peak \( u_1 \), the ratio with another peak \( j \) cycles apart \( u_{j+1} \) is given by

\[
\frac{u_1}{u_{j+1}} = \frac{u_1 u_2 u_3 \cdots u_j}{u_2 u_3 u_4 \cdots u_{j+1}} = e^{j\delta}
\]  

(2.17)

where \( \delta \) is the natural logarithm of the ratio, and denoted the logarithmic decrement which provides the exact solution for \( \zeta \) according to Equation (2.18). An approximate solution for the damping ratio which applies for small damping ratios, sets the denominator of the multiplicand in Equation (2.18) to unity, and is thus \( \delta \simeq 2\pi \zeta \)

\[
\delta = \frac{1}{j} \frac{2\pi \zeta}{\sqrt{1 - \zeta^2}}
\]  

(2.18)

The frequency response curve which relates the response of the system to the forcing frequency entails important properties from which the damping ratio can be evaluated. The half-power bandwidth is defined in Figure 2.4. The maximum response is obtained at the natural frequency \( \omega_n \). Let \( \omega_a \) and \( \omega_b \) be the forcing frequencies at which the amplitude is \( 2^{-1/2} \) times the resonant amplitude on both sides of the resonant frequency. For small damping ratios, which is representative of practical structures, the damping ratio can be approximately related to the forcing frequencies \( \omega_a \), \( \omega_b \) and the natural frequency \( \omega_n \) according to

\[
\zeta = \frac{\omega_b - \omega_a}{2\omega_n}
\]  

(2.19)
CHAPTER 2. THEORETICAL BACKGROUND

2.3 Damping

As mentioned in previous sections, damping is the fundamental contributor to reduced amplitude of vibrations. In damping, various mechanisms constitutes the energy dissipation of the vibrating system. These are, amongst others, for instance friction at supports and along structural boundaries, opening and closing of micro-cracks in concrete and energy dissipation through bending and shear effects, i.e. internal material damping. However, it is difficult to describe these mathematically. Therefore, the damping in a structure is idealized by a linear viscous dashpot in the elastic region. The damping coefficient is chosen which is equivalent all energy dissipating mechanisms combined, i.e. an equivalent viscous damping is used. The result of energy dissipation of this kind is the decay of amplitude of the free vibration. Often, damping is measured by exciting a real structure on which measurements are performed. As previously described, the damping can be evaluated with the logarithmic decrement and the Half-Power Method in the time- and frequency domain, respectively (Chopra, 2013).

It is generally accepted that the main sources of vibration attenuation are material and geometrical (radiation) damping. The former reflects the energy dissipation in the soil by hysteretic action while the latter refers to energy carried away by wave propagation (Gazetas, 1991). However, there exists differences in opinions by several authors regarding the importance of material damping. One opinion is that geometrical damping is the main source in the attenuation of Rayleigh waves. This constitutes the generally implemented viewpoint in engineering applications because it assumes that the soil is a perfectly elastic medium and the neglection of material damping. Contrary, some authors propose that material damping might have the same significance as geometrical damping. Furthermore, for shallow layers of soil, the geometrical damping might be reduced significantly, causing the material damping to be the primary source of dissipation (Ambrosini, 2006). Shortly, the justification of the assumption of elasticity theory of soil will be discussed and the fundamentals of wave propagation theory will be presented in Section 2.8.
2.3 DAMPING

2.3.1 Rate-Dependent Damping

In this section, a form of rate-dependent damping will be explained, i.e. the Rayleigh damping. Moreover, the concept of a rate-dependent constitutive material model will be presented, which forms the ground of understanding the structural damping that is rate-independent in the following section.

Rayleigh Damping

A rather simple mathematical representation of material damping, is to combine the mass matrix and stiffness matrix to create a proper damping matrix, i.e. Rayleigh damping, shown in Equation (2.20). The mass proportional damping diminishes with increase in natural vibration frequencies and on the contrary, the stiffness proportional damping grows with increase in natural vibration frequencies. Furthermore, the modes of interest, i.e. mode \(i\) and \(j\), are assigned a modal damping ratio \(\zeta\) in order to evaluate the parameters \(\alpha\) and \(\beta\) from Equation (2.21). Thus, the Rayleigh damping matrix is now fully represented and is by definition frequency dependent.

\[
c = \alpha m + \beta k \tag{2.20}
\]

\[
\alpha = \zeta \frac{2\omega_i\omega_j}{\omega_i + \omega_j} \quad \beta = \zeta \frac{2}{\omega_i + \omega_j} \tag{2.21}
\]

Rate-Dependent Constitutive Material Models

The inclusion of inertial forces in dynamic analysis as opposed to static analysis requires the consideration of infinitesimal strains. Despite small strains, even as small as the \(10^{-6}\), the strains can not be disregarded as increased rapidity of motion might yield significant effect. It has been proven that deformation characteristics of soils depend on the shear strains exhibited by the soil. Furthermore, elastic models are justified for small strains. In the stress range below the order of \(10^{-5}\), deformations are elastic and recoverable. Such small strains are characterized by vibration or wave propagation through the soil. For medium shear strain range, i.e. in the order of magnitude of \(10^{-5}\) to \(10^{-3}\), the soil behavior is considered elasto-plastic. In this region, a constitutive model based on the classical theory of linear viscoelasticity reflects the behavior of the soil reasonably accurate (Ishihara, 1996). Although soil materials preferably are described by the theory of plasticity, Ishihara (1996) suggests choice of material models with respect to the strain range. Essentially, the energy dissipation in soils are of hysteretic nature and rate-independent. In the following, two material models will be described as presented by Ishihara, from which one is rate dependent and the other is not.

Materials exhibiting storage of strain energy as well as energy dissipation over time, i.e. elastic and viscous behavior respectively, need proper idealization to represent the effects of damping. Viscoelastic materials can be idealized mathematically by springs and dashpots connected in parallel or series. One such model is the Kelvin solid which has a parallel connection. The total shear stress is governed by

\[
\tau = G\gamma + G'\frac{d\gamma}{dt} \tag{2.22}
\]
where $\gamma$ denotes the shear strain while $G$ and $G'$ indicates the spring and dashpot coefficients, respectively. Equation (2.22) is derived from a general relationship applicable to viscoelastic bodies. Thus, it is of interest to relate the spring and dashpot coefficient to the elastic modulus $\mu$ and loss modulus $\mu'$, respectively. To evaluate the stress-strain relation in a compact manner, one should preferably use the method of complex variables. By comparing the real and imaginary part separately, the relations shown in Equation (2.23) is obtained. The stress-strain relation in the time domain thus becomes as shown in Equation (2.22)

$$\mu = G, \quad \mu' = G' \omega$$

$$\tau = \mu \gamma + \mu' \dot{\gamma} = \mu [\gamma + \eta \dot{\gamma}]$$

where $\eta = \mu / \mu'$ is the loss factor. The relation between the loss factor and the damping ratio $\zeta$ of the material model is

$$\eta = 2 \zeta$$

The Fourier transformation of Equation (2.22) gives the stress-strain relation in the frequency domain:

$$\tau = \mu [1 + i \omega \eta] \gamma$$

From Equation (2.26) it is evident that the viscoelastic response through Fourier transform can be obtained from the elastic response by using the complex modulus of the viscoelastic material rather than the real modulus of the elastic material. This is, what is referred to as the correspondence principle (Bland, 2016).

In the Kelvin solid representation, the loss coefficient $\eta$ increase linearly with increased frequency for a viscoelastic body subjected to cyclic loading for the simple reason that the elastic modulus $\mu$ is a shear constant, while the loss modulus $\mu'$ is a linear function of angular frequency.

A viscoelastic body undergoing harmonic loading with circular frequency $\omega$, the oscillating strain is expressed as shown in Equation (2.27) where $\gamma_0$ is the amplitude in strain and $\phi$ is the phase lag in strain response due to the application of stress.

$$\gamma = \gamma_0 \sin(\omega t - \phi)$$

Combining Equation (2.22) and Equation (2.27), the hysteresis loop for the rate-dependent Kelvin solid is obtained, see Figure 2.5. The hysteresis loop becomes rounder as the loss modulus increase, indicating greater damping during the cyclic loading.

### 2.3.2 Rate-Independent Damping

As mentioned, the frequency-dependent nature of the loss factor is a result of the utilization of a viscous dashpot, correlating the stress with the strain rate. However, soils exhibit material damping independent of cyclic nature of the loading, i.e. the frequency or strain rate. By simply eliminating the frequency $\omega$ in Equation (2.23), the relation between the dashpot coefficient and the loss modulus becomes frequency independent. Let $G'_0$ denote a dashpot coefficient, the stress
state in a non-viscous Kelvin model thus can then be expressed

\[ \tau = (G + iG')\gamma \]  \hspace{1cm} (2.28)

Analogically the rate-dependent model, the Fourier transformation is obtained, yielding

\[ \tau = \mu[1 + i\eta]\gamma \]  \hspace{1cm} (2.29)

Hysteretic damping is a widely used notation to account for the damping that is rate-independent and used in the rate-independent material model. This kind of damping is also denoted structural damping, and is what has been used in this thesis along with Rayleigh damping.

![Hysteresis loop in viscoelastic material model. From Ishihara (1996).](image)

### 2.4 Signal Analysis

The derivations in this section are made from several sources, amongst authors as Chopra (2013), Bodén et al. (2014) and Fiche (2008), to which the reader is referred for further in-depth reading. A signal or a time history indicates a recording of a physical quantity varying with time. The analysis of such signals is essential in the field of science and engineering. Basically, three types of signal classes with different nature can be discussed, those are periodic-, random- and transient signals. A periodic signal evaluated at integer multiples of the period \( T \) of a given time instance \( t \) will be equivalent, e.g. a sine wave. Random signals as the name implies, are signal which have arbitrary variation. Fully accurate future description of such signals is not possible from knowledge of the past signal. Transient signals are contrary to periodic and random signals of short duration. As a signal is applied to a system, it has a transient response before it settles to a steady-state response. A signal is regarded transient, if the time of duration is of the same magnitude as the transient response.

The theory of the description of periodic signals was stipulated by Jean-Baptiste Fourier. Essentially, a periodic function may be represented by a sum of sine and cosine terms. This finding allows for the transformation of time histories of signals into the frequency domain, which is advantageous in dynamic applications. The concepts of transformation between domains will be presented subsequent some fundamentals of Fourier analysis.
2.4.1 Fourier Analysis

In this section, the real and complex Fourier series will be briefly discussed.

**Fourier Series**

An arbitrary periodic function can be represented by its harmonics, i.e. an infinite sum of sine and cosine functions as

\[
x(t) = a_0 + \sum_{m=1}^{\infty} a_m \cos(m\omega t) + b_m \sin(m\omega t)
\]  

(2.30)

where the functions have \( m \) whole periods over period \( T \). Theoretically, infinite numbers of terms are required for convergence between the Fourier series representation and the function \( x(t) \). However, few terms are adequate for acceptable convergence in practice. Due to the orthogonality of the sine and cosine functions, the coefficients in the Fourier series can be expressed with dependency of \( x(t) \), see Equation (2.31). The average value of \( x(t) \) is \( a_0 \), while the coefficients \( a_m \) and \( b_m \) indicates the amplitudes of the \( m \)th harmonics of frequency \( m\omega \).

\[
a_0 = \frac{1}{T} \int_0^T x(t) dt \quad a_m = \frac{2}{T} \int_0^T x(t) \cos(m\omega t) dt \quad b_m = \frac{2}{T} \int_0^T x(t) \sin(m\omega t) dt
\]  

(2.31)

**Complex Fourier Series**

A more convenient way to represent an arbitrary excitation is obtained by introducing the complex Fourier series. Recalling Euler’s identity, the relation between complex exponentials and sinusoids is

\[
e^{i\phi} = \cos \phi + i \sin \phi
\]  

(2.32)

representing a point in the complex plane with a real component \( \cos \phi \) and an imaginary component \( \sin \phi \). Euler’s identity describes a point on the unit circle in the complex plane at an angle \( \phi \) to the positive real axis. In practical applications, any signal may be evaluated in the complex plane, containing information of magnitude and phase which are vector components in the complex plane, see Figure 2.6. The real and imaginary component are denoted \( u_r \) and \( u_i \), respectively with the phase \( \phi \) relating the magnitude \( u \) to the positive real axis (MSC Software Corporation, 2016).

![Fig. 2.6 The complex plane. From MSC Software Corporation (2016).](image-url)
The Fourier series may be written in a compact manner as seen in Equation (2.33) with the Fourier coefficients denoted in Equation (2.34).

\[
x(t) = \sum_{-\infty}^{\infty} c_m e^{im\omega t}
\]  

\[
c_m = \frac{1}{T} \int_0^T x(t)e^{-im\omega t} dt
\]  

2.4.2 Fast Fourier Transform

Signals are traditionally observed in the time domain, i.e. the variation of some parameter with time. However, as mentioned in Section 2.4.1, any signal can be generated by summation of infinite number of sinusoids, enabling decomposition of a time signal to a frequency spectrum. This allows for clear distinction of frequency components of the signal that can be eliminated at filtering stage. The frequency domain analysis of the dynamic response of systems will be developed in Section 2.5. The response to arbitrary excitation of a system can not be evaluated analytically except for simple functions applied to the system. Thus, numerical evaluation is a necessity. Introducing numerical evaluation requires truncation of the Fourier integrals over infinite range to finite range. This is equivalent to approximating the arbitrarily time-varying excitation by a periodic function.

Prior digital processing of signals, sampling is mandatory as the information to be manipulated need to be in the form of a discrete sequence of finite values. Signals are generally sampled at regular time intervals, \( \Delta t \). The sampling interval should be small enough both in relation to the periods of the significant harmonics of the excitation and to the natural period \( T_n \) of the system. The former requirement ensures accurate representation of the excitation and of the forced vibrational component of the response, while the latter requirement need to be satisfied to allow the free vibrational response component to be accurately represented. Furthermore, the time period over which the signal is sampled should be extended beyond the duration of the excitation since the peak response of a system may be obtained after the excitation has passed. The duration of the excitation force is denoted \( t_d \) with \( T_n \) being the natural period.

\[
T_0 \geq t_d + \frac{T_n}{2}
\]  

In such case, it will be attained in the first half-cycle of the free vibrations due decay of motion caused by damping. This quiet zone is necessary to attain the desired attenuation of the free vibration (Chopra, 2013); (Kausel and Roësset, 1992).

The inverse of the sampling interval is defined as the sampling frequency, \( f_s = 1/\Delta t \), i.e. the number of samples in one time unit. The sampling frequency need to be chosen with care, as to few samples might introduce the phenomenon of aliasing and improper representation of the time signal. The Nyquist-Shannon theorem describes conditions under which a series of samples correctly represents the original signal. As the sampling function (see Fiche (2008); Bodén et al.
(2014)) introduces a periodicity and thus a replication in the frequency spectrum, the sampling frequency must be great enough to prevent the repeated spectra to overlap. This is referred to as aliasing, which is prevented by applying the Nyquist-Shannon theorem which states that a signal with frequency component less than \( f \) can be entirely restituted provided that the sampling frequency is at least \( 2f \). The minimum sampling frequency fulfilling the condition, is in signal analysis referred to as the Nyquist frequency (Fiche, 2008). Furthermore, to prevent aliasing, the use of anti-aliasing filter may be used in the preprocessing of the signal to attenuate high frequency components of the original signal.

Recall the complex Fourier series from Equation (2.33). By truncation of the series to include only a finite number of harmonic functions as well as numerical evaluation using the trapezoidal rule applied to the values of the integrand in Equation (2.34) at discrete time instants, one can in Equation (2.33) replace the continuous signal \( x(t) \) with an array \( x_n \) describing the discretized signal as a superposition of \( N \) harmonic functions. Thus, Equation (2.33) becomes

\[
x_n = \sum_{m=0}^{N-1} c_m e^{im\omega_0 t}
\]  

in which \( \omega_0 = 2\pi/T_0 \) is the frequency of the first harmonic of the extension of \( x(t) \). The circular frequency of the \( m \)th harmonic is described by \( m\omega \), and \( c_m \) is a complex-valued coefficient defining the amplitude and phase of the \( m \)th harmonic. Furthermore, the coefficients \( c_m \) associated with the array \( x_n \) can be expressed

\[
c_m = \frac{1}{N} \sum_{n=0}^{N-1} x_n e^{-2\pi inm/N}
\]

Equation (2.36) and Equation (2.37) define a discrete Fourier transform (DFT) pair. Note that only positive frequencies are included in the DFT pair, this renders a so called one-sided Fourier expansion. Additionally, the complex Fourier series contains negative frequencies in the two-sided expansion. However, the negative frequencies does not have a physical significance, analogical to the insignificance of frequencies \( f \in (N/2, N-1) \).

Computing the complex Fourier series with \( N \) samples would require \( N^2 \) operations, where one operation refer to one complex multiplication followed by one complex addition. By the use of symmetry of the trigonometric functions involved in the computation of the DFT, computational cost may be reduced since the Fast Fourier Transform (FFT) algorithm enables \( rN \log_r N \) computations. Most efficient calculations are obtained for \( r = 3 \), which is approximately 6% quicker than \( r = 2 \) or 4. However, other advantages are found when \( N \) is a power of 2 or 4 for computers using binary arithmetic, why one should strive to choose \( N \) as a power of 2 (Cooley and Tukey, 1965).
2.5 Frequency Domain Analysis

In the following section, the mathematical expressions to properly describe the conceptual understanding of tools used in the frequency domain are described. Herein, the frequency domain method is described. The derivations are made in accordance with Chopra (2013) and Bodén et al. (2014)

The frequency-domain method for analysis of response in accordance with the theory of linear elasticity is a fast, alternative way of evaluating response due to dynamic excitations. The complex frequency-response function (FRF) obtained, contains the steady-state response. For non-periodic excitations, the response is represented by the Fourier integral that involves the Fourier transform of the excitation. To understand the conceptual meaning of the FRF, a viscously damped SDOF system subjected to an external force \( p(t) \) is considered. Recall the EOM from Section 2.2 together with the compact representation of sinusoidal and cosine forces together given by Euler’s identity as shown in Equation (2.38).

\[
m\dddot{u} + c\dot{u} + ku = e^{i\omega t} \quad (2.38)
\]

The steady-state response of the system is harmonic motion at forcing frequency \( \omega \) is shown in Equation (2.39). The first and second derivatives are shown in Equation (2.40)

\[
\begin{align*}
    u(t) &= H(\omega)e^{i\omega t} \quad (2.39) \\
    \dot{u}(t) &= i\omega H(\omega)e^{i\omega t}, \quad \ddot{u}(t) = -\omega^2 H(\omega)e^{i\omega t} \quad (2.40)
\end{align*}
\]

Differentiation of Equation (2.39) twice, and insertion to Equation (2.38) results in

\[
H(\omega)e^{i\omega t}(-\omega^2 m + i\omega c + k) = e^{i\omega t} \quad (2.41)
\]

from which the complex frequency-response function \( H(\omega) \) is obtained:

\[
H(\omega) = \frac{1}{-\omega^2 m + i\omega c + k} \quad (2.42)
\]

The complex frequency-response function describes the steady-state response of the system enforced by a harmonic force of unit amplitude. Furthermore, it depends on the system parameters \( k, \omega_n \) and \( \zeta \).

Periodic excitation can be written as a Fourier series in which the appearing frequencies are multiples of the basic frequency \( 1/T \). The basic frequency is denoted the fundamental frequency while its multiples are called the harmonics. Thus, a periodic excitation may be described by its fundamental and harmonics. However, for arbitrary excitation, i.e. non-periodic excitation, the excitation is represented by the Fourier integral. The response of a linear system to such a excitation seen in Equation (2.44) is determined by superposition of the individual harmonic excitation terms of the Fourier integral, see Equation (2.43)

\[
p(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P(\omega)e^{i\omega t}d\omega \quad \text{where} \quad P(\omega) = \int_{-\infty}^{\infty} p(t)e^{-i\omega t}dt \quad (2.43)
\]
\[ u(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} U(\omega)e^{i\omega t} d\omega \quad \text{where} \quad U(\omega) = H(\omega)P(\omega) \quad (2.44) \]

### 2.6 Time Domain Analysis

In this section, the mathematical expressions describe the procedure used in modal analysis and direct time integration schemes. The derivations are made in accordance with Chatzi and Abiatti (2017), Chopra (2013) and Dassault Systèmes (2014).

#### 2.6.1 Mode Superposition

To determine the time-history response of a MDOF, subjected to a dynamic load, it is appropriate to perform a modal analysis. In this chapter, this method will briefly be explained and is the fundamentals of any method containing modal superposition.

Recall the EOM for MDOF in Equation (2.3) for a classically damped system, i.e. Rayleigh damping, subjected to a time varying load vector \( P(t) \). The general procedure to compute the total time history response is based on solving the structural eigenvalue problem. Thus, the eigenfrequencies \( \omega_n \) and mode shape vector \( \phi \) of the system are obtained. The mode shape vectors can be assigned to a matrix column-wise, i.e. modal matrix \( \Phi \), where the number of columns represent the number of eigenmodes. In order to decompose the system, the vectors in Equation (2.45) are essential to state, where \( i \) denotes the mode number. Furthermore, the notation \( q \) indicates moving from DOF to modal coordinates.

\[
\begin{align*}
\dot{u}(t) &= \sum_{i=1}^{N} \phi_i(q_i(t) = \Phi q(t) \\
\ddot{u}(t) &= \sum_{i=1}^{N} \phi_i\ddot{q}_i(t) = \Phi \ddot{q}(t) \\
\end{align*}
\]  
(2.45)

Then, the problem is separated into each mode by decomposing the system. Inside these modes, the time history is evaluated for each mode expressed in modal coordinates. Eventually, the total response of the system is composed by a linear combination of the individual modal response shown in Equation (2.45). This is possible due to the orthogonality theorem of modes, i.e. linear independence, and is shown in Equation (2.46) for mode number \( i \) and \( j \).

\[
\omega_i \neq \omega_j \quad \rightarrow \quad \begin{cases} 
\phi_i^T M \phi_j = 0 \\
\phi_i^T C \phi_j = 0 \\
\phi_i^T K \phi_j = 0
\end{cases}
\]  
(2.46)

The mathematical derivations behind mode superposition is based on creating a set of uncoupled SDOF from the coupled MDOF. By inserting Equation (2.45) in the original Equation (2.3), and premultiplication with \( \phi_j^T \) will render Equation (2.47).


\[ \sum_{i=1}^{N} \phi_j^T M \ddot{\phi}_i(t) + \sum_{i=1}^{N} \phi_j^T C \dot{\phi}_i(t) + \sum_{i=1}^{N} \phi_j^T K \phi_i(t) = \phi_j^T P(t) \]  

(2.47)

Note that the mode shape vectors, whenever \( i \neq j \), are orthogonal which leads to the following simplified expression in Equation (2.48). This expression is equivalent to the coupled EOM mentioned previously. The modal stiffness, damping, mass and load are denoted with subscript \( n \) for each mode and are emphasized in Equation (2.48).

\[ \phi_j^T M \ddot{\phi}_j(t) + \phi_j^T C \dot{\phi}_j(t) + \phi_j^T K \phi_j(t) = \phi_j^T P(t), \quad \text{where} \quad n = 1, 2, ..., N \]  

(2.48)

In order to represent the modal SDOF in a compact notation, another significant consequence of the orthogonality theorem implies that square matrices such as \( M \), \( C \) and \( K \) multiplied with the modal matrix \( \Phi \), and premultiplied with the transpose of it, is a diagonal matrix.

\[ \hat{M} \equiv \Phi^T M \Phi, \quad \hat{C} \equiv \Phi^T C \Phi \quad \text{and} \quad \hat{K} \equiv \Phi^T K \Phi \]  

(2.49)

Furthermore, if they are positive definite, the diagonal terms will be positive which, indeed, is a great consequence. In the literature, the diagonal matrices are usually referred to as the modal stiffness-, damping- and mass matrix, which are distinguished with a hat (\( \hat{\cdot} \)) in this thesis. Thus, the uncoupled EOM can be expressed as in Equation (2.50).

\[ \hat{M} \ddot{q} + \hat{C} \dot{q} + \hat{K} q = \hat{P}(t) \]  

(2.50)

Note that the excitation vector is changed, and is represented by a modal excitation vector \( \hat{P}(t) \). The modal excitation depends on the spatial distribution and time variation of the load. An intuitive way of describing this is that, symmetric loads may never cause antisymmetric modes, independent of the forcing frequency.

### 2.6.2 Direct Time Integration Methods

Direct time integration (DTI) methods aims at approximating continuous problem similar to FEM. However, instead of discretizing space, DTI methods attempts to discretize time as the name implies. Thus, the aim is to satisfy the EOM at discrete points in time. In between the time intervals, a particular variation of displacements, velocities and accelerations is assumed with interpolation functions. The two main categories of DTI are the implicit and explicit method. An explicit method uses only information from the previous time step for solving the discretized EOM. On the contrary, implicit methods additionally include information from the current time instance to solve the problem. The major benefit of using an implicit integration scheme compared to explicit is stability. Many of the implicit schemes are able to run with any arbitrarily large time step, i.e they are unconditionally stable. Explicit algorithms usually have a critical time step that ensures stability of the solution. Stability of numerical procedures ensures that the solutions will not diverge (blow-up), albeit accuracy of the solutions is not guaranteed.
CHAPTER 2. THEORETICAL BACKGROUND

The Newmark Method

Below, discretization of the IVP with a time step $\Delta t = t_{k+1} - t_k$ is shown in Equation (2.51) where subscript $u_{k+1}$ indicates displacement vector at $t_{k+1}$. Furthermore, the initial displacements and velocities must be determined.

$$\begin{cases} M\dddot{u}_{k+1} + C\dot{u}_{k+1} + Ku_{k+1} = P(t_{k+1}) \\ u(t_0) = u_0, \quad \dot{u}(t_0) = \dot{u}_0 \end{cases} \quad (2.51)$$

One of the most widely used time stepping algorithms for structural dynamics is the Newmark’s method. It is based on the interpolation equations shown in Equation (2.52) derived by Newmark in 1959. Furthermore, it is the basis for the Hilber-Hughes Taylor (HHT) algorithm (1977), which is implemented in the commercial FEM software Abaqus (Dassault Systèmes, 2014).

$$\begin{cases} u_{k+1} = u_k + \dot{u}_k \Delta t + \ddot{u}_k (\frac{1}{2} - \beta) \Delta t^2 + \dddot{u}_{k+1} \beta \Delta t^2 \\ \dot{u}_{k+1} = \dot{u}_k + \ddot{u}_k (1 - \gamma) \Delta t + \dddot{u}_{k+1} \gamma \Delta t \end{cases} \quad (2.52)$$

The notations $\beta$ and $\gamma$ are parameters of the Newmark’s method. In fact, they act as weights for approximating the acceleration in between two discrete time instances. These parameters can be tuned in order to balance stability and accuracy. However, $\gamma$ is usually given as $1/2$ which provides second order accuracy of the procedure, i.e. quadratic convergence. Instead, $\beta$ is usually adjusted in order to make the algorithm explicit or implicit in accordance to Equation (2.53).

$$\begin{cases} \beta = 0 : \text{explicit algorithm} \\ \beta \neq 0 : \text{implicit algorithm} \rightarrow \text{inversion of } K \end{cases} \quad (2.53)$$

There are two specific combinations of these parameters which are of particular interest. The first is when $\gamma = 1/2$ and $\beta = 1/4$, which makes the algorithm implicit and unconditionally stable. The second is $\gamma = 1/2$ and $\beta = 1/6$, which also is an implicit method, albeit there is a critical time step which ensures stability.

Considering linear elastic problems, the Newmark algorithm can be interpreted in a multi-step scheme by the means of predictors and correctors. Predictors depend on previous time steps and correctors determine the current time step solution. Recalling the interpolation formulas in Equation (2.52), one can simply estimate the displacement and velocities by predictors. These are denoted with tilde ($\tilde{\cdot}$) and are based on information from the previous time step, i.e. $\tilde{u}_{k+1}$ and $\tilde{\dot{u}}_{k+1}$ according to Equation (2.54). This, indeed, is the first step of Newmark’s algorithm and are known a priori.

$$\begin{cases} \tilde{u}_{k+1} = u_k + \dot{u}_k \Delta t + \ddot{u}_k (\frac{1}{2} - \beta) \Delta t^2 \\ \tilde{\dot{u}}_{k+1} = \dot{u}_k + \ddot{u}_k (1 - \gamma) \Delta t \end{cases} \quad (2.54)$$
Further, the governing discretized IVP in Equation (2.51) can be rewritten with help of predictors and addition of the unknown current acceleration quantity $\ddot{u}_{k+1}$ shown in Equation (2.55).

$$M\ddot{u}_{k+1} + C(\dot{u}_{k+1} + \ddot{u}_{k+1}\gamma\Delta t) + K(\dot{\ddot{u}}_{k+1} + \ddot{u}_{k+1}\beta\Delta t^2) = P(t_{k+1})$$  \hspace{1cm} (2.55)

The second step consists of solving the linear problem by rearranging Equation (2.55) and inverting the mass-, damping- and stiffness matrices in accordance to Equation (2.56) to solve for the current acceleration. The load vector $P(t_{k+1})$ is given for all time instances.

$$\ddot{u}_{k+1} = \left( M + C\gamma\Delta t + K\beta\Delta t^2 \right)^{-1} \left( P(t_{k+1}) - K\ddot{u}_{k+1} - C\dot{u}_{k+1} \right)$$  \hspace{1cm} (2.56)

The stiffness matrix depends on the parameter $\beta$ and is excluded to invert for explicit methods. Hence, only the mass- and damping matrix will be inverted. Furthermore, if damping is neglected and a lumped mass matrix is formulated, the solution procedure will only consist of inverting a diagonal matrix which indeed is a trivial process. Thus, explicit methods are generally faster to compute the acceleration response for linear systems.

The final step in Newmark’s method is to correct the estimates, i.e. the predictors, by addition of the computed current acceleration. This is shown in Equation (2.57) and is referred to as correctors. The correctors are then used as prior information for next time instances in the integration scheme.

$$\begin{cases} u_{k+1} = \ddot{u}_{k+1} + \ddot{u}_{k+1}\beta\Delta t^2 \\ \ddot{u}_{k+1} = \dot{u}_{k+1} + \ddot{u}_{k+1}\gamma\Delta t \end{cases}$$  \hspace{1cm} (2.57)

### Stability and Distortions

The analytical solution of an undamped continuous system is only dependent of the properties of the system, e.g. mass and stiffness which renders the eigenfrequency. However, for a numerical integration algorithm the solution is not only dependent of the system properties but also the discretization, i.e. time step $\Delta t$. Thus, one may introduce distortions of the eigenfrequency and damping ratio of the discretized system depending on the chosen time step. For zero damping, Newmark’s method is conditionally stable if and only if Equation (2.58) is fulfilled, where $\omega_{max}$ is the maximum frequency of interest. Furthermore, unconditional stability is ensured when Equation (2.59) is valid.

$$\gamma \geq \frac{1}{2}, \quad \beta \leq \frac{1}{2} \text{ and } \quad \Delta t = \frac{1}{\omega_{max}\sqrt{\frac{\gamma}{2} - \beta}}$$  \hspace{1cm} (2.58)

$$2\beta \geq \gamma \geq \frac{1}{2}$$  \hspace{1cm} (2.59)

Errors are introduced in numerical integration schemes such as the Newmark method for $\gamma \geq 1/2$. These are referred to as numerical damping and period elongation, i.e. larger period of oscillation and damping than analytically. However, because of the unconditional stability it is widely used due to its robustness.
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Hilber-Hughes-Taylor (α-method)

The HHT-α algorithm is based on the Newmark method. However, it shifts the EOM with the parameter α which modulates the algorithmic damping, where α ∈ [−1/3, 0]. The parameters governing Newmark’s method are reformulated according to Equation (2.60). The HHT method is unconditionally stable, albeit damping and frequency bias are introduced depending on the choice of α. The largest benefit of using the HHT method compared to Newmark’s method is that the α-method introduces less damping in the lower modes. Nevertheless, the α-method manages to damp the higher modes similar to the Newmark method. Also, it ensures second order accuracy, meaning there is a quadratic convergence rate as the time step is refined.

\[ \beta = \frac{(1 - \alpha)^2}{4}, \quad \gamma = \frac{(1 - 2\alpha)}{2} \quad (2.60) \]

2.7 Nonlinearity

Linear problems provide proper approximations of structural systems for practical purposes. However, they are limited to fundamental assumptions regarding small displacements and linear elastic behavior. On the contrary, behavior of nonlinear structural problems includes a wide range of phenomena, which includes material nonlinearity, contact nonlinearity and large displacements. This, indeed, provides a more proper representation of reality. A significant drawback of nonlinear analysis is that the principle of superposition no longer is applicable, i.e. results can no longer be scaled nor be superimposed from different load cases. In the following chapter about nonlinearity, the derivations are made in accordance with Chatzi and Abiatti (2017), Chopra (2013), Cook et al. (2007) and Dassault Systèmes (2014).

2.7.1 The Newton-Raphson Method

The Newton-Raphson Method, also referred to as Newtons Method, is a numerical method for approximating the roots of a polynomial, i.e. finding the zeros of a function. The most generic form of the algorithm for multiple dimensions will be described in the following section. Given an arbitrary nonlinear real valued vector depending on \( x \), the aim is to find the roots denoted \( \hat{x} \) which satisfies Equation (2.61).

\[ \hat{x} : f(\hat{x}) = 0 \quad (2.61) \]

The solution does not necessarily have an analytical expression and is found by following an iterative procedure. Furthermore, it is based on linearization around a given trial \( x_j \) based on prior information, i.e. expansion by its Taylor series where second and higher order terms are neglected. Then, the linearized function is assumed to be zero at the updated value \( x_{j+1} \). This is done in order to find the increment \( \Delta x_j \) of the solution with respect to the initial guess. The mathematical expressions are shown in Equation (2.62) where the partial derivative of the function \( f \) with respect to \( x \) is called the Jacobian.

\[ f(x_j + \Delta x_j) \approx f(x_j) + \frac{\partial f}{\partial x}|_{x_j} \Delta x_j = 0 \quad \Rightarrow \quad \Delta x_j = -\left( \frac{\partial f}{\partial x}|_{x_j} \right)^{-1} f(x_j) \quad (2.62) \]
Then, the estimate of $\hat{x}$ can be updated with respect to the increment according to Equation (2.63). These steps are repeated until the increment $\Delta x_j$ is smaller than a given tolerance. Furthermore, the Newton-Raphson method converges with a quadratic rate near the end of the iteration process, i.e. the error in the $j + 1$ iterate is less than the square of the error in the previous step $j$.

$$x_{j+1} = x_j + \Delta x_j$$  \hspace{1cm} (2.63)

### 2.7.2 Nonlinear Dynamics

In nonlinear dynamics problems, the nonlinearity usually encounters dealing with nonlinear stiffness terms. For implicit time stepping algorithms, the Newton-Raphson method is coupled together with the Newmark method and is appropriate to analyze the nonlinear time history response. Recall the discretized IVP with a time step $\Delta t = t_{k+1} - t_k$ in Equation (2.51) and introduce the nonlinear terms depending on displacements in an internal restoring force vector $r(u_{k+1})$. Thus, the discretized IVP ends up as a nonlinear problem in the form of a minimization problem at each load step.

$$\begin{bmatrix}
M\ddot{u}_{k+1} + C\dot{u}_{k+1} + r(u_{k+1}) = P(t_{k+1}) \\
u(t_0) = u_0, \quad \dot{u}(t_0) = \dot{u}_0, \quad \ddot{u}(t_0) = \ddot{u}_0
\end{bmatrix}$$  \hspace{1cm} (2.64)

At the global level, a residual vector $\varepsilon$ is assigned which is set to zero in accordance to Equation (2.65). This is the residual of the balance EOM, in which the aim is to find the acceleration increment $\Delta \ddot{u}_{k+1}$ based on linearization, similar to the Newton-Raphson method.

$$\varepsilon_{k+1} = P(t_{k+1}) - r(u_{k+1}) - Cu_{k+1} - Mu_{k+1} = 0$$  \hspace{1cm} (2.65)

In Equation (2.66) the acceleration increment $\Delta \ddot{u}_{k+1}$ is solved by multiplying the residual and the inverted Jacobian, which is updated in every step for implicit schemes. The solutions are then corrected with respect to the acceleration increment. In Equation (2.67), the superscript $j$ denotes the iteration number in the Newton-Raphson method. The residual in Equation (2.65) is then updated with the corrected solutions, and the procedure continues while the residual is greater than a given tolerance.

$$\Delta \ddot{u}_{k+1}^{(j)} = \left( M + C\gamma \Delta t + K\beta \Delta t^2 \right)^{-1} \varepsilon_{k+1}^{(j)}$$  \hspace{1cm} (2.66)

$$\begin{bmatrix}
u_{k+1}^{(j+1)} = u_{k+1}^{(j)} + \Delta \ddot{u}_{k+1}^{(j)} \beta \Delta t^2 \\
_{k+1}^{(j+1)} = \dot{u}_{k+1}^{(j)} + \Delta \ddot{u}_{k+1}^{(j)} \gamma \Delta t \\
_{k+1}^{(j+1)} = \ddot{u}_{k+1}^{(j)} + \Delta \ddot{u}_{k+1}^{(j)}
\end{bmatrix}$$  \hspace{1cm} (2.67)

Given a displacement trial $u_{k+1}^{(j)}$ to the local elements, the response of the global nonlinear system depends upon the input parameter. The outputs, i.e. restoring force and local tangent stiffness, are used at the global element level. They are assembled to the global tangent stiffness $K$ which enters the Jacobian. Furthermore, the global restoring force vector $r(u_{k+1})$ enters the residual.
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balance EOM. These steps are essential in order to continue the minimization process. The aforementioned distinction between implicit and explicit methods, i.e. the value of $\beta$, indicates if the inverted Jacobian needs to include the global tangent stiffness. The computational cost will, indeed, be affected depending on the choice of time integration method. For explicit methods, the modified Newton-Raphson is used which only requires inversion of the global tangent stiffness once.

Generally, there are different convergence criteria for nonlinear dynamic analysis. However, the most fundamental one is that the norm of the force residual should be near the computer (machine) tolerance because small quantities are being dealt with. The force residual criteria is emphasized in Equation (2.68).

$$10^{-8} \leq \| \mathbf{e}^{(j)} \| \leq 10^{-3}$$

(2.68)

The schematics of an implicit nonlinear dynamic analysis including the full Newton-Raphson iteration is visualized in Figure 2.7.

![Fig. 2.7 Full Newton-Raphson Iteration. From Chatzi and Abiatti (2017).](image)

2.7.3 Contact Nonlinearity

The contact problem arises when different parts of structures separate, slide or collide. Contact forces are transmitted along the boundary of adjacent structures and solids, very much like soil-structure interaction during dynamic analysis of bridges. Depending on the type of relative motion, different forces are transmitted such as normal and tangential forces. Essential for contact analysis of abutment-backfill interaction is that both elements must be regarded as deformable bodies. Contact phenomena in FEM are indeed nonlinear, as a matter of fact they are highly so due to sudden and large changes in stiffnesses.

A simple academic problem will be described in order to emphasize the computational difficulty with contact nonlinearity, which is shown in Figure 2.8. Consider a massless rigid L-shaped block hanging on a spring, subjected to a force $P$, which makes contact with a rigid horizontal surface. The mathematical model makes contact with the surface whenever $u = g$. Thus, an increase in the force $P$ will not cause a further displacement $u$ in the mathematical model. However,
computational models are approximations and dealing with infinite stiffness will evidently become a problem. In the computational model the contact problem is being dealt with by adding a nonlinear spring with very large stiffness \( k_g \) for \( u > g \) and zero stiffness for \( u < g \). The load displacement relation is shown in Equation (2.69).

\[
ku = P \quad \text{for} \quad P < kg \\
(k + k_g)u = P \quad \text{for} \quad P > kg
\]  

(2.69)

These constraint \( u \leq g \) can be enforced approximately by the means of penalty fashion, or exactly by the method of Lagrange multipliers. Considering this example, the method of Lagrange multipliers is shown in Equation (2.70), which is enforced when \( P > kg \). The force constraint is denoted \( \lambda \). In commercial FE-software the augmented Lagrange method is used, which couples the penalty method with the Lagrange method.

\[
\begin{bmatrix}
k & 1 \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
u \\
\lambda
\end{bmatrix}
= \begin{bmatrix}
P \\
g
\end{bmatrix}
\Rightarrow
\begin{align*}
u &= g \\
\lambda &= P - kg
\end{align*}
\]  

(2.70)

Generally, nodes along the surface of adjacent finite elements does not coincide during the analysis. Thus, penetration may occur depending on choice of contact algorithm. However, zero penetration can be enforced by an impenetrability condition in an average sense. The commercial FE-software Abaqus handles this with help of Lagrange multipliers, penalty algorithms, or a combination of these, i.e. augmented Lagrange method.

![Diagram](image)

(a) Mathematical model. (b) Computational model. (c) Force vs. displacement for computational model.

Fig. 2.8 Contact nonlinearity. From Cook et al. (2007).

## 2.8 Wave Propagation Theory in Elastic Media

Wave motion is described by a collective phenomenon, where the individual material particles oscillate about a state of equilibrium and to which it returns after the passage of a wave (Bodare, 1996). As shown in Section 2.3.1, hysteresis damping is a type of damping that is inherent in the soil. Additionally, soil materials also contribute with geometric (radiation) damping due to diffusion of vibratory energy as waves originated from the source propagate to infinity. Thus, wave propagation theory is essential when constructing models including radiation damping. Generally, soil materials are characterized by non-linearity. However, for railway induced vibrations, relatively low strain levels in the soil prevails. Thus, the constitutive behavior of the soil material can be represented by a linear elastic relation. In the derivations to follow, it is assumed that the soil is homogeneous and isotropic due to the absence of body forces (Houbrechts et al., 2011).
The fundamentals of wave propagation theory have been established by cross reading the work of authors such as Graff (2012); Houbrechts et al. (2011); Bodare (1996) and thus no further citation will be made.

Using a Cartesian frame of reference, the components of the displacement vector at position \( x \) at time instance \( t \) can be denoted as \( u_i(x, t) \). The components of the small strain tensor are related to the displacement through

\[
\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{2.71}
\]

where \( u_{i,j} \) is the differentiation of \( u_i \) with respect to the \( j \)-th coordinate. In an elastic medium, the dynamic equilibrium is satisfied as

\[
\sigma_{ji,i} + \rho b_i = \rho \ddot{u}_i \tag{2.72}
\]

with \( \rho b_i \) and \( \rho \) are the body forces and density, respectively. The constitutive relation between the Cauchy stress tensor \( \sigma_{ij} \) and the small strain tensor \( \varepsilon_{ij} \) is expressed

\[
\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{i,j} + 2\mu \varepsilon_{i,j} \tag{2.73}
\]

where \( \varepsilon_{kk} \) is the volumetric strain. The Kronecker delta is denoted \( \delta_{ij} \) and \( \lambda \) and \( \mu \) are the Lamé constants. These constants are related to the modulus of elasticity and Poisson’s ratio as follows.

\[
\lambda = \frac{E\nu}{(1 + \nu)(1 - 2\nu)}, \quad \mu = \frac{E}{2(1 + \nu)} \tag{2.74}
\]

In the frequency domain, frequency independent hysteretic material damping in the soil can be modeled by the use of the correspondence principle by introducing energy dissipation through complex Lamé coefficients. Navier’s equations, see Equation (2.75), originate by introducing the constitutive equations in Equation (2.73) and the linearized strain-displacement relations in Equation (2.71) into the equilibrium equations in Equation (2.72).

\[
(\lambda + \mu)u_{i,ij} + \mu u_{i,jj} + \rho b_i = \rho \ddot{u}_i \tag{2.75}
\]

Complemented with initial and boundary conditions, the elastodynamic problem is defined. Introducing vector notation, the Navier’s equation takes the form

\[
(\lambda + \mu)\nabla \nabla \cdot \mathbf{u} + \mu \nabla^2 \mathbf{u} = \rho \ddot{\mathbf{u}} \tag{2.76}
\]

The solution to Navier’s equation is in classical elastodynamics attained by Helmholtz decomposition into a set of uncoupled partial differential equations which define the propagation of dilatation and shear waves, respectively. The dilatation waves refers to the volumetric component and the shear waves corresponds to the distortional component, and they travel at different speeds. In seismological literature, the dilatational waves and shear waves may be referred to as P- and S-waves, respectively. Their respective wave velocities are defined in Equation (2.77) and
2.8 WAVE PROPAGATION THEORY IN ELASTIC MEDIA

Equation (2.78).

\[ C_p = \sqrt{\frac{\lambda + 2\mu}{\rho}} \]  \hspace{1cm} (2.77)

\[ C_s = \sqrt{\frac{\mu}{\rho}} \]  \hspace{1cm} (2.78)

The particle motion in dilatational waves are parallel, whilst for shear waves it is perpendicular the direction of wave propagation. In an infinite soil, the dilatation- and shear waves exists independent of each other and it is evident that the dilatation wave velocity is greater. A schematic illustration of the different wave types are seen in Figure 2.9. As the waves propagate further away from the source of vibration, the energy dissipates from the source. This is known as Sommerfeld’s radiation condition.

Previously presented types of waves are present in infinite soil medium. For semi-infinite media, as for instance half spaces or layered soils, another type of wave appears. Semi-infinite models have a free edge which is stress-free. As the previously mentioned body waves, i.e. the dilatational and shear waves, hits the free edge, particles displace in three directions and the wave reflects. Hence, two types of issues arise. One type is caused by the horizontal component of the shear wave which governs the particle motion at the free edge. The second type of issue is originated from the dilatation wave as well as the vertical component of the shear wave. A wave propagating towards the free edge with acute angle will reflect against the edge. The reflection of an impinging dilatational wave which converts to a vertical shear wave is denoted mode conversion.

Besides body waves, surface waves exist in semi-infinite medium. Similar to wave propagation on water surfaces, the Rayleigh wave propagates on the surface of the soil media. The Rayleigh waves travels at speed below the shear wave velocity. The particle motion of Rayleigh waves is characterized by an ellipse where the minor axis is parallel the direction of wave propagation. This elliptical motion is governed by the phase shift of \(\pi/2\) between the vertical and horizontal translations. Furthermore, in the interface between two elastic materials, surface waves may appear. These are denoted as Stoneley waves and is similar to the Rayleigh wave.
2.9 Impedance Functions

Foundations exhibit oscillatory response when subjected to dynamic loads. The oscillation depends on the nature and deformability of the underlaying supporting ground, the geometry and inertia of the foundation and evidently, the characteristics of the dynamic excitation. As mentioned in Section 2.2, it is essential to estimate the dynamic spring and dashpot coefficients in dynamic response analysis. There exist various computational methods to obtain these parameters for the foundation, amongst which are analytical and semi-analytical methods; dynamic finite-element methods; combined analytical-numerical methods and approximate techniques. This section is dedicated to briefly introduce the concept of impedance function. The following derivations are based on Fang (1991).

Assume the foundation as a rigid body possessing three translational and three rotational degrees of freedom in the three-dimensional space. Each of these degrees of freedom can be represented by a complex-valued impedance function as will become apparent. Initially, consider a single degree of freedom system shown in Figure 2.10a allowing translation in the vertical direction. The system is subjected to a harmonic excitation $p(t)$. The rigid block of mass $m$ is supported by a horizontal linearly deforming soil layer, which generates equal and opposite reactions distributed in some unknown manner with the unknown resultant $f(t)$. As the foundation experience a vertical harmonic displacement $u(t)$, the supporting ground have identical displacement since the bodies are in contact. From Figure 2.10a, the dynamic equilibrium of the block and the supporting ground is given as Equation (2.79) and Equation (2.80), respectively.

\[ f(t) + m\ddot{u}(t) = p(t) \]  
\[ f(t) = Zu(t) \]

where $Z$ is the dynamic vertical impedance which defines the dynamic force-over-displacement ratio. By combining the dynamic equilibria, one obtains

\[ m\ddot{u}(t) + Zu(t) + p(t) = 0 \]  
(2.81)

The steady-state solution $u(t)$ for a harmonic excitation with frequency $\omega$ is harmonic, and theoretical and experimental results reveal that response of the block is out of phase with the excitation with same frequency. Thus, the introduction of complex notation is motivated accompanied by its computational ease. From Equation (2.80), the dynamic impedance may be expressed in complex form with respect to the frequency $\omega$ as

\[ Z(\omega) = \frac{\bar{f}}{\bar{u}} = \bar{k}(\omega) + i\omega c(\omega) \]  
(2.82)

Combining Equation (2.82) and Equation (2.81) yields the equation of motion of a simple oscillator with mass, spring and dashpot. The response is then obtained as

\[ u(t) = \frac{p(t)}{-m\omega^2 + \bar{k}(\omega) + i\omega c(\omega)} \]  
(2.83)
2.10 The Standard Viscous Boundary

The real component of the impedance is denoted the dynamic stiffness and reflects the stiffness and inertia of the supporting soil where its frequency dependency is attributed to the influence that frequency exerts on inertia. To a good approximation, soil properties are frequency independent. The imaginary component of the impedance is composed as the product of the circular frequency and the dashpot coefficient. This reflects both radiation damping and the material damping in the system. The material damping is considered by an approximation based on the correspondence principle of viscoelasticity, see Equation (2.84), where $\zeta$ is the damping ratio of the soil material.

$$C_{\text{hystereic}} = \frac{2k}{\omega \zeta}$$

The soil response to a vertically oscillating foundations can thus be presented by the complex frequency-dependent dynamic impedance $Z(\omega)$, or equivalently by the frequency-dependent spring and dashpot coefficients $\bar{k}(\omega)$ and $c(\omega)$, respectively. The elaborated concept of impedance functions is, evidently applicable for the remaining degrees of freedom. Along with lateral and longitudinal impedances, there exists rocking impedances about the long and short axis of the foundation, as well as torsional impedance about the vertical axis. In embedded foundations and piles, there also exists cross-coupling impedances which for shallow foundations are negligibly small. However, these may not be neglected in piles.

Fig. 2.10 Dynamic Equilibrium and Impedance Function. From Fang (1991).

2.10 The Standard Viscous Boundary

Considering dynamic problems as wave propagation problems in an infinite solid may be advantageous. For such problems, the radiation of energy from the vibratory source into the far field can be properly accounted for. Furthermore, when aiming to find closed form analytical solutions for homogeneous systems, the concept of infinity is mathematically convenient. A method developed by Lysmer and Kuhlemeyer (1969) enables the approximation of an infinite system by a finite system by implementing viscous boundary conditions. Considering a boundary enclosing all sources of disturbances and irregularities in geometry, the energy dissipation will dissipate from the interior of the enclosed boundary towards an exterior region, surpassing the boundaries. From
this simple easy-to visualize model, it becomes apparent to define a method from which the dynamic response of the interior region of a finite model, constituted solely by an interior region, subjected to boundary conditions is able to ensure that all energy dissipated to the boundary is absorbed. This prevents waves from reflecting back into the computational domain, which is essential in dynamic analysis. The boundary conditions can analytically be expressed

\[
\begin{align*}
\sigma &= a \rho C_p \dot{w} \\
\tau &= b \rho C_s \dot{u}
\end{align*}
\]  

(2.85)

where \(\sigma\) and \(\tau\) are the normal and shear stress, respectively; \(a\) and \(b\) are some dimensionless constants and \(\rho\) is the mass density; \(\dot{w}\) and \(\dot{u}\) are the normal and tangential velocities, respectively and \(C_p\) and \(C_s\) are the dilatation and shear wave velocities, respectively. This type of boundary conditions corresponds to a situation where dashpots normal and tangential to the boundary is applied. The parameters \(a\) and \(b\) is defined by studying the reflection of elastic waves at the viscous boundary. Thus, the ability of the viscous boundary to absorb impinging elastic waves are studied. The ratio between the transmitted energy of the reflected waves and the transmitted energy of an incident wave constitutes a good measure for measuring the absorption. Different values of \(a\) and \(b\) renders different energy ratios which depends on the incident angle \(\theta\) and Poisson’s ratio \(\nu\). Lysmer and Kuhlemeyer (1969) showed that the maximum absorption is obtained when \(a = b = 1\), and that the absorption can not be perfect for all ranges of incident angles. Almost perfect absorption is obtained when \(\theta > \pi/6\) for \(a = b = 1\) while it for smaller angles appear some reflections. The viscous boundary is more efficient in absorbing the energy carried by dilatation waves rather than shear waves, although both have close to full absorption effectivity. Boundary waves have not been considered in previously proposed boundary conditions, and some irregularities is to be expected. The relative effect of the boundary waves decreases with increased frequency and increased computational domain. This is referred to as the standard viscous boundary, and it provides a method which absorb elastic wave to great accuracy.

Reflected surface waves may as mentioned induce irregularities which is not absorbed by the standard viscous boundary. Such waves, i.e. Rayleigh waves can not be completely absorbed by the standard viscous boundary. Hence, to increase the accuracy, the Rayleigh wave boundary may be implemented. However, it may not be implemented for transient problems, as it is frequency dependent and by increasing the length of the boundary, the effect of Rayleigh waves diminishes (Lysmer and Kuhlemeyer, 1969).

### 2.11 Simplified Modeling of Confining Stratum

Dynamic response of earth retaining structures, such as the abutment-backfill interaction of portal frame bridges, involves a complex soil-wall interaction problem. A model representing the dynamic behavior of a soil-retaining wall system within the theory of elasticity was proposed by Scott (1973). In this simplified model, the soil is regarded as a one-dimensional shear beam attached to the wall by Winkler springs representing the soil-wall interaction (Jain and Scott, 1989). Scott’s model was applicable for stratum of finite extension retained by straight vertical walls at the ends.
The far-field action of the stratum was adapted through a vertical cantilever shear-beam along with massless linear springs connecting the shear-beam to the wall. The material and geometrical properties of the shear-beam corresponds to those of the soil, where the springs are defined by their stiffness according to Equation (2.86).

\[
k = \frac{0.8(1 - \nu) G}{1 - 2\nu H}
\]

The height of the shear-beam, i.e. the stratum, is defined as \( H \) and where the shear modulus and Poisson’s ratio are defined as \( G \) and \( \nu \), respectively. The characteristics of ground motion is not included in the spring stiffness, and the only damping for the model is inherent in the shear-beam itself. Furthermore, as \( \nu \to 0.5 \), \( k \) tends to infinity, which introduce a major drawback in the model. As Scott’s model is unable to transfer forces vertically by horizontal shearing for the medium between the wall and far field, Veletsos and Younan (1995) proposed a model for systems with straight walls. In this model, the medium may correctly be modeled by a series of semi-ininitely long, elastically supported horizontal bars with distributed mass. A bar in harmonic motion provides a dynamic stiffness which represent the amplitude of the harmonic force required to yield a steady-state displacement of unit amplitude. The dynamic stiffness of such bars is complex and is dependent of the bar characteristics and the forcing frequency. For a viscoelastic bar with frequency-independent damping, the dynamic stiffness \( k_n \) and the coefficient of the viscous damper \( c_n \) in the normal direction are expressed (Veletsos and Younan, 1994)

\[
\begin{align*}
k_n &= \frac{(2n-1)\pi G}{\sqrt{2(1-\nu)}} H \left[ \alpha_n + i \frac{\omega}{\omega_n} \beta_n \right] \\
c_n &= \beta_n \sqrt{\frac{2}{1-\nu}} G \rho
\end{align*}
\]

where \( n \) is the \( n \)th mode of vibration, \( \rho \) denotes the density and \( \alpha_n \) and \( \beta_n \) are some dimensionless stiffness and damping coefficients dependent on the frequency ratio \( \omega/\omega_n \) and the material damping \( \zeta \). For quasi-static frequencies, i.e. as \( \omega/\omega_n \to 0 \), \( \alpha_n = 1 \) and the terms within the brackets becomes unity. The dashpot coefficient \( c_n \) in the normal direction can be expressed as seen in Equation (2.88). Linear springs and dashpots are also included in the two tangential directions to consider the transmission of frictional forces between the walls and the soil, see Equation (2.88). The values in the tangential directions are assumed as a fraction of the values in the normal direction (Zangeneh, 2018). \( C_s \) is the shear wave velocity of the soil medium, and \( C_{La} \) is Lysmer’s analog wave velocity.

\[
\begin{align*}
k_n &= \frac{\pi G}{\sqrt{2(1-\nu)}} H \\
c_n &= \rho C_{La}
\end{align*}
\]

\[
\begin{align*}
k_t &= \frac{C_s}{C_{La}} k_n \\
c_t &= \frac{C_s}{C_{La}} c_n
\end{align*}
\]

\[
C_{La} = \frac{3.4}{\pi(1-\nu)} C_s
\]
Chapter 3

Method

3.1 Modeling Procedure

The models presented in this report were implemented in Abaqus and BRIGADE/Plus, which are software for conducting FEA. Performing parametric studies of FEA may indeed introduce high amount of modeling time. Thus, the regular Graphical User Interface (GUI) in Abaqus might be a cumbersome choice of method when generating the FE-models. With the power of programming, it is more feasible to generate the FE-models with the Abaqus scripting interface, which enables the user to also create models that vary in some parameters, which ease the parametric study. The scripting interface of Abaqus is an application programming interface (API) to the models and data used by Abaqus, which is an extension of the object-oriented programming language Python. Thus, Python scripts can be used to generate the FE-models in Abaqus. In this report, Python 3.6.4 was used to create the scripts which was implemented in Abaqus 6.14-2 and BRIGADE/Plus 6.1. The former includes an integrated Abaqus FEA Solver and has additional modules such as the Dynamic Live Load module which enables analysis of the dynamic response of moving loads. Throughout this thesis, the terms Abaqus and BRIGADE/Plus may be used interchangeably. Train simulations in BRIGADE/Plus is computationally expensive when comparing to train simulations performed in MATLAB which is a powerful programming environment and was thus preferably chosen for analysis of train passages. Furthermore, postprocessing of acquired data was performed in MATLAB as it comes handy when great amount of data is to be analyzed. In this thesis, MATLAB 2017a was used.

In BRIGADE/Plus, the modeling was performed following the modules within the software. The used modules are seen in Figure 3.1. Initially, the individual parts of the model were sketched. Section and material definitions were created and assigned to the regions of the parts followed by the creation of instances of the parts which then were assembled. The steps were defined, which govern the method of analysis for which associated output requests were demanded. Subsequently, the interactions were defined, for instance, contact between regions of models. The load and boundary conditions were then applied. The dynamic live load module enables dynamic response analysis for live load. Finally, the finite element mesh was generated and jobs were created for analysis. In the post-processing, the analysis result was visualized. (Scanscot Technology AB,
2015). More in-depth discussion of the modeling procedure will be described in the remainder of this chapter.

![Modeling workflow in BRIGADE/Plus](image)

**Fig. 3.1 Modeling workflow in BRIGADE/Plus.**

### 3.2 Geometry

The geometrical properties for portal frame bridges governing SSI are essential to capture the modal damping ratio at resonance. Thus, the parametric study of this report aims at investigating the effect of span lengths and associated bridge- and soil geometry based on engineering drawings from Sweden.

#### 3.2.1 Bridge

In this thesis, the considered theoretical span lengths were 5-, 10-, 15- and 20 meters, which interchangeably may be denoted $L_5$, $L_{10}$, $L_{15}$ and $L_{20}$. They were chosen based on assumptions integrated with the railway bridge stock of Botniabanan (Johansson et al., 2013), and covers an upper-, intermediate- and lower bound. Furthermore, portal frame bridges shorter than 5 meters usually include a closed bottom plate, which interferes with the concept of keeping constancy for the models within this parametric study. However, the associated geometry considering height, thickness, and width of the bridge were based on engineering drawings with smaller adjustments. Following bridges on Botniabanan were considered as the basis; Nyåkersviken, Käringbärget, Håknäs, and Norbäcksvägen. The abutment height $H$ for the shorter span $L_5$ was set to 5 meters, and for $L_{10}$, -15 and -20, set to 7 meters for the longer span lengths. This was made to emphasize the frame effect which is essential for portal frame bridges. Furthermore, the deck width $d_1$ was fixed to 7 meters for all bridges which correspond to single-track railway bridges. This is important to notice because a 2D analysis was performed in this report. Thus, eccentric deck twist and plate effects were neglected for the train simulations. Furthermore, the influence of wing walls was neglected in both the 2- and 3D FEA.

Considering the portal frame bridges, the geometrical properties are summarized in Table 3.1, given as variables in Figure 3.2. Note that a tapered deck with haunches $t_2$ were only given for the longer span lengths, i.e. $L_{15}$ and $L_{20}$, which was given as an equivalent thickness. Moreover, a ballast layer on the bridge deck was assumed with thickness of 0.6 meters. The geometry, and thus the stiffness of the ballast, was not included in the models. However, the mass of the ballast was included by assigning an equivalent bridge deck density.
### 3.2 GEOMETRY

Table 3.1 Geometric properties of portal frame bridges.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$H$</th>
<th>$a$</th>
<th>$t_1$</th>
<th>$t_2$</th>
<th>$t_3$</th>
<th>$t_4$</th>
<th>$b_1$</th>
<th>$b_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[m]</td>
<td>[m]</td>
<td>[m]</td>
<td>[m]</td>
<td>[m]</td>
<td>[m]</td>
<td>[m]</td>
<td>[m]</td>
</tr>
<tr>
<td>L5</td>
<td>5.00</td>
<td>1.00</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.70</td>
<td>1.75</td>
<td>1.35</td>
</tr>
<tr>
<td>L10</td>
<td>7.00</td>
<td>2.00</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>1.00</td>
<td>2.00</td>
<td>1.60</td>
</tr>
<tr>
<td>L15</td>
<td>7.00</td>
<td>3.00</td>
<td>1.00</td>
<td>1.25</td>
<td>1.00</td>
<td>1.20</td>
<td>2.25</td>
<td>1.85</td>
</tr>
<tr>
<td>L20</td>
<td>7.00</td>
<td>4.00</td>
<td>1.25</td>
<td>1.56</td>
<td>1.25</td>
<td>1.45</td>
<td>2.50</td>
<td>2.10</td>
</tr>
</tbody>
</table>

Fig. 3.2 The bridge geometry.

#### 3.2.2 Soil

The modeling of soil with finite elements, considering geometrical properties, is a tedious work. In 2D, it was a choice based on verifications with 3D FE-models. In this study, the findings have shown that the choice of finite element thickness was essential. By observing the engineering drawings, it was seen that the backfill was designed with a slope 1:1.5 in both directions normal to the plane. Obviously, this geometry could not be accurately reproduced in a 2D FE-model. Nevertheless, a thickness must be given for a plane stress element, and was finally set to an equivalent constant thickness according to Table 3.2 and Figure 3.3. This assumption was based on keeping constant mass between the 2- and 3D FE-model and introduced modeling errors, which indeed was troublesome. Furthermore, it implies that the whole FE-model behaves as its mid-surface, which is not the case in the mathematical model.

The convergence study on the plane stress thickness in 2D is shown in Appendix B.1, and indicates greater stiffness, but also less modal damping, with increased plane stress thickness. Some authors such as Kok (2000); Vega et al. (2012) and Mellat et al. (2014) modeled the soil with varying thickness, where the latter calculated an effective width calibrated on a reference 3D model. In this thesis, the backfill length $l$ was based on convergence studies with respect to Sommerfeld’s radiation condition. However, the results presented in Appendix B.2 shows a somewhat erratic behavior and final conclusions in 2D could not be made, regarding the length...
of the backfill. The half-space radius $R$ was not as sensitive to variations as depicted in Figure B.6.

Physically speaking, wave propagation in soils, is most certainly a 3D-problem. Thus, 3D FE-models with backfill were developed, e.g. with slope and a rectangular block. These were essential to understand the actual SSI for portal frame bridges. To validate the 2D models, regarding the radiation condition, further convergence studies were performed in 3D. These are presented in Appendix B.2, and facilitated the choice of backfill length $l$, and plane stress thickness $d_2$, in 2D. Finally, it was crucial to model a circular boundary, contrary to a rectangular boundary to disallow spurious wave reflection at the edges of the model. The geometries of the backfill and half-space are depicted in Figure 3.3 and Figure 3.4, respectively.

Table 3.2 The geometric properties of the soil.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
<th>$l$</th>
<th>$R$</th>
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<td>$L5$</td>
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<td>14.50</td>
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<td>14.50</td>
</tr>
<tr>
<td>$L10$</td>
<td>7.00</td>
<td>17.50</td>
<td>10.50</td>
<td>23.00</td>
<td>28.00</td>
</tr>
<tr>
<td>$L15$</td>
<td>7.00</td>
<td>17.50</td>
<td>10.50</td>
<td>21.00</td>
<td>28.50</td>
</tr>
<tr>
<td>$L20$</td>
<td>7.00</td>
<td>17.50</td>
<td>10.50</td>
<td>23.50</td>
<td>33.50</td>
</tr>
</tbody>
</table>

Fig. 3.3 The backfill geometry.
3.3 Material Models

The material models assigned to the FE-models will be described in the following sections and are mainly related to the rate-independent material model as described previously in Section 2.3.1. However, this is only possible in the frequency domain and must be adjusted to frequency-dependent damping, i.e. Rayleigh damping, when a nonlinear dynamic analysis is performed in the time domain. Furthermore, when introducing soil material, additional parameters such as P- and S wave velocities must be accurately defined to capture the radiation damping.

3.3.1 Concrete

In ultimate limit state (ULS) design, reinforced concrete is seldom referred to as a linear elastic material. Nevertheless, during serviceability limit state (SLS) where acceleration levels are evaluated, concrete bridges are associated with cracks and linear elasticity. Therefore, these investigations were performed in accordance to SLS, i.e. considering a linear elastic material with cracked concrete, which governs the material parameters for this study.

A concrete class of C20/25 was assumed, which renders an elastic modulus of 30 GPa. The effect of changes in elastic modulus was negligible, particularly for the full models where soil and concrete were modeled. However, a frequency shift was observed, but there were no remarkable changes in modal damping ratios for the full soil models, which is seen in Appendix B.3 for the curious reader. Furthermore, the Poisson’s ratio for cracked concrete was set to 0 (CEN, 2004). Additionally, the concrete E-modulus was reduced by a factor 0.8 which is assumed to be conservative for cracked concrete during dynamic train analysis (Tahershamsi, 2011). The concrete bridge deck was given an equivalent density to include the mass of the ballast, without adding any stiffness per se (CEN, 2002). Finally, a material damping ratio of 1.5 % was assigned in accordance with Eurocode (CEN, 2003). The damping ratios were either applied as a loss factor $\eta$, i.e. twice the damping ratio, or as a Rayleigh damping by determining the $\alpha$ and $\beta$ values.
given the modes of interest. The concrete material parameters are summarized in Table 3.3\(^{i}\). The Rayleigh parameters are not presented in Table 3.3, instead, they are shown in Appendix B.8.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>(E) [GPa]</th>
<th>(\nu) [-]</th>
<th>(\rho) [kg/m(^3)]</th>
<th>(\zeta) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>24.0</td>
<td>0.0</td>
<td>2500.0</td>
<td>1.5</td>
</tr>
</tbody>
</table>

### 3.3.2 Soil

Considering railway-induced vibrations, in which the response is governed by low amplitude oscillations and small strains, soil can be regarded as a rate-independent material in the computational model. Similarly, to concrete, the loss factor only applies for frequency domain analysis and must be adjusted with Rayleigh damping when dealing with nonlinear dynamics. Nevertheless, a material damping must be determined and is according to Houbrechts et al. located between 2-4 \% for soils. Thus, a lower bound was chosen to not overestimate the material damping effects. Furthermore, the dynamic properties of soil are mainly governed by the shear modulus \(G\), which affects the P- and S wave velocity, i.e. radiation damping. In this thesis, the shear modulus was determined by assigning shear wave velocities from tabulated values by Bodare (1996), e.g. sand, gravel, and moraine. However, densities and Poisson’s ratios were assumed, to render the elastic modulus. These were obtained from the norms CEN (2004) and previous studies by Zangeneh (2018) and Hall (2000).

Soil is not a homogeneous medium, which causes ambiguity during FEA. The choice of material parameters are essential, considering SSI, and it is important to ensure the reliability of the models. In this thesis, no calibration towards physical experiments was made, thus, the results lack an external validity. However, variations of the material parameters were performed to emphasize the robustness of these FE-models. This corresponds to a qualitative sensitivity analysis and is presented in Appendix B.3. Naturally, the half-spaces models were most volatile to changes in material parameters. The shear wave velocities, for backfill and subsoil, were increased with 50 \% and 28 \%, respectively. As a consequence, the modal damping reduced substantially, where a decrease of 21 \% and 33 \% for Case- III and V was observed.

The material properties for soil and ballast are shown in Table 3.4. Furthermore, the P-wave velocity is largely dependent on the water content, i.e. the saturation of the soil is influential. In this thesis, the presence of groundwater was neglected. Therefore, the P-wave velocities are associated with unsaturated tabular data from Bodare (1996).

\(^{i}\)The tabulated modulus of elasticity corresponds to cracked concrete according to Tahershamsi (2011).
3.4 LOADS

Proper replication of a train-induced load is a complicated matter and depends on several parameters. Initially, it is a choice of analysis method, which decides the computational procedure. As presented briefly in the theoretical background, during dynamic analysis, there are two alternative domains, i.e. the time- and frequency domain. These two are essential during train-induced vibrations on bridges and offer different features considering, e.g. central processing unit (CPU) time, and limitations of the analysis, i.e. linear or nonlinear. Furthermore, it is important to state that only point loads were idealized in this thesis, i.e. neglecting the inertial effect of the train as a moving mass load.

According to CEN (2003), the general HSLM, can be defined as Figure 3.5. The boogie axle spacing \( d \), point force \( P \), coach length \( D \) (denoted \( \lambda \) in this thesis) and the number of coaches \( N \) distinguish the different trains HSLM-A1 to -A10. Furthermore, the longitudinal distribution of a point load can be distributed over three sleeper distances. However, this distribution was not considered in this thesis.

\[
\begin{array}{cccccc}
\text{Parameters} & E & \nu & \rho & \zeta & C_p & C_s & C_{La} \\
\text{[MPa]} & [-] & [kg/m}\text{³}] & [%] & [m/s] & [m/s] & [m/s] \\
\text{Backfill} & 270 & 0.2 & 1800 & 2.5 & 408 & 250 & 338 \\
\text{Subsoil} & 691 & 0.2 & 1800 & 2.5 & 653 & 400 & 541 \\
\text{Ballast} & - & - & 1800 & - & - & - & - \\
\end{array}
\]

Fig. 3.5 The HSLM-A. From CEN (2003).
CHAPTER 3. METHOD

Table 3.5 The HSLM train load formulation. From CEN (2003).

<table>
<thead>
<tr>
<th>HSLM-</th>
<th>N</th>
<th>D [m]</th>
<th>d [m]</th>
<th>P [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>18</td>
<td>18</td>
<td>2.0</td>
<td>170</td>
</tr>
<tr>
<td>A2</td>
<td>17</td>
<td>19</td>
<td>3.5</td>
<td>200</td>
</tr>
<tr>
<td>A3</td>
<td>16</td>
<td>20</td>
<td>2.0</td>
<td>180</td>
</tr>
<tr>
<td>A4</td>
<td>15</td>
<td>21</td>
<td>3.0</td>
<td>190</td>
</tr>
<tr>
<td>A5</td>
<td>14</td>
<td>22</td>
<td>2.0</td>
<td>170</td>
</tr>
<tr>
<td>A6</td>
<td>13</td>
<td>23</td>
<td>2.0</td>
<td>180</td>
</tr>
<tr>
<td>A7</td>
<td>13</td>
<td>24</td>
<td>2.0</td>
<td>190</td>
</tr>
<tr>
<td>A8</td>
<td>12</td>
<td>25</td>
<td>2.5</td>
<td>190</td>
</tr>
<tr>
<td>A9</td>
<td>11</td>
<td>26</td>
<td>2.0</td>
<td>210</td>
</tr>
<tr>
<td>A10</td>
<td>11</td>
<td>27</td>
<td>2.0</td>
<td>210</td>
</tr>
</tbody>
</table>

The significant parameters, governing the resonance for train-induced loads, are the characteristic coach length $D$ and speed $v$, regarding the train. The procedure to simulate the train load will be described in the following. For this thesis, the rail, sleeper nor ballast were modeled. Thus, the idealized point load was directly applied to the bridge deck, which obviously was a simplification in the computational model.

3.4.1 Gravity

The effect of statics during dynamic analysis for linear systems can be determined by combining the response of the time-varying excitation and the dead load, i.e. the law of superposition applies. However, during nonlinear analysis, this separation cannot be made, and it is crucial to apply the static load prior to the dynamic analysis.

The gap formation between abutment and backfill has a significant static effect from the lateral earth pressure. To capture this, a gravity load was applied to the FE-model which caused lateral loads, due to the Poisson’s effect of the soil. In this thesis, this was the simplest way of approximating an at-rest lateral earth pressure. However, it is a significant simplification compared to the mathematical model, which underestimates the lateral loads. Unfortunately, the gravity load in the simplified model only included dead loads from the bridge, thus, it was not a proper way of representing the at-rest lateral earth pressure between backfill and abutment. Therefore, the simplified nonlinear model, i.e. Case VI, was prone to modeling error when including dead loads.

3.4.2 Time Domain Analysis

Sine Sweep Load

In order to simulate an FRF in the time domain, it is possible to apply a sine sweep load, covering the frequencies of interest. In this thesis, it was mainly performed on the nonlinear FE-models, by creating a linear swept-frequency cosine signal using the MATLAB function Chirp, which is shown in Figure 3.6.
3.4 LOADS

The sine sweep load was applied at a given point, e.g. mid- or quarter point, and the output acceleration response was extracted at the same point. Then, by applying the FFT algorithm in MATLAB on the response and load, the FRF was rendered by dividing the response spectrum with the load spectrum. The schematics of the procedure are emphasized in Figure 3.7. Furthermore, this was used to verify the direct-solution steady state (DSS) method in Abaqus which is a step to obtain an FRF for linear FE-models. A comparison was done for Case I and -II, which is shown in Figure C.1. Regarding the nonlinear models, additional investigations were made considering the amplitude dependency during the sine sweep load.

**HSLM**

To subject the structure to a load-time history, amplitude functions are needed when dealing with train loads in the time domain. These are referred to as $f_n(t)$, for the discretized nodes $n$ along the bridge deck, according to Figure 3.8. The functions represent a load time history during the passage of a train given a velocity and are mathematically expressed as a series of triangular loads. Furthermore, they depend on the discretization, i.e. the mesh of the bridge deck. Each triangular pulse is associated with an axle load and the time shift between them are determined by the axle length $\lambda$ and train velocity $v_{train}$. Every node along the bridge deck have a unique amplitude function, which in relation to the adjacent node are shifted in time, with respect to the element size $\Delta_{el}$ and train velocity $v_{train}$. As the mesh discretization was refined,
CHAPTER 3. METHOD

the triangular pulses become spikier. Thus, they converge towards a Dirac-delta pulse, which had some important consequences in the frequency domain. However, in the time domain, a refined mesh did not have any significant effects regarding the load model.

In this thesis, two procedures in the time domain were used to simulate the response of HSLM trains. One included the Dynamic Live Load module provided in BRIGADE/Plus, which calculates the time history response, given an HSLM train and velocity. The second procedure uses amplitude functions, which was scripted in Python and pre-processed into BRIGADE/Plus. Then, an implicit DTI was performed, i.e. HHT ($\alpha$-method), which calculates the time history response. Finally, this was post-processed into an acceleration envelope, which represents the maximum acceleration obtained for an HSLM train and velocity. The illustration of the train analysis in the time domain is shown in Figure 3.9.

Fig. 3.8 Discretized nodes along the bridge deck and the amplitude functions.

In this thesis, two procedures in the time domain were used to simulate the response of HSLM trains. One included the Dynamic Live Load module provided in BRIGADE/Plus, which calculates the time history response, given an HSLM train and velocity. The second procedure uses amplitude functions, which was scripted in Python and pre-processed into BRIGADE/Plus. Then, an implicit DTI was performed, i.e. HHT ($\alpha$-method), which calculates the time history response. Finally, this was post-processed into an acceleration envelope, which represents the maximum acceleration obtained for an HSLM train and velocity. The illustration of the train analysis in the time domain is shown in Figure 3.9.

Fig. 3.9 Flow chart describing the procedure to obtain the acceleration envelope in the time domain.

The Dynamic Live Load module in BRIGADE/Plus generates live loads on the model for dynamic response analysis. A track was used to define the position of vehicles as they move along the structure. The track was simply selected as a geometrical edge of the model. BRIGADE/Plus provides vehicle formulations from design codes such as the HSLM, as well as user-defined number of axle load with user-defined load intensities, and internal distances. Train speeds were defined
followed by the step definition. Procedures were chosen, such as modal dynamics or DTI where the latter requires damping as a material property whilst for the modal dynamics the damping was set in the step, either by direct- or composite damping.

### 3.4.3 Frequency Domain Analysis

**FRF**

As briefly described in the theoretical background, an FRF denoted $H(\omega)$, is a transfer function applicable to linear systems. Abaqus offers a step which provides the amplitude and phase of the response from a harmonic excitation, given the frequencies of interest. In fact, this procedure corresponds to a sine sweep load. However, the DSS procedure in Abaqus calculates the steady state response due to a harmonic excitation directly. The DSS was a time-consuming step in Abaqus for large FE-models. Nevertheless, it is the most accurate procedure and was used throughout this thesis for the 2D FE-models. Concerning the 3D models, a modal procedure was also used, which is based on mode superposition and was computationally cheaper than the DSS. The frequency domain method for train analysis, i.e. combining the FRF from Abaqus and the FFT in MATLAB, allowed for time-efficient analysis of linear structural systems. In this thesis, it was an essential procedure due to its effectiveness, contrary to methods in the time domain and will be described in the following.

**HSLM**

The discretized nodes along the bridge deck in Figure 3.8 were in the frequency domain method subjected to a harmonic load by the DSS. Consequently, this rendered an FRF in an output point, e.g. mid- and quarter point, which was investigated in this thesis. This procedure was repeated for all nodes along the bridge deck, in return, a frequency response map was produced which provided the acceleration amplitude given a load position and forcing frequency. An example of this influence function is shown in Figure 3.10 and is referred to as a frequency response map.
The FRF is a complex function and during the post-processing (PP) in MATLAB it was important to not operate with the magnitude to not lose information regarding the phase. Instead, the real and imaginary parts were considered, which contains this information. Furthermore, the findings showed the significance of applying a real instead of imaginary load to the system when performing the FRF. The real load captured the response amplitude accurately. However, it was shifted in time, which was not the case for the imaginary load.

The amplitude functions mentioned in Section 3.4.2 were created for all HSLM trains, in the frequency domain, and transformed into a load spectrum with the help of the FFT. The load spectrum $F_n(\omega)$ involves information about the frequency content from the amplitude functions. Eventually, the acceleration spectrum $\ddot{U}_n(\omega)$ was computed by multiplying the FRF with the load spectrum. Then, by using the inverse fast Fourier transform (IFFT), the acceleration time history response was determined for a given HSLM train and speed. It has been observed in the findings that the load spectrum was mesh dependent, i.e. the frequency domain method converged when the approximation of the triangular pulse behaved as a Dirac-delta pulse. This had significant consequences considering the final time history response in the frequency domain and will be described more thoroughly.

The amplitude functions for the midpoint are shown in Appendix B.4 for different mesh sizes. It was observed that the load spectrum changed as the approximation of the triangular pulse converged towards a Dirac-delta pulse. Furthermore, the solution converged towards the calculated response in the time domain method, i.e. it improved with refined mesh. The DFT solution represents the steady-state response of a system due to periodic excitation. Thus, errors were introduced if these conditions were violated. According to Chopra (2013) the DFT solution of an SDOF increases in accuracy as the duration of the free vibration becomes longer. During a train analysis, i.e. a series of impulses, the free vibration response occurs frequently between...
the axle loads. Furthermore, it is known that an arbitrary excitation can be interpreted as a periodic excitation with an infinitely long period. However, this was not possible to capture for a coarse mesh because of the short free vibration response between impulses. Thus, the at-rest initial conditions were violated during the DFT solutions and the computed response was inaccurate. These findings were puzzling, however, further literature studies, e.g. Ülker Kaustell (2009) strengthened the beliefs regarding the mesh dependency in the frequency domain analysis. Note that the convergence studies were performed on Case I with span length 5 meters, which is a lightly damped model. Therefore, it requires substantial number of cycles to reach at-rest initial conditions during the train simulation. However, for the FE-models where SSI has been included, the damping ratios are significant which decreases the number of cycles required to reach at-rest. Hence, these models in the frequency domain ought not to be that sensitive for the mesh discretization.

The concepts of the frequency domain method for train analysis is shown in Figure 3.11. Despite its complexity, it offers an effective way of computing the acceleration response for HSLM trains. Compared to the time domain analysis, which was rather time-consuming, the frequency domain analysis was used on all linear FE-models in this thesis due to its time effectiveness. The subscript \( n \) denotes the frequency functions evaluated at the discretized nodes illustrated in Figure 3.8.

![Flow chart](image.png)

Fig. 3.11 Flow chart describing the procedure to obtain the acceleration envelope in the frequency domain.
3.5 Boundary Conditions

In dynamic analysis, the choice of boundary conditions in the model constitutes a fundamental parameter when investigating the effect of the dynamic behavior of portal frame bridges. Different boundary conditions were applied for the models with surrounding soil and for simplified models. Initially, the model without any type of SSI was created, followed by the model with confining backfill which finally was developed to the half-space model. Subsequently, simplified models were used to enable faster computations for the cases of backfill and subsoil. Beyond boundary conditions in the linear elastic domain, boundary conditions in the nonlinear domain were applied. The modeling of the boundary conditions applied within this thesis will be described in following sections and those are either (see section reference for theoretical background):

- clamped which prohibit translation and rotation of the specified DOFs
- defined according to the standard viscous boundary (q.v. Section 2.10)
- defined by linear springs and dashpots using a Kelvin-Voigt model (q.v. Section 2.11)
- frequency-dependent impedance functions (q.v. Section 2.9) or
- by nonlinear gap elements (q.v. Section 2.7.3). The nonlinear contact will be elaborated in Section 3.7.1.

3.5.1 Clamped Boundaries

When the stiffness of the bedrock layer underlying the bridge is assumed to be infinitely high, clamped boundary conditions were defined. Such boundary conditions do not allow the DOFs to translate nor rotate. Clamped boundaries were applied to the footing of the bridge for the model with only the bridge where no SSI was included. For the model with backfill, the bottom of the backfill and the bridge footings are clamped, as well as the model with nonlinear gap elements.

CASE I

Fig. 3.12 Clamped boundary conditions applied to the bridge footings.

3.5.2 Implementation of the Standard Viscous Boundary

Since wave reflection at the boundaries of the model is not desired to obtain an accurate steady-state response, the extension of the finite domain of the soil along with transmitting boundaries is necessary. The transmitting boundary was implemented with ease by the viscous boundary method discussed in Section 2.10. The special case when $a = b = 1$ in Equation (2.85) is referred
to as the standard viscous boundary and perfectly absorbs waves at normal incidence.

From Equation (2.85), damping coefficients $D_t$ and $D_n$ in the tangential direction and normal direction can be derived which absorb dilatation and shear waves, respectively. Thus, damping coefficients were assigned to the nodes of the boundaries in the FE-model. The damping coefficients are defined by the density of the soil medium $\rho$, the dilation and shear wave velocities $C_p$ and $C_s$, and the tributary area $A_{trib}$ which is the area of the discretized mesh which corresponds to one node, see Figure 3.13. Hence, the tributary area is dependent on the mesh size, type of finite element as well as the order of the interpolation function of the element. Note also that in 2D the tributary area has one side equal to the plane stress thickness of the soil medium contrary to 3D modeling where the meshed area governs the tributary area.

$$
\begin{aligned}
D_t &= \rho C_s A_{trib} \\
D_n &= \rho C_p A_{trib}
\end{aligned}
$$

\[3.1\]

In the FE-model, dashpots were defined along the boundaries, which aim is to absorb the waves spreading in the soil. The damping coefficients of Equation (3.1) were inserted as the dashpot coefficients, where it was important to specify the dashpots with a coordinate system which enables damping in the tangential and normal direction of the boundaries. Thus, for the backfill models, a Cartesian coordinate systems was defined, while as for the boundary of the half-space, a cylindrical coordinate systems was defined. This is necessary since the efficiency of the absorbing capacity of the dashpots diminishes for waves impinging at non-perpendicular angles to the boundary (Zangeneh, 2018). The clamped boundary conditions applied to the bridge is seen in Figure 3.12. The implementation of the standard viscous boundary method by dashpots along transmitting boundaries are visualized in Figure 3.14 and Figure 3.15 for cases with backfill and half-space, respectively. Notice that in 2D, only one tangential dashpot is activated while there exist two tangential dashpots in 3D.
Chapter 3. Method

3.5.3 Kelvin-Voigt Model

The backfill in the simplified models was substituted by Kelvin-Voigt models with linear springs and dashpots, a concept which was discussed in Section 2.11. In the same manner as the standard viscous boundary, the spring- and dashpot coefficients were assigned to the nodes of the boundary where the coefficients depend on the tributary area, recall Figure 3.13. A modeling approach which represents the abutment-backfill interaction is thus to define distributed linear springs and dashpots for the nodes on the mesh grid along the abutment. Recalling Equation (2.87), the stiffness of a viscoelastic bar with frequency-independent damping governs the parameters inserted as normal springs in the FE-model. The values used in this thesis were assumed solely for the first mode and assumed frequency-independent, i.e. \( n = 1 \) and \( \omega / \omega_n \rightarrow 0 \). Furthermore, the value of the dashpot coefficient in Equation (2.87) was approximately equal to the frequency-independent vertical impedance of the dashpot coefficient. These were derived from foundations on the surface of a homogeneous half-space found in Fung (1991). The corresponding translational values are assumed as a fraction of the values applied in the normal direction. Thus, the stiffness and damping in the normal and tangential direction are given as \( k_n, c_n, k_t \) and \( c_t \), recall Equation (2.88). These presented stiffness- and dashpot coefficient are set for a unitary area, thus the normal stiffness- and dashpot coefficients are multiplied with the tributary area, \( A_{trib} \) to distribute the coefficients over the discretization area covered by a single node. This modeling approach was applied in the
simplified models where backfill is present, see Figure 3.16 and Figure 3.18. The simplification of
the underlying soil was modeled by impedances, described in Section 3.5.4.

3.5 BOUNDARY CONDITIONS

CASE IV

Fig. 3.16 Linear springs and dashpots as a substitute for the backfill in the simplified modeling.

3.5.4 Frequency Dependent Impedance Functions

When seeking modeling approaches to consider the confinement of a half-space, there exist different
methods as discussed in Section 2.9. One method which contributes to the modal damping of the
structure is the dynamic stiffness function expressed in Equation (2.82), recapitulated as

\[ Z(\omega) = \bar{k}(\omega) + i\omega c(\omega) \]

Fang provides charts and analytical expressions to obtain dynamic stiffness and dashpot coefficients
in three directions, as well as for rocking and torsional modes of vibration. In this thesis, the
impedances were obtained from a case which describes the dynamic stiffness function of arbitrarily
shaped foundations on the surface of homogeneous half-space. The static stiffness \( k_{st} \) is defined
as a function of solely the geometrical properties of the foundation, i.e. the footings of the portal
frame bridge. By multiplying the static stiffness with a dynamic stiffness coefficient \( k(\omega) \), the
dynamic stiffness \( \bar{k}(\omega) \) was obtained, see Equation (3.2). The dynamic stiffness coefficient is
obtained from design charts based on simple physical models calibrated with results of boundary-
element formulations and data from pioneers within the subject. These design charts consider
the forcing frequency of the external excitation \( \omega \), the geometry of the foundation \( B \) as well as
the shear wave velocity of the stratum \( C_s \), namely by \( a_0 = \omega B/C_s \) where \( B \) is length of the
shorter side of a rectangle circumscribing the arbitrarily shaped foundation. Analogically the
dynamic stiffness coefficients, there also exists dynamic dashpots coefficients \( \tilde{c} \) which implemented
in analytical expressions renders the radiation damping coefficient \( c_r \). To incorporate hysteretic
damping, the material dashpot coefficient \( 2\bar{k}\zeta/\omega \) from Equation (2.84) is added. Thus, the total
damping impedance \( c(\omega) \) can be defined as Equation (3.3).

\[ \bar{k}(\omega) = k_{st} \cdot k(\omega) \]  \hspace{1cm} (3.2)
\[ c(\omega) = c_r(\tilde{c}) + \frac{2\bar{k}\zeta}{\omega} \]  \hspace{1cm} (3.3)

The dynamic stiffness and damping coefficient are by nature frequency dependent as mentioned.
Thus, the analyst may choose frequency-independent impedance functions by simply letting \( \omega \to 0 \).
Frequency-dependent impedances may be implemented by choosing to the evaluate the dynamic
stiffness function at the natural frequencies of the structure. In this thesis, a Python script was
implemented to evaluate the dynamic impedance function for a large range of frequencies with
dispersion. The design curves provided in Fang was discretized in linear functions allowing
evaluation for any given frequency within a specified range up to 50 Hz. From the script, the
dynamic stiffness and damping coefficients were calculated and used in the analytical expressions
provided in Fang (1991). The dynamic stiffness function was then inserted by the script as
stiffness and dashpot coefficients in BRIGADE/Plus. The procedure to obtain the impedance
functions are illustrated in Figure 3.17.

\[
\bar{k}(\omega) + i\omega c(\omega)
\]

In BRIGADE/Plus, the dynamic stiffness function \( Z(\omega) \) was modeled by coupling the nodes of
the footings of the bridge to a reference point underlying the footing, i.e. a rigid footing seen in
Figure 3.18. This reference point was in turn coupled to another closely positioned reference point
beneath, for which the rotation and translation were prohibited. This means that the specified
nodes imitate the behavior of the reference point. A connector behavior between the reference
points was defined and governed the motion of the nodes. The behavior was defined by specifying
the dynamic stiffness and damping in Equation (3.2) and Equation (3.3) into frequency-dependent
springs and dashpots in the vertical, horizontal and rotational directions. The reference point
may be positioned at an arbitrary vertical offset of the footing since a convergence study showed
its negligible effect when the distance between the reference points less was than 0.5 meters.
Furthermore, the lateral placement of the reference point was selected to coincide of with the
center of gravity of the footing in 2D as well as in 3D.

\[
Z(\omega) = k(\omega) + i\omega c(\omega)
\]

Fig. 3.18 Complex impedance functions applied to the bridge footings by connectors.

3.5.5 Implementation of Simplified Backfill Nonlinear Model

Previously mentioned springs and dashpots show linear elastic behavior. However, to capture the
full interaction between abutment and soil, a nonlinear spring was implemented in series with the
discretized Kelvin-Voigt model. To simulate a rigid connection, the nonlinear spring was given a
stiffness \( 10^3 \) larger than the Kelvin-Voigt type spring, in the active direction. Large differences in
stiffness were avoided, which may cause a problem to the FE-model, due to ill-conditioning. In
the inactive direction, the nonlinear spring stiffness was set to zero. Also, the tangential springs
were removed, simply because they did not have a significant contribution for the first and second bending mode, and modeling this coupling would over complicate this complex system. The concepts of defining a gap element in series with the Kelvin-Voigt type spring and dashpot are shown in Figure 3.19.

\[ k_g = 1000k_{KV} \]

Fig. 3.19 Force definition of the nonlinear spring.

The implementation of the nonlinear gap elements are shown in Figure 3.20, and was made via connector elements which allow the GUI in BRIGADE/Plus. Nevertheless, it was a tedious procedure due to limitations in BRIGADE/Plus when assigning the connector elements to the abutment vertices. Initially, vertices, must be specified by the analyst prior to the discretization step. This means that the analyst, himself, affect the discretization of the structure by partitioning it at desired points. This method was somewhat cumbersome and was indeed done with ease by scripting. Generally, this is not recommended during FE-modeling. However, the alternative was to use the spring and dashpot feature which may be modified to include nonlinear behavior by editing the input file, which is not allowed in the GUI. This procedure allows the assemblage of nodes of edges and was therefore practically convenient in this sense. Both approaches were performed and agreement was observed. It is the author’s recommendation to use connector element for such cases, where the scripting environment of BRIGADE/Plus enables the implementation of nonlinear springs from a more practical point of view.

Nonlinear dynamics is a complicated procedure, where capturing the physical response was struggling. An ambitious attempt was made to avoid the nonlinear gap element, which was to only model a nonlinear spring and dashpot in parallel. This was an incorrect assumption due
to the $\pi/2$ phase shift between displacements and velocities. Thus, the nonlinear springs and
dashpots where generating forces continuously, albeit, not simultaneously.

The improved simplified nonlinear model was able to simulate the gap formed between the
abutments and the Kelvin-Voigt model. Before simulating the HSLM trains, sine sweep loads were
applied to obtain FRFs. The force-time history from a sine sweep load, shown in Appendix B.7,
emphasizes the anticipated gap behavior, i.e. the nonlinear gap element force was constantly in
equilibrium with the sum of linear spring and dashpot forces. Furthermore, the forces during
the first bending mode were in phase, thus, they were active at same time instances. However,
during the second bending mode, the abutments were phase shifted with $\pi/2$. Considering the
time step $\Delta t$, convergence studies in Appendix B.7 showed that 0.001 seconds were sufficient for
the simplified nonlinear model. Mainly, the nonlinear studies were carried out for span length 10
meters with the acceleration response output in quarter point, to include the response from the
second bending mode. Moreover, the simplified backfill nonlinear model was validated towards a
full backfill model including a contact algorithm, with mixed success. The response agreed well
when gravity was excluded, however, the response differed significantly when gravity loads were
applied the nonlinear backfill contact model.

### 3.6 Elements and Mesh

The choice of finite elements and discretization is vital in FEA. A too coarse mesh will introduce
discretization errors, i.e. the difference between the discretized finite element model and the
mathematical model. As mentioned in Section 2.1, FEA only renders approximations of the true
solution. A sufficiently refined mesh ought to produce results converging towards results implicit
in the mathematical model. To obtain a sufficient convergence rate, h-, p- and r-refinement was
made. H-refinement consists of enriching the mesh by adding more of the same mesh type by
repeated subdivision of the element size of existing elements. P-refinement increases the degree of
the highest complete polynomial in the element field quantity. This was obtained by adding nodes
on existing interelement boundaries and keeping the same number of elements. R-refinement
refers to the relocation of nodes. The polynomial degree of the field quantity, as well as the
number of elements were unchanged (Cook et al., 2007). In BRIGADE/Plus, this was obtained
by controlling the mesh technique.

#### 3.6.1 Element Size

Convergence studies on the FRF with respect to different mesh sizes were performed in 2D for
the bridge solely and the model with backfill. It was shown that the system was rather insensitive
to the mesh. Thus, any choice of mesh apart from the coarsest one would be appropriate
regarding the accuracy of the study. However, one would seek to choose a coarser mesh to
reduce computational cost. It was noted that the mesh size had a greater effect on shorter span
lengths and models without soil, as it was observed that the second mode of the backfill model
displayed good precision, see Figure C.8 and Figure C.9. Despite the agreement of different mesh
sizes regarding the FRFs, the chosen mesh was based on a convergence study of the maximum
acceleration of a train passage analyzed in the frequency domain, see Figure B.20. Regarding
3.6 ELEMENTS AND MESH

The soil in the models, the geometry of the mesh needs to be chosen in a way such that the propagation of shear waves at a frequency is enabled to resolve. This is ensured by choosing a mesh type, such that a sufficient number of nodes fit within the minimum shear wavelength. Lysmer (1978) suggests eight linear elements, i.e., nine nodes per wavelength. The minimum wavelength is given by \( \lambda_{\text{min}} = C_s / f_{\text{max}} \), where \( C_s \) is the shear wave velocity, and \( f_{\text{max}} \) is the highest frequency regarded. The maximum size of the element size thus becomes

\[
\Delta_{\text{el}} = \frac{\lambda_{\text{min}}}{8} = \frac{C_s}{8f_{\text{max}}}
\]  

Mesh studies of only the bridge in 3D showed perfect agreement between the studied mesh sizes with quadratic element formulation for the first mode of vibration, see Figure C.10. The studies were aimed to be extended to include the backfill soil in 3D, however, too large computational domain disallowed the study. Including backfill, the study was thus limited to two linear element formulations and one quadratic element formulation, see Figure C.11. It was observed that the linear elements showed better precision for the first mode of vibration compared to the second mode. The quadratic element formulation showed close agreement to the finer linear element formulation for both peaks. The quadratic element formulation also fulfilled the requirement proposed by Lysmer.

3.6.2 Element Shape

The choice of element type and interpolation functions of the chosen elements need to be chosen with care. One should aim to ensure interelement compatibility to assure the quality of the model. This was done by choosing elements and size, such that the nodes along a shared edge between adjacent elements coincided. Alongside the increased accuracy, perfect alignment of the mesh between elements reduced the computational time drastically, particularly for 3D analysis as will be explained subsequently. Throughout this thesis, a comparison was made between linear and quadratic formulations of the chosen element types.

Choice of Elements in 2D-analysis

In 2D, the bridge was modeled with beam elements. A beam within Timoshenko beam theory will render the natural frequency for a slender Euler-Bernoulli beam. Furthermore, Timoshenko beams will include the rotational inertia as well as shear deformations. For higher modes of vibration, these properties tend to reduce the natural frequency. For a portal frame bridge model in 2D, the behavior is anticipated to act something in between a simply supported beam and a beam clamped at both ends. It is noticeable in Figure C.12, that the bridge shifts its behavior from simply supported to fixed-fixed as the span length increases. This is rather intuitive due to the increase in relative deck-to-abutment stiffness as the span increases. Consequently, Timoshenko beams were considered in the 2D-analysis. In BRIGADE/Plus, those are implemented as B21 and B22 beams with linear and quadratic interpolation functions, respectively. The backfill and half-space was modeled using plane stress quadrilaterals, which for bilinear and biquadratic interpolation functions are described by CPS4 and CPS8, respectively. The choice of element types in 2D is summarized in Figure 3.21. One issue of 2D-modeling is the assumption of the
plane stress thickness. It was shown in Appendix B.1 that the dynamic response of the systems
was sensitive to the plane stress thickness.

![Fig. 3.21 Choices of finite elements in the 2D mesh.](image)

**Choice of Elements in 3D-analysis**

In the extension to 3D, the bridge was modeled with shell elements. For the soil, solid continuum
elements were chosen. These elements are more accurate if not distorted, especially for hexahedrons.
Tetrahedral elements are however less sensitive to distortion. The tetrahedral elements are
moreover geometrically versatile, which are feasible for complex geometries. One drawback of
the tetrahedral with first order interpolation function is that they are generally overstiff, for
which high mesh density is required for accurate results. Because of the overstiffness, the linear
tetrahedral element should be avoided in stress analysis since they exhibit slow convergence
rate with mesh refinement. However, for the purpose of this thesis, they were compared to the
hexahedral elements. A good mesh of hexahedral elements does however usually provide an
accurate result at less cost. When choosing the order of interpolation function of the tetrahedral
and hexahedral elements, both first and second order were considered. Thus, the elements chosen
for analysis in 3D were linear and quadratic tetrahedral and hexahedral elements (Dassault
Systèmes, 2014). The combinations of elements are presented in Figure 3.22.

![Fig. 3.22 Choices of finite elements in the 3D mesh.](image)

Altering the type of continuum elements and, caeteris paribus, the hexahedral and tetrahedral
elements were put to a benchmark test. The results from the two combinations are precise, see
Figure C.7, however, there is a remarkable difference in the CPU-time, Figure C.6. The test was
performed on an Intel® Core™ i7-4790 processor clocking at 3.20GHz. For the shortest bridge,
the difference in CPU-time was negligibly small. However, it was clearly demonstrated that as the computational domain of the model increases, the relative gain in CPU time when using hexahedral elements in favor of tetrahedral elements yields drastic decrease in CPU-time with marginal difference in precision of the results. For the longest bridge, less than half the time was required for hexahedral elements.

Furthermore it is in Figure C.7 noticeable that the order of the interpolation function is significant for shorter bridges where some difference in the response is observable. However, for longer bridges, the solution seems to be independent to the choice of first- or second order elements.

The chosen element type and mesh size in 2- and 3D are summarized in Table 3.6. A mesh size of 0.25 meters was chosen for $L_5$ due to the relatively short span length. Linear interpolation functions in 2D allow for faster CPU-time, without the expense of accuracy compared to quadratic element formulation. In 3D, plate bending becomes present why it was desired to choose quadratic element formulations which is more feasible in bending dominated problems. It was moreover shown that the hexahedral elements were more suitable from an economic point of view.

Table 3.6 Properties of the mesh in 2- and 3D.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Model</th>
<th>Element</th>
<th>$L_5$</th>
<th>$L_10$</th>
<th>$L_15$</th>
<th>$L_20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D Bridge</td>
<td>B21</td>
<td>0.25</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>2D Soil</td>
<td>CPS4</td>
<td>0.25</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>2D Nonlinear</td>
<td>B21/CPS4</td>
<td>0.25</td>
<td>0.25</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>3D Bridge</td>
<td>S8R</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>3D Soil</td>
<td>C3D20</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>

3.7 Connections in Structures

Connections in the FE-model are essential, involving combining different elements or geometries, and treating contact phenomena. In this thesis, the fundamental study considering connections, was to investigate the contact problem between abutment and backfill during dynamic analysis. However, further topics will be explained, such as perfectly matched nodes.

3.7.1 Implementation of the Backfill Contact Nonlinear Model

Contrary to Case II where a tie constraint was assigned between the structure and backfill, the contact model, Case VII, allows for separation. The interface was modeled with a hard contact, i.e. zero penetration in an average sense, and the master- and slave surfaces were selected to the abutments and backfill respectively, see Figure 3.24. The choice of the master- and slave surface are crucial and decides which surface that is allowed to penetrate the other, which is the master surface. Furthermore, an attempt was made to model a tangential contact by the penalty algorithm, with a friction coefficient of 0.7 (Plaxis, 2018), which had a slight effect on the
outcome. The illustration of Case VII is shown in Figure 3.23 where the red area highlights the contact region.

**CASE VII**

![Fig. 3.23 Implementation of the backfill contact in the nonlinear FE-model.](image)

Regarding the implicit DTI in BRIGADE/Plus, the $\alpha$-method with moderate dissipation was assigned, which is applicable for systems including contact nonlinearity (Dassault Systèmes, 2014). However, it introduces some algorithmic damping due to $\alpha < 0$, nevertheless, it provides accurate results for low-frequency vibrations which was the case studied in this thesis. The moderate dissipation application assumes values of Newmark’s parameters, i.e. $\beta$ and $\gamma$ given the value of $\alpha$ by recalling Equation (2.60). The values are shown in Table 3.7. The unconditional stability for the $\alpha$-method is only valid for linear systems. Thus, a thorough convergence study regarding the time step and element size was performed in Appendix B.8. These were performed for the acceleration response by a sine sweep load and the train analysis. Similar to the Case VI, the time step $\Delta t$ was chosen to 0.001 seconds for the nonlinear backfill contact model, which proved to be an accurate and computationally economic time increment.

### Table 3.7 Parameters for the $\alpha$-method.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>-0.41421</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.91421</td>
</tr>
</tbody>
</table>

![Fig. 3.24 Contact between the master and slave surface.](image)

**3.7.2 Perfectly Matched Nodes**

In exact solutions, differential elements in a material are in equilibrium, the boundary conditions are met, and compatibility prevails. Due to the approximate nature of an FE-solution, these conditions are not always met. Thus, it was important to ensure model quality by taking measures that favor these conditions. Compatibility should prevail at nodes where the displacement components are shared. This means that the elements chosen must be considered due to the order of interpolation and discretization size. For instance, see the incompatibility of elements in Figure 3.26. Furthermore, particularly for complex geometries, the FE-discretization at connections between different parts should be revised, as these may be a source of reduced
3.8 Verification

To assure the quality of the model and analysis procedure, some verifications and validations were made and will be described in the upcoming sections. By the means of the dynamic analysis performed in this thesis, the parameters to verify against analytical solutions are the natural frequencies of the systems.

3.8.1 Model Verification

As the complexity of the models increase with the inclusion of soil, difficulties arise in trying to verify the FE-models when one seeks to find simple analytical solutions to evaluate the natural frequency of the system. The structural response of a portal frame bridge solely is likely to lie between the response of a simply supported beam and a clamped-clamped beam. The natural frequencies obtained from FEA was compared to analytical solutions. For pinned-pinned uniform beams with torsion springs at pinned joints, the natural frequencies are expressed as

$$f_n = \frac{\beta^2}{2\pi L^2} \sqrt{\frac{EI}{m}}$$

where $\beta = \pi$ for a simply supported beam and $\beta = 3\pi/2$ for a clamped-clamped beam (Blevins, 2016). The analytical solutions do indeed form an upper and lower bound in between which the
CHAPTER 3. METHOD

FEA-solution lies, as depicted in Figure C.12. As the span length of the bridge increases, it was noticed that the behavior resembles the clamped-clamped beam. An attempt was made trying to pinpoint the natural frequency of a frame. To include the stiffness governed by the abutments, the structural system was idealized as a partially clamped beam, i.e. a simply supported beam with rotational springs. The stiffness for the rotational springs was obtained as the stiffness induced in a clamped-clamped beam subjected to a unit rotation, see Figure 3.27. However, simplifications were made for the analytical solution, i.e. considering the abutments as massless. The corresponding frequency parameter $\beta$ is tabulated in Table 3.8. Along with the verification of the structural behavior of the portal frame bridge, this verification also assures correct models generated from the Python script in some sense.

<table>
<thead>
<tr>
<th>Case</th>
<th>SS</th>
<th>L5</th>
<th>L10</th>
<th>L15</th>
<th>L20</th>
<th>CC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>$\pi$</td>
<td>1.16$\pi$</td>
<td>1.21$\pi$</td>
<td>1.28$\pi$</td>
<td>1.36$\pi$</td>
<td>1.50$\pi$</td>
</tr>
</tbody>
</table>

Fig. 3.27 Idealization of the portal frame bridge as a partially clamped beam.

3.8.2 Model Validation

In 2D analyses, the DSS analysis procedures are affordable regarding computational cost. However, the cost increases drastically in 3D models. Thus, an analysis procedure based on modal superposition is feasible for linear problems. The DSS-solution was more accurate than the mode-based steady-state dynamics, especially when frequency dependent material damping or viscoelastic material behavior was present. Thus, a need to compare the two analysis procedures emerged. An analysis made solely on the bridge indicated a perfect match between the two methods, see Figure C.2. However, one must not jump to hasty conclusions when including the soil and the viscous boundary. As it has come to the author’s belief, the inclusion of the soil and dashpots at the boundary of the models yield minor differences in the solution obtained by the two procedures, see Figure C.3. For mode-based steady-state analysis, BRIGADE/Plus requires an eigenvalue evaluation step after which the modal superposition is enabled. It is to the judgment of the analyst of how many modes that ought to be included in the analysis. The cutoff frequency in the eigenvalue extraction step is recommended to vary with a factor 1.5 - 4 times the highest frequency content of the loading. Modes above the upper limit tend to respond in a static fashion (Cook et al., 2007) and hence, a factor of 1.5 was chosen for the eigenvalue extraction step to obtain a dynamic response.

To reduce computational time, the train simulations in the frequency- and time domain were performed in MATLAB and Python, respectively. However, the codes which simulate the train passage need to be validated. Thus, the Dynamic Live Load module in BRIGADE/Plus was
The different analysis procedures were compared by reviewing the time histories from the simulation of a train passage of HSLM-A10 at 300 km/h. As seen in Figure C.4 and Figure C.5, there was a perfect match between the time domain analyses. In BRIGADE/Plus, both modal superposition techniques, as well as direct time integration were used. The frequency domain method in MATLAB succeeded to capture the amplitudes accurately which is of the essence. However, a smaller phase shift was visible.

The impedances of the footings can be modeled as described in Section 3.5.4 by assigning elastic and damping behavior to a reference point which governs the behavior of the entire footing through a connector. The benefit of using connectors is that frequency dependent behavior may be implemented, and thus dynamic impedances could be applied rather than only a static impedance function. Static here refers to the implementation of stiffness and damping coefficients for only one frequency. Moreover, the connector allows the inclusion of nonlinear force versus displacement behavior in the unconstrained relative motion components, which was defined for the nonlinear analysis conducted in this thesis. By assigning the nodes of the footing to adapt the behavior of one single reference point, the flexibility of the footing was prevented. Thus, it arises a demand to validate the method of connectors with a corresponding Winkler based spring and dashpot bed, which allows for a flexible foundation. The maximum difference observed in the FRF between the two methods was below 2% and no significant CPU cost was distinguished, although the connector behavior was faster.
Chapter 4

Results

The results presented in this thesis have been deliberately selected in order to emphasize the nature of the dynamic SSI on portal frame bridges. Nevertheless, it is important to recall that these simulations have been performed on virtual models, which indeed are idealizations of the physical systems they represent. Considering the results, the main focus is to present the FRFs and acceleration envelopes for the train simulations. Furthermore, the influence of the span lengths and different boundary conditions on the modal parameters will be distinguished.

4.1 Comparison of 2- and 3D FE-Models

In this section, the comparison between 2- and 3D models is available, where an agreement has been observed particularly for the longer bridges $L_{15}$ and $L_{20}$. Hence, in the following figures, the comparison in between the spatial dimensions will be presented mainly for the shorter bridges. Regarding the 3D-models, only the response in the midpoint has been evaluated. In Figure 4.1, fine agreement between modeling solely the bridge is visible. Notice the maximum peak for $L_{5}$ of Case VIII, indicating a plate mode which evidently is not visible in 2D and disappears for longer span lengths. NB, the ordinator in the figures presented side by side are not normalized, hence, the reader should be observant of the magnitude rather than the direct ocular comparison between curves.

![Fig. 4.1 Comparison of the frame model in 2- and 3D.](image-url)
CHAPTER 4. RESULTS

It has been observed that some discrepancies are present for the models with backfill modeling for L5 and L10. Figure 4.2 illustrates a closeup of the first vertical bending mode for these lengths. Notice that for the shorter span length, the FRF in 3D is not as smooth as in 2D, albeit good match in the peak response is observed. Furthermore, a period shortening is observable in 2D. For L10, the curves show more resemblance to the shape, although the difference in maximum response governed the modal damping ratios. Recall that the choice of the plane stress thickness in the 2D-model influences the result significantly. Moreover, the stiffness varies due to the rigid nature of the 2D-space. To obtain finer agreement, a trade-off between the stiffness and mass in 2- and 3D must be considered. An alternative method, as mentioned in Section 3.2.2, is to apply the principle of energy conservation between models. However, this method has not been investigated in this thesis.

![Fig. 4.2 Comparison of the backfill models in 2- and 3D.](image)

Between the simplified models, the precision increase as the length increases. For L5 in Figure 4.3 it is observed that the 2D simplified model show a tendency of higher natural frequencies than the corresponding counterpart in 3D. The magnitude has been captured to a good degree. For L10 the opposite relation is observed, i.e. the frequency agrees well, however some difference in the magnitude has been noticed.

![Fig. 4.3 Comparison of the simplified backfill models in 2- and 3D.](image)
4.2 COMPARISON OF FULL- AND SIMPLIFIED SOIL MODELS

The simplified half-space models have shown fine agreement regarding the modal damping ratio and the natural frequency for the intermediate and longer bridges. A visible discrepancy has been identified for \( L_5 \), for which the FRF is presented in Figure 4.4. Analogous the simplified backfill, the simplified half-space model has a natural frequency shift as well as a minor increase in response magnitude. Generally, the 2D models tend to show a stiffer and less damped behavior. This ought to be explained by the addition of the impedance functions and the Kelvin-Voigt models in the extended spatial dimension that follows with the 3D-models.

Fig. 4.4 Comparison of the simplified half-space models in 2- and 3D.

4.2 Comparison of Full- and Simplified Soil Models

4.2.1 Linear FE-Models

In the following section, the 2- and 3D soil models will be presented, as seen in Figure 4.5. Besides the obvious difference in modal damping ratios, a period lengthening has been observed for the half-space models compared to the models including backfill. Furthermore, the simplified backfill model somewhat overestimates the modal damping for \( L_{15} \) and is therefore detrimental. However, for \( L_5 \), the simplified backfill shows to be conservative, i.e. it underestimates the modal damping ratio and lengthens the period of vibration. Moreover, the full half-space model is generally in good agreement with the simplified half-space model. Nevertheless, there is a small period lengthening observed for Case V, compared to Case III. Additionally, the modal damping has been increased for the simplified half-space \( L_5 \) in comparison to the full half-space model.
4.2.2 Nonlinear FE-Models

Considering the nonlinear FE-models, FRFs which are shown in Figure 4.6 were simulated by applying a sine sweep load. As can be seen, the modal damping ratio for the nonlinear simplified backfill model is substantially reduced. Seemingly, the nonlinear contact does not affect the first peak, instead, the damping of the second bending mode decreases remarkably for Case VII. Due to the nonlinearity, the FRFs are amplitude dependent and cannot be used to superimpose the results. Thus, they are more feasible for linear systems. Regarding nonlinear systems, the concept of classic resonance is not necessarily descriptive. Instead, these systems are associated with nonlinear resonance which depends on the amplitude of vibration. Nevertheless, an indication for L5 and L10 has been given, were a significant period shortening and a decrease in the modal damping ratio have been observed for L5.
4.3 Parametric Results of Modal Properties

In this section, the modal damping ratios of the 1st and 2nd bending modes are presented. The results illustrate the modal damping ratio assessed with the Half-Power Bandwidth method applied to the FRFs for all lengths of the linear systems. Furthermore, the corresponding natural frequencies are presented for the 1st and 2nd bending mode, respectively. NB, no values are presented for L5 for the 2nd bending mode, as those occurred at frequencies outside the interval of interest. To illustrate the deformed shape during vibration, the associated mode shapes for L10 are shown in Figure 4.7.

Fig. 4.6 The FRFs for the nonlinear FE-models.

Fig. 4.7 The mode shapes of the 1st- and 2nd vertical bending mode.
CHAPTER 4. RESULTS

1st Vertical Bending Mode

In Figure 4.8, the soil models in 2- and 3D are presented in a two-way logarithmic plot. This is partly done to emphasize the relative error between the spatial dimensions for the direct FE-approach, i.e. full backfill models. However, it also illustrates the logarithmic behavior of SSI depending the span length for the simplified FE-models. The half-space models are shifted upwards linearly compared to the simplified backfill models for all lengths. Thus, including the subsoil contributes equally for all span lengths. Generally, there has been a good agreement between the simplified models, considering the shapes of the FRFs in 2- and 3D for all span lengths except the half-space model for 5 meters which deviated. Nevertheless, a smaller linear shift has been observed for the simplified backfill models in 2- and 3D. Finally, it has been noticed that the full backfill model in 2D renders a significantly higher increase of the modal damping ratio than the corresponding 3D model considering $L_{10}$.

![Modal Damping Ratio 1st Vertical Bending Mode](image_url)

Fig. 4.8 Two-way logarithmic plot of the modal damping ratio of the soil models in 2- and 3D.

Regarding the 2D models, the modal damping ratios and the natural frequencies are illustrated in Figure 4.9 and presented in Table 4.1. Moreover, the 3D models are also illustrated in Figure 4.10. The simplified backfill models in both 2- and 3D overestimates the modal damping ratios for the longer span lengths, i.e. $L_{15}$ and $L_{20}$. However, discrepancies are observed for $L_{10}$ where the modal damping ratio for the simplified backfill model has been underestimated, which has not been the case in 2D.
For $L_5$ and $L_{10}$, the natural frequency of the 1st bending mode in the soil models increase in descending order from Case V to II, as seen in Table 4.1. However, the portal frame bridge solely has shown to yield the lowest natural frequency. The full backfill models for $L_{15}$ exhibit the highest natural frequency, followed by full models considering half-space and simplified backfill models. Furthermore, the models with no soil show higher natural frequency than the simplified half-space model. For $L_{20}$, the full backfill models and the simplified half-space models show the highest and lowest natural frequencies, respectively. In 3D, the natural frequency of the frame only and simplified backfill model coincide, whereas for the corresponding models in 2D, the simplified backfill model shows greater natural frequency. The full half-space model in 2D has a natural frequency somewhat lower than the frame only. Thus, period elongation is inevitable for the longer span lengths due to the flexibility of the subsoil. Nevertheless, this has not been observed for the shorter span lengths.
CHAPTER 4. RESULTS

Table 4.1 The modal damping ratios $\zeta[-]$ and the natural frequencies $f$ [Hz] of the 1st bending mode in 2D.

<table>
<thead>
<tr>
<th>Case</th>
<th>Parameter</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VIII</th>
<th>IX</th>
<th>X</th>
<th>XI</th>
</tr>
</thead>
<tbody>
<tr>
<td>L5</td>
<td>$\zeta_n$</td>
<td>0.014</td>
<td>0.106</td>
<td>0.122</td>
<td>0.081</td>
<td>0.179</td>
<td>0.015</td>
<td>0.093</td>
<td>0.075</td>
<td>0.151</td>
</tr>
<tr>
<td></td>
<td>$f_n$</td>
<td>27.1</td>
<td>33.1</td>
<td>32.4</td>
<td>30.5</td>
<td>30.2</td>
<td>27.1</td>
<td>30.9</td>
<td>29.6</td>
<td>28.5</td>
</tr>
<tr>
<td>L10</td>
<td>$\zeta_n$</td>
<td>0.015</td>
<td>0.045</td>
<td>0.099</td>
<td>0.050</td>
<td>0.094</td>
<td>0.016</td>
<td>0.067</td>
<td>0.045</td>
<td>0.093</td>
</tr>
<tr>
<td></td>
<td>$f_n$</td>
<td>13.5</td>
<td>14.6</td>
<td>14.4</td>
<td>14.3</td>
<td>13.8</td>
<td>13.5</td>
<td>14.6</td>
<td>14.1</td>
<td>13.5</td>
</tr>
<tr>
<td>L15</td>
<td>$\zeta_n$</td>
<td>0.015</td>
<td>0.020</td>
<td>0.062</td>
<td>0.031</td>
<td>0.058</td>
<td>0.016</td>
<td>0.018</td>
<td>0.028</td>
<td>0.057</td>
</tr>
<tr>
<td></td>
<td>$f_n$</td>
<td>8.9</td>
<td>9.3</td>
<td>9.1</td>
<td>9.1</td>
<td>8.8</td>
<td>8.9</td>
<td>9.2</td>
<td>9.1</td>
<td>8.6</td>
</tr>
<tr>
<td>L20</td>
<td>$\zeta_n$</td>
<td>0.016</td>
<td>0.017</td>
<td>0.046</td>
<td>0.023</td>
<td>0.044</td>
<td>0.015</td>
<td>0.017</td>
<td>0.022</td>
<td>0.044</td>
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<td></td>
<td>$f_n$</td>
<td>6.9</td>
<td>7.1</td>
<td>6.8</td>
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<td>6.6</td>
<td>6.9</td>
<td>7.1</td>
<td>6.9</td>
<td>6.5</td>
</tr>
</tbody>
</table>

2nd Bending Mode

By observing Figure 4.11, it is noticeable that the consideration of half-space in the models yields substantially greater modal damping ratio, for increasing span lengths, compared to the backfill models. Remarkably, the modal damping ratios for the half-space models increase as the span length increases. This has not been the case for the prior evaluated response in midpoint and ought to depend on the structural compatibility of the abutments for the 2nd bending mode. There, the abutments are oscillating with a phase shift of $\pi/2$. Thus, the longitudinal (swaying) and rotational (rocking) impedances govern the modal damping ratios more drastically. Furthermore, a significant period shortening has been observed when including soil for L10. Considering the simplified backfill models, the modal damping ratio is underestimated for the longer bridges, i.e. L15 and L20. However, the simplified half-space models overestimate the modal damping compared to the full models for L10 and L15. The damping ratios and natural frequencies evaluated in Figure 4.11 are presented in Table 4.2. NB, no comparisons have been made with 3D models considering the 2nd bending mode.

![Fig. 4.11 Two-way logarithmic plot of the modal parameters of the 2nd bending mode.](image-url)
4.4 TRAIN ANALYSIS

In Table 4.2, it can be seen that the soil models have increased the natural frequency drastically for $L_{10}$ in comparison to the portal frame bridge solely. However, this effect diminishes as the span lengths are increased.

Table 4.2 The modal damping ratios $\zeta[-]$ and the natural frequencies $f$ [Hz] of the 2\textsuperscript{nd} bending mode in 2D.

<table>
<thead>
<tr>
<th>Case</th>
<th>Parameter</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>L5</td>
<td>$\zeta_n$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$f_n$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>L10</td>
<td>$\zeta_n$</td>
<td>0.014</td>
<td>0.075</td>
<td>0.120</td>
<td>0.105</td>
<td>0.159</td>
</tr>
<tr>
<td></td>
<td>$f_n$</td>
<td>34.8</td>
<td>42.2</td>
<td>44.4</td>
<td>41.9</td>
<td>45.1</td>
</tr>
<tr>
<td>L15</td>
<td>$\zeta_n$</td>
<td>0.014</td>
<td>0.078</td>
<td>0.165</td>
<td>0.059</td>
<td>0.191</td>
</tr>
<tr>
<td></td>
<td>$f_n$</td>
<td>24.2</td>
<td>24.2</td>
<td>26.2</td>
<td>24.9</td>
<td>25.8</td>
</tr>
<tr>
<td>L20</td>
<td>$\zeta_n$</td>
<td>0.014</td>
<td>0.057</td>
<td>0.211</td>
<td>0.043</td>
<td>0.182</td>
</tr>
<tr>
<td></td>
<td>$f_n$</td>
<td>18.7</td>
<td>18.3</td>
<td>19.1</td>
<td>19.0</td>
<td>18.6</td>
</tr>
</tbody>
</table>

4.4 Train Analysis

4.4.1 Linear FE-Models

The HSLM train analysis has been performed for Case I-VII, i.e. only the 2D FE-models, and the acceleration envelopes are presented for both mid- and quarter point. It can be observed in Figure 4.12 and Figure 4.13 that the acceleration response is governed by the first bending mode for all investigated bridges. Thus, a downward shift has been observed for the acceleration envelope in the quarter point, compared to the response in the midpoint. Furthermore, the acceleration response can be observed in Figure 4.16. Notice that the acceleration response for longer bridges including soil increases with increasing span lengths. Hence, they eventually reach the magnitude of Case I.
Fig. 4.12 The acceleration envelope obtained in the midpoint for Case I-V.

Considering the FE-models including backfill, the critical speed \( v_{cr} \) is shifted towards higher values, for which resonance is observed most clearly for \( L_{15} \) in Figure 4.12. This, ought to be a consequence of the period shortening shown in Table 4.1. Moreover, the acceleration response for shorter span lengths including SSI, have a significant modal damping mentioned a priori, i.e. a minimum of 8% modal damping ratio for \( L_5 \). Therefore, the acceleration response is reduced drastically for these models compared to the portal frame bridge solely. Thus, they are clustered close to each other. Regarding the longer span lengths, i.e. \( L_{15} \) and \( L_{20} \), the models including half-space contributes remarkably to reduce the acceleration response. As can be seen in Figure 4.8, the half-space models increase the modal damping ratios linearly independent of the span lengths, compared to the backfill models. Thus, they have a significant effect, even when the backfill models have converged towards the solution of the portal frame bridge solely. Finally, the underestimation of the modal damping ratio for the simplified backfill models are visible for \( L_{15} \) and \( L_{20} \), i.e. the response is reduced compared to the full backfill model. Consequently, this renders detrimental models for longer span lengths. Nevertheless, the simplified backfill model for \( L_5 \) proves to be conservative.
4.4 TRAIN ANALYSIS

4.4.2 Nonlinear FE-Models

Regarding the nonlinear train analysis, the results are presented for $L_5$ and $L_{10}$ simulated with HSLM A10 and HSLM A1, respectively. Prior results in Section 4.3, emphasize the decrease in SSI for longer span lengths. Therefore, the nonlinear FEA was only performed on the shorter span lengths. Furthermore, the chosen HSLM trains for Case- VI and VII, were based on the trains governing the corresponding global maxima observed for the linear FE-models. This approach circumvents the high computational cost that is associated with nonlinear FE-analysis.

Overall, the nonlinear simplified backfill model generates the highest accelerations. These results represent an upper bound and do somewhat overestimate the gap formation response due to the neglection of static effects. However, the nonlinear backfill contact model (Case VII), where the lateral pressure is simulated, shows good agreement with the corresponding linear FE-models for $L_5$. Nonetheless, a small downward shift is observed for $L_{10}$ between the nonlinear- and linear backfill model. Moreover, no significant contribution from the 2\textsuperscript{nd} bending mode was observed. Thus, the response in the quarter points are shifted downwards as mentioned previously in Section 4.4.
Regarding the HSLM trains, a detailed investigation has been made for the nonlinear backfill contact model with span length $L_{10}$ to understand the decrease in the acceleration response. Further simulations, e.g. HSLM A1-A10, did not generate any drastic changes. Nevertheless, HSLM A10 proved to cause a slight acceleration increase around 500 km/h, however, not as much as expected. Additionally, a time domain analysis was performed for Case II on $L_{10}$, simulated with HSLM A1, to exclude discrepancy from the approach of analysis. It is known from Section 3.7, that the HHT-method used in BRIGADE/Plus renders algorithmic damping, due to $\alpha = -0.41421$. The acceleration spectrum for Case II and VII are shown in Figure 4.15. Notice that the frequency content in the spectral decomposition of the acceleration time signal for Case II mainly includes low frequencies. Thus, the filtered signal was not affected. However, the response for Case VII was governed by high frequencies because these proved to be less damped in Figure 4.6. Therefore, the final response was highly influenced by a low-pass filter (LPF) with a cutoff frequency of 60 Hz, which caused the final reduction that has been seen in Figure 4.14.
4.4 TRAIN ANALYSIS

4.4.3 Parametric Results of Acceleration Response

In this section, the maximum acceleration response in the midpoint, obtained from the HSLM train analysis, are summarized in Figure 4.16. It can be observed, that the response converges towards the portal frame bridge solely. However, the soil models are converging at different rates, whereas the full backfill model reaches the frame only response quickest. Furthermore, there is a large difference in $L10$ between the acceleration response for the nonlinear models, which indicates the significance of including the lateral loads in the full nonlinear model.

Fig. 4.16 The acceleration response summarized from the train analysis.
Chapter 5

Discussion and Conclusion

In this chapter, the parametric results of the work on the dynamic effects of SSI of railway portal frame bridges will be deliberated, followed by some concluding remarks. Within the work presented in this thesis, portal frame bridges of four different span lengths, different boundary conditions, and approaches to include the surrounding soil have been studied. The bridge substructure-soil system has been investigated by direct FE-modeling of the soil and frequency-dependent impedance functions and linear Kelvin-Voigt type springs and dashpots. Mainly, the analyses have been performed within the domain of linear elasticity. The work has also been expanded to include behavior described by nonlinearity in the bridge-soil interface.

5.1 Dynamic Effect of SSI on Portal Frame Bridges

In this thesis, the dynamic effect of SSI on portal frame bridges has been numerically studied. Regarding the simplified models, it has been shown that the backfill models in comparison to the half-space models, yields a significant difference concerning the modal damping ratio. The effect of the span length has shown to be insignificant in this matter. Hence, approximately the same relative increase in the modal damping ratio for the half-space models has been observed. Consequently, this implies that the acceleration response, particularly at resonance regime, decreases for all spans lengths considered herein. Regarding the backfill models, the modal damping ratio converged towards that of the portal frame bridge solely as the span length increased. However, the period shortening implied that the critical speed increased. Considering the half-space models, the observed period lengthening for longer span lengths has been rather detrimental. However, the severe contribution to the modal damping ratio still yields favorable effects regarding the acceleration response.

5.2 Modeling Issues

One of the main aspects of this thesis has been to quantitatively assess the effects of SSI on portal frame bridges in 2D. This allows for fast analysis and becomes practically convenient, provided that accurate results can be collected. The main issue when modeling in 2D with the inclusion of soil as continuum elements has been the assumption of the plane stress thickness of the soil medium. The findings have indicated a high sensitivity of the chosen element thickness. With
increasing thickness, the damping ratio has been reduced significantly together with an increase in the natural frequency, which indeed is legitimated with the associated stiffness increase of the element. Another troublesome aspect of modeling the soil, which has been encountered for both 2- and 3D analysis, were the longitudinal extension of the soil in the model. The radiation condition needs to be fulfilled such that no energy scatters back from the far field into the source, hence the effect of changes in soil length has been investigated. Some erratic behavior in the FRFs has been observed, particularly for shorter bridges with confining backfill both in 2- and 3D. For longer bridges, the results converged, emphasizing that shorter bridges have been dependent on the surrounding soil conditions. The same inconsistent behavior of the response has not, however, been found to a high degree for the shortest half-space model in 2D. It is the authors’ belief that the behavior of the bridge is probably governed by the soil-modes, specifically in the near vicinity of the bridge. However, these modes may be constrained by the inclusion of wing walls or with slopes in the transverse direction.

The previous statements demonstrate the difficulty of formulating a realistic, robust 2D model which can reproduce the results from 3D models when direct modeling approach of the soil is considered. One must be aware that the results of 3D models do not directly imply accurate results. Agreement may be obtained between models which only would mean precise results, while the accuracy of the results is to be justified by validation through field measurements.

5.3 Consideration of Full and Simplified FE-Models

Simplified models, which are economically cheaper and advantageous in practical applications, have been put in contrast to direct modeling approaches. For short span lengths with only the backfill, the simplified models have been more conservative regarding the modal damping ratio of the first fundamental bending, while a tendency of overestimation has been observed for longer span lengths. For the half-space models, the inverse relation emerged from the analysis, however, with much greater precision. A clear trend of period lengthening has been observed for the simplified models, both the backfill and the half-space models in 2D as well as in 3D. Very fine agreement has been found between simplified models in 2- and 3D of the modal damping ratio for the 1\textsuperscript{st} vertical bending. Evidently, some modes of vibrations, e.g. plate modes, could not be captured in 2D.

5.4 Some Aspects of Nonlinearity in SSI

Dealing with contact nonlinearity in dynamic SSI has, indeed, been a tough challenge. Within the scope of this thesis, the aim has been to model the contact interaction between abutment and backfill during simulation of HSLM trains. Two attempts have been made, i.e. simplified- and full soil models, with mixed success. Considering the simplified models, where a nonlinear spring element governs the response, difficulties arose when the importance of adding an at-rest lateral pressure was discovered. Since the lateral pressure has been applied by the means of Poisson’s effect and gravity loads in the full soil model, no corresponding application has been found to
simulate this behavior in the simplified models.

Considering the full nonlinear backfill contact model, some interesting findings have been made. Initially, the sine sweep loads have shown similar modal damping ratio in the 1st vertical bending mode compared to the linear backfill model. Nevertheless, differences have been observed for the higher modes. During the train analysis for $L10$ the acceleration response in both mid- and quarter point decreased compared to the linear case. These, rather non-intuitive results, ought to depend on the decrease in the modal damping ratio of the higher modes. Thus, the acceleration response including higher frequencies, got eliminated when an LPF was applied.

The influence of lateral loads has clearly affected the nonlinear response in the full backfill model. Considering the 1st bending mode, the contact between abutment and backfill was retained and only a small reduction of the modal damping ratio and natural frequency has been observed. However, regarding the higher modes, the separation proved to be substantial. In reality, the at-rest lateral earth pressures are significantly larger than the lateral loads that have been generated during these simulations. Thus, the gap formation ought to have an even smaller effect when studying the higher modes in the physical system.

5.5 Conclusion

The conclusions from this thesis are summarized in the following:

- It has been shown that the inclusion of the surrounding soil drastically increases the modal damping ratio and natural frequency, illuminating the essence of including SSI in dynamic analysis. The effect of SSI has proven to be severe and beneficial for shorter span lengths, although longer span lengths benefit as well. For shorter bridges, the effect of backfill has been prominent while the modal damping ratio for the longer span lengths has been more affected by the inclusion of the subsoil. These parameters does indeed reduce the response of train-induced vibrations.

- Numerically, it has for the simplified models been observed that the SSI of portal frame bridges yields a logarithmic trend of the modal damping ratio, depending on the span length. These relations have been investigated for full- and simplified FE-models including backfill and half-space. Moreover, the logarithmic behavior propose an indication of the modal parameters for the 1st and 2nd bending modes of portal frame bridges with intermediate span lengths. Nevertheless, these modal properties are very much dependent on the boundary conditions, which are a consequence of the soil parameters.

- FE-modeling of soil in 2D, with plane stress elements, should be avoided unless no validations are made with 3D models. Nevertheless, simplified models such as, frequency-dependent impedance functions and linear Kelvin-Voigt type springs and dashpots has proven to be successful when comparing with 3D FE-models. Further, this implies that the implementation of 2D simplified models can be used to a greater extent. Subsequently, verification between full and simplified models in 3D and validation against field measurements, the 2D simplified models may be implemented in the future.
Contact nonlinearity has been difficult to deal with in the simplified models. Instead, full soil models have been advantageous in order to include the static and dynamic effects. In this thesis, the complicated contact interaction between abutment and backfill has been investigated by the means of gravity loads and Poisson’s effect. This approach has proven to be enlightening due to the significant changes in acceleration response, simply by including static effects. Furthermore, the train-induced vibrations of portal frame bridges have been governed by the 1st bending mode. Thus, no significant differences have been observed between the linear and nonlinear full soil FE-models.

5.6 Future Research

Concluding this thesis, there are some topics to consider for further investigation based on the presented findings, where the following are of interest:

- Within the thesis, quantitative results were presented which are indicative of trends governed by SSI and results in deeper understanding of the phenomena. Yet, the current design code stipulates too conservative values of damping ratios. In order to present a trendline which may be implemented in design, the authors humbly want to shed some light on in-situ measurements to validate the numerical analyzes. Hence, accurate result can be obtained. One important aspect of numerical models used to simulate the behavior of physical systems is the similarity of the model. The reliability of the simplified models used within this thesis should be verified against other numerical models, for instance where wing-walls, edge beams, ballast and the SSI between the bridge superstructure and ballasted track are included.

- Concerning the nonlinear contact phenomena, a broader understanding of the physical system needs to be grounded in the academia. Nevertheless, more sophisticated numerical models can be developed which accurately capture the lateral earth pressure. Preferably, these models could simulate the variation of lateral earth pressure, i.e. the active or passive earth pressure, depending on the motion of the abutments.
References


REFERENCES


Appendix A

Impedance Functions

In the following subsections, the dynamic stiffness functions as described in Section 3.5.4 are presented in Appendix A.1 and Appendix A.2 for the spring and dashpot coefficients used in 2D and 3D, respectively.

A.1 2D Impedance Functions

In Figure A.1 and Figure A.2, the spring coefficient $k$ and dashpot coefficient $c$ are presented. The subscripts denotes the direction of action, where $x$ and $y$ denotes the longitudinal and vertical direction of the bridge, respectively. The rotational components about the transverse axis is denoted $zz$. 
APPENDIX A. IMPEDANCE FUNCTIONS

Fig. A.1 2D Impedance functions of $L_5$ and $L_{10}$ bridge.
Fig. A.2 2D Impedance functions of L15 and L20 bridge.
A.2 3D Impedance functions

In addition to the longitudinal, vertical and rocking impedance about the transverse axis in 2D presented in Figure A.1 and Figure A.2, six more impedances are present in 3D. These impedances concerns the transverse direction $z$, as well rocking about the longitudinal $xx$ and torsional mode of vibration $yy$.

Fig. A.3 3D Impedance functions of $L5$ and $L10$ bridge.
Fig. A.4 3D Impedance functions of $L_{15}$ and $L_{20}$ bridge.
Appendix B

Convergence Studies

In this appendix, convergence studies performed throughout the thesis will be presented. The convergence studies are mainly based on L5 and the FRFs have been evaluated at midpoint.

- **Appendix B.1**: the effect of the plane stress thickness chosen in 2D is presented.
- **Appendix B.2**: illustrates the effect of the radiation condition for Case II and Case III in 2D as well as for Case VIII and Case IX in 3D.
- **Appendix B.3**: presents a qualitative sensitivity analysis on the difference in concrete classes, i.e. C20/25 and C50/60 as well as different shear wave velocities of the soil medium and Poisson’s ratio.
- **Appendix B.4**: demonstrates how the time history behaves as a Dirac-Delta pulse and the effect on the load spectrum as the mesh size decrease. The effect of the load spectrum on the acceleration time history is presented.
- **Appendix B.5**: the effect of the mesh size on the acceleration time history in time domain is presented.
- **Appendix B.6**: the effect of imaginary and real load the the acceleration time history in the frequency domain is presented.
- **Appendix B.7**: for the simplified nonlinear model, physical interpretation of nonlinear springs and dashpots are presented as well as time step convergence. Furthermore a study of the discretization size as well as the amplitude dependency is presented. The effect of the inclusion of gravity in nonlinear models is studied.
- **Appendix B.8**: for the nonlinear backfill contact model, the same studies are presented as in Appendix B.7 with the addition of the investigation of contact behavior specified in normal and tangential direction.

B.1 Plane Stress Thickness

Figure B.1 illustrates the effect of the chosen plane stress thickness $d_2$ of the backfill soil, see Figure 3.3 for to recall the geometry of the backfill.
Effect of Plane Stress Thickness

Forcing Frequency, $f$ [Hz]

Response Magnitude, $|H|$ [m/s$^2$/kN]

Convergence Study - Case II

Plane stress $d_z = 7.0$ m
Plane stress $d_z = 10.8$ m
Plane stress $d_z = 14.5$ m
Plane stress $d_z = 18.3$ m
Plane stress $d_z = 22.0$ m

Fig. B.1 Study of the plane stress thickness in 2D for 5 m bridge.
B.2 Radiation Condition

B.2.1 2D Radiation Condition

Fig. B.2 Convergence study of radiation condition in 2D of backfill length for 5 m bridge.

Fig. B.3 Convergence study of radiation condition in 2D of backfill length for 10 m bridge.
Fig. B.4 Convergence study of radiation condition in 2D of backfill length for 15 m bridge.

Fig. B.5 Convergence study of radiation condition in 2D of backfill length for 20 m bridge.
B.2 RADIATION CONDITION

**Effect of Change in Halfspace Radius - Case III**

![Graph showing response magnitude vs forcing frequency for different halfspace radii.](image)

- $R_{\text{halfspace}} = 8 \text{ m}$
- $R_{\text{halfspace}} = 12 \text{ m}$
- $R_{\text{halfspace}} = 16 \text{ m}$
- $R_{\text{halfspace}} = 20 \text{ m}$
- $R_{\text{halfspace}} = 24 \text{ m}$

Fig. B.6 Convergence study of radiation condition in 2D of halfspace radius for 5 m bridge.

**B.2.2 3D Radiation Condition**

**Effect of Change in Backfill Length - Case IX - L5**

![Graph showing response magnitude vs forcing frequency for different backfill lengths.](image)

- $L_{\text{backfill}} = 8 \text{ m}$
- $L_{\text{backfill}} = 12 \text{ m}$
- $L_{\text{backfill}} = 16 \text{ m}$
- $L_{\text{backfill}} = 20 \text{ m}$
- $L_{\text{backfill}} = 24 \text{ m}$

Fig. B.7 Convergence study of radiation condition in 3D of backfill length for 5 m bridge.
Effect of Change in Backfill Length - Case IX - L10

Forcing Frequency, $f$ [Hz]

Response Magnitude, $|H|$ [m/s$^2$/kN]

Radiation Condition

$L_{\text{backfill}} = 9$ m
$L_{\text{backfill}} = 14$ m
$L_{\text{backfill}} = 19$ m
$L_{\text{backfill}} = 23$ m
$L_{\text{backfill}} = 28$ m

Fig. B.8 Convergence study of radiation condition in 3D of backfill length for 10 m bridge.
B.3 Qualitative Sensitivity Analysis

Fig. B.9 Concrete elastic modulus $E_c = 24$ GPa.

Fig. B.10 Concrete elastic modulus $E_c = 29.6$ GPa.
APPENDIX B. CONVERGENCE STUDIES

Effect of S-Wave Velocity

![Graph showing the effect of S-wave velocity on response magnitude.]

Fig. B.11 Effect of 50 m/s increase in S-wave velocity for soil material.

![Graph showing the effect of S-wave velocity on response magnitude.]

Fig. B.12 Effect of 50 m/s decrease in S-wave velocity for soil material.
B.3 QUALITATIVE SENSITIVITY ANALYSIS

Effect of Poisson’s Ratio in Soil

\( \nu_{bf} = 0.3 \) & \( \nu_{ss} = 0.3 \)

Fig. B.13 Effect of increase in Poisson’s ratio in soil material \( \nu = 0.3 \).

Effect of Poisson’s Ratio in Soil

\( \nu_{bf} = 0.2 \) & \( \nu_{ss} = 0.2 \)

Fig. B.14 Effect of Poisson’s ratio in soil material for reference parameter \( \nu = 0.2 \).
B.4 Mesh Dependency in Frequency Domain Method

Fig. B.15 Amplitude function and load spectrum for mesh: 1.0.

Fig. B.16 Amplitude function and load spectrum for mesh: 0.5.
Fig. B.17 Amplitude function and load spectrum for mesh: 0.25.

Fig. B.18 Amplitude function and load spectrum for mesh: 0.10.
Fig. B.19 Mesh convergence of acceleration time history from frequency domain method.

Fig. B.20 Mesh convergence of maximum acceleration frequency domain method.
B.5 Mesh Dependency in Time Domain Method

Fig. B.21 Mesh convergence of acceleration time history from time domain method.

Fig. B.22 Mesh convergence of maximum acceleration from time domain method.
B.6 Load Dependency in Frequency Domain Method

Fig. B.23 Real- vs. Imaginary Load of train analysis in frequency domain method 0-1.5 sec.

Fig. B.24 Real- vs. Imaginary Load of Train Analysis in Frequency Domain Method 4-5.5 sec.
B.7 Simplified Backfill Nonlinear FE-Model

Fig. B.25 Force time history for simplified backfill nonlinear FE-model.
Fig. B.26 Convergence of time step for simplified nonlinear FE-Model.
Convergence of Element Size

Case VI - L10 - Quarterpoint

![Graph showing the convergence of element size for Case VI.](image)

Fig. B.27 Convergence of element size for simplified backfill nonlinear Case VI.

Amplitude Dependancy of FRF by Sine Sweep Load

Case VI - L10

![Graph showing the amplitude dependency of FRF for Case VI.](image)

Fig. B.28 Amplitude dependency of FRF for Case VI.
Comparison Between FRF for Case VI and Case VII

Fig. B.29 FRF comparison between Case VI and Case VII for L10 without gravity.
B.8 Nonlinear Backfill Contact FE-Model

![Rayleigh Damping - L5](image1)

**Concrete:** $\alpha = 3.0845$, $\beta = 4.3406\times10^{-5}$

**Backfill:** $\alpha = 5.1408$, $\beta = 7.2343\times10^{-5}$

Fig. B.30 Rayleigh parameters of backfill and concrete, $L5$.

![Rayleigh Damping - L10](image2)

**Concrete:** $\alpha = 1.885$, $\beta = 7.6394\times10^{-5}$

**Backfill:** $\alpha = 3.1416$, $\beta = 0.00012732$

Fig. B.31 Rayleigh parameters of backfill and concrete, $L10$.  

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Convergence of Element Size With Gravity Load

**Case VII - L5 - Midpoint**

| Forcing Frequency, $f$ [Hz] | Response Magnitude, $|H|$ [m/s$^2$/kN] |
|-----------------------------|------------------------------------------|
| 0                           | 0.05                                      |
| 10                          | 0.1                                       |
| 20                          | 0.15                                      |
| 30                          | 0.2                                       |
| 40                          | 0.25                                      |
| 50                          | 0.3                                       |

Fig. B.32 Convergence of element size for backfill contact nonlinear Case VII L5.

Convergence of Time Step $\Delta t$ - Case VII - L5 - LPF (60 Hz)

**First Bending Mode: 4.85-5.35 sec**

<table>
<thead>
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<th>Time, $t$ [sec]</th>
<th>Vertical Acceleration, $a$ [m/s$^2$]</th>
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</thead>
<tbody>
<tr>
<td>4.85</td>
<td>10</td>
</tr>
<tr>
<td>4.9-5.35</td>
<td>0-10</td>
</tr>
</tbody>
</table>

Fig. B.33 Convergence of time step for backfill contact nonlinear Case VII L5.
Convergence of Time Step $\Delta t$ - Case VII - L10 - LPF (60 Hz)

Fig. B.34 Convergence of time step for backfill contact nonlinear Case VII L10.
APPENDIX B. CONVERGENCE STUDIES

Convergence of Element Size With Gravity Load

Forcing Frequency, $f$ [Hz]

Response Magnitude, $|H|$ [m/s$^2$/kN]

Case VII - L10 - Quarterpoint

Fig. B.35 Convergence of element size for backfill contact nonlinear Case VII L10.

Amplitude Dependency of FRF by Sine Sweep Load Including Gravity

Forcing Frequency, $f$ [Hz]

Response Magnitude, $|H|$ [m/s$^2$/kN]

Case VII - L10 - Quarterpoint

Fig. B.36 Amplitude dependency of FRF including gravity loads for Case VII.
Effect of Contact Directions and Including Gravity

Case VII - L10 - Quarterpoint

Normal Contact
Normal and Tangential Contact

Influence of normal and tangential contact for Case VII L10.

Influence of Gravity Load

Case VII - L10 - Quarterpoint

Gravity
No Gravity

Influence of gravity for Case VII L10.
APPENDIX B. CONVERGENCE STUDIES

Influence of Gravity Between Case I - II and VII

Fig. B.39 Influence of gravity for Case I - II and VII L10.

Convergence of Time Step $\Delta t$ - LPF (62 Hz)

Fig. B.40
Appendix C

Method Validation

Throughout this Appendix, intermediate results used to validate the model is presented.

- **Appendix C.1:** illustrates a comparison between the applied sine sweep load generated from MATLAB and the step procedure DSS in BRIGADE/Plus.

- **Appendix C.2:** validates the choice of eigensolver for modal analysis. The solutions are compared to the direct-solution steady state. Validation performed in 3D for Case VIII and Case IX.

- **Appendix C.3:** validates the frequency- and time domain analysis performed in MATLAB and Python, respectively, with the time domain analysis performed with the Dynamic Live Load module in BRIGADE/Plus.

- **Appendix C.4:** presents the effect of different element choice in 3D. A comparison between hexahedral and tetrahedral elements is presented as well as mesh convergence analyses of Case I, II, VIII and IX.

- **Appendix C.5:** verifies the model compared to analytical solutions of a frame regarding the natural frequency of the first mode of vibration.
C.1 Validation of FRF

Validation of FRF in BRIGADE/Plus

Forcing Frequency, $f$ [Hz]
Response Magnitude, $|H|$ [m/s$^2$/kN]

Direct-Solution Steady-State vs. Sine Sweep Load
Case I - SSL
Case I - DSS
Case II - SSL
Case II - DSS

Fig. C.1 Sine Sweep Load vs. DSS.
C.2 Validation of Modal Eigensolver

Validation of Analysis procedure in BRIGADE/Plus

Modal Eigensolver vs. Direct-Solution Steady State - Case VIII

Fig. C.2 Comparison between modal eigensolver and DSS - Case VIII

Validation of Analysis procedure in BRIGADE/Plus

Modal Eigensolver vs. Direct-Solution Steady State - Case IX

Fig. C.3 Comparison between modal eigensolver and DSS - Case IX
C.3 Validation of HSLM and Analysis Procedures

Train Validation - HSLM A10 - Velocity 300 km/h - LPF (62Hz)

Case I Frame Only L5: 0 - 3 sec

BRIGADE/Plus Modal
BRIGADE/Plus DTI
Python TD
MATLAB FD

Fig. C.4 HSLM Validation 0-3 sec.

Case I Frame Only L5: 3 - 6 sec

BRIGADE/Plus Modal
BRIGADE/Plus DTI
Python TD
MATLAB FD

Fig. C.5 HSLM Validation 3-6 sec.
C.4 Mesh Validation

Fig. C.6 Time comparison between quadratic hexahedral and tetrahedral elements in 3D.
Fig. C.7 Comparison of FRF between HEX and TET elements in 3D - Case IX.
Fig. C.8 Mesh convergence of frame model with linear elements in 2D.
Fig. C.9 Mesh convergence of backfill model with linear elements in 2D.
**C.4 MESH VALIDATION**

Fig. C.10 Mesh convergence of frame only with quadratic elements in 3D.

Fig. C.11 Mesh and element study of backfill in 3D.
C.5 Verification to Analytical Solution

verification of FE-model and analytical solution

Forcing Frequency, $f$ [Hz]

Response Magnitude, $|H|$ [m/s^2/kN]

Fig. C.12 Verification of FE-model and analytical solutions.