Structural Concrete Elements Subjected to Air Blast Loading

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TRITA-BKN. Bulletin 92, 2007
ISSN 1103-4270
ISRN KTH/BKN/B--92--SE

Licentiate Thesis
Preface

This thesis presents experimental and theoretical research on concrete beams subjected to static and air blast loads. The experimental research has been carried out at the Swedish Defence Research Agency (FOI), Department of Physical Protection and Weapons Effects, with financial support received from the Swedish Armed Forces Headquarters. The theoretical studies were financially supported by Fortifikationskåren, the Division of Concrete Structures, Royal Institute of Technology (KTH) and FOI and carried out at KTH. The completion of the thesis was performed partly at KTH and partly at FOI.

I wish to express my deep gratitude to my supervisor Prof. Jonas Holmgren for his guidance and support during the first period of time I was a doctoral student. I also especially thank my supervisor Assoc. Prof. Anders Ansell for his encouragements and invaluable support during the second period as a doctoral student and upon completion of this thesis. I wish also to express my gratitude to Assoc. Prof. Mikael Hallgren at Ramböll for invaluable discussions and cooperation during the experimental research and for revising the reports and papers involving the conventionally reinforced concrete beams. Special thanks also to Mr. Rickard Forsén at FOI, especially for revising the chapters including blast loads and iso-damage curves in the thesis and for his valuable comments and sharing of knowledge on these subjects. I thank Mr. Anders Carlberg and Mr. Klas-Göran Bolling at FOI who assisted the tests for their valuable contributions and friendly cooperation. Also, I thank Mr. Ingvar Anglevik at the Swedish Armed Forces Headquarters for his never-ending enthusiasm and support during the experimental work. I also thank Assoc. Prof. Raid Karoumi at the Division of Structural Design and Bridges for revising the thesis. I am also grateful to the personnel and my fellow doctoral students at the Department of Civil and Architectural Engineering and especially the Divisions of Concrete Structures and Structural Design and Bridges at KTH, and also to my colleagues at FOI for contributing to friendly and fruitful atmospheres. Finally, I also wish to express my deepest gratitude to my Angelica for her understanding and patience during my completion of this thesis.

Stockholm, May 2007

Johan Magnusson

“Explosions can be both awesome and devastating”
Kinney & Graham (1985)
Abstract

In the design of structures to resist the effects of air blast loading or other severe dynamic loads it is vital to have large energy absorbing capabilities, and structural elements with large plastic deformation capacities are therefore desirable. Structures need to be designed for ductile response in order to prevent partial or total collapse due to locally failed elements. The research in this thesis considers experimental and theoretical studies on concrete beams of varying concrete strength. The nominal concrete compressive strength varied between 30 MPa and 200 MPa. A total of 89 beams were tested of which 49 beams were reinforced with varying amounts of tensile reinforcement. These beams were also reinforced with stirrups and steel fibres were added to a few beams. The remaining 40 beams were only reinforced with steel fibres with a fibre content of 1.0 percent by volume. Two different fibre lengths having constant length-to-diameter ratio were employed. The tests consisted of both static and air blast tests on simply supported beams. The blast tests were performed within a shock tube with a detonating explosive charge. All experimental research focused on deflection events, failure modes and loads transferred to the supports. The dynamic analyses involve single-degree-of-freedom (SDOF) modelling of the beam response and the use of iso-damage curves. Also, the dynamic support reactions were calculated and compared with test results.

For beams with tensile reinforcement, the failure mode of some beam types was observed to change from a flexural failure in the static tests to a flexural shear failure in the dynamic tests. Beams with a high ratio of reinforcement and not containing steel fibres failed in shear, whereas beams with a lower ratio of reinforcement failed in flexure. The introduction of steel fibres prevented shear cracks to develop, thus increasing the shear strength of the beams. The presence of steel fibres also increased the ductility and the residual load capacity of the beams. Beams subjected to air blast loading obtained an increased load capacity when compared to the corresponding beams subjected to static loading. The SDOF analyses showed good agreement with the experimental results regardless of concrete strength and reinforcement amount. The results of using iso-damage curves indicate conservative results with larger load capacities of the beams than expected. The theoretical evaluations of the dynamic reactions were in agreement with the measured average reactions, both in amplitudes and in general shape.

The experimental results with steel fibre reinforced concrete beams indicate that the dynamic strength was higher than the corresponding static strength and that the toughness was reduced when increasing the compressive strength. Beams of normal strength concrete failed by fibre pull-out while a few beams of high strength concrete partly failed by fibre ruptures. It may be favourable to use shorter fibres with smaller aspect ratios in structural elements of high strength concrete and subjected to large dynamic loads.

Further research should involve studies on the size effect, on different boundary conditions, on different types of structural elements and on the combination of blast and fragment loads. The theoretical work should involve analyses both with the use of SDOF modelling and finite element analysis.
Sammanfattning

Vid dimensionering av konstruktioner som skall motstå effekterna av explosionslaster eller andra stora dynamiska laster är det viktigt att dimensionera för stor energiupptagningsförmåga. Det är därför önskvärt att konstruktionselementen har stor plastisk deformationsförmåga för att motverka ras då brott uppstår för enskilda element. Forskningen i föreliggande avhandling beaktar experimentella och teoretiska studier av betongbalkar med en nominell tryckhållfasthet varierande mellan 30 MPa och 200 MPa. Totalt genomfördes experiment på 89 balkar varav 49 stycken var armerade med varierande mängder dragarmering. Dessa balkar var även armerade med byggar och i vissa fall innehöll de även stålffibrer. Återstående 40 balkar var enbart armerade med 1.0 volymprocent stålffibrer. Två olika fiberlängder med samma slankhetstal användes. Experimenten bestod av både statiska försök och försök med explosionslaster på fritt upplagda balkar. Försöken med explosionslaster genomfördes i en stötvågstub med en detonerande sprängladdning. Forskningen fokuserade på utböjningar, brottmoder och lastöverföring till stöd. Den dynamiska analysen involverade en frihetsgradsberäkningar av balkresponsen samt användningen av skadekurvor. Även de dynamiska upplagsreaktionerna beräknades och jämfördes med resultaten från försöken.


Försöken med balkar enbart armerade med stålffibrer visade på högre bärfrämjade vid dynamisk belastning än vid statisk belastning och det framgick att segheten minskar med ökande betonghållfasthet. Balkar med normalhållfast betong gick till brott genom att fibrerna drogs ut ur betongen medan balkar av högsta hållfastheten delvis gick till brott genom avslitning av fibrenna. Försöken indikerar att det är fördelaktigt med kortare fibrer med lägre slankhetstal för konstruktionselement bestående av höghållfast betong och exponerade för stora dynamiska laster.

Vidare forskning bör innehålla studier av skalaffekter, olika typer av konstruktionselement, olika upplagsförhållanden och kombinationen av luftsjuvbåg och splitter. De teoretiska studierna bör innefatta analyser med både enfrihetsgradssystem och modellering med finita elementmetoden.
Enclosed papers

The following papers are included in this thesis:


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List of symbols

$A$ area of beam cross-section
$A_s$ cross-sectional area of steel reinforcement
$b$ beam width
$c$ damping coefficient
$c$ constant
$C$ constant
$d$ effective depth of beam
$D$ constant
$E$ Modulus of elasticity
$E_c$ Modulus of elasticity for concrete
$E_s$ Modulus of elasticity for steel
$EI$ flexural stiffness
$f_{cc}$ concrete compressive strength
$f_t$ concrete tensile strength
$f_y$ yield strength of steel
$G$ constant
$G_F$ fracture energy of concrete
$h$ beam height
$H$ constant
$i$ impulse density
$i_c$ characteristic impulse density
$I$ impulse
$I_c$ characteristic impulse
$k$ stiffness
$K$ kinetic energy
$l$ beam span
$l_{ch}$ characteristic length
$L$ beam span
$m$ mass
$M$ bending moment capacity
$M_{dyn}$ dynamic bending moment capacity
$p$ overpressure
$p_c$  characteristic overpressure  
$P$  load  
$P_1$  maximum load  
$P_c$  characteristic load  
$Q$  mass of explosive charge  
$R$  distance  
$R_s$  spring resistance  
$R_1$  maximum spring resistance  
$t$  time  
$T$  natural period  
$x$  height of concrete compressive block  
$U$  strain energy  
$W$  work  
$y$  displacement  
$y_1$  maximum displacement  
$\dot{y}$  velocity  
$\ddot{y}$  acceleration  
$Z$  scaled distance  
$\alpha$  coefficient  
$\chi_P$  constant  
$\chi_R$  constant  
$\dot{\varepsilon}$  strain rate  
$\kappa_L$  load factor  
$\kappa_M$  mass factor  
$\kappa_R$  resistance factor  
$\rho$  density  
$\tau$  parameter  
$\omega$  circular frequency  
$\omega_s$  mechanical ratio of reinforcement
1 Introduction

1.1 Background

A large number of reinforced concrete structures exist as a part of the urban environment, as a part of the infrastructure or as different types of civilian and military facilities. Dynamic events of different sorts are common in our every day life, such as gusts of winds making trees bend and structures deflect, traffic loads on a bridge and objects falling and impacting the roof of a structure, or other catastrophic events. Explosions due to civilian accidents, detonating high explosives or weapons effects result in extreme loading conditions on all objects nearby. Explosions occurring in urban areas or close to facilities such as buildings and protective structures may cause tremendous damage and loss of life. The immediate effects of such explosions are blast overpressures propagating through the atmosphere, fragments generated by the explosion and ground shock loads resulting from the energy imparted to the ground. However, the effects of the blast overpressures are usually the governing load when considering the dynamic response of structures and only these effects on reinforced concrete structures are considered in this thesis. There may be situations where the effects of fragments or ground shock loads are just as important, but these situations are not addressed herein.

In the design of buildings to resist the effects of air blast loading or other severe dynamic loads, it is not economical to consider the structural response in its elastic range only. If that was the case, such structures would become unproportionally strong. Therefore, the structural elements should be allowed for certain plastic deformations, which better utilises their energy absorbing capabilities if exposed to dynamic loads. It is vital to design for ductile response in order to prevent partial or total collapse of a structure due to locally failed elements. In a case where a structure has been damaged by explosive loading the residual strength of the individual elements may be very important in order to prevent a progressive collapse.

Normal strength concrete (NSC) is a widely used material in structures of the society. However, in recent years the use of high strength concrete (HSC) has increased for different structures such as bridges and high rise buildings. HSC is also an interesting material that may be used in the design of blast-resistant structures and shelters. In this context, HSC refers to a concrete with a compressive strength exceeding 80 MPa. Research on HSC is carried out all around the world and compressive strengths of over 200 MPa is now possible to obtain with conventional methods. It is well known that plain concrete of higher strength leads to a more brittle failure compared to that of plain conventional concrete. However, the introduction of steel fibres into the concrete matrix results in an enhanced ductility. Steel fibre reinforce concrete (SFRC) elements are known to have larger energy absorbing capabilities compared to plain concrete elements. Thus, adding steel fibres in combination with the use of a proper amount of bar reinforcement results in a ductile structural element. A national research program concerning HSC was carried out in Sweden during the years 1991–1998. The program was divided into three main areas, namely material properties, structural design and construction. The project resulted in about 150 research reports and two handbooks, *Betonhandbok* (2000) and *Design Handbook*.
More research is however needed regarding the structural behaviour of HSC subjected to dynamic loading in order to properly use this material. A number of investigations have studied the behaviour of dynamically loaded reinforced concrete beams and slabs with a conventional concrete strength and it is also of interest to study the behaviour of HSC structural elements subjected to air blast loading. The combination of well-defined experiments and theoretical analyses is a necessity for a better understanding of this complex problem and that approach is used herein. A long-term goal may be to generate and achieve the knowledge necessary to enable response predictions of complicated structures loaded by severe dynamic loads. In order to be able to predict the dynamic response of different kinds of structures it is necessary to, as a first step, achieve enough knowledge and understanding of the dynamic behaviour of less complex cases. These cases are the response of individual structural elements such as simply supported concrete beams. When sufficient knowledge has been theoretically and experimentally acquired it is then possible to move on to more complicated cases.

The single-degree-of-freedom (SDOF) method is commonly used for analysing the dynamic flexural response of structural elements and in the design of protective structures subjected to air blast loading. This is still the case even though more powerful methods like the finite element method are available nowadays. The SDOF method has several advantages such as ease of use, low design costs and due to that this method is incorporated in design manuals for blast resistant design, e.g. TM5-1300 (1990). Due to this wide applicability it was of interest to use the SDOF method in a dynamic analysis of the air blast tests reported in Paper I and II. The Swedish guidelines for design of fortifications and other concrete structures were based on research available at that time (Publikation nr 25, 1973). Several parameters in the guidelines are based on empirical data and should be updated. Boverket (1994) is a handbook where accidental loads such as explosions among others are considered in the design of structures. This handbook gives merely a brief description of the loads from explosions and diagrams of maximum deformations at different ratios between load duration and natural period of vibrations of an SDOF system.

1.2 Aim and scope

The main objective of the research presented in this thesis is to theoretically and experimentally study the structural behaviour of HSC beams subjected to air blast loading. These beams can be regarded as part of a slab or a wall in a real structure. The work focus on load and deflection capacity as well as failure modes of the beams. The experimental series will comprise both statically and dynamically loaded beams with varying concrete strength and varying reinforcement amounts. The behaviour of solely steel fibre reinforced beams is also investigated. The theoretical studies involve SDOF response calculations of the air blast tested reinforced beams and the use of iso-damage curves. Similar calculations are not performed on the SFRC beams due to the fact that these are not able to obtain such large deformations as beams containing rebars. The dynamic support reactions are also theoretically determined. Other dynamic loads such as the impact of falling masses or impacting fragments are not considered in the analysis. A secondary objective is to give a background to and describe the features of air blast loading, which serves as a basis for the present and future research.
1.3 Outline of thesis

This thesis is based on knowledge obtained in several experimental studies performed at the Swedish Defence Research Agency (FOI), which are complemented with theoretical studies. The outline of this thesis is as follows: A literature review of the loads originating from detonating high explosives is given in chapter 2. Some aspects on high strength concrete are given in section 3.1. This section also presents strain rate effects in concrete and reinforcement steel due to dynamic loads. The general structural dynamic response is discussed in section 3.2. A theoretical review of the SDOF method is made in section 3.3 also involving iso-damage curves and the dynamic support reactions. The theoretical analysis is presented in chapter 4 starting in section 4.1 with a discussion of the major findings from the experimental programs. Section 4.2 incorporates the dynamic analysis of the experiments with the use of analytical solutions and numerical solutions in some cases, iso-damage curves and also evaluations of the support reactions. This dynamic analysis is not performed in any of the listed papers I–III. The experimental and theoretical results are commented and discussed in chapter 5 and the conclusions along with suggestions for further research are presented in chapter 6.

1.4 Enclosed papers

Paper I:

Paper I involves the results of static and air blast tests on reinforced concrete beams of different concrete grades. The nominal concrete compressive strength varied between 40 MPa and 200 MPa. One type of beam consisted of two concrete layers with grade 40 in the tensile zone and grade 200 in the compression zone of the beam. A total of 33 beams were tested, of which 26 were subjected to air blast loading and 7 were tested with a static load. The amount of tensile reinforcement was varied in order to study the mechanical properties of the beams and some beams also contained steel fibres. The paper was written by the author of this thesis, and reviewed by Assoc. Prof. Mikael Hallgren.

Paper II:

This paper is a continuation of the experimental series discussed in Paper I and involves beams of nominal concrete grades 140 MPa and 200 MPa. Some beams consisted of two concrete layers. The reinforcement content varied between the beams of different concrete grade and steel fibres were used in a majority of the beams. The results from the experimental investigation included in Paper I are also included in Paper II. This paper involves testing of an extra total of 16 beams, of which 12 were exposed to air blast loading and the remaining 4 beams were tested statically. The paper was written by the author of this thesis, and reviewed by Assoc. Prof. Mikael Hallgren.

Paper III:

Paper III involves testing of 40 steel fibre reinforced concrete (SFRC) beams subjected to static and air blast loads. A total of 22 beams were subjected to air blast loading and the remaining 18 beams were loaded statically. The concrete compressive
strength varied between 36 MPa and 189 MPa with a fibre content of 1.0 percent by volume. Two different fibre lengths having a length-to-diameter ratio of 80 were employed.
2 Air blast loads

An explosion is a result of a sudden release of energy. There are a number of possible situations that may cause an explosion such as that of a sudden relief of compressed air in a tire, a sudden release of pressurised steam in a boiler or that from detonation of high explosives. The contents of this thesis deals only with the severe dynamic loads that arise from the detonation of high explosives, which at close range generates high pressures and temperatures. Detonation involves a quick chemical reaction that propagates through the explosive as a shock wave at a supersonic velocity. A shock wave is defined as a discontinuity in pressure, temperature and density (Meyers, 1994). The pressures immediately behind the detonation front in the explosive can be 10–30 GPa (Mays and Smith, 1995).

An explosion in air creates a blast wave as a result of the large accumulation of energy pushing back the surrounding atmosphere. This violent release of energy will be followed by the spherical expansion of the blast wave into the surrounding air. As the blast wave propagates through the medium, nearly discontinuous increases in pressure, temperature and density are obtained. As the shock front propagates the air particles are accelerated in the direction of the travelling front, which results in a net particle velocity. Figure 2.1 shows an idealistic representation of a blast wave profile at a given distance from the centre of explosion. The blast wave is illustrated with the time axis at ambient pressure in the figure. The arrival of the blast wave creates an almost instant increase from the ambient pressure to the peak overpressure. The arrival of the shock front is immediately followed by pressure decay down to the ambient pressure. This first part of the blast wave is termed the positive phase. Then, the pressure will continue to decay below the ambient pressure until the minimum negative pressure is reached, after which the ambient pressure is obtained once more. The second part of the blast wave is termed the negative phase and usually exhibits a longer duration than that of the positive phase. In design, the negative phase is normally less important compared to the positive phase. However, in some cases it is important to also consider the effects of the negative phase, e.g. when considering damage of windows or in a case with heavy structures having a response time longer than the positive phase. Besides the peak overpressure it is also of importance to consider the impulse density. The impulse density of the positive and negative phase, respectively, is defined by (Baker, 1973):

\[
i^+_p = \int_{t_p}^{t_p+t^*_p} p(t) \, dt
\]

\[
i^-_p = \int_{t_p+t^*_p}^{t_p+t^*_p+t^-} p(t) \, dt
\]

where \( p \) denotes the overpressure. The time endpoints of the intervals are chosen as referred to in Figure 2.1.
Figure 2.1  Ideal blast wave profile at a given distance from the centre of explosion. The blast wave is illustrated with the time axis at ambient pressure.

Scaling of the properties of blast waves can be done by Hopkinson-Cranz “cube-root” scaling (Conrath et al., 1999). This scaling principle states that when two explosive charges of identical geometry and of the same explosive but of different size are detonated in the same atmosphere and at identical scaled distances, then the same blast overpressures are produced. A scaled distance is commonly used when considering different blast wave parameters in free air as stated by

\[ Z = \frac{R}{Q^{\frac{1}{3}}} \]  

where \( R \) is the distance from the centre of the explosive charge and \( Q \) is the mass of a standard explosive such as TNT (Trinitrotoluene), which is widely used for military purposes. The blast scaling law has been verified experimentally but may not be applicable for very small scaled distances. The blast wave characteristics are similar from different high explosives, which enable calculations of an equivalent TNT mass of an explosive required to produce the same pressure and impulse density as a TNT explosive. The equivalent TNT mass may differ substantially for the same explosive regarding the pressure and impulse density. Lists on the equivalent TNT mass for different explosives can be found in the literature, e.g. TM5-1300 (1990) and Conrath et al. (1999).

As the incident blast wave strikes a solid surface the wave is reflected, which brings the particle velocity to zero while the pressure, density and temperature are reinforced. The actual pressure that develops is determined by various factors such as the peak overpressure of the incident blast wave and the angle between the direction of motion of the wave and the face of the structure, i.e. the angle of incidence. The largest increase will be for normal reflection where the direction of motion of the wave is perpendicular to the surface at the point of incidence. Normal reflection of weak blast waves will result in a reflected overpressure \( p_r \) of about twice the value of the incident wave. However, the reflected pressure will for stronger blast waves reach
values several times larger in magnitude compared to that of the incident pressure, see Figure 2.2. In extreme cases the reflected pressure has the ability to reach up to twenty times the incident pressure values (Mays and Smith, 1995). Figure 2.2 presents the incident and reflected pressure values for both the positive and the negative phase of a blast wave at different scaled distances. The pressure increase during reflection is due to the conversion of the kinetic energy of the air immediately behind the shock front into internal energy as the moving air particles are decelerated at the surface (Glasstone and Dolan, 1977). The impulse density will in a similar way as the incident pressure be reinforced by reflection. The spherical expansion of the blast wave results in a continuous reduction of the peak pressure and impulse density while, on the contrary, the duration of the blast wave increases. A shock front striking a solid surface at an angle differing from that of normal reflection is termed oblique reflection.

Should an explosive charge detonate on or very near the ground surface, the blast wave will be reflected and reinforced by the ground. In this case the incident wave will merge with the reflected wave and form a single wave, which propagates hemispherically into the undisturbed atmosphere. Due to the interaction of the incident and reflected waves the surface burst creates higher pressures and impulse densities compared to the case of a free air explosion. Theoretically, with no energy

---

**Figure 2.2** Incident and reflected (normal reflection) overpressure and negative pressure, respectively, as a function of the scaled distance. Detonation of a spherical TNT charge in free air. Reproduced from Balazs (1998).
dissipation due to the reflection, the energy from the explosion would have to be released over half the volume compared to the case of an explosion in free air. This means that the generated blast wave would have the same properties as from an explosive charge twice the size in free air. However, in reality, the released energy from the explosion will be partly converted into ground movements and cratering and an increase of the explosive charge by a factor of 1.8 can be used according to Conrath et al. (1999). Estimations on how to determine when an explosive charge is close enough to the ground surface to account for ground reflections can be made according to FortH 2 (1987).

An explosive charge detonating inside a tunnel will initially give rise to a complicated event due to the blast wave reflections against the tunnel walls, roof and floor. However, at a certain distance from the centre of explosion the propagation of the two shock fronts propagating in opposite longitudinal directions will be mainly one-dimensional and planar. In a tunnel with a constant cross section this one-dimensional propagation of the blast waves leads to higher pressures, higher impulse densities and longer durations of the blast waves compared to the case with a spherical expansion of the shock front. A shock tube, which can be considered as a small tunnel, was used in the dynamic tests with concrete beams in order to generate blast loads with relatively small amounts of explosive charges as described in section 4.1 and Paper I–III. The air blast parameters for explosions in tunnels can be estimated according to FortH 2 (1987).

As a plane shock front strikes a structure without openings the pressure and impulse density are reinforced at the front wall the same way as previously discussed in this section. As the shock front continues to propagate over the roof and over the sides of the building these parts will be loaded by the incident pressure (Baker, 1973), see Figure 2.3. Differences in pressure will arise at the intersection of the front wall, the roof and side walls, respectively, which leads to the propagation of rarefaction waves from the corners of the roof and side walls towards the centre of the front wall. At the same time the shock front continues over the roof and end walls and finally over the back wall. This phenomenon when the blast wave bends around an object is termed diffraction. Vortices are created as the shock front passes over the edges and the corners of the structure. The rarefaction waves at the front wall will not affect the peak reflected pressure but can substantially affect the magnitude of the impulse density because the duration of the blast wave may be strongly reduced. From this it is evident that the structure geometry may influence the magnitude of the blast loads acting on the structure. The previous description relates to a situation where a blast wave is formed relatively far away and, thus, a planar shock front strikes the structure. Should instead an explosive charge detonate at relatively close range to a solid surface the shock front will be curved as it strikes the front wall, see Figure 2.4. Consequently, the pressure load will vary across the surface because the curved blast wave will strike the surface at different angles and at different distances from the explosive charge across the surface. The largest pressures arise at locations where normal reflection occurs.

Figure 2.5 shows two explosive charges of different size where the larger charge is positioned at twice the distance from the front face of a structure in relation to the position of the smaller charge. A charge at twice the distance to the structure needs to be eight times larger than that of the closer charge to obtain the same overpressure at
$A$ in Figure 2.5. Note that only a part of the expanding shock front is shown in the figure. However, the larger charge will produce a significantly larger area with reflected pressure on the structure and the magnitude of the impulse density at $A$ will be twice that generated by the small charge. Consequently, even if the structure will be loaded with the same overpressure from the two explosive charges in Figure 2.5 the situation of a larger area loaded with reflected overpressure and a doubled impulse density may contribute to more damage to the structure.

![Diagram](image)

Figure 2.3  Diffraction of a blast wave by a building without openings, viewed from the side. Based on Baker (1973).

![Diagram](image)

Figure 2.4  Detonation of an explosive charge relatively close to a solid surface (a), illustrating the shock front at different points of time ($t_1$–$t_3$). The reflected overpressure on the surface (b).
Figure 2.5 Small and large charge ($Q$ and $8Q$, respectively) that give the same overpressure at $A$. Based on course notes from Baker Engineering and Risk Consultants, Inc (2003).
3 Dynamic loading and analysis

The behaviour of structures that are subjected to rapidly changing loads may change significantly from that under static or quasi-static situations. Common for all dynamic events of a structure is the presence of inertial effects due to accelerating masses. Due to the inertial effects the elements that comprise the structure will try to resist any change in velocity and these effects need to be considered in a dynamic analysis. The dynamic load will bring about a certain degree of deformation rate in different parts of the structural elements as these deform, commonly referred to as strain rate effects. This will affect the response in such a way that the material strength increases. Some aspects on high strength concrete (HSC) are given in the first section, followed by discussions of the dynamic properties of concrete and steel reinforcement. The second section gives a general review of the dynamic events from different investigations. The dynamic analysis using the single-degree-of-freedom model and the use of iso-damage curves will be presented in the last section. This section also gives a review of a model to predict the dynamic support reactions.

3.1 Material properties

3.1.1 High strength concrete

A large amount of research on concrete is carried out throughout the world. The increased knowledge has enabled the development of stronger and denser concrete, which can be used in different applications. In this context, HSC refers to a concrete with a compressive cube strength exceeding 80 MPa. It is possible to reach a compressive strength of over 200 MPa with conventional methods. HSC has throughout the world been used in different types of structures such as bridges, high-rise buildings and road pavements. The increased strength enables the design of more slender structural elements, the transfer of higher loads at a fixed geometry and a higher durability with reduced maintenance costs. The rapid strength development of HSC the first days after casting also enables a higher production rate. Earlier research at the Swedish Research Agency (FOI) has also shown that HSC beams exhibit higher resistance against blast loading (Magnusson and Hallgren, 2000 and 2003).

Compared to normal strength concrete (NSC), the higher strength is reached by reducing the water content, by careful grading of the aggregates and by adding a filler material such as silica fume. The silica fume contributes to a denser material structure due to a particle size only one hundredth of the size of cement particles. This will effectively fill the space between aggregates and cement particles. Since silica fume is also puzzolanic, i.e. reacts with the calcium hydroxide and forms cement gel, it will also contribute to a denser material. Silica fume will also contribute to a strength increase of the hardened concrete. Simply reducing the water content, which is necessary for higher strength, would make casting very difficult or practically impossible. This problem is overcome by adding a superplasticizer during the mixing of the concrete, which separates the cement particles. Enclosed water is thereby released and is made accessible for further reactions with the cement grains. Thus, reducing water is possible at a constant workability by adding a proper amount of...
superplasticizer to the concrete mix. It is also of great importance to consider the natural moisture content of the aggregates and sand because a relatively large portion of the total water added may originate from these constituents. This moisture content will affect the workability and the strength of the hardened concrete.

The strength of the aggregates will be more crucial in HSC compared to NSC and careful grading is also important to attain a dense material structure. The maximum aggregate size of HSC needs to be limited and there are several reasons for this. Larger aggregates result in weaker interface between the aggregate and the cement paste due to the separation of water that usually occurs beneath coarse aggregates. As the material is loaded, stress concentrations that are present due to stiffness differences between the cement paste and the aggregate will be more evenly distributed as the maximum aggregate size is reduced (Betonghandbok, 2000; Walraven, 2002). This contributes to a more homogeneous material structure. Reduced aggregate size also leads to the probability of a slight strength increase of the aggregates due to statistics stating that there is a higher probability of a larger number of weakness zones existing in specimens of larger volumes (Betonghandbok – Material, 1994). The maximum aggregate size should not exceed 16 mm. This maximum size was used in the casting of beams for concrete grades 100 and 140, see section 4.1.1. A maximum aggregates size of 8 mm was used for concrete grade 200 where the aggregate consisted of bauxite.

Energy is consumed in the process of crack initiation and propagation in concrete. Microcracks are initiated in the interface zone of the aggregates and the cement paste (the transition zone) at relatively low stress levels. As the stress is increased the microcracks grow and link up into larger cracks and which eventually forms the fracture zone. In NSC, the fracture zone usually propagates through the transition zone of the aggregates and the cement paste. Thus, the cement paste is the weakest link of the composite material while the aggregates are normally stronger. Coarser aggregates may arrest crack growth, which produces meandering and branching of the cracks. The fracture of HSC, however, is mainly characterised by fracture through the cement paste and the aggregates. The fracture surface of HSC is for that reason relatively plane compared to that of NSC, Figure 3.1. Additional energy is consumed for every change in direction of the fracture surface and the total fracture energy of HSC is therefore proportionally smaller compared to that of NSC. The definition of the fracture energy ($G_F$) is the total energy consumed per unit area of the fracture surface in a fracture zone during failure. However, $G_F$ is not an adequate energy measure of the fracture toughness of a material. A material can prove to have large fracture energy but still exhibit a brittle behaviour. A more adequate parameter to describe the behaviour of materials is the so-called characteristic length, $l_{ch}$. This parameter can be calculated according to

$$l_{ch} = \frac{E \cdot G_F}{f_t^2}$$  \hspace{1cm} (3.1)

where $E$ is the elastic modulus and $f_t$ the tensile strength. Generally, the fracture energy increases with concrete strength. However, this does not lead to higher toughness because the concrete tensile strength also increases. The increase of tensile strength is larger in relation to the increase of both the fracture energy and the elastic modulus. Thus, the value of the characteristic length decreases with increasing
concrete strength. This can be interpreted as an increasing brittleness. For HSC, $I_{ch}$ can decrease to about one third of that of NSC (Betonghandbok, 2000). The ductility of HSC, however, can be improved considerably by introducing steel fibres to the matrix.

![NSC and HSC](image)

*Figure 3.1 Fracture of conventional concrete and high strength concrete. Reproduced from Betonghandbok – Material (1994).*

### 3.1.2 Dynamic properties of concrete

Reinforced concrete structures subjected to blast loading will respond by deforming over a relatively short period of time and the strain rates in the concrete and reinforcement reach magnitudes considerably higher than that of a statically loaded structure. A static compressive load is defined at a strain rate of $3 \cdot 10^{-5} \text{s}^{-1}$ according to the *CEB-FIP Model Code 1990* (CEB, 1993). As a comparison, strain rates of around $1 \text{s}^{-1}$ can be expected for concrete elements subjected to blast loading (Palm, 1989; Magnusson and Hallgren, 2000). The material strength increase is normally referred to as a dynamic increase factor (DIF), which is the ratio of the dynamic to the static value.

Several researchers have experimentally studied the compressive and tensile strength of concrete at different strain rates. A selection of their commonly published results is presented in Figure 3.2–3.3. For strain rates exceeding static loads there is an increasingly larger scatter in test results for increasing strain rates. Bischoff and Perry (1991) suggest that this relatively large scatter for the compressive behaviour of concrete may depend on factors such as experimental techniques used and methods of analysis employed. Other factors that can influence the results are specimen size, geometry and aspect ratio as well as moisture content in the specimens. Therefore, care should be taken when comparing the results of concrete strength properties from different research programs.
Figure 3.2 Strain rate effects on the concrete compressive strength. Reproduced from Bischoff and Perry (1991).

Figure 3.3 Strain rate effects on the concrete tensile strength. Reproduced from Johansson (2000) and based on Malvar and Crawford (1998-a).

Clearly, for both compressive and tensile loading, there exist two intervals with different strain rate dependencies and with a relatively sharp transition zone between them. The first more moderate increase in strength has been explained by different
authors and is summarised by Johansson (2000). The increase in strength for this interval can be explained by viscous effects with free water in the micropores. Several investigations point in the direction that wet specimens are more strain rate sensitive than dry specimens. A specimen loaded in compression will force the free water to move inside the specimen, which results in the build-up of an internal pressure. This pressure would then help the material in resisting the external load, delay the crack initiation and thereby contributing to a higher compressive strength. The water also has positive effects on the tensile strength. The presence of water will work in a similar way as when a thin film of water is trapped between two plane and parallel plates moving apart, which will give rise to a resisting force. In concrete, the walls in the micropores would then in a similar way resist movements trying to separate the material and, consequently, help increasing the tensile strength.

At strain rates exceeding the transition zone there will be a more sharp increase in both tensile and compressive strength. Explanations for this behaviour can mainly be ascribed to inertia effects and lateral confinement. Weerheijm (1992) studied strain rate effects on the tensile strength of concrete by using linear elastic fracture mechanics. His studies showed that changed stress and energy distributions due to inertia effects around the crack tips were the cause of the rapid strength increase. At an increasing static compressive loading the behaviour will be affected by the propagation of microcracks (Zielinski, 1984). When the specimen instead is exposed to a rapid load, the time available for initiation and propagation of microcracks will be reduced (Bishoff and Perry, 1991). This could be an explanation of the strain-rate dependent behaviour at higher strain rates. Johansson (2000) reasons that the effects of the inertia effects around the crack tips, which explains tensile strain rate sensitivity, also is a reasonable explanation since the compressive failure is also governed by cracking. Bishoff and Perry (1991) suggest that the sudden increase in compressive strength also can be ascribed to lateral inertia confinement. This effect can be explained as follows. An elastic material statically loaded in compression will exhibit lateral expansion due to the effect of Poisson’s ratio. Should, however, a cylindrical specimen be exposed to a rapid compressive load in the axial direction the specimen will not be able to expand in the radial direction at an instant due to inertial restraint. This will initially leave the specimen in a similar stress state as that of uniaxial strain with corresponding lateral stresses. This inertial effect will only act over a finite time until the material accelerates in the radial direction. As concrete is known to be sensitive to lateral confinement, the inertial effect would result in a substantial increase in compressive strength.

A model for the strain rate dependence of concrete in compression and tension is presented in the *CEB-FIP Model Code 1990* (CEB, 1993). The model is valid at strain rates up to 300 s⁻¹. Figure 3.4 presents this model for both NSC and HSC. In compression the model appears to properly fit the test data as shown in Figure 3.2. The change in a moderate strength increase into the more dramatic strength enhancement is set to a strain rate of 30 s⁻¹ for both compressive and tensile loading. However, Malvar and Crawford (1998-a) argue that available test data reveal that the change in slope should instead occur around 1 s⁻¹. Therefore, they proposed a formulation similar to the CEB-FIP model, which was fit against the test data and their result is also shown in Figure 3.4. This modified formulation seems to show a better fit to the test data in Figure 3.3.
Steel fibre reinforced concrete (SFRC) loaded in tension also shows strain rate dependence. Research by Gopalaratnam and Shah (1986) and Stevens et al. (1995) has shown that the resistance against pull-out of the fibres increases with an increasing loading rate. Lok and Lu (2001) investigated the dynamic splitting strength of NSC cylinders containing steel fibres with volume fractions of 0.3 % and 0.6 %, respectively. They found that the splitting strength could reach values up to 4.5 times the static value at a strain rate of about 10–15 s⁻¹ when hooked-end fibres were used. No significant difference in strength increase between the SFRC and the plane concrete specimens could be observed at these strain rates.

The elastic modulus of concrete is also affected by changes in strain rate. According to Bishoff and Perry (1991), the enhancement of the elastic modulus at dynamic loading can be ascribed to the decrease in internal microcracking at a given stress level with an increasing strain rate. This behaviour results in a stress-strain curve that remains linear up to higher stress values. Bishoff and Perry also reason that the tangent modulus should not be affected by strain rate since there is no significant microcracking during this initial response stage. The strain-rate dependence of the elastic modulus is also included in the CEB-FIP Model Code 1990 as presented in Figure 3.5.

Model-scale experiments are commonly used to investigate the behaviour of structural elements subjected to dynamic loading. Elements of smaller size loaded dynamically give rise to proportionally higher accelerations and strain rates, which may need to be considered (Lidén et al., 1994). The increase in strain rate is inversely proportional to the length scale, so a case where the specimen is tested in half-scale gives twice as high strain rates during the test compared to a similar element in full scale.

![Figure 3.4 Model for the strain rate dependence of concrete in compression and tension according to the CEB-FIP Model Code (CEB, 1993) and with the modified model according to Malvar and Crawford (1998-a).](image-url)
Similarly to concrete, steel reinforcement bars are also affected by dynamic loads and will exhibit different degrees of strength increases depending on steel grade. Both the yield strength and ultimate strength increase, but the increase of the former is usually more significant. The elastic modulus, however, is usually found to remain constant as stated by Malvar and Crawford (1998-b). The reasons for the strength increase of reinforcement steels are explained to be due to dislocation effects originating from the crystalline structure of the steel under shear stresses, Palm (1989) and Meyers (1994). Other effects will be dominant as the strain rates are increased but these effects will not be discussed here. Malvar and Crawford proposed a formulation of the dynamic increase factor (DIF) for both the yield strength and ultimate strength for reinforcement bars at different strain rates as follows:

\[ DIF = \left( \frac{\dot{\varepsilon}}{10^{-4}} \right)^{\alpha} \]  

(3.2)

where for the yield stress

\[ \alpha = 0.074 - 0.040 \frac{f_y}{414} \]  

(3.3)

and for the ultimate stress

\[ \alpha = 0.019 - 0.009 \frac{f_y}{414} \]  

(3.4)
and where $\dot{e}$ is the strain rate (s$^{-1}$), and $f_y$ is the yield strength (MPa) of the reinforcing bar. This formulation is valid for bars with yield strengths between 290 MPa and 710 MPa and for strain rates of 0.0001–225 s$^{-1}$. According to this formulation the magnitude of the DIF will decrease with an increasing reinforcement yield strength, which is the case for real bars according to Malvar and Crawford (1998-b). Figure 3.6 illustrates the yield and ultimate strength increase, respectively, of the reinforcement grade B500BT. This reinforcement grade was used in the blast wave experiments as presented in Paper I–II, see also section 4.1.

![Figure 3.6](image)

**Figure 3.6** Strain effects on the yield and ultimate strength of reinforcement steel of grade B500BT according to Malvar and Crawford (1998-b). The static yield strength was in this case 550 MPa.

### 3.2 Structural response under dynamic loading

A structure subjected to dynamic loading may exhibit a different behaviour compared to a structure loaded statically, especially if the applied load has a high peak value and is of short duration. Such loads could be from detonating explosives creating air blast waves or falling masses impacting on a structure. The abrupt changes in the applied load give rise to accelerations of the structural elements and, consequently, the effects of inertia and kinetic energy must be considered in the dynamic analysis. It is not relevant to design a structure for severe dynamic loading such as blast loading only by considering its elastic behaviour. Such a structure would be too robust and uneconomical. Instead, the structural elements should be designed to allow for plastic deformations, thereby better utilising the energy absorbing capabilities of the structure. Thus, it is of utmost importance to design ductile elements that can undergo large plastic deformations and brittle failure modes need to be prevented when considering severe dynamic loads. Figure 3.7 schematically illustrates typical behaviours of brittle and ductile response modes, respectively. Instead of designing for stiff elements with a large load capacity it can be better to use less stiff elements.
that allow for larger deformations and, thereby, obtain larger energy absorption. The behaviour of reinforced concrete elements will be studied in the following.

Reinforced concrete beams and one-way concrete slabs subjected to air blast loading can fail in a variety of mechanisms. They can fail in flexure where plastic hinges form at locations where the ultimate bending moment capacity is obtained, i.e. at mid-span for a simply supported beam where a symmetric load is applied. This failure mode is characterised by initial cracking of the concrete, subsequent yielding of the tensile reinforcement and, ultimately, compression failure of the concrete. The nature of the flexural failure is normally ductile and energy absorbing. The flexural shear failure mode, on the other hand, is abrupt and brittle in nature, which severely limits the capacity of the element. The flexural shear mode is characterised by initial flexural cracks that develop where the maximum bending moment is obtained and then, ultimately, the formation of an inclined diagonal tension crack close to one or both supports. Thus, this is a premature failure mode where the element is unable to develop its ultimate bending moment with corresponding deformation and therefore undesirable. Figure 3.8 schematically illustrates the development of a diagonal tension crack in a concrete beam with an incipient shear failure.

Beams and slabs can also fail in a direct shear mode under the action of a uniformly distributed impulsive load (Ross, 1983). Shear failures generally occur at locations near the supports or at the joints of elements that comprise the structure where the maximum shear stresses occur and are possible even in elements designed for flexural shear. Direct shear failure of an element is characterized by the rapid propagation of a vertical crack through the depth of the element. According to Ross, shear failures can occur at times soon after the transmitted part of the shock wave has propagated through the thickness of the structural element. As a comparison, the flexural response of the element is not initiated until much later when the element has attained some momentum. Thus, a direct shear failure mode is a premature failure mechanism where the element has had no time to deflect and therefore it is a very brittle response. This failure mode, however, as well as other dynamic loading conditions such as fragment impact and detonating charges in contact with the structural element do not fall within the scope of this thesis and will not be discussed further herein. Thus, focus will be put upon dynamic events such as air blast loading at a certain distance and impacting masses on concrete elements in this section.

Figure 3.7 Schematic representation of brittle and ductile response of a structural element.
The free vibration of an elastic beam is expressed as

$$EI \frac{\partial^4 y}{\partial x^4} + \rho A \frac{\partial^2 y}{\partial t^2} = 0$$  \hspace{1cm} (3.5)$$

where $EI$ is the flexural stiffness, $\rho A$ denotes the mass per unit length. Here, both the rigidity and the mass are assumed uniform along the span. Eq. 3.5 describes the elastic beam behaviour and allows only for flexure and translatory inertia, thus, the transverse vibration of the beam. The effects of shear deformation and rotary inertia have not been considered in eq. 3.5 but these effects become more significant as the number of modes excited during the loading increases (Hughes and Speirs, 1982). The natural frequencies of a simply supported beam is given by

$$\omega_n = \frac{n^2 \pi^2}{L^2} \sqrt{\frac{EI}{\rho A}}$$  \hspace{1cm} (3.6)$$

where $n$ is the number of the mode excited and $L$ is the span. Figure 3.9 shows the first three vibration modes of a pin-ended simply supported beam.
In the case of large dynamic loads it is important that the deformation capacity of structural elements joined together can be utilised without failing prematurely in a shear failure mode. A structural element subjected to transient dynamic loading will experience higher modes of vibration depending on the nature of the load. The amplitudes of the higher modes are relatively small and their contributions to the overall deformations are therefore limited. However, the higher vibration modes will give rise to larger shear forces in the element. Hughes and Speirs (1982) have performed a theoretical and experimental investigation on reinforced concrete beams subjected to mid-span impact of a falling mass, see also Hughes and Beeby (1982). They made a comparison between the response of the static mode versus first and third free vibration modes for pin-ended beams. The mode profiles for equal potential energies are schematically illustrated in Figure 3.10. This figure shows reductions in displacements and bending moments for the first and third vibration modes compared to the corresponding properties for the static mode. The shear forces, on the other hand, are greater for both vibration modes in relation to the static mode and this is especially the case for the third mode response. Larger shear forces can therefore be associated with higher vibration modes of a beam under impact loading compared to the shear forces in a corresponding static loading case. As a consequence, a beam could show flexural failure under static loading, whereas an identical beam might fail in flexural shear under dynamic loading.

Niklasson (1994) also performed impact experiments on simply supported reinforced concrete beams where the amount of reinforcement as well as the concrete strength was varied. The load from the impact of the falling mass was transmitted to the concrete beam as two point loads symmetrically positioned on both sides of mid-span. Load cells were positioned directly beneath the point loads and also at each support. The conditions of the impact zone could be varied by placing zero, one or two rubber pads between the striking mass and the impact zone. This way the rise time and amplitude of the applied load were controlled. Thus, at the same applied energy level, the magnitude of the applied load could be reduced substantially and the rise time be increased by this configuration.
Another way of describing the variations in time by the applied load is to refer to its frequency content. Accordingly, a short rise time can be regarded as a load containing high frequencies as opposed to a load with a longer rise time. Niklasson (1994) reasons like Hughes and Speirs (1982), that a dynamic load induces higher vibration modes in an element. Thus, the failure mode of a reinforced concrete beam may depend on the frequency content of the applied load, or in other words, the number of vibration modes that are excited by the load. A load containing low frequencies, such as a quasi-static load, can not affect the beam any more than in its fundamental mode of vibration. One consequence of high-frequency dynamic loads producing shear failures of reinforced concrete beams is that lower energy contents may be enough to cause this type of failure whereas loads with lower frequency contents but with a higher energy may lead to a flexural response. Ansell (2005) also points out that a concrete structure designed to fail in flexure under static loads may fail in shear when loaded dynamically. The response of a structural element subjected to rapidly changing loads such as those from impacting masses and explosions, will besides the element’s ductility also be governed by its natural period of vibration. Consequently, there is a strong relationship between the load duration, rise time of the load and the natural period of vibration and this is discussed further in section 3.3.

![Figure 3.10 Comparisons between different modes with equal potential energies. Reproduced from Hughes and Speirs (1982).](image-url)
In general terms, the probability of shear failures is highest when the loading pulse has short duration and a high peak value. However, the beam stiffness also plays an important role on the change in failure modes. Palm (1989) conducted a literature survey concerning reinforced concrete structures under dynamic loading conditions. This survey showed that investigations where beams of different reinforcement contents were exposed to dynamic loading exhibited in majority shear failures. Only beams with the lowest reinforcement content failed in flexure. According to Palm, it is suggested that to be able to absorb an impulsive load the reinforcement content and its yield strength should be comparatively low. Kishi et al. (2002) studied reinforced beams without stirrups impacted at mid-span by a falling mass, which show similar results. That is to say, beams with a relatively low amount of reinforcement failed in flexure whereas beams failed in shear as the reinforcement content was more than doubled when subjected to the same dynamic load. However, as the impact velocity was further increased the more flexible beams also failed in shear. This can be compared to a loading situation where an explosive charge is detonated at a reduced distance.

These different findings show that apart from the load intensity determining the mode of failure of an element, the stiffness also plays an important role. Consequently, abrupt shear failures can be prevented by designing less stiff structural elements, which thereby will respond in a ductile manner. Structures exposed to severe dynamic loading such as a nearby explosion will give rise to substantially higher strain rates compared to the situation with a static load. This strength increase will give rise to a stiffer element with a larger bending moment capacity, which may influence the susceptibility of shear failures. The influence of strain rates on the response are further discussed in section 3.3.

The investigations mentioned above in this section were all conducted with falling masses striking concrete beams at mid-span. This loading situation differs from that of air blast loading. In tests with a falling mass, the beam is loaded along one or two distinct positions with limited area whereas blast loading generally gives rise to a distributed pressure across the beam surface. The impact will generate bending moment waves and shear waves travelling from the impact zone and towards each support as described by Hughes and Speirs (1982). This means that, initially, the beam does not “know” its end conditions until some time after impact. Large bending moments and shear forces may appear at different locations along the beam, which differs from the shear and moment distributions in a static loading case. In a case where the falling mass has a high enough energy there is a risk of a local failure with the mass punching a cone out of the beam due to the concentration of large shear forces in that area usually referred to as punching shear failure (Zielinski, 1984; Ansell, 2005). Tests where a mass impacted on plain and fibre reinforced concrete beams were investigated by Ågårdh, Bolling and Laine (1997). Their results also show that the plain beams could fail at several locations along the span due to bending, tensile and shear crack failure. A similar effect as of a striking weight is obtained with explosions very close to a structural element where large pressures are concentrated over a relatively limited area. A beam loaded by a blast load distributed over the surface will respond by building up bending moment and shear forces at the support, which then propagate towards mid-span of the beam, as described in Publikation nr 42 (1975). Thus, similar waves opposite in direction appear as in the case with a falling mass striking at mid-span. Structural elements tested with an
impacting mass may be related to a similar element loaded by blast. Ansell and Svedbjörk (2003) performed tests with an impacting mass on concrete beams and pointed out that it would be of interest to compare the loads from such tests with the loads obtained from air blast loading. To obtain similar deformations and damage level in these two loading situations the velocities and strain rates of the element should be similar. In situations with dynamic concentrated loads or blast loads the bending waves and shear waves that appear initially within structural elements are also transferred to the rest of the structure.

Abrahamsson (1961) studied the behaviour of reinforced concrete slabs subjected to air blast loading. In the experiments he found that the slabs, which all were clamped around the edges during the tests, were able to stand considerably higher loads than predicted. According to Abrahamsson, this high load capacity could not be ascribed to an enhanced material strength due to strain rate effects. During the test, the slabs were unable to move in its plane due to the boundary conditions, which enabled compressive stresses to develop a compressive membrane. Adequate lateral restraint at the supports is a necessary condition for the slab to act as a shallow dome rather than a plane slab. The hypothesis of compression membrane action was checked in a static test by Abrahamsson, which seemed to confirm the presence of this mechanism in the slab. In this test, the ultimate failure was sudden in some sort of stability collapse or a snap-through failure. Further theoretical and experimental studies of this structural behaviour was carried out, where a beam is first regarded as an arch and then presents an expression for the maximum sustainable peak pressure for a beam and a slab, respectively (Abrahamsson, 1967). The investigations showed that compression membrane action was in fact present in the previous tests, being a more predominant mechanism the thicker the slab is. With the same reasoning, the arch action of a beam should in this context become more predominant for a deeper beam. The compression membrane and arch actions are limited to a certain deflection of the element, as schematically shown in Figure 3.11. Only the vertical components of the forces are shown in this figure. When looking at half the beam, i.e. from mid-span to one support, it is evident that the resisting moment caused by the force couple creating an arch will in fact have a maximum at a certain deflection but will be reduced as the displacement grows.

Figure 3.11  Schematic view of a beam with edges restrained and a developed arch. Based on Birke (1975).
Schematically, the variation in displacement with resistance of an edge-restrained slab with tensile reinforcement anchored at the edges and subjected to a quasi-static load is shown in Figure 3.12. In the elastic stage, the load is mainly absorbed through flexural action (Birke, 1975). As cracks develop in the slab a compressive membrane action is built up at an increasing displacement and this continues up to point \( A \) in the figure. Thereafter, as the displacement grows the moment created by the force couple will be reduced and, subsequently, the membrane forces in the slab are gradually transformed from compression to tensile action up to point \( B \). From this point on a tensile membrane action will be built up and again the load capacity of the slab is enhanced, providing the tensile reinforcement can sustain the tensile forces needed. During this last stage the load can be increased until failure of the reinforcement. Similar to compressive membrane and arch action, a tension membrane of the structural element can be provided only when full lateral restraint is provided at the supports. It is probable that real structures will not have completely unyielding lateral resistance at the supports and Birke (1975) studied the arch action of concrete slabs with elastically restrained supports. In comparison to a case with full lateral restraint, the support configuration studied by Birke resulted in a decreasing ultimate load and also in an increased deflection at that load, thus, reducing the effects of the arch action.

Other investigations have studied the behaviour of load bearing walls in a multi-storey building that are subjected to blast loading. As such a wall deflects due to the blast load it will force the large mass from storeys above to move upwards and due to the inertia of that mass this will create an increase of the in-plane compressive forces in the wall. Investigations of this behaviour have been reported by Jonasson (1986), Kihlström (1988), Forsén (1989) and Edin and Forsén (1991). Their findings were that an arch was built up in the specimens and that these exhibited an enhanced resistance to blast loads compared to a simply supported specimen without edge restraints.

![Figure 3.12 The behaviour of a slab with edge constraints. Based on Birke (1975).](image-url)
3.3 Dynamic analysis

3.3.1 The single-degree-of-freedom system

The basic differences between structures under static and dynamic loads are the presence of inertia in the equation of motion and that of kinetic energy in the equation of energy conservation. Both terms are related to the mass of the structure and, hence, the mass of the structure becomes an important consideration in dynamic analysis. Structural elements can in many cases be reduced to an equivalent dynamic system having a similar behaviour as the real element. A beam for instance can then be reduced to a single-degree-of-freedom (SDOF) system, which is defined as a system in which only one type of motion is possible. A typical SDOF system is the mass-spring-damper system shown in Figure 3.13 where the mass can move in a vertical direction only. In this system, the spring and damper are considered to have no mass and it is also assumed that the spring is linear. The force of gravity is omitted in this figure because the displacement of the mass \( y \) is measured from its neutral position, which the mass assumes in a static case where no load is applied.

\[
\begin{align*}
    c & \quad k \\
    m & \quad P(t) \\
    y & \quad P(t)
\end{align*}
\]

Figure 3.13 Single-degree-of-freedom (SDOF) system.

The dynamics of this system is developed in textbooks on mechanical vibration and elementary physics. The equation of motion of the SDOF damped system subjected to an applied force \( P(t) \) is

\[
m\ddot{y} + c\dot{y} + ky = P(t)
\]

with \( m \), \( c \) and \( k \) being system mass, damping coefficient and system stiffness, respectively. The time derivatives of the displacement are denoted \( \dot{y} \) for the velocity and \( \ddot{y} \) for the acceleration. The response of all structural dynamic systems involves a certain degree of damping and if a continuing state of vibration is studied then damping may be very important to consider. However, the effects of damping are rarely considered in the analysis of structures subjected to blast loads because damping has very little effect on the maximum deflection, which usually is the only response of interest (Baker et al., 1983). The reason for this is that structural damping
is much lower than for other systems where critical damping needs to be considered. In addition, the energy absorbed due to plastic deformations of the element is much greater than that absorbed by structural damping. Plastic response may be modelled with the use of SDOF analysis as shown in this section. Thus, the equation of motion can be reduced to

\[ m\ddot{y} + ky = P(t) \]  

(3.8)

If the applied force is of constant amplitude \( P_1 \), suddenly applied and remaining constant thereafter indefinitely, then the general solution of this equation is

\[ y = C \cos(\omega t) + D \sin(\omega t) + \frac{P_1}{k} \]  

(3.9)

where \( C \) and \( D \) are constants and can be determined from the initial conditions, i.e. if the system has an initial displacement and initial velocity. The natural circular frequency is given by

\[ \omega = \sqrt{\frac{k}{m}} \]  

(3.10)

with the natural period of the system given by

\[ T = 2\pi \sqrt{\frac{m}{k}} = \frac{2\pi}{\omega} \]  

(3.11)

If the system is initially at rest, then the solution of eq. (3.9) is

\[ y = \frac{P_1}{k} \left( 1 - \cos(\omega t) \right) \]  

(3.12)

The maximum response for an elastic system subjected to a suddenly applied and thereafter constant load is determined by the maximum value of eq. (3.12). The result is that the maximum displacement is twice that of the static response:

\[ y_{\text{max}} = \frac{2P}{k} = 2y_{\text{stat}} \]  

(3.13)

Consider a case where a system at rest is subjected to a constant force of very short duration \( t_d \) compared to the natural period of the system. Such load is referred to as an impulse \( I \) with the magnitude

\[ I = \int_0^{t_d} P(t) \, dt \]  

(3.14)

In principal, if \( t_d \) is short enough compared to the natural period of the system then little spring resistance will develop during the time \( t_d \). If the resistance is negligible compared to \( P \), then the acceleration can be considered as constant giving the mass an
initial velocity. Biggs (1964) suggests that the loading can be treated as an impulse if $t_d$ is less than about 0.1 times the natural period of the system. Thus, if this condition is fulfilled the value of the velocity at time $t_d$ imparted to the mass will be

$$\dot{y} = \frac{I}{m}$$  \hspace{1cm} (3.15)

A system loaded with an impulsive load as shown in Figure 3.21 in section 3.3.3 results in a response as

$$y = \frac{I}{m\omega} \sin(\omega t)$$  \hspace{1cm} (3.16)

The actual shape of the load-time function during the time of duration of the impulse is clearly of no importance since most of the response of the system occurs during its free vibration face and little spring resistance is developed during time $t_d$. This behaviour is schematically shown in Figure 3.14 where the load duration to natural period ratio of a system is small.

![Figure 3.14 Schematic illustration of the relation between load and response time for an impulsive load.](image)

It is of interest to obtain a general solution applicable to an arbitrary load function for an elastic SDOF system. In the case where $P(t)$ is a general load function the solution of the equation of motion for an elastic system is (Chopra, 1995)

$$y = \frac{1}{m\omega} \int_0^t P(\tau) \sin \omega(t - \tau) \, d\tau$$  \hspace{1cm} (3.17)

This solution is commonly referred to as Duhamel’s integral and is based on the assumption that the external load varies arbitrarily in time can be made up of a large number of infinitesimally short impulses. These impulses are then added up to the time of interest.

A closed solution of the equation of motion is possible only when the loading and the resistance functions can be expressed in relatively simple mathematical terms. In other cases a numerical analysis can be used. In the analysis of blast loaded structures it is common practice to account for large plastic deformations of the elements
comprising the structure. This can be done in a simplified manner in the SDOF calculations by using a resistance function, which is linearly elastic, perfectly plastic with a maximum resistance of $R_1$ as shown in Figure 3.15. An air blast load on a structure is essentially a single pulse and can usually be idealised by a simple geometric shape that gives a good resemblance of the actual blast wave. The type of load considered here is the triangular load with zero rise time. Thus, the system is subjected to an initial suddenly applied load $P_1$, which then decreases linearly to zero at time $t_d$, see Figure 3.15.

![Figure 3.15](image)

**Figure 3.15** Resistance function of a linear elastic, perfectly plastic SDOF system subjected to a triangular load.

A spring-mass system with an elastic perfectly plastic response has two discontinuities and therefore needs to be computed in three separate stages, i.e. in the elastic range, in the plastic range and then in the elastic behaviour after the maximum response has been attained.

The response in the elastic range and considering the triangular load may be computed by eq. (3.17). The load function is given by

$$ P(t) = P_1 \left(1 - \frac{t}{t_d}\right) \quad (3.18) $$

and the solution in the elastic stage is (Biggs, 1964)

$$ y = \frac{P_1}{k} \left(1 - \cos(\omega t)\right) + \frac{P_1}{kt_d} \left(\frac{\sin(\omega t)}{\omega} - t\right) \quad (0 \leq t \leq t_{el}) \quad (3.19) $$

The response in the plastic range is then computed by direct integration of the equation of motion

$$ m\ddot{y} + R_i = P(t) \quad (3.20) $$
and the solution

\[ y = -\frac{P_1}{6m t_d} t_1^3 + \left(\frac{P_1 - R_1}{2m} - \frac{P_{tl}}{2mt_d}\right) t_1^2 + G t_1 + H \quad (t_{el} \leq t \leq t_m) \quad (3.21) \]

where \( t_1 = t - t_{el} \). The time of the maximum elastic response is denoted \( t_{el} \). The initial conditions of the problem are then used to solve for the constants \( G \) and \( H \) at \( t_1 = 0 \) and the final solution for this stage is obtained by substituting back into eq. (3.21). According to Biggs (1964), the final stage with elastic behaviour after the time of maximum displacement \((y_1)\) is then given by

\[ y = y_1 + \frac{R_1 - P_1}{k} \left( \cos(\omega t_2) - 1 \right) \quad (3.22) \]

where \( t_2 = t - t_1 \) with \( t_1 \) being the time at maximum displacement.

In structural dynamics, it is convenient to refer the duration of the load \( t_d \) to the natural period \( T \) of the system. The important parameter here is the ratio of load duration to natural period \( t_d/T \) rather than the actual value of either quantity. Diagrams have been developed for the maximum response of SDOF systems subjected to different types of simple load functions and with a bilinear resistance function. One such chart is presented in Figure 3.16 (Biggs, 1964). The ratio \( t_d/T \) and the internal resistance to applied load ratio \( R_m/F_1 \) are sufficient for a solution of the maximum response, which is presented as the maximum to elastic response ratio \( y_m/y_{el} \).

![Figure 3.16](image-url) Maximum deflection of elastic-perfectly plastic SDOF systems subjected to a triangular load. Reproduced from Biggs (1964).
It is not always possible to obtain closed-form solutions and numerical solutions can instead be used where the equation of motion is solved step by step starting at zero time. The numerical integration method below is referred by Biggs (1964) as the constant velocity procedure where the time scale is divided into discrete intervals $\Delta t$. Suppose the deformations are known up to time $s$. The determination of the next displacement is then determined as

$$y^{(s+1)} = y^{(s)} + \dot{y}_m \Delta t$$ \hspace{1cm} (3.23)

where $\dot{y}_m$ is the average velocity between the point of times $s$ and $s+1$, and expressed by

$$\dot{y}_m = \frac{y^{(s)} - y^{(s-1)}}{\Delta t} + \ddot{y}^{(s)} \Delta t$$ \hspace{1cm} (3.24)

where $y^{(s-1)}$ and $y^{(s)}$ are the displacements at the corresponding time intervals. The second term in eq. (3.24) is the increase in velocity between the two time intervals, where it is assumed that $\ddot{y}_s$ is the average acceleration. Thus, the acceleration curve is approximated by a series of straight lines as shown in Figure 3.17 and the shaded area represents the change in average velocity between the two adjacent time intervals. The acceleration at time $s$ is calculated by rewriting the equation of motion.

$$\ddot{y}^{(s)} = \frac{P^{(s)}(t) - ky^{(s)}}{m}$$ \hspace{1cm} (3.25)

Using the acceleration from eq. (3.25) in eq. (3.24) and then substituting eq. (3.24) into eq. (3.23) the following recurrence formula is obtained:

$$y^{(s+1)} = 2y^{(s)} - y^{(s-1)} + \ddot{y}^{(s)} \left(\Delta t\right)^2$$ \hspace{1cm} (3.26)

The numerical analysis of an SDOF system with the constant velocity procedure is an approximate method that gives sufficiently accurate results for the purposes considered herein provided that $\Delta t$ is taken small compared to the variations in acceleration. Biggs (1964) suggests that sufficiently accurate results are obtained if the time interval is taken not larger than one tenth of the natural period of the system.

Figure 3.17 Numerical integration with the constant velocity procedure (Biggs, 1964).
3.3.2 Modelling of structural elements as equivalent systems

In reality, beams and slabs have a continuously distributed mass along the span and these elements show idealistically an infinite number of degrees of freedom. However, only a few of the lower modes have responses of any significance and in some cases only the fundamental mode is of importance. Higher modes are of importance when considering shear forces in the elements as discussed previously in section 3.2. In the analysis of blast loaded structures it is convenient to model the real structure as an equivalent SDOF system. The equivalent system is determined on the principle of kinematic equivalency, i.e. the acceleration, velocity and displacement are the same as that for a significant point on the actual structure such as at mid-span of a beam. This requires the assumption of a deformed shape of the real structure.

In order to define this mass-spring system in Figure 3.18 the equivalent mass \( m \), force \( P \) and stiffness \( k \), respectively, need to be evaluated and these properties are derived from energy relationships. Thus, the external work, the strain energy and the kinetic energy of the simplified system equals the corresponding quantities of the real structure (Granström, 1958; Biggs, 1964). It is then convenient to introduce transformation factors to convert the properties of the real structure into the equivalent system. The equivalent mass, for instance, is determined so that the kinetic energy of the equivalent system and the actual structure are equal. The equivalent stiffness and equivalent force are then determined in a similar way, related to the strain energy and external work, respectively. The derivations of the transformation factors \( \kappa_M \), \( \kappa_R \), and \( \kappa_L \) are reported and listed for beams and slabs with different end conditions and deformed shapes by Granström (1958) and Biggs (1964). It should be noted that forces in the equivalent system are not directly comparable to those in the real structure. The dynamic support reactions for beams are further discussed in section 3.3.4.

\[ p(t) \]

\[ m, EI \]

\[ x \]

\[ L \]

\[ y \]

\[ k_e \]

\[ m_e \]

\[ P_e(t) \]

**Figure 3.18** Dynamically loaded beam idealised as a mass-spring system.

To evaluate the spring constant \( k \) the known relationship in eq. (3.27) can be used. This expression holds for a simply supported beam of constant flexural rigidity \( EI \) and subjected to a uniformly distributed load.
As a concrete beam responds to a dynamic load it deflects and progresses through different stages before coming to rest. The beam stiffness varies considerably through the different stages as schematically shown in Figure 3.19. State 1 represents the elastic stage and state 2 represents a cracked beam. As yielding of the reinforcement commences around mid-span and a plastic hinge is formed the stiffness is reduced once more and all the deformations will take place within the vicinity of the formed hinge. This behaviour is valid for the case studied herein, i.e., a simply supported beam with a uniformly distributed load. The flexural stiffness of a reinforced concrete beam may be difficult to determine because the amount of cracking has a considerable effect on the stiffness. The amount of cracking varies along the span of the beam and the cracking also depends on the extent of the deformations due to the applied load and an estimate must be made in order to evaluate the stiffness. The flexural stiffness in the elastic state is determined by

\[ k = \frac{384EI}{5L^2} \]  

(3.27)

where \( E_c \) is the modulus of elasticity of concrete, and \( b \) and \( h \) are the width and height of the beam cross section, respectively. The flexural stiffness of the cracked beam in state 2 is obtained according to (Mårtensson, 1996)

\[ EI_{II} = 0.5bd^3E_c\left(\frac{x^2}{d}\left(1-\frac{x}{3d}\right)\right) \]  

(3.29)

where \( x \) and \( d \) are the depth of the compression zone and the effective depth of the beam, respectively. The ratio between these two parameters is determined as

\[ \frac{x}{d} = \frac{A_s}{bd} \frac{E_s}{E_c} \left(1 + \frac{2bd}{A_s} \frac{E_c}{E_s} - 1\right) \]  

(3.30)

where \( E_s \) is the modulus of elasticity of steel. Biggs (1964) suggests that an approximate expression of the flexural stiffness is used, which is based on the average values of the uncracked and cracked sections as follows:

\[ EI = E_c \frac{bd^3}{2} \left(5.5 \frac{A_s}{bd} + 0.083\right) \]  

(3.31)

However, it has been found that this expression gives a too stiff response for the beams studied herein. The load-deflection curves from the static tests reveal that the beams were predominantly in a cracked state (state 2) during a continuously increasing load up to the load where yielding of the tensile reinforcement commenced. The beams were in state 1 during approximately 20% of the total load span up to yielding of the reinforcement. Herein, a modified average beam stiffness was
determined where both states were taken into account with weighting factors as follows:

$$EI_{\text{mod}} = \frac{1}{5} EI_1 + \frac{4}{5} EI_2$$

(3.32)

Thus, in the calculations the two states were substituted with a single modified stiffness and the beam response was regarded as elastic up to deformations where the plastic hinge forms.

Since the system progresses through these different stages, the response in each stage generally needs to be computed separately using the transformation factors for each specific range. However, Biggs (1964) suggests that if the response is expected to reach beyond the elastic range, an average value of the factors may be employed. Here, the chosen transformation factors were taken as the average of these in the elastic and the plastic region, respectively. Thus, the mass factor $\kappa_M = 0.41$, the load factor $\kappa_L = 0.57$ and stiffness factor $\kappa_R = 0.57$ were used.

![Figure 3.19 Schematic view of the reduction in stiffness at an increasing cracking of a reinforced concrete beam with a modified stiffness.](image)

3.3.3 Iso-damage curves

The principal of energy conservation during dynamic response states that at any time the external work $W$ equals the sum of the internal strain energy $U$ and the kinetic energy $K$. If an elastic-perfectly plastic mass-spring system is subjected to a suddenly applied and constant load of long duration, the mass will be accelerated in the direction of the force as long as this force is larger than the internal resistance, see Figure 3.20–3.21. This type of load will be referred to as a quasi-static load in the following and is used as a representation of a dynamic load with long duration in relation to the natural period of the system. The acceleration will diminish as the increasing resistance approaches the same magnitude as the applied load, and will eventually subside as the external and internal forces are equal. At this point of time the system has reached its maximum velocity. As the internal resistance continues to increase and becomes larger than the applied load, the system decelerates and
eventually is reduced to zero velocity at the point where the external work equals the internal work. At this moment the system has reached its maximum displacement.

Should the response of the SDOF system be chosen as elastic only and with the same type of load, then the maximum displacement is exactly twice that of the same force applied statically. Thus, at twice the static displacement the strain energy equals the work done on the system as shown mathematically in section 3.3.1, see eq. (3.13). Should the applied load instead be of short duration in relation to the natural period of the system, Figure 3.21, then the magnitude of the load may considerably exceed the resistance of the element. Very little resistance will develop during the load duration and the impulsive load will give the system an initial velocity and then leave the system to decelerate.

![Diagram](image)

**Figure 3.20** The response of an SDOF system with a bilinear resistance function and subjected to a suddenly applied load $P_1$ of long duration. Based on Granström (1958).

![Diagram](image)

**Figure 3.21** Principal concepts of quasi-static loading (a) and impulsive loading (b).
Granström (1958) presents a method of analysing structural beams and slabs subjected to air blast loading by considering energy relations. Two extreme types of loads can be defined, i.e. the characteristic quasi-static load $P_c$ (N) and the characteristic impulsive load $I_c$ (Ns). These two quantities constitute the maximum pressure a structural element can resist when subjected to a quasi-static load and the maximum impulse the element can resist when exposed to an impulsive load, respectively. The characteristic quasi-static load is derived from the relationship stating that the work done on the system $W$ equals the strain energy $U$ at maximum displacement. Thus, the work done on the system is

$$W = P_c y_1$$

(3.33)

where $y_1$ denotes the maximum displacement. The strain energy of the system is

$$U = \int_0^{y_1} R_s(y) \, dy$$

(3.34)

At maximum response and all kinetic energy has been converted into strain energy the following applies after solving for $P_c$.

$$P_c = \frac{\int_0^{y_1} R_s(y) \, dy}{y_1}$$

(3.35)

An impulsive load is typically of very high magnitude and of very short duration as schematically shown in Figure 3.21. The character of the impulsive load is that an initial velocity is imparted to the system during a very short period of time and that little deformation occurs before the load duration is over. The initial velocity may be determined by eq. (3.15) in section 3.3.1. The kinetic energy associated with the initial velocity when no strain energy is stored in the system is

$$K = \frac{I_c^2}{2m}$$

(3.36)

As the system has reached its peak deformation the strain energy equals the initially imparted kinetic energy and solving for $I_c$ gives

$$I_c = \sqrt{2m \int_0^{y_1} R_s(y) \, dy}$$

(3.37)

This expression can also be expressed in terms of $P_c$:

$$I_c = \sqrt{2 P_c y_1 m}$$

(3.38)

If values per unit area are sought, which usually is the case for blast loaded structures, the characteristic parameters are instead denoted $p_c$ (Pa) and $i_c$ (Pas). The two
characteristic parameters are referred to as the pressure and the impulse density, which can both be related to the pressure-time curve from an air blast as discussed in chapter 2. The following expressions are used to determine the characteristic pressure and impulse density for a beam deforming plastically (Balazs, 1998).

\[ p_c = \frac{\kappa_R}{\kappa_I} \frac{8\bar{c} M}{b l^2} \]  
\[ i_c = 4 \sqrt{\frac{\kappa_R \kappa_M}{\kappa_I^2} \cdot \frac{\bar{c} M A \rho y}{b^3 l^2}} \]

where \( M \) is the bending moment capacity, \( y \) the deflection and \( \rho \) the density. \( A, b \) and \( l \) denote the area of the beam cross-section, the beam width and span between the supports, respectively. The transformation factors \( \kappa_M, \kappa_R, \) and \( \kappa_I \) were defined in the previous section. Finally, \( \bar{c} \) is a constant relating the area beneath the resistance function to a purely plastic resistance function as shown in Figure 3.22. The constant \( \bar{c} \) is used when considering the resistance function of the system and is defined as (Granström, 1958)

\[ \bar{c} = \int_0^{\gamma_1} R_i(y) dy \]

This expression relates the strain energy according to the approximated resistance function to the strain energy in a case where the system has a rigid perfectly plastic resistance function.

From these basic equations combined with the transformation factors to reduce the real system into an equivalent SDOF system, the characteristic values \( p_c \) and \( i_c \) can be determined for different structural elements. Apart from the resistance of the element and its dimensions, \( p_c \) and \( i_c \) also depend on assumed deformation shape, support conditions and type of load. Lindqvist et al. (1994) list \( p_c \) and \( i_c \) values for the collapse of different types of walls of dwelling houses.

The expressions above, for both extreme loading situations, were derived with the use of energy relationships. Granström (1958) presented a method to determine different combinations of pressures and impulse densities and that give identical displacement of a structural element. These combinations form a continuous iso-deformation or iso-damage curve graphically presented as shown in Figure 3.23. Thus, any point on the curve represents a unique combination of pressure and impulse density that gives the structural element identical deformations. The values of \( p_c \) and \( i_c \) define the asymptotes to the iso-damage curve. The following expression may be used for a plastic response:

\[ \frac{p_c}{p} + \frac{i_c}{i} = 1 \]
This expression is only an approximation and Granström (1958) points out that the error is less than 10–15 % for a triangular load on the condition that the calculations are conservative. It can also be shown that the position of the iso-damage curves change as the applied load changes shape as shown in Figure 3.24 (Granström, 1958; Balazs, 1998). Structural elements of different size and resistance all have different iso-damage curves. The iso-damage curve clearly indicates that two parameters are necessary in determining a structure’s dynamic load capacity. Iso-damage curves are also useful when graphically comparing the resistance of different structural elements and it is a practical method for performing parametric studies. Iso-damage curves can be useful in performing vulnerability assessment where the dynamic load is known in general terms only. Iso-damage curves may also be useful prior to performing air blast experiments to simply determine the size of the explosive charge.

![Diagram](image-url)

*Figure 3.22 Definition of the constant $\bar{c}$ for a perfectly plastic (a) and elastic-perfectly plastic (b) resistance function, respectively (Granström, 1958).*

Furthermore, three distinct regimes of the iso-damage curves can be observed, namely the impulsive loading regime, the quasi-static loading regime and the dynamic loading regime. In the quasi-static loading regime the deformation depends only on the peak load and the structural resistance. The load duration and system mass have no influence on the response. On the other hand, the response in the impulsive loading domain depends on the resistance and mass as well as the impulse density. The transition zone connecting the impulsive loading and quasi-static loading regime is termed the dynamic loading regime. This regime is characterised by deformations depending on the entire load history and no approximate idealisations can be applied. The deflections depend on the pressure and the impulse density as well as the system resistance and mass. Mays and Smith (1995) suggest that a certain load falls in one of the three regimes depending on the product between the natural circular frequency of the system $\omega$ and the load duration $t_d$ as indicated below:

- Impulsive loading $\omega t_d < 0.4$
- Dynamic $0.4 < \omega t_d < 40$
- Quasi-static loading $\omega t_d > 40$
As discussed in section 3.3.1, Biggs (1964) suggests that the loading can be treated as an impulse if $t_d$ is less than about 0.1 times the natural period of the system. This gives a $\omega t_d$ of around 0.6, which is relatively close to the impulsive loading domain according to Mays and Smith (1995).

Iso-damage curves of different structural elements may not only be derived from analytical solutions as previously discussed in this section. These curves may also be determined from experiments. Forsén (1985) presents experimentally determined iso-damage curves for different types of wall panels, such as walls made of reinforced concrete, lightweight concrete and brick masonry.

![Figure 3.23](image)

*Figure 3.23 Schematic representation of an iso-damage curve and corresponding regimes. Based on Baker et al. (1983).*
3.3.4 Dynamic support reactions

A comparison of registered and calculated dynamic support reactions is of interest because these can be related to the maximum shear forces that arise in the structural element during its response. The dynamic reactions are also of interest in the design of supporting elements and different kinds of joints. It is important to recognise that the dynamic reactions can not be evaluated with the equivalent SDOF system because forces in the equivalent system are not directly comparable to those in the real structure. This is true because the parameters of the mass-spring system were derived on the assumption that the system has the same displacement as that of a characteristic point of the real element, rather than the same stress characteristics. This is also further discussed in section 3.3.2. Thus, the dynamic reactions can not be evaluated from the spring force of the simplified system. Instead there is a need to consider the dynamic equilibrium of the actual element as shown in Figure 3.25 (Biggs, 1964). This figure shows a simply supported beam subjected to a uniformly distributed dynamic load. The expression for the reactions is obtained by considering the dynamic equilibrium of the beam. For a beam with evenly distributed mass the inertia has the same distribution as the assumed deflected shape. When considering only half of the beam and taking the moments about the resultant inertia force results in a relationship as follows (TM5-1300, 1990):

\[ V(t) = \chi_R R_b(t) + \chi_P P(t) \]  

Thus, the dynamic reaction is a function of both the resistance of the beam \( R_b \) and the applied load \( P \). The constants \( \chi_R \) and \( \chi_L \) depend on the assumed deflected shape of the beam. For elastic response, the values of these constants are 0.39 and 0.11,
respectively. During the plastic response where the beams were assumed to deform as two rigid bodies with a plastic hinge at mid-span the corresponding values change slightly into 0.38 and 0.12. The dynamic reactions for beams and slabs with varying support conditions and applied loads are listed in Biggs (1964) and TM5-1300 (1990).

Cracking of a blast loaded beam at increasing displacement will reduce its stiffness and for flexural response the plastic hinge forms where the maximum bending moment is developed and the resistance-displacement curve levels off. At this time where the hinge is formed at mid-span this part of the beam will experience a sudden jump in velocity (van Wees and Peters, 1995). This velocity jump will excite free vibrations of the two beam parts between the hinge and each support, which will show itself as an oscillation about a mean value that also affects the shear forces at the supports.

Figure 3.25  Determination of the dynamic support reactions for a simply supported beam subjected to a uniformly distributed dynamic load (Biggs, 1964).
4 Beams subjected to air blast loading

Structural elements consisting of simply supported concrete beams of varying concrete strength and reinforcement content subjected to air blast loading are studied herein. The beam can be regarded as part of a slab or a wall in a real structure. The findings of the experimental investigations are briefly discussed and commented on and also serve as reference in the dynamic analyses. It is of great interest to be able to predict the structural response when subjected to severe dynamic loading such as blast loads. The dynamic analyses involve SDOF modelling of the beam response, and the use of iso-damage curves. Also, the dynamic support reactions are calculated and compared with test results.

4.1 Experimental investigations

Several concrete beams of varying strength and amounts of reinforcement have been tested both to static loading as well as air blast loading. As a whole, 89 concrete beams were tested in the different investigations of which 49 beams were reinforced with tensile reinforcement and stirrups. Reinforcement steel grade B500BT with a nominal yield strength of 500 MPa was used in the beams. The remaining 40 beams were solely reinforced with steel fibres of content 1.0 percent by volume. The investigations on the reinforced beams are presented in Papers I–II, with corresponding reports by Hallgren and Balazs (1999) and Magnusson and Hallgren (2000, 2003). The tested fibre reinforced beams are presented in Paper III, with corresponding reports by Magnusson (1998, under evaluation).

4.1.1 Reinforced concrete beams

In the case with beams reinforced with rebars the mechanical ratio of reinforcement $\omega_s$ was mostly used as a measure of the reinforcement content. The mechanical ratio of reinforcement is given by

$$\omega_s = \frac{A_s}{b d} \cdot \frac{f_{sy}}{f_{cc}}$$  \hspace{1cm} (4.1)

where $f_{sy}$ and $f_{cc}$ denote the static yield strength of the reinforcement and concrete compressive strength. The compressive strength values on cylinders with a 150×300 mm specimen size were used on in the determination of $\omega_s$. $A_s$ is the cross-sectional area of the tensile reinforcement, and $b$ and $d$ denote the beam width and effective depth, respectively. The small amount of compression reinforcement gives, theoretically, an improvement of the strength of the compressive zone. However, as the beam deflects and approaches a compressive failure these rebars do not contribute to the strength because the concrete compression zone is not as deep. The rebars may even be subjected to small tensile stresses at this point and their contribution to the load capacity was therefore not accounted for herein. Table 4.1 summarises the total amount of beams in the experimental programs. The concrete grade, as referred to in this table is, based on the nominal compressive strength of 150 mm cubes. Except for
beams of grade 40 and 100(16), the aim was to keep $\omega_s$ constant but variations in actual concrete strength, yield strength of the reinforcement and effective depth resulted in variations of the actual $\omega_s$. Post-test photos of beams from the static tests and one representative beam of each type in the air blast tests are presented in the Appendix.

All reinforced beams loaded statically failed in flexure. The static tests showed that the stiffness and the load capacity were enhanced for beams of higher concrete strength and where the reinforcement content was kept constant. The maximum deformations were also observed to increase for these beams, which is the result of the mechanical reinforcement ratio $\omega_s$ decreasing with an increasing concrete strength. This can also be expressed as the ductility of the beams increasing with a decreasing $\omega_s$. The static tests also showed that the ductility and deformation capacity seemed to be independent of concrete strength as $\omega_s$ was kept constant, which was also observed by Hallgren (1994) and Fransson (1997).

It was also observed that in a case where steel fibres were introduced to the concrete the ductility could be further increased. This was the case for beams of concrete grade 140F where the compression failure of the beams was considerably more ductile compared to beams without steel fibres, see Figure 4.1. Due to the ductile failure process in compression the loss in load bearing capacity of the beam post-failure response was not as severe for the beam containing steel fibres. Also as presented in paper II, beams containing steel fibres were generally more ductile in comparison to beams without fibres when loaded statically.

The residual strength was also investigated experimentally for beams of grade 140, 140F and 200F/40. These tests were performed for some of the dynamically tested beams to quantify their load capacity after they had been damaged to different degrees due to air blast loading. The residual load capacity just about reached the descending branch of the load-deflection curve of the statically tested beam, which is also shown in Figure 4.1. This figure also shows that the beam stiffness is reduced at an increasing initial damage. Trial calculations were performed to determine the residual bending moment capacity of the beams by accounting for the damage in the compression zone and thereby reducing the effective depth of the beam (Magnusson and Hallgren, 2003).

The air blast tests were performed within a shock tube, which allows for considerably smaller explosive charges compared to the case with a free air explosion having the same load intensity. In the shock tube and at a distance of ten metres from the explosive charge to the assembled concrete beam, the beams may be assumed to be exposed to a planar blast wave. As the blast wave strikes the closed end of the shock tube with the assembled beam, the wave reflects and a uniformly distributed load spreads across the surface of the beam. A typical pressure-time registration is shown in Figure 4.2. Figure 4.3 illustrates the reflected pressures and impulse densities that were obtained in the blast tests for the different test series.
Table 4.1  Beam types included in the different experimental programs.

<table>
<thead>
<tr>
<th>Nominal concrete grade</th>
<th>Tensile reinforcement / Fibre length (mm)</th>
<th>$\omega_s$</th>
<th>Number of beams</th>
<th>Incl. in paper no.</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Static</td>
<td>Blast</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>5φ16</td>
<td>0.34</td>
<td>1</td>
<td>5</td>
<td>I, II</td>
</tr>
<tr>
<td>100(16)</td>
<td>5φ16</td>
<td>0.14</td>
<td>1</td>
<td>3</td>
<td>I, II</td>
</tr>
<tr>
<td>100(12)</td>
<td>4φ12</td>
<td>0.087</td>
<td>1</td>
<td>3</td>
<td>I, II</td>
</tr>
<tr>
<td>140</td>
<td>6φ12</td>
<td>0.11</td>
<td>1</td>
<td>3</td>
<td>II</td>
</tr>
<tr>
<td>140F</td>
<td>6φ12</td>
<td>0.11</td>
<td>1</td>
<td>3</td>
<td>II</td>
</tr>
<tr>
<td>140F/40</td>
<td>6φ12</td>
<td>0.089</td>
<td>1</td>
<td>3</td>
<td>II</td>
</tr>
<tr>
<td>150</td>
<td>6φ12</td>
<td>0.079</td>
<td>1</td>
<td>3</td>
<td>I, II</td>
</tr>
<tr>
<td>200</td>
<td>5φ16</td>
<td>0.088</td>
<td>1</td>
<td>3</td>
<td>I, II</td>
</tr>
<tr>
<td>200F</td>
<td>5φ16</td>
<td>0.0722 / 0.0782</td>
<td>1</td>
<td>6</td>
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<td>5φ16</td>
<td>0.088</td>
<td>1</td>
<td>3</td>
<td>I, II</td>
</tr>
<tr>
<td>200F/40</td>
<td>5φ16</td>
<td>0.10</td>
<td>1</td>
<td>3</td>
<td>II</td>
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Steel fibre reinforced beams

<p>| | | | | | | |</p>
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<td>III</td>
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<td>-</td>
<td>3</td>
<td>4</td>
<td>III</td>
<td>-</td>
</tr>
</tbody>
</table>

1  Steel fibres were of length 12.5 mm and a diameter of 0.4 mm.
2  $\omega_s = 0.072$ for beam tested statically, and $\omega_s = 0.078$ for beams tested dynamically to failure, see Magnusson and Hallgren (2000).
Figure 4.1 Static tests and residual static tests on beams B140 and B140F (Magnusson and Hallgren, 2003).

Based on the support reactions it was noted that larger load capacity was obtained in the air blast tests in relation to the corresponding capacity in the static tests for all types of beams. All reinforced beams subjected to static loading failed in flexure. However, beams subjected to air blast loading showed that the failure mode in some cases changed from a flexural failure in the static tests to a shear failure in the dynamic tests. These beams contained a relatively large amount of tensile reinforcement. Beams with a reduced reinforcement content failed in flexure due to reduced stiffness and bending moment capacity. The same result was observed as a HSC beam that originally failed in shear was cast in two concrete layers with the bottom layer consisting of conventional concrete. The reduced stiffness resulted in a flexural failure even though diagonal shear cracks appeared, see further paper I–II. The investigations also showed that the flexural shear failures exhibited by a beam could be prevented by introducing steel fibres to the concrete and thereby increasing
The experimental results show that apart from the load intensity determining the mode of failure of an element, its stiffness also plays an important role. A less stiff element will respond to a dynamic load of high intensity by deflecting readily and obtaining a larger magnitude of kinetic energy compared to a case with a much stiffer element. In this case the element will respond by building up larger internal resistance and thereby attaining a larger portion of strain energy compared to the softer element. This will also give rise to larger shear forces in the element, see further section 3.2. The experiments also showed that beams that eventually failed in flexural shear responded in a bending mode at relatively low pressure levels.

Figure 4.2 Registration of the variations in reflected pressure over time from blast test in the shock tube with a spherical 4.0 kg plastic explosive.

Figure 4.3 Reflected pressures and impulse densities obtained in the air blast tests.
4.1.2 Steel fibre reinforced beams

A flexural shear failure mode was prevented by introducing steel fibres to a reinforced beam as discussed in section 4.1.1. The static tests with steel fibre reinforced concrete (SFRC) beams showed that all combinations of concrete strength and fibre length exhibited strain hardening characteristics. The main flexural failure mechanism was by fibre pull-out for all beams. Beams of the highest concrete grade (150) exhibited, however, a combination of fibre pull-out and fibre fractures. The probable cause for the fibres fracturing is the increase in bond strength for higher concrete grade. The load capacity increased with an increasing concrete strength. However, the positive effects of the fibres in beams of higher strength were reduced in relation to beams of conventional concrete. There was a substantial decrease in ductility as the concrete strength was increased and this was especially noticeable for the beams of the highest strength that also contained the long fibres.

All the air blast loaded beams failed in a flexural mode due to their relatively low bending moment capacity in relation to their flexural shear capacity. The lower bending strength of the SFRC beams compared to beams with reinforcement bars resulted in much lower magnitudes of pressure and impulse density to obtain failures, Figure 4.3. Similarly to the static tests, the fibres were pulled out of the matrix for beams of the two lower concrete grades, while a combination of fibre pull-out and fibre fractures was observed in beams of concrete grade 150. For beams of this grade the relative amount of fibre fractures in beams containing the long fibres was larger compared to beams containing the short fibres. The dynamic tests showed a tendency towards an increased amount of fibre fractures compared to beams loaded statically.

4.1.3 Support conditions and compressive arch action

In the blast tests, the supports consisted of homogeneous steel with a half-circular cross section. The beams were then fixed to each support with two bolts with a diameter of 10 mm and nuts in order to ensure that the beam stayed in place during the tests, see further paper I–III. Due to the relatively low flexural strength of the bolts the beams were considered as simply supported. In addition, the holes for the bolts close to the beam ends were rectangular in order to enable movements and rotations of beams. A compressive arch could appear if the lateral movements of the beams would be restricted to some degree during the tests, see section 3.2.

It is interesting to study the registrations during the tests and sort out the behaviour of the beams for the continuing discussion. Figure 4.4 illustrates a combination of registered quantities, which are overpressure, mid-span acceleration, support reactions and deflection at mid-span. At this stage, it is not important to know the amplitudes of each quantity and instead the sequences in time are studied. Initially, the shock load strikes the beam over its front surface and shortly thereafter accelerations are built up over the beam span. The acceleration grows over time and after a certain time delay there is the build-up of dynamic support reactions. The support reactions also increase over time and slowly the beam attains some deflection, which does not occur until the inertia of the beam has been overcome. This sequence of events was similar for all beams tested within the shock tube. It is clear that the blast load is transmitted through the beam to the supports before any deflection has occurred and therefore no flexural resistance has been built up. Parts of the pressure load near the beam ends will clearly
be transmitted directly to the supports during the whole event. But it is also possible that friction at the supports may initially contribute to some degree of restraint and thereby contributing to an arching action. However, as inertia and friction at the supports is overcome and the beam starts to deflect this initial arch mechanism will be lost and the load will mainly be carried by flexural resistance.

![Figure 4.4](image)

*Figure 4.4* Comparisons of response time for different quantities. Values from blast test with beam B140F-D3, Magnusson and Hallgren (2003).

### 4.2 Dynamic analysis of experiments

As already discussed in section 3.3, the SDOF method is designed to predict deflections by assuming that the main response is in the first bending mode. The method of using iso-damage curves, which is also based on the SDOF method, can be a quick means of performing parametric studies and performing vulnerability assessment. The shear forces can be derived from the assumed shape of the element. Hence, the SDOF method, use of iso-damage curves and evaluation of shear forces will be discussed in the following sections in the assessment of dynamically loaded concrete beams.

#### 4.2.1 Single-degree-of-freedom system

To account for strain rate effects both in the concrete and in the tensile reinforcement the maximum resistance of the beams should be increased. For B500BT reinforcement can an increase of the yield strength of 1.2 be expected at a strain rate of 1 s\(^{-1}\), according to Figure 3.6. Strain rates of this magnitude were measured in earlier blast tests according to Magnusson and Hallgren (2000). In reality, the elements experience variations in strain rates in the concrete and reinforcement and, consequently, the corresponding strength increase due to strain rate effects will also
vary. The measured strain histories in the concrete compressive zone and in the rebars indicate that the maximum strain rate values appear around the time where the plastic hinge forms. The strength capacity of the beams is almost reached at this point where yielding of the reinforcement commences. This indicates that the assumption of using the maximum strain rate in the SDOF analyses is a good approximation. The concrete compressive strength can be expected to show an increase of about 1.35 at the same strain rate for conventional concrete but only around 1.1 for concrete of higher strength, according to Figure 3.4. However, when considering the maximum bending moment capacity, the yield strength of the tensile reinforcement is the dominant parameter and, thus, an increase by a factor of 1.2 times the static bending moment capacity was used in the analysis. The resistance of each beam was evaluated from the bending moment capacity in the static tests.

The load-time function plays an important role in the dynamic response of structural elements. The registrations of the blast loads from the tests needed to be approximated into triangular loads within the time of interest in the SDOF calculations, i.e. up to the time of maximum response. In the approximations of the loads it was necessary also to compare the respective impulse density obtained from the registrations to represent that of the real registration. Figure 4.5 illustrates a triangular approximation and the corresponding variations in impulse densities over time for one test. The impulse density of the triangular load shows good agreement with that of the real registration up to about 0.013 s, which is just the point of time beyond maximum deflection. A linearly piecewise approximation is also included in Figure 4.5, which was done in order to study the difference in calculated beam response compared to when using a triangular load. The piecewise linear curve was used as a better fit to the registered pressure curve and for this type of problem numerical analysis was more appropriate considering the more complicated load-time function. Thus, the constant velocity method was used as discussed in section 3.3.1.

Several parameters needed to be determined for the SDOF calculations in order to evaluate the response of the different beams in the test series. For this purpose, two tested beams with a relatively large difference in maximum response were chosen, namely beams B140F-D2 and B140F-D3 with maximum deflections of 36.9 mm and 69.8 mm, respectively (Magnusson and Hallgren, 2003). The influence of the beam mass, load function, stiffness, dynamic increase factor for the maximum resistance and the conversion factors were evaluated. The results from the parameter study are presented in Figure 4.6, which also includes the response for the final model. This response is denoted ‘final’ in Figure 4.6. The variations of the parameter values are summarised in Table 4.2.

The results from beams B140F-D2 and -D3 were similar so the results for the latter are not presented herein. The variations in deflections were proportionally similar for these two beams as the parameters were varied. The real beam mass was reduced by a factor 0.87 in the calculations due to the span being smaller than the entire beam length and that way the deflecting mass should be reduced compared to the total mass of the beam. Thus, the mass of the beam parts outside each support was subtracted. The influence on the beam response when instead the whole mass is accounted for is shown in Figure 4.6 and the curve denoted ‘mass’. Reduced deflections are obtained as well as a somewhat increased natural period of vibration. The linearly piecewise load function gave results that correspond well with the result when using a triangular
load approximation. Thus, the approximation of the triangular load seems to correspond well to the registered load up to the time of maximum response. Using the modified stiffness seems to be a more appropriate description compared to using the stiffness in state 2, which gives a too soft response. Variations in the dynamic increase factor resulted in large differences in response. The results in Figure 4.6 show that it is important to consider the strain rate effects. Furthermore, the use of conversion factors where two straight beam parts are assumed and a plastic hinge forms at mid-span throughout the whole response was also investigated. The conversion factors for a plastic response have the values $\kappa_M = 0.33$ and $\kappa_L = \kappa_R = 0.5$ were used, which resulted in a slightly larger deflection. In all, it is clear that it is vital to carefully choose the right values of the different parameters involved.

Figure 4.5 Linearly piecewise approximation of pressure curve for test with beam B140F-D2 (Magnusson and Hallgren, 2003).
Figure 4.6 Parameter study of tests with beam B140F-D2 as reference (Magnusson and Hallgren, 2003)
Table 4.2  Variations of values in the parameter study with beam B140F-D2.

<table>
<thead>
<tr>
<th>Notation</th>
<th>$P_1$ (kN)</th>
<th>$t_d$ (ms)</th>
<th>$R_m$ (kN)</th>
<th>$m$ (kg)</th>
<th>$k$ (MN/m)</th>
<th>$y_m$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final ¹</td>
<td>258</td>
<td>13</td>
<td>194</td>
<td>75.1</td>
<td>24.2</td>
<td>35.0</td>
</tr>
<tr>
<td>Piecewise</td>
<td>405</td>
<td>n.a.</td>
<td>194</td>
<td>75.1</td>
<td>24.2</td>
<td>33.6</td>
</tr>
<tr>
<td>Mass</td>
<td>258</td>
<td>13</td>
<td>194</td>
<td>86.1</td>
<td>24.2</td>
<td>32.5</td>
</tr>
<tr>
<td>Stiffness state 2</td>
<td>258</td>
<td>13</td>
<td>194</td>
<td>75.1</td>
<td>15.8</td>
<td>42.7</td>
</tr>
<tr>
<td>DIF = 1.0 / 1.4</td>
<td>258</td>
<td>13</td>
<td>162 / 226</td>
<td>75.1</td>
<td>24.2</td>
<td>49.9 / 26.6</td>
</tr>
<tr>
<td>Plastic ²</td>
<td>226</td>
<td>13</td>
<td>170</td>
<td>61.0</td>
<td>21.2</td>
<td>36.5</td>
</tr>
</tbody>
</table>

$P_1$ = Peak load  
$t_d$ = Load duration  
$R_m$ = Maximum resistance  
$m$ = Equivalent mass  
$k$ = Stiffness  
$y_m$ = Maximum deflection  

¹ Values for the final model of beam B140F.  ² $\kappa_M = 0.33$ and $\kappa_L = \kappa_R = 0.5$

Beams with varying reinforcement amounts and concrete strength were calculated with the SDOF model and the results are presented in Figure 4.7 as the ratio between the calculated and the test results. The numerals of the beams in this figure denote the nominal concrete compressive strength. A slash between two numerals indicate that the beams consisted of two concrete layers, i.e. high strength concrete in the compression zone and conventional concrete in the tensile zone, see Table 4.1. As noticed in Figure 4.7, a majority of the results from the calculations using the modified stiffness as described by eq. (3.32) ended up below the test values. This indicates that the theoretical model in this case gave a too stiff response. On the other hand, in the case where the state 2 stiffness was used according to eq. (3.29) the calculated deformations were in a majority of cases larger than the deformations from the tests.

In the case for beam B200F-D3, it is probable that the registration of the mid-span deflection was not correct and it was re-evaluated by integration of the accelerometer signals. It is a well-known problem that these signals may experience a zero-drift during dynamic events and this was compensated for in the integrations. The assumption was made that the error of the signal was assumed as increasing linearly over time, starting at the first rise of the acceleration. The slope of the linear correction was fitted against the acceleration signal so that the registration ended up with oscillations around the time axis, see Figure 4.8. The same procedure was performed with the velocity curve so that the beam velocity was set to zero at the assumed peak response, which occurs at a time where the acceleration shows a maximum deceleration. Finally, a corrected deflection curve was obtained as presented in Figure 4.8 and a comment is made in Figure 4.7. A similar procedure with a linear correction was used by Ansell and Svedbjörk (2003).

The deflection-time curves for six beams are presented in Figure 4.9a–b. In most cases there is a relatively good agreement between the deflection registrations and the calculated curves when considering the peak deflection and rise time. The parameters used in the SDOF analyses are summarised in Table 4.3.
* Probable that the deflection registration was not correct.

*Figure 4.7 Results of the SDOF calculations.*

*Figure 4.8 Corrected value for the acceleration and deflection from integration of the accelerations for beam B200F-D3.*
Figure 4.9a Results from the SDOF calculations with tests results as reference.
Figure 4.9 b  Results from the SDOF calculations with tests results as reference.
<table>
<thead>
<tr>
<th>Beam no</th>
<th>$P_1$ (kN)</th>
<th>$t_d$ (ms)</th>
<th>$R_m$ (kN)</th>
<th>$m$</th>
<th>$k^2$ (MN/m)</th>
<th>$y_m^2$ (mm)</th>
<th>$t_d/T^2$ (-)</th>
<th>$y_m/y_{el}^2$ (-)</th>
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<tr>
<td>B40-D1</td>
<td>128</td>
<td>9.5</td>
<td>226</td>
<td>71.5</td>
<td>26.4/20.1</td>
<td>7.4/9.3</td>
<td>0.92/0.80</td>
<td>0.86/0.83</td>
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<tr>
<td>B40-D2</td>
<td>237</td>
<td>8.5</td>
<td>226</td>
<td>71.5</td>
<td>26.4/20.1</td>
<td>15.6/18.7</td>
<td>0.82/0.72</td>
<td>1.81/1.67</td>
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<tr>
<td>B100(12)-D2</td>
<td>191</td>
<td>12</td>
<td>120</td>
<td>72.2</td>
<td>21.1/12.9</td>
<td>20.2/26.7</td>
<td>1.03/0.81</td>
<td>3.55/2.90</td>
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<tr>
<td>B100(16)-D2</td>
<td>282</td>
<td>8.5</td>
<td>266</td>
<td>73.3</td>
<td>28.7/21.5</td>
<td>17.6/21.3</td>
<td>0.85/0.73</td>
<td>1.89/1.72</td>
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<td>B140F-D2</td>
<td>258</td>
<td>13</td>
<td>194</td>
<td>75.1</td>
<td>24.2/15.8</td>
<td>35.0/42.7</td>
<td>1.17/0.95</td>
<td>4.38/3.47</td>
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<td>B140F-D3</td>
<td>322</td>
<td>13</td>
<td>194</td>
<td>74.0</td>
<td>24.2/15.8</td>
<td>68.1/77.3</td>
<td>1.18/0.96</td>
<td>8.51/6.28</td>
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<tr>
<td>B140F/40-D1</td>
<td>248</td>
<td>13</td>
<td>194</td>
<td>74.2</td>
<td>21.4/13.9</td>
<td>33.4/41.6</td>
<td>1.11/0.90</td>
<td>3.67/3.0</td>
</tr>
<tr>
<td>B140F/40-D2</td>
<td>279</td>
<td>13</td>
<td>194</td>
<td>74.3</td>
<td>21.4/13.9</td>
<td>46.5/55.5</td>
<td>1.11/0.89</td>
<td>5.11/3.96</td>
</tr>
<tr>
<td>B200-D2</td>
<td>322</td>
<td>8.5</td>
<td>257</td>
<td>86.5</td>
<td>40.3/26.2</td>
<td>18.7/23.6</td>
<td>0.92/0.75</td>
<td>2.92/2.41</td>
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<tr>
<td>B200F-D3</td>
<td>384</td>
<td>6.0</td>
<td>313</td>
<td>89.4</td>
<td>40.5/24.9</td>
<td>15.6/20.5</td>
<td>0.64/0.50</td>
<td>2.03/1.64</td>
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<td>B200F-D6</td>
<td>546</td>
<td>7.0</td>
<td>313</td>
<td>88.7</td>
<td>40.5/24.9</td>
<td>39.5/46.3</td>
<td>0.75/0.59</td>
<td>5.13/3.70</td>
</tr>
<tr>
<td>B200/40-D1</td>
<td>293</td>
<td>10</td>
<td>246</td>
<td>79.3</td>
<td>26.9/20.8</td>
<td>23.8/27.6</td>
<td>0.93/0.82</td>
<td>2.62/2.34</td>
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<tr>
<td>B200/40-D3</td>
<td>310</td>
<td>11</td>
<td>246</td>
<td>80.3</td>
<td>26.9/20.8</td>
<td>29.7/34.0</td>
<td>1.01/0.89</td>
<td>3.26/2.88</td>
</tr>
</tbody>
</table>

$P_1$ = Peak load  
$R_m$ = Maximum resistance  
$m$ = Mass (reduced with a factor 0.87)  
$k$ = Stiffness  
$t_d$ = Load duration  
$y_m$ = Maximum deflection  
$T$ = Natural period of vibration  
$y_{el}$ = Elastic deflection  
$m^1$ = The equivalent mass was reduced by a factor 0.87 due to the span being smaller than the entire beam length.  
$y_m/y_{el}^2$ = Parameters due to the modified stiffness according to eq. (3.32) and stiffness in state 2 according to eq. (3.29), respectively.
4.2.2 Iso-damage curves

Iso-damage curves were set up with the test results of the reinforced concrete beams of different strength according to section 4.1 as reference. The use of iso-damage curves gives a good graphical representation of the capacity of the beams when exposed to blast loads. The same conversion factors were used for conversion of the real system into an SDOF system as in the analysis performed in section 4.2.1. The resistance of each beam was also increased with a factor of 1.2 due to strain rate effects. The product \( \omega t_d \), which can also be expressed as the ratio of the natural period of vibration of the system to the load duration, was in the order of 3–7 for the beams. This means that the load can be categorised as a dynamic load according to Mays and Smith (1995), see also Figure 3.23. The combinations of pressure and impulse densities obtained in the tests also shows that the load is in the dynamic regime for these beams, see Figure 4.10.

The results of the analysis for a selection of beams are shown in Figure 4.10. In each diagram, the iso-damage curves for the calculated deflections in the tests are presented. The combinations of pressures and impulse densities obtained from the tests are also inserted in these diagrams as data points. Thus, each data point corresponds to a specific combination of pressure and impulse density giving the beam a certain maximum deflection, which is also given by the respective iso-damage curve. Beams that failed in flexural shear are omitted in Figure 4.10 because the iso-damage curves were based on the assumption of a flexural failure with the formation of a plastic hinge at mid-span. The parameters used in the iso-damage curves are summarised in Table 4.4.

### Table 4.4 Parameters used for the iso-damage curves.

<table>
<thead>
<tr>
<th>Beam no</th>
<th>( M_{dyn} ) (kNm)</th>
<th>( \rho ) (kg/m³)</th>
<th>( y ) (mm)</th>
<th>( \bar{\epsilon} )</th>
<th>( p_c ) (kPa)</th>
<th>( i_c ) (kPas)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B100(12)-D2</td>
<td>39.4</td>
<td>2550</td>
<td>23</td>
<td>0.83</td>
<td>399</td>
<td>2.32</td>
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<tr>
<td>B100(12)-D3</td>
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<td></td>
<td>45</td>
<td>0.91</td>
<td>440</td>
<td>3.41</td>
</tr>
<tr>
<td>B100(16)-D1</td>
<td>87.6</td>
<td>2550</td>
<td>19</td>
<td>0.66</td>
<td>683</td>
<td>2.76</td>
</tr>
<tr>
<td>B100(16)-D2</td>
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<td></td>
<td>25</td>
<td>0.74</td>
<td>768</td>
<td>3.36</td>
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<tr>
<td>B140F-D1</td>
<td>63.8</td>
<td>2650</td>
<td>42</td>
<td>0.90</td>
<td>708</td>
<td>4.26</td>
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<tr>
<td>B140F-D2</td>
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<td></td>
<td>37</td>
<td>0.89</td>
<td>698</td>
<td>3.97</td>
</tr>
<tr>
<td>B140F-D3</td>
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<td></td>
<td>70</td>
<td>0.94</td>
<td>738</td>
<td>5.61</td>
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<td>B200-D2</td>
<td>84.5</td>
<td>3000</td>
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<td>0.78</td>
<td>806</td>
<td>3.16</td>
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<td>B200F-D3</td>
<td>102.8</td>
<td>3140</td>
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<td>841</td>
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<td>B200F-D6</td>
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<td>38</td>
<td>0.89</td>
<td>1128</td>
<td>5.57</td>
</tr>
</tbody>
</table>

\( M_{dyn} \) = Dynamic bending moment capacity  \( \bar{\epsilon} \) = Constant
\( \rho \) = Average density of beam  \( p_c \) = Characteristic pressure
\( y \) = Mid-span maximum deflection  \( i_c \) = Characteristic impulse density
Figure 4.10a Calculated iso-damage curves and inserted test results for beam types B100(12), B100(16) and B140F (Magnusson and Hallgren, 2000 and 2003).
### 4.2.3 Dynamic support reactions

The dynamic support reactions are a function of the beam resistance $R_b$ and applied load $P$ according to eq. (3.43), see section 3.3.4. The values of the constants $\chi_R$ and $\chi_P$ depend on the assumed deflected shape of the beam and vary thereby when changing from elastic to plastic response. These constants are given the values 0.39 and 0.11, respectively, for elastic deformations and the corresponding values during the plastic response are 0.38 and 0.12 (Biggs, 1964). Thus, the variations for a simply supported beam are relatively small. Nevertheless, these variations were accounted for in the calculations of the dynamic reactions.

The registrations from the load cells consisted of low frequency oscillations, which were filtered to obtain smoother load-deflection curves, see Magnusson and Hallgren (2003). Data as vectors were transformed from the time domain into the frequency domain by using a Fast Fourier Transform (FFT) operation. The first data corresponding to the lowest frequencies in the new frequency vector were set to zero.
Then, an inverse FFT operation was carried out from which a vector was obtained consisting of the high frequency vibrations in the time domain. This vector was then subtracted from the original data vector resulting in a final vector only consisting of the lower frequencies.

![Graph](image1.png)

**Figure 4.11** Comparisons between the registered and calculated support reactions for beams B140F-D2 and B140F-D3 (Magnusson and Hallgren, 2003).

The results from the calculations along with the registrations from the tests are presented for two of the beams in Figure 4.11. Each of the calculated reactions consists of four linear parts and three discontinuities. The first part is vertical and corresponds to the suddenly applied triangular load at zero time. The reactions then increase linearly up to the point in time where the bilinear resistance curve levels off into a perfectly plastic response. This point of time was taken from the previous SDOF calculations as discussed in section 4.2.1. At this point the calculated reaction has reached its peak value and will decline linearly over time. The last part of the calculated reaction was an approximation based on the SDOF calculations. The point of time where the third part of the reaction ends, which actually is where the whole response of the system ends, was chosen to be the time as the maximum deflection
was attained in the SDOF calculations. The dynamic reaction was assumed to end at the point of time when the SDOF system had rebound completely, i.e. at exactly half the natural period of vibration. Naturally, the different points of times taken from the SDOF calculations will depend upon the chosen equivalent stiffness and peak resistance and may for this reason vary depending on the made assumptions. The points in time were taken from the SDOF calculations using the modified stiffness according to eq. (3.32). The linear parts of the support reactions are associated with the linear relationships of both the approximated triangular load as well as the assumed resistance function of the beams in the SDOF calculations.
5 Discussion

5.1 General comments

The dynamic tests were performed in a shock tube with a detonating charge generating a blast wave. For the same pressure, the duration of the blast wave in such a tube is considerably larger than the case of explosions in free air at the same distance. For instance, the test with beam B200F-D6 generated a reflected pressure on the beam of 3078 kPa and a reflected impulse density of 11.1 kPas for a spherical explosive charge consisting of 6 kg TNT and plastic explosive. The charge mass would have to be increased to about 600 kg TNT in a spherical free-air burst at the same distance to generate similar peak reflected pressures. However, the duration of the positive phase in the free-air burst is about half the duration obtained in the shock tube and the corresponding impulse density is only about one third of what is obtained in the tube. In addition, the blast wave would not be planar for the case with a free-air burst at that distance. To obtain the same pressure and impulse density and a planar blast wave, both the charge and the distance to the target would need to be increased substantially. In this context it is interesting to note that the beam in question failed in flexure but was still fully intact, see further section 4.2.1. However, it is probable that the beam would exhibit less damage in the case with an air burst of 600 kg TNT due to the shorter duration of the positive phase. Furthermore, the blast load within the shock tube will load the element with its full duration due to the tunnel inclusion and without any effects from free boundaries. The duration of the blast load would otherwise be affected by rarefaction waves from the boundaries in a case with free edges, see Figure 2.3.

The behaviour explained by van Wees and Peters (1995) when the plastic hinge is formed at mid-span and where the velocity of the beam experiences a sudden jump can be observed in the registrations of the acceleration at mid-span, see Figure 5.1. The first part of the acceleration curve is due to the beam accelerating and generating a velocity and an increasing displacement. As this first part of the acceleration curve diminishes another sharp acceleration will take place, thereby inducing a sudden increase in velocity. This takes place at 0.004 s in Figure 5.1 where the second acceleration pulse is clearly visible. At this time the beam had reached a deflection of 7–8 mm and when studying the static test with this beam type it is clear that the plastic hinge forms at this deflection. Consequently, the beam will now be divided into two oscillating beam parts between each support and the plastic hinge. The behaviour of a sudden jump in velocity of the beam as the plastic hinge forms was clearly visible in the acceleration signals from all tests on beams with tensile reinforcement.
5.2 Experimental investigations

In the design of structures to resist the effects of air blast loading or other severe dynamic loads it is vital to have large energy absorbing capabilities, and structural elements with large plastic deformation capacities are therefore desirable. Structures need to be designed for ductile response in order to prevent partial or total collapse due to locally failed elements. Such design also needs to include strong connections between different elements in order to allow these to develop their ultimate strength.

Reinforced HSC beams loaded statically proved to have an enhanced stiffness and load capacity compared to similar beams of NSC. These tests also indicated that the deformation capacity and ductility appear to be independent of concrete strength as the mechanical reinforcement ratio $\omega_s$ is kept constant. This is an interesting feature of HSC beams, which was also observed by Hallgren (1994) and Fransson (1997). Thus, even if plain high strength concrete is known to be more brittle in comparison to conventional concrete, it is possible to obtain ductile structural elements of HSC with proper reinforcement detailing. As steel fibres were used in the concrete the ductility of HSC beams appeared to increase further. The presence of fibres resulted in a compressive failure with increased ductility, thereby preventing the sudden drop in strength observed in beams without fibres. It is most likely that the presence of the fibres provided for a certain degree of confinement of the concrete in the compression zone during the failure process. The fibres would then partly prevent the dilatation of the concrete during failure, which enables certain enhancements of the ductility. This has been extensively studied in biaxial and triaxial compressive tests on concrete specimen by several researchers, e.g. Imran and Pantazopoulou (1996). A similar confinement effect occurs when stirrups with a small enough spacing are present, which enhances the rotation capacity of the beam (Hillerborg, 1988). However, in the beams considered herein the stirrup spacing was too large to give the same
confinement of the compression zone. In all, besides enhancements of concrete ductility in tension, steel fibres also contribute to an enhanced ductility in the compression zone during a flexural failure. This also has a significant effect on the residual strength of an element, which previously has been damaged. The residual bending moment capacity of elements is an important feature of concrete structures exposed to explosive loading. The residual strength of damaged beams or other elements may be predicted by accounting for the extent of damage in the compression zone as shown by Magnusson and Hallgren (2003).

All statically tested reinforced beams failed in a ductile flexural mode. However, the dynamic tests showed that in some cases the failure modes could change from a flexural failure in the static tests into a flexural shear failure mode in the air blast tests, see Paper I–II. This has also been observed in other investigations, e.g. Hughes and Speirs (1982), Palm (1989), Niklasson (1994) and in the literature review by Ansell (2005). The failure mode of a reinforced concrete beam may depend on the number of vibration modes that are excited by the dynamic load. Considerably larger shear forces will be present if vibration modes of higher order than the first mode are excited in the beam. The dynamic load also needs to be of a certain magnitude to cause a brittle failure. The beams in the experimental investigations were reinforced with stirrups but their contribution to the shear strength was limited due to the relatively large spacing between them.

Apart from the load intensity determining the mode of failure of an element, the stiffness of the element also plays an important role. A stiff element will respond to a dynamic load by building up large internal resistance at relatively small deformations and thereby attaining a large portion of strain energy. This will give rise to large shear forces in the element. An element with a reduced stiffness will respond by deflecting more readily than a stiffer counterpart and will not be able to develop the same amount of strain energy for identical deformations. The softer element will instead attain a proportionally larger amount of kinetic energy and the shear forces in the element will thereby be limited. Hence, whereas a soft element fails by large deformations in flexure a stiff element may fail in a brittle shear mode for the same load intensity. This was observed in the tests with beams of two concrete layers where the stiffness was reduced by casting the bottom half of the beams with a weaker concrete grade and thereby preventing shear failures to occur, see Paper I–II. A similar effect was obtained by reducing the reinforcement content for beams of the same concrete grade. The introduction of steel fibres prevented shear cracks to develop, thus increasing the shear strength of the beams. This also agrees with research according to Gustafsson and Noghabai (1999). In all, concrete structures designed to fail in flexure may fail in a shear mode when loaded dynamically. This is especially the case where an explosive charge detonates at close range from a structure and the load will in this case be impulsive.

Even though steel fibre reinforced concrete (SFRC) has many benefits, the traditional way of reinforcing concrete structural elements with steel rebars is still preferred in the design of blast resistant structures. SFRC elements simply do not have the ability of large deformations required for absorbing blast loads. However, the presence of steel fibres has the advantages of increasing the shear capacity, the ductility and the residual strength of reinforced concrete elements as previously discussed. At a high enough content the fibres will increase the tensile strength of the concrete and,
especially, will increase the fracture toughness of the material. In the studies with SFRC beams it is experimentally shown that the positive effects of the used fibres are reduced in relation to conventional concrete, see Paper III. The strain hardening characteristics and the ductility of the beams were reduced at an increasing concrete strength.

The dynamic tests showed a tendency towards an increased amount of fibre fractures compared to beams loaded statically, which also has been observed in other investigations (Gopalaratnam and Shah, 1986; Stevens et al., 1995; Hallgren and Balazs, 1996). A larger amount of fibre fractures were observed in dynamically loaded HSC beams with the long fibres compared to similar beams containing the short fibres even if both fibre lengths were of the same aspect ratio. Dynamically loaded NSC beams were observed to fail by fibre pull-out. The tests clearly indicate that the fibre length should be reduced in combination with higher concrete strength and this is especially the case in dynamic events. The bond between the concrete increases for an increasing concrete strength and this enhances the bond further. It may therefore be recommended to use shorter fibres with smaller aspect ratios for these purposes.

5.3 Dynamic analyses

The results from the dynamic analyses show the abilities to theoretically predict the dynamic structural response of reinforced concrete beams when exposed to explosive loading. The SDOF method may be used for the analysis and design of blast resistant structures by assuming that the response is in the first flexural mode without the risk of flexural shear failures. The results from the analyses show that it is possible to obtain a dynamic response that agrees well with the experimental results. However, it is vital to carefully choose appropriate values of the different structural parameters and load characteristics involved. The calculations appeared to be independent of large variations in both concrete strength and amounts of tensile reinforcement.

Structural elements undergo several stages during the deformations, which changes the flexural rigidity of the element and an average value of the stiffness needs to be used in the SDOF calculations. Due to the varying deflected shapes the transformation factors also need to be determined based on this behaviour. As noticed in Figure 4.7, a majority of the results from the calculations using the modified stiffness as described by eq. (3.32) ended up below the test values. This indicates that the theoretical model in this case gave a too stiff response. On the other hand, in the case where the state 2 stiffness was used according to eq. (3.29) the calculated deformations were in a majority of cases larger than the deformations from the tests. Thus, an average stiffness somewhere between the modified stiffness and that of a cracked beam may be an appropriate approximation. The determination of an appropriate stiffness is especially difficult when beams consist of two concrete layers. It is clear that proper determination of the approximated stiffness is important in SDOF calculations. Other factors also contribute to variations in agreement between the theoretical and experimental results. One unknown factor can be ascribed to possible variations in the vertical position of the tensile reinforcement for beams of concrete grades 40, 100 and 200. The exact position of the rebars was not measured for these beams, which leaves uncertainties of the real bending moment capacity. Also variations in the yield
strength of the reinforcement may contribute to uncertainties regarding the real strength of the beams. Furthermore, the use of a dynamic increase factor for the bending moment capacity is an idealisation in the SDOF model because the strain rate will vary throughout the response. In addition, the theoretical assumption of a perfectly plastic resistance function is naturally idealised because the static tests showed slight increases in strength during their plastic response. However, this behaviour can be modelled in an SDOF analysis by introducing a corresponding increase in resistance. These effects together with natural variations in material strength and specimen cross sections, as well as the vertical position of the tensile reinforcement, will give rise to scatter between theoretical calculations and test results.

In most cases there is a relatively good agreement between the deflection registrations and the calculated curves when considering the peak deflection and rise time. However, the theoretical curves exhibit a post-peak response with both higher frequencies and larger amplitudes in relation to the registrations, see Figure 4.9. This is simply due to the fact that no damping was introduced in the calculations and that the stiffness was left unaffected throughout the whole response, which is not the case for a real beam as stated above. The stiffness is greatly affected by the amount of cracking and damage. However, more experiments would be necessary to more exactly determine the parameters in the theoretical model for other types of beams and support conditions.

The duration of the triangular load was approximately the same as the natural period of vibration of the SDOF system, which categorises the response as dynamic according to section 3.3.3. This can also be observed from the combinations of pressure and impulse densities obtained in the tests. The analyses by using iso-damage curves show tendencies being conservative, i.e. the beams were in most cases stronger than predicted. The exception is beam type B100(16) where the data points with test values ended up to the left of each corresponding iso-damage curve. The SDOF calculations also show lower deflections compared to the test values for this beam type, indicating that the beams had a softer response than expected, see Figure 4.7 and Figure 4.9. In some cases the beams exhibited considerably larger load capacities than expected by the iso-damage curves. The derivation of the iso-damage curves are derived from the assumptions of two extreme loads with the dynamic loading regime between these extremes. The deformations in this regime depend on the entire load history as well as the resistance and mass of the system. Thus, for more correct solutions, the assessment of the beam response may require a complete solution of the equation of motion as performed in the SDOF analyses. Also, the method used herein for generating the iso-damage curves may be 10–15 % conservative as stated by Granström (1958). Using a exponentially decaying load function results in damage curves with higher pressure and impulse densities, see Figure 3.24. This means that if the characteristic values of the pressure and impulse density are increased accordingly, these curves approach the test results. Nevertheless, iso-damage curves give a good graphical representation of the capacity of beams and other structural elements that are exposed to blast loads, which makes this method useful in performing parametric studies and vulnerability assessment. The use of iso-damage curves may also serve as a tool when determining the magnitude of the explosive load in the planning of air blast tests on structural elements.
The forces transmitted to the supports relate to the shear forces and stresses that appear in the element, which also gives input for the loading on adjacent elements in a structure. The air blast tests showed that relatively large support reactions were obtained before any noticeable deflections were reached suggesting that the beam will initially be subjected to large shear forces. Thus, a premature flexural shear failure may be initiated early in the response of the beam with a subsequent catastrophic failure. The results from the calculations of the support reactions is in relatively good agreement with the general shape of the registered reactions and seem to represent the mean values of the dynamic reactions. Furthermore, the tests showed that as similar beams were exposed to an increasingly larger blast load the maximum dynamic reactions did not change, on the condition that the beams obtained sufficiently large deformations so that a plastic hinge formed. The beams did not transfer any more load to the supports. The larger blast load was instead absorbed by larger plastic deformations of the beam as shown in Figure 5.2. The simplified bilinear resistance function used in the SDOF analyses then appears to be a good approximation of the real beam response. The experiments also show that the dynamic reactions increased for stiffer beams subjected to similar loads as softer beams, indicating higher shear forces in stiff beams, as previously discussed in this section.

A striking mass or a blast loads at close range to a wall gives rise to bending moment waves and shear waves of large amplitudes, propagating from the loading point towards the supports. This may result in early local failures before any significant deformations in the element have taken place. The equivalent SDOF system is generated by the use of conversion factors, which are evaluated on the basis of an assumed deformed shape of the element in question as discussed in section 3.3.2. This shape is usually taken from the static deformation mode and, therefore, the effects of wave propagation can not be accounted for by using an SDOF model. This model may be unsuitable for prediction of element response to impulsive loading because wave propagation effects become significant. The prediction of the maximum shear force at the supports may be very inaccurate for impulsive loads as stated by van Wees and Peters (1995). Finite element analyses may instead be a much more suitable tool to use in these cases.

Figure 5.2 Dynamic reaction-deflection curves from the air blast tests with beams of type B140F (Magnusson and Hallgren, 2003).
6 Conclusions and further research

6.1 Conclusions

Reinforced beams of high strength concrete (HSC) subjected to static loading exhibited an enhanced stiffness and load capacity compared to similar beams of normal strength concrete (NSC). The deformation capacity and ductility seem to be independent of concrete strength as the mechanical reinforcement ratio $\omega_s$ is kept constant. Thus, it is possible to obtain ductile structural elements of HSC with proper reinforcement detailing. The ductility of the HSC beams could be increased further by adding steel fibres, which enhanced the toughness of the compression zone during the compressive failure. This prevented the sudden drop in bending moment capacity, which otherwise occurred as beams without fibres failed. The presence of fibres also resulted in an enhanced residual strength of the beams, which had previously been damaged by blast loads.

In general, beams of HSC exhibited higher resistance against blast loading compared to NSC beams. All reinforced concrete beams subjected to static loading failed in flexure, whereas similar beams could fail in flexural shear in the air blast tests. A suddenly applied dynamic load with large amplitude which can excite vibration modes of higher order than the first mode in the beam introduces large shear forces in the element. This may in some cases lead to flexural shear failures and this is especially the case for stiff elements. A soft element responds by deflecting more readily than the stiffer counterpart and the shear forces are thereby limited. Hence, whereas a soft element fails by large deformations in flexure a stiff element may fail in a brittle shear mode for the same load intensity. Furthermore, the presence of steel fibres in the concrete increased the shear strength of the beams and the development of flexural shear cracks was thereby prevented.

The tests with steel fibre reinforced concrete (SFRC) beams clearly indicate that the fibre length should be reduced in combination with higher concrete strength and this is especially the case in dynamic events. Beams of NSC failed by fibre pull-out while a few beams of HSC partly failed by fibre ruptures. The bond between the steel fibres and the concrete matrix increases for an increasing concrete strength and as the beam is loaded dynamically the bond is enhanced even further. This may lead to unfavourable fibre ruptures. It may therefore be favourable to use shorter fibres with smaller aspect ratios in structural elements of HSC and subjected to large dynamic loads.

The theoretical studies show the abilities to predict the dynamic structural response of reinforced concrete beams when exposed to explosive loading with the use of the single-degree-of-freedom (SDOF) method. The calculated response agreed well with the experimental results regardless of concrete strength and reinforcement amount. The results of using iso-damage curves as used herein indicate conservative results on the ‘safe side’ with larger load capacities of the beams than expected. For more correct solutions in the dynamic loading regime, the assessment of the beam response possibly requires a complete solution of the equation of motion as performed in the SDOF analyses.
The forces transmitted to the supports during dynamic loading relate to the shear forces that appear in the element. The theoretical evaluations of the dynamic reactions were in agreement with the measured average reactions both in amplitudes and in general shape. The experiments also show that the dynamic reactions increase in a case where a stiffer beam is subjected to a similar load as a softer beam, which indicates higher shear forces in the stiff beam.

6.2 Further research

It has been shown herein that it is possible to predict the response of dynamically loaded simply supported reinforced concrete beams with the use of an SDOF system. There are however a number of unanswered questions regarding the response of other structural elements and entire structures.

The beams studied herein have a relatively small depth to width ratio and can therefore be considered as part of a slab or a wall. It is well known that the formal shear strength of a reinforced concrete beam is reduced with an increased beam height. The effect when the shear strength of a beam is reduced as the height is increased is commonly referred to as a size effect. Hillerborg (1978) also states that an increase in dimensions of a concrete beam is unfavourable in many respects. He reasons that also the flexural failure with concrete crushing in the compressive zone is size dependent. The average strains at failure should therefore be lower and the failure more brittle at increasing dimensions of the beam and thereby resulting in a reduced rotational capacity. Consequently, it is of interest to include studies on the dynamic capacity of HSC beams with a larger depth to span ratio at a constant span in future research. It is also of interest to study other elements such as slabs and columns with different boundary conditions. Studies on the residual strength should also be included. In this context it may also be valuable to expose already damaged beams to additional explosive loads.

In order to theoretically study the structural response under large plastic deformations, the use of finite element analysis is a powerful tool. This permits for detailed analyses of failure modes, amount of cracking of the concrete as well as strains and stresses in different parts of an element during the deformation event. In the numerical analyses, the influence of parameters can be considered such as concrete strength, amount and placement of reinforcement, boundary conditions and varying geometries of the cross section. Alternative dynamic load cases may also be studied such as a falling mass striking the element. When possible, SDOF analyses may also be used as a comparison to experiments and simulations.

In many cases an explosion generates fragments of different types and shapes from casings. The combined load by fragments and blast pressures on a concrete wall can result in considerably larger damage and the combined loading from steel fragments and blast waves of reinforced HSC beams or slabs are therefore of interest to investigate further.
Due to the large shear forces that are induced in structures during blast loads, it is interesting to highlight the criteria when there is a risk of shear failure modes appearing. This should be incorporated in further research.
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Appendix – Post-test photos of beams

Beams that failed in the static tests and one representative beam of each type that failed in the air blast tests are shown below, see further Magnusson and Hallgren (2000, 2003).

Static tests

Figure A1  Beams B40-S1 (bottom), B100-S1(16) (middle) and B200F-S1 (top).

Figure A2  Beam B100-S1(12) (bottom) and B150-S1 (top).

Figure A3  Beam B200/40-S1 (bottom) and B200-S1 (top).
Figure A5 From the top: Beams B140-S1, B140F-S1, B140F/40-S1, B200F/40-S1 and B200F/40-S1.
Air blast tests

Figure A6  Beam B40-D4.

Figure A7  Beam B100-D3(16) at the top and Beam B100-D3(12) at the bottom

Figure A9  Beams B140-D3 (top), B140F-D3 (middle) and B140F/40-D3 (bottom).
Figure A12  Beam B150-D3.

Figure A13  Beams B200-D1 (top), B200F-D5 (middle top), B200/40-D2 (middle bottom) and B200F/40-D3 (bottom).