Adaptation in Multiple Input Multiple Output Systems with Channel State Information at Transmitter

JINLIANG HUANG

Licentiate Thesis
Stockholm, Sweden 2007
Abstract

The thesis comprises two parts: the first part presents channel-adaptive techniques to achieve high spectral efficiency in a single user multiple-input multiple-output (MIMO) system; the second part exhibits a programmable and reconfigurable software-defined-radio workbench (SDR-WB) in the Matlab/Octave environment that accommodates a variety of wireless applications.

In an attempt to achieve high spectral efficiency, an adaptive modulation technique is applied at the transmitter to vary the data rate depending on the channel state information (CSI). To further enhance the spectral efficiency, adaptive power allocation schemes are applied in the spatial domain to adjust the power on every transmit antenna. We analyze several power control schemes subject to a peak power constraint to maximize the spectral efficiency given an instantaneous target bit-error-rate (BER). A novel power allocation strategy is proposed to achieve high spectral efficiency with relatively low complexity. In addition, adaptive techniques that switch across different MIMO schemes enables even higher spectral efficiency by choosing the scheme with the highest spectral efficiency for a given signal to noise ratio (SNR). We propose a new method to switch between spatial multiplexing with zero-forcing (ZF) detection and orthogonal space-time block coding (OSTBC). This is done by exploiting closed form expressions of the spectral efficiencies—discrete rate spectral efficiency (DRSE)—and finding the crossing point of the two curves of spectral efficiency. The proposed adaptation scheme adds little complexity to the transmitter since it requires only statistical information of the channel, which does not change as time evolves.

Software Defined Radio (SDR) has received more and more interest recently as a promising multi-band multi-standard solution for transceiver design. In order to support various wireless applications, we build a programmable and reconfigurable workbench, namely the SDR-WB, in Matlab/Octave environment. The workbench is functionally modularized into generic blocks to facilitate fast development and verification of new algorithms and architectures. The modulation formats that are currently supported by the SDR-WB are MIMO, Orthogonal frequency-division multiplexing (OFDM), MIMO-OFDM, DS-CDMA.

keywords: MIMO, adaptive modulation, power allocation, spectral efficiency, CSI
Acknowledgement

I would like to express my sincere gratitude to my advisor, Docent Svante Signell, for giving me a chance to be a graduate student in the Department of ECS and leading me into this wonderland of wireless communications, for his enlightening guidance and constant support during these years, especially when I experienced the toughness of setbacks somewhere along the line.

I owe many thanks to Prof. Mohammed Ismail, Prof. Lirong Zheng and Docent Ana Rusu for bringing me into the RaMSiS group. I gratefully acknowledge all my former and present colleagues in RaMSiS group and ECS lab, Dr. Zhonghai Lu, Dr. Steffen Albrecht, Dr. Xinzhong Duo, Dr. Wim Michielsen, Jad Atallah, Sleiman Bou Sleiman, Saul Rodriguez, Dr. Adam Strak, Majid Baghaei Nejad, Martin Gustafsson, Delia Rodriguez, Yajie Qin, Jun Zhu, Dr. Yiran Sun, Xiaolong Yuan, Roshan Weerasekera, for creating a pleasant working environment. Grateful acknowledgement to Jinfeng Du for all the inspiring suggestions and discussions in the work, and also for proofreading the thesis manuscript.

Many thanks to the Swedish Foundation for Strategic Research (SSF) for financially supporting me under the RaMSiS program.

With no less respect, I gratefully thank Lena Beronius, Robert Röngren, Agneta Herling for being patient with me and my long list of questions.

Also, I’m grateful to all my friends in Sweden, without them, life would be as dark as the winter of Stockholm.

Last, but not the least, I’m especially grateful to my parents for their encouragement and unconditioned support to me in all my decisions.
Notation and used symbols

Throughout this thesis, the following notations will be used:

- **x**        bold face lower-case letters denote column vectors
- **A**        bold face upper-case letters denote matrices
- **a_i**     the $i$th column vector of **A**
- **a_j**     the $j$th row vector of **A**
- **[A]_{ji}, A_{ji}**  the $(j, i)$th element of **A**
- **I**        the identity matrix
- **(·)_***   the complex conjugate transpose (Hermitian)
- **(·)̄**    the complex conjugate
- **(·)^T**   the transpose
- **∥x∥_2, ∥x∥** the Euclidean norm of **x**
- **∥A∥_F**   the Frobenius norm of **A**, $∥A∥_F^2 = \text{tr}(AA^*)$
- **vec(\(A\))** the vectorization operator, vec stacks the columns of **A** into a vector, i.e.

$$
\text{vec}(A) = \begin{bmatrix}
  a_1 \\
  \vdots \\
  a_n
\end{bmatrix}
$$
Notation and used symbols

\( A^\dagger \) the Moore-Penrose pseudoinverse of \( A \)
If the columns of \( A \) are linearly independent, then
\( A^\dagger = (A^*A)^{-1}A^* \)

\( A^{-1} \) the inversion of a non-singular square matrix, \( A^{-1}A = I \)

\( (x)_+ \) \( \max\{x,0\} \)

\( \text{tr} \) the trace of matrix

\( \text{diag}(x_1,\ldots,x_n) \) a diagonal matrix with \( x_1,\ldots,x_n \) on the main diagonal.

\( \mathbb{E}(\cdot) \) the expected value of a random variable

\( X \otimes Y \) the Kronecker product, \( X \otimes Y = \begin{bmatrix} X_{11}Y & \cdots & X_{1n}Y \\ \vdots & \ddots & \vdots \\ X_{m1}Y & \cdots & X_{mn}Y \end{bmatrix} \)

\( \mathcal{CN}(\mu,\sigma^2) \) the circularly symmetric complex Gaussian random variable with mean \( \mu \) and variance \( \sigma^2 \).

Here follows a list of some commonly used symbols in the thesis:

\( N_t \) the number of transmit antennas

\( N_r \) the number of receive antennas

\( N_{\text{min}} \) \( N_{\text{min}} = \min\{N_t, N_r\} \)

\( N_{\text{max}} \) \( N_{\text{max}} = \max\{N_t, N_r\} \)

\( N_s \) the number of singular value channels that are used to deliver data

\( P_T \) the total transmit power

\( \sigma_n^2 \) the variance of Gaussian white noise

\( \mathbf{H}_w \) the complex valued i.i.d. Rayleigh fading channel

\( K \) the Ricean \( K \)-factor

\( \rho_{tx}, \rho_{rx} \) the spatial correlation coefficients at the transmitter and the receiver
\( \gamma_0 \) the system SNR defined as \( \gamma_0 = \frac{P_T}{\sigma_n^2} \)

\( \lambda_i \) the \( i \)th eigenvalue of \( HH^* \)

\( \Gamma_k \) the SNR thresholds for adaptive modulation
Abbreviations and Acronyms

MIMO  Multiple Input Multiple Output
SISO  Single Input Single Output
OFDM  Orthogonal Frequency Division Multiplexing
DMMT  Discrete Matrix Multitone
CSI   Channel State Information
CSIT  Channel State Information at Transmitter
CSIR  Channel State Information at Receiver
CRSE  Average Continuous-rate Spectral Efficiency
DRSE  Average Discrete-rate Spectral Efficiency
SVD   Singular Value Decomposition
OSTBC Orthogonal Space-Time Block Coding
ZF    Zero Forcing
SDR-WB Software Defined Radio WorkBench
TAS   Transmit Antenna Selection
i.i.d. Independent and Identically Distributed
p.d.f. Probability Density Function
d.o.f. Degree Of Freedom.
Contents

Notation and used symbols vii

Abbreviations and Acronyms xi

Contents xii

List of Figures xiv

1 Introduction 1
   1.1 Background ........................................... 1
   1.2 Contributions and outline ............................ 4

2 Overview of MIMO Technology 7
   2.1 Signal model .............................................. 7
   2.2 MIMO schemes ............................................ 11
   2.3 Capacity .................................................. 14
   2.4 Conclusions .............................................. 15
   2.A Least square method for ZF ............................ 16

3 Adaptive schemes for MIMO systems with SVD 19
   3.1 Constant-power variable-rate techniques ............... 20
   3.2 Variable-power variable-rate techniques ................ 24
   3.3 Further discussion ....................................... 32
   3.4 Concluding Remarks ..................................... 34

4 Adaptive schemes for MIMO systems with OSTBC and ZF 37
   4.1 DRSEs of MIMO systems with OSTBC ........................ 38
   4.2 DRSEs of MIMO systems with ZF detection ............... 42
   4.3 A low complexity adaptation scheme ..................... 45
   4.4 Conclusions .............................................. 49
   4.A The p.d.f. of the effective SNR of OSTBC in correlated Rayleigh fading channel ......................... 50
List of Figures

1.1 Block diagram of MIMO systems ........................................................................ 2
1.2 A comparison of capacities with CSIT and without CSIT ............................ 3
2.1 Capacities of MIMO by using different schemes in $2 \times 2$ i.i.d. Rayleigh fading channel ............................................................ 14
3.1 A block diagram of adaptive MIMO ................................................................. 20
3.2 Discrete-rate spectral efficiencies achieved by svd and beamforming in $2 \times 2$ i.i.d. Rayleigh channel with target BER $10^{-3}$ .................................. 24
3.3 A comparison of the capacity and the spectral efficiencies ............................. 25
3.4 DRSEs using capacity-optimal WF and continuous-rate optimal WF in a $4 \times 4$ i.i.d. Rayleigh fading channel ....................................................... 27
3.5 Spectral efficiencies of power control schemes in a $4 \times 4$ i.i.d Rayleigh fading channel ................................................................. 31
3.6 Power allocated to the 1st singular value channel in $4 \times 4$ i.i.d. Rayleigh fading channel ........................................................................ 32
3.7 Spectral efficiencies of power control schemes in a $4 \times 4$ spatially correlated Rayleigh fading channel, with $\rho_{tx} = \rho_{rx} = 0.5$ ............................. 33
3.8 Spectral efficiencies of power control schemes in a $4 \times 4$ Ricean fading channel ........................................................................ 34
3.9 Comparison of greedy allocation and QoS-based WF in different channel conditions ................................................................. 35
4.1 A block diagram of adaptive MIMO switching between different schemes 38
4.2 spectral efficiencies achieved by OSTBC in $2 \times N_r$ i.i.d Rayleigh fading channel with $N_r = 2, 4, 6$ ....................................................... 40
4.3 spectral efficiencies achieved by OSTBC in $2 \times N_r$ spatially correlated Rayleigh fading channel with $\rho_{tx} = 0.5$ ............................. 42
4.4 spectral efficiencies achieved by ZF in $2 \times N_r$ i.i.d. Rayleigh fading channel with $N_r = 2, 4, 6$ ....................................................... 44
4.5 spectral efficiencies achieved by ZF in $2 \times N_r$ spatially correlated Rayleigh fading channel with $\rho_{tx} = 0.5$ ............................. 45
4.6 Switching point of ZF-OSTBC in a $2 \times 2$ i.i.d. Rayleigh fading channel 46
List of Figures

4.7 Spectral efficiencies achieved by static adaptation and dynamic adaption in $2 \times N_r$ i.i.d. Rayleigh fading channel ........................................ 47
4.8 Crossing points of the spectral efficiencies in $2 \times N_r$ i.i.d. Rayleigh fading channel ......................................................... 48
4.9 Spectral efficiencies achieved by OSTBC and ZF in $2 \times 2$ Rayleigh fading channel with different correlation coefficients .................... 49
4.10 Spectral efficiencies achieved by OSTBC and ZF in $2 \times 6$ Rayleigh fading channel with different correlation coefficients .................... 50
4.11 The p.d.f. of effective SNR when $\gamma_0 = 20$dB .............................. 51

5.1 Block diagram of the modularized workbench .................................... 56
5.2 File structure of the workbench ...................................................... 57
5.3 Flowcharts in the “run” phase ...................................................... 60
5.4 $4 \times 4$ BER in frequency-selective Rayleigh fading channel ............... 62
5.5 $4 \times 4$ throughputs in frequency-selective Rayleigh fading channel ...... 62
5.6 Number of bits loaded on sub-carriers .......................................... 63
5.7 $2 \times 2$ BER in flat Rayleigh fading channel .................................. 63
Chapter 1

Introduction

1.1 Background

Multiple antenna systems

In point-to-point wireless links, multiple-input multiple-output (MIMO) systems that utilize multiple antennas at transmitter and receiver can considerably increase link capacity as well as link reliability compared to conventional single-input single-output (SISO) systems [1–6]. The advantages come from spatial diversity which are provided by the multiple antennas together with the scattering environment surrounding the transmitter and the receiver. If the transmitter and the receiver are equipped with $N_t$ and $N_r$ antennas respectively, the number of scalar channels (sub-channels) that are created by singular value decomposition (SVD) [1] is

$$N_{\text{min}} = \min\{N_t, N_r\}$$  \hspace{1cm} (1.1)

in a rich scattering environment\(^1\). If separate streams of data are transmitted across the sub-channels or singular value channels, the link capacity achieved by having multiple antennas is $N_{\text{min}}$ times the capacity of SISO. This is called spatial multiplexing gain [1, 2, 7]. A definition of the spatial multiplexing gain is given in [8]:

$$r = \lim_{\text{SNR} \rightarrow \infty} \frac{R}{\log \text{SNR}},$$  \hspace{1cm} (1.2)

where $R$ is the data rate. On the other hand, one can deliver the same data stream on different antennas to explore high diversity gain [4, 5], which is defined as [8]:

$$d = -\lim_{\text{SNR} \rightarrow \infty} \frac{\log P_e}{\log \text{SNR}},$$  \hspace{1cm} (1.3)

\(^1\)otherwise, $N_{\text{min}} = \min\{N_t, N_r, N_p\}$, where $N_p$ is the number of multipath provided by the scatters [12].
where $P_e$ is error ratio. For instance, orthogonal space-time block codes (OSTBC) [4] can provide a diversity gain of 4 in a $2 \times 2$ system.

Furthermore, the spatial multiplexing gain and the diversity gain can be achieved simultaneously in a given channel, but there is a tradeoff between how much gain can one get [8,9].

A general block diagram of a MIMO system is illustrated in Figure 1.1, where MIMO encoder and MIMO decoder accommodate various MIMO schemes, such as SVD and OSTBC that were mentioned previously.

### Channel adaptive technologies

It is well-known that the channel capacity $C$ gives a threshold value that for any rate $R < C$, there exists at least one channel encoder and channel decoder that achieves arbitrarily small error probability. If channel state information (CSI) is available at the receiver and the transmitter does not know the channel information, it is best to distribute the transmit power $P_T$ equally among the antennas and the ergodic capacity is written as [1]:

$$C_{\text{CSIT}} = \mathbb{E} \left\{ \sum_{i=1}^{N_{\text{min}}} \log \left( 1 + \frac{P_T \lambda_i}{N_i \sigma_n^2} \right) \right\},$$

where $\lambda_i$ is the $i$th eigenvalue of $\mathbf{HH}^*$, $\sigma_n^2$ is the noise power. On the other hand, if the CSI is also available at the transmitter (CSIT), the optimal power allocation can be derived by applying the well-known waterfilling. Let’s assume that the channel coherence time is larger than the interval of updating CSI at the transmitter, hence the transmitter has perfect CSI and the power allocated on every sub-channel is adjusted based on the instantaneous CSIT. Then the ergodic capacity can be written as [1]:

$$C_{\text{CSIT}} = \mathbb{E} \left\{ \sum_{i=1}^{N_{\text{min}}} \log \left( 1 + \frac{P_T \lambda_i}{\sigma_n^2} \right) \right\},$$

Figure 1.1: Block diagram of MIMO systems
1.1 Background

![Graph showing comparison of capacities with CSIT and no CSIT](image)

Figure 1.2: A comparison of capacities with CSIT and without CSIT

where $P_i$ is the power allocated on the $i$th sub-channel obtained by using the well-known waterfilling:

$$P_i = \left( \xi - \frac{\sigma_n^2}{X_i} \right)_+, \quad (1.6)$$

where $(x)_+ = \max\{x, 0\}$ and $\xi = \frac{1}{\mu \ln 2}$ is the “water-level” that is given by the criterion

$$\sum_{i=1}^{N_{\text{max}}} P_i = P_T. \quad (1.7)$$

The improvement by having the CSIT is shown in Figure 1.2, in which an evident enhancement is shown at low SNRs.

Unfortunately, these capacities are not achievable in practice due to that they assume Gaussian signals and infinite coding length for the channel encoder. To get a close performance to the committed capacities shown in Figure 1.2, the transmitter must assign a proper rate and suitable amount of power on every sub-channel based on the CSIT, e.g., sub-channel gains and noise variance. Furthermore, a powerful channel coding scheme have to be applied, e.g., LDPC code. In this work, we do not consider the channel coding, but focus on adaptive modulation and power schemes.
Introduction

Besides the rate and power adaptation, transmitter can also select a suitable MIMO scheme based on the CSIT to obtain either higher rate or lower error ratio \cite{61-63}.

1.2 Contributions and outline

In this thesis, we aim at maximizing spectral efficiency for a point-to-point wireless link equipped with multiple antennas. Adaptive modulation and adaptive power control schemes are considered and a novel diversity/multiplexing switching method is proposed to further enhance the spectral efficiencies.

This dissertation is organized into six chapters. In more detail, the outline of each chapter is as follows:

Chapter 2

We present the baseband signal models as well as the channel models for both narrowband MIMO systems and wideband MIMO-OFDM systems. Three MIMO schemes are briefly reviewed, i.e., SVD, spatial multiplexing with ZF detection and OSTBC.

Part of the material was submitted to


Chapter 3

In this chapter, adaptive modulation and adaptive power control schemes are reviewed. A novel power allocation is suggested in the context of peak power constraint and instantaneous BER constraint, which exhibits near-optimal performance with relatively low computational complexity.

Most of the material was published in


Chapter 4

This chapter presents DRSEs of ZF and OSTBC, based on which an original low complexity adaptation scheme is suggested to switch between ZF and OSTBC in an attempt to explore high spectral efficiency.

The results of algorithm switching in an i.i.d. Rayleigh fading channel were published in
1.2 Contributions and outline


When the channel is spatially correlated Rayleigh fading, the results were submitted to


The results of DRSE in both i.i.d. Rayleigh fading channel and spatially correlated Rayleigh fading channel can be extended to the case with arbitrary number of receive antennas, which were submitted to


Chapter 5

A reconfigurable software-defined-radio workbench (SDR-WB) in Matlab/Octave environment is presented and it supports diverse wireless link applications, e.g. OFDM, MIMO, MIMO-OFDM, WCDMA.

Part of the workbench architecture was published in


Part of the simulation results were published in


Chapter 6

This chapter summaries the results of this dissertation and points out the possible improvements as well as several open problems for future work.
Chapter 2

Overview of MIMO Technology

Digital communication using multiple antennas has been receiving much attention recently [1–4] due to its substantial benefits on spectral efficiency and link reliability. A number of schemes have been suggested for MIMO systems and they mainly fall into two categories: spatial multiplexing-based [1, 2] and diversity-based [4, 5]. The spatial multiplexing-based schemes, e.g. VBLAST\(^1\) [2], SVD [1], are highly spectrum efficient: they take advantage of spatial diversity of MIMO channels and creates parallel sub-channels over which separate data streams can be transmitted; whereas diversity-based algorithms dedicate to build up channels with high diversity gain, e.g. OSTBC [4, 5]. A hybrid class of scheme that trades off between spatial multiplexing gain and diversity gain is also available, e.g. double space-time transmit diversity (D-STTD) [9].

2.1 Signal model

Narrowband

In a point to point narrowband MIMO system with \(N_t\) transmit antennas and \(N_r\) receive antennas, the channel is assumed to be flat fading. The discrete-time baseband equivalent signal model can be written as:

\[
y(n) = H(n)x(n) + z(n)
\]

(2.1)

where \(H(n)\) is an \(N_r \times N_t\) complex-valued channel matrix. \(x(n)\) is an \(N_t \times 1\) transmitted vector at time \(n\) subject to a peak power \((P_T)\) constraint:

\[
\sum_{i=1}^{N_t} |x_i|^2 \leq P_T.
\]

(2.2)

\(^1\)Vertical Bell Laboratories Layered Space-Time Architecture
Overview of MIMO Technology

\( y(n) \) is a \( N_r \times 1 \) received vector. \( z(n) \) is the vector of additive white Gaussian noise with covariance \( \sigma_n^2I_{N_r} \). The signal-to-noise ratio (SNR) is defined as:

\[
\gamma_0 = \frac{P_T}{\sigma_n^2}
\]  

(2.3)

Wideband

Orthogonal Frequency Division Multiplexing (OFDM) [10] can be employed in MIMO systems to increase the data rate by allowing data transmission over a wide range of bandwidth [11]. Let \( M \) denotes the number of sub-carriers of one OFDM symbol, then \( M + \nu \) samples are transmitted at each antenna during one OFDM symbol, where \( \nu \) is the length of cyclic prefix. The \( N_t \times (M + \nu) \) transmitted samples for the \( n \)th OFDM symbol can be stacked as follows:

\[
x_T(n) = [x_T^1(n), \ldots, x_{N_t}^T(n)],
\]

where

\[
x_T^i(n) = \left[ x_i(nM - \nu + 1), \ldots, x_i(nM - M + 1), \ldots, x_i(nM) \right]_x = \tilde{x}_o^i(n),
\]

\( \tilde{x}_o \) is a replica of the last \( \nu \) samples of \( x_o \). Identically, the output samples of the channel are stacked as

\[
y_T(n) = [y_T^1(n), \ldots, y_{N_t}^T(n)],
\]

where \( y_T^i(n) \) is the received signal for the \( n \)th OFDM symbol at the \( i \)th receive antenna. The channel can be expressed as a vector equation:

\[
y(n) = H(n)x(n) + z(n),
\]

(2.4)

where the spatio-temporal channel matrix is composed of \( (M + \nu) \times (M + \nu) \) SISO subblocks

\[
H = \begin{bmatrix}
H_{1,1} & \cdots & H_{1,N_t} \\
\vdots & \ddots & \vdots \\
H_{N_t,1} & \cdots & H_{N_t,N_t}
\end{bmatrix} \in \mathcal{C}^{(M+\nu) \times (M+\nu) \times N_t}.
\]

(2.5)

If the multipath delay spread of the channel is smaller than the length of cyclic prefix, the spatio-temporal channel can be reduced to \( M \) non-interfering parallel flat fading channels by using Discrete Matrix Multitone (DMMT) [12]. The simplified channel may be expressed as:

\[
y_m(n) = H_m(n)x_m(n) + z(n), \quad m = 1, \ldots, M
\]

(2.6)
2.1 Signal model

where

\[ x_m = [x_1(nM - M + m), \ldots, x_{N_t}(nM - M + m)] \]

is the data vector transmitted on the \( m \)th frequency bin, \( \mathcal{H}_m \) is an \( N_r \times N_t \) space-frequency channel evaluated on the \( m \)th frequency bin and \( y_m \) is the received data on this frequency. With the DMMT approach, any MIMO schemes suggested for narrowband systems can now be extended to MIMO-OFDM systems by applying the coding in space-frequency domain \([12, 13]\).

In this dissertation, we will mainly focus on narrowband MIMO systems since an OFDM system can be easily converted into a group of parallel narrowband systems.

Channel model

A number of channel models for narrowband and wideband wireless links have been discussed in many works \([12, 14, 15]\) and \([16]\) gives a complete review of the channel models.

Flat fading channel

There are three types of channel under consideration in this thesis: independent and identically distributed (i.i.d.) Rayleigh fading, spatially correlated Rayleigh fading and Ricean fading channel.

Let us denote by \( \mathbf{H}_w \) the i.i.d. Rayleigh fading channel, the entries of which are assumed to be independent zero mean unit variance circularly symmetric complex Gaussian random variable, i.e., \([\mathbf{H}_w]_{ij} \sim \mathcal{CN}(0, 1)\).

In spatially correlated Rayleigh fading channel, the fading correlation \( \mathbf{R} \) can be evaluated by:

\[ \mathbf{R} = \mathbf{E} \{ \text{vec} (\mathbf{H}) \text{vec}^*(\mathbf{H}) \} . \tag{2.7} \]

The correlation may be separated into two parts \([16]\): the transmitter spatial correlation \( \mathbf{R}_t \) and the receiver spatial correlation \( \mathbf{R}_r \), which are evaluated by:

\[ \mathbf{R}_t = \mathbf{E} \left\{ (\mathbf{h}^j)^T \mathbf{h}^j \right\} , \quad 1 \leq j \leq N_r \tag{2.8} \]

and

\[ \mathbf{R}_r = \mathbf{E} \{ \mathbf{h}_i^j \mathbf{h}_i^j \} , \quad 1 \leq j \leq N_t \tag{2.9} \]

respectively. \( \mathbf{h}_i \) is the \( i \)th column of \( \mathbf{H} \) and \( \mathbf{h}_i^j \) is the \( j \)th row of \( \mathbf{H} \). They are related to \( \mathbf{R} \) by a Kronecker product:

\[ \mathbf{R} = \mathbf{R}_t \otimes \mathbf{R}_r . \tag{2.10} \]

Then the channel model is written as:

\[ \mathbf{H} = \mathbf{R}_r^{1/2} \mathbf{H}_w \mathbf{R}_t^{1/2} . \tag{2.11} \]
Overview of MIMO Technology

$(\cdot)^{1/2}$ denotes any matrix square root that $R^{1/2}(R^{1/2})^* = R$. Furthermore, the transmitter and the receiver spatial correlation can be modeled by correlation coefficients $\rho_{tx}$ and $\rho_{rx}$ as:

$$[R_{t}]_{ij} = \rho_{tx}^{i-j},$$

$$[R_{r}]_{ij} = \rho_{rx}^{i-j}. \quad (2.12)$$

A semi-correlated fading channel, e.g., a Rayleigh fading channel with spatial correlation at the transmitter only, can be obtained by setting $\rho_{rx} = 0$.

In case there exists a line-of-sight (LOS) component between the transmitter and the receiver, the channel is Ricean distributed [23]:

$$H = \sqrt{\frac{K}{K+1}} a_{tx}(\theta_{tx}) a_{rx}^*(\theta_{tx}) + \sqrt{\frac{1}{K+1}} H_w, \quad (2.14)$$

where the first term corresponds to the LOS component with an angle of departure $\theta_{tx}$ and an angle of arrival $\theta_{rx}$. $a_{tx}(\theta_{tx})$ is the normalized transmit antenna array response vector due to the angle of departure, and $a_{rx}(\theta_{rx})$ is the normalized receive antenna array response vector due to the angle of arrival. $K$ is the Ricean $K$–factor defined as the ratio of deterministic to scattered power. The second term is the non-line-of-sight (NLOS) part representing the elements of scattering component, which can be assumed to be i.i.d. Raleigh fading in case of a rich scattering environment.

Frequency-selective fading channel

A wideband communication system is usually subject to frequency-selective fading channel, which can be modeled by a tapped-delay-line model with each tap representing a multipath component [17]. The channel on each tap is flat fading and can be modeled by any of the statistical models mentioned in the preceding section.

An alternative approach is to generate a spatio-temporal representation of the channel as $(2.5)$, which can be obtained by using a ray-based discrete spatio-temporal channel model [18].

$$H = \sum_{l=1}^{N_r} \beta_l A_{rx}(\theta_{rx,l}) \cdot G_l \cdot A_{tx}^T(\theta_{tx,l}) \quad (2.15)$$

where

$$A_{rx} = \begin{bmatrix}
a_{rx,1}(\theta_{rx,l})I_{(M+\nu)} \\
\vdots \\
a_{rx,N_r}(\theta_{rx,l})I_{(M+\nu)}
\end{bmatrix} \quad (2.16)$$

and

$$A_{tx} = \begin{bmatrix}
a_{tx,1}(\theta_{tx,l})I_{(M+\nu)} \\
\vdots \\
a_{tx,N_t}(\theta_{tx,l})I_{(M+\nu)}
\end{bmatrix}. \quad (2.17)$$
2.2 MIMO schemes

\( a_{tx,j}(\theta_{tx,l}) \) is the \( j \)th transmitting antenna gain response due to the angle of departure of multipath \( l \). \( a_{rx,i}(\theta_{rx,l}) \) is the \( i \)th receiving antenna gain response due to the angle of arrival of multipath \( l \). \( I_{(M+\nu)} \) is an \( M + \nu \) identity matrix. \( \beta_l \) is the complex gain of multipath \( l \), \( N_P \) is the number of resolvable multipaths with different delays.

\[
G_l = \begin{bmatrix}
0 & \cdots & \cdots & \cdots & 0 \\
1 & 0 & \cdots & \cdots & 0 \\
0 & 1 & 0 & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & 1 & 0
\end{bmatrix}
\]

is the \( (M + \nu) \times (M + \nu) \) multipath delay matrix with all ones on the main diagonal or the \( d \)th lower subdiagonal. \( d \) is the quantised delay defined as \( d = \left\lfloor \frac{\tau_l}{T_s} \right\rfloor \) where \( \tau_l \) is the relative delay of multipath \( l \) and \( T_s \) is the sample period, which is 50nS in case of WLAN 802.11a.

Furthermore, to introduce time correlation to the channel coefficients, Jake’s model [19] or modified Jake’s model [20] can be employed to generate a series of time-correlated gains for each multipath component \( \beta_l \) and the time-correlated channel coefficients are obtained by substituting \( \beta_l \) into (2.15).

2.2 MIMO schemes

Spatial multiplexing based schemes transmit separate streams of data across multiple antennas. At the receiver, there exist several decoding schemes, e.g., joint maximum likelihood (ML) [25], sphere decoding [26], linear receiver with zero-forcing (ZF) [7, 25], linear receiver with minimum mean-square error (MMSE) [7, 25], successive interference cancellation (SIC) with ZF [7, 25], SIC with MMSE\(^2\) [7, 25] and etc. Additionally, if precoding is feasible at the transmitter, SVD can be adopted to convert the interfering MIMO channel into a set of parallel sub-channels, or singular value channels, over which separate data streams are transmitted.

In diversity based schemes, structured codes are applied across space and time domain [4, 5, 24] to combine the diversity gains provided by the two dimensions. Alternatively, the codes can be employed across the space and frequency domain [13] to combine the diversity gains from space and frequency in MIMO-OFDM systems. In narrowband systems, the most frequently used space-time codes are Alamouti codes [4] for two transmit antennas and generalized space-time block codes for three or four transmit antennas [5].

Linear receiver with ZF detection

We can write the channel matrix \( H \) in (2.1) column-wisely:

\[
H = [h_1, h_2, \ldots, h_N]
\]

\(^2\)this is also referred to as VBLAST.
Overview of MIMO Technology

where \( h_i \) is the \( i \)th column of \( H \). Then the received signal can be rewritten as:

\[
y = h_i x_i + \sum_{j \neq i} h_j x_j + z
\]

which can be viewed as the sum of the desired signal \( x_i \) together with the interferences \( x_j \) and the noise. To extract \( x_i \) out of the received signal, \( y \) is projected onto a subspace orthogonal to the one spanned by the vectors \( h_1, \ldots, h_{i-1}, h_{i+1}, \ldots, h_N \). The linear operation of the projection can be represented by a \( d_i \times N_r \) matrix \( Q_i \), where \( d_i \) is found to be \( N_r - N_i + 1 \) \([7]\). The resulting signal after the projection is:

\[
y_i = Q_i y = Q_i (h_i x_i + \sum_{j \neq i} h_j x_j + z)
\]  \hspace{1cm} (2.19)

Then Maximum Ratio Combining (MRC) is used to maximize the received SNR:

\[
(Q_i h_i)^* Q_i y_i = \|Q_i h_i\|^2 x_i + (Q_i h_i)^* Q_i w_i
\]  \hspace{1cm} (2.20)

It is known that this is equivalent to the least square solution \([7]\)

\[
x = H^* y = (H^* H)^{-1} H^* y,
\]  \hspace{1cm} (2.21)

for an overdetermined system

\[
y = H x
\]

where \( H \) is an \( N_r \times N_t \) matrix with \( N_r \geq N_t \). The proof of equivalence is provided in Appendix 2.A.

The SNR of the \( i \)th stream can be obtained from (2.20)

\[
\text{SNR}_i = \frac{P_r \|Q_i h_i\|^2}{N_t \sigma_n^2}.
\]  \hspace{1cm} (2.22)

If the channel is i.i.d. Rayleigh fading, then \( \|Q_i h_i\|^2 \) is Chi-square distributed with degrees of freedom (d.o.f) equal to \( 2(N_r - N_i + 1) \). The capacity that can be achieved by using ZF detection is written as:

\[
C_{zf} = E \left\{ \sum_{i=1}^{N_t} \log \left( 1 + \frac{\gamma_0}{N_t} \|Q_i h_i\|^2 \right) \right\},
\]  \hspace{1cm} (2.23)

**Singular value decomposition**

The channel matrix \( H \) can be decomposed by using SVD:

\[
H = U \Lambda V^*
\]  \hspace{1cm} (2.24)
2.2 MIMO schemes

where U and V are unitary matrices, Λ is an \(N_r \times N_t\) matrix with \(\sqrt{\lambda_1}, \ldots, \sqrt{\lambda_{N_{\min}}}\) on the diagonal and zeros otherwise. \(\lambda_i\) are the eigenvalues of \(HH^*\) and \(N_{\min} = \min\{N_t, N_r\}\).

By premultiplying with \(V\) at the transmitter and postmultiplying with \(U^*\) at the receiver, see (2.25), the channel is converted into a set of parallel sub-channels, coined as singular value channels.

\[
\hat{s} = U^* y = U^* Hv s + \bar{w} = \Lambda s + \bar{w} \tag{2.25}
\]

Then the effective SNR on the \(i\)th singular value channel is given as:

\[
\gamma_i = \frac{P_T \lambda_i}{N_{\min} \sigma_n^2} \tag{2.26}
\]

The diversity gain on the \(i\)th singular value channel is derived in [27, 28] as:

\[
d_i = (N_t - i + 1)(N_r - i + 1). \tag{2.27}
\]

The capacity by using SVD is:

\[
C_{\text{svd}} = E \left\{ \sum_{i=1}^{N_{\min}} \log \left( 1 + \frac{\gamma_i \lambda_i}{N_{\min}} \right) \right\}, \tag{2.28}
\]

To further enhance the capacity, waterfilling can be adopted to calculate the power to be assigned on every singular value channel and the resulting capacity is given by (1.5). Application of the waterfilling power allocation in adaptive modulation systems is investigated in Chapter 3.

**OSTBC**

Full diversity full rate OSTBC is only available for MIMO systems with two transmit antennas, where the symbols to be transmitted are structured as a 2×2 block [4]:

\[
\begin{bmatrix}
    s_1 \\
    s_2
\end{bmatrix}
\]

where \(x^*\) represents the complex conjugate of \(x\). The signals in the first row are the symbols to be transmitted through antenna 1 in two consecutive time slots, and the second row contains the data for antenna 2. The channel is assumed to be constant over two symbol intervals. Then the relationship of received signal \(y\) and symbols \(s\) can be written as [22]:

\[
\begin{bmatrix}
    y_1 \\
    y_2
\end{bmatrix} = \begin{bmatrix}
    h_1 \\
    h_2
\end{bmatrix} \begin{bmatrix}
    s_1 \\
    s_2
\end{bmatrix} + \begin{bmatrix}
    z_1 \\
    z_2
\end{bmatrix} \tag{2.29}
\]
where $\mathbf{h}_i$ is an $N_r \times 1$ vector denoting the channel coefficients of transmit antenna $i$ to all receive antennas and $\hat{\mathbf{h}}_i$ is the complex conjugate of $\mathbf{h}_i$. At the receiver of OSTBC, MRC is applied to maximize the received SNR:

$$\hat{s} = \hat{\mathbf{H}}^* \hat{\mathbf{y}} = \begin{bmatrix} \|\mathbf{H}\|_F^2 & 0 \\ 0 & \|\mathbf{H}\|_F^2 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \hat{\mathbf{H}}^* \tilde{\mathbf{z}}$$

(2.30)

where $\| \cdot \|_F$ is the Frobenius norm. The effective SNR $\gamma$ after the MRC is:

$$\gamma = \frac{\gamma_0 \|\mathbf{H}\|_F^2}{2}.$$ 

(2.31)

The diversity gain is $2N_r$ for every symbol [4] and the capacity achieved by OSTBC is

$$C_{ostbc} = \mathbb{E} \left\{ \log \left( 1 + \frac{\gamma_0}{2} \|\mathbf{H}\|_F^2 \right) \right\},$$

(2.32)

2.3 Capacity

The capacities achieved by using various MIMO schemes are shown in Figure 2.1. It is observed that the performances of different MIMO schemes differ significantly

Figure 2.1: Capacities of MIMO by using different schemes in $2 \times 2$ i.i.d. Rayleigh fading channel
from each other. Similarly, the practical spectral efficiencies that can be achieved in real systems differ from each other. This motivates our research work on how to adapt the transmission schemes to maximize the spectral efficiency. The results shown in Figure 2.1 provide us two hints on how to do this:

1. adopt power allocation at the transmitter to vary the transmit power across multiple antennas,

2. switch across different MIMO schemes based on the crossing point of the spectral efficiencies achieved by the candidate schemes.

The adaptive power allocation requires instantaneous CSIT, which fits in the structure of SVD algorithm, where the instantaneous CSI is available at the transmitter either through a feedback channel in Frequency Division Duplex (FDD) systems or by estimation in receive mode in Time Division Duplex (TDD) systems. Throughout this literature, we assume CSI is obtained at the transmitter via a feedback channel in FDD mode. On the other hand, we can switch between the diversity based scheme and the spatial multiplexing based scheme to make benefits from both algorithms. For instance, the crossing point of the spectral efficiencies of ZF and OSTBC can be used as the switching point between ZF and OSTBC. Furthermore, the crossing point is a function of the channel statistical information which does not change as time elapses, so the selection of the MIMO scheme is done once and for all.

2.4 Conclusions

In this chapter, we presented the base-band signal models for both narrowband MIMO systems and wideband MIMO-OFDM systems. The associated channel models for flat fading and frequency-selective fading environment are studied. Since MIMO-OFDM can be decomposed into a set of interference-free narrowband MIMO subblocks, we restrict our work to narrowband MIMO systems.

Several MIMO schemes were reviewed, i.e., SVD, OSTBC, Spatial multiplexing with ZF receiver, and they exhibited distinct performances in terms of capacity, which motivated our work in finding an adaptation technique based on CSI to achieve the maximal spectral efficiency.
Appendix

2. A Least square method for ZF

An over-determined system is written as:

\[ y = Hx \]  

(2.33)

where \( H \) is an \( N_r \times N_t \) matrix with \( N_r \geq N_t \). The least square method is to find an estimate \( \hat{x} \) that minimizes

\[ \xi = \{ \| y - H\hat{x} \| ^2 \}. \]  

(2.34)

This is solved by multiplying by a pseudoinverse matrix:

\[ \hat{x} = H^\dagger y = (H^*H)^{-1}H^*y \]  

(2.35)

In this section, it is shown that the pseudoinverse of \( H \) (2.35) is equivalent to the filter’s solution in (2.20).

Without loss of generality, we look at the \( i \)th filter’s coefficients:

\[ c_i^* = (Q_i h_i)^* Q_i = h_i^* Q_i^* Q_i, \]  

(2.36)

where \( Q_i \) is composed of orthonormal basis \( \{ q_i^T, q_i^2, \ldots, q_i^T \} \), which spans the subspace \( S^\perp \) that is orthogonal to the one spanned by the vectors

\[ \{ h_1, \ldots, h_{i-1}, h_{i+1}, \ldots, h_{N_t} \}. \]

Note that \( Q_i^* Q_i \) is both hermitian and idempotent: i.e. \((Q_i^* Q_i)^* = (Q_i^* Q_i)^2 = Q_i^* Q_i \), it can be viewed as a projection matrix onto subspace \( S^\perp \), with the following properties:

1. \( Q_i^* Q_i h_i = h_i \)

2. \( Q_i^* Q_i h_j = 0 \) where \( j = 1, \ldots, i-1, i+1, \ldots, N_t \)

By stacking the coefficients \( c_i^* \):

\[ C = \begin{bmatrix}
    h_1^* Q_1^* Q_1 \\
    \vdots \\
    h_{N_t}^* Q_{N_t}^* Q_{N_t}
\end{bmatrix} \]

the product of \( C \) and \( H \) are:

\[ CH = \begin{bmatrix}
    h_i^* Q_i^* Q_i \\
    \vdots \\
    h_{N_t}^* Q_{N_t}^* Q_{N_t}
\end{bmatrix} [h_1, \ldots, h_{N_t}] \]  

(2.37)

\[ = \begin{bmatrix}
    \| h_1 \|^2 & \cdots & 0 \\
    \vdots & \ddots & \vdots \\
    0 & \cdots & \| h_{N_t} \|^2
\end{bmatrix} \]
2. A Least square method for ZF

With properly chosen scaling factors for each row of $C$, i.e. $1/||h_i||^2$, we have $CH = I$ and hence

$$C = H^\dagger$$

Due to the uniqueness of pseudoinverse matrix,

$$C = (H^*H)^{-1}H^*$$
Chapter 3

Adaptive schemes for MIMO systems with SVD

Adaptive transmission schemes that adjust the transmission parameters relatively to the time-varying channel enable robust and spectrally efficient data transmission over the wireless fading channel. The idea of channel-adaptive transmission is to feedback the CSI from the receiver to the transmitter, then the transmission parameters are adjusted based on the feedback information with respect to the channel conditions. This technique was first investigated in [36], but it was short-lived maybe due to hardware constraints. Then it was re-visited in [37], where variable-rate, variable-power was suggested to approach channel capacity over a Rayleigh fading channel. So far, adaptive transmission schemes have been suggested for both SISO [37–40] and MIMO applications [35, 41–45, 47–49, 52] and they mainly fall into three categories [35]:

- maximize the link spectral efficiency with fixed BER performance subject to a total power constraint [35, 41–45, 47, 48],
- minimize BER with fixed rate subject to a total power constraint [35, 49, 50],
- minimize transmit power with fixed rate and fixed BER [51, 52].

Throughout this work, we target to maximizing the spectral efficiency while keeping BER under a predefined level.

In adaptive transmission technology, the adaptation of parameters is based on the CSI, which can be either fed back to the transmitter in frequency division duplex (FDD) systems or can be estimated in the receiver mode in time division duplex (TDD) systems. We assume that channel estimation is perfect and there is no feedback delay or error. Thus the instantaneous channel information is available at the transmitter and it can be employed to adapt the parameters for every channel realization. This is applicable for a slowly fading wireless channel where the coherence time is larger than the feedback delay. In practice, however, due
Figure 3.1: A block diagram of adaptive MIMO

to imperfect channel estimation and feedback, the CSIT obtained at the transmi-
ter is noise contaminated [54–56], and partial CSI can be employed to control the
parameters [57, 58].

We start our investigation by assuming the rate on every singular value chan-
nel is variable but the power is constant; then different power control policies are
introduced to further increase the potential rate achievable in multi-antenna sys-
tems. A block diagram of the adaptive system is shown in Figure 3.1. Both the
modulation order and the transmit power are controlled by the link adaptation unit
through a feedback channel. In order to minimize the data transmitted over the
feedback channel and the resulting bandwidth, the modulation order and the power
are calculated in the link adaptation unit and directly fed back to the transmitter.

3.1 Constant-power variable-rate techniques

As suggested in [37, 39], the rate on every scalar channel is adjusted by applying
adaptive modulation that dynamically determines the constellation size depending
on the channel gain. In the following, we will review adaptive modulation with con-
tinuous rate, i.e., the modulation order can take any nonnegative real value. Then
we will look into the discrete rate case, where the modulation order is restricted to
some certain integers. The reason for studying the continuous rate modulation is
that it upper bounds the performance of the practical modulation schemes and it
is more analytically tractable due to the rate continuity.
3.1 Constant-power variable-rate techniques

Continuous rate
In [37], the relationship of the effective SNR (\(\gamma\)), BER (\(P_b\)) and the constellation size (\(M_k\)) of MQAM modulation for coherent detection with Gray bit mapping is approximated as:

\[
P_b \approx 0.2 \exp \left( -1.5 \frac{\gamma}{M_k - 1} \right),
\]

(3.1)

which is tight within 1 dB when \(M_k \geq 4\) and \(P_b \leq 10^{-3}\).

\(M_k\) can be rewritten as a function of \(P_b\) and \(\gamma\) based on (3.1). If we assume each of the MQAM symbol uses a Nyquist data pulse, i.e. \(B = 1/T_s\), where \(B\) is the signal bandwidth and \(T_s\) is the symbol duration. Then the continuous-rate spectral efficiency (CRSE) is derived as:

\[
\text{CR} = E \{ \log M_k \} = E \{ \log(1 + g_o \gamma) \},
\]

(3.2)

where

\[
g_o = \frac{-1.5}{\ln(5P_b)}
\]

is the SNR gap [12] between the CRSE and the Shannon capacity. This gap arises from the discrepancies in the coding length and the signal space. Infinite channel coding length and Gaussian signals are assumed in Shannon theory whereas uncoded QAM modulation is used here.

Furthermore, the continuous-rate adaptive modulation scheme can be applied independently on every singular value channel and the ergodic CRSE of MIMO with SVD is:

\[
\text{CR}_{svd} = E \left\{ \sum_{i=1}^{N_{\min}} \log \left( 1 + \frac{g_o \gamma_i A_i}{N_{\min}} \right) \right\},
\]

(3.3)

Recall the ergodic capacity in (2.28), an analogy can be drawn between the capacity and the CRSE, with the only difference given by the SNR gap \(g_o\).

Discrete rate
Now, we consider the situation in a real system that the modulation orders can only be drawn from a finite set of integers, e.g., \(M_k = \{0, 1, 2, 4, \ldots \}\). Let \(A_k\) denotes the SNR region for \(M_k\)-QAM, i.e., the \(M_k\)-QAM is selected if the effective SNR falls within \(A_k\). The SNR region \(A_k\) is defined as \(A_k = \{ x : \Gamma_k \leq x < \Gamma_{k+1} \}\) where \(\Gamma_k\) denote the region boundaries or SNR thresholds. The key point of the issue is to determine the SNR thresholds and relate it to the desired BER.

In general, there exist two BER constraints for adaptive modulation systems, i.e., average BER constraint [40]¹:

\[
\text{BER}_t \leq \text{BER}_i
\]

(3.4)

¹It is originally defined for rate adaptation in SISO, we extend it to MIMO application by assuming that the same target BER applies for every sub-channel
and instantaneous BER constraint [40]:

\[ \text{BER}_i \leq \text{BER}_t, \]  

(3.5)

where BER\(_i\) is the predefined target BER. BER\(_i\) and BER\(_t\) are the average and instantaneous BER on the \(i\)th singular value channel, respectively. With average BER constraint, the average BER over a sufficient number of channel realizations must be limited by the target BER. The instantaneous BER implies that the BER for every transmission must be subject to the target BER, which is obviously more restrictive than the average BER constraint and the corresponding spectral efficiency is certainly lower than the spectral efficiency achieved by applying average BER constraint. For instantaneous BER constraint, the SNR thresholds can be obtained from (3.1), i.e.,

\[ \Gamma_k = \frac{M_k - 1}{1.5} \ln \frac{1}{5 \text{BER}_t} \]  

(3.6)

For average BER, the optimal solution for the SNR thresholds are hard to find and a suboptimal solution was suggested in [40], where the SNR thresholds from (3.6) were scaled by a properly selected factor such that the average BER constraint (3.4) is satisfied. Considering the computational complexity, we only focus on instantaneous BER constraint hereafter.

However, we should notice that the adaptive modulation scheme with the SNR thresholds suggested for the instantaneous BER can achieve an error ratio that is lower than the target BER. This is because for every \(\gamma\) that falls in \((\Gamma_k, \Gamma_{k+1})\), the actual BER is lower than the target BER.

By imposing the SNR thresholds (3.6) to every singular value channel of MIMO system with SVD, the discrete-rate spectral efficiency (DRSE) can be written as:

\[
\text{DR}_{\text{svd}} = \mathbb{E} \left\{ \sum_{i=1}^{N_{\text{min}}} r_i \right\} = \sum_{i=1}^{N_{\text{min}}} \mathbb{E} \left\{ \mathcal{Q}(\gamma_i|\Gamma) \right\} 
= \sum_{i=1}^{N_{\text{min}}} \left( \sum_{k=1}^{K} d_k \cdot \int_{\Gamma_k}^{\Gamma_{k+1}} p(\gamma_i) d\gamma_i \right),
\]

(3.7)

where \(r_i\) is the rate allocated to singular value channel \(i\). \(\mathcal{Q}(\gamma_i|\Gamma)\) is the slicing function that outputs \(r_i\) with input \(\gamma_i\) conditioned on the modulation order thresholds, \(\Gamma = \{\Gamma_1, \Gamma_2, \ldots, \Gamma_K\}\). \(d_k = \log_2 M_k\) is the number of bits assigned when the effective SNR falls in the interval: \([\Gamma_k, \Gamma_{k+1})\). \(p(\gamma_i)\) is the p.d.f. of \(\gamma_i\).

Let’s assume that the channel is i.i.d. Rayleigh fading, i.e., \(H = H_w\). Then the joint distribution function of the eigenvalues \((\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{N_{\text{min}}} )\) of matrix \(HH^*\) is given in [4] as:

\[
p(\lambda_1, \lambda_2, \ldots, \lambda_{N_{\text{min}}}) = C \prod_{i=1}^{N_{\text{min}}} e^{-\lambda_i} \lambda_i^{N_{\text{max}} - N_{\text{min}}} \prod_{i<j} (\lambda_i - \lambda_j)^2
\]

(3.8)
3.1 Constant-power variable-rate techniques

where

\[
C = \frac{\pi^{N_{\text{min}}(N_{\text{min}} - 1)}}{\Gamma_{N_{\text{min}}}(N_{\text{max}}) \Gamma_{N_{\text{min}}}(N_{\text{min}})}
\]

with

\[
\Gamma_{N_{\text{min}}}(l) = \pi^{N_{\text{min}}(N_{\text{min}} - 1)/2} \prod_{i=1}^{N_{\text{min}}}(l - i)!
\]

and \(N_{\text{max}} = \max\{N_{t}, N_{r}\}\). In a 2 × 2 system, the marginal p.d.f. of \(\lambda_1\) and \(\lambda_2\) can be obtained from (3.8). Further, \(p(\gamma_i)\) can be derived based on the marginal p.d.f.:

\[
p(\gamma_1) = \left(\frac{8\gamma_1^2}{\gamma_0^2} - \frac{8\gamma_1}{\gamma_0} + 4\right) e^{-\frac{2\gamma_1}{\gamma_0}} - \frac{4}{\gamma_0} e^{-\frac{\gamma_1}{\gamma_0}}
\]

(3.9)

\[
p(\gamma_2) = \frac{4}{\gamma_0} e^{-\frac{4\gamma_2}{\gamma_0}}
\]

(3.10)

By plugging the p.d.f.s into (3.7), we get the DRSE of SVD with equal power allocation as a function of \(\gamma_0\):

\[
\text{DR}_{\text{svd}}(\gamma_0) = \sum_{k=1}^{K} \Delta d_k e^{-\frac{2\gamma_k}{\gamma_0}} \left(\frac{2\Gamma_k}{\gamma_0}\right)^2 + 2,
\]

(3.11)

where \(\Delta d_k = d_k - d_{k-1}\) and \(d_0 = 0\).

Similarly, the DRSE of BF by utilizing only one singular value out of two is:

\[
\text{DR}_{\text{bf}}(\gamma_0) = \sum_{k=1}^{K} \Delta d_k e^{-\frac{2\gamma_k}{\gamma_0}} \left[\frac{\Gamma_k}{\gamma_0}^2 + 2 - e^{-\frac{\gamma_k}{\gamma_0}}\right].
\]

(3.12)

The DRSEs as well as the empirical spectral efficiencies attained from the Software-Defined-Radio Workbench (SDR-WB) [32] are shown in Figure 3.2. The result at each point is averaged over 10,000 channel realizations and the target BER is set to 0.1%. It is observed that the DRSEs match with the empirical results very well. Furthermore, we observe that BF outperforms SVD in the low SNR region. This is due to that the 2nd singular value channel is rather weak in the low SNR region, where it is optimal to put all of the transmit power onto the stronger one (as suggested by waterfilling). However, as SNR increases, the spatial multiplexing gain plays a more important role and SVD achieves a higher spectral efficiency.

DRSEs of SVD for MIMO systems with more than 2 antennas are provided in [43]. Nevertheless, all of the results shown here and from [43] are obtained based on the assumption of i.i.d. Rayleigh fading channel and to the best knowledge of author, the DRSE in case of spatially correlated Rayleigh fading channel is still unsolved.

A comparison is made between the capacity and the corresponding CRSE, DRSE of an uncoded adaptive modulation \(^2\) 4 × 4 MIMO system in Figure 3.3, where the

\(^2\)the available constellation sizes of MQAM modulation are \(\{2, 4, 16, 64\}\).
channel is assumed to be i.i.d. Rayleigh fading and the results are averaged over 10,000 channel realizations. As expected, the SNR gap between the CRSE and the capacity is a constant that depends on the target BER only, which, in our case, is about 5.5 dB for a target BER 0.1%. This coincides with the theoretical result:

\[ 10 \log_{10} \frac{-1.5}{\ln(5 \text{BER}_t)} \]

On the other hand, there exists a gap between the CRSE and DRSE due to that discrete rate is used in practice.

### 3.2 Variable-power variable-rate techniques

Thus far, adaptive modulation is applied independently across the transmit antennas assuming equal power allocation. Now, we take into account adaptive power allocation to further enhance the spectral efficiency.

It is well-known that the channel capacity of MIMO system is maximized by utilizing waterfilling power allocation [1] if full CSI is available at the transmitter. It is of great interest to examine the performance of waterfilling-based power allocation schemes in practice.
3.2 Variable-power variable-rate techniques

![Graph showing capacity and spectral efficiencies vs SNR](image)

Figure 3.3: A comparison of the capacity and the spectral efficiencies

In case of continuous-rate MQAM modulation under instantaneous BER constraint and average power constraint, the optimum power allocation can be calculated by Lagrange’s multiplier [35] and the solution is similar to the waterfilling that maximizes channel capacity. However, the optimality of waterfilling does not hold for the discrete-rate modulation case. Table 3.1 summarizes the optimal solutions for power and SNR thresholds using discrete-rate QAM signals under different constraints. By imposing the average constraints on BER and power, the optimal solution for power control and a suboptimal solution for SNR thresholds are derived in [40], although it was designed for SISO channels using temporal adaptation, it can be extended to MIMO applications based on the unordered eigenvalue distribution [43]. Substituting instantaneous BER constraint for average BER constraint, the problem is more tractable and closed form expressions were obtained for the power and the SNR thresholds [43]. However, As far as we know, the optimal solution subject to an average BER constraint and peak power constraint is not available. In this chapter, we investigate more practical control schemes subject to instantaneous BER constraint and peak power constraint. Although the optimal solution is found to be an exhaustive search [35], a variety of other power control schemes have been suggested to reduce the computational complexity, e.g., QoS-based waterfilling [44], greedy allocation [53] and uniform power allocation with transmit antenna selection (TAS).
Adaptive schemes for MIMO systems with SVD

<table>
<thead>
<tr>
<th></th>
<th>average power</th>
<th>peak power</th>
</tr>
</thead>
<tbody>
<tr>
<td>average BER</td>
<td>Lagrangian [40]</td>
<td>?</td>
</tr>
<tr>
<td>instantaneous BER</td>
<td>Lagrangian [43]</td>
<td>exhaustive search [35]</td>
</tr>
</tbody>
</table>

*where the optimal power allocation is obtained, but optimal solution for SNR thresholds is not available.

Table 3.1: Optimal power control schemes for discrete-rate QAM signals

Note that the results in Table 3.1 do not hold for spatial multiplexing algorithms, in which the signal to interference and noise power ratio (SINR) after the data streams are separated is not proportional to the corresponding transmit power and this non-linearity leads to a more complicated problem \[45,46\].

**Power control policies in discrete-rate modulation scheme**

Subject to the instantaneous BER constraint, the SNR thresholds of the modulation order regions are given by (3.6). Then the optimization problem of power control is stated as:

\[
\max_{P_i} \sum_{i=1}^{N_{\text{min}}} \sum_{k=1}^{K} d_k \int_{\Gamma_k} p(\gamma_i(P_i)) d\gamma_i(P_i)
\]

subject to \(\sum_{i=1}^{N_{\text{min}}} P_i \leq P_T\)

with \(P_i \geq 0, i = 1, 2, \ldots, N_{\text{min}}\)

The optimization involves a mixture of discrete and continuous variables for which no closed form expression exists. However, we can use exhaustive search to find the optimal power allocation together with associated bit-loading that maximizes the spectral efficiency \[35\].

In order to reduce the complexity while achieving a relatively good performance as exhaustive search, several other power control schemes are considered here, e.g., direct waterfilling \[12\], QoS-based waterfilling \[44\], greedy allocation \[53\] and uniform power allocation with TAS.

Unlike the case of constant power allocation, closed form expression of the DRSE using adaptive power allocation is not available due to the intractability of \(P_i\).

**A direct waterfilling method**

The optimal power allocation policy in case of continuous-rate is shown to be the waterfilling method \[12,35\]:

\[
P_i = \left( \xi - \frac{\sigma_n^2}{g_0 \lambda_i} \right)_+, \quad i = 1, 2, \ldots, N_s
\]

(3.14)
3.2 Variable-power variable-rate techniques

![Graph showing capacity-optimal and continuous-rate optimal DRSEs](image)

Figure 3.4: DRSEs using capacity-optimal WF and continuous-rate optimal WF in a 4 × 4 i.i.d. Rayleigh fading channel

where \( \xi \) is chosen to satisfy the peak power constraint. The number of non-zero power allocation is \( N_s \leq N_{\min} \). This policy is different from (1.6) by including a noise-amplifying factor \( g_o \) where \( g_o < 1 \). This is intuitively reasonable because there exists an SNR gap between the CRSE and the capacity and this gap can be viewed as a result of noise amplification. The improvement in DRSE by taking into account the noise amplification is shown in Figure 3.4. By applying power allocation given by (3.14), the resulting effective SNR is written as:

\[
\gamma_i = \frac{P_i \lambda_i}{\sigma_n^2}, \quad i = 1, \ldots, N_s \tag{3.15}
\]

and the bit-loading is obtained by:

\[
r_i = Q(\gamma_i | \Gamma), \quad i = 1, \ldots, N_s. \tag{3.16}
\]

QoS-based waterfilling method

In a sense, the direct waterfilling is essentially an SNR quantization where the transmit power may not make the possibly maximal contribution. A modified waterfilling method, referred to as QoS-based WF, is proposed in [44] to enhance the spectral efficiency on basis of the direct waterfilling. The idea of it is to execute
QoS-based waterfilling

1. Calculate the residual power allocated to every singular value channel:
   \[
   \Delta P_i = P_i - \bar{P}_i
   \]
   where \( P_i \) is the initial power calculated from waterfilling (3.14) and
   \[
   \bar{P}_i = \frac{\sigma^2 \Gamma_k}{\lambda_i}
   \]
   is the desired power that achieves the SNR threshold \( \Gamma_k \). Then the sum of the residual power is:
   \[
   P_r = \sum_{i=1}^{N_s} \Delta P_i
   \]

2. Let \( \Delta P = P_r, \ i = 1 \)

3. Calculate the possibly maximal rate that can be achieved with \( \Delta P \) on the \( i \)th singular value channel:
   \[
   \hat{r}_i = Q \left( \frac{(\bar{P}_i + \Delta P)\lambda_i}{\sigma^2 n |\Gamma|} \right)
   \]
   if \( \hat{r}_i > r_i \), \( \Delta P = \Delta P - \frac{\sigma^2 \Gamma_k}{\lambda_i} \), where \( \hat{\Gamma}_k \) corresponds to the SNR threshold for the updated modulation order.

4. \( i = i + 1 \), if \( i \leq N_s \), go to step 3; otherwise, go to step 5

5. The end.

Table 3.2: QoS-based waterfilling

the direct waterfilling in the first step, and then collect the residual power from each singular value channel that is not used to improve the rate. Lastly, re-distribute the residual power on all singular value channels to achieve potential increase in data rate. A detailed procedure is shown in Table 3.2.

Based on the previous discussions, there are two versions of waterfilling, one is capacity-optimal, the other is continuous-rate optimal, both of which can be adopted here, but the performances are almost the same due to the re-distribution.

Greedy power allocation

The execution of QoS-based waterfilling can be divided into two steps: the first step is to calculate the ideal power allocation over all singular value channels; the
3.2 Variable-power variable-rate techniques

Greedy power allocation

1. Let $\Delta P = P_T$ and $i = 1$

2. Find the maximum rate $r_i$ that the singular value channel $i$ can support with $\Delta P$:

$$r_i = Q\left(\frac{\Delta P \lambda_i}{\sigma^2_n \Gamma} \right)$$

3. Calculate the desired power to achieve the rate $r_i$:

$$P_i = \frac{\sigma^2_n \Gamma_{\min}}{\lambda_i}$$

where $\Gamma_{\min}$ is the SNR threshold for the modulation order $r_i$. Re-calculate the residual power:

$$\Delta P = \Delta P - P_i$$

4. $i = i + 1$, if $i \leq N_{\min}$ and $\Delta P > 0$, go to step 2; otherwise, go to step 5

5. The end.

Table 3.3: Greedy power allocation

The second step is to redistribute the residual power. Notice that power allocation at the first step is optimal only under the assumption of Gaussian signal or continuous-rate QAM modulation. Therefore, an alternative strategy, referred to as greedy allocation, is proposed by considering this discrete effect on the power allocation from the beginning.

The idea of greedy power allocation is to assign as much power as possible onto a singular value channel to achieve the possibly maximal constellation size. Considering that:

$$\lambda_1 > \lambda_2 > \ldots > \lambda_{N_{\min}},$$

the power allocation can be done in an order from the strongest singular value channel ($i = 1$) to the weakest one ($i = N_{\min}$). The process is illustrated in Table 3.3.

From computer simulations, the CPU time of greedy power allocation is about 56% of that of the QoS-based waterfiling and the complexity is significantly reduced.

Uniform power allocation with TAS

Although greedy power allocation has considerably lowered the computational complexity, the power allocation needs to be calculated for every transmit antenna. To
1. Initialize the number of selected tx: $N_s = N_{\text{min}}$

2. calculate the effective SNR on the smallest singular value channel:

$$\gamma_{N_s} = \frac{P_T \lambda_{N_s}}{N_s \sigma^2_s}$$

if the achievable rate of the smallest singular value channel:

$$r_i = Q(\gamma_{N_s} | \Gamma) > 0$$

go to step 4, otherwise go to step 3

3. Reduce the number of selected antennas:

$N_s = N_s - 1$. If $N_s > 0$, go to step 2, otherwise go to step 4

4. The end.

Table 3.4: Uniform power allocation with TAS

further simplify the computation, the transmit antennas with strong gains are selected and the power is then uniformly distributed among the selected antennas. The criterion for antenna selection is that the effective SNR on every singular value channel must be larger than the smallest SNR threshold $\Gamma_1$ and the process is presented in Table 3.4.

Other suboptimal and low-complexity power allocation strategies, like channel inversion (CI) and truncated channel inversion (TCI), are proposed in [21]. With CI, the power is poured on every sub-channel to maintain the same gain on every sub-channel. The most important advantage of CI is that the effective SNR is identical for all sub-channels and a uniform constellation size is applied to all sub-channels, which simplifies the code design as well as the decoding process. However, CI is subject to substantial rate loss when the sub-channel is invertible, i.e., $\lambda_i \sim 0$. One way to solve this problem is by using TCI, where a cut-off level $\lambda_0$ is set for $\lambda_i$ so that CI is applied only for sub-channel $i$ whose $\lambda_i > \lambda_0$.

A comparison of power control schemes

We have reviewed four power allocation policies, direct waterfilling, QoS-based waterfilling, greedy power allocation and uniform power allocation with TAS. The performance of them are compared in three different channel environment: i.i.d. Rayleigh fading, spatially correlated Rayleigh fading and Ricean fading channel.

In a $4 \times 4$ i.i.d. Rayleigh fading channel, the spectral efficiencies achieved by utilizing different power control schemes are shown in Figure 3.5. It is observed that
3.2 Variable-power variable-rate techniques

![Graph showing spectral efficiencies of power control schemes](image)

Figure 3.5: Spectral efficiencies of power control schemes in a $4 \times 4$ i.i.d Rayleigh fading channel

the QoS-based waterfilling almost achieves the maximal spectral efficiency reached by using exhaustive search. Greedy allocation has a very close performance as QoS-based waterfilling in most cases. QoS-based waterfilling has about 2 dB gain over the direct waterfilling, which comes from the residual power re-distribution. Equal power allocation with TAS, on the other hand, obtain the same performance as QoS-based waterfilling in the low SNR region, but as SNR increases, it only attains similar performance as direct waterfilling. This is because only one singular value channel is activated at low SNRs, as the case in QoS-based waterfilling, whereas at high SNRs, direct waterfilling and uniform power allocation with TAS are asymptotic to equal power allocation.

Furthermore, the power allocated to the first singular value channel by using different power control schemes are shown in Figure 3.6. The behavior of greedy allocation follows that of QoS-based waterfilling in both low SNR region and high SNR region, except in the middle SNR region where greedy allocation allocates a higher portion of power than QoS-based waterfilling, which explains the discrepancy in spectral efficiency between the greedy allocation and QoS-based waterfilling. On the other hand, uniform power allocation with TAS behaves similarly to the direct waterfilling. As SNR increases, the waterfilling tends to allocate equal power on all sub-channels.
Figure 3.6: Power allocated to the $1^\text{st}$ singular value channel in $4 \times 4$ i.i.d. Rayleigh fading channel

In case there are not sufficient scatters around the transmitter and the receiver, correlation exists between any pair of the transmit and the receive antennas. Under this condition, the attainable spectral efficiencies of different power control schemes are shown in Figure 3.7. The spatial correlation is further emphasized by a LOS component in a Ricean fading channel and the performance is given in Figure 3.8. We notice that the discrepancy between the QoS-based waterfilling and the greedy allocation is vanishing as the spatial correlation increases.

A more noticeable performance contrast is made in Figure 3.9, where ratios of the spectral efficiency achieved by the greedy allocation and that of QoS-based waterfilling in different channel environment are plotted. As the spatial correlation increases, the performance of greedy allocation is approaching that of the QoS-based waterfilling, and even outperforms the latter one at some SNRs in highly spatially correlated environment.

3.3 Further discussion

Thus far, all of the power control schemes considered are assumed to have the same power “target” for all singular value channels. Specifically, the power allocated on the $i$th sub-channel, $P_i = h_i(\lambda_i)$, is independent of $i$, i.e., $\forall i, h_i \equiv h$. A new class of
multi-target power control schemes are proposed in [59], where different targets are applied to channels with different fading statistics, i.e., the power allocated onto sub-channel $i$ does not only depend on $\lambda_i$, but also on $h_i$ itself.

Waterfilling is claimed to be capacity-optimal under the assumption of Gaussian coding and infinite coding length. In practice, however, the M-PSK and M-QAM modulation are used instead of Gaussian signals and the waterfilling turns out not to be optimal any more, as shown in previous discussions. Although the impact caused by practical modulation schemes are partly compensated by the SNR gap $g_o$, a better solution that explores the optimal power allocation catering to M-QAM and M-PSK modulation is expected. By taking into account the exact modulation schemes, a mercury/waterfilling power control strategy is suggested in [60], where the gap between Gaussian signal and the practical modulation scheme on each sub-channel is filled by “mercury” first and “water (power)” is poured on top of the mercury. In case of Gaussian signals, the gap is zero and this method boils down to waterfilling. This method can also be regarded as a multi-target power allocation scheme: the gap between Gaussian codes and the desired modulation scheme depends on the exact modulation scheme and the modulation order used by the channel, which tends to differ from sub-channel to sub-channel, then the power targets of different sub-channels are different.

Figure 3.7: Spectral efficiencies of power control schemes in a $4 \times 4$ spatially correlated Rayleigh fading channel, with $\rho_{tx} = \rho_{rx} = 0.5$
Figure 3.8: Spectral efficiencies of power control schemes in a $4 \times 4$ Ricean fading channel

Although we assume perfect CSI at the transmitter and the receiver, the imperfection in channel estimation and feedback is inevitable in practice due to hardware constraints. This has been extensively studied in [43, 54, 56].

3.4 Concluding Remarks

In this chapter, we reviewed adaptive power adaptive rate techniques for high data rate transmission. In the case of constant power variable rate, the CRSE and DRSE were derived for continuous-rate QAM modulation and discrete-rate QAM modulation, respectively. DRSE was obtained based on the SNR thresholds that satisfied the instantaneous BER constraint. In case that the power and the rate are both adaptable, we concentrated on how to achieve a relatively good performance subject to a peak power constraint and an instantaneous BER constraint with comparably low complexity. Several power control policies were reviewed (direct waterfilling, QoS-based waterfilling and uniform power allocation with TAS) and a new power control method—greedy power allocation—was suggested, which reduced the computational complexity while achieving a comparably good performance compared to the QoS-based waterfilling.
3.4 Concluding Remarks

Figure 3.9: Comparison of greedy allocation and QoS-based WF in different channel conditions
Chapter 4

Adaptive schemes for MIMO systems with OSTBC and ZF

In order to achieve the possibly maximal spectral efficiency, adaptive modulation and adaptive power techniques are applied to approach the theoretical upper bound of a single MIMO scheme. On the other hand, a new adaptation policy to switch across different MIMO schemes have been suggested to further enhance the spectral efficiency promised by multiple antenna systems [62,63], where they switch between a diversity scheme and a spatial multiplexing scheme based on the BER performance to explore the highest spectral efficiency while maintaining a fixed BER. Alternatively, there exists another type of adaptation which employs diversity/multiplexing switching method in an attempt to minimize BER for fixed spectral efficiency [61].

In this chapter, we will consider applying two MIMO schemes in an adaptive modulation system: OSTBC and spatial multiplexing with ZF detection. To adopt adaptive modulation into the systems, the effective SNR of every sub-channel is estimated at the receiver and a proper modulation order is selected and fed back to the transmitter through a feedback channel in FDD mode. The variation rate of the fading channel is assumed to be lower than the link adaptation rate. Similarly to Chapter 3, closed form expressions of the DRSEs are obtained for OSTBC and spatial multiplexing with ZF. To improve the potential spectral efficiency, a low complexity adaptation scheme is proposed to switch between the diversity scheme and the spatial multiplexing scheme. Unlike [62] and [63] where the switching is based on the BER performance, we employ the DRSEs here to find out the switching point for the two schemes. The block diagram of the system is illustrated in Figure 4.1, where the modulation order is updated for each channel realization based on the instantaneous CSI. The scheme is selected based on the statistical information of the channel.
4.1 DRSEs of MIMO systems with OSTBC

Because OSTBC [4] is dedicated for two transmit antennas, we confine our work in this chapter to a $2 \times N_r$ MIMO system, where the channel environment is assumed to be either i.i.d. Rayleigh fading without spatial correlation or spatially correlated Rayleigh fading with correlation at transmitter only.

Similar to SVD, the DRSE of OSTBC can be evaluated by:

$$
\text{DR}_{\text{ostbc}} = \mathbb{E}\{r\} = \mathbb{E}\{Q(\gamma)\gamma\}
= \left(\sum_{k=1}^{K} d_k \cdot \int_{\Gamma_{k+1}}^{\Gamma_k} p(\gamma)d\gamma \right),
$$

(4.1)

where the key to evaluating DRSE is to find $p(\gamma)$, the p.d.f. of the effective SNR.

**DRSEs of OSTBC in i.i.d. Rayleigh fading channel**

Recall that the effective SNR for a $2 \times N_r$ i.i.d. Rayleigh fading channel is written as:

$$
\gamma = \frac{\gamma_0 \| H_w \|^2_F}{2},
$$

(4.2)

where $H_w$ denotes the i.i.d. Rayleigh fading channel. Therefore the distribution of $\gamma$ depends on the p.d.f. of $\| H_w \|^2_F$, which is Chi-square distributed with p.d.f. [7]:

$$
p(x) = \frac{1}{(L-1)!} x^{L-1} e^{-x}
$$

(4.3)
4.1 DRSEs of MIMO systems with OSTBC

Here $x = \|H_u\|_F^2$, $2L$ is the degree of freedom (d.o.f.), i.e., the number of independent real-valued random variables, and $L = 2N_r$ in this case. With simple transformations, $p(\gamma)$ can be derived from (4.3):

$$p(\gamma) = \frac{2}{\gamma_0 (2N_r - 1)!} \left( \frac{2\gamma}{\gamma_0} \right)^{2N_r - 1} e^{-\frac{2\gamma}{\gamma_0}}, \quad (4.4)$$

Then the DRSE of OSTBC can be derived by inserting (4.4) into (4.1):

$$\text{DR}_{\text{ostbc}}(\gamma_0) = \sum_{k=1}^{K} \Delta d_k e^{-\frac{2\gamma_k}{\gamma_0}} \sum_{j=0}^{2N_r-1} \frac{1}{(2N_r - j - 1)!} \left( \frac{2\Gamma_k}{\gamma_0} \right)^{2N_r - j - 1} \quad (4.5)$$

With the Taylor series expansion: $e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \ldots$, it is obvious that

$$\sum_{j=0}^{2N_r-1} \frac{1}{(2N_r - j - 1)!} \left( \frac{2\Gamma_k}{\gamma_0} \right)^{2N_r - j - 1} = e^{\frac{2\gamma_k}{\gamma_0}} - O \left( (2\Gamma_k/\gamma_0)^{2N_r} \right), \quad (4.6)$$

where $O \left( x^{2N_r} \right)$ denotes the terms of order higher than $2N_r$. Therefore, the DRSE is upper bounded by

$$\text{DR}_{\text{ostbc}} \leq \sum_{k=1}^{K} \Delta d_k e^{-\frac{2\gamma_k}{\gamma_0}} \left( e^{\frac{2\gamma_k}{\gamma_0}} - O \left( (2\Gamma_k/\gamma_0)^{2N_r} \right) \right)$$

$$< \sum_{k=1}^{K} \Delta d_k = d_K \quad (4.7)$$

The empirical spectral efficiencies of OSTBC and the corresponding DRSEs when $N_r = 2, 4, 6$ are shown in Figure 4.2, where the target BER is set to 0.1%. As expected, the DRSEs match very well with the empirical results that are obtained from the SDR-WB based on 10,000 channel realizations.

**DRSEs of OSTBC in spatially correlated Rayleigh fading channel**

In spatially correlated Rayleigh fading channel with correlation at the transmitter, the channel transfer function is written as:

$$\mathbf{H} = \mathbf{H}_u \mathbf{R}_t^{1/2}. \quad (4.8)$$

In case of two transmit antennas, the spatial correlation matrix $\mathbf{R}_t$ can be written as:

$$\mathbf{R}_t = \begin{bmatrix} 1 & \rho_{tx} \\ \rho_{tx} & 1 \end{bmatrix}, \quad (4.9)$$

which can be diagonalized as:

$$\mathbf{R}_t = \mathbf{U}_t \Omega_t \mathbf{U}_t^* \quad (4.10)$$
Adaptive schemes for MIMO systems with OSTBC and ZF

Figure 4.2: spectral efficiencies achieved by OSTBC in $2 \times N_r$ i.i.d Rayleigh fading channel with $N_r = 2, 4, 6$

where $U_t$ is a unitary matrix. $\Omega_t$ is a diagonal matrix with $\omega_1 = 1 + \rho_{tz}$ and $\omega_2 = 1 - \rho_{tz}$ on the main diagonal. The effective SNR is related to $H$ which is the product of $H_w$ and $R_t^{1/2}$. By substituting (4.8) into (2.31), the effective SNR given in (2.31) can be rewritten:

$$
\gamma = \frac{\gamma_0}{2} \| H \|_F^2
= \frac{\gamma_0}{2} \text{tr} \left( H_w R_t^{1/2} \left( H_w R_t^{1/2} \right)^* \right)
= \frac{\gamma_0}{2} \text{tr} \left( H_w U_t \underbrace{\Omega_t U_t^*}_{H_w} H_w^* \right)
= \frac{\gamma_0}{2} \text{tr} \left( H_w^* H_w \Omega_t \right)
= \frac{\gamma_0}{2} \sum_{i=1}^{2} \omega_i \| h_{\omega_i} \|^2
$$

(4.11)
4.1 DRSEs of MIMO systems with OSTBC

Here $\text{tr}(A)$ denotes the trace of $A$. The third equation comes from (4.10). The fourth equation follows from $\text{tr}(AB) = \text{tr}(BA)$. $h_{wi}$ is the $i$th column vector of $H_{w}$ and $\| h_{wi} \|_2$ is the Euclidean norm of it.

Because $U_t$ is a unitary matrix, the distribution of $H_w = H_w U_t$ is the same as $H_w$ [1]. Hence, $\| h_{wi} \|_2^2$ is Chi-square distributed with the d.o.f. equal to $2N_r$, whose p.d.f. is given in (4.3) with $L = N_r$:

$$ p(x_i) = \frac{1}{(N_r - 1)!} x_i^{N_r-1} e^{-x_i}, $$

where $x_i = \| h_{wi} \|_2^2$. Then $\gamma$ is the sum of two weighted Chi-square distributed variables:

$$ \gamma = \gamma_1 + \gamma_2 $$

where $\gamma_i = \gamma_i \omega_i / 2$ and it’s p.d.f. is given as follows:

$$ p(\gamma_i) = \frac{2N_r}{(N_r - 1)!} \left( \frac{\gamma_i}{\gamma_0 \omega_i} \right)^{N_r-1} e^{-\frac{\gamma_i}{\gamma_0 \omega_i}} $$

(4.12)

From the theory of probability [31], the p.d.f. of $\gamma$ is the convolution of the p.d.f. of $\gamma_1$ and $\gamma_2$, provided that $\gamma_1$ and $\gamma_2$ are independent variables:

$$ p(\gamma) = b e^{-\frac{a}{\omega_2^2}} \sum_{i=0}^{N_r-1} \frac{(-1)^{N_r-i} (N_r-1) \gamma_1^i}{a^{N_r-i+1}} \left[ e^{-a \gamma_1} \sum_{j=0}^{M_i} \mathcal{P}_m^{n_i} (a \gamma_1)^{M_i-j} - M_i! \right] $$

(4.13)

where $M_i = 2N_r - i - 2$ and

$$ a = 2 \left( \frac{1}{\omega_1} - \frac{1}{\omega_2} \right), $$

$$ b = \frac{4^{N_r}}{(N_r - 1)! (\gamma_0^2 \omega_1 \omega_2)^{N_r}}, $$

$$ \mathcal{P}_m^n = \frac{m!}{(m-n)!} \left[ 1 - \frac{m!}{(m-n)!} \right] $$

The proof can be found in Appendix 4.A. As $\rho_{tx} \to 0$, (4.13) approaches to the result of i.i.d. Rayleigh fading channel, as can be seen in Appendix 4.B. Furthermore, the DRSE of OSTBC with $N_r$ receive antennas can be derived by plugging
(4.13) into (4.1):

\[
\text{DR}_{\text{ostbc}}(\gamma_0, \rho_{tx}) = b \sum_{i=0}^{N_r-1} \frac{(-1)^{N_r-i}(N_r-1)_i}{a^{M_i+1}} \left[ \sum_{j=0}^{M_i} \mu_1^{M_i-j} \sum_{k=1}^{K} \Delta d_k e^{-\frac{\Delta d_i}{\mu_1}} + \sum_{l=0}^{M_i} \mathcal{P}^{i}_{M_j} \left( \frac{\Gamma_k}{\mu_1} \right)^{M_j-l} - \mu_2^{i+1} M_i! \sum_{k=1}^{K} \Delta d_k e^{-\frac{\Gamma_k}{\mu_2}} \sum_{l=0}^{i} \mathcal{P}^{i}_{l} \left( \frac{\Gamma_k}{\mu_2} \right)^{i-l} \right] \]

(4.14)

where \( M_j = 2N_r - j - 2 \) and \( \mu_i = \gamma_0 \omega_i / 2 \).

The DRSEs of OSTBC when \( N_r = 2, 4, 6 \) and the empirical results of spectral efficiencies obtained from simulation are shown in Figure 4.3, where we assume \( \rho_{tx} = 0.5 \) and the target BER is 0.1%. It is observed that the DRSEs coincide with the empirical results.

![Figure 4.3: spectral efficiencies achieved by OSTBC in \( 2 \times N_r \) spatially correlated Rayleigh fading channel with \( \rho_{tx} = 0.5 \)](image)

4.2 DRSEs of MIMO systems with ZF detection

By using linear ZF detection at the receiver, a group of non-interfering sub-channels are established at the cost of amplified noise power. Applying uncoded adaptive
4.2 DRSEs of MIMO systems with ZF detection

modulation on every such sub-channels, the DRSE can be computed as:

$$\text{DR}_{zf} = E \left\{ \sum_{i=1}^{N_t} r_i \right\} = E \left\{ \sum_{i=1}^{N_t} Q(\gamma_i | \Gamma) \right\}$$

$$= \sum_{i=1}^{N_t} \sum_{k=1}^{K} \Delta_k e^{-2\gamma_k \sigma^2_t / \gamma_0} \frac{\Gamma_{k+1}}{\Gamma_k} p(\gamma_i) d\gamma_i,$$

(4.15)

To be consistent with OSTBC, we restrict ZF to a $2 \times N_t$ system, in which $N_r \geq 2$ to guarantee the existence of solution for the linear system.

**DRSEs of Spatial-multiplexing with ZF receiver in i.i.d. Rayleigh fading channel**

It is known from [7] that the sub-channel gain $|Q(h_i)|^2$ (2.22) of the ZF receiver is a Chi-square distributed random variable in case of i.i.d. Rayleigh fading channel. Since the d.o.f. of the Chi-square distributed random variable is $2(N_r - N_t + 1) = 2(N_r - 1)$ [7]. The p.d.f. of $\gamma_i$ is:

$$p(\gamma_i) = \frac{2(2\gamma_i / \gamma_0)^{N_r-2}}{\gamma_0(N_r-2)!} e^{-2\gamma_i / \gamma_0},$$

(4.16)

Then the DRSE of ZF can be obtained as:

$$\text{DR}_{zf}(\gamma_0) = 2 \sum_{k=1}^{K} \Delta_k e^{-2\gamma_k \sigma^2_t / \gamma_0} \frac{\Gamma_{k+1}}{\Gamma_k} \frac{(2\gamma_k / \gamma_0)^{N_r-j-2}}{(N_r-j-2)!}$$

(4.17)

The DRSEs of ZF are shown in Figure 4.4, where both empirical results from computer simulation and the theoretical results (4.17) are plotted.

**DRSEs of Spatial-multiplexing with ZF receiver in spatially correlated Rayleigh fading channel**

In spatially correlated Rayleigh fading environment, the effective SNR on the $i$th sub-stream is distributed as [66]:

$$p(\gamma_i) = \frac{\sigma_i^2 e^{-\gamma_i \sigma_i^2 / \gamma_0}}{N_r! (N_r - N_t)!} \left( \frac{\gamma_i \sigma_i^2}{\gamma_0 / \gamma_t} \right)^{N_r-N_t}$$

(4.18)

where $\sigma_i^2$ is the $i$th diagonal entry of $\mathbf{R}_v^{-1}$. The DRSE can be obtained by substituting (4.18) into (4.15):

$$\text{DR}_{zf}(\gamma_0, \sigma^2_t) = \sum_{i=1}^{N_t} \sum_{k=1}^{K} \Delta_k e^{-2\gamma_k \sigma^2_t / \gamma_0} \frac{\Gamma_{k+1}}{\Gamma_k} \frac{1}{(N_r-l-2)!} \left( \frac{2\Gamma_k \sigma^2_t}{\gamma_0} \right)^{N_r-l-2}$$

(4.19)
Figure 4.4: spectral efficiencies achieved by ZF in $2 \times N_r$ i.i.d. Rayleigh fading channel with $N_r = 2, 4, 6$

In a $2 \times N_r$ Rayleigh fading channel with transmit spatial correlation given by (4.9), $\sigma_i^2 = 1/(1 - \rho_{tx}^2)$ for $i = 1, 2$. Then (4.19) can be rewritten as:

$$DR_{zf}(\gamma_0, \rho_{tx}) = 2 \sum_{k=1}^K \Delta d_k e^{-\frac{2\Gamma_k}{\gamma_0(1-\rho_{tx})}} \sum_{l=0}^{N_r-2} \frac{\left(\frac{2\Gamma_k}{\gamma_0(1-\rho_{tx})}\right)^{N_r-l-2}}{(N_r-l-2)!}$$

(4.20)

In the special case where there is no spatial correlation, $\rho_{tx} = 0$ and (4.20) reduces to (4.17). Therefore, (4.20) holds for:

$$0 \leq \rho_{tx} < 1.$$

However, it is not possible to apply the same extension to the DRSE of OSTBC because the derivation of the p.d.f. of the effective SNR in the case of spatially correlated channel assumes $\rho_{tx} \neq 0$ and $\rho_{tx} = 0$ would result in a different p.d.f.. This will be further discussed in Appendix 4.A.

As shown in Figure 4.5, the DRSE turns out to be an accurate estimation of the spectral efficiencies achieved in practice.
4.3 A low complexity adaptation scheme

In conventional adaptive systems [37,48], the channel is time-varying so that the constellation size has to be updated over time. The complexity of the system mainly lies in the link adaptation, where the constellation size is decided based on the estimated effective SNR and fed back to the transmitter through a feedback channel in an FDD system.

Besides adaptive modulation, the proposed adaptation scheme employs algorithm switching between OSTBC and ZF to further enhance the spectral efficiency. This is facilitated by the closed form expressions of the spectral efficiencies derived for OSTBC and ZF. Since the DRSE is a function of SNR and the spatial correlation coefficient (in case of spatial correlation) which are statistical information that would not change as time elapses, the selection of the optimal MIMO scheme only needs to be done once. This adds limited complexity to the current existing adaptive modulation system. Additionally, some extra space is needed to store all schemes, but only the one that is selected is activated.

With the closed form expressions of the spectral efficiency derived in (4.5) and (4.17) for i.i.d. Rayleigh fading channel and the target BER is set to be 0.1%, we
are able to find the switching point by solving the equation for $\gamma_0$:

$$ DR_{\text{ostbc}}(\gamma_0) = DR_{zf}(\gamma_0). \quad (4.21) $$

Unfortunately, the equation is intractable and only numerical results can be found by using Newton-Raphson method [67], which are listed in Table 4.1.

Furthermore, the decrease of the target BER results in a raise of the switching point, as shown in Figure 4.6. This is due to the fact that ZF has a higher spatial multiplexing gain and the rate is more affected by the decreased target BER.

The selection of the optimal algorithm depends on the SNR only, not the exact channel realization, which can be regarded as static adaptation. On the other hand, if we compute the constellation size by assuming OSTBC and ZF, respectively, and choose the one with higher modulation order for every channel realization, a higher spectral efficiency is obtained as shown in Figure 4.7. This can be viewed as dynamic adaptation. The gap between static adaptation and dynamic adaptation is maximally 2dB which happens at the switching point when $N_r = 2$. This is due
4.3 A low complexity adaptation scheme

to the fact that the two schemes have similar performance and it really depends on the instantaneous channel gain to decide which one is better. At low SNRs, dynamic adaptation has almost the same spectral efficiency as static adaptation since OSTBC outperforms ZF most of the time. The situation is similar at high SNRs where ZF is favored. However, the price we have to pay for dynamic adaptation is a higher system complexity. As the number of receive antennas increases, the difference between the two adaptation strategies is vanishing to negligibly small. This is because OSTBC and ZF have similar performance around the switching point in the $2 \times 2$ case, the benefit we can get from dynamic adaptation is maximal due to the uncertainty. As $N_r$ increases, the performance of OSTBC differs distinctly from ZF around the switching point, as can be seen in Figure 4.8, and the benefit of adaptation becomes negligible.

![Graph showing spectral efficiencies for dynamic and static adaptation]

Figure 4.7: Spectral efficiencies achieved by static adaptation and dynamic adaptation in $2 \times N_r$ i.i.d. Rayleigh fading channel

If the channel is spatially correlated Rayleigh fading, the switching point can be derived from:

$$ DR_{ostbc}(\gamma_0, \rho_{tx}) = DR_{zf}(\gamma_0, \rho_{tx}). \tag{4.22} $$

The solutions for $\rho_{tx} = 0.5$ are derived numerically for $N_r = 2, 4, 6$ in Table 4.2.

It has been shown in many contributions, e.g. [33, 34], that the spatial correlation has an effect on the capacity. As a result, the spectral efficiency also depends on the spatial correlation. The impact of spatial correlation on the spectral efficiency
is shown in Figure 4.9 and Figure 4.10 for $2 \times 2$ and $2 \times 6$ systems, respectively, where $\rho_{tx}$ is used to quantify the spatial correlation at the transmitter. It is noticed that OSTBC is more robust to the variation of the spatial correlation compared to ZF. The spectral efficiency of ZF degrades dramatically as $\rho_{tx}$ increases.

The idea of ZF is to project the received signal, consisting of two parts—the interference and the desired signal—onto a subspace that is orthogonal to the interference while keeping as much as possible of the desired signal. If high spatial correlation exists at the transmitter, it means the interference is aligned in a direction close to the desired signal and the effective SNR after the projection would inevitably suffer a lot. The detection of OSTBC, on the other hand, combines the received signal constructively to cancel out the interference, so there is no SNR degradation in the process. The theoretical p.d.f. of the effective SNR is illustrated in Figure 4.11 to present the SNR degradation due to spatial correlation. We notice that the shape and peak of the p.d.f. does not change much in case of OSTBC as the

<table>
<thead>
<tr>
<th>$N_r$</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_{sp}$</td>
<td>21.8dB</td>
<td>8.8dB</td>
<td>6.6dB</td>
</tr>
</tbody>
</table>

Table 4.2: SNR switching point in spatially correlated Rayleigh fading channel
4.4 Conclusions

Figure 4.9: Spectral efficiencies achieved by OSTBC and ZF in 2×2 Rayleigh fading channel with different correlation coefficients.

correlation coefficient increases, which means there is only a small variation of the average effective SNR. However, the SNR of ZF experiences serious degradation as the correlation coefficient increases.

4.4 Conclusions

Closed form expressions of the spectral efficiency of an uncoded adaptive modulation system, namely DRSE, are derived for OSTBC and spatial multiplexing with ZF receiver. By predefining the modulation scheme and the target BER, the DRSEs only depend on the SNR and possibly the spatial correlation coefficient in case of spatially correlated Rayleigh fading channel. the DRSEs of OSTBC are provided in (4.5) and (4.14) for a 2×N_R MIMO system in i.i.d. Rayleigh fading channel and spatially correlated Rayleigh fading channel, respectively. (4.17) and (4.20) present the DRSEs of ZF in i.i.d. Rayleigh fading channel and spatially correlated fading channel with transmit spatial correlation. Furthermore, the DRSEs of ZF receiver in the two types of channel can be generalized to (4.20) with 0 ≤ ρ_{tx} < 1.

To further enhance the spectral efficiency, a low complexity adaptation scheme that switches between OSTBC and ZF is suggested based on the DRSEs. The switching points for different antenna setups in different channel environment are
Adaptive schemes for MIMO systems with OSTBC and ZF

Figure 4.10: Spectral efficiencies achieved by OSTBC and ZF in $2 \times 6$ Rayleigh fading channel with different correlation coefficients

found numerically by the Newton-Raphson method.

Appendices

4.A The p.d.f. of the effective SNR of OSTBC in correlated Rayleigh fading channel

In this appendix, the p.d.f. of the effective SNR are derived for $2 \times N_r$ OSTBC in spatially correlated Rayleigh fading channel with spatial correlation on the transmitter side only.

Since $\gamma_1$ and $\gamma_2$ (4.2) are independent random variables, the p.d.f. of $\gamma$ is the convolution of $p(\gamma_1)$ and $p(\gamma_2)$, where $p(\gamma_1)$ is given by

$$p(\gamma_i) = \frac{2^{N_r}\gamma_i^{N_r-1}}{(N_r - 1)!(\gamma_0\omega_i)^{N_r}} e^{-\frac{\gamma_i}{\gamma_0\omega_i}}.$$ 

50
4.4 The p.d.f. of the effective SNR of OSTBC in correlated Rayleigh fading channel

![Graph of p.d.f. of effective SNR](image)

Figure 4.11: The p.d.f. of effective SNR when $\gamma_0 = 20$dB

The p.d.f. of $\gamma$ can then be obtained:

\[
p(\gamma) = \int_{\gamma_1} \nu(\gamma_1)\nu(\gamma - \gamma_1)d\gamma_1
\]

\[
= \frac{4^{M_i}e^{-\frac{\gamma}{\gamma_0^2}}}{(N_r - 1)!^2(\gamma_0^2\omega_1\omega_2)^{N_r}} \int_{\gamma_1}^{\gamma} (\gamma_1\gamma - \gamma_1^2)^{N_r-1}e^{-\frac{2\lambda_1 - 2\lambda_2}{\gamma_0^2}} \gamma_1^2 \cdot \gamma_1 d\gamma_1
\]  \hspace{1cm} (4.23)

For convenience, we repeat the notations here:

\[
M_i = 2N_r - i - 2,
\]

\[
a = 2\left(\frac{1}{\omega_1} - \frac{1}{\omega_2}\right),
\]

\[
b = \frac{4^{N_r}}{(N_r - 1)!^2(\gamma_0^2\omega_1\omega_2)^{N_r}},
\]

\[
\nu_m^n = \frac{m!}{(m-n)!n!},
\]

\[
\binom{m}{n} = \frac{\nu_m^n}{n!} = \frac{m!}{(m-n)!n!},
\]
and the integration can be rewritten as:

\[ p(\gamma) = b e^{-\frac{2\gamma}{\rho_{tx}} \sum_{i=0}^{N_r-1} (-1)^{(N_r-1) i} \gamma^i \frac{M_i}{a^{M_i+1}}} \left[ e^{-a\gamma} \sum_{j=0}^{M_i} \mathcal{P}_{M_i}^{j}(a\gamma)^{M_i-j} - M_i! \right] \]

The third equation follows the indefinite integral of exponential function, in which the exponent \( a \neq 0 \). Recall that \( a = -4\rho_{tx}/\gamma_0(1 - \rho_{tx}^2) \), it is equivalent to \( \rho_{tx} \neq 0 \). Insert (4.25) into (4.24), the p.d.f. of effective SNR is written as:

\[ p(\gamma) = b e^{-\frac{2\gamma}{\rho_{tx}} \sum_{i=0}^{N_r-1} (-1)^{(N_r-1) i} \gamma^i \frac{M_i}{a^{M_i+1}}} \left[ e^{-a\gamma} \sum_{j=0}^{M_i} \mathcal{P}_{M_i}^{j}(a\gamma)^{M_i-j} - M_i! \right] \]

4.B Asymptotic p.d.f. of the effective SNR of OSTBC

In this appendix, the p.d.f. of the effective SNR of OSTBC is derived as \( \rho_{tx} \to 0 \). By denoting \( x = a\gamma \), we can rewrite (4.13) as:

\[ p(\gamma) = b e^{-\frac{2\gamma}{\rho_{tx}} \gamma^{2N_r-1} \sum_{i=0}^{N_r-1} (-1)^{(N_r-1) i} \frac{M_i}{x^{M_i+1}}} \left[ e^{-x} \sum_{j=0}^{M_i} \frac{M_i!}{(M_i-j)!} - M_i! \right] \]
4. B Asymptotic p.d.f. of the effective SNR of OSTBC

Recall the Taylor series expansion: $e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \ldots$. (4.27) can be rewritten as:

$$p(\gamma) = be^{-\frac{2\gamma}{\pi\sigma^2}}\gamma^{2N_r-1} \sum_{i=0}^{N_r-1} \left( \frac{(-1)^{N_r-i}(N_r-1)}{i!} \right)$$

$$M_i!e^{-x} \left( e^x - \frac{x^{M_i+1}}{(M_i+1)!} - \mathcal{O}(x^{M_i+2}) \right) - M_i!$$

$$= be^{-\frac{2\gamma}{\pi\sigma^2}}\gamma^{2N_r-1} \sum_{i=0}^{N_r-1} \left( \frac{(-1)^{N_r-i+1}(N_r-1)}{i!} \right)$$

$$M_i!e^{-x} \left( \frac{x^{M_i+1}}{(M_i+1)!} + \mathcal{O}(x^{M_i+2}) \right)$$

As $\rho_{tx} \to 0$,

$$p(\gamma) \to 2e^{-\frac{2\gamma}{\pi\sigma^2}}(2\gamma/\gamma_0)^2 \sum_{i=0}^{N_r-1} \left( \frac{(-1)^{N_r-i+1}(N_r-1)}{i!} \right) \frac{(N_r-1)}{M_i + 1}$$

(4.28)

where $\sum_{i=0}^{N_r-1} \left( \frac{(-1)^{N_r-i+1}(N_r-1)}{i!} \right)$ is shown to be equal to $\frac{(N_r-1)^2}{(2N_r-1)!}$ from computer simulation. Therefore (4.13) is approaching to (4.4) and the resulting discrete-rate spectral efficiency (4.14) reduces to (4.5) as $\rho_{tx} \to 0$. 

53
Chapter 5

Software Defined Radio Workbench (SDR-WB)

As a promising solution for the fast developing radio communications, Software Defined Radio (SDR) has received more and more interests recently. The idea of SDR is to put the AD/DA converters as close as possible to the antennas and operate as much as possible in the digital domain by software; the hardware at the front end is parameterized so that they can be operated in the right mode, e.g. frequency band, sampling frequency, etc.

In this chapter, we present a generic workbench for SDR that is supportable to both narrow-band and wideband systems with either single antenna or multiple antennas.

5.1 Introduction

The SDR-WB mainly deals with the digital signal processing in MATLAB/OCTAVE simulation environment. In order to accommodate various communications standards, the workbench is functionally modularized into generic blocks with a standardized interface. Furthermore, the numbers, e.g. number of transmit and receive antennas, are parameterized so that the workbench can be easily reconfigured. These properties facilitate the modification and extension of the workbench and new transmission schemes can be easily adopted.

5.2 Workbench architecture

The main function of the workbench is called 'SDR.m', the execution of which is divided into three phases: setup (initializations of all functions and parameters), run (data transmission) and wrapup (results calculation).

Inside every phase, the modules are called in a specified order. There are five main modules in the workbench: SOURCE, TX, CH, RX and SINK, as shown in
Figure 5.1. They indicate the signal source, the transmitter, the wireless channel, the receiver and the signal sink. Within every module, there exist sub-modules to split the task into several sub-tasks and have them accomplished separately by the models, e.g., BPU\_T in TX calls scrambler, channelcoder and interleaver to deal with all bit processing tasks. Alternatively, modules can call models directly to handle the basic signal processing. For example, nsource in SOURCE generates random data bits to be transmitted.

![Block diagram of the modularized workbench](image)

Figure 5.1: Block diagram of the modularized workbench

The main file structure of the workbench in case of MIMO-OFDM with SVD is illustrated in Figure 5.2. The top level include the main function SDR and the phase functions, namely Setup, Run and Wrapup. On the second level are the modules that are directly called by the top level functions, i.e. SOURCE, TX, RX, CH and SINK. Modules can be further broken down into either sub-modules, e.g. BPU\_T, SPU\_T, or models, e.g. nsource, displayresults.

**Kernel functions**

The phase functions mentioned in the preceding section, namely, Setup, Run, Wrapup, are kernel functions that define the data flows of the workbench. They are located in a right limited directory that not supposed to be viewed or modified by the users.
5.2 Workbench architecture

![Diagram of the Workbench Architecture]

Figure 5.2: File structure of the workbench

Modules

The **SOURCE** generates data bits that are to be transmitted. Then the data bits are encoded and mapped into symbols in the **TX** before they are sent to the antennas. Through **CH**, the discrete-time signal is received at the **RX**, where the data bits are detected. The results such as **BER** and spectral efficiency are computed and shown in the **SINK**, both in table- and graphical format.

The **TX** and **RX** consist of several generic sub-modules: **BPU** for the Bit Processing Unit, **SPU** for the Symbol Processing Unit, **DRU** for the Digital Radio Units, **ARU** for the Analog Radio Units and **EAU** for the Estimation and Adaptation Units, which are dedicated for channel estimation and link adaptation for the purpose of adaptation.

The **CH** is made up of four sub-modules: **GCH** to generate the complex valued channel matrix, **FCH** to feed the transmitted signal through the channel generated by **GCH**, **RCH** to feedback CSFI from **RX** to **TX** which is used in an adaptive system that needs CSI at the transmitter and **ECH** to obtain perfect channel information directly from **GCH**.
Sub-modules

Sub-modules in TRX

Since TX and RX have a complicated signal processing procedure as shown in Figure 5.2, it is necessary to divide them into sub-procedures that are treated by sub-modules, i.e., $BPU\_T$, $SPU\_T$, $DRU\_T$, $ARU\_T$, and $EAU\_T$.

$BPU\_T$ includes all the bit processing units, like data bit scrambler, convolutional channel coding and block interleaver.

$SPU\_T$ deals with the symbol processing part. It maps the bits into symbols and performs some application-specific functions, like MIMO encoding.

$DRU\_T$ consists of the blocks that are located before the DA converter, such as OFDM modulation, interpolation and channel filtering.

$ARU\_T$ includes the frequency translation, power amplifier, and transmitter filter. They have not been implemented yet.

$EAU\_R$ is composed of two parts, channel estimation and link adaptation. Link adaptation is used in adaptive systems to update the transmission parameters depending on the CSI from the channel estimation. $EAU\_T$ is designed to manage the adaptive transmission based on full or partial CSI.

Sub-modules in CH

$GCH$ calls various models to generate a channel with desired statistics. Then the generated channel is called in $FCH$ to compute the received signal. At the receiver, channel information is required to detect the transmitted signal, which is obtained either from channel estimation using preamble, or from $ECH$ to get the error-free CSI. In case of an adaptive system, $RCH$ is applied to feedback the partial or full CSI from RX to TX.

Models

The basic units of the workbench that perform the signal processing to realize an atomic functionality are called models. All of the models are parameterized to ease reconfiguration. The values of the parameters are pre-defined in a corresponding ini-file of the model, where they are systematically organized and can be easily altered. For users’ convenience, these parameters are alternatively specified in the upmost system-level ini-file such as $SDR\_ini$, so that one doesn’t need to go through the hierarchy into every ini-file to change the parameter values. Furthermore, all models are written independently with each other so that they can be arbitrarily assembled as long as they make sense.
5.3 Control flows

Other functions

Besides the kernel, modules, submodules and models functions, there are other supporting user-defined functions. For instance, readParam.m helps each modules/models read in the parameter values set in the ini-files; some commonly-used functions which are called by different models are modularized into library functions, such as root-raised-cosine filter; template files for sub-modules and models are available to provide guidelines to users who want to extend or reconfigure the workbench. Furthermore, the system parameters for different transmission schemes, e.g., WCDMA, MIMO-SVD, OFDM, MIMO-OFDM-GSTBC, are stored in application-specific ini-files.

5.3 Control flows

The applications supported by SDR-WB can be categorized into two classes: the channel-adaptive transmission schemes (Scenario_CSIT) and non-adaptive schemes (Scenario_Blind).

**Scenario_CSIT** treats the applications that adapt the transmission parameters such as rate and power depending on the partial or full CSI.

**Scenario_Blind** does not have CSIT and the transmission parameters are fixed regardless of the variation of the wireless channel.

As a result, the process of them in the Run phase distinguishes from each other, as illustrated in Figure 5.3. The loop is a parameter of Monte Carlo simulation used in both flowcharts to get an average measure of the performance for each SNR point.

In Scenario_CSIT, since the transmission depends on the estimated CSI from the receiver, the execution of data transmission is placed after the channel estimation. To improve the program efficiency and get error-free channel estimation, the channel estimation by using preamble is substituted by ECH. Based on the perfect CSI obtained from ECH, the transmission parameters such as modulation order are computed by EAU_R. If the effective SNR is lower than the cut-off threshold, i.e., $\gamma < \gamma_0$, then no data is going to be transmitted and a new channel realization starts. Otherwise the transmission parameters are fed back through RCH to TX and a conventional transmission is carried out. The process of Scenario_Blind is similar to the Scenario_CSIT except that EAU_R and RCH are omitted in this case.

Both control flows discussed here assume a quasi-static fading channel, i.e. the channel remains constant for the current loop and changes independently for the next channel realization (loop). Therefore, the channel applied in data transmission is the same as the channel generated at the beginning of the loop. Alternatively, if a delay exists for the two cases, the channel varies within one interval and FCH has to employ a new channel realization generated by GCH.
5.4 Case study

Thus far, the transmission schemes that can be simulated by SDR-WB include SISO, OFDM, MIMO, MIMO-OFDM and WCDMA. Different algorithms are available for MIMO technology, e.g., SVD, OSTBC, D-STTD, BF. In this section,
5.4 Case study

we present two examples of simulating a MIMO-OFDM system with SVD and a MIMO-STBC in the SDR-WB.

MIMO-OFDM

There are four transmit antennas and four receive antennas, the number of sub-carriers for OFDM is 64, 48 out of which are used to carry the information-bearing signals. We assume a frequency-selective Rayleigh fading channel generated by a deterministic spatial-temporal channel model [18]. Perfect CSI is available at the transmitter so that adaptive modulation is applied for every scalar channel. Furthermore, adaptive power control utilizing the uniform on/off power allocation is considered in spatial domain for the sub-channels at the same frequency bin.

- Setup
  In the setup phase, the system parameters are read from MIMOOFDM-SVD.ini, an application-specific ini-file on the top level. Then the workbench goes through all models that are going to be called later in the run phase to initialize the parameters. A list of models that are called is already given in Figure 5.2.

- Run
  Scenario_CSIT is called by Run to control the data flow as shown in Figure 5.3 (the left one). In Monte-Carlo simulations, the results are averaged over a large number of channel realizations.

- Wrapup
  The simulation results are computed based on 10,000 channel realizations, and shown in Figure 5.4, 5.5 and 5.6. Figure 5.4 shows the BER performance of the system, which is around $10^{-4}$, less than the target BER, $10^{-3}$. The throughput is shown in Figure 5.5, where a large SNR gap exists between the theoretical capacity and the throughput [12]. Figure 5.6 illustrates the average number of bits loaded on every frequency index as a function of SNR. There is no difference between the average number of bits loaded on every frequency index since the average sub-channel gain is equivalent to each other over a large number of channel realizations.

MIMO-STBC

If the CSI is not available at the transmitter, fixed transmission is applied regardless of the channel quality. The control flow is given in Figure 5.3 (the right one) and the BER performance of MIMO-OSTBC in a $2 \times 2$ flat Rayleigh fading channel assuming 16-QAM modulation is shown in Figure 5.7.
Figure 5.4: $4 \times 4$ BER in frequency-selective Rayleigh fading channel

Figure 5.5: $4 \times 4$ throughputs in frequency-selective Rayleigh fading channel
Figure 5.6: Number of bits loaded on sub-carriers

Figure 5.7: $2 \times 2$ BER in flat Rayleigh fading channel
5.5 Conclusions

This chapter described a generic simulation workbench for multiple antenna SDR systems in MATLAB or OCTAVE, for both Windows and Unix/Linux operating systems. The workbench is functionally modularized into blocks and sub-blocks with a common interface for the convenience of modification and reconfiguration. Currently, it accommodates a variety of transmission schemes, including single-carrier multiple-input multiple-output (MIMO), Orthogonal Frequency-Division Multiplexing (OFDM), multi-carrier (OFDM) MIMO, Wideband Code Division Multiple Access (WCDMA), filtered multitone (FMT).

In future, more features are going to be included to the SDR-WB, e.g., preamble design for MIMO-OFDM, wideband channel models for 802.11n, multiuser scenario, MAC layer design, and etc. New applications such as RFID, UWB OFDM/OQAM and more are going to be realized.
Chapter 6

Conclusions and future work

6.1 Conclusions

In this thesis, we investigated adaptive transmission strategies to maximize the spectral efficiency in a communication system with multiple antennas. Our particular focus was on the scenario that the receiver has perfect CSI and adapts the transmitter accordingly by employing a non-delayed feedback channel. The channel used throughout the thesis was assumed to be quasi-static flat fading either with spatial correlation or without spatial correlation.

We began our investigation with a description of baseband-equivalent signal models for wireless communication systems where both the transmitter and the receiver were equipped with multiple antennas. Three statistical channel models were provided to simulate i.i.d. Rayleigh fading channel, spatially correlated Rayleigh fading channel and Ricean fading channel, respectively. Moreover, three MIMO schemes were described, namely SVD, spatial multiplexing with ZF and OSTBC.

With SVD, the MIMO channel can be converted into a set of parallel sub-channels over which separate data streams were transmitted. To achieve the highest spectral efficiency, adaptive modulation and adaptive power control were employed on every singular value channel subject to a peak power constraint and an instantaneous target BER. We started with a discussion on adaptive modulation and constant power, where the SNR thresholds for maintaining the instantaneous BER under a predefined target were obtained and the closed form expression of the spectral efficiency, namely DRSE was acquired. Then we extended the discussions by including adaptive power control policies. In this occasion, waterfilling can no longer provide the maximal spectral efficiency, but a modified version of waterfilling can still achieve a relatively high spectral efficiency. To reduce the computational complexity of waterfilling-based schemes, the greedy allocation was suggested and it can achieve comparable performance to the modified waterfilling. To further simplify the power control policy, the uniform power allocation with TAS was adopted and it was found to reach the same spectral efficiency as the modified waterfilling.
Conclusions and future work

in low SNR regions.

By utilizing adaptive modulation in ZF and OSTBC, we obtained closed form expressions of the spectral efficiencies under different channel conditions. With the DRSEs, we were able to find the crossing point of the two curves numerically and an adaptation strategy switching between ZF and OSTBC was proposed based on the crossing point. In contrast to conventional adaptation strategies, the proposed adaptation added more flexibility to the system design and enhanced the spectral efficiency by trading off the two candidate schemes.

Last but not the least, a reconfigurable versatile SDR workbench that accommodates a variety of wireless communication systems was described. The workbench included the transmitter, the wireless channel and the receiver. It has a hierarchical structure consisting of generic blocks and each block is parameterized for ease of reconfiguration.

6.2 Future work

As stated in Chapter 3, the dynamic power allocation has been evolved from “single target” to “multi-target”. However, the optimal multi-target power allocation in an adaptive modulation system that maximizes the spectral efficiency is still not available.

Chapter 4 propose a novel approach to adapt the transmitter based on the CSI, which exhibited a promising gain in spectral efficiency. However, the adaptation is based upon the assumption that the channel is either i.i.d. Rayleigh fading or semi spatially correlated with correlation on the transmitter side. An immediate proposition for future work is to extend the channel environment to Ricean fading and spatially correlated Rayleigh fading with correlation on both sides. Furthermore, other schemes, e.g. spatial multiplexing with MMSE receiver, D-STTD, can be considered.

The adaptation strategies in both chapter 3 and 4 assume perfect CSI at the receiver and the transmitter. It is an interesting topic to take into account imperfect CSI and analyze its impact on the system performance. Furthermore, channel coding should be taken into account in future study, where a pronounced improvement in the performance of spectral efficiency is expected.

The main functionalities of the SDR workbench is already finished, but more blocks should be added to cover more cases, for instance, channel models used in 3GPP and 802.11n should be incorporated. On the other hand, more wireless applications can be applied in SDR-WB, e.g. WiMAX, Bluetoot, GSM and etc.

Finally, implementation of the SDR-WB with special focus on adaptation strategies is of great interest. FPGA is a promising candidate to implement the baseband signal processing in the digital domain.
Bibliography


68


