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Towards Immortal Wireless Sensor Networks by Optimal Energy Beamforming and Data Routing

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Abstract—The lifetime of a wireless sensor network (WSN) determines how long the network can be used to monitor the area of interest. Hence, it is one of the most important performance metrics for WSN. The approaches used to prolong the lifetime can be briefly divided into two categories: reducing the energy consumption, such as designing an efficient routing, and providing extra energy, such as using wireless energy transfer (WET) to charge the nodes. Contrary to the previous line of work where only one of those two aspects is considered, we investigate these two together. In particular, we consider a scenario where dedicated wireless chargers transfer energy wirelessly to sensors. The overall goal is to maximize the minimum sampling rate of the nodes while keeping the energy consumption of each node smaller than the energy it receives. This is done by properly designing the routing of the sensors and the WET strategy of the chargers. Although such a joint routing and energy beamforming problem is non-convex, we show that it can be transformed into a semi-definite optimization problem (SDP). We then prove that the strong duality of the SDP problem holds, and hence the optimal solution of the SDP problem is attained. Accordingly, the optimal solution for the original problem is achieved by a simple transformation. We also propose a low-complexity approach based on pre-determined beamforming directions. Moreover, based on the alternating direction method of multipliers (ADMM), the distributed implementations of the proposed approaches are studied. The simulation results illustrate the significant performance improvement achieved by the proposed methods. In particular, the proposed energy beamforming scheme significantly out-performs the schemes where one does not use energy beamforming, or one does not use optimized routing. A thorough investigation of the effect of system parameters, including the number of antennas, the number of nodes, and the number of chargers, on the system performance is provided. The promising convergence behaviour of the proposed distributed approaches is illustrated.

I. Introduction

Wireless Sensor Networks (WSNs) enable several monitoring use cases of major societal importance. For example,
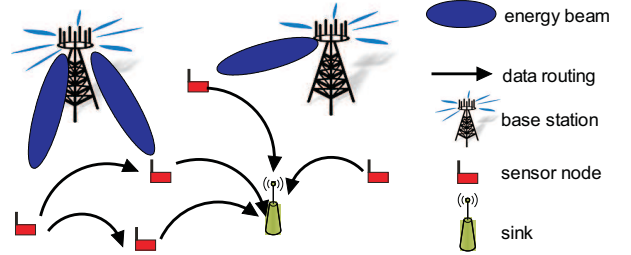

Fig. 1: A wireless sensor network with dedicated wireless energy chargers (base stations). The chargers form energy beams such that the sensors can receive more energy, whereas the sensors use proper routing to reduce their energy consumptions.

the realization of the emerging smart city vision is based on various WSNs monitoring applications, such as electrical grid monitoring, structural health monitoring, and pollution detection. However, affordable WSNs consist of energy limited battery powered sensor nodes. With the ever increasing number of such applications, a growing concern is how to achieve longer WSN lifetime without the need of changing batteries.

A promising framework to prolong the lifetime of such WSNs is given by the recent paradigm of energy harvesting [1], [2]. Sensor nodes with energy harvesting capabilities can harvest the energy from the environment, for instance from vibrations or solar radiations, and store the energy into their rechargeable batteries. Therefore, it is possible to make a WSN immortal if the energy harvested by each node is larger than the energy it consumes. However, the major limitation of this approach is the fact that the ambient energy is intermittent, which potentially makes the network performance degraded and inconsistent.

To overcome the problem above, wireless energy transfer (WET) that transfers energy remotely to the sensor nodes provides an attractive alternative to harvesting ambient energy [3]. Dedicated transmitters enable us to control the charging to the sensor nodes, and to optimize the network operations. Therefore, in this paper, we consider such a wirelessly powered WSN (WPSN) with multiple wireless chargers as shown in Fig. 1. We propose to optimize the network performance in terms of the minimum sampling rate of the sensors, under the condition that the energy consumed by each node is less than the energy it harvests. Therefore, the performance depends on not only how the wireless chargers transmit the energy to the

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nodes, but also how the nodes consume the received energy.

Regarding the energy transmission task, the chargers form energy beams [4] to improve the WET efficiency, such that more energy could be harvested by the nodes. To maximize the power received by the nodes, the existing solutions are based on transmitting the energy beam according to the dominant eigenvector of the effective channel [5], [6]. However, even when the energy beamforming is optimized, if the nodes do not consume energy efficiently, the energy would be wasted and the network performance would degrade. Therefore, the energy consumption of the nodes should be also well designed.

The major energy consumption of a sensor node comes from data transmission. Consequently, we adopt a multi-hop transmission scheme to reduce the transmission distance. Thus, the routing of the WSN should be optimized. Based on the motivation above, we propose to jointly reduce the energy consumption of nodes by optimizing the data routing among the sensor nodes and to improve the WET efficiency by optimizing the energy beamforming of the chargers. This gives us a novel joint energy beamforming and data routing problem. Different from the one with only energy beamforming [5], [6], the routing part of the problem introduces an additional linear constraint to the optimization problem. Under such a scenario, the optimal result is no longer the dominant eigenvector of the effective channel. Thus, the solution of the new problem is not trivial.

To summarize, the main contributions of this paper are as follows:

- We jointly consider the energy beamforming on the wireless chargers side and the data routing on the WSN side to maximize the monitoring performance and guarantee the immortality of the WSN. To the best of our knowledge, this technical problem has not been studied before, except our preliminary work [7].
- We propose an algorithm that finds the optimal solution by transforming the original optimization problem into a semi-definite programming (SDP) problem. For the sake of rigorousness, we prove the strong duality proposition of the SDP problem (which, unlike linear programming, does not always hold for SDPs), such that the optimal value is achievable [8]. The simulations show that the performance of the joint optimization is significantly better than the case where only energy beamforming or only routing is optimized.
- We propose a scheme with low complexity where time-sharing among pre-determined beamforming vectors is adopted. We show by simulation that, the selection of the pre-determined beamforming vectors plays an important role in the network performance, and the proposed selection of beamforming vectors yields a near optimal solution.
- Based on the alternating direction method of multipliers (ADMM) [9], we provide a new hierarchical distributed approach that offloads the computations to the wireless chargers, such that communication burden in the backbone network and the overall computing complexity is smaller.

The rest of the paper is organized as follows. In Section II, we summarize the prior work on wireless energy beamforming and data routing. The joint energy beamforming and data routing problem is formulated in Section III. We propose the centralized solution method for the optimization problem in Section IV, then the distributed version in Section V. The simulations are presented in Section VI. We conclude the paper and discuss the future work in Section VII.

Notation: We denote $\boldsymbol{X} = \{x_{ij}\}$ a matrix whose ith row and jth column element is given by x_{ij}. For a vector $\boldsymbol{x}$ or a matrix $\boldsymbol{X}$, $(\cdot)^T$ is the transpose of the vector or matrix, and $(\cdot)^H$ is the conjugate transpose of the vector or matrix. $\mathrm{tr}[\boldsymbol{X}] = \sum_i x_{ii}$ is the trace of square matrix $\boldsymbol{X}$. For a Hermitian matrix $\boldsymbol{X}$, the notation $\boldsymbol{X} \succeq 0$ means that $\boldsymbol{X}$ is positive semi-definite. Given a vector $\boldsymbol{x}$, the diagonalization $\mathrm{diag}[\boldsymbol{x}]$ constructs a matrix whose diagonal elements are $x_1, \ldots, x_n$.

II. RELATED WORK

The battery of wireless devices can be charged by WET to improve the performance on throughput or lifetime [10], [11]. In a WET system, the energy of electro-magnetic waves transmitted from the wireless chargers can be harvested by the rectifying antenna on the wireless devices, which are the sensor nodes in our case. To improve the energy transmission efficiency, the chargers can form energy beams to make the energy more concentrated at certain directions. Thus, by knowing the channel to the nodes, the chargers are able to control the energy that will be received by the nodes. From this point of view, WET will provide a more consistent performance in energy provision than harvesting energy from ambient environment. As a result, providing energy to wireless devices by WET have been widely studied.

Most of the studies in this direction typically focus on the optimization of throughput or similar communication theory metrics [4], [12–14]. Reference [12] has considered a throughput maximization problem over the energy allocation and time of WET. Optimal beamforming with simultaneous information transfer is considered under an interference scenario in [13]. A joint problem of designing energy beamforming vectors, energy allocation and scheduling of WET durations among different devices to maximize the minimum throughput has been considered in [4]. The benefits of massive MIMO arrays for WET are investigated in [14]. However, in these studies, the energy receivers transmit the data or information to the sink directly, and the possibility of using data routing is not considered. Thus, the energy consumption part is not optimized from a network perspective, and the results cannot be applied directly to WSNs, where data routing can greatly reduce the energy consumptions.

For sensor networks, the work in [15] has investigated a throughput maximization problem for a WPSN by controlling the duration of energy transmission. The authors formulated a convex optimization and provided a closed form solution. However, the data of the sensor nodes are transmitted directly to the sink due to the fact that data routing is infeasible in the underground model considered in this work. In [16], the authors assumed that the base station forms a sharp

energy beam to charge a sensor node in a given timeslot. Then, they studied the problem of scheduling the energy beams to maximize the WSN lifetime, and provided a greedy algorithm to achieve the optimal solution. They also provided the necessary conditions for a WPSN to be immortal. Based on this result, the authors of [17] have considered a node deployment problem, i.e., the problem of using the minimum number of nodes while satisfying a condition that is necessary for the WSN to be immortal. However, it is not clear whether the selected energy beams, i.e., the dominant eigenvector of the channel to each sensor, is optimal or not. To make the WSN immortal, the authors of [18], [19] have studied an application where a mobile charger charges sensor nodes at different locations. They have formulated a path planning problem for the mobile charger and proposed a solution approach. Although data routing in considered in these papers, the wireless charging is assumed to be done over short ranges over direct links, and thus energy beamforming is not considered.

Optimizing data routing is a common approach to reduce energy consumptions of the nodes. The seminal work of [20] has modelled the energy consumption of the sensor nodes as a linear function of the traffic flows, and we adopt such a model in this paper. Based on this model, the work in [21] has investigated an optimal routing and sampling problem in a WSN such that the network lifetime is maximized and the estimation error based on the measured data of the nodes is within a given threshold. The authors also provided a distributed solution approach based on primal-dual decomposition.

Although optimal routing in WSN networks is a fundamental concern, only a limited number of studies jointly consider routing together with energy harvesting, and even less with WET. For the case where the nodes can harvest energy from ambient environment, the work in [22] has proposed a system where several rovers are used to harvest energy from environment and to charge the sensor nodes to maximize the data flow of the sensor network. The authors of [23] have considered optimizing the routing and sensing for a WSN with energy harvesting capabilities in order to maximize the quality of monitoring. They formulated the problem as a resource allocation problem and presented an algorithm that provides a near-optimal solution. The work in [24] has investigated a sampling rate and routing optimization problem for a WSN, where the nodes can harvest solar energy. The sensor nodes are assumed to be able to predict the energy that they can harvest. Based on such predictions, the sensor nodes allocate their energy consumptions for the subsequent period and change their sampling rate and routings. The authors also provide a distributed approach based on dual-decomposition and sub-gradient approach.

Different from the aforementioned work, we consider the case where dedicated energy transmitters charge the WSN. Thus, we jointly optimize the routing of the WSN, and the wireless energy transmission part, in terms of the energy beamforming vectors and their time durations. Different from some existing works on WET where energy is isotropically broadcasted in all directions with a fixed power [15], [25], the energy transmitters that we consider form sharp beams to improve the received energy at the sensor nodes. This set-up

makes the optimization problem more challenging. To the best of our knowledge, our previous work [7] is the first paper that jointly considers the energy beamforming and the data routing problem. Here, we provide the full proofs and we further extend the centralized solution in the conference version to a novel distributed approach based on ADMM [9]. In addition, we have extensive simulations to show the performance, in terms of sampling rate, of different energy beamforming schemes under different system parameters, such as number of antennas, number of sensors, number of chargers. We also compare the convergence performance of the proposed ADMM based algorithm to a block descent algorithm, which is another widely used distributed approach.

III. System Model and Problem Formulation

We consider a WPSN in the paper. Specifically, we have a WSN with N sensor nodes and a sink node to monitor an area of interest, and N_{ET} wireless chargers to supply energy to the nodes using energy beamforming, as shown in Fig. 1. Each node v_i makes measurements with the sampling rate w_i, and transmits the data to the sink in a multi-hop manner to save energy. Then, the vector $\boldsymbol{w} = [w_1, \ldots, w_N]^T$ denotes the sampling rate of the nodes. The sensor nodes have rechargeable batteries to store the energy received from the wireless chargers. Such a network structure can be applied to a wide range of applications, such as smart agriculture, smart pipeline monitoring, and smart warehouse.

We use q_{ij} to represent the data flow from v_i to v_j, and e_{ij}^O the energy cost of sending a unit size of data. For a node v_i, its neighbor nodes are represented by a set $\mathcal{S}_i$. We denote $v_j \in \mathcal{S}_i^{\mathrm{out}} \subseteq \mathcal{S}_i$ if there is an out-going data link from v_i to v_j. Similarly, $v_j \in \mathcal{S}_i^{\mathrm{in}} \subseteq \mathcal{S}_i$ if there is an incoming data link from v_j to v_i. Then, $\mathcal{S}_i^{\mathrm{in}}$ and $\mathcal{S}_i^{\mathrm{out}}$ represent the preceding neighbor nodes and succeeding neighbor nodes of v_i, respectively. Similar to many studies that consider the routing of WSNs [19–22], the relationship of energy consumption and data flow is considered as linear[1]. Then, the energy consumption of a node v_i is given by

$$E_i^U = \sum_{j \in S_i^{\mathrm{out}}} e_{ij}^O q_{ij}. \qquad (1)$$

Here, the energy consumption for data communication over the wireless channel between node i and j, e_{ij}^O, is based on the distance between them and other fading factors.

Recall that we have N_{ET} wireless chargers. Each charger has M antennas. Note that the energy beam of a charger can be time-varying, i.e., a charger l can form energy beam vectors $\boldsymbol{u}_{l,1}, \boldsymbol{u}_{l,2}, \ldots, \boldsymbol{u}_{l,j}, \ldots$, where $\boldsymbol{u}_{l,j} \in \mathbb{C}^{M \times 1}$. We assume here that the number of the beam vectors is larger or equal to M. We denote $t_{l,j}$ the average time charger l transmits beam $\boldsymbol{u}_{l,j}$. Let the channel from charger l to node i be $\boldsymbol{g}_{l,i} \in \mathbb{C}^{M \times 1}$.

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[1]This model is widely used for WSNs because the power that can be used for data transmission by the sensor nodes is very limited, compared to other wireless devices such as mobile phones.

Then, using the similar model in [4], we have that the average energy received by node i is

$$E_i^R = \sum_{l=1}^{N_{\mathrm{ET}}} \sum_j \eta t_{l,j} \mathbb{E}_g[\boldsymbol{g}_{l,i}^H \boldsymbol{u}_{l,j} \boldsymbol{u}_{l,j}^H \boldsymbol{g}_{l,i}], \quad (2)$$

where the expectation is over the channels $\boldsymbol{g}_{l,i}$, η is the energy conversion efficiency. The receiver noise is ignored, since its energy is too little to be harvested [4]. We assume that the frequency bands used for data transmission and energy transmission are different, such that the nodes can harvest energy and transmit data concurrently [6], [26]. However, for the cases where the frequencies are the same, the results of the paper are still valid if one applies a time-division scheme [15], [27], which schedules the transmission of data and energy to different time slots.

We assume that the sensor nodes have large enough battery capacities. Then, the requirement that the WSN is immortal can be expressed as $E_i^R \geq E_i^U, \forall i$. Thus, we should determine each node's sampling rate and the route based on the total energy that each node receives. In regard to WSN performance, we want to have as much sampled data as possible, but we also do not want to have many sensors with very low sampling rates (balancing issue). Therefore, we aim to set the minimum sampling rate of the nodes as large as possible. We denote the monitoring performance of the WSN by $F(\boldsymbol{w}) = \min_i\{w_i\} \triangleq w_{\min}$, which is the minimum sampling rate among the nodes. We stack $q_{ij}, \forall i, j$ to form the column vector $\boldsymbol{q} \in \mathbb{R}^L$, where L is the number of candidate data routing links of the whole WSN[2]. Then, the considered problem can be formulated as:

$$\max_{w_{\min}, \boldsymbol{w}, \boldsymbol{q}, \boldsymbol{u}, \boldsymbol{t}} \quad w_{\min} \quad (3a)$$

$$\text{s.t.} \quad w_i + \sum_{j \in S_i^{\mathrm{in}}} q_{ji} - \sum_{k \in S_i^{\mathrm{out}}} q_{ik} = 0, \quad \forall i, \quad (3b)$$

$$E_i^U \leq E_i^R, \quad \forall i, \quad (3c)$$

$$\sum_j t_{l,j} \boldsymbol{u}_{l,j}^H \boldsymbol{u}_{l,j} \leq P_l, \quad \forall l, \quad (3d)$$

$$\sum_j t_{l,j} = 1, \quad \forall l, \quad (3e)$$

$$w_{\min}, \boldsymbol{q}, \boldsymbol{t} \geq 0, w_i \geq w_{\min}, \quad \forall i, \quad (3f)$$

where $w_{\min}, \boldsymbol{q}, \boldsymbol{t}$ are all non-negative, Constraint (3b) is the flow conservation constraint, Constraint (3c) ensures the immortality of the WSN, Constraint (3d) provides the power constraint for each charger, and Constraint (3e) means that, for each charger l, the summation of the percentages of the time that each energy beam $\boldsymbol{u}_{l,j}$ is used is 1. We note that the problem is non-convex due to the quadratic constraints (3c) even if $t_{l,j}$ were given. However, we propose an original transformation of this optimization problem into a SDP problem, as shown in the next section. Moreover, we will show that the strong duality holds for the SDP, such that we will be able to find the optimal solution efficiently.

[2] A simple example for a network with two nodes v_1, v_2, and a sink v_3 with $L = 3$ candidate links: $\langle v_1, v_2 \rangle$, $\langle v_2, v_3 \rangle$, and $\langle v_1, v_3 \rangle$. Then, $\boldsymbol{q} = [q_{12}, q_{13}, q_{23}]^T$.

TABLE I: Major notations used in the paper

Symbols	Meanings
$\boldsymbol{A}$	candidate routing tables of the sensor nodes
$\boldsymbol{B}$	energy consumption matrix of the nodes
$\boldsymbol{I}_i$	a matrix with 1 only at the i-th element of its diagonal and with 0 for all the other entries
$\boldsymbol{K}_{l,i}$	second moment of the channel from charger l to v_i, i.e., $\mathbb{E}_g[\boldsymbol{g}_{l,i} \boldsymbol{g}_{l,i}^H]$
L	number of candidate links
M	number of antennas of each wireless charger
N	number of sensor nodes
N_{ET}	number of wireless chargers
P_l	the WET power constraint of charger l
$\boldsymbol{U}_l$	auxiliary variable to represent $\sum_j t_{l,j} \boldsymbol{u}_{l,j} \boldsymbol{u}_{l,j}^H$
$\boldsymbol{W}$	auxiliary matrix to represent $\mathrm{diag}(\boldsymbol{w})$
e_{ij}^O	energy cost of sending one data unit from v_i to v_j
$\boldsymbol{g}_{l,i}$	channel from charger l to node i
q_{ij}	data flow from v_i to v_j
$t_{l,j}$	average time charger l transmits beam $\boldsymbol{u}_{l,j}$
$\boldsymbol{u}_{l,j}$	the j-th energy beam vector of charger l
v_i	sensor node i
w_i	sampling rate of v_i
η	energy conversion efficiency

To improve readability, we provide the major notational conventions of the paper in Table I.

IV. CENTRALIZED SOLUTION APPROACH

In this section, we will provide a centralized solution method for Problem (3), and then focus on the pre-determined beamforming scenario that provides a simpler but computationally more efficient approach for the optimization of energy transfer.

A. Algorithm based on SDP

The idea of the solution algorithm is to first transform the original problem to a new convex optimization problem that is easy to solve. Then we convert the optimal solution of the new problem back. We will show that the solution achieved from the optimal solution of the new convex problem is also the optimal solution for the original problem.

Recall that L is the number of candidate data routing links. To make the problem more concise, we construct a matrix $\boldsymbol{A} = \{a_{ij}\} \in \mathbb{R}^{N \times L}$ that corresponds to the candidate routing table of the nodes, where $a_{i,j} = 1$ if link j goes into node i; $a_{i,j} = -1$ if link j starts with node i; otherwise, $a_{i,j} = 0$. This indicates that for each column of A, there is always one -1, and at most one 1 (the column with no 1 corresponds the case that the link goes into the sink node). Then, we re-write Constraint (3b) as $\boldsymbol{w} + \boldsymbol{A}\boldsymbol{q} = \boldsymbol{0}$. Similarly, $\boldsymbol{B}_i = \{b_{i,j}\} \in \mathbb{R}^{1 \times L}$ denotes the energy consumption vector for each candidate link that starts with node v_i, i.e., $b_{i,j} = e_{i,k}^O$ if the candidate link j is from v_i to v_k. Then, $E_i^U = \boldsymbol{B}_i \boldsymbol{q}$. By stacking up the row vectors $\boldsymbol{B}_i$, we construct the energy

consumption matrix of the nodes, denoted by $\boldsymbol{B} \in \mathbb{R}^{N \times L}$. We re-write Constraint (3c) as follows:

$$\boldsymbol{B}_i \boldsymbol{q} \leq \sum_{l=1}^{N_{\mathrm{ET}}} \sum_j \eta t_{l,j} \mathbb{E}_g [\boldsymbol{g}_{l,i}^H \boldsymbol{u}_{l,j} \boldsymbol{u}_{l,j}^H \boldsymbol{g}_{l,i}]$$

$$\overset{(a)}{=} \sum_{l=1}^{N_{\mathrm{ET}}} \sum_j \eta t_{l,j} \, \mathrm{tr}[\boldsymbol{K}_{l,i} \boldsymbol{u}_{l,j} \boldsymbol{u}_{l,j}^H]$$

$$= \eta \sum_{l=1}^{N_{\mathrm{ET}}} \mathrm{tr}\Big[\boldsymbol{K}_{l,i} \sum_j t_{l,j} \boldsymbol{u}_{l,j} \boldsymbol{u}_{l,j}^H\Big] \,,$$

where $\boldsymbol{K}_{l,i} = \mathbb{E}_g[\boldsymbol{g}_{l,i} \boldsymbol{g}_{l,i}^H]$ and it is a Hermitian positive semi-definite matrix. Step (a) comes from that the trace is invariant under cyclic permutations, and it commutes with expectation. Similarly, Constraint (3d) can be re-written as $\mathrm{tr}\Big[\sum_j t_{l,j} \boldsymbol{u}_{l,j} \boldsymbol{u}_{l,j}^H\Big] \leq P_l, \forall l$. Now, we substitute $\sum_j t_{l,j} \boldsymbol{u}_{l,j} \boldsymbol{u}_{l,j}^H$ by a Hermitian positive semi-definite matrix $\boldsymbol{U}_l$. Then, the original Problem (3) is equivalent to the following one:

$$\max_{w_{\min}, \boldsymbol{w}, \boldsymbol{q}, \boldsymbol{u}, \boldsymbol{t}} \quad w_{\min} \tag{4a}$$

$$\text{s.t.} \quad \boldsymbol{w} + \boldsymbol{A}\boldsymbol{q} = \boldsymbol{0}, \tag{4b}$$

$$\boldsymbol{B}_i \boldsymbol{q} \leq \eta \sum_{l=1}^{N_{\mathrm{ET}}} \mathrm{tr}[\boldsymbol{K}_{l,i} \boldsymbol{U}_l], \quad \forall i, \tag{4c}$$

$$\mathrm{tr}[\boldsymbol{U}_l] \leq P_l, \quad \forall l, \tag{4d}$$

$$\boldsymbol{U}_l \succeq 0, \quad \forall l, \tag{4e}$$

$$w_i \geq w_{\min}, \quad \forall i, \tag{4f}$$

$$w_{\min} \geq 0, \boldsymbol{q} \geq 0, \boldsymbol{t} \geq 0, \quad \forall i, \tag{4g}$$

$$\boldsymbol{U}_l = \sum_j t_{l,j} \boldsymbol{u}_{l,j} \boldsymbol{u}_{l,j}^H, \quad \forall l, \tag{4h}$$

$$\sum_j t_{l,j} = 1, \quad \forall l. \tag{4i}$$

Notice that if we relax Constraints (4h) - (4i), Problem (4) can be expressed as the following relaxed problem:

$$\min_{w_{\min}, \boldsymbol{w}, \boldsymbol{q}, \boldsymbol{U}_l, \forall l} \quad - w_{\min} \tag{5a}$$

$$\text{s.t.} \quad (4b), (4c), (4d), (4e), (4f), (4g). $$

Problem (5) is formed by relaxing the Constraint (4h) - (4i) and keeping the objective function and the other constraints of Problem (4) unchanged. Thus, the feasible region of Problem (4) is a subset of the feasible region of Problem (5). Recall that both of these problems are minimization problems, thus we have that the optimum value of Problem (4) must be no less than the optimum value of Problem (5). Therefore, if we achieve the optimal solution $(w_{\min,\mathrm{relax}}^*, \boldsymbol{w}_{\mathrm{relax}}^*, \boldsymbol{q}_{\mathrm{relax}}^*, \boldsymbol{U}_{\mathrm{relax}}^*)$ for Problem (5), and also $\boldsymbol{t}^*$ and $\boldsymbol{u}^*$ that satisfy Constraints (4h) - (4i), i.e., $\boldsymbol{U}_{l,\mathrm{relax}}^* = \sum_j t_{l,j}^* \boldsymbol{u}_{l,j}^* \boldsymbol{u}_{l,j}^{*H}, \forall l$ and $\sum_j t_{l,j}^* = 1, \forall l$, then $(w_{\min,\mathrm{relax}}^*, \boldsymbol{w}_{\mathrm{relax}}^*, \boldsymbol{q}_{\mathrm{relax}}^*, \boldsymbol{u}^*, \boldsymbol{t}^*)$ is the optimal solution of Problem (4). The idea here is to first solve Problem (5), and then find $\boldsymbol{t}^*$ and $\boldsymbol{u}^*$. To begin with, we show the convexity of Problem (5) by the following proposition:

Proposition 1: Problem (5) is equivalent to a convex semi-definite programming problem.

Proof: Let $\boldsymbol{W} = \mathrm{diag}(\boldsymbol{w})$, $\boldsymbol{Q} = \mathrm{diag}(\boldsymbol{q})$. Since $\boldsymbol{w} \geq 0$, we have that $\boldsymbol{W} \succeq 0$. Constraint (4b) can be written as $\mathrm{tr}[\boldsymbol{I}_i \boldsymbol{W}] + \mathrm{tr}[\mathrm{diag}(\boldsymbol{A}_i)\boldsymbol{Q}] = 0, \forall i$, where $\boldsymbol{A}_i$ is the i-th row of $\boldsymbol{A}$, $\boldsymbol{I}_i$ is a matrix with 1 only at the i-th element of its diagonal, and with 0 for the other elements. Constraints (4c) are equivalent to $\mathrm{tr}[\mathrm{diag}(\boldsymbol{B}_i)\boldsymbol{Q}] - \sum_{l=1}^{N_{\mathrm{ET}}} \mathrm{tr}[\boldsymbol{K}_{l,i}\boldsymbol{U}_l] \leq 0, \forall i$. To summarize, Problem (5) is equivalent to the following formulation:

$$\min_{w_{\min}, \boldsymbol{W}, \boldsymbol{Q}, \boldsymbol{U}_l, \forall l} \quad - w_{\min} \tag{6a}$$

$$\text{s.t.} \quad \mathrm{tr}[\boldsymbol{I}_i \boldsymbol{W}] + \mathrm{tr}[\mathrm{diag}(\boldsymbol{A}_i)\boldsymbol{Q}] = 0, \quad \forall i, \tag{6b}$$

$$\mathrm{tr}[\mathrm{diag}(\boldsymbol{B}_i)\boldsymbol{Q}] - \eta \sum_l \mathrm{tr}[\boldsymbol{K}_{l,i}\boldsymbol{U}_l] \leq 0, \quad \forall i, \tag{6c}$$

$$\mathrm{tr}[\boldsymbol{U}_l] \leq P_l, \quad \forall l, \tag{6d}$$

$$\mathrm{tr}[\boldsymbol{I}_i \boldsymbol{W}] - w_{\min} \geq 0, \quad \forall i, \tag{6e}$$

$$\boldsymbol{Q} \succeq \boldsymbol{0}, \boldsymbol{U}_l \succeq \boldsymbol{0}, \boldsymbol{W} \succeq \boldsymbol{0}, w_{\min} \geq 0. \tag{6f}$$

Recall that $\boldsymbol{I}_i$, $\mathrm{diag}(\boldsymbol{A}_i)$, $\mathrm{diag}(\boldsymbol{B}_i)$, and $\boldsymbol{K}_{l,i}$ are Hermitian. Hence the formulation is now cast as a standard form SDP [28]. Since the objective function is linear and the feasible region is convex, Problem (5) is a convex SDP formulation. ∎

Since Problem (6) is a SDP problem, we have that if the strong duality of the problem holds, the duality gap is zero and therefore we can achieve the optimal solution of Problem (6) with any sufficiently small error $\epsilon > 0$ in time $O((L+N+N_{\mathrm{ET}}M)^{4.5} \log(1/\epsilon))$ [29]. However, unlike linear programming, the strong duality of a SDP does not always hold. Although the existing results [6] have shown that, when routing is not considered, the strong duality of the problem holds, it is unknown for the case with routing constraints. Here, for the sake of rigorousness, we will prove that, with the constraints introduced by routing, the strong duality still holds, which means that the optimal value is achievable [8], [28]. To show the strong duality, we first write the dual problem of Problem (6) as follows:

$$\max_{\boldsymbol{y}_1, \boldsymbol{y}_2, \boldsymbol{y}_3, \boldsymbol{y}_4} \quad \sum_l P_l y_{3l} \tag{7a}$$

$$\text{s.t.} \quad \sum_i y_{1i} \boldsymbol{I}_i - \sum_i y_{4i} \boldsymbol{I}_i \preceq \boldsymbol{0}, \tag{7b}$$

$$\sum_i y_{1i} a_{ij} + \sum_i y_{2i} b_{ij} \leq 0, \quad \forall j \tag{7c}$$

$$\eta \sum_{i=1}^N y_{2i} \boldsymbol{K}_{l,i} - y_{3l} \boldsymbol{I} \succeq 0, \quad \forall l \tag{7d}$$

$$\sum_i y_{4i} \leq -1, \tag{7e}$$

$$y_{2i} \leq 0, y_{3l} \leq 0, y_{4i} \leq 0, \quad \forall i, l, \tag{7f}$$

where y_{1i} corresponds to Constraint (6b), y_{2i} corresponds to Constraint (6c), y_{3l} corresponds to Constraint (6d), y_{4i} corresponds to Constraint (6e). Then, we have the following proposition:

Proposition 2: Consider Problem (6) and its dual (7), where $\boldsymbol{A}, \boldsymbol{B}$ are constructed according to the topology of a connected WSN[3], $P_l > 0$. Strong duality holds, i.e., for both problems there exist strictly feasible solutions.[4]

Proof: The proof consists of checking the existence of the strictly feasible solutions for each problem. Since the WSN is connected, the elements in $\boldsymbol{B}_i$ are bounded and positive.

For Problem (6), we can set $\boldsymbol{U}_l = (1 - \varepsilon)P_l\boldsymbol{I}, \forall l$. As $\boldsymbol{K}_{l,i}$ is positive semi-definite, and $\boldsymbol{K}_{l,i} \neq \boldsymbol{0}$, we have $\text{tr}[\boldsymbol{K}_{l,i}] > 0$. Thus, it is straightforward that we can find a small enough routing decision $\boldsymbol{Q}$, such that $0 < \text{tr}[\text{diag}(\boldsymbol{B}_i)\boldsymbol{Q}] < \eta \sum_l (1 - \varepsilon) \text{tr}[\boldsymbol{K}_{l,i}], \forall i$. We can set $w_{\min} < (1 - \varepsilon) \min_i \{\text{tr}[\boldsymbol{I}_i\boldsymbol{W}]\}$, such that $\text{tr}[\boldsymbol{I}_i\boldsymbol{W}] - w_{\min} > 0$. It means that there exist strictly feasible solutions for Problem (6).

For Problem (7), Constraints (7b) require that $y_{1i} < y_{4,i}, \forall i$. We can set $y_{1i} = -1 - \varepsilon_1 < -1 - \varepsilon_2 = y_{4,i}, \forall i$, where $\varepsilon_2 > \varepsilon_1 > 0$ such that $\sum_i y_{4i} = -N - N\epsilon_2 < -1$ strictly holds. Since $\boldsymbol{A}$ corresponds to the candidate routing table, each column of which has at most one 1 and one -1, Then, we have that $\sum_i y_{1i}a_{ij} \leq 1 + \varepsilon_1, \forall j$. Since $\boldsymbol{B}$ corresponds to the energy consumption for each candidate link, we have that $b_{ij} \geq 0$, and $\sum_i b_{ij} > 0$. Thus, we can set $y_{2i} = -(1 + \varepsilon_1)/\min_j \sum_k b_{kj} - \varepsilon_3, \forall i$, where $\varepsilon_3 > 0$, such that $\sum_i y_{2i}b_{ij} < -1 - \varepsilon_1$ and Constraint (7c) strictly holds. For Constraint (7d), it is also possible to find a small enough y_{3l}, such that y_{3l} is smaller than the smallest eigenvalue of $\eta \sum_{i=1}^{N} y_{2i}\boldsymbol{K}_{l,i}, \forall l$, which makes Constraints (7d) strictly hold. Thus, there exists a strictly feasible solution for Problem (7).

Thus, we have that for Problem (6) and its dual Problem (7), there exist strictly feasible solutions. Thus, strong duality holds [8], [28]. This completes the proof. ∎

Based on this proposition, we conclude that we can achieve a solution, using interior-point methods, with a sufficient small error $\epsilon > 0$ to the optimum of Problem (6) in time $\log(1/\epsilon)$ [29]. Thus, this solution is taken as the global optimal solution. Consequently, the main idea of the centralized approach is to first find the optimal solution for Problem (6) (equivalently the optimal solution for Problem (5)), denoted by $(w^*_{\min,\text{relax}}, \boldsymbol{w}^*_{\text{relax}}, \boldsymbol{q}^*_{\text{relax}}, \boldsymbol{U}^*_{\text{relax}})$. Then in the second step, based on $\boldsymbol{U}^*_{\text{relax}}$, we find $\boldsymbol{u}^*, \boldsymbol{t}^*$ that satisfy Constraints (4h) - (4i) (and do not cause a change in the optimum). If there exists such a $\boldsymbol{u}^*, \boldsymbol{t}^*$, then $(w^*_{\min,\text{relax}}, \boldsymbol{w}^*_{\text{relax}}, \boldsymbol{q}^*_{\text{relax}}, \boldsymbol{u}^*, \boldsymbol{t}^*)$ is also an optimal solution for Problem (4). Next, we are going to convert the optimal solution of Problem (6) to a candidate solution for Problem (4), and then show its optimality.

Recall that $\boldsymbol{U}_l, \forall l$ is positive semi-definite. Hence, all the eigenvalues of $\boldsymbol{U}^*_{l,\text{relax}}$ are non-negative. Let us denote the j-th eigenvalue of $\boldsymbol{U}^*_{l,\text{relax}}$ by $\lambda_{l,j}$ and the corresponding eigenvector by $\boldsymbol{d}_{l,j}$. Then, $t_{l,j} = \lambda_{l,j}/\sum_i \lambda_{l,i}$ and $\boldsymbol{u}_{l,j} = \sqrt{\sum_i \lambda_{l,i}}\boldsymbol{d}_{l,j}$ is a feasible point of Problem (4), such that Constraints (4h)- (4i) are satisfied. Thus, we can let $\boldsymbol{t}^* = \{t_{l,j}\}$ and $\boldsymbol{u}^* = \{\boldsymbol{u}_{l,j}\}$. Therefore, Problem (4) can be solved in 3 steps: 1) turning it into a convex SDP problem, 2) expressing

the solution in terms of the spectral decomposition, and 3) forming the final solution by re-scaling, as summarized in Algorithm 1. The following proposition shows that Algorithm 1 achieves the optimal solution of Problem (3).

Proposition 3: Consider a feasible optimization Problem (3), where $\boldsymbol{K}_{l,i}$ is positive semi-definite. Then, Algorithm 1 provides a global optimal solution for Problem (3).

Proof: Denote $(w_{\min,\text{ts}}, \boldsymbol{w}_{\text{ts}}, \boldsymbol{q}_{\text{ts}}, \boldsymbol{u}_{\text{ts}}, \boldsymbol{t}_{\text{ts}})$ the output of Algorithm (1). Recall that Problem (3) is equivalent to Problem (4). Thus, we first prove that $(w_{\min,\text{ts}}, \boldsymbol{w}_{\text{ts}}, \boldsymbol{q}_{\text{ts}}, \boldsymbol{u}_{\text{ts}}, \boldsymbol{t}_{\text{ts}})$ is feasible for Problem (4), and then prove its optimality.

Feasibility: According to Propositions 1 and 2, Problem (5) is convex and strong duality holds. Thus, the result of Step 1 of Algorithm 1, $\left(w^*_{\min,\text{relax}}, \boldsymbol{w}^*_{\text{relax}}, \boldsymbol{q}^*_{\text{relax}}, \boldsymbol{U}^*_{\text{relax}}\right)$, is achievable, feasible, and optimal [8], [28]. Therefore, from the output of the algorithm $w_{\min,\text{ts}} = w^*_{\min,\text{relax}}$, $\boldsymbol{w}_{\text{ts}} = \boldsymbol{w}^*_{\text{relax}}$, $\boldsymbol{q}_{\text{ts}} = \boldsymbol{q}^*_{\text{relax}}$, we know that $(w_{\min,\text{ts}}, \boldsymbol{w}_{\text{ts}}, \boldsymbol{q}_{\text{ts}}, \boldsymbol{u}_{\text{ts}}, \boldsymbol{t}_{\text{ts}})$ satisfies Constraints (4b), (4f), and (4g). From Constraints (5c)-(5d), we have that $\boldsymbol{B}_i\boldsymbol{q}^*_{\text{relax}} \leq \eta \sum_l \text{tr}[\boldsymbol{K}_{l,i}\boldsymbol{U}^*_{l,\text{relax}}], \forall i$, and $\text{tr}[\boldsymbol{U}^*_{l,\text{relax}}] \leq P_l, \forall l$. Lines 3-4 in Algorithm 1 give us that $\boldsymbol{u}_{l,j,\text{ts}} = \sqrt{\sum_i \lambda_{l,i}}\boldsymbol{d}_{l,j}$, $t_{l,j,\text{ts}} = \lambda_{l,j}/\sum_i \lambda_{l,i}$, where $\lambda_{l,i}$ and $\boldsymbol{d}_{l,i}$ are the eigenvalue and corresponding eigenvector of $\boldsymbol{U}^*_{l,\text{relax}}$. Thus, we have that

$$P_l \geq \text{tr}[\boldsymbol{U}^*_{l,\text{relax}}] = \sum_i \lambda_{l,i}\boldsymbol{d}^H_{l,i}\boldsymbol{d}_{l,i}$$
$$= \sum_i \frac{\lambda_{l,i}\boldsymbol{u}^H_{l,i,\text{ts}}\boldsymbol{u}_{l,i,\text{ts}}}{\sum_i \lambda_{l,i}} = \sum_i t_{l,i,\text{ts}}\boldsymbol{u}^H_{l,i,\text{ts}}\boldsymbol{u}_{l,i,\text{ts}} \forall l,$$

where the first equality holds due to the fact that $\boldsymbol{U}^*_{l,\text{relax}}$ is positive semi-definite, which is diagonalizable. Therefore, $(\boldsymbol{t}_{\text{ts}}, \boldsymbol{u}_{\text{ts}})$ satisfies Constraints (4d), (4h) and (4i). Similarly, we have that

$$\boldsymbol{B}_i\boldsymbol{q}^*_{\text{ts}} = \boldsymbol{B}_i\boldsymbol{q}^*_{\text{relax}} \leq \eta \, \text{tr}[\boldsymbol{K}_{l,i}\boldsymbol{U}^*_{l,\text{relax}}]$$
$$= \eta \, \text{tr}[\boldsymbol{K}_{l,i} \sum_i t_{l,i,\text{ts}}\boldsymbol{u}_{l,j,\text{ts}}\boldsymbol{u}^H_{l,i,\text{ts}}],$$

which means that $(\boldsymbol{w}^*_{\text{ts}}, \boldsymbol{q}^*_{\text{ts}}, \boldsymbol{t}_{\text{ts}}, \boldsymbol{u}_{\text{ts}})$ satisfies Constraints (4c).

Furthermore, since $\boldsymbol{U}^*_{l,\text{relax}}$ is positive semi-definite, its eigenvalue $\lambda_{l,i}$ is nonnegative and real, which means that $t_{l,i,\text{ts}}$ is nonnegative and real. Thus, $\boldsymbol{t}_{\text{ts}}$ satisfies $\boldsymbol{t} \geq 0$. Also, $\boldsymbol{U}_l = \sum_i t_{l,i,\text{ts}}\boldsymbol{u}^H_{l,i,\text{ts}}\boldsymbol{u}_{l,i,\text{ts}}$ is positive semi-definite, which satisfies Constraint (4e). Therefore, we have that $\left(w^*_{\min,\text{relax}}, \boldsymbol{w}^*_{\text{relax}}, \boldsymbol{q}^*_{\text{relax}}, \boldsymbol{t}_{\text{ts}}, \boldsymbol{u}_{\text{ts}}\right)$ satisfies all Constraints of Problem (4), thus it is a feasible solution of Problem (4).

Optimality: It is easy to show by contradiction. Suppose that there exists a feasible solution $(w_{\min,o}, \boldsymbol{w}_o, \boldsymbol{q}_o, \boldsymbol{t}_o, \boldsymbol{u}_o)$ for Problem (4), such that $w_{\min,o} > w_{\min,\text{ts}}$. Then, we can construct $\boldsymbol{U}_{l,o} = \sum_i t_{l,i,o}\boldsymbol{u}_{l,i,o}\boldsymbol{u}^H_{l,i,o}$, such that $(w_{\min,o}, \boldsymbol{w}_o, \boldsymbol{q}_o, \boldsymbol{U}_{l,o})$ is feasible for Problem (5). Then, $w_{\min,o} > w_{\min,\text{ts}} = w^*_{\min,\text{relax}}$ contradicts the assumption that $\left(w^*_{\min,\text{relax}}, \boldsymbol{w}^*_{\text{relax}}, \boldsymbol{q}^*_{\text{relax}}, \boldsymbol{U}^*_{\text{relax}}\right)$ is an optimal solution for Problem (5). Thus, the assumption is not valid and $(w_{\min,\text{ts}}, \boldsymbol{w}_{\text{ts}}, \boldsymbol{q}_{\text{ts}}, \boldsymbol{t}_{\text{ts}}, \boldsymbol{u}_{\text{ts}})$ is an optimal solution for Problem (4). This completes the proof of the optimality for Problem (4).

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[3]It means that, for each column of $\boldsymbol{A}$, there exists one -1 and at most one 1, whereas the other elements are 0. For $\boldsymbol{B}$, all its elements are non-negative.

[4]It is also sufficient to show that the proposition holds for the case where $F(\boldsymbol{w}) = \sum \alpha_i w_i$.

Recall that Problem (4) is equivalent to Problem (3). Therefore, Algorithm 1 achieves a global optimal solution of Problem (3). ∎

Consequently, we can achieve the optimal sampling rate and routing of the sensor nodes, as well as the beamformings of the chargers by Algorithm 1 based on SDP. Next, we will consider an alternative approach, which is suboptimal but of lower-complexity.

B. Pre-determined beamforming vectors

In this subsection, we will discuss a special case of Problem (3). Specifically, the beamforming vectors of the chargers are pre-determined, whereas the power and the time duration for each beam should be optimized. The major motivation of doing so is to reduce the complexity of Problem (3). Also, we are interested in how the selection of the pre-determined beams affects the network performance, which will be discussed in the simulation section. We refer this scheme as pre-determined beamforming. In this scheme, each charger l has M_{B} pre-determined beams, which are denoted by $\boldsymbol{u}_{l,j}, 1 \leq j \leq M_{\mathrm{B}}$, and $\|\boldsymbol{u}_{l,j}\|^2 = 1$. We denote the power and average time of beam $\boldsymbol{u}_{l,j}$ by $p_{l,j}$ and $t_{l,j}$. Then, the optimization problem is

$$\max_{w_{\min},\boldsymbol{w},\boldsymbol{q},\boldsymbol{p},\boldsymbol{t}} w_{\min} \tag{8a}$$

$$\text{s.t.} \quad \boldsymbol{w} + \boldsymbol{A}\boldsymbol{q} = \boldsymbol{0} \tag{8b}$$

$$\boldsymbol{B}_i\boldsymbol{q} \leq \sum_{l=1}^{N_{\mathrm{ET}}}\sum_{j=1}^{M_{\mathrm{B}}} \eta p_{l,j}t_{l,j}\operatorname{tr}[\boldsymbol{K}_{l,i}\boldsymbol{u}_{l,j}\boldsymbol{u}_{l,j}^H], \quad \forall i, \tag{8c}$$

$$\sum_{j=1}^{M_{\mathrm{B}}} t_{l,j} = 1, \quad \forall l, \tag{8d}$$

$$\sum_{j=1}^{M_{\mathrm{B}}} p_{l,j}t_{l,j} \leq P_l, \quad \forall l, \tag{8e}$$

$$\boldsymbol{w} \geq w_{\min} \geq 0, \boldsymbol{q} \geq 0, \boldsymbol{p} \geq 0, \boldsymbol{t} \geq 0. \tag{8f}$$

Problem (8) is non-convex, due to the multiplication of the variables in the constraints. However, we can introduce a new variable $y_{l,j}$ to represent $p_{l,j}t_{l,j}$. Based on this, we can find the optimal solution of Problem (8) as follows: The approach consists of two steps. First, we temporarily relax Constraints (8d), and solve the following linear optimization problem:

$$\max_{w_{\min},\boldsymbol{w},\boldsymbol{q},\boldsymbol{y}} w_{\min} \tag{9a}$$

$$\text{s.t.} \quad \boldsymbol{w} + \boldsymbol{A}\boldsymbol{q} = \boldsymbol{0} \tag{9b}$$

$$\boldsymbol{B}_i\boldsymbol{q} \leq \sum_{l=1}^{N_{\mathrm{ET}}}\sum_{j=1}^{M_{\mathrm{B}}} \eta y_{l,j}\operatorname{tr}[\boldsymbol{K}_{l,i}\boldsymbol{u}_{l,j}\boldsymbol{u}_{l,j}^H], \quad \forall i, \tag{9c}$$

$$\sum_{i=1}^{M_{\mathrm{B}}} y_{il} \leq P_l, \quad \forall l, \tag{9d}$$

$$\boldsymbol{w} \geq w_{\min} \geq 0, \boldsymbol{q} \geq 0, \boldsymbol{y} \geq 0. \tag{9e}$$

Suppose the optimal solution for Problem (9) is $(w_{\min}^*, \boldsymbol{w}^*, \boldsymbol{q}^*, \boldsymbol{y}^*)$. Then, in the second step, we need

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Algorithm 1 Time-splitting beamforming algorithm

Require: $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{K}_{il}$, $\boldsymbol{P}_l$
Ensure: $w_{\min,\mathrm{ts}}, \boldsymbol{w}_{\mathrm{ts}}, \boldsymbol{q}_{\mathrm{ts}}, \boldsymbol{u}_{\mathrm{ts}}, \boldsymbol{t}_{\mathrm{ts}}$
1: Find the optimal solution $\left(w_{\min,\mathrm{relax}}^*, \boldsymbol{w}_{\mathrm{relax}}^*, \boldsymbol{q}_{\mathrm{relax}}^*, \boldsymbol{U}_{\mathrm{relax}}^*\right)$ for Problem (5).
2: **for** $l = 1$ to N_{ET} **do**
3:     Find the eigenvalues $\boldsymbol{\lambda}_l = \{\lambda_{l,i}\}$ and the corresponding eigenvectors $\boldsymbol{d}_l = \{\boldsymbol{d}_{l,i}\}$ of $\boldsymbol{U}_{l,\mathrm{relax}}^*$.
4:     Construct $\boldsymbol{u}_{l,j} = \sqrt{\sum_i \lambda_{l,i}}\boldsymbol{d}_{l,j}$, and $t_{l,j} = \lambda_{l,j}/\sum_i \lambda_{l,i}$.
5: **end for**
6: **return** $\boldsymbol{u}_{\mathrm{ts}} = \{\boldsymbol{u}_{l,j}\}$, $w_{\min,\mathrm{ts}} = w_{\min,\mathrm{relax}}^*$, $\boldsymbol{w}_{\mathrm{ts}} = \boldsymbol{w}_{\mathrm{relax}}^*$, $\boldsymbol{q}_{\mathrm{ts}} = \boldsymbol{q}_{\mathrm{relax}}^*$, $\boldsymbol{t}_{\mathrm{ts}} = \{t_{l,j}\}$.

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to find the feasible $t_{l,j}$ and $p_{l,j}$, $\forall l, j$, such that the following equations are satisfied:

$$\begin{cases} p_{l,j}t_{l,j} = y_{l,j}^*, & \forall l, j & \text{(10a)} \\ \sum_{i=1}^{M_{\mathrm{B}}} t_{l,j} = 1, & \forall l & \text{(10b)} \\ 0 \leq p_{l,j}, 0 \leq t_{l,j}, & \forall l, j. & \text{(10c)} \end{cases}$$

There may exist several solutions for Equations (10). However, one simple solution is given by $p_{l,j} = M_{\mathrm{B}}y_{l,j}^*, t_{l,j} = 1/M_{\mathrm{B}}, \forall l, j$. Then, we have the following proposition:

Proposition 4: Consider feasible optimization Problem (8). If $(w_{\min}^*, \boldsymbol{w}^*, \boldsymbol{q}^*, \boldsymbol{y}^*)$ is an optimal solution for Problem (9), then $(w_{\min,\mathrm{pd}} = w_{\min}^*, \boldsymbol{w}_{\mathrm{pd}} = \boldsymbol{w}^*, \boldsymbol{q}^*, \boldsymbol{p}_{\mathrm{pd}} = M_{\mathrm{B}}\boldsymbol{y}^*, \boldsymbol{t}_{\mathrm{pd}} = (1/M_{\mathrm{B}})\boldsymbol{1}^T)$ is one of the optimal solutions for Problem (8).

Proof: The proof is similar to the one of Proposition 3. Please refer to our technical report [30] for the complete proof. ∎

One approach to set $\boldsymbol{u}_{l,i}$ is $\boldsymbol{u}_{l,i} = \hat{\boldsymbol{g}}_{l,i}/\|\hat{\boldsymbol{g}}_{l,i}\|$, where $\hat{\boldsymbol{g}}_{li}$ is the estimation of channel $\boldsymbol{g}_{l,i}$. This approach requires the knowledge of instantaneous channel information. An alternative approach is to set it as the dominant eigenvector of $\mathbb{E}_g[\boldsymbol{g}_{l,i}\boldsymbol{g}_{l,i}^H]$. This can be interpreted as the charger l serving node i by beamforming vector $u_{l,i}$. This approach does not require the instantaneous channel information and only depends on the channel covariance matrix. We use such beamforming vectors as the pre-determined beamforming vectors in the simulations, and will compare the performance with other selections of $\boldsymbol{u}_{l,i}$ in our numerical results.

V. DISTRIBUTED APPROACH

In the previous section, we have provided algorithms to solve the optimal beamforming and routing problems for the general case (Problem (3)) and for the pre-determined case (Problem (8)). These approaches are centralized, and require the collection of large amount of state information such as channel covariance matrices $\boldsymbol{K}_{l,i}, \forall l, i$ at the central decision maker. This makes the method not scalable for networks with large size and for the large number of antennas the chargers may have. Moreover, the centralized approach needs to solve the SDP problem with N_{ET} variables of size $M \times M$. The time complexity of such a problem is growing rapidly with N_{ET} and M as $O((L + N + N_{\mathrm{ET}}M)^{4.5})$ [29], which further hinders the scalability of the centralized approach. Thus, it is necessary to find a distributed approach that is scalable with

the size of network, especially with the number of the chargers and the antennas.

In this section, we provide distributed methods for the optimal beamforming and routing problems, based on the results achieved in the previous section. We will begin with the general case, and then move on to the pre-determined beamforming case.

A. Distributed solution for optimal beamforming

Recall that the optimal solution of Problem (4) can be found using the optimal solution of Problem (5). Therefore, we can achieve a distributed solution for Problem (4) if we can solve Problem (5) in a distributed manner. Notice that the overall network consists of two type of nodes: energy providers (i.e. chargers) and energy consumers (i.e. sensor nodes). Utilizing the ADMM method [9], we first decompose the problem into two parts based on this classification as described in the following.

We first introduce slack variables $z = \{z_i\}(z_i \geq 0)$ for Constraint (4d), such that the constraint becomes $B_i q - \eta \sum_{l=1}^{N_{\text{ET}}} \text{tr}[K_{l,i}, U_l] + z_i = 0, \forall i$. Thus, the partial augmented Lagrangian (using the scaled dual variable) is $L_\rho = -w_{\min} + 0.5 \sum_{i=1}^{N} \left(B_i q - \eta \sum_{l=1}^{N_{\text{ET}}} \text{tr}[K_{l,i} U_l] + z_i + v_i \right)^2$, where v are the scaled dual variables. The updates of the optimization variables are as follows:

For the energy consumers side, the updates are given by:

$$\left(w_{\min}^{(k+1)}, w^{(k+1)}, q^{(k+1)}, z^{(k+1)} \right)$$
$$= \arg \min_{w_{\min}, w, q, z} L_\rho(w_{\min}, w, q, z, U^{(k)}, v^{(k)}) \quad (11a)$$
$$\text{s.t.} \, w + A q = 0, \quad (11b)$$
$$w_{\min}, q, z \geq 0, w_i \geq w_{\min}, \forall i. \quad (11c)$$

This problem is a convex quadratic optimization problem with linear constraints, which can be efficiently solved by the off-the-shelf optimization tools [31–33] [5].

For the energy providers side, the updates are as follows:

$$U^{(k+1)} = \arg \min_U L_\rho(w_{\min}^{(k+1)}, w^{(k+1)}, q^{(k+1)}, z^{(k+1)}, U, v^{(k)})$$
$$(12a)$$
$$\text{s.t.} \, \text{tr}[U_l] \leq P_l, \forall l \quad (12b)$$
$$U_l \succeq 0, \forall l. \quad (12c)$$

Lastly, the scaled dual variables are updated at the sink node as follows:

$$v_i^{(k+1)} = v_i^k + B_i q^{(k+1)} + z_i^{(k+1)} - \eta \sum_{l=1}^{N_{\text{ET}}} \text{tr}[K_{l,i} U_l^{(k+1)}],$$
$$(13)$$

[5] Due to the convexity and differentiability, we can also apply primal-dual decomposition and sub-gradient approach to solve the problem distributedly [21], [24]. However, such an approach may suffer from low convergence rate and consume more energy while exchanging information among sensor nodes. However, for the proposed ADMM method, sensor nodes only update information in the outer loop, and it takes approximately 100 iterations of outer loops for convergence to optimal. Thus, the energy spent by the sensor nodes for information exchange is comparatively small, as will be further discussed in the simulations.

Notice that the decision variables of Problem (12) are N_{ET} semi-positive definite matrices of size $L \times L$. The dimension of the decision variables makes the problem complicated. We naturally hope to further decompose it into several subproblems such that each wireless charger makes beamforming decisions locally based on some shared information. We rewrite Problem (12) in the following form:

$$\min_U \, \sum_{i=1}^{N} \left(\eta \sum_{l=1}^{N_{\text{ET}}} \text{tr}[K_{l,i} U_l] - c_{\text{tr,i}}^{(k)} \right)^2 \quad (14a)$$
$$\text{s.t.} \, \text{tr}[U_l] \leq P_l, \forall l, \quad (14b)$$
$$U_l \succeq 0, \forall l, \quad (14c)$$

where $c_{\text{tr,i}}^{(k+1)} \triangleq B_i q^{(k)} + z_i^{(k+1)} + v_i^{(k)}$. Further, we introduce auxiliary variables $D \triangleq [d_1, d_2, \ldots, d_{N_{\text{ET}}}] \in \mathbb{R}_{N \times N_{\text{ET}}}^+$, whose element $d_{i,l}$ denotes the energy that node i should receive from base station l. Then, we can re-write Problem (14) as follows:

$$\min_{D,U} \, \|\sum_{l=1}^{N_{\text{ET}}} d_l - c_{\text{tr}}^{(k)}\|_2^2 \quad (15a)$$
$$\text{s.t.} \, \text{tr}[U_l] \leq P_l, \quad \forall l, \quad (15b)$$
$$\eta \, \text{tr}[K_{l,i} U_l] = d_{i,l}, \quad \forall l, i, \quad (15c)$$
$$U_l \succeq 0, d_{i,l} \geq 0, \quad \forall l, i. \quad (15d)$$

We further use ADMM to solve Problem (15) by relaxing Constraint (15c). The partial augmented Lagrangian of Problem (14) is as follows:

$$L_\rho(D, U, \mu) = \|\sum_{l=1}^{N_{\text{ET}}} d_l - c_{\text{tr}}^{(k)}\|_2^2$$
$$+ \frac{\rho}{2} \sum_{l=1}^{N_{\text{ET}}} \sum_{i=1}^{N} \left(\eta \, \text{tr}[K_{l,i} U_l^{(t)}] - d_{i,l} + \mu_{i,l}^{(t)} \right)^2,$$

where $\mu \in \mathbb{R}_{N \times N_{\text{ET}}}$ is the scaled dual variable.

Then, the sink node updates d by solving the following quadratic problem:

$$D^{(t+1)} = \arg \min_{d \geq 0} \left\{ \|\sum_{l=1}^{N_{\text{ET}}} d_l - c_{\text{tr}}^{(k)}\|_2^2 \right.$$
$$\left. + \frac{\rho}{2} \sum_{l=1}^{N_{\text{ET}}} \sum_{i=1}^{N} \left(\eta \, \text{tr}[K_{l,i} U_l^{(t)}] - d_{i,l} + \mu_{i,l}^{(t)} \right)^2 \right\}. \quad (16)$$

The problem is a quadratic optimization problem that can be efficiently solved with standard numerical techniques [31–33]. The sink node broadcasts the result $d^{(t+1)}$ to the chargers, and each charger l updates y_l by:

$$U_l^{(t+1)} = \arg \min_{U_l \succeq 0} \, \sum_{i=1}^{N} \left(\eta \, \text{tr}[K_{l,i} U_l] - d_{i,l}^{(t+1)} + \mu_{i,l}^{(t)} \right)^2$$
$$(17a)$$
$$\text{s.t.} \, \text{tr}[U_l] = P_l \quad (17b)$$

which is a convex problem and can be solved by off-the-shelf optimization tools [31–33]. Then, the charger l uploads
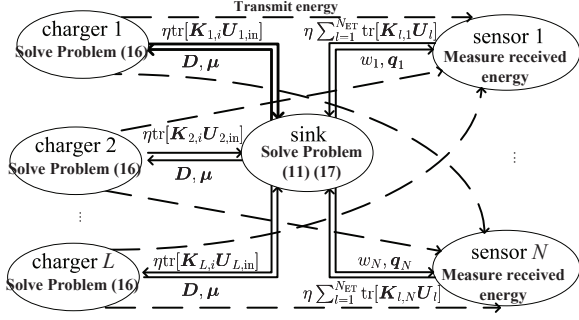
Fig. 2: Operations of the distributed approach for Problem (3), where the solid lines represent the transmission of the variables, and the dashed lines represent the transmission of energy.

$\eta \operatorname{tr}[\boldsymbol{K}_{l,i} \boldsymbol{U}_l^{(t+1)}], \forall i$ to the sink node, and the sink node updates $\mu_{i,l}$ as

$$\mu_{i,l}^{(t+1)} = \mu_{i,l}^{(t)} + \eta \operatorname{tr}[\boldsymbol{K}_{l,i} \boldsymbol{U}_l^{(t+1)}] - d_{i,l}^{(t+1)}. \quad (18)$$

The update of $\boldsymbol{d}$, $\boldsymbol{U}$, and $\boldsymbol{\mu}$ repeats until convergence. To summarize, the distributed algorithm is hierarchical and it consists of an outer loop and an inner loop. In the outer loop, the sink node determines sampling rate $w_{\min}, \boldsymbol{w}$, routing $\boldsymbol{q}$, and the variables $\boldsymbol{z}, \boldsymbol{v}$, whereas in the inner loop, the sink node updates $\boldsymbol{d}$ and $\boldsymbol{\mu}$, and each charger updates $\boldsymbol{U}_l$ locally. For the inner loop, we consider it converged if the difference of the values of the objective function in two consecutive iterations is smaller than a threshold, or the number of iterations exceeds a given threshold.

Recall the inner loop. Although we decompose it into more than two subproblems, the objective function of each subproblem is strongly convex. Thus, $\boldsymbol{U}^{(t)}$ achieved by the inner loops following the ADMM approach converges to the optimal solution for Problem (12). Furthermore, for the outer loops, the problem is decomposed into two parts, and each part achieves the optimal solution. Thus, $w_{\min}^{(k)}$ converges to the optimal solution, which gives us the following result:

Result 1: Assume that Problem (3) is feasible. $w_{\min}^{(k)}$ in the iteration of Algorithm 2 will converge to the $w_{\min}^*$, the optimal solution of Problem (3).

To summarize, the distributed approach is given in Algorithm 2, where Lines 11 to 16 are the inner loop for solving Problem (14) by ADMM to get $\boldsymbol{U}^{(k+1)}$. We can see that the chargers do not need to transmit the channel matrix $\boldsymbol{K}_{l,i}$ to the sink node, and the beamforming matrix $\boldsymbol{U}_l$ is determined locally by each charger. The operation of the algorithm in terms of variables exchanges and energy transfer is presented in Fig. 2. Due to the limited space, please refer to our technical report [30] for more discussions on the computation complexity and the amount of information exchange.

B. Distributed solution for pre-determined beamforming

The basic idea of the distributed solution to solve Problem (8) is similar to the one in Section V-A. Due to the limited space, we skip the details, which can be found in our technical report [30].

Algorithm 2 Distributed time-splitting beamforming algorithm

1: Sink node initializes $w_{\min}^{(0)}, \boldsymbol{w}^{(0)}, \boldsymbol{q}^{(0)}, \boldsymbol{z}^{(0)}, \boldsymbol{v}^{(0)}, \boldsymbol{d}^{(0)}$
2: Each charger l initializes $\boldsymbol{U}_l^{(0)}$, and transmits energy according to $\boldsymbol{U}_l^{(0)}$
3: Sensor nodes upload received energy $\eta \sum_{l=1}^{N_{\mathrm{ET}}} \operatorname{tr}[\boldsymbol{K}_{l,i} \boldsymbol{U}_l^{(0)}]$ to the sink
4: Sink node finds the optimal solution $\left(w_{\min,\mathrm{relax}}^*, \boldsymbol{w}_{\mathrm{relax}}^*, \boldsymbol{q}_{\mathrm{relax}}^*, \boldsymbol{U}_{\mathrm{relax}}^*\right)$ for Problem (5).
5: $k = 0$
6: **while** not converge **do**
7:     Sink node updates $w_{\min}^{(k+1)}, \boldsymbol{w}^{(k+1)}, \boldsymbol{q}^{(k+1)}, \boldsymbol{z}^{(k+1)}$ by solving convex quadratic Problem (11), and broadcasts $\boldsymbol{w}^{(k+1)}, \boldsymbol{q}^{(k+1)}$ to the sensors.
8:     Sink node updates $c_{\mathrm{tr},i}^{(k+1)} \leftarrow \boldsymbol{B}_i \boldsymbol{q}^{(k)} + z_i^{(k+1)} + v_i^{(k)}$
9:     Sink node initializes $\boldsymbol{\mu}^{(0)} \leftarrow \boldsymbol{0}, t \leftarrow 0$
10:     Each charger l initializes $\boldsymbol{U}_{l,\mathrm{in}}^{(t)} \leftarrow \boldsymbol{U}_l^{(k)}$
11:     Reset $t \leftarrow 0$
12:     **while** $\exists l, \boldsymbol{U}_{l,\mathrm{in}}$ not converge or $t \leq t_{\mathrm{threshold}}$ **do**
13:         Sink node updates $\boldsymbol{D}^{(t+1)}$ by solving Problem (16), and broadcasts $\boldsymbol{D}^{(t+1)}$ to the chargers
14:         Each charger l updates $\boldsymbol{U}_{l,\mathrm{in}}^{(t+1)}$ in parallel by solving Problem (17) and transmits $\eta \operatorname{tr}[\boldsymbol{K}_{l,i} \boldsymbol{U}_{l,\mathrm{in}}^{(t+1)}], \forall i$ to the sink
15:         Sink updates $\boldsymbol{\mu}^{(t+1)}$ according to (18), and broadcasts it to the chargers
16:         $t \leftarrow t + 1$
17:     **end while**
18:     Each charger l sets $\boldsymbol{U}_l^{(k+1)} \leftarrow \boldsymbol{U}_{l,\mathrm{in}}^{(t)}$ and transmits energy according to $\boldsymbol{U}_l^{(k+1)}$
19:     Sensor nodes upload received energy $\eta \sum_{l=1}^{N_{\mathrm{ET}}} \operatorname{tr}[\boldsymbol{K}_{l,i} \boldsymbol{U}_l^{(k+1)}]$ to the sink
20:     Sink node updates $\boldsymbol{v}^{(k+1)}$ according to (13)
21:     $k \leftarrow k + 1$
22: **end while**

VI. NUMERICAL RESULTS

In this section, we evaluate the monitoring performance of the WSN with the optimal energy beamforming (by Algorithm 1), the optimal pre-determined energy beamforming (see Section IV.B), and the method that does not use beamforming (omni-directional WET). Also, we will test the distributed approach (Algorithm 2). We use Matlab for the simulations.

In the simulations, we randomly deploy N sensor nodes in a region of size 30 meters by 30 meters, and a sink at the center to collect data. The distance between each pair of nodes $d_{i,j}$ is then determined by the position of the nodes. For a node v_i and its neighbor node v_j, the transmission power e_{ij}^O is $10^{-7} d^2$ Watts, and the data rate is 250kbps. The energy conversion efficiency η is 0.01^6. For the charger, its transmission power in WET is 1 Watts, and it has M antennas. The carrier frequency of the energy transmission is 915MHz. The channel model of energy transmission is considered as Rician fading model as [14], and is described as $\boldsymbol{g}_{l,i} = \sqrt{\beta_{l,i} K/(K+1)} \boldsymbol{g}_{l,i}^d + \sqrt{\beta_{l,i}/(K+1)} \boldsymbol{g}_{l,i}^s$, where $\boldsymbol{g}_{l,i}^d$ is a deterministic normalized vector representing the line-of-sight (LOS) path from charger l to node i, $\boldsymbol{g}_{l,i}^s \sim \mathcal{CN}(0, \|\boldsymbol{g}_{l,i}^d\|_2^2 \boldsymbol{I})$ is the scatter component,

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[6]We choose η as this value in order to not be over-optimistic, especially since in our case where the received power of the node is in the order of 10^{-4} to 10^{-5} Watts. This efficiency is consistent with the efficiency for low power levels [34].

TABLE II: Simulation Parameters

Parameter	Area size	Data rate	Frequency of WET	e_{ij}^O	P_l	η
Value	30 m × 30 m	250 kbps	915 MHz	$10^{-7}d^2$ Watts / (m^2·bit)	1 Watt(s)	1%

$\beta_{l,i}$ denotes the path loss that is determined by the distance $d_{l,i}$, K is the Rician factor, which is set to be either 100 to show the cases where LOS path is much stronger than the scatter component, or 4.5db (approximately 2.8) for the cases where the strength of LOS path is similar to that of the scatter component. Then, $\boldsymbol{K}_{l,i} = \beta K \boldsymbol{g}_{l,i}^d \boldsymbol{g}_{l,i}^{dH}/(K+1) + \beta \boldsymbol{I}/(K+1)$. The monitoring performance is defined as $F(\boldsymbol{w}) = \min w_i$. The parameters are summarized in Table II. In the following, we will discuss the simulation results in different cases.

A. Centralized case

1) Single charger with fixed N: To begin with, we consider the case where there is only one charger that transmits energy. The charger is also the sink that collects data. We fix the number of sensor nodes to be 15 and vary the number of antennas M that the charger has from 50 to 100. For each M, we simulate 200 times for different node locations. The results are shown in Fig. 3, where x-axis is the number of antennas and y-axis is the average minimum sampling rate of the sensor nodes. The blue line with circles represents the case of the optimal energy beamforming achieved by Algorithm 1. The green line with squares, and the red line with crosses represents the case of the pre-determined energy beamforming, and the case of non beamforming, respectively. We observe that, for both cases ($K = 100$ and $K = 2.8$), as the number of antenna increases, the monitoring performance of all schemes increases. The reason is that, with more antennas, the charger can form sharper beams. Thus, more energy can be received by the sensor nodes. It also shows that, the performance of the optimal beamforming approach is much better than that of the case without beamforming. This result illustrates the benefits of using beamforming instead of broadcasting the energy. Moreover, the performance of the optimal beamforming achieved by Algorithm 1 is slightly better than that of the pre-determined beamforming found by solving Problem (8), where the pre-determined beamforming vectors are the dominant eigenvector of $\boldsymbol{K}_{l,i}$s. Recall that Problem (8) can be turned into a linear optimization, whereas the optimal beamforming requires solving an SDP. The time complexity for the pre-determined beamforming is much lower than that of the optimal beamforming. Thus, pre-determined beamforming is a promising alternative for the optimal beamforming problem.

By comparing the performance with different values of Rician factor K, we observe that the minimum sampling rate is higher when K is smaller. The reason is that, with smaller K values, more energy could be harvested from scattering. Moreover, we also simulated the case of using the pre-determined beamforming without optimizing their power, as shown by the green dashed line with diamond markers in Fig. 3(b). We can see that, in such a case, the sampling rate is only slightly better than the energy broadcasting case. Thus, the power of the energy beams needs be optimized for a good performance.

2) Single charger with fixed M: In this case, we fix the number of antennas of the charger as 100, and change the number of sensors, N, from 15 to 100. For each N, we run the simulation 800 times with different sensor node locations. The simulation results are shown in Fig. 4, where x-axis is the number of sensor nodes, and y-axis is the minimum sampling rate of the sensor nodes. The solid lines correspond to the cases where the routing is optimized, whereas the dashed lines correspond to the cases where every node transmits data to the sink directly without any data relaying.

In general, the monitoring performance obtained by using energy beamforming is much better than the case of WET by broadcasting. Pre-determined energy beamforming is slightly worse than the optimal energy beamforming, which is similar to the case where we vary the number of antennas of the charger. If we check the monitoring performance ($F(\boldsymbol{w}) = \min_i\{w_i\}$) of the optimal routing cases, we can see that the trends of the optimal beamforming for both cases of K are decreasing with the number of sensor nodes when N is small, but increasing when N is large. The reason is that, with more sensor nodes, the energy received by each node in average decreases, whilst the energy consumption of a node may also reduce due to the decrease in the average data transmission distance. When N is small, the routing choices of the nodes are limited. Thus, the reduced received energy of each node is the major factor. However, when N is large, the network becomes dense. Consequently, each node has more choices for routing and the reduced energy consumption becomes the major factor that makes the minimum sampling rate increase. This explains why the performance of the optimal beamforming and the pre-determined beamforming scheme in no routing cases decreases with N.

We now consider the scenarios where no beamforming is performed. In these scenarios, the minimum sampling rate increases with N if routing is allowed. This is because the energy consumption is slightly reduced by using node relaying whereas the received energy does not change. When the nodes are only allowed to transmit data directly to the sink, the minimum sampling rate does not show any clear trend of increase with increasing N.

We also note that, in the case of $K = 2.8$, when N is very small, such as $N = 6$, the gap between the sampling rate achieved by the pre-determined beamforming and the sampling rate achieved by the optimal beamforming is not small. The possible reason is that, the chosen pre-determined beams are suboptimal. However, when N is large, it is more likely that the chosen beams are close to the optimal beams, which makes the gap smaller when N is large.

3) Multiple chargers with fixed M: We also test the case with multiple chargers. The setting is similar to the case with single charger. The difference is that we have four chargers, which are located at $(30,0)$, $(0,30)$, $(-30,0)$, and $(0,-30)$, respectively. Each charger has 100 antennas and has a power
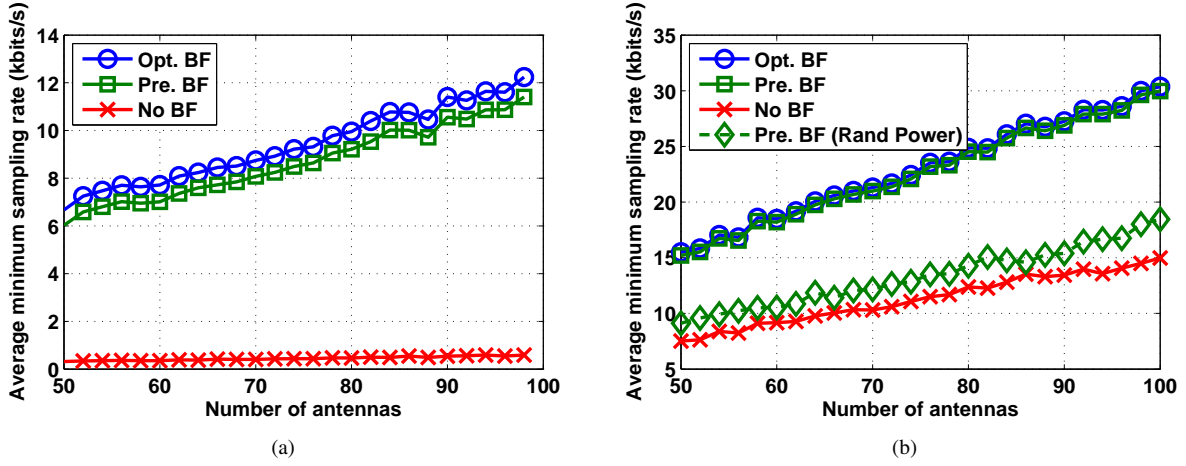
(a)

(b)

Fig. 3: Comparison of minimum sampling rate with varying number of antennas, achieved by optimal energy beamforming, pre-determined beamforming, and no beamforming, with the following parameters: (a) $N = 15, K = 100$, (b) $N = 15, K = 2.8$.
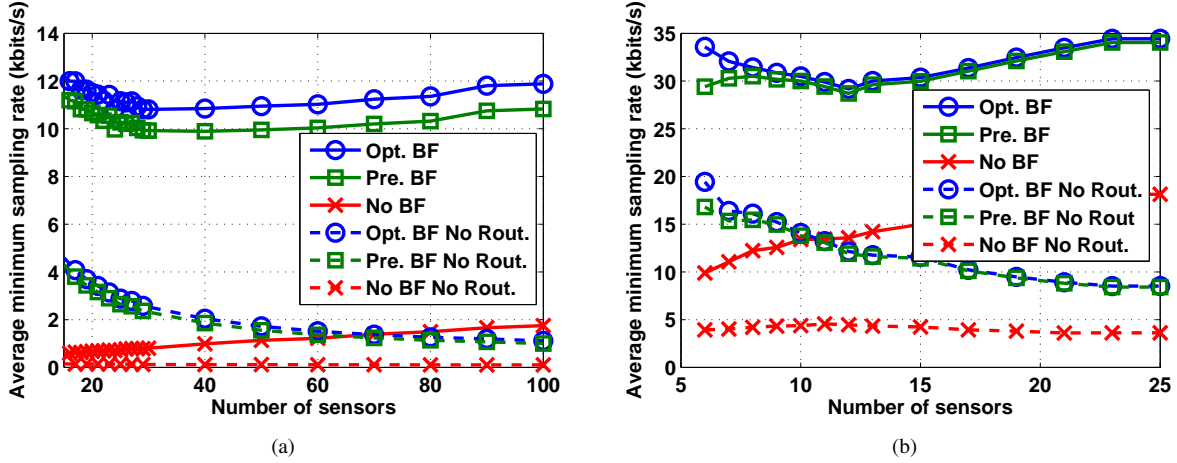

(a)

(b)

Fig. 4: Comparison of minimum sampling rate with varying number of sensors, achieved by optimal energy beamforming, pre-determined beamforming, and no beamforming, with the following parameters: (a) $M = 100, K = 100$, (b) $M = 100, K = 2.8$.

limit of $P_l = 1$ Watt. Due to the limited space, we only show the results for $K = 100$ from here on. The result, as shown in Fig. 5(a), is similar to the case with a single charger. More specifically, the performance of pre-determined beamforming is close to that of optimal beamforming, and both of them are much better than the case with only energy broadcasting. When data relaying is allowed, the average minimum sampling rate first decreases with N, and then increases later, for both beamforming scenarios. However, when data relaying is not used, the minimum sampling rate decreases with N.

To gain insight into the case where the number of antennas is small, i.e. $M \leq N$, we also run some simulations where the locations of the chargers are kept the same but the number of antennas are reduced to 10. The number of sensor nodes ranges from 15 to 60, and the result is shown in Fig. 5(b). We also present the performance of the scenario with pre-determined beams (Section IV-B). The green solid line with squares represents the cases where $u_{l,i}$ corresponds to the dominant eigenvector of the covariance matrix $K_{l,i}$. In this case, we have more pre-determined beams than the number of antennas. In such a case, the performance of the pre-determined beamforming is close to that of the optimal beam-forming. When we have an upper limit on the number of beams

that we can use, we set the pre-determined orthonormal basis beams (green dashed lines with squares) as the orthonormal basis for the range of $[v_{l,1}, \dots, v_{l,N}], \forall l$ using singular value decomposition (SVD), where $v_{l,i}$ is the dominant eigenvector of $K_{l,i}$. We also use random normalized vectors as the beam vectors for the pre-determined random beams case (green dotted line with squares). We observe that, if we limit the number of beam patterns that each base station can have as 10, the performance degrades. The performance achieved by the pre-determined orthonormal beams is approximately 3/4 of the performance achieved by the optimal beamforming, but the performance of the case with randomly chosen pre-determined beamformers is much worse. This indicates that, the performance of the pre-determined beamforming scheme greatly depends on the selection of the pre-determined beam vectors. A good choice of pre-determined vectors will lead to a smaller gap with the optimal solution, whereas a bad choice may lead to a performance more closer to the performance of the energy broadcasting case. Thus, how to select the pre-determined beams, especially for the cases where we have an upper bound on the number of such beams that can be used, is an interesting research direction to explore.

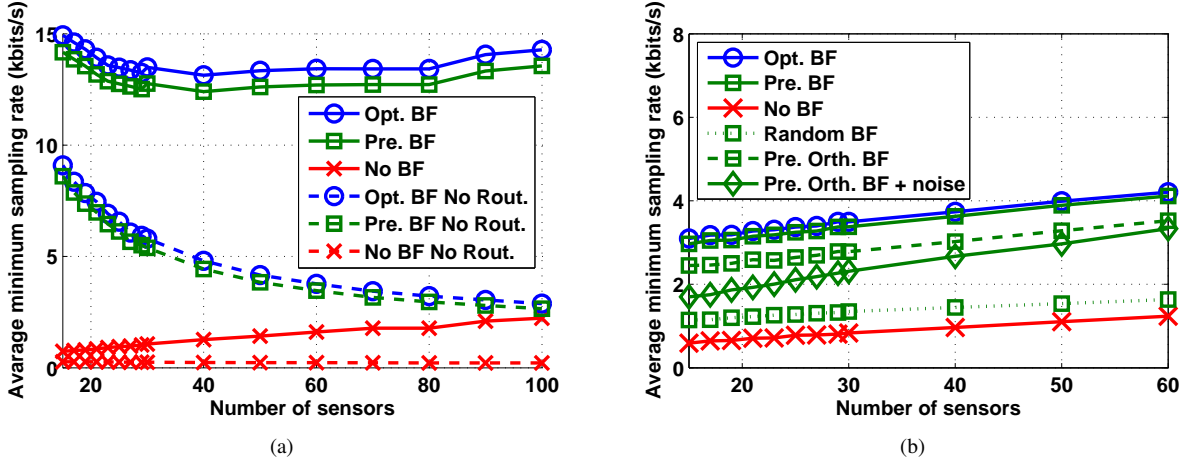We should also mention that, the performance of the pre-

(a)

(b)

Fig. 5: Comparison of the minimum sampling rates with varying number of sensors, achieved by optimal energy beamforming, pre-determined beamforming, and no beamforming from four chargers, with the following parameters: (a) M=100, K=100; (b) M=10, K=100.
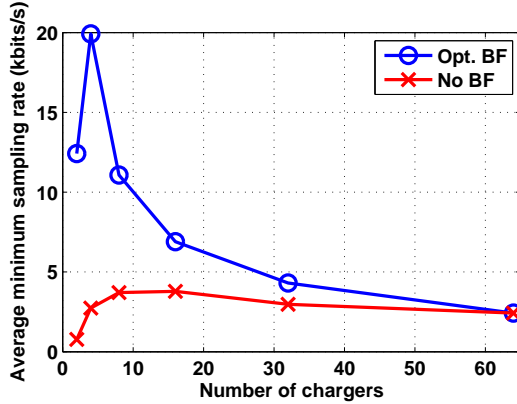

Fig. 6: Comparison of the minimum sampling rates with varying number of chargers and antennas achieved by optimal energy beamforming and no beamforming.

determined beamforming depends on the accuracy of the estimation of the covariance matrix of the channel, as shown by the green solid line with diamond shaped markers in Fig. 5(b). For the inaccurate covariance matrix cases, let $\hat{\boldsymbol{K}}_{l,i} \neq \boldsymbol{K}_{l,i}$ be the estimation of the channel covariance matrix in our hands, and $\hat{\boldsymbol{v}}_{l,i}$ be the dominant eigenvector of $\hat{\boldsymbol{K}}_{l,i}$. The line represents the case where $\|\boldsymbol{K}_{l,i} - \hat{\boldsymbol{K}}_{l,i}\|_F^2/\|\boldsymbol{K}_{l,i}\|_F^2 = 1/10$. We observe that, when $N = 15$, the sampling rate of the case is only approximately 60% of the case where the estimation of the channel covariance matrix is exact. However, such a performance degradation that dues to the inaccurate channel covariance matrix estimation becomes smaller when we have more sensor nodes.

4) Varying number of chargers and their antennas: Furthermore, we are interested to see whether we should use more chargers with a smaller number of antennas or a smaller number of chargers with a larger number of antennas. Consequently, we set the number of chargers as 2, 4, 8, 16, 32, and 64, and the corresponding number of antennas per charger as 32, 16, 8, 4, 2, and 1, respectively. This set of number of antennas and the number of chargers pairs are chosen for fair comparison between scenarios such that the total number of antennas of the wireless chargers is constant, which is

64, in all scenarios. All chargers are deployed on the circle with radius 30 meters centered at $(0,0)$ with equal angular difference, e.g., for the case with 8 chargers, they are located at $(0, 30), (15\sqrt{2}, 15\sqrt{2}), (30, 0), \ldots, (-15\sqrt{2}, 15\sqrt{2})$. For fairness, we set the total power the chargers transmit, $N_{\mathrm{ET}}P_l$, to be 8 Watts. In total 30 sensor nodes are deployed at a disk region with center $(0,0)$ and radius 15 meters. The result is shown in Fig. 6, where the blue curve with circles shows the minimum sampling rate of the optimal beamforming cases and the red curve with crosses shows the minimum sampling rate of the energy broadcasting cases. We can see that, when N_{ET} is small, the performance improves with increasing N_{ET}. The reason is that increasing N_{ET} makes the chargers cover the sensor nodes better, i.e., it is less likely that there is a node that is far away from all the chargers and become the bottleneck of the network. However, when N_{ET} exceeds a threshold, the performance degrades with the number of chargers. Let us consider the scenario with $N_{\mathrm{ET}} = 64$ chargers as an example. Since the number of antennas is 1, no beamforming can be performed and both strategies give the same performance. We conclude that, when N_{ET} is large, the effect of having fewer antennas on each charger becomes the dominating factor that reduces the energy a node can harvest. Consequently, when N_{ET} is larger than a certain number, which is 4 for the optimal beamforming in our case, the sampling rate reduces with N_{ET}.

To summarize, compared to the case without using energy beamforming, the monitoring performance improves significantly by using the optimal energy beamforming. Also, the performance of the pre-determined energy beamforming is slightly worse than the optimal energy beamforming, and the set of pre-determined beams affects the performance significantly. We also show that, the configurations of the wireless chargers, in terms of the locations and the number of the antennas, also play an important role in the network performance.

B. Distributed approach

We now focus on the distributed approach in this subsection. As expected, in our simulations we have observed that it is

possible to obtain the same optimum (approximately 10^{-5} relative gap) with the distributed approach. We now take a closer look to the convergence properties of the distributed approach.

1) Convergence of the distributed algorithms: In the simulations, ρ is chosen to be 0.2 to have a fast convergence. We first test Algorithm 2 for the optimal beamforming case. In the simulations, the number of wireless chargers is 4, and they are located at $(30, 0)$, $(0, 30)$, $(-30, 0)$, and $(0, -30)$. Each charger has 50 antennas. There are in total 15 sensor nodes deployed in the field. We first use Algorithm 1 to find the optimal solution $(w_{\min})$ for Problem (3), and use it as a reference for Algorithm 2. Then, we apply Algorithm 2 and record the relative difference of minimum sampling rates in each iteration (defined as $\|w_{\min}^{(k)} - w_{\min}^*\|/w_{\min}^*$). The result is shown in Fig. 7(a). The markers in circle correspond to the updates of in the outer loops (Lines 6 to 21) and the iterations count between two circles corresponds to the inner loop updates (Lines 11 to 16). For the inner loop, we consider it as converged if the values of the objective function in two consecutive iterations are smaller than 10^{-3} during the first 10 outer loop iterations and 10^{-4} for the remaining iterations. To have a faster convergence, we set the maximum number of iterations of inner loops per outer loop as 50. Thus, we can see that the sampling rates and routing update 12 times (and the beamformings update in total 600 times) such that relative difference of the sampling rate to the optimal sampling rate is approximately below 0.01. In the last several updates, the inner loops converge very fast (it takes one or two iterations). Therefore, the markers are very dense at the tail of the curve. After approximately 1200 updates of beamformings, the relative difference of the sampling rate achieved by Algorithm 2 to the optimum rate is below 10^{-4}. We can see that the resulting sampling rate converges to the optimum value.

Then, with the same parameter setting, we test the distributed approach for the pre-determined beamforming case. The convergence of the resulting sampling rate to the optimal value of Problem (8) is shown in Fig. 7(b). Here, the maximum number of iterations in an inner loop is set to be 100. We can see that, it takes about 500 updates of beamformings and 10 updates of sampling rates and routings, such that the difference is within 10^{-2}. After approximately 880 updates of beamformings and 25 updates of sampling rates and routings, the difference is below 10^{-4}. The convergence speed of the pre-determined beamforming case is faster than that of the optimal beamforming case. Recall that the sampling rate of the pre-determined case is close to that of the optimal case. Therefore, although the pre-determined case is suboptimal, it is still useful for us to achieve a close-to-optimal solution fast and distributedly.

2) Comparison of different distributed approaches: We also compare the convergence of the proposed ADMM based approach to a block descent based approach. More specifically, the block descent based approach iteratively updates as follows: 1) by fixing the energy beamforming U_l, update sampling rate w and routing q; 2) by fixing the sampling rate and routing, update energy beamforming. The result is shown
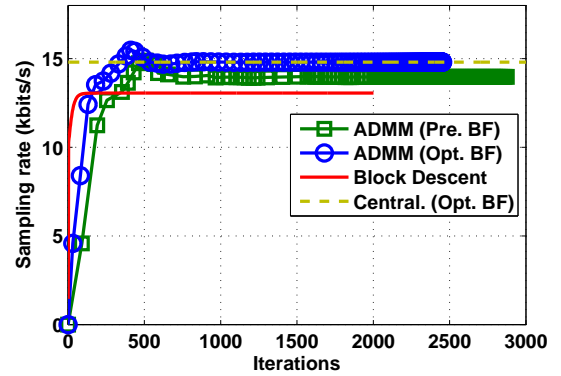

Fig. 8: Comparison of the convergence of different distributed approaches.

in Fig. 8. The blue line with circles represents the resulting sampling rates achieved by Algorithm 2 in each iteration; the green line with squares is the result of the ADMM approach for Problem (8) (see Section V-B); the red line is the result of the block descent based approach, where the wireless chargers iteratively update local energy beamforming and the sensor nodes update sampling rate and routing; the dashed line in yellow is the optimum achieved by the centralized approach. The result shows that, the sampling rate achieved by Algorithm 2 converges to the optimal solution, whereas the one achieved by Algorithm 2 adopted for Problem (8) converges to a suboptimal solution. We should also mention that, the sampling rate achieved by the block descent approach keeps increasing with the number of iterations. However, the increment is too small, which indicates a slow convergence rate. Recall that, for the ADMM approaches, the sensor nodes only exchange information in the outer loop of the distributed approach, and the number of outer loop iterations for convergence is small. However, for the block descent based approach, the sensors share informations in each iteration to update their sampling rates. Consequently, the sensor nodes with the ADMM approaches spend much less energy in exchanging information to update sampling rates than the case where they use block descent based approach. To summarise, Algorithm 2 is an efficient distributed approach for the joint beamforming and routing problem.

VII. CONCLUSIONS AND FUTURE WORK

We considered a wireless sensor network whose energy comes from wireless energy transmission from multiple wireless chargers. We investigated the problem of maximizing the minimum sampling rate of the nodes, by jointly considering routing and energy beamforming for wireless energy transmission. This set-up led to a non-convex problem formulation. We transformed the original problem to a semi-definite programming problem, for which strong duality holds. We proved that the optimal solution of the new problem is also the optimal solution of the original problem. Thus, we proposed an efficient algorithm to solve the original problem. We also provided a low-complexity scheme where the beamforming vectors are pre-determined. To offload the computation of the optimal solution, we further developed an
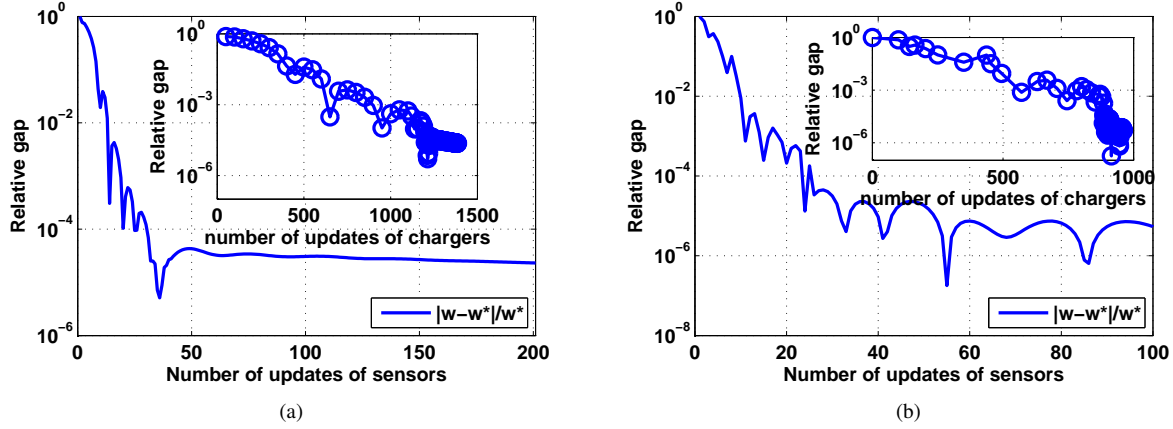
(a)

(b)

Fig. 7: The relative difference of the minimum sampling rate achieved by (a) the distributed approach (Algorithm 2); (b) the distributed approach (Algorithm 2 adopted for Problem (8)) with the optimum in each iteration.

efficient distributed algorithm. The simulation results showed that significant performance gains can be obtained by using the proposed optimal energy beamforming compared to non-optimized energy broadcasting. Moreover, the performance of the pre-determined energy beamforming scheme is observed to be only slightly worse than the optimal case, which suggests that it could be a good substitute for the optimal beamforming.

An interesting future research direction is to consider the scenario where the energy beam vectors have further restrictions, such as the number of different beam vectors that can be employed. In addition, we will also consider energy sharing between sensor nodes and optimization of the location of the wireless chargers.

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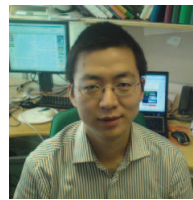

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