Seakeeping enhancement by lengthening a ship

RÉMI CLAUDEL
MASTER DEGREE PROJECT
SEAKEEPING ENHANCEMENT BY LENGTHENING A SHIP

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Abstract

In this study, a tentative assessment of a passive solution for pitch decrease, namely the increase in length of the studied ship, is made. The hull form of the lengthened version of the ship is derived from the reference hull form after utilization of Lackenby’s sectional area curve transformation through a prismatic coefficient change (Reference [3]), and utilization of a sectional area curve “swinging” induced by a change of longitudinal position of the centre of buoyancy. Following this, and after a complementary mass estimate of the lengthened version, seakeeping calculations are made and show a significant decrease in pitch, from almost 35% for low sea states to 20% for relatively high sea states. To conclude this study, operability for classic NATO frigate missions have been calculated and the decrease in pitch induces a slight gain in operability for the lengthened version.
Acknowledgments

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## Table of contents

1 LIST OF SYMBOLS ................................................................................................................. 7

2 INTRODUCTION .................................................................................................................... 9
  2.1 Project Topic ....................................................................................................................... 9
  2.2 Motivation ........................................................................................................................ 9
  2.3 State of the art .................................................................................................................. 9
  2.4 Study steps ....................................................................................................................... 10

3 LENGTHENED HULL FORM .............................................................................................. 11
  3.1 Reference ship ................................................................................................................... 11
  3.2 “Frigate+” hullform main characteristics calculations ....................................................... 11
  3.3 Sectional area curve transformation .................................................................................. 13
     3.3.1 Transformation by variation of the prismatic coefficient ............................................ 13
     3.3.2 Transformation by variation of the centre of buoyancy ............................................. 16
  3.4 Hull form transformations .................................................................................................. 18
     3.4.1 Modification of cross section shapes ........................................................................ 19
     3.4.2 Modification of the cross section longitudinal positions ......................................... 21
  3.5 Structure .......................................................................................................................... 22
     3.5.1 Midship section strengthening .................................................................................. 22
     3.5.2 Displacement estimate ............................................................................................. 24

4 SEAKEEPING AND OPERABILITY ..................................................................................... 26
  4.1 Precal_R seakeeping results ............................................................................................. 26
     4.1.1 Transfer functions .................................................................................................... 27
     4.1.2 Pitch ......................................................................................................................... 31
  4.2 Calculations et Formulas used in Precal_R output data post processing ............................ 35
     4.2.1 Wave spectrum ........................................................................................................ 35
     4.2.2 Motions and derivatives ........................................................................................... 37
     4.2.3 Emergence and Slamming ....................................................................................... 38
     4.2.4 Motion Sickness Incidence (MSI) .......................................................................... 40
     4.2.5 Motion Induced Interruptions (MII) ....................................................................... 41
     4.2.6 Lateral Force Estimator .......................................................................................... 42
  4.3 PRECAL_R post processing Excel tool ............................................................................ 43
  4.4 Software limitations .......................................................................................................... 47
     4.4.1 Transfer functions values ....................................................................................... 47
     4.4.2 Roll damping fins implementation ........................................................................... 49
4.5 Operability results
4.5.1 Pitch
4.5.2 Transit and Patrol mission
4.5.3 Speed maintaining in heavy seas
4.5.4 Helicopter and drone operations
4.5.5 Helicopter and drone handling and helicopter armament
4.5.6 Crafts launch by the side
4.5.7 Replenishment at sea
4.5.8 Replenishment at sea by helicopter
4.5.9 Vertical launchers operation
4.5.10 Torpedo tubes operation
4.5.11 Main gun operation

5 RESISTANCE
5.1 Using existing reference measures
5.2 Fung Method
5.3 Speed and Power

6 COSTS

7 CONCLUSION

8 REFERENCES

9 APPENDICES
9.1 Appendix 1: Proof of the variation of LCB via a «swinging» of the sectional area curve
9.2 Appendix 2: Formulas for the deformation of a section
9.3 Appendix 3: Hull girder strengthening effective length
1 List of Symbols

\( A_m \) Midship section underwater area
\( B \) Waterline breadth
\( B_{oa} \) Overall breadth
\( C \) Depth
\( C_b \) Block coefficient
\( C_m \) Midship section coefficient
\( C_p \) Prismatic coefficient
\( C_{WS} \) Wetted surface coefficient
\( D_1 \) Longitudinal motion
\( D_2 \) Lateral motion
\( D_3 \) Vertical motion
\( F_n \) Froude number
\( g \) Gravitational acceleration
\( h \) Height of an operator’s centre of gravity
\( H \) Significant wave height
\( IE \) Deadrise angle
\( KG \) Vertical position of the centre of gravity of the ship relatively to the keel
\( l \) Half width between an operator’s feet
\( L \) Waterline length
\( L_{aft} \) Length aft of the midship section
\( L_{for} \) Length forward of the midship section
\( L_{oa} \) Overall length
\( L_{pp} \) Length between perpendiculars
\( LCB \) Longitudinal position of the centre of buoyancy from the aft perpendicular
\( LCG \) Longitudinal position of the centre of gravity from the aft perpendicular
\( LFE \) Lateral Force Estimator
\( M_f \) Bending moment
\( MII \) Motion Induced Interruptions
\( MSI \) Motion Sickness Incidence
\( P \) Power
\( R_f \) Viscous resistance
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<tbody>
<tr>
<td>Rₚ</td>
<td>Residuary resistance</td>
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<tr>
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<td>Specific resistance</td>
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<td>Root Mean Square</td>
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<td>S</td>
<td>Midship section structure cross section area</td>
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<td>Wave spectrum</td>
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<td>ωₑ</td>
<td>Encounter frequency</td>
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2 Introduction

2.1 Project Topic

The main objective of this project is to assess the seakeeping, operational and cost impact of increasing the length of a ship beyond that which results from a traditional spiral design process whereby a ship inner volume is solely defined by its systems physical integration requirements.

It is proposed that this increase in length is achieved by adding to a ship platform a forward empty section which sole purposes are to increase the ship waterline length and its forward reserve of buoyancy. It is not to be outfitted; the ship internal arrangement remains within the length required for physical integration and constraints.

This study has been undertaken by establishing a comparison between a parent (reference) ship and a lengthened version such as described above. The other ship parameters are not changed or, if it cannot be avoided, the least possible.

This length increase leads to a decrease in heave and pitch and so to better seakeeping characteristics (less slamming, propeller emergence, green water on deck…) which allows the ship to operate at higher speeds in heavy seas. However, this modification incurs a building cost increase. So, in addition to quantifying the seakeeping characteristics enhancement of the modified ship; construction and operational cost increase are also estimated in order to answer the question: does seakeeping improvement justify cost increases?

2.2 Motivation

Speed at sea is imperative for the diverse missions of the French Navy as it is detailed by Frigate Captain Laurent Célérier (Reference [1]). Be it for maritime traffic protection via anti-piracy patrols, for natural resources protection via the interception of poachers and terrorists, for the fight against illegal traffic, for the reaction to crisis outbreaks or during stabilization stages or simply for self-defense, the maximum speed that a naval ship can achieve is a key operational factor. Right now, according to him, this speed is satisfactory in calm water. In heavy seas, however, ships cannot reach this same maximum speed. This explains the interest for a ship that would have a better seakeeping and could therefore travel faster in heavy sea and keep on operating in heavier seas than they currently can. This internship project aims to respond to French Navy demand.

2.3 State of the art

Studies on the effect of the increase of the length of a reference ship on its operability have previously been undertaken and their outcome published. Keuning has done a lot of research around the “Enlarged Ship Concept” (Reference [6], [7] and [8]) and presents the results of a trade-off analysis between seakeeping characteristics and combined production and operational costs. The results are encouraging; in spite of the expected construction cost increase incurred by increasing the ship length, overall life costs have been found to be reduced due to the reduction of the operational cost which exceeds the initial investment.
No publication of similar studies for frigates and corvettes has been found or with very few data with the exception of Reference [10] which details the lengthening of a destroyer, through length to breadth ratio adjustment, which leads to a significant decrease in heave and pitch of 20% for head seas at full speed. A Naval Group study on the “jumboisation” of a frigate, i.e. the increase of its length and internal volume by the addition of a watertight section amidships included a comparative seakeeping assessment of the pre and post jumboisation versions of the ship. However, the jumboisation relative length increase was much lower than those described in Reference [4] and the study outcome was not found to be useful for this project.

2.4 Study steps

The different steps of the study are the following:

- Choice of a reference ship, of the new increased length and of the reference ship characteristics that should not be modified for the lengthened ship

- Re-designing the enlarged ship: of its main characteristics (other than the ones that should not be modified when compared to the reference ship), hull shape transformation (sectional area curve and sections modifications), displacement re-estimation via the necessary structure strengthening of the hull

- Seakeeping and operability evaluation

- Resistance evaluation

- Costs (preliminary) evaluation

Due to the confidentiality of Naval Group data used for this study, all results provided in this report have been normalised. Regression formulas used are not given in details either, only the variables in the formula are given. For example, if $y$ is given by a regression formula containing $x_1$ and $x_2$, the regression formula will be written $y = f(x_1, x_2)$. 
3 Lengthened hull form

3.1 Reference ship

The reference ship, specified by the head of Naval Group surface ship design department, is a frigate built by the Company which principal characteristics are the following:

- Overall length: 125 m (approximately)
- Overall breadth: 15 m (approximately)
- Displacement: 4000 t (approximately)

This reference ship will be referred to as “Frigate” in the following sections of this report and the lengthened version as “Frigate+”.

3.2 “Frigate+” hull form main characteristics calculations

An increase in length is decided for Frigate+ but there are still several other parameters that need to be before obtaining its hull form. Some of these parameters are taken to be the same as those of the reference ship. As for the others, they are either calculated through naval architecture equations or through regression formulas.

The Frigate+ main characteristics that are set are the following:

- Waterline length $L = \text{Reference frigate waterline length} + \text{added length}$
- Waterline breadth $B = \text{Reference frigate waterline breadth}$
- Depth $C = \text{Reference frigate depth}$
- Midship coefficient $C_m = \text{Reference frigate } C_m$
- Midship section area $A_m = \text{Reference frigate } A_m$

$B$, $C_m$ and $A_m$ being the same as those of the reference ship, the draught $T$ is also unchanged since $C_m = \frac{A_m}{BT}$

The goal is to define a new hull form which is done by using Lackenby’s method (Reference [3]). This method modifies the sectional area curve once a new prismatic coefficient is defined. It is this new prismatic coefficient that requires to be determined.

$$C_p = \frac{\nabla}{A_mL}$$

The new length is set and the amidships sectional area is taken to be that of the reference ship; the displaced volume $\nabla$ is thus the only parameter left to be estimated in order to calculate $C_p$.

Since the length is the only main characteristics of reference ship that is modified, the only change to the weight breakdown of the reference ship is the “hull and structure” Weight Group (WG) which is designated $\Delta_{2110}$ (it is the Weight Group 2110 in Naval Group documents). This is true because only the waterline length has been increased and not the length of accommodated part of the ship (the lengthened part remains empty in this concept). The new displacement will then simply be the reference ship displacement minus the “old” $\Delta_{2110}$ to which is added the “new” $\Delta_{2110}$. This new $\Delta_{2110}$ is calculated by a regression formula:
\[ \Delta_{2110} = f(L, B_{oa}, C, \Delta) \]

Since this regression formula depends on the total displacement, it is necessary to iterate the calculations until the total displacement value converges. It is then given by:

\[ \Delta_{new} = \Delta_{old} - \Delta_{2110\ old} + \Delta_{2110\ new} \]

After transforming the sectional area curve with a required Cp it is also necessary to transform it with a required longitudinal position of the centre of buoyancy LCB to match the variation of LCG. It is thus necessary to determine the LCG and the KG for subsequent stability analyses. As for the mass, only the block 2110 changes so we only need to calculate LCG_{2110} et KG_{2110}. KG_{2110} is considered to be unchanged since the regression formula used depends only on the depth C. LCG_{2110} is calculated thanks to by a regression formula:

\[ LCG_{2110} = f(L_{pp}) \]

We can then obtain KG and LCG by

\[
\begin{align*}
KG_{new} &= \frac{KG_{old}\Delta_{old} - KG_{2110\ old}\Delta_{2110\ old} + KG_{2110\ new}\Delta_{2110\ new}}{\Delta_{new}} \\
KG_{new} &= \frac{KG_{old}\Delta_{old} + KG_{2110\ old}(\Delta_{2110\ new} - \Delta_{2110\ old})}{\Delta_{new}} \\
LCG_{new} &= \frac{LCG_{old}\Delta_{old} - LCG_{2110\ old}\Delta_{2110\ old} + LCG_{2110\ new}\Delta_{2110\ new}}{\Delta_{new}}
\end{align*}
\]

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<th>Frigate+</th>
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<td>B (m)</td>
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<td>Boa (m)</td>
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<td>1.00</td>
</tr>
<tr>
<td>T (m)</td>
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<td>1.00</td>
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<td>C (m)</td>
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<td>C_{p}</td>
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### Normalised Characteristics

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<td>KG2110 (m)</td>
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</tr>
<tr>
<td>LCG (m)</td>
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</tr>
</tbody>
</table>

**Table 1: Reference frigate and lengthened frigate normalised main characteristics**

### 3.3 Sectional area curve transformation

An Excel VBA (Visual Basic for Applications, the Excel programming language) software routine has been developed in order to obtain the Frigate+ sectional area curve using Lackenby’s method (Reference [3]). The routine main input data are the reference frigate sectional area curve coordinates; its main output data are the modified sectional area curve coordinates in a “.txt” file.

#### 3.3.1 Transformation by variation of the prismatic coefficient

This transformation, only applicable for ships without a parallel middle body, is detailed in Reference [3]. (Reference [3]). It comprises the following three steps:

- The first: split and normalization of the sectional area curve
- The second: adjustment of the sectional area curve
- The third: setting back to scale of the sectional area curve

The validation of this method is provided in Lackenby’s publication.

**Step 1**

First of all, the sectional area curve is split into two parts: from x = 0 to amidships and from amidships to x = L. For each of these two halves, the y coordinates are divided by the midship section underwater area Am so that the y coordinates for each half vary from 0 to 1. The x coordinates are divided by the length corresponding to each half so that the x coordinates for each half vary between 0 and 1.

So we have the following transformations:

\[
x_1 = \frac{x}{L_{aft}}
\]
For $x$ forward of the midship section: $x_1 = \frac{x - L_{aft}}{L_{for}}$

For all $x$: $y_1 = \frac{y}{A_m}$

With:

- $L_{aft}$ = length of the first part of the sectional area curve (which corresponds to the longitudinal position of the amidships section)
- $L_{for}$ = length of the second one ($L_{for} + L_{aft} = L$).

**Step 2**

For each of the two parts of the sectional area curve, the transformation is the same:

$$x_2 = x_1 + \frac{\delta \Phi}{\Phi} x_1 (1 - x_1) \frac{1}{(1 - 2\bar{x}_1)}$$

With:

- $\bar{x}_1$ = $x$-coordinate of the centroid of the corresponding half of the sectional area curve (after normalization)
- $\Phi$ = area under the curve of the normalized sectional area curve ($\Phi_{aft}L_{aft}A_m + \Phi_{for}L_{for}A_m = V$)
- $\delta \Phi$ = the required variation of area under the normalized curve.

![Figure 1: Lackenby's sectional area curve variation method illustration (forward part)](image)

We choose $\delta \Phi_{aft} = \delta \Phi_{for}$.

By choosing the change of aft and forward lengths proportional to the one of the total length, that is to say $L_{aft_{nuew}} = \frac{L_{new}}{L} L_{aft}$ and $L_{for_{nuew}} = \frac{L_{new}}{L} L_{for}$, we have:
So we choose \( \delta \Phi_{\text{aft}} = \delta \Phi_{\text{for}} = \delta C_p \).

It is to be noted that for a \( \delta C_p > 0 \), we have, as expected, an increase in the area under the curve forward of the amidships section as shown in Figure 1. It also occurs aft of the amidships section. Indeed, aft we have \( x_2 - x_1 = \delta x < 0 \) since \( (1 - 2 \bar{x}_{1\text{ aft}}) < 0 \) whereas \( (1 - 2 \bar{x}_{1\text{ for}}) > 0 \).

**Step 3**

The sectional area curve is put back to scale by the following operations:

- For \( x \) aft of the midship section: \( x_3 = x_2 L_{\text{aft new}} \)
- For \( x \) forward of the midship section: \( x_3 = x_2 L_{\text{for new}} + L_{\text{aft new}} \)
- For all \( x \): \( y_3 = y_2 A_{m \text{ new}} (= y_2 A_m \text{ in our case}) \)

**Figure 2:** Illustration of the sectional area curve transformation by \( C_p \) variation
Figure 3 shows the Excel tool I coded to use this sectional area curve transformation. There are four command buttons on this sheet:

- “Clear Excel Sheet” deletes the curves coordinates, the plot and the input parameters. It clears all data but the layout of the sheet is retained.
- “Import sectional area curve” is used to import the old sectional area curve (on the left) from a text file.
- “Calculate new sectional area curve” is used to calculate the coordinates of the new sectional area curve. The input data for this calculation are the reference ship sectional area curve coordinates and the Cp variation required. The red parameters (Cp variation) are mandatory whilst the blue ones (new length, new midship section area) are not as they are set to a default value. The new sectional area curve coordinates are displayed to the right of the screen and both the old and the new sectional area curves are shown on the same graph.
- “Export new sectional area curve” is used to export the new sectional area curve coordinates to a text file.

3.3.2 Transformation by variation of the centre of buoyancy

This transformation is necessary to adjust the variation of the LCB compared to the variation of Frigate+ LCG. The LCG is determined by a regression formula and the variation of LCB is obtained by the transformation of the sectional area curve to meet the new prismatic coefficient.

This transformation, called « swinging » of the sectional area curve is:

\[ x_{\text{new}} = x + \delta x = x + y \tan \theta \]

Where:
- $y$ is $y$ coordinate of the sectional area curve
- $\tan \theta = \frac{\delta LCB}{\bar{y}}$
- $\bar{y}$ is the $y$ coordinate of the centroid of the sectional area curve

$\theta$ corresponds to the sweeping of angle of each point going from $x$ to $x_{\text{new}}$ (see Figure 4).

**Figure 4: Sectional area curve swinging illustration**

The proof of this method is not provided in Reference [3]; one is thus proposed in Appendix 1.

**Figure 5: Illustration of the sectional area curve transformation by LCB variation**
Figure 6: Excel tool for sectional area curve transformation by LCB variation

Figure 6 shows the Excel tool I coded to use this sectional area curve transformation. It is identical to the tool for the Cp variation transformation.

This method has the property of conserving the amidships section area (it only induces a δx), the length (δx null for x = 0 and x = L) and the displacement. Since these three characteristics are unchanged, so is the prismatic coefficient.

This transformation does not interfere with the previous one. However, the order in which the two transformations are applied is important. If this transformation was first applied and the transformation related to the variation of Cp applied in second, then:

- The resulting $C_p$ would be the one required since the transformation related to the LCB does not affect $C_p$.
- The resulting LCB would not be, however, the one required as the $C_p$ variation related transformation also leads to a change in LCB.

3.4 Hull form transformations

Once the new sectional area curve has been obtained by application of Lackenby’s method, the reference hull form shape needs to be modified so that its sectional area curve corresponds to this new sectional area curve. This can be done by modifying the cross sections representing the hull. These cross sections are defined by their shapes and by their longitudinal positions. There are thus two possible choices:

- Modifying the sections shapes
- Modifying the sections positions
A possible third choice would be modifying both at the same time but we would have to choose which relative weight to give to the modification of the cross section shapes compared to the modification of positions. Moreover, that would make two transformations instead of one and there does not seem to be real value to it in our case so this method has not been studied.

3.4.1 Modification of cross section shapes

It is the first method that has been studied since a study on this transformation had already been undertaken at Naval Group by Benoît Quesson (Reference [2]) in 2104. Improvements of this method have been made in this study in order to:

- Enhance the accuracy of the new cross sectional shapes obtained by the calculations
- Ensure that there are no restrictions to the possible cross sectional shapes
- And, most of all, ensure that the required shape is correctly generated (e.g. no discontinuity or local aberrations)

The idea behind this deformation is to make it so that the underwater part of the new section is a transformation of only the underwater part of the old section and, similarly, that the above water part of the new section is a transformation of only the above water part of the old section.

So this method takes as input a reference section with its draught and depth, a new waterline breadth, a new overall breadth, a new draught, a new depth and a new underwater area and gives as output the new section.

This transformation of a section consists in a multiplication of the two parts (underwater and above water) of the old section by a polynomial in \( z \) of degree 2 for the underwater part and degree 1 for the above water part and in an adjustment of the \( z \) coordinates related to the new values \( T_{\text{new}} \) et \( C_{\text{new}} \). This can be written:

- For \( z \leq T \): \( B_{\text{new}}(z_{\text{new}}) = B(z)(c_1z_{\text{new}}^2 + c_2z_{\text{new}} + c_3) \)
- For \( z \geq T \): \( B_{\text{new}}(z_{\text{new}}) = B(z)(az_{\text{new}} + b) \)

The three conditions used to determine \( c_1 \), \( c_2 \) and \( c_3 \) are related to the function \( B_{\text{new}}(z_{\text{new}}) \) while \( z_{\text{new}} \) varies between 0 and \( T_{\text{new}} \). They are:

1. Underwater area of the new section equals the required area (improvement of existing work)
2. Continuity in \( z_{\text{new}} = T_{\text{new}} \) of the new section that is to say no discontinuity in the hull (condition already used in existing work)
3. Continuity in \( z_{\text{new}} = T_{\text{new}} \) of the slope of the new section that is to say no hard chine in the hull (improvement of existing work)

The two conditions used to determine \( a \) and \( b \) are:

1. Waterline breadth satisfied: \( B_{\text{new}}(T_{\text{new}}) = B_{\text{new}} \)
2. Overall breadth satisfied: \( B_{\text{new}}(C_{\text{new}}) = B_{oa \text{ new}} \)

For this method, the overall breadth \( B_{oa} \) refers to the breadth at a height equal to the depth of the ship.
Figure 7: Illustration of a section transformation

The complete calculations and equations regarding this transformation are detailed in Appendix 2.

Figure 8: Excel tool for section transformation

Figure 8 shows the Excel tool I coded to use this section transformation. It is similar to the tools for the sectional area curve transformations except that there is an additional import command button. Indeed, this transformation requires not only the old section coordinates but the new sectional area required which is calculated here by interpolation using the new sectional area curve coordinates (which is imported with the additional import command button). Modification of the cross section longitudinal positions.
3.4.2 Modification of the cross section longitudinal positions

This second method consists in changing the longitudinal position of a cross section so that its sectional area matches the area given by the new sectional area curve at this new longitudinal position. In Section 3.3, the sectional area curve longitudinal positions have been modified. If we apply these longitudinal positions modifications to the corresponding cross sections we will obtain a hull form which sectional area curve corresponds to the new sectional area curve.

This second method is the one used in this study. Indeed, the first one is far too time consuming since the reference hull shape has to be modified point by point whereas for the second method, only longitudinal translations of whole sections are used. However, this second method can only be used in this study because the breadth, draught and depth are unchanged between Frigate and Frigate+. Otherwise, the first method would need to be used.

Figure 9: Frigate and Frigate+ hull shape views
3.5 Structure

3.5.1 Midship section strengthening

The ship structure study has been conducted using “MarsMili”, a software application developed by Bureau Veritas. It only deals with the hull girder midship section since it is the part of the ship to which the highest longitudinal bending moment is applied and, subsequently, where the highest longitudinal stresses develop. It is serves as a validation of the mass estimate made earlier in the study.

Half of the hull girder midship section of the (reference) Frigate is modelled, which the software then mirrors about its vertical axis for the calculations. However, there exists a port-starboard dissymmetry in the Frigate midship section; there is a 2380 mm wide port opening and an 800 mm wide starboard opening in the main deck and weather deck. This also leads to there being two additional stiffeners starboard when compared to port. We were only interested in the neutral axis position and the structure cross-sectional area and 2nd moment of inertia with respect to the y-axis and so it has been chosen that a mean of the two openings would be used in the half midship section model. Overall, between the reference frigate structure and its model the structure is conserved, only the y coordinates of parts of decks and of one stiffener change compared to the real midship section which has no effect on the neutral axis, structural area and the moment of inertia with respect to the y-axis.

![Hull girder midship cross-section scantlings](image)

**Figure 10 : Hull girder midship cross-section scantlings**

The design maximum stress criteria for the Frigate midship section is 100 MPa. For the design of the Frigate+ hull girder cross-section it was decided to keep the same structural design criteria (stress lower than 100 Mpa) and the position of Frigate cross-section neutral axis.
The bending moment is calculated with the formula:

\[ M_f = \frac{\Delta L_{oa} g}{m} \]

Where \( m \) is a coefficient computed using formulas defined in Naval Group structural calculation manual and which is usually around 25. The moment of inertia and the resisting surface are given by MarsMili. The stresses in the lowest part and the highest part of the section are then given by:

\[ \sigma_{lp} = \frac{M_f}{I} D \]
\[ \sigma_{hp} = \frac{M_f}{I} (C - D) \]

Where:
- \( \sigma_{lp} \) is the stress in the lowest part
- \( \sigma_{hp} \) is the stress in the highest part
- \( C \) is the depth of the ship
- \( D \) is the height of the neutral axis.

It was also decided to only change the thickness of the plates to strengthen the midship section and to keep the stiffeners unchanged.

Table 2 shows the different structural data for Frigate and Frigate+ (Designed Frigate, i.e not the actual Frigate but its design stage, has a displacement lower than the actual Frigate that is why the displacement is 0.936 and not 1).

<table>
<thead>
<tr>
<th></th>
<th>Designed Frigate</th>
<th>Frigate+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement used for the bending</td>
<td>0.936</td>
<td>1.136</td>
</tr>
<tr>
<td>moment calculation ( \Delta ) (t)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bending moment / g (t.m)</td>
<td>0.936</td>
<td>1.404</td>
</tr>
<tr>
<td>Midship section structure cross</td>
<td>1</td>
<td>1.576</td>
</tr>
<tr>
<td>section area ( S ) (m²)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neutral axis height ( D ) (m)</td>
<td>1</td>
<td>1.003</td>
</tr>
<tr>
<td>Transverse moment of inertia with</td>
<td>1</td>
<td>1.496</td>
</tr>
<tr>
<td>respect to the neutral axis ( I ) (m²)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stress in lowest part (MPa)</td>
<td>1</td>
<td>1.005</td>
</tr>
</tbody>
</table>

Table 2 : Midship section normalised structural data
3.5.2 Displacement estimate

This structural strengthening due the increase in length and displacement (estimated with a regression formula at the beginning of study, as explained in Section 3.2) can then be used to calculate the new displacement (design iteration). If this new displacement calculation is different from the one previously used to calculate the bending moment then an iteration on the structural strengthening is necessary with this new displacement as a basis for the bending moment calculation.

It is estimated that the increase in mass due to strengthening is proportional to the increase in midship section structure cross section area. The ratio of the new structure mass over the old one is then taken as the ratio of the new midship section structure cross section area over the old one. If we consider the structure strengthening over the whole length of the ship then:

\[ \Delta_{\text{new}} = (\Delta - \Delta_{2110}) + \Delta_{2110} \frac{L_{\text{oa new}}}{L_{\text{oa}}} S_{\text{new}} \]

However, only the longitudinal structural members have been reinforced since they are the only ones modelled in our MarsMili midship section. Their overall weight approximately amounts to 70% of Weight Group 2110 which leads to the following modified formula:

\[ \Delta_{\text{new}} = (\Delta - \Delta_{2110}) + \Delta_{2110} \frac{L_{\text{oa new}}}{L_{\text{oa}}} \left(0.3 + 0.7 \frac{S_{\text{new}}}{S}\right) \]

Nevertheless, this is still an overestimation because the ship structure is not strengthened over its entire length as much as in the amidships area where the bending moment is maximal. At the fore and aft ends, for instance, longitudinal strength is not the critical structural criteria and the structure does not require strengthening. A coefficient lower than 1 needs to be added before \( \frac{S_{\text{new}}}{S} \) leading to the final formula:

\[ \Delta_{\text{new}} = (\Delta - \Delta_{2110}) + \Delta_{2110} \frac{L_{\text{oa new}}}{L_{\text{oa}}} \left(0.3 + 0.7 \left( \frac{S_{\text{new}}}{S} + 1 - \alpha \right) \right) \]

This coefficient \( \alpha \) corresponds to an equivalent percentage of the ship length which is reinforced as much as the midship section. This coefficient has been calculated to be 0.47 for Frigate. This calculation is detailed in Appendix 3.

The new displacement is obtained by applying this formula. If necessary, an iteration on the structure strengthening is done until the input and output displacements converge (here a 1% maximal difference is chosen).
<table>
<thead>
<tr>
<th></th>
<th>Frigate</th>
<th>Frigate+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement at the beginning of study (t)</td>
<td>1</td>
<td>1.177</td>
</tr>
<tr>
<td>Displacement after 1st strengthening of structure (t)</td>
<td>-</td>
<td>1.136</td>
</tr>
<tr>
<td>Number of additional iterations necessary for a 1 % convergence</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>Displacement after iterations (t)</td>
<td>-</td>
<td>1.129</td>
</tr>
<tr>
<td>Convergence of the last iteration: difference between last and second last iteration (%)</td>
<td>-</td>
<td>0.56</td>
</tr>
</tbody>
</table>

Table 3 : Frigate+ normalised displacement estimate
4 Seakeeping and Operability

The seakeeping software used is Precal_R (Reference [17]), developed within the Cooperative Research Ships (CRS) framework. It uses the frequency domain linear 3D diffraction theory and can be used to calculate the motion response in regular waves at arbitrary speed and heading. RAOs (Response Amplitude Operator) and RMS (Root Mean Square) of ship motions can be given as output.

For each combination of ship speed, wave direction and wave frequency the ship-wave interaction is described as the superposition of the forces on a fixed ship in incoming waves (the diffraction part) and the forces on an oscillating ship in calm water (the radiation part).

The wave excitation forces are composed of the incoming wave forces and the diffraction wave forces which are obtained by integration of the corresponding pressures over the hull surface. In the same way, the reaction forces are expressed in terms of added mass and damping coefficients. The assumption of linearity implies that in principle the results are valid for small wave amplitudes and small motion amplitudes only.

4.1 Precal_R seakeeping results

The hull shapes used for the seakeeping calculations are bare hulls without appendices such as roll stabilising fins or bilge keel. The linear equivalent roll damping coefficient $B_{eqv}$ is taken to be a percentage $\delta$ of the critical roll damping coefficient $B_{crit}$ (see Figure 11).

\[
B_{eqv} = \delta B_{crit} \\
B_{crit} = 2\sqrt{(M_{44} + A_{44}^{(nat)})C_{44}} 
\]

Where:

- $M_{44}$ is the roll moment of inertia
- $A_{44}^{(nat)}$ is the roll-roll added mass coefficient at the roll natural frequency
- $C_{44}$ is the roll restoring coefficient.

Precal_R makes this percentage (also called damping ratio) vary with the ship speed:

$$\delta = \delta_0\left(1 + 0.8\left(1 - e^{-10\, F_n}\right)\right)$$

$\delta_0$ is the input parameter, it has been chosen as 10 % (value usually used by Naval Group hydrodynamics specialists).
Figure 11 : Roll damping ratio as a function of the Froude number

4.1.1 Transfer functions

Only the heave, roll and pitch transfer functions are plotted here because heave and pitch, cause slamming, which we want to reduce, and roll is usually found to be critical for compliance with warship operating criteria. These transfer functions are plotted for different speeds but only for a unique wave heading corresponding to the most critical situation. So, the heave and pitch transfer functions are plotted for a wave relative (compared to the ship forward direction) heading of 180° and the roll transfer functions are plotted for a wave relative heading of 90°.

Figure 12 : Frigate heave transfer function for a 180° wave heading
We can observe a significant decrease in the heave response peak for speeds of 25 and 30 kn when increasing the ship length, since for a speed of 25 kn a peak does not even appear and for a speed of 30 kn it is decreased by 30%. This is consistent with our expectations of heave reduction achieved by hull lengthening.
Figure 15: Frigate+ roll transfer function for a 90° wave heading

For roll we can see that Frigate+ peak responses at all speeds are much higher than those of Frigate. They increase significantly with increasing ship speeds. Indeed, for a speed of 0 kn the increase is of the order of 20% and for a speed of 30 kn it is approximately 80%.

No explanation for these incoherent results has been found by Naval Group hydrodynamics specialists based in Lorient. Consequently, the operability comparison presented in following sections of this report use the Frigate roll results for Frigate+ to measure the heave and pitch variation influence on the operability (not perturbated by this puzzling roll results).
We can see a decrease in the pitch response peak at all speeds. This decrease varies between 10 and 12 % throughout the speed range considered except at 30 kn for which it varies by 17 %. This pitch reduction, like the heave reduction, is consistent with our expectations.
4.1.2 Pitch

Pitch is the ship motion we are especially interested in as it is highly dependent of the length of a ship. The transfer functions for the most critical case (180° wave heading) have already been presented in Section 4.1.1. This section presents Frigate+ pitch RMS values as percentages of those of Frigate for 3 speeds (0 kn, 15 kn usual operational speed and 30 kn possible maximum speed) for 3 sea states: sea state 4/5 (Figure 18), sea state 5 (Figure 19) and sea state 5/6 (Figure 20).

We can see a decrease in the RMS value for Frigate+ at all speeds, wave headings and sea states. The normalised pitch RMS at 90° and 270° wave headings have been set at 100% in order to avoid representing the “freak” normalised pitch RMS results calculated at these values which distort the overall illustration of the results. Factually, at these wave headings, pitch is almost zero but the variation in percentage between the RMS calculated for Frigate and Frigate+ can be extremely important compared to those for other wave headings. For instance, for sea state 4/5, 90° wave heading and 15 kn speed, the RMS value for Frigate is 0.021 °/m and 0.078 °/m for Frigate+ which corresponds to 367% of Frigate RMS value. As Frigate+ RMS is between 60 and 100% of Frigate RMS for the other headings, having a value of 367% in the graph “hides” the other results due to the change of scale necessary to plot the 367% value.

![Figure 18](#): Frigate+ pitch RMS as percentage of Frigate pitch RMS for sea state 4/5
Figure 19: Frigate+ pitch RMS as percentage of Frigate pitch RMS for sea state 5

Figure 20: Frigate+ pitch RMS as percentage of Frigate pitch RMS for sea state 5/6
We can see a 20 to 24 % (depending on the speed) decrease for head seas at sea state 4/5, a 15 to 18 % decrease at sea state 5 and a 7 to 12 % decrease at sea state 5/6. This relative decrease diminishes as the sea state increases. To confirm this trend, calculations have been undertaken at higher and lower sea states. The results, are presented in Figures 20, 21 and 22. For an easier reading of the graphs, one graph corresponds to only one speed. Below are the 0, 15 and 30 kn results and, as previously, the 90° (and 270°) results have been set at 100 %.

Figure 21 : Frigate+ pitch RMS as percentage of Frigate pitch RMS for V = 0 kn and for different sea states
Figure 22: Frigate+ pitch RMS as percentage of Frigate pitch RMS for V = 15 kn and for different sea states

Figure 23: Frigate+ pitch RMS as percentage of Frigate pitch RMS for V = 30 kn and for different sea states

These results confirm that the higher the sea state is, the lesser is the relative (%) pitch reduction achieved by lengthening the ship. For head seas, for a speed of 0 kn, the decrease in pitch is 33% for sea states 2 and 3 and only 5% for sea states 7 and 8. There also appears to be a convergence.
toward this 5% value when the sea state increases. For a speed of 30 kn, the decrease varies between 33% and 1% depending on the sea state.

Precal_R output includes RAOS (Response Amplitude Operator) and RMS (Root Mean Square) from which it is possible to plot graphs such as the ones above. However, it does not include operability assessments based on the seakeeping characteristics of a ship against a set of limiting seakeeping criteria (motions/velocities/accelerations) such as those defined in Reference [5] for naval ships.

Moreover, Precal_R does not provide RAO and RMS for relative vertical motions, velocities and accelerations nor does it calculate Motion Sickness Incidence (MSI), Motion Induced Interruption (MII) and Lateral Force Estimator (LFE).

Thus, the development of a tool for postprocessing Precal_R RAOs in order to calculate slamming and emergence, MSI, MII and LFE as long as operability graphs was needed. This tool is an Excel VBA routine.

4.2 Calculations et Formulas used in Precal_R output data post processing

4.2.1 Wave spectrum

The wave spectrum chosen for this study is the JONSWAP (Joint North Sea Wave Project) wave spectrum which is defined as follows (Reference [17]):

$$S(\omega) = \frac{ag^2}{\omega^5} \exp \left( -\frac{\beta \omega_p^4}{\omega^4} \right) \gamma^a$$

Where:

- $g$ is the gravitational acceleration 9.81 m.s$^{-2}$
- $\omega$ is the (angular) frequency
- $\beta = \frac{5}{4}$
- $\omega_p$ is the spectrum peak frequency
- $\gamma = 3.3$
- $a = \exp \left( -\frac{(\omega-\omega_p)^2}{2\omega_p^2\sigma^2} \right)$
- $\sigma = \begin{cases} 0.07 & \text{if } \omega \leq \omega_p \\ 0.09 & \text{if } \omega > \omega_p \end{cases}$
- $\alpha$ is so that $16 \int_0^\infty S(\omega) d\omega = H^2$
- $H$ is the significant wave height
With this spectrum and the RAO transfer functions we can calculate the RMS (Root Mean Square) of this motion and check if it meets a criteria (most criteria are expressed in RMS).

\[
\sigma = \sqrt{\int_0^{+\infty} |RAO(\omega)|^2 S(\omega) d\omega}
\]

The sea states concerned by this study are sea states 4/5, 5 and 5/6 which significant wave heights and peak periods, defined in accordance with the values specified in Reference [4], are given in the table below.

<table>
<thead>
<tr>
<th>Sea State</th>
<th>H (m)</th>
<th>T (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.3</td>
<td>7.5</td>
</tr>
<tr>
<td>3</td>
<td>0.88</td>
<td>7.5</td>
</tr>
<tr>
<td>4</td>
<td>1.88</td>
<td>8.8</td>
</tr>
<tr>
<td>4/5</td>
<td>2.5</td>
<td>9.2</td>
</tr>
<tr>
<td>5</td>
<td>3.25</td>
<td>9.7</td>
</tr>
<tr>
<td>5/6</td>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>12.4</td>
</tr>
<tr>
<td>7</td>
<td>7.5</td>
<td>15</td>
</tr>
<tr>
<td>8</td>
<td>11.5</td>
<td>16.4</td>
</tr>
</tbody>
</table>

Table 4 : Sea states significant wave heights and peak periods

The JONSWAP wave spectra for sea states 4/5, 5 and 5/6 are plotted in Figure 23Error ! Source du renvoi introuvable.. These are the spectra used for this study.
4.2.2 Motions and derivatives

The 6 motions of a ship, i.e. surge, sway, heave, roll, pitch and yaw, respectively \( \eta_1, \eta_2, \eta_3, \eta_4, \eta_5 \) and \( \eta_6 \) in the following motion transfer functions, are calculated at its centre of gravity. The motions at a given point (longitudinal, lateral and vertical motions \( D_1, D_2 \) and \( D_3 \)) are then calculated from the ship motions and the position of the given point (the 3 rotations are the same). These motions transfer functions are given by:

\[
D_1 = \eta_1 - (X - X_G)(1 - \cos \eta_6) - (Y - Y_G) \sin \eta_6 - (X - X_G)(1 - \cos \eta_5) + (Z - Z_G) \sin \eta_5 \\
D_2 = \eta_2 - (Y - Y_G)(1 - \cos \eta_6) + (X - X_G) \sin \eta_6 - (Y - Y_G)(1 - \cos \eta_4) - (Z - Z_G) \sin \eta_4 \\
D_3 = \eta_3 - (Z - Z_G)(1 - \cos \eta_4) + (Y - Y_G) \sin \eta_4 - (Z - Z_G)(1 - \cos \eta_5) - (X - X_G) \sin \eta_5
\]

The rotation motions being rather small, the first order approximation in rotations can be made. This approximation is often used, it is the case for the PreCal_R software for instance.

\[
D_1 \approx \eta_1 - (Y - Y_G)\eta_6 + (Z - Z_G)\eta_5 \\
D_2 \approx \eta_2 + (X - X_G)\eta_6 - (Z - Z_G)\eta_4 \\
D_3 \approx \eta_3 + (Y - Y_G)\eta_4 - (X - X_G)\eta_5
\]

To determine the derivatives, that is to say longitudinal, lateral and vertical velocities and accelerations, the encounter frequency between the waves and the ship must be used.

\[
\dot{D}_i = -i\omega_e D_i
\]
Where $\omega_e$ is the wave encounter frequency of the ship given by:

$$\omega_e = \omega - \frac{\omega^2 V \cos \mu}{g}$$

Where $\omega$ is the wave frequency, $V$ the ship speed, $\mu$ the wave heading and $g$ the gravitational acceleration.

4.2.3 Emergence and Slamming

The emergence of a part of a ship (hull, propeller…) depends on its vertical motion relative to the sea surface, i.e. the vertical motion of the ship to which the vertical wave motion is substracted. The slamming depends both on the vertical relative motion and the vertical relative velocity (given by multiplying the vertical relative motion by $-i\omega_e$).

Most seakeeping operating criteria are defined by RMS motion thresholds that are not to be exceeded (Reference [5]). Emergence and slamming criteria are not, however. They are defined by a maximum number of occurrences per hour that must not be exceeded. This number of occurrences is calculated from the probability of occurrence per wave which is then multiplied by the number of waves encountered in one hour. The cumulative Rayleigh probability distribution function is used to calculate the probability of emergence per wave. It is defined as follows:

$$f(x) = \frac{x e^{-\frac{x^2}{2\sigma^2}}}{\sigma^2}$$

Where $\sigma$ is a parameter which is taken as the RMS of the seakeeping parameter concerned (the vertical relative motion or vertical relative velocity). The cumulative distribution function which gives the probability of being less than $x$ is given by:

$$F(x) = 1 - e^{-\frac{x^2}{2\sigma^2}}$$
The probability of exceeding a given value $x_0$ is then given by $P(x \geq x_0) = \exp \left( -\frac{x_0^2}{2\sigma^2} \right)$.

For the emergence to occur at a location which vertical position is $Z$, the vertical relative motion needs to exceed $T-Z$ where $T$ is the draught of the ship. So, the probability that $Z$ exceeds $T-Z$ is multiplied by the number of waves encountered per hour to get the number of occurrences per hour:

$$N_{b_{emergence}} = \exp \left( -\frac{(T-Z)^2}{2\sigma_{vertical \ relative \ motion}^2} \right) \frac{3600}{T_m}$$

Where $T_m$ is the mean period of wave encountering and since it is expressed in s, the number of waves encountered per hour is $\frac{3600}{T_m}$.

$$T_m = 2\pi \frac{\sigma_{vertical \ relative \ motion}}{\sigma_{vertical \ relative \ velocity}}$$

Where $\sigma_x$ is the parameter $x$ RMS.

Concerning slamming, in addition to emergence, the vertical relative velocity needs to exceed a minimum value which is taken as $3.66 \frac{L}{\sqrt{158.5}}$ which is the adaptation of the 12 feet per second for a 520 feet long vessel threshold using Froude’s law (Reference [12]). So, the two probabilities need to be multiplied and we get:

$$N_{b_{slamming}} = \exp \left( -\frac{(T-Z)^2}{2\sigma_{vertical \ relative \ motion}^2} \right) \exp \left( -\frac{\left( 3.66 \frac{L}{\sqrt{158.5}} \right)^2}{2\sigma_{vertical \ relative \ velocity}^2} \right) \frac{3600}{T_m}$$
4.2.4 Motion Sickness Incidence (MSI)

The Motion Sickness Incidence index represents the peak percentage of a ship crew being seasick. It usually occurs after four hours because after several hours the crew tends to accommodate and the percentage of the crew affected by seasickness decreases. The MSI as a function of the encountered frequency is given in Reference [13] by:

$$MSI(\omega_e) = \int_{-\infty}^{\log_{10}(\bar{a})} \frac{100}{\sigma \sqrt{2}} \exp\left(-\frac{(x - \mu(\omega_e))^2}{2\sigma^2}\right) dx$$

Which can be re-written (Reference [15])

$$MSI(\omega_e) = 100 \left(1 + \frac{1}{2} \operatorname{erf}\left(\frac{\log_{10}(\bar{a}) - \mu(\omega_e)}{\sigma \sqrt{2}}\right)\right)$$

$\sigma$ is a parameter which value has been empirically determined in Reference [13] and $\sigma = \frac{0.4}{\sqrt{2}}$.

$\bar{a}$ refers to the mean vertical acceleration which has been normalised by $g$. For an acceleration of the form $A \sin \omega t$ ($A > 0$) the mean absolute value normalised by $g$ equals to:

$$\bar{a} = \frac{A}{g \pi}$$

$\mu(\omega_e)$ is given by the formula (Reference [13]):

$$\mu(\omega_e) = 0.654 + 3.697 \log_{10} \left(\frac{\omega_e}{2\pi}\right) + 2.320 \left(\log_{10} \left(\frac{\omega_e}{2\pi}\right)\right)^2$$

There also exists a second formula (Reference [14] and [15]) but both formulas sensibly give the same values of $\mu(\omega_e)$.

$$\mu(\omega_e) = -0.819 + 2.320 \left(\log_{10}(\omega_e)\right)^2$$

$\operatorname{erf}$ is the error function defined as:

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-u^2) du$$

The approximation used in this study for the error function is:

$$\operatorname{erf}(x) \sim \text{Sign}(x) \sqrt{1 - \exp\left(-\frac{4x^2}{\pi}\right)}$$
It is a very satisfying approximation since the difference between the \( erf \) function and this approximation never exceeds 0.7\% as shown in Figure 26.

\[
\text{Figure 26} : \text{erf function approximation precision}
\]

This calculation of \( MSI(\omega_e) \) gives a MSI transfer function from which we can calculate the total MSI (Reference [14]):

\[
MSI = \int_0^{+\infty} MSI(\omega_e)S(\omega)d\omega
\]

4.2.5 Motion Induced Interruptions (MII)

The MII calculation consists in calculating the risk of an operator tipping due to the ship motions which would lead to an interruption of his activities. To evaluate the risk of tipping, we look at the sum of moments at the operator’s feet (Reference [16]).

\[
\text{Figure 27} : \text{Operator’s geometry for MII calculations}
\]
The forces (in the plane (y, z)) applied to an operator are his weight, the deck reaction force and the fictitious forces due to the ship lateral and vertical accelerations. The deck reaction force consists of two components, each one applied to one of the operator’s feet. The one applied at the foot at which the sum of the moments of the forces applied to the operator is calculated does not induce any moment. The force applied to the other foot becomes null as soon as there is tipping since the foot is not in contact with the deck any longer. This explains why the deck reaction forces do not appear in the following calculations. The sum of moments at \( O_L \) and \( O_R \) (see Figure 27) are:

\[
\sum M_{O_L} = mgl \cos \eta_4 + mgh \sin \eta_4 - (-m \ddot{D}_2)h - (-m \ddot{D}_3)l
\]

\[
\sum M_{O_R} = -mgl \cos \eta_4 + mgh \sin \eta_4 - (-m \ddot{D}_2)h + (-m \ddot{D}_3)l
\]

There is tipping if \( \sum M_{O_L} < 0 \) or if \( \sum M_{O_R} > 0 \). Using the assumption of small angles, we get tipping if:

\[-g\eta_4 - \ddot{D}_2 - \ddot{D}_3 \frac{l}{h} > \frac{gl}{h}\]

Or if

\[g\eta_4 + \ddot{D}_2 - \ddot{D}_3 \frac{l}{h} > \frac{gl}{h}\]

The MII are then simply given by the sum of probabilities that \(-g\eta_4 - \ddot{D}_2 - \ddot{D}_3 \frac{l}{h}\) exceeds \(\frac{gl}{h}\) and that \(g\eta_4 + \ddot{D}_2 - \ddot{D}_3 \frac{l}{h}\) exceeds \(\frac{gl}{h}\). As opposed to the slamming calculations, here it is the sum of probabilities and not the product because it is the occurrence of an event OR the other that we are interested in and not an event AND the other. Like previously, it is the Rayleigh probability that is used.

\[MII = \left( P \left( -g\eta_4 - \ddot{D}_2 - \ddot{D}_3 \frac{l}{h} > \frac{gl}{h} \right) + P \left( g\eta_4 + \ddot{D}_2 - \ddot{D}_3 \frac{l}{h} > \frac{gl}{h} \right) \right) \frac{3600}{T_m}\]

\[MII = \left( \exp \left( -\frac{(\frac{gl}{h})^2}{2\sigma^2} \right) + \exp \left( -\frac{(\frac{gl}{h})^2}{2\sigma^2} \right) \right) \frac{3600}{T_m}\]

4.2.6 Lateral Force Estimator

The Lateral Force Estimator (LFE) is, as its name implies, a means to estimate the lateral forces (by unit of mass). It takes into account the fictitious force due to the lateral motion and the lateral component of the weight. By making the small angles assumption, we get:

\[LFE = -\ddot{D}_2 - g\eta_4\]
The criteria associated to LFE are expressed in RMS so there is no calculation using Rayleigh probabilities for the LFE.

4.3 PRECAL_R post processing Excel tool

The Excel VBA tool I coded is an Excel file with one main sheet, six imported data sheets and one total operability sheet. The main sheet is shown in Figure 28. In this sheet there are:

- Two command buttons: one to import the surge, sway, heave, roll, pitch and yaw RAOs Precal_R output files (they are stored in different sheets one for each motion) and the other one to launch the calculations.

- One table with the list of all available operability parameters for calculations. The user may fill in three columns. The first column is to select what parameters are to be used. The second one is to specify the operability criteria which should be used for the selected parameters. The third one is to select the parameters for which, in addition to a global operability graph which is generated by default, specific operability graphs should be produced.

- Several input parameters tables: one for the position of the centre of gravity of the ship and the position of the point where the calculations should be made, one for the significant wave height and peak period used for the wave spectrum and one (only used if the user select the emergence or slamming parameters) with the waterline length and draught of the ship.

- Additional information displayed through two tables: one with the number of wave frequencies, wave headings and ship speeds contained in the imported RAOs and one with significant wave height and peak periods corresponding to different sea states.

Figure 28: Excel post processing tool main sheet
Of the seven other sheets, six are used to store the RAOs for surge, sway, heave, roll, pitch and yaw (one for each motion). Their format is similar to the original files given as output by Precal_R see Figure 29.

Figure 29 : Excel post processing tool surge sheet

For each ship speed, for each wave heading, there are three columns of data. The first one corresponds to the wave frequency, the second one to the RAO amplitude and the last one to the RAO phase angle. Before each set of three columns, the ship speed and wave heading are given.

Once the RAOs have been imported and the user has filled in all that needs to be in the main sheet, he can press the calculations command button. After this, an additional sheet is created for each parameter the user has selected for the operability calculations (for surge, sway, heave, roll, pitch and yaw the existing sheet is used, no additional sheet needs creating). Figure 29 is an example of the roll sheet after calculations without the user having selected an additional operability graph.
As in Figure 28, there are still the RAOs for different ship speeds and wave headings with the three columns of data but now there are also two additional tables. The first one is the RMS table with the different lines corresponding to different ship speeds and the different columns corresponding to different wave headings (for emergence, slamming and MII it is not an RMS table but an occurrence table and for MSI a percentage table). The second table is an operability table with the same lines and columns as the RMS table but filled with “1” and “0”: 1 if the RMS value does not exceed the criteria chosen by the user and 0 if it does. Just below the operability table, there is also a preliminary operability percentage which is calculated as the percentage of boxes of the operability table filled with a “1”.

If the user selects an additional operability graph, the only changes (see Figure 30) are the generation of this graph and its data table. This data corresponds to the different boundaries of the blue and red areas on the graph corresponding respectively to areas where the criteria is satisfied and where the criteria is exceeded. These boundaries are calculated as the point where the parameter is equal to the criteria (through linear interpolation).
Figure 31: Excel post processing tool vertical motion sheet after calculations with graph

An example of an operability radar graph is given in Figure 31. This graph has a radial axis corresponding to the different ship speeds and an angular axis corresponding to the different wave headings. As previously said, blue indicates compliance with the operability criteria and red non-compliance. In addition, the graph title provides the name of the operability criteria that the graph represents and the ship operability percentage calculated using the boundaries of the blue and red areas.

Figure 32: Excel post processing tool operability graph example
The final sheet is the total operability sheet and is similar to the sheet of a selected parameter after calculations. In this sheet the user can find (see Figure 33)

- The wave spectrum

- A total operability table which is a combination of all the operability tables of the parameters selected for the calculations. A “1” is present if a “1” is present for every parameter and a “0” is present if a “0” is present for at least one of the parameters

- All the operability tables of the parameters selected

- The total operability graph (along with the table from which it is plotted)

![Excel post processing tool total operability sheet](image)

**Figure 33**: Excel post processing tool total operability sheet

## 4.4 Software limitations

Precal_R is still being developed, with a new version being released every few months, and is not a final product. As such it has some shortcomings that are detailed in the following sections.

### 4.4.1 Transfer functions values

Depending on the wave frequency range used (here from 0.2 rad.s\(^{-1}\) to 2.5 rad.s\(^{-1}\) with a 0.025 rad.s\(^{-1}\) step, outside this range the wave spectrum is equal to 0, see Figure 23), the wave encounter frequency can approach 0. When this encounter frequency is “too small”, Precal_R calculates absurd ship motion responses. In the case of the transfer functions calculated for Frigate, such unrealistic results occur for surge, sway and yaw. Some are presented in Table 5.
### Table 5: Examples of Precal_R RAO calculations for Frigate

Such results distort the results of some of the operational assessments such as those related to lateral motions which calculations include surge, sway and yaw. For instance, for sea state 4/5, a speed of 25 kn and a wave heading of 60° (see Table 5, “configuration 3” column), the lateral motion RMS is calculated to be 5 652 m which, of course, is not realistic.

It has thus been decided, which is standard practice, to manually « smooth » the surge, sway and yaw transfer functions in order to remove these abnormal peaks and correct the transfer functions. An example of such a transfer function, before and after “smoothing” is given in Figure 34 alongside its smoothed version.
The smoothing process used simply consists of removing all the peaks and then applying a linear interpolation between the two points at their base. It is a basic correction of the transfer functions but it is sufficient as the wave frequency range of the peaks is narrow.

Thanks to this “smoothing”, corrected RMS values can be calculated and used for operability results. Table 6 is an example of Frigate+ surge RMS values. The RMS in coloured cells are RMS calculated from transfer functions which will be modified by “smoothing” (other uncorrected RMS are not shown for confidentiality purpose).

<table>
<thead>
<tr>
<th>RMS tab</th>
<th>μ = 0°</th>
<th>μ = 15°</th>
<th>μ = 30°</th>
<th>μ = 45°</th>
<th>μ =60°</th>
<th>μ = 75°</th>
<th>μ = 90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>V = 0 Nd</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V = 10 Nd</td>
<td>207.241</td>
<td>49.239</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V = 15 Nd</td>
<td>6.876</td>
<td>12.391</td>
<td>143.467</td>
<td>1.009</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V = 20 Nd</td>
<td>9.178</td>
<td>1133.815</td>
<td>848.521</td>
<td>17.274</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V = 25 Nd</td>
<td>19.591</td>
<td>74.749</td>
<td>297.21</td>
<td>6401.296</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V = 30 Nd</td>
<td>157.609</td>
<td>168.437</td>
<td>5872.585</td>
<td>315.706</td>
<td>13.662</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6 : Uncorrected example of RMS calculations from Precal_R RAOs

4.4.2 Roll damping fins implementation

In order to reflect the actual active stabilisation of all modern naval ships, it was decided to add active stabiliser fins to both Frigate and Frigate+ Precal_R hull form models. These fins are active which means there is a control law to adjust their incidence angle as a function of roll amplitude and its derivatives in order to better reduce the ship rolling motions. Fin incidence
angle upper and lower limits are to be defined in order to reflect the actual mechanical limits of active stabiliser fin systems and to avoid the stalling of the fins at high ship speeds. The method used by Naval Group to bound the fins incident angle is to iterate on the magnitude of the coefficients of the control law which is a function of the roll angle, its derivative and its second derivative until the fins incident angle is always between acceptable limits.

However, even though all these calculations are already implemented in Precal_R, the command to export the fins incidence angle value does not yet exist. So, it is impossible to check whether the incidence angle exceeds the required limits or not and to iterate on it.

It was then decided to model passive fins which means their incidence angle remains null. It is not realistic compared to reality but it still reduces roll and is closer to reality than the bare hull. However, inexplicably, with passive fins all transfer functions output results were “NaN” (Not A Number) for all motions, all speeds, all wave headings and all wave frequencies within the range studied.

In conclusion, the active and passive stabiliser fins modelling and results being obviously unusable, seakeeping calculations have been limited to the bare hulls. The results obtained are presented in the following sections of this report.

4.5 Operability results

The operability results computed are pure pitch operability to show the effect of pitch reduction, and classic NATO criteria for generic frigate missions (Reference [5]). These results consist of figures with two operability graphs: on the left the Frigate graph and on the right the Frigate+ graph. For pure pitch, several sea states have been studied and for classic NATO criteria all graphs correspond to sea state 4/5. The operability percentage is given for the range of speed from 0 to 15 kn which corresponds to the frigate reaching its cruise speed and another operability percentage is given for the range from 0 to 30 kn which corresponds to the frigate reaching a usual frigates maximum speed.

Since the increase in roll for Frigate+ has not been explained and the implementation of roll damping fins in the calculations is not possible (fins which would be bigger for Frigate+ than Frigate and thus have a greater effect on its roll), it has been decided for a better comparison to use Frigate roll results as Frigate+ roll results for the following operability graphs.

4.5.1 Pitch

In this section, the graphs presented are operability graphs calculated for only one criteria which is the pitch RMS being lower than 1.5 deg (the lowest pitch NATO criteria). This criteria is only exceeded by Frigate and Frigate+ for sea state 5/6 and above, lower sea states graphs are thus not shown here.
Figure 35: Frigate (left) and Frigate+ (right) pitch operability graph for sea state 5/6

Figure 35 shows that there is a real improvement of the maximum admissible speed since for the 180° wave heading, the maximum speed doubles (from 9 kn to 18 kn) between Frigate and Frigate+. For sea state 6 (Figure 36), the improvement is not as much noticeable but the maximum speed still goes from almost zero to 5 kn for head seas.

Figure 36: Frigate (left) and Frigate+ (right) pitch operability graph for sea state 6
For sea state 7 (Figure 37), there is no improvement (in operability no in pitch RMS values) for head seas. Nevertheless, we can see that following seas also become critical and an improvement is seen for these following seas.

**Figure 37 : Frigate (left) and Frigate+ (right) pitch operability graph for sea state 7**

### 4.5.2 Transit and Patrol mission

The point in which this operability is calculated is the Bridge (same position relative to the aft perpendicular for Frigate and Frigate+ since the general arrangement remains the same in this study).

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Limit value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll</td>
<td>4</td>
<td>° RMS</td>
</tr>
<tr>
<td>Pitch</td>
<td>1.5</td>
<td>° RMS</td>
</tr>
<tr>
<td>Vertical acceleration</td>
<td>0.2</td>
<td>g RMS</td>
</tr>
<tr>
<td>Lateral acceleration</td>
<td>0.1</td>
<td>g RMS</td>
</tr>
</tbody>
</table>

**Table 7 : Transit and Patrol mission operability criteria**
The critical criteria is roll, all other criteria are below their limit value.

4.5.3 Speed maintaining in heavy seas

The point in which this operability is calculated is the below the keel at 15% of the forward perpendicular (different for Frigate and Frigate+ because of the length difference) for the slamming criteria and at the superior quarter of the propeller (same position for Frigate and Frigate+ since the general arrangement is kept the same in this study) for the emergence criteria.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Limit value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slamming at 15% of forward perpendicular, under the keel</td>
<td>20</td>
<td>Nb/h</td>
</tr>
<tr>
<td>Emergence of the superior quarter of the propeller</td>
<td>90</td>
<td>Nb/h</td>
</tr>
</tbody>
</table>

Table 8: Speed maintaining in heavy seas operability criteria
The critical criteria is the propeller emergence, the slamming criteria is always met.

4.5.4 Helicopter and drone operations

The point in which the operability is calculated is the helicopter spot.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Limit Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll</td>
<td>2.5</td>
<td>° RMS</td>
</tr>
<tr>
<td>Pitch</td>
<td>1.5</td>
<td>° RMS</td>
</tr>
<tr>
<td>Yaw</td>
<td>0.8</td>
<td>° RMS</td>
</tr>
<tr>
<td>Lateral Motion</td>
<td>0.825</td>
<td>m RMS</td>
</tr>
<tr>
<td>Vertical speed</td>
<td>1.08</td>
<td>m/s RMS</td>
</tr>
<tr>
<td>Lateral speed</td>
<td>0.65</td>
<td>m/s RMS</td>
</tr>
</tbody>
</table>

Table 9: Helicopter and drone operations operability criteria
The critical criteria for head seas is the vertical speed for Frigate, for Frigate+ all criteria are met. For red areas between 285° and 75°, they are due to lateral motion, roll and yaw, yaw being critical for a wider area for Frigate+ compared to Frigate.

4.5.5 Helicopter and drone handling and helicopter armament

The point in which the operability is calculated is the helicopter spot (same position for Frigate and Frigate+ since the general arrangement is kept the same in this study).

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Limit value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll</td>
<td>1.8</td>
<td>° RMS</td>
</tr>
<tr>
<td>Pitch</td>
<td>1.8</td>
<td>° RMS</td>
</tr>
<tr>
<td>Lateral Force Estimator</td>
<td>0.08</td>
<td>g RMS</td>
</tr>
<tr>
<td>Vertical acceleration</td>
<td>0.1</td>
<td>g RMS</td>
</tr>
</tbody>
</table>

Table 10: Helicopter and drone handling and helicopter armament operability criteria
The critical criteria for head seas is the vertical acceleration. The other red areas are due to roll. Sometimes LFE also exceeds its limit value but always while roll also does so the LFE inoperability area is comprised inside the roll inoperability area.

4.5.6 Crafts launch by the side

The point in which the operability is calculated is lateral crafts (RHIB, same position for Frigate and Frigate+ since the general arrangement is kept the same in this study).

Table 11: Crafts launch operability criteria

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Limit Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll</td>
<td>4</td>
<td>° RMS</td>
</tr>
<tr>
<td>Pitch</td>
<td>1.5</td>
<td>° RMS</td>
</tr>
<tr>
<td>Vertical acceleration</td>
<td>0.2</td>
<td>g RMS</td>
</tr>
<tr>
<td>Lateral acceleration</td>
<td>0.1</td>
<td>g RMS</td>
</tr>
<tr>
<td>MSI</td>
<td>20</td>
<td>%/4hr</td>
</tr>
<tr>
<td>MII</td>
<td>1</td>
<td>nb/min</td>
</tr>
</tbody>
</table>

Figure 41: Frigate (left) and Frigate+ (right) helicopter and drone handling and helicopter armament operability
Figure 42: Frigate (left) and Frigate+ (right) crafts launch by the side operability

The critical criteria is roll.

4.5.7 Replenishment at sea

The point in which the operability is calculated is Replenishment At Sea (RAS) area (port, same position for Frigate and Frigate+ since the general arrangement is kept the same in this study).

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Limit value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll</td>
<td>2.2</td>
<td>° RMS</td>
</tr>
<tr>
<td>Pitch</td>
<td>2.2</td>
<td>° RMS</td>
</tr>
<tr>
<td>MII RAS area</td>
<td>0.5</td>
<td>nb/min</td>
</tr>
<tr>
<td>MSI RAS area</td>
<td>20</td>
<td>%/4hr</td>
</tr>
<tr>
<td>Lateral motion</td>
<td>0.8</td>
<td>m RMS</td>
</tr>
<tr>
<td>Lateral Force Estimator</td>
<td>0.08</td>
<td>g RMS</td>
</tr>
</tbody>
</table>

Table 12: Replenishment at sea operability criteria
The critical criteria are roll and LFE for wave headings between 45° and 75° (and between 285° and 315°) and for wave headings between 30° and 45° for low speeds. Between 30° and 45° for high speeds the critical criteria is the lateral motion. MII also exceeds its limiting value but for an area included in the one where LFE exceeds its limiting value.

4.5.8 Replenishment at sea by helicopter

The point in which the operability is calculated is the helicopter spot (same position for Frigate and Frigate+ since the general arrangement is kept the same in this study).

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Limit Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll</td>
<td>1.6</td>
<td>° RMS</td>
</tr>
<tr>
<td>Pitch</td>
<td>1.6</td>
<td>° RMS</td>
</tr>
<tr>
<td>Vertical Motion</td>
<td>0.7</td>
<td>m RMS</td>
</tr>
<tr>
<td>Vertical Velocity</td>
<td>1.05</td>
<td>m/s RMS</td>
</tr>
<tr>
<td>Lateral Motion</td>
<td>0.924</td>
<td>m RMS</td>
</tr>
<tr>
<td>Lateral Velocity</td>
<td>0.65</td>
<td>m/s RMS</td>
</tr>
<tr>
<td>MII</td>
<td>0.5</td>
<td>nb/min</td>
</tr>
</tbody>
</table>
The critical criteria is vertical velocity: for head seas for Frigate, between 105° and 150° and between 210° and 255° for Frigate+.

For the two other areas, the critical criteria are roll and lateral motion. MII and lateral velocity also exceeds their limiting values at times but always while roll and lateral motion also exceed theirs.

4.5.9 Vertical launchers operation

The point in which the operability is calculated is vertical launchers (same position for Frigate and Frigate+ since the general arrangement is kept the same in this study).
Crit 

eria Limit Value Unit

Roll 8.8 ° RMS

Pitch 1.5 ° RMS

Yaw 0.8 ° RMS

Longitudinal acceleration 0.15 g RMS

Lateral acceleration 0.35 g RMS

Vertical acceleration 0.3 g RMS

Table 14: Vertical launchers operation operability criteria

![Overall Operability](image)

Figure 45: Frigate (left) and Frigate+ (right) vertical launchers operation operability

The critical criteria is yaw.

4.5.10 Torpedo tubes operation

The three rotations (roll, pitch and yaw) not being affected by the point of calculation, any point can be used for this operability calculation (the centre of gravity is taken in this case).
Criteria | Limit Value | Unit
--- | --- | ---
Roll | 3.8 | ° RMS
Pitch | 3.8 | ° RMS

Table 15: Torpedo tubes operation operability criteria

Figure 46: Frigate (left) and Frigate+ (right) torpedo tubes operation operability

The critical criteria is roll.

4.5.11 Main gun operation

The point at which the operability is calculated is the main gun (same position for Frigate and Frigate+ since the general arrangement is kept the same in this study).

Criteria | Limit Value | Unit
--- | --- | ---
Roll | 3.8 | ° RMS
Pitch | 3.8 | ° RMS
Vertical velocity | 0.5 | m/s RMS

Table 16: Main gun operation operability criteria
The critical criteria for a wave heading between 45° and 75° (and between 285° and 315°) is roll.

For all other wave headings it is vertical velocity.
5 Resistance

Two different methods have been used to calculate the resistance of the two ships and thus calculate the increase in maximum speed, with the same propulsive power, following lengthening.

The first method consists in interpolating the ship specific resistance (total resistance divided by the displacement) from already existing reference measures. In this instance, this interpolation is made using the specific resistance of Figure 48 as a function of its Froude number.

The second method is the Fung method.

5.1 Using existing reference measures

Since the lengthened version has a length different from Frigate’s, its Froude number is different at the same speed and so is its specific resistance.

![Figure 48: Frigate normalised specific resistance](image)

So for a given speed there is a new Froude number and from this new Froude number and from the reference curve (Figure 48), the new specific resistance is (linearly) interpolated. We can then compare the specific resistance as a function of the speed (Figure 49).
Figure 49: Frigate and Frigate+ normalised specific resistance comparison

The difference we observe is decreased when going from specific resistance to total resistance since we multiply by the displacement and Frigate+ displacement is greater than Frigate’s.

\[ R_t = R_{ts} \times \Delta \]

When we then look at the required propulsive power we simply multiply by the speed and divide by the total propulsive system efficiency (which is taken for Frigate+ as equal to Frigate’s). So the difference in percentage between the two curves does not change.

\[ P = \frac{R_t \times V}{\eta} \]

Figure 50: Frigate and Frigate+ normalised required power comparison
5.2 Fung Method

The Fung method (Reference [10]) consists in determining a series of coefficients which then are summed up to obtain a total coefficient. To each of these coefficients corresponds a parameter. Fung has calculated the value of each coefficient depending on its corresponding parameter value and on the additional parameter V/L (which gives a table of coefficient values). From Fung’s measures we can then calculate by interpolation the coefficient value in our case and obtain the resistance. The unit system used in Fung’s method is the imperial units system and not the international units system. These different parameters (of which some are not non-dimensional) are:

- The displacement to length ratio \( DL = \frac{\Delta}{(0.01L)^3} \)
- The breadth to draught ratio \( BT = \frac{B}{T} \)
- The prismatic coefficient \( C_p \)
- The midship section coefficient \( C_m \)
- The deadrise angle \( IE \)
- The transom area ratio \( TA = \frac{A_{20}}{A_m} \) which is the ratio between the underwater area of station 20 (see below for station definition) and the midship section underwater area
- The transom breadth ratio \( TW = \frac{B_{20}}{B} \) which is the ratio between waterline breadth of station 20 and the midship section waterline breadth
- The transom depth ratio \( TT = \frac{T_{20}}{T} \) which is the ratio between the draught of station 20 and the midship section draught
- The wetted surface coefficient \( CWS = \frac{WS}{\sqrt{L}} \) with WS the ship wetted surface
- The bow area ratio \( BA = \frac{A_0}{A_x} \) which is the ratio between the underwater area of station 0 and the midship section underwater area

The « station i » used as index in some of Fung’s parameters definition corresponds to the longitudinal position \( \frac{L(21-i)}{21} \). Station 0 is thus at position L and station 21 at position 0.

For the parameters TT, IE and CWS, Fung gives the following regression formulas:

\[
TT = 0.0067 + 0.3502 \, TW - 0.4948 \, TW^2 + 1.8239 \, TA - 1.9415 \, TA^2
\]
\[
IE = 120.0192 + \frac{69.2808}{DL} + 0.1131 \, \frac{L}{B} + 104.864 \, \frac{B}{L} - 28.5611 \, C_p - \frac{26.5287}{C_p}
- 24.8166 \, C_m - \frac{16.2033}{C_m} + 2.6398 \, TW - 49.657 \, FB
\]
\[
CWS = -6.2263 - 0.0094 \, DL + 16.0209 \, \frac{B}{L} + 0.9207 \, BT + \frac{5.463}{BT} + 9.8528 \, C_m
+ 7.1592 \, C_m
+ 0.0201 \, IE + \frac{2.1857}{IE} + 1.0359 \, TW + 2.4925 \, BA - 3.7181 \, FB
\]
With FB being the ratio $\frac{L CB}{L}$.

Once these parameters have been determined, we can use Fung’s data to interpolate the coefficients. For instance, below is the Fung’s table to determine the coefficient associated to the parameter BT.

<table>
<thead>
<tr>
<th>Bn/fo</th>
<th>0.00</th>
<th>0.20</th>
<th>0.80</th>
<th>1.00</th>
<th>1.20</th>
<th>1.30</th>
<th>1.40</th>
<th>1.50</th>
<th>1.60</th>
<th>1.70</th>
<th>1.80</th>
<th>1.90</th>
<th>2.00</th>
<th>2.10</th>
<th>2.20</th>
<th>2.30</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.000</td>
<td>-0.214</td>
<td>1.237</td>
<td>1.371</td>
<td>1.648</td>
<td>2.037</td>
<td>3.000</td>
<td>2.760</td>
<td>2.605</td>
<td>2.403</td>
<td>1.738</td>
<td>1.124</td>
<td>0.992</td>
<td>0.744</td>
<td>0.432</td>
<td>0.374</td>
<td>0.352</td>
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<tr>
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Table 17: Fung’s residuary resistance coefficient component table

After the different coefficients are determined, they are summed to get a total coefficient $C_R$. From this coefficient we can calculate the residuary resistance $R_R$ to which we will have to add the viscous resistance in order to obtain the ship total resistance.

$$ R_R = \frac{1}{2} \rho WS V^2 C_R $$
5.3 Speed and Power

The viscous resistance is not given by Fung’s method and is calculated with a regression formula:

\[ R_f = f(WS, L, \Delta, V) \]

We then get the required engine power with the propulsive system efficiency:

\[ P = \frac{(R_R + R_f)V}{\eta} \]

We can now plot the curves of the two ships propulsive power, calculated with the two methods, as a function of their speed. This allows us to determine the maximum speed that the ship can reach for a given installed power.

Figure 51: Frigate and Frigate+ normalised residuary resistance comparison
If we choose as installed power the power of Frigate’s actual propulsion system, we get the following maximum speed results:

<table>
<thead>
<tr>
<th>Maximum Speed (kn)</th>
<th>Measures</th>
<th>Fung</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frigate</td>
<td>24.0</td>
<td>25.1</td>
</tr>
<tr>
<td>Frigate+</td>
<td>25.2</td>
<td>26.7</td>
</tr>
</tbody>
</table>

So we can increase the speed by 1 kn in both cases (and even 1.5 kn with Fung’s method). It is also possible to choose to keep the same maximum speed requirement (25 kn) and to reduce the power to be installed.
Table 19: Normalised engine power comparison for same speed

For the cruise speed of 15 kn, the measures data gives a reduction of the required power of 13% while the Fung method gives a required power approximately equal. For higher speeds, possible maximum speeds for the lengthened frigate, the reduction given by the two methods is similar. For 25 kn, the current maximum speed, the reduction is almost 20% and for even higher speeds it is even more important, it is over 25% and approaches 30%.
6 Costs

A Naval Group cost engineer has made a first cost estimate of the lengthened hull as 25% more expensive than the reference one. Usually, hull costs represent about 15% of the total frigate purchase cost which gives us an increase of total cost of 3.8%. This estimation is close to the calculations made for a patroller (Reference [7]) which give an increase of 3% of building costs.

Besides this increase in building cost, there is a reduction in operational costs. Indeed, the reduction in required power for a same speed leads to a decrease in fuel consumption (and may also lead to the purchase of a smaller and cheaper engine). Knowing the price of fuel, the engine fuel consumption and the operational profile of the reference frigate it is possible to calculate the cost reduction. However, at the time of this report, the reference frigate operational profile had not yet been acquired. Thus, this decrease in operational cost, which may or may not compensate the increase in building cost overall the frigate lifespan, has not been estimated.
7 Conclusion

The concept of lengthened version of a reference ship developed in this study requires a few transformations on the hull form and not just a basic homothety. After having modified the hull form, and having estimated the lengthened ship mass, seakeeping and operability were calculated. The seakeeping results showed a significant decrease in pitch, from 35% to 15% depending on the sea state. However, the operability results do not really show an improvement which seems to indicate that despite the fact that the sailors regard pitch as the worst motion which thus has to be reduced, it is not the critical criteria for operability. Moreover, the lack of roll damping fins implementation in the seakeeping model leads to roll being higher than it is supposed to be and thus to the appearance of red areas in operability graphs where roll exceeds its limit value whereas it should not.

With regard to the costs of such a lengthened ship, the building costs increase by a few percent. The lengthened ship’s resistance is lower than the reference ship’s which means a decrease in fuel consumption and thus a reduction of operational costs. This reduction still needs to be estimated and compared to the increase in building costs to see whether or not they balance each other during the lifespan of the ship. This potential through life cost reduction would come as a “bonus” to the increased seakeeping performance of the lengthened ship.
8 References


9 Appendices

9.1 Appendix 1: Proof of the variation of LCB via a « swinging » of the sectional area curve

For a required $\delta$LCB, the transformation given is: $x_{\text{new}} = x + \delta x = x + y \tan \theta$ where $y$ is the y coordinate of the sectional area curve and $\tan \theta = \frac{\delta \text{LCB}}{\bar{y}}$ where $\bar{y}$ is the y coordinate of the centroid of the sectional area curve. This transformation has the property to give $\delta x = 0$ for $x = 0$ and $x = L$ (so $L_{\text{new}} = L$) since $y = 0$ for these positions.

A necessary preliminary calculation, which is detailed below, is the proof of the displacement conservation. Knowing that $y_{\text{new}}(x_{\text{new}}) = y(x)$ (which will simply be noted $y$ below), we can write:

\[
\nabla_{\text{new}} = \int_{0}^{L_{\text{new}}} y_{\text{new}}(x_{\text{new}})dx_{\text{new}} \\
= \int_{0}^{L} y(1 + y'tan \theta)dx \\
= \int_{0}^{L} ydx + \int_{0}^{L} yy'tan \theta dx \\
= \nabla + \tan \theta \left[\frac{y^2}2\right]_0^L \\
= \nabla
\]

$$LCB_{\text{new}} = \frac{\int_{0}^{L_{\text{new}}} y_{\text{new}}(x_{\text{new}})x_{\text{new}}dx_{\text{new}}}{\int_{0}^{L_{\text{new}}} y_{\text{new}}(x_{\text{new}})dx_{\text{new}}}$$

\[
= \frac{1}{\nabla_{\text{new}}} \int_{0}^{L_{\text{new}}} y_{\text{new}}(x_{\text{new}})x_{\text{new}}dx_{\text{new}} \\
= \frac{1}{\nabla} \int_{0}^{L} y(x + y\tan \theta)(1 + y'tan \theta)dx \\
= \frac{1}{\nabla} \int_{0}^{L} (yx + yxy'tan \theta + y^2\tan \theta + y^2y'tan^2 \theta)dx \\
= \frac{1}{\nabla} \int_{0}^{L} yxdx + \frac{\tan \theta}{\nabla} \int_{0}^{L} (yxy' + y^2 + y^2y'tan \theta)dx \\
= LCB + \frac{\tan \theta}{\nabla} \int_{0}^{L} (yxy' + y^2 + y^2y'tan \theta)dx
\]

Since $y(x = 0) = y(x = L) = 0$. we have:

\[
\int_{0}^{L} yxy'dx = \left[\frac{xy^2}{2}\right]_0^L - \int_{0}^{L} \frac{y^2}{2}dx = -\int_{0}^{L} \frac{y^2}{2}dx
\]
\[ LCB_{\text{new}} = LCB + \frac{\tan \theta}{V} \int_0^L \left( \frac{y^2}{2} + y^2 y' \tan \theta \right) dx \]

\[ = LCB + \frac{\tan \theta}{V} \int_0^L \frac{y^2}{2} dx + \frac{\tan^2 \theta}{V} \left[ \frac{y^3}{3} \right]_0^L \]

\[ = LCB + \frac{\tan \theta}{V} \int_0^L \frac{y^2}{2} dx \]

And

\[ \bar{y} = \frac{\int_0^L \int_0^L y(x) y^2 dx \, dy}{\int_0^L \int_0^L y^2 dx \, dy} \]

\[ = \frac{\int_0^L \frac{y^2}{2} dx}{\int_0^L y^2 dx} \]

\[ = \frac{\int_0^L \frac{y^2}{2} dx}{V} \]

So:

\[ LCB_{\text{new}} = LCB + \frac{\tan \theta}{V} \bar{y}V \]

\[ = LCB + \delta LCB \]

After transformation, the variation in LCB is really the required \( \delta LCB \).
9.2 Appendix 2: Formulas for the deformation of a section

The idea behind this deformation is to ensure that the new underwater cross-section is a transformation of the old underwater cross-section only and, similarly, that the new above water cross-section is a transformation of the old above water cross-section only.

This transformation of a section consists in a multiplication of the two parts (underwater and above water) of the old section by a polynomial of degree 2 in \( z \) for the underwater cross-section and degree 1 for the above water cross-section and in an adjustment of the \( z \) coordinates related to the new values \( T_{\text{new}} \) et \( C_{\text{new}} \). This can be written:

- For \( z \leq T \):
  \[
  B_{\text{new}}(z_{\text{new}}) = B(z)(c_1 z_{\text{new}}^2 + c_2 z_{\text{new}} + c_3)
  \]

  \( z_{\text{new}} \) must verify:
  \[
  \begin{cases}
  z_{\text{new}}(z = 0) = 0 \\
  z_{\text{new}}(z = T) = T_{\text{new}}
  \end{cases}
  \]

  By choosing \( z_{\text{new}} \) as a linear function of \( z \), we get:
  \[
  z_{\text{new}} = \frac{z T_{\text{new}}}{T}
  \]

- For \( z \geq T \):
  \[
  B_{\text{new}}(z_{\text{new}}) = B(z)(a z_{\text{new}} + b)
  \]

  \( z_{\text{new}} \) must verify:
  \[
  \begin{cases}
  z_{\text{new}}(z = T) = T_{\text{new}} \\
  z_{\text{new}}(z = C) = C_{\text{new}}
  \end{cases}
  \]

  By choosing \( z_{\text{new}} \) as a linear function of \( z \), we get:
  \[
  z_{\text{new}} = \frac{(z-T)(C_{\text{new}}-T_{\text{new}})}{C-T} + T_{\text{new}}
  \]

The improvement I personally added to this method is as follow. Previously only the \( a \) and \( b \) coefficients were determined as well as \( c_2 \) as a function of \( c_1 \) and \( c_3 \). \( c_1 \) and \( c_3 \) were then given an initial default value which was then modified by arbitrary increment until reaching, theoretically, a satisfying section. This method restricted the range of possible values taken by \( c_1 \) and \( c_3 \) (and so \( c_2 \) too) and the variation of these coefficients at each increment was done arbitrarily. For instance, if the underwater area of the section was too high \( c_1 \) was modified until it reached a fixed limit after which \( c_3 \) was modified and then \( c_1 \) again. After the improvement, all five coefficients are directly determined. There no longer is an incremental search for a section which may not be found. The satisfying section is now directly determined which results in a substantial calculations time reduction.

**Determination of \( a \) and \( b \)**

The two conditions used to determine \( a \) and \( b \) are:

\[
\begin{cases}
  B_{\text{new}}(T_{\text{new}}) = B_{\text{new}} \\
  B_{\text{new}}(C_{\text{new}}) = B_{oa\text{ new}}
\end{cases}
\]

This corresponds to making it so that the above water part of the section respects the new dimensions required so making the breadth equal to the waterline breadth required for a height equal to the required draught and the breadth equal to the overall breadth required for a height equal to the required depth.

For this method, the overall breadth \( B_{oa}\) refers to the breadth at a height equal to the depth of the ship even though for fighting ships the overall breadth is at a height lower than the depth.
\[ a(C_{\text{new}} - T_{\text{new}}) = \frac{B_{o.a, \text{new}}}{B_{o.a}} - \frac{B_{\text{new}}}{B} \]

\[ b = \frac{B_{\text{new}}}{B} - a \frac{T_{\text{new}}}{C_{\text{new}} - T_{\text{new}}} \]

\[ = \frac{1}{C_{\text{new}} - T_{\text{new}}} \left( \frac{B_{\text{new}}}{B} C_{\text{new}} - T_{\text{new}} \right) \left( \frac{B_{o.a, \text{new}}}{B_{o.a}} - T_{\text{new}} \right) + T_{\text{new}} \frac{B_{\text{new}}}{B} \]

For \( z \geq T \) we then have:

\[ B_{\text{new}}(z_{\text{new}}) = \frac{B(z)}{C_{\text{new}} - T_{\text{new}}} \left( z_{\text{new}} \left( \frac{B_{o.a, \text{new}}}{B_{o.a}} - \frac{B_{\text{new}}}{B} \right) + \frac{B_{\text{new}}}{B} C_{\text{new}} - \frac{B_{o.a, \text{new}}}{B_{o.a}} T_{\text{new}} \right) \]

With

\[ z_{\text{new}} = \frac{(z-T)(C_{\text{new}} - T_{\text{new}})}{C-T} + T_{\text{new}} \]

**Determination of \( c_1, c_2 \) and \( c_3 \)**

The three conditions used to determine \( c_1, c_2 \) and \( c_3 \) are:

1. Underwater area of the new section equal to the required area (improvement)
2. Continuity in \( z_{\text{new}} = T_{\text{new}} \) of the new section that is to say no discontinuity in the hull (condition already used in a previous work)
3. Continuity in \( z_{\text{new}} = T_{\text{new}} \) of the slope of the new section that is to say no hard chine in the hull (improvement)

It is not possible to use the condition: same value of \( z \) at a breadth equal to 0 for the new and the old section. This condition corresponds to both sections having the same point on the axis \( y = 0 \). However, for most sections, the first coordinate in \( B(z) \) is 0. The equation would then become \( 0 = 0 \times (c_1 z_{\text{new}}^2 + c_2 z_{\text{new}} + c_3) \) and would not give any condition on the coefficients.

- Underwater area condition

\[ A_x = \int_0^{T_{\text{new}}} B_{\text{new}}(z_{\text{new}}) \, dz_{\text{new}} \]

\[ = \int_0^{T_{\text{new}}} B(z) \left( c_1 \left( \frac{z T_{\text{new}}}{T} \right)^2 + c_2 \left( \frac{z T_{\text{new}}}{T} \right) + c_3 \right) \frac{T_{\text{new}}}{T} \, dz \]

\[ c_1 \left( \frac{T_{\text{new}}}{T} \right)^3 \int_0^T B(z) z^2 \, dz + c_2 \left( \frac{T_{\text{new}}}{T} \right)^2 \int_0^T B(z) z \, dz + c_3 \frac{T_{\text{new}}}{T} \int_0^T B(z) \, dz = A \]
• Continuity condition

\[ B_{\text{new}}(T_{\text{new}}) = B(T)(c_1 T_{\text{new}}^2 + c_2 T_{\text{new}} + c_3) \]
\[ c_1 T_{\text{new}}^2 + c_2 T_{\text{new}} + c_3 = \frac{B_{\text{new}}}{B} \]

• Slope continuity condition

\[
\begin{align*}
\frac{d B_{\text{new}}}{d z} (z \leq T) &= B(z) \left( 2 c_1 z \left( \frac{T_{\text{new}}}{T} \right)^2 + c_2 \left( \frac{T_{\text{new}}}{T} \right) \right) \\
&+ B'(z) \left( c_1 \left( \frac{z T_{\text{new}}}{T} \right)^2 + c_2 \left( \frac{z T_{\text{new}}}{T} \right) + c_3 \right) \\
\frac{d B_{\text{new}}}{d z} (z \geq T) &= B(z) \left( \frac{1}{C_{\text{new}} - T_{\text{new}}} \left( \frac{B_{\text{oa new}}}{B_{oa}} - \frac{B_{\text{new}}}{B} \right) C_{\text{new}} - T_{\text{new}} \right) \\
&+ \frac{B'(z)}{C_{\text{new}} - T_{\text{new}}} \left( \frac{B_{\text{oa new}}}{B_{oa}} - \frac{B_{\text{new}}}{B} \right) \left( (z - T) \left( \frac{C_{\text{new}} - T_{\text{new}}}{C - T} \right) + T_{\text{new}} \right) \\
&+ \frac{B_{\text{new}}}{B} C_{\text{new}} - \frac{B_{\text{oa new}}}{B_{oa}} T_{\text{new}} \right)
\end{align*}
\]

These two expressions must be equal for \( z = T \) which gives us:

\[
\begin{align*}
B \left( 2 c_1 \frac{T_{\text{new}}^2}{T} + c_2 \frac{T_{\text{new}}}{T} \right) + B'(T) \left( c_1 T_{\text{new}}^2 + c_2 T_{\text{new}} + c_3 \right) &= B \left( \frac{B_{\text{oa new}}}{B_{oa}} - \frac{B_{\text{new}}}{B} \right) \frac{C - T}{B} \\
&+ \frac{B'(T)}{C_{\text{new}} - T_{\text{new}}} \left( T_{\text{new}} \left( \frac{B_{\text{oa new}}}{B_{oa}} - \frac{B_{\text{new}}}{B} \right) + \frac{B_{\text{new}}}{B} C_{\text{new}} - \frac{B_{\text{oa new}}}{B_{oa}} T_{\text{new}} \right) \\
c_1 \left( 2 \frac{B_{\text{new}}}{T} T_{\text{new}}^2 + B'(T) T_{\text{new}}^2 \right) + c_2 \left( \frac{B_{\text{new}}}{T} T_{\text{new}} + B'(T) T_{\text{new}} \right) + c_3 B'(T) &= B \left( \frac{B_{\text{oa new}}}{B_{oa}} - \frac{B_{\text{new}}}{B} \right) \frac{C - T}{B} + \frac{B'(T) B_{\text{new}}}{B}
\end{align*}
\]

These three conditions can be written in matrix format \( AC = B \):
\[
\begin{pmatrix}
\left(\frac{T_{\text{new}}}{T}\right)^3 \int_0^T B(z)z^2dz & \left(\frac{T_{\text{new}}}{T}\right)^2 \int_0^T B(z)dz & \left(\frac{T_{\text{new}}}{T}\right) \int_0^T B(z)dz \\
T_{\text{new}}^2 & T_{\text{new}} & 1 \\
2B\frac{T_{\text{new}}^2}{T} + B'(T)T_{\text{new}}^2 & B\frac{T_{\text{new}}}{T} + B'(T)T_{\text{new}} & B'(T)
\end{pmatrix}
\begin{pmatrix}
c_1 \\
c_2 \\
c_3
\end{pmatrix}
= \begin{pmatrix}
A_x \\
\frac{B_{\text{new}}}{B} \\
\frac{B}{C - T} \left( \frac{B_{\text{oa new}}}{B_{\text{oa}}} - \frac{B_{\text{new}}}{B} \right) + \frac{B'(T)B_{\text{new}}}{B}
\end{pmatrix}
\]

We then get the coefficients \(c_1, c_2\) and \(c_3\) by simply inverting the matrix \(A\) (\(C = A^{-1}B\)).

The mathematical proof that matrix \(A\) is always invertible, and that the coefficients \(c_1, c_2\) and \(c_3\) can therefore always be calculated, is provided below. This is true in the case where the underwater cross-section part exists, of course. This corresponds to \(T > 0\) and the existence of at least one point of the section with \(z < T\).

\[
\det(A) = \frac{T_{\text{new}}^4}{T^3} B'(T) \int_0^T B(z)z^2dz + \left( B\frac{T_{\text{new}}}{T} + B'(T)T_{\text{new}} \right) \frac{T_{\text{new}}^3}{T} \int_0^T B(z)dz \\
+ \left( 2B\frac{T_{\text{new}}^2}{T} + B'(T)T_{\text{new}}^2 \right) \frac{T_{\text{new}}^2}{T^2} \int_0^T B(z)dz \\
- \frac{T_{\text{new}}^2}{T} \left( 2B\frac{T_{\text{new}}^2}{T} + B'(T)T_{\text{new}}^2 \right) \int_0^T B(z)dz \\
- \left( B\frac{T_{\text{new}}}{T} + B'(T)T_{\text{new}} \right) \frac{T_{\text{new}}^3}{T^3} \int_0^T B(z)dz \\
+ T_{\text{new}}^4 \int_0^T B(z)z^2dz \left( B\frac{T_{\text{new}}}{T} + B'(T)T_{\text{new}} \right) \frac{T_{\text{new}}^3}{T^3} \\
+ \int_0^T B(z)dz \left( 2B\frac{T_{\text{new}}^2}{T} + B'(T)T_{\text{new}}^2 \right) \frac{T_{\text{new}}^2}{T^2} - \frac{T_{\text{new}}^4}{T^2} B'(T) \\
+ \int_0^T B(z)dz \left( B\frac{T_{\text{new}}}{T} + B'(T)T_{\text{new}} \right) \frac{T_{\text{new}}^3}{T^3} - \frac{T_{\text{new}}^2}{T} \left( 2B\frac{T_{\text{new}}^2}{T} + B'(T)T_{\text{new}}^2 \right) \\
= -B\frac{T_{\text{new}}^4}{T^4} \int_0^T B(z)z^2dz + 2B\frac{T_{\text{new}}^4}{T^3} \int_0^T B(z)dz \\
= B\frac{T_{\text{new}}^4}{T^4} \int_0^T B(z)(-z^2 - T^2 + 2z)dz
\]
\[-B \left(\frac{T_{\text{new}}}{T}\right)^4 \int_0^T B(z)(z - T)^2 dz < 0\]

The determinant of the matrix is not null; the matrix is thus always invertible.
9.3 Appendix 3: Hull girder strengthening effective length

The effective length $L_{eff}$, as illustrated below, is defined as the length for which the area between the two bending moment curves $M_{f_{new}}(x)$ and $M_f(x)$ is equal to the area between the curve $M_f(x)$ and the curve equal to $M_f(x)$ to which is added $M_{f_{new\ max}} - M_{f_{max}}$ over that length i.e. equality of the two orange areas shown in Figure 53.

\[
\int_0^L \left(M_{f_{new}}(x) - M_f(x)\right) dx = L_{eff} \left(M_{f_{new\ max}} - M_{f_{max}}\right)
\]

We then make the assumption that $M_{f_{new}}(x) = \frac{M_{f_{new\ max}}}{M_{f_{max}}} M_f(x)$ so we have:
\[ L_{\text{eff}} \left( M_{f_{\text{new max}}} - M_{\text{max}} \right) = \int_{0}^{L} \left( \frac{M_{f_{\text{new max}}}}{M_{f_{\text{max}}}} - 1 \right) M_{f}(x) \, dx \]

\[ L_{\text{eff}} = \int_{0}^{L} \frac{M_{f}(x)}{M_{f_{\text{max}}}} \, dx \]

The effective length does not depend on the variation applied to the bending moment curve anymore. To one curve we can associate one effective length independent of the applied variation.

We then study the relation between effective lengths of two curves 1 and 2 so that \( M_{f_1} \leq M_{f_2} \) and \( M_{f_{1 \text{max}}} = M_{f_{2 \text{max}}} \)

![Figure 54: Comparison of two bending moments]

\[ L_{\text{eff}_1} = \int_{0}^{L} \frac{M_{f_1}(x)}{M_{f_{1 \text{max}}}} \, dx \]

\[ \leq \int_{0}^{L} \frac{M_{f_2}(x)}{M_{f_{1 \text{max}}}} \, dx = L_{\text{eff}_2} \]

So, if the Frigate bending moment curve fits between two bending moment curves (having the same maximum bending moment as Frigate) then we will have a lower and upper boundary for the effective length associated to the Frigate bending moment curve.

The bending moment is null at 0 and L and its maximum is \( M_{f_{\text{max}}} \) at \( \frac{L}{2} \). The simplest function verifying these conditions is \( f(x) = M_{f_{\text{max}}} x(L - x) \frac{4}{L^2} \)

We use the indexed family of functions \( f_n(x) = M_{f_{\text{max}}} x^n(L - x)^n \left( \frac{2}{L} \right)^{2n} \) to “surround” the Frigate bending moment curve.
The Frigate bending moment curve used is the following one (in light green). It corresponds to the load case which induces the highest bending moment. Bending moment curves induced by other load cases are thus not shown on the following graphs.

![Frigate highest bending moment curve](image)

**Figure 55: Frigate highest bending moment curve**

![Frigate highest bending moment curve and the family of functions curves](image)

**Figure 56: Frigate highest bending moment curve and the family of functions curves**

We can then calculate:

\[
L_{eff,n} = \left(\frac{2}{L}\right)^2 \int_0^L x^n (L - x)^n \, dx
\]

<table>
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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha = \frac{L_{eff}}{L})</td>
<td>0.666</td>
<td>0.533</td>
<td>0.457</td>
<td>0.406</td>
</tr>
</tbody>
</table>

**Table 20: Effective length to length ratio**

Since the Frigate curve is between curves 2 and 3 and much closer to 3, we choose \(\alpha_{FLF} = 0.47\)