COMBINED CURRENT/VOLTAGE MODEL WITH INDUCTANCE MISMATCH COMPENSATION FOR ON-LINE ESTIMATION OF FLUX LINKAGES IN SYNCHRONOUS MOTOR DRIVES

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Abstract

A flux linkage estimator for synchronous machines which combines a voltage and a current model is analysed, based on an original scheme with some modifications. The proposed schemes assure convergence at steady-state and better transient performances. Simulation and experimental results were performed for different combinations of control schemes and flux linkage estimators, validating the discussed theory.

1 Introduction

On-line parameter estimation for synchronous motor drives is still an interesting research topic. Whether the machine is a surface/interior permanent magnet machine (SPMM or IPMM) or a synchronous reluctance motor (SynRM), the problems of the estimation are quite similar. The stator resistance changes with temperature [4], the magnet flux linkage (if any) is influenced by the temperature and ageing [4], [5], [9], while inductances are subjected to saturation [5]. Different techniques apply to the on-line parameter estimation: Model-reference adaptive systems [1], Kalman filters [1], artificial neural networks [8], recursive least square error methods [2], signal injection methods [7], observers based on Lyapunov theory [5], estimators based on machine equations [3], [6], [10]. Some approaches consider constant inductances, as [4] and [9], limiting their utilisation to systems on which there exists some prior knowledge of the machine characteristics. Other solutions, e.g. like [6], combine the machine equations to very specific machine features, like the amplitude of the main harmonics in the back electromotive force. Others, like [3] and [10], rely on the knowledge of the stator resistance and try to perform a flux linkage estimation even in saturating conditions.

In this paper, the solutions proposed in [3] and [10] are analysed and modified in order to obtain improved convergence performances. The proposed estimator is based on the machine equations, but it aims to compensate the inductance value mismatch caused by the magnetic saturation. The paper is organised as follows. Section 2 recalls the basic theoretical background of both the machine equations and of

the original flux estimator. The proposed modifications are discussed in subsection 2.3. Simulation results are shown in Section 3, while experimental measurements are reported in Section 5. Some conclusive remarks and acknowledgements conclude the paper.

2 Theory of operation

2.1 Machine model

The equations describing the dynamics of a synchronous machine in a stator-fixed $\alpha\beta$ reference frame are as follows:

$$u_\alpha = Ri_\alpha + \frac{d\lambda_\alpha}{dt}, \quad u_\beta = Ri_\beta + \frac{d\lambda_\beta}{dt} \quad (1)$$

where R is the stator resistance, u_α and u_β are the stator voltages on the α and β axes respectively, i_α and i_β are the stator currents on the α and β axes respectively, λ_α and λ_β are the stator flux linkages on the α and β axes respectively. A more convenient description in the dq rotating reference frame is given, by using the following transformations of a generic space vector $\mathbf{g} = g_\alpha + jg_\beta$:

$$\begin{bmatrix} g_\alpha \\ g_\beta \end{bmatrix} = \mathbf{T}_{dq/\alpha\beta} \begin{bmatrix} g_d \\ g_q \end{bmatrix}, \quad \begin{bmatrix} g_d \\ g_q \end{bmatrix} = \mathbf{T}_{\alpha\beta/dq} \begin{bmatrix} g_\alpha \\ g_\beta \end{bmatrix} \quad (2)$$

$$\mathbf{T}_{dq/\alpha\beta} = \begin{bmatrix} \cos(\vartheta_{me}) & -\sin(\vartheta_{me}) \\ \sin(\vartheta_{me}) & \cos(\vartheta_{me}) \end{bmatrix} \quad (3)$$

$$\mathbf{T}_{\alpha\beta/dq} = \begin{bmatrix} \cos(\vartheta_{me}) & \sin(\vartheta_{me}) \\ -\sin(\vartheta_{me}) & \cos(\vartheta_{me}) \end{bmatrix}$$

where ϑ_{me} is the electro-mechanical angle of the machine with respect to the fixed stator reference frame. By applying Equation (3) to (1), one obtains:

$$u_d = Ri_d + \frac{d\lambda_d}{dt} - \omega_{me}\lambda_q \quad (4)$$

$$u_q = Ri_q + \frac{d\lambda_q}{dt} + \omega_{me}\lambda_d$$

where u_d and u_q are the stator voltages on the d and q axes respectively, i_d and i_q are the stator currents on the d and q axes respectively, λ_d and λ_q are the stator flux linkages on the d and q axes respectively, and ω_{me} is the electro-mechanical speed. In order to complete the description of the drive system, the torque equation and the mechanical equation should be added:

$$\tau = \frac{3}{2}p(\lambda_d i_q - \lambda_q i_d) \quad (5)$$

$$\tau = J_m \frac{d\omega_m}{dt} + B_m \omega_m + \tau_L$$

where τ is the motor torque, p is the number of pole pairs, J_m is the mechanical inertia, B_m is the viscous friction of the mechanical system, and ω_m is the mechanical speed equal to ω_{me}/p . It is worth to note that in general the flux linkages λ_d and λ_q are non-linear functions of the current i_d and i_q , due to the saturation of the magnetic paths.

2.1 Flux linkage estimation

In order to perform an on-line estimation of the flux linkages (whether in the $\alpha\beta$ or in the dq reference frames), it is necessary to have a reliable mathematical model. Typically, there are two ways of estimating flux linkages, as reported in Equation (6) and Equation (7).

$$\lambda_\alpha = \int (u_\alpha - R i_\alpha) dt, \quad \lambda_\beta = \int (u_\beta - R i_\beta) dt \quad (6)$$

$$\lambda_d = L_d i_d + \lambda_{mg}, \quad \lambda_q = L_q i_q \quad (7)$$

Equation (6) is based on the reversal form of Equation (1), and thus it is performed in the $\alpha\beta$ reference frame. The stator resistance R is the crucial parameter. Actually, the literature offers smart solutions to the problem of the estimation and the on-line tracking of the stator resistance. Therefore, in the following the stator resistance will be considered, without loss of generality, as a constant and known parameter.

Opposite to Equation (6), Equation (7) is performed in the dq reference frame, setting L_d and L_q as the synchronous inductances in the d and q axes respectively, and λ_{mg} as the magnet flux linkage. As known, the values of L_d and L_q are subjected to changes due to the different load conditions (which forces different operating points in the magnetic circuit of the machine). Thus, their value cannot be considered neither known a-priori, nor constant. The magnet flux linkage can be considered known with a certain degree of confidence, although its value is not of foremost importance for a correct convergence of the flux linkage estimator.

The flux linkage estimation performed by Equation (6) suffers of the discrete digital implementation of the integrator. Many solutions have been proposed in order to mitigate the problem, but the ultimate solution has still to be found. On the top of that, it is also recognised that inaccuracies in the stator resistance estimation, or in the voltage or current measurements, have a larger impact when the machine is rotating at low speed and lighter loads – that is, voltages and currents are small, if compared to their nominal values. For the aforementioned reasons, it is a good practice to use the Equation (7) in the low speed region of the machine, for the absence of an integral action and of the voltage terms.

2.2 The original flux linkage estimator

An example of flux linkage estimator that mixes Equation (6) and Equation (7) is reported in [3], and further elaborated in a

successive publication [10]. The basic flux linkage estimator proposed in [3] and [10] is the following:

$$\begin{aligned} \hat{\lambda}_{\alpha\beta} &= \frac{s^2}{s^2 + 2\xi\omega_0 s + \omega_0^2} \hat{\lambda}_{\alpha\beta,v} + \\ &+ \frac{2\xi\omega_0 s + \omega_0^2}{s^2 + 2\xi\omega_0 s + \omega_0^2} \hat{\lambda}_{\alpha\beta,c} \\ &= G_{\alpha\beta,v} \hat{\lambda}_{\alpha\beta,v} + G_{\alpha\beta,c} \hat{\lambda}_{\alpha\beta,c} \end{aligned} \quad (8)$$

where $\hat{\lambda}_{\alpha\beta}$ is the estimated flux linkage vector, $\hat{\lambda}_{\alpha\beta,v}$ is the estimated flux linkage vector with the Equation (6) (so-called the voltage model), and $\hat{\lambda}_{\alpha\beta,c}$ is the estimated flux linkage vector with the Equation (7) (called the current model) and then back-transformed in the $\alpha\beta$ reference frame. The discrimination between the voltage and the current model is obtained by means of a high-pass filter for the voltage model, and a low-pass filter for the current model (second-order filters in Equation (8)). The two filters (called $G_{\alpha\beta,v}$ and $G_{\alpha\beta,i}$) are complementary, which means that their sum is equal to one.

As described in [3] and [10], however, the presence of magnetic saturation and the inaccuracy of the inductance values lead to incorrect flux linkage estimation. Thus, the solution in [10] proposes to compensate the inaccuracy by means of an additive term $\Delta\lambda_{dq} = \lambda_{dq} - \hat{\lambda}_{dq}$, as shown in Fig. 1, where $\mathbf{R}$ and $\hat{\mathbf{L}}$ are equal to:

$$\mathbf{R} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}, \quad \hat{\mathbf{L}} = \begin{bmatrix} \hat{L}_d & 0 \\ 0 & \hat{L}_q \end{bmatrix} \quad (9)$$

Fig. 1. Flux estimation scheme proposed by [10].

The scheme of Fig. 1 shows that the additional compensating term is processed through the same transfer function of the current model. In other words, the output is expressed by:

$$\hat{\lambda}_{\alpha\beta} = G_{\alpha\beta,v} \hat{\lambda}_{\alpha\beta,v} + G_{\alpha\beta,c} \hat{\lambda}_{\alpha\beta,c} + G_{\alpha\beta,c} \Delta\lambda_{\alpha\beta} \quad (10)$$

where:

$$\Delta\lambda_{\alpha\beta} = \mathbf{T}_{dq/\alpha\beta} \Delta\lambda_{dq} \quad (11)$$

The solution of [10] obtains the value of $\Delta\lambda_{dq}$ from a conventional FOC with PI current regulators and decoupling terms, where the latter are used to remove the dependence of the d axis to the q axis and vice versa (see Equation (4)). With the hypothesis of perfect estimation of the flux linkages, it is recognised that during steady-state operation (no derivative terms in Equation (4)) the PI current regulators generate a value that compensates the resistive drop, while the terms $-\omega_{me}\lambda_q$ and $\omega_{me}\lambda_d$ disappear due to the decoupling terms. When the flux linkage is not estimated correctly, [10] shows that it is possible to obtain $\Delta\lambda_{dq}$ by arranging Equation (4) (for more details, refer to [10]):

$$\Delta\lambda_{dq} = \frac{1}{\omega_{me}} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} (\mathbf{u}_{dq,int} - \mathbf{R}\mathbf{i}_{dq}) \quad (12)$$

Where $\mathbf{u}_{dq,int}$ is the space vector containing the integral part of the PI regulators output. Equation (12) becomes null in case of perfect flux linkage estimation.

2.3 The modified flux linkage estimator

In order to understand the proposed modifications, Fig. 2 reports a simplified block scheme of the original estimator.

Fig. 2. Simplification of the original scheme.

It is now assumed that the inductances in the current model are not accurate, thus the current model provides an incorrect flux linkage estimation which will affect the whole flux linkage estimation. With the compensating term as in Fig. 2, the following conditions occur:

- If $\Delta\lambda_{dq} \neq 0$ then $\lambda_{dq} \neq \hat{\lambda}_{dq}$, and the estimator is not converging to the correct flux linkage;
- If $\Delta\lambda_{dq} = 0$, the contribution from the upper branch in Fig. 2 is zeroed because the overall transfer function does not contain any integral action. Thus, the scheme of Fig. 2 falls back to the case of Equation (8). Since it was assumed that the current model was not estimating correctly, the total flux linkage estimation will also be incorrect.

As a consequence, the scheme of Fig. 1 and Fig. 2 do not assure perfect convergence to the actual flux linkage. However, a correct convergence can be re-established with a slight modification of the compensation term. Maintaining its origin from the integral part of the PI regulators output, its transfer function to the flux linkage estimation could be modified as shown in Fig. 3.

Fig. 3. Proposed flux linkage estimator, case 1.

The main difference with respect to the scheme of Fig. 1 is represented by the presence of another PI regulator, with proportional gain K_p and integral gain K_i . The overall flux linkage estimation algorithm is:

$$\hat{\lambda}_{dq} = G_{dq,v}\hat{\lambda}_{dq,v} + G_{dq,c}\hat{\lambda}_{dq,c} + \left(K_p + \frac{K_i}{s}\right)(\lambda_{dq} - \hat{\lambda}_{dq}) \quad (13)$$

which leads to the following expression:

$$\hat{\lambda}_{dq} = \frac{sK_p + K_i}{s + sK_p + K_i}\lambda_{dq} + \frac{sG_{dq,v}}{s + sK_p + K_i}\hat{\lambda}_{dq,v} + \frac{sG_{dq,c}}{s + sK_p + K_i}\hat{\lambda}_{dq,c} \quad (14)$$

By applying the final value theorem to Equation (14) with the hypothesis of a step change of λ_{dq} (and thus a step change of $\hat{\lambda}_{dq,v}$ and $\hat{\lambda}_{dq,c}$ as well), it is immediate to obtain $\hat{\lambda}_{dq} = \lambda_{dq}$ at steady state in any condition.

Another possible modification is shown in Fig. 4.

Fig. 4. Proposed flux linkage estimator, case 2.

In the latter case, the inductances of the matrix $\hat{\mathbf{L}}$ are recalculated with a constant sample time, larger than the PWM and current sample, following the formula:

$$\hat{\mathbf{L}} = \frac{\lambda_{dq} - \hat{\lambda}_{pm}}{i_{dq}} \quad (15)$$

The output of the current model is consequently updated to the last output of the total flux linkage estimator. If the voltage model produces a correct output, it is possible to obtain a correct convergence of the overall scheme. Such consideration limits the exploitation of the scheme in Fig. 4 only to the cases in which the voltage estimator is working perfectly, actually making the current model redundant. Nevertheless, its utilisation can be of interest when merged with the scheme of Fig. 3, as shown in Fig. 5.

Fig. 5. Proposed flux linkage estimator, case 3.

In this last case, the presence of the inductance update is meant to help the convergence of the overall flux estimator, particularly during transients.

3 Simulation results

Simulations of the three proposed schemes were performed on the IPM machine whose parameters are reported in Tab. 1, with the current-flux linkage relation shown in Fig. 6.

Name	Value
DC-bus voltage	550 V
PWM sample time	10 kHz
Nominal speed	94,3 rad/s
Nominal torque	7 Nm
Nominal current	4,5 A _{rms}
Pole pairs	2
Stator resistance	6,2 Ω
Magnet flux linkage	0,0419 V · s
Mechanical inertia	3,1 · 10 ⁻³ kg · m ²
Viscous friction	1,3 · 10 ⁻³ N · m · s/rad

Tab. 1. Parameters of the simulated machine.

Fig. 6. i_{dq} -to- λ_{dq} relations for the simulated machine.

Two different control schemes were tested: a conventional FOC with decoupling terms, and a hysteresis-based current control in the abc reference frame, with outer PI control for speed regulation. The basic differences between the two options are:

- the hysteresis control operates at faster sample times (25 μ s in the simulations, 4 times smaller than the FOC current sample time);
- the hysteresis control do not have a PI integral action, thus the term in Equation (12) must be calculated from the hysteresis blocks output by removing the coupling terms and by low-pass filtering.

Different combinations of estimators have been tested on the typical simulation profile shown in Fig. 7. The profile is obtained with a no-load speed step from 0 to 100 rpm at $t = 0$ s, a rated-torque load step at 100 rpm at $t = 1$ s, and a speed step from 100 rpm to 200 rpm at rated torque at $t = 2$ s. Steady-state errors e_∞ were calculated before a new transient, as well as transient errors e_{RMS} with the following error formula:

$$e_{RMS} = \sqrt{\frac{\sum_{k=n}^m (\lambda_{dq,k} - \hat{\lambda}_{dq,k})^2}{m - n}} \quad (16)$$

where n and m denote the starting and ending sample of the transient, respectively. The errors were subsequently normalised to the nominal flux linkage value.

Fig. 8 and Fig. 9 report the steady-state and transient errors for the d -axis flux linkage estimation. The twelve simulation combinations were obtained by FOC or hysteresis control, absence or presence of inverter dead-time compensation (denoted as NoIC or IC), original estimator or estimators of

case 1, 2 or 3 (denoted as Or, c1, c2, c3). Steady-state error results show a decrease of estimation error with the proposed schemes, with a clear benefit from the inverter compensation (when present) or from the increase of sampling time of the hysteresis control. Transient error results are higher than the steady-state ones, especially for no-load and low-speed transients. However, it is worth to highlight that the original estimator in [10] is based on steady-state equations (see Equation (12)).

Another batch of simulations, here omitted, showed that the scheme is completely robust against variations of the magnet flux linkage value.

Fig. 7. Typical simulation for the evaluation of the estimators.

Fig. 8. Steady-state error on d -axis flux linkage estimation.

Fig. 9. Transient error on d -axis flux linkage estimation.

4 Experimental results

To test the wide applicability of the method, a batch of experimental results were performed on a new motor, namely a 15-kW SynRM with nominal speed of 1500 rpm. The control strategy had a sample time equal to 250 μ s. Results are shown from Fig. 10 to Fig. 15 considering the scheme of Fig. 3.

The actual flux linkages λ_d and λ_q have been calculated from look-up tables obtained from a finite-element analysis of the

machine. The currents i_d and i_q are used as input of the look-up tables. The estimated flux linkages of the proposed estimator are reported as $\hat{\lambda}_d$ and $\hat{\lambda}_q$, while the output of the estimator without the proposed modifications (that of Fig. 1) is reported as $\hat{\lambda}_{d,orig}$ and $\hat{\lambda}_{q,orig}$. The transient described in the pictures refers to a load torque inset from 0% to 97% of the nominal one at $t = 0,3$ s, at a reference speed of 50 rpm. The speed was chosen in order to be lower than the frequency set by ω_0 in the filters of Equation (8).

Fig. 10 and Fig. 11 show the performances of the flux linkage estimator, when the stator resistance is set properly and the position/speed measurements are correct. The inductances of the current model are set to a value related to the operation in the linear magnetic circuit condition (low-load condition). It is visible the correct convergence of the proposed estimator, after a transient that is mainly determined by the parameters of the PI regulator of Fig. 3 and the lower limitation for the speed in the calculation of $\Delta\lambda_{dq}$ (see Equation (12), this issue was also mentioned in [10]). The relatively slowness of the convergence (about one-two seconds) is related to the estimator assumption of steady-state condition, which leads to the calculations of Equation (12). For this reason, the original and the proposed schemes are not intended to be instantaneous real-time estimators. Indeed, they could be exploited as an aiding tool for the on-line update and store of flux-linkage look-up tables, useful to map the operating conditions of the machine. The results of the update process could then be used in the control scheme.

Fig. 12 and Fig. 13 report the behaviour of the estimators in presence of a stator resistance mismatch, when the control scheme is using a value equal to -10% the actual one (this condition resembles an increase of stator resistance value due to an increase in temperature in the machine windings). It is clearly visible the dependence of the whole estimation scheme upon the value of this parameter, although the sensibility has been reduced with the proposed estimator. As already mentioned, the results suggest that the stator resistance is a critical parameter that should be estimated with great attention, possibly avoiding the use of machine models that include flux linkages or inductances.

Fig. 14 and Fig. 15 report the behaviour of the scheme in presence of an artificial error introduced in the electrical position measurement performed by the drive. An error of +5 electrical degrees was added to the measure in order to resemble the case of an inaccurate sensorless control scheme, in which the position of the d and q axes are not estimated correctly. The results show a greater sensibility of the estimators on the q axis, while the d axis estimation remains quite unaffected (although the original estimator scheme improved by comparing Fig. 10, but it is believed that the introduced position error compensated the steady-state error of the estimator). These results suggest that a complete sensorless scheme with flux estimation scheme should be thoroughly tested in order to avoid instabilities in case of non-perfect alignment of the estimated dq reference frame with respect to the actual one.

5 Conclusions

A flux linkage estimator based on a mixed voltage and current model was analysed and modifications have been proposed to improve its performances in terms of convergence at steady-state. Simulation results were performed on an IPMM and experimental test were performed on a SynRM. Results show an increase of performances both in steady-state and in transient conditions as well, while the dependence upon the stator resistance value is reduced. Future works should focus on the implementation within a speed-sensorless estimation scheme.

Fig. 10. Estimation on the d axis, correct parameters.

Fig. 11. Estimation on the q axis, correct parameters.

Fig. 12. Estimation on the d axis, estimated $R = -10\%$.

Fig. 13. Estimation on the q axis, estimated $R = -10\%$.

Fig. 14. Estimation on the d axis, position error=+5 el.deg..

Fig. 15. Estimation on the q axis, position error=+5 el.deg..

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