Modeling and control of a line-commutated HVDC transmission system interacting with a VSC STATCOM

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Doctoral Dissertation
Royal Institute of Technology
Department of Electrical Engineering
To:

my professor Åke Ekström,
my wife Yoshimi and my son Alexandre,
my mother Katalin, brothers and sister,
and my friend Mats Lagerkvist,
(in memoriam)
Abstract

The interaction of an HVDC converter with the connected power system is of complex nature. An accurate model of the converter is required to study these interactions. The use of analytical small-signal converter models provides useful insight and understanding of the interaction of the HVDC system and the connected system components.

In this thesis analytical models of the HVDC converters are developed in the frequency-domain by calculating different transfer functions for small superimposed oscillations of voltage, current, and control signals. The objective is to study the dynamic properties of the combined AC-DC interaction and the interaction between different HVDC converters with small signal analysis.

It is well known that the classical Bode/Nyquist/Nichols control theory provides a good tool for this purpose if transfer functions that thoroughly describe the ‘plant’ or the ‘process’ are available. Thus, there is a need for such a frequency-domain model.

Experience and theoretical calculation have shown that voltage/power stability is a very important issue for an HVDC transmission link based on conventional line-commutated thyristor-controlled converters connected to an AC system with low short circuit capacity. The lower the short circuit capacity of the connected AC system as compared with the power rating of the HVDC converter, the more problems related to voltage/power stability are expected.

Low-order harmonic resonance is another issue of concern when line-commutated HVDC converters are connected to a weak AC system. This resonance appears due to the presence of filters and shunt capacitors together with the AC network impedance. With a weak AC system connected to the HVDC converter, the system impedances interact through the converter and create resonances on both the AC- and DC-sides of the converter. In general, these resonance conditions may impose limitations on the design of the HVDC controllers.

In order to improve the performance of the HVDC transmission system when it is connected to a weak AC system network, a reactive compensator with a voltage source converter has been closely connected to the inverter bus. In this thesis it is shown that the voltage source converter, with an appropriate control strategy, will behave like a rotating synchronous condenser and can be used in a similar way for the dynamic compensation of power transmission systems, providing voltage support and increasing the transient stability of the converter.

Keywords: HVDC transmission, LCC – Line-commutated Current-source Converter, STATCOM, VSC – Voltage Source Converter, Space-Vector (complex vector), transfer-function, frequency-domain analysis.
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Modeling and control of an HVDC transmission system in the frequency domain is a big subject, and I should say that this was a difficult subject as well. Because of that this is not a short monograph. The goal was to make it as useful as possible to the reader. Therefore I have put effort in trying to detail all the important parts of the work, and combined theory with validation examples. Hopefully this will be valuable and used in future research. To achieve this Lennart Ängquist have been helpful in formulating the approach to system modeling and analysis used in the thesis. I am deeply grateful that Lennart Ångquist had imparted to me his strong and solid knowledge and expertise, not only the in the application field of HVDC and FACTS, but also in the mathematical and scientific field. He has been an excellent supervisor and a mentor as well.

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For all of these, I have been privileged to have been student from both Lennart Ångquist and Hans Peter Nee.

Professor Åke Ekström, former professor in High Power Electronics at KTH, brought up the idea and encouraged me to perform this research work. The idea for this research grew along the years. During the 1970’ies Åke Ekström together with Erik Persson from ASEA (now ABB) worked together in the theoretical investigation of some phenomena observed in different HVDC projects. The result of the investigation was presented by Erik Persson in an IEE paper 1970. Professor Åke Ekström suggested the continuation of that development work, expanding the modeling to different types of HVDC converters in a systematic way, and also applying modern mathematics. I am sincerely grateful for this suggestion. Professor Åke Ekström has been and always will be a constant source of inspiration.

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Paulo Fischer de Toledo
Ludvika, 11 July 2007
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<table>
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<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>AC</td>
<td>Alternate Current</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>ESCR</td>
<td>Effective Short-Circuit Ratio</td>
</tr>
<tr>
<td>HVDC</td>
<td>High Voltage Direct Current transmission</td>
</tr>
<tr>
<td>LCC</td>
<td>Line-Commutated Current-Source Converter</td>
</tr>
<tr>
<td>PLL</td>
<td>Phase-Locked-Loop</td>
</tr>
<tr>
<td>PWM</td>
<td>Pulse-Width Modulation</td>
</tr>
<tr>
<td>rms</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>SCC</td>
<td>Short-Circuit Capacity</td>
</tr>
<tr>
<td>SCR</td>
<td>Short-Circuit Ratio</td>
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<tr>
<td>SSR</td>
<td>Subsynchronous Resonance</td>
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<tr>
<td>SSTI</td>
<td>Subsynchronous Torsional Interaction</td>
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<tr>
<td>STATCOM</td>
<td>Static Synchronous Compensator</td>
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<tr>
<td>SVC</td>
<td>Static Var Compensator</td>
</tr>
<tr>
<td>VSC</td>
<td>Voltage Source Converter</td>
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CHAPTER 1
INTRODUCTION

1.1 HVDC TRANSMISSION

1.1.1 HVDC-LCC converters

The first commercial HVDC transmission link was commissioned in 1954. It utilized line-commutated current-source converters (LCC) with mercury-arc valves. They were followed during the late 1960’s by similar converters using semiconductor thyristor valves. Thyristor valves have now become standard equipment for DC converter stations.

The turn-on of the thyristors in an LCC is controlled by a gate signal while the turn-off occurs at the zero crossing of the AC current which is determined by the AC network voltage (‘line commutation’).

During forward conduction of the thyristor devices, charges are internally stored, and at turnoff these charges must be removed before the valve can reestablish a forward voltage blocking capability. Therefore, the HVDC inverters require a certain minimum time, the recovery time, with negative voltage before forward blocking voltage is applied. In order to achieve this condition the inverter is operated with a certain commutation margin angle. When sufficient recovery time is not provided the converter may suffer a commutation failure. Most often commutation failures originate from voltage disturbances due to AC system faults. A commutation failure can, however, also be caused if the connected system has limited short circuit capability and cannot provide the required voltage for the commutation process.

As the current is always lagging the voltage, this type of converter, either in rectifier or inverter operation, continuously absorbs reactive power as a part of the conversion process. Therefore, shunt compensation is required to compensate for some or the entire converter VAR requirement. The amount of reactive power
absorbed is determined by the DC control strategy, and typically is in the order of 50% of the active power flow through the converter.

1.1.2 Voltage source converters

A new generation of HVDC converters based on forced-commutated voltage-source converters (VSC) was developed during 1990’s.

The fundamental difference between the conventional LCC and the new VSC is that the VSC makes use of components that can turn-off the current, and not just turn it on. Typical turn-off devices as switching elements are IGBTs.

Since the current in a VSC can be turned off there is no need for commutation voltage in the connected AC network. Therefore, in this type of converter the AC current can be leading or lagging the AC voltage, which means that the converter can consume or supply reactive power to the connected AC network.

A VSC operating as a static compensator (STATCOM) can support the system in providing the necessary commutation voltage for the operation of the LCC. It can also provide reactive power compensation to the network during steady-state operation, dynamic and transient condition.

1.2 LINE-COMMUTATED HVDC CONVERTERS CONNECTED TO A WEAK AC SYSTEM

The performance of various components in a power system depends on the strength of the system. The strength of the system reflects the sensitivity of the system voltage to various disturbances in the system. In a strong system, disturbances caused, for example, by a change in the power of a load, do not cause any significant change in the voltage and angle in the power system. However, in a weak system even a small disturbance can cause so large deviations in the voltages that the operation of the system is jeopardized. The short circuit level or equivalent impedance at a bus is a good measure of the strength of the system at that particular point.

HVDC converters based on line-commutated converters (HVDC-LCC) can be seen as loads connected to the network with special characteristics such as voltage and angle dependence. They may offer controllability due to the possibility to control the firing instants of the valves.

The strength of an AC system connected to an HVDC converter is often described in terms of the Effective Short-Circuit Ratio (ESCR) [20]. In general, system planners were using an ESCR of 2.5 as the lower limit for considering interconnection of systems by HVDC [5]. However, advanced control techniques have made it possible to successfully operate DC links located at terminals with ESCR's less than the limit of 2.5. As an example, the Itaipu HVDC transmission link in Brazil has been designed to operate under certain system conditions having ESCR of 1.7.
1.2.1 Voltage/power stability

Experience and theoretical calculations have shown that the voltage/power stability is a critical issue for an HVDC transmission link based on conventional LCCs, if the receiving end of the transmission link is connected to an AC system having low short circuit capacity. The lower the short circuit capacity of the connected AC system as compared with the power rating of the HVDC converter, the more problems related to voltage/power stability can be expected. The physical mechanism causing this voltage instability is the inability of the power system to provide the reactive power needed by the converters to maintain an acceptable system voltage level.

1.2.2 Low order harmonic resonance

Low-order harmonic resonance is another issue of concern when HVDC-LCC converters are connected to weak AC systems. This resonance appears due to the presence of filters and shunt capacitors (designed to prevent harmonics generated by the HVDC system entering the AC system and to provide reactive power needed by the operation of the converters) with the AC network impedance. When the weak AC system is connected to an HVDC converter terminal, the system impedances interact through the converter to create resonances on both the AC and DC sides of the converter [6-7]. This can create a highly oscillatory power system, which could be close to the point of instability [8-11]. In general, this resonance condition imposes limitations on the design of the HVDC controllers. The weaker the network, the more challenging the control problem becomes.

1.3 THE STATCOM USED TO SUPPORT THE HVDC-LCC

The reactive power supply from fixed capacitors to HVDC-LCCs can be critical for weak systems since the resulting effective short circuit (ESCR) will be lower, which makes the AC system even weaker.

To reduce the amount of fixed capacitors for reactive power compensation an alternative is to use Static Var Compensators (SVC) as an alternative for AC transmission systems. However, experience from existing installations has shown that an SVC at the HVDC inverter terminal where the AC system has a very low SCR (SCR = 1.5) can cause an increased number of commutation failures during the recovery from single line to ground faults [12]. This behavior can be attributed to the natural dynamics of the SVC, which maintains most of the capacitors during the fault and after the fault is cleared as it is sensing the low voltage, reducing the ESCR even further, resulting in repeated commutation failures.

A synchronous condenser is a compensation device which provides a real contribution to the system strength. It helps to reduce the magnitude of the terminal voltage excursions and the frequency of the resulting commutation failures, and to
improve the DC recovery. The Itaipu receiving end [13] and the Nelson River System upgrade [14] are two examples of where synchronous condensers are preferable and have been chosen to provide the reactive compensation after considering overall factors. However, the synchronous condenser has a large response time as compared to other compensation options, and also exhibits low frequency electromechanical oscillations.

The expected performance of a VSC operating as a STATCOM is that it should be analogous to that of the rotating synchronous condenser and can be used in a similar way for the dynamic compensation of power transmission systems, providing voltage support, increased transient stability, and improved damping [15]. One important contribution from this work is to qualify and quantify the support that a VSC can give to improve the operation of the LCC when connected to a system with limited short circuit capacity.

1.4 HVDC CONVERTER MODEL

The interaction of an HVDC converter with the connected power system is of complex nature. An accurate model of the converter is required to study these interactions. The use of an analytical small-signal converter model will provide a useful insight and understanding of the HVDC system behavior.

In this thesis an analytical model of the HVDC converter has been developed in the frequency-domain by calculating different transfer functions for small superimposed oscillations in voltage, current, and control signals. The objective is to study the dynamic proprieties of the combined AC-DC interaction and the interaction between different HVDC converters.

It is well known that the classical Bode/Nyquist/Nichols control theory provides a good tool for this purpose if transfer functions that thoroughly describe the ‘plant’ or the ‘process’ are available. Thus, there is a need for such a frequency-domain model.

The models have been developed in such a way that they use a number of subsystems, each having its own linearized model. Examples of such subsystems are: the LCC itself, the controllers of the converter, the VSC, the controllers of the VSC, the components of the AC and DC networks. For all of these subsystems transfer functions are individually determined, and then they are interconnected to form the complete system. This approach relies on the principle of superposition for linear systems. The same has been made for the VSC model.

1.4.1 LCC model

Studies that require an exact representation of the non-linear behavior of the thyristor converter, including its commutations, can only be performed in time-domain simulations. However, it is well known that a small-signal model in the frequency domain can be used to predict the dynamic performance and stability.
1.4 HVDC Converter model

Such a model in most cases is very advantageous as it requires much less computation time than a corresponding time-domain simulation. It also provides more insight and understanding of the interaction between AC and DC sides caused by the converter.

In a paper [1] (1970) Erik Persson presented a small-signal frequency-domain model of an HVDC converter. A constant non-varying overlap angle was considered in this model. The embodiment of this work was the derivation of transfer functions between the DC side quantities and the control signal by the use of conversion functions. Various elementary transfer functions were combined in order to analyze the current control system using the Nyquist stability criterion. Several resonance conditions were identified, which imposed limits on the gain in the current controller, thereby limiting the speed of control.

An objective of the present work is to pursue the method outlined in [1]. The formulation of the derivation of the transfer functions, however, has been modified by introducing (complex-valued) space vectors to represent the three-phase quantities. The space-vector concept was introduced already in the 1950’s by Kovacs [2] and others and it offers several advantages: a concise representation of the system as the zero-sequence model is not necessary when the neutral is not connected; representation of three time-varying quantities with one space vector complex variable; transfer functions can be formally defined for the space-vector variables making it possible to apply classical control theory.

As in reference [1] the technique with conversion functions to describe the operation of the converter bridge will be used. It will be shown that only the fundamental frequency component of the conversion function needs to be considered since the harmonics could be disregarded as they have little influence on the dynamics for low frequency oscillations.

An important contribution from this work is an improvement of the model in [1] with regard to the overlap angle during commutation. In the model in [1] a fixed overlap angle was assumed. However, experience shows that in reality a certain variation of the overlap angle may occur, partly due to varying current and partly due to changes in the commutating voltage. This condition arises in particular when the converter is connected to a weak network in which case the AC current may excite a resonance in the grid. The phenomenon can introduce a significant damping in the transfer functions. The inclusion of the varying overlap angle significantly increases the complexity of the model. A separate specific chapter in this thesis is dedicated to describe this part of the model.

The model also includes the Phase-Locked Loop (PLL) used for synchronization of the controllers of the HVDC converter, which also may have a significant impact on system stability.

1.4.2 VSC model

Similar to the HVDC-LCC transmission link model an analytical model of the VSC which includes transfer functions for superimposed oscillations of small amplitude
in voltage, current, and control signals has been developed. With such a model it is possible to predict the dynamic properties and understand the interaction between the AC- and DC-sides of the converter. It is also possible to synthesize different control systems of the converter.

The model has been developed in the $d-q$ frame, which is the usual way to configure the controllers of the VSC. The $d-q$ system results from $\alpha-\beta$ transformation of the space-vector coordinate system into a synchronously rotating coordinate system.

The VSC model includes the basic inner-loop control that controls the voltage from the current regulator and the outer loop controller that controls the voltage of the DC side of the converter and the filter bus voltage. The model also includes the dynamics of the PLL.

## 1.5 MAIN CONTRIBUTIONS

The main contributions of this thesis are listed below:

- An analytical model of the HVDC-LCC transmission link in the frequency domain has been developed. It can be used to study the interaction of an HVDC system with a connected power system. The objective is to obtain useful insight and understanding of how the HVDC system interacts with the connected system component. Contrary to earlier models this model considers the variation of the overlap angle.

- Similar as for the HVDC-LCC transmission link an analytical model of the VSC-STATCOM has been developed in the frequency domain.

- The frequency-domain model of the VSC-STATCOM has been integrated in the HVDC-LCC transmission link in order to study the behavior when they are connected to a common bus in a weak power system.

- A method to evaluate the performance of the HVDC-LCC transmission link in the frequency domain has been developed based on analysis of classical control theory (Nyquist theory), and a number of new voltage/current sensitivity factors have been defined.

- It was shown that the VSC-STATCOM with inner AC current control did not improve the transient performance of the closely connected HVDC-LCC converter due to the AC side resonance, which limits the available bandwidth of the current control system.

- A new control strategy for the VSC is proposed, which makes the converter less sensitive to resonances in the AC network. Its behavior is similar to that of an equivalent synchronous condenser.
1.6  THESIS ORGANIZATION

Chapter 2: This chapter introduces two general theories that are used in the thesis:

1) The classical control theory based on the analysis of transfer functions of the models in the frequency domain.

2) The space vector concept that makes a concise representation of the model, describing the three-phase quantities by a complex vector, when the zero-sequence model is not necessary.

Chapters 3 and 4: The frequency domain model of an HVDC-LCC transmission is developed.

The model is described in two parts: In Part I (Chapter 3) it is assumed that the overlap angle during the commutation remains constant. A similar assumption was made in Erik Persson’s model [1]. However, it was shown in this thesis that this assumption introduces resonances that cause severe errors at certain network conditions.

In Part II (Chapter 4) the model is extended so as to cope with a varying overlap angle in order to bring the frequency domain model into agreement with the results obtained from time-domain simulations.

Chapter 5: A frequency domain model of the VSC with current controller is developed. The control system is based on controlling the converter AC current through the phase reactor. The standard control strategy is applied, i.e., AC and DC voltages are controlled by regulators that give references to the inner current loop.

Chapter 6: This chapter describes how the model of the VSC-STATCOM can be integrated into the model of the HVDC-LCC transmission link.

Chapter 7: In Chapter 7 a methodology for doing stability analysis of an HVDC transmission system in the frequency domain is introduced. This method is an analogy of the classical method of analyzing the power/voltage stability developed by J.D. Ainsworth, A.E. Hammad, G. Andersson, O.B. Nayak [16-19] and others, where they establish the concept of ‘Maximum Available Power’ curves and ‘Voltage Sensitivity Factors’. In this thesis these concepts are used, together with some new ‘Sensitivity Indices’ defined in the thesis to be used in the frequency domain.

The analysis of these ‘Sensitivity Indices’ and analysis of different frequency-domain measures used in the classical control theory were performed in order to evaluate the interaction between the VSC-STATCOM and the HVDC-LCC transmission system.
**Chapter 8**: Chapter 7 has demonstrated that the conventional VSC-STATCOM which is based on controlling the converter AC current through a phase reactor to control the AC voltage cannot improve the performance of the HVDC-LCC voltage/power stability conditions, if the connected AC network is weak in relation to the rating of the converters.

A new control strategy for the VSC-STATCOM is proposed and the frequency domain model of this type of converter is formed in Chapter 8.

**Chapter 9**: In chapter 9 the analysis of cooperation between VSC-STATCOM and the HVDC-LCC transmission is made, with the new control strategy used for the VSC-STATCOM.

The result obtained from this analysis shows that with the new VSC-STATCOM, the converter transiently behaves similar to a synchronous condensed, having equivalent short circuit impedance equal to the impedance of the phase reactor.

**Chapter 10**: Finally, the results from work are summarized.
CHAPTER 2

BASIC CONCEPTS: INTRODUCTION OF THE SPACE VECTOR AND REVIEW OF THE CLASSICAL FEEDBACK CONTROL THEORY IN THE FREQUENCY DOMAIN

This chapter introduces general theory that will be relied upon in subsequent chapters. As the Space Vector concept and the classical feedback control theory should be known already, then the presentation of them will be kept brief.

This chapter starts with the classical feedback control theory. Many powerful methods for analysis and design of control systems are based on modeling of the controlled system (the ‘plant’) in the frequency domain. The main idea is to use the fact that a linear time-invariant system can be completely characterized by its steady-state response to sinusoidal signals. The classical feedback control theory formulated in the frequency domain will be reviewed in this chapter for the analysis of a single-input single-output system.

The chapter then ends with the concept of Space Vector. The models developed in this thesis will be described using complex vector representation of the corresponding three-phase quantities. This representation has advantages as compared to the conventional three-phase representation as it makes a concise representation of the system when the zero-sequence model is not necessary.

2.1 REVIEW OF CLASSICAL FEEDBACK CONTROL PERFORMED IN THE FREQUENCY DOMAIN

A linear system can be characterized by its transfer function $G(s)$, and that the practical study of $G(s)$ can be performed by studying the $G(j\omega)$, where $\omega$ is real. The concepts of frequency response and transfer functions enable the use of graphic methods which are the bases for the classical control theory.
The frequency domain $G(j\omega)$ analysis of the transfer function $G(s)$ is very useful due to the following reasons:

- $G(j\omega)$ gives the response to a sinusoidal input of frequency $\omega$.
- Invaluable insights are obtained from simple frequency-dependent plots.
- Stability conditions and performance of the system can easily be determined by applying classical control theory.
- Important concepts for feedback such as bandwidth and peaks of closed-loop transfer functions can be defined.
- A series interconnection of systems corresponds in the frequency domain to multiplication of the individual system transfer functions, whereas in the time domain the evaluation of complicated convolution integrals is required.

### 2.1.1 Frequency Response

Assume that a linear, time-invariant system exhibits an input/output behavior that is governed by a set of ordinary linear differential equations with constant coefficients. Let $u(t)$ and $y(t)$ represent the input and output signals, respectively. A simple example of such a system then is

$$y(t) + a_1y(t) + a_2y(t) = b_1u(t) + b_2u(t)$$

(2-1)

Applying the Laplace transform, and making $y(0) = \dot{y}(0) = u(0) = 0$, the following corresponding transfer function is obtained

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_1s + b_2}{s^2 + a_1s + a_2}$$

(2-2)

On replacing $s$ by $j\omega$ in the transfer function $G(s)$ the so-called frequency response description is obtained. The frequency response describes a system’s response to sinusoidal input signals of varying frequency. It has a direct link to the time domain, and at each angular frequency $\omega$ the complex number $G(j\omega)$ gives the response to an input sinusoid with the angular frequency $\omega$.

Let us apply a sinusoidal input signal with the angular frequency $\omega$ [rad/s] and magnitude $u_0$, such that

$$u(t) = u_0 \sin(\omega t + \alpha)$$

(2-3)

This signal is persistent and the output signal is also a persistent sine wave with the same frequency

$$y(t) = y_0 \sin(\omega t + \beta)$$

(2-4)

Here $u_0$ and $y_0$ are the magnitudes. It should be noted that the sinusoidal output is shifted in phase from the input by $\phi = \beta - \alpha$. 

The relations \( y_0/u_0 \) and \( \phi = \beta - \alpha \) can be obtained directly from the Laplace transform of \( G(s) \) after inserting the imaginary number \( s = j\omega \). The magnitude and phase of the \( G(j\omega) \) are evaluated by

\[
\frac{y_0}{u_0} = |G(j\omega)| \quad \text{and} \quad \phi = \angle G(j\omega) \quad [\text{rad}]
\]  

Both \( |G(j\omega)| \) and \( \angle G(j\omega) \) depend on the angular frequency \( \omega \).

### 2.1.2 A Feedback structure and corresponding closed-loop transfer functions

A classical negative feedback structure having one degree-of-freedom is shown in Figure 2.1. Let \( r \) denote the reference value for the output \( y \).

![Figure 2.1: Block diagram of a feedback control system](image)

The input to the controller \( K(s) \) is \( r - y \). The input to the plant is

\[
u = K(s) (r - y)
\]  

The objective of the control is to manipulate \( u \), that is, design a controller \( K \) such that the control error, defined by

\[
e = r - y
\]  

remains small.

The plant model is written as

\[
y = G(s) u
\]  

The closed-loop response is then given by

\[
y = GK (r - y)
\]  

yielding

\[
y = \left(1 + GK\right)^{-1} GK r
\]  

The control error is

\[
e = r - y = \left(1 + GK\right)^{-1} r
\]
The following notation and terminology is usually used in the classical control theory:

\[ L = GK : \text{loop transfer function} \] (2-12)
\[ S = (1+ GK)^{-1} = (1+ L)^{-1} : \text{sensitivity function} \] (2-13)
\[ T = (1+ GK)^{-1} GK = (1+ L)^{-1} L : \text{complementary sensitivity function} \] (2-14)

Hence, the equation for the closed-loop system can be re-written as

\[ y = T r \] (2-15)
\[ e = r - y = S r \] (2-16)

These relations show that the objective is to have the output signal close to the reference, by making \( T \approx 1 \), while keeping the control error small, by making \( S \approx 0 \).

Typical Bode plots for \( L \), \( S \) and \( T \) are shown in Figure 2.2, where an example of a system with the loop transfer function given by

\[ L = KG = \frac{0.3(-s + 1)}{s(s + 1)^2} \] (2-17)

is used.

![Bode Diagram](image)

> Figure 2.2: Bode magnitude and phase plots of \( L = KG \), \( S \) and \( T \) when \( L = \frac{0.3(-s + 1)}{s(s + 1)^2} \)

### 2.1.3 Closed-loop stability

From the transfer function of the closed-loop

\[ y = (1+ GK)^{-1} GK = (1+ L)^{-1} L = T r \] (2-18)
the stability of the system is determined by the roots of the closed-loop characteristic equation given by

$$1 + GK = 0$$  \hspace{1cm} (2-19)

provided that the system has no hidden unstable modes, that is, when forming the model of the system there is no cancellation of right half-plane poles and zeros.

One method commonly used to determine the closed-loop stability is to plot the frequency response of \(L(j\omega)\) in the complex plane and the number of encirclement it makes of the critical point \(-1\) is counted. By the Nyquist’s stability criterion the closed-loop stability is inferred by equating the number of encirclements to the number of open-loop unstable poles.

Equivalently, using the Bode’s stability condition, for the open-loop stable systems where \(\angle L(j\omega)\) falls with frequency such that \(\angle L(j\omega)\) crosses \(-180^\circ\) only once (from above at frequency \(\omega_{\ell 80}\)), the closed-loop system is stable if and only if the loop gain \(|L(j\omega)|\) is less than 1 at this frequency. Hence, there is stability when

$$|L(j\omega_{\ell 80})| < 1$$  \hspace{1cm} (2-20)

where \(\omega_{\ell 80}\) is the phase crossover frequency defined by \(\angle L(j\omega_{\ell 80}) = -180^\circ\).

In Figure 2.3 the Nyquist locus and Bode plot of a closed-loop stable system are shown by the solid line, while that of an unstable system is shown by the dashed line; in each case only half locus (for \(0 \leq \omega \leq +\infty\)) is shown.

![Figure 2.3: (A) Nyquist locus and (B) Bode plot of closed-loop stable system (solid curve) and unstable system (dashed curve); it is assumed that the open-loop system is stable.](image-url)
2.1.4 Time/Frequency domain performance

In time-domain simulations, the performance of a control system is typically evaluated by looking at the response to a step in the reference input, and considers the following characteristics:

- **Rise time**: is the time it takes for the output to first reach 90% of its final value. Usually the required time must be small.
- **Settling time**: the time after which the output remains within ±5% of its final value. Usually the required time must be small.
- **Overshoot**: the peak value divided by the final value, which should typically be 1.2 (20%) or less.
- **Decay time**: the ratio of the second and first peaks, which should typically be 1.3 (30%) or less.
- **Steady-state offset**: the difference between the final value and the desired final value. Usually this difference must be small.

Rise time and settling time are measure of the speed of the response, whereas the overshoot, decay time and steady-state offset are related to the quality of the response.

The frequency-response of the open-loop transfer function, \( L(j\omega) \), and of the various closed loop transfer functions, sensitivity function \( S(j\omega) \) and complementary sensitivity function \( T(j\omega) \), may also be used to characterize the closed-loop performance.

An advantage of the frequency domain compared to a step response analysis, is that it considers a broader class of signals (that is, sinusoids of any frequency). This makes it easier to characterize feedback properties, and in particular the system behavior in the crossover (bandwidth) region.

In the following some of the important frequency-domain measures are used to assess the performance.

**Gain and phase margins**

In the Bode plot and the Nyquist diagram for the loop transfer function \( L(j\omega) \) the gain margin (GM) and phase margin (PM) can easily be obtained (see Figure 2.4).

The gain margin is defined as

\[
GM = \frac{1}{|L(j\omega_{80})|}
\]  

(2-21)

where the phase crossover angular frequency \( \omega_{80} \) is where the Nyquist curve of \( L(j\omega) \) crosses the negative real axis between −1 and 0, that is

\[
\angle L(j\omega_{80}) = -180^\circ
\]  

(2-22)
2.1 Review of classical feedback control performed in the frequency domain

On a Bode plot, usually having logarithmic axis for $|L|$, then the GM (in logarithmic or [dB]) is the vertical distance from the unit magnitude line down to $|L(j\omega)|$.

The phase margin is defined as

$$PM = 180^\circ + \angle L(j\omega_c)$$

(2-23)

where the gain crossover angular frequency $\omega_c$ is where $|L(j\omega_c)|$ first crosses 1 from above, that is,

$$|L(j\omega_c)| = 1$$

(2-24)

Figure 2.4: (A) Nyquist locus and (B) Bode plot for $L = \frac{0.3(-s+1)}{s(s+1)^2}$
The gain margin \( GM \) is the factor by which the loop gain \( |L(j\omega)| \) may be increased before the closed loop system becomes unstable. In general, the GM is thus a direct safeguard against steady-state gain uncertainty. Typically, a \( GM > 1.5 \) is required.

In case the Nyquist plot of \( L \) crosses the negative real axis between \(-1\) and \(-\infty\) then a gain reduction margin can be similarly defined from the smallest value of \( |L(j\omega_{180})| \) of such a crossing.

The phase margin \( PM \) tells how much negative phase (phase lag) that can be added to \( L(s) \) at the angular frequency \( \omega_c \) before the phase at this frequency becomes \(-180^\circ\) which corresponds to closed-loop instability. In general, the PM is a direct safeguard against time delay uncertainty; the system becomes unstable if a time delay of \( \theta_{\text{max}} = PM/\omega_c \) is included in the system. Typically, a PM larger than \( 30^\circ \) or more is required.

It should be noted that by decreasing the value of \( \omega_c \) (lowering the closed-loop bandwidth, resulting in a slower response) the system can tolerate larger time delay errors.

**Maximum peak criteria**

The maximum peaks of the sensitivity and complementary sensitivity functions are defined as

\[
M_s = \max_\omega |S(j\omega)| \quad (2-25)
\]

\[
M_t = \max_\omega |T(j\omega)| \quad (2-26)
\]

It can be shown that a large value of \( M_s \) occurs if and only if \( M_t \) is large.

The following discussion is applicable for any real system:

In a system including a feedback control the control error is given by \( e = Sr \).

Usually, \( |S| \) is small at low frequencies (in fact, for a system with integral action, \( |S(0)| = 0 \)).

At high frequencies, \( L \to 0 \) or equivalently \( S \to 1 \).

Therefore, at intermediate frequencies it is not possible to avoid a peak value, resulting in that \( M_s \) will become larger than 1. Thus, there is an intermediate frequency range where a feedback control in reality degrades the performance, and the value of \( M_s \) is a measure of the worst-case performance or stability degradation.

Hence, for both stability and performance reason, it is desired that \( M_s \) is close to 1. A large value of \( M_s \) indicates poor performance.
2.1 Review of classical feedback control performed in the frequency domain

**Bandwidth and crossover frequency**

The angular frequency $\omega_c$ is the gain crossover frequency. It is also used to define the closed-loop bandwidth. The angular frequency $\omega_{\phi_0}$ is the phase crossover frequency.

### 2.1.5 M-circles

An alternative way of specifying stability margins is to require the Nyquist locus to remain outside some neighborhood of the point $-1 + j0$. Such neighborhoods are usually defined by M-circles. These are the loci of points $z$ in the complex plane, for which

$$\frac{z}{1+z} = M$$

(2.27)

where $M$ is some positive real number.

The previous examples illustrating a stable and an unstable system are repeated in Figure 2.5, but now some M-circles are shown in the figure. It can be seen that all M-circles for which $M > 1$ (or $M > 0 [\text{dB}]$) enclose the point $-1 + j0$, and that the circles become smaller as $M$ becomes larger.

M-circles give a useful stability margin specification. In case the Nyquist locus of $L(s)$ penetrates a high-valued M-circle over some range of frequencies, then the closed-loop frequency response will exhibit a large peak at these frequencies. This in turn indicates the presence of resonance in the closed-loop system, which is usually undesirable. This is normally caused by the presence of some poorly damped closed-loop poles, close to the stability boundary.

It is practical to specify that the Nyquist locus should remain outside the $M = \sqrt{2}$ (or $M = 3 [\text{dB}]$) M-circle. This implies some minimum degree of damping on the closed-loop poles.

It is often useful to display the Nyquist locus on the Nichols chart, namely with $\log|G(j\omega)|$ plotted against $\arg G(j\omega)$. Figure 2.5 also shows the stable and unstable examples on the Nichols chart, with some M-circles superimposed. The M-circles are now distorted into non-circular shapes.
Chapter 2. Basic Concepts: Introduction of the Space Vector and review of the classical feedback control theory in the frequency domain

Figure 2.5: (A) Nyquist locus showing M-circles and (B) Nichols chart showing the Nyquist locus for the closed-loop stable system (solid curve) and unstable system (dashed curve)
2.2 REPRESENTATION OF THREE-PHASE QUANTITIES: SPACE VECTOR / COMPLEX VECTOR

The Space-Vector representation of three-phase quantities has been in general use for analysis of machines and power electronics for the last twenty-five years. It goes back to the works of Kovacs [2] and others, starting already in the 1950’ies.

Many electrical devices are fed from a three-phase supply. Their instantaneous state of operation can be described by triples of scalar values \( \{s_a(t), s_b(t), s_c(t)\} \), each representing the instantaneous value of the corresponding quantity. However, very often the phase quantities are not independent of each other as the main circuit imposes some restrictions. If the phase terminals are the only electrical connection to a device then the following apply

\[
s_a + s_b + s_c = 0
\]  

(2-28)

This means that the system is ‘zero-sequence free’. This is a condition that generally applies for most three-phase quantities (currents, voltages, fluxes, etc.) of an electrical circuit.

In a ‘zero-sequence free’ system the triple quantities, \( \{s_a(t), s_b(t), s_c(t)\} \), has in reality only two degrees of freedom since one quantity can be expressed by the other two, like, for example, \( s_b = -s_a - s_c \). For this reason, this system can be described by an equivalent instantaneous vector, which also exhibits two degrees of freedom. This vector is characterized by two perpendicular axes, denoted by \( \alpha \) and \( \beta \), which are conveniently considered as the real and imaginary axes in the complex plane. As shown in Figure 2.6 below, the three axes along the directions \( \frac{2\pi}{3}, \frac{4\pi}{3}, \pi \) are defined in the complex plane. Each one is associated with specific phase \( a \), \( b \) and \( c \).

![Figure 2.6: Definition of space vector](image)

Using the complex two-phase representation, the transformation of a three-phase system is given by:
If the phase quantities are sinusoids with a constant angular frequency $\omega$, this vector rotates anti-clockwise with the nominal synchronous angular frequency $\omega$. The superscript ‘S’ indicates stationary coordinates.

It should be noted that a power invariant transformation is often used in the literature. In such case, the factor $2/3$ in the above definition is replaced by $\sqrt{2/3}$.

From a complex vector it is possible to obtain the corresponding three-phase instantaneous components, since they are the projections of the vector in those three directions:

$$s_a(t) = \text{Re}(\bar{S}(t)) = \frac{\bar{S}(t)}{2} + \frac{\bar{S}^*(t)}{2}$$

$$s_b(t) = \text{Re}\left\{\bar{S}(t)e^{-j\frac{2\pi}{3}}\right\} = \frac{\bar{S}(t)e^{-j\frac{2\pi}{3}}}{2} + \frac{\bar{S}^*(t)e^{j\frac{2\pi}{3}}}{2}$$

$$s_c(t) = \text{Re}\left\{\bar{S}(t)e^{j\frac{2\pi}{3}}\right\} = \frac{\bar{S}(t)e^{j\frac{2\pi}{3}}}{2} + \frac{\bar{S}^*(t)e^{-j\frac{2\pi}{3}}}{2}$$

If one needs to consider separately the real and imaginary parts of the complex vector we can represent the complex-valued space vector by the real-valued vector as:

$$\bar{S}^S(t) = s_a + js_b \equiv \bar{s} = \begin{bmatrix} s_a \\ s_b \end{bmatrix}$$

(2-31)
CHAPTER 3
FREQUENCY DOMAIN MODEL OF A LINE-COMMUTATED CURRENT-SOURCE CONVERTER. PART I: FIXED OVERLAP

In this and the next chapters a frequency-domain model of an HVDC line-commutated current-source converter is presented. Using the space-vector concept transfer functions between superimposed oscillations in the control signal and the output, the AC and DC side voltages and currents are derived. The dynamic properties of the HVDC converter can be studied by applying classical Bode/Nyquist/Nichols control methods taking the characteristics of the networks on both the AC and the DC side into consideration. The resulting model has been validated by time-domain studies in PSCAD/EMTDC.

The model is described in two parts. In Part I it has been assumed that the overlap angle during commutation remains constant. It was shown in the validation that this assumption introduces resonances that cause severe errors at certain network conditions. In Part II the model is extended so as to cope with the varying overlap angle in order to bring the frequency-domain model into agreement with the results obtained from time-domain simulations.

Part I is presented in this chapter and Part II is presented in the next chapter.

3.1 SPACE VECTOR REPRESENTATION OF LINE-COMMUTATED CURRENT SOURCE CONVERTERS OPERATING IN SIX-STEP MODE

Consider a six-pulse line-commutated current-source converter (LCC) operating at steady state in six-step mode. The AC-side voltage is assumed to be symmetrical and the DC side current is assumed to be stiff. Moreover, it is assumed that the commutation of the current between the valves occurs instantaneously. The stiff DC side current then becomes distributed among the phases in the connected AC
system depending on which valves that are conducting in the converter bridge. The valves can then be considered to operate as switching units, which periodically switch between different AC phases (Figure 3.1).

The operation of the converter can be described by three conversion functions \( k_a(t) \), \( k_b(t) \), and \( k_c(t) \), which can take the values \(-1\), \(0\), or \(+1\): the value is \(-1\) when connected to the lower DC rail, \(+1\) when connected to the upper DC rail, and \(0\) when the phase is isolated. The space vector representing the AC side current is defined as

\[
\bar{i}_v(t) = \frac{2}{3} \left( i_{V_a}(t) 1 + i_{V_b}(t) \bar{a} + i_{V_c}(t) \bar{a}^2 \right)
\]  

where, \( a = e^{\frac{2\pi}{3}} \), and it can be expressed in terms of the DC side current as

\[
\bar{i}_v(t) = \frac{2}{3} \left( k_a(t) \cdot 1 + k_b(t) \cdot \bar{a} + k_c(t) \cdot \bar{a}^2 \right) \cdot i_d(t) = \frac{2}{3} K(t) \cdot i_d(t)
\]  

or

\[
\bar{i}_v(t) = \frac{2}{3} \left( k_a(t) \cdot 1 + k_b(t) \cdot \bar{a} + k_c(t) \cdot \bar{a}^2 \right) \cdot i_d(t) = \frac{2}{3} K(t) \cdot i_d(t)
\]  

or

\[
\bar{i}_v(t) = \frac{2}{3} \left( k_a(t) \cdot 1 + k_b(t) \cdot \bar{a} + k_c(t) \cdot \bar{a}^2 \right) \cdot i_d(t) = \frac{2}{3} K(t) \cdot i_d(t)
\]
3.1 Space Vector Representation of Line-Commutated Current Source Converters operating in six-step mode

\[ i_v(t) = \frac{2}{3} \overline{K}(t) \cdot i_d(t) \]  

(3-3)

In the equation the time dependent conversion vector function (or simply the complex conversion function) has been defined as the complex-valued expression

\[ \overline{K}(t) = k_a(t) \cdot 1 + k_b(t) \cdot \overline{a} + k_c(t) \cdot \overline{a}^2 \]  

(3-4)

At each instant one phase conversion function takes the value +1 and another one the value −1. The conversion vector then can take one of six values, which all have the magnitude \( \sqrt{3} \), and which are distributed in the complex plane in six different directions, as can be seen in Table 3.1.

<table>
<thead>
<tr>
<th>State</th>
<th>( k_a(t) )</th>
<th>( k_b(t) )</th>
<th>( k_c(t) )</th>
<th>( \overline{K} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>( \sqrt{3} \exp(j \pi/6) )</td>
</tr>
<tr>
<td>II</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>( \sqrt{3} \exp(j \pi/6) )</td>
</tr>
<tr>
<td>III</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>( \sqrt{3} \exp(j 2\pi/3) )</td>
</tr>
<tr>
<td>IV</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>( \sqrt{3} \exp(j \pi/3) )</td>
</tr>
<tr>
<td>V</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>( \sqrt{3} \exp(j 5\pi/6) )</td>
</tr>
<tr>
<td>VI</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>( \sqrt{3} \exp(j \pi/2) )</td>
</tr>
</tbody>
</table>

Table 3.1: Phase conversion function and corresponding complex coupling function

Considering that only one phase conversion function is +1 and only one is −1 at a given time, the resulting DC side voltage for all possible phase conversion functions is found to be

\[ u_d(t) = k_a(t) u_{v_a}(t) + k_b(t) u_{v_b}(t) + k_c(t) u_{v_c}(t) = \text{Re} \left[ \overline{K}^*(t) \overline{u}_v(t) \right] \]  

(3-5)

or

\[ u_d(t) = \text{Re} \left[ \overline{K}^*(t) \overline{u}_v(t) \right] \]  

(3-6)

In the equation \( \overline{K}^*(t) \) is the conjugate of the complex coupling function that has been used.

Figure 3.2 shows the time domain model of the LCC represented by its complex quantities.

\[ i_v(t) = \frac{2}{3} \mathcal{K}(t) \cdot i_d(t) \]
\[ u_d(t) = \text{Re} \left[ \mathcal{K}^*(t) u_v(t) \right] \]

\[ \mathcal{K}(t) = k_a(t) \cdot 1 + k_b(t) \cdot a + k_c(t) \cdot a^2 \]
\[ k_a(t) = \{-1, 0, +1\}, \]
\[ |\mathcal{K}(t)| = \sqrt{3}, \quad \text{arg} \mathcal{K} = \frac{\pi}{6} + (0...5) \pm \frac{\pi}{6} \]

Figure 3.2: The time domain model of a six-step LCC represented by its complex quantities

Assume that the AC side voltage has the argument \( \omega_t \), i.e. if the voltage vector can be written as

\[ \mathcal{U}_v(t) = \mathcal{U}_v e^{i \omega_t} \] (3-7)

If the thyristors in the bridge are fired as soon as they are becoming forward biased then the conversion functions in the angle interval \( 0 < \omega t < \pi / 3 \) will be \( k_a = +1, k_b = 0, k_c = -1 \) and the conversion vector takes the constant value \( \sqrt{3} e^{j \pi / 6} \) during that angle interval. It should be noted that the average value of the argument of the voltage vector also is \( \pi / 6 \) during this interval, which means that the fundamental frequency component of the AC side current is in phase with the AC side voltage in this case.

The control of the HVDC converter is carried out by the current regulator, which introduces a delay \( \alpha \), \( 0 < \alpha < \pi \), from the instant when the valve becomes forward biased until it is being fired. This means that the conversion state mentioned above, i.e. \( k_a = +1, k_b = 0, k_c = -1 \) will appear in a delayed angle interval \( \alpha < \omega t < \frac{\pi}{3} + \alpha \).

At steady state the conversion vector remains constant during each \( \pi / 3 \) angle interval. At the end of each such interval it instantaneously rotates forward by the angle \( \pi / 3 \). This mechanism means that the function \( \mathcal{K}(t) \cdot e^{-j \omega t} \) becomes periodical with period \( \pi / 3 \) (fundamental angular frequency \( 6 \omega_t \)).

If,

\[ \mathcal{L}(t) = \mathcal{K}(t) \cdot e^{-j \omega t} \] (3-8)

is introduced, Figure 3.3 illustrates the real and imaginary parts of this function.
3.1 Space Vector Representation of Line-Commutated Current Source Converters operating in six-step mode

Accordingly, it can be expanded into a Fourier series in the form

$$\bar{K}(t) \cdot e^{-j\omega_0 t} = \sum_{m=-\infty}^{m=\infty} c_m e^{j6m\omega_0 t}$$  \hspace{1cm} (3-9)$$

where

$$c_m = \frac{1}{\pi} \int_{\theta_0}^{\theta_0 + \pi} (\bar{K}(t)e^{-j\omega_0 t}) e^{-j6m\omega_0 t} \omega_0 dt$$  \hspace{1cm} (3-10)$$

for any arbitrary angle $\theta_0$. Specifically $\theta_0 = \alpha$ yields

$$c_m = \frac{3}{\pi} \int_{\alpha}^{\alpha + \pi/6} \sqrt{3} e^{j\theta} e^{-j(6m+1)\theta} d\theta = \frac{3\sqrt{3}}{\pi} \frac{1}{6m+1} e^{-j(6m+1)\alpha}$$  \hspace{1cm} (3-11)$$

The corresponding Fourier series expansion for the complex conversion function $\bar{K}(t)$ is

$$\bar{K}(t) = \sum_{m=-\infty}^{\infty} c_m e^{j(6m+1)\omega_0 t} = \frac{3\sqrt{3}}{\pi} \sum_{m=-\infty}^{\infty} \frac{1}{6m+1} e^{-j(6m+1)\alpha}$$  \hspace{1cm} (3-12)$$

The equation shows that the complex conversion function $\bar{K}(t)$ has the fundamental frequency component and the harmonics of order $+7, +13, +19, \ldots$ all rotating with positive sequence while the harmonics of order $-5, -11, -17, \ldots$ rotate with negative sequence.

For small low frequency oscillations the harmonics can be disregarded as they have very little influence on the dynamics of the converter. This means that only the fundamental frequency part of the conversion function needs to be taken into consideration. Then, the above defined conversion function can be simplified to
\[ K(t) = \frac{3\sqrt{3}}{\pi} e^{-j\alpha} e^{j\omega t} \]  

Classical LCC formulas

When the AC-side voltage is given as
\[ \bar{u}_v(t) = \tilde{u}_v e^{j\omega t} \]  
and the DC-side current is \( I_d \), the line current is given by
\[ \tilde{i}_v(t) = \frac{2}{3} K(t) \cdot i_d(t) = \tilde{i}_v e^{-j\alpha} e^{j\omega t} = I_{y,rms} \sqrt{2} e^{-j\alpha} e^{j\omega t} \]  
where,
\[ I_{y,rms} = \frac{\sqrt{6}}{\pi} i_d \approx 0.78 \cdot I_d \]

Correspondingly, the DC-side voltage given by
\[ u_d(t) = \text{Re}[\bar{K}^*(t) \cdot \bar{u}_v(t)] = U_{dio} \cdot \cos \alpha \]
where
\[ U_{dio} = \frac{3\sqrt{3}}{\pi} \tilde{u}_v = \frac{3\sqrt{6}}{\pi} U_{y,rms} = \frac{3\sqrt{2}}{\pi} U_{y,1-1, rms} \approx 1.35 \cdot U_{y,1-1, rms} \]

These formulas are at once recognized as the formulas obtained in elementary textbooks about LCCs.

The description above also shows that the main control mechanism in an LCC is that it directly, through the control angle \( \alpha \), controls the phase angle of the AC-side current relative to the AC-side voltage.

The description of the converter using the complex conversion function \( K(t) \) is valid for both rectifier and inverter operation. In rectifier operation the firing angle \( \alpha \) should be less than \( \pi/2 \) (in relation to the converter commutating voltage, i.e. the converter bus). At firing angles beyond \( \pi/2 \) the converter operates as an inverter. The DC side current direction remains unchanged, since the valve can only carry current in one direction. However, the polarity of the voltage across the bridge will be changed. The power direction then also changes.

Figure 3.4 illustrates the DC-side voltage of an LCC in rectifier and inverter operation.
3.2 Calculation of the Conversion Function Considering the Influence of Commutation Overlap

The complex conversion function in the previous section has been determined assuming that the commutation from one valve to the next one occurs instantaneously with no overlap. However, in the real line-commutated current-source converter the commutation of the current from one phase to the succeeding one takes a specific time due to the inductances included in the main circuit, mainly from the converter transformer. In this section this fact will be taken into consideration.

Formulas to calculate the steady state overlap value – classical LCC theory

The dynamic model needs the steady state overlap angle as a function of the DC-current and of the transformer leakage reactance as a parameter. The classical LCC theory is used for this calculation. Below the formulas for the overlap are derived.

Traditionally, the overlap is being considered in the LCC literature by assuming that the DC-side current is stiff and that the AC voltage behind the “commutation reactance” is a stiff three-phase symmetric sinusoidal with nominal frequency.

Assume that valves 1 and 2 are conducting and that valve 3 is triggered at the instant \( \frac{\pi}{3} + \alpha \). This corresponds to the circuit shown in Figure 3.5, with valves 1, 2, and 3 conducting. The emf voltage is \( \overline{u}_c \), which is assumed to be stiff.
After valve 3 has been fired, the commutation of the direct current from valve 1 to valve 3 is determined by the total commutation reactance in the circuit, $2X_c$, and the commutation voltage, $u_k$. This commutation voltage is the voltage, which at constant direct current would have occurred across the valve 3, if this valve had not been fired. This corresponds to the emf inside the loop. The commutation voltage from thyristor 1 to thyristor 3 during the interval $\frac{\pi}{3} + \alpha < \omega_N t < \frac{\pi}{3} + \alpha + \mu$ is given by

$$
\begin{align*}
    u_{cb} - u_{ca} &= -\sqrt{3}\hat{u}_c \cos \left(2\omega_t + \frac{\pi}{3} \right) - 3\hat{u}_c \cos \left(\omega_N t + \frac{\pi}{6} \right)
\end{align*}
$$

The corresponding current derivative in phase current $i_y$ becomes

$$
\frac{di_y}{dt} = \frac{-\sqrt{3}\hat{u}_c}{2X_c} \cos \left(\omega_N t + \frac{\pi}{6} \right) = \frac{-\sqrt{3}\hat{u}_c}{2X_c} \omega_N \cos \left(\omega_N t + \frac{\pi}{6} \right)
$$

The current in valve 1 is at the beginning of the commutation interval, $t = \frac{\pi}{3} + \alpha$, equal to the converter bridge current $i_d$. The current in valve 3 is then zero. At the end of the commutation interval, $t = \frac{\pi}{3} + \alpha + \mu$, the current in valve 1 is zero, and in valve 3 the current equals the converter bridge current $i_d$. Assuming constant converter bridge current $i_d$, constant commutating voltage amplitude and phase, and negligible resistive losses, the current in valve 3 after the commutation is found to be
3.2 Calculation of the Conversion Function Considering the Influence of Commutation Overlap

\[ i_{vb}\left(\frac{\pi}{3} + \alpha + \mu\right) - i_{vb}\left(\frac{\pi}{3} + \alpha\right) = i_d - 0 = \]

\[ = \int_{\frac{\pi}{3} + \alpha}^{\frac{\pi}{3} + \alpha + \mu} \frac{di_{vb}}{dt} dt = -\frac{\sqrt{3} \hat{u}_v}{2 X_C} \int_{\frac{\pi}{3} + \alpha}^{\frac{\pi}{3} + \alpha + \mu} \cos\left(\theta + \frac{\pi}{6}\right) d\theta = \]

\[ = \frac{\sqrt{3} \hat{u}_v}{2 X_C} \left[\cos \alpha - \cos(\alpha + \mu)\right] \]

(3-21)

The result is

\[ i_d = \frac{\sqrt{3} \hat{u}_v}{2 \omega_N L_C} \left[\cos(\alpha) - \cos(\alpha + \mu)\right] \]

(3-22)

or,

\[ \cos(\alpha + \mu) = \cos \alpha - \frac{2X_C i_d}{\sqrt{3} \hat{u}_v} \]

(3-23)

This is another equation from the classical theory used in the textbooks about LCCs, which associates the direct current with the firing angle and the commutation angle.

In the textbooks about LCCs an expression for the relative inductive voltage droop is also frequently defined. This expression reads

\[ d_s = \frac{\omega_N L_C i_d}{\sqrt{3} \hat{u}_v} = \frac{3 i_d X_C}{\pi U_{dio}} \]

(3-24)

Consequently,

\[ d_s = \frac{1}{2} \left[\cos(\alpha) - \cos(\alpha + \mu)\right] \]

(3-25)

With rated direct current and nominal AC voltage a nominal relative inductive voltage drop is obtained. This value is given by

\[ d_{sN} = \frac{\omega_N L_C I_{d\text{N}}}{\sqrt{3} \hat{u}_{v\text{N}}} = \frac{3 I_{d\text{N}} X_{CN}}{\pi U_{dioN}} \]

(3-26)

In the following section two approximations are made when deriving the equations for the dynamic model. When the linear transfer of the conversion vector between its consecutive constant values has been introduced, the duration of the transfer interval (commutation interval \(\mu\)) that has been calculated, will be assumed constant. The firing angle \(\alpha\) is also assumed to be constant. Having made these two assumptions the derivation of the equations is simplified considerably.
3.2.1 Conversion function for current transfer

In this section the conversion function for current transfer, \( K'(t) \) will be calculated. In the upper commutation group the transfer of the current from phase \( a \) to phase \( b \) is initiated when the upper thyristor in phase \( b \) is triggered and the transfer lasts during an angle \( \mu \). The commutation interval is thus given by

\[
\frac{\pi}{3} + \alpha < \omega_N t < \frac{\pi}{3} + \alpha + \mu
\]  

(3-27)

The initial value of \( K'(t) \) is \( \sqrt{3} e^{j \pi/6} \) and the final value is \( \sqrt{3} e^{j \pi/2} \). Using the linear approximation, the conversion function during this particular commutation interval is then given by the expression

\[
K'_{1 \rightarrow 3}(t) = \sqrt{3} e^{j \pi/6} + \sqrt{3} \left( e^{j \pi/6} - e^{j \pi/2} \right) \frac{\omega_N t - \pi/3 - \alpha}{\mu} 
\]  

(3-28)

Now the conversion function can be defined in a complete interval with a duration corresponding to the angle \( \pi/3 \). In the interval \( \alpha + \mu < \omega_N t < \pi/3 + \alpha + \mu \) the following formulas apply (see the corresponding coupling vector in the complex plane in Figure 3.6):

\[
K'(t) = \begin{cases} 
\sqrt{3} e^{j \pi/6}, & \alpha + \mu < \omega_N t < \pi/3 + \alpha \\
\sqrt{3} e^{j \pi/6} + \sqrt{3} \left( e^{j \pi/6} - e^{j \pi/2} \right) \frac{\omega_N t - \pi/3 - \alpha}{\mu}, & \frac{\pi}{3} + \alpha < \omega_N t < \pi/3 + \alpha + \mu 
\end{cases}
\]  

(3-29)

As before, the function \( K'(t) \cdot e^{-j \omega_N t} \) is periodical with the period \( \pi/3 \) (fundamental angular frequency \( 6\omega_N \)) and it can be expanded as the Fourier series

\[
K'(t) \cdot e^{-j \omega_N t} = \sum_{m=-\infty}^{\infty} \xi_m^t e^{j 6m \omega_N t} 
\]  

(3-30)

where

\[
\xi_m^t = \frac{3 \sqrt{3}}{\pi} \frac{\sin \left( \frac{(m+1)\mu}{2} \right)}{(6m+1)(6m+1)\mu} e^{-j(6m+1)\left( \frac{\alpha + \mu}{2} \right)}
\]  

(3-31)

Considering only the fundamental frequency, and disregarding the contribution from the harmonics, the following expression is obtained for the conversion function

\[
\xi_m^t = \frac{3 \sqrt{3}}{\pi} \frac{\sin \left( \frac{(m+1)\mu}{2} \right)}{(6m+1)\mu} e^{-j(6m+1)\left( \frac{\alpha + \mu}{2} \right)}
\]  

(3-31)
3.2 Calculation of the Conversion Function Considering the Influence of Commutation Overlap

\[ \mathbf{K}'(t) = \ell' e^{j\omega t} \]  

(3-32)

with

\[ \ell' = \frac{3\sqrt{3}}{\pi} \frac{\sin \frac{\mu}{2}}{e^{-\left(\frac{\alpha + \mu}{2}\right)}} \]  

(3-33)

\[ \text{Coupling Function, including commutation period} \]

\[ \text{Linear transition between states during interval: } \frac{\pi}{3} + \alpha < \omega_\lambda t < \frac{\pi}{3} + \alpha + \mu \]

\[ \text{State during interval: } \alpha + \mu < \omega_\lambda t < \frac{\pi}{3} + \alpha \]

\[ \text{Figure 3.6: Conversion function } \mathbf{K}' \text{ in the complex plane, highlighting State I, one of the six possible states and the linear transfer of the coupling function between two states (State I and State II)} \]

**Classical LCC formulas**

Initially it is assumed that \( L_c = 0 \), and that the DC current is perfectly smooth, (smoothing reactor is assumed very large), yielding constant instantaneous DC-current \( (i_d = I_d) \). Having a pure sinusoidal AC-input voltage the AC current in one of the phases is shown in Figure 3.7-(A). The rms value of the fundamental frequency component \( I_{a1} \) of the AC current obtained from the Fourier components of the current waveform is given by

\[ I_{a1} = \frac{2\sqrt{3}}{\sqrt{2} \pi} I_d \approx 0.78 I_d \]  

(3-34)
The total rms value of the phase current (considering all the harmonics) can be calculated as

\[
I_a = \sqrt{\frac{2}{3}} I_d \approx 0.816 I_d \quad (3-35)
\]

Therefore in the absence of commutation reactance \((L_C = 0)\) the fundamental frequency component corresponds to approximately 95.5% of the total current, that is

\[
\frac{I_{a1}}{I_a} = \frac{3}{\pi} \approx 0.955 \quad (3-36)
\]

In the case with non-zero commutation inductance \((L_C \neq 0)\) and finite single overlap angle (overlap angle in the range \(0 < \mu < 60^\circ\)), the above equations are also approximately valid. See Figure 3.7-(B). In reference [21, Appendix B], an extensive Fourier analysis has been made to determine the fundamental component of the AC-current component considering the current in an AC-phase of the converter transformer. The result of this calculation has shown that the effective value of the AC current for \(\alpha = 15^\circ\) and \(\mu = 20^\circ\) is equal to 0.975, that is

\[
\frac{I_{a\text{ effective}}}{\sqrt{2} I_d} = 0.975 \quad (3-37)
\]

In Figure 3.7-(C) a linear approximation of the current waveform during the commutation interval is shown. This trapezoidal waveform was considered when deducing the Complex Coupling Function, in which a linear transfer of the coupling function between two states was made. Considering this approximation, and assuming an overlap \(\mu = 20^\circ\), the rms value of the fundamental frequency component \(I_{a1}\) of the AC current is equal to

\[
I_{a1} = \frac{1}{\sqrt{2}} \frac{2}{\pi} \sin \frac{0.349}{2} I_d \approx 0.776 I_d \quad (3-38)
\]

It should be noted that this result is approximately the same as the result given by Eq. (3-34), indicating that the accuracy of the model has not been affected by the linear approximation of the current waveform.
3.2 Calculation of the Conversion Function Considering the Influence of Commutation Overlap

<table>
<thead>
<tr>
<th>Rectifier Operation</th>
<th>Inverter Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Rectifier Diagram" /></td>
<td><img src="image2" alt="Inverter Diagram" /></td>
</tr>
</tbody>
</table>

Figure 3.7: AC-current waveform: (A) neglecting commutation reactance \( L_c = 0 \); (B) finite commutation reactance \( L_c \neq 0 \); (C) trapezoidal approximation, assuming linear transfer of the coupling function

### 3.2.2 Conversion function for voltage transfer

The general principle for modeling the LCC in this thesis is that we calculate the AC-side current based on the DC-side current by applying the conversion function for current transfer derived in the preceding section. Then the AC-side voltage at the converter bridge terminals, i.e. \( u_{v_a} \), \( u_{v_b} \) and \( u_{v_c} \), is calculated taking the impedances in the network into consideration. If, in this process, the overlap interval length and the current conversion function waveform were perfect, no voltage difference should appear between the commutating terminals during the commutation interval. However, an approximation of the current conversion function (linear transfer) was made and further the overlap angle is assumed to be constant, and therefore a small voltage deviation occurs between the converter terminals of two commutating phases. This small imperfectness will be treated by using the average of the involved line-line voltages when the DC-side voltage is calculated from the AC-side voltage in below.

The conversion function \( K^U(t) \) for voltage transfer between the AC and the DC sides of the converter can be derived in a similar way as the conversion function for current transfer.

The derivation of the conversion function will be made assuming commutation from valve 1 to valve 3 (see Figure 3.8). Assume that valve 1 and valve 2 are both conducting. In this case the DC side voltage is given by

\[
u_d(t) = u_{v_a}(t) - u_{v_c}(t)
\]  

(3-39)
During the commutation the line-line voltage of the short-circuited phases is zero causing the two line-neutral voltages to be equal. This voltage is the average of the corresponding open circuit voltages. During the commutation interval, when current is transferred between phase $a$ and phase $b$, i.e. in the interval $\frac{\pi}{3} + \alpha < \omega_N t < \frac{\pi}{3} + \alpha + \mu$, the DC side voltage is given by

$$u_d(t) = \frac{u_{va}(t) + u_{vb}(t)}{2} - u_{vc}(t) = -\frac{3}{2} u_{vc}(t)$$  \hspace{1cm} (3-40)

For one interval with angle length $\frac{\pi}{3}$ the following formulas apply

$$u_d(t) = \begin{cases} 
    u_{va}(t) - u_{vc}(t), & \alpha + \mu < \omega_N t < \frac{\pi}{3} + \alpha \\
    -\frac{3}{2} u_{vc}(t), & \frac{\pi}{3} + \alpha < \omega_N t < \frac{2\pi}{3} + \alpha + \mu
\end{cases}$$  \hspace{1cm} (3-41)

The lower part of Figure 3.9 shows the DC-side voltage of the converter. Additionally, the line-voltages are shown, highlighting with a solid line the AC-voltages corresponding to the analyzed two intervals: one interval defined by the driving AC voltage where the valves 1 and 2 are conducting and the second interval where the commutation between valve 1 and valve 3 is occurs. The resulting DC voltage is obtained from the combination of the waveforms given by $u_{va}(t) - u_{vc}(t)$ and $-\frac{3}{2} u_{vc}(t)$. 

---

Figure 3.8: Commutation from valve 1 to valve 3 – derivation of conversion function for voltage transfer
3.2 Calculation of the Conversion Function Considering the Influence of Commutation Overlap

As was shown in Section 3.1 the DC voltage can be calculated by the use of a complex conversion function \( \overline{K}^U \) using the formula

\[
u_d(t) = \text{Re}\left[ \overline{K}(t) \overline{\pi}_\nu(t) \right]
\]  

(3-42)

Here, however, \( \overline{K} = \overline{K}^U \) has been modified in order to take into account the effect of the commutation. Thus, during the commutation from valve 1 to valve 3,

\[
\overline{K}^U(t) = \begin{cases} 
\sqrt{3} e^{\frac{\pi}{6}}, & \alpha + \mu < \omega_s t < \frac{\pi}{3} + \alpha \\
\frac{3}{2} e^{\frac{\pi}{3}}, & \frac{\pi}{3} + \alpha < \omega_s t < \frac{\pi}{3} + \alpha + \mu 
\end{cases}
\]  

(3-43)

The above defined complex conversion function \( \overline{K}^U(t) \) representing the voltage transfer between the AC side and the DC side voltages of the LCC can be expanded into a Fourier series in the same way as was done for the current transfer function. The result is

\[
\overline{K}^U(t) \cdot e^{-j\omega_s t} = \sum_{m=-\infty}^{+\infty} \ell_m e^{j6m\omega_s t}
\]  

(3-44)

where

\[
\ell_m = \frac{3\sqrt{3}}{\pi} \frac{1}{(6m+1)} \cos\left(\frac{6m+1}{2}\mu\right) e^{-j(6m+1)\left(\frac{\alpha + \mu}{2}\right)}
\]  

(3-45)

Similar to the conversion function for current transfer, only the fundamental frequency is taken into consideration. Accordingly,
\[ R^U(t) = \ell^U e^{j\omega t} \] (3-46)

with

\[ \ell^U = \frac{3\sqrt{3}}{\pi} \cos \frac{\mu}{2} e^{-j\frac{\alpha + \mu}{\pi}} \] (3-47)

The coefficient \( \ell^U \) corresponds to the classical ideal no-load direct voltage \( U_{dio} \) times two factors depending on \( \mu \) and \( \alpha \). The direct voltage at zero angle of overlap, no losses and zero firing angle is commonly defined as,

\[ U_{dio} = \frac{3\sqrt{3}}{\pi} \hat{u}_v \] (3-48)

Using Eq. (3-40) for the DC-side voltage, the steady state mean value of the DC voltage is obtained as

\[ U_d = U_{dio} \cos \frac{\mu}{2} \cos \left( \alpha + \frac{\mu}{2} \right) = U_{dio} \cos \alpha + \cos \left( \alpha + \mu \right) \] (3-49)

This is a well-known expression from classical HVDC textbooks.

### 3.3 THE LCC MODEL CONNECTED TO A NETWORK

#### 3.3.1 Circuit arrangement

Let the LCC converter be connected to an infinite source through a network with given impedance. Furthermore, let a shunt filter be connected to the converter bus. Figure 3.10 outlines this circuit arrangement.

The phase impedance between the converter bus and the infinite bus is \( Z_L \), the impedance in the converter bus filter is \( Z_F \), and the commutation reactance in the LCC transformer is represented by \( Z_C \). The voltage source that energizes the transmission system is defined by the vector \( \vec{u}_b \), and contains only a fundamental frequency component.

In the analysis the starting point is often a given voltage at the filter bank and an operating control angle \( \alpha_c \) with respect to the source voltage bus voltage. Obviously, the system illustrated in Figure 3.10-(A) could be transformed into an equivalent circuit diagram according to Figure 3.10-(B), having included all the Thevenin impedances and Thevenin voltage.
When the DC current and the filter bank voltage are known together with the commutation reactance, the overlap angle $\mu$ can be calculated. Assuming that the filter bank voltage is given by

$$\bar{u}_c(t) = \hat{U}_c e^{j\omega_it}$$  \hspace{1cm} (3-50)

It is assumed the argument is zero at $t = 0$. The approximate current conversion function is found to be

$$\bar{K}'(t) = \ell' e^{j\omega_it} = \frac{3\sqrt{3}}{\pi} \frac{\sin \frac{\mu}{2}}{\mu} e^{-j \left( \frac{\alpha_c + \frac{\mu}{2}}{2} \right)} e^{j\omega_it}$$  \hspace{1cm} (3-51)

From this formula the fundamental component of the line current is obtained as

$$\bar{i}_r(t) = \frac{2}{3} \bar{K}'(t) \bar{i}_d = \frac{2}{3} \ell' \bar{i}_d e^{j\omega_it}$$  \hspace{1cm} (3-52)

Accordingly, the driving voltage is given as below

$$\bar{u}_s(t) = \left( \hat{U}_c + \frac{2}{3} \ell' \bar{i}_d Z'_F \right) e^{j\omega_it}$$  \hspace{1cm} (3-53)

The driving voltage as above can be resolved into its amplitude and phase

$$\bar{u}'_s(t) = \hat{U}'_s e^{j(\omega_st + \beta)}$$

$$\hat{U}'_s = \left| \hat{U}_c + \frac{2}{3} \ell' \bar{i}_d Z'_F \right|$$  \hspace{1cm} (3-54)

$$\beta = \arg \left( \frac{\hat{U}_c + \frac{2}{3} \ell' \bar{i}_d Z'_F}{\hat{U}'_s} \right)$$

Accordingly, the firing angle given with respect to the fixed driving voltage source becomes

$$\alpha = \alpha_c + \beta$$  \hspace{1cm} (3-55)
Now, the interaction between the LCC and the connected AC system can be studied by calculating the transfer functions for superimposed oscillations with small amplitudes in voltages and currents. It is assumed that the source voltage is constant. The impact of the overlap angle variation $\Delta \mu$ will be studied in Chapter 4.

### 3.3.2 Calculation of the transfer function $G_3 = \Delta u_d / \Delta i_d$

To calculate the transfer function between $\Delta i_d$ and $\Delta u_d$, the DC current is assumed to incorporate a small variation with the amplitude $A$ and the angular frequency $\Omega$. Then

$$\Delta i_d(t) = \text{Re}[A e^{j\mu}] = \frac{A}{2} e^{j\mu} + \frac{A^*}{2} e^{-j\mu}$$

(3-56)

Now, recall that the AC side current can be calculated using the conversion function $K^l(t)$

$$\Delta i_l(t) = \frac{2}{3} K^l(t) \cdot \Delta i_d(t)$$

(3-57)

with

$$K^l(t) = \ell e^{j\omega_l t}$$

(3-58)

Combining these equations yields

$$\Delta i_l(t) = \frac{A}{3} \ell e^{j\omega_l t} + \frac{A^*}{3} \ell e^{-j\omega_l t}$$

(3-59)

For practical reasons, the following notation is used

$$\begin{cases} \omega^+ = \omega_N + \Omega \\ \omega^- = \omega_N - \Omega \end{cases}$$

(3-60)

The current is injected into the network impedance, which has the following impedances at the positive and negative sidebands

$$Z^+ = Z'_I(j\omega^+) + Z_C(j\omega^+)$$
$$Z^- = Z'_I(j\omega^-) + Z_C(j\omega^-)$$

(3-61)

As a result the following variation is obtained in the AC side voltage

$$\Delta u_l(t) = \left( \frac{A}{3} Z^+ \ell e^{j\omega_l t} + \frac{A^*}{3} Z^- \ell e^{-j\omega_l t} \right)$$

(3-62)

However, a change in the AC side voltage will be reflected back into the DC side voltage according to the relation

$$\Delta u_d = \text{Re}\left(\left(K^U(t)^*\right)^\Delta \Delta u_l(t)\right)$$
$$K^U(t) = \ell e^{j\omega_N t}$$

(3-63)
The variation of the DC voltage can now be written as

\[
\Delta u_d(t) = -\text{Re}\left\{ \frac{A}{3} \ell \ell^* e^{j\omega t} \left( Z^* \ell^l e^{j\omega t} + \frac{A^*}{3} Z^* \ell^l e^{j\omega t} \right) \right\} = \\
= -\text{Re}\left\{ \frac{A}{3} \ell \ell^* Z^* \ell^l e^{j\omega t} + \frac{A^*}{3} \ell \ell^* Z^* \ell^l e^{j\omega t} \right\}
\]

(3-64)

Noting that any complex number and its conjugate have identical real parts it is found that

\[
\Delta u_d(t) = -\text{Re}\left\{ \frac{A}{3} \ell \ell^* Z^* \ell^l e^{j\omega t} + \frac{A^*}{3} \ell \ell^* Z^* \ell^l e^{j\omega t} \right\} = \\
= \text{Re}\left\{ -\frac{1}{3} \left( \ell \ell^* Z^* \ell^l + \ell \ell^* Z^* \ell^l \right) (A e^{j\omega t}) \right\}
\]

(3-65)

The expression yields the transfer function from the DC side current to the DC side voltage in the converter

\[
G_3(j\Omega) = -\frac{1}{3} \left( \ell \ell^* Z^* \ell^l + \ell \ell^* Z^* \ell^l \right)
\]

(3-66)

Note that the complex exponentials in the coefficients cancel such that the formula can be simplified to

\[
G_3(j\Omega) = -\frac{9}{\pi} \frac{\sin \mu \left( Z^* + (Z^*)^* \right)}{\mu}
\]

(3-67)

### 3.3.3 Calculation of the transfer function \( G_2 = \Delta u_d / \Delta \alpha \)

The transfer function between \( \Delta \alpha \) and \( \Delta u_d \) can be derived by a similar procedure as the one detailed in Section 3.3.2. The transfer function consists of two terms:

\[
G_2 = H_{11} + H_{12}
\]

(3-68)

Recall that the following equations describe the operation of the converter

\[
\bar{i}_v(t) = \frac{2}{3} \bar{K}'(t) i_d(t)
\]

(3-69)

and

\[
u_d(t) = \text{Re}\left\{ \bar{K}_0^*(t) \bar{u}_v(t) \right\}
\]

(3-70)

Applying a general differentiation on these expressions yields

\[
\Delta \bar{i}_v(t) = \frac{2}{3} \Delta \bar{K}'(t) I_d(t) + \frac{2}{3} \bar{K}'(t) \Delta i_d(t)
\]

(3-71)

The conversion functions are functions of time $t$ with the firing angle $\alpha$ and the overlap angle $\mu$ as parameters. It is assumed that the overlap angle remains constant. In the formulas for the differentiation above, the variation is therefore exclusively caused by the variation of the firing angle $\alpha$. Accordingly,

$$\Delta K^I(t) = \frac{\partial K^I(t; \alpha, \mu)}{\partial \alpha} \Delta \alpha(t)$$  \hspace{1cm} (3-73)

and

$$\Delta K^U(t) = \frac{\partial K^U(t; \alpha, \mu)}{\partial \alpha} \Delta \alpha(t)$$  \hspace{1cm} (3-74)

The first term $H_{II}$

The first term describes the impact of the varying control angle $\alpha$ on the DC voltage caused by the variation of the voltage conversion function. This transfer function reflects the basic principle of HVDC control, which simply is to control the DC side voltage by means of manipulating the firing angle.

To derive the transfer function, assume that the control angle $\alpha$ incorporates a small variation

$$\Delta \alpha = \text{Re}[\Delta \hat{\alpha} e^{i\hat{\alpha} t}] = \frac{\Delta \hat{\alpha}}{2} e^{i\hat{\alpha} t} + \frac{\Delta \hat{\alpha}^*}{2} e^{-i\hat{\alpha} t}$$  \hspace{1cm} (3-75)

The variation of the conversion function for the voltage due to the variation of the control angle $\alpha$ becomes

$$\Delta K^U = \frac{\partial K^U}{\partial \alpha} \frac{\Delta \hat{\alpha}}{2} e^{i\hat{\alpha} t} + \frac{\partial K^U}{\partial \alpha} \frac{\Delta \hat{\alpha}^*}{2} e^{-i\hat{\alpha} t}$$  \hspace{1cm} (3-76)

The varying control angle $\alpha$ causes the following DC voltage variation

$$\Delta U_y = \text{Re}\left\{ \Delta K^U \ast U_{y0} \right\}$$  \hspace{1cm} (3-77)

where $U_{y0}$ is the steady-state valve voltage, which can be calculated from

$$U_{y0}(\omega_n t) = \hat{u}_{y0} e^{i\omega_n t} - \frac{2}{3} Z K^I(\omega_n t)$$  \hspace{1cm} (3-78)

where,

$$Z = Z_L(j\omega_n) + Z_C(j\omega_n)$$  \hspace{1cm} (3-79)

From the DC voltage variation the following transfer function is obtained
The second term $H_{12}$

A variation of the control angle also impacts the transformation of the DC side current into the AC side current. The variation in the AC side current produces a voltage drop in the valve voltage if the network is weak. The AC side voltage drop in turn causes a reduction of the DC side voltage.

If it is assumed that the control angle $\alpha$ incorporates a small variation, the following AC current variation is obtained

$$
\Delta i_v = I_{d0} \frac{\partial K^I}{\partial \alpha} \Delta \alpha
$$

(3-81)

where, the variation of the conversion function for the current due to variation of the control angle $\alpha$ is given by

$$
\Delta K^I = \frac{\partial I^I}{\partial \alpha} \Delta \hat{\alpha} + \frac{\partial i^I}{\partial \alpha} \Delta \hat{\alpha}^* e^{j\omega t}
$$

(3-82)

The current $\Delta i_v$ is injected into the AC network creating an AC voltage variation which is then transferred into the DC side of converter as DC side voltage variation given by

$$
\Delta u_d = -\frac{2}{3} I_{d0} \left( u^* e^{j\omega t} \frac{\partial \hat{\alpha}}{\partial \alpha} \Delta \hat{\alpha} + \frac{\partial \hat{\alpha}^*}{\partial \alpha} \Delta \hat{\alpha}^* e^{j\omega t} \right)
$$

(3-83)

Here, the side-band impedances include the network and converter transformer impedances

$$
Z^+ = Z_L(j\omega) + Z_C(j\omega)
$$

$$
Z^- = Z_L(j\omega) + Z_C(j\omega)
$$

(3-84)

From this expression the transfer function is then obtained as

$$
H_{12}(j\Omega) = -\frac{1}{3} I_{d0} \left( u^* Z^+ \frac{\partial \hat{\alpha}}{\partial \alpha} + \frac{\partial \hat{\alpha}^*}{\partial \alpha} \right)
$$

(3-85)

### 3.3.4 Validation of the transfer functions

A program has been developed in MATLAB to calculate the values of the transfer functions in the frequency domain using the formulas derived above. In order to validate the results a specific case was studied and compared with results obtained by simulation in the time-domain. The simulated system that has been modeled contained an HVDC converter rated 1500 MW connected to a weak network.
(short-circuit power of 3000 MVA). The converter transformer leakage reactance is 16% and the total shunt filter reactive power generation is 810 MVAr.

Figure 3.11 presents the results for the transfer function $G_3$ (Figure 3.11-(A)) and for $G_2$ (Figure 3.11-(B)). The solid curves show the results obtained from the calculation in the frequency domain (MATLAB) while the dotted ones are the results obtained from the time-domain simulation.

The results depicted in Figure 3.11 were obtained in the case mentioned above in which the HVDC converter was connected to a weak network. In this case the shunt filter banks and the line inductance produce a resonance at a fairly low frequency (90 Hz) with a quite high Q value. From the DC side this resonance appears at the frequencies $50\pm90$ Hz, i.e. at 40 Hz and 140 Hz. It can be seen that at these frequencies the model in the frequency domain predicts a much higher gain than what was obtained by simulation in the time domain. It has been found that this discrepancy mainly emerges from the inadequate assumption that the overlap angle remains constant when the DC side quantities and the control signal vary.

Comparison of an unrealistic case lacking the big filter bank, i.e. an HVDC converter fed from a resistive-inductive line, does not show any deviation between the models in frequency and time domains.

The impact of the variation of the overlap angle is the subject of the next chapter, Chapter 4.
3.3 The LCC Model Connected to a Network

3.3.5 Block diagram of the complete converter model

The complete model of the line-commutated current source converter for smallsignal analysis is represented in the form of a block diagram as shown in Figure 3.12. The current control system has been simplified and only contains an ordinary PI regulator represented by the transfer function $G_1$. The output from the current controller is the control angle $\alpha$. The transfer function $G_2$ describes the relationship between the variation in the firing angle and the corresponding variation in voltage.
across the DC side terminals for constant DC current. The transfer function $G_3$ describes how a small variation in DC side current causes a variation in the DC side voltage due to the variation in the AC side voltage produced by the corresponding AC side current variation. In the block diagram $G_f$ represents the measured filter time constant for the DC current.

$$\Delta i_d^{ref} \quad \Delta e \quad -1 \quad G_1 \quad \Delta \alpha_{csc} \quad G_2 \quad \Delta u_d$$

Figure 3.12: The Complete model of a line commutated converter including the DC current controller

**Classical LCC formulas**

The steady-state operation of an LCC having a constant feeding AC voltage can be represented by the following equivalent static characteristic equation

$$U_d = U_{dso} \cdot \cos \alpha - R_c \cdot i_d$$  (3-86)

where, $R_c = \frac{3}{\pi} X_c$ is an equivalent resistive voltage drop due to the commutation reactance. The above equation gives the DC voltage across converter when operating as a rectifier. When operating as an inverter a similar equation is used, but in the formula instead of using firing angle $\alpha$ the extinction angle $\gamma$ should be used.

The combining transfer functions $G_3 = \frac{\Delta u_d}{\Delta i_d}$ and $G_2 = \frac{\Delta u_d}{\Delta \alpha}$ that were derived in the previous section, represent the characteristics of the static equation of an LCC for zero frequency (steady-state conditions). The transfer function $G_3 = \frac{\Delta u_d}{\Delta i_d}$ represents the voltage drop due to commutation inductance. Assuming a stiff AC voltage connected to the converter bus, that is, assuming that $Z_L = 0$ it is found that

$$G_3 = \frac{\Delta u_d}{\Delta i_d} \approx R_c = \frac{3}{\pi} X_c$$  (3-87)
The transfer function $G_2 = \frac{\Delta u_d}{\Delta \alpha}$ represents how the DC voltage across the converter changes due to a change in the firing angle, after the system has reached the steady-state condition. Accordingly,

$$
\Delta u_d = U_{dio} \cdot \cos \Delta \alpha = -U_{dio} \cdot \sin \alpha \cdot \Delta \alpha
$$

or

$$
G_2 = \frac{\Delta u_d}{\Delta \alpha} \approx -U_{dio} \cdot \sin \alpha \cdot \Delta \alpha
$$

### 3.4 MODEL OF A TWO-TERMINAL SYSTEM

#### 3.4.1 Schematic diagram

The schematic diagram of a two terminal system is presented in Figure 3.13. Whenever applicable the same notation has been used both for the rectifier and for the inverter. The same names are used in both the rectifier and the inverter terminals. However, the variables in the inverter have primed names.

**Figure 3.13:** Block Diagram of a two-terminal HVDC system including a rectifier in current control and an inverter operating with constant DC voltage control
The following control system is used:

1) In the rectifier the firing of the valves is determined by the current control loop via the current controller represented by the transfer function $G_1$.

2) In the inverter the firing of the valves is determined by the voltage control loop represented by the transfer function $G'_5$.

Both $G_1$ and $G'_5$ are ordinary PI controllers with appropriate gains and time constants.

Between the two converter terminals there is a model of the DC line that includes the DC reactors and possibly DC filters. The modeling of the main circuit representing the DC side of the converter is described in the next sub-section.

**Transfer function to verify the performance of the control system**

Four transfer functions are of interest to qualify the performance of the control system:

**Rectifier current controller:**

1) The closed-loop transfer function for the current controller

$$\frac{\Delta I_d}{\Delta I_{d\text{-ref}}}(3-90)$$

2) The open-loop transfer function for the current controller

$$\frac{\Delta I_d}{\Delta \varepsilon}(3-91)$$

**Inverter voltage controller**

3) The closed loop transfer function for the voltage controller

$$\frac{\Delta U'_d}{\Delta U_{d\text{-ref}}}(3-92)$$

4) The open-loop transfer function for the voltage controller

$$\frac{\Delta U'_d}{\Delta \varepsilon'}(3-93)$$

**3.4.2 Main circuit representation of the DC side of the converter**

A two-port form representation of the electric circuit is used having the two DC voltages as dependent variable and the two DC currents as independent variables (see Figure 3.14). This pair of equations can be written as
3.4 Model of a Two-terminal System

\[
\begin{bmatrix}
\Delta i_d \\
\Delta i_d'
\end{bmatrix} = \begin{bmatrix}
\frac{1}{Z_k} & G_k \\
G_k & \frac{1}{Z_k'}
\end{bmatrix}
\begin{bmatrix}
\Delta u_d \\
\Delta u_d'
\end{bmatrix}
\]

(3-94)

Figure 3.14: Two-port model with voltage and current definitions

The four functions in the square matrix characterize the network. They are computed by conventional circuit analysis. The circuit includes the representation of the DC line model and smoothing reactors. DC filter may also be included.

The corresponding short-circuit transfer admittances and short-circuit impedances are calculated assuming

\[
G_k = \frac{\Delta i_d}{\Delta u_d} \quad \text{for} \quad \Delta u_d = 0
\]

(3-95)

\[
G_k' = \frac{\Delta i_d'}{\Delta u_d} \quad \text{for} \quad \Delta u_d = 0
\]

and

\[
Z_k = \frac{\Delta u_d}{\Delta i_d} \quad \text{for} \quad \Delta u_d' = 0
\]

(3-96)

\[
Z_k' = \frac{\Delta u_d'}{\Delta i_d} \quad \text{for} \quad \Delta u_d = 0
\]

DC line model

The DC line model used in the program corresponds to the Pole-Mode PI equivalent (see Figure 3.15) where the parameters of the equivalent are computed based on hyperbolic functions. This is preferred for the representation of long transmission lines.

It should be noted that a bipolar DC line has two modal impedances involved: the pole mode and the ground mode. As the present model consists of an equivalent
monopolar representation of the transmission, only the pole mode will be included, although the model could easily be upgraded to have the ground mode as well, in the case of a complete bipolar representation of the system.

![Figure 3.15: Equivalent PI Circuit for a long transmission line (only the Pole Mode is considered)](image)

The parameters are calculated as

\[
Z_\pi = Z_C \sinh(\gamma l)
\]

\[
\frac{Y_\pi}{2} = \frac{1}{Z_C} \tanh\left(\frac{\gamma l}{2}\right)
\]

where

\[
Z_C = \sqrt{\frac{R + j\omega L}{j\omega L}}
\]

is the Characteristic Impedance of the line. The parameters \( R \), \( L \), and \( C \) are the electrical characteristics of the line per length (per km), and \( l \) is the total length of the line.

Typical values of these parameters are: \( R \) is in the order of 0.01 – 0.02 \( \Omega / km \), \( L \) is in the order of 1 \( mH / km \) and \( C \) is in the order of 10 \( nF / km \).

The quantity

\[
\gamma = \sqrt{(R + j\omega L)(j\omega C)}
\]

is called the propagation constant.
3.5 APPLICATION

Assume an integrated AC-DC system with the system parameters, operating conditions, and control parameters as specified in Table 3.2. This is a 1500 MW HVDC transmission link connected between two AC networks having a short-circuit power of 3000 MVA at both the rectifier and the inverter side of the HVDC transmission link.

Table 3.2: Data of the system used in the application

<table>
<thead>
<tr>
<th></th>
<th>Rectifier</th>
<th>Inverter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Network System</td>
<td>3000 MVA, 85°</td>
<td>3000 MVA, 85°</td>
</tr>
<tr>
<td>Shunt filters</td>
<td>819 MVA</td>
<td>819 MVA</td>
</tr>
<tr>
<td>Converter transformer</td>
<td>1785 MVA, leakage reactance = 0.16 pu</td>
<td></td>
</tr>
<tr>
<td>Smoothing reactor</td>
<td>0.29 H</td>
<td></td>
</tr>
<tr>
<td>Nominal DC quantities</td>
<td>500 kV ; 3 kA</td>
<td>500 kV</td>
</tr>
<tr>
<td>Operating DC Voltage</td>
<td>500 kV</td>
<td>500 kV</td>
</tr>
<tr>
<td>Operating DC Current</td>
<td>3 kA</td>
<td>3 kA</td>
</tr>
<tr>
<td>Firing (extinction) angle initial condition</td>
<td>15 deg</td>
<td>18 deg</td>
</tr>
<tr>
<td>Current Controller – Prop-gain</td>
<td>0.63 rad/pu</td>
<td>-</td>
</tr>
<tr>
<td>Current Controller – Int-gain</td>
<td>65.61 rad/pu</td>
<td>-</td>
</tr>
<tr>
<td>Voltage Controller – Prop-gain</td>
<td>-</td>
<td>0.436 rad/pu</td>
</tr>
<tr>
<td>Voltage Controller – Int-gain</td>
<td>-</td>
<td>15.87 rad/pu</td>
</tr>
<tr>
<td>Vol. Ctrl – smoothing filter</td>
<td>-</td>
<td>16 msec</td>
</tr>
</tbody>
</table>

The Bode plot of the closed loop transfer function $\Delta I_d / \Delta I_{d-ref}$ and the Nyquist plot of the open-loop transfer function $\Delta I_d / \Delta \alpha$ for the rectifier current controller have been checked versus time-domain simulation performed in PSCAD/EMTDC. The results are presented in Figure 3.16-(A) and Figure 3.16-(B), respectively.

A critical resonance frequency seen from the DC side of the converter appears at about 70 Hz in both simulations. However, the model in the frequency domain has much higher gain than what is obtained in the time-domain simulation. This was envisaged in Section 3.3.4 when validating the individual transfer functions included in the model.

Another critical resonance frequency that could only be observed in the mathematical model implemented in MATLAB is 15 Hz. This frequency is not observed in the time-domain simulation due to a significantly higher damping in the time-domain simulation as compared to the mathematical model.

All these discrepancies mainly arise from the assumption that the overlap angle is constant.

Figure 3.16: (A) Bode Plot Closed Loop \( \frac{\Delta I_d}{\Delta I_{d-ref}} \); (B) Nyquist Plot Open Loop \( \frac{\Delta I_d}{\Delta \varepsilon} \)
3.6 CONCLUSION

This chapter presents a frequency-domain model of an HVDC line-commutated current-source converter. Transfer functions between superimposed oscillations in the control signal and the AC and DC voltage and the current have been derived. With this model the classical Bode/Nyquist/Nichols control theory is well suitable to study the impact of the connected AC and DC networks on system performance and stability.

The derivations of the transfer functions are made using the space-vector concept. This gives a concise representation of the system as the zero-sequence is cancelled when the neutral is not connected. The result is a representation of the three time-varying quantities by means of one complex space vector.

A program developed in MATLAB to calculate the values of the transfer function in the frequency domain using the derivation described in this paper has been validated against a time-domain simulations. Individual transfer functions included the in the model and overall transfer functions of a two terminal system have been compared. It was observed that the two models show similar resonance frequencies. However, the frequency-domain model has much higher gains at the resonances than those obtained by simulation in the time domain. This discrepancy is mainly due to the assumption made in the derivation of the frequency-domain model that the overlap angle remains constant when the DC side quantities and the control signal vary.

In Chapter 4 the model is extended to deal with the varying overlap angle in order to bring the frequency-domain model into agreement with the results obtained from time-domain simulations.
CHAPTER 4
FREQUENCY DOMAIN MODEL OF A LINE-COMMUTATED CURRENT-SOURCE CONVERTER. PART II: VARYING OVERLAP

This chapter presents the second part of the description of a frequency-domain model of an HVDC line-commutated current-source converter. The model is based on transfer functions between superimposed oscillations in the control signal and the AC and DC side voltages and currents.

In the description in Chapter 3 the overlap angle was assumed to be constant during the commutation period. However, in the derivation it was mentioned that a certain variation of overlap may occur due to varying current and commutation voltage. The calculated transfer functions become poorly damped around the resonance frequencies if the overlap angle is assumed to be constant.

In this chapter the derivation of the model will be completed, including the influence of a varying commutation period.

Furthermore, the algebraic transfer function of the Phase Locked Loop (PLL) will be included. It has an impact on the overall stability of the system.

4.1 LCC CONVERTER MODEL ASSUMING VARIABLE OVERLAP ANGLE

4.1.1 Circuit arrangement

Assume the same circuit arrangement as presented in Chapter 3. The LCC is fed from an infinite source through network impedance. A shunt filter is connected to the converter bus. Figure 4.1 outlines the circuit setup.

The impedance connecting each phase of the LCC and the feeding infinite source is represented by $Z_L$. The impedance $Z_F$ represents the converter bus filter and the
The commutation reactance of the LCC converter transformer is represented by $Z_C$. The voltage source that energizes the transmission system is defined by $\hat{U}_{b0}(t) = \hat{U}_{b0} e^{j\omega_0 t}$, which only contains a fundamental frequency component and the argument is zero at zero time.

![Figure 4.1](image)

**Figure 4.1:** (A) System configuration and (B) corresponding Thevenin equivalent

The analysis is made by replacing the network impedances and the filter bus impedances by the corresponding Thevenin equivalents:

$$Z_L = Z_L \parallel Z_F$$

$$\hat{U}_{b0}(t) = \hat{U}_{b0} e^{j\omega_0 t}$$

$$\hat{U}_{b0} = \frac{Z_F(j\omega_N)}{Z_F(j\omega_N) + Z_L(j\omega_N)} \hat{U}_{b0}$$

Below the interaction between the LCC and the connected ac system is studied by calculating transfer functions for superimposed small-signal oscillations in voltages and currents.

Recall from Chapter 3 the following set of basic equations, which applies to an LCC:

$$\tilde{I}_V(t) = \frac{2}{3} \vec{K}'(t) i_d(t)$$

$$u_d(t) = \text{Re}\{\vec{K}'(t) \vec{I}_V(t)\}$$  \hspace{1cm} (4-2)

In Chapter 3 the conversion function $\vec{K}'(t)$ was introduced, which associates the DC side current $i_d(t)$ and the AC current $\tilde{I}_V(t)$ (see Chapter 3, Section 3.2.1). The conversion function $\vec{K}'(t)$ associates the AC valve voltage $\vec{I}_V(t)$ and DC voltage $u_d(t)$ (Chapter 3, Section 3.2.2).

The overlap angle is influenced by two factors: the DC side current and the commutating voltage. The latter depends on the value of the AC voltage at the filter bus during the commutation interval. In order to represent it, a new scalar function $u_{\text{comm}}(t)$ has been introduced in the model. This scalar function is defined as the commutating voltage which is applicable for a specific commutation during the $60^\circ$
interval surrounding this specific commutation. Assume the same definitions as the
ones used in the preceding chapter, i.e. that the filter bus voltage has the argument
zero at $t = 0$ such that $\pi_{ca}(t) = \hat{U}_c e^{j\omega_N t}$. In the interval $\frac{\pi}{3} + \alpha < \omega_N t < \frac{\pi}{3} + \alpha + \mu$ the
commutation from thyristor 1 to thyristor 3 occurs. The applicable commutation
equation in this case is $u_{cb} - u_{ca}$ and the $60^\circ$ interval, which symmetrically surrounds
the commutation interval, is

$$
\left( \frac{\pi}{3} + \alpha + \frac{\mu}{2} \right) - \frac{\pi}{6} < \omega_N t < \left( \frac{\pi}{3} + \alpha + \frac{\mu}{2} \right) + \frac{\pi}{6}
$$

(4-3)

i.e.,

$$
\frac{\pi}{6} + \alpha + \frac{\mu}{2} < \omega_N t < \frac{\pi}{2} + \alpha + \frac{\mu}{2}
$$

(4-4)

Accordingly, the commutation voltage is defined as

$$
u_{\text{comm}}(t) = u_{cb}(t) - u_{ca}(t), \quad \frac{\pi}{6} + \alpha + \frac{\mu}{2} < \omega_N t < \frac{\pi}{2} + \alpha + \frac{\mu}{2}
$$

(4-5)

This commutation voltage can be extracted by means of a complex conversion
function $\overline{K}(t)$ using the same principle as the one used for extracting the DC
equation $u_d(t)$. Thus,

$$
u_{\text{comm}}(t) = \text{Re}\{\overline{K}(t)\pi_c(t)\}
$$

(4-6)

From the discussion above it is found that $\overline{K}(t)$, in the interval defined by Eq. (4-4), is given by

$$
\overline{K}(t) = e^{j\frac{2\pi}{3}} - 1 = -\sqrt{3} e^{-j\frac{\pi}{6}}, \quad \frac{\pi}{6} + \alpha + \frac{\mu}{2} < \omega_N t < \frac{\pi}{2} + \alpha + \frac{\mu}{2}
$$

(4-7)

The commutation voltage in consecutive $60^\circ$ intervals can be obtained by rotating
the conversion function $\overline{K}(t)$ forward by the factor $e^{j\frac{\pi}{3}}$ in each consecutive
interval. The commutation voltage function is illustrated in Figure 4.2.

In reality it is of course only the commutation voltage during the commutation
interval that has a real impact on the overlap angle. It is represented by the voltage
in the midpoint of each commutation interval. In the calculations here the value of
the commutation voltage in the whole $60^\circ$ interval will have a certain influence.
Chapter 4. Frequency domain model of a Line-Commutated Current-Source Converter. Part II: Varying overlap

Figure 4.2: The new scalar function $u_{\text{comm}}(t)$ calculated using the conversion function $K^c(t)$. The figure shows the space vector of the filter bus voltage, the DC side voltage of the converter and the scalar $u_{\text{comm}}(t)$ in the same graph where the line-to-line voltage at the filter bus is presented: (A) rectifier operation, assuming $\alpha = 15^\circ$, converter transformer reactance $x_C = 0.16 \text{pu}$ and nominal values for all other parameters; (B) inverter operation assuming $\alpha = 140^\circ$.

The basic relation that governs the overlap angle expresses that a certain voltage-time area is required in order to commutate the DC current from one phase to another one. The voltage-time area is determined by the leakage impedance $Z_C$ of the transformer that feeds the LCC. Here, a reasonable approximation has been made, which is to simply consider the reactance part, $x_C$. The following equation holds

$$2L_{C}i_{d} = u_{\text{comm}} \frac{\mu}{\omega_N} \quad (4-8)$$

Thus

$$2x_{C}i_{d} = u_{\text{comm}} \mu \quad (4-9)$$

where, $u_{\text{comm}}$ is the average commutation voltage during the commutation interval. This equation will be used to determine the variation of the overlap angle as a function of the DC current and the commutation voltage.
4.1.2 Conversion function used to extract commutation voltage

Similarly to $\mathbf{K}^I$ and $\mathbf{K}^U$, the function $\mathbf{K}^C \cdot e^{-j\theta}$ is periodical and it can be expanded as a Fourier series, yielding

$$\mathbf{K}^C(\theta) = \sum_m \ell_m^C e^{j(6m+1)\theta}$$

(4-10)

with,

$$\ell_m^C = j\frac{3\sqrt{3}}{\pi} \frac{(-1)^m}{6m+1} e^{-j(6m+1)\left(\frac{\alpha+\mu}{2}\right)}$$

(4-11)

Again, similar to $\mathbf{K}^I$ and $\mathbf{K}^U$, the assumption to consider only the fundamental frequency will be used, as the harmonics have little influence on the dynamics for small low frequency oscillations. Then, the conversion function is simplified to

$$\mathbf{K}^C(t) = \ell^C e^{j\omega t}$$

(4-12)

with

$$\ell^C = j\frac{3\sqrt{3}}{\pi} e^{-j\left(\frac{\alpha+\mu}{2}\right)}$$

(4-13)

4.1.3 Small signal relations applied to the basic set of equations

Summing up, the following set of equations is considered

$$\begin{cases}
\Delta \bar{i}_p(t) = \frac{2}{3} (\mathbf{K}^I(t) i_d(t)) \\
\Delta u_d(t) = \text{Re}\{\mathbf{K}^C(t) \bar{u}_d(t)\} \\
\Delta u_{\text{comm}}(t) = \text{Re}\{\mathbf{K}^C(t) \bar{u}_c(t)\}
\end{cases}$$

(4-14)

A general differentiation of these expressions yields

$$\begin{cases}
\Delta \bar{i}_p(t) = \frac{2}{3} \Delta \mathbf{K}^I(t) I_d(t) + \frac{2}{3} \mathbf{K}^I_0(t) \Delta i_d(t) \\
\Delta u_d(t) = \text{Re}\{\Delta \mathbf{K}^U(t) \bar{U}_d(t) + \mathbf{K}^U_0(t) \Delta \bar{u}_d(t)\} \\
\Delta u_{\text{comm}}(t) = \text{Re}\{\Delta \mathbf{K}^C(t) \bar{U}_c(t) + \mathbf{K}^C_0(t) \Delta \bar{u}_c(t)\}
\end{cases}$$

(4-15)

The independent variables included in the conversion functions are the time $t$, the firing angle $\alpha$, and the overlap angle $\mu$. In the formulas the differentiation will be made for variation of the firing angle and the overlap angle. Accordingly,
Chapter 4. Frequency domain model of a Line-Commutated Current-Source Converter. Part II: Varying overlap

\[
\begin{align*}
\Delta R^i(\theta) &= \frac{\partial R^i(\theta, \alpha, \mu)}{\partial \alpha} \Delta \alpha(\theta) + \frac{\partial R^i(\theta, \alpha, \mu)}{\partial \mu} \Delta \mu(\theta) \\
\Delta R^u(\theta) &= \frac{\partial R^u(\theta, \alpha, \mu)}{\partial \alpha} \Delta \alpha(\theta) + \frac{\partial R^u(\theta, \alpha, \mu)}{\partial \mu} \Delta \mu(\theta) \\
\Delta R^c(\theta) &= \frac{\partial R^c(\theta, \alpha, \mu)}{\partial \alpha} \Delta \alpha(\theta) + \frac{\partial R^c(\theta, \alpha, \mu)}{\partial \mu} \Delta \mu(\theta)
\end{align*}
\]  

(4-16)

4.1.4 The transfer function \( G_3 = \Delta u_d / \Delta i_d \)

Consider the case that there is no action taken by the control signals (i.e. \( \alpha \) is maintained constant). A variation in the DC current then results in a mutual interaction between the different quantities. The mutual interaction from the different quantities can then be represented by the transfer function block diagram shown in Figure 4.3.

![Block diagram](image)

Figure 4.3: Block diagram describing mutual the interactions between different quantities representing the transfer function \( G_3 = \Delta U_d / \Delta I_d \)

The transfer function obtained from the block diagram from a variation in direct current \( \Delta i_d \) to a variation in the direct voltage \( \Delta u_d \) is given by:
4.1 LCC converter model assuming variable overlap angle

\[
G_3 = \frac{\Delta u_d}{\Delta i_d} = H_1 + \frac{\left[ (H_2 + H_3) \times (H_7 + H_8 \times H_4) \right]}{\left[ 1 - H_8 \times (H_5 + H_6) \right]}
\]

(4-17)

Below the individual transfer functions included in the block diagram will be discussed.

**Transfer function H₁**

The transfer function \(H_1\) expresses the direct impact of the DC side current on the DC side voltage when the varying overlap is disregarded. This transfer function is derived in APPENDIX A. Accordingly,

\[
H_1(j\Omega) = -\frac{1}{3} \left( e^{\omega^*} Z^* \xi^l + e^{\omega} Z^- \xi^l \right)
\]

\[
Z^* = Z_L'(j\omega^*) + Z_C(j\omega^*)
\]

\[
Z^- = Z_L'(j\omega^-) + Z_C(j\omega^-)
\]

(4-18)

The following notation has been used

\[
\begin{align*}
\omega^+ &= \omega_N + \Omega \\
\omega^- &= \omega_N - \Omega
\end{align*}
\]

(4-19)

**Transfer function H₂**

Variation of the overlap angle introduces a variation, \(\Delta \overline{K}'\), of the conversion function \(\overline{K}'\). This deviation acts on the steady-state DC current. The resulting change of the AC side current causes a voltage drop on the AC side. The effect is a modification of the DC voltage.

This transfer function is derived in APPENDIX B. Accordingly,

\[
H_2(j\Omega) = -\frac{1}{3} I_d^o \left( e^{\omega^*} Z^* \frac{\partial \xi^l}{\partial \mu} + e^{\omega} Z^- \frac{\partial \xi^l}{\partial \mu} \right)
\]

\[
Z^* = Z_L'(j\omega^*) + Z_C(j\omega^*)
\]

\[
Z^- = Z_L'(j\omega^-) + Z_C(j\omega^-)
\]

(4-20)

**Transfer function H₃**

This transfer function describes the impact of the varying overlap angle on the DC voltage caused by the variation of the voltage conversion function. The angle variation acts on the steady state valve voltage \(\overline{U}_{\ell 0}\).

This transfer function is derived in APPENDIX C. Accordingly,
\[ H_3(j\Omega) = \frac{1}{2} \left( \frac{\partial \ell^U}{\partial \mu} \hat{U}_{v0} + \frac{\partial \ell^U}{\partial \mu} \hat{U}^*_{v0} \right) \]  

(4-21)

**Transfer function \( H_4 \)**

The transfer function \( H_4 \) expresses the direct impact of the DC side current on the commutation voltage, when the variation of the overlap angle is disregarded. The formula is derived in a similar way as the corresponding formula for \( H_1 \) (see APPENDIX A). It should be noted that the voltage drop on the AC side in this case only depends on the line impedance. Accordingly, only \( Z_L' \) occurs in the formula. Furthermore, the voltage conversion function \( \overline{K_c} \) extracts the commutation voltage instead of the DC voltage.

\[ H_4(j\Omega) = -\frac{1}{3} \left( \ell \epsilon^c \ell^l \ell^l + \ell \epsilon^c \ell^l \ell^l \right) \]

\[ Z^+ = Z_L'(j\omega^+) \]

\[ Z^- = Z_L'(j\omega^-) \]  

(4-22)

**Transfer function \( H_5 \)**

The variation of the overlap causes a deviation of the conversion function \( \overline{K}^l \). At constant DC side current this gives a contribution to the deviation in the AC side current and as a result the commutation voltage becomes disturbed. The transfer function \( H_5 \) expresses this impact of the overlap angle variation on the commutation voltage. The formulation is similar to \( H_2 \) (see APPENDIX B) except that the impedance only includes \( Z_L' \) and that the conversion function for the commutation voltage has been used.

\[ H_5(j\Omega) = -\frac{1}{3} I_{\ell^l} \left( \ell \epsilon^c \ell^l \frac{\partial \ell^l}{\partial \mu} + \ell \epsilon^c \ell^l \frac{\partial \ell^l}{\partial \mu} \right) \]

\[ Z^+ = Z_L'(j\omega^+) \]

\[ Z^- = Z_L'(j\omega^-) \]  

(4-23)

**Transfer function \( H_6 \)**

The transfer function \( H_6 \) is similar to transfer function \( H_3 \) (see APPENDIX C). It expresses the impact of the overlap angle variation on the commutation voltage caused by the variation of the voltage conversion function. It acts on the steady-state commutating voltage \( U_{c0} \).
\[ H_6(j\Omega) = \frac{1}{2} \left( \frac{\partial \dot{\epsilon}^C}{\partial \mu} \dot{U}_v^* + \frac{\partial \dot{\epsilon}^C}{\partial \mu} \dot{U}_v^* \right) \] (4-24)

**Transfer functions \( H_7 \) and \( H_8 \)**

These transfer functions reflect the direct connection between the overlap angle variation and the DC side current variation and commutation voltage variation, respectively.

The basic relation that governs the overlap angle has been discussed above. It can be expressed as follows

\[ 2x_c i_d = u_{\text{comm}} \mu \] (4-25)

\( H_7 \) can be defined as:

\[ H_7 = \frac{\partial \mu}{\partial i_d} = \frac{2x_c}{u_{c_0}} \] (4-26)

The other transfer function \( H_8 \) that reflects the direct connection between the variation in the commutating voltage and the overlap angle variation is defined as:

\[ H_8 = \frac{\partial \mu}{\partial u_{\text{comm}}} = -\frac{2x_c I_{d_0}}{(u_{\text{comm}})^2} \] (4-27)

The steady-state commutation voltage included in the above transfer functions is calculated by

\[ \Delta u_{\text{comm}} = \Delta \overline{U}_{d_0} - Z_L^* \Delta \overline{i}_y = \dot{U}_{d_0} e^{j\theta} - \frac{2}{3} I_{d_0} Z e^{j\theta} \]

\[ Z = Z_L(j\omega_N) \] (4-28)

**4.1.5 The transfer function \( G_2 = \Delta u_d / \Delta \alpha \)**

Below the variation in the DC voltage \( U_d \) due to a small variation in the firing angle \( \alpha \) caused by control action of the LCC converter will be derived. It is assumed that the DC current is constant.

The mutual interaction between the different quantities for a small variation in the control signal \( \alpha \), can be represented by the transfer function block diagram shown in Figure 4.4.

The transfer function obtained from the block diagram is:

\[ G_2 = \Delta u_d / \Delta \alpha = H_{11} + H_{12} + \frac{H_8 \cdot (H_2 + H_3) \times (H_{13} + H_{14})}{1 - H_8 \cdot (H_5 + H_6)} \] (4-29)
Below the additional transfer functions included in the block diagram are discussed.

Transfer function $H_{11}$

This transfer function describes the impact of the varying control angle $\alpha$ on the DC voltage caused by the variation of the voltage conversion function. The angle variation acts on the steady-state valve voltage.

This transfer function has already been calculated in Chapter 3, Section 3.3.3. The result is

$$H_{11}(j\Omega) = \frac{1}{2} \left( \frac{\partial \phi_u^*}{\partial \alpha} \hat{U}_{Y_0} + \frac{\partial \phi_u^*}{\partial \alpha} \hat{U}_{Y_0}^* \right)$$

(4-30)

Transfer function $H_{12}$

This transfer function translates a scalar variation in the control angle $\alpha$ into a variation in DC voltage caused by modulation of the current conversion function $\hat{R}'$. This transfer function has also been calculated in Chapter 3, Section 3.3.3, which is given by
4.1 LCC converter model assuming variable overlap angle

\[
H_{12}(j\Omega) = -\frac{1}{3} I_{d0} \left( \ell_{u*} Z^+ \frac{\partial \ell^l}{\partial \alpha} + \ell_{u} Z^- \frac{\partial \ell^l}{\partial \alpha} \right)
\]

\[
Z^+ = Z_L^+(j\omega^+) + Z_C^+(j\omega^+)
\]

\[
Z^- = Z_L^-(j\omega^-) + Z_C^-(j\omega^-)
\]  

\[\text{(4-31)}\]

**Transfer function \(H_{13}\)**

This transfer function expresses the impact of a variation in the control angle \(\alpha\) on the commutation voltage caused by the variation of the voltage conversion function. It acts on the steady-state commutating voltage \(\bar{U}_{c0}\).

The transfer function \(H_{13}\) has a similar form as compared to the transfer function \(H_{11}\). This transfer function makes use of the coupling function for the commutation voltage instead.

\[
H_{13}(j\Omega) = \frac{1}{2} \left( \frac{\partial \ell^c}{\partial \alpha} \bar{U}_{v0} + \frac{\partial \ell^c}{\partial \alpha} \bar{U}_{v0}^* \right)
\]  

\[\text{(4-32)}\]

**Transfer function \(H_{14}\)**

This transfer function expresses the impact of a variation in the control angle \(\alpha\) on the commutation voltage caused by the variation of the current conversion function \(\bar{K}^l\).

The formula is similar to that for \(H_{12}\) except that here the coupling function for the commutation voltage is used:

\[
H_{14}(j\omega) = -\frac{1}{3} I_{d0} \left( \ell_{c*} Z^+ \frac{\partial \ell^l}{\partial \alpha} + \ell_{c} Z^- \frac{\partial \ell^l}{\partial \alpha} \right)
\]

\[
Z^+ = Z_L^+(j\omega^+)
\]

\[
Z^- = Z_L^-(j\omega^-)
\]  

\[\text{(4-33)}\]

### 4.1.6 Validation of the transfer functions

The transfer function has been checked versus time-domain simulations performed in PSCAD/EMTDC. The same case studied in Chapter 3 has been used in this test validation, which includes an HVDC converter rated 1500 MW which is connected to a weak AC network (short-circuit power 3000 MVA). The converter transformer leakage reactance is 16% and the total shunt reactive power compensation is 810 MVA.

In Figure 4.5-(A) the results obtained for the transfer function \(G_3\) calculated in the frequency-domain model using MATLAB (solid curve) are compared to the time-
domain simulation using PSCAD/EMTDC (dotted curve). In Figure 4.5-(B) the corresponding results for $G_2$ are shown.

\[ G_3 = \frac{\Delta u_d}{\Delta I_d} \] (rectifier operation, network SCR=2)

\[ G_2 = \frac{\Delta u_d}{\Delta \alpha} \] (rectifier operation, network SCR=2)

\[ \text{MATLAB} - \text{solid} \]

\[ \text{EMTDC} - \text{- - - dotted} \]

**Figure 4.5:** (A) transfer function $G_3 = \frac{\Delta u_d}{\Delta I_d}$; (B) transfer function $G_2 = \frac{\Delta u_d}{\Delta \alpha}$

In contrast to the results obtained from the model derived in Part I, Chapter 3, much better agreement has now been obtained in the comparison. When introducing the varying overlap in the mathematical model the amount of damping has significantly increased, approaching the levels obtained in the time-domain simulation.
4.2 INTRODUCING THE PHASE-LOCK-LOOP (PLL) COORDINATED SYSTEM

4.2.1 Introduction

The control system operates in the PLL coordinate system. Only in steady-state conditions the control is aligned with the system reference which is normally the filter bus voltage side of the converter transformer. The PLL detects the argument of the voltage. During dynamic conditions the dynamics of the PLL have to be included, shifting the control angle $\alpha$ to the PLL coordinate.

As the PLL operates on the filter bus side of the converter transformer, the argument of the filter bus voltage is calculated from $\bar{U}_{c0} = \hat{U}_{c0} e^{j(\phi_c + \theta)}$. For small signal variations the corresponding argument of the voltage to be used as input to the PLL can be approximated by

$$\Delta \theta_{pu}^{in} = \arg\left(\frac{(U_{c0} + \Delta U_{t})}{U_{c0} e^{j(\phi_c + \theta)}}\right) \approx \Re\left[-j \cdot \frac{\Delta U_{t}}{U_{c0} e^{j(\phi_c + \theta)}}\right] \quad (4-34)$$

From above it is now possible to define a new conversion function,

$$\bar{K}^C_{PLL} = \frac{+j}{\hat{U}_{c0} e^{-j(\phi_c + \theta)}} \quad (4-35)$$

This will be used to extract the argument from the PLL coordinate system. Define:

$$\bar{K}^C_{PLL}(\theta) = \hat{e}_{PLL} e^{j\theta} \quad (4-36)$$

with

$$\hat{e}_{PLL} = \frac{+j}{\hat{U}_{c0} e^{-j\phi_c}} \quad (4-37)$$

The output from the PLL is then calculated from

$$\Delta \theta_{PLL}^{out} = \frac{\bar{K}^C_{PLL}}{1 + \bar{K}^C_{PLL}} \cdot \Delta \theta_{PLL}^{in} \quad (4-38)$$

where $\bar{K}^C_{PLL}$ is the transfer function of the PLL control, which usually has the form

$$\bar{K}^C_{PLL} = k_{PLL} (1 + j\Omega_{PLL}) \cdot \frac{1}{j\Omega_{PLL}} \quad (4-39)$$

In the equation $k_{PLL}$ is the PLL gain and $t_{PLL}$ is the PLL integral time constant.
4.2.2 The transfer function $G_{41} = \Delta \theta_C / \Delta i_d$

This is a transfer function that takes into consideration the input contribution to the PLL control resulting from a small variation on the DC-side current of the converter.

Assuming no action from the control signals (constant firing angle $\alpha$), the input argument to the PLL control resulting from a small perturbation in the DC current can be represented by the transfer function block diagram shown in Figure 4.6.

The transfer function obtained from the block diagram from a variation in the direct current $\Delta i_d$ to a variation in the input argument to the PLL control $\Delta \theta_{PLL}$ is:

$$G_{41} = \frac{\Delta \theta_C}{\Delta i_d} = H_{21} + \frac{H_{22} \times (H_8 \times H_4 + H_7)}{[1 - H_6 \times (H_5 + H_6)]}$$

(4-40)

Below the additional transfer functions included in the block diagram are discussed.
4.2 Introducing the phase-lock-loop (PLL) coordinated system

Transfer function $H_{21}$

The transfer function $H_{21}$ expresses the direct impact of the DC-side current on the input argument to the PLL control, when the variation of the overlap angle is disregarded. The formula is similar to $H_4$ (see APPENDIX A) except that here the conversion function is given that extracts the argument of the obtained voltage variation of filter bus:

$$H_{21}(j\Omega) = -\frac{1}{3}\left(\ell_{PLL}^* Z^+ \ell^+ + \ell_{PLL}^* Z^- \ell^-\right)$$

$$Z^+ = Z'_I(j\omega^+)$$

$$Z^- = Z'_I(j\omega^-) \quad (4-41)$$

Transfer function $H_{22}$

This transfer function translates a scalar variation in overlap angle to the input argument of the PLL control caused by the variation in the current conversion function $K'$. The calculation procedure is similar to that of $H_5$ (see APPENDIX B), except that the conversion function that is used extracts the argument of the obtained voltage variation of the filter bus:

$$H_{22}(j\Omega) = -\frac{1}{3I_d}\left(\ell_{PLL}^* \frac{\partial\ell^+}{\partial\mu} + \ell_{PLL}^* \frac{\partial\ell^-}{\partial\mu}\right)$$

$$Z^+ = Z'_I(j\omega^+)$$

$$Z^- = Z'_I(j\omega^-) \quad (4-42)$$

4.2.3 The transfer function $G_{42} = \Delta \theta_c / \Delta \alpha$

This transfer function takes into consideration the input contribution to the PLL control resulting from a small variation in the control angle $\alpha$.

Assuming that the DC current is unchanged, the input argument to the PLL control resulting from a small variation in the control angle $\alpha$ can be represented by the transfer function block diagram shown in Figure 4.7.

The transfer function obtained from the block diagram is:

$$G_{42} = \frac{\Delta \theta_c}{\Delta \alpha} = H_{23} + \left(\frac{H_{22} \times H_8}{H_8 - H_5 \times (H_{13} + H_{14})}\right) \left(\frac{H_{13} + H_{14}}{1 - H_8 \times (H_5 + H_6)}\right) \quad (4-43)$$

Below the additional transfer function included in the block diagram is discussed.
Figure 4.7: Transfer function block diagram to obtain \( G_{42} = \Delta \theta_c / \Delta \alpha \)

Transfer function \( H_{23} \)

This transfer function translates a scalar variation in the control angle \( \alpha \) into a variation in the argument to the PLL caused by the variation of the current conversion function \( \bar{K}' \). The expression is similar to \( H_{14} \) except that here the conversion function is given that extracts the argument of the obtained voltage variation of the filter bus:

\[
H_{23}(j\Omega) = -\frac{1}{3} \int_{\alpha_0}^{\alpha} \left( I_{PLL}^* Z^* \frac{\partial \ell}{\partial \alpha} + I_{PLL}^* Z^{-} \frac{\partial \ell^*}{\partial \alpha} \right) d\theta
\]

\[
Z^* = Z_l(j\omega^*)
\]

\[
Z^- = Z_l(j\omega^-)
\]

(4.44)

4.3 BLOCK DIAGRAM OF THE COMPLETE MODEL

The complete small-signal model of the line-commutated current-source converter is shown in the form of a block diagram in Figure 4.8.

As described in Chapter 3, the dynamics of the current control system has been represented by the transfer function \( G_i \) which typically is an ordinary PI regulator. The output from the current regulator is the control angle, which receives the contribution from the PLL control. The transfer function \( G_2 \) describes the
relationship between the variation in firing angle and the corresponding variation in voltage across the converter terminal, for constant DC current. The transfer function $G_3$ describes how a small variation in direct current via the converter and a corresponding voltage variation on the AC side will give a variation in the direct voltage across the converter under the assumption of constant firing angle $\alpha$.

For the PLL control the following transfer functions are included: $G_{41}$ and $G_{42}$, which extract the phase-angle variation of the converter bus voltage due to variation in DC current and control angle $\alpha$, respectively. The transfer function indicated by the block $G_{PLL}$ includes the dynamics of the PLL control.

Figure 4.8: The complete model of a line-commutated converter including the DC current controller
### 4.4 MODEL OF A TWO-TERMINAL SYSTEM

The schematic diagram of a two-terminal system is given in Figure 4.9.

![Block diagram of a two-terminal HVDC system including current control for the rectifier and inverter operating with constant DC voltage control](image)

**Figure 4.9:** Block diagram of a two-terminal HVDC system including current control for the rectifier and inverter operating with constant DC voltage control

The block diagram was made in a similar fashion to that presented in Chapter 3: when applicable, the same notation has been given to both the rectifier and the inverter model of the converter, with the primed names for the symbols and variables used for the inverter terminal.

It is assumed that the rectifier controls the DC current (dynamics of the controller represented by $G_1$), and the inverter controls the DC voltage (dynamics of the controller represented by $G'_3$). The two converters are interconnected by a four-
pole represented by $G_k$, $G_k'$, $Z_k$, and $Z_k'$, which represents the DC line, the DC reactors, and the DC filters.

### 4.5 APPLICATION

Assume the integrated AC-DC system with the system parameters, operating conditions and control parameters presented in the Table 4.1. This is an 800 km monopolar HVDC transmission link, rated 1500 MW, connecting two AC networks. The short circuit power of the connected AC network at the rectifier side is 7500 MVA, and at the inverter side is 3500 MVA.

#### Table 4.1: Data of the system used in the application

<table>
<thead>
<tr>
<th></th>
<th>Rectifier</th>
<th>Inverter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal frequency</td>
<td>50 Hz</td>
<td>50 Hz</td>
</tr>
<tr>
<td>System Impedance</td>
<td>7500 MVA, 85°</td>
<td>3500 MVA, 85°</td>
</tr>
<tr>
<td>Shunt filters</td>
<td>819 MVA</td>
<td>819 MVA</td>
</tr>
<tr>
<td>Converter transformer</td>
<td>1785 MVA, leakage reactance = 0.16 pu</td>
<td></td>
</tr>
<tr>
<td>Smoothing reactor</td>
<td>0.29 H</td>
<td></td>
</tr>
<tr>
<td>Length of Transmission line</td>
<td>800 km</td>
<td></td>
</tr>
<tr>
<td>Transmission line Resistance R</td>
<td>0.01864 Ω/km</td>
<td></td>
</tr>
<tr>
<td>Transmission line Inductance L</td>
<td>0.98888 mH/km</td>
<td></td>
</tr>
<tr>
<td>Transmission line Capacitance C</td>
<td>11.867 nF/km</td>
<td></td>
</tr>
<tr>
<td>Nominal DC quantities</td>
<td>500 kV ; 3 kA</td>
<td>460 kV ; 3 kA</td>
</tr>
<tr>
<td>Operating DC Voltage</td>
<td>500 kV</td>
<td>460 kV</td>
</tr>
<tr>
<td>Operating DC Current</td>
<td>3 kA</td>
<td>3 kA</td>
</tr>
<tr>
<td>Firing (extinction) angle initial condition</td>
<td>15 deg</td>
<td>18 deg</td>
</tr>
<tr>
<td>Current Controller – Prop-gain</td>
<td>0.63 rad/pu</td>
<td>-</td>
</tr>
<tr>
<td>Current Controller – Int-gain</td>
<td>65.61 rad/pu</td>
<td>-</td>
</tr>
<tr>
<td>Voltage Controller – Prop-gain</td>
<td>-</td>
<td>0.436 rad/pu</td>
</tr>
<tr>
<td>Voltage Controller – Int-gain</td>
<td>-</td>
<td>15.87 rad/pu</td>
</tr>
<tr>
<td>Vol. Ctrl – smoothing filter</td>
<td>-</td>
<td>16 msec</td>
</tr>
<tr>
<td>PLL – Prop-gain</td>
<td>20 (rad/s)/rad</td>
<td>5 (rad/s)/rad</td>
</tr>
<tr>
<td>PLL – Int-Tcete</td>
<td>5 sec</td>
<td>5 sec</td>
</tr>
</tbody>
</table>

The Bode plot of the closed current control loop, $\Delta I_d / \Delta I_{d-ref}$, and Nyquist plot of the open current control loop, $\Delta I_d / \Delta \xi$, for the rectifier current controller have been checked versus time-domain simulations performed in PSCAD/EMTDC. The results are present in Figure 4.10-(A) and 4.10-(B), respectively.
Chapter 4. Frequency domain model of a Line-Commutated Current-Source Converter. Part II: Varying overlap

Figure 4.10: (A) Bode Plot Closed Loop $\frac{\Delta I_d}{\Delta I_{d-ref}}$; (B) Nyquist Plot Open Loop $\frac{\Delta I_d}{\Delta \epsilon}$
4.6 Conclusion

It should be noted that not much effort has been made to perfectly tune the control parameters for the system that has been set up, as the purpose of this study was simply to validate the mathematical model against a time-domain simulation.

The results show reasonable agreement between the frequency-domain and time-domain simulations. In the mathematical model the gain margin at the resonance frequency is slightly lower than in the time-domain simulation. However, the difference is considered to be acceptable.

4.6 CONCLUSION

This chapter completes the description of the frequency-domain model of an HVDC line-commutated current-source converter. The description of the model was initiated in Chapter 3. In that part of the description, a fixed overlap was assumed. That model was tested against time-domain simulations, showing that the transfer function obtained from the mathematical model was found to have significantly less damping as compared to the time-domain simulations. The time-domain simulation also shows that a certain variation of the overlap may occur, partly due to varying current and partly due to changes in the commutation voltage. These phenomena can introduce significant damping in the transfer functions around the resonance frequency.

In this chapter the model was upgraded by including new transfer functions that take the variation of the overlap into consideration. Additionally, the algebraic transfer functions of the phase-locked loop (PLL), which may impact the system stability, were included.

Typical transfer functions were calculated and compared with time-domain simulations showing very good agreement.
CHAPTER 5
FREQUENCY DOMAIN MODEL OF A VOLTAGE SOURCE CONVERTER WITH CURRENT CONTROLLER

This chapter presents a frequency-domain model of a Voltage Source Converter (VSC) with current controller operating as a STATCOM. The model can be analyzed in a straightforward manner by means of classical methods like the Bode, Nyquist, and Nichols methods.

As for the LCC model, this model is based on calculation of different transfer functions for small, superimposed oscillations in the voltage, the current, and the control signal. The objective is to study the small-signal dynamic properties of the combined AC-DC interaction.

As for the LCC model, space vectors are used to represent the three-phase quantities.

The derivation of the transfer functions is valid for arbitrary impedances on both the AC- and the DC-sides. It is then possible to analyze the behavior of the converter connected to a weak AC-system or even when it is connected to a system that includes another active device, such as a synchronous machine or another HVDC converter, e.g. a terminal of an HVDC transmission system based on line-commutated converters.

The model has been developed in the \(d-q\) frame, which is a rotating coordinate system, running synchronously with the undisturbed space-vector quantities representing the connected power system. In order to obtain the \(d-q\) components of the measured quantities, their space-vector representation, in fixed coordinates, are first obtained through \(\alpha-\beta\) transformation of the measured phase quantities followed by a transformation from the fixed coordinate system to the synchronously rotating one.

The technique with switching functions, which was used for the LCC model, could have been used to describe the operation of the VSC as well. However, when a high
switching frequency is used and the modulator compensates for the DC variations, it is possible to disregard from the harmonic voltages produced by the converter and simply assume that the produced (averaged) voltage output from the converter follows the voltage reference given from the control system. But, even if the distortion of the voltage waveform can be neglected, the limited switching frequency together with the required calculation time in the control system introduces a delay, which is important for the control performance. Accordingly, the delay has been represented in the model.

As long as the converter uses the ideal PWM modulator to produce the switching pattern of the valves, it perfectly compensates for the DC voltage variation. Consequently, the DC voltage will not appear in the description of the AC side dynamics. Nevertheless, modulation of the AC side quantities causes modulation of the DC side current injected into the DC side. Each frequency $\Omega$ appearing on the DC-side corresponds to two frequencies $\omega_n \pm \Omega$ on the AC-side [23].

The control system of the VSC implemented in the model includes the traditional inner-loop AC current control and outer loop controllers that control the voltage of the DC side of the converter and the AC filter bus voltage as shown in Figure 5.1. The model also includes the dynamics of the Phase Locked Loop (PLL).

Since the model is a small-signal representation, it is assumed that the system is linearized around a given operating point.

---

**Figure 5.1:** General overview of the controls for the VSC-STATCOM
5.1 OUTLINE OF THE SYSTEM – AC NETWORK ARRANGEMENT

Consider the model of a system depicted in Figure 5.2. The VSC is connected to a filter bus with the voltage $\bar{u}_F$ through a phase reactor with the impedance $z_v$. A shunt filter having the impedance $z_F$ is connected to the filter bus which is connected to a common bus with the voltage $\bar{u}_C$ via a transformer with the impedance $z_T$.

In order to test and validate the mathematical model of the VSC converter it is assumed that the common bus is energized from a network having inner impedance $z_s$. As a result the analyzed system becomes quite simple making it easy to study the interaction between the VSC and the connected AC system.

\[ \Delta \bar{u}_C \quad \Delta \bar{u}_F \quad \Delta \bar{u}_V \]

\[ \Delta i_C \quad \Delta i_F \quad \Delta i_V \]

\[ z_T \quad z_V \quad z_F \]

\[ \Delta \bar{u}_S \quad \Delta i_S \quad z_S \]

Figure 5.2: System outline
5.2 CURRENT CONTROL OF THE VOLTAGE SOURCE CONVERTER – SPACE VECTOR REPRESENTATION

5.2.1 The physical equation that governs the current controller

The control of the VSC is based on controlling the current $I_v$ through a phase reactor represented by the resistance $R_v$ and the inductance $L_v$. The control of the converter current is performed by manipulating the voltage $U_v$. Consider the system shown in Figure 5.3. From basic equations of physics it is found that the dynamics of the system can be expressed as

$$\Delta U_v^{\alpha\beta} = \Delta U_v^{\alpha\beta} + R_v \Delta I_v^{\alpha\beta} + L_v \frac{d \Delta I_v^{\alpha\beta}}{dt}$$  \hspace{1cm} (5-1)

This basic relationship, between the complex quantities, representing the converter current $\Delta I_v$, the bridge voltage $\Delta U_v$, and the ac line voltage $\Delta U_F$, governs the controller of the VSC.

However, the control system is developed using quantities that have been transformed from the fixed $\alpha-\beta$ coordinate system to a synchronous rotating coordinate system, that removes the rotation of the vectors and results in fixed complex phasors (at steady-state), as the $d-q$ coordinates rotates with the nominal frequency $\omega_N$.

Now, if the equation that governs the dynamic model is transformed into the synchronously rotating $d-q$ reference frame, it is found that

$$\Delta U_v^{dq} = \Delta U_v^{dq} + R_v \Delta I_v^{dq} + j \omega_N L_v \Delta I_v^{dq} + L_v \frac{d \Delta I_v^{dq}}{dt}$$  \hspace{1cm} (5-2a)

This voltage equation is used as the core of the control law to be included in the basic control of the VSC. The following two terms of the dynamic equation deserve special attention:

1) The “steady-state” term $j \omega_N L_v \Delta I_v^{dq}$ (resulting from the transformation to the synchronous reference frame);
5.2 Current control of the voltage source converter – Space vector representation

2) The “transient” term

\[ L_i \frac{d \Delta i^q_{PLL}}{dt} \].

Equation (5-2a) shows that a current change in a certain direction \( \Delta i^q \) requires a voltage component in the same direction as the current change to cover the inductive counter-voltage \( L_i \frac{d \Delta i^q_{PLL}}{dt} \), and the resistive voltage drop \( R_i \Delta i^q_{PLL} \), but also an additional voltage contribution in the perpendicular direction to cover the rotational voltage \( j \omega_N L_i \Delta i^q_{PLL} \). These terms will be feed-forwarded, using the estimated impedance parameters of the phase reactor. Finally, a feed-forward of the measured filter bus voltage, \( \Delta v^q_{PLL} \), completes the description of the control law that describes the dynamic model.

5.2.2 The implementation of the current controller

5.2.2.1 Synchronization with the PLL controller – general

The control relies upon alignment of the rotating \( d-q \) reference in relation to the measured space vector quantity \( \bar{u}_F \), which is the measured voltage at the filter bus ‘F’. A crucial component in the control is the PLL control that performs the synchronization with \( \bar{u}_F \). The synchronization controller extracts the time derivative of the transformation angle \( \dot{\theta} = \Omega \) from the filter bus voltage \( \bar{u}_F \), which is available from on-line measurements. The PLL mechanism is described in detail in Section 5.2.6.

5.2.2.2 Impact of the rotating coordinate system of the PLL on input/output signals to the current controller

All variables in this analytical model are represented in a rotating and undisturbed coordinate system, the \( d-q \) frame. The rotating coordinate system is generated by the PLL. This PLL detects the argument of the voltage vector in the filter bus. At steady state the PLL system coincides with the undisturbed rotating coordinate system. As a result, the vectors that represent steady-state operation are identical in the two coordinate systems.

During transients the filter bus voltage is disturbed which influences the argument produced by the PLL. If we assume that the phase from the output of this PLL is given by the angle \( \Delta \theta_{PLL}(t) \) in relation to the undisturbed coordinates, then, the AC current controller, which operates in the PLL coordinates, receives the current response calculated by:

\[
(\bar{i}^q_{PLL,0} + \Delta \bar{i}^q_{PLL}(t)) e^{-j \Delta \theta_{PLL}(t)} \approx \left[ \bar{i}^q_{PLL,0} + \Delta \bar{i}^q_{PLL}(t) \right] \left[ 1 - j \Delta \bar{\theta}_{PLL}(t) \right] \approx \bar{i}^q_{PLL,0} + \Delta \bar{i}^q_{PLL}(t) - j \bar{i}^q_{PLL,0} \Delta \bar{\theta}_{PLL}(t)
\]

In the undisturbed system the deviation of the current response is therefore given by

\[ (5-3) \]
\[ \Delta \tilde{v}_{dq}(t) - j \tilde{v}_{dq} \Delta \overline{\theta}_{PLL}(t) \]

(5-4)

Now, a notation with an upper superscript ‘PLL’ for the quantities in the PLL coordinate system is introduced. Thus,

\[ \Delta \tilde{v}_{d}^{PLL}(t) = \Delta \tilde{v}_{dq}(t) - j \tilde{v}_{dq} \Delta \overline{\theta}_{PLL}(t) \]

(5-5)

In a similar way, the derivation of the formula for the deviation of the filter bus voltage can be made:

\[ \Delta u_{F}^{PLL}(t) = \Delta u_{dq}(t) - j \mu_{dq} \Delta \overline{\theta}_{PLL}(t) \]

(5-6)

5.2.2.3 Current controller

The desired VSC voltage is synthesized in the PLL coordinate system by the current controller that takes the current reference and the current response vectors as input signals together with the measured filter bus voltage.

The current control law is derived from the physical equation governing the current flow through the phase reactor of the VSC derived in Section 5.2.1. The implementation of the control law will then comprise the following contributions:

A) Contribution from the feed-forward voltage term:

The first contribution is a replica of the filter bus voltage which forms a feed forward compensation of the network voltage response, given by:

\[ \Delta u_{F,A}^{PLL} = K_{UF} \Delta u_{F}^{PLL} \]

(5-7)

where \( K_{UF} \) is a transfer function which shall ideally be unity, but which may, for example, represent time-delays or measurement filters.

B) Contribution from the compensation of the phase reactor voltage:

For convenience two different contributions for this compensation are separated.

B.1) Contribution from the steady-state term:

This contribution refers to the compensation related to the resistive voltage drop \( R_{r} \Delta \tilde{v}_{PLL}^{r} \) and the voltage contribution in the perpendicular direction to cover the rotational voltage \( j \omega_{k} L_{r} \Delta \tilde{v}_{PLL}^{r} \).

The compensation may be based either on the measured converter current or on the current reference. In order to keep both these alternatives open, the current used here is a linear combination of the current reference and the measured current

\[ \Delta \tilde{v}_{r,comp}^{PLL} = \kappa \Delta \tilde{v}_{ref}^{r} + (1 - \kappa) \Delta \tilde{v}_{PLL}^{r} \]

(5-8)
The coefficient $0 < \kappa < 0$ determines the portion of the compensation that is based on the current reference and the portion $(1 - \kappa)$ that will be based on the measured current. It was found that the combination was especially beneficial when the VSC was connected to a weak network. In Section 5.6.2 will show that, when VSC is connected to a weak system the coefficient $\kappa$ influences not only the bandwidth but also the damping of the current controller at the resonance frequency.

Hence, the contribution from this steady-state term is given by

$$\Delta \mathbb{u}^{PLL}_{V, \beta i} = \mathbb{z}_v \left[ \kappa \Delta \tilde{i}_{v, \text{ref}} + (1 - \kappa) K_{iv} \Delta i^*_{v, \text{PLL}} \right] \quad (5-9)$$

where

$$\mathbb{z}_v = \mathbb{R}_v + \mathbb{L}_v j \omega_n$$ \quad (5-10)

is the estimated steady-state impedance of the phase reactor (in fixed coordinates $\omega_n$) and $K_{iv}$ represents time-delays and measurement filters for the current response.

B.2) Contribution from the transient term:

This contribution refers to the current change (derivate of the current) to cover the inductive counter-voltage $L_v \frac{d \Delta i^*_{v, \text{PLL}}}{dt}$.

In a computer-controlled system this term corresponds to the control strategy called dead-beat control. In a dead-beat control the converter current is obtained from a change of the current reference by introducing the following term in the control law

$$\frac{\mathbb{L}_v}{T_s} \left( \tilde{i}_{v, \text{ref}}(k) - \tilde{i}_{v, \text{ref}}(k - 1) \right)$$ \quad (5-11)

where, $T_s$ is the computer sampling time and $\tilde{i}_{v, \text{ref}}(k)$ and $\tilde{i}_{v, \text{ref}}(k - 1)$ are the values of the current reference in the actual and previous sampling time.

This dead-beat control strategy is equivalent to the following continuous frequency term

$$\mathbb{L}_v j \Omega \Delta \tilde{i}_{v, \text{ref}}$$ \quad (5-12)

It should be noted that the derivate operator $\frac{d}{dt}$ corresponds to the Laplace $j \Omega$.

A coefficient $\lambda$ is introduced, that dictates to what degree the dead-beat control should be used. In a perfect dead-beat control $\lambda$ could be chosen to be equal to one, which means that 100 percent of the transient term is compensated for.

However, from experience (see section 5.6.5) it is known that a lower value than one is usually a wiser choice, in particular when the VSC is connected to a weak network, which requires a reduced contribution from this derivative term.
The contribution related to the transient compensation of the voltage across the reactor can now be written as 

$$\Delta u_{V,B1}^{PLL} = \lambda \left( L_v^* \, j\Omega \right) \Delta i_{ref}$$

(5-13)

If the transient reactance of the phase reactor of the VSC is defined as 

$$x_v^* = \tilde{L}_v^* \, j\Omega$$

(5-14)

then the transient term will be given by 

$$\Delta u_{V,B2}^{PLL} = \lambda \, x_v^* \, \Delta i_{ref}$$

(5-15)

C) Contribution from the current regulator:

The third contribution is a common PI regulator, which generates the voltage

$$\Delta u_{V,C}^{PLL} = K_I \left( \Delta i_{ref} - K_i \Delta i_{PLL} \right)$$

(5-16)

where $K_i$ is the transfer function above mentioned for a measuring filter applied to the current response. The current regulator itself is given by

$$K_I(s) = \frac{k_{ip} \left( 1 + s \cdot \frac{1}{t_i} \right)}{s \cdot t_i}$$

or simply $K_I(j\Omega) = \frac{k_{ip} \left( 1 + j\Omega \cdot t_i \right)}{j\Omega \cdot t_i}$

(5-17)

The total voltage reference for the VSC can now be obtained, by adding the contributions given by Eq. (5-10), (5-12), (5-18) and (5-19):

$$\Delta u_{V,PLL} = \Delta u_{V,0}^{PLL} + \Delta u_{V,1}^{PLL} + \Delta u_{V,2}^{PLL} + \Delta u_{V,C}^{PLL}$$

(5-18)

The steady-state VSC voltage vector (in both coordinate systems) is $\bar{u}_{V,0}$, so the total output voltage in the PLL coordinate system is:

$$\bar{u}_{V,PLL} = \bar{u}_{V,0} + \Delta \bar{u}_{V}^{PLL}$$

(5-19)

When the voltage is transformed from the PLL coordinate system to the undisturbed rotating coordinates the phase angle of the PLL coordinate system must once again be considered. Consequently,

$$\bar{u}_v = \bar{u}_{V,PLL} \, e^{j\Delta \theta_{PLL}(t)} \approx \bar{u}_{V,0} + \Delta \bar{u}_{V}^{PLL}(t) + j\bar{u}_{V,0} \Delta \bar{u}_{PLL}(t)$$

(5-20)

Thus, the total VSC output voltage deviation is given by

$$\Delta \bar{u}_v = \left( K_I + \kappa \, z_v^* + \lambda \, x_v^* \right) \Delta i_{ref} + \left( -K_I + (1 - \kappa) \, z_v^* \right) K_i \Delta i_{PLL} +$$

$$+ K_{ip} \Delta \bar{u}_{V}^{PLL} + j\bar{u}_{V,0} \Delta \bar{u}_{PLL}$$

(5-21)
Eq. (5-21) can now be re-written as
\[
\Delta \bar{u}_v = a_1 \Delta \bar{i}_{V_{ref}} + a_2 \Delta \bar{i}_v + a_3 \Delta \bar{u}_v + a_4 \Delta \bar{\theta}_{PLL}
\]
(5-22)

where,
\[
a_1 = \left( K_I + \kappa \bar{z}_v + \lambda \bar{x}_v \right) \\
a_2 = \left[ - K_I + (1 - \kappa) \bar{z}_{vd} \right] K_{Iv} \\
a_3 = K_{UF} \\
a_4 = -j \left[ - K_I + (1 - \kappa) \bar{z}_{vd} \right] K_{Iv} \bar{i}_v + j K_{UF} \bar{u}_F + j \bar{u}_0
\]

Note: These coefficients and others that may be defined from here on are complex valued transfer functions, and for simplicity no special symbols will be used, as they are inherently self-explained.

Figure 5.4 shows the small-signal linear model used for the inner-loop current controller of the VSC.

5.2.3 Preliminary set of equations calculated from the main circuit

According to the system depicted in Figure 5.2, the analyzed network is up to the common bus ‘C’, considering that the contribution from the network will be included later. Seen from bus ‘C’ (designated as the common bus, where the VSC
will be connected to the grid), and without the contribution from the system ‘S’, from a simple analysis of the circuit it is possible to obtain the following transfer functions defined by Eq. (5-23). Here two independent variables $\Delta \overline{u}_C$ and $\Delta \overline{u}_V$ and three dependent variables $\Delta \overline{v}_i$, $\Delta \overline{c}_i$ and, $\Delta \overline{f}_u$ have been selected.

\[
\begin{align*}
\Delta \overline{v}_i &= b_1 \Delta \overline{u}_C + b_2 \Delta \overline{u}_V \\
\Delta \overline{c}_i &= b_3 \Delta \overline{u}_C + b_4 \Delta \overline{u}_V \\
\Delta \overline{f}_u &= b_5 \Delta \overline{u}_C + b_6 \Delta \overline{u}_V
\end{align*}
\]

where,

\[
\begin{align*}
b_1 &= -\frac{1}{1 + \frac{z_F}{z_T + z_V / z_F}} \\
b_2 &= \frac{1}{z_V + z_T / z_F} \\
b_3 &= -\frac{1}{z_T + z_V / z_F} \\
b_4 &= \frac{1}{1 + \frac{z_f}{z_V + z_T / z_F}} \\
b_5 &= \frac{z_F / z_F}{z_T + z_V / z_F} \\
b_6 &= \frac{z_V / z_F}{z_V + z_T / z_F}
\end{align*}
\]

It should be noted that the impedances used in the above coefficients are impedances in the synchronously rotating coordinate system.

As is described in Appendix L, when using complex-valued space vectors in a fixed coordinate system, the models for passive R, L, and C components are similar to scalar models. In the synchronous coordinate system, the models are given by

\[
\overline{u}^{dq}_{\omega} (j\Omega) = R \overline{\gamma}^{dq}_{\omega} (j\Omega) \\
\overline{u}^{dq}_{\omega} (j\Omega) = j\omega_C L \overline{\gamma}^{dq}_{\omega} + L \frac{d\overline{u}^{dq}_{\omega} (j\Omega)}{dt} \\
\overline{\gamma}^{dq}_{\omega} (j\Omega) = j\omega_C \overline{u}^{dq}_{\omega} + C \frac{d\overline{u}^{dq}_{\omega} (j\Omega)}{dt}
\]

This is due to the relation $j\Omega \rightarrow j\Omega + j\omega_N$ when transforming from the fixed to rotating coordinate system. Therefore, in the synchronous coordinate system the impedances are given by

\[
\begin{align*}
Z_R (j\Omega) &= R \\
Z_L (j\Omega) &= (j\omega_N + j\Omega) L \\
Z_C (j\Omega) &= \frac{1}{(j\omega_N + j\Omega) C}
\end{align*}
\]
5.2.4 Closing the current control loop

Converter path

The variables \( \Delta \tilde{i}_v \) and \( \Delta \tilde{u}_v \) in Eq. (5-22) cease to be independent when the current control loop is closed since, from (5-23), they both depend on \( \Delta \tilde{u}_v \). Thus,

\[
\Delta \tilde{u}_v = a_1 \Delta \tilde{u}_{v,ref} + a_2 (b_1 \Delta \tilde{u}_c + b_2 \Delta \tilde{u}_v ) + a_3 (b_3 \Delta \tilde{u}_c + b_4 \Delta \tilde{u}_v ) + a_4 \Delta \tilde{\theta}_{PLL}
\]

(5-24)

Solving Eq. (5-24) for \( \Delta \tilde{u}_v \), and then, replacing the resulting \( \Delta \tilde{u}_v \) in the set of Eq. (5-23), yields

\[
\begin{align*}
\Delta \tilde{i}_v &= d_1 \Delta \tilde{u}_c + d_2 \Delta \tilde{i}_{v,ref} + d_3 \Delta \tilde{\theta}_{PLL} \\
\Delta \tilde{i}_c &= d_4 \Delta \tilde{u}_c + d_5 \Delta \tilde{i}_{v,ref} + d_6 \Delta \tilde{\theta}_{PLL} \\
\Delta \tilde{u}_e &= d_7 \Delta \tilde{u}_c + d_8 \Delta \tilde{i}_{v,ref} + d_9 \Delta \tilde{\theta}_{PLL}
\end{align*}
\]

(5-25)

where:

\[
\begin{align*}
d_1 &= b_1 + b_2 \frac{a_2 b_1 + a_3 b_3}{1 - a_2 b_2 - a_3 b_6} \\
d_2 &= b_2 \frac{a_1}{1 - a_2 b_2 - a_3 b_6} \\
d_3 &= b_2 \frac{a_4}{1 - a_2 b_2 - a_3 b_6} \\
d_4 &= b_3 + b_1 \frac{a_2 b_1 + a_3 b_3}{1 - a_2 b_2 - a_3 b_6} \\
d_5 &= b_4 \frac{a_1}{1 - a_2 b_2 - a_3 b_6} \\
d_6 &= b_4 \frac{a_4}{1 - a_2 b_2 - a_3 b_6} \\
d_7 &= b_5 + b_6 \frac{a_2 b_1 + a_3 b_3}{1 - a_2 b_2 - a_3 b_6} \\
d_8 &= b_6 \frac{a_1}{1 - a_2 b_2 - a_3 b_6} \\
d_9 &= b_6 \frac{a_4}{1 - a_2 b_2 - a_3 b_6}
\end{align*}
\]

Contribution from the feeding network

Now, considering the feeding network, the following equation applies:

\[
\Delta \tilde{i}_c^f = e_1 \Delta \tilde{u}_c + e_2 \Delta \tilde{u}_e
\]

(5-26)
where,
\[
e_1 = -\frac{1}{\bar{z}_S}
\]
\[
e_2 = \frac{1}{\bar{z}_S}
\]

**Solution at the common bus**

The equation that interconnects the VSC with the feeding network, implies that the sum of injected current at the connection point is zero. Hence,

\[
\Delta I_c + \Delta I_c^s = (d_4 \Delta \bar{u}_c + d_5 \Delta \bar{I}_{vref} + d_6 \Delta \bar{\theta}_{PLL}) + (e_1 \Delta \bar{u}_c + e_2 \Delta \bar{u}_s) = 0 \tag{5-27}
\]

The solution of (5-27) for \( \Delta \bar{u}_c \) is given by

\[
\Delta \bar{u}_c = f_1 \Delta \bar{I}_{vref} + f_2 \Delta \bar{u}_s + f_3 \Delta \bar{\theta}_{PLL} \tag{5-28}
\]

where,
\[
f_1 = -\frac{d_5}{d_4 + e_1}
\]
\[
f_2 = -\frac{e_2}{d_4 + e_1}
\]
\[
f_3 = -\frac{d_6}{d_4 + e_1}
\]

Replacing the independent variable \( \Delta \bar{u}_c \) in (5-25) by the expression in (5-28) yields

\[
\Delta \bar{I}_v = g_1 \Delta \bar{I}_{vref} + g_2 \Delta \bar{u}_s + g_3 \Delta \bar{\theta}_{PLL}
\]
\[
\Delta \bar{I}_c^s = g_4 \Delta \bar{I}_{vref} + g_5 \Delta \bar{u}_s + g_6 \Delta \bar{\theta}_{PLL}
\]
\[
\Delta \bar{u}_F = g_7 \Delta \bar{I}_{vref} + g_8 \Delta \bar{u}_s + g_9 \Delta \bar{\theta}_{PLL} \tag{5-29}
\]

where,
\[
g_1 = d_1 f_1 + d_2
\]
\[
g_2 = d_1 f_2
\]
\[
g_3 = d_1 f_3 + d_3
\]
\[
g_4 = d_1 f_1 + d_5
\]
\[
g_5 = d_4 f_2
\]
\[
g_6 = d_4 f_3 + d_6
\]
\[
g_7 = d_7 f_1 + d_8
\]
\[
g_8 = d_7 f_2
\]
\[
g_9 = d_7 f_3 + d_9
\]
It is also convenient to determine $\Delta \bar{u}_v$. This can be done by means of Eq. (5-23), and the result is
\begin{equation}
\Delta \bar{u}_v = \frac{1}{b_2} \Delta \bar{t}_v - \frac{b_1}{b_2} \Delta \bar{u}_c \tag{5-30}
\end{equation}

Since in Eq. (5-30) $\Delta \bar{t}_v$ and $\Delta \bar{u}_c$ are known from (5-29) and (5-28), respectively, it can be rewritten as
\begin{equation}
\Delta \bar{u}_v = g_{10} \Delta \bar{v}_{ref} + g_{11} \Delta \bar{u}_s + g_{12} \Delta \bar{\theta}_{PLL} \tag{5-31}
\end{equation}

where,
\begin{align*}
g_{10} &= \frac{g_1}{b_2} - \frac{b_1}{b_2} f_1 \\
g_{11} &= \frac{g_2}{b_2} - \frac{b_1}{b_2} f_2 \\
g_{12} &= \frac{g_3}{b_2} - \frac{b_1}{b_2} f_3
\end{align*}

Now, all the complex transfer functions have been obtained. The set of equations defined by (5-29) and (5-31) has $\Delta \bar{t}_v$, $\Delta \bar{u}_s$, and $\Delta \bar{\theta}_{PLL}$ as independent variables, and $\Delta \bar{t}_v$, $\Delta \bar{u}_c$, $\Delta \bar{u}_s$, and $\Delta \bar{u}_v$ are the dependent variables.

At this point all the independent variables are formally considered complex-valued, even the variable $\Delta \bar{\theta}_{PLL}$. As mentioned above, later, only the real part of this variable will be used.

### 5.2.5 Transfer functions between components

Eq. (5-29) and (5-31) provide all the transfer functions $g_1 \ldots g_{12}$ that have been derived so far. It is now possible to resolve all variables into $d$- and $q$- components, considering the following $\alpha - \beta$ to $d - q$ transformation formulas (see Appendix L where these formulas are derived):
\begin{align*}
g_{x^{dq}}(j\omega) &= \frac{g_x(j\omega^+) + (g_x(j\omega^-))^*}{2} \\
g_{x^{dq}}(j\omega) &= \frac{-g_x(j\omega^+) - (g_x(j\omega^-))^*}{2j}
\end{align*}

where,
\begin{align*}
\omega^+ &= \omega_N + \Omega \\
\omega^- &= \omega_N - \Omega
\end{align*}

are the two side bands on the AC side of a disturbance frequency $\Omega$ that appears on the DC side of the converter.
As $\Delta \theta_{PLL}$ is a real-valued quantity, only the d-component is used and its q-component is zero. Then the following matrices of transfer functions are obtained:

$$
\begin{pmatrix}
\Delta i_{Vd} \\
\Delta i_{Vq} \\
\Delta i_{Cd} \\
\Delta i_{Cq} \\
\Delta u_{Fd} \\
\Delta u_{Fq} \\
\Delta u_{Vd} \\
\Delta u_{Vq}
\end{pmatrix} = \begin{pmatrix}
G_{u1} & -G_{u2} \\
G_{u3} & G_{u4} \\
G_{u5} & -G_{u6} \\
G_{u7} & G_{u8} \\
G_{u9} & -G_{u10} \\
G_{u11} & G_{u12}
\end{pmatrix} \begin{pmatrix}
\Delta i_{Vref} \\
\Delta i_{Vref} \\
\Delta u_{Sd} \\
\Delta u_{Sd} \\
\Delta u_{Sd} \\
\Delta u_{Sd}
\end{pmatrix} + G_{V} \Delta \theta_{PLL}
$$

(5-33)

where,

$$
G_{u} = \begin{pmatrix}
g_{1}^{dd} & -g_{1}^{dq} & g_{2}^{dd} & -g_{2}^{dq} \\
g_{3}^{dd} & g_{4}^{dd} & g_{5}^{dd} & -g_{5}^{dq} \\
g_{6}^{dd} & g_{7}^{dd} & g_{8}^{dd} & -g_{8}^{dq} \\
g_{9}^{dd} & g_{10}^{dd} & g_{11}^{dd} & -g_{11}^{dq} \\
g_{12}^{dd} & g_{13}^{dd} & g_{14}^{dd} & -g_{14}^{dq}
\end{pmatrix}
$$

and

$$
G_{V} = \begin{pmatrix}
g_{5}^{dd} \\
g_{6}^{dd} \\
g_{7}^{dd} \\
g_{8}^{dd} \\
g_{9}^{dd} \\
g_{10}^{dd} \\
g_{11}^{dd} \\
g_{12}^{dd}
\end{pmatrix}
$$

### 5.2.6 Closing the PLL loop

The PLL operates on the filter bus voltage. In order to simplify the expressions it is preferable to select the rotating coordinate system in such a way that it is synchronous with the undisturbed filter bus voltage. In that case the steady state voltage vector is real-valued, i.e. its q-component is zero.

The input signal to the PLL is derived in the following manner. The filter bus voltage is transformed to the PLL’s coordinate system, which has the angle $\Delta \theta_{PLL}(t)$. This filter bus voltage is calculated by

$$
\bar{u}_{F}^{PLL}(t) = u_{F}(t) e^{-j\Delta \theta_{PLL}(t)} \approx \bar{u}_{F0} + \Delta \bar{u}_{F}(t) - j \bar{u}_{F0} \Delta \bar{\theta}_{PLL}(t)
$$

(5-34)

Using the coordinate system defined above $u_{F0}$ is a real-valued quantity, and therefore the argument of $\Delta \bar{u}_{F}^{PLL}$ can be written as:

$$
\epsilon_{PLL} = \arctan \frac{\Delta u_{Fq} - u_{F0} \Delta \theta_{PLL}}{u_{F0d} + \Delta u_{Fd}} \approx \frac{\Delta u_{Fq}}{u_{F0}} - \Delta \theta_{PLL}
$$

(5-35)

This quantity now enters into the PLL frequency regulator. This is a PI controller, providing the PLL frequency. The PLL angle argument then is obtained by integration of the PLL frequency. The total transfer function is given by:
\[ \Delta \theta_{PLL} = K_{PLL} \left( \frac{\Delta u_{Fq}}{u_{F0}} - \Delta \theta_{PLL} \right) \] (5-36)

where, \( K_{PLL} \) captures the dynamics of the PLL. Here we assume an ordinary PI regulator, acting on the angle \( \dot{\theta}_{PLL} = \Omega \) extracted from the bus voltage. Then, the dynamics of the PLL will be represented by

\[ K_{PLL} = \frac{k_{PLL} (1 + j\Omega t_{PLL})}{j\Omega t_{PLL}} \frac{1}{j\Omega} \] (5-37)

where \( k_{PLL} \) is the PLL gain and \( t_{PLL} \) is the PLL integral time constant.

Eq. (5-36) can be solved directly yielding

\[ \Delta \theta_{PLL} = \frac{K_{PLL}}{1 + K_{PLL} u_{F0}} \frac{\Delta u_{Fq}}{u_{F0}} = K_{PLL}^{CL} \Delta u_{Fq} \] (5-38)

where,

\[ K_{PLL}^{CL} = \frac{k_{PLL}}{1 + K_{PLL} u_{F0}} \]

Introducing the matrix,

\[ P_{C8} = \begin{pmatrix} 0 & 0 & 0 & 0 & K_{PLL}^{CL} & 0 & 0 \end{pmatrix} \] (5-39)

Eq. (5-38) becomes

\[ \Delta \theta_{PLL} = P_{C8} \begin{pmatrix} \Delta i_{vd} \\ \Delta i_{vq} \\ \Delta i_{cd} \\ \Delta i_{cq} \\ \Delta u_{Fd} \\ \Delta u_{Fq} \\ \Delta u_{vd} \\ \Delta u_{vq} \end{pmatrix} \] (5-40)

Inserting (5-40) into (5-33) yields

\[ \begin{pmatrix} \Delta i_{vd} \\ \Delta i_{vq} \\ \Delta i_{cd} \\ \Delta i_{cq} \\ \Delta u_{Fd} \\ \Delta u_{Fq} \\ \Delta u_{vd} \\ \Delta u_{vq} \end{pmatrix} = G_{V} \begin{pmatrix} \Delta i_{vd}^{\text{ref}} \\ \Delta i_{vq}^{\text{ref}} \\ \Delta u_{Sd} \\ \Delta u_{Sq} \end{pmatrix} + G_{V} P_{C8} \begin{pmatrix} \Delta i_{vd} \\ \Delta i_{vq} \\ \Delta i_{cd} \\ \Delta i_{cq} \\ \Delta u_{Fd} \\ \Delta u_{Fq} \\ \Delta u_{vd} \\ \Delta u_{vq} \end{pmatrix} \] (5-41)
The solution of Eq. (5-41) provides the matrix of all transfer functions:

\[
S = (I - G_0 P_{cs})^{-1} G_u
\]  

(5-42)

where \( I = \text{eye}(8) \) is the identity matrix in MATLAB.

### 5.2.7 Validation by comparing results with time-domain simulation in PSCAD/EMTDC

A key transfer function that can be extracted from the above matrix of transfer functions is the closed current control loop, defined by the relation between converter current, \( \Delta i_v \) and the corresponding reference, \( \Delta i_{v ref} \). In order to validate the calculation method described in the preceding sections this transfer function has been calculated for a specific case. The same transfer function was extracted from a time-domain simulation of the same system and the results were compared. The studied case was a 350 MVA STATCOM connected to a network with a short circuit power of 700 MVA. Detailed system data and corresponding control parameters of the converter are given in Table 5.1, see Section 5.4.

In PSCAD/EMTDC simulations the control variables are being calculated from the latest circuit solution and the obtained control signals are then being applied in the circuit calculation in next time-step. Therefore, a time-delay of one PSCAD time-step should be used in the frequency-domain calculation in order to obtain comparable results. In the comparison the PSCAD time-step was 50 µs.

Bode plots of the individual transfer function elements are shown in Figure 5.5.

It should be noted that the analyzed quantities in the Bode plots are given in the undisturbed rotating \( d-q \) frame, and not in the ‘PLL’ coordinates. This is the reason to why the Bode plots of the transfer functions \( \Delta i_{vd} / \Delta i_{vd ref} \) and \( \Delta i_{vq} / \Delta i_{vq ref} \) are not starting at 1 at zero frequency.

Figure 5.5-(B) shows cross-coupling between the d- and q-components. To understand how this was originated, it is necessary to return to the basic Eq. (5-2a) defining the current control law. If the equation is split into its real and imaginary parts, the following real-valued model is obtained:

\[
\begin{align*}
\Delta u^d_v &= \Delta u^d_f + R_v \Delta i^d_v - \omega_N L_v \Delta i^q_v + L_v \frac{d \Delta i^q_v}{dt} \\
\Delta u^q_v &= \Delta u^q_f + R_v \Delta i^q_v + \omega_N L_v \Delta i^d_v + L_v \frac{d \Delta i^d_v}{dt}
\end{align*}
\]  

(5-2b)

In this set of equations the cross-coupling terms \( \omega_N L_v \Delta i^q_v \) and \( \omega_N L_v \Delta i^d_v \) are found. These are resulting from the transformation to synchronous coordinates \( j \Omega \rightarrow j \Omega + j \omega_N \), which is included in the model.
5.3 Solution for the DC side of the converter and inclusion of the AC and DC side voltage controllers

5.3.1 General

The equations that govern the VSC interaction with the transmission system on the AC side have been derived. Using the fact that the active power on the AC and DC sides of the converter is equal, it is found that

\[ u_{dc}\Delta i_{dc} = \frac{3}{2} \text{Re} (u_{v} \Delta i_{y}) = \frac{3}{2} u_{v_{d}} \Delta i_{d} + \frac{3}{2} u_{v_{q}} \Delta i_{q} \]  (5-43)

A general differentiation of the above expression yields

\[ u_{dc,0} \Delta i_{dc} + \Delta u_{dc} \Delta i_{dc,0} = \frac{3}{2} u_{v_{d}} \Delta i_{d,0} + \frac{3}{2} \Delta u_{v_{d}} i_{d,0} + \frac{3}{2} u_{v_{q}} \Delta i_{q,0} + \frac{3}{2} \Delta u_{v_{q}} i_{q,0} \]  (5-44)

The DC side of the converter is characterized by its impedance \( z_{dc} \). If the VSC is operating as a STATCOM, the only main circuit component that influences \( z_{dc} \) is the DC link capacitor. Accordingly,

\[ \Delta u_{dc} = -z_{dc} \Delta i_{dc} \]  (5-45)

For a STATCOM the steady-state DC current is zero, but this variable is kept for the HVDC case. Thus,
\[
(u_{dq0} - z_{dq} i_{dq0}) \Delta i_{dc} = \frac{3}{2} u_{q0} \Delta i_{q0} + \frac{3}{2} \Delta u_{q0} i_{q0} + \frac{3}{2} u_{vq0} \Delta i_{vq} + \frac{3}{2} \Delta u_{vq} i_{vq0}
\]

or

\[
\begin{align*}
\Delta i_{dc} &= \frac{3}{2} \left( u_{dq0} \Delta i_{dq} + \Delta u_{dq} i_{dq0} + u_{q0} \Delta i_{q} + \Delta u_{q} i_{q0} \right) \\
\Delta u_{dc} &= \frac{3}{2} \left( u_{dq0} \Delta i_{dq} + \Delta u_{dq} i_{dq0} + u_{q0} \Delta i_{q} + \Delta u_{q} i_{q0} \right)
\end{align*}
\]

(5-46)

Or in matrix form

\[
\begin{align*}
\Delta i_{dc} &= \left( \begin{array}{c} \Delta i_{dq} \\
\Delta i_{q} \end{array} \right) \\
\Delta u_{dc} &= \left( \begin{array}{c} \Delta u_{dq} \\
\Delta u_{q} \end{array} \right)
\end{align*}
\]

(5-47)

The new matrices of coefficients are here defined as

\[
\begin{align*}
\left[ r_1 \right] &= \left( \begin{array}{c} r_1 \\
r_2 \end{array} \right) \\
\left[ r_2 \right] &= \left( \begin{array}{c} r_3 \\
r_4 \end{array} \right) \\
\left[ r_3 \right] &= \left( \begin{array}{c} r_1 \\
r_1 \end{array} \right) \\
\left[ r_4 \right] &= \left( \begin{array}{c} r_2 \\
r_2 \end{array} \right)
\end{align*}
\]

where

\[
\begin{align*}
r_1 &= \frac{3}{2} \frac{u_{dq0}}{(u_{dq0} - z_{dq} i_{dq0})} \\
r_2 &= \frac{3}{2} \frac{u_{q0}}{(u_{dq0} - z_{dq} i_{dq0})} \\
r_3 &= \frac{3}{2} \frac{u_{vq0} z_{dq}}{(u_{dq0} - z_{dq} i_{dq0})} \\
r_4 &= \frac{3}{2} \frac{i_{vq0} z_{dq}}{(u_{dq0} - z_{dq} i_{dq0})}
\end{align*}
\]

Another approach to obtain the solution for the DC side of the VSC converter is derived in APPENDIX K, which makes use of complex coupling functions to
5.3 Solution for the DC side of the converter and inclusion of the AC and DC side voltage controllers

describe the converter. Using that approach it is possible to study the interaction between the AC and DC sides of the converter.

5.3.2 Introducing the dynamics of the DC voltage regulator

The DC voltage is entered into the DC voltage regulator which is designed as an ordinary PI regulator acting on \( \Delta i_{d_{\text{ref}}} \). Thus,

\[
\Delta i_{\text{vdr}_{\text{ref}}} = -K^{U_{dc}} \left( \Delta u_{d_{\text{dc}}} - \Delta u_{d_{\text{dc}}^*} \right) \tag{5-48}
\]

where \( K^{U_{dc}} \) represents the dynamics of the PI regulator of the DC voltage controller, given by

\[
K^{U_{dc}} = \frac{K^{U_{dc}} (1 + j \Omega U_{dc})}{j \Omega U_{dc}} \tag{5-49}
\]

Combining (5-48) and (5-47), results in

\[
\Delta i_{\text{vdr}_{\text{ref}}} = -K^{U_{dc}} \left[ \Delta u_{d_{\text{dc}_{\text{ref}}}} - \begin{pmatrix} n_1 & n_2 \end{pmatrix} \begin{pmatrix} \Delta i_{v_d} \\ \Delta i_{v_q} \end{pmatrix} \right] \tag{5-50}
\]

Now, the following new matrices of coefficients can be defined as

\[
\Delta i_{\text{vdr}_{\text{ref}}} = n_1^1 \Delta u_{d_{\text{dc}_{\text{ref}}}} + \begin{pmatrix} n_1^2 & n_2^2 \end{pmatrix} \begin{pmatrix} \Delta i_{v_d} \\ \Delta i_{v_q} \end{pmatrix} + \begin{pmatrix} n_1^3 & n_2^3 \end{pmatrix} \begin{pmatrix} \Delta u_{v_d} \\ \Delta u_{v_q} \end{pmatrix} \tag{5-51}
\]

where

\[
\begin{aligned}
n_1^1 &= -K^{U_{dc}} \\
n_1^2 &= \begin{pmatrix} n_1^2 & n_2^2 \end{pmatrix} = +K^{U_{dc}} \begin{pmatrix} r_1^3 & r_2^3 \\ r_1^4 & r_2^4 \end{pmatrix} \\
n_1^3 &= \begin{pmatrix} n_1^3 & n_2^3 \end{pmatrix} = +K^{U_{dc}} \begin{pmatrix} r_1^4 & r_2^4 \\ r_1^1 & r_2^1 \end{pmatrix}
\end{aligned}
\]

Eq. (5-51) has:

1) A new independent variable, \( \Delta u_{d_{\text{dc}_{\text{ref}}}} \), which replaces \( \Delta i_{v_{d_{\text{ref}}}} \);

2) The variables \( \Delta i_{v_d} \) and \( \Delta i_{v_q} \) which are known;

3) The variables \( \Delta u_{v_d} \) and \( \Delta u_{v_q} \) which are also known.

5.3.3 Introducing the dynamics of the AC voltage regulator

Let assume here an ordinary PI regulator for the AC voltage controller, acting on \( \Delta i_{q_{\text{ref}}} \)

\[
\Delta i_{vq_{\text{ref}}} = -K^{U_{ac}} F \left( \Delta u_{d_{\text{dc}_{\text{ref}}}} - \Delta u_{d_{\text{dc}_{\text{ref}}}}^* \right) \tag{5-52}
\]

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where \( K^{U_{ac F}} \) represents the dynamics of the PI regulator of the AC voltage controller, given by

\[
K^{U_{ac}} = \frac{K^{U_{ac}} (1 + j \Omega f^{U_{ac}})}{j \Omega f^{U_{ac}}} \tag{5-53}
\]

The controlled variable \( \Delta u_{ac F} \) is the amplitude of the filter bus voltage, is obtained from

\[
\Delta u_{ac F} = \Delta u_{fd} \tag{5-54}
\]

This means that the controller will operate directly on the d-component of the measured filter bus voltage.

Combining (5-52) and (5-54) yields

\[
\Delta i_{iq \text{ ref}} = -K^{U_{ac F}} (\Delta u_{ac F \text{ ref}} - \Delta u_{fd}) \tag{5-55}
\]

Or in matrix form,

\[
\Delta i_{iq \text{ ref}} = q_1^1 \Delta u_{ac F \text{ ref}} + q_1^2 \Delta u_{fd} \tag{5-56}
\]

where

\[
q_1^1 = -K^{U_{ac F}} \quad q_1^2 = +K^{U_{ac F}}
\]

Eq. (5-56) has:

1) A new independent variable, \( \Delta u_{ac \text{ ref}} \), which replaces \( \Delta i_{iq \text{ ref}} \);

2) The variable \( \Delta u_{fd} \), which is known.

### 5.3.4 Rearranging the transfer function matrix to include two new dependent variables, \( \Delta u_{dc \text{ ref}} \) and \( \Delta u_{ac \text{ ref}} \)

Eq. (5-41) has the independent variables \( \Delta i_{v \text{ ref}} \) and \( \Delta i_{v \text{ ref}} \) that can now be replaced by the new independent variables \( \Delta u_{dc \text{ ref}} \) and \( \Delta u_{ac \text{ ref}} \), resulting

\[
\begin{pmatrix}
\Delta i_{v \text{ ref}} \\
\Delta i_{v \text{ ref}} \\
\Delta u_{sd} \\
\Delta u_{sq}
\end{pmatrix} = S_1
\begin{pmatrix}
\Delta u_{dc \text{ ref}} \\
\Delta u_{ac \text{ ref}} \\
\Delta u_{sd} \\
\Delta u_{sq}
\end{pmatrix} + S_2
\begin{pmatrix}
\Delta i_{v \text{ ref}} \\
\Delta i_{v \text{ ref}} \\
\Delta u_{sd} \\
\Delta u_{sq}
\end{pmatrix} \tag{5-57}
\]

where the following auxiliary matrices were used:
5.4 Validation of the model

Assume the system parameters and control parameters of the system presented in Table 5.1. This is a 350 MVA STATCOM connected to a 700 MVA network. This system has also been modeled by means of time-domain simulations with EMTDC/PSCAD, which has been used to validate the mathematical model.

As previously mentioned in Section 5.2.7, in the PSCAD/EMTDC simulations the control variables are being calculated from the latest circuit solution and the obtained control signals are then being applied in the circuit calculation in the next time-step. Therefore, a time-delay of one PSCAD time-step has been included in the frequency-domain calculation in order to obtain comparable results. In the comparison the PSCAD time-step was 50 µs.
Chapter 5. Frequency domain model of a voltage source converter with current controller

Bode plots of the transfer function between the DC voltage reference $\Delta u_{dc\text{ ref}}$ and the DC voltage response $\Delta u_{DC}$ obtained from the mathematical model and from the time domain simulation are presented in Figure 5.6-(A).

In the second test case the Bode plot of the transfer function between AC voltage reference $\Delta u_{ac\text{ ref}}$ and the AC voltage response $\Delta u_{F_a}$ is presented in Figure 5.6-(B).

The comparisons illustrated in the figures show very good agreement between the mathematical model and the three-phase representation in EMTDC.

Table 5.1: System details

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>converter rating</td>
<td>350 MVA</td>
</tr>
<tr>
<td>source voltage</td>
<td>422.5 kV rms line-line voltage</td>
</tr>
<tr>
<td>source impedance</td>
<td>$z_S=44.3+j251$ Ω/ph (SCR=2.0)</td>
</tr>
<tr>
<td>converter transformer impedance</td>
<td>$z_T=2.049+j0.13$ Ω/ph (0.004+j0.12) pu</td>
</tr>
<tr>
<td>shunt filter</td>
<td>$z_F=-j3498$ Ω/ph ($y_F=j0.146$ pu)</td>
</tr>
<tr>
<td>converter reactor</td>
<td>$z_V=2.64+j79.2$ Ω/ph (0.005+j0.155 pu)</td>
</tr>
<tr>
<td>current controller gains</td>
<td>$k_h=0.049$ pu/ pu, $t_h=0.2$ sec</td>
</tr>
<tr>
<td>DC Voltage controller gains</td>
<td>$k_{dc}=10.2$ pu/ pu, $t_{dc}=0.025$ sec</td>
</tr>
<tr>
<td>AC Voltage controller gains</td>
<td>$k_{ac}=0.064$ pu/ pu, $t_{ac}=0.001$ sec</td>
</tr>
<tr>
<td>PLL controller gains</td>
<td>$k_{PLL}=20$ (rad/s)/ rad, $t_{PLL}=6$ sec</td>
</tr>
<tr>
<td>Time-delay in $u_F$ measurement equals the PSCAD time-step</td>
<td>50 µs</td>
</tr>
<tr>
<td>coefficient ‘$\kappa$’; current compensation</td>
<td>$\kappa = 0.5$</td>
</tr>
<tr>
<td>coefficient ‘$\lambda$’; dead-beat control strategy</td>
<td>$\lambda = 1$</td>
</tr>
</tbody>
</table>
5.5 Impact of the network resonance on the available bandwidth of the current controller

An important factor limiting the performance of the controls is the resonance frequency due to the AC network impedance and the AC filter. Figure 5.7 shows the transfer function of the relationship between converter current and the bridge AC voltage. The transfer function corresponds to an impedance seen by the converter. The magnitude of the impedance as a function of frequency has been evaluated assuming fixed \( \alpha - \beta \) coordinates. As the analyzed network is quite simple, this transfer function is given by

\[
\frac{\Delta u_L}{\Delta i_L} = z_v + (z_f + z_s)/z_f
\]  

(5-60)

The plot shows that the resonance frequency is about 170 Hz. The maximum value of impedance is measured close to the resonance frequency. This resonance frequency influences the performance not only of the inner current control loop, but also the outer DC voltage and AC voltage controllers.

The 170 Hz resonance frequency can be observed in the Bode plots of the individual transfer functions of the closed inner current control loop (see Figure 5.5-(A) and (B)) which have been evaluated in synchronous \( d-q \) coordinates. In this coordinate system the resonance frequency corresponds to \( \omega_0 \pm \omega_N \), which is approximately 120 and 220 Hz respectively. These two side-bands correspond to...
Chapter 5. Frequency domain model of a voltage source converter with current controller

170 Hz appearing on the DC side of the converter (see the Bode plot of the closed outer loop voltage controllers, Figure 5.6).

This resonance peak and the associated phase shift limit the available bandwidth of the current controller.

5.6 IMPACT OF THE TIME-DELAYS, COEFFICIENTS INCLUDED IN THE CONTROLLERS AND CONTROL PARAMETERS

5.6.1 Impact of the ‘\(T_d = 50 \mu s\)’ time-delay introduced to emulate the PSCAD 50\(\mu s\) time-step

As mentioned above, in PSCAD/EMTDC simulations the control variables are being calculated from the latest circuit solution and the obtained control signals are then applied in the circuit calculation in the next time-step. Therefore, a time-delay of one PSCAD time-step has been used in the frequency-domain calculation in order to obtain comparable results. In the comparison the PSCAD time-step was 50\(\mu s\) (see solid-line case shown in Figure 5.8).

In the actual control system this time-delay should not be included in the controls. The case without this time delay is shown as the dashed line in Figure 5.8. The results show that the time-delay introduces damping around the resonance frequency.

The results also show that the ‘\(T_d = 50 \mu s\)’ time-delay reduces the bandwidth of the current controller (see Figure 5.8-A) as well as affects the cross coupling between the d- and q- axes in the current control loop (see Figure 5.9-B).
One way to increase the damping around resonance frequency is to include a low pass filter in the feed-forward filter bus voltage. The impact of having this filter is shown by the dotted line in Figure 5.8. As the damping around the resonance frequency is increased by the filter, the gains of the outer-loop controllers could have been increased, for both the DC and AC voltage controllers.

Table 5.2: Set up to verify the impact of the $T_d = 50 \mu s$ time-delay introduced to emulated the PSCAD 50µs time-step

<table>
<thead>
<tr>
<th>parameter</th>
<th>Solid line</th>
<th>Dashed Line</th>
<th>Dotted line</th>
</tr>
</thead>
<tbody>
<tr>
<td>current controller gains</td>
<td>$k^b = 0.049 , pu , pu$</td>
<td>$k^b = 0.049 , pu , pu$</td>
<td>$k^b = 0.049 , pu , pu$</td>
</tr>
<tr>
<td></td>
<td>$t^b = 0.2 , sec$</td>
<td>$t^b = 0.2 , sec$</td>
<td>$t^b = 0.2 , sec$</td>
</tr>
<tr>
<td>DC Voltage controller gains</td>
<td>$k^{ca} = 10.2 , pu , pu$</td>
<td>$k^{ca} = 10.2 , pu , pu$</td>
<td>$k^{ca} = 10.2 , pu , pu$</td>
</tr>
<tr>
<td></td>
<td>$t^{ca} = 0.025 , sec$</td>
<td>$t^{ca} = 0.025 , sec$</td>
<td>$t^{ca} = 0.025 , sec$</td>
</tr>
<tr>
<td>AC Voltage controller gain</td>
<td>$k^{ac} = 0.064 , pu , pu$</td>
<td>$k^{ac} = 0.064 , pu , pu$</td>
<td>$k^{ac} = 0.064 , pu , pu$</td>
</tr>
<tr>
<td></td>
<td>$t^{ac} = 0.001 , sec$</td>
<td>$t^{ac} = 0.001 , sec$</td>
<td>$t^{ac} = 0.001 , sec$</td>
</tr>
<tr>
<td>PLL controller gains</td>
<td>$k^{PLL} = 20 , (rad/s)/rad$</td>
<td>$k^{PLL} = 20 , (rad/s)/rad$</td>
<td>$k^{PLL} = 20 , (rad/s)/rad$</td>
</tr>
<tr>
<td></td>
<td>$t^{PLL} = 6 , sec$</td>
<td>$t^{PLL} = 6 , sec$</td>
<td>$t^{PLL} = 6 , sec$</td>
</tr>
<tr>
<td>$T_d$ time-delay due to PSCAD time-step</td>
<td>$T_d = 50 \mu s$</td>
<td>$T_d = 0$</td>
<td>$T_d = 0$</td>
</tr>
<tr>
<td>$T_{del}$ time-delay due to control delay and modulation</td>
<td>$T_{del} = 0$</td>
<td>$T_{del} = 0$</td>
<td>$T_{del} = 0$</td>
</tr>
<tr>
<td>Low Pass filter included in the feed-forward filter bus voltage</td>
<td>LP: 0 sec time constant</td>
<td>LP: 0 sec time constant</td>
<td>0.00025 sec time constant</td>
</tr>
<tr>
<td>coefficient ‘$\kappa$’: current compensation</td>
<td>$\kappa = 0.5$</td>
<td>$\kappa = 0.5$</td>
<td>$\kappa = 0.5$</td>
</tr>
<tr>
<td>coefficient ‘$\lambda$’: dead-beat control strategy</td>
<td>$\lambda = 1$</td>
<td>$\lambda = 1$</td>
<td>$\lambda = 1$</td>
</tr>
</tbody>
</table>
Chapter 5. Frequency domain model of a voltage source converter with current controller

Figure 5.8: Impact of time-delay 50 μs introduced in the model to emulate the PSCAD time-step of 50 μs. (A) Inner current control $\Delta i_{vd} / \Delta i_{vd\text{ref}}$ and $\Delta i_{vq} / \Delta i_{vq\text{ref}}$ and (B) $\Delta i_{vd} / \Delta i_{vq\text{ref}}$ and $\Delta i_{vq} / \Delta i_{vd\text{ref}}$. (C) the DC voltage controller transfer function $\Delta u_{dc} / \Delta u_{dc\text{ref}}$ and (D) the AC voltage controller transfer function $\Delta u_{vd} / \Delta u_{ac\text{ref}}$

- Solid line: case having ‘50 μs’ time-delay included
- Dashed line: case without the ‘50 μs’ time-delay
- Dotted line: case without the ‘50 μs’ time-delay and a 0.25 ms LP filter is introduced in the feed-forward filter bus voltage signal

5.6.2 Impact of the coefficient ‘$\kappa$’ that determines the portion of the compensation that is based on the current reference and the portion that is based on the measured current

The compensation of the voltage across the phase reactor has been made based on either the measured converter current $(1 - \kappa)$ or on the current reference $(\kappa)$ (here we assume $0 < \kappa < 1$). This linear combination between the current reference and the measured current is studied in this section.
Three different compensation levels are studied:

- Case solid line: $\kappa = 0.2$
- Case dotted line: $\kappa = 0.5$
- Case dashed line: $\kappa = 0.8$

The control parameters are given in Table 5.3, and the results are shown in Figure 5.9.

The results show that the bandwidth of the inner-loop current control increases with the increase of the coefficient ‘$\kappa$’. However, the Bode plot of the outer AC voltage controller indicates that the case having a higher coefficient ‘$\kappa$’ is closer to instability at the resonance frequency. This is because the damping decreases with the increase of the coefficient ‘$\kappa$’.

A reasonable compromise between damping level and bandwidth is achieved by having the coefficient $\kappa = 0.5$.

### Table 5.3: Setup to verify the impact of the coefficient ‘$\kappa$’

<table>
<thead>
<tr>
<th></th>
<th>Solid line</th>
<th>Dashed Line</th>
<th>Dotted line</th>
</tr>
</thead>
<tbody>
<tr>
<td>current controller gains</td>
<td>$k^c = 0.049 \ pu/\ pu$</td>
<td>$k^c = 0.049 \ pu/\ pu$</td>
<td>$k^c = 0.049 \ pu/\ pu$</td>
</tr>
<tr>
<td></td>
<td>$t^c = 0.2\ sec$</td>
<td>$t^c = 0.2\ sec$</td>
<td>$t^c = 0.2\ sec$</td>
</tr>
<tr>
<td>DC Voltage controller gains</td>
<td>$k^{vdc} = 10.2 \ pu/\ pu$</td>
<td>$k^{vdc} = 10.2 \ pu/\ pu$</td>
<td>$k^{vdc} = 10.2 \ pu/\ pu$</td>
</tr>
<tr>
<td></td>
<td>$t^{vdc} = 0.025\ sec$</td>
<td>$t^{vdc} = 0.025\ sec$</td>
<td>$t^{vdc} = 0.025\ sec$</td>
</tr>
<tr>
<td>AC Voltage controller gain</td>
<td>$k^{vac} = 0.064 \ pu/\ pu$</td>
<td>$k^{vac} = 0.064 \ pu/\ pu$</td>
<td>$k^{vac} = 0.064 \ pu/\ pu$</td>
</tr>
<tr>
<td></td>
<td>$t^{vac} = 0.001\ sec$</td>
<td>$t^{vac} = 0.001\ sec$</td>
<td>$t^{vac} = 0.001\ sec$</td>
</tr>
<tr>
<td>PLL controller gains</td>
<td>$k^{PLL} = 20(rad/s)/rad$</td>
<td>$k^{PLL} = 20(rad/s)/rad$</td>
<td>$k^{PLL} = 20(rad/s)/rad$</td>
</tr>
<tr>
<td></td>
<td>$t^{PLL} = 6\ sec$</td>
<td>$t^{PLL} = 6\ sec$</td>
<td>$t^{PLL} = 6\ sec$</td>
</tr>
<tr>
<td>$T_d$: time-delay due to PSCAD time-step</td>
<td>$T_d = 50\ \mu s$</td>
<td>$T_d = 50\ \mu s$</td>
<td>$T_d = 50\ \mu s$</td>
</tr>
<tr>
<td>$T_{del}$: time-delay due to control delay and modulation</td>
<td>$T_{del} = 0$</td>
<td>$T_{del} = 0$</td>
<td>$T_{del} = 0$</td>
</tr>
<tr>
<td>Low Pass filter included in the feed-forward filter bus voltage</td>
<td>LP: 0 time constant</td>
<td>LP: 0 time constant</td>
<td>LP: 0 time constant</td>
</tr>
<tr>
<td>coefficient ‘$\kappa$’: current compensation</td>
<td>$\kappa = 0.2$</td>
<td>$\kappa = 0.5$</td>
<td>$\kappa = 0.8$</td>
</tr>
<tr>
<td>coefficient ‘$\lambda$’: dead-beat control strategy</td>
<td>$\lambda = 1$</td>
<td>$\lambda = 1$</td>
<td>$\lambda = 1$</td>
</tr>
</tbody>
</table>
Figure 5.9: Impact of the coefficient ‘κ’ that determines the portion of the compensation that is based on the current reference and the portion that will be based on the measured current. (A) Inner current control \( \frac{\Delta i_d}{\Delta i_{d\text{ref}}} \) and \( \frac{\Delta i_q}{\Delta i_{q\text{ref}}} \) and (B) \( \frac{\Delta i_d}{\Delta i_{d\text{ref}}} \) and \( \frac{\Delta i_q}{\Delta i_{q\text{ref}}} \). (C) the DC voltage controller transfer function \( \frac{\Delta u_{dc}}{\Delta u_{dc\text{ref}}} \) and (D) the AC voltage controller transfer function \( \frac{\Delta u_{ac}}{\Delta u_{ac\text{ref}}} \).

5.6.3 Impact of the coefficient ‘\( \lambda \)’ that dictates the degree of dead-beat control

In general the performance obtained in a computer-controlled system is superior to a continuous-time system. The periodic nature of the control actions can be used to obtain control strategies with superior performance like a dead-beat control. With a dead-beat control the necessary voltage to be impressed by the converter bridge to get the desired current is calculated which is used to reach faster the desired status.

The coefficient ‘\( \lambda \)’ in the mathematical model dictates the degree of dead-beat control.

A perfect dead-beat control assumes that the derivative of the current is obtained in two sampling periods. In this case \( \lambda = 1 \). This means that the contribution of the
5.6 Impact of the time-delays, coefficients included in the controllers and control parameters

Inductive counter-voltage \( \tilde{I}_v \frac{d\Delta I_{PLL}}{dt} \), corresponds to \( \Delta u_{PLL, b2} = 1 \cdot (\tilde{I}_v \cdot j\Omega) \Delta I_{Vref} \) included in the control law.

Assuming now the other extreme, that the control law does not include the dead-beat control; then, \( \lambda = 0 \). In this case the inductive counter-voltage \( \tilde{I}_v \frac{d\Delta I_{PLL}}{dt} \) is not included in the calculation, and the contribution of the term \( \Delta u_{PLL, b2} = 0 \cdot (\tilde{I}_v \cdot j\Omega) \Delta I_{Vref} \) is null.

By setting \( 0 < \lambda < 1 \) intermediate conditions are considered, where different implantation of the dead-beat control may be tested.

To verify the influence of the inductive counter-voltage \( \tilde{I}_v \frac{d\Delta I_{PLL}}{dt} \) term three different cases are compared in Figure 5.10:

- **Solid line** - An implementation having a poor dead-control \( \lambda = 0.6 \);
- **Dashed line** - An intermediate dead-beat control \( \lambda = 0.8 \) is assumed;
- **Dotted line** – An ideal dead-beat control, \( \lambda = 1.0 \) is considered

By looking at the Bode plot of the inner current control it is possible to observe that the bandwidth of the current controller increases with the increase of \( \lambda \), which means that a control strategy that is closer to a perfect dead-beat control will respond faster.

However, a faster controller yields a system closer to instability at the resonance frequency as seen in the Bode plots of the outer loop controls; both AC and DC voltage controllers are closer to instability at higher values of \( \lambda \).

The conclusion is that a too fast current control, obtained by a perfect dead-beat control, would require lower control gains of the PI regulator to avoid instability of the outer loop controllers.
### Table 5.4: Setup to verify the impact of the coefficient λ

<table>
<thead>
<tr>
<th></th>
<th>Solid line</th>
<th>Dashed Line</th>
<th>Dotted line</th>
</tr>
</thead>
<tbody>
<tr>
<td>current controller gains</td>
<td>( k^c = 0.049 ; \text{pu} / \text{pu} ) ( t^c = 0.2 ; \text{sec} )</td>
<td>( k^c = 0.049 ; \text{pu} / \text{pu} ) ( t^c = 0.2 ; \text{sec} )</td>
<td>( k^c = 0.049 ; \text{pu} / \text{pu} ) ( t^c = 0.2 ; \text{sec} )</td>
</tr>
<tr>
<td>DC Voltage controller gains</td>
<td>( k^{\text{DC}} = 10.2 ; \text{pu} / \text{pu} ) ( t^{\text{DC}} = 0.025 ; \text{sec} )</td>
<td>( k^{\text{DC}} = 10.2 ; \text{pu} / \text{pu} ) ( t^{\text{DC}} = 0.025 ; \text{sec} )</td>
<td>( k^{\text{DC}} = 10.2 ; \text{pu} / \text{pu} ) ( t^{\text{DC}} = 0.025 ; \text{sec} )</td>
</tr>
<tr>
<td>AC Voltage controller gain</td>
<td>( k^{\text{AC}} = 0.064 ; \text{pu} / \text{pu} ) ( t^{\text{AC}} = 0.001 ; \text{sec} )</td>
<td>( k^{\text{AC}} = 0.064 ; \text{pu} / \text{pu} ) ( t^{\text{AC}} = 0.001 ; \text{sec} )</td>
<td>( k^{\text{AC}} = 0.064 ; \text{pu} / \text{pu} ) ( t^{\text{AC}} = 0.001 ; \text{sec} )</td>
</tr>
<tr>
<td>PLL controller gains</td>
<td>( k^{\text{PLL}} = 20 ; (\text{rad/s}) / \text{rad} ) ( t^{\text{PLL}} = 6 ; \text{sec} )</td>
<td>( k^{\text{PLL}} = 20 ; (\text{rad/s}) / \text{rad} ) ( t^{\text{PLL}} = 6 ; \text{sec} )</td>
<td>( k^{\text{PLL}} = 20 ; (\text{rad/s}) / \text{rad} ) ( t^{\text{PLL}} = 6 ; \text{sec} )</td>
</tr>
<tr>
<td>( T_s ); time-delay due to PSCAD time-step</td>
<td>( T_s = 50 ; \mu s )</td>
<td>( T_s = 50 ; \mu s )</td>
<td>( T_s = 50 ; \mu s )</td>
</tr>
<tr>
<td>( T_{\text{del}} ); time-delay due to control delay and modulation</td>
<td>( T_{\text{del}} = 0 )</td>
<td>( T_{\text{del}} = 0 )</td>
<td>( T_{\text{del}} = 0 )</td>
</tr>
<tr>
<td>Low Pass filter included in the feed-forward filter bus voltage</td>
<td>LP; 0 time constant</td>
<td>LP; 0 time constant</td>
<td>LP; 0 time constant</td>
</tr>
<tr>
<td>coefficient ( \kappa ): current compensation</td>
<td>( \kappa = 0.5 )</td>
<td>( \kappa = 0.5 )</td>
<td>( \kappa = 0.5 )</td>
</tr>
<tr>
<td>coefficient ( \lambda ): dead-beat control strategy</td>
<td>( \lambda = 0.6 )</td>
<td>( \lambda = 0.8 )</td>
<td>( \lambda = 1 )</td>
</tr>
</tbody>
</table>
5.6 Impact of the time-delays, coefficients included in the controllers and control parameters

The sampling time of the time-discrete control system and the PWM modulation can be approximated as a time-delay. In addition, time-delays will also occur because the switching valves do not switch on and off instantaneously, and for current measurement, synchronous sampling often requires pre-sampling anti-aliasing filters. Such filters will also affect signals and can also be approximated as a time-delays. Therefore, when implementing the controllers, it is important to study the impact of these time-delays.

It is possible to include the effect of the carrier based PWM modulation used for the realization of the converter bridge AC voltage. In the model this will be made by

5.6.4 Impact of the time-delay $T_{del}$ caused by the calculation time and limited switching frequency

Figure 5.10: Impact of the coefficient $\lambda$ that dictates the degree of dead-beat control: (A) Inner current control $\Delta i_d / \Delta i_d \text{ref}$ and $\Delta i_q / \Delta i_q \text{ref}$ and (B) $\Delta i_d / \Delta i_q \text{ref}$ and $\Delta i_q / \Delta i_d \text{ref}$; (C) the DC voltage controller transfer function $\Delta u_{dc} / \Delta u_{dc \text{ref}}$ and (D) the AC voltage controller transfer function $\Delta u_{ac} / \Delta u_{ac \text{ref}}$
introducing a delay in the control signal. This is necessary as it is known that the
transfer functions are dependent on the sample period used in the control.

The sample period is inversely proportional to the pulse number, which for a carrier
based PWM is the ratio between the frequency of the triangular carrier wave and the
fundamental frequency. It is given by

\[ T_s = \frac{T}{2 \cdot p} \]  

(5-61)

An approximation to consider the influence of the sampling period on the transfer
functions is to introduce a control delay equal to one sample period in the reference
voltage of the converter bridge to be realized by the PWM (which means that the
calculation time required by the control computer is taken as one whole sampling
period). The realization of the bridge AC voltage with PWM can also be
approximated by an extra delay of \( \frac{1}{2} T_s \) (which means that the delay related to the
PWM switching is approximated by an average delay of half a switching period). For
a carrier based PWM with the pulse number \( p = 33 \) this corresponds to a delay of
\[ \frac{3}{2} T_s = 455 \mu s \]  

(see in Figure 5.11 the referred time instants).

Figure 5.11: Time instants of the samplings and corresponding time for the calculation and
modulation

This approximation has been included in the model and its influence is shown in
Figure 5.12. It can be noted that introducing the delay significantly degrades the
performance of the controls.

One possible way to mitigate the effect of the sampling period and modulation
delay is to advance the phase reference voltage value of the converter bridge to be
realized by the PWM with a constant value. However, this mitigation measure was
not investigated in this study.
### Table 5.5: Set up to verify the impact of the Time-delay ‘\( T_{del} \)

<table>
<thead>
<tr>
<th></th>
<th>Solid line</th>
<th>Dashed Line</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>current controller gains</strong></td>
<td>( k^i = 0.049 \text{ pu/pu} )</td>
<td>( k^i = 0.049 \text{ pu/pu} )</td>
</tr>
<tr>
<td></td>
<td>( t^i = 0.2 \text{ sec} )</td>
<td>( t^i = 0.2 \text{ sec} )</td>
</tr>
<tr>
<td><strong>DC Voltage controller gains</strong></td>
<td>( k^{Udc} = 10.2 \text{ pu/pu} )</td>
<td>( k^{Udc} = 10.2 \text{ pu/pu} )</td>
</tr>
<tr>
<td></td>
<td>( t^{Udc} = 0.025 \text{ sec} )</td>
<td>( t^{Udc} = 0.025 \text{ sec} )</td>
</tr>
<tr>
<td><strong>AC Voltage controller gain</strong></td>
<td>( k^{Uac} = 0.064 \text{ pu/pu} )</td>
<td>( k^{Uac} = 0.064 \text{ pu/pu} )</td>
</tr>
<tr>
<td></td>
<td>( t^{Uac} = 0.001 \text{ sec} )</td>
<td>( t^{Uac} = 0.001 \text{ sec} )</td>
</tr>
<tr>
<td><strong>PLL controller gains</strong></td>
<td>( k^{PLL} = 20 \text{ (rad/s)/rad} )</td>
<td>( k^{PLL} = 20 \text{ (rad/s)/rad} )</td>
</tr>
<tr>
<td></td>
<td>( t^{PLL} = 6 \text{ sec} )</td>
<td>( t^{PLL} = 6 \text{ sec} )</td>
</tr>
<tr>
<td><strong>( T_d )</strong> time-delay due to ( T_d = 50 \mu\text{s} ) ( \text{PSCAD time-step} )</td>
<td>( T_d = 50 \mu\text{s} )</td>
<td>( T_d = 50 \mu\text{s} )</td>
</tr>
<tr>
<td><strong>( T_{del} )</strong> time-delay due to control delay and modulation</td>
<td>( \text{LP: 0 time constant} )</td>
<td>( \text{LP: 0 time constant} )</td>
</tr>
<tr>
<td><strong>Low Pass filter included in the feed-forward filter bus voltage</strong></td>
<td>( \kappa = 0.5 )</td>
<td>( \kappa = 0.5 )</td>
</tr>
<tr>
<td><strong>coefficient ‘( \kappa )’: current compensation</strong></td>
<td>( \lambda = 1 )</td>
<td>( \lambda = 1 )</td>
</tr>
</tbody>
</table>
Figure 5.12: Impact of the Time-delay ‘$T_{del}$’ caused by the calculation time and limited switching frequency: (A) Inner current control $\Delta i_d / \Delta i_{d,ref}$ and $\Delta i_q / \Delta i_{q,ref}$ and (B) $\Delta i_d / \Delta i_{d,ref}$ and $\Delta i_q / \Delta i_{q,ref}$ (C) the DC voltage controller transfer function $\Delta u_{dc} / \Delta u_{dc,ref}$ and (D) the AC voltage controller transfer function $\Delta u_{vd} / \Delta u_{ac,ref}$
- Solid line $T_{del} = 0$,
- Dotted line $T_{del} = 455\mu s$

5.6.5 Performance of the VSC having the actual time-delays and possible mitigation measures to enhance its performance

In this case the $455\mu s$ time-delay is included in the controls to simulate the computer sampling time and PWM modulation.

The control parameters, the gains of the inner current control, of the outer AC and DC voltage control were optimized in combination with different values of the coefficients ‘$\kappa$’ and ‘$\lambda$’ that have been introduced in the model. The enhanced performance obtained from this optimization is illustrated in Figure 5.13, which uses the control adjustments provided in Table 5.6.
5.6 Impact of the time-delays, coefficients included in the controllers and control parameters

It should be noted that the bandwidths of the controllers do not exceed the resonance frequency of the network. This resonance is the main limiting factor that restricts the performance. Another important limiting factor is the time-delay in the computer sampling time and from the PWM.

Table 5.6: Setup including all time-delays and mitigation measures to enhance the performance

<table>
<thead>
<tr>
<th></th>
<th>Solid line</th>
</tr>
</thead>
<tbody>
<tr>
<td>current controller gains</td>
<td>$k^c = 0.1pu / pu$</td>
</tr>
<tr>
<td></td>
<td>$t^c = 0.02$ sec</td>
</tr>
<tr>
<td>DC Voltage controller gains</td>
<td>$k^{Vdc} = 5.0pu / pu$</td>
</tr>
<tr>
<td></td>
<td>$t^{Vdc} = 1.0$ sec</td>
</tr>
<tr>
<td>AC Voltage controller gains</td>
<td>$k^{Vac} = 0.25pu / pu$</td>
</tr>
<tr>
<td></td>
<td>$t^{Vac} = 0.001$ sec</td>
</tr>
<tr>
<td>PLL controller gains</td>
<td>$k^{PLL} = 200(rad / s) / rad$</td>
</tr>
<tr>
<td></td>
<td>$t^{PLL} = 0.6$ sec</td>
</tr>
<tr>
<td>$T_d$ time-delay due to PSCAD time-step</td>
<td>$T_d = 0$</td>
</tr>
<tr>
<td>$T_{del}$ time-delay due to control delay and modulation</td>
<td>$T_{del} = 455\mu s$</td>
</tr>
<tr>
<td>Low Pass filter included in the feed-forward filter bus voltage</td>
<td>LP: 0 sec time constant</td>
</tr>
<tr>
<td>coefficient ‘$\kappa$’: current compensation</td>
<td>$\kappa = 1.0$</td>
</tr>
<tr>
<td>coefficient ‘$\lambda$’: dead-beat control strategy</td>
<td>$\lambda = 0.5$</td>
</tr>
</tbody>
</table>
Figure 5.13: Set up including time-delays due to computer sampling time and modulation and mitigation measures to enhance the performance. (A) Inner current control $\Delta i_{\text{d}} / \Delta i_{\text{d ref}}$ and $\Delta i_{\text{q}} / \Delta i_{\text{q ref}}$ and (B) $\Delta i_{\text{d}} / \Delta i_{\text{q ref}}$ and $\Delta i_{\text{q}} / \Delta i_{\text{d ref}}$; (C) the DC voltage controller transfer function $\Delta u_{\text{dc}} / \Delta u_{\text{dc ref}}$ and (D) the AC voltage controller transfer function $\Delta u_{\text{ac}} / \Delta u_{\text{ac ref}}$. 
CHAPTER 6
INTEGRATION OF THE VSC-STATCOM INTO HVDC-LCC FREQUENCY DOMAIN MODEL

The small-signal models of the individual HVDC-LCC and VSC-STATCOM systems have been described in the previous chapters. In this chapter those models will be integrated to provide the total transfer function of the complete system.

The models for each one of the systems have been developed in such a way that it is represented by a subsystem that is connected to the common bus. This bus also is connected to the AC power system.

For each subsystem, transfer functions have been derived, which express the current changes from each subsystem to the common bus as a function of the voltage changes on that same bus. Thus, when the VSC-STATCOM is operating with its controllers having fixed reference values, it will appear towards the HVDC-LCC as linear impedance connected to the common bus. This principle will be used in the following as a way to derive the impact of the VSC-STATCOM on the control system of the HVDC-LCC.

6.1 OUTLINE OF THE SYSTEM

The circuit according to Figure 6.1 outlines the studied network configuration. It consists of an HVDC-LCC transmission, having a VSC-STATCOM connected at the common bus (Bus ‘C’) of the HVDC transmission. At this common bus an AC voltage source is connected through an impedance.

The converters are both connected to the common bus via transformer and filter arrangements.

The feeding AC voltage is assumed to be a stiff, symmetrical and sinusoidal voltage. Disturbances in the network voltage can be represented in the $d-q$ frame as indicated in the figure. The reason for this is that a disturbance with a certain
frequency $\Omega$, which is applied to either of the source voltage components, generates disturbances with the same frequency $\Omega$ on the DC-sides of both HVDC-LCC converters and of the VSC-STATCOM converter.

Figure 6.1: Structure of the main circuit configuration

6.2 MODEL OF THE HVDC-LCC TRANSMISSION LINK

6.2.1 The derivation that has been described in chapter 4

The HVDC-LCC model described in Chapter 4 is used below, where the transfer functions governing the DC current control in the HVDC-LCC rectifier is studied.

The model takes the variation of the overlap angle into account. This is important mainly when resonances are excited in the AC grid.

The HVDC-LCC model comprises a number of different transfer functions (see Figure 4.8, Chapter 4):

The transfer function $G_3$ (see Figure 4.3, Chapter 4) describes how a small variation in direct current $\Delta i_d$ results in a small deviation in voltage across the converter under the assumption of constant firing angle ($\Delta \alpha = 0$).

The transfer function $G_2$ (see Figure 4.4, Chapter 4) describes how a small deviation in the firing angle $\Delta \alpha$ results in a small deviation in voltage across the converter bridge, under the assumption of constant DC current ($\Delta i_d = 0$).

Both $G_3$ and $G_2$ depend on the commutation overlap angle and of the AC-side impedance seen by the converter.
The PLL has also been considered in the model. The transfer functions $G_{41}$ and $G_{42}$ extract the phase angle variation of the converter bus voltage due to variations in DC current and control angle $\alpha$, respectively, which is the input to the PLL control.

A schematic diagram of a two terminal system is presented Figure 4.9, Chapter 4. In that model the following control system has been used:

The rectifier controls the DC current. The firing of the valves is determined by the current control loop via the current controller represented by the transfer function $G_1$.

The inverter controls the DC voltage. The firing of the valves is determined by the voltage control loop represented by the transfer function $G'_s$.

Both $G_1$ and $G'_s$ are ordinary PI controllers with appropriate gains and time constants.

### 6.2.2 The HVDC-LCC model having the AC side represented by d- and q- component impedances

In the derivation of the transfer functions in Chapter 4 it was assumed that the impedances on the AC side were symmetrical. This means that the complex voltage vector is proportional to the complex current vector, $\Delta \tilde{u} = Z \cdot \Delta \tilde{I}$, with a complex constant $Z$.

Typically transfer functions, like e.g. $H_1$, (calculated in detail in Appendix A), have the form:

$$H_1(j\Omega) = -\frac{1}{3} \left( \epsilon^{U*}Z^*_S \epsilon^I + \epsilon^U Z^*_S \epsilon^{I*} \right)$$

$$Z^*_S = Z'_L(j\omega^*) + Z_c(j\omega^*)$$

$$Z^*_S = Z'_L(j\omega^-) + Z_c(j\omega^-)$$

where the following notation has been used

$$\omega^* = \omega_N + \Omega$$

$$\omega^- = \omega_N - \Omega$$

In order to integrate the VSC-STATCOM into the description it is necessary to generalize this expression to the non-symmetrical conditions, in which case the HVDC-LCC voltage and current d- and q- components are related through the full matrix relation

$$\begin{bmatrix} \Delta u^d_d \\ \Delta u^q_d \\ \Delta u^d_q \\ \Delta u^q_q \end{bmatrix} = \begin{bmatrix} z^{'dd}_{Net} & z^{'dq}_{Net} \\ z^{'qd}_{Net} & z^{'qq}_{Net} \end{bmatrix} \begin{bmatrix} \Delta I^d_d \\ \Delta I^q_d \\ \Delta I^d_q \\ \Delta I^q_q \end{bmatrix}$$

As an example, to obtain the $d - q$ component impedances of a symmetrical AC system the following formulas should be applied (see Appendix L)
Chapter 6. Integration of the VSC-STATCOM into HVDC-LCC frequency domain model

\[ z_s^{sd}(j\Omega) = z_s^{dq}(j\Omega) = \frac{Z_s(j\omega^+) + (Z_s(j\omega^-))}{2} \]
\[ z_s^{dq}(j\Omega) = -z_s^{sd}(j\Omega) = -\frac{Z_s(j\omega^+) - (Z_s(j\omega^-))}{2j} \]  (6-4)

For the above referred transfer function \( H_1 \), the details about how this transfer function can be calculated is described in Appendix D, and here only the result is given as

\[ H_1 = \frac{\Delta u_d}{\Delta i_d} = -\frac{2}{3} \left[ \ell_d U_{s}^{dd}(j\Omega)i_d^l + \ell_q U_{s}^{dq}(j\Omega)i_q^l + \ell_q U_{s}^{qd}(j\Omega)i_d^l + \ell_d U_{s}^{qq}(j\Omega)i_q^l \right] \]  (6-5)

It should be noted that in the formula the real and imaginary parts of the coefficients of the conversion functions are obtained from

\[
\begin{align*}
\ell^l &= \ell_d^l + j\ell_q^l \\
\ell^u &= \ell_d^u + j\ell_q^u
\end{align*}
\]  (6-6)

### 6.3 MODEL OF THE VSC-STATCOM CONVERTER

The VSC-STATCOM model described in Chapter 5 is used below for the transfer functions governing the AC-side voltage impressed by the converter.

The model has been developed in the \( d-q \) frame, which is a rotating coordinate system, running synchronously with the undisturbed space-vector quantities representing the connected power system. However, the control relies upon alignment of the rotating \( d-q \) reference in relation to the measured space vector \( \vec{u}_r \), which is the measured voltage at the filter bus. This is performed by the PLL control.

The control system of the VSC includes the traditional inner loop AC current control and outer loop controllers that control the voltage of the DC side of the converter and the AC filter bus voltage.

The inner loop current control of VSC converter is based on controlling the current \( i_r \) through a phase reactor represented by the resistance \( R_r \) and the inductance \( L_r \). The control of the converter current is performed by manipulating the impressed voltage \( \vec{u}_r \) generated by the converter bridge. As the control makes use of the estimated resistance and inductance values of the phase reactor in the model (see Eq. (5-10), in Chapter 5), an additional term is added in the control law which is obtained from a PI regulator based on the feedback of the measured AC current.

The DC voltage regulator and the AC voltage regulator act on the \( d \)- and \( q \)-components of the current reference of the inner current controller, respectively.
A general overview of the controllers is presented in Figure 5.1, in Chapter 5.

The VSC is connected to the common bus ‘C’ according to Figure 6.1. Seen from bus ‘C’, transfer functions have been derived, which express the current changes as function of the voltage changes on this bus. When the VSC-STATCOM is operating with its controllers having fixed reference values, it will appear towards the HVDC-LCC as linear impedance connected to this common bus ‘C’. This principle is used below as a way to derive the impact of the VSC-STATCOM on the control system of the HVDC-LCC.

In Chapter 5 the VSC was assumed to be connected to a network with the impedance $Z_s$ (see the outline of the system in Figure 5.2, Chapter 5). Now, if the network impedance is removed, and if it is assumed that the common bus ‘C’ is energized from the source voltage, according to Figure 6.2 below, then the different transfer functions for a small, superimposed oscillation in the voltage, the current, and the control signal are derived, obtaining the following set of equations (similar to that obtained by Eq. (5-58) in Chapter 5), where, $T_F$ is the corresponding matrix of transfer functions representing both the inner-loop and outer-loop controllers of the VSC converter. Thus,

$$
\begin{pmatrix}
\Delta i_{q_d}
\\Delta i_{q_q}
\\Delta i_{C_d}
\\Delta i_{C_q}
\Delta u_{F_d}
\Delta u_{F_q}
\Delta u_{V_d}
\Delta u_{V_q}
\end{pmatrix} = T_F
\begin{pmatrix}
\Delta u_{ac \text{ ref}}
\Delta u_{ac \text{ ref}}
\Delta u_{C_d}
\Delta u_{C_q}
\end{pmatrix}
\times
\begin{pmatrix}
\Delta u_{ac \text{ ref}}
\Delta u_{ac \text{ ref}}
\Delta u_{C_d}
\Delta u_{C_q}
\end{pmatrix}
\quad (6-7)
$$

The way of representing the VSC in the frequency domain is the result of the relation between the two variables $\Delta i_{C_d} = \Delta i_{C_d} + j \Delta i_{C_q}$ and $\Delta u_{C_d} = \Delta u_{C_d} + j \Delta u_{C_q}$. This relation corresponds to the equivalent $d-q$ component admittances, $y_{VSC}^{dd}$, $y_{VSC}^{dq}$, $y_{VSC}^{qd}$, and $y_{VSC}^{qq}$ seen from the common bus ‘C’.

These admittances are extracted directly from the transfer function matrix $T_F$,

$$
y_{VSC}^{d-q} = \left( \begin{array}{c}
\Delta u_{C_d} \\
\Delta u_{C_q}
\end{array} \right)^{-1} \cdot \left( \begin{array}{c}
\Delta i_{C_d} \\
\Delta i_{C_q}
\end{array} \right) = \begin{pmatrix}
tf_{VSC}^{1-1} & tf_{VSC}^{1-2} \\
tf_{VSC}^{1-2} & tf_{VSC}^{2-2}
\end{pmatrix} = \begin{pmatrix}
y_{VSC}^{dd} & y_{VSC}^{dq} \\
y_{VSC}^{dq} & y_{VSC}^{qq}
\end{pmatrix}
\quad (6-8)
$$

The integration of the VSC converter into the AC system will be made with the use of these $d-q$ component admittances.
Figure 6.2: Seen from bus 'C' the VSC is represented by the equivalent d- and q-component admittances

6.4 INTERACTION OF THE LCC-CONVERTER WITH THE CONNECTED AC SYSTEM

Figure 6.3 details the connected AC network at the inverter bus of the HVDC-LCC transmission: the AC network represented by the impedance $Z_s$, (this is the Thevenin impedance that includes the network and the filters connected to the common bus 'C'); and the VSC-STATCOM, which appears towards the converter as an impedance representing the VSC-converter. All these impedances are represented in their generalized form as the non-symmetrical conditions of the VSC-representation in their voltage and current d- and q- components which relates through a full matrix relation.

According to Figure 6.3, the total admittance seen by the LCC is calculated by simply adding the network admittance and the admittance of the VSC converter, represented by the corresponding d- and q-component admittances,

$$\frac{1}{z_{Net}^{d-q}} = y_{Net}^{d-q} = y_s^{d-q} + y_{VSC}^{d-q}$$ (6-9)

This gives the combined impedance interacting with the HVDC-LCC.
6.5 APPLICATION

Assume an HVDC link based on line commutated converters (main circuit data and control parameters are presented in Table 6.1). This is a 1500 MW, 800 km HVDC transmission link connecting two AC networks. The AC network connected at the rectifier side of the HVDC transmission link has a short-circuit power of 7500 MVA and the short-circuit power of the AC network at the inverter side is 3000 MVA.

Now, assume that a VSC operating as a STATCOM is connected to the inverter AC network close to the AC converter bus of the HVDC transmission link. The VSC converter is rated 350 MVA (main circuit parameters and control parameters are presented in Table 6.2).

The studied system is outlined in Figure 6.4.

With the mathematical model of this system it is possible to calculate the Bode diagram of the closed current-control loop of the rectifier, (see Figure 6.5) and the Nyquist Plot of the open current-control loop of the rectifier (see Figure 6.6). In order to verify the influence of the VSC on the performance of the HVDC transmission link, a case having the VSC-STATCOM connected (solid curve) is compared to a case where the VSC-STATCOM was disconnected (dotted curve).

The results show very little influence of the VSC converter on the calculated transfer function representing the closed loop DC current controller of the HVDC transmission link. This will be investigated in more detail in the next chapter.
Table 6.1: System data and control parameters used for the HVDC LCC link

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Rectifier</th>
<th>Inverter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal frequency</td>
<td>50 Hz</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Connected Network</td>
<td>7500 MVA, 85°</td>
<td>3000 MVA, 85°</td>
</tr>
<tr>
<td>Shunt filters</td>
<td>819 MVA</td>
<td>819 MVA</td>
</tr>
<tr>
<td>Converter transformer</td>
<td>1785 MVA, leakage reactance = 0.16 pu</td>
<td>819 MVA</td>
</tr>
<tr>
<td>Smoothing reactor</td>
<td>0.29 H</td>
<td></td>
</tr>
<tr>
<td>Length of Transmission line L</td>
<td>800 km</td>
<td></td>
</tr>
<tr>
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<td>11.867 nF/km</td>
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<td>500 kV, 3 kA</td>
<td>460 kV, 3 kA</td>
</tr>
<tr>
<td>Operating DC Voltage</td>
<td>500 kV</td>
<td>460 kV</td>
</tr>
<tr>
<td>Operating DC Current</td>
<td>3 kA</td>
<td>3 kA</td>
</tr>
<tr>
<td>Firing (extinction) angle initial condition</td>
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<td>18 deg</td>
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<tr>
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<td>0.63 rad/pu</td>
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<td>65.61 rad/pu</td>
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<td>Voltage Controller – Proportional-gain</td>
<td></td>
<td>0.436 rad/pu</td>
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<td>PLL – Gain $k_{PLL}$</td>
<td>20 (rad/s)/rad</td>
<td>5 (rad/s)/rad</td>
</tr>
<tr>
<td>PLL – Time Constant $t_{PLL}$</td>
<td>5 sec</td>
<td>5 sec</td>
</tr>
</tbody>
</table>
Table 6.2: Details of the VSC-STATCOM having controllers according to the description given in Chapter 5

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Converter rating</td>
<td>350 MVA</td>
</tr>
<tr>
<td>Source voltage</td>
<td>422.5 kV rms line-line voltage</td>
</tr>
<tr>
<td>Converter transformer impedance</td>
<td>$x_T=0.004+j0.12$ pu</td>
</tr>
<tr>
<td>Shunt filter</td>
<td>$y_F=j0.146$ pu</td>
</tr>
<tr>
<td>Converter reactor</td>
<td>$x_C=0.005+j0.155$ pu</td>
</tr>
<tr>
<td>DC-side capacitor: time cte</td>
<td>18 ms</td>
</tr>
<tr>
<td>Current controller gains</td>
<td>$k^c = 0.1 pu / pu$</td>
</tr>
<tr>
<td></td>
<td>$r^c = 0.02$ sec</td>
</tr>
<tr>
<td>DC Voltage controller gains</td>
<td>$k^{dc} = 5.0 pu / pu$</td>
</tr>
<tr>
<td></td>
<td>$r^{dc} = 1.0$ sec</td>
</tr>
<tr>
<td>AC Voltage controller gains</td>
<td>$k^{ac} = 0.25 pu / pu$</td>
</tr>
<tr>
<td></td>
<td>$r^{ac} = 0.001$ sec</td>
</tr>
<tr>
<td>PLL controller gains</td>
<td>$k^{PLL} = 200 (rad/s)/rad$</td>
</tr>
<tr>
<td></td>
<td>$r^{PLL} = 0.6$ sec</td>
</tr>
<tr>
<td>$T_d$: time-delay due to PSCAD time-step</td>
<td>$T_d = 0$</td>
</tr>
<tr>
<td>$T_{del}$: time-delay due to control delay and modulation</td>
<td>$T_{del} = 455 \mu s$</td>
</tr>
<tr>
<td>Low Pass filter included in the feed-forward filter bus voltage</td>
<td>LP: 0 sec time constant</td>
</tr>
<tr>
<td>Coefficient ‘$\kappa$’: current compensation</td>
<td>$\kappa = 1.0$</td>
</tr>
<tr>
<td>Coefficient ‘$\lambda$’: dead-beat control strategy</td>
<td>$\lambda = 0.5$</td>
</tr>
</tbody>
</table>
Chapter 6. Integration of the VSC-STATCOM into HVDC-LCC frequency domain model

\[ P_{dc} = 1500 MW \]
\[ U_d = 500 kV \]
\[ I_d = 3 kA \]

\[ Q = 800 MVA \]
\[ S = 350 MVA \]

Figure 6.4: Outline of the system studied

Figure 6.5: Bode plot of the closed loop, \( \frac{\Delta I_d}{\Delta I_{d-ref}} \) having the VSC disconnected (solid line) or VSC connected (dashed line) at the inverter AC network.
6.6 CONCLUSION

In this chapter the small-signal model of the VSC-STATCOM has been integrated into the small-signal model of the HVDC-LCC, to provide the total transfer function of the complete system. The models for each one of the systems have been developed in such a way that they are represented by subsystems which are connected to the common bus. This bus also is connected to the AC power system.

For each subsystem, transfer functions have been derived, which express the current changes from each subsystem to the common bus as function of the voltage changes on that same bus. When the VSC-STATCOM is operating with its controllers having fixed reference values, the STATCOM will appear towards the HVDC-LCC as linear impedance connected to the common bus. This principle has been used to derive the impact of the VSC-STATCOM on the control system of the HVDC-LCC.

Application of this analysis technique to a monopolar HVDC transmission system connected to an AC system including a VSC-STATCOM has shown that this small-signal model is accurate and can be used to perform stability analysis of system dynamics.
CHAPTER 7
ANALYSIS OF INTERACTION BETWEEN VSC-STATCOM AND HVDC-LCC

There are several methods for doing stability analysis on HVDC converters terminating at AC system locations having low short circuit capacities [references 16-19]. They are usually based on steady-state models, assuming fundamental frequency operation. These methods can also provide some insight into the basic mechanisms behind voltage instability phenomena.

‘Maximum Available Power’, or simply MAP, is one method for the case of HVDC transmission systems with rectifiers operating in constant power control mode and the inverter in constant extinction angle control mode.

Another method is based on the concept of ‘Voltage Sensitivity Factor’ (VSF) which is the incremental change in the AC busbar voltage at the converter bus due to a change in reactive power supply at that bus, i.e., $dV/dQ$. Normally the VSF is positive (the voltage increases with reactive power increase). A negative value, however, indicates voltage instability.

These methods are described in detail in [20].

In reference [19], O.B. Nayak introduced the ‘Control Sensitivity Index’ (CSI) approach based on analyzing the sensitivity of the power system response to a change in an output quantity of the controller under consideration. The CSI approach focuses on the characteristics of the system which are directly associated with the controller. The formulation of the CSI does not use controller parameters, but instead, the system characteristics are analyzed from the point of view of a controller output quantity.

These three methods, MAP, VSF and CSI, have in common that they are based on the steady-state network equations, assuming fundamental frequency AC voltages. They only consider the steady-state instability associated with the control mode.

In this chapter new sensitivity functions, which are generalized as characteristics defined in the whole frequency-domain, will be introduced to study the
voltage/power stability of the HVDC converter. Thus, contrary to the other methods, not only the steady-state conditions (low frequency values) are studied.

The analysis is performed considering the presence of a VSC-STATCOM closely connected to the HVDC-LCC inverter bus. The initial expectation is that the voltage/power stability of the system is improved due to the reactive power support from the VSC-STATCOM.

### Chapter 7. Analysis of interaction between VSC-STATCOM and HVDC-LCC

#### 7.1 THE FUNDAMENTALS OF VOLTAGE/POWER STABILITY OF HVDC CONVERTERS BASED ON STEADY-STATE MODELS

##### 7.1.1 General

Experience and theoretical calculations have shown that the voltage/power stability of an HVDC-LCC transmission link connected to an AC system with low short circuit capacity is a very critical issue [17, 18, 20]. The lower the short circuit capacity of the connected AC system as compared with the power rating of the HVDC converter, the more problems related to voltage/power stability are expected. The physical mechanism that causes this voltage stability problem is the inability of the power system to provide the reactive power needed by the converters to maintain an acceptable system voltage level.

This instability is most likely to occur during constant power control operation for the HVDC link, resulting in that the desired amount of power cannot be transmitted, which is the motivation for the term voltage/power stability in this case [18].

Reference [20] presents basically two analytical methods for analysis of the transient voltage/power stability. One is based on the Maximum Power Curve (MPC) method and the other is based on the Voltage Sensitivity Factor (VSF). These two methods use the same mathematical equations for their formulation. For this reason both methods give the same results.

In [20] the standard model for studying transient voltage/power stability of HVDC converters is defined. It is reproduced in Figure 7.1. This model states a number of assumptions that are used for the calculation of the Maximum Power Curve:

1) A fundamental frequency phasor representation can be used for the AC system.

2) The AC system can be modeled by an equivalent impedance and a fixed voltage source.

3) The HVDC converters are represented by their steady state equations.
7.1 The fundamentals of voltage/power stability of HVDC converters based on steady-state models

![Figure 7.1: Model of an HVDC converter connected to an AC system suitable for analysis of voltage/power stability](image)

These assumptions take into consideration that the mechanism of relevance arises in the time frame where the current controllers of the HVDC converters have already responded, which motivates the use of steady state equations for the converters, whereas the voltage controllers of the synchronous machines in the AC system have not yet reacted. This assumption for the machines leads to a static model of the AC network. The impedance $Z$ indicated in the figure, which is referred to as short-circuit impedance, is calculated assuming that the synchronous machines are modeled by a constant emf behind the transient reactance, $X'$. If there are closely connected active devices with fast voltage control, like SVC or VSC-STATCOM, they should be modeled explicitly.

The model in Figure 7.1 is thus relevant when studying variations of the DC current with time constants in the domain of, say 100 ms to 1 sec.

A basic parameter used in the analysis is the Short-Circuit Ratio (SCR) of the system, which is defined as

$$SCR = \frac{S_{SC}}{P_{dN}} = \frac{1}{Z}$$

(7-1)

were $S_{SC}$ is the short circuit capacity measured at the AC converter bus and $Z$ is the impedance of the system expressed in pu of $Z_{base} = \frac{U_{CN}^2}{P_{dN}}$.

Another quantity often used is the Equivalent SCR (ESCR) defined by

$$ESCR = \frac{S_{SC} - Q_{CN}}{P_{dN}}$$

(7-2)

which takes the (weakening) effect of the reactive shunt compensation into account.
7.1.2 Maximum Power Curve

The Maximum Power Curve (MPC) was developed to analyze the characteristics of HVDC-LCC inverters operating with constant extinction angle, $\gamma$. However, it can be generalized to be applied to converters with other operating modes, like constant firing angle control or constant DC voltage control.

When applied to the standard constant extinction angle $\gamma$ inverter, the MPC indicates the maximum amount of active power that can be transmitted when varying only the DC current from an initial operating point. This means that the DC power is varied by only changing the DC current of the converter.

The MPC is achieved by plotting the DC power as a function of the DC current. As this curve shows the inverter power capability, it is assumed that the rectifier provides no limitation to the supply of DC current at rated DC voltage. The points along the curve are calculated from steady-state equations.

The use of MPC for analysis of voltage/power stability is demonstrated for an HVDC converter, assuming three different connected AC systems:

- strong system, having $SCR = 4$
- weak system, having $SCR = 2.1$
- very weak system, having $SCR = 1.7$

The converter data is: $d_x = 0.08 \, pu; \gamma = 17^\circ; Q_{dc} = 0.54 \, pu$. The results are shown in Figure 7.2. In the figure the converter bus voltage $U_i$ is also plotted as a function of $I_d$.

The criterion for voltage/power stability at a given point is that the slope of the MPC, that is, $dP_d/dI_d$, is positive, indicating that an increase in DC current will result in an increase in DC power.

Of particular interest is the behavior of the MPC at the operating point. The necessary condition for voltage/power stability is that $dP_d/dI_d > 0$ at that operating point. Unstable conditions are encountered if $dP_d/dI_d < 0$.

The DC power at the maximum point of the MPC is denoted Maximum Available Power and the associated DC current is denoted Maximum Available DC Current. The Maximum Available Power tells how much power that can be transmitted into the AC system by varying only the DC current from the operating point.

When the nominal DC current coincides with the Maximum Available Power operating point, the corresponding SCR or ESCR value to this situation is referred as being just on the limit giving stable operation at nominal conditions. In the case that is illustrated in Figure 7.2, the connected AC network having $SCR = 1.7$ corresponds to this situation.
7.2 Sensitivity indices used in the frequency domain analysis

7.2.1 Definition of new sensitivity indices

In the previous section an overview of classical methods to analyze the voltage/power stability of HVDC converters was given. The analysis was based on a quasi-static approach, where the steady-state equations for the converters are used and a static model of the connected AC network is utilized. Thus, it is assumed that the active voltage control systems included in the network act slowly compared to the current control of the HVDC converters.

It was shown that a criterion for voltage/power stability at a given point is that the slope of the MPC, that is, $\frac{dP_d}{dl_d}$, is positive, indicating that an increase in DC current will result in an increase in DC power.

In contrast to the MPC, the voltage/power stability of the converter is now studied considering the dynamic small signal model of the converter which was developed in the previous chapters. Together with the dynamic model of the converter, the influence of the associated control is also taken into account. As for the MPC criterion, the characterization of the voltage/power stability conditions of the dynamic model is made by analyzing the slope of a number of sensitivity indices calculated in the frequency domain.

Below a list of sensitivity indices that will be used in the analysis is given:
Chapter 7. Analysis of interaction between VSC-STATCOM and HVDC-LCC

Sensitivity index nr. 1: \( S_{\text{I:U-I}}^1 = \frac{\Delta U_{d\text{-inv}}}{\Delta I_{d\text{-inv}}} \)  

(7-3)

Sensitivity index nr. 2: \( S_{\text{I:P-I}}^2 = \frac{\Delta P_{I\text{-inv}}}{\Delta I_{d\text{-inv}}} \)  

(7-4)

Sensitivity index nr. 3: \( S_{\text{R:U-I}}^3 = \frac{\Delta U_{d\text{-ref}}}{\Delta I_{d\text{-ref}}} \)  

(7-5)

Sensitivity index nr. 4: \( S_{\text{R:P-I}}^4 = \frac{\Delta P_{d\text{-ref}}}{\Delta I_{d\text{-ref}}} \)  

(7-6)

Sensitivity index nr. 5: \( S_{\text{I:U-e}}^5 = \frac{\Delta u_{e\text{-inv}}}{\Delta I_{d\text{-ref}}} \)  

(7-7)

Sensitivity index nr. 6: \( S_{\text{I:U-e}}^6 = \frac{\Delta \theta_{e\text{-inv}}}{\Delta I_{d\text{-ref}}} \)  

(7-8)

7.2.2 Voltage based sensitivity indices

The first index, \( S_{\text{I:U-I}}^1 = \frac{\Delta U_{d\text{-inv}}}{\Delta I_{d\text{-inv}}} \), tells how the inverter DC voltage changes as a result of a change in the inverter DC current. It can be interpreted as the sensitivity of inverter DC voltage due to changes in the impressed inverter DC current. Only the inverter and the associated connected AC system are considered for this sensitivity index. The contributions from the rectifier converter and the DC line are not included. It should be pointed out that the index takes into consideration the dynamics of the controllers associated with the inverter. Depending on how the control system is designed the following typical controllers can be used:

- Constant gamma control
- Constant DC voltage control
- Constant alpha control
- Constant current control, etc.

This study focuses on the constant DC voltage control for the inverter side of the HVDC link.

The third index, \( S_{\text{R:U-I}}^3 = \frac{\Delta U_{d\text{-ref}}}{\Delta I_{d\text{-ref}}} \), tells how the DC voltage measured at the rectifier changes as a result of a change in the reference of the DC current of the rectifier. As a result, all components included in the system contribute to the index: the contribution from both the rectifier and inverter controllers, the contribution from the DC line, the connected AC systems, etc.
The fifth and sixth Sensitivity indices, $S_{I,Uc}^{V}$ and $S_{I,\theta c}^{\theta}$, indicate the sensitivity of AC voltage (amplitude and argument, respectively) with respect to changes in the reference of the DC current of the rectifier. A high magnitude of these indices indicates that the connected AC system on the inverter side is weak. It also gives an indication that the inverter would be prone to commutation failures if connected to such a system.

Details of the calculation of the sensitivity indices are given in Appendixes E-J. The calculation is made using the system shown in the block diagram in Figure 7.3.

### 7.2.3 Extending the DC voltage sensitivity indices to DC power sensitivity indices

It is possible to extend the first and the third sensitivity indices related to DC voltage variation to the sensitivity related to DC power variation as a result of DC current variation. Considering that
Chapter 7. Analysis of interaction between VSC-STATCOM and HVDC-LCC

$$P_d = U_d I_d$$  \hfill (7-9)

Performing a general differentiation of this expression yields

$$\Delta P_d = \Delta U_d I_{d0} + U_{d0} \Delta I_d$$  \hfill (7-10)

If both sides of the expression are divided by $\Delta I_d$, it is found that

$$\frac{\Delta P_d}{\Delta I_d} = I_{d0} \frac{\Delta U_d}{\Delta I_d} + U_{d0}$$  \hfill (7-11)

If the quantity $\frac{\Delta P_d}{\Delta I_d}$ is positive, i.e. when a positive variation of the DC current results in a positive variation of the transmitted power, then, this indicates a stable condition. On the other hand, when the quantity $\frac{\Delta P_d}{\Delta I_d}$ is negative, this indicates an unstable condition.

The magnitude of the quantity is a measure of how sensitive the response of the system is to changes in the DC current. A high magnitude indicates that very small changes in DC current would result a large change in DC power. This might also indicate an unstable condition.

The DC power sensitivity indices are derived from the DC voltage sensitivity indices as

$$S_{1,P-I}^2 = \frac{\Delta P_{d-inv}}{\Delta I_{d-inv}} = I_{d0} S_{1,U-I}^1 + U_{d0} = I_{d0} \frac{\Delta U_{d-inv}}{\Delta I_{d-inv}} + U_{d0}$$  \hfill (7-12)

$$S_{R,P-I}^4 = \frac{\Delta P_{d-rec}}{\Delta I_{d-ref}} = I_{d0} S_{R,U-I}^3 + U_{d0} = I_{d0} \frac{\Delta U_{d-rec}}{\Delta I_{d-ref}} + U_{d0}$$  \hfill (7-13)

It should be noted that, from these indices, the low frequency values give an indication similar to that provided by the MPC curves proposed in [20] (assuming that the systems are operating in the same control mode).

7.3 APPLICATION 1: COMBINED OPERATION OF THE HVDC-LCC TRANSMISSION LINK AND A VSC-STATCOM WITH CURRENT CONTROL

7.3.1 System setup

Assume the same system model used in Chapter 6; the receiving end of a 1500 MW, 800 km HVDC transmission link based on line commutated converters (main circuit data and control parameters are presented in Table 7.1) and a VSC operating as a STATCOM (main circuit data and control parameters are presented in Table 7.2) are connected to the same AC system bus.
The studied system is outlined in Figure 7.4.

![Figure 7.4: Outline of the system studied](image)

In order to demonstrate the applicability of using the different sensitivity indices for analyzing the stability of the HVDC transmission link, the influence of the VSC-STATCOM on the operation of the HVDC link is studied. The inverter is controlled with constant DC voltage control and the rectifier is controlled with constant DC current control.

The plots are calculated in the frequency interval 1-250 Hz, making it possible to identify resonances in the AC network. The plots are then repeated in the frequency interval 0.1-50 Hz which typically covers the bandwidth of the controllers included in both the HVDC link and the VSC-STATCOM.

Note that in the formulation of the model, the dynamics of the connected AC system are neglected, i.e., the HVDC link and the VSC-STATCOM are assumed to be much faster than the AC voltage controllers of the system equivalent. It is assumed that the system is represented by its constant internal voltage behind its equivalent transient impedance.

Figures 7.5-7.12 compare the performance of the HVDC link between two different cases:

- Solid lines: case with VSC-STATCOM disconnected
- Dashed lines: case with the VSC-STATCOM connected close to the HVDC-LCC inverter bus
A summary of the performance evaluation based on different criteria is presented in Table 7.3.

Table 7.1: System data and control parameters used for the HVDC LCC link

<table>
<thead>
<tr>
<th>Parameter</th>
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</tr>
<tr>
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</tr>
<tr>
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<td></td>
</tr>
<tr>
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<td>18 deg</td>
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</tr>
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<tr>
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<td>16 msec</td>
</tr>
<tr>
<td>PLL – Gain $k_{PLL}$</td>
<td>20 (rad/s)/rad</td>
<td>5 (rad/s)/rad</td>
</tr>
<tr>
<td>PLL – Time Constant $t_{PLL}$</td>
<td>5 sec</td>
<td>5 sec</td>
</tr>
</tbody>
</table>
7.3 Application 1: combined operation of the HVDC-LCC transmission link and a VSC-STATCOM with current control

Table 7.2: Details of VSC-STATCOM

<table>
<thead>
<tr>
<th>Solid line</th>
</tr>
</thead>
<tbody>
<tr>
<td>current controller gains</td>
</tr>
<tr>
<td>$k^b = 0.1, \text{pu} / \text{pu}$</td>
</tr>
<tr>
<td>$t^b = 0.02, \text{sec}$</td>
</tr>
<tr>
<td>DC Voltage controller gains</td>
</tr>
<tr>
<td>$k^{I_{dc}} = 5.0, \text{pu} / \text{pu}$</td>
</tr>
<tr>
<td>$t^{I_{dc}} = 1.0, \text{sec}$</td>
</tr>
<tr>
<td>AC Voltage controller gains</td>
</tr>
<tr>
<td>$k^{U_{ac}} = 0.25, \text{pu} / \text{pu}$</td>
</tr>
<tr>
<td>$t^{U_{ac}} = 0.001, \text{sec}$</td>
</tr>
<tr>
<td>PLL controller gains</td>
</tr>
<tr>
<td>$k^{PLL} = 200(\text{rad/s})/\text{rad}$</td>
</tr>
<tr>
<td>$t^{PLL} = 0.6, \text{sec}$</td>
</tr>
<tr>
<td>$T_d$, time-delay due to PSCAD time-step</td>
</tr>
<tr>
<td>$T_d = 0$</td>
</tr>
<tr>
<td>$T_{ac}$, time-delay due to control delay and modulation</td>
</tr>
<tr>
<td>$T_{ac} = 455, \mu\text{sec}$</td>
</tr>
<tr>
<td>Low Pass filter included in the feed-forward filter bus voltage</td>
</tr>
<tr>
<td>LP: 0 sec time constant</td>
</tr>
<tr>
<td>coefficient <code>$\kappa$</code>: current compensation</td>
</tr>
<tr>
<td>$\kappa = 1.0$</td>
</tr>
<tr>
<td>coefficient <code>$\lambda$</code>: dead-beat control strategy</td>
</tr>
<tr>
<td>$\lambda = 0.5$</td>
</tr>
</tbody>
</table>

7.3.2 Analysis of the performance of the system in the frequency domain

Criteria 1 and 2: Gain and Phase margins of the closed loop rectifier DC current controller, $\frac{\Delta I_d}{\Delta I_{d-ref}}$

The gain and phase margins are the simplest tools to characterize the performance of a system in the frequency domain. They give a measure of the stability condition of the feedback control system in the crossover (bandwidth) region. From the Bode plot of the loop transfer function ‘$T = \frac{\Delta I_d}{\Delta I_{d-ref}}$’ of the system, the gain margin at the phase crossover frequency $\omega_{80}$ and the phase margin at unity gain crossover frequency $\omega_c$ are measured. These quantities can also be obtained from the Nyquist plot of the loop transfer function ‘$L = \frac{\Delta I_d}{\Delta \xi}$’ of the system (see Chapter 2 for a more detailed definition of these and other basic classic control concepts).

The dynamics of the DC current controller included in the HVDC rectifier basically results from the gains of an ordinary PI regulator and the synchronizing PLL control that uses the rectifier AC converter bus as the reference bus. The gains have been adjusted in order to obtain reasonable time domain performance of the link, based on the analysis of the response to a step in the control reference.
Chapter 7. Analysis of interaction between VSC-STATCOM and HVDC-LCC

The main limitation factor that has determined the gains has been the resonance between AC network and connected AC filters at the converter bus on both the rectifier and inverter sides. At the rectifier side this resonance frequency can be observed on the Bode plot of the closed loop rectifier current controller (Figure 7.5). This resonance frequency is approximately 115-120 Hz.

On the inverter side, as the connected AC network is weak, the resonance frequency occurs at a lower frequency. This can be observed in Figure 7.7, where the resonance on the AC side of the converter is reflected to the DC side. The DC-side resonance is between 40 and 45 Hz.

These resonances impose limitations on the gain of the DC current controller. To avoid instability of the controllers, the gains were adjusted such that the obtained gain margin was 1.2-1.5 (measured a 118 Hz) and the phase margin was of the order of $55^\circ - 60^\circ$, measured between 3 and 5 Hz.

The case having the VSC-STATCOM connected showed almost the same performance as if the VSC was disconnected.

**Criterion 3: Bandwidth or Crossover frequency of the rectifier DC current controller**

The crossover frequency $\omega_c$ is defined as the frequency where the magnitude of the loop transfer function $L = \frac{\Delta I_d}{\Delta \epsilon}$ crosses 1 the first time. This quantity will be used to quantify the bandwidth of the controller.

Mainly, due to the low resonance frequency at the inverter AC network, the bandwidth was found quite small. Having or not having the VSC connected the bandwidth is approximately 6 Hz.

**Criterion 4: Maximum peak value of $T = \frac{\Delta I_d}{\Delta I_{d-ref}}$**

The maximum peak was measured on the complementary sensitivity function $T = \frac{\Delta I_d}{\Delta I_{d-ref}}$. A large value of this quantities indicates poor performance.

Both setups have shown high peak values. There is a peak value at low frequency (approximately 4 Hz) and another at high frequency (approximately 115 Hz). The high peak value at low frequency indicates a high overshoot in the step response to a change in the control reference. The high peak value at high frequency indicates a possible stability problem of the controller, and high frequency oscillations during disturbances.

This abnormal behavior is not much improved by having the VSC connected.
7.3 Application 1: combined operation of the HVDC-LCC transmission link and a VSC-STATCOM with current control

Criterion 5: Sensitivity indices related to the inverter DC voltage,

\[ S_{I:d-inv}^1 = \frac{\Delta U_{d-inv}}{\Delta I_{d-inv}} \text{ and } S_{I:p-inv}^2 = \frac{\Delta P_{d-inv}}{\Delta I_{d-inv}} \]

The inverter DC voltage index \( S_{I:d-inv}^1 \) tells us how the inverter DC voltage changes due to a change in the impressed current into the inverter. If the converter is connected to a stiff AC network, the DC voltage change is related to the inverter DC current change only by the internal commutation reactance of the converter transformer.

In the particular case having the converter connected to a weak AC system, and due to the presence of shunt filter compensation connected at the converter bus, the resulting high impedance around the resonance frequency can significantly affect this index, as is shown in Figure 7.7. In the case without the VSC-STATCOM connected, the index has a peak of approximately 0.5 pu at approximately 45 Hz.

In the case having the VSC-STATCOM connected the index has a slightly higher peak, in the order of 0.7 pu and at a lower frequency, approximately 39 Hz.

The conclusion is that, when the VSC converter is connected close to inverter of the HVDC link, the system will be more sensitive to DC current variations. The VSC-STATCOM is in reality de-stabilizing the system instead of making it more robust. It was expected that with the VSC-STATCOM included in the system, the AC voltage of the system would be maintained constant. However, due to the low frequency resonance in the AC network it was not possible to make the AC voltage controller included in the VSC converter sufficiently fast, and the result was that the VSC-STATCOM degraded the performance of the HVDC transmission link.

At very low frequency (this corresponds to the situation where the controllers have already reached their steady-state condition) the case having the VSC-STATCOM connected shows slightly superior performance as compared to the case without the VSC connected. This was expected considering that one of the objectives of the VSC converter was to maintain a constant AC voltage close to the converter bus.

Similar observations can be made for the inverter DC power index \( S_{I:p-inv}^2 \), as is illustrated in Figure 7.8.

Criterion 6: Sensitivity indices related to the rectifier DC voltage,

\[ S_{R:d-ref}^3 = \frac{\Delta U_{d-ref}}{\Delta I_{d-ref}} \text{ and } S_{R:p-ref}^4 = \frac{\Delta P_{d-ref}}{\Delta I_{d-ref}} \]

Similar to the indices related to inverter side, these two additional indices qualify the voltage stability and power stability of the HVDC transmission link seen from the rectifier side of the HVDC transmission link. Here, the rectifier, the corresponding rectifier DC current controller, and the connected AC system are also taken into consideration.
Regarding the voltage sensitivity index $S_{R;U-1}^3$, this is not only influenced by possible resonances in the connected inverter AC network but also resonances in the connected rectifier AC network of the HVDC link.

Ideally $S_{R;U-1}^3$ should be small for all frequencies, and only be dependent on the converter transformer reactance. However, in the non-ideal situation, due to the influence of the connected AC system at the rectifier bus, and in particular at resonance frequencies, $S_{R;U-1}^3$ can be high, indicating possible instability conditions. This condition puts high demands on the controllers. This can be seen in Figure 7.9, where it is possible to read $S_{R;U-1}^3$ equal to 2.5 at approximately 115 Hz.

The impact of the connected inverter AC system in the $S_{R;U-1}^3$ index can be seen at lower frequency. It is approximately 0.2 pu at approximately 6 Hz. This value can be read in both cases, with the VSC-STATCOM connected or with the VSC disconnected.

The corresponding indices related to DC power, $S_{R;P-1}^4$, follow the same behavior as the DC voltage related index.

**Criteria 7 and 8: Sensitivity indices related to the inverter AC voltage,**

\[ S_{I;Uc}^5 = \frac{\Delta U_{c-inv}}{\Delta I_{d-ref}} \quad \text{and} \quad S_{I;\theta c}^6 = \frac{\Delta \theta_{c-inv}}{\Delta I_{d-ref}} \]

These indices tell how the filter bank AC voltage changes due to changes in the DC current reference of the HVDC transmission link. These indices will show small values in a strong AC system. This is valid for all frequencies; in a weak network these indices will show high values. In particular around resonance frequencies these indices are expected to show high values.

Figure 7.11 and 7.12 present the results for these indices for the two studied systems.

Considering the sensitivity related to the amplitude of the AC voltage, the system having the VSC-STATCOM connected is slightly worse as compared to the system without the VSC connected.

Now, considering the sensitivity related to the argument of the AC voltage, the case having VSC connected gives lower values at very low frequency (measured at 1 Hz). This was expected considering the final action of the VSC-STATCOM controllers. However, in the transient region (measured at 10 Hz and 50 Hz) the case without VSC-STATCOM connected shows better results. This indicates inadequate dynamic behavior of the controllers of the VSC.

The reason for the undesired characteristics of the VSC controller is that it was not possible to achieve a good tuning of the controllers due to the low frequency resonance in the AC network.
Table 7.3: Evaluating the performance; summary results

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Performance Evaluation</th>
<th>Case VSC Disconnected</th>
<th>Case VSC Connected</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Rectifier Current Control: Gain Margin ( GM )</td>
<td>1.17 (at 118 Hz)</td>
<td>1.43 (at 118 Hz)</td>
</tr>
<tr>
<td>2</td>
<td>Rectifier Current Control: Phase Margin ( PM )</td>
<td>58°</td>
<td>54°</td>
</tr>
<tr>
<td>3</td>
<td>Rectifier Current Control: Crossover Frequency ( \omega_c )</td>
<td>6.0 Hz</td>
<td>6.2 Hz</td>
</tr>
<tr>
<td>4</td>
<td>Rectifier Current Control: Maximum Peak Value ( M_T )</td>
<td>1.28 (at 4 Hz)</td>
<td>1.32 (at 4.5 Hz)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.97 (at 117 Hz)</td>
<td>1.48 (at 115 Hz)</td>
</tr>
<tr>
<td>5</td>
<td>Sensitivity Index: Inverter DC Voltage (nr.1)</td>
<td>0.51 (at 43 Hz)</td>
<td>0.69 (at 39 Hz)</td>
</tr>
<tr>
<td>6</td>
<td>Sensitivity Index: Rectifier DC Voltage (nr.3)</td>
<td>0.17 (at 5.5 Hz)</td>
<td>0.18 (at 6 Hz)</td>
</tr>
<tr>
<td>7</td>
<td>Sensitivity Index: Inverter AC Voltage, amplitude (nr.5)</td>
<td>0.06 (at 5 Hz)</td>
<td>0.06 (at 5 Hz)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.28 (at 115 Hz)</td>
<td>0.2 (at 117 Hz)</td>
</tr>
<tr>
<td>8</td>
<td>Sensitivity Index: Inverter AC Voltage, argument (nr.6)</td>
<td>9.7° (at 1 Hz)</td>
<td>9.7° (at 1 Hz)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.0° (at 10 Hz)</td>
<td>5.1° (at 10 Hz)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10.3° (at 50 Hz)</td>
<td>6.9° (at 50 Hz)</td>
</tr>
</tbody>
</table>

Figure 7.5: Bode plot of the closed loop rectifier current controller \( \frac{\Delta I_d}{\Delta I_{d-ref}} \): VSC is disconnected (solid line) and VSC is connected (dashed line)
Figure 7.6: Nyquist diagram of the open loop rectifier current controller $\frac{\Delta I_{d}}{\Delta \epsilon}$: VSC is disconnected (solid line) and VSC is connected (dashed line)

Figure 7.7: Sensitivity Index nr. 1 $S_{\epsilon:U-I}^{1} = \frac{\Delta U_{d-inv}}{\Delta I_{d-inv}}$: VSC is disconnected (solid line) and VSC is connected (dashed line)
7.3 Application 1: combined operation of the HVDC-LCC transmission link and a VSC-STATCOM with current control

Figure 7.8: Sensitivity index nr. 2 $S^2_{I: \text{PLL}} = \frac{\Delta P_{\text{d-inv}}}{\Delta I_{\text{d-inv}}}$: VSC is disconnected (solid line) and VSC is connected (dashed line)

Figure 7.9: Sensitivity index nr. 3 $S^3_{R:U-I} = \frac{\Delta U_{\text{d-rec}}}{\Delta I_{\text{d-ref}}}$: VSC is disconnected (solid line) and VSC is connected (dashed line)
Chapter 7. Analysis of interaction between VSC-STATCOM and HVDC-LCC

Figure 7.10: Sensitivity index 4 \( S_{4}^{P} = \frac{\Delta P_{d\text{-rec}}}{\Delta I_{d\text{-ref}}} \): VSC is disconnected (solid line) and VSC is connected (dashed line)

Figure 7.11: Sensitivity index 5 \( S_{5}^{U/e} = \frac{\Delta U_{e\text{-inv}}}{\Delta I_{d\text{-ref}}} \): VSC is disconnected (solid line) and VSC is connected (dashed line)

Figure 7.12: Sensitivity index 6 \( S_{6}^{\theta/e} = \frac{\Delta \theta_{e\text{-inv}}}{\Delta I_{d\text{-ref}}} \): VSC is disconnected (solid line) and VSC is connected (dashed line)
7.4 CONCLUSIONS

The interaction of the VSC-STATCOM with the HVDC-LCC transmission link was studied.

The results have shown that the VSC-STATCOM cannot properly control the AC voltage during transients as the bandwidth of the controllers had to be limited due to the low resonance frequency in the inverter AC network. The low resonance frequency is due to the presence of large amounts of capacitors which are installed at the AC inverter bus combined with a weak AC network. This imposes limitations not only on the bandwidth of the inner current controller of the VSC, but also on the outer AC and DC voltage controllers which rely on the inner current control.
CHAPTER 8
MODEL OF VSC WITH MODIFIED CONTROLS

The conventional VSC described in Chapter 5 is based on controlling the AC current through a phase reactor by manipulating the converter AC voltage. This is the core of the inner current control loop. The AC voltage and DC voltage controllers act on the reference value of the current controller. With this control strategy the VSC-STATCOM is in reality a current source converter.

The performance of this type of VSC in supporting the operation of the LCC inverter was investigated in the preceding chapter. There, it was shown that the VSC does not provide the necessary transient voltage support to the LCC inverter. The reason to this was that the large amount of shunt capacitors in the HVDC-LCC filter bank combined with the weak AC network form a parallel resonance circuit seen from the VSC-STATCOM. The consequence of this is a high AC impedance value at a fairly low frequency. This resonance in the AC network imposes limitations on the VSC controllers. The bandwidths of these controllers were therefore limited below this resonance frequency, reducing its ability to have a fast response.

When comparing different transfer functions calculated for the HVDC-LCC transmission, having the VSC-STATCOM connected or not connected at the inverter bus of the HVDC transmission link, the results show that, in reality, the VSC did not improve the performance of the HVDC-LCC link since this converter was too sensitive to the resonance in the AC network.

Alternatively, if the controls of the VSC are made comparably slow, and the inner current control loop is eliminated, it is expected that the VSC converter would become less sensitive to the resonance condition of the network, and would impress a constant AC voltage behind the series reactor. This would yield the characteristics of a constant voltage source seen from the DC side of the converter and a constant AC voltage source behind the series reactor of the VSC.

The proposed modified control for the VSC converter is illustrated in Figure 8.1. The reference value of the converter bridge AC voltage is obtained from the
feedback AC voltage controller, and is realized with PWM. This voltage is synchronized with the filter bus voltage via the PLL. The realization of the bridge AC voltage is affected by the DC voltage control, acting on the argument of this voltage. In this chapter the revised frequency domain model of this modified VSC converter is described.

\[
\Delta \overline{u}_f, ref, \Delta \overline{u}_c ref, \Delta \mu_V, ref, \Delta \mu_f, ref, \Delta \mu_{\text{ref}}\nu, \Delta \phi_{\text{ref}}\nu
\]

\[
\text{Figure 8.1: Overview of the modified control system of the VSC-STATCOM}
\]

### 8.1 SYSTEM OUTLINE

Recall from Chapter 5 the model of the system shown in Figure 8.2. The VSC is connected to a filter bus with voltage $\overline{u}_f$ via a phase reactor with the impedance $z_f$. A shunt filter with the impedance $z_f$ is connected to the filter bus, which is connected to a common bus with the voltage $\overline{u}_c$ via a transformer with the impedance $z_f$. The common bus ‘C’ will be the connection point to the inverter side of the HVDC-LCC transmission that will be studied in Chapter 9.
8.2 THE VSC CONTROLLER

8.2.1 Synchronization with the PLL controller

As with the model described in Chapter 5, this controller relies upon alignment of the rotating $d-q$ reference frame in relation to the measured space vector $\vec{u}_F$, which is the voltage at the filter bus ‘F’. This alignment is made by the PLL controller (the Phase Locked Loop control). The synchronization controller extracts the transformation angle $\hat{\theta} = \Omega$ from the filter bus $\vec{u}_F$. The argument of the PLL coordinate system is given by the angle $\Delta \theta_{PLL}(t)$ in relation to the undisturbed rotating coordinate system.

The AC quantities, represented in the rotating and undisturbed coordinate system, are transformed to the rotating coordinate system, which is generated by the PLL.

8.2.2 The AC voltage controller

The AC voltage controller operates in the PLL coordinates. In the PLL coordinate system (with the superscript ‘PLL’ for the quantities) the deviation of the voltage response is given by

$$\Delta \vec{u}_F^{PLL}(t) = \Delta \vec{u}_F^{dq}(t) - j \vec{u}_F^{dq} \Delta \vec{\theta}_{PLL}(t)$$  \hspace{1cm} (8-1)

If the AC voltage controller has the transfer function $K^{UAC}$, the voltage reference of the VSC is given by

$$\Delta \vec{u}_V^{PLL}_{ref} = K^{UAC}(\Delta \vec{u}_F^{ref} - K_{UF} \Delta \vec{u}_F^{PLL})$$  \hspace{1cm} (8-2)

where $K_{UF}$ is a transfer function which represents the time delays included in the control loop and possibly the measurement filters. The dynamics of the control voltage from the AC voltage regulator is given by

$$K^{UAC} = \frac{k^{UACP} (1 + j \Omega t^{UAC})}{j \Omega t^{UAC}}$$  \hspace{1cm} (8-3)
The voltage reference for the VSC can now be written as

$$\Delta V_{\text{reg}} = K^{\text{UAC}} [\Delta u_{\text{ref}} - K_{\text{U}} \Delta \theta_{\text{PLL}}]$$  \hspace{1cm} (8-4)$$

When the voltage is transformed from the PLL coordinate system to the undisturbed rotating coordinates, the phase angle of the PLL coordinate system must be considered. Accordingly,

$$\Delta u_{\text{reg}} = K^{\text{UAC}} [\Delta u_{\text{ref}} - j K_{\text{F}} \Delta \theta_{\text{PLL}}] + j \Delta u_{\text{ref}} \Delta \theta_{\text{PLL}}$$  \hspace{1cm} (8-5)$$

As the DC voltage controller acts on the argument of the VSC voltage a new independent variable must here be anticipated, \(\Delta \theta_{\text{U}_{\text{d}} \text{cont}}(t)\), which later will be considered when closing the DC voltage control loop. Consequently, the total VSC output voltage deviation is given by

$$\Delta u_{\text{v}} = K^{\text{UAC}} [\Delta u_{\text{ref}} - K_{\text{U}} \Delta u_{\text{F}}] + j K_{\text{F}} \Delta \theta_{\text{U}_{\text{d}} \text{cont}} + j \Delta u_{\text{ref}} \Delta \theta_{\text{PLL}}$$  \hspace{1cm} (8-6)$$

To simplify the notation the superscript ‘dq’ is omitted when referring to the undisturbed rotating coordinate system. Thus,

$$\Delta u_{\text{v}} = K^{\text{UAC}} [\Delta u_{\text{ref}} - j K_{\text{F}} \Delta \theta_{\text{PLL}}] + j \Delta u_{\text{ref}} \Delta \theta_{\text{PLL}}$$  \hspace{1cm} (8-7)$$

Equation (8-7) can now be re-written as

$$\Delta u_{\text{v}} = a_1 \Delta u_{\text{ref}} + a_2 \Delta u_{\text{F}} + a_3 \Delta \theta_{\text{U}_{\text{d}} \text{cont}} + a_4 \Delta \theta_{\text{PLL}}$$  \hspace{1cm} (8-8)$$

where,

$$a_1 = K^{\text{UAC}}$$
$$a_2 = -K^{\text{UAC}} K_{\text{U}}$$
$$a_3 = j \Delta u_{\text{F}}$$
$$a_4 = j \Delta u_{\text{F}} + K^{\text{UAC}} K_{\text{U}} j \Delta u_{\text{F}}$$

### 8.2.3 Preliminary set of equations calculated from the main circuit

According to the system depicted in Figure 8.2, the following impedance transfer functions, relating initially two independent variables \(\Delta \bar{u}_{\text{v}}\) and \(\Delta \bar{u}_{\text{F}}\) and three dependent variables \(\Delta \bar{u}_{\text{v}}\), \(\Delta \bar{i}_{\text{v}}\) and \(\Delta \bar{i}_{\text{F}}\), can be defined

$$\Delta \bar{u}_{\text{v}} = b_1 \Delta \bar{u}_{\text{v}} + b_2 \Delta \bar{u}_{\text{F}}$$
$$\Delta \bar{i}_{\text{v}} = b_3 \Delta \bar{u}_{\text{v}} + b_4 \Delta \bar{u}_{\text{F}}$$
$$\Delta \bar{i}_{\text{F}} = b_5 \Delta \bar{u}_{\text{v}} + b_6 \Delta \bar{u}_{\text{F}}$$  \hspace{1cm} (8-9)$$

where,
8.2 The VSC controller

\[
\begin{align*}
\frac{b_1}{z_f} &= \frac{1}{1 + \frac{z_f}{z_T + z_v / / z_f}} \\
\frac{b_2}{z_f} &= \frac{1}{z_v + z_T / / z_f} \\
\frac{b_3}{z_f} &= \frac{1}{z_T + z_v / / z_f} \\
\frac{b_4}{z_f} &= \frac{1}{1 + \frac{z_T}{z_T + z_v / / z_f}} \\
\frac{b_5}{z_f} &= \frac{z_T / / z_f}{z_T + z_v / / z_f} \\
\frac{b_6}{z_f} &= \frac{z_T / / z_f}{z_v + z_T / / z_f}
\end{align*}
\]

It should be noted that the impedances used in the above coefficients are impedances in the synchronously rotating coordinate system. They are calculated using

\[
\begin{align*}
Z_R(j\Omega) &= R \\
Z_L(j\Omega) &= (j\omega_N + j\Omega) L \\
Z_C(j\Omega) &= \frac{1}{(j\omega_N + j\Omega) C}
\end{align*}
\]

8.2.4 The AC voltage control loop

The variable \( \Delta u_v \) in Eq. (8-8) ceases to be independent when the current control loop becomes closed since, from (8-9), it depends on \( \Delta u_v \). Thus,

\[
\Delta u_v = a_1 \Delta u_{pref} + a_2 (b_5 \Delta u_c + b_6 \Delta u_v ) + a_3 \Delta \theta_{tdc ctrl} + a_4 \Delta \theta_{PLL} 
\]

(8-10)

Solving Eq. (8-10) for \( \Delta u_v \) yields

\[
\Delta u_v = c_1 \Delta u_{pref} + c_2 \Delta u_c + c_3 \Delta \theta_{tdc ctrl} + c_4 \Delta \theta_{PLL}
\]

(8-11)

where,

\[
\begin{align*}
c_1 &= \frac{a_1}{1 - a_2 b_6} \\
c_2 &= \frac{a_2 b_5}{1 - a_2 b_6} \\
c_3 &= \frac{a_3}{1 - a_2 b_6} \\
c_4 &= \frac{a_4}{1 - a_2 b_6}
\end{align*}
\]

Now, replacing the resulting \( \Delta u_v \) from the set of equations defined in (8-9), it is found that
\[
\Delta i_v = d_1 \Delta u_{\text{ref}} + d_2 \Delta u_c + d_3 \Delta \theta_{\text{Ud.crl}} + d_4 \Delta \theta_{\text{PLL}}
\]
\[
\Delta i_c = d_5 \Delta u_{\text{ref}} + d_6 \Delta u_c + d_7 \Delta \theta_{\text{Ud.crl}} + d_8 \Delta \theta_{\text{PLL}}
\]
\[
\Delta u_r = d_9 \Delta u_{\text{ref}} + d_{10} \Delta u_c + d_{11} \Delta \theta_{\text{Ud.crl}} + d_{12} \Delta \theta_{\text{PLL}}
\]

where,
\[
d_1 = b_2 c_1
\]
\[
d_2 = b_1 + b_2 c_2
\]
\[
d_3 = b_2 c_3
\]
\[
d_4 = b_2 c_4
\]
\[
d_5 = b_4 c_1
\]
\[
d_6 = b_3 + b_2 c_2
\]
\[
d_7 = b_4 c_3
\]
\[
d_8 = b_4 c_4
\]
\[
d_9 = b_6 c_1
\]
\[
d_{10} = b_5 + b_2 c_2
\]
\[
d_{11} = b_6 c_3
\]
\[
d_{12} = b_6 c_4
\]

It is convenient to include the variable \( \Delta u_r \) as a new dependent variable. This can be made by solving for \( \Delta u_r \) in (8-9) and combing it with (8-12). Accordingly,
\[
\Delta u_r = \frac{1}{b_2} \left( d_1 \Delta u_{\text{ref}} + d_2 \Delta u_c + d_3 \Delta \theta_{\text{Ud.crl}} + d_4 \Delta \theta_{\text{PLL}} \right) - \frac{b_1}{b} \Delta u_c
\]

This equation can be re-written with a new set of coefficients. Namely,
\[
\Delta u_r = d_{13} \Delta u_{\text{ref}} + d_{14} \Delta u_c + d_{15} \Delta \theta_{\text{Ud.crl}} + d_{16} \Delta \theta_{\text{PLL}}
\]

where,
\[
d_{13} = \frac{d_1}{b_2}
\]
\[
d_{14} = \frac{d_2}{b_2} - \frac{b_1}{b_2}
\]
\[
d_{15} = \frac{d_3}{b_2}
\]
\[
d_{16} = \frac{d_4}{b_2}
\]

All the complex transfer functions have now been derived. At this point all the independent variables are formally considered complex-valued, even the variables \( \Delta \theta_{\text{Ud.crl}} \) and \( \Delta \theta_{\text{PLL}} \). Later, only the real part of these two variables will be used.
8.2.5 Transfer functions between components

All the equations are written in the $d - q$ reference frame. To be able to resolve them separately into $d$- and $q$-components, it is necessary to calculate the individual components of each transfer function defined in (8-12) and (8-14), considering the $\alpha - \beta$ to $d - q$ transformation formulas given by

\[
\begin{align*}
d_{x}^{dd}(j\Omega) &= \frac{d_x(j\omega^+) + \left(d_x(j\omega^-)\right)}{2} \\
d_{x}^{dq}(j\Omega) &= \frac{d_x(j\omega^+) - \left(d_x(j\omega^-)\right)}{2j}
\end{align*}
\]  

(8-15)

where,

\[
\begin{align*}
\omega^+ &= \omega_N + \Omega \\
\omega^- &= \omega_N - \Omega
\end{align*}
\]

As $\Delta\theta_{ld_{ref}}$ and $\Delta\theta_{PLL}$ are real-valued quantities, only the $d$-component is used and its $q$-component is zero. Then the following matrices of transfer functions are obtained

\[
\begin{align*}
\begin{bmatrix}
\Delta i_{ld} \\
\Delta i_{q} \\
\Delta i_{cl} \\
\Delta i_{cq} \\
\Delta u_{ld} \\
\Delta u_{q} \\
\Delta u_{cld} \\
\Delta u_{cq}
\end{bmatrix}
= G_U \begin{bmatrix}
\Delta u_{ld_{ref}} \\
\Delta u_{q_{ref}} \\
\Delta u_{cld} \\
\Delta u_{cq}
\end{bmatrix} + G_x \Delta\theta_{ld_{ref}} + G_Y \Delta\theta_{PLL}
\end{align*}
\]

(8-16)

where,

\[
G_U = \begin{bmatrix}
d_1^{dd} & d_1^{dq} & d_2^{dd} & d_2^{dq} \\
d_3^{dd} & d_3^{dq} & d_4^{dd} & d_4^{dq} \\
d_5^{dd} & d_5^{dq} & d_6^{dd} & d_6^{dq} \\
d_7^{dd} & d_7^{dq} & d_8^{dd} & d_8^{dq} \\
d_9^{dd} & d_9^{dq} & d_{10}^{dd} & d_{10}^{dq} \\
d_{11}^{dd} & d_{11}^{dq} & d_{12}^{dd} & d_{12}^{dq} \\
d_{13}^{dd} & d_{13}^{dq} & d_{14}^{dd} & d_{14}^{dq} \\
d_{15}^{dd} & d_{15}^{dq} & d_{16}^{dd} & d_{16}^{dq}
\end{bmatrix},
\]

\[
G_x = \begin{bmatrix}
d_4^{dd} \\
d_4^{dq} \\
d_8^{dd} \\
d_8^{dq}
\end{bmatrix}
\]

and

\[
G_Y = \begin{bmatrix}
d_4^{dd} \\
d_4^{dq} \\
d_8^{dd} \\
d_8^{dq}
\end{bmatrix}
\]
8.2.6 The PLL

Similar to the model described in Chapter 5, the PLL operates on the filter bus voltage. If the angle $\Delta \theta_{PLL}(t)$ is the phase of the PLL coordinate system in relation to the undisturbed coordinate system, this angle enters into the PLL frequency regulator, which is an ordinary PI controller, providing the PLL argument. The total transfer function is given by

$$
\Delta \theta_{PLL} = K_{PLL} \left( \frac{\Delta u_{Fq}}{u_{F0}} - \Delta \theta_{PLL} \right) = \frac{K_{PLL}}{1 + K_{PLL}} \frac{\Delta u_{Fq}}{u_{F0}} = K_{PLL} \Delta u_{Fq}
$$

and $K_{PLL}$ represents the dynamics of the PLL, that has the following form

$$
K_{PLL} = \frac{k_{PLL}(1 + j\Omega t_{PLL})}{j\Omega t_{PLL}} \frac{1}{j\Omega}
$$

With the use of the following auxiliary matrix,

$$
P_{CS} = \begin{bmatrix} 0 & 0 & 0 & 0 & K_{CL}^{PLL} & 0 & 0 \end{bmatrix}
$$

Eq. (8-16) becomes

$$
\begin{pmatrix}
\Delta i_{ld} \\
\Delta i_{q} \\
\Delta i_{d} \\
\Delta i_{c} \\
\Delta u_{Fq} \\
\Delta u_{Vq} \\
\Delta u_{Vd}
\end{pmatrix} = \begin{pmatrix}
\Delta u_{Fq}^{ref} \\
\Delta u_{Fq}^{ref} \\
\Delta u_{Fq}^{ref} \\
\Delta u_{Fq}^{ref} \\
\Delta u_{Fq}^{ref} \\
\Delta u_{Fq}^{ref} \\
\Delta u_{Fq}^{ref}
\end{pmatrix} + G_{U} \Delta \theta_{U_{dc,ref}} + G_{V} P_{CS} \begin{pmatrix}
\Delta i_{ld} \\
\Delta i_{q} \\
\Delta i_{d} \\
\Delta i_{c} \\
\Delta u_{Fq} \\
\Delta u_{Vq} \\
\Delta u_{Vd}
\end{pmatrix}
$$

8.2.7 Solution for the DC side and including the DC-side voltage controller

Solution for the DC side

Similar to the model described in Chapter 5, the solution for the DC side can be made considering that the active powers on both the AC and DC sides of the converter are equal. Thus,

$$
u_{dc} i_{dc} = \frac{3}{2} \text{Re}(u_{dc}^{*} i_{dc}^{*}) = \frac{3}{2} u_{dc} i_{d} + \frac{3}{2} u_{dc} i_{q}
$$

The DC side of the converter is characterized by its impedance $z_{dc}$, and if the VSC is operating as a STATCOM, then this impedance is characterized by the DC link capacitor. Then the following applies

$$
\Delta u_{dc} = -z_{dc} \Delta i_{dc}
$$
Making a general differentiation of Eq. (8-20) and using the expression (8-21), the DC voltage and DC current are found to be

$$
\begin{align*}
\Delta i_{dc} &= \frac{3}{2} \left( \frac{u_{d0}}{u_{d0} - z_{dc} i_{d0}} \right) \left( u_{d0} \Delta i_{vd} + \Delta u_{d0} i_{d0} + u_{q0} \Delta i_{vq} + \Delta u_{q0} i_{q0} \right) \\
\Delta u_{dc} &= \frac{3}{2} \left( \frac{z_{dc}}{u_{d0} - z_{dc} i_{d0}} \right) \left( u_{d0} \Delta i_{vd} + \Delta u_{d0} i_{d0} + u_{q0} \Delta i_{vq} + \Delta u_{q0} i_{q0} \right)
\end{align*}
$$

or in matrix form

$$
\begin{align*}
\Delta i_{dc} &= \begin{bmatrix} r_1^1 & r_2^1 \end{bmatrix} \begin{bmatrix} \Delta i_{vd} \\ \Delta i_{vq} \end{bmatrix} + \begin{bmatrix} r_1^2 & r_2^2 \end{bmatrix} \begin{bmatrix} \Delta u_{vd} \\ \Delta u_{vq} \end{bmatrix} \\
\Delta u_{dc} &= \begin{bmatrix} r_1^3 & r_2^3 \end{bmatrix} \begin{bmatrix} \Delta i_{vd} \\ \Delta i_{vq} \end{bmatrix} + \begin{bmatrix} r_1^4 & r_2^4 \end{bmatrix} \begin{bmatrix} \Delta u_{vd} \\ \Delta u_{vq} \end{bmatrix}
\end{align*}
$$

The elements of the coefficient matrices are given by

<table>
<thead>
<tr>
<th>$r_1^1$</th>
<th>$r_2^1$</th>
<th>$r_1^2$</th>
<th>$r_2^2$</th>
<th>$r_1^3$</th>
<th>$r_2^3$</th>
<th>$r_1^4$</th>
<th>$r_2^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{2} \frac{u_{d0}}{u_{d0} - z_{dc} i_{d0}}$</td>
<td>$\frac{3}{2} \frac{u_{q0}}{u_{d0} - z_{dc} i_{d0}}$</td>
<td>$\frac{3}{2} \frac{i_{d0}}{u_{d0} - z_{dc} i_{d0}}$</td>
<td>$\frac{3}{2} \frac{i_{q0}}{u_{d0} - z_{dc} i_{d0}}$</td>
<td>$\frac{3}{2} \frac{u_{d0}}{u_{d0} - z_{dc} i_{d0}}$</td>
<td>$\frac{3}{2} \frac{u_{q0}}{u_{d0} - z_{dc} i_{d0}}$</td>
<td>$\frac{3}{2} \frac{i_{d0}}{u_{d0} - z_{dc} i_{d0}}$</td>
<td>$\frac{3}{2} \frac{i_{q0}}{u_{d0} - z_{dc} i_{d0}}$</td>
</tr>
</tbody>
</table>

### Dynamics of the DC voltage regulator

The DC voltage is entered into the DC voltage regulator, which is an ordinary PI regulator, acting on the argument of the output voltage of the VSC. Thus,

$$
\Delta \theta_{d, ctrl} = -K^{Udc} \left( \Delta u_{dc\text{ ref}} - \Delta u_{dc} \right)
$$

where $K^{Udc}$ represents the dynamics of the PI regulator, given by

$$
K^{Udc} = \frac{k^{Udc}}{j \Omega t^{Udc}}
$$
Making use of (8-23), Eq. (8-24) can be re-written as
\[
\Delta\theta_{Udc,ref} = -K^{Udc} \cdot \left( \begin{array}{c} \Delta u_{dc,ref} - \left( r_1^3 \right) \left( \Delta i_{vd} \right) - \left( r_2^3 \right) \left( \Delta i_{iq} \right) \\ \Delta i_{vd} \\ \Delta i_{iq} \\ \Delta i_{cd} \\ \Delta i_{cq} \\ \Delta u_{fd} \\ \Delta u_{fq} \\ \Delta u_{fd} \\ \Delta u_{fq} \end{array} \right) \]
\]
(8-26)

Eq. (8-26) can be re-written by defining new coefficient matrices:
\[
\Delta\theta_{Udc,ref} = n_1 \Delta u_{dc,ref} + n_2 \left( \Delta i_{vd} \right) + n_3 \left( \Delta u_{vd} \right)
\]
(8-27)

where,
\[
n_1 = -K^{Udc} \\
\left[ n_2 \right] = +K^{Udc} \cdot \left( r_1^3 \right) \\
\left[ n_3 \right] = +K^{Udc} \cdot \left( r_1^4 \right)
\]

Now, (8-19) and (8-27) can be combined eliminating the variable \( \Delta\theta_{Udc,ref} \).

Accordingly,
\[
\left( \begin{array}{c} \Delta i_{vd} \\ \Delta i_{iq} \\ \Delta i_{cd} \\ \Delta i_{cq} \\ \Delta u_{fd} \\ \Delta u_{fq} \\ \Delta u_{vd} \\ \Delta u_{iq} \end{array} \right) = G_U \left( \begin{array}{c} \Delta u_{fd,ref} \\ \Delta u_{cd} \\ \Delta u_{cq} \end{array} \right) + G_x \left[ n_1 \Delta u_{dc,ref} + n_2 \left( \Delta i_{vd} \right) + n_3 \left( \Delta u_{vd} \right) \right] + G_f \left( \begin{array}{c} \Delta i_{vd} \\ \Delta i_{iq} \\ \Delta i_{cd} \\ \Delta i_{cq} \\ \Delta u_{fd} \\ \Delta u_{fq} \\ \Delta u_{vd} \\ \Delta u_{iq} \end{array} \right)
\]
(8-28)

A new independent variable \( \Delta u_{dc,ref} \) is included in the set of equations, and the following auxiliary matrices are introduced:
\[
P_{AS} = \left( \begin{array}{cccc} 0 & 0 & 0 & n_1 \end{array} \right)
\]
\[
P_{BS} = \left( \begin{array}{cccc} n_2(1) & n_2(2) & 0 & 0 & 0 & n_3(1) & n_3(2) \end{array} \right)
\]
\[
G_{UL} = \left( \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right)
\]
Eq. (8-28) can now be rewritten as

\[
\begin{pmatrix}
\Delta i_{vd} \\
\Delta i_{vq} \\
\Delta i_{cd} \\
\Delta i_{cq} \\
\Delta u_{fd} \\
\Delta u_{fq} \\
\Delta u_{id} \\
\Delta u_{iq}
\end{pmatrix} = G_{UL} \begin{pmatrix}
\Delta u_{Fd\text{ref}} \\
\Delta u_{Fq\text{ref}} \\
\Delta u_{Cd} \\
\Delta u_{Cq} \\
\Delta u_{dc\text{ref}}
\end{pmatrix} + G_{X} P_{A5} \begin{pmatrix}
\Delta u_{Fd\text{ref}} \\
\Delta u_{Fq\text{ref}} \\
\Delta u_{Cd} \\
\Delta u_{Cq} \\
\Delta u_{dc\text{ref}}
\end{pmatrix} + P_{BS} \begin{pmatrix}
\Delta i_{vd} \\
\Delta i_{vq} \\
\Delta i_{cd} \\
\Delta i_{cq} \\
\Delta u_{fd} \\
\Delta u_{fq} \\
\Delta u_{id} \\
\Delta u_{iq}
\end{pmatrix}
\]

\[\text{Eq. (8-29)}\]

The solution of Eq. (8-29) represents the closing of the AC voltage control loop, the PLL loop and the DC voltage control loop. The solution of Eq. (8-29) is given by

\[
\begin{pmatrix}
\Delta i_{vd} \\
\Delta i_{vq} \\
\Delta i_{cd} \\
\Delta i_{cq} \\
\Delta u_{fd} \\
\Delta u_{fq} \\
\Delta u_{id} \\
\Delta u_{iq}
\end{pmatrix} = \text{inv} (\text{eye}(8) - G_{X} P_{BS} - G_{Y} P_{CS}) \begin{pmatrix}
\Delta u_{Fd\text{ref}} \\
\Delta u_{Fq\text{ref}} \\
\Delta u_{Cd} \\
\Delta u_{Cq} \\
\Delta u_{dc\text{ref}}
\end{pmatrix}
\]

\[\text{Eq. (8-30)}\]

where the desired matrix of transfer function is

\[
TF = \text{inv} (\text{eye}(8) - G_{X} P_{BS} - G_{Y} P_{CS})
\]

\[\text{Eq. (8-31)}\]

It is convenient to expand Eq. (8-30) with a new dependent variable \(\Delta u_{dc}\). This can easily be made by the used of Eq. (8-23).
Section 8.3 FREQUENCY DOMAIN RESPONSE

A 350 MVA STATCOM connected to a network having a short-circuit power of 500 MVA was modeled in MATLAB. The main circuit values and control signal parameters are given in Table 8.1.

Figure 8.3-(A) illustrates the closed loop transfer function for the AC voltage controller. The solid line is the closed loop transfer function for d-d component of the controller and the dashed line the q-q component. It should be noted that the analyzed quantities in the Bode plots are given in the fixed undisturbed rotating coordinate system, and not in the ‘PLL’ coordinate system.

Figure 8.3-(B) shows the closed loop transfer function of the DC voltage controller.

The control parameters were chosen such that the controller becomes insensitive to resonances in the network. Therefore, the bandwidth of either the AC or the DC voltage controller is quite low, indicating that the response to a disturbance or a change in the set point is not very fast.

As in Chapter 5, the influence of the sampling time of the discrete controls and the PWM modulation has been studied. Considering a PWM pulse number $p = 33$, an estimated total delay of $455 \mu \text{sec}$ was included in the realization of bridge AC voltage by the PWM. This represents $\frac{3}{2} T_s$, where $T_s$ is the sampling time of the computer.

The influence of this $455 \mu \text{sec}$ delay included in the model is shown in Figure 8.4.

The impact of the sampling time of the computer and the PWM modulation is considerably smaller as compared to the conventional model described in Chapter 5.
Table 8.1 Main circuit parameters and control system values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Converter rating</td>
<td>350 MVA</td>
</tr>
<tr>
<td>Source voltage</td>
<td>422.5 kV rms line-line voltage</td>
</tr>
<tr>
<td>Source impedance</td>
<td>$z_S = 62.0 + j357 \Omega/\text{ph} \ (500 \text{ MVA})$</td>
</tr>
<tr>
<td>Converter transformer impedance</td>
<td>$z_T = 2.049 + j61.3 \Omega/\text{ph} \ (0.004 + j0.12 \text{ pu})$</td>
</tr>
<tr>
<td>Shunt filter</td>
<td>$z_F = -j3498 \Omega/\text{ph} \ (y_F = j0.146 \text{ pu})$</td>
</tr>
<tr>
<td>Converter reactor</td>
<td>$z_V = 2.64 + j79.2 \Omega/\text{ph} \ (0.005 + j0.155 \text{ pu})$</td>
</tr>
<tr>
<td>PLL controller gains</td>
<td>$k^{\text{PLL}} = 20 \text{rad/ sec}/ \text{rad}$ and $t^{\text{PLL}} = 6 \text{sec}$</td>
</tr>
<tr>
<td>AC Voltage controller gain</td>
<td>$k^{\text{Uac}} = 0.02 \text{pu}/ \text{pu}$ and $t^{\text{Uac}} = 0.001 \text{sec}$</td>
</tr>
<tr>
<td>DC Voltage controller gains</td>
<td>$k^{\text{Udc}} = 0.1 \text{rad}/ \text{pu}$ and $t^{\text{Udc}} = 1 \text{sec}$</td>
</tr>
<tr>
<td>Time delay in $u_c$ measurement</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 8.3: Bode plot of the two basic controllers included in the VSC model

(A) Closed loop AC voltage controller, $\Delta u_{\text{ac}} / \Delta u_{\text{ac-ref}}$, the solid line corresponds to the d-d part and the dashed line to the q-q part of the AC voltage controller

(B) Bode plot of the closed loop DC Voltage controller, $\Delta u_{\text{dc}} / \Delta u_{\text{dc-ref}}$
Figure 8.4: Case having a 455 μsec delay for realization of bridge AC voltage by the PW
CHAPTER 9

IMPROVING THE PERFORMANCE OF THE HVDC TRANSMISSION SYSTEM HAVING THE VSC WITH MODIFIED CONTROLS

In Chapter 7 it was shown that a VSC-STATCOM with a conventional type of control did not show benefits in supporting the operation of the inverter of a line-commutated HVDC transmission system. The VSC did neither enhance the transient conditions of the converter AC bus voltage nor the control of the AC voltage.

The low resonance frequency of the filter capacitors at the converter station and the weak connected AC system limits the performance of both the inner AC current controller and the outer-loop AC and DC voltage controllers of the VSC-STATCOM. The low bandwidth of these controllers did not allow the VSC converter be sufficiently fast to support the operation of the line-commutated HVDC inverter.

In Chapter 8 a new control strategy for the VSC-STATCOM was proposed. In this alternative the controllers of the VSC converter were made comparably slow and the inner current control loop was removed.

With such a control strategy it is expected that the VSC would become less sensitive to the resonance in the AC network, and would impress a constant AC voltage behind the series reactor. This gives the VSC-STATCOM the characteristics of a constant AC voltage source behind the series inductance.

The performance of the modified VSC-STATCOM will be investigated in this chapter in two different applications.
9.1 APPLICATION 2: COMBINED OPERATION OF THE HVDC TRANSMISSION LINK AND A VSC WITH MODIFIED CONTROLS

Application 1 studied in Chapter 7 is now re-examined with the modified VSC control.

9.1.1 System setup

The system to be studied in this section is similar to that studied in Application 1, Section 7.3. It consists of a 1500 MW HVDC transmission combined with a 350 MVA VSC-STATCOM connected at the inverter side of the transmission.

The HVDC transmission system has a transmission line with a length of 800 km. The connected AC network at the rectifier side of the HVDC transmission link is relatively strong. Measured at the converter bus it provides a short-circuit power of 7500 MVA. At the inverter side the network is relatively weak, providing a short-circuit power of 3000 MVA. The main circuit parameters of the HVDC transmission link and corresponding control parameters are presented in Table 9.1.

A slow VSC operating as STATCOM is modeled according to the description in Chapter 8. This VSC is closely connected to the HVDC-LCC inverter bus. The VSC-STATCOM converter has a rating of 350 MVA. The main circuit parameters and control parameters are presented in Table 9.2.

The studied system is outlined in Figure 9.1.
### Table 9.1: System data and control parameters used for the HVDC LCC link

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Rectifier</th>
<th>Inverter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connected Network</td>
<td>7500 MVA, 85°</td>
<td>3000 MVA, 85°</td>
</tr>
<tr>
<td>Shunt filters</td>
<td>819 MVA</td>
<td>819 MVA</td>
</tr>
<tr>
<td>Converter transformer</td>
<td>2x892.5 MVA, leakage reactance = 0.16 pu</td>
<td></td>
</tr>
<tr>
<td>Smoothing reactor</td>
<td>0.29 H (each side of link)</td>
<td></td>
</tr>
<tr>
<td>Length of Transmission line L</td>
<td>800 km</td>
<td></td>
</tr>
<tr>
<td>Transmission line Resistance R</td>
<td>0.01864 Ω/km</td>
<td></td>
</tr>
<tr>
<td>Transmission line Inductance L</td>
<td>0.98888 mH/km</td>
<td></td>
</tr>
<tr>
<td>Transmission line Capacitance C</td>
<td>11.867 nF/km</td>
<td></td>
</tr>
<tr>
<td>Nominal DC quantities</td>
<td>500 kV ; 3.0 kA : 1500 MW</td>
<td></td>
</tr>
<tr>
<td>Operating DC Voltage</td>
<td>500 kV</td>
<td>500 kV</td>
</tr>
<tr>
<td>Operating DC Current</td>
<td>3.0 kA</td>
<td>3.0 kA</td>
</tr>
<tr>
<td>Firing (extinction) angle initial condition</td>
<td>15 deg</td>
<td>17 deg</td>
</tr>
<tr>
<td>Current Controller – Proportional-gain</td>
<td>0.63 rad/pu</td>
<td>-</td>
</tr>
<tr>
<td>Current Controller – Integral-gain</td>
<td>65.61 rad/pu</td>
<td>-</td>
</tr>
<tr>
<td>Voltage Controller – Proportional-gain</td>
<td>-</td>
<td>0.436 rad/pu</td>
</tr>
<tr>
<td>Voltage Controller – Integral-gain</td>
<td>-</td>
<td>15.87 rad/pu</td>
</tr>
<tr>
<td>Vol. Ctrl – Low Pass (smoothing filter)</td>
<td>-</td>
<td>16 msec</td>
</tr>
<tr>
<td>PLL – Gain $k_{PLL}$</td>
<td>20 (rad/s)/rad</td>
<td>5 (rad/s)/rad</td>
</tr>
<tr>
<td>PLL – Time Constant $t_{PLL}$</td>
<td>5 sec</td>
<td>5 sec</td>
</tr>
</tbody>
</table>
Chapter 9. Improving the performance of the HVDC transmission system with modified VSC control

Table 9.2: Details of VSC-STATCOM having modified controllers according to the description given in Chapter 8

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Converter rating</td>
<td>350 MVA</td>
</tr>
<tr>
<td>Converter transformer impedance</td>
<td>$z_T = 2.049 + j61.3 \ \Omega/\text{ph}$ (0.004 + j0.12 pu)</td>
</tr>
<tr>
<td>Shunt filter</td>
<td>$z_F = -j3498 \ \Omega/\text{ph}$ ($y_F = j0.146 \text{ pu}$)</td>
</tr>
<tr>
<td>Converter reactor</td>
<td>$z_V = 2.64 + j79.2 \ \Omega/\text{ph}$ (0.005 + j0.155 pu)</td>
</tr>
<tr>
<td>PLL controller gains</td>
<td>$k_{PLL}^{rad} = 20 \text{ rad/sec/ rad}$ and $t^{\text{licc}} = 6 \text{ sec}$</td>
</tr>
<tr>
<td>AC Voltage controller gain</td>
<td>$k^{licc} = 0.02 \text{ pu/ pu}$ and $t^{\text{licc}} = 0.001 \text{ sec}$</td>
</tr>
<tr>
<td>DC Voltage controller gains</td>
<td>$k^{Udc} = 0.1 \text{ rad/ pu}$ and $t^{\text{Udc}} = 1 \text{ sec}$</td>
</tr>
<tr>
<td>Time delay to emulate the PWM sampling time</td>
<td>455 $\mu$s</td>
</tr>
<tr>
<td>and Zero Order Hold</td>
<td></td>
</tr>
<tr>
<td>Time delay in $u_B$ measurement</td>
<td>0</td>
</tr>
</tbody>
</table>

As already mentioned, this VSC is slower than the one studied in Chapter 7 and has no fast inner current control loop. Only a DC voltage controller and an AC voltage controller are included in the basic control system. They have been adjusted in such a way that they provide reasonable step responses with respect to the control references (the response times are approximately 10-50 ms).

From the examination of the performance of the new VSC-STATCOM it is expected that the transient behavior would be almost equivalent to a constant source voltage behind an impedance given by the sum of the converter reactor impedance and converter transformer impedance. This is similar to a synchronous condenser where its transient behavior is given by a constant internal emf source behind the transient impedance of the machine and step-up transformer impedance. In order to illustrate this expected behavior, a faked setup case has been created, which consists of the same 3000 MVA inverter network in parallel with an additional network having short-circuit capacity of 1225 MVA.

This 1225 MVA network represents the 350 MVA-STATCOM. The 1225 MVA network corresponds to a fixed emf source voltage behind an equivalent impedance representing the reactor and transformer of the VSC.

Hence three cases will be compared:

- Case 1 (solid line shown in the diagrams) includes a 3000 MVA network (85 deg) connected at the inverter (SCR = 2) and the VSC-STATCOM disconnected;
9.1 Application 2: combined operation of the HVDC transmission link and a VSC with modified controls

- Case 2 (dashed line): similar to Case 1, which includes a 3000 MVA network (85 deg) connected at the inverter, but in this case a 350 MVA VSC-STATCOM is connected close to the inverter AC bus of the HVDC transmission;
- Case 3 (dotted line): a combined 3000 + 1225 MVA = 4225 MVA (85deg) network connected at the inverter (SCR = 2.82) representing the combined 3000 MVA network from Case 1 and the 1225 MVA network as an equivalent representation of the 350 MVA-STATCOM included in Case 2.

As the HVDC system includes a DC line, new poles and zeros are introduced in the closed loop control system. This is because a transmission line modal impedance is alternatively capacitive and inductive with increasing frequency.

In Figure 9.2 the magnitudes and arguments of the short circuit driving point impedance \( Z_k \) and the short circuit admittance \( G_k \) of the 800 km 500 kV DC line are plotted as functions of frequency. Smoothing reactors are connected at the terminations of the line (each smoothing reactor has the inductance 0.29 H). No DC filters are considered in the studied cases. The following electrical parameters are assumed for the DC line:

\[
R = 0.01864 \Omega / km \\
L = 0.98888 mH / km \\
C = 11.867 nF / km
\]

The fact that the DC side impedance seen by the converter includes resonances at different frequencies will introduce different poles and zeros in the closed loop control system of the HVDC-LCC transmission system, affecting the performance of the controllers. Figure 9.2 shows a resonance at approximately 70 Hz in the short circuit driving point impedance \( Z_k \) and at approximately 120 Hz in the short circuit admittance \( G_k \).

![Figure 9.2: Short circuit driving impedance and short circuit admittance of the DC system main circuit of the HVDC transmission](image-url)
Chapter 9. Improving the performance of the HVDC transmission system with modified VSC control

9.1.2 Frequency domain analysis

Please refer to Figures 9.3-9.10 and the summary of the results based on different performance criteria to evaluate the performance of the different case setups presented in Table 9.3.

Criteria 1 and 2: Gain and Phase margins of the closed loop rectifier DC current controller

Case 2 and Case 3 have increased phase margins as compared with Case 1. This is due to the fact that having a strong inverter (either by having a 4225 MVA network or a combined 3000 MVA network with a 350 MVA VSC-STATCOM) improves the performance of the rectifier current controller within its bandwidth (which is in the order 6-7 Hz).

However, the gain margin is not much influenced by having a stronger inverter. In fact, Case 2 and 3 have slightly lower gain margins as compared with Case 1. The reason is that the critical gain margin was measured at approximately 120 Hz, which coincides with a resonance in the short circuit admittance $G_k$. It also coincides with the resonance frequency between the network impedance connected to the AC side of the rectifier and the AC filter. When this resonance is demodulated by the converter, the frequency at the DC side is approximately 120 Hz. Having a strong inverter AC network connected to converter reduces the damping in the closed loop current control included in the rectifier controller.

This problem motivates the introduction of a low pass filter in the current control to reduce the gain of the controller around this frequency. With such a low pass filter it is probably necessary to have some kind of phase-compensation in order to prevent worsening the performance of the controller around its bandwidth region (this will be investigated in more detail in Section 9.3).

Criteria 3 and 4: Bandwidth defined by the crossover frequency of the rectifier DC current controller

The bandwidth defined by the crossover frequency of the current controller is approximately in the range of 6-7 Hz. It should be noted that the small difference that has been measured between the cases was obtained without retuning the DC current control parameters.

The Nyquist diagram corresponding to the open loop DC current control and the Bode plot of the closed DC current control indicate that it is possible to increase the gains of the DC current control in Case 2 and Case 3, but not that much in Case 1. Doing so, the bandwidths in Case 2 and 3 increase, speeding up the performance of the system.

It should be pointed out that to be able to increase the gains of the DC current controller the 120 Hz resonance problem should be resolved first. Otherwise, the control system would become unstable (see Section 9.3).
9.1 Application 2: combined operation of the HVDC transmission link and a VSC with modified controls

Criterion 5: Maximum peak value

There are two decisive peak values to be considered in the frequency response of the closed loop DC current control transfer function: one at the DC resonance frequency (which is affected by the AC resonance frequency between AC network and AC filters connected to the rectifier side); the other one, which occurs at a much lower frequency, is within the bandwidth of the DC current controller. In the former case the strength of the connected inverter AC network plays a stronger role in the stability of the controller.

Considering now only the low frequency maximum peak value, Case 1 (weaker inverter side AC side) shows a higher peak value. This means that, in case of applying steps in the control reference, the system will have higher overshoots in the DC current response as compared to Case 2 or Case 3. The reason is that Case 2 and Case 3 have stiffer inverter AC networks as compared to Case 1. This implies that they are less sensitive to variations in the DC current.

In relation to the maximum peak value around 129 Hz, Case 2 and Case 3 are more critical as compared to Case 1. This means that with a stronger inverter as in Case 2 and Case 3 the resonance condition at the rectifier AC network and DC line has less damping as compared with a case having a weaker network on the inverter side like in Case 1.

Criterion 6: Sensitivity Indices related to the inverter DC voltage, inv

\[ S_{1:U-I}^1 = \frac{\Delta U_{d-inv}}{\Delta I_{d-inv}} \quad \text{and power,} \quad S_{1:P-I}^2 = \frac{\Delta P_{d-inv}}{\Delta I_{d-inv}} \]

These indices show that a VSC-STATCOM can improve the operation of the inverter by strengthening the AC network. The critical value without the VSC connected is observed at 43 Hz in Case 1 and 60 Hz in both Cases 2 and 3. This shows that the resonance frequency between network and filter is moved to a higher frequency.

This result also indicates an improvement of the voltage/power stability conditions of the HVDC inverter.

Criterion 7: Sensitivity indices related to the rectifier DC voltage, rec

\[ S_{3:U-I}^3 = \frac{\Delta U_{d-rec}}{\Delta I_{d-ref}} \quad \text{and power,} \quad S_{3:P-I}^4 = \frac{\Delta P_{d-rec}}{\Delta I_{d-ref}} \]

These sensitivity indices indicate that Case 2 and Case 3, having a stiffer AC network connected to the inverter as compared to Case 1, show better performance. This can be concluded by observing amplitude values, where the variation of the DC voltage due to a variation in DC current is lower in Cases 2 and 3 up to the bandwidth of the current controller. This means that in order to obtain a certain DC current variation, the required variation in the driving voltage is lower when the inverter is connected to a stiffer AC voltage.
These indices also show that at about 120 Hz, due to the resonance problem in the DC line and resonance between connected AC impedance and AC filter impedance at the rectifier side in Cases 2 and 3, a poor performance is expected as compared to Case 1. A weak inverter AC network improves the damping around this resonance frequency.

Criteria 8 and 9: Sensitivity indices related to the inverter AC voltage,

\[ S_{I:Uc}^5 = \frac{\Delta u_{c-inv}}{\Delta I_{d-ref}} \quad \text{and} \quad S_{I:\theta c}^6 = \frac{\Delta \theta_{c-inv}}{\Delta I_{d-ref}} \]

These indices show that the VSC-STATCOM, when connected to the inverter side of the HVDC transmission, will make the AC system stiffer, resulting in lowered variations in either amplitude or argument of the AC voltage for all frequencies.

Conclusion

The conclusion is that the 350 MVA VSC-STATCOM with modified controllers, as described in Chapter 8, provides an enhancement of the system performance equivalent to a system having a short circuit power of 1225 MVA. This means that this 350 MVA-STATCOM will transiently behave like a synchronous condenser having a transient short circuit power of 1225 MVA.
### Table 9.3: Evaluating the performance; summary results observed in Application 2

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Performance Evaluation</th>
<th>Case VSC Disconnected</th>
<th>Case VSC Connected</th>
<th>Without VSC, 3500 MVA net</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Rectifier Current Control: Gain Margin ( GM )</td>
<td>1.17 (at 119 Hz)</td>
<td>0.98 (at 120 Hz)</td>
<td>1.01 (at 121 Hz)</td>
<td></td>
</tr>
<tr>
<td>2 Rectifier Current Control: Phase Margin ( PM )</td>
<td>57.6°</td>
<td>62.2°</td>
<td>63.4°</td>
<td></td>
</tr>
<tr>
<td>3 Rectifier Current Control: Crossover Frequency ( \omega_c )</td>
<td>6.0 Hz</td>
<td>7.2 Hz</td>
<td>7.0 Hz</td>
<td></td>
</tr>
<tr>
<td>4 Rectifier Current Control: Maximum Peak Value ( M_T )</td>
<td>1.28 (at 4.1 Hz)</td>
<td>1.17 (at 4.6 Hz)</td>
<td>1.16 (at 4.5 Hz)</td>
<td></td>
</tr>
<tr>
<td>5 Sensitivity Index: Inverter DC Voltage (nr. 1)</td>
<td>0.51 (at 43 Hz)</td>
<td>0.44 (at 60 Hz)</td>
<td>0.42 (at 61 Hz)</td>
<td></td>
</tr>
<tr>
<td>6 Sensitivity Index: Rectifier DC Voltage (nr. 3)</td>
<td>0.317 (at 5.4 Hz)</td>
<td>0.13 (at 7.0 Hz)</td>
<td>0.13 (at 6.5 Hz)</td>
<td></td>
</tr>
<tr>
<td>7 Sensitivity Index: Inverter AC Voltage, amplitude (nr. 5)</td>
<td>0.13 (at 1 Hz)</td>
<td>0.06 (at 1 Hz)</td>
<td>0.06 (at 1 Hz)</td>
<td></td>
</tr>
<tr>
<td>8 Sensitivity Index: Inverter AC Voltage, argument (nr. 6)</td>
<td>6.9° (at 1 Hz)</td>
<td>3.4° (at 1 Hz)</td>
<td>4.0° (at 1 Hz)</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 9.3:** Bode plot of the closed loop current controller \( \frac{\Delta i_d}{\Delta i_{d-ref}} \) of the rectifier: VSC is disconnected (solid line), VSC is connected (dashed line) and case without VSC but strength of the AC network increased from 3000 MVA to 4225 MVA (dotted line)
Chapter 9. Improving the performance of the HVDC transmission system with modified VSC control

Figure 9.4: Nyquist diagram of the open loop rectifier current controller $\frac{\Delta I_d}{\Delta \varepsilon}$: VSC is disconnected (solid line), VSC is connected (dashed line) and case without VSC but strength of the AC network increased from 3000 MVA to 4225 MVA (dotted line)

Figure 9.5: Sensitivity index nr. 1 $S_{1:U_{inv}} = \frac{\Delta U_{d-inv}}{\Delta I_{d-inv}}$: VSC is disconnected (solid line), VSC is connected (dashed line) and case without VSC but strength of the AC network increased from 3000 MVA to 4225 MVA (dotted line)
9.1 Application 2: combined operation of the HVDC transmission link and a VSC with modified controls

Figure 9.6: Sensitivity index nr. 2 $S_{I_{dc}-inv}^2 = \frac{\Delta P_{d-inv}}{\Delta I_{d-inv}}$: VSC is disconnected (solid line), VSC is connected (dashed line) and case without VSC but strength of the AC network increased from 3000 MVA to 4225 MVA (dotted line)

Figure 9.7: Sensitivity index nr. 3 $S_{R,U-ref}^3 = \frac{\Delta U_{d-ref}}{\Delta I_{d-ref}}$: VSC is disconnected (solid line), VSC is connected (dashed line) and case without VSC but strength of the AC network increased from 3000 MVA to 4225 MVA (dotted line)
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Figure 9.8: Sensitivity index 4 $S_{R,1}^4 = \frac{\Delta P_{d-rec}}{\Delta I_{d-ref}}$: VSC is disconnected (solid line), VSC is connected (dashed line) and case without VSC but strength of the AC network increased from 3000 MVA to 4225 MVA (dotted line)

Figure 9.9: Sensitivity index 5 $S_{U,c}^5 = \frac{\Delta U_{inv}}{\Delta I_{d-ref}}$: VSC is disconnected (solid line), VSC is connected (dashed line) and case without VSC but strength of the AC network increased from 3000 MVA to 4225 MVA (dotted line)
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Figure 9.10: Sensitivity index $S_{\theta:uc}^6 = \frac{\Delta \theta_{uc \rightarrow inv}}{\Delta I_{d-ref}}$: VSC is disconnected (solid line), VSC is connected (dashed line) and case without VSC but strength of the AC network increased from 3000 MVA to 4225 MVA (dotted line)

9.1.3 Time domain simulation - Performance of the VSC-STATCOM during a commutation failure of the HVDC transmission link

Section 9.1.2 has demonstrated that a slow VSC-STATCOM would behave like a synchronous condenser, improving the performance of the HVDC transmission link. The AC voltage support provided by the VSC converter can enhance the transient stability conditions of the HVDC inverter.

A question that is now raised is how the VSC-STATCOM behaves during disturbances in the AC system or during a commutation failure of the HVDC inverter. Commutation failure is one of the critical issues inherent in an LCC that strongly impacts the connected AC system. Below only on the commutation failure issue is considered.

This phenomenon has been studied by means of a time domain simulation. The same system described in the previous section has now been modeled in EMTDC/PSCAD. As this is a three-phase representation of the system, having detailed models of the converter valves, converter transformers, and the complete control system including the firing pulse generation, the transient behavior of the VSC-STATCOM can be verified. The model used for the VSC-STATCOM is according to that described in Chapter 8, still assuming that the PWM modulator is ideal, which perfectly compensates for the DC voltage variation.

In order to provoke a commutation failure in the inverter of the HVDC transmission link, a step in the argument of the AC source voltage is applied. A sudden shift in the phase angle of the AC voltage results in an insufficient voltage-time area during the commutation to complete the commutation, provoking the commutation failure.
Figure 9.11 below illustrates the dynamic behavior of both the HVDC transmission link and the VSC during the commutation failure.

Considering the dynamic performance of the VSC-STATCOM there are two key issues to be considered:

**Overcurrent during disturbance**

It was mentioned that, in the modified VSC-STATCOM the traditional fast inner current controller has been removed. The converter without the current control may be exposed to high current during disturbances in the AC system. This has been observed in the test case performed in EMTDC/PSCAD where a simple commutation failure in the HVDC inverter converter will heavily disturb the connected VSC converter, resulting in high overcurrents.

The result shows that in this particular setup case high overcurrent has been measured, which exceeded the nominal rating of the converter (0.673 kA, peak value). This type of disturbance should be considered when designing the converter valves, converter reactor and converter transformer.

In order to limit the overcurrent during disturbances one can consider increase the impedance of the converter reactor and converter transformer. However, this will limit the performance of the converter.

Other alternative to prevent high overcurrent is to temporary block the valves. This is a common protective action used for this type of converter. Within the normal current levels the converter operates with the normal controls; at high current levels the corresponding valves connected to phase where the high current is measured the valves are temporary blocked, preventing that they would be exposed to high stresses during these periods of high transient current.

This type of control or protective action will not affect the performance of the converter when the converter is operating under normal operating conditions. Only at the severe disturbance condition, when the temporary blocking takes place, the converter would not provide the required support.

**Control of the DC voltage within the design operating limits**

It is required that the DC voltage level should be maintained within acceptable level. In this case the gains of the controls have to be tuned to not result in high voltage excursion, say, within the $+20/−50\%$. This has been assumed just as a design criterion; however other levels can be used depending on the rating of the main circuit components.

The energy size of the DC capacitor plays an important role in this analysis. In this case, the DC capacitor was dimensioned to obtain a time constant of 10 ms. If smaller capacitor is used a faster control would be needed to maintain the DC voltage within the $+20/−50\%$ as a target.
9.2 APPLICATION 3: IMPROVING THE PERFORMANCE OF THE 1500 MW HVDC TRANSMISSION BY MODIFYING THE CURRENT CONTROLLER

9.2.1 General

In section 9.2 we have studied a 1500 MW HVDC transmission link connecting a strong rectifier AC network into a weak inverter AC network. A 350 MVA VSC-STATCOM has been included in the system electrically close connected to the inverter converter bus.

The analysis of different transfer function have been calculated in the frequency domain and it has been concluded that the VSC converter performs like a
Chapter 9. Improving the performance of the HVDC transmission system with modified VSC control

synchronous condenser machine in terms of transient performance, enhancing the performance of the inverter LCC converter.

However, a more detailed frequency analysis of those different transfer functions has shown that the HVDC transmission link still had a poor performance due to DC resonances frequencies in the system, which is also affected by the AC resonance existing between the AC system impedance and AC filters connected to the rectifier side of the HVDC transmission link. Due to these resonances the control gains have to be low, slowing down the regulators.

Let us recall how DC/AC system resonances may affect the operation and performance of the converter.

Due to the switching action of the converter a frequency transformation of voltage and current between the AC side and DC side is established. The converter can be seen as a modulator of the DC side oscillating quantities when they are transformed to the AC side. Assume that the carrier frequency $f_N$ is the fundamental frequency of the modulating voltage and that a modulation frequency $f_\Omega$ describes the DC side oscillation. New side-band frequencies $f_N \pm f_\Omega$ are generated in the AC side phase currents. The converse is also valid, that is, the converter acts as a demodulator for non-fundamental frequencies voltages on the AC side.

This frequency transformation means that an oscillation of the current at certain frequency on the DC side of the converter sees impedances at the corresponding side-band frequencies on the AC side of the converter.

In application 2, studied in section 9.1, the studied HVDC transmission link includes an 800 km long DC line. Measurement of the short circuit driving point impedance $kZ$ made on this transmission line including the smoothing reactor connected at each of its terminal shows a resonance at about 70 Hz, and about 120 Hz when measuring the short circuit admittance $kG$.

Now, looking at the AC side of the converter, the combined AC network and AC filters impedances can affect the DC side resonance frequencies. In that particular case the 7500 MVA AC network connected to the rectifier side of the HVDC transmission and the 800 MVA shunt filters has a resonance frequency at about 160 Hz. This resonance frequency, when transforming it to the DC of the converter, becomes very close to the frequency where the DC resonance frequency of the DC line occurred. The result shows that this high impedance due to AC side resonance strongly affects the DC side resonance frequency.

In a similar way, the 3000 MVA AC network connected to the inverter side of the HVDC transmission and the 800 MVA shunt filters have a resonance frequency at about 100 Hz. This corresponds to a side-band frequency for a 50 Hz oscillation on the DC side of the converter.

From this simple analysis of the main circuit elements connected to the both AC and DC side of the converter it was possible to predict possible constraints in the operation of system due to these resonance conditions. These constraints have been observed when calculating different transfer function in the frequency domain.
The objective of this section is to study the stability of the closed-loop of the Current Controller and also to evaluate and improve the closed-loop performance of the HVDC transmission link by compensating and tuning the controls, considering the impact of different resonance frequencies included in the AC and DC networks. Robust stability condition and robust performance will also be evaluated by assuming uncertainty in the connected AC system impedance on both rectifier and inverter side of the HVDC transmission link.

### 9.2.2 Improving the stability and performance by modifying the open loop transfer function

#### Standard PI current controller

Let us consider the system that has been studied in section 9.1: a strong AC network connected to the rectifier bus (short circuit power of 7500MVA) and a weak AC network connected to the inverter bus (short circuit power 3000 MVA). A 350 MVA VSC-STATCOM is connected close to the inverter bus. It has been shown in the previous section that this VSC converter performs like an infinite source behind an impedance that can transiently provide 1225 MVA of short circuit power. The system is outlined in Figure 9.12.

![Figure 9.12: Outline of the system studied](image)

The basic DC current controller used in the model consists of an ordinary PI regulator. In section 9.1 we verified poor gain and phase margins of the DC current controller due to the resonance condition in the main circuit components included in the AC and DC side.
We will now make an attempt to mitigate these resonance problems by modifying the current controller characteristics.

In order to design a more robust controller let us assume uncertainties in the connected AC system on both sides of the HVDC transmission link. Let us consider 4 different combinations: strong and weak rectifier AC network combined with strong and weak inverter AC network. The following 4 cases to be studied, which are listed below:

- **Case 1** – (solid line): Rectifier 7500 MVA; Inverter 2275 MVA+ 350VSC (this combined inverter system is equivalent to 3500 MVA network)
- **Case 2** – (dotted line): Rectifier 7500 MVA; Inverter 4775MVA+350VSC (equivalent to 6000 MVA)
- **Case 3** – (dashdot line): Rectifier 4500 MVA; Inverter 2275 MVA+ 350VSC (equivalent to 3500 MVA)
- **Case 4** – (dashed line): Rectifier 4500 MVA; Inverter 4775MVA+350VSC (equivalent to 6000 MVA)

The Bode plot of the closed-loop DC current controller and Nyquist plot of the open-loop current controller are presented in Figure 9.14 for the four different combinations of the AC network. In these cases a standard DC current Controller, that is, an ordinary PI regulator has been used.

Depending on which combination of connected AC network that is used, the following performance was evaluated from the closed loop transfer function (see Table 9.4):

- Bandwidth varies from 6 Hz to 9 Hz.
- Maximum peak value varies from 1.02 up to 1.23.

These figures show that current controller is not too fast; it also shows that it is not expected too high overshooting of the current response (see Figure 9.21) when a step in the DC current reference in the regulator is applied.

However, when observing the Nyquist plot for the open loop current controller (see Figure 9.14) one can see that the gain margin at about 120 Hz is very low. The system is close to an unstable condition. The critical cases are: Case 1, 2 and 3. The poor gain margin is due to the resonance condition of the DC line and the resonance condition between the connected AC network and the shunt filters at the rectifier converter bus. If this frequency is excited, for example, under system faults, it is expect severe oscillatory behavior would occur.

A possible way to mitigate the stability problem of the controller is to increase roll-off rate of regulator by adding Low-Pass filters in the controller, reducing the gain at the resonance frequencies, as we will do next.
9.2 Application 3: Improving the performance of the 1500 MW HVDC transmission by modifying the current controller

**DC current controller, alternative 1**

To improve the design of the current controller let us make use of the Nichols chart technique which was developed to aid both in understanding design and to support the regulator design.

In the Nichols chart the magnitude of the open-loop transfer function \( L = \frac{\Delta I_d}{\Delta \epsilon} \) is plotted versus its phase, similar to the concept of plotting the real and imaginary part of \( L \), which formed the basis for the Nyquist plots. However, in the Nichols chart a log scale is used for the magnitude. Then, contours of unity-feedback closed-loop transfer function, \( T = \frac{L}{1 + L} \) are included in the Nichols chart. The indicated contours are the closed-loop magnitude and phase in this relationship.

From the Nichols chart the following can be determined:

1) The bandwidth of a closed-loop system from the plot of the open-loop data on a Nichols chart by noting where the open-loop curve crosses the 0.70 contour (-3dB) of the closed loop magnitude and determining the frequency of the corresponding data point;

2) The resonant peak amplitude by noting the value of the magnitude of the highest closed-loop contour touched by the open-loop curve. The frequency associated with the magnitude and phase at the point of contact is usually referred to as the resonant frequency;

3) The gain margin by observing the value of the gain where the Nichols plot crosses the \(-180^\circ\) line;

4) The phase margin by observing the phase where the plot crosses the amplitude 1 line (0 dB).

A MATLAB program called `RegDsgn`, reference [22], provides an easy drawing of Nichols chart and includes a number of ‘controller links’ (Low Pass filters, Lead-Lag filters, Washout function, etc) that are used to define links that will be cascaded with transfer function ‘\( L \)’ in order to form or to improve the regulator that are under design. This MATLAB program has been used to calculated different alternatives of compensation to be included in the control to improve the performance of the system. The results will be presented in the following.

Figures 9.17-9.20 part (A) present the Nichols chart for the open-loop current controller for the four combinations of AC network configurations. In (A) it is assumed that the Standard PI regulator of the DC current control is used. It can be observed that the curve crosses the +6 dB contour of the closed-loop magnitude between 600 and 800 rad/s. This is the critical region that we need to look after.

Let us introduce Low-Pass filters to roll-off the control gain in this region, and at the same time try to avoid that the curve approaches the +3 dB contour in the 20 rad/s region. By improving the open-loop characteristics we can now speed up the controller by increasing the overall gain in order to increase the bandwidth of the
controller. The following compensation is suggested (here we designate as Compensation Alternative 1):

- Gain multiplication factor: 1.8
- Second order Low-Pass filter:
  - Characteristic frequency $\omega_{3dB} = 300 \text{ rad} / \text{s} \ (47.7 \text{ Hz})$
  - Damping ratio $\zeta = 0.5$

The second order Low-Pass has the following transfer function:

$$LP2 = \frac{1}{\left(\frac{s}{\omega_{3dB}}\right)^2 + 2 \zeta \frac{s}{\omega_{3dB}} + 1}$$

In Figures 9.17-9.20, Part (B) the Nichols chart for the open-loop current controller that includes the compensation according to Alternative 1 are shown. Now, the curve touches the -3 dB contour in the region between 600 and 800 rad/s, only in few combination of the AC network, indicating a significant improvement.

However, when analyzing Case 4, the curve tangents the +3 dB contour at about 100 rad/s, indicating that this is the new critical resonant frequency. The magnitude is little too high. This indicates a tendency of oscillatory behavior at this particular frequency.

To confirm the finding observed from Nichols chart we present in Figure 9.15 the Bode Plot of the new closed-loop transfer function and Nyquist Plot of the open-loop transfer function, having the original controller including the standard PI controller and the suggested Compensation Alternative 1. All 4 AC network configurations have been calculated. The results confirm that the proposed Compensation Alternative 1, will improve the performance, but still problems are expected with the controller in some of the AC network configurations. Therefore the solution was found not robust for all studied combinations of the AC network.

**DC current controller Alternative 2**

In this alternative (designated as Alternative 2) we are using the same principle as in alternative 1: we include Low Pass filter to roll-off the control gain to mitigate the resonance problem at about 120 Hz.

The drawback of using Low Pass filters is that we introduce phase lagging at lower frequencies degrading the performance of the control within its bandwidth. To compensate this phase lagging we combine the Low Pass filters with additional Lead-Lag filters, advancing the phase at lower frequency. Below details of the parameter settings of the proposed compensation Alternative 2:

- Gain multiplication factor: 2.2
- First order Low-Pass filter: $\omega_{3dB} = 175 \text{ rad} / \text{s} \ (27.9 \text{ Hz})$
Another first order Low-Pass filter: \( \omega_{3dB} = 1250 \text{ rad/s} \ (198.9 \text{ Hz}) \)

Lead-Lag filter:

- Characteristic frequency \( \omega_c = 50 \text{ rad/s} \ (7.96 \text{ Hz}) \)
- Frequency ratio \( \omega_{ratio} = 2 \)

where, a Low-Pass filter and a Lead-Lag filter have the following transfer functions:

\[
\begin{align*}
LP1 &= \frac{1}{s + \frac{\omega}{\omega_{3dB}}} \\
LL &= \frac{\frac{s}{\omega_c} + 1}{\frac{s}{\omega_c} + \phi} ; \quad \phi = \sqrt{\omega_{ratio}} , \text{ respectively}
\end{align*}
\]

Figure 9.13 below compares the frequency response of the three different controls: the original controller having the standard PI regulator (Solid Line), the PI regulator having the Compensation Alternative 1 (Dotted line) and finally the PI regulator having Compensation Alternative 2 (Dashed line) that we are now studying.
Figures 9.17-9.20, Part (C), present the Nichols chart for the open-loop current controller for each of the studied AC network configurations. The curve tangents the -6 dB contour in the region between 600 and 800 rad/s, which suggests better improvement as compared to Alternative 1.

The Nichols chart also shows that the curve passes far from the +3 dB contour for both studied Cases 1 and 4. This indicates that the resonance problem in the region of 100 rad/s observed in Alternative 1 is not a problem any longer.

To confirm the finding observed from Nichols chart we present in Figure 9.16 the Bode Plot of the new closed-loop transfer function and Nyquist Plot of the open-loop transfer function, having the PI controller with Compensation Alternative 2. The results confirm that the proposed Compensation Alternative 2, is the best choice since the performance has improved in all aspects. The compensation was also found robust: similar and acceptable performance can be observed in all combinations of the AC network.

Table 9.4 compares different measured quantities to qualify the performance between the three control designs that have been studied.

In order to confirm the frequency domain analysis time domain simulations were performed in EMTDC. Figure 9.21 compares the step response in Current Order between the three control designs. Table 9.5 summarizes the results of these time-domain simulation tests, showing improvement in the performance characteristic for the design with Compensation Alternative 2.

Figure 9.14: Frequency response characteristic assuming a Standard rectifier current controller using an ordinary PI regulator (without compensation)
9.2 Application 3: Improving the performance of the 1500 MW HVDC transmission by modifying the current controller

Figure 9.15: Frequency response characteristic for a compensated controller, **Compensation Alternative 1** (gain $G = 1.8$; a second order Low Pass filter with the following parameters: $\omega_{rad} = 300 \text{rad/s}$ and damping $\zeta = 0.5$)

Figure 9.16: Frequency response characteristic for a compensated controller, **Compensation Alternative 2** (gain $G = 2.2$; a Lead Lag filter with the following parameters: center frequency $\omega_c = 50 \text{rad/s}$ and frequency ratio $\omega_{ratio} = 2$)

Table 9.4: Summary of performance at different AC network configurations and control alternatives

<table>
<thead>
<tr>
<th>Ordinary PI regulator</th>
<th>$M_f$</th>
<th>$\omega_c$</th>
<th>$PM$</th>
<th>$GM$</th>
</tr>
</thead>
<tbody>
<tr>
<td>value Hz Hz deg.</td>
<td></td>
<td></td>
<td></td>
<td></td>
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### Symbols

$M_T$ : Maximum peak value of complementary sensitivity transfer function $T$

$\omega_c$ : Gain crossover frequency, used as closed-loop bandwidth

$PM$ : Phase Margin

$GM$ : Gain Margin
9.2 Application 3: Improving the performance of the 1500 MW HVDC transmission by modifying the current controller

(A): Standard PI control
Gain: 1

(B) Compensation Alt. 1
Gain: 1.8
LP2 filter:
$\omega_{3db} = 300 \text{ rad/s}$
$\zeta = 0.5$

(C) Compensation Alt. 2
Gain: 2.2
LP1 filter: $\omega_{3db} = 175 \text{ rad/s}$
LP1 filter: $\omega_{3db} = 1250 \text{ rad/s}$
LL filter:
$\omega_c = 50 \text{ rad/s}$
$\omega_{ratio} = 2$

Figure 9.17: Use the Nichols Diagram to improve the current controller loop; AC Network according to configuration 1 (strong rectifier against weak inverter)
Chapter 9. Improving the performance of the HVDC transmission system with modified VSC control

(A): Standard PI control
Gain: 1

(B) Compensation Alt. 1
Gain: 1.8
LP2 filter:
\[ \omega_{3db} = 300 \text{ rad/s} \]
\[ \zeta = 0.5 \]

(C) Compensation Alt. 2
Gain: 2.2
LP1 filter: \[ \omega_{3db} = 175 \text{ rad/s} \]
LP1 filter: \[ \omega_{3db} = 1250 \text{ rad/s} \]
LL filter:
\[ \omega_c = 50 \text{ rad/s} \]
\[ \omega_{ratio} = 2 \]

Figure 9.18: Use the Nichols Diagram to improve the current controller loop; AC Network according to configuration 2 (weak rectifier against strong inverter)
9.2 Application 3: Improving the performance of the 1500 MW HVDC transmission by modifying the current controller

(A): Standard PI control
Gain: 1

(B) Compensation Alt. 1
Gain: 1.8
LP2 filter:
ω_{3db} = 300 rad/s
ζ = 0.5

(C) Compensation Alt. 2
Gain: 2.2
LP1 filter: ω_{3db} = 175 rad/s
LP1 filter: ω_{3db} = 1250 rad/s
LL filter:
ω_\text{C} = 50 rad/s
ω_\text{ratio} = 2

Figure 9.19: Use the Nichols Diagram to improve the current controller loop; AC Network according to configuration 3 (weak rectifier against strong inverter)
Chapter 9. Improving the performance of the HVDC transmission system with modified VSC control

(A): Standard PI control
Gain: 1

(B) Compensation Alt. 1
Gain: 1.8
LP2 filter:
\( \omega_{3dB} = 300 \text{ rad/s} \)
\( \zeta = 0.5 \)

(C) Compensation Alt. 2
Gain: 2.2
LP1 filter:
\( \omega_{3dB} = 175 \text{ rad/s} \)
LP1 filter:
\( \omega_{3dB} = 1250 \text{ rad/s} \)
LL filter:
\( \omega_c = 50 \text{ rad/s} \)
\( \omega_{ratio} = 2 \)

Figure 9.20: Use the Nichols Diagram to improve the current controller loop; AC Network according to configuration 4 (weak rectifier against strong inverter)
9.2 Application 3: Improving the performance of the 1500 MW HVDC transmission by modifying the current controller

Case Configuration 1 (strong rectifier against weak inverter)

Case Configuration 2 (strong rectifier against strong inverter)

Standard Control

Compensation Alternative 1

Compensation Alternative 2

Figure 9.21: Time Domain simulation (step in current order) run in EMTDC/PSCAD comparing the performance between different control schemes
Chapter 9. Improving the performance of the HVDC transmission system with modified VSC control

Case Configuration 3 (weak rectifier against weak inverter)

Case Configuration 4 (weak rectifier against strong inverter)

Standard Control

Compensation Alternative 1

Compensation Alternative 2

Figure 9.22: Time Domain simulation (step in current order) run in EMTDC/PSCAD comparing the performance between different control schemes
9.2 Application 3: Improving the performance of the 1500 MW HVDC transmission by modifying the current controller

Table 9.5: Time Domain simulation (step in current order), summary of results

<table>
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<tr>
<th></th>
<th>Config. 1</th>
<th>Config. 2</th>
<th>Config. 3</th>
<th>Config. 4</th>
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<td>9 %</td>
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<td>Compensation Alternative 1</td>
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<tr>
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<td>Overshoot [%]</td>
<td>27 %</td>
<td>25 %</td>
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CHAPTER 10

SUMMARY OF THE THESIS, CONCLUSIONS AND FUTURE RESEARCH

10.1 SUMMARY AND CONCLUSIONS

In this thesis a study of the interaction between a Voltage Source Converter operating as a STATCOM and a HVDC transmission system with line-commutated current-source converters connected to a common bus to a weak inverter AC system has been performed.

The study was based on small signal analysis of the system in the frequency domain. A large number of transfer functions of the system have been derived and they have then been combined to analyze the system stability. Regulators of the converters have been developed in such a way that the interaction between the two converters can be used to enhance the system performance especially if the grid is weak.

From this, the following summary and conclusions can be made:

Chapters 3 and 4:

The derivation of the transfer functions was made using the space-vector concept. This gives a concise representation of the system as the zero-sequence is canceled when the neutral is not connected, resulting in that the representation of the three time-varying quantities can be represented with one space-vector complex variable.

The model of HVDC-LCC transmission link that has been implemented in MATLAB to calculate the values of the transfer function in the frequency domain using the transfer functions described in this thesis has been validated against time-domain simulations. Such comparisons have been performed both for individual transfer functions included in the model as well as for the overall transfer function of a two terminal system. Large deviations were obtained when it was assumed that the overlap angle remains constant irrespective of the variation of the DC side quantities or the control signal.
Then, the model was extended to deal with the varying overlap angle in order to bring the frequency-domain model into agreement with the results obtained from time-domain simulations. Additionally, the transfer functions of the Phase Locked Loop (PLL), which may impact the system stability, were also included. Typical transfer functions were calculated for the extended model and compared with time domain simulations and it was shown that the agreement was very good. It was concluded that, in order to get correct results, it is necessary to consider the variation of the overlap angle.

Chapter 5:

A frequency-domain model of a Voltage Source Converter (VSC) with current controller operating as a STATCOM was developed. The current control law was derived from the physical equation governing the current flow through the phase reactor of the VSC. A PI-controller was used to cope with the uncertainties in the control law and in the realization of the bridge voltage by the pulse width modulation (PWM). The model also includes the AC and DC voltage controllers.

The model was developed in the \( d - q \) frame, which is a rotating coordinate system, running synchronously with the undisturbed space-vector quantities representing the connected power system. The control relies upon alignment of the rotating \( d - q \) reference in relation to the measured voltage space vector at the filter bus. This alignment is made by the PLL.

Similar to the HVDC-LCC model, the model of the VSC-STATCOM was developed based on the use of the space vector concept.

The frequency domain model was tested against time-domain simulations made in EMTDC/PSCAD. The results show very good agreement between the transfer functions derived in the frequency domain and the ones extracted from the time-domain simulation. In the model the PWM modulation was assumed to be ideal. The finite sampling frequency was modeled as a time delay between the reference and the execution of the ideal modulation. It was shown that the time delay plays an important role for system stability.

Another important factor limiting the performance of the controllers is the resonance frequency due to the AC network impedance and the connected AC filters. The magnitude of the impedance at the resonance frequency and the associated phase shift limit the available bandwidths of the controllers.

Chapter 6:

The model of the VSC-STATCOM was integrated into the small-signal model of the HVDC-LCC, in order to provide the total transfer function of the complete system.

For each subsystem, transfer functions were derived, which express the current changes from each subsystem to the common bus as functions of the voltage changes on that same bus. When the VSC-STATCOM is operating with fixed
references it will appear to the HVDC-LCC as linear impedance connected to the common bus. This principle was used to derive the impact of the VSC-STATCOM on the control system for the HVDC-LCC.

As the VSC model was developed in the \( d-q \) reference frame, the HVDC-LCC model had to be extended by generalizing the expressions to the non-symmetrical conditions, in which case the HVDC-LCC voltage and current \( d \)- and \( q \)-components are related through a full matrix relation.

Application of this analysis technique to a monopolar HVDC transmission system connected to an AC system that includes a VSC-STATCOM proved that this small-signal model is accurate and can be used to perform control stability analysis of the system.

**Chapter 7:**

The conventional methods of studying voltage/power stability of HVDC-LCC transmission links connected to a weak inverter AC network, such as analysis of the Maximum Power Curve (MPC) and Voltage Sensitivity Factor (VSF), were extended by defining new sensitivity indices to be applied in the frequency domain. The performance of the system was evaluated using these new sensitivity indices in combination with the classical frequency domain performance measures (gain and phase margins, bandwidths, and maximum peak values measured on the transfer functions of the feedback loop controllers).

Studies of the impact of the VSC-STATCOM with inner AC current control on the transient performance of the inverter of the HVDC-LCC transmission link showed that the influence was very small. The reason was that the converter was operating in an AC system having a low resonance frequency. This resonance was due to the presence of large amounts of filter capacitors in the HVDC-LCC system combined with the high network impedance. This resonance condition imposes severe limitations on the bandwidth of the VSC controllers, reducing their ability to properly control the AC voltage and give the necessary support to the LCC. The results showed that in reality the VSC slightly degraded the performance of the LCC as the VSC was very sensitive to the network resonance condition.

It thus was concluded that the VSC-STATCOM with inner current control cannot improve the transient performance of the HVDC transmission system.

**Chapter 8:**

A new control strategy for the VSC-STATCOM was proposed, which does not include the conventional inner AC current control loop.

In the proposed modified control system for the VSC converter the reference value of the converter bridge AC voltage obtained from the feedback AC voltage controller is directly realized with the PWM modulator. This voltage is synchronized with the filter bus voltage via the PLL. The DC voltage control is carried out by controlling the phase of the voltage produced by the VSC-STATCOM.
In this chapter the development of the VSC model with the modified control described above was described.

The control parameters for the VSC were chosen such that the VSC became insensitive to the resonance conditions in the AC network. The result was that the controllers had to have quite low bandwidths. Having such low bandwidths the influence of the 455 μsec time delay to simulate the PWM and sampling time of the discrete controllers was considerably smaller as compared to the conventional VSC model with current control as described in Chapter 5.

Chapter 9:

1) Tests of the new VSC-STATCOM with modified controllers described in Chapter 8 were performed in this chapter. With the new control strategy implemented in the VSC it became insensitive to the resonance in the AC network, impressing a constant AC voltage behind the series reactor. This gives a characteristic of constant voltage source converter on the DC side of the converter and a constant AC voltage source behind the series inductance. The results showed that a 350 MV A VSC performed like a synchronous condenser having a transient short circuit power of 1225 MVA. This means that a 1500 MW HVDC-LCC transmission link, if connected to a 3000 MVA inverter AC network will behave as if it had been connected to a system with a 4225 MVA short circuit capacity.

The performance of this system that has been evaluated in the frequency domain, and has been confirmed in time domain simulations made in EMTDC/PSCAD.

2) In the study of the modified VSC-STATCOM the traditional fast inner current controller was removed. The converter without the current control may be exposed to high currents during disturbances in the AC system. This was observed in time domain simulations, where a simple commutation failure in the HVDC-LCC converter heavily disturbed the closely connected VSC, resulting in high overcurrents. However, if the converter includes overcurrent protection this would prevent damage of the converter valves.

3) The small-signal models of both the HVDC-LCC transmission link and the VSC-STATCOM give new opportunities to develop and optimize the controllers of the system. In the final part of the thesis it was shown that the performance of a 1500 MW HVDC transmission was significantly improved by simple modification of its current controllers.
10.2 FUTURE RESEARCH

10.2.1 Feasibility of using the VSC-STATCOM and the VSC-HVDC control structure without the inner AC current control

In the thesis it was proposed to modify the basic control structure of the VSC. In the traditional control structure, the P/Q or Ud/Uac outer loop controllers are used as a reference for the inner AC current control loop. In the new proposed approach the inner AC current control is removed and the VSC voltage simply is obtained from the reference voltage (to be synthesized through the carrier based PWM) based on the P/Q or Ud/Uac controllers.

The problem with the inner current control loop is that having the VSC connected to a weak system, it is not possible to get a bandwidth of the controller that exceeds the parallel resonance frequency between the AC filters in the HVDC-LCC and the network impedance. This limits the performance of the VSC.

With the modified controls (without the inner current controller) the following benefits may be obtained:

- It becomes more robust in terms of different connected AC networks;
- It is possible to obtain reasonable performance with reduced pulse number in the PWM modulator.

However, since the VSC does not include a fast inner AC current control it becomes more vulnerable to overcurrents. This is the drawback that needs to be investigated very carefully.

The following studies should be made:

- Design a robust control structure assuming the new control concept design
- Evaluate the new control concept in either a VSC-HVDC transmission or in a VSC-STATCOM converter
- Propose different approaches to protect the converter from overcurrent during system faults
- Qualify the performance of the new VSC control concept by comparing results obtained with the traditional controllers
- Evaluate the impact on performance using different types of carrier based PWM
- Verify the impact of using different PWM pulse numbers

10.2.2 Analysis of resonance interaction between the AC and DC sides of the HVDC converters

This is a critical issue in the design of an HVDC transmission, in particular when there is a resonance at the second harmonic on the AC network combined with
fundamental frequency resonance on the DC side network. In such a situation there is a strong risk for core instability in the converter transformer during disturbances.

Usually the analysis of these resonances is made by means of time domain simulations like in EMTDC/PSCAD having a detailed modeling of the system and then faults are applied in the system to tune the controllers and design DC filters to cope with the resonances.

Having the frequency domain model of HVDC converters that was developed in this thesis it is then possible to study this phenomenon in the frequency domain, instead of with time domain simulations.

With the correct representation of the converters in the frequency domain it is possible to evaluate how the resonance conditions in the network impact the performance of the HVDC transmission.

### 10.2.3 AC/DC interaction in the VSC model

In the thesis the solution for the DC side of the VSC was obtained using power equations on the AC and DC sides respectively. Using this approach a very simple derivation was obtained. This implies, however, that the PWM modulator perfectly compensates for the DC voltage variation, and therefore, there is no direct interaction with the DC quantities. Consequently, the DC voltage will not appear in the description of the AC side dynamics. It also assumes that the DC voltage is sufficiently high to avoid limitations.

However, if one would like to study interaction between the AC and DC sides of the converter an alternative derivation was proposed, which is described in Appendix K.

The aim of the proposed study is to verify if this interaction can be made in the frequency domain, if this affects the PWM, and if this affects the performance of the VSC.

### 10.2.4 Modulation and sampling delays

In the thesis it is assumed that the PWM is ideal, calculating the desired AC voltage to be generated by the converter bridge. The influence of the sampling time of the computer and the PWM was included by introducing time delays in the equations.

The suggested delay used in the thesis was 455 $\mu$sec which is equivalent to $\frac{3}{2} T_i$, assuming a pulse number of $p = 33$, at 50 Hz.

The question that needs to be investigated is if this approach satisfies the modeling of different types of PWM patterns.
10.2.5  Extension of VSC-STATCOM to HVDC-VSC

In the thesis a VSC operating as STATCOM has been modeled. This model can be extended to an HVDC-VSC transmission link in a future work.

10.2.6  Sub-synchronous torsional interaction

Sub-synchronous torsional interaction (SSTI) is an instability phenomenon associated with synchronous machines, where torsional resonance in the turbine-generator shaft is destabilized through interaction with the network. Depending on the controllers used in the HVDC converters they may behave as a circuit with negative resistance in various frequency regions, including sub-synchronous. This will influence the damping of sub-synchronous oscillations of the shaft of the machines.

A way to analyze this phenomenon is by doing frequency scanning of the system, where the complex torque of the machine is analyzed. With the frequency domain models developed in this thesis for the HVDC-LCC transmission link and for the VSC it is now possible to perform analytically the sub-synchronous torsional interaction analysis between the converters and the machines in the frequency domain.
**APPENDIX A**

**CALCULATION OF TRANSFER FUNCTION $H_1$**

Assume that the DC current incorporates a small variation with amplitude $\mathcal{A}$ and frequency $\Omega$. Then

$$i(t) = \text{Re}[\mathcal{A}e^{j\Omega t}] = \frac{\mathcal{A}}{2}e^{j\Omega t} + \frac{\mathcal{A}^*}{2}e^{-j\Omega t} \quad (A-1)$$

Now recall that the AC side current can be calculated using the conversion function $\mathcal{K}'(t)$

$$\Delta i_n(t) = \frac{2}{3} \mathcal{K}'(t) \cdot \Delta i_d(t) \quad (A-2)$$

with

$$\mathcal{K}'(t) = \ell' e^{j\omega_n t} \quad (A-3)$$

Combining these equations the result is

$$\Delta i_n(t) = \frac{\mathcal{A}}{3} \ell' e^{j\omega'_n t} + \frac{\mathcal{A}^*}{3} \ell' e^{j\omega''_n t} \quad (A-4)$$

For practical reasons, the following notation have been used

$$\begin{cases} 
\omega' = \omega_n + \Omega \\
\omega'' = \omega_n - \Omega 
\end{cases} \quad (A-5)$$

The current is injected into the network impedance, which has the following impedances at the positive and negative sidebands

$$\begin{align*}
Z' &= Z_i(j\omega') + Z_c(j\omega') \\
Z'' &= Z_i(j\omega'') + Z_c(j\omega'') 
\end{align*} \quad (A-6)$$

As a result the following AC side voltage variation is obtained
\[ \Delta \tilde{u}_d(t) = -\left( \frac{A}{3} Z^+ \ell^t e^{j\omega t} + \frac{A^*}{3} Z^- \ell^t e^{j\omega t} \right) \]  

(A-7)

However, a change in the AC side voltage will be reflected back into the DC side according to the relation

\[ \Delta u_d = \text{Re} \left[ \left( K^U(t) \right)^* \Delta \tilde{u}_r(t) \right] \]

\[ K^U(t) = \ell^U e^{j\omega t} \]  

(A-8)

The variation of the DC voltage can be written as

\[ \Delta u_d(t) = -\text{Re} \left\{ \left( \ell U^* e^{-j\omega t} \left( \frac{A}{3} Z^+ \ell^t e^{j\omega t} + \frac{A^*}{3} Z^- \ell^t e^{j\omega t} \right) + \frac{A^*}{3} \ell U^* Z^- \ell^t e^{-j\omega t} \right) \right\} = \]

\[ = -\text{Re} \left\{ \frac{A}{3} \ell U^* Z^+ \ell^t e^{j\omega t} + \frac{A^*}{3} \ell U^* Z^- \ell^t e^{-j\omega t} \right\} \]  

(A-9)

Noting that any complex number and its conjugate have identical real parts it is found that

\[ \Delta u_d(t) = -\text{Re} \left\{ \frac{A}{3} \ell U^* Z^+ \ell^t e^{j\omega t} + \frac{A^*}{3} \ell U^* Z^- \ell^t e^{-j\omega t} \right\} = \]

\[ = -\text{Re} \left\{ \frac{1}{3} (\ell U^* Z^+ \ell^t + \ell U^* Z^- \ell^t) A e^{j\omega t} \right\} \]  

(A-10)

The expression yields the transfer function from the DC side current to the DC side voltage in the converter

\[ H_c(j\Omega) = -\frac{1}{3} (\ell U^* Z^+ \ell^t + \ell U^* Z^- \ell^t) \]

\[ Z^+ = Z_L^t(j\omega^+ ) + Z_C(j\omega^+ ) \]

\[ Z^- = Z_L^t(j\omega^- ) + Z_C(j\omega^- ) \]  

(A-11)
APPENDIX B

CALCULATION OF TRANSFER FUNCTION $H_2$

Assume that the overlap angle incorporates a small variation

$$\Delta \mu = \text{Re} \left\{ \Delta \hat{\mu} e^{j\alpha} \right\} = \frac{\Delta \hat{\mu}}{2} e^{j\alpha} + \frac{\Delta \hat{\mu}^*}{2} e^{-j\alpha}$$  \hspace{1cm} (B-1)

The varying overlap angle causes the following AC current variation

$$\Delta I_v = I_{d0} \frac{\partial R_l}{\partial \mu} \Delta \mu = I_{d0} \left( \frac{\partial \ell^l}{\partial \mu} \Delta \hat{\mu} e^{j\alpha} + \frac{\partial \ell^l}{\partial \mu} \Delta \hat{\mu}^* e^{-j\alpha} \right)$$  \hspace{1cm} (B-2)

This current is injected into the AC network. The resulted AC voltage variation is given by

$$\Delta U_v = -\frac{1}{3} I_{d0} \left( \frac{\partial \ell^l}{\partial \mu} Z^* \Delta \hat{\mu} e^{j\alpha} + \frac{\partial \ell^l}{\partial \mu} Z \Delta \hat{\mu}^* e^{-j\alpha} \right)$$

The corresponding DC side voltage variation is calculated from

$$\Delta U_d = -\text{Re} \left\{ \frac{1}{3} I_{d0} \left( e^{U^*} Z^* \frac{\partial \ell^l}{\partial \mu} + e^{U^*} Z \frac{\partial \ell^l}{\partial \mu} \right) \Delta \hat{\mu} e^{j\alpha} \right\}$$  \hspace{1cm} (B-3)

The transfer function is immediately obtained from this expression

$$H_2(j\omega) = -\frac{1}{3} I_{d0} \left( e^{U^*} Z^* \frac{\partial \ell^l}{\partial \mu} + e^{U^*} Z \frac{\partial \ell^l}{\partial \mu} \right)$$

$$Z^+ = Z'_l(j\omega^*) + Z_c(j\omega^*)$$  \hspace{1cm} (B-4)

$$Z^- = Z'_l(j\omega^-) + Z_c(j\omega^-)$$


**APPENDIX C**

**CALCULATION OF TRANSFER FUNCTION H₃**

From a small variation applied to the overlap angle results the following variation of the voltage conversion function

\[
\Delta \tilde{K}^U(\theta) = \left( \frac{\partial \ell^U}{\partial \mu} \frac{\Delta \tilde{\mu}}{2} e^{j\omega t} + \frac{\partial \ell^U}{\partial \mu} \Delta \tilde{\mu}^* e^{j\omega t} \right) \tag{C-1}
\]

The varying overlap angle causes the following DC voltage variation

\[
\Delta u_d = \text{Re}\{\Delta \tilde{K}^U(\theta) \cdot U_{V0}(\theta)\} \tag{C-2}
\]

The result is

\[
\Delta u_d = \text{Re}\left\{ \frac{1}{2} \left( \frac{\partial \ell^U}{\partial \mu} \frac{\Delta \tilde{\mu}}{2} U_{V0} + \frac{\partial \ell^U}{\partial \mu} U_{V0}^* \Delta \tilde{\mu} e^{j\omega t} \right) \right\} \tag{C-3}
\]

This expression provides the transfer function from variation of the voltage conversion function to the DC side voltage in the converter. Accordingly,

\[
H_3(j\Omega) = \frac{1}{2} \left( \frac{\partial \ell^U}{\partial \mu} \frac{\Delta \tilde{\mu}}{2} U_{V0} + \frac{\partial \ell^U}{\partial \mu} U_{V0}^* \Delta \tilde{\mu} e^{j\omega t} \right) \tag{C-4}
\]

Note: It should be recalled the steady stated valve voltage \( U_{V0} \) is calculated from the undisturbed equivalent Thevenin source voltage by

\[
U_{V0}(\theta) = U_{B0} e^{j\theta} - \frac{2}{3} (Z_s + Z_t) I_{d0} \tilde{K}^I(\theta) =
\]

\[
= U_{B0} e^{j\theta} - \frac{2}{3} I_{d0} Z \ell^I e^{j\theta} \tag{C-5}
\]

where,

\[
Z = Z_s(j\omega_e) + Z_c(j\omega_e) \tag{C-6}
\]

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APPENDIX D

CALCULATION OF TRANSFER FUNCTION $H_1$, ASSUMING THE AC SYSTEM IS MODELED IN THE $d-q$ FRAME

The calculation of the transfer function $H_1$ (recall that this transfer function expresses the direct impact of the DC side current on the DC side voltage when the varying overlap is disregarded) has already been calculated in APPENDIX A. In this appendix the transfer function will be re-calculated, considering that the AC system will be modeled in the $d-q$ frame.

Let assume that a small variation is applied to the LCC’s DC current. Then

$$
\Delta i_d(t) = \text{Re}[A e^{i\omega t}] = \frac{A}{2} e^{i\omega t} + \frac{A^*}{2} e^{-i\omega t}
$$

(D-1)

The AC-side current disturbance from the LCC converter becomes:

$$
\Delta i_v(t) = -\frac{2}{3} \kappa'(t) \Delta i_d
$$

(D-2)

Since $\kappa'(t) = \ell' e^{j\omega_{v} t}$, then we get

$$
\Delta i_v(t) = -\frac{2}{3} \ell' e^{j\omega_{v} t} \left( \frac{A}{2} e^{i\omega t} + \frac{A^*}{2} e^{-i\omega t} \right)
$$

(D-3)

The current $\Delta i_v$ is referred to $\alpha-\beta$ fixed frame. By using the $\alpha-\beta$ to $d-q$ transformation, that is, $\Delta i^d_v(t) = \Delta i_v(t) e^{-j\omega_{v} t}$, we can get the corresponding value in the $d-q$ rotating reference frame, which rotates synchronously with the vector. Then we get

$$
\Delta i^d_v(t) = -\frac{2}{3} \ell' \left( \frac{A}{2} e^{i\omega t} + \frac{A^*}{2} e^{-i\omega t} \right)
$$

(D-4)
For convenience, let us introduce the following notation:

\[
\ell^l = \ell^l_d + j \ell^l_q \\
\ell^U = \ell^U_d + j \ell^U_q
\]  
(D-5)

The d- and q- components of the converter current will be now calculated, knowing that,

\[
\Delta i^d_q(t) = \Delta i^d_q(t) + j \Delta i^q_q(t) = \text{Re}\{\Delta i^d_q(t)\} + j \text{Im}\{\Delta i^d_q(t)\}
\]  
(D-6)

d- component:

\[
\Delta i^d_d(t) = -\frac{2}{3} \text{Re}\left\{\frac{A}{2} \ell^l e^{jut} + \frac{A^*}{2} \ell^l e^{-jut}\right\} = \\
= -\frac{2}{3} \text{Re}\left\{\frac{A}{2} \ell^l e^{+jut} + \frac{A^*}{2} \ell^l e^{-jut}\right\} = \\
= -\frac{2}{3} \text{Re}\left\{A \text{Re}\{\ell^l\} e^{jut}\right\} = \\
= -\frac{2}{3} \text{Re}\{A l^l e^{+jut}\}
\]  
(D-7)

q- component:

\[
\Delta i^q_d(t) = -\frac{2}{3} \text{Im}\left\{\frac{A}{2} \ell^l e^{jut} + \frac{A^*}{2} \ell^l e^{-jut}\right\} = \\
= -\frac{2}{3} \text{Re}\left\{-j \frac{A}{2} \ell^l e^{jut} - j \frac{A^*}{2} \ell^l e^{-jut}\right\} = \\
= -\frac{2}{3} \text{Re}\left\{-j \frac{A}{2} \ell^l e^{+jut} + j \frac{A^*}{2} \ell^l e^{-jut}\right\} = \\
= -\frac{2}{3} \text{Re}\left\{A \left(\frac{-j \ell^l + j \ell^l}{2j}\right) e^{jut}\right\} = \\
= -\frac{2}{3} \text{Re}\left\{A \text{Im}\{\ell^l\} e^{+jut}\right\} = \\
= -\frac{2}{3} \text{Re}\{A l^l e^{+jut}\}
\]  
(D-8)
The converter current is injected into the network $\bar{Z}^{(ac)} = \bar{Z}_s$. The network is represented by its d- and q-component impedances, $Z_s^{dq} \leftrightarrow z_{s-dd}, z_{s-qd}, z_{s-dq}$ and $z_{s-qq}$. It includes the VSC modeled by its d- and q-component admittance matrix (see chapter 6).

The result of the AC current variation $\Delta i^{ac}(t)$ injected into the network $Z_s^{dq}$ gives the following AC side voltage variation

$$\Delta \bar{u}_v^{dq} = Z_s^{dq} \cdot \Delta i_v^{dq} \quad \text{(D-9)}$$

This is equivalently to write as

$$\Delta u_v^d + j\Delta u_v^q = Z_s^{dq} \cdot \left(\Delta i_v^d + j\Delta i_v^q\right) \quad \text{(D-10)}$$

or in matrix notation,

$$\begin{bmatrix} \Delta u_v^d \\ \Delta u_v^q \end{bmatrix} = \begin{bmatrix} z_s^{dd} & z_s^{dq} \\ z_s^{qd} & z_s^{qq} \end{bmatrix} \cdot \begin{bmatrix} \Delta i_v^d \\ \Delta i_v^q \end{bmatrix} \quad \text{(D-11)}$$

The set of equation for the AC side voltage variation is

$$\begin{align*}
\Delta u_v^d &= z_s^{dd} \cdot \Delta i_v^d + z_s^{dq} \cdot \Delta i_v^q \\
\Delta u_v^q &= z_s^{qd} \cdot \Delta i_v^d + z_s^{qq} \cdot \Delta i_v^q
\end{align*} \quad \text{(D-12)}$$

The d- and q-components of the AC side voltage variation are

$$\begin{align*}
\Delta u_v^d &= z_s^{dd}(j\Omega) \cdot \left(\frac{-2}{3} \text{Re}\left\{A l'_d e^{j\varphi_d}\right\}\right) + z_s^{dq}(j\Omega) \cdot \left(\frac{-2}{3} \text{Re}\left\{A l'_q e^{j\varphi_q}\right\}\right) = \\
&= -\frac{2}{3}\left[\text{Re}\left\{(Az_s^{dd}(j\Omega)l'_d + Az_s^{dq}(j\Omega)l'_q) e^{j\varphi_d}\right\}\right] \\
\Delta u_v^q &= z_s^{qd}(j\Omega) \cdot \left(\frac{-2}{3} \text{Re}\left\{A l'_q e^{j\varphi_q}\right\}\right) + z_s^{qq}(j\Omega) \cdot \left(\frac{-2}{3} \text{Re}\left\{A l'_q e^{j\varphi_q}\right\}\right) = \\
&= -\frac{2}{3}\left[\text{Re}\left\{(Az_s^{qd}(j\Omega)l'_d + Az_s^{qq}(j\Omega)l'_q) e^{j\varphi_d}\right\}\right]
\end{align*} \quad \text{(D-13)}$$

The voltage $\Delta \bar{u}_v^{dq}$ is now transformed back to the $\alpha - \beta$ static frame, by using the following inverse transformation

$$\Delta \bar{u}_v(t) = \Delta \bar{u}_v^{dq} e^{j\omega_d t} = \left(\Delta u_v^d + j\Delta u_v^q\right) e^{j\omega_d t} \quad \text{(D-15)}$$

A change in AC-side voltage is reflected back into the DC-side according to

$$\Delta u_D = + \text{Re}\left\{\bar{K}^{U}(t)\right\} \Delta \bar{u}_v(t) \quad \text{(D-16)}$$

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where,
\[ \mathcal{K}^U(t) = \ell^U e^{j \omega t} = (\ell^U_d + j \ell^U_q) e^{j \omega t} \]  
(D-17)

Then we get
\[
\Delta u_p = \Re \left( \ell^U_d - j \ell^U_q \right) e^{-j \omega t} \Delta \Pi_v(t) = \\
= \Re \left( \ell^U_d - j \ell^U_q \right) e^{-j \omega t} \left( \Delta u_p^d + j \Delta u_p^q \right) e^{j \omega t} = \\
= \ell^U_d \cdot \Delta u_p^d + \ell^U_q \cdot \Delta u_p^q = \\
= \ell^U_d \cdot \left\{ - \frac{2}{3} \Re \left[ (Az^d_z(j \Omega) l^I_d + Az^d_{q q}(j \Omega) l^I_q) e^{j \Omega t} \right] \right\} + \\
+ \ell^U_q \cdot \left\{ - \frac{2}{3} \Re \left[ (Az^q_z(j \Omega) l^I_d + Az^q_{q q}(j \Omega) l^I_q) e^{j \Omega t} \right] \right\} = \\
= - \frac{2}{3} \Re \left[ \ell^U_d z^d_z(j \Omega) l^I_d + \ell^U_q z^q_z(j \Omega) l^I_q + \ell^U_d z^d_{q q}(j \Omega) l^I_q + \ell^U_q z^q_{q q}(j \Omega) l^I_q \right] e^{j \Omega t} \\ \]  
(D-18)

The resulting final transfer function is then given by
\[ H_1 = \frac{\Delta u_D}{\Delta l_D} = \frac{-2}{3} \left[ \ell^U_d z^d_z(j \Omega) l^I_d + \ell^U_q z^q_z(j \Omega) l^I_q + \ell^U_d z^d_{q q}(j \Omega) l^I_q + \ell^U_q z^q_{q q}(j \Omega) l^I_q \right] \]  
(D-19)

This procedure has been used to calculate all the other transfer functions defined in chapters 3 and 4.
APPENDIX E

CALCULATION OF THE TRANSFER FUNCTION – CLOSED LOOP RECTIFIER

CURRENT CONTROL: \( T = CL = \frac{\Delta I_{d_{\text{rec}}}}{\Delta I_{d_{\text{ref}}}} \)

To simplify the presentation we will omit in this and in the following appendixes the symbol \( \Delta \) since all variables represents deviations from the operating point condition.

To derive the desired final transfer function intermediate transfer functions are calculated considering the different loops included in the Block Diagram shown in Figure 7.3. The technique used is that start from the output or from an end of a loop at the inverter, writing down the block as we meet then when moving backwards against the signal flow, taking the most direct path towards the input.

1)
\[
\alpha' = -\alpha'_{\text{PLL}} + \alpha'_{\text{V,ref}} = -K'_{\text{PLL}} (G_{a1} i_{d} + G_{a2} \alpha') - G_{5} (-1)G_{F} u_{d} \\
= \frac{-K'_{\text{PLL}} G_{a1}}{1 + K'_{\text{PLL}} G_{a2}} i_{d} + \frac{-G_{5} (-1)G_{F}}{1 + K'_{\text{PLL}} G_{a2}} u_{d} \\
\alpha' = X_{1} i_{d} + X_{2} u_{d} \\
\]

where
\[
X_{1} = \frac{-K'_{\text{PLL}} G_{a1}}{1 + K'_{\text{PLL}} G_{a2}} \\
X_{2} = \frac{-G_{5} (-1)G_{F}}{1 + K'_{\text{PLL}} G_{a2}} \\
\]

(E-3)

(E-1)

(E-2)
2) 

\[ u'_d = G'_2 \alpha + G'_3 i'_d \]

\[ = G'_2 \left( X'_1 i'_d + X'_2 u'_d \right) + G'_3 i'_d \]

\[ = \frac{G'_2 X'_1 + G'_3}{1 - G'_2 X'_2} \]

\[ u'_d = X'_3 i'_d \tag{E-5} \]

where

\[ X'_3 = \frac{G'_2 X'_1 + G'_3}{1 - G'_2 X'_2} \tag{E-6} \]

3) 

\[ i'_d = \frac{1}{Z_K} u'_d + G_K u_d \]

\[ = \frac{1}{Z_K} \left( X'_3 i'_d \right) + G_K u_d \tag{E-7} \]

\[ = \frac{G_K}{1 - \frac{1}{Z_K} X'_3} u_d \]

\[ i'_d = X_4 u_d \tag{E-8} \]

where

\[ X_4 = \frac{G_K}{1 - \frac{1}{Z_K} X'_3} \tag{E-9} \]

4) 

\[ i_d = \frac{1}{Z_K} u_d + G_K u'_d \]

\[ = \frac{1}{Z_K} u_d + G_K \left[ X'_3 i'_d \right] \]

\[ = \frac{1}{Z_K} u_d + G_K \left[ X'_3 \left( X_4 u_d \right) \right] \tag{E-10} \]

\[ = \left( \frac{1}{Z_K} + G_K X'_3 X_4 \right) u_d \]

\[ i_d = X_5 u_d \tag{E-11} \]
where,

\[ X_5 = \frac{1}{\frac{G_K}{Z_K}} + G_K X_3 X_4 \]  \hspace{1cm} (E-12)

5)

\[ u_d = G_3 i_d + G_2 \alpha \]

\[ \frac{i_d}{X_5} = G_3 i_d + G_2 \alpha \]

\[ \therefore \quad i_d = \frac{G_2}{X_5 - G_3} \alpha \]

\[ i_d = X_6 \alpha \]  \hspace{1cm} (E-14)

where

\[ X_6 = \frac{G_3}{X_5 - G_3} \]  \hspace{1cm} (E-15)

6)

\[ \alpha = \alpha_{set} - \alpha_{PLL} \]

\[ = G_f (-1)(i_{d,ref} - G_F i_d) - K_{PLL} (G_{a41} i_d + G_{a42} \alpha) \]

\[ = \frac{G_f (-1)}{1 + K_{PLL} G_{a42}} i_{d,ref} + \frac{-K_{PLL} G_{a41} - G_f (-1)G_F}{1 + K_{PLL} G_{a42}} i_d \]

\[ \alpha = X_7 i_{d,ref} + X_8 i_d \]  \hspace{1cm} (E-17)

where

\[ X_7 = \frac{G_f (-1)}{1 + K_{PLL} G_{a42}} \]

\[ X_8 = \frac{-K_{PLL} G_{a41} - G_f (-1)G_F}{1 + K_{PLL} G_{a42}} \]  \hspace{1cm} (E-18)
7)  
\[ \alpha = X_7 i_{d \text{ ref}} + X_8 i_d \]
\[ \frac{i_d}{X_6} = X_7 i_{d \text{ ref}} + X_8 i_d \]  \hspace{1cm} (E-19)
\[ \therefore i_d = \frac{X_7}{1} i_{d \text{ ref}} \]
\[ \frac{1}{X_6 - X_8} \]

Thus,

\[ T = CL = \frac{\Delta I_{d - \text{ ref}}}{\Delta I_{d \text{ ref}}} = \frac{\Delta i_d}{\Delta i_{d \text{ ref}}} = \frac{X_7}{1} \]
\[ \frac{1}{X_6 - X_8} \]  \hspace{1cm} (E-20)
APPENDIX F

CALCULATION OF THE TRANSFER FUNCTION – OPEN LOOP RECTIFIER

CURRENT CONTROL: \( L = OL = \frac{\Delta I_{d-rec}}{\Delta \varepsilon} \)

1)
\[
\alpha' = -\alpha_{PLL}' + \alpha_{F, ctrl}' = -K_{PLL}'(G'_{d1} \dot{i}_d + G'_{d2} \alpha') - G'_{s}(-1)G_F' \dot{u}_d
\]
\[
= -\frac{K_{PLL}' G'_{d1}}{1 + K_{PLL}' G'_{d2}} \dot{i}_d + \frac{-G'_{s}(-1)G_F'}{1 + K_{PLL}' G'_{d2}} \dot{u}_d
\]
\[
\alpha' = X_1 \dot{i}_d + X_2 \dot{u}_d \tag{F-1}
\]

where
\[
X_1 = -\frac{K_{PLL}' G'_{d1}}{1 + K_{PLL}' G'_{d2}} \tag{F-3}
\]
\[
X_2 = \frac{-G'_{s}(-1)G_F'}{1 + K_{PLL}' G'_{d2}} \tag{F-2}
\]

2)
\[
u_d = G_2 \alpha' + G_3 \dot{i}_d
\]
\[
= G_2 \left( X_1 \dot{i}_d + X_2 \dot{u}_d \right) + G_3 \dot{i}_d
\]
\[
= \frac{G_2 X_1}{1 - G_2 X_2} \dot{i}_d \tag{F-4}
\]
\[
u_d = X_3 \dot{i}_d \tag{F-5}
\]

where
\[ X_3 = \frac{G'X_1 + G_s}{1 - G_sX_2} \quad \text{(F-6)} \]

3)

\[ i_d' = \frac{1}{Z_K}u_d' + G_Ku_d \]
\[ = \frac{1}{Z_K}\left(X_3i_d'\right) + G_Ku_d \quad \text{(F-7)} \]
\[ = \frac{G_K}{1 - \frac{1}{Z_K}X_3}u_d \]

\[ i_d' = X_4u_d \quad \text{(F-8)} \]

where

\[ X_4 = \frac{G_K}{1 - \frac{1}{Z_K}X_3} \quad \text{(F-9)} \]

4)

\[ i_d = \frac{1}{Z_K}u_d + G_Ku_d \]
\[ = \frac{1}{Z_K}u_d + G_K\left[X_3i_d'\right] \quad \text{(F-10)} \]
\[ = \frac{1}{Z_K}u_d + G_K\left[X_3\left(X_4u_d\right)\right] \]
\[ = \left(\frac{1}{Z_K} + G_KX_3X_4\right)u_d \]

\[ i_d = X_5u_d \quad \text{(F-11)} \]

where,

\[ X_5 = \frac{1}{Z_K} + G_KX_3X_4 \quad \text{(F-12)} \]
5)  
\[ u_d = G_3 i_d + G_2 \alpha \]  
\[ \frac{i_d}{X_5} = G_3 i_d + G_2 \alpha \]  
\[ \therefore \quad i_d = \frac{G_2}{1} \frac{\alpha}{X_5 - G_3} \]  
\[ i_d = X_6 \alpha \]  
\[ \text{where} \]  
\[ X_6 = \frac{G_2}{1} \frac{1}{X_5 - G_3} \]  

6)  
\[ \alpha = \alpha_{con} - \alpha_{PLL} \]  
\[ = G_1 (-1) \varepsilon - K_{PLL} (G_{41} i_d + G_{42} \alpha) \]  
\[ = \frac{G_1 (-1)}{1 + K_{PLL} G_{42}} \varepsilon + \frac{-K_{PLL} G_{41} i_d}{1 + K_{PLL} G_{42}} \]  
\[ \alpha = X_9 \varepsilon + X_{10} i_d \]  
\[ \text{where} \]  
\[ X_9 = \frac{G_1 (-1)}{1 + K_{PLL} G_{42}} \]  
\[ X_{10} = \frac{-K_{PLL} G_{41}}{1 + K_{PLL} G_{42}} \]  

7)  
\[ \alpha = X_9 \varepsilon + X_{10} i_d \]  
\[ \frac{i_d}{X_6} = X_9 \varepsilon + X_{10} i_d \]  
\[ \therefore \quad i_d = \frac{X_9}{X_6 - X_{10}} \varepsilon \]  

Thus,
\[ L = OL = \frac{\Delta I_{d-rec}}{\Delta \varepsilon} = \frac{\Delta I_2}{\Delta \varepsilon} = \frac{X_0}{1 - \frac{X_0}{X_6}} \]  

(F-20)
APPENDIX G

SENSITIVITY INDEX NR. 1: $S_{I_i:U-I}^1 = \frac{\Delta U_{d-inv}}{\Delta I_{d-inv}}$

1)

\[ u_d' = G_3 i_d' + G_2 \alpha' \]  \hspace{1cm} (G-1)

\[ \alpha' = -K'_{PLL} (G_{41} i_d' + G_{42} \alpha') + \alpha'_{v_{ctrl}} \]  \hspace{1cm} (G-2)

\[ \alpha'_{v_{ctrl}} = -G_5' (-1) G_F' u_d' \]  \hspace{1cm} (G-3)

2)

From

\[ \alpha' = -K'_{PLL} G_{41} i_d' - K'_{PLL} G_{42} \alpha' - G_5' (-1) G_F' u_d' \]

\[ = \frac{- K'_{PLL} G_{41} i_d'}{1 + K'_{PLL} G_{42}} - \frac{G_5' (-1) G_F'}{1 + K'_{PLL} G_{42}} u_d' \]  \hspace{1cm} (G-4)

\[ \alpha' = R_1 i_d' + R_2 u_d' \]  \hspace{1cm} (G-5)

where

\[ R_1 = \frac{- K'_{PLL} G_{41}'}{1 + K'_{PLL} G_{42}'} \]  \hspace{1cm} (G-6)

\[ R_2 = \frac{- G_5' (-1) G_F'}{1 + K'_{PLL} G_{42}'} \]

3)

\[ u_d' = G_3 i_d' + G_2 (R_1 i_d' + R_2 u_d') \]

\[ = \frac{G_3' + G_2' R_1 i_d'}{1 - G_2' R_2} \]  \hspace{1cm} (G-7)
Thus,

\[
S_{U_{d-1}}^{1} = \frac{\Delta U_{d-\text{inv}}}{\Delta i_{d-\text{inv}}} = \frac{\Delta u'_{d}}{\Delta i'_{d}} = \frac{G_{3} + G_{2} R_{2}}{1 - G_{2} R_{2}}
\]  

(G-9)
APPENDIX H

SENSITIVITY INDEX NR. 3: \( S_{R/U-I}^3 = \frac{\Delta U_{d-rec}}{\Delta I_{d-ref}} \)

1) \[ \alpha' = -\alpha'_{\text{PLL}} + \alpha'_{\text{ctrl}} = -K'_{\text{PLL}} \left( G'_{41} \alpha_d + G'_{42} \alpha' \right) - G'_5 (-1) G'_{F} \ u_d' \]
   \[ = - \frac{K'_{\text{PLL}} G'_{41}}{1 + K'_{\text{PLL}} G'_{42}} \ i_d' + - \frac{G'_5 (-1) G'_{F}}{1 + K'_{\text{PLL}} G'_{42}} \ u_d' \]  
   \( \alpha' = X_1 i_d' + X_2 u_d' \)  
   \( \text{(H-1)} \)

where
   \( X_1 = \frac{- K'_{\text{PLL}} G'_{41}}{1 + K'_{\text{PLL}} G'_{42}} \)
   \( X_2 = \frac{- G'_5 (-1) G'_{F}}{1 + K'_{\text{PLL}} G'_{42}} \)  
   \( \text{(H-3)} \)

2) \[ u'_d = G'_2 \alpha' + G'_3 i_d' \]
   \[ = G'_2 \left( X_1 i_d' + X_2 u_d' \right) + G'_3 i_d' \]
   \[ = G'_2 \frac{X_1 + G'_3 i_d'}{1 - G'_2 X_2} \]
   \[ u'_d = X_3 i_d' \]  
   \( \text{(H-4)} \)

where
   \[ X_3 = \frac{G'_2 X_1 + G'_3}{1 - G'_2 X_2} \]  
   \( \text{(H-5)} \)

\( \text{(H-2)} \)
Appendix H

3)

\[ i_d' = \frac{1}{Z_K} u_d' + G_K u_d \]

\[ = \frac{1}{Z_K} (X_3 i_d') + G_K u_d \]

\[ = \frac{G_K}{1 - \frac{1}{Z_K} X_3} u_d \]

\[ i_d' = X_4 u_d \] (H-8)

where

\[ X_4 = \frac{G_K}{1 - \frac{1}{Z_K} X_3} \] (H-9)

4)

\[ i_d = \frac{1}{Z_K} u_d + G_K u_d' \]

\[ = \frac{1}{Z_K} u_d + G_K \left[ X_3 i_d' \right] \]

\[ = \frac{1}{Z_K} u_d + G_K \left[ X_3 (X_4 u_d) \right] \]

\[ = \left( \frac{1}{Z_K} + G_K X_3 X_4 \right) u_d \]

\[ i_d = X_5 u_d \] (H-11)

where,

\[ X_5 = \frac{1}{Z_K} + G_K X_3 X_4 \] (H-12)

5)

\[ \alpha = \alpha_{ctrl} - \alpha_{PLL} \]

\[ = G_i \left( -1 \right) \left( i_d_{ref} - G_F i_d \right) - K_{PLL} \left( G_{a1} i_d + G_{a2} \alpha \right) \]

\[ = \frac{G_i \left( -1 \right)}{1 + K_{PLL} G_{a2}} i_{d_{ref}} + \frac{-K_{PLL} G_{a1} - G_i \left( -1 \right) G_F}{1 + K_{PLL} G_{a2}} i_d \]

\[ \alpha = X_7 i_{d_{ref}} + X_8 i_d \] (H-14)
where

\[ X_7 = \frac{G_t(-1)}{1 + K_{PLL} G_{42}} \]

\[ X_8 = \frac{-K_{PLL} G_{41} - G_t(-1)G_P}{1 + K_{PLL} G_{42}} \]  

\( \text{(H-15)} \)

6)

\[ u_d = G_3 i_y + G_2 \alpha \]

\[ = G_3 (X_5 u_d) + G_2 [X_7 i_{d \text{ref}} + X_8 i_d] \]  

\[ = G_3 (X_5 u_d) + G_2 [X_7 i_{d \text{ref}} + X_8 (X_5 u_d)] \]  

\[ u_d = \frac{G_2 X_7}{1 - G_3 X_5 - G_2 X_8 X_5} i_{d \text{ref}} \]  

\( \text{(H-16)} \)

Thus,

\[ S_{\Delta U/d \text{ref}}^3 = \frac{\Delta U_{d \text{ref}}}{\Delta I_{d \text{ref}}} = \frac{\Delta u_d}{\Delta i_{d \text{ref}}} = \frac{G_2 X_7}{1 - G_3 X_5 - G_2 X_8 X_5} \]  

\( \text{(H-17)} \)
APPENDIX I

SENSITIVITY INDEX NR. 5: $S_{I:uc}^5 = \frac{\Delta u_{c-inv}}{\Delta I_{d-ref}}$

For definition of variables $X_1 - X_8$ see APPENDIX E

1) $u_c' = G_{51}' \dot{i}_d' + G_{52}' \alpha' = G_{51}' \dot{i}_d' + G_{52}' \left(X_1 \dot{i}_d' + X_2 u_d'\right)$ = $(G_{51}' + G_{52}' X_1) \dot{i}_d' + G_{52}' X_2 u_d'$ $u_c' = S_1 \dot{i}_d' + S_2 u_d'$ (I-1)

where

$S_1 = (G_{51}' + G_{52}' X_1)$
$S_2 = +G_{52}' X_2$ (I-2)

2) $u_c' = S_1 \dot{i}_d' + S_2 u_d'$ $= S_1 \dot{i}_d' + S_2 X_3 \dot{i}_d'$ $= (S_1 + S_2 X_3) \dot{i}_d'$ $u_c' = S_3 \dot{i}_d'$ (I-3)

where

$S_3 = S_1 + S_2 X_3$ (I-4)
3) 

\[ u'_c = S_3 i'_d \]

\[ = S_3 X_4 u_d \]

\[ = S_3 X_4 \frac{1}{X_5} i_d \]

\[ = S_3 X_4 \frac{1}{X_5} X_6 \alpha \]

\[ u'_c = S_4 \alpha \]  

(\text{I-7}) 

Where

\[ S_4 = S_3 X_4 \frac{1}{X_5} X_6 \]  

(\text{I-8}) 

4) 

From

\[ i_d = \frac{X_7}{X_6} i_{d \text{ ref}} \]  

(\text{I-10}) 

Let define

\[ S_5 = \frac{X_7}{1 - X_6 X_8} \]  

(\text{I-11}) 

5) 

\[ u'_c = S_4 \alpha \]

\[ = S_4 \left( X_7 i_{d \text{ ref}} + X_8 i_d \right) \]

\[ = S_4 \left( X_7 i_{d \text{ ref}} + X_8 S_5 i_{d \text{ ref}} \right) \]

\[ u'_c = S_4 \left( X_7 + X_8 S_5 \right) i_{d \text{ ref}} \]

(\text{I-12}) 

Thus,

\[ S_{Iac}^S = \frac{\Delta u_{c \text{ inv}}}{\Delta I_{d \text{ ref}}} = \frac{\Delta u'_c}{\Delta i_{d \text{ ref}}} = S_4 \left( X_7 + X_8 S_5 \right) \]  

(\text{I-13}) 

\[ S_{Iac}^S = \frac{\Delta u_{c \text{ inv}}}{\Delta I_{d \text{ ref}}} = \frac{\Delta u'_c}{\Delta i_{d \text{ ref}}} = S_4 \left( X_7 + X_8 S_5 \right) \]  

(\text{I-14})
APPENDIX J
SENSITIVITY INDEX NR. 6: \( S_{I: \theta_c}^6 = \Delta \frac{\theta_{c-\text{inv}}}{\Delta I_{d-\text{ref}}} \)

For definition of variables \( X_1 - X_8 \) see APPENDIX E

1)  
\[
\theta_c = G'_{41} \dot{i}_d + G'_{42} \alpha' \\
= G'_{41} \dot{i}_d + G'_{42} \left( -K_{PLL} \theta_c - G'_F (-1) G'_F u_d \right) \\
= \frac{G'_{41}}{1 + G'_{42} K_{PLL}} \dot{i}_d + \frac{-G'_{42} G'_F (-1) G'_F}{1 + G'_{42} K_{PLL}} u_d \\
\]

\[
\theta_c = T_1 \dot{i}_d + T_2 u_d \\
\]

where  
\[
T_1 = \frac{G'_{41}}{1 + G'_{42} K_{PLL}} \\
T_2 = \frac{-G'_{42} G'_F (-1) G'_F}{1 + G'_{42} K_{PLL}} \\
\]

2)  
\[
\theta_c = T_1 \dot{i}_d + T_2 u_d \\
= T_1 \dot{i}_d + T_2 X_3 \dot{i}_d \\
= \left( T_1 + T_2 X_3 \right) \dot{i}_d \\
\]

\[
\theta_c = T_3 \dot{i}_d \\
\]

Where
\[ T_3 = (T_1 + T_2 X_3) \]  

(J-6)

3)

\[ \theta_c = T_3 i_d \]
\[ = T_3 X_4 u_d \]
\[ = T_3 X_4 \frac{1}{X_5} i_d \]
\[ = T_3 X_4 \frac{1}{X_5} X_6 \alpha \]

\[ \theta_c = T_4 \alpha \]  

(J-7)

(J-8)

Where

\[ T_4 = T_3 X_4 \frac{1}{X_5} X_6 \]  

(J-9)

4)

From

\[ i_d = \frac{X_5}{1} \frac{i_{d, \text{ref}}}{X_6 - X_8} \]  

(J-10)

Let define

\[ T_5 = \frac{X_5}{1} \frac{1}{X_6 - X_8} \]  

(J-11)

5)

\[ \theta_c' = T_4 \alpha \]
\[ = T_4 \left( X_7 i_{d, \text{ref}} + X_8 i_d \right) \]
\[ = T_4 \left( X_7 i_{d, \text{ref}} + X_8 T_5 i_{d, \text{ref}} \right) \]
\[ = T_4 \left( X_7 + X_8 T_5 \right) i_{d, \text{ref}} \]

Thus,

\[ S_{\theta_c = \Delta \theta_c}^{\Delta \theta_c - \Delta i_{d, \text{ref}}} = \frac{\Delta \theta_c - \Delta i_{d, \text{ref}}}{\Delta i_{d, \text{ref}}} = T_4 \left( X_7 + X_8 T_5 \right) \]  

(J-13)
APPENDIX K

ALTERNATIVE DERIVATION TO OBTAIN THE SOLUTION FOR THE DC SIDE OF THE VSC

In Chapter 5 the solution for the DC side of the Voltage Source Converter (VSC) has been obtained using power equations between the AC and DC sides respectively. Using this approach very simple derivation has been obtained.

This implies however that the PWM modulator perfectly compensates the DC voltage variation, and therefore, there is no direct interaction with the DC quantities. It also assumes that the DC voltage is sufficiently high to avoid limitations.

However, if one would like to study interaction between the AC and DC sides of the converter an alternative derivation is needed and one solution is presented in this appendix.

Before going in more detail, let start first describing the VSC by the coupling function technique to represent the operation of the converter.

K.1 SPACE VECTOR REPRESENTATION OF VSC IN SQUARE WAVE OPERATION

K.1.1 VSC representation by complex-coupling function

The Voltage Source Converter (VSC) can be modeled as three two-way switches connected between two DC rails as shown in Figure K.1. In Figure K.1-(A) the switches have been drawn together with their control functions $k_a(t)$, $k_b(t)$, $k_c(t)$. The Voltage Source Converter can also be represented by its complex function as shown in Figure K.1-(B).
Appendix K

The VSC comprises a capacitor between its DC rails, which prevents the DC voltage from being instantaneously changed. The VSC thus force the AC side voltage to a certain value determined by the given coupling functions.

Let the coupling functions take the value \( k_x = 0 \), whenever phase \( x \) is switched to the lower DC rail and the value \( k_x = 1 \) when it is switched to the upper DC rail. Then the phase voltages relative the lower DC rail are given by

\[
\begin{align*}
    u_a(t) &= k_a(t)u_d(t) \\
    u_b(t) &= k_b(t)u_d(t) \\
    u_c(t) &= k_c(t)u_d(t)
\end{align*}
\]

Now the zero-sequence-free part of the AC side voltage can easily be obtained just by inserting the phase quantities in the general formula for the space vector. This yields

\[
\bar{u}_y(t) = \frac{2}{3} \left[ k_a(t) + k_b(t)e^{\frac{2\pi}{3}} + k_c(t)e^{-\frac{2\pi}{3}} \right] u_d(t)
\]

We define the complex-valued expression within the brackets to be the coupling vector time function

\[
\bar{K}^{VSC}(t) = k_a(t) + k_b(t)e^{\frac{2\pi}{3}} + k_c(t)e^{-\frac{2\pi}{3}}
\]

Thus,

\[
\bar{u}_y(t) = \frac{2}{3} \bar{K}^{VSC}(t)u_d(t)
\]

The coupling vector can have one of the seven values according to Table K-1. The table shows that the coupling vector has unit amplitude if it is not zero vector.
Table K.1: Phase coupling functions and complex coupling vector for a VSC

<table>
<thead>
<tr>
<th>( k_a(t) )</th>
<th>( k_b(t) )</th>
<th>( k_c(t) )</th>
<th>State a-b-c</th>
<th>( \vec{K}^{VSC}(\omega_Nt) )</th>
<th>Time Interval: ((\omega_Nt))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>- - -</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>+ + +</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>+ - -</td>
<td>( e^{j0\pi/3} = 1 \arg(0^\circ) )</td>
<td>([-1\pi/6; 1\pi/6])</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>+ + -</td>
<td>( e^{j1\pi/3} = 1 \arg(60^\circ) )</td>
<td>([1\pi/6; 3\pi/6])</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>- + -</td>
<td>( e^{j2\pi/3} = 1 \arg(120^\circ) )</td>
<td>([3\pi/6; 5\pi/6])</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>- + +</td>
<td>( e^{j3\pi/3} = 1 \arg(180^\circ) )</td>
<td>([5\pi/6; 7\pi/6])</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>- - +</td>
<td>( e^{j4\pi/3} = 1 \arg(240^\circ) )</td>
<td>([7\pi/6; 9\pi/6])</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>+ - +</td>
<td>( e^{j5\pi/3} = 1 \arg(300^\circ) )</td>
<td>([9\pi/6; 11\pi/6])</td>
</tr>
</tbody>
</table>

The switching of the converter also establishes a certain connection between the impressed AC side current vector and the DC side current. Using the definition of the phase coupling functions we can immediately write the following expression as each phase only contributes to the DC side current when it is connected to the upper DC rail. Thus

\[
i_a(t) = k_a(t)i_a(t) + k_b(t)i_b(t) + k_c(t)i_c(t)
\]  

(K-5)

But the phase currents can be expressed as the projections of the current vector. We get

\[
i_a(t) = k_a(t)\text{Re}\left[\vec{i}_a(t)\right] + k_b(t)\text{Re}\left[\vec{i}_b(t)e^{-j2\pi/3}\right] + k_c(t)\text{Re}\left[\vec{i}_c(t)e^{j2\pi/3}\right]
\]  

(K-6)

\[
= \text{Re}\left[\left(k_a(t) + k_b(t)e^{-j2\pi/3} + k_c(t)e^{j2\pi/3}\right)\vec{i}_a(t)\right] = \text{Re}\left[\vec{K}^{VSC*}(\omega_Nt)\vec{i}_a(t)\right]
\]

The description of the VSC thus can be summarized as follows:
\[ \bar{u}_j(t) = \frac{2}{3} K^{vsc}(t) u_j(t) \]
\[ i_d(t) = \text{Re} \left[ K^{vsc}*(t) \bar{I}_p(t) \right] \]
\[ K^{vsc}(t) = k_a(t) + k_b(t)e^{j\frac{2\pi}{3}} + k_c(t)e^{-j\frac{2\pi}{3}} \]
\[ k_z(t) = \{0,1\} \quad |K^{vsc}(t)| = \{0,1\}, \quad \text{arg} \ K^{vsc} = (0...5) \times \frac{\pi}{3} \]

**K.1.2 Spectrum of coupling vector function for VSC**

If the PWM modulation is three-phase symmetric the coupling vector switching pattern in each consecutive \( \theta \) subinterval of length \( \frac{\pi}{3} \) repeats itself with sequentially changing phase legs and DC-rails \(+a, -c, +b, -a, +c, -b\). Figure K.2 shows the simplest switching pattern, i.e. fundamental frequency switching.

\[ \theta(t) = \omega_c t \]

We select the time zero instant at the point where the argument of the generated voltage fundamental frequency component zero-crosses. Then we may write the vector function as follows

\[ K^{vsc}(\theta) = \hat{K} e^{j\frac{\pi}{3} \text{round}(\frac{3\theta}{\pi})} \]

where the MATLAB function ‘round(\(x\))’ returns the integer closest to ‘\(x\)’. The coupling vector steps forward \( \pi/3 \) after each subinterval of length \( \pi/3 \). Therefore, if we define \( L(\theta) \) as

\[ L^{vsc}(\theta) = e^{-j\theta} K^{vsc}(\theta) \]
this latter function will be periodic with period $\pi/3$. Accordingly it can be Fourier expanded as

$$L^{VSC}(\theta) = \sum_{m=-\infty}^{\infty} \ell^{VSC}_m e^{j6m\theta}$$  \hspace{1cm} (K-11)

where the Fourier coefficients can be obtained by integration over any interval of length $\pi/3$. We select the interval $-\pi/6 < \theta < +\pi/6$.

This yields

$$\ell^{VSC}_m = \frac{1}{\pi} \int_{\pi/6}^{\pi/6} L^{VSC}(\theta)e^{-j6m\theta} d\theta = \frac{3}{\pi} \int_{\pi/6}^{\pi/6} K^{VSC}(\theta)e^{-j\theta}e^{-j6m\theta} d\theta = \frac{3}{\pi} \int_{\pi/6}^{\pi/6} K^{VSC}(\theta)e^{-j(6m+1)\theta} d\theta$$

$$\ell^{VSC}_m = \frac{3}{\pi} \frac{(-1)^m}{6m+1}$$  \hspace{1cm} (K-12)

In the special case of fundamental frequency switching the coupling vector is constant $K^{VSC}(\theta) = \hat{K}^{VSC}$ = 1 within the selected interval and we get

$$\ell^{VSC}_m = \frac{3}{\pi} \frac{(-1)^m}{6m+1}$$  \hspace{1cm} (K-13)

Thus we get the following spectrum for the coupling function in the coordinate system shown in Figure K.2.

$$\bar{K}_m^{VSC} = \frac{3}{\pi} \sum_{m=-\infty}^{\infty} \frac{(-1)^m}{6m+1} e^{j(6m+1)\theta}$$

$$\bar{K}_m^{VSC} = \frac{3}{\pi} \sum_{m=-\infty}^{\infty} \frac{(-1)^m}{6m+1} e^{j(6m+1)\theta}$$

Considering only the fundamental frequency component and neglecting all the harmonics then the Fourier coefficient for the modulator for fundamental frequency switching is

$$\ell^0_{\text{VSC}} = \frac{3}{\pi}$$

It should be noted that the fundamental frequency voltage amplitude then becomes

$$\hat{u}_{\text{ph,ffsw}} = \frac{2}{3} \ell^0_{\text{VSC}} u_d = \frac{2}{\pi} u_d$$

This is the maximum voltage amplitude that the VSC may produce with a given DC voltage. However, the voltage at this amplitude level always is distorted. The maximum undistorted voltage level that can be produced by a PWM at given DC voltage is

$$\hat{u}_{\text{vsc,undist}} = \frac{u_d}{\sqrt{3}} = \frac{\pi}{2\sqrt{3}} \hat{u}_{\text{ph,ffsw}} = 0.9069 \hat{u}_{\text{ph,ffsw}}$$

(K-17)
K.2 INTERACTION BETWEEN THE AC AND DC SIDES OF THE CONVERTER

K.2.1 General

Let the AC current control for $i_v$, the feed forward signal from the filter bus voltage and the voltage that compensates for the voltage drop in the phase reactor and the PLL, all contribute to the total VSC voltage reference signal that is sent to the PWM modulator and the latter performs the modulation task (in chapter 5 is mentioned that possibly a certain delay determined by the switching frequency might be need to have included in the loop), independent of the variations in the DC voltage.

Due to the variation in the VSC voltage and the AC side current the DC current that enters into the DC side will vary. The DC current is given by the formula

$$i_{dc} = \text{Re}\{K_{VSC}^* i_v\} = K_d^* i_{vd} + K_q^* i_{vq} \quad (K-18)$$

where the coupling function $K_{VSC}$ has been introduced.

For small signal analysis, then the following equation is applicable

$$\Delta i_{dc} = K_{d0}^* \Delta i_{vd} + K_{q0}^* \Delta i_{vq} + i_{vd0} \Delta K_d^* + i_{vq0} \Delta K_q^* \quad (K-19)$$

In this equation the variables $\Delta i_{vd}$ and $\Delta i_{vq}$ are known from the analysis on the AC side. It is still needed to determine the variables $\Delta K_d$ and $\Delta K_q$.

The coupling function also describes the relation between the AC side voltage and the DC side voltage according to the equation

$$u_v = \frac{2}{3} K_{VSC} u_{dc} = u_{vd} + ju_{vq} \quad (K-20)$$

This equation immediately provides the relation between the steady state voltage and the coupling functions

$$K_{d0}^{VSC} = \frac{3}{2} \frac{u_{vd0}}{u_{dc0}} \quad (K-21)$$

$$K_{q0}^{VSC} = \frac{3}{2} \frac{u_{vq0}}{u_{dc0}}$$

The small signal relations then become

$$\Delta u_{vd} = \frac{2}{3} K_{d0}^{VSC} \Delta u_{dc} + \frac{2}{3} u_{dc0} \Delta K_d^{VSC} \quad (K-22)$$

$$\Delta u_{vq} = \frac{2}{3} K_{q0}^{VSC} \Delta u_{dc} + \frac{2}{3} u_{dc0} \Delta K_q^{VSC}$$

Here the variables $\Delta u_{vd}$ and $\Delta u_{vq}$ are known from the analysis on the AC side.
K.2.2 Solution for DC side current and voltage

The DC side of the converter is characterized by its impedance \( z_{dc} \). If the VSC is operating as a STATCOM then the only main circuit component that influences \( z_{dc} \) is the DC link capacitor. Then the following applies

\[
\Delta u_{dc} = -z_{dc} \Delta i_{dc}
\]  

(K-23)

In the following the variables \( \Delta K_d \) and \( \Delta K_q \) and thereby the change in the DC side voltage will be determined. The latter variable is required as it will be the response to the DC voltage regulator, which will finally determine the d-component of the current reference.

\[
\begin{align*}
\frac{3}{2} \Delta u_{vd} &= -K_{d0}^{VSC} z_{dc} \Delta i_{dc} + u_{dc0} \Delta K_d^{VSC} = \\
&-K_{d0}^{VSC} z_{dc} \left( i_{vd0} \Delta K_d^{VSC} + i_{q0} \Delta K_q^{VSC} + K_{d0}^{VSC} \Delta i_{vd} + K_{q0}^{VSC} \Delta i_{q} \right) + \\
&+ u_{dc0} \Delta K_d^{VSC} \\
\frac{3}{2} \Delta u_{vq} &= -K_{q0}^{VSC} z_{dc} \Delta i_{dc} + u_{dc0} \Delta K_q^{VSC} = \\
&-K_{q0}^{VSC} z_{dc} \left( i_{vd0} \Delta K_d^{VSC} + i_{q0} \Delta K_q^{VSC} + K_{d0}^{VSC} \Delta i_{vd} + K_{q0}^{VSC} \Delta i_{q} \right) + \\
&+ u_{dc0} \Delta K_q^{VSC}
\end{align*}
\]  

(K-24)

Solving the equation for \( \Delta K_d^{VSC} \) and \( \Delta K_q^{VSC} \) yields

\[
\begin{pmatrix} \Delta K_d^{VSC} \\ \Delta K_q^{VSC} \end{pmatrix} = \frac{3}{2} \begin{pmatrix} l_1 \end{pmatrix}^{-1} \begin{pmatrix} \Delta u_{vd} \\ \Delta u_{vq} \end{pmatrix} + \begin{pmatrix} l_2 \end{pmatrix}^{-1} \begin{pmatrix} \Delta i_{vd} \\ \Delta i_{q} \end{pmatrix}
\]  

(K-25)

where,

\[
l_1 =\begin{pmatrix} -K_{d0}^{VSC} z_{dc} i_{vd0} + u_{dc0} & -K_{q0}^{VSC} z_{dc} i_{q0} \\ -K_{q0}^{VSC} z_{dc} i_{vd0} & -K_{q0}^{VSC} z_{dc} i_{q0} + u_{dc0} \end{pmatrix}
\]  

(K-26)

and

\[
l_2 =\begin{pmatrix} K_{d0}^{VSC} z_{dc} K_{q0}^{VSC} & K_{q0}^{VSC} z_{dc} K_{d0}^{VSC} \\ K_{q0}^{VSC} z_{dc} K_{d0}^{VSC} & K_{d0}^{VSC} z_{dc} K_{q0}^{VSC} \end{pmatrix}
\]  

(K-27)

The following coefficients can now be defined

\[
(m_1) = \frac{3}{2} (l_1)^{-1}
\]  

(K-28)

and

\[
(m_2) = (l_1)^{-1} (l_2)^{-1}
\]  

(K-29)

Then, the DC side current is given by the expression,
\[ \Delta i_{dc} = \begin{pmatrix} K_{d0}^{FSC} & K_{q0}^{FSC} \end{pmatrix} \begin{pmatrix} \Delta i_{vd} \\ \Delta i_{vq} \end{pmatrix} + \begin{pmatrix} i_{vd0} \\ i_{vq0} \end{pmatrix} \begin{pmatrix} \Delta K_{d}^{FSC} \\ \Delta K_{q}^{FSC} \end{pmatrix} \]  

(K-30)

Here, we will introduce \( \Delta K_{d}^{FSC} \) and \( \Delta K_{q}^{FSC} \) from (K-25), and then the following result is obtained for DC voltage and DC current

\[
\begin{align*}
\Delta i_{dc} &= (K_{d0}^{FSC} & K_{q0}^{FSC}) \begin{pmatrix} \Delta i_{vd} \\ \Delta i_{vq} \end{pmatrix} + (i_{vd0} & i_{vq0}) \begin{pmatrix} \Delta u_{vd} \\ \Delta u_{vq} \end{pmatrix} + (m_1) \begin{pmatrix} \Delta i_{vd} \\ \Delta i_{vq} \end{pmatrix} + (m_2) \begin{pmatrix} \Delta i_{vd} \\ \Delta i_{vq} \end{pmatrix} \\
\Delta u_{dc} &= -z_{dc} \times (K_{d0}^{FSC} & K_{q0}^{FSC}) \begin{pmatrix} \Delta i_{vd} \\ \Delta i_{vq} \end{pmatrix} - z_{dc} \times (i_{vd0} & i_{vq0}) \begin{pmatrix} \Delta u_{vd} \\ \Delta u_{vq} \end{pmatrix} + (m_1) \begin{pmatrix} \Delta i_{vd} \\ \Delta i_{vq} \end{pmatrix} + (m_2) \begin{pmatrix} \Delta i_{vd} \\ \Delta i_{vq} \end{pmatrix}
\end{align*}
\]  

(K-31)
APPENDIX L
TRANSFORMATION OF COORDINATES

Coordinate transformation

In the $\alpha - \beta$ frame the coordinate is stationary. Let define another coordinate system which synchronously rotates with the nominal synchronous frequency $\omega_s$, the so called $d - q$ system. It is possible to transform the system into this synchronously rotating coordinate system with the usage of $d - q$ transformation.

Let assume $m^{\alpha \beta}$ is a general positive sequence, zero-sequence free space vector. Then, the $d - q$ transformation is given by

$$m^{dq}(t) = m_d + j m_q = m^{\alpha \beta} e^{-j \omega_s t} \quad (L-1)$$

This representation removes the rotation of the vector, which becomes constant in steady state.

The inverse transformation is called $\alpha - \beta$ transformation, and is given by

$$m^{\alpha \beta}(t) = m_d + j m_q = m^{dq} e^{j \omega_s t} \quad (L-2)$$

If one needs to consider separately the real and imaginary parts of the complex vector the corresponding real-valued vector is

$$m^{dq}(t) = m_d + j m_q \equiv m^{dq} = \begin{bmatrix} m_d \\ m_q \end{bmatrix} \quad (L-3)$$

Complex-valued dynamic system – transformation to synchronous coordinates

Particular attention should be given to the transforming the time derivative of space vector from stationary coordinates to synchronous coordinate. The time derivative of the $m^{\alpha \beta}$ is transformed as
\[
\frac{d \bar{m}^{aq}}{dt} = \frac{d \left( e^{+j\omega_N} \bar{m}^{dq} \right)}{dt} = e^{+j\omega_N} \left( j \omega_N \bar{m}^{dq} + \frac{d \bar{m}^{dq}}{dt} \right) \quad (L-4)
\]

It should be noted the derivative operator \( \frac{d}{dt} \), which corresponds to the Laplace ‘s’, then the transformation can be expressed as

\[
\frac{d}{dt} \left( \bar{m}^{aq} \right) = e^{+j\omega_N} \left( \frac{d}{dt} + j \omega_N \right) \bar{m}^{dq} \quad (L-5)
\]

This means that the transformation produces \( \frac{d}{dt} \rightarrow \frac{d}{dt} + j \omega_N \). Going from stationary to synchronous coordinated the following substitution has to be made

\[
s \rightarrow s + j \omega_N \quad (L-6)
\]

where, \( s = j \Omega \) is the Laplace operator.

The added term \( j \omega_N \) accounts for the synchronous rotation associated with the transformation of coordinates.

Example 1: Let consider an inductor; its Laplace impedance in synchronous coordinates is

\[
Z_{dq}^{\omega} \rightarrow (s + j \omega_N)L \quad (L-7)
\]

Example 2: Let assume the space vector representation of the dynamic model for the system in chapter 5, Figure 5.3. In the stationary reference frame, the system is represented by

\[
\Delta \bar{u}_{\alpha\beta} = \Delta \bar{u}_{\alpha\beta}^{\omega} + R_v \Delta \bar{i}_{\gamma}^{\omega} + L_v \frac{d \Delta \bar{i}_{\gamma}^{\omega}}{dt} \quad (L-8)
\]

If we now transform the basic derived relationship to the synchronous rotating reference frame, using the \( d - q \) transformation, we get:

\[
\Delta \bar{u}_{d} = \Delta \bar{u}_{d}^{\omega} + R_v \Delta \bar{i}_{\gamma}^{d} + j \omega_N L_v \Delta \bar{i}_{\gamma}^{d} + L_v \frac{d \Delta \bar{i}_{\gamma}^{d}}{dt} \quad (L-9)
\]

It should be noted the “steady state” term \( j \omega_N L_v \Delta \bar{i}_{\gamma}^{dq} \) (added due the synchronous rotation associated with the transformation of coordinates) and the “transient” term \( L_v \frac{d \Delta \bar{i}_{\gamma}^{dq}}{dt} \) in the dynamic equation.
d- and q- component impedance

Let have a system represented by the real-valued space vectors voltage and current. Let assume that the system is represented in the $d-q$ synchronously rotating coordinate system. Let $\Delta u_q^{dq} = \Delta u_{q-d} + j \Delta u_{q-q}$ is the output voltage and $\Delta i_q^{dq} = \Delta i_{q-d} + j \Delta i_{q-q}$ is the input current and these component functions are sinusoidal at frequency $\Omega$. Assume that the relation between these two functions at that frequency is given by the transfer function matrix

$$\Delta \vec{u}_q^{dq} = \Delta \vec{u}_{q-d} - \Delta \vec{u}_{q-q}$$

If we express this relation in terms of corresponding real-valued vectors we have:

$$\begin{bmatrix} \Delta u_{q-d} \\ \Delta u_{q-q} \end{bmatrix} = \begin{bmatrix} z_{S-dd} & z_{S-dq} \\ z_{S-qd} & z_{S-qq} \end{bmatrix} \begin{bmatrix} \Delta i_{q-d} \\ \Delta i_{q-q} \end{bmatrix}$$

This means that the network has been represented by its d- and q- component impedances $z_{S-dd}$, $z_{S-dq}$, $z_{S-qd}$, and $z_{S-qq}$.

To derive these impedances, let assume $\Delta i_{q-d}^{dq}$ is the driven sinusoidal function at frequency $\Omega$, which can be expressed as

$$\begin{aligned}
\Delta i_{q-d} &= \Re\{A e^{j\omega t}\} = \frac{A}{2} e^{j\omega t} + \frac{A^*}{2} e^{-j\omega t} \\
\Delta i_{q-q} &= \Re\{B e^{j\omega t}\} = \frac{B}{2} e^{j\omega t} + \frac{B^*}{2} e^{-j\omega t}
\end{aligned}$$

where A and B are some constant.

The output function then becomes

$$\Delta u_{q-d} + j \Delta u_{q-q} = \frac{(A + jB)(-\Omega + j\Omega)}{2} e^{j\omega t} + \frac{(A^* + jB^*)(-j\Omega)}{2} e^{-j\omega t}$$

For the individual components we get

$$\begin{aligned}
\Delta u_{q-d} &= \Re\left\{\frac{(A + jB)(-\Omega + j\Omega)}{2} e^{j\omega t} + \frac{(A^* + jB^*)(-j\Omega)}{2} e^{-j\omega t}\right\} = \\
&= \Re\left\{\frac{A + jB}{2} \left[\frac{Z_{ac}^{\infty}(+j\Omega)}{2} e^{j\omega t} + \left[A - jB\right] \frac{Z_{ac}^{\infty}(-j\Omega)}{2} e^{-j\omega t}\right]\right\} = \\
&= \Re\left\{A \frac{Z_{ac}^{\infty}(+j\Omega)}{2} e^{j\omega t} - B \frac{Z_{ac}^{\infty}(+j\Omega)}{2j} e^{-j\omega t}\right\}
\end{aligned}$$

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and

\[
\Delta u_{y-q} = \text{Re} \left\{ \frac{(A + jB)Z^{ac}(+ j\Omega)}{2} e^{+j\Omega} + \frac{(A^* + jB^*)Z^{ac}(- j\Omega)}{2} e^{-j\Omega} \right\} = \\
= \text{Re} \left\{ \frac{(A + jB)Z^{ac}(+ j\Omega)}{2j} e^{+j\Omega} + \frac{(A^* + jB^*)Z^{ac}(- j\Omega)}{2j} e^{-j\Omega} \right\} = \\
= \text{Re} \left\{ \frac{(A + jB)Z^{ac}(+ j\Omega)}{2j} e^{+j\Omega} - \frac{(A - jB)(Z^{ac}(- j\Omega))^*}{2j} e^{-j\Omega} \right\} = \\
= \text{Re} \left\{ \frac{A}{2j} Z^{ac}(+ j\Omega) - \frac{(Z^{ac}(- j\Omega))^*}{2} + \frac{B}{2} Z^{ac}(+ j\Omega) + \frac{(Z^{ac}(- j\Omega))^*}{2} e^{-j\Omega} \right\} \\
\tag{L-15}
\]

From these components results the following formulas for the d- and q-component impedance transfer functions:

\[
z_{s-dd}(j\Omega) = \frac{Z^{ac}(+ j\Omega) + (Z^{ac}(- j\Omega))^*}{2} \\
z_{s-qq}(j\Omega) = \frac{Z^{ac}(+ j\Omega) + (Z^{ac}(- j\Omega))^*}{2} \\
z_{s-dq}(j\Omega) = -\frac{Z^{ac}(+ j\Omega) - (Z^{ac}(- j\Omega))^*}{2j} \\
z_{s-qd}(j\Omega) = +\frac{Z^{ac}(+ j\Omega) - (Z^{ac}(- j\Omega))^*}{2j} \\
\tag{L-16}
\]
REFERENCES


[22] L. Angquist, ‘User’s Guide for MATLAB function RegDsgn’, a MATLAB function used to support the design a regulator in the frequency domain using Nichol’s chart, 2006-1-06.

# List of Symbols

## Coordinate systems

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIXED</td>
<td>Coordinate system with phases a,b,c</td>
</tr>
<tr>
<td>$\alpha - \beta$</td>
<td>Stationary coordinate, coordinate system aligned with the undisturbed system</td>
</tr>
<tr>
<td>$d - q$</td>
<td>Coordinate system which synchronously rotates with nominal frequency $\omega_N$</td>
</tr>
</tbody>
</table>

## General

<table>
<thead>
<tr>
<th>Capital letters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{A}$</td>
<td>rms values or magnitude of phasors</td>
</tr>
<tr>
<td>$\hat{A}$</td>
<td>phasor/vector</td>
</tr>
<tr>
<td>$\tilde{A}$</td>
<td>peak value</td>
</tr>
<tr>
<td>$\hat{\tilde{A}}$</td>
<td>estimated value</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lower case letters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{a}(t)$</td>
<td>instantaneous values</td>
</tr>
<tr>
<td>$\hat{a}(t)$</td>
<td>three-dimensional vectors</td>
</tr>
<tr>
<td>$\tilde{a}(t)$</td>
<td>two-dimensional vectors, “space vectors”</td>
</tr>
<tr>
<td>$\hat{\tilde{a}}(t)$</td>
<td>normalized quantity</td>
</tr>
</tbody>
</table>

$\Delta$ deviation from steady state operating point
List of Symbols

Subscripts

a,b,c  phase quantities in FIXED coordinates
α,β  components in the FIXED coordinates, zero sequence free
d,q  components in the rotating coordinates, zero sequence free
l-l  line-line
meas  measured value
N  nominal/rated value
ref  reference value

Superscripts

*  complex conjugate

Some Specific Symbols

α  firing angle
γ  commutation margin angle
μ  overlap angle
ωN  network angular frequency
Ω  angular frequency deviation