Preface

The work presented in this thesis has been carried out between October 2002 and October 2007 at the department of Solid Mechanics, School of Engineering Sciences, Royal Institute of Technology (KTH).

First of all I would like to express my sincere gratitude to my supervisors Prof. Bertil Storåkers and Prof. Per – Lennart Larsson for their excellent guidance, encouragement and support. Thank you for giving me the opportunity to work as your student.

Messrs Hans Öberg, Bengt Möllerberg and Martin Öberg are acknowledged for very helpful discussions on the experimental part of the work and design of the experimental setup. Help of Messrs Bertil Dolk and Johan Wikström in preparation of test specimens is also greatly acknowledged.

I would also like to express my gratitude to all my colleagues for providing a very nice atmosphere at the department.

Finally, I want to thank my family for their love and never ending support that made this thesis possible.

Stockholm, Oktober 2007

Denis Jelagin
Dissertation
This dissertation contains an introduction to the subject and the following appended papers:

Paper A

Paper B

Paper C

Paper D
Abstract

This thesis addresses normal axisymmetric contact of dissimilar elastic solids at finite interfacial friction. It is shown that in the case of smooth and convex but otherwise arbitrary contact profiles and monotonically increasing loading a single stick-slip contour evolves being independent of loading and profile geometry. This allows developing an incremental procedure based on a reduced problem corresponding to frictional rigid flat punch indentation of an elastic half-space. The reduced problem, being independent of loading and contact region, was solved by a finite element method based on a stationary contact contour and characterized by high accuracy. Subsequently, a tailored cumulative superposition procedure was developed to resolve the original problem to determine global and local field values for two practically important geometries: flat and conical profiles with rounded edges and apices. Results are given for relations between force, depth and contact contours together with surface stress distributions and maximum von Mises effective stress, in particular to predict initiation of fracture and plastic flow. It is also observed that the presence of friction radically reduces the magnitude of the maximum surface tensile stress, thus retarding brittle fracture initiation.

Hertzian fracture through indentation of flat float glass specimens by steel balls has been examined experimentally for a full load cycle. It has been observed that if the specimen survived during loading to a maximum level it frequently failed at decreasing load. It has been proposed by Johnson et al. (1973) that the underlying physical cause of Hertzian fracture initiation during load removal is that at unloading frictional tractions reverse their sign over part of the contact region. Guided by these considerations a robust computational procedure has been developed to determine global and local field values in particular at unloading at finite friction. In contrast to the situation at monotonically increasing loading, at unloading invariance properties are lost and stick-slip regions proved to be severely history dependent and in particular with an opposed frictional shear stress at the contact boundary region. This causes an increase of the maximum tensile stress at the contour under progressive unloading. It is shown that the experimental observations concerning Hertzian fracture initiation at unloading are at least in qualitative correlation with the effect friction has on the maximum surface tensile stress.

A contact cycle between two dissimilar elastic bodies at finite Coulomb friction has been further investigated analytically and numerically for a wider range of material parameters and contact geometries. With the issue of Hertzian fracture initiation in mind, results concerning the influence of the friction coefficient and compliance parameters on the absolute maximum surface tensile stress during a frictional contact cycle are reported along with the magnitudes of the relative increase of maximum tensile stresses at unloading. Based on a critical stress fracture criterion it is discussed how the predicted increases will influence the critical loads required for crack initiation.

Fracture loads are measured with steel and tungsten carbide spherical indenters in contact with float glass specimens at monotonically increasing loading and during a load cycle. Computational predictions concerning the fracture loads are given based on Hertz and frictional contact theories combined with a critical stress fracture criterion. The computational results obtained for frictional contact are shown to be in better agreement with experimental findings as compared to the predictions based on the Hertz theory. The remaining quantitative discrepancy was attributed to the well-known fact that a Hertzian macro-crack initiates from pre-existing defects on the specimen’s surface. In
order to account for the influence of the random distribution of these defects on the fracture loads at monotonic loading, Weibull statistics was introduced. The predicted critical loads corresponding to 50% failure probability were found to be in close agreement with experimentally observed ones.

*Keywords:* Contact mechanics; Elastic material; Friction; Nanoindentation; Fracture
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Introduction

The problem of elastic contact between two bodies of different curvature radii has originally been solved by Hertz (1882). The theory he developed stands as a landmark in linear elasticity and is based on the following assumptions:

- the surfaces are continuous and non-conforming;
- the strains are small;
- each solid can be considered as an elastic half-space;
- frictional tractions are absent.

The last assumption is fully justified only in case when the contacting bodies are elastically similar, as at dissimilar elastic contact there will be a relative radial slip between the surfaces which will be opposed by frictional tractions. Presence of these shear tractions at the contact interface will alter the whole stress field as compared to the one given by Hertz theory.

The present thesis addresses two problems of axisymmetric frictional elastic contact; normal contact of dissimilar elastic bodies with arbitrary profiles at monotonically increasing load and Hertzian fracture initiation under a frictional contact cycle.

Frictional elastic contact at monotonically increasing load

Already in the Hertz formulation based on linear kinematics, the contact problem is essentially nonlinear as a moving boundary is present, and a further nonlinearity evolves when finite friction is introduced at the interface. With Coulomb’s law of friction assumed to be valid point wise, the frictional boundary conditions at the interface may be formulated as follows; at points where frictional tractions are not sufficient to initiate tangential slip:

\[ \mu |p(r)| - |q(r)| > 0, \quad \frac{\partial u_r(r,a)}{\partial a} = 0 \]  

(1)

and at points where slip occurs:

\[ \mu |p(r)| - |q(r)| = 0, \quad \frac{\partial u_r(r,a)}{\partial a} \left| \frac{\partial u_r(r,a)}{\partial a} \right| = -q(r)/|q(r)|. \]  

(2)

Here \( p(r) \) and \( q(r) \) are surface normal and tangential tractions respectively, \( u_r \) is the radial component of displacement and \( \mu \) is the friction coefficient. The regions of stick, where eq. (1) holds, and slip, where eq. (2) holds, are in general history and geometry dependent and not known \textit{a priori} and unknown internal boundaries are to be determined as part of the analysis.

For the case of a monotonically increasing load, substantial progress was made by Spence (1975a,b) who showed that in this case a single stick-slip contour will evolve being independent of loading and the contact profile provided the indenter has a polynomial shape.

Mossakovskii (1963) seems to be the first to propose that normal contact problems at adhesive behaviour may be attacked in two steps, by first solving the problem at an incremental advance and subsequently apply superposition. By emphasizing self-similarity for power-law profiles, further advances were made by Spence (1968, 1975a, b). It has also been pointed out by Mossakovskii.
and Spence that the contact tractions arising at the interface between two dissimilar elastic solids may be determined from a single half-space solution by using a tailored combination of material parameters for two dissimilar solids.

Mossakovskii and Spence were mainly concerned with determination of surface tractions and displacements. In case of axisymmetric and frictionless contact it was later shown by Hill and Storåkers (1990) that complete field values may readily be determined by a solution for incremental fields followed by cumulative superposition along radial paths. Numerically, such a procedure is at advantage as only a stationary mesh is required when finite element methods are to be used. In view of some similarity principles applied by Hill et al. (1989) the strategy was utilized in full by Storåkers and Larsson (1994) for Norton creep by combining a finite element procedure with cumulative superposition. For the case of linear viscosity, the approach corresponds to linear elasticity though with incompressibility anticipated. In subsequent work, hereditary material behaviour was treated in the same spirit for the case of plastic flow theory, Biwa and Storåkers (1995), and viscoplastcility, Storåkers et al. (1997), with also finite friction included Carlsson et al. (2000), and obliquity, Larsson and Storåkers (2002).

Recently, cf. Ciavarella and Hills (1999), Ciavarella (1999), contact of various nonstandard profiles such as blunted cones and flat indenters with rounded edges, see Fig. 1, has been investigated. These profiles are of a significant practical importance. Indentation testing of brittle material using notionally sharp indenters (Vickers, Berkovich and a conical indenter) is usually modeled by a shallow cone indentation. However, no real indenter is atomically sharp but rather has some finite apex radius. Indenters having a nominally flat end are frequently found both in experiments and in engineering practice. Examples are readily found in fretting fatigue experiments, in indentation testing of glasses and ceramics, in support feet for all sorts of equipment. In each of these problems contact arise that characteristically have a flat base, with some kind of round-off at the edge, which is either pre-existing or initiated by localized plasticity during first application of load.

In paper A in this thesis, a consistent and robust method is developed to solve frictional contact problems with smooth and convex but otherwise arbitrary profiles at monotonically increasing load. The presence of friction introduces history dependence into the problem. It is shown, however, that at monotonically increasing loading a single stick-slip contour evolves being independent of loading and profile geometry. Thus, the history dependence is only fictitious and the problem might be reduced to an intermediate one of

Fig. 1. Geometry and notation of the problem: (a) flat indenter with a rounded corner and (b) conical indenter with a rounded tip.
frictional indentation of a rigid flat punch into a linear elastic half-space, Fig. 2. Commercial finite element software ABAQUS (2002) was employed to solve the reduced problem and subsequently a cumulative superposition procedure was applied to obtain full field solutions for the two practically important geometries shown in Fig. 1.

**Influence of elastic mismatch on Hertzian fracture**

Since Hertz first investigated cone shaped fractures produced in contacts between glass lenses in the 1880s (Hertz (1896)), a great amount of research has been done on brittle Hertzian fracture. For a detailed account of the work done in the field the interested reader is referred to review papers by Lawn and Wilshaw (1975) or Lawn (1998). There are several reasons to study Hertzian fracture properties. First there is a need to provide a theoretical basis for the Hertzian fracture test — a simple way to measure the strength of brittle materials, cf. e.g. Frank and Lawn (1967), Wilshaw (1971) and Warren (1995). A more practical reason is the need to model indentation damage which is now recognized to be of great importance in a wide range of engineering applications where brittle materials are involved, from laminated window glasses (Flocker and Dharani (1997)) to biomechanical applications (Deng et al. (2002)), as the presence of such cracks on the material surface may cause significant reduction of strength, cf. Evans (1973), Chantikul et al. (1978).

The essential step in an analysis of Hertzian fracture initiation is an accurate description of the stress field induced by an indenter. This is usually done by assuming that the indentation takes place without influence of friction, i.e. by the Hertz theory of elastic contact. The maximum tensile stress which is a governing parameter for fracture initiation is then the surface radial stress at the contact boundary:

\[
\sigma_{r} = \frac{1-2\nu}{2\pi} P^{\nu/3} \left( \frac{4E^*}{3R} \right)^{2/3}
\]

(3)
where $R$ is the curvature radii of the indenter and $E^*$ is the combined elasticity modulus given by

$$E^* = \left( \frac{1-x_1^2}{E_1} + \frac{1-x_2^2}{E_2} \right)^{-1}$$

(4)

where $E_1, \nu_1$ are Youngs moduli and Poissons ratios of the contacting bodies.

Based on the critical stress fracture criterion Hertzian fracture might be expected to initiate at the contact boundary and furthermore a tougher indenter would result in a lower fracture load as compared to a softer one provided that the indenter radii are the same. Both these predictions are at variance with experimental observations.

Hertzian fracture initiates outside the contact contour at a radius approximately 10 to 40% higher than the contact radius. Furthermore as it has been pointed out by Chaudhri and Yoffe (1981) and by Chaudhri and Phillips (1990) for the tests performed on float glass, the critical loads observed for steel and tungsten carbide balls were approximately equal. Also it has been found by Johnson et al. (1973) in experiments done with steel and glass indenters on float glass specimens that the fracture load for the steel indenter was approximately twice as high as compared to the glass one. Also the ring crack radii measured at steel to glass contact were consistently higher as compared to the ones observed at glass to glass contact. The comparison was made in their study of indenters with the curvature radii chosen so as that the same force would result in the same contact radii for both indenter materials, i.e.

$$R_{steel} \cong \frac{2}{3} R_{glass}.$$  (5)

In such a situation the Hertz theory would predict identical fracture loads for both indenters.

It has been originally suggested by Johnson et al. (1973) that the experimental observations discussed above might be explained by the presence of interfacial friction. It was shown by Johnson et al. (1973) based on asymptotic solutions of full stick and full slip that during monotonically increasing load, friction induces outward shear tractions on the surface of a more compliant body. As a result, the maximum tensile stress will be reduced and shifted away from the contact contour. Thus, it might be expected that if the specimen is more compliant than the indenter conical fracture will be initiated at a higher load and have a larger initiation radius compared to the case when elastically similar materials are used.

The influence of interfacial friction on the surface tensile stress distribution at monotonic loading has been further examined by Hills and Sackfield (1987), Andersson (1996a) and in Paper A in this thesis. In general, it has been observed that even the presence of moderate friction radically reduces the maximum tensile stress in the more compliant body and shifts its location from the contact contour, the relative distance depending on the profile geometry, the coefficient of friction and relative difference of elastic stiffness of contacting bodies.
Another experimental finding which may not be explained on the basis of the Hertz contact theory alone is the Hertzian fracture initiation during unloading. It was noticed by several investigators for dissimilar contact pairs that if a brittle specimen has not fractured during indentation to maximum load it frequently fails during unloading, cf. e.g. Argon et al. (1960), Wilshaw (1971), Chaudhri and Phillips (1990) and Geandier et al. (2003). Argon et al. (1960) carried out a series of indentation experiments performed on polished crown glass with steel indenters during a complete load cycle. In two of the series the load was first raised to its maximum value, maintained at this level for 5 minutes and then removed at the same rate as it was applied. The results of these experiments are summarized in Table 1. One may observe that the specimens which survived during loading and while the load was maintained at constant level very frequently failed during unloading.

Table 1. Experimental results by Argon et al. (1960)

<table>
<thead>
<tr>
<th>Maximum load (kg)</th>
<th>Number of experiments</th>
<th>Number of fractures during loading</th>
<th>Number of fractures at delay</th>
<th>Number of fractures during unloading</th>
</tr>
</thead>
<tbody>
<tr>
<td>140</td>
<td>20</td>
<td>9</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>150</td>
<td>40</td>
<td>12</td>
<td>14</td>
<td>14</td>
</tr>
</tbody>
</table>

It has been originally suggested by Johnson et al. (1973) that friction may be an underlying physical cause for Hertzian fracture initiation during unloading. Namely at unloading shear tractions of opposite sign arise at the edge of the contact area. Accordingly the frictional effect which is protective during loading gives rise to a damaging peak tension during unloading.

In Fig. 3, taken from paper B, a typical crack formed during monotonically increasing load is depicted at loading and after unloading. In Fig. 3(a), at a load 3.0 kN, a photograph was taken immediately after formation of a cone crack, the initial surface ring is shown with a dash-dotted line. It may be seen that also several radial cracks form. In Fig. 3 (b) a picture was taken of the
same specimen after unloading to 0.5 kN, the initial surface ring is again denoted by a dash-dotted line. It may be noticed that during unloading multiple secondary ring cracks form closer to the contact boundary.

The analysis of the stress field induced by frictional unloading is complicated. While at monotonically increasing load it is possible to specify in advance how the contact region divides into stick and slip zones, at unloading invariance properties are lost and stick-slip regions proved to be severely history dependent. Turner (1979) obtained a numerical solution for the problem of unloading of an elastic half-space indented by a circular flat die accounting for interfacial friction. It was observed that two separate moving slip zones evolved during load removal with reversal of the frictional shear stress inside the contact contour. As a consequence, at a slightly reduced load, the maximum tensile stress started to increase at progressive unloading.

In Paper B in the present thesis, the influence of interfacial friction on Hertzian fracture initiation at loading and unloading was investigated experimentally and theoretically. Fracture experiments for a full contact cycle were performed on float glass with steel balls and characteristics of crack formation were recorded and analyzed. In a number of experiments the contact region was continuously observed directly through a microscope in order to study the details of the Hertzian crack formation process. Hertzian cracks formed, whether during loading or unloading, had the same characteristic shape of the frustum of a cone, as depicted in Fig. 3, propagating rapidly and with no precursory surface ring cracks.

A robust computational procedure is laid down to simulate local and global fields numerically and to predict fracture inception at interfacial finite friction between a spherical indenter and a flat specimen for a full contact cycle.
From computations it is found that at monotonically increasing load due to the presence of friction, the maximum tensile stress will be reduced and shifted from the contact boundary. At unloading shear tractions will change sign over part of the contact region and as a result will give rise to additional surface tension near the contact boundary. Thus the surface tensile stress distribution changes qualitatively at unloading having a sharper shape similar to the one for frictionless contact. Furthermore, the location of the tensile stress maximum moves to the contact contour.

In Fig. 4 the computed normalized radial stress distribution in a glass specimen surface is shown at maximum load and at 50% unloading for the case of a steel indenter and the friction coefficient being 0.1. By adopting a critical stress criterion governing fracture it is argued then that the experimental observations of the Hertzian fracture initiation are at least in qualitative correlation with computational results.

It was also pointed out in Paper B, that the size of the regions of opposing shear tractions, their magnitude and as an effect their impact on the surface tensile stress distribution were found to be highly sensitive to the elastic properties of the contacting bodies and on the coefficient of friction, $\mu$. Specifically the maximum tensile stress was found to be higher at unloading for $\mu < 0.30$ at rigid-glass contact and only for $\mu < 0.15$ at steel-glass contact.

In Paper C, a thorough investigation of the effect of the indenter elasticity and of the friction coefficient on the Hertzian fracture initiation at frictional contact cycles was performed for three practically important indenter geometries: flat, spherical and flat with rounded corners. The contact pressure and shear stress distribution depend on the friction coefficient, $\mu$, and on the Dundur’s parameter, $\beta$, which is a measure of dissimilarity of stiffness of the contacting bodies. The surface tensile stress distribution in a brittle specimen also depends on the specimen’s Poisson’s ratio, $\nu$. 

![Figure 5. Maximum tensile stress during loading (solid lines) and unloading (dashed lines) as a function of $\mu$ and $\beta$. Flat indenter with rounded corners, $\nu_{spec}=0.23$.](image)
Generally, in contrast to the one invariant stick-slip radii found at monotonically increasing load, three moving stick-slip boundaries were observed at unloading, being severely dependent on contact geometry as well as \( \mu \) and \( \beta \). Furthermore, the stick-slip regime was essentially different for complete and incomplete contacts. Namely, in the case of incomplete contact the region of an outward slip formed immediately at the very beginning of load removal while for the case of complete contact the initiation of outward slip was postponed to the later stages of unloading. It was also shown that the specimen’s Poisson’s ratio has a strong influence on the magnitude of the maximum tensile stress. However, its influence on the relative increase of tensile stresses during unloading is less pronounced.

In Fig. 5 the effect of the Dundur’s parameter on the maximum surface tensile stresses during loading and unloading is illustrated for the case of a flat indenter with rounded corners in contact with a flat specimen. The Poisson’s ratio of the specimens is set to 0.23 which corresponds to the float glass and the surface maximum tensile stress is given as a function of friction, normalized with nominal pressure. In Fig. 6 the influence of the specimens Poisson’s ratio on the maximum tensile stress is illustrated in an analogous way.

Finally it is discussed how the observed increase in tensile stress at unloading would influence predictions concerning the critical load for the Hertzian fracture initiation. In Fig. 7 the evolution of the maximum tensile stress is presented for steel spherical and flat-rounded indenters in contact with float glass specimens with the frictional coefficient set to 0.05. For each of the geometries two different loading scenarios resulting in the same maximum tensile stress were attempted: monotonically increasing loading and a full contact cycle. With the critical stress fracture criterion assumed it might be concluded immediately that the same maximum tensile stress level should result in the same fracture response of the specimen. It may be noted that the predicted
critical load for the cyclic case is 30%-40% of the one found at monotonic loading. Furthermore fracture might be expected in the contact cycle characterized by a maximum load which is approximately between 50% and 60% of the corresponding value found at monotonic loading.

**Table 3. Ratios of fracture loads; experimental vs computational**

<table>
<thead>
<tr>
<th></th>
<th>Hertz theory</th>
<th>Frictional contact</th>
<th>Hertz + Weibull statistics</th>
<th>Friction + Weibull statistics</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{P_{f}^{gt}}{P_{f}^{steel}}$, same $a(P)$</td>
<td>1</td>
<td>1.25</td>
<td>1</td>
<td>1.15</td>
<td>1.20</td>
</tr>
<tr>
<td>$\frac{P_{f}^{steel}}{P_{f}^{glass}}$, same $a(P)$</td>
<td>1</td>
<td>6.33</td>
<td>1</td>
<td>2.7</td>
<td>2.3</td>
</tr>
<tr>
<td>$\frac{P_{f}^{gt}}{P_{f}^{steel}}$, same $R$</td>
<td>0.66</td>
<td>0.87</td>
<td>0.85</td>
<td>0.93</td>
<td>0.96</td>
</tr>
</tbody>
</table>

It is well-known that Hertzian fracture initiates from pre-existing flaws on the specimen surface, cf. e.g. Langitan and Lawn (1969), Wilshaw (1971). These initial surface flaws are randomly distributed in a specimen; this applies to their size, shape and location. This fact brings up a statistical issue, which has been addressed in several works, e.g. Oh and Finnie (1967), Hamilton and Rawson (1970) based on Weibull (1939) statistics and Hertz theory of elastic contact. However there seems to exist only one study of statistical effects based on frictional contact theory. Andersson (1996b) used a solution developed by Hills and Sackfield (1987) coupled with Weibull statistics to investigate the size effect of the indenter and the location of the first ring crack. In Paper D, Weibull
statistics is used to account for the flaw distribution effect on fracture loads at monotonic loading. A comparison between the experimentally observed and computationally predicted fracture loads was performed in Paper D, for the particular cases of a float glass specimen in contact with a spherical indenter of different materials.

In Table 3 experimental findings are summarized and compared with computational predictions. Here $P_{f_{\text{glass}}}$, $P_{f_{\text{steel}}}$, $P_{f_{\text{glass}}}$ denote the critical loads for glass, steel and tungsten carbide indenters correspondingly and experimental results concerning the change of fracture loads with the change of indenter material are given and compared with the numerical predictions based on Hertz and frictional contact theories. It may be seen that the frictional results are in better agreement with the experimental observations. In particular the predictions concerning the change of fracture loads with a change from steel to tungsten carbide indenters are in very close agreement with the experimental observations. Analogous results concerning the change from steel to glass indenters are at least in qualitative agreement with the experimental findings, but obviously there is a big quantitative discrepancy, which might be attributed to the influence of the random surface flaw distribution. Also in Table 3, the fracture loads are given predicted using a critical stress criterion combined with a Weibull statistical approach. It might be seen in Table 3, that when the flaw distribution is taken into account, the results based on frictional contact theory are both in qualitative and quantitative correlation with the experimental observations.

The effect of indenter elasticity on Hertzian fracture initiation at unloading has also been studied experimentally and numerically in Paper D. In

![Figure 8](image.png)

Figure 8. Failure probabilities at unloading as function of maximum load, steel and tungsten carbide indenters, $R = 5$ mm.
experiments performed with steel and tungsten carbide indenters the critical load required to initiate fracture at unloading were consistently lower for the tungsten carbide indenter as compared to the steel one, as it is illustrated in Fig. 8. In particular, the 50% critical loads were measured to be 1.20 kN and 0.68 kN for steel and tungsten carbide indenters respectively.

Numerically, it was found that the maximum surface tensile stress at unloading increases for both indenter materials, as it is shown in Figure 9. In particular for tungsten carbide indenters tensile stresses increase up to 54% higher as compared to the maximum load, while for steel indenters the increase is only up to 14%. Based on the results given in Fig. 9 and the critical stress fracture criterion it was predicted that the critical load at tungsten carbide/glass unloading contact should be approximately 3.7 times lower during a full contact cycle as compared to the case of increasing loading, and for the steel/glass contact the corresponding value is 1.48 times lower. As the 50% fracture loads measured at monotonically increasing load were 1.68 kN and 1.62 kN for steel and tungsten carbide indenters correspondingly, the predicted critical loads at unloading are then 1.14 kN and 0.44 kN respectively. Thus it might be concluded that numerical results are at least in qualitative agreement with experimental observations reported above. The remaining quantitative discrepancy might, analogous to the case of monotonic loading, be attributed to the influence of flaw statistics. However this feature is not studied presently.

Summary of appended papers

Paper A: Hertz contact at finite friction and arbitrary profiles

Axisymmetric contact at finite Coulomb friction and arbitrary profiles is examined analytically and numerically for dissimilar elastic solids. Invariance and generality are aimed at and an incremental procedure is developed resulting
in a reduced benchmark problem corresponding to a rigid flat indentation of an elastic half-space. The reduced problem being independent of loading and contact region, was solved by a finite element method based on a stationary contact contour and characterized by high accuracy. Subsequently, a tailored cumulative superposition procedure was developed to resolve the original problem to determine global and local field values. Save for the influence of the coefficients of friction and contraction ratio, it is shown that at partial slip the evolving relative stick-slip contour is independent of any convex and smooth contact profile at monotonic loading. For flat and conical profiles with rounded edges and apices, results are illustrated for relations between force, depth and contact contours together with surface stress distributions. The solution for dissimilar solids in a full space is transformed to a half-space problem and solved for a combination of material parameters in order to first determine interface traction distributions. Subsequently, full field values for the two solids were computed individually. In order to predict initiation of fracture and plastic flow, results are reported for the location and magnitude of maximum tensile stress and effective stress, respectively, for a range of geometrical and material parameters. In two illustrations, predicted results are compared with experimental findings related to initiation of brittle fracture and load-depth relation at nanoindentation.

Paper B: On Hertzian fracture at unloading

Hertzian fracture through indentation of flat float glass specimens by steel balls has been examined experimentally. Initiation of cone cracks has been observed and failure loads together with contact and fracture radii determined at monotonically increasing load but also during unloading phases. Contact of dissimilar elastic solids under decreasing load may cause crack inception triggered by finite interface friction and accordingly the coefficient of friction was determined by two different methods. In order to make relevant predictions of experimental findings, a robust computational procedure has been developed to determine global and local field values in particular at unloading at finite friction. It was found that at continued loading it is possible to specify in advance how the contact domain divides into invariant regions of stick and slip. The maximum tensile stress was found to occur at the free surface just outside the contact contour, the relative distance depending on the different elastic compliance properties and the coefficient of friction. In contrast, at unloading invariance properties are lost and stick/slip regions proved to be severely history dependent and in particular with an opposed frictional shear stress at the contact boundary region. This causes an increase of the maximum tensile stress at the contour under progressive unloading. Predictions of loads to cause crack initiation during full cycles were made based on a critical stress fracture criterion and proved to be favourable as compared to the experimental results.

Paper C: Hertzian Fracture at Finite Friction: a Parametric Study

Friction has a profound influence on Hertzian fracture initiation when dissimilar materials are involved. Experimental studies show that the presence of friction results in higher fracture loads and fracture radii as compared to the frictionless case. It has also been shown recently that the experimental observations concerning Hertzian fracture initiation at unloading may be
explained by the effect friction has on a surface tensile stress distribution. Presently a contact cycle between two dissimilar elastic bodies at finite Coulomb friction has been investigated numerically for a wide range of material parameters and contact geometries. Emphasis has been given to the surface tensile stress distribution which is assumed to be a governing parameter for Hertzian fracture initiation. In particular it was found that during loading the contact region divides into invariant stick and inward slip regions and the presence of outward frictional shear tractions reduces the maximum surface tensile stress and shifts it away from the contact contour as compared to the frictionless case. At unloading, the distributions of stick-slip zones were found to be severely history and geometry dependent and shear tractions reversed their direction over part of the contact area. Consequently, tensile stresses were found to grow at unloading. Results concerning the influence of the friction coefficient, Dundur’s parameter and the specimen’s Poisson’s ratio on the absolute maximum surface tensile stress obtained at a frictional contact cycle are reported along with the magnitudes of the relative increase of maximum tensile stresses at unloading. Based on a critical stress fracture criterion it is discussed how the predicted increases will influence the critical loads required for crack initiation.

**Paper D: On indentation and initiation of fracture in glass**

The influence of indenter elasticity on Hertzian fracture initiation at frictional dissimilar elastic contact has been examined experimentally and numerically. In flat float glass specimens initiation of cone cracks has been observed and fracture loads measured with steel and tungsten carbide indenters at monotonically increasing loading and during a load cycle. The observed effect of indenter elasticity on fracture loads was found to be qualitatively different from the one predicted by the Hertz contact theory. This discrepancy may be explained by the presence of interfacial friction. The friction coefficient between the indenters and the specimen was measured and a contact cycle at finite Coulomb friction has been analyzed numerically. The influence of the indenter elasticity and the friction coefficient on the surface maximum tensile stress has been investigated and the results concerning the influence of these parameters on the fracture loads were given based on a critical stress fracture criterion. The obtained computational results were found to be in better agreement with experimental findings as compared to the predictions based on the frictionless contact theory. A remaining quantitative discrepancy was attributed to the well-known fact that a Hertzian macro-crack initiates from pre-existing defects on the specimen’s surface. In order to account for the influence of the random distribution of these defects a Weibull statistics was introduced. The predicted critical loads corresponding to the 50% failure probability were found to be in close agreement with experimentally observed ones.

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